Computer algebra independent integration tests

3-Logarithms/3.4-u-a+b-log-c-d+e-x^m-^n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [641]. This is test number [63].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (641)	% 0.00 (0)
Mathematica	% 96.88 (621)	% 3.12 (20)
Maple	% 52.57 (337)	% 47.43 (304)
Maxima	% 60.69 (389)	% 39.31 (252)
Fricas	% 61.31 (393)	% 38.69 (248)
Sympy	% 27.61 (177)	% 72.39 (464)
Giac	% 54.60 (350)	% 45.40 (291)
Mupad	% 50.86 (326)	% 49.14 (315)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

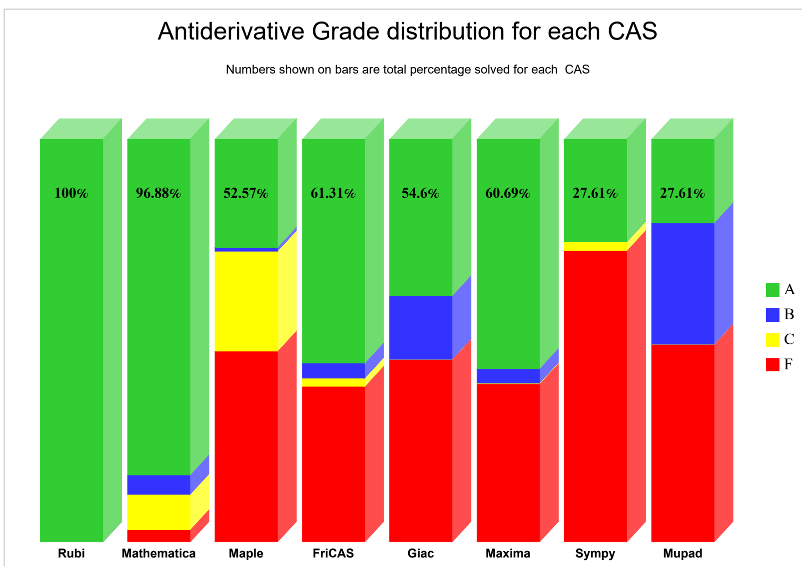
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

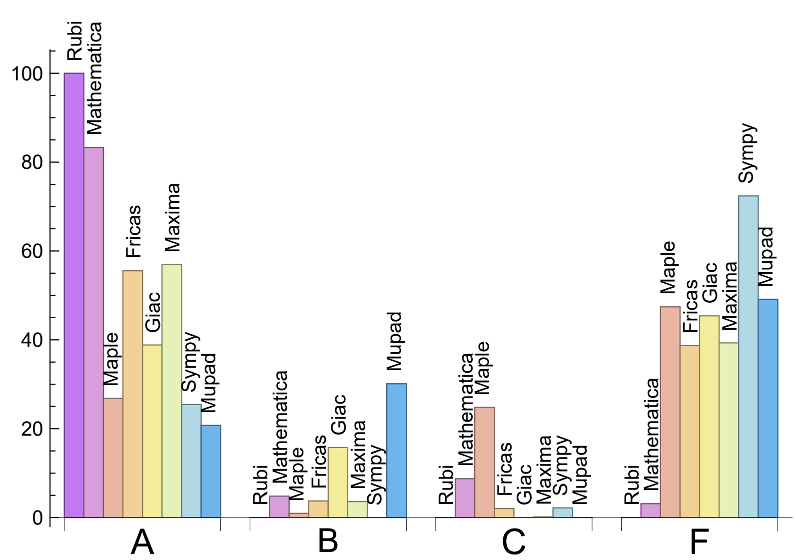
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	83.31	4.84	8.74	3.12
Maple	26.83	0.94	24.80	47.43
Maxima	56.94	3.59	0.16	39.31
Fricas	55.54	3.74	2.03	38.69
Sympy	25.43	0.00	2.18	72.39
Giac	38.85	15.76	0.00	45.40
Mupad	20.75	30.11	0.00	49.14

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	20	100.00 %	0.00 %	0.00 %
Maple	304	97.04 %	2.96 %	0.00 %
Maxima	252	94.84 %	0.79 %	4.37 %
Fricas	248	100.00 %	0.00 %	0.00 %
Sympy	464	35.56 %	62.07 %	2.37 %
Giac	291	98.63 %	0.34 %	1.03 %
Mupad	315	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

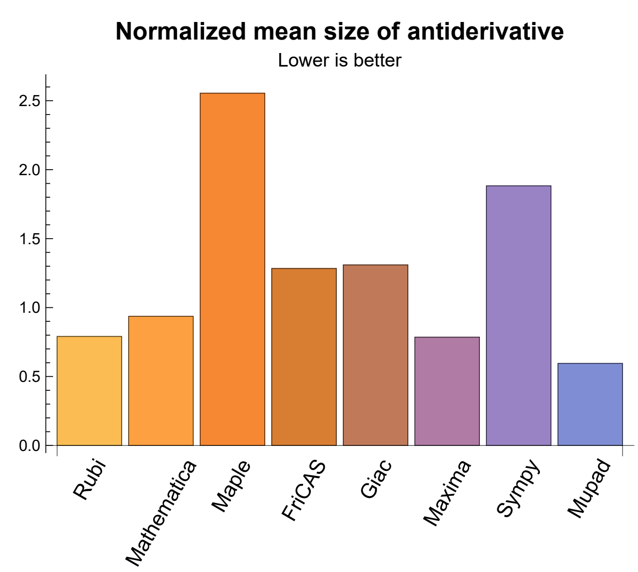
1.3 Performance

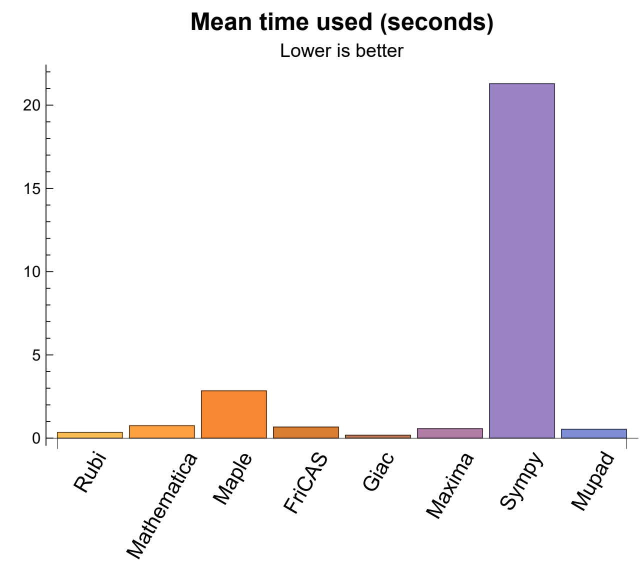
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.35	220.12	0.79	121.00	1.00
Mathematica	0.75	236.42	0.94	121.00	0.89
Maple	2.85	367.70	2.55	113.00	1.24
Maxima	0.57	130.06	0.79	82.00	0.83
Fricas	0.67	288.05	1.28	80.00	1.00
Sympy	21.29	186.12	1.88	82.00	1.12
Giac	0.18	261.96	1.31	90.00	0.91
Mupad	0.53	121.17	0.59	48.00	0.79

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{98, 99, 100, 101, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 211, 216, 217, 218, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 381, 382, 383, 384, 385, 386, 387, 388, 398, 485, 486, 487, 488, 528, 529, 530, 531, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 625, 626, 627, 635, 640, 641}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {98, 99, 100, 101, 158, 159, 277, 298, 299, 485, 486, 487, 488, 528, 529, 530, 531}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {99, 158, 159, 168, 262, 271, 277, 292, 298, 299, 343, 344, 345, 346, 347, 376}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

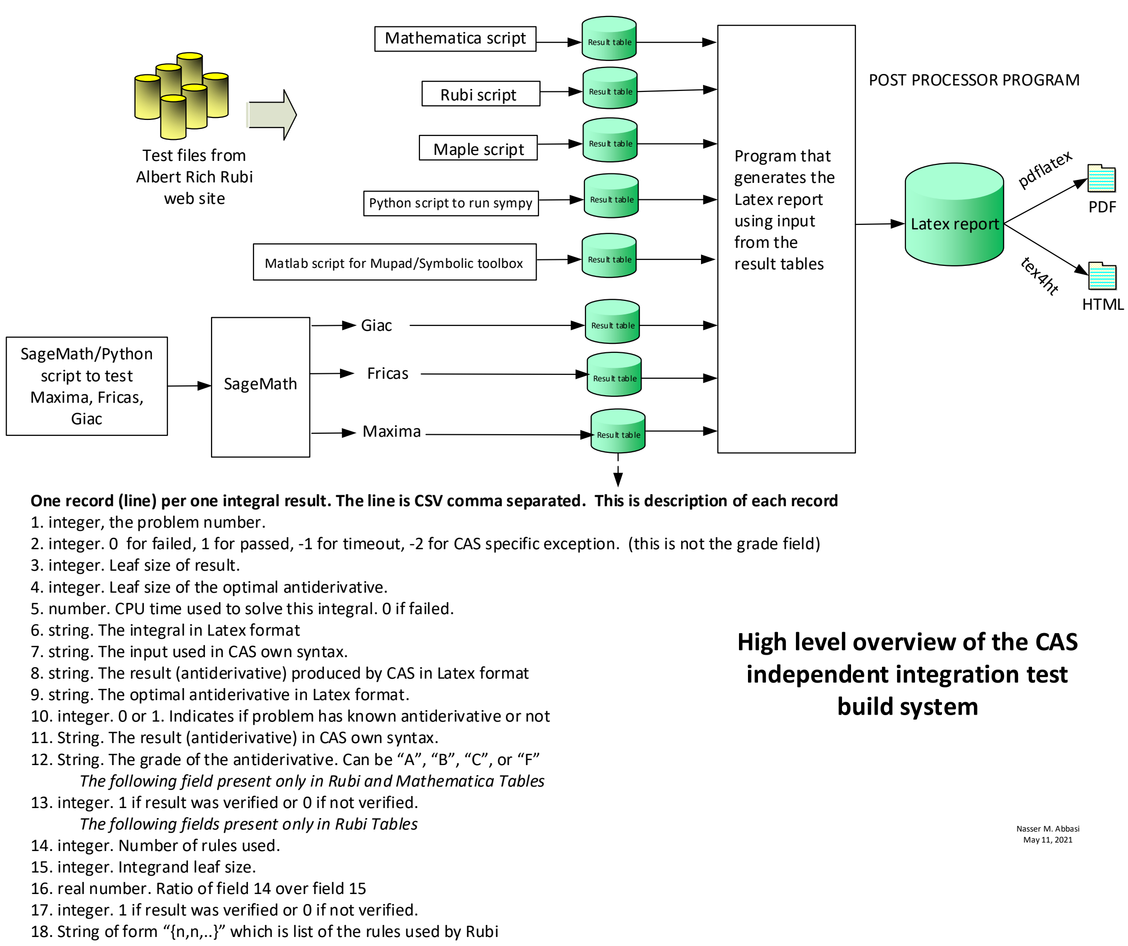
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 15, 16, 18, 19, 21, 22, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 93, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 240, 241, 242, 243, 244, 245, 246, 248, 249, 250, 251, 252, 253, 257, 258, 261, 262, 263, 264, 265, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 371, 372, 375, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 416, 417, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 455, 456, 457, 458, 459, 461, 462, 463, 464, 465, 466, 467, 469, 471, 472, 474, 475, 476, 477, 478, 481, 482, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 503, 504, 506, 507, 508, 510, 512, 513, 514, 515, 522, 523, 524, 525, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 556, 557, 558, 559, 560, 561, 566, 567, 568, 569, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 611, 612, 613, 616, 617, 618, 619, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 635, 638, 639, 640, 641 }

B grade: { 45, 80, 94, 95, 96, 131, 174, 175, 376, 390, 411, 418, 419, 432, 438, 453, 460, 473, 483, 500, 505, 516, 517, 518, 526, 620, 621, 622, 623, 636, 637 }

C grade: { 9, 11, 14, 17, 20, 23, 24, 36, 38, 89, 90, 133, 137, 191, 192, 193, 196, 197, 233, 234, 238, 239, 247, 254, 255, 256, 259, 260, 266, 267, 321, 322, 335, 336, 337, 347, 359, 433, 434, 435, 468, 470, 479, 480, 501, 502, 509, 511, 519, 520, 521, 609, 610, 614, 615, 634 }

F grade: { 370, 373, 374, 377, 538, 539, 540, 541, 553, 554, 555, 562, 563, 564, 565, 575, 576, 591, 592, 593 }

2.1.3 Maple

A grade: { 4, 5, 16, 18, 30, 32, 40, 43, 45, 49, 54, 98, 99, 100, 101, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 124, 126, 128, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 169, 172, 178, 179, 186, 187, 194, 205, 211, 216, 217, 218, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 299, 302, 303, 304, 305, 306, 307, 308, 309, 358, 359, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 395, 396, 397, 398, 403, 424, 426, 445, 466, 485, 486, 487, 488, 492, 528, 529, 530, 531, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 625, 626, 627, 630, 631, 632, 635, 639, 640, 641 }

B grade: { 170, 171, 391, 394, 511, 633 }

C grade: { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 19, 20, 21, 22, 23, 24, 25, 67, 73, 77, 78, 79, 81, 82, 83, 91, 92, 93, 103, 110, 117, 129, 130, 132, 140, 150, 166, 173, 174, 175, 176, 177, 180, 181, 182, 183, 184, 185, 188, 189, 190, 191, 192, 193, 195, 196, 197, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 261, 262, 263, 268, 269, 270, 271, 288, 289, 290, 291, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 619, 620, 624, 629, 634 }

F grade: { 26, 27, 28, 29, 31, 33, 34, 35, 36, 37, 38, 39, 41, 42, 44, 46, 47, 48, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 74, 75, 76, 80, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 102, 109, 116, 123, 125, 127, 131, 133, 134, 135, 136, 137, 138, 139, 148, 149, 160, 163, 164, 165, 167, 168, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 212, 213, 214, 215, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 264, 265,

266, 267, 272, 273, 274, 292, 293, 294, 295, 296, 297, 298, 300, 301, 353, 354, 355, 356, 357, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 489, 490, 491, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 621, 622, 623, 628, 636, 637, 638
}

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 46, 47, 48, 49, 51, 52, 53, 54, 64, 65, 66, 68, 69, 77, 78, 79, 80, 81, 82, 83, 91, 92, 93, 95, 96, 97, 98, 99, 101, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 129, 130, 132, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 211, 216, 217, 218, 223, 224, 225, 243, 244, 245, 246, 264, 268, 269, 270, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 296, 300, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 348, 349, 350, 351, 352, 358, 371, 372, 375, 376, 393, 396, 398, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 415, 416, 417, 421, 422, 423, 424, 426, 427, 428, 433, 434, 435, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 481, 482, 484, 485, 486, 487, 488, 489, 490, 491, 492, 494, 495, 496, 501, 502, 506, 507, 508, 509, 510, 511, 513, 514, 515, 519, 520, 527, 528, 529, 530, 531, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 625, 626, 627, 630, 631, 632, 633, 635, 639, 640, 641 }

B grade: { 6, 19, 31, 41, 45, 50, 94, 180, 222, 340, 378, 379, 390, 391, 394, 404, 425, 439, 446, 467, 483, 493, 512 }

C grade: { 263 }

F grade: { 55, 56, 57, 58, 59, 60, 61, 62, 63, 67, 70, 71, 72, 73, 74, 75, 76, 84, 85, 86, 87, 88, 89, 90, 100, 102, 103, 109, 110, 116, 117, 123, 124, 125, 126, 127, 128, 131, 133, 134, 135, 136, 137, 138, 139, 140, 148, 149, 150, 160, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 188, 195, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 265, 266, 267, 271, 272, 273, 274, 291, 292, 293, 294, 295, 297, 298, 299, 301, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 373, 374, 377, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 392, 395, 397, 399, 411, 412, 413, 414, 418, 419, 420, 429, 430, 431, 432, 436, 437, 438, 453, 454, 455, 460, 461, 462, 474, 475, 476, 477, 478, 479, 480, 497, 498, 499, 500, 503, 504, 505, 516, 517, 518, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 629, 634, 636, 637, 638 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 64, 65, 66, 67, 68, 69, 73, 77, 78, 79, 91, 92, 93, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 167, 168, 169, 172, 173, 177, 178, 179, 181, 184, 185, 186, 187, 189, 194, 198, 199, 200, 202, 211, 216, 217, 218, 268, 269, 270, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 334, 335, 336, 337, 350, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 390, 391, 395, 398, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 415, 416, 421, 422, 423, 424, 426, 427, 428, 433, 434, 435, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 463, 464, 465, 466, 468, 469, 470, 471, 472, 481, 482, 485, 486, 487, 488, 489, 490, 491, 492, 494, 495, 496, 501, 502, 507, 508, 509, 510, 513, 514, 515, 519, 520, 527, 528, 529, 530, 531, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 624, 625, 626, 627, 630, 631, 632, 635, 639, 640, 641 }

B grade: { 170, 171, 176, 182, 183, 190, 203, 204, 392, 417, 439, 506, 511, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 633 }

C grade: { 191, 192, 193, 196, 197, 389, 619, 620, 621, 622, 623, 629, 634 }

F grade: { 6, 19, 31, 41, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 70, 71, 72, 74, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 131, 132, 133, 134, 135, 136, 137, 160, 166, 174, 175, 180, 188, 195, 201, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 271, 272, 273, 274, 291, 292, 293, 294, 295, 313, 314, 326, 327, 328, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 370, 371, 372, 373, 374, 375, 376, 377, 393, 394, 396, 397, 399, 404, 411, 412, 413, 414, 418, 419, 420, 425, 429, 430, 431, 432, 436, 437, 438, 446, 453, 454, 455, 460, 461, 462, 467, 473, 474, 475, 476, 477, 478, 479, 480, 483, 484, 493, 497, 498, 499, 500, 503, 504, 505, 512, 516, 517, 518, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 616, 617, 618, 628, 636, 637, 638 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 12, 13, 16, 17, 18, 22, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 46, 47, 48, 49, 51, 54, 56, 58, 59, 77, 78, 79, 91, 92, 93, 98, 99, 100, 101, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 124, 126, 128, 129, 130, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 176, 177, 178, 179, 183, 184, 185, 186, 187, 191, 193, 194, 198, 199, 200, 202, 203, 216, 217, 269, 270, 275, 277, 278, 280, 281, 282, 284, 285, 286, 290, 298, 299, 302, 303, 306, 307, 311, 312, 318, 319, 320, 321, 325, 333, 334, 386, 398, 400, 401, 402, 403, 405, 421, 422, 423, 424, 442, 443, 444, 445, 466, 490, 491, 492, 511, 625, 631, 632, 635, 639, 640, 641 }

B grade: { }

C grade: { 45, 70, 71, 72, 74, 75, 76, 169, 170, 171, 212, 213, 214, 215 }

F grade: { 6, 11, 14, 15, 19, 20, 21, 23, 24, 31, 41, 50, 52, 53, 55, 57, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 73, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 102, 103, 109, 110, 116, 117, 123, 125, 127, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 148, 149, 150, 158, 163, 164, 165, 166, 167, 168, 172, 173, 174, 175, 180, 181, 182, 188, 189, 190, 192, 195, 196, 197, 201, 204, 205, 206, 207, 208, 209, 210, 211, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 271, 272, 273, 274, 276, 279, 283, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 300, 301, 304, 305, 308, 309, 310, 313, 314, 315, 316, 317, 322, 323, 324, 326, 327, 328, 329, 330, 331, 332, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 399, 404, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 626, 627, 628, 629, 630, 633, 634, 636, 637, 638 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 11, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 36, 37, 38, 39, 40, 42, 44, 54, 77, 78, 79, 91, 92, 93, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 178, 179, 181, 184, 185, 186, 187, 189, 191, 192, 193, 194, 196, 216, 217, 218, 268, 269, 270, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 312, 318, 319, 320, 321, 322, 332, 333, 334, 335, 336, 337, 381, 382, 383, 384, 385, 386, 387, 388, 398, 421, 423, 424, 464, 465, 466, 468, 469, 470, 472, 485, 486, 487, 488, 489, 490, 491, 492, 494, 495, 496, 508, 509, 510, 511, 513, 514, 515, 528, 529, 530, 531, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 625, 626, 627, 629, 630, 631, 635, 640, 641 }

B grade: { 10, 12, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 43, 45, 46, 47, 48, 49, 51, 52, 53, 109, 116, 117, 148, 149, 176, 177, 182, 183, 190, 197, 198, 199, 200, 202, 203, 204, 310, 311, 315, 316, 317, 323, 324, 325, 329, 330, 331, 350, 390, 393, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 415, 416, 417, 422, 426, 427, 428, 433, 434, 435, 439, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 463, 471, 481, 482, 501, 502, 506, 507, 632, 633, 639 }

C grade: { }

F grade: { 6, 19, 41, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 131, 132, 133, 134, 135, 136, 137, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 180, 188, 195, 201, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 271, 272, 273, 274, 291, 292, 293, 294, 295, 313, 314, 326, 327, 328, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 389, 391, 392, 394, 395, 396, 397, 399, 404, 411, 412, 413, 414, 418, 419, 420, 425, 429, 430, 431, 432, 436, 437, 438, 446, 453, 454, 455, 460, 461, 462, 467, 473, 474, 475, 476, 477, 478, 479, 480, 483, 484, 493, 497, 498, 499, 500, 503, 504, 505, 512, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 634, 636, 637, 638 }

2.1.8 Mupad

A grade: { 98, 99, 100, 101, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 211, 216, 217, 218, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 381, 382, 383, 384, 385, 386, 387, 388, 398, 485, 486, 487, 488, 528, 529, 530, 531, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 625, 626, 627, 635, 640, 641 }

B grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 77, 78, 79, 91, 92, 93, 124, 126, 128, 129, 130, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 202, 203, 204, 268, 269, 270, 288, 289, 290, 310, 311, 312, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 334, 335, 336, 337, 350, 379, 390, 391, 393, 394, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 415, 416, 417, 421, 422, 423, 424, 426, 427, 428, 433, 434, 435, 439, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 463, 465, 466, 469, 471, 472, 481, 482, 489, 490, 491, 492, 494, 495, 496, 501, 502, 506, 507, 508, 510, 511, 514, 519, 520, 527, 629, 630, 631, 632, 633, 634, 639 }

C grade: { }

F grade: { 6, 19, 31, 41, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 102, 103, 109, 110, 116, 117, 123, 125, 127, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 148, 149, 150, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 180, 188, 195, 201, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 271, 272, 273, 274, 291, 292, 293, 294, 295, 313, 314, 326, 327, 328, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 380, 389, 392, 395, 396, 397, 399, 404, 411, 412, 413, 414, 418, 419, 420, 425, 429, 430, 431, 432, 436, 437, 438, 446, 453, 454, 455, 460, 461, 462, 464, 467, 468, 470, 473, 474, 475, 476, 477, 478, 479, 480, 483, 484, 493, 497, 498, 499, 500, 503, 504, 505, 509, 512, 513, 515, 516, 517, 518, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 636, 637, 638 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	74	229	72	188	136	71	62
normalized size	1	1.00	0.92	2.86	0.90	2.35	1.70	0.89	0.78
time (sec)	N/A	0.047	0.047	0.483	1.270	0.466	104.663	0.179	0.221

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	183	55	57	70	97	51
normalized size	1	1.00	1.00	3.10	0.93	0.97	1.19	1.64	0.86
time (sec)	N/A	0.049	0.015	0.321	0.560	0.451	6.461	0.173	0.224

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	62	217	59	152	121	59	50
normalized size	1	1.00	0.94	3.29	0.89	2.30	1.83	0.89	0.76
time (sec)	N/A	0.037	0.024	0.352	1.573	0.482	28.762	0.204	0.228

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	50	44	40	56	43	39
normalized size	1	1.00	0.97	1.43	1.26	1.14	1.60	1.23	1.11
time (sec)	N/A	0.024	0.009	0.074	0.645	0.453	2.098	0.161	0.226

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	45	107	90	41	37
normalized size	1	1.00	1.00	0.84	1.00	2.38	2.00	0.91	0.82
time (sec)	N/A	0.018	0.013	0.051	1.417	0.462	7.657	0.163	0.224

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	232	80	0	0	0	-1
normalized size	1	1.00	0.98	5.27	1.82	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.007	0.247	0.704	0.464	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	195	36	105	377	40	36
normalized size	1	1.00	1.00	4.43	0.82	2.39	8.57	0.91	0.82
time (sec)	N/A	0.020	0.010	0.263	1.548	0.453	25.687	0.168	0.221

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	45	45	173	44	43	82	58	41
normalized size	1	1.18	1.18	4.55	1.16	1.13	2.16	1.53	1.08
time (sec)	N/A	0.037	0.003	0.254	0.649	0.454	4.662	0.166	0.245

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	49	211	49	135	763	58	46
normalized size	1	1.00	0.82	3.52	0.82	2.25	12.72	0.97	0.77
time (sec)	N/A	0.030	0.003	0.354	1.421	0.480	111.551	0.200	0.252

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	198	54	58	102	132	56
normalized size	1	1.00	0.88	3.09	0.84	0.91	1.59	2.06	0.88
time (sec)	N/A	0.052	0.041	0.260	0.791	0.448	14.046	0.164	0.259

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	49	235	62	170	0	71	61
normalized size	1	1.00	0.66	3.18	0.84	2.30	0.00	0.96	0.82
time (sec)	N/A	0.037	0.003	0.395	1.531	0.483	0.000	0.185	0.261

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	68	206	69	71	116	191	68
normalized size	1	1.00	0.87	2.64	0.88	0.91	1.49	2.45	0.87
time (sec)	N/A	0.060	0.069	0.346	0.684	0.461	32.461	0.166	0.257

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	183	55	57	70	97	51
normalized size	1	1.00	1.00	3.10	0.93	0.97	1.19	1.64	0.86
time (sec)	N/A	0.049	0.014	0.428	0.625	0.440	19.679	0.165	0.239

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	69	196	147	161	0	162	157
normalized size	1	1.00	0.43	1.23	0.92	1.01	0.00	1.02	0.99
time (sec)	N/A	0.129	0.004	0.423	1.484	0.462	0.000	0.205	2.503

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	147	194	144	144	0	160	129
normalized size	1	1.00	0.94	1.24	0.92	0.92	0.00	1.02	0.82
time (sec)	N/A	0.114	0.053	0.420	1.452	0.452	0.000	0.176	2.555

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	50	44	40	56	43	39
normalized size	1	1.00	0.97	1.43	1.26	1.14	1.60	1.23	1.11
time (sec)	N/A	0.029	0.010	0.046	0.506	0.442	4.701	0.177	0.232

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	53	184	131	150	260	150	121
normalized size	1	1.00	0.36	1.25	0.89	1.02	1.77	1.02	0.82
time (sec)	N/A	0.085	0.003	0.452	1.329	0.475	142.411	0.203	2.378

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	129	113	125	110	231	143	134
normalized size	1	1.00	0.97	0.85	0.94	0.83	1.74	1.08	1.01
time (sec)	N/A	0.082	0.038	0.052	1.361	0.471	63.407	0.190	0.458

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	180	80	0	0	0	-1
normalized size	1	1.00	0.98	4.09	1.82	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.008	0.406	0.659	0.465	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	47	184	119	126	0	137	149
normalized size	1	1.00	0.35	1.38	0.89	0.95	0.00	1.03	1.12
time (sec)	N/A	0.078	0.003	0.342	1.542	0.471	0.000	0.184	0.810

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	134	197	120	150	0	138	115
normalized size	1	1.00	0.96	1.42	0.86	1.08	0.00	0.99	0.83
time (sec)	N/A	0.075	0.035	0.339	1.448	0.472	0.000	0.181	2.606

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	173	44	43	82	58	41
normalized size	1	1.00	1.00	3.84	0.98	0.96	1.82	1.29	0.91
time (sec)	N/A	0.039	0.003	0.252	0.637	0.473	10.815	0.162	0.259

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	49	215	127	138	0	153	125
normalized size	1	1.00	0.32	1.42	0.84	0.91	0.00	1.01	0.83
time (sec)	N/A	0.095	0.003	0.365	1.465	0.466	0.000	0.207	2.361

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	49	216	128	172	0	149	156
normalized size	1	1.00	0.32	1.43	0.85	1.14	0.00	0.99	1.03
time (sec)	N/A	0.091	0.003	0.359	1.271	0.483	0.000	0.211	2.446

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	198	54	58	102	132	56
normalized size	1	1.00	0.88	3.09	0.84	0.91	1.59	2.06	0.88
time (sec)	N/A	0.051	0.039	0.261	0.685	0.476	45.917	0.177	0.284

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	85	0	74	89	122	308	77
normalized size	1	1.00	0.96	0.00	0.83	1.00	1.37	3.46	0.87
time (sec)	N/A	0.055	0.049	0.139	0.668	0.445	12.376	0.195	0.241

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	74	0	64	77	109	257	65
normalized size	1	1.00	0.99	0.00	0.85	1.03	1.45	3.43	0.87
time (sec)	N/A	0.040	0.033	0.066	0.659	0.461	7.248	0.185	0.214

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	62	0	51	64	95	210	53
normalized size	1	1.00	1.02	0.00	0.84	1.05	1.56	3.44	0.87
time (sec)	N/A	0.032	0.026	0.087	0.714	0.483	4.121	0.198	0.228

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	40	0	40	50	82	152	41
normalized size	1	1.00	0.85	0.00	0.85	1.06	1.74	3.23	0.87
time (sec)	N/A	0.021	0.018	0.083	0.636	0.420	2.187	0.184	0.246

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	37	30	27	33	48	96	27
normalized size	1	1.00	1.37	1.11	1.00	1.22	1.78	3.56	1.00
time (sec)	N/A	0.009	0.002	0.049	0.705	0.448	1.118	0.163	0.205

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	0	83	0	0	152	-1
normalized size	1	1.00	1.02	0.00	2.08	0.00	0.00	3.80	-0.02
time (sec)	N/A	0.037	0.003	0.109	0.675	0.485	0.000	0.518	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	48	50	36	39	63	40
normalized size	1	1.00	1.00	1.60	1.67	1.20	1.30	2.10	1.33
time (sec)	N/A	0.021	0.005	0.049	0.647	0.477	2.215	0.176	0.681

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	63	55	66	150	53
normalized size	1	1.00	1.00	0.00	1.07	0.93	1.12	2.54	0.90
time (sec)	N/A	0.035	0.015	0.091	0.649	0.468	3.929	0.191	0.340

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	74	66	80	234	65
normalized size	1	1.00	1.00	0.00	1.01	0.90	1.10	3.21	0.89
time (sec)	N/A	0.050	0.018	0.061	0.646	0.476	6.434	0.225	0.321

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	85	79	94	317	78
normalized size	1	1.00	1.00	0.00	0.98	0.91	1.08	3.64	0.90
time (sec)	N/A	0.056	0.021	0.088	0.696	0.474	10.470	0.204	0.388

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	49	0	59	178	162	75	56
normalized size	1	1.00	0.68	0.00	0.82	2.47	2.25	1.04	0.78
time (sec)	N/A	0.038	0.007	0.223	1.465	0.511	92.914	0.214	0.263

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	56	0	44	56	87	59	45
normalized size	1	1.00	1.10	0.00	0.86	1.10	1.71	1.16	0.88
time (sec)	N/A	0.034	0.022	0.059	0.517	0.446	9.550	0.168	0.245

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	47	0	48	141	146	63	44
normalized size	1	1.00	0.81	0.00	0.83	2.43	2.52	1.09	0.76
time (sec)	N/A	0.027	0.003	0.057	1.500	0.470	33.145	0.173	0.241

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	45	0	33	42	71	47	33
normalized size	1	1.00	1.22	0.00	0.89	1.14	1.92	1.27	0.89
time (sec)	N/A	0.014	0.003	0.078	0.588	0.438	3.351	0.167	0.209

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	43	38	33	107	109	42	33
normalized size	1	1.00	1.05	0.93	0.80	2.61	2.66	1.02	0.80
time (sec)	N/A	0.015	0.008	0.050	1.848	0.461	11.318	0.190	0.106

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	45	0	89	0	0	0	-1
normalized size	1	1.00	1.02	0.00	2.02	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.003	0.104	0.632	0.448	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	52	0	49	119	129	54	42
normalized size	1	1.00	1.04	0.00	0.98	2.38	2.58	1.08	0.84
time (sec)	N/A	0.027	0.014	0.060	1.411	0.445	25.447	0.172	0.241

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	50	54	41	58	65	47
normalized size	1	1.00	0.97	1.43	1.54	1.17	1.66	1.86	1.34
time (sec)	N/A	0.026	0.009	0.051	0.647	0.435	5.648	0.172	0.236

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	70	0	62	154	177	73	55
normalized size	1	1.00	1.03	0.00	0.91	2.26	2.60	1.07	0.81
time (sec)	N/A	0.036	0.022	0.088	1.579	0.470	72.060	0.193	0.254

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	34	9	35	11	8	110	8
normalized size	1	1.00	4.25	1.12	4.38	1.38	1.00	13.75	1.00
time (sec)	N/A	0.008	0.004	0.046	0.707	0.453	2.780	0.350	0.257

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	134	0	120	129	146	339	121
normalized size	1	1.00	0.88	0.00	0.78	0.84	0.95	2.22	0.79
time (sec)	N/A	0.118	0.137	0.141	0.625	0.462	27.233	0.224	0.334

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	112	0	98	105	119	255	97
normalized size	1	1.00	0.91	0.00	0.80	0.85	0.97	2.07	0.79
time (sec)	N/A	0.088	0.055	0.082	0.680	0.471	8.446	0.180	0.277

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	88	0	76	80	92	171	73
normalized size	1	1.00	0.95	0.00	0.82	0.86	0.99	1.84	0.78
time (sec)	N/A	0.060	0.038	0.056	0.647	0.479	2.910	0.222	0.267

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	46	50	51	61	97	47
normalized size	1	1.00	1.00	0.87	0.94	0.96	1.15	1.83	0.89
time (sec)	N/A	0.028	0.025	0.049	0.585	0.458	1.502	0.176	0.124

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	0	79	0	0	0	-1
normalized size	1	1.00	1.02	0.00	1.72	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.003	0.057	0.638	0.447	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	55	0	53	55	461	132	49
normalized size	1	1.00	0.87	0.00	0.84	0.87	7.32	2.10	0.78
time (sec)	N/A	0.045	0.041	0.050	0.618	0.484	40.891	0.177	0.660

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	90	0	76	84	0	232	72
normalized size	1	1.00	0.90	0.00	0.76	0.84	0.00	2.32	0.72
time (sec)	N/A	0.063	0.048	0.052	0.654	0.474	0.000	0.182	0.460

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	114	0	98	109	0	324	97
normalized size	1	1.00	0.88	0.00	0.75	0.84	0.00	2.49	0.75
time (sec)	N/A	0.079	0.067	0.052	0.584	0.505	0.000	0.188	0.563

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	33	40	31	28	133	31	33
normalized size	1	1.00	1.03	1.25	0.97	0.88	4.16	0.97	1.03
time (sec)	N/A	0.018	0.010	0.042	0.581	1.129	0.715	0.169	0.276

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.029	1.114	0.000	0.584	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	0	0	359	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	4.43	0.00	-0.01
time (sec)	N/A	0.042	0.028	1.122	0.000	0.874	95.987	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	56	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.026	0.958	0.000	0.743	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	201	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	3.00	0.00	-0.01
time (sec)	N/A	0.041	0.017	180.000	0.000	0.831	21.400	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	76	0	0	0	348	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	4.24	0.00	-0.01
time (sec)	N/A	0.054	0.030	180.000	0.000	1.365	77.641	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.032	8.851	0.000	0.477	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	76	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.035	0.135	0.000	0.477	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	77	0	0	0	0	0	-1
normalized size	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.039	0.174	0.000	0.518	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	77	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.038	1.535	0.000	0.475	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	92	0	115	112	0	0	-1
normalized size	1	1.00	0.65	0.00	0.82	0.79	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.080	1.569	0.767	0.479	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	74	0	95	92	0	0	-1
normalized size	1	1.00	0.66	0.00	0.85	0.82	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.041	1.497	0.695	0.493	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	48	0	70	57	0	0	-1
normalized size	1	1.00	0.70	0.00	1.01	0.83	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.034	1.438	0.753	0.471	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	201	0	63	0	0	-1
normalized size	1	1.00	0.92	4.02	0.00	1.26	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.011	1.793	0.000	0.467	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	57	0	71	75	0	0	-1
normalized size	1	1.00	0.71	0.00	0.89	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.018	1.528	0.725	0.495	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	76	0	99	104	0	0	-1
normalized size	1	1.00	0.63	0.00	0.82	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.048	1.544	0.738	0.485	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	104	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	1.60	0.00	-0.02
time (sec)	N/A	0.028	0.033	1.139	0.000	0.487	13.628	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	104	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	1.60	0.00	-0.02
time (sec)	N/A	0.024	0.033	1.290	0.000	0.442	6.631	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	48	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.89	0.00	-0.02
time (sec)	N/A	0.020	0.030	1.042	0.000	0.463	3.429	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	177	0	60	0	0	-1
normalized size	1	1.00	0.98	4.02	0.00	1.36	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.003	1.708	0.000	0.476	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	59	0	0	0	46	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.70	0.00	-0.02
time (sec)	N/A	0.032	0.034	1.004	0.000	0.486	7.711	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	62	0	0	0	51	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.71	0.00	-0.01
time (sec)	N/A	0.031	0.027	1.002	0.000	0.451	15.285	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	62	0	0	0	51	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.73	0.00	-0.01
time (sec)	N/A	0.029	0.026	1.008	0.000	0.514	32.217	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	175	200	1436	145	189	267	325	126
normalized size	1	0.81	0.93	6.68	0.67	0.88	1.24	1.51	0.59
time (sec)	N/A	0.299	0.065	0.524	0.696	0.460	22.718	0.188	0.308

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	105	1242	120	148	209	207	100
normalized size	1	1.00	0.72	8.57	0.83	1.02	1.44	1.43	0.69
time (sec)	N/A	0.153	0.059	0.473	0.772	0.478	8.639	0.187	0.259

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	63	1034	97	96	139	96	70
normalized size	1	1.00	1.03	16.95	1.59	1.57	2.28	1.57	1.15
time (sec)	N/A	0.050	0.010	0.532	0.727	0.480	3.132	0.191	0.229

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	163	0	118	0	0	0	-1
normalized size	1	1.00	2.26	0.00	1.64	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.065	0.788	0.752	0.468	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	93	841	118	0	0	0	-1
normalized size	1	1.00	1.16	10.51	1.48	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.025	0.380	0.689	0.452	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	147	137	1080	142	0	0	0	-1
normalized size	1	1.14	1.06	8.37	1.10	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.269	0.081	0.387	0.977	0.453	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	211	205	1289	173	0	0	0	-1
normalized size	1	1.09	1.06	6.68	0.90	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.407	0.058	0.437	0.813	0.441	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	248	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.408	0.205	0.439	0.000	0.473	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	223	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.323	0.137	0.957	0.000	0.441	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	193	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.267	0.083	0.723	0.000	0.435	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	173	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.054	0.864	0.000	0.478	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	207	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.285	0.095	0.888	0.000	0.448	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	277	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	0.286	0.968	0.000	0.477	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	334	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.377	0.236	0.922	0.000	0.432	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	309	5905	239	359	561	595	187
normalized size	1	1.00	0.93	17.68	0.72	1.07	1.68	1.78	0.56
time (sec)	N/A	0.358	0.200	1.132	0.721	0.470	37.188	0.200	0.356

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	237	4942	203	275	450	360	144
normalized size	1	1.00	1.12	23.42	0.96	1.30	2.13	1.71	0.68
time (sec)	N/A	0.205	0.092	1.064	0.759	0.475	15.110	0.219	0.305

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	87	3925	164	176	301	169	103
normalized size	1	1.00	0.94	42.20	1.76	1.89	3.24	1.82	1.11
time (sec)	N/A	0.066	0.013	1.032	0.748	0.491	5.332	0.170	0.240

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	279	0	217	0	0	0	-1
normalized size	1	1.00	2.63	0.00	2.05	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.105	0.664	0.646	0.447	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	302	0	202	0	0	0	-1
normalized size	1	1.00	2.54	0.00	1.70	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.298	1.189	1.133	0.474	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	236	478	0	270	0	0	0	-1
normalized size	1	1.08	2.18	0.00	1.23	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.434	0.367	1.505	1.356	0.470	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	331	571	0	338	0	0	0	-1
normalized size	1	0.94	1.62	0.00	0.96	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.745	0.416	1.206	1.269	0.458	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	0	909	0	0	0	0	0	-1
normalized size	1	0.00	2.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.834	3.895	47.763	0.000	0.482	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	290	0	789	0	0	0	0	0	-1
normalized size	1	0.00	2.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.434	3.532	0.727	0.000	0.481	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	0	505	0	0	0	0	0	-1
normalized size	1	0.00	9.90	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.827	7.525	0.000	0.455	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	0	851	0	0	0	0	0	-1
normalized size	1	0.00	3.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.351	2.710	22.774	0.000	0.441	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	96	0	0	68	0	73	-1
normalized size	1	1.00	0.90	0.00	0.00	0.64	0.00	0.68	-0.01
time (sec)	N/A	0.154	0.146	0.517	0.000	0.426	0.000	0.172	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	272	0	29	0	31	-1
normalized size	1	1.00	1.00	5.33	0.00	0.57	0.00	0.61	-0.02
time (sec)	N/A	0.058	0.045	1.259	0.000	0.445	0.000	0.164	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.191	0.501	0.000	0.430	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.351	0.582	0.000	0.485	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.018	0.308	0.514	0.000	0.466	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.004	0.010	0.477	0.000	0.483	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.018	0.416	0.495	0.000	0.458	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	157	0	0	141	0	298	-1
normalized size	1	1.00	1.14	0.00	0.00	1.02	0.00	2.16	-0.01
time (sec)	N/A	0.207	0.160	6.551	0.000	0.462	0.000	0.200	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	97	421	0	78	0	141	-1
normalized size	1	1.00	1.17	5.07	0.00	0.94	0.00	1.70	-0.01
time (sec)	N/A	0.072	0.047	1.223	0.000	0.458	0.000	0.182	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.300	1.839	0.000	0.470	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	1.551	3.763	0.000	0.438	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.350	3.517	0.000	0.432	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.004	0.384	3.490	0.000	0.471	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	1.252	3.521	0.000	0.447	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	185	0	0	270	0	840	-1
normalized size	1	1.00	0.91	0.00	0.00	1.32	0.00	4.12	-0.00
time (sec)	N/A	0.289	0.213	6.592	0.000	0.444	0.000	0.237	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	113	716	0	157	0	406	-1
normalized size	1	1.00	0.99	6.28	0.00	1.38	0.00	3.56	-0.01
time (sec)	N/A	0.093	0.058	1.215	0.000	0.447	0.000	0.180	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.588	3.736	0.000	0.450	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	3.203	3.916	0.000	0.465	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.563	3.651	0.000	0.441	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.004	0.488	3.728	0.000	0.428	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	2.256	4.751	0.000	0.442	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	0	0	54	0	38	-1
normalized size	1	1.00	0.91	0.00	0.00	1.20	0.00	0.84	-0.02
time (sec)	N/A	0.095	0.090	0.060	0.000	0.430	0.000	0.165	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	23	0	19	27	19	18
normalized size	1	1.00	1.00	1.15	0.00	0.95	1.35	0.95	0.90
time (sec)	N/A	0.027	0.015	0.050	0.000	0.414	2.155	0.159	0.348

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	0	0	99	0	89	-1
normalized size	1	1.00	0.93	0.00	0.00	1.39	0.00	1.25	-0.01
time (sec)	N/A	0.125	0.117	0.074	0.000	0.427	0.000	0.173	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	43	59	0	55	49	45	46
normalized size	1	1.00	0.91	1.26	0.00	1.17	1.04	0.96	0.98
time (sec)	N/A	0.043	0.021	0.051	0.000	0.409	2.163	0.163	0.359

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	87	0	0	142	0	141	-1
normalized size	1	1.00	0.69	0.00	0.00	1.12	0.00	1.11	-0.01
time (sec)	N/A	0.170	0.131	0.074	0.000	0.428	0.000	0.174	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	55	94	0	79	70	68	74
normalized size	1	1.00	0.75	1.29	0.00	1.08	0.96	0.93	1.01
time (sec)	N/A	0.059	0.024	0.050	0.000	0.447	2.195	0.176	0.451

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	105	1242	120	148	206	221	100
normalized size	1	1.00	0.70	8.28	0.80	0.99	1.37	1.47	0.67
time (sec)	N/A	0.156	0.066	0.669	0.664	0.485	31.918	0.172	0.275

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	63	1036	97	96	160	104	71
normalized size	1	1.00	0.95	15.70	1.47	1.45	2.42	1.58	1.08
time (sec)	N/A	0.056	0.010	0.563	0.662	0.443	7.879	0.168	0.232

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	163	0	0	0	0	0	-1
normalized size	1	1.00	2.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.101	0.719	0.000	0.450	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	84	771	118	0	0	0	-1
normalized size	1	1.00	0.98	8.97	1.37	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.039	0.533	0.821	0.445	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1294	1300	823	0	0	0	0	0	-1
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.920	1.235	0.563	0.000	0.448	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1304	1310	1090	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.791	0.727	0.908	0.000	0.479	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1137	1143	742	0	0	0	0	0	-1
normalized size	1	1.01	0.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.328	0.869	0.918	0.000	0.467	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1170	1176	745	0	0	0	0	0	-1
normalized size	1	1.01	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.344	0.872	0.981	0.000	0.450	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1328	1334	847	0	0	0	0	0	-1
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.723	1.645	1.052	0.000	0.459	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	146	0	0	116	0	108	-1
normalized size	1	1.00	0.89	0.00	0.00	0.71	0.00	0.66	-0.01
time (sec)	N/A	0.237	0.247	0.728	0.000	0.470	0.000	0.181	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	96	0	0	68	0	72	-1
normalized size	1	1.00	0.90	0.00	0.00	0.64	0.00	0.67	-0.01
time (sec)	N/A	0.147	0.125	0.625	0.000	0.464	0.000	0.177	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	272	0	29	0	31	-1
normalized size	1	1.00	1.00	5.33	0.00	0.57	0.00	0.61	-0.02
time (sec)	N/A	0.062	0.043	1.275	0.000	0.465	0.000	0.181	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.190	0.542	0.000	0.445	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.018	0.346	0.634	0.000	0.419	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.018	0.278	0.546	0.000	0.436	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.010	0.241	0.548	0.000	0.443	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.004	0.010	0.475	0.000	0.427	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.384	0.529	0.000	0.454	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.018	0.402	0.549	0.000	0.441	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	290	0	0	211	0	494	-1
normalized size	1	1.00	1.49	0.00	0.00	1.08	0.00	2.53	-0.01
time (sec)	N/A	0.381	0.281	3.984	0.000	0.452	0.000	0.226	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	157	0	0	141	0	323	-1
normalized size	1	1.00	1.11	0.00	0.00	1.00	0.00	2.29	-0.01
time (sec)	N/A	0.208	0.141	7.069	0.000	0.460	0.000	0.224	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	97	421	0	78	0	154	-1
normalized size	1	1.00	1.17	5.07	0.00	0.94	0.00	1.86	-0.01
time (sec)	N/A	0.082	0.049	1.333	0.000	0.427	0.000	0.176	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.306	2.055	0.000	0.433	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	1.534	4.339	0.000	0.420	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.344	3.768	0.000	0.419	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.010	0.497	4.053	0.000	0.415	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.004	0.382	3.952	0.000	0.415	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	1.307	4.245	0.000	0.448	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	1.244	4.252	0.000	0.413	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	0	994	0	0	0	0	0	-1
normalized size	1	0.00	12.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	2.199	0.874	0.000	0.442	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	466	0	0	0	0	0	-1
normalized size	1	0.00	6.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	1.080	1.267	0.000	0.466	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	0	0	359	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	4.43	0.00	-0.01
time (sec)	N/A	0.043	0.025	0.047	0.000	0.423	94.766	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.018	0.336	3.060	0.000	0.455	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	0.569	6.747	0.000	0.423	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	278	171	0	239	266	0	0	-1
normalized size	1	0.75	0.46	0.00	0.64	0.72	0.00	0.00	-0.00
time (sec)	N/A	0.321	0.173	1.816	0.558	0.447	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	140	0	200	204	0	0	-1
normalized size	1	1.00	0.55	0.00	0.78	0.80	0.00	0.00	-0.00
time (sec)	N/A	0.187	0.126	1.721	0.566	0.450	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	74	0	146	121	0	0	-1
normalized size	1	1.00	0.73	0.00	1.45	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.026	1.638	0.544	0.435	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	168	1473	0	0	0	0	-1
normalized size	1	1.00	1.91	16.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.107	3.518	0.000	0.443	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	148	0	0	197	0	0	-1
normalized size	1	1.00	1.19	0.00	0.00	1.59	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.094	1.812	0.000	0.450	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	238	288	0	0	279	0	0	-1
normalized size	1	1.19	1.44	0.00	0.00	1.40	0.00	0.00	-0.00
time (sec)	N/A	0.316	0.317	1.683	0.000	0.486	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	0	12	14	0	-1
normalized size	1	1.00	1.00	1.08	0.00	0.92	1.08	0.00	-0.08
time (sec)	N/A	0.009	0.003	0.065	0.000	0.467	3.116	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	56	0	40	78	0	-1
normalized size	1	1.00	1.00	2.67	0.00	1.90	3.71	0.00	-0.05
time (sec)	N/A	0.027	0.003	0.065	0.000	0.444	3.723	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	57	0	41	78	0	-1
normalized size	1	1.00	1.00	2.71	0.00	1.95	3.71	0.00	-0.05
time (sec)	N/A	0.028	0.004	0.068	0.000	0.431	3.868	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	39	41	0	54	0	0	-1
normalized size	1	1.00	0.95	1.00	0.00	1.32	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.008	0.137	0.000	0.446	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	177	0	60	0	0	-1
normalized size	1	1.00	0.98	4.02	0.00	1.36	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.005	0.077	0.000	0.441	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	164	1356	0	0	0	0	-1
normalized size	1	1.00	2.08	17.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.067	3.154	0.000	0.418	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	270	6131	0	0	0	0	-1
normalized size	1	1.00	2.39	54.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.108	3.939	0.000	0.469	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	185	766	214	269	369	558	208
normalized size	1	1.00	1.32	5.47	1.53	1.92	2.64	3.99	1.49
time (sec)	N/A	0.077	0.213	0.537	0.449	0.446	6.404	0.200	0.314

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	121	537	136	172	223	313	131
normalized size	1	1.00	1.08	4.79	1.21	1.54	1.99	2.79	1.17
time (sec)	N/A	0.073	0.114	0.450	0.444	0.416	3.019	0.179	0.267

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	83	74	91	116	142	68
normalized size	1	1.00	0.98	0.99	0.88	1.08	1.38	1.69	0.81
time (sec)	N/A	0.037	0.048	0.089	0.440	0.426	1.377	0.165	0.254

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	30	35	32	37	39	29
normalized size	1	1.00	1.00	1.25	1.46	1.33	1.54	1.62	1.21
time (sec)	N/A	0.009	0.007	0.067	0.431	0.416	0.459	0.160	0.070

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	242	118	0	0	0	-1
normalized size	1	1.00	0.98	4.17	2.03	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.004	0.317	0.468	0.421	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	52	329	65	80	0	91	70
normalized size	1	1.00	0.76	4.84	0.96	1.18	0.00	1.34	1.03
time (sec)	N/A	0.027	0.049	0.453	0.444	0.433	0.000	0.163	1.072

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	80	582	120	236	0	266	96
normalized size	1	1.00	0.76	5.54	1.14	2.25	0.00	2.53	0.91
time (sec)	N/A	0.059	0.100	0.571	0.447	0.446	0.000	0.172	0.638

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	105	873	232	443	6069	495	145
normalized size	1	1.00	0.79	6.56	1.74	3.33	45.63	3.72	1.09
time (sec)	N/A	0.077	0.150	0.656	0.470	0.468	39.661	0.186	0.757

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	249	1330	177	498	422	273	222
normalized size	1	1.00	1.40	7.47	0.99	2.80	2.37	1.53	1.25
time (sec)	N/A	0.163	0.775	0.720	0.986	0.466	47.873	0.206	0.456

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	211	965	131	320	309	173	263
normalized size	1	1.00	1.50	6.84	0.93	2.27	2.19	1.23	1.87
time (sec)	N/A	0.132	0.480	0.599	0.990	0.446	24.023	0.196	3.276

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	83	93	80	198	160	100	81
normalized size	1	1.00	0.84	0.94	0.81	2.00	1.62	1.01	0.82
time (sec)	N/A	0.079	0.027	0.072	0.982	0.446	11.677	0.185	1.133

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	45	107	90	41	37
normalized size	1	1.00	1.00	0.84	1.00	2.38	2.00	0.91	0.82
time (sec)	N/A	0.019	0.014	0.069	0.991	0.448	5.842	0.163	0.081

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	201	366	0	0	0	0	-1
normalized size	1	1.00	1.00	1.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.266	0.083	0.375	0.000	0.408	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	137	1233	108	261	0	158	337
normalized size	1	1.00	1.15	10.36	0.91	2.19	0.00	1.33	2.83
time (sec)	N/A	0.090	0.078	0.611	1.508	0.470	0.000	0.215	1.255

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	217	2684	206	744	0	420	272
normalized size	1	1.00	1.25	15.43	1.18	4.28	0.00	2.41	1.56
time (sec)	N/A	0.145	0.571	0.917	1.075	0.549	0.000	0.220	0.981

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	264	738	332	8840	265	407	536
normalized size	1	1.00	0.82	2.31	1.04	27.62	0.83	1.27	1.68
time (sec)	N/A	0.742	0.512	0.756	1.021	11.378	79.675	0.244	0.948

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	218	537	249	5799	0	298	358
normalized size	1	1.00	0.87	2.15	1.00	23.20	0.00	1.19	1.43
time (sec)	N/A	0.483	0.330	0.774	1.020	2.764	0.000	0.213	0.322

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	204	335	187	2284	520	220	210
normalized size	1	1.00	0.89	1.46	0.82	9.97	2.27	0.96	0.92
time (sec)	N/A	0.317	0.070	0.735	1.014	1.249	141.242	0.189	0.310

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	129	113	125	110	231	143	134
normalized size	1	1.00	0.97	0.85	0.94	0.83	1.74	1.08	1.01
time (sec)	N/A	0.087	0.042	0.063	1.009	0.456	60.148	0.188	0.004

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	313	261	0	0	0	0	-1
normalized size	1	1.00	1.02	0.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.517	0.162	0.595	0.000	0.413	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	202	1068	311	7010	0	398	736
normalized size	1	1.00	0.69	3.66	1.07	24.01	0.00	1.36	2.52
time (sec)	N/A	0.549	0.639	0.830	1.011	1.534	0.000	0.320	0.487

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	303	4085	517	13236	0	790	2227
normalized size	1	1.00	0.77	10.45	1.32	33.85	0.00	2.02	5.70
time (sec)	N/A	0.713	0.691	0.841	1.028	7.227	0.000	0.724	0.884

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	114	0	166	239	484	1659	184
normalized size	1	1.00	0.82	0.00	1.19	1.72	3.48	11.94	1.32
time (sec)	N/A	0.126	0.149	0.409	0.463	0.415	9.451	0.300	0.343

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	86	0	102	153	298	918	111
normalized size	1	1.00	0.84	0.00	1.00	1.50	2.92	9.00	1.09
time (sec)	N/A	0.094	0.085	0.373	0.455	0.431	4.943	0.262	0.321

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	85	0	55	80	156	394	57
normalized size	1	1.00	1.09	0.00	0.71	1.03	2.00	5.05	0.73
time (sec)	N/A	0.056	0.029	0.143	0.457	0.415	2.354	0.190	0.304

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	114	0	159	0	0	0	-1
normalized size	1	1.00	1.01	0.00	1.41	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.027	0.443	0.492	0.416	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	0	85	148	585	192	85
normalized size	1	1.00	1.00	0.00	1.05	1.83	7.22	2.37	1.05
time (sec)	N/A	0.077	0.068	0.423	0.458	0.523	7.623	0.206	0.532

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	113	0	160	428	4527	805	217
normalized size	1	1.00	0.89	0.00	1.26	3.37	35.65	6.34	1.71
time (sec)	N/A	0.119	0.195	0.413	0.479	1.174	23.003	0.240	1.082

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	164	0	299	818	0	1841	662
normalized size	1	1.00	0.94	0.00	1.71	4.67	0.00	10.52	3.78
time (sec)	N/A	0.172	0.277	0.407	0.520	6.459	0.000	0.320	1.853

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	80	114	82	0	0	0	-1
normalized size	1	1.00	0.76	1.09	0.78	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.040	0.136	0.479	0.414	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	239	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.756	0.676	1.170	0.000	0.417	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	176	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.248	0.246	1.359	0.000	0.438	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	77	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.060	1.229	0.000	0.422	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	123	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.083	180.000	0.000	0.424	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	211	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.533	0.471	180.000	0.000	0.418	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.011	0.561	1.773	0.000	0.456	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	224	0	0	0	415	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	1.77	0.00	-0.00
time (sec)	N/A	0.233	0.490	1.796	0.000	0.418	32.591	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	178	0	0	0	284	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	1.57	0.00	-0.01
time (sec)	N/A	0.176	0.251	1.882	0.000	0.451	20.058	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	130	0	0	0	162	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	1.23	0.00	-0.01
time (sec)	N/A	0.136	0.127	2.187	0.000	0.422	12.862	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	48	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.89	0.00	-0.02
time (sec)	N/A	0.017	0.031	0.050	0.000	0.411	3.434	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.012	1.714	1.638	0.000	0.445	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.012	0.194	1.370	0.000	0.442	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.012	0.215	1.536	0.000	0.448	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	183	919	0	0	0	0	-1
normalized size	1	1.00	0.73	3.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.246	0.197	0.332	0.000	0.416	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	127	666	0	0	0	0	-1
normalized size	1	1.00	0.80	4.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.095	0.325	0.000	0.421	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	79	427	0	0	0	0	-1
normalized size	1	1.00	0.87	4.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.034	0.322	0.000	0.420	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	242	118	0	0	0	-1
normalized size	1	1.00	0.98	4.17	2.03	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.004	0.100	0.473	0.419	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	98	420	123	0	0	0	-1
normalized size	1	1.00	1.01	4.33	1.27	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.022	0.266	0.647	0.420	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	139	615	156	0	0	0	-1
normalized size	1	1.00	0.95	4.21	1.07	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.052	0.254	0.650	0.447	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	188	850	216	0	0	0	-1
normalized size	1	1.00	0.83	3.74	0.95	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.223	0.180	0.252	1.009	0.442	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	338	1083	0	0	0	0	-1
normalized size	1	1.00	0.86	2.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.426	0.350	0.331	0.000	0.430	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	271	825	0	0	0	0	-1
normalized size	1	1.00	0.87	2.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.334	0.170	0.306	0.000	0.435	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	225	576	0	0	0	0	-1
normalized size	1	1.00	0.88	2.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.273	0.123	0.314	0.000	0.593	0.000	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	201	366	0	0	0	0	-1
normalized size	1	1.00	1.00	1.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.185	0.026	0.104	0.000	0.498	0.000	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	232	624	0	0	0	0	-1
normalized size	1	1.00	0.94	2.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.303	0.073	0.248	0.000	0.692	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	268	831	0	0	0	0	-1
normalized size	1	1.00	0.88	2.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.350	0.207	0.280	0.000	0.801	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	320	1071	0	0	0	0	-1
normalized size	1	1.00	0.86	2.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.395	0.223	0.276	0.000	0.520	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	692	692	497	912	0	0	0	0	-1
normalized size	1	1.00	0.72	1.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.891	0.650	0.560	0.000	0.646	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	643	643	504	704	0	0	0	0	-1
normalized size	1	1.00	0.78	1.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.774	0.413	0.543	0.000	0.653	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	430	500	0	0	0	0	-1
normalized size	1	1.00	0.94	1.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.632	0.215	0.539	0.000	0.663	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	313	261	0	0	0	0	-1
normalized size	1	1.00	1.02	0.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.387	0.057	0.094	0.000	0.724	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	358	461	0	0	0	0	-1
normalized size	1	1.00	1.02	1.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.565	0.056	0.488	0.000	0.796	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	510	510	424	732	0	0	0	0	-1
normalized size	1	1.00	0.83	1.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.677	0.068	0.493	0.000	0.743	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	674	674	542	1025	0	0	0	0	-1
normalized size	1	1.00	0.80	1.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.793	0.393	0.495	0.000	0.881	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	251	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.321	0.228	0.430	0.000	0.490	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	183	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.266	0.134	0.401	0.000	0.551	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	149	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.059	0.424	0.000	0.582	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	114	0	159	0	0	0	-1
normalized size	1	1.00	1.01	0.00	1.41	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.019	0.063	0.735	0.703	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	139	0	179	0	0	0	-1
normalized size	1	1.00	0.87	0.00	1.13	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.246	0.061	0.417	1.053	0.692	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	166	0	230	0	0	0	-1
normalized size	1	1.00	0.84	0.00	1.16	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.277	0.097	0.408	0.877	0.625	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	241	0	307	0	0	0	-1
normalized size	1	1.00	0.84	0.00	1.07	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	0.237	0.426	1.014	1.021	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	375	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.586	0.440	0.420	0.000	0.678	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	319	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.492	0.232	0.445	0.000	1.138	0.000	0.000	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	271	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.438	0.167	0.448	0.000	0.576	0.000	0.000	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	242	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.334	0.061	0.423	0.000	0.541	0.000	0.000	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	264	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.461	0.122	0.418	0.000	0.645	0.000	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	320	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.507	0.208	0.418	0.000	0.681	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	364	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.547	0.297	0.410	0.000	0.881	0.000	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	714	714	505	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.921	0.421	0.458	0.000	0.700	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	666	666	443	0	0	0	0	0	-1
normalized size	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.731	0.234	0.421	0.000	0.906	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	488	488	403	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.601	0.125	0.415	0.000	0.809	0.000	0.000	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	350	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.409	0.092	0.409	0.000	0.847	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	395	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.538	0.099	0.415	0.000	0.550	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	429	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.668	0.216	0.433	0.000	0.710	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	737	737	520	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.795	0.302	0.420	0.000	0.643	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	749	749	867	327	0	0	0	0	-1
normalized size	1	1.00	1.16	0.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.931	0.616	0.775	0.000	0.653	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	533	533	564	504	0	0	0	0	-1
normalized size	1	1.00	1.06	0.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.509	0.374	1.477	0.000	0.503	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	178	419	309	0	0	0	-1
normalized size	1	1.00	0.78	1.83	1.35	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	0.128	0.668	1.290	0.864	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	373	0	377	0	0	0	-1
normalized size	1	1.00	1.04	0.00	1.05	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.437	0.257	0.475	1.266	0.625	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	597	597	706	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.839	0.392	0.421	0.000	0.656	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	541	541	422	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.813	0.301	0.402	0.000	0.817	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	561	561	912	0	0	0	0	0	-1
normalized size	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.107	0.581	0.407	0.000	0.767	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	215	995	227	596	0	309	298
normalized size	1	1.00	0.64	2.94	0.67	1.76	0.00	0.91	0.88
time (sec)	N/A	0.259	0.280	0.520	1.015	0.923	0.000	0.205	0.380

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	151	686	150	404	415	201	193
normalized size	1	1.00	0.68	3.10	0.68	1.83	1.88	0.91	0.87
time (sec)	N/A	0.171	0.127	0.498	1.014	0.665	89.769	0.194	0.338

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	416	85	220	228	109	97
normalized size	1	1.00	1.00	3.56	0.73	1.88	1.95	0.93	0.83
time (sec)	N/A	0.087	0.038	0.493	1.038	0.781	23.523	0.177	0.324

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	533	533	564	504	0	0	0	0	-1
normalized size	1	1.00	1.06	0.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.459	0.269	0.070	0.000	0.680	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	751	751	1236	0	0	0	0	0	-1
normalized size	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.024	3.880	1.068	0.000	0.692	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	945	945	435	0	0	0	0	0	-1
normalized size	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.251	0.525	0.749	0.000	0.585	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	548	281	0	0	0	0	0	-1
normalized size	1	1.00	0.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.720	0.281	1.517	0.000	1.021	0.000	0.000	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.029	2.841	18.029	0.000	0.456	0.000	0.000	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	8.954	41.744	0.000	0.566	0.000	0.000	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	683	0	1460	0	0	0	0	0	-1
normalized size	1	0.00	2.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.386	4.569	76.675	0.000	0.836	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.027	4.123	4.945	0.000	0.606	0.000	0.000	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	18.321	32.637	0.000	0.875	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.492	0.840	0.000	0.628	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.014	0.313	0.568	0.000	0.955	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	0.566	0.939	0.000	0.850	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.027	2.630	1.035	0.000	0.659	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.899	4.072	0.000	0.750	0.000	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.013	0.650	4.250	0.000	0.747	0.000	0.000	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	4.604	5.280	0.000	0.632	0.000	0.000	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	7.728	4.734	0.000	0.625	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	258	1311	278	708	0	354	316
normalized size	1	1.00	0.70	3.58	0.76	1.93	0.00	0.97	0.86
time (sec)	N/A	0.308	0.254	0.711	0.998	0.821	0.000	0.227	3.328

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	178	869	178	454	0	225	317
normalized size	1	1.00	0.77	3.76	0.77	1.97	0.00	0.97	1.37
time (sec)	N/A	0.177	0.214	0.544	0.995	0.756	0.000	0.247	2.769

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	402	92	250	175	117	94
normalized size	1	1.00	1.00	3.65	0.84	2.27	1.59	1.06	0.85
time (sec)	N/A	0.097	0.049	0.549	0.996	0.607	48.265	0.193	0.923

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1165	1165	990	1180	0	0	0	0	-1
normalized size	1	1.00	0.85	1.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.604	0.835	0.895	0.000	0.647	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1861	1863	2168	0	0	0	0	0	-1
normalized size	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.887	7.142	1.287	0.000	0.829	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1221	1139	1020	0	0	0	0	0	-1
normalized size	1	0.93	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.642	0.993	1.634	0.000	1.023	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	835	835	475	0	0	0	0	0	-1
normalized size	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.079	0.568	2.489	0.000	0.947	0.000	0.000	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	415	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.507	0.165	1.831	0.000	0.560	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.027	13.832	180.000	0.000	1.410	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-1)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	22.938	180.000	0.000	0.553	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-2)	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1126	0	2727	0	0	0	0	0	-1
normalized size	1	0.00	2.42	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.649	9.323	180.000	0.000	0.675	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	518	0	1146	0	0	0	0	0	-1
normalized size	1	0.00	2.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.768	4.540	175.459	0.000	0.798	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	17.402	180.000	0.000	0.697	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-1)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	39.129	180.000	0.000	0.615	0.000	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.382	0.992	0.000	0.660	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.014	0.338	0.683	0.000	1.282	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.029	2.912	1.066	0.000	0.562	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.027	8.767	1.211	0.000	0.732	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	0.716	4.557	0.000	0.742	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.014	0.562	4.747	0.000	0.857	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	8.147	4.979	0.000	0.659	0.000	0.000	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	14.479	5.957	0.000	0.613	0.000	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	170	413	132	152	0	308	127
normalized size	1	1.00	1.20	2.91	0.93	1.07	0.00	2.17	0.89
time (sec)	N/A	0.229	0.044	0.507	0.465	0.511	0.000	0.189	0.324

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	140	387	108	128	170	235	103
normalized size	1	1.00	1.18	3.25	0.91	1.08	1.43	1.97	0.87
time (sec)	N/A	0.179	0.027	0.500	0.456	0.694	162.652	0.227	0.308

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	98	361	99	99	139	148	78
normalized size	1	1.00	1.04	3.84	1.05	1.05	1.48	1.57	0.83
time (sec)	N/A	0.093	0.046	0.479	0.470	0.764	44.852	0.166	0.313

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	80	419	91	0	0	0	-1
normalized size	1	1.00	0.98	5.11	1.11	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.022	0.323	1.261	0.692	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	92	421	93	0	0	0	-1
normalized size	1	1.00	0.99	4.53	1.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.032	0.260	1.265	1.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	105	392	77	97	0	322	85
normalized size	1	1.00	1.13	4.22	0.83	1.04	0.00	3.46	0.91
time (sec)	N/A	0.137	0.041	0.404	0.471	0.675	0.000	0.203	0.369

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	130	428	104	129	0	515	113
normalized size	1	1.00	1.04	3.42	0.83	1.03	0.00	4.12	0.90
time (sec)	N/A	0.163	0.070	0.409	0.448	0.695	0.000	0.240	0.371

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	158	448	132	155	0	674	134
normalized size	1	1.00	1.07	3.03	0.89	1.05	0.00	4.55	0.91
time (sec)	N/A	0.202	0.118	0.512	0.455	0.632	0.000	0.204	0.399

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	118	453	112	300	277	138	126
normalized size	1	1.00	0.77	2.94	0.73	1.95	1.80	0.90	0.82
time (sec)	N/A	0.130	0.065	0.546	0.985	0.775	90.172	0.184	0.318

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	416	85	220	228	109	97
normalized size	1	1.00	1.00	3.56	0.73	1.88	1.95	0.93	0.83
time (sec)	N/A	0.086	0.039	0.094	0.993	0.853	24.130	0.175	0.002

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	93	62	403	61	199	262	78	83
normalized size	1	1.29	0.86	5.60	0.85	2.76	3.64	1.08	1.15
time (sec)	N/A	0.084	0.049	0.770	0.994	0.706	45.995	0.192	0.327

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	96	430	65	191	1454	92	65
normalized size	1	1.00	0.89	3.98	0.60	1.77	13.46	0.85	0.60
time (sec)	N/A	0.100	0.043	0.658	0.989	0.798	105.075	0.177	0.365

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	101	483	88	259	0	122	88
normalized size	1	1.00	0.72	3.45	0.63	1.85	0.00	0.87	0.63
time (sec)	N/A	0.121	0.007	0.624	1.006	0.858	0.000	0.208	0.377

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	205	687	223	262	0	534	224
normalized size	1	1.00	0.82	2.74	0.89	1.04	0.00	2.13	0.89
time (sec)	N/A	0.471	0.186	0.481	0.471	0.573	0.000	0.224	0.364

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	173	643	185	224	0	418	184
normalized size	1	1.00	0.82	3.06	0.88	1.07	0.00	1.99	0.88
time (sec)	N/A	0.361	0.143	0.472	0.474	0.719	0.000	0.191	0.342

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	135	599	152	180	260	288	142
normalized size	1	1.00	1.09	4.83	1.23	1.45	2.10	2.32	1.15
time (sec)	N/A	0.141	0.114	0.464	0.480	0.669	173.995	0.220	0.335

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	121	652	161	0	0	0	-1
normalized size	1	1.00	0.79	4.26	1.05	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.091	0.301	1.298	0.679	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	126	642	155	0	0	0	-1
normalized size	1	1.00	0.93	4.76	1.15	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.085	0.310	0.749	0.662	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	148	663	166	0	0	0	-1
normalized size	1	1.00	0.86	3.85	0.97	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.223	0.121	0.258	1.305	0.775	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	141	656	137	183	0	791	151
normalized size	1	1.00	1.08	5.05	1.05	1.41	0.00	6.08	1.16
time (sec)	N/A	0.208	0.133	0.465	0.488	0.821	0.000	0.226	0.415

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	184	713	183	230	0	1089	190
normalized size	1	1.00	0.85	3.30	0.85	1.06	0.00	5.04	0.88
time (sec)	N/A	0.290	0.178	0.509	0.491	0.755	0.000	0.239	0.427

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	215	748	223	268	0	1340	225
normalized size	1	1.00	0.85	2.96	0.88	1.06	0.00	5.30	0.89
time (sec)	N/A	0.333	0.239	0.529	0.497	0.666	0.000	0.305	0.458

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	188	761	189	492	0	246	235
normalized size	1	1.00	0.68	2.74	0.68	1.77	0.00	0.88	0.85
time (sec)	N/A	0.236	0.182	0.535	1.019	0.640	0.000	0.250	0.358

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	151	686	150	404	415	201	193
normalized size	1	1.00	0.68	3.10	0.68	1.83	1.88	0.91	0.87
time (sec)	N/A	0.163	0.137	0.094	1.016	1.135	95.794	0.193	0.002

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	112	742	112	366	510	168	180
normalized size	1	1.00	0.63	4.17	0.63	2.06	2.87	0.94	1.01
time (sec)	N/A	0.155	0.147	0.588	1.042	0.935	172.739	0.183	0.371

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	113	740	105	350	0	154	108
normalized size	1	1.00	0.67	4.38	0.62	2.07	0.00	0.91	0.64
time (sec)	N/A	0.147	0.135	0.615	1.012	0.981	0.000	0.184	0.392

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	156	753	116	351	0	181	115
normalized size	1	1.00	0.78	3.76	0.58	1.76	0.00	0.90	0.58
time (sec)	N/A	0.171	0.068	0.797	1.032	0.561	0.000	0.191	0.393

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	161	784	151	429	0	222	149
normalized size	1	1.00	0.64	3.11	0.60	1.70	0.00	0.88	0.59
time (sec)	N/A	0.206	0.030	0.533	1.031	0.860	0.000	0.188	0.426

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	143	902	182	0	0	0	-1
normalized size	1	1.00	0.76	4.80	0.97	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.276	0.126	1.116	1.305	0.559	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	91	672	123	0	0	0	-1
normalized size	1	1.00	0.81	6.00	1.10	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.043	0.768	1.320	0.638	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	64	472	138	0	0	0	-1
normalized size	1	1.00	0.91	6.74	1.97	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.008	0.686	0.486	0.642	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	92	732	140	0	0	0	-1
normalized size	1	1.00	0.77	6.15	1.18	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	0.039	0.635	1.259	0.789	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	147	942	178	0	0	0	-1
normalized size	1	1.00	0.84	5.35	1.01	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.267	0.074	0.675	1.260	0.722	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	667	667	691	1011	0	0	0	0	-1
normalized size	1	1.00	1.04	1.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.718	0.589	0.497	0.000	0.732	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	585	585	680	746	0	0	0	0	-1
normalized size	1	1.00	1.16	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.594	0.293	0.495	0.000	1.063	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	533	533	564	504	0	0	0	0	-1
normalized size	1	1.00	1.06	0.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.425	0.151	0.092	0.000	1.092	0.000	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	673	755	0	0	0	0	-1
normalized size	1	1.00	1.16	1.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.597	0.289	0.404	0.000	0.703	0.000	0.000	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	651	651	754	1005	0	0	0	0	-1
normalized size	1	1.00	1.16	1.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.650	0.257	0.586	0.000	0.766	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	166	985	337	0	0	0	-1
normalized size	1	1.00	0.83	4.95	1.69	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.283	0.221	0.765	0.799	0.821	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	131	732	181	0	0	0	-1
normalized size	1	1.00	0.85	4.72	1.17	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.095	0.759	1.292	0.656	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	63	371	74	91	0	182	80
normalized size	1	1.00	0.76	4.47	0.89	1.10	0.00	2.19	0.96
time (sec)	N/A	0.074	0.050	0.543	0.465	0.699	0.000	0.211	1.458

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	170	984	197	0	0	0	-1
normalized size	1	1.00	0.85	4.90	0.98	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	0.108	0.716	1.270	0.691	0.000	0.000	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	208	1216	295	0	0	0	-1
normalized size	1	1.00	0.83	4.84	1.18	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.341	0.173	0.676	0.736	0.666	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	802	802	1349	0	0	0	0	0	-1
normalized size	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.688	4.544	1.191	0.000	1.132	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	746	746	1231	0	0	0	0	0	-1
normalized size	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.513	3.517	1.140	0.000	0.685	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	751	751	1236	0	0	0	0	0	-1
normalized size	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.825	3.187	0.059	0.000	0.830	0.000	0.000	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	803	803	1438	0	0	0	0	0	-1
normalized size	1	1.00	1.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.542	4.824	1.209	0.000	0.651	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	128	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.049	1.500	0.000	0.557	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	248	214	208	0	0	0	-1
normalized size	1	1.00	1.04	0.90	0.87	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.290	0.128	0.342	1.064	0.891	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	468	282	0	0	0	0	-1
normalized size	1	1.00	2.16	1.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.253	0.133	0.086	0.000	0.699	0.000	0.000	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	118	428	0	148	0	0	-1
normalized size	1	1.00	0.82	2.97	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.173	3.014	0.000	0.808	0.000	0.000	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	100	410	0	133	0	0	-1
normalized size	1	1.00	0.81	3.31	0.00	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.116	2.912	0.000	0.625	0.000	0.000	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	68	376	0	100	0	0	-1
normalized size	1	1.00	0.82	4.53	0.00	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.059	3.500	0.000	0.491	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	87	423	0	114	0	0	-1
normalized size	1	1.00	0.90	4.36	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.100	3.413	0.000	0.455	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	104	448	0	150	0	0	-1
normalized size	1	1.00	0.83	3.56	0.00	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.200	3.441	0.000	0.451	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	209	795	0	291	0	0	-1
normalized size	1	1.00	0.64	2.43	0.00	0.89	0.00	0.00	-0.00
time (sec)	N/A	0.326	0.306	4.040	0.000	0.474	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	171	734	0	242	0	0	-1
normalized size	1	1.00	0.67	2.89	0.00	0.95	0.00	0.00	-0.00
time (sec)	N/A	0.267	0.268	3.793	0.000	0.471	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	124	665	0	192	0	0	-1
normalized size	1	1.00	0.70	3.78	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.192	3.801	0.000	0.460	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	150	693	0	210	0	0	-1
normalized size	1	1.00	0.78	3.59	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.242	0.393	3.434	0.000	0.482	0.000	0.000	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	188	755	0	265	0	0	-1
normalized size	1	1.00	0.73	2.94	0.00	1.03	0.00	0.00	-0.00
time (sec)	N/A	0.317	0.584	3.638	0.000	0.468	0.000	0.000	0.000

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	0	695	0	0	0	0	-1
normalized size	1	1.00	0.00	2.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.424	5.132	0.594	0.000	0.455	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	92	532	154	0	0	0	-1
normalized size	1	1.00	0.76	4.40	1.27	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.076	0.586	0.891	0.450	0.000	0.000	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	64	298	112	0	0	0	-1
normalized size	1	1.00	0.91	4.26	1.60	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.024	0.595	0.863	0.437	0.000	0.000	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	0	461	0	0	0	0	-1
normalized size	1	1.00	0.00	2.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.412	1.446	0.671	0.000	0.474	0.000	0.000	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	0	1036	0	0	0	0	-1
normalized size	1	1.00	0.00	2.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.565	8.115	0.603	0.000	0.454	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	171	805	233	0	0	0	-1
normalized size	1	1.00	0.84	3.95	1.14	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.268	0.165	0.708	0.897	0.448	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	433	589	209	0	0	0	-1
normalized size	1	1.00	2.78	3.78	1.34	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.278	1.507	0.616	0.883	0.460	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	0	810	0	0	0	0	-1
normalized size	1	1.00	0.00	2.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.609	1.258	0.709	0.000	0.467	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	26	23	106	25	0	0	-1
normalized size	1	1.00	1.04	0.92	4.24	1.00	0.00	0.00	-0.04
time (sec)	N/A	0.157	0.072	0.090	0.879	0.451	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	26	23	109	25	0	0	21
normalized size	1	1.00	1.04	0.92	4.36	1.00	0.00	0.00	0.84
time (sec)	N/A	0.098	0.022	0.079	0.517	0.459	0.000	0.000	0.647

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	34	24	0	30	0	0	-1
normalized size	1	1.00	1.31	0.92	0.00	1.15	0.00	0.00	-0.04
time (sec)	N/A	0.155	0.073	0.084	0.000	0.438	0.000	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	608	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.085	0.431	50.015	0.000	0.473	0.000	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	307	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.079	0.306	47.947	0.000	0.440	0.000	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.107	0.449	39.879	0.000	0.446	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.110	0.382	40.446	0.000	0.441	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.091	2.440	11.839	0.000	0.456	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.086	1.960	19.944	0.000	0.435	0.000	0.000	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.105	1.960	26.703	0.000	0.492	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.110	0.251	17.188	0.000	0.477	0.000	0.000	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	75	66	0	76	0	0	-1
normalized size	1	1.00	1.09	0.96	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.060	1.783	0.000	0.435	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	12	34	9	59	11	0	68	8
normalized size	1	1.50	4.25	1.12	7.38	1.38	0.00	8.50	1.00
time (sec)	N/A	0.010	0.003	0.072	0.461	0.415	0.000	0.266	0.314

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	76	69	15	0	0	10
normalized size	1	1.00	1.00	6.33	5.75	1.25	0.00	0.00	0.83
time (sec)	N/A	0.018	0.003	0.133	0.473	0.420	0.000	0.000	0.289

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	0	61	0	0	-1
normalized size	1	1.00	1.00	1.07	0.00	4.36	0.00	0.00	-0.07
time (sec)	N/A	0.019	0.005	0.078	0.000	0.427	0.000	0.000	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	36	34	67	0	0	204	37
normalized size	1	1.00	1.03	0.97	1.91	0.00	0.00	5.83	1.06
time (sec)	N/A	0.056	0.005	0.079	0.484	0.430	0.000	0.660	0.406

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	108	77	0	0	0	33
normalized size	1	1.00	1.03	2.77	1.97	0.00	0.00	0.00	0.85
time (sec)	N/A	0.047	0.004	0.092	0.475	0.413	0.000	0.000	0.440

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	46	0	67	0	0	-1
normalized size	1	1.00	0.94	0.98	0.00	1.43	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.017	0.079	0.000	0.450	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	80	114	124	0	0	0	-1
normalized size	1	1.00	0.76	1.09	1.18	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.031	0.092	0.495	0.417	0.000	0.000	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	228	335	0	0	0	0	-1
normalized size	1	1.00	1.00	1.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.378	0.111	0.136	0.000	0.426	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	0.502	1.677	0.000	0.417	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	82	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.055	2.215	0.000	0.424	0.000	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	159	0	128	148	155	357	137
normalized size	1	1.00	0.96	0.00	0.77	0.89	0.93	2.15	0.83
time (sec)	N/A	0.136	0.137	0.164	0.497	0.438	24.204	0.198	0.514

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	131	0	106	122	128	271	111
normalized size	1	1.00	0.98	0.00	0.79	0.91	0.96	2.02	0.83
time (sec)	N/A	0.099	0.094	0.100	0.486	0.426	9.657	0.208	0.421

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	107	0	84	95	100	185	85
normalized size	1	1.00	1.05	0.00	0.82	0.93	0.98	1.81	0.83
time (sec)	N/A	0.072	0.034	0.091	0.496	0.438	4.683	0.199	0.409

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	53	57	65	66	107	52
normalized size	1	1.00	1.00	0.88	0.95	1.08	1.10	1.78	0.87
time (sec)	N/A	0.040	0.030	0.080	0.471	0.417	1.864	0.166	0.431

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	107	0	0	0	-1
normalized size	1	1.00	1.04	0.00	2.10	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.003	0.126	1.141	0.437	0.000	0.000	0.000

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	67	0	61	65	554	187	58
normalized size	1	1.00	0.96	0.00	0.87	0.93	7.91	2.67	0.83
time (sec)	N/A	0.058	0.046	0.089	0.481	0.444	63.061	0.184	0.802

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	104	0	84	97	0	366	83
normalized size	1	1.00	0.95	0.00	0.77	0.89	0.00	3.36	0.76
time (sec)	N/A	0.074	0.040	0.091	0.489	0.441	0.000	0.191	0.634

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	132	0	106	124	0	542	110
normalized size	1	1.00	0.94	0.00	0.75	0.88	0.00	3.84	0.78
time (sec)	N/A	0.092	0.139	0.089	0.493	0.445	0.000	0.208	0.756

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	355	295	0	324	487	0	956	434
normalized size	1	0.74	0.61	0.00	0.68	1.01	0.00	1.99	0.90
time (sec)	N/A	0.476	0.332	0.086	0.518	0.477	0.000	0.246	1.782

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	263	223	0	257	357	0	642	420
normalized size	1	0.77	0.65	0.00	0.75	1.04	0.00	1.88	1.23
time (sec)	N/A	0.361	0.211	0.090	0.554	0.452	0.000	0.252	0.563

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	150	0	179	225	0	361	186
normalized size	1	1.00	0.77	0.00	0.92	1.15	0.00	1.85	0.95
time (sec)	N/A	0.184	0.085	0.085	0.540	0.440	0.000	0.191	0.474

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	195	0	0	0	0	0	-1
normalized size	1	1.00	2.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.142	0.092	0.000	0.425	0.000	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	176	188	0	0	0	0	0	-1
normalized size	1	1.14	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.351	0.194	0.092	0.000	0.451	0.000	0.000	0.000

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	318	353	0	0	0	0	0	-1
normalized size	1	1.09	1.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.667	0.362	0.086	0.000	0.444	0.000	0.000	0.000

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	432	538	0	0	0	0	0	-1
normalized size	1	1.06	1.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.034	0.294	0.091	0.000	0.445	0.000	0.000	0.000

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	907	907	577	0	666	1197	0	2223	976
normalized size	1	1.00	0.64	0.00	0.73	1.32	0.00	2.45	1.08
time (sec)	N/A	1.007	0.482	0.099	0.564	0.527	0.000	0.356	8.180

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	595	595	433	0	536	861	0	1483	840
normalized size	1	1.00	0.73	0.00	0.90	1.45	0.00	2.49	1.41
time (sec)	N/A	0.619	0.305	0.087	0.570	0.539	0.000	0.304	0.771

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	241	0	381	527	0	763	350
normalized size	1	1.00	0.85	0.00	1.34	1.86	0.00	2.69	1.23
time (sec)	N/A	0.251	0.212	0.089	0.902	0.464	0.000	0.264	0.602

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	333	0	0	0	0	0	-1
normalized size	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.196	0.168	0.090	0.000	0.427	0.000	0.000	0.000

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	283	536	0	0	0	0	0	-1
normalized size	1	1.08	2.04	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.595	0.753	0.088	0.000	0.450	0.000	0.000	0.000

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	573	550	841	0	0	0	0	0	-1
normalized size	1	0.96	1.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.497	1.167	0.092	0.000	0.433	0.000	0.000	0.000

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	158	0	118	179	162	272	140
normalized size	1	1.00	0.92	0.00	0.69	1.05	0.95	1.59	0.82
time (sec)	N/A	0.132	0.138	0.164	0.734	0.445	134.389	0.380	0.977

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	130	0	96	153	134	236	106
normalized size	1	1.00	0.94	0.00	0.69	1.10	0.96	1.70	0.76
time (sec)	N/A	0.096	0.089	0.099	0.711	0.451	49.020	0.387	0.694

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	102	0	74	126	88	81	86
normalized size	1	1.00	0.95	0.00	0.69	1.18	0.82	0.76	0.80
time (sec)	N/A	0.073	0.029	0.092	0.710	0.463	17.257	0.338	0.831

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	62	94	48	90	76	56	44
normalized size	1	1.00	1.17	1.77	0.91	1.70	1.43	1.06	0.83
time (sec)	N/A	0.034	0.033	0.085	0.665	0.459	8.420	0.251	0.380

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	123	0	0	0	-1
normalized size	1	1.00	1.04	0.00	2.41	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.003	0.184	2.687	0.417	0.000	0.000	0.000

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	68	63	75	70	0	162	60
normalized size	1	1.00	1.05	0.97	1.15	1.08	0.00	2.49	0.92
time (sec)	N/A	0.051	0.035	0.094	0.812	0.422	0.000	0.222	0.425

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	109	0	95	96	0	349	87
normalized size	1	1.00	1.05	0.00	0.91	0.92	0.00	3.36	0.84
time (sec)	N/A	0.076	0.071	0.128	0.622	0.421	0.000	0.272	0.417

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	133	0	117	123	0	535	113
normalized size	1	1.00	0.98	0.00	0.86	0.90	0.00	3.93	0.83
time (sec)	N/A	0.097	0.095	0.095	0.807	0.422	0.000	0.272	0.442

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	428	540	0	0	0	0	0	-1
normalized size	1	1.06	1.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.007	0.273	0.096	0.000	0.447	0.000	0.000	0.000

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	311	321	0	0	0	0	0	-1
normalized size	1	1.08	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.637	0.232	0.094	0.000	0.456	0.000	0.000	0.000

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	174	170	0	0	0	0	0	-1
normalized size	1	1.14	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.354	0.134	0.091	0.000	0.463	0.000	0.000	0.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	386	0	0	0	0	0	-1
normalized size	1	1.00	4.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.361	0.100	0.000	0.435	0.000	0.000	0.000

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	298	0	248	235	0	503	193
normalized size	1	1.00	1.53	0.00	1.27	1.21	0.00	2.58	0.99
time (sec)	N/A	0.198	0.352	0.104	0.786	0.435	0.000	0.406	0.474

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	263	473	0	321	361	0	1071	424
normalized size	1	0.77	1.39	0.00	0.94	1.06	0.00	3.14	1.24
time (sec)	N/A	0.366	0.391	0.138	0.880	0.444	0.000	0.439	0.559

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	355	692	0	387	490	0	1639	440
normalized size	1	0.74	1.44	0.00	0.81	1.02	0.00	3.41	0.92
time (sec)	N/A	0.471	0.351	0.170	0.667	0.499	0.000	0.451	1.772

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	569	546	777	0	0	0	0	0	-1
normalized size	1	0.96	1.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.492	1.035	0.103	0.000	0.450	0.000	0.000	0.000

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	281	476	0	0	0	0	0	-1
normalized size	1	1.08	1.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.616	0.729	0.104	0.000	0.455	0.000	0.000	0.000

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	532	0	0	0	0	0	-1
normalized size	1	1.00	3.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.310	0.104	0.000	0.462	0.000	0.000	0.000

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	558	0	568	541	0	1127	357
normalized size	1	1.00	1.96	0.00	1.99	1.90	0.00	3.95	1.25
time (sec)	N/A	0.272	0.674	0.100	0.822	0.466	0.000	0.651	0.625

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	595	595	766	0	732	869	0	2389	846
normalized size	1	1.00	1.29	0.00	1.23	1.46	0.00	4.02	1.42
time (sec)	N/A	0.642	1.096	0.102	0.893	0.457	0.000	0.725	0.800

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	907	907	950	0	864	1203	0	3651	989
normalized size	1	1.00	1.05	0.00	0.95	1.33	0.00	4.03	1.09
time (sec)	N/A	1.011	1.795	0.099	1.068	0.481	0.000	0.855	8.179

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	219	0	172	201	216	529	189
normalized size	1	1.00	0.94	0.00	0.74	0.86	0.92	2.26	0.81
time (sec)	N/A	0.188	0.239	0.155	0.869	0.467	81.574	0.203	0.652

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	176	0	140	161	173	400	150
normalized size	1	1.00	0.95	0.00	0.76	0.87	0.94	2.16	0.81
time (sec)	N/A	0.134	0.141	0.125	0.722	0.438	19.484	0.194	0.506

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	133	0	106	122	131	271	111
normalized size	1	1.00	0.98	0.00	0.78	0.90	0.96	1.99	0.82
time (sec)	N/A	0.095	0.099	0.088	0.677	0.456	5.801	0.187	0.416

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	66	70	77	82	135	65
normalized size	1	1.00	1.00	0.86	0.91	1.00	1.06	1.75	0.84
time (sec)	N/A	0.053	0.044	0.069	0.809	0.460	1.710	0.173	0.342

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	166	0	0	0	-1
normalized size	1	1.00	1.04	0.00	3.25	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.003	0.125	1.500	0.422	0.000	0.000	0.000

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	84	0	75	81	0	280	74
normalized size	1	1.00	0.97	0.00	0.86	0.93	0.00	3.22	0.85
time (sec)	N/A	0.068	0.037	0.085	0.620	0.445	0.000	0.216	0.592

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	134	0	106	125	0	542	109
normalized size	1	1.00	0.94	0.00	0.74	0.87	0.00	3.79	0.76
time (sec)	N/A	0.093	0.145	0.086	0.717	0.448	0.000	0.209	0.657

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	177	0	139	163	0	808	154
normalized size	1	1.00	0.92	0.00	0.72	0.85	0.00	4.21	0.80
time (sec)	N/A	0.125	0.198	0.097	0.972	0.480	0.000	0.223	0.612

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	491	411	0	424	674	0	1427	608
normalized size	1	0.72	0.60	0.00	0.62	0.99	0.00	2.10	0.89
time (sec)	N/A	0.697	0.572	0.087	0.588	0.522	0.000	0.279	4.703

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	355	301	0	323	484	0	956	431
normalized size	1	0.74	0.63	0.00	0.67	1.01	0.00	1.99	0.90
time (sec)	N/A	0.462	0.391	0.091	0.553	0.523	0.000	0.266	1.720

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	210	197	0	217	287	0	479	290
normalized size	1	0.79	0.74	0.00	0.81	1.07	0.00	1.79	1.09
time (sec)	N/A	0.290	0.132	0.087	0.567	0.463	0.000	0.206	0.511

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	195	0	0	0	0	0	-1
normalized size	1	1.00	2.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.111	0.119	0.000	0.420	0.000	0.000	0.000

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	253	274	0	0	0	0	0	-1
normalized size	1	1.10	1.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.501	0.231	0.087	0.000	0.447	0.000	0.000	0.000

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	430	533	0	0	0	0	0	-1
normalized size	1	1.06	1.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.013	0.286	0.103	0.000	0.474	0.000	0.000	0.000

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1835	1835	1009	0	1064	2183	0	4443	1802
normalized size	1	1.00	0.55	0.00	0.58	1.19	0.00	2.42	0.98
time (sec)	N/A	2.267	1.307	0.088	0.676	0.780	0.000	0.511	8.472

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1357	1357	808	0	867	1688	0	3333	1386
normalized size	1	1.00	0.60	0.00	0.64	1.24	0.00	2.46	1.02
time (sec)	N/A	1.573	0.876	0.102	0.645	0.669	0.000	0.428	8.254

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	907	907	589	0	668	1190	0	2223	979
normalized size	1	1.00	0.65	0.00	0.74	1.31	0.00	2.45	1.08
time (sec)	N/A	0.979	0.510	0.100	0.644	0.557	0.000	0.356	8.059

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	362	0	455	690	0	1105	558
normalized size	1	1.00	0.83	0.00	1.04	1.58	0.00	2.52	1.27
time (sec)	N/A	0.444	0.225	0.090	0.613	0.481	0.000	0.285	0.688

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	333	0	0	0	0	0	-1
normalized size	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.181	0.096	0.000	0.468	0.000	0.000	0.000

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	414	733	0	0	0	0	0	-1
normalized size	1	0.94	1.67	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.006	0.752	0.108	0.000	0.438	0.000	0.000	0.000

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	765	742	1074	0	0	0	0	0	-1
normalized size	1	0.97	1.40	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.053	1.872	0.118	0.000	0.438	0.000	0.000	0.000

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	135	0	108	129	0	266	113
normalized size	1	1.00	0.98	0.00	0.78	0.93	0.00	1.93	0.82
time (sec)	N/A	0.107	0.114	0.167	0.486	0.456	0.000	0.335	0.446

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	135	0	104	337	0	104	-1
normalized size	1	1.00	1.04	0.00	0.80	2.59	0.00	0.80	-0.01
time (sec)	N/A	0.078	0.101	0.094	1.004	0.474	0.000	0.321	0.000

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	94	0	76	83	0	82	74
normalized size	1	1.00	1.06	0.00	0.85	0.93	0.00	0.92	0.83
time (sec)	N/A	0.065	0.028	0.171	0.476	0.451	0.000	0.384	0.390

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	62	66	231	133	68	56
normalized size	1	1.00	1.00	0.86	0.92	3.21	1.85	0.94	0.78
time (sec)	N/A	0.053	0.027	0.082	0.997	0.470	7.612	0.259	0.394

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	113	0	0	0	-1
normalized size	1	1.00	1.00	0.00	2.05	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.012	0.144	1.008	0.482	0.000	0.000	0.000

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	59	0	59	208	0	61	-1
normalized size	1	1.00	0.87	0.00	0.87	3.06	0.00	0.90	-0.01
time (sec)	N/A	0.040	0.018	0.093	1.009	0.477	0.000	0.284	0.000

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	91	0	77	85	0	95	74
normalized size	1	1.00	0.97	0.00	0.82	0.90	0.00	1.01	0.79
time (sec)	N/A	0.070	0.035	0.124	0.484	0.462	0.000	0.278	0.610

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	65	0	94	313	0	94	-1
normalized size	1	1.00	0.53	0.00	0.76	2.54	0.00	0.76	-0.01
time (sec)	N/A	0.077	0.014	0.084	1.017	0.487	0.000	0.326	0.000

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	355	328	0	330	508	0	953	440
normalized size	1	0.74	0.68	0.00	0.68	1.05	0.00	1.98	0.91
time (sec)	N/A	0.488	0.382	0.130	0.524	0.532	0.000	0.712	1.754

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	217	239	0	231	304	0	316	299
normalized size	1	0.79	0.87	0.00	0.84	1.11	0.00	1.15	1.09
time (sec)	N/A	0.305	0.165	0.091	0.510	0.472	0.000	0.954	0.529

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	199	0	148	0	0	0	-1
normalized size	1	1.00	2.09	0.00	1.56	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.124	0.147	0.789	0.456	0.000	0.000	0.000

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	261	264	0	0	0	0	0	-1
normalized size	1	1.10	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.500	0.314	0.092	0.000	0.471	0.000	0.000	0.000

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	436	539	0	0	0	0	0	-1
normalized size	1	1.06	1.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.016	0.350	0.122	0.000	0.515	0.000	0.000	0.000

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	547	547	438	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.769	0.526	0.125	0.000	0.489	0.000	0.000	0.000

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	319	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.447	0.226	0.092	0.000	0.454	0.000	0.000	0.000

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	247	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.406	0.192	0.126	0.000	0.445	0.000	0.000	0.000

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	473	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.620	0.551	0.091	0.000	0.485	0.000	0.000	0.000

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	640	640	678	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.944	1.236	0.130	0.000	0.476	0.000	0.000	0.000

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	913	913	598	0	680	1241	0	2224	992
normalized size	1	1.00	0.65	0.00	0.74	1.36	0.00	2.44	1.09
time (sec)	N/A	1.032	1.070	0.098	0.588	0.627	0.000	1.346	8.104

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	428	0	484	720	0	778	575
normalized size	1	1.00	0.95	0.00	1.08	1.60	0.00	1.73	1.28
time (sec)	N/A	0.459	0.430	0.125	0.570	0.518	0.000	1.869	0.723

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	339	0	282	0	0	0	-1
normalized size	1	1.00	2.44	0.00	2.03	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.259	0.097	0.799	0.416	0.000	0.000	0.000

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	428	764	0	740	0	0	0	-1
normalized size	1	0.95	1.69	0.00	1.64	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.008	0.874	0.129	1.009	0.489	0.000	0.000	0.000

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	794	0	3146	0	0	0	0	0	-1
normalized size	1	0.00	3.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.006	9.253	0.089	0.000	0.454	0.000	0.000	0.000

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	0	598	0	0	0	0	0	-1
normalized size	1	0.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.079	1.283	0.128	0.000	0.450	0.000	0.000	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	0	646	0	0	0	0	0	-1
normalized size	1	0.00	2.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.487	2.660	0.092	0.000	0.453	0.000	0.000	0.000

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	632	0	803	0	0	0	0	0	-1
normalized size	1	0.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.870	2.937	0.129	0.000	0.462	0.000	0.000	0.000

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	218	0	162	232	0	161	191
normalized size	1	1.00	0.91	0.00	0.68	0.97	0.00	0.67	0.80
time (sec)	N/A	0.171	0.238	0.262	0.473	0.477	0.000	0.442	0.796

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	175	0	128	192	180	131	153
normalized size	1	1.00	0.92	0.00	0.67	1.01	0.95	0.69	0.81
time (sec)	N/A	0.130	0.135	0.092	0.471	0.457	102.006	0.382	0.677

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	132	0	96	153	119	101	112
normalized size	1	1.00	0.94	0.00	0.68	1.09	0.84	0.72	0.79
time (sec)	N/A	0.092	0.090	0.099	0.472	0.484	27.062	0.364	0.792

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	79	115	59	107	92	66	59
normalized size	1	1.00	1.13	1.64	0.84	1.53	1.31	0.94	0.84
time (sec)	N/A	0.050	0.048	0.094	0.452	0.441	7.104	0.285	0.470

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	185	0	0	0	-1
normalized size	1	1.00	1.04	0.00	3.63	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.003	0.197	1.916	0.431	0.000	0.000	0.000

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	85	0	86	107	0	95	73
normalized size	1	1.00	1.04	0.00	1.05	1.30	0.00	1.16	0.89
time (sec)	N/A	0.068	0.037	0.109	0.469	0.487	0.000	0.378	0.430

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	135	0	117	165	0	123	113
normalized size	1	1.00	0.98	0.00	0.85	1.20	0.00	0.89	0.82
time (sec)	N/A	0.097	0.094	0.103	0.474	0.471	0.000	0.387	0.464

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	178	0	150	213	0	153	152
normalized size	1	1.00	0.95	0.00	0.80	1.14	0.00	0.82	0.81
time (sec)	N/A	0.133	0.156	0.109	0.476	0.448	0.000	0.412	0.526

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	572	596	738	0	0	0	0	0	-1
normalized size	1	1.04	1.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.715	0.437	0.099	0.000	0.470	0.000	0.000	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	423	546	0	0	0	0	0	-1
normalized size	1	1.06	1.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.024	0.246	0.158	0.000	0.468	0.000	0.000	0.000

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	248	237	0	0	0	0	0	-1
normalized size	1	1.09	1.04	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.533	0.192	0.126	0.000	0.479	0.000	0.000	0.000

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	389	0	0	0	0	0	-1
normalized size	1	1.00	4.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.218	0.153	0.000	0.462	0.000	0.000	0.000

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	212	374	0	284	357	0	787	299
normalized size	1	0.79	1.39	0.00	1.06	1.33	0.00	2.93	1.11
time (sec)	N/A	0.311	0.377	0.095	0.519	0.478	0.000	0.398	0.564

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	355	698	0	387	597	0	1639	439
normalized size	1	0.74	1.46	0.00	0.81	1.25	0.00	3.42	0.92
time (sec)	N/A	0.480	0.354	0.102	0.533	0.505	0.000	0.589	1.762

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	759	736	1006	0	0	0	0	0	-1
normalized size	1	0.97	1.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.975	1.665	0.101	0.000	0.464	0.000	0.000	0.000

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	410	675	0	0	0	0	0	-1
normalized size	1	0.94	1.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.042	0.778	0.092	0.000	0.459	0.000	0.000	0.000

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	527	0	0	0	0	0	-1
normalized size	1	1.00	3.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.325	0.143	0.000	0.461	0.000	0.000	0.000

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	666	0	640	814	0	1758	570
normalized size	1	1.00	1.52	0.00	1.46	1.86	0.00	4.01	1.30
time (sec)	N/A	0.452	0.896	0.395	0.592	0.517	0.000	0.700	0.740

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	907	907	962	0	864	1404	0	3651	992
normalized size	1	1.00	1.06	0.00	0.95	1.55	0.00	4.03	1.09
time (sec)	N/A	1.001	1.866	0.105	0.640	0.580	0.000	0.893	8.196

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	134	0	98	160	0	103	112
normalized size	1	1.00	0.94	0.00	0.69	1.12	0.00	0.72	0.78
time (sec)	N/A	0.105	0.116	0.292	0.470	0.496	0.000	0.502	0.702

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	65	0	92	399	0	97	-1
normalized size	1	1.00	0.54	0.00	0.76	3.30	0.00	0.80	-0.01
time (sec)	N/A	0.072	0.016	0.095	0.997	0.504	0.000	0.519	0.000

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	91	0	63	113	0	72	73
normalized size	1	1.00	0.97	0.00	0.67	1.20	0.00	0.77	0.78
time (sec)	N/A	0.062	0.025	0.091	0.466	0.481	0.000	0.438	0.592

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	168	57	279	61	57	51
normalized size	1	1.00	0.82	2.58	0.88	4.29	0.94	0.88	0.78
time (sec)	N/A	0.042	0.014	0.283	0.985	0.507	52.162	0.329	0.429

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	126	0	0	0	-1
normalized size	1	1.00	1.00	0.00	2.29	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.012	0.186	1.743	0.444	0.000	0.000	0.000

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	80	0	72	235	0	73	-1
normalized size	1	1.00	1.04	0.00	0.94	3.05	0.00	0.95	-0.01
time (sec)	N/A	0.051	0.050	0.094	0.992	0.454	0.000	0.457	0.000

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	94	0	88	84	0	104	74
normalized size	1	1.00	1.06	0.00	0.99	0.94	0.00	1.17	0.83
time (sec)	N/A	0.068	0.036	0.091	0.469	0.455	0.000	0.515	0.439

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	137	0	105	339	0	103	-1
normalized size	1	1.00	1.04	0.00	0.80	2.57	0.00	0.78	-0.01
time (sec)	N/A	0.088	0.066	0.095	0.998	0.477	0.000	0.456	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	436	968	0	0	0	0	0	-1
normalized size	1	1.06	2.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.017	0.448	0.096	0.000	0.444	0.000	0.000	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	264	542	0	0	0	0	0	-1
normalized size	1	1.10	2.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.484	0.461	0.096	0.000	0.458	0.000	0.000	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	199	0	0	0	0	0	-1
normalized size	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.153	0.110	0.000	0.418	0.000	0.000	0.000

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	217	691	0	298	307	0	0	302
normalized size	1	0.79	2.50	0.00	1.08	1.11	0.00	0.00	1.09
time (sec)	N/A	0.301	0.545	0.101	0.520	0.477	0.000	0.000	0.573

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	355	1021	0	397	513	0	0	440
normalized size	1	0.74	2.12	0.00	0.82	1.06	0.00	0.00	0.91
time (sec)	N/A	0.476	0.862	0.214	0.536	0.463	0.000	0.000	1.809

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	735	0	0	0	0	0	-1
normalized size	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.807	2.193	0.182	0.000	0.443	0.000	0.000	0.000

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	523	0	0	0	0	0	-1
normalized size	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.445	1.069	0.095	0.000	0.482	0.000	0.000	0.000

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	598	0	0	0	0	0	-1
normalized size	1	1.00	1.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.594	1.272	0.095	0.000	0.430	0.000	0.000	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	773	746	1014	0	0	0	0	0	-1
normalized size	1	0.97	1.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.019	2.585	0.195	0.000	0.454	0.000	0.000	0.000

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	428	683	0	0	0	0	0	-1
normalized size	1	0.95	1.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.003	1.421	0.186	0.000	0.480	0.000	0.000	0.000

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	341	0	0	0	0	0	-1
normalized size	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	0.264	0.108	0.000	0.441	0.000	0.000	0.000

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	692	0	684	725	0	0	578
normalized size	1	1.00	1.54	0.00	1.52	1.61	0.00	0.00	1.29
time (sec)	N/A	0.463	1.531	0.173	0.616	0.452	0.000	0.000	0.745

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1278	0	764	0	0	0	0	0	-1
normalized size	1	0.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.169	4.953	0.125	0.000	0.433	0.000	0.000	0.000

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	738	0	475	0	0	0	0	0	-1
normalized size	1	0.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.297	3.218	0.100	0.000	0.462	0.000	0.000	0.000

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	483	0	1097	0	0	0	0	0	-1
normalized size	1	0.00	2.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.383	2.315	0.128	0.000	0.430	0.000	0.000	0.000

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	784	0	2726	0	0	0	0	0	-1
normalized size	1	0.00	3.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.625	8.705	0.208	0.000	0.430	0.000	0.000	0.000

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	730	730	435	0	0	0	0	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.336	1.017	0.185	0.000	0.445	0.000	0.000	0.000

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	325	0	0	0	0	0	-1
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.864	0.896	0.085	0.000	0.439	0.000	0.000	0.000

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	229	0	0	0	0	0	-1
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.543	0.434	0.083	0.000	0.471	0.000	0.000	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	130	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.129	0.097	0.000	0.461	0.000	0.000	0.000

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	0.272	0.072	0.000	0.489	0.000	0.000	0.000

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	0.502	0.075	0.000	0.459	0.000	0.000	0.000

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	907	907	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.404	0.568	0.245	0.000	0.458	0.000	0.000	0.000

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	677	677	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.982	0.387	0.074	0.000	0.456	0.000	0.000	0.000

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	445	445	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.627	0.249	0.072	0.000	0.453	0.000	0.000	0.000

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.261	0.124	0.091	0.000	0.455	0.000	0.000	0.000

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	0.120	0.074	0.000	0.477	0.000	0.000	0.000

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	0.119	0.082	0.000	0.456	0.000	0.000	0.000

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	1.163	0.123	0.000	0.753	0.000	0.000	0.000

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.021	0.244	0.079	0.000	0.612	0.000	0.000	0.000

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	0.331	0.083	0.000	0.804	0.000	0.000	0.000

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	131	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.247	0.176	0.082	0.000	0.700	0.000	0.000	0.000

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	325	0	0	0	0	0	-1
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.848	0.792	0.081	0.000	0.642	0.000	0.000	0.000

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	926	926	525	0	0	0	0	0	-1
normalized size	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.557	3.313	0.079	0.000	0.664	0.000	0.000	0.000

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	0.233	0.233	0.000	0.810	0.000	0.000	0.000

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	0.107	0.079	0.000	0.915	0.000	0.000	0.000

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.060	0.172	0.079	0.000	0.980	0.000	0.000	0.000

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	213	0	0	0	0	0	0	-1
normalized size	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.288	0.137	0.082	0.000	0.861	0.000	0.000	0.000

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	676	676	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.995	0.144	0.084	0.000	0.688	0.000	0.000	0.000

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1141	1141	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.738	0.146	0.080	0.000	1.056	0.000	0.000	0.000

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1121	1121	670	0	0	0	0	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.868	2.707	0.166	0.000	0.854	0.000	0.000	0.000

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	831	831	501	0	0	0	0	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.352	1.001	0.067	0.000	1.897	0.000	0.000	0.000

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	325	0	0	0	0	0	-1
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.846	0.951	0.073	0.000	0.798	0.000	0.000	0.000

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	174	0	0	0	0	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.393	0.206	0.069	0.000	0.807	0.000	0.000	0.000

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	0.261	0.076	0.000	0.656	0.000	0.000	0.000

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	0.622	0.071	0.000	0.867	0.000	0.000	0.000

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1363	1363	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.133	0.707	0.203	0.000	0.807	0.000	0.000	0.000

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1035	1035	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.566	0.504	0.070	0.000	0.805	0.000	0.000	0.000

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	673	673	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.971	0.371	0.073	0.000	0.768	0.000	0.000	0.000

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.460	0.136	0.068	0.000	0.616	0.000	0.000	0.000

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	0.121	0.069	0.000	0.909	0.000	0.000	0.000

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	0.122	0.071	0.000	0.784	0.000	0.000	0.000

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	325	0	0	0	0	0	-1
normalized size	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.869	0.942	0.178	0.000	0.741	0.000	0.000	0.000

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	181	0	0	0	0	0	-1
normalized size	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.381	0.216	0.071	0.000	0.808	0.000	0.000	0.000

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	0.299	0.074	0.000	0.691	0.000	0.000	0.000

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	0.624	0.078	0.000	0.627	0.000	0.000	0.000

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	0.742	0.079	0.000	0.731	0.000	0.000	0.000

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.032	0.178	0.067	0.000	1.204	0.000	0.000	0.000

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	0.490	0.071	0.000	1.307	0.000	0.000	0.000

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	678	675	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.971	0.521	0.237	0.000	0.770	0.000	0.000	0.000

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	350	347	0	0	0	0	0	0	-1
normalized size	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.453	0.300	0.083	0.000	1.232	0.000	0.000	0.000

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	0.162	0.072	0.000	1.218	0.000	0.000	0.000

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	0.159	0.069	0.000	0.707	0.000	0.000	0.000

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.058	0.183	0.070	0.000	0.535	0.000	0.000	0.000

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	0.097	0.070	0.000	0.783	0.000	0.000	0.000

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	0.129	0.073	0.000	0.667	0.000	0.000	0.000

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	1.705	0.108	0.000	0.611	0.000	0.000	0.000

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.033	0.287	0.071	0.000	0.813	0.000	0.000	0.000

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	0.330	0.078	0.000	0.666	0.000	0.000	0.000

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	175	0	0	0	0	0	-1
normalized size	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.392	0.250	0.078	0.000	0.684	0.000	0.000	0.000

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	554	554	325	0	0	0	0	0	-1
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.833	0.791	0.073	0.000	0.706	0.000	0.000	0.000

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	832	832	502	0	0	0	0	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.303	0.867	0.071	0.000	0.885	0.000	0.000	0.000

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	0.242	0.199	0.000	0.932	0.000	0.000	0.000

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	0.116	0.073	0.000	0.694	0.000	0.000	0.000

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	0.173	0.079	0.000	0.590	0.000	0.000	0.000

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	342	339	0	0	0	0	0	0	-1
normalized size	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.478	0.146	0.074	0.000	0.790	0.000	0.000	0.000

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	673	673	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.972	0.142	0.073	0.000	0.801	0.000	0.000	0.000

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1036	1036	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.519	0.139	0.077	0.000	0.719	0.000	0.000	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	1.497	0.109	0.000	0.702	0.000	0.000	0.000

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	0.753	0.074	0.000	0.765	0.000	0.000	0.000

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	0.822	0.076	0.000	0.609	0.000	0.000	0.000

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.033	0.269	0.076	0.000	0.846	0.000	0.000	0.000

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	0.515	0.079	0.000	0.720	0.000	0.000	0.000

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	0.516	0.085	0.000	0.733	0.000	0.000	0.000

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.061	0.305	0.225	0.000	1.594	0.000	0.000	0.000

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	0.212	0.076	0.000	0.859	0.000	0.000	0.000

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	0.252	0.081	0.000	0.864	0.000	0.000	0.000

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	0.119	0.074	0.000	1.231	0.000	0.000	0.000

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	0.175	0.089	0.000	0.841	0.000	0.000	0.000

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	0.151	0.077	0.000	1.106	0.000	0.000	0.000

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	631	344	0	572	1196	0	514	-1
normalized size	1	1.00	0.55	0.00	0.91	1.90	0.00	0.81	-0.00
time (sec)	N/A	0.895	0.473	1.005	1.088	1.136	0.000	0.299	0.000

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	603	603	316	0	548	1162	0	431	-1
normalized size	1	1.00	0.52	0.00	0.91	1.93	0.00	0.71	-0.00
time (sec)	N/A	0.795	0.485	0.866	1.085	0.761	0.000	0.360	0.000

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	588	588	271	0	524	1236	0	443	-1
normalized size	1	1.00	0.46	0.00	0.89	2.10	0.00	0.75	-0.00
time (sec)	N/A	0.741	0.444	0.845	1.079	0.596	0.000	0.370	0.000

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	620	620	309	0	541	1348	0	478	-1
normalized size	1	1.00	0.50	0.00	0.87	2.17	0.00	0.77	-0.00
time (sec)	N/A	0.799	0.195	0.832	1.078	1.326	0.000	0.413	0.000

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	641	641	100	0	557	1369	0	488	-1
normalized size	1	1.00	0.16	0.00	0.87	2.14	0.00	0.76	-0.00
time (sec)	N/A	0.827	0.072	0.863	1.079	1.268	0.000	0.376	0.000

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1002	1002	588	0	893	2178	0	820	-1
normalized size	1	1.00	0.59	0.00	0.89	2.17	0.00	0.82	-0.00
time (sec)	N/A	1.309	1.481	1.052	1.151	1.043	0.000	0.412	0.000

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	949	949	436	0	844	2118	0	649	-1
normalized size	1	1.00	0.46	0.00	0.89	2.23	0.00	0.68	-0.00
time (sec)	N/A	1.261	0.872	1.051	1.148	1.786	0.000	0.581	0.000

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	932	932	503	0	830	2112	0	638	-1
normalized size	1	1.00	0.54	0.00	0.89	2.27	0.00	0.68	-0.00
time (sec)	N/A	1.215	0.983	1.050	1.149	1.752	0.000	0.487	0.000

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	935	935	340	0	813	2205	0	660	-1
normalized size	1	1.00	0.36	0.00	0.87	2.36	0.00	0.71	-0.00
time (sec)	N/A	1.183	1.083	1.053	1.147	1.111	0.000	0.557	0.000

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	968	968	294	0	838	2283	0	674	-1
normalized size	1	1.00	0.30	0.00	0.87	2.36	0.00	0.70	-0.00
time (sec)	N/A	1.232	0.269	1.067	1.156	1.052	0.000	0.490	0.000

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1680	1680	1471	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.085	1.406	1.007	0.000	0.956	0.000	0.000	0.000

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1361	1361	1297	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.831	0.395	0.907	0.000	0.965	0.000	0.000	0.000

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1659	1659	1336	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.529	1.617	0.883	0.000	1.333	0.000	0.000	0.000

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	191	0	35	0	0	-1
normalized size	1	1.00	1.00	5.79	0.00	1.06	0.00	0.00	-0.03
time (sec)	N/A	0.026	0.012	2.060	0.000	0.702	0.000	0.000	0.000

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	210	1373	0	105	0	0	-1
normalized size	1	1.00	2.80	18.31	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.141	0.484	0.000	0.815	0.000	0.000	0.000

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	659	0	0	417	0	0	-1
normalized size	1	1.00	4.09	0.00	0.00	2.59	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.278	0.199	0.000	1.291	0.000	0.000	0.000

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	456	0	0	281	0	0	-1
normalized size	1	1.00	3.45	0.00	0.00	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.248	0.187	0.000	0.717	0.000	0.000	0.000

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	265	0	0	161	0	0	-1
normalized size	1	1.00	2.60	0.00	0.00	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.193	0.186	0.000	0.743	0.000	0.000	0.000

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	189	0	69	0	0	-1
normalized size	1	1.00	1.00	3.86	0.00	1.41	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.015	3.187	0.000	0.765	0.000	0.000	0.000

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.292	0.562	0.181	0.000	0.688	0.000	0.000	0.000

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.120	2.159	0.182	0.000	0.705	0.000	0.000	0.000

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	11.061	0.217	0.000	1.262	0.000	0.000	0.000

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	65	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.028	2.056	0.000	0.868	0.000	0.000	0.000

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	147	145	0	1357	0	265	192
normalized size	1	1.00	0.87	0.86	0.00	8.03	0.00	1.57	1.14
time (sec)	N/A	0.206	0.104	0.477	0.000	6.179	0.000	0.455	0.280

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	98	100	206	0	96	82
normalized size	1	1.00	1.00	1.56	1.59	3.27	0.00	1.52	1.30
time (sec)	N/A	0.048	0.035	0.167	1.480	0.771	0.000	0.199	0.134

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	47	57	54	46	80	69	46
normalized size	1	1.00	1.34	1.63	1.54	1.31	2.29	1.97	1.31
time (sec)	N/A	0.016	0.032	0.118	0.512	0.697	1.521	0.163	0.307

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	56	74	65	65	109	172	67
normalized size	1	1.00	1.24	1.64	1.44	1.44	2.42	3.82	1.49
time (sec)	N/A	0.027	0.049	0.134	0.639	0.841	2.079	0.206	0.177

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	61	115	100	287	0	137	163
normalized size	1	1.00	1.03	1.95	1.69	4.86	0.00	2.32	2.76
time (sec)	N/A	0.036	0.048	0.129	1.645	0.847	0.000	0.468	0.522

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	66	157	0	1392	0	0	499
normalized size	1	1.00	0.40	0.95	0.00	8.44	0.00	0.00	3.02
time (sec)	N/A	0.173	0.327	0.477	0.000	3.899	0.000	0.000	0.633

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.006	0.420	0.296	0.000	0.456	0.000	0.000	0.000

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	739	0	0	0	0	0	-1
normalized size	1	1.00	3.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.278	1.418	0.203	0.000	0.431	0.000	0.000	0.000

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	415	0	0	0	0	0	-1
normalized size	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.724	0.097	0.000	0.438	0.000	0.000	0.000

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	219	0	0	0	0	0	-1
normalized size	1	1.00	1.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.265	0.155	0.000	0.443	0.000	0.000	0.000

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	70	81	70	76	114	177	61
normalized size	1	1.00	1.40	1.62	1.40	1.52	2.28	3.54	1.22
time (sec)	N/A	0.038	0.042	0.102	0.674	0.443	2.047	0.210	0.419

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.006	0.556	0.154	0.000	0.441	0.000	0.000	0.000

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.006	0.876	0.156	0.000	0.438	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [134] had the largest ratio of [1.429]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	16	0.188
2	A	4	3	1.00	16	0.188
3	A	4	3	1.00	16	0.188
4	A	3	3	1.00	14	0.214
5	A	3	3	1.00	12	0.250
6	A	3	3	1.00	16	0.188
7	A	2	2	1.00	16	0.125
8	A	5	5	1.18	16	0.312
9	A	3	3	1.00	16	0.188
10	A	4	3	1.00	16	0.188
11	A	4	3	1.00	16	0.188
12	A	4	3	1.00	16	0.188
13	A	4	3	1.00	16	0.188
14	A	9	8	1.00	16	0.500
15	A	9	8	1.00	16	0.500
16	A	3	3	1.00	16	0.188
17	A	8	8	1.00	14	0.571
18	A	8	8	1.00	12	0.667
19	A	3	3	1.00	16	0.188
20	A	7	7	1.00	16	0.438
21	A	7	7	1.00	16	0.438
22	A	5	5	1.00	16	0.312
23	A	8	8	1.00	16	0.500
24	A	8	8	1.00	16	0.500
25	A	4	3	1.00	16	0.188
26	A	4	3	1.00	16	0.188
27	A	4	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	4	3	1.00	16	0.188
29	A	4	3	1.00	14	0.214
30	A	3	3	1.00	12	0.250
31	A	3	3	1.00	16	0.188
32	A	3	3	1.00	16	0.188
33	A	4	3	1.00	16	0.188
34	A	4	3	1.00	16	0.188
35	A	4	3	1.00	16	0.188
36	A	5	4	1.00	16	0.250
37	A	5	4	1.00	16	0.250
38	A	4	4	1.00	16	0.250
39	A	3	3	1.00	14	0.214
40	A	3	3	1.00	12	0.250
41	A	3	3	1.00	16	0.188
42	A	4	4	1.00	16	0.250
43	A	3	3	1.00	16	0.188
44	A	5	4	1.00	16	0.250
45	A	1	1	1.00	12	0.083
46	A	4	3	1.00	18	0.167
47	A	4	3	1.00	18	0.167
48	A	4	3	1.00	16	0.188
49	A	4	3	1.00	14	0.214
50	A	3	3	1.00	18	0.167
51	A	4	3	1.00	18	0.167
52	A	4	3	1.00	18	0.167
53	A	4	3	1.00	18	0.167
54	A	3	3	1.00	16	0.188
55	A	3	3	1.00	18	0.167
56	A	3	3	1.00	18	0.167
57	A	2	2	1.00	16	0.125
58	A	4	4	1.00	18	0.222
59	A	4	4	1.00	18	0.222
60	A	4	4	1.00	18	0.222
61	A	4	4	1.00	20	0.200
62	A	5	5	1.00	20	0.250
63	A	3	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
64	A	5	4	1.00	22	0.182
65	A	5	4	1.00	22	0.182
66	A	5	4	1.00	20	0.200
67	A	4	4	1.00	19	0.210
68	A	6	6	1.00	22	0.273
69	A	5	4	1.00	22	0.182
70	A	2	2	1.00	16	0.125
71	A	2	2	1.00	14	0.143
72	A	2	2	1.00	12	0.167
73	A	3	3	1.00	16	0.188
74	A	2	2	1.00	16	0.125
75	A	2	2	1.00	16	0.125
76	A	2	2	1.00	16	0.125
77	A	8	8	0.81	18	0.444
78	A	9	8	1.00	18	0.444
79	A	4	4	1.00	16	0.250
80	A	5	5	1.00	18	0.278
81	A	4	4	1.00	18	0.222
82	A	10	10	1.14	18	0.556
83	A	14	12	1.09	18	0.667
84	A	20	13	1.00	18	0.722
85	A	16	13	1.00	18	0.722
86	A	12	11	1.00	14	0.786
87	A	7	8	1.00	18	0.444
88	A	11	10	1.00	18	0.556
89	A	14	11	1.00	18	0.611
90	A	18	11	1.00	18	0.611
91	A	15	8	1.00	18	0.444
92	A	11	8	1.00	18	0.444
93	A	5	4	1.00	16	0.250
94	A	6	6	1.00	18	0.333
95	A	6	6	1.00	18	0.333
96	A	13	12	1.08	18	0.667
97	A	22	16	0.94	18	0.889
98	A	0	0	0.00	0	0.000
99	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
100	A	0	0	0.00	0	0.000
101	A	0	0	0.00	0	0.000
102	A	9	7	1.00	18	0.389
103	A	4	4	1.00	16	0.250
104	A	0	0	0.00	0	0.000
105	A	0	0	0.00	0	0.000
106	A	0	0	0.00	0	0.000
107	A	0	0	0.00	0	0.000
108	A	0	0	0.00	0	0.000
109	A	13	8	1.00	18	0.444
110	A	5	5	1.00	16	0.312
111	A	0	0	0.00	0	0.000
112	A	0	0	0.00	0	0.000
113	A	0	0	0.00	0	0.000
114	A	0	0	0.00	0	0.000
115	A	0	0	0.00	0	0.000
116	A	18	9	1.00	18	0.500
117	A	6	5	1.00	16	0.312
118	A	0	0	0.00	0	0.000
119	A	0	0	0.00	0	0.000
120	A	0	0	0.00	0	0.000
121	A	0	0	0.00	0	0.000
122	A	0	0	0.00	0	0.000
123	A	8	7	1.00	16	0.438
124	A	3	3	1.00	14	0.214
125	A	11	8	1.00	16	0.500
126	A	4	4	1.00	14	0.286
127	A	15	9	1.00	16	0.562
128	A	5	4	1.00	14	0.286
129	A	9	8	1.00	18	0.444
130	A	4	4	1.00	18	0.222
131	A	5	5	1.00	18	0.278
132	A	4	4	1.00	18	0.222
133	A	49	19	1.00	16	1.187
134	A	49	20	1.00	14	1.429
135	A	39	11	1.01	18	0.611

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	39	11	1.01	18	0.611
137	A	48	18	1.00	18	1.000
138	A	12	7	1.00	18	0.389
139	A	9	7	1.00	18	0.389
140	A	4	4	1.00	18	0.222
141	A	0	0	0.00	0	0.000
142	A	0	0	0.00	0	0.000
143	A	0	0	0.00	0	0.000
144	A	0	0	0.00	0	0.000
145	A	0	0	0.00	0	0.000
146	A	0	0	0.00	0	0.000
147	A	0	0	0.00	0	0.000
148	A	21	8	1.00	18	0.444
149	A	13	8	1.00	18	0.444
150	A	5	5	1.00	18	0.278
151	A	0	0	0.00	0	0.000
152	A	0	0	0.00	0	0.000
153	A	0	0	0.00	0	0.000
154	A	0	0	0.00	0	0.000
155	A	0	0	0.00	0	0.000
156	A	0	0	0.00	0	0.000
157	A	0	0	0.00	0	0.000
158	A	0	0	0.00	0	0.000
159	A	0	0	0.00	0	0.000
160	A	3	3	1.00	18	0.167
161	A	0	0	0.00	0	0.000
162	A	0	0	0.00	0	0.000
163	A	9	9	0.75	24	0.375
164	A	10	9	1.00	24	0.375
165	A	5	5	1.00	22	0.227
166	A	6	6	1.00	21	0.286
167	A	5	5	1.00	24	0.208
168	A	11	11	1.19	24	0.458
169	A	1	1	1.00	12	0.083
170	A	3	3	1.00	12	0.250
171	A	3	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
172	A	3	3	1.00	14	0.214
173	A	3	3	1.00	16	0.188
174	A	5	5	1.00	18	0.278
175	A	6	6	1.00	18	0.333
176	A	3	2	1.00	18	0.111
177	A	3	2	1.00	18	0.111
178	A	3	2	1.00	16	0.125
179	A	2	2	1.00	10	0.200
180	A	3	3	1.00	18	0.167
181	A	4	3	1.00	18	0.167
182	A	3	2	1.00	18	0.111
183	A	3	2	1.00	18	0.111
184	A	6	5	1.00	20	0.250
185	A	6	5	1.00	20	0.250
186	A	6	5	1.00	18	0.278
187	A	3	3	1.00	12	0.250
188	A	9	6	1.00	20	0.300
189	A	6	5	1.00	20	0.250
190	A	6	5	1.00	20	0.250
191	A	13	11	1.00	20	0.550
192	A	12	11	1.00	20	0.550
193	A	11	10	1.00	18	0.556
194	A	8	8	1.00	12	0.667
195	A	12	6	1.00	20	0.300
196	A	11	10	1.00	20	0.500
197	A	11	10	1.00	20	0.500
198	A	4	3	1.00	20	0.150
199	A	4	3	1.00	20	0.150
200	A	4	3	1.00	18	0.167
201	A	8	7	1.00	20	0.350
202	A	4	3	1.00	20	0.150
203	A	4	3	1.00	20	0.150
204	A	4	3	1.00	20	0.150
205	A	8	7	1.00	16	0.438
206	A	6	3	1.00	20	0.150
207	A	5	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	2	2	1.00	18	0.111
209	A	5	5	1.00	20	0.250
210	A	9	6	1.00	20	0.300
211	A	0	0	0.00	0	0.000
212	A	8	4	1.00	20	0.200
213	A	7	4	1.00	20	0.200
214	A	6	4	1.00	18	0.222
215	A	2	2	1.00	12	0.167
216	A	0	0	0.00	0	0.000
217	A	0	0	0.00	0	0.000
218	A	0	0	0.00	0	0.000
219	A	13	8	1.00	21	0.381
220	A	10	8	1.00	21	0.381
221	A	7	7	1.00	19	0.368
222	A	3	3	1.00	18	0.167
223	A	7	8	1.00	21	0.381
224	A	11	10	1.00	21	0.476
225	A	14	10	1.00	21	0.476
226	A	21	15	1.00	23	0.652
227	A	17	13	1.00	23	0.565
228	A	14	10	1.00	21	0.476
229	A	9	6	1.00	20	0.300
230	A	14	9	1.00	23	0.391
231	A	16	11	1.00	23	0.478
232	A	21	15	1.00	23	0.652
233	A	33	20	1.00	23	0.870
234	A	30	17	1.00	23	0.739
235	A	22	15	1.00	21	0.714
236	A	12	6	1.00	20	0.300
237	A	17	9	1.00	23	0.391
238	A	24	16	1.00	23	0.696
239	A	31	17	1.00	23	0.739
240	A	21	14	1.00	23	0.609
241	A	17	14	1.00	23	0.609
242	A	13	11	1.00	21	0.524
243	A	8	7	1.00	20	0.350

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
244	A	13	9	1.00	23	0.391
245	A	16	11	1.00	23	0.478
246	A	20	13	1.00	23	0.565
247	A	25	14	1.00	23	0.609
248	A	21	12	1.00	23	0.522
249	A	18	11	1.00	21	0.524
250	A	13	7	1.00	20	0.350
251	A	18	9	1.00	23	0.391
252	A	22	13	1.00	23	0.565
253	A	25	15	1.00	23	0.652
254	A	37	18	1.00	23	0.783
255	A	34	18	1.00	23	0.783
256	A	26	16	1.00	21	0.762
257	A	16	7	1.00	20	0.350
258	A	21	9	1.00	23	0.391
259	A	30	18	1.00	23	0.783
260	A	39	19	1.00	23	0.826
261	A	16	9	1.00	22	0.409
262	A	12	8	1.00	22	0.364
263	A	8	4	1.00	20	0.200
264	A	12	12	1.00	22	0.546
265	A	18	12	1.00	22	0.546
266	A	19	8	1.00	24	0.333
267	A	20	10	1.00	24	0.417
268	A	17	6	1.00	22	0.273
269	A	13	6	1.00	22	0.273
270	A	9	6	1.00	20	0.300
271	A	12	8	1.00	22	0.364
272	A	26	13	1.00	22	0.591
273	A	50	15	1.00	24	0.625
274	A	30	15	1.00	22	0.682
275	A	0	0	0.00	0	0.000
276	A	0	0	0.00	0	0.000
277	A	0	0	0.00	0	0.000
278	A	0	0	0.00	0	0.000
279	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
280	A	0	0	0.00	0	0.000
281	A	0	0	0.00	0	0.000
282	A	0	0	0.00	0	0.000
283	A	0	0	0.00	0	0.000
284	A	0	0	0.00	0	0.000
285	A	0	0	0.00	0	0.000
286	A	0	0	0.00	0	0.000
287	A	0	0	0.00	0	0.000
288	A	17	9	1.00	22	0.409
289	A	13	9	1.00	22	0.409
290	A	9	7	1.00	20	0.350
291	A	29	7	1.00	22	0.318
292	A	47	11	1.00	22	0.500
293	A	55	29	0.93	24	1.208
294	A	47	23	1.00	24	0.958
295	A	23	20	1.00	22	0.909
296	A	0	0	0.00	0	0.000
297	A	0	0	0.00	0	0.000
298	A	0	0	0.00	0	0.000
299	A	0	0	0.00	0	0.000
300	A	0	0	0.00	0	0.000
301	A	0	0	0.00	0	0.000
302	A	0	0	0.00	0	0.000
303	A	0	0	0.00	0	0.000
304	A	0	0	0.00	0	0.000
305	A	0	0	0.00	0	0.000
306	A	0	0	0.00	0	0.000
307	A	0	0	0.00	0	0.000
308	A	0	0	0.00	0	0.000
309	A	0	0	0.00	0	0.000
310	A	5	5	1.00	23	0.217
311	A	5	5	1.00	23	0.217
312	A	4	3	1.00	21	0.143
313	A	7	7	1.00	23	0.304
314	A	9	9	1.00	23	0.391
315	A	5	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
316	A	5	5	1.00	23	0.217
317	A	5	5	1.00	23	0.217
318	A	10	4	1.00	23	0.174
319	A	9	6	1.00	20	0.300
320	A	7	5	1.29	23	0.217
321	A	7	4	1.00	23	0.174
322	A	9	4	1.00	23	0.174
323	A	5	5	1.00	25	0.200
324	A	5	5	1.00	25	0.200
325	A	4	3	1.00	23	0.130
326	A	10	8	1.00	25	0.320
327	A	11	11	1.00	25	0.440
328	A	12	10	1.00	25	0.400
329	A	5	5	1.00	25	0.200
330	A	5	5	1.00	25	0.200
331	A	5	5	1.00	25	0.200
332	A	14	4	1.00	25	0.160
333	A	13	6	1.00	22	0.273
334	A	11	6	1.00	25	0.240
335	A	10	6	1.00	25	0.240
336	A	11	4	1.00	25	0.160
337	A	14	4	1.00	25	0.160
338	A	11	9	1.00	25	0.360
339	A	8	8	1.00	25	0.320
340	A	4	4	1.00	23	0.174
341	A	8	9	1.00	25	0.360
342	A	12	11	1.00	25	0.440
343	A	21	13	1.00	25	0.520
344	A	17	11	1.00	25	0.440
345	A	12	8	1.00	22	0.364
346	A	16	10	1.00	25	0.400
347	A	19	11	1.00	25	0.440
348	A	12	11	1.00	25	0.440
349	A	10	9	1.00	25	0.360
350	A	5	4	1.00	23	0.174
351	A	12	10	1.00	25	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
352	A	16	11	1.00	25	0.440
353	A	43	16	1.00	25	0.640
354	A	40	14	1.00	25	0.560
355	A	26	13	1.00	22	0.591
356	A	42	15	1.00	25	0.600
357	A	6	7	1.00	22	0.318
358	A	12	8	1.00	18	0.444
359	A	11	7	1.00	18	0.389
360	A	8	7	1.00	25	0.280
361	A	8	7	1.00	25	0.280
362	A	7	7	1.00	23	0.304
363	A	9	9	1.00	25	0.360
364	A	8	7	1.00	25	0.280
365	A	11	7	1.00	27	0.259
366	A	11	7	1.00	27	0.259
367	A	10	8	1.00	25	0.320
368	A	12	11	1.00	27	0.407
369	A	11	9	1.00	27	0.333
370	A	13	11	1.00	27	0.407
371	A	8	9	1.00	25	0.360
372	A	5	5	1.00	27	0.185
373	A	9	7	1.00	27	0.259
374	A	19	14	1.00	27	0.518
375	A	12	10	1.00	25	0.400
376	A	10	10	1.00	27	0.370
377	A	17	14	1.00	27	0.518
378	A	4	4	1.00	33	0.121
379	A	3	3	1.00	29	0.103
380	A	4	4	1.00	33	0.121
381	A	0	0	0.00	0	0.000
382	A	0	0	0.00	0	0.000
383	A	0	0	0.00	0	0.000
384	A	0	0	0.00	0	0.000
385	A	0	0	0.00	0	0.000
386	A	0	0	0.00	0	0.000
387	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
388	A	0	0	0.00	0	0.000
389	A	4	4	1.00	14	0.286
390	A	1	1	1.50	12	0.083
391	A	2	2	1.00	14	0.143
392	A	2	2	1.00	16	0.125
393	A	4	4	1.00	14	0.286
394	A	4	4	1.00	16	0.250
395	A	4	4	1.00	18	0.222
396	A	9	8	1.00	18	0.444
397	A	14	8	1.00	20	0.400
398	A	0	0	0.00	0	0.000
399	A	3	3	1.00	22	0.136
400	A	4	3	1.00	22	0.136
401	A	4	3	1.00	22	0.136
402	A	4	3	1.00	20	0.150
403	A	5	3	1.00	18	0.167
404	A	3	3	1.00	22	0.136
405	A	4	3	1.00	22	0.136
406	A	4	3	1.00	22	0.136
407	A	4	3	1.00	22	0.136
408	A	8	8	0.74	24	0.333
409	A	8	8	0.77	22	0.364
410	A	10	8	1.00	20	0.400
411	A	5	5	1.00	24	0.208
412	A	10	10	1.14	24	0.417
413	A	18	12	1.09	24	0.500
414	A	26	12	1.06	24	0.500
415	A	28	8	1.00	24	0.333
416	A	20	8	1.00	22	0.364
417	A	12	8	1.00	20	0.400
418	A	6	6	1.00	24	0.250
419	A	13	12	1.08	24	0.500
420	A	35	17	0.96	24	0.708
421	A	4	3	1.00	22	0.136
422	A	4	3	1.00	22	0.136
423	A	4	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
424	A	6	4	1.00	18	0.222
425	A	3	3	1.00	22	0.136
426	A	4	3	1.00	22	0.136
427	A	4	3	1.00	22	0.136
428	A	4	3	1.00	22	0.136
429	A	26	12	1.06	24	0.500
430	A	18	12	1.08	22	0.546
431	A	11	11	1.14	20	0.550
432	A	5	5	1.00	24	0.208
433	A	10	8	1.00	24	0.333
434	A	8	8	0.77	24	0.333
435	A	8	8	0.74	24	0.333
436	A	35	17	0.96	22	0.773
437	A	14	13	1.08	20	0.650
438	A	6	6	1.00	24	0.250
439	A	12	8	1.00	24	0.333
440	A	20	8	1.00	24	0.333
441	A	28	8	1.00	24	0.333
442	A	4	3	1.00	22	0.136
443	A	4	3	1.00	22	0.136
444	A	4	3	1.00	20	0.150
445	A	5	3	1.00	18	0.167
446	A	3	3	1.00	22	0.136
447	A	4	3	1.00	22	0.136
448	A	4	3	1.00	22	0.136
449	A	4	3	1.00	22	0.136
450	A	8	8	0.72	24	0.333
451	A	8	8	0.74	22	0.364
452	A	8	8	0.79	20	0.400
453	A	5	5	1.00	24	0.208
454	A	14	12	1.10	24	0.500
455	A	26	12	1.06	24	0.500
456	A	52	8	1.00	24	0.333
457	A	40	8	1.00	24	0.333
458	A	28	8	1.00	22	0.364
459	A	16	8	1.00	20	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
460	A	6	6	1.00	24	0.250
461	A	22	16	0.94	24	0.667
462	A	73	17	0.97	24	0.708
463	A	4	3	1.00	22	0.136
464	A	5	4	1.00	22	0.182
465	A	4	3	1.00	20	0.150
466	A	6	4	1.00	18	0.222
467	A	3	3	1.00	22	0.136
468	A	4	4	1.00	22	0.182
469	A	4	3	1.00	22	0.136
470	A	7	4	1.00	22	0.182
471	A	8	8	0.74	24	0.333
472	A	8	8	0.79	22	0.364
473	A	5	5	1.00	24	0.208
474	A	14	12	1.10	24	0.500
475	A	26	12	1.06	24	0.500
476	A	30	14	1.00	24	0.583
477	A	18	14	1.00	20	0.700
478	A	12	11	1.00	24	0.458
479	A	24	12	1.00	24	0.500
480	A	45	12	1.00	24	0.500
481	A	28	8	1.00	24	0.333
482	A	16	8	1.00	22	0.364
483	A	6	6	1.00	24	0.250
484	A	22	16	0.95	24	0.667
485	A	0	0	0.00	0	0.000
486	A	0	0	0.00	0	0.000
487	A	0	0	0.00	0	0.000
488	A	0	0	0.00	0	0.000
489	A	4	3	1.00	22	0.136
490	A	4	3	1.00	22	0.136
491	A	4	3	1.00	20	0.150
492	A	6	4	1.00	18	0.222
493	A	3	3	1.00	22	0.136
494	A	4	3	1.00	22	0.136
495	A	4	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
496	A	4	3	1.00	22	0.136
497	A	38	12	1.04	24	0.500
498	A	26	12	1.06	22	0.546
499	A	15	13	1.09	20	0.650
500	A	5	5	1.00	24	0.208
501	A	8	8	0.79	24	0.333
502	A	8	8	0.74	24	0.333
503	A	73	17	0.97	22	0.773
504	A	23	17	0.94	20	0.850
505	A	6	6	1.00	24	0.250
506	A	16	8	1.00	24	0.333
507	A	28	8	1.00	24	0.333
508	A	4	3	1.00	22	0.136
509	A	6	5	1.00	22	0.227
510	A	4	3	1.00	20	0.150
511	A	6	5	1.00	18	0.278
512	A	3	3	1.00	22	0.136
513	A	6	5	1.00	22	0.227
514	A	4	3	1.00	22	0.136
515	A	9	5	1.00	22	0.227
516	A	26	12	1.06	24	0.500
517	A	14	12	1.10	22	0.546
518	A	5	5	1.00	24	0.208
519	A	8	8	0.79	24	0.333
520	A	8	8	0.74	24	0.333
521	A	28	17	1.00	24	0.708
522	A	14	13	1.00	20	0.650
523	A	19	14	1.00	24	0.583
524	A	73	17	0.97	24	0.708
525	A	22	16	0.95	22	0.727
526	A	6	6	1.00	24	0.250
527	A	16	8	1.00	24	0.333
528	A	0	0	0.00	0	0.000
529	A	0	0	0.00	0	0.000
530	A	0	0	0.00	0	0.000
531	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
532	A	27	7	1.00	22	0.318
533	A	21	7	1.00	22	0.318
534	A	15	7	1.00	20	0.350
535	A	9	7	1.00	18	0.389
536	A	0	0	0.00	0	0.000
537	A	0	0	0.00	0	0.000
538	A	27	7	1.00	24	0.292
539	A	21	7	1.00	24	0.292
540	A	15	7	1.00	22	0.318
541	A	9	7	1.00	20	0.350
542	A	0	0	0.00	0	0.000
543	A	0	0	0.00	0	0.000
544	A	0	0	0.00	0	0.000
545	A	0	0	0.00	0	0.000
546	A	0	0	0.00	0	0.000
547	A	9	7	1.00	22	0.318
548	A	21	7	1.00	22	0.318
549	A	33	7	1.00	22	0.318
550	A	0	0	0.00	0	0.000
551	A	0	0	0.00	0	0.000
552	A	0	0	0.00	0	0.000
553	A	9	7	0.99	24	0.292
554	A	21	7	1.00	24	0.292
555	A	33	7	1.00	24	0.292
556	A	39	7	1.00	22	0.318
557	A	30	7	1.00	22	0.318
558	A	21	7	1.00	20	0.350
559	A	12	7	1.00	18	0.389
560	A	0	0	0.00	0	0.000
561	A	0	0	0.00	0	0.000
562	A	39	7	1.00	24	0.292
563	A	30	7	1.00	24	0.292
564	A	21	7	1.00	22	0.318
565	A	12	7	1.00	20	0.350
566	A	0	0	0.00	0	0.000
567	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
568	A	21	7	1.00	22	0.318
569	A	12	7	1.00	20	0.350
570	A	0	0	0.00	0	0.000
571	A	0	0	0.00	0	0.000
572	A	0	0	0.00	0	0.000
573	A	0	0	0.00	0	0.000
574	A	0	0	0.00	0	0.000
575	A	21	7	1.00	24	0.292
576	A	12	7	0.99	22	0.318
577	A	0	0	0.00	0	0.000
578	A	0	0	0.00	0	0.000
579	A	0	0	0.00	0	0.000
580	A	0	0	0.00	0	0.000
581	A	0	0	0.00	0	0.000
582	A	0	0	0.00	0	0.000
583	A	0	0	0.00	0	0.000
584	A	0	0	0.00	0	0.000
585	A	12	7	1.00	22	0.318
586	A	21	7	1.00	22	0.318
587	A	30	7	1.00	22	0.318
588	A	0	0	0.00	0	0.000
589	A	0	0	0.00	0	0.000
590	A	0	0	0.00	0	0.000
591	A	12	7	0.99	24	0.292
592	A	21	7	1.00	24	0.292
593	A	30	7	1.00	24	0.292
594	A	0	0	0.00	0	0.000
595	A	0	0	0.00	0	0.000
596	A	0	0	0.00	0	0.000
597	A	0	0	0.00	0	0.000
598	A	0	0	0.00	0	0.000
599	A	0	0	0.00	0	0.000
600	A	0	0	0.00	0	0.000
601	A	0	0	0.00	0	0.000
602	A	0	0	0.00	0	0.000
603	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
604	A	0	0	0.00	0	0.000
605	A	0	0	0.00	0	0.000
606	A	26	12	1.00	29	0.414
607	A	25	12	1.00	29	0.414
608	A	23	10	1.00	29	0.345
609	A	24	11	1.00	29	0.379
610	A	25	11	1.00	29	0.379
611	A	38	13	1.00	31	0.419
612	A	36	12	1.00	31	0.387
613	A	35	12	1.00	31	0.387
614	A	34	11	1.00	31	0.355
615	A	35	11	1.00	31	0.355
616	A	39	20	1.00	31	0.645
617	A	25	11	1.00	31	0.355
618	A	37	19	1.00	31	0.613
619	A	2	2	1.00	18	0.111
620	A	3	3	1.00	23	0.130
621	A	6	5	1.00	28	0.179
622	A	5	5	1.00	28	0.179
623	A	4	4	1.00	26	0.154
624	A	3	3	1.00	20	0.150
625	A	0	0	0.00	0	0.000
626	A	0	0	0.00	0	0.000
627	A	0	0	0.00	0	0.000
628	A	3	3	1.00	16	0.188
629	A	9	9	1.00	16	0.562
630	A	4	4	1.00	16	0.250
631	A	3	3	1.00	14	0.214
632	A	4	4	1.00	16	0.250
633	A	4	4	1.00	16	0.250
634	A	9	9	1.00	16	0.562
635	A	0	0	0.00	0	0.000
636	A	8	8	1.00	22	0.364
637	A	7	7	1.00	22	0.318
638	A	5	5	1.00	22	0.227
639	A	5	4	1.00	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
640	A	0	0	0.00	0	0.000
641	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int x^4 \log\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=80

$$\frac{2a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5b^{5/2}}-\frac{2a^2px}{5b^2}+\frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)+\frac{2apx^3}{15b}-\frac{2px^5}{25}$$

```
[Out] -2/5*a^2*p*x/b^2+2/15*a*p*x^3/b-2/25*p*x^5+2/5*a^(5/2)*p*arctan(x*b^(1/2)/a
^(1/2))/b^(5/2)+1/5*x^5*ln(c*(b*x^2+a)^p)
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2455, 302, 205}

$$-\frac{2a^2px}{5b^2}+\frac{2a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5b^{5/2}}+\frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)+\frac{2apx^3}{15b}-\frac{2px^5}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (-2*a^2*p*x)/(5*b^2) + (2*a*p*x^3)/(15*b) - (2*p*x^5)/25 + (2*a^(5/2)*p*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(5*b^(5/2)) + (x^5*Log[c*(a + b*x^2)^p])/5
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^4 \log\left(c\left(a+bx^2\right)^p\right)dx &= \frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{5}(2bp)\int\frac{x^6}{a+bx^2}\,dx\\ &= \frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{5}(2bp)\int\left(\frac{a^2}{b^3}-\frac{ax^2}{b^2}+\frac{x^4}{b}-\frac{a^3}{b^3\left(a+bx^2\right)}\right)dx\\ &= -\frac{2a^2px}{5b^2}+\frac{2apx^3}{15b}-\frac{2px^5}{25}+\frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)+\frac{\left(2a^3p\right)\int\frac{1}{a+bx^2}\,dx}{5b^2}\\ &= -\frac{2a^2px}{5b^2}+\frac{2apx^3}{15b}-\frac{2px^5}{25}+\frac{2a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5b^{5/2}}+\frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 74, normalized size = 0.92

$$\frac{1}{75}\left(\frac{30a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{30a^2px}{b^2}+15x^5\log\left(c\left(a+bx^2\right)^p\right)+\frac{10apx^3}{b}-6px^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[c*(a + b*x^2)^p],x]
```

```
[Out] ((-30*a^2*p*x)/b^2 + (10*a*p*x^3)/b - 6*p*x^5 + (30*a^(5/2)*p*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/b^(5/2) + 15*x^5*Log[c*(a + b*x^2)^p])/75
```

fricas [A] time = 0.47, size = 188, normalized size = 2.35

$$\left[\frac{15\,b^2px^5\log\left(bx^2+a\right)-6\,b^2px^5+15\,b^2x^5\log(c)+10\,abpx^3+15\,a^2p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-30\,a^2px}{75\,b^2},\ \frac{15\,b^2p}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [1/75*(15*b^2*p*x^5*log(b*x^2 + a) - 6*b^2*p*x^5 + 15*b^2*x^5*log(c) + 10*a
*b*p*x^3 + 15*a^2*p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
a)) - 30*a^2*p*x)/b^2, 1/75*(15*b^2*p*x^5*log(b*x^2 + a) - 6*b^2*p*x^5 + 15
*b^2*x^5*log(c) + 10*a*b*p*x^3 + 30*a^2*p*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a)
 - 30*a^2*p*x)/b^2]
```

giac [A] time = 0.18, size = 71, normalized size = 0.89

$$\frac{1}{5}\,px^5\log\left(bx^2+a\right)-\frac{1}{25}\left(2\,p-5\,\log(c)\right)x^5+\frac{2\,apx^3}{15\,b}+\frac{2\,a^3p\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{5\,\sqrt{ab}\,b^2}-\frac{2\,a^2px}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 1/5*p*x^5*log(b*x^2 + a) - 1/25*(2*p - 5*log(c))*x^5 + 2/15*a*p*x^3/b + 2/5
*a^3*p*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 2/5*a^2*p*x/b^2
```

maple [C] time = 0.48, size = 229, normalized size = 2.86

$$-\frac{i\pi\,x^5\operatorname{csgn}(ic)\operatorname{csgn}\left(i\left(b\,x^2+a\right)^p\right)\operatorname{csgn}\left(ic\left(b\,x^2+a\right)^p\right)}{10}+\frac{i\pi\,x^5\operatorname{csgn}(ic)\operatorname{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2}{10}+\frac{i\pi\,x^5\operatorname{csgn}\left(i\left(b\,x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(b*x^2+a)^p),x)
```

```
[Out] 1/5*x^5*ln((b*x^2+a)^p)+1/10*I*Pi*x^5*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a
)^p)^2-1/10*I*Pi*x^5*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/
10*I*Pi*x^5*csgn(I*c*(b*x^2+a)^p)^3+1/10*I*Pi*x^5*csgn(I*c*(b*x^2+a)^p)^2*c
sgn(I*c)+1/5*ln(c)*x^5-2/25*p*x^5+2/15*a*p*x^3/b+1/5/b^3*(-a*b)^(1/2)*a^2*p
*ln(-(-a*b)^(1/2)*x+a)-1/5/b^3*(-a*b)^(1/2)*a^2*p*ln((-a*b)^(1/2)*x+a)-2/5*
a^2*p*x/b^2
```

maxima [A] time = 1.27, size = 72, normalized size = 0.90

$$\frac{1}{5}\,x^5\log\left(\left(bx^2+a\right)^p c\right)+\frac{2}{75}\,bp\left(\frac{15\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}-\frac{3\,b^2x^5-5\,abx^3+15\,a^2x}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*log((b*x^2 + a)^p*c) + 2/75*b*p*(15*a^3*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*b^3) - (3*b^2*x^5 - 5*a*b*x^3 + 15*a^2*x)/b^3)
```

mupad [B] time = 0.22, size = 62, normalized size = 0.78

$$\frac{x^5\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{5}-\frac{2\,p\,x^5}{25}+\frac{2\,a^{5/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5\,b^{5/2}}+\frac{2\,a\,p\,x^3}{15\,b}-\frac{2\,a^2\,p\,x}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*log(c*(a + b*x^2)^p),x)
```

```
[Out] (x^5*log(c*(a + b*x^2)^p))/5 - (2*p*x^5)/25 + (2*a^(5/2)*p*atan((b^(1/2)*x)
/a^(1/2)))/(5*b^(5/2)) + (2*a*p*x^3)/(15*b) - (2*a^2*p*x)/(5*b^2)
```

sympy [A] time = 104.66, size = 136, normalized size = 1.70

$$\begin{cases} \frac{ia^{\frac{5}{2}}p\log\left(a+bx^2\right)}{5b^3\sqrt{\frac{1}{b}}}-\frac{2ia^{\frac{5}{2}}p\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{5b^3\sqrt{\frac{1}{b}}}-\frac{2a^2px}{5b^2}+\frac{2apx^3}{15b}+\frac{px^5\log\left(a+bx^2\right)}{5}-\frac{2px^5}{25}+\frac{x^5\log\left(c\right)}{5} & \text{for}\ b\neq 0\\ \frac{x^5\log\left(a^pc\right)}{5} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((I*a**(5/2)*p*log(a + b*x**2)/(5*b**3*sqrt(1/b)) - 2*I*a**(5/2)*p
*log(-I*sqrt(a)*sqrt(1/b) + x)/(5*b**3*sqrt(1/b)) - 2*a**2*p*x/(5*b**2) + 2
*a*p*x**3/(15*b) + p*x**5*log(a + b*x**2)/5 - 2*p*x**5/25 + x**5*log(c)/5,
Ne(b, 0)), (x**5*log(a**p*c)/5, True))
```

3.2 $\int x^3 \log\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=59

$$-\frac{a^2 p\log\left(a+bx^2\right)}{4b^2}+\frac{1}{4}x^4\log\left(c\left(a+bx^2\right)^p\right)+\frac{apx^2}{4b}-\frac{px^4}{8}$$

```
[Out] 1/4*a*p*x^2/b-1/8*p*x^4-1/4*a^2*p*ln(b*x^2+a)/b^2+1/4*x^4*ln(c*(b*x^2+a)^p)
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2395, 43}

$$-\frac{a^2 p\log\left(a+bx^2\right)}{4b^2}+\frac{1}{4}x^4\log\left(c\left(a+bx^2\right)^p\right)+\frac{apx^2}{4b}-\frac{px^4}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (a*p*x^2)/(4*b) - (p*x^4)/8 - (a^2*p*Log[a + b*x^2])/(4*b^2) + (x^4*Log[c*(
a + b*x^2)^p])/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\log\left(c\left(a+bx^2\right)^p\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int x\log\left(c(a+bx)^p\right)\,dx,x,x^2\right)\\
&= \frac{1}{4}x^4\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{4}(bp)\,\text{Subst}\left(\int \frac{x^2}{a+bx}\,dx,x,x^2\right)\\
&= \frac{1}{4}x^4\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{4}(bp)\,\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{apx^2}{4b}-\frac{px^4}{8}-\frac{a^2p\log\left(a+bx^2\right)}{4b^2}+\frac{1}{4}x^4\log\left(c\left(a+bx^2\right)^p\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 1.00

$$-\frac{a^2 p \log \left(a+b x^2\right)}{4 b^2}+\frac{1}{4} x^4 \log \left(c \left(a+b x^2\right)^p\right)+\frac{a p x^2}{4 b}-\frac{p x^4}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (a*p*x^2)/(4*b) - (p*x^4)/8 - (a^2*p*Log[a + b*x^2])/(4*b^2) + (x^4*Log[c*(
a + b*x^2)^p])/4
```

fricas [A] time = 0.45, size = 57, normalized size = 0.97

$$-\frac{b^2 p x^4-2 b^2 x^4 \log(c)-2 a b p x^2-2 \left(b^2 p x^4-a^2 p\right) \log \left(b x^2+a\right)}{8 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] -1/8*(b^2*p*x^4 - 2*b^2*x^4*log(c) - 2*a*b*p*x^2 - 2*(b^2*p*x^4 - a^2*p)*lo
g(b*x^2 + a))/b^2
```

giac [A] time = 0.17, size = 97, normalized size = 1.64

$$\frac{\frac{\left(2 \left(b x^2+a\right)^2 \log\left(b x^2+a\right)-4 \left(b x^2+a\right) a \log\left(b x^2+a\right)-\left(b x^2+a\right)^2+4 \left(b x^2+a\right) a\right) p}{b}+\frac{2 \left(\left(b x^2+a\right)^2-2 \left(b x^2+a\right) a\right) \log(c)}{b}}{8 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 1/8*((2*(b*x^2 + a)^2*log(b*x^2 + a) - 4*(b*x^2 + a)*a*log(b*x^2 + a) - (b*
x^2 + a)^2 + 4*(b*x^2 + a)*a)*p/b + 2*((b*x^2 + a)^2 - 2*(b*x^2 + a)*a)*log
(c)/b)/b
```

maple [C] time = 0.32, size = 183, normalized size = 3.10

$$-\frac{i \pi x^4 \mathrm{csgn}\left(i c\right) \mathrm{csgn}\left(i \left(b x^2+a\right)^p\right) \mathrm{csgn}\left(i c \left(b x^2+a\right)^p\right)}{8}+\frac{i \pi x^4 \mathrm{csgn}\left(i c\right) \mathrm{csgn}\left(i c \left(b x^2+a\right)^p\right)^2}{8}+\frac{i \pi x^4 \mathrm{csgn}\left(i \left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x^2+a)^p),x)
```

```
[Out] 1/4*x^4*ln((b*x^2+a)^p)+1/8*I*Pi*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)^2-1/8*I*Pi*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/8*
I*Pi*x^4*csgn(I*c*(b*x^2+a)^p)^3+1/8*I*Pi*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(
I*c)+1/4*ln(c)*x^4-1/8*p*x^4+1/4*a*p*x^2/b-1/4*a^2*p*ln(b*x^2+a)/b^2
```

maxima [A] time = 0.56, size = 55, normalized size = 0.93

$$\frac{1}{4} x^4 \log \left(\left(b x^2+a\right)^p c\right)-\frac{1}{8} b p \left(\frac{2 a^2 \log \left(b x^2+a\right)}{b^3}+\frac{b x^4-2 a x^2}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((b*x^2 + a)^p*c) - 1/8*b*p*(2*a^2*log(b*x^2 + a)/b^3 + (b*x^4 -
 2*a*x^2)/b^2)
```

mupad [B] time = 0.22, size = 51, normalized size = 0.86

$$\frac{x^4\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{4}-\frac{p\,x^4}{8}-\frac{a^2\,p\,\ln\left(b\,x^2+a\right)}{4\,b^2}+\frac{a\,p\,x^2}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(c*(a + b*x^2)^p),x)
```

```
[Out] (x^4*log(c*(a + b*x^2)^p))/4 - (p*x^4)/8 - (a^2*p*log(a + b*x^2))/(4*b^2) +
 (a*p*x^2)/(4*b)
```

sympy [A] time = 6.46, size = 70, normalized size = 1.19

$$\begin{cases} -\frac{a^2p\log\left(a+bx^2\right)}{4b^2}+\frac{apx^2}{4b}+\frac{px^4\log\left(a+bx^2\right)}{4}-\frac{px^4}{8}+\frac{x^4\log\left(c\right)}{4} & \text{for}\: b \neq 0 \\ \frac{x^4\log\left(a^pc\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((-a**2*p*log(a + b*x**2)/(4*b**2) + a*p*x**2/(4*b) + p*x**4*log(a
 + b*x**2)/4 - p*x**4/8 + x**4*log(c)/4, Ne(b, 0)), (x**4*log(a**p*c)/4, Tr
ue))
```

3.3 $\int x^2 \log\left(c\left(a+bx^2\right)^p\right) dx$

Optimal. Leaf size=66

$$-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}}+\frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)+\frac{2apx}{3b}-\frac{2px^3}{9}$$

```
[Out] 2/3*a*p*x/b-2/9*p*x^3-2/3*a^(3/2)*p*arctan(x*b^(1/2)/a^(1/2))/b^(3/2)+1/3*x
^3*ln(c*(b*x^2+a)^p)
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2455, 302, 205}

$$-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}}+\frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)+\frac{2apx}{3b}-\frac{2px^3}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (2*a*p*x)/(3*b) - (2*p*x^3)/9 - (2*a^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
3*b^(3/2)) + (x^3*Log[c*(a + b*x^2)^p])/3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2\log\left(c\left(a+bx^2\right)^p\right)dx &= \frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{3}(2bp)\int\frac{x^4}{a+bx^2}\,dx\\
&= \frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{3}(2bp)\int\left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2\left(a+bx^2\right)}\right)dx\\
&= \frac{2apx}{3b}-\frac{2px^3}{9}+\frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)-\frac{\left(2a^2p\right)\int\frac{1}{a+bx^2}\,dx}{3b}\\
&= \frac{2apx}{3b}-\frac{2px^3}{9}-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}}+\frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.94

$$\frac{1}{9}\left(-\frac{6a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}+3x^3\log\left(c\left(a+bx^2\right)^p\right)+\frac{6apx}{b}-2px^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b*x^2)^p],x]
```

```
[Out] ((6*a*p*x)/b - 2*p*x^3 - (6*a^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
+ 3*x^3*Log[c*(a + b*x^2)^p])/9
```

fricas [A] time = 0.48, size = 152, normalized size = 2.30

$$\left[\frac{3\,bpx^3\log\left(bx^2+a\right)-2\,bpx^3+3\,bx^3\log(c)+3\,ap\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\,apx}{9\,b},\ \frac{3\,bpx^3\log\left(bx^2+a\right)-2\,bp\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [1/9*(3*b*p*x^3*log(b*x^2 + a) - 2*b*p*x^3 + 3*b*x^3*log(c) + 3*a*p*sqrt(-a
/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 6*a*p*x)/b, 1/9*(3*b*
p*x^3*log(b*x^2 + a) - 2*b*p*x^3 + 3*b*x^3*log(c) - 6*a*p*sqrt(a/b)*arctan(
b*x*sqrt(a/b)/a) + 6*a*p*x)/b]
```

giac [A] time = 0.20, size = 59, normalized size = 0.89

$$\frac{1}{3}\,px^3\log\left(bx^2+a\right)-\frac{1}{9}\left(2\,p-3\,\log(c)\right)x^3-\frac{2\,a^2p\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b}+\frac{2\,apx}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 1/3*p*x^3*log(b*x^2 + a) - 1/9*(2*p - 3*log(c))*x^3 - 2/3*a^2*p*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b) + 2/3*a*p*x/b
```

maple [C] time = 0.35, size = 217, normalized size = 3.29

$$-\frac{i\pi\,x^3\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)}{6}+\frac{i\pi\,x^3\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2}{6}+\frac{i\pi\,x^3\mathrm{csgn}\left(i\left(b\,x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^2+a)^p),x)
```

```
[Out] 1/3*x^3*ln((b*x^2+a)^p)+1/6*I*Pi*x^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)^2-1/6*I*Pi*x^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/6*
I*Pi*x^3*csgn(I*c*(b*x^2+a)^p)^3+1/6*I*Pi*x^3*csgn(I*c*(b*x^2+a)^p)^2*csgn(
I*c)+1/3*ln(c)*x^3-2/9*p*x^3+1/3/b^2*(-a*b)^(1/2)*a*p*ln(-(-a*b)^(1/2)*x-a)
-1/3/b^2*(-a*b)^(1/2)*a*p*ln((-a*b)^(1/2)*x-a)+2/3*a*p*x/b
```

maxima [A] time = 1.57, size = 59, normalized size = 0.89

$$\frac{1}{3}\,x^3\log\left(\left(bx^2+a\right)^pc\right)-\frac{2}{9}\,bp\left(\frac{3\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{bx^3-3\,ax}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((b*x^2 + a)^p*c) - 2/9*b*p*(3*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*b^2) + (b*x^3 - 3*a*x)/b^2)
```

mupad [B] time = 0.23, size = 50, normalized size = 0.76

$$\frac{x^3\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{3}-\frac{2\,p\,x^3}{9}-\frac{2\,a^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3\,b^{3/2}}+\frac{2\,a\,p\,x}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(a + b*x^2)^p),x)
```

```
[Out] (x^3*log(c*(a + b*x^2)^p))/3 - (2*p*x^3)/9 - (2*a^(3/2)*p*atan((b^(1/2)*x)/
a^(1/2)))/(3*b^(3/2)) + (2*a*p*x)/(3*b)
```

sympy [A] time = 28.76, size = 121, normalized size = 1.83

$$\begin{cases} -\frac{ia^{\frac{3}{2}}p\log\left(a+bx^2\right)}{3b^2\sqrt{\frac{1}{b}}}+\frac{2ia^{\frac{3}{2}}p\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{3b^2\sqrt{\frac{1}{b}}}+\frac{2apx}{3b}+\frac{px^3\log\left(a+bx^2\right)}{3}-\frac{2px^3}{9}+\frac{x^3\log\left(c\right)}{3} & \text{for } b\neq 0 \\ \frac{x^3\log\left(a^pc\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((-I*a**(3/2)*p*log(a + b*x**2)/(3*b**2*sqrt(1/b)) + 2*I*a**(3/2)*
p*log(-I*sqrt(a)*sqrt(1/b) + x)/(3*b**2*sqrt(1/b)) + 2*a*p*x/(3*b) + p*x**3
*log(a + b*x**2)/3 - 2*p*x**3/9 + x**3*log(c)/3, Ne(b, 0)), (x**3*log(a**p*
c)/3, True))
```

3.4 $\int x \log\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=35

$$\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2b}-\frac{px^2}{2}$$

```
[Out] -1/2*p*x^2+1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2454, 2389, 2295}

$$\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2b}-\frac{px^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b*x^2)^p],x]
```

```
[Out] -(p*x^2)/2 + ((a + b*x^2)*Log[c*(a + b*x^2)^p])/(2*b)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int x\log\left(c\left(a+bx^2\right)^p\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx, x, x^2\right)\\ &= \frac{\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, a+bx^2\right)}{2b}\\ &= -\frac{px^2}{2}+\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.97

$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}-px^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b*x^2)^p],x]
[Out] (-(p*x^2) + ((a + b*x^2)*Log[c*(a + b*x^2)^p])/b)/2
```

fricas [A] time = 0.45, size = 40, normalized size = 1.14

$$-\frac{bpx^2 - bx^2\log(c) - \left(bpx^2 + ap\right)\log\left(bx^2 + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] -1/2*(b*p*x^2 - b*x^2*log(c) - (b*p*x^2 + a*p)*log(b*x^2 + a))/b
```

giac [A] time = 0.16, size = 43, normalized size = 1.23

$$-\frac{\left(bx^2 - \left(bx^2 + a\right)\log\left(bx^2 + a\right) + a\right)p - \left(bx^2 + a\right)\log(c)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] -1/2*((b*x^2 - (b*x^2 + a)*log(b*x^2 + a) + a)*p - (b*x^2 + a)*log(c))/b
```

maple [A] time = 0.07, size = 50, normalized size = 1.43

$$-\frac{p\,x^2}{2} + \frac{x^2\ln\left(c\left(b\,x^2 + a\right)^p\right)}{2} - \frac{ap}{2b} + \frac{a\ln\left(c\left(b\,x^2 + a\right)^p\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^2+a)^p),x)
[Out] 1/2*x^2*ln(c*(b*x^2+a)^p)-1/2*p*x^2+1/2/b*ln(c*(b*x^2+a)^p)*a-1/2/b*a*p
```

maxima [A] time = 0.65, size = 44, normalized size = 1.26

$$-\frac{1}{2}\,bp\left(\frac{x^2}{b} - \frac{a\log\left(bx^2 + a\right)}{b^2}\right) + \frac{1}{2}\,x^2\log\left(\left(bx^2 + a\right)^p c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] -1/2*b*p*(x^2/b - a*log(b*x^2 + a)/b^2) + 1/2*x^2*log((b*x^2 + a)^p*c)
```

mupad [B] time = 0.23, size = 39, normalized size = 1.11

$$\frac{x^2\,\ln\left(c\left(b\,x^2 + a\right)^p\right)}{2} - \frac{p\,x^2}{2} + \frac{a\,p\,\ln\left(b\,x^2 + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(a + b*x^2)^p),x)
[Out] (x^2*log(c*(a + b*x^2)^p))/2 - (p*x^2)/2 + (a*p*log(a + b*x^2))/(2*b)
```

sympy [A] time = 2.10, size = 56, normalized size = 1.60

$$\begin{cases} \frac{ap\log\left(a+bx^2\right)}{2b} + \frac{px^2\log\left(a+bx^2\right)}{2} - \frac{px^2}{2} + \frac{x^2\log\left(c\right)}{2} & \text{for } b \neq 0 \\ \frac{x^2\log\left(a^pc\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((a*p*log(a + b*x**2)/(2*b) + p*x**2*log(a + b*x**2)/2 - p*x**2/2
+ x**2*log(c)/2, Ne(b, 0)), (x**2*log(a**p*c)/2, True))
```

3.5 $\int \log\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=45

$$x\log\left(c\left(a+bx^2\right)^p\right)+\frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}-2px$$

```
[Out] -2*p*x+x*ln(c*(b*x^2+a)^p)+2*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2448, 321, 205}

$$x\log\left(c\left(a+bx^2\right)^p\right)+\frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}-2px$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p],x]
```

```
[Out] -2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + b
*x^2)^p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \log\left(c\left(a+bx^2\right)^p\right)dx &= x\log\left(c\left(a+bx^2\right)^p\right)-(2bp)\int\frac{x^2}{a+bx^2}\,dx\\
&= -2px+x\log\left(c\left(a+bx^2\right)^p\right)+(2ap)\int\frac{1}{a+bx^2}\,dx\\
&= -2px+\frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}+x\log\left(c\left(a+bx^2\right)^p\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 45, normalized size = 1.00

$$x\log\left(c\left(a+bx^2\right)^p\right)+\frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}-2px$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p],x]
[Out] -2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + b
*x^2)^p]
```

fricas [A] time = 0.46, size = 107, normalized size = 2.38

$$\left[px\log\left(bx^2+a\right)+p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-2\,px+x\log(c),px\log\left(bx^2+a\right)+2\,p\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] [p*x*log(b*x^2 + a) + p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^
2 + a)) - 2*p*x + x*log(c), p*x*log(b*x^2 + a) + 2*p*sqrt(a/b)*arctan(b*x*s
qrt(a/b)/a) - 2*p*x + x*log(c)]
```

giac [A] time = 0.16, size = 41, normalized size = 0.91

$$px\log\left(bx^2+a\right)+\frac{2\,ap\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\left(2\,p-\log(c)\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] p*x*log(b*x^2 + a) + 2*a*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - (2*p - log(c))
*x
```

maple [A] time = 0.05, size = 38, normalized size = 0.84

$$\frac{2ap\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-2px+x\ln\left(c\left(b\,x^2+a\right)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p),x)
[Out] x*ln(c*(b*x^2+a)^p)-2*p*x+2*p*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

maxima [A] time = 1.42, size = 45, normalized size = 1.00

$$2\,bp\left(\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}-\frac{x}{b}\right)+x\log\left(\left(bx^2+a\right)^p c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] 2*b*p*(a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - x/b) + x*log((b*x^2 + a)^p*c
)
```

mupad [B] time = 0.22, size = 37, normalized size = 0.82

$$x\,\ln\left(c\left(b\,x^2+a\right)^p\right)-2\,p\,x+\frac{2\,\sqrt{a}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p),x)
```

```
[Out] x*log(c*(a + b*x^2)^p) - 2*p*x + (2*a^(1/2)*p*atan((b^(1/2)*x)/a^(1/2)))/b^
(1/2)
```

sympy [A] time = 7.66, size = 90, normalized size = 2.00

$$\begin{cases} \frac{i\sqrt{a}p\log\left(a+bx^2\right)}{b\sqrt{\frac{1}{b}}} - \frac{2i\sqrt{a}p\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{b\sqrt{\frac{1}{b}}} + px\log\left(a+bx^2\right) - 2px + x\log\left(c\right) & \text{for } b \neq 0 \\ x\log\left(a^p c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((I*sqrt(a)*p*log(a + b*x**2)/(b*sqrt(1/b)) - 2*I*sqrt(a)*p*log(-I
*sqrt(a)*sqrt(1/b) + x)/(b*sqrt(1/b)) + p*x*log(a + b*x**2) - 2*p*x + x*log
(c), Ne(b, 0)), (x*log(a**p*c), True))
```

3.6 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}\,dx$

Optimal. Leaf size=44

$$\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)+\frac{1}{2}p\text{Li}_2\left(\frac{bx^2}{a}+1\right)$$

```
[Out] 1/2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)+1/2*p*polylog(2,1+b*x^2/a)
```

Rubi [A] time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2394, 2315}

$$\frac{1}{2}p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)+\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x,x]
```

```
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/2 + (p*PolyLog[2, 1 + (b*x^2)/a])/
2
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x}\,dx,x,x^2\right)\\
&= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{2}(bp)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)+\frac{1}{2}p\text{Li}_2\left(1+\frac{bx^2}{a}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.98

$$\frac{1}{2}\left(\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)+p\text{Li}_2\left(\frac{bx^2+a}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x,x]
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2, (a + b*x^2)/a])/2
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x,x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^p*c)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x,x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)/x, x)
```

maple [C] time = 0.25, size = 232, normalized size = 5.27

$$-\frac{i\pi\,\text{csgn}(ic)\,\text{csgn}\left(i\left(bx^2+a\right)^p\right)\text{csgn}\left(ic\left(bx^2+a\right)^p\right)\ln(x)}{2}+\frac{i\pi\,\text{csgn}(ic)\,\text{csgn}\left(ic\left(bx^2+a\right)^p\right)^2\ln(x)}{2}+\frac{i\pi\,\text{csgn}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x,x)
[Out] ln(x)*ln((b*x^2+a)^p)-p*ln(x)*ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-p*ln(x)*
ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-p*dilog((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))-p*dilog((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+1/2*I*ln(x)*Pi*csgn(I*(b*x^2+a)
^p)*csgn(I*c*(b*x^2+a)^p)^2-1/2*I*ln(x)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*
x^2+a)^p)*csgn(I*c)-1/2*I*ln(x)*Pi*csgn(I*c*(b*x^2+a)^p)^3+1/2*I*ln(x)*Pi*c
sgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+ln(c)*ln(x)
```

maxima [B] time = 0.70, size = 80, normalized size = 1.82

$$\frac{1}{2}bp\left(\frac{2\log\left(bx^2+a\right)\log(x)}{b}-\frac{2\log\left(\frac{bx^2}{a}+1\right)\log(x)+\text{Li}_2\left(-\frac{bx^2}{a}\right)}{b}\right)-p\log\left(bx^2+a\right)\log(x)+\log\left(\left(bx^2+a\right)^p c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*log(b*x^2 + a)*log(x)/b - (2*log(b*x^2/a + 1)*log(x) + dilog(-b*
x^2/a))/b) - p*log(b*x^2 + a)*log(x) + log((b*x^2 + a)^p*c)*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/x,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)/x, x)
```

3.7 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=44

$$\frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}$$

[Out] -ln(c*(b*x^2+a)^p)/x+2*p*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(1/2)

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2455, 205}

$$\frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}$$

Antiderivative was successfully verified.

[In] Int[Log[c*(a + b*x^2)^p]/x^2,x]

[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - Log[c*(a + b*x^2)^p]/x

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx &= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}+(2bp)\int \frac{1}{a+bx^2}\,dx\\ &= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$\frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c*(a + b*x^2)^p]/x^2,x]

[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - Log[c*(a + b*x^2)^p]/x

fricas [A] time = 0.45, size = 105, normalized size = 2.39

$$\left[\frac{px\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-p\log\left(bx^2+a\right)-\log(c)}{x}, \frac{2\,px\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-p\log\left(bx^2+a\right)-\log(c)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2,x, algorithm="fricas")
```

```
[Out] [(p*x*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - p*log(b*
x^2 + a) - log(c))/x, (2*p*x*sqrt(b/a)*arctan(x*sqrt(b/a)) - p*log(b*x^2 +
a) - log(c))/x]
```

giac [A] time = 0.17, size = 40, normalized size = 0.91

$$\frac{2\,bp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{p\log\left(bx^2+a\right)}{x}-\frac{\log(c)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2,x, algorithm="giac")
```

```
[Out] 2*b*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - p*log(b*x^2 + a)/x - log(c)/x
```

maple [C] time = 0.26, size = 195, normalized size = 4.43

$$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)}{x}-\frac{-i\pi a\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)+i\pi a\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^2,x)
```

```
[Out] -1/x*ln((b*x^2+a)^p)-1/2*(I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^
2-I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*a*csgn(I*
c*(b*x^2+a)^p)^3+I*Pi*a*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-2*(-a*b)^(1/2)*p*
ln(-b*x-(-a*b)^(1/2))*x+2*(-a*b)^(1/2)*p*ln(-b*x+(-a*b)^(1/2))*x+2*ln(c)*a)
/a/x
```

maxima [A] time = 1.55, size = 36, normalized size = 0.82

$$\frac{2\,bp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{\log\left(\left(bx^2+a\right)^pc\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2,x, algorithm="maxima")
```

```
[Out] 2*b*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - log((b*x^2 + a)^p*c)/x
```

mupad [B] time = 0.22, size = 36, normalized size = 0.82

$$\frac{2\,\sqrt{b}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\ln\left(c\left(b\,x^2+a\right)^p\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/x^2,x)
```

```
[Out] (2*b^(1/2)*p*atan((b^(1/2)*x)/a^(1/2)))/a^(1/2) - log(c*(a + b*x^2)^p)/x
```

sympy [**A**] time = 25.69, size = 377, normalized size = 8.57

$$\begin{cases} -\frac{\log(0^p c)}{x} \\ -\frac{p\log(b)}{x} - \frac{2p\log(x)}{x} - \frac{2p}{x} - \frac{\log(c)}{x} \\ -\frac{\log(a^p c)}{x} \\ \frac{ia^{\frac{3}{2}}px\sqrt{\frac{1}{b}}\log(a+bx^2)}{\frac{a^2x}{b}+ax^3} - \frac{2ia^{\frac{3}{2}}px\sqrt{\frac{1}{b}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{\frac{a^2x}{b}+ax^3} + \frac{ia^{\frac{3}{2}}x\sqrt{\frac{1}{b}}\log(c)}{\frac{a^2x}{b}+ax^3} + \frac{i\sqrt{a}bpx^3\sqrt{\frac{1}{b}}\log(a+bx^2)}{\frac{a^2x}{b}+ax^3} - \frac{2i\sqrt{a}bpx^3\sqrt{\frac{1}{b}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{\frac{a^2x}{b}+ax^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**2,x)
```

```
[Out] Piecewise((-log(0**p*c)/x, Eq(a, 0) & Eq(b, 0)), (-p*log(b)/x - 2*p*log(x)/
x - 2*p/x - log(c)/x, Eq(a, 0)), (-log(a**p*c)/x, Eq(b, 0)), (I*a**(3/2)*p*
x*sqrt(1/b)*log(a + b*x**2)/(a**2*x/b + a*x**3) - 2*I*a**(3/2)*p*x*sqrt(1/b
)*log(-I*sqrt(a)*sqrt(1/b) + x)/(a**2*x/b + a*x**3) + I*a**(3/2)*x*sqrt(1/b
)*log(c)/(a**2*x/b + a*x**3) + I*sqrt(a)*b*p*x**3*sqrt(1/b)*log(a + b*x**2)
/(a**2*x/b + a*x**3) - 2*I*sqrt(a)*b*p*x**3*sqrt(1/b)*log(-I*sqrt(a)*sqrt(1
/b) + x)/(a**2*x/b + a*x**3) + I*sqrt(a)*b*x**3*sqrt(1/b)*log(c)/(a**2*x/b
+ a*x**3) - a**2*p*log(a + b*x**2)/(a**2*x + a*b*x**3) - a**2*log(c)/(a**2*
x + a*b*x**3) - a*p*x**2*log(a + b*x**2)/(a**2*x/b + a*x**3) - a*x**2*log(c
)/(a**2*x/b + a*x**3), True))
```

3.8 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^3}\,dx$

Optimal. Leaf size=38

$$\frac{bp\log(x)}{a}-\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2ax^2}$$

```
[Out] b*p*ln(x)/a-1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/a/x^2
```

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2454, 2395, 36, 29, 31}

$$-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2x^2}-\frac{bp\log\left(a+bx^2\right)}{2a}+\frac{bp\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x^3,x]
```

```
[Out] (b*p*Log[x])/a - (b*p*Log[a + b*x^2])/(2*a) - Log[c*(a + b*x^2)^p]/(2*x^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^3}dx = \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^2}dx, x, x^2\right)$$
$$= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2x^2} + \frac{1}{2}(bp)\text{Subst}\left(\int \frac{1}{x(a+bx)}dx, x, x^2\right)$$
$$= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2x^2} + \frac{(bp)\text{Subst}\left(\int \frac{1}{x}dx, x, x^2\right)}{2a} - \frac{\left(b^2p\right)\text{Subst}\left(\int \frac{1}{a+bx}dx, x, x^2\right)}{2a}$$
$$= \frac{bp\log(x)}{a} - \frac{bp\log\left(a+bx^2\right)}{2a} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{2x^2}$$

Mathematica [A] time = 0.00, size = 45, normalized size = 1.18

$$-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2x^2} - \frac{bp\log\left(a+bx^2\right)}{2a} + \frac{bp\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^3,x]
[Out] (b*p*Log[x])/a - (b*p*Log[a + b*x^2])/(2*a) - Log[c*(a + b*x^2)^p]/(2*x^2)
```

fricas [A] time = 0.45, size = 43, normalized size = 1.13

$$\frac{2\,bpx^2\log(x) - \left(bpx^2+ap\right)\log\left(bx^2+a\right) - a\log(c)}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3,x, algorithm="fricas")
[Out] 1/2*(2*b*p*x^2*log(x) - (b*p*x^2 + a*p)*log(b*x^2 + a) - a*log(c))/(a*x^2)
```

giac [A] time = 0.17, size = 58, normalized size = 1.53

$$-\frac{\frac{b^2p\log\left(bx^2+a\right)}{a} - \frac{b^2p\log\left(bx^2\right)}{a} + \frac{bp\log\left(bx^2+a\right)}{x^2} + \frac{b\log(c)}{x^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3,x, algorithm="giac")
[Out] -1/2*(b^2*p*log(b*x^2 + a)/a - b^2*p*log(b*x^2)/a + b*p*log(b*x^2 + a)/x^2
+ b*log(c)/x^2)/b
```

maple [C] time = 0.25, size = 173, normalized size = 4.55

$$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)}{2x^2} - \frac{-4bp\,x^2\ln(x) + 2bp\,x^2\ln\left(b\,x^2+a\right) - i\pi a\,\text{csgn}\left(ic\right)\text{csgn}\left(i\left(b\,x^2+a\right)^p\right)\text{csgn}\left(ic\left(b\,x^2+a\right)^p\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^3,x)
[Out] -1/2/x^2*ln((b*x^2+a)^p)-1/4*(I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)^2-I*Pi*a*csgn(I*c)*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)-I*Pi*a*csg
```

```
n(I*c*(b*x^2+a)^p)^3+I*Pi*a*csgn(I*c)*csgn(I*c*(b*x^2+a)^p)^2-4*b*p*ln(x)*x
^2+2*b*p*ln(b*x^2+a)*x^2+2*a*ln(c))/a/x^2
```

maxima [A] time = 0.65, size = 44, normalized size = 1.16

$$-\frac{1}{2}\,bp\left(\frac{\log\left(bx^2+a\right)}{a}-\frac{\log\left(x^2\right)}{a}\right)-\frac{\log\left(\left(bx^2+a\right)^p c\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3,x, algorithm="maxima")
[Out] -1/2*b*p*(log(b*x^2 + a)/a - log(x^2)/a) - 1/2*log((b*x^2 + a)^p*c)/x^2
```

mupad [B] time = 0.24, size = 41, normalized size = 1.08

$$\frac{b\,p\,\ln(x)}{a}-\frac{b\,p\,\ln\left(b\,x^2+a\right)}{2\,a}-\frac{\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/x^3,x)
[Out] (b*p*log(x))/a - (b*p*log(a + b*x^2))/(2*a) - log(c*(a + b*x^2)^p)/(2*x^2)
```

sympy [A] time = 4.66, size = 82, normalized size = 2.16

$$\begin{cases} -\frac{p\log\left(a+bx^2\right)}{2x^2}-\frac{\log\left(c\right)}{2x^2}+\frac{bp\log\left(x\right)}{a}-\frac{bp\log\left(a+bx^2\right)}{2a} & \text{for}\ a\neq 0 \\ -\frac{p\log\left(b\right)}{2x^2}-\frac{p\log\left(x\right)}{x^2}-\frac{p}{2x^2}-\frac{\log\left(c\right)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**3,x)
[Out] Piecewise((-p*log(a + b*x**2)/(2*x**2) - log(c)/(2*x**2) + b*p*log(x)/a - b
*p*log(a + b*x**2)/(2*a), Ne(a, 0)), (-p*log(b)/(2*x**2) - p*log(x)/x**2 -
p/(2*x**2) - log(c)/(2*x**2), True))
```

3.9 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^4} \, dx$

Optimal. Leaf size=60

$$-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{3x^3}-\frac{2bp}{3ax}$$

```
[Out] -2/3*b*p/a/x-2/3*b^(3/2)*p*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)-1/3*ln(c*(b*x^
2+a)^p)/x^3
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2455, 325, 205}

$$-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{3x^3}-\frac{2bp}{3ax}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x^4,x]
```

```
[Out] (-2*b*p)/(3*a*x) - (2*b^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*a^(3/2)) -
Log[c*(a + b*x^2)^p]/(3*x^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^4} \, dx &= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{3x^3}+\frac{1}{3}(2bp)\int \frac{1}{x^2\left(a+bx^2\right)} \, dx\\
&= -\frac{2bp}{3ax}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{3x^3}-\frac{\left(2b^2p\right)\int \frac{1}{a+bx^2} \, dx}{3a}\\
&= -\frac{2bp}{3ax}-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{3x^3}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 49, normalized size = 0.82

$$-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{3x^3}-\frac{2bp\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{bx^2}{a}\right)}{3ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^4,x]
```

```
[Out] (-2*b*p*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^2)/a)])/(3*a*x) - Log[c*(a +
 b*x^2)^p]/(3*x^3)
```

fricas [A] time = 0.48, size = 135, normalized size = 2.25

$$\left[\frac{bpx^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2\,bpx^2-ap\log\left(bx^2+a\right)-a\log(c)}{3\,ax^3},-\frac{2\,bpx^3\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,bpx^2+ap\,\mathrm{l}}{3\,ax^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^4,x, algorithm="fricas")
```

```
[Out] [1/3*(b*p*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) -
2*b*p*x^2 - a*p*log(b*x^2 + a) - a*log(c))/(a*x^3), -1/3*(2*b*p*x^3*sqrt(b/
a)*arctan(x*sqrt(b/a)) + 2*b*p*x^2 + a*p*log(b*x^2 + a) + a*log(c))/(a*x^3)
]
```

giac [A] time = 0.20, size = 58, normalized size = 0.97

$$-\frac{2\,b^2p\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,a}-\frac{p\log\left(bx^2+a\right)}{3\,x^3}-\frac{2\,bpx^2+a\log(c)}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^4,x, algorithm="giac")
```

```
[Out] -2/3*b^2*p*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/3*p*log(b*x^2 + a)/x^3 -
 1/3*(2*b*p*x^2 + a*log(c))/(a*x^3)
```

maple [C] time = 0.35, size = 211, normalized size = 3.52

$$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)}{3x^3}-\frac{-2a\,x^3\,\mathrm{RootOf}\left(a^3\_Z^2+b^3p^2\right)\ln\left(\mathrm{RootOf}\left(a^3\_Z^2+b^3p^2\right)a^2bp+\left(3\,\mathrm{RootOf}\left(a^3\_Z^2+b^3p^2\right)^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^4,x)
```

```
[Out] -1/3/x^3*ln((b*x^2+a)^p)-1/6*(I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)^2-I*Pi*a*csgn(I*c)*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)-I*Pi*a*csg
n(I*c*(b*x^2+a)^p)^3+I*Pi*a*csgn(I*c)*csgn(I*c*(b*x^2+a)^p)^2-2*sum(_R*ln((
3*_R^2*a^3+2*b^3*p^2)*x+a^2*b*p*_R),_R=RootOf(_Z^2*a^3+b^3*p^2))*a*x^3+4*x^
2*p*b+2*a*ln(c))/a/x^3
```

maxima [A] time = 1.42, size = 49, normalized size = 0.82

$$-\frac{2}{3}\,bp\left(\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{1}{ax}\right)-\frac{\log\left(\left(bx^2+a\right)^pc\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -2/3*b*p*(b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/(a*x)) - 1/3*log((b*x^2
 + a)^p*c)/x^3
```

mupad [B] time = 0.25, size = 46, normalized size = 0.77

$$-\frac{\ln\left(c\left(b\,x^2+a\right)^p\right)}{3\,x^3}-\frac{2\,b^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3\,a^{3/2}}-\frac{2\,b\,p}{3\,a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/x^4,x)
```

```
[Out] - log(c*(a + b*x^2)^p)/(3*x^3) - (2*b^(3/2)*p*atan((b^(1/2)*x)/a^(1/2)))/(3
*a^(3/2)) - (2*b*p)/(3*a*x)
```

sympy [A] time = 111.55, size = 763, normalized size = 12.72

$$\begin{cases} -\frac{\log\left(0^p c\right)}{3x^3} \\ -\frac{\log\left(a^p c\right)}{3x^3} \\ -\frac{p\log\left(b\right)}{3x^3}-\frac{2p\log\left(x\right)}{3x^3}-\frac{2p}{9x^3}-\frac{\log\left(c\right)}{3x^3} \\ -\frac{ia^{\frac{5}{2}}p\sqrt{\frac{1}{b}}\log\left(a+bx^2\right)}{3ia^{\frac{5}{2}}x^3\sqrt{\frac{1}{b}}+3ia^{\frac{3}{2}}bx^5\sqrt{\frac{1}{b}}}-\frac{ia^{\frac{5}{2}}\sqrt{\frac{1}{b}}\log\left(c\right)}{3ia^{\frac{5}{2}}x^3\sqrt{\frac{1}{b}}+3ia^{\frac{3}{2}}bx^5\sqrt{\frac{1}{b}}}-\frac{ia^{\frac{3}{2}}px^2\sqrt{\frac{1}{b}}\log\left(a+bx^2\right)}{\frac{3ia^{\frac{5}{2}}x^3\sqrt{\frac{1}{b}}}{b}+3ia^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}}-\frac{2ia^{\frac{3}{2}}px^2\sqrt{\frac{1}{b}}}{\frac{3ia^{\frac{5}{2}}x^3\sqrt{\frac{1}{b}}}{b}+3ia^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}}-\frac{ia^{\frac{3}{2}}x^2\sqrt{\frac{1}{b}}\log\left(c\right)}{\frac{3ia^{\frac{5}{2}}x^3\sqrt{\frac{1}{b}}}{b}+3ia^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**4,x)
```

```
[Out] Piecewise((-log(0**p*c)/(3*x**3), Eq(a, 0) & Eq(b, 0)), (-log(a**p*c)/(3*x*
*3), Eq(b, 0)), (-p*log(b)/(3*x**3) - 2*p*log(x)/(3*x**3) - 2*p/(9*x**3) -
log(c)/(3*x**3), Eq(a, 0)), (-I*a**(5/2)*p*sqrt(1/b)*log(a + b*x**2)/(3*I*a
**(5/2)*x**3*sqrt(1/b) + 3*I*a**(3/2)*b*x**5*sqrt(1/b)) - I*a**(5/2)*sqrt(1
/b)*log(c)/(3*I*a**(5/2)*x**3*sqrt(1/b) + 3*I*a**(3/2)*b*x**5*sqrt(1/b)) -
I*a**(3/2)*p*x**2*sqrt(1/b)*log(a + b*x**2)/(3*I*a**(5/2)*x**3*sqrt(1/b)/b
+ 3*I*a**(3/2)*x**5*sqrt(1/b)) - 2*I*a**(3/2)*p*x**2*sqrt(1/b)/(3*I*a**(5/2
)*x**3*sqrt(1/b)/b + 3*I*a**(3/2)*x**5*sqrt(1/b)) - I*a**(3/2)*x**2*sqrt(1/
b)*log(c)/(3*I*a**(5/2)*x**3*sqrt(1/b)/b + 3*I*a**(3/2)*x**5*sqrt(1/b)) - 2
*I*sqrt(a)*b*p*x**4*sqrt(1/b)/(3*I*a**(5/2)*x**3*sqrt(1/b)/b + 3*I*a**(3/2)
*x**5*sqrt(1/b)) + a*p*x**3*log(a + b*x**2)/(3*I*a**(5/2)*x**3*sqrt(1/b)/b
+ 3*I*a**(3/2)*x**5*sqrt(1/b)) - 2*a*p*x**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(
3*I*a**(5/2)*x**3*sqrt(1/b)/b + 3*I*a**(3/2)*x**5*sqrt(1/b)) + a*x**3*log(c
)/(3*I*a**(5/2)*x**3*sqrt(1/b)/b + 3*I*a**(3/2)*x**5*sqrt(1/b)) + b*p*x**5*
log(a + b*x**2)/(3*I*a**(5/2)*x**3*sqrt(1/b)/b + 3*I*a**(3/2)*x**5*sqrt(1/b
)) - 2*b*p*x**5*log(-I*sqrt(a)*sqrt(1/b) + x)/(3*I*a**(5/2)*x**3*sqrt(1/b)/
b + 3*I*a**(3/2)*x**5*sqrt(1/b)) + b*x**5*log(c)/(3*I*a**(5/2)*x**3*sqrt(1/
b)/b + 3*I*a**(3/2)*x**5*sqrt(1/b)), True))
```

3.10 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx$

Optimal. Leaf size=64

$$\frac{b^2p\log\left(a+bx^2\right)}{4a^2}-\frac{b^2p\log(x)}{2a^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4}-\frac{bp}{4ax^2}$$

```
[Out] -1/4*b*p/a/x^2-1/2*b^2*p*ln(x)/a^2+1/4*b^2*p*ln(b*x^2+a)/a^2-1/4*ln(c*(b*x^
2+a)^p)/x^4
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2395, 44}

$$\frac{b^2p\log\left(a+bx^2\right)}{4a^2}-\frac{b^2p\log(x)}{2a^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4}-\frac{bp}{4ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x^5,x]
```

```
[Out] -(b*p)/(4*a*x^2) - (b^2*p*Log[x])/(2*a^2) + (b^2*p*Log[a + b*x^2])/(4*a^2)
- Log[c*(a + b*x^2)^p]/(4*x^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4} + \frac{1}{4}(bp)\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, x^2\right)$$
$$= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4} + \frac{1}{4}(bp)\operatorname{Subst}\left(\int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right)dx, x, x^2\right)$$
$$= -\frac{bp}{4ax^2} - \frac{b^2p\log(x)}{2a^2} + \frac{b^2p\log\left(a+bx^2\right)}{4a^2} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.88

$$\frac{1}{4}bp\left(\frac{b\log\left(a+bx^2\right)}{a^2} - \frac{2b\log(x)}{a^2} - \frac{1}{ax^2}\right) - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^5,x]
[Out] (b*p*(-(1/(a*x^2)) - (2*b*Log[x])/a^2 + (b*Log[a + b*x^2])/a^2))/4 - Log[c*
(a + b*x^2)^p]/(4*x^4)
```

fricas [A] time = 0.45, size = 58, normalized size = 0.91

$$-\frac{2\,b^2px^4\log(x) + abpx^2 + a^2\log(c) - \left(b^2px^4 - a^2p\right)\log\left(bx^2+a\right)}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^5,x, algorithm="fricas")
[Out] -1/4*(2*b^2*p*x^4*log(x) + a*b*p*x^2 + a^2*log(c) - (b^2*p*x^4 - a^2*p)*log
(b*x^2 + a))/(a^2*x^4)
```

giac [B] time = 0.16, size = 132, normalized size = 2.06

$$-\frac{\frac{b^3p\log\left(bx^2+a\right)}{\left(bx^2+a\right)^2 - 2\left(bx^2+a\right)a + a^2} - \frac{b^3p\log\left(bx^2+a\right)}{a^2} + \frac{b^3p\log\left(bx^2\right)}{a^2} + \frac{\left(bx^2+a\right)b^3p - ab^3p + ab^3\log(c)}{\left(bx^2+a\right)^2a - 2\left(bx^2+a\right)a^2 + a^3}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^5,x, algorithm="giac")
[Out] -1/4*(b^3*p*log(b*x^2 + a)/((b*x^2 + a)^2 - 2*(b*x^2 + a)*a + a^2) - b^3*p*
log(b*x^2 + a)/a^2 + b^3*p*log(b*x^2)/a^2 + ((b*x^2 + a)*b^3*p - a*b^3*p +
a*b^3*log(c))/((b*x^2 + a)^2*a - 2*(b*x^2 + a)*a^2 + a^3))/b
```

maple [C] time = 0.26, size = 198, normalized size = 3.09

$$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)}{4x^4} - \frac{4b^2p\,x^4\ln(x) - 2b^2p\,x^4\ln\left(-b\,x^2-a\right) - i\pi\,a^2\operatorname{csgn}(ic)\operatorname{csgn}\left(i\left(b\,x^2+a\right)^p\right)\operatorname{csgn}\left(ic\left(b\,x^2+a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^5,x)
```

```
[Out] -1/4/x^4*ln((b*x^2+a)^p)-1/8*(4*b^2*p*ln(x)*x^4-2*b^2*p*ln(-b*x^2-a)*x^4+I*
Pi*a^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*a^2*csgn(I*(b*x^2+a
)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*a^2*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*
a^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*a*b*p*x^2+2*ln(c)*a^2)/a^2/x^4
```

maxima [A] time = 0.79, size = 54, normalized size = 0.84

$$\frac{1}{4}\,bp\left(\frac{b\log\left(bx^2+a\right)}{a^2}-\frac{b\log\left(x^2\right)}{a^2}-\frac{1}{ax^2}\right)-\frac{\log\left(\left(bx^2+a\right)^pc\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*b*p*(b*log(b*x^2 + a)/a^2 - b*log(x^2)/a^2 - 1/(a*x^2)) - 1/4*log((b*x^
2 + a)^p*c)/x^4
```

mupad [B] time = 0.26, size = 56, normalized size = 0.88

$$\frac{b^2\,p\,\ln\left(b\,x^2+a\right)}{4\,a^2}-\frac{\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{4\,x^4}-\frac{b^2\,p\,\ln(x)}{2\,a^2}-\frac{b\,p}{4\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/x^5,x)
```

```
[Out] (b^2*p*log(a + b*x^2))/(4*a^2) - log(c*(a + b*x^2)^p)/(4*x^4) - (b^2*p*log(
x))/(2*a^2) - (b*p)/(4*a*x^2)
```

sympy [A] time = 14.05, size = 102, normalized size = 1.59

$$\begin{cases} -\frac{p\log\left(a+bx^2\right)}{4x^4}-\frac{\log\left(c\right)}{4x^4}-\frac{bp}{4ax^2}-\frac{b^2p\log\left(x\right)}{2a^2}+\frac{b^2p\log\left(a+bx^2\right)}{4a^2} & \text{for}\ a\neq 0\\ -\frac{p\log\left(b\right)}{4x^4}-\frac{p\log\left(x\right)}{2x^4}-\frac{p}{8x^4}-\frac{\log\left(c\right)}{4x^4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**5,x)
```

```
[Out] Piecewise((-p*log(a + b*x**2)/(4*x**4) - log(c)/(4*x**4) - b*p/(4*a*x**2) -
 b**2*p*log(x)/(2*a**2) + b**2*p*log(a + b*x**2)/(4*a**2), Ne(a, 0)), (-p*l
og(b)/(4*x**4) - p*log(x)/(2*x**4) - p/(8*x**4) - log(c)/(4*x**4), True))
```

3.11 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^6}\,dx$

Optimal. Leaf size=74

$$\frac{2b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5a^{5/2}}+\frac{2b^2p}{5a^2x}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5}-\frac{2bp}{15ax^3}$$

```
[Out] -2/15*b*p/a/x^3+2/5*b^2*p/a^2/x+2/5*b^(5/2)*p*arctan(x*b^(1/2)/a^(1/2))/a^(
5/2)-1/5*ln(c*(b*x^2+a)^p)/x^5
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2455, 325, 205}

$$\frac{2b^2p}{5a^2x}+\frac{2b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5a^{5/2}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5}-\frac{2bp}{15ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x^6,x]
```

```
[Out] (-2*b*p)/(15*a*x^3) + (2*b^2*p)/(5*a^2*x) + (2*b^(5/2)*p*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(5*a^(5/2)) - Log[c*(a + b*x^2)^p]/(5*x^5)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^6}\,dx = -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5} + \frac{1}{5}(2bp)\int \frac{1}{x^4\left(a+bx^2\right)}\,dx$$
$$= -\frac{2bp}{15ax^3} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5} - \frac{\left(2b^2p\right)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{5a}$$
$$= -\frac{2bp}{15ax^3} + \frac{2b^2p}{5a^2x} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5} + \frac{\left(2b^3p\right)\int \frac{1}{a+bx^2}\,dx}{5a^2}$$
$$= -\frac{2bp}{15ax^3} + \frac{2b^2p}{5a^2x} + \frac{2b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5a^{5/2}} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5}$$

Mathematica [C] time = 0.00, size = 49, normalized size = 0.66

$$-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5} - \frac{2bp\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{bx^2}{a}\right)}{15ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^6,x]
```

```
[Out] (-2*b*p*Hypergeometric2F1[-3/2, 1, -1/2, -((b*x^2)/a)])/(15*a*x^3) - Log[c*
(a + b*x^2)^p]/(5*x^5)
```

fricas [A] time = 0.48, size = 170, normalized size = 2.30

$$\left[\frac{3\,b^2px^5\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,b^2px^4-2\,abpx^2-3\,a^2p\log\left(bx^2+a\right)-3\,a^2\log(c)}{15\,a^2x^5}, \frac{6\,b^2px^5\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^6,x, algorithm="fricas")
```

```
[Out] [1/15*(3*b^2*p*x^5*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a
)) + 6*b^2*p*x^4 - 2*a*b*p*x^2 - 3*a^2*p*log(b*x^2 + a) - 3*a^2*log(c))/(a^
2*x^5), 1/15*(6*b^2*p*x^5*sqrt(b/a)*arctan(x*sqrt(b/a)) + 6*b^2*p*x^4 - 2*a
*b*p*x^2 - 3*a^2*p*log(b*x^2 + a) - 3*a^2*log(c))/(a^2*x^5)]
```

giac [A] time = 0.18, size = 71, normalized size = 0.96

$$\frac{2\,b^3p\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{5\,\sqrt{ab}\,a^2} - \frac{p\log\left(bx^2+a\right)}{5\,x^5} + \frac{6\,b^2px^4-2\,abpx^2-3\,a^2\log(c)}{15\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^6,x, algorithm="giac")
```

```
[Out] 2/5*b^3*p*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/5*p*log(b*x^2 + a)/x^5
+ 1/15*(6*b^2*p*x^4 - 2*a*b*p*x^2 - 3*a^2*log(c))/(a^2*x^5)
```

maple [C] time = 0.40, size = 235, normalized size = 3.18

$$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)}{5x^5} - \frac{-6\sqrt{-ab}\;b^2p\,x^5\ln\left(-bx-\sqrt{-ab}\right)+6\sqrt{-ab}\;b^2p\,x^5\ln\left(-bx+\sqrt{-ab}\right)-12a\,b^2p\,x^4-3i\pi\,a^3\mathrm{csgr}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^6,x)
```

```
[Out] -1/5/x^5*ln((b*x^2+a)^p)-1/30*(-6*(-a*b)^(1/2)*p*b^2*ln(-b*x-(-a*b)^(1/2))*
x^5+6*(-a*b)^(1/2)*p*b^2*ln(-b*x+(-a*b)^(1/2))*x^5+3*I*Pi*a^3*csgn(I*(b*x^2
+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-3*I*Pi*a^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x
^2+a)^p)*csgn(I*c)-3*I*Pi*a^3*csgn(I*c*(b*x^2+a)^p)^3+3*I*Pi*a^3*csgn(I*c*(
b*x^2+a)^p)^2*csgn(I*c)-12*a*b^2*p*x^4+4*a^2*b*p*x^2+6*ln(c)*a^3)/a^3/x^5
```

maxima [A] time = 1.53, size = 62, normalized size = 0.84

$$\frac{2}{15}\,bp\left(\frac{3\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{3\,bx^2-a}{a^2x^3}\right)-\frac{\log\left(\left(bx^2+a\right)^p c\right)}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^6,x, algorithm="maxima")
```

```
[Out] 2/15*b*p*(3*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + (3*b*x^2 - a)/(a^2*
x^3)) - 1/5*log((b*x^2 + a)^p*c)/x^5
```

mupad [B] time = 0.26, size = 61, normalized size = 0.82

$$\frac{2\,b^{5/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5\,a^{5/2}}-\frac{\frac{2\,b\,p}{3\,a}-\frac{2\,b^2\,p\,x^2}{a^2}}{5\,x^3}-\frac{\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/x^6,x)
```

```
[Out] (2*b^(5/2)*p*atan((b^(1/2)*x)/a^(1/2)))/(5*a^(5/2)) - ((2*b*p)/(3*a) - (2*b
^2*p*x^2)/a^2)/(5*x^3) - log(c*(a + b*x^2)^p)/(5*x^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**6,x)
```

```
[Out] Timed out
```

3.12 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx$

Optimal. Leaf size=78

$$-\frac{b^3p\log\left(a+bx^2\right)}{6a^3}+\frac{b^3p\log(x)}{3a^3}+\frac{b^2p}{6a^2x^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{6x^6}-\frac{bp}{12ax^4}$$

```
[Out] -1/12*b*p/a/x^4+1/6*b^2*p/a^2/x^2+1/3*b^3*p*ln(x)/a^3-1/6*b^3*p*ln(b*x^2+a)
/a^3-1/6*ln(c*(b*x^2+a)^p)/x^6
```

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2395, 44}

$$\frac{b^2p}{6a^2x^2}-\frac{b^3p\log\left(a+bx^2\right)}{6a^3}+\frac{b^3p\log(x)}{3a^3}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{6x^6}-\frac{bp}{12ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x^7,x]
```

```
[Out] -(b*p)/(12*a*x^4) + (b^2*p)/(6*a^2*x^2) + (b^3*p*Log[x])/(3*a^3) - (b^3*p*L
og[a + b*x^2])/(6*a^3) - Log[c*(a + b*x^2)^p]/(6*x^6)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{6x^6} + \frac{1}{6}(bp)\,\mathrm{Subst}\left(\int \frac{1}{x^3(a+bx)}\,dx, x, x^2\right)$$
$$= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{6x^6} + \frac{1}{6}(bp)\,\mathrm{Subst}\left(\int \left(\frac{1}{ax^3} - \frac{b}{a^2x^2} + \frac{b^2}{a^3x} - \frac{b^3}{a^3(a+bx)}\right) dx, x, x^2\right)$$
$$= -\frac{bp}{12ax^4} + \frac{b^2p}{6a^2x^2} + \frac{b^3p\log(x)}{3a^3} - \frac{b^3p\log\left(a+bx^2\right)}{6a^3} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{6x^6}$$

Mathematica [A] time = 0.07, size = 68, normalized size = 0.87

$$-\frac{\frac{bpx^2\left(2b^2x^4\log\left(a+bx^2\right)+a\left(a-2bx^2\right)-4b^2x^4\log(x)\right)}{a^3} + 2\log\left(c\left(a+bx^2\right)^p\right)}{12x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^7,x]
```

```
[Out] -1/12*((b*p*x^2*(a*(a - 2*b*x^2) - 4*b^2*x^4*Log[x] + 2*b^2*x^4*Log[a + b*x
^2]))/a^3 + 2*Log[c*(a + b*x^2)^p])/x^6
```

fricas [A] time = 0.46, size = 71, normalized size = 0.91

$$\frac{4\,b^3px^6\log(x) + 2\,ab^2px^4 - a^2bpx^2 - 2\,a^3\log(c) - 2\left(b^3px^6 + a^3p\right)\log\left(bx^2+a\right)}{12\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^3*p*x^6*log(x) + 2*a*b^2*p*x^4 - a^2*b*p*x^2 - 2*a^3*log(c) - 2*(
b^3*p*x^6 + a^3*p)*log(b*x^2 + a))/(a^3*x^6)
```

giac [B] time = 0.17, size = 191, normalized size = 2.45

$$-\frac{\frac{2\,b^4p\log\left(bx^2+a\right)}{\left(bx^2+a\right)^3 - 3\left(bx^2+a\right)^2a + 3\left(bx^2+a\right)a^2 - a^3} + \frac{2\,b^4p\log\left(bx^2+a\right)}{a^3} - \frac{2\,b^4p\log\left(bx^2\right)}{a^3} - \frac{2\left(bx^2+a\right)^2b^4p - 5\left(bx^2+a\right)ab^4p + 3\,a^2b^4p - 2\,a^2b^4\log(c)}{\left(bx^2+a\right)^3a^2 - 3\left(bx^2+a\right)^2a^3 + 3\left(bx^2+a\right)a^4 - a^5}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(2*b^4*p*log(b*x^2 + a)/((b*x^2 + a)^3 - 3*(b*x^2 + a)^2*a + 3*(b*x^2
 + a)*a^2 - a^3) + 2*b^4*p*log(b*x^2 + a)/a^3 - 2*b^4*p*log(b*x^2)/a^3 - (2
*(b*x^2 + a)^2*b^4*p - 5*(b*x^2 + a)*a*b^4*p + 3*a^2*b^4*p - 2*a^2*b^4*log(
c))/((b*x^2 + a)^3*a^2 - 3*(b*x^2 + a)^2*a^3 + 3*(b*x^2 + a)*a^4 - a^5))/b
```

maple [C] time = 0.35, size = 206, normalized size = 2.64

$$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)}{6x^6} - \frac{-4b^3p\,x^6\ln(x) + 2b^3p\,x^6\ln\left(b\,x^2+a\right) - 2a\,b^2p\,x^4 - i\pi\,a^3\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^7,x)
```

```
[Out] -1/6/x^6*ln((b*x^2+a)^p)-1/12*(-4*b^3*p*ln(x)*x^6+2*b^3*p*ln(b*x^2+a)*x^6+I
*Pi*a^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*a^3*csgn(I*(b*x^2+
a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*a^3*csgn(I*c*(b*x^2+a)^p)^3+I*Pi
*a^3*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-2*a*b^2*p*x^4+a^2*b*p*x^2+2*a^3*ln(c
))/a^3/x^6
```

maxima [A] time = 0.68, size = 69, normalized size = 0.88

$$-\frac{1}{12}\,bp\left(\frac{2\,b^2\log\left(bx^2+a\right)}{a^3}-\frac{2\,b^2\log\left(x^2\right)}{a^3}-\frac{2\,bx^2-a}{a^2x^4}\right)-\frac{\log\left(\left(bx^2+a\right)^pc\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*b*p*(2*b^2*log(b*x^2 + a)/a^3 - 2*b^2*log(x^2)/a^3 - (2*b*x^2 - a)/(a
^2*x^4)) - 1/6*log((b*x^2 + a)^p*c)/x^6
```

mupad [B] time = 0.26, size = 68, normalized size = 0.87

$$\frac{b^2\,p}{6\,a^2\,x^2}-\frac{b^3\,p\,\ln\left(b\,x^2+a\right)}{6\,a^3}-\frac{\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{6\,x^6}+\frac{b^3\,p\,\ln(x)}{3\,a^3}-\frac{b\,p}{12\,a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/x^7,x)
```

```
[Out] (b^2*p)/(6*a^2*x^2) - (b^3*p*log(a + b*x^2))/(6*a^3) - log(c*(a + b*x^2)^p)
/(6*x^6) + (b^3*p*log(x))/(3*a^3) - (b*p)/(12*a*x^4)
```

sympy [A] time = 32.46, size = 116, normalized size = 1.49

$$\begin{cases} -\frac{p\log\left(a+bx^2\right)}{6x^6}-\frac{\log\left(c\right)}{6x^6}-\frac{bp}{12ax^4}+\frac{b^2p}{6a^2x^2}+\frac{b^3p\log\left(x\right)}{3a^3}-\frac{b^3p\log\left(a+bx^2\right)}{6a^3} & \text{for}\ a\neq 0\\ -\frac{p\log\left(b\right)}{6x^6}-\frac{p\log\left(x\right)}{3x^6}-\frac{p}{18x^6}-\frac{\log\left(c\right)}{6x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**7,x)
```

```
[Out] Piecewise((-p*log(a + b*x**2)/(6*x**6) - log(c)/(6*x**6) - b*p/(12*a*x**4)
+ b**2*p/(6*a**2*x**2) + b**3*p*log(x)/(3*a**3) - b**3*p*log(a + b*x**2)/(6
*a**3), Ne(a, 0)), (-p*log(b)/(6*x**6) - p*log(x)/(3*x**6) - p/(18*x**6) -
log(c)/(6*x**6), True))
```

3.13 $\int x^5 \log\left(c\left(a+bx^3\right)^p\right) dx$

Optimal. Leaf size=59

$$-\frac{a^2 p \log\left(a+bx^3\right)}{6b^2}+\frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right)+\frac{apx^3}{6b}-\frac{px^6}{12}$$

```
[Out] 1/6*a*p*x^3/b-1/12*p*x^6-1/6*a^2*p*ln(b*x^3+a)/b^2+1/6*x^6*ln(c*(b*x^3+a)^p
)
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2395, 43}

$$-\frac{a^2 p \log\left(a+bx^3\right)}{6b^2}+\frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right)+\frac{apx^3}{6b}-\frac{px^6}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (a*p*x^3)/(6*b) - (p*x^6)/12 - (a^2*p*Log[a + b*x^3])/(6*b^2) + (x^6*Log[c*
(a + b*x^3)^p])/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int x^5 \log\left(c\left(a+bx^3\right)^p\right) dx &= \frac{1}{3}\text{Subst}\left(\int x\log\left(c(a+bx)^p\right) dx, x, x^3\right)\\
&= \frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{6}(bp)\,\text{Subst}\left(\int \frac{x^2}{a+bx}\,dx, x, x^3\right)\\
&= \frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{6}(bp)\,\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right) dx, x, x^3\right)\\
&= \frac{apx^3}{6b}-\frac{px^6}{12}-\frac{a^2 p\log\left(a+bx^3\right)}{6b^2}+\frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 1.00

$$-\frac{a^2 p \log\left(a+bx^3\right)}{6b^2}+\frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right)+\frac{apx^3}{6b}-\frac{px^6}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (a*p*x^3)/(6*b) - (p*x^6)/12 - (a^2*p*Log[a + b*x^3])/(6*b^2) + (x^6*Log[c*
(a + b*x^3)^p])/6
```

fricas [A] time = 0.44, size = 57, normalized size = 0.97

$$-\frac{b^2px^6-2\,b^2x^6\log(c)-2\,abpx^3-2\left(b^2px^6-a^2p\right)\log\left(bx^3+a\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] -1/12*(b^2*p*x^6 - 2*b^2*x^6*log(c) - 2*a*b*p*x^3 - 2*(b^2*p*x^6 - a^2*p)*l
og(b*x^3 + a))/b^2
```

giac [A] time = 0.16, size = 97, normalized size = 1.64

$$\frac{\frac{\left(2\left(bx^3+a\right)^2\log\left(bx^3+a\right)-4\left(bx^3+a\right)a\log\left(bx^3+a\right)-\left(bx^3+a\right)^2+4\left(bx^3+a\right)a\right)p}{b}+\frac{2\left(\left(bx^3+a\right)^2-2\left(bx^3+a\right)a\right)\log(c)}{b}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] 1/12*((2*(b*x^3 + a)^2*log(b*x^3 + a) - 4*(b*x^3 + a)*a*log(b*x^3 + a) - (b
*x^3 + a)^2 + 4*(b*x^3 + a)*a)*p/b + 2*((b*x^3 + a)^2 - 2*(b*x^3 + a)*a)*lo
g(c)/b)/b
```

maple [C] time = 0.43, size = 183, normalized size = 3.10

$$-\frac{i\pi\,x^6\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(b\,x^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^3+a\right)^p\right)}{12}+\frac{i\pi\,x^6\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(b\,x^3+a\right)^p\right)^2}{12}+\frac{i\pi\,x^6\mathrm{csgn}\left(i\left(b\,x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*ln(c*(b*x^3+a)^p),x)
```

```
[Out] 1/6*x^6*ln((b*x^3+a)^p)+1/12*I*Pi*x^6*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a
)^p)^2-1/12*I*Pi*x^6*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-1/
12*I*Pi*x^6*csgn(I*c*(b*x^3+a)^p)^3+1/12*I*Pi*x^6*csgn(I*c*(b*x^3+a)^p)^2*c
sgn(I*c)+1/6*ln(c)*x^6-1/12*p*x^6+1/6*a/b*p*x^3-1/6*a^2*p*ln(b*x^3+a)/b^2
```

maxima [A] time = 0.62, size = 55, normalized size = 0.93

$$\frac{1}{6}\,x^6\log\left(\left(bx^3+a\right)^p c\right)-\frac{1}{12}\,bp\left(\frac{2\,a^2\log\left(bx^3+a\right)}{b^3}+\frac{bx^6-2\,ax^3}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log((b*x^3 + a)^p*c) - 1/12*b*p*(2*a^2*log(b*x^3 + a)/b^3 + (b*x^6
- 2*a*x^3)/b^2)
```

mupad [B] time = 0.24, size = 51, normalized size = 0.86

$$\frac{x^6\,\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{6}-\frac{p\,x^6}{12}-\frac{a^2\,p\,\ln\left(b\,x^3+a\right)}{6\,b^2}+\frac{a\,p\,x^3}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*log(c*(a + b*x^3)^p),x)
```

```
[Out] (x^6*log(c*(a + b*x^3)^p))/6 - (p*x^6)/12 - (a^2*p*log(a + b*x^3))/(6*b^2)
+ (a*p*x^3)/(6*b)
```

sympy [A] time = 19.68, size = 70, normalized size = 1.19

$$\begin{cases} -\frac{a^2 p \log\left(a+bx^3\right)}{6b^2}+\frac{apx^3}{6b}+\frac{px^6 \log\left(a+bx^3\right)}{6}-\frac{px^6}{12}+\frac{x^6 \log\left(c\right)}{6} & \text{for}\ b \neq 0 \\ \frac{x^6 \log\left(a^p c\right)}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Piecewise((-a**2*p*log(a + b*x**3)/(6*b**2) + a*p*x**3/(6*b) + p*x**6*log(a
 + b*x**3)/6 - p*x**6/12 + x**6*log(c)/6, Ne(b, 0)), (x**6*log(a**p*c)/6, T
rue))
```

3.14 $\int x^4 \log \left(c \left(a+b x^3\right)^p\right) dx$

Optimal. Leaf size=159

$$-\frac{a^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{10b^{5/3}}+\frac{a^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5b^{5/3}}+\frac{\sqrt{3}a^{5/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{5b^{5/3}}+\frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)$$

```
[Out] 3/10*a*p*x^2/b-3/25*p*x^5+1/5*a^(5/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(5/3)-1/10*
a^(5/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(5/3)+1/5*x^5*ln(c*(b
*x^3+a)^p)+1/5*a^(5/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*
3^(1/2)/b^(5/3)
```

Rubi [A] time = 0.13, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2455, 302, 292, 31, 634, 617, 204, 628}

$$-\frac{a^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{10b^{5/3}}+\frac{a^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5b^{5/3}}+\frac{\sqrt{3}a^{5/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{5b^{5/3}}+\frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (3*a*p*x^2)/(10*b) - (3*p*x^5)/25 + (Sqrt[3]*a^(5/3)*p*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(5*b^(5/3)) + (a^(5/3)*p*Log[a^(1/3) + b^(1/
3)*x])/(5*b^(5/3)) - (a^(5/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(10*b^(5/3)) + (x^5*Log[c*(a + b*x^3)^p])/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^4 \log\left(c\left(a+bx^3\right)^p\right) dx &= \frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{5}(3bp)\int \frac{x^7}{a+bx^3}\,dx\\
&= \frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{5}(3bp)\int\left(-\frac{ax}{b^2}+\frac{x^4}{b}+\frac{a^2x}{b^2\left(a+bx^3\right)}\right)dx\\
&= \frac{3apx^2}{10b}-\frac{3px^5}{25}+\frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)-\frac{\left(3a^2p\right)\int\frac{x}{a+bx^3}\,dx}{5b}\\
&= \frac{3apx^2}{10b}-\frac{3px^5}{25}+\frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)+\frac{\left(a^{5/3}p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{5b^{4/3}}-\frac{\left(a^{5/3}p\right)\int\frac{\cdots}{a^{2/3}-\cdots}}{5b\cdots}\\
&= \frac{3apx^2}{10b}-\frac{3px^5}{25}+\frac{a^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5b^{5/3}}+\frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)-\frac{\left(a^{5/3}p\right)\int\frac{\cdots}{a^{2/3}\cdots}}{1\cdots}\\
&= \frac{3apx^2}{10b}-\frac{3px^5}{25}+\frac{a^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5b^{5/3}}-\frac{a^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{10b^{5/3}}+\cdots\\
&= \frac{3apx^2}{10b}-\frac{3px^5}{25}+\frac{\sqrt{3}a^{5/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{5b^{5/3}}+\frac{a^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5b^{5/3}}-\frac{a^{5/3}p\log\cdots}{\cdots}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 69, normalized size = 0.43

$$\frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)-\frac{3apx^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right)}{10b}+\frac{3apx^2}{10b}-\frac{3px^5}{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (3*a*p*x^2)/(10*b) - (3*p*x^5)/25 - (3*a*p*x^2*Hypergeometric2F1[2/3, 1, 5/
3, -((b*x^3)/a)])/(10*b) + (x^5*Log[c*(a + b*x^3)^p])/5
```

fricas [A] time = 0.46, size = 161, normalized size = 1.01

$$\frac{10\,bpx^5\log\left(bx^3+a\right)-6\,bpx^5+10\,bx^5\log(c)+15\,apx^2-10\,\sqrt{3}\,ap\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3\,a}\right)-5\,ap\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\,\mathrm{l}}{50\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] 1/50*(10*b*p*x^5*log(b*x^3 + a) - 6*b*p*x^5 + 10*b*x^5*log(c) + 15*a*p*x^2
- 10*sqrt(3)*a*p*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3)
- sqrt(3)*a)/a) - 5*a*p*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a
*(a^2/b^2)^(1/3)) + 10*a*p*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b
```

giac [A] time = 0.20, size = 162, normalized size = 1.02

$$\frac{1}{10}\,a^2b^4p\left(\frac{2\,\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab^5}+\frac{2\,\sqrt{3}\,\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^7}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right.\right.}{ab^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] 1/10*a^2*b^4*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5) + 2*sqrt(
3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*
b^7) - (-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^7)) + 1
/5*p*x^5*log(b*x^3 + a) - 1/25*(3*p - 5*log(c))*x^5 + 3/10*a*p*x^2/b
```

maple [C] time = 0.42, size = 196, normalized size = 1.23

$$-\frac{i\pi\,x^5\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(b\,x^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^3+a\right)^p\right)}{10}+\frac{i\pi\,x^5\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(b\,x^3+a\right)^p\right)^2}{10}+\frac{i\pi\,x^5\mathrm{csgn}\left(i\left(b\,x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(b*x^3+a)^p),x)
```

```
[Out] 1/5*x^5*ln((b*x^3+a)^p)-1/10*I*Pi*x^5*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a
)^p)*csgn(I*c)+1/10*I*Pi*x^5*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+1/10*I*Pi*x^
5*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-1/10*I*Pi*x^5*csgn(I*c*(b*x^3
+a)^p)^3+1/5*x^5*ln(c)-3/25*p*x^5+3/10*a/b*p*x^2-1/5/b^2*a^2*p*sum(1/_R*ln(
-_R+x),_R=RootOf(_Z^3*b+a))
```

maxima [A] time = 1.48, size = 147, normalized size = 0.92

$$\frac{1}{5}\,x^5\log\left(\left(bx^3+a\right)^p c\right)-\frac{1}{50}\,bp\left(\frac{10\,\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\,a^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{10\,a^2\log\left(x+\left(\frac{a}{b}\right.\right.}{b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*log((b*x^3 + a)^p*c) - 1/50*b*p*(10*sqrt(3)*a^2*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1/3)) + 5*a^2*log(x^2 - x*(a/b
)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/3)) - 10*a^2*log(x + (a/b)^(1/3))/(b^3
*(a/b)^(1/3)) + 3*(2*b*x^5 - 5*a*x^2)/b^2)
```

mupad [B] time = 2.50, size = 157, normalized size = 0.99

$$\frac{x^5\ln\left(c\left(b\,x^3+a\right)^p\right)}{5}-\frac{3\,p\,x^5}{25}+\frac{a^{5/3}\,p\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{5\,b^{5/3}}+\frac{3\,a\,p\,x^2}{10\,b}+\frac{a^{5/3}\,p\,\ln\left(\frac{9\,a^4\,p^2\,x}{25\,b}+\frac{9\,a^{13/3}\,p^2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{25\,b^{4/3}}\right)\left(-\frac{1}{2}+\right.}{5\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*log(c*(a + b*x^3)^p),x)
```

```
[Out] (x^5*log(c*(a + b*x^3)^p))/5 - (3*p*x^5)/25 + (a^(5/3)*p*log(b^(1/3)*x + a^
(1/3)))/(5*b^(5/3)) + (3*a*p*x^2)/(10*b) + (a^(5/3)*p*log((9*a^4*p^2*x)/(25
*b) + (9*a^(13/3)*p^2*((3^(1/2)*1i)/2 - 1/2)^2)/(25*b^(4/3)))*((3^(1/2)*1i)
/2 - 1/2))/(5*b^(5/3)) - (a^(5/3)*p*log((9*a^4*p^2*x)/(25*b) + (9*a^(13/3)*
p^2*((3^(1/2)*1i)/2 + 1/2)^2)/(25*b^(4/3)))*((3^(1/2)*1i)/2 + 1/2))/(5*b^(5
/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Timed out
```

3.15 $\int x^3 \log\left(c\left(a+bx^3\right)^p\right)\,dx$

Optimal. Leaf size=157

$$\frac{a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{8b^{4/3}}-\frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{4b^{4/3}}+\frac{\sqrt{3}\,a^{4/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{4b^{4/3}}+\frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right)+$$

```
[Out] 3/4*a*p*x/b-3/16*p*x^4-1/4*a^(4/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(4/3)+1/8*a^(4
/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(4/3)+1/4*x^4*ln(c*(b*x^3
+a)^p)+1/4*a^(4/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1
/2)/b^(4/3)
```

Rubi [A] time = 0.11, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2455, 302, 200, 31, 634, 617, 204, 628}

$$\frac{a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{8b^{4/3}}-\frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{4b^{4/3}}+\frac{\sqrt{3}\,a^{4/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{4b^{4/3}}+\frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right)+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (3*a*p*x)/(4*b) - (3*p*x^4)/16 + (Sqrt[3]*a^(4/3)*p*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(4*b^(4/3)) - (a^(4/3)*p*Log[a^(1/3) + b^(1/3)*
x])/(4*b^(4/3)) + (a^(4/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(8*b^(4/3)) + (x^4*Log[c*(a + b*x^3)^p])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3 \log\left(c\left(a+bx^3\right)^p\right)dx &= \frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{4}(3bp)\int\frac{x^6}{a+bx^3}\,dx\\
&= \frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{4}(3bp)\int\left(-\frac{a}{b^2}+\frac{x^3}{b}+\frac{a^2}{b^2\left(a+bx^3\right)}\right)dx\\
&= \frac{3apx}{4b}-\frac{3px^4}{16}+\frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right)-\frac{\left(3a^2p\right)\int\frac{1}{a+bx^3}\,dx}{4b}\\
&= \frac{3apx}{4b}-\frac{3px^4}{16}+\frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right)-\frac{\left(a^{4/3}p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{4b}-\frac{\left(a^{4/3}p\right)\int\frac{2}{a^{2/3}-\sqrt[3]{}}}{4b}\\
&= \frac{3apx}{4b}-\frac{3px^4}{16}-\frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}}+\frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right)+\frac{\left(a^{4/3}p\right)\int\frac{-}{a^{2/3}-}}{8}\\
&= \frac{3apx}{4b}-\frac{3px^4}{16}-\frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}}+\frac{a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8b^{4/3}}+\frac{1}{4}\\
&= \frac{3apx}{4b}-\frac{3px^4}{16}+\frac{\sqrt{3}a^{4/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4b^{4/3}}-\frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}}+\frac{a^{4/3}p\log}{}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 147, normalized size = 0.94

$$\frac{2a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-4a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+4\sqrt{3}a^{4/3}p\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4b^{4/3}x^4\log\left(c\left(a\right.\right.}{16b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (12*a*b^(1/3)*p*x - 3*b^(4/3)*p*x^4 + 4*Sqrt[3]*a^(4/3)*p*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]] - 4*a^(4/3)*p*Log[a^(1/3) + b^(1/3)*x] + 2*a^(4/
3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 4*b^(4/3)*x^4*Log[c*(
a + b*x^3)^p])/(16*b^(4/3))
```

fricas [A] time = 0.45, size = 144, normalized size = 0.92

$$\frac{4\,bpx^4\log\left(bx^3+a\right)-3\,bpx^4+4\,bx^4\log(c)+4\,\sqrt{3}\,ap\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)-2\,ap\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(\right.\right.}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b*p*x^4*log(b*x^3 + a) - 3*b*p*x^4 + 4*b*x^4*log(c) + 4*sqrt(3)*a*p
*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) - 2*a*
p*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) + 4*a*p*(-a/b)^(1/3
)*log(x - (-a/b)^(1/3)) + 12*a*p*x)/b
```

giac [A] time = 0.18, size = 160, normalized size = 1.02

$$\frac{1}{8}\,a^2b^3p\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab^4}-\frac{2\,\sqrt{3}\,\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^5}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)\right.}{ab^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] 1/8*a^2*b^3*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) - 2*sqrt(3
)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b
^5) - (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^5)) + 1/
4*p*x^4*log(b*x^3 + a) - 1/16*(3*p - 4*log(c))*x^4 + 3/4*a*p*x/b
```

maple [C] time = 0.42, size = 194, normalized size = 1.24

$$-\frac{i\pi\,x^4\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(b\,x^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^3+a\right)^p\right)}{8}+\frac{i\pi\,x^4\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(b\,x^3+a\right)^p\right)^2}{8}+\frac{i\pi\,x^4\mathrm{csgn}\left(i\left(b\,x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x^3+a)^p),x)
```

```
[Out] 1/4*x^4*ln((b*x^3+a)^p)+1/8*I*Pi*x^4*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)
^p)^2-1/8*I*Pi*x^4*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-1/8*
I*Pi*x^4*csgn(I*c*(b*x^3+a)^p)^3+1/8*I*Pi*x^4*csgn(I*c*(b*x^3+a)^p)^2*csgn(
I*c)+1/4*ln(c)*x^4-3/16*p*x^4-1/4/b^2*a^2*p*sum(1/_R^2*ln(-_R+x),_R=RootOf(
_Z^3*b+a))+3/4*a/b*p*x
```

maxima [A] time = 1.45, size = 144, normalized size = 0.92

$$\frac{1}{4}\,x^{4}\log\left(\left(bx^{3}+a\right)^{p}c\right)-\frac{1}{16}\,bp\left(\frac{4\,\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\,a^{2}\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{4\,a^{2}\log\left(x+\left(\right.\right.}{b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((b*x^3 + a)^p*c) - 1/16*b*p*(4*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) - 2*a^2*log(x^2 - x*(a/b)
^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 4*a^2*log(x + (a/b)^(1/3))/(b^3*(
a/b)^(2/3)) + 3*(b*x^4 - 4*a*x)/b^2)
```

mupad [B] time = 2.55, size = 129, normalized size = 0.82

$$\frac{x^{4}\,\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{4}-\frac{3\,p\,x^{4}}{16}+\frac{3\,a\,p\,x}{4\,b}-\frac{a^{4/3}\,p\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{4\,b^{4/3}}+\frac{a^{4/3}\,p\,\ln\left(2\,b^{1/3}\,x-a^{1/3}-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right.}{4\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(c*(a + b*x^3)^p),x)
```

```
[Out] (x^4*log(c*(a + b*x^3)^p))/4 - (3*p*x^4)/16 + (3*a*p*x)/(4*b) - (a^(4/3)*p*
log(b^(1/3)*x + a^(1/3)))/(4*b^(4/3)) + (a^(4/3)*p*log(2*b^(1/3)*x - 3^(1/2
)*a^(1/3)*1i - a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(4*b^(4/3)) - (a^(4/3)*p*lo
g(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(4*b^
(4/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Timed out
```

3.16 $\int x^2 \log\left(c\left(a+bx^3\right)^p\right)\,dx$

Optimal. Leaf size=35

$$\frac{\left(a+bx^3\right)\log\left(c\left(a+bx^3\right)^p\right)}{3b}-\frac{px^3}{3}$$

[Out] -1/3*p*x^3+1/3*(b*x^3+a)*ln(c*(b*x^3+a)^p)/b

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2389, 2295}

$$\frac{\left(a+bx^3\right)\log\left(c\left(a+bx^3\right)^p\right)}{3b}-\frac{px^3}{3}$$

Antiderivative was successfully verified.

[In] Int[x^2*Log[c*(a + b*x^3)^p],x]

[Out] -(p*x^3)/3 + ((a + b*x^3)*Log[c*(a + b*x^3)^p])/(3*b)

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\log\left(c\left(a+bx^3\right)^p\right)\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx,x,x^3\right)\\ &= \frac{\operatorname{Subst}\left(\int \log\left(cx^p\right)\,dx,x,a+bx^3\right)}{3b}\\ &= -\frac{px^3}{3}+\frac{\left(a+bx^3\right)\log\left(c\left(a+bx^3\right)^p\right)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.97

$$\frac{1}{3}\left(\frac{\left(a+bx^3\right)\log\left(c\left(a+bx^3\right)^p\right)}{b}-px^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b*x^3)^p],x]
[Out] (-(p*x^3) + ((a + b*x^3)*Log[c*(a + b*x^3)^p])/b)/3
```

fricas [A] time = 0.44, size = 40, normalized size = 1.14

$$-\frac{bpx^3 - bx^3 \log(c) - \left(bpx^3 + ap\right) \log\left(bx^3 + a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^3+a)^p),x, algorithm="fricas")
[Out] -1/3*(b*p*x^3 - b*x^3*log(c) - (b*p*x^3 + a*p)*log(b*x^3 + a))/b
```

giac [A] time = 0.18, size = 43, normalized size = 1.23

$$-\frac{\left(bx^3 - \left(bx^3 + a\right) \log\left(bx^3 + a\right) + a\right)p - \left(bx^3 + a\right) \log(c)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^3+a)^p),x, algorithm="giac")
[Out] -1/3*((b*x^3 - (b*x^3 + a)*log(b*x^3 + a) + a)*p - (b*x^3 + a)*log(c))/b
```

maple [A] time = 0.05, size = 50, normalized size = 1.43

$$-\frac{p\,x^3}{3} + \frac{x^3 \ln\left(c\left(b\,x^3 + a\right)^p\right)}{3} - \frac{ap}{3b} + \frac{a \ln\left(c\left(b\,x^3 + a\right)^p\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^3+a)^p),x)
[Out] 1/3*x^3*ln(c*(b*x^3+a)^p)-1/3*p*x^3+1/3/b*ln(c*(b*x^3+a)^p)*a-1/3/b*a*p
```

maxima [A] time = 0.51, size = 44, normalized size = 1.26

$$\frac{1}{3}\,x^3 \log\left(\left(bx^3 + a\right)^p c\right) - \frac{1}{3}\left(\frac{x^3}{b} - \frac{a \log\left(bx^3 + a\right)}{b^2}\right)bp$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^3+a)^p),x, algorithm="maxima")
[Out] 1/3*x^3*log((b*x^3 + a)^p*c) - 1/3*(x^3/b - a*log(b*x^3 + a)/b^2)*b*p
```

mupad [B] time = 0.23, size = 39, normalized size = 1.11

$$\frac{x^3\,\ln\left(c\left(b\,x^3 + a\right)^p\right)}{3} - \frac{p\,x^3}{3} + \frac{a\,p\,\ln\left(b\,x^3 + a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(a + b*x^3)^p),x)
[Out] (x^3*log(c*(a + b*x^3)^p))/3 - (p*x^3)/3 + (a*p*log(a + b*x^3))/(3*b)
```

sympy [A] time = 4.70, size = 56, normalized size = 1.60

$$\begin{cases} \frac{ap\log\left(a+bx^3\right)}{3b} + \frac{px^3\log\left(a+bx^3\right)}{3} - \frac{px^3}{3} + \frac{x^3\log\left(c\right)}{3} & \text{for } b \neq 0 \\ \frac{x^3\log\left(a^pc\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Piecewise((a*p*log(a + b*x**3)/(3*b) + p*x**3*log(a + b*x**3)/3 - p*x**3/3
+ x**3*log(c)/3, Ne(b, 0)), (x**3*log(a**p*c)/3, True))
```

3.17 $\int x \log \left(c \left(a+bx^3\right)^p\right) dx$

Optimal. Leaf size=147

$$\frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{4b^{2/3}}-\frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2b^{2/3}}-\frac{\sqrt{3}\,a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}}+\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)$$

```
[Out] -3/4*p*x^2-1/2*a^(2/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)+1/4*a^(2/3)*p*ln(a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)+1/2*x^2*ln(c*(b*x^3+a)^p)-1/2*a
^(2/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)
```

Rubi [A] time = 0.08, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2455, 321, 292, 31, 634, 617, 204, 628}

$$\frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{4b^{2/3}}-\frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2b^{2/3}}-\frac{\sqrt{3}\,a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}}+\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (-3*p*x^2)/4 - (Sqrt[3]*a^(2/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a
^(1/3))])/(2*b^(2/3)) - (a^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*b^(2/3)) +
(a^(2/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*b^(2/3)) + (x
^2*Log[c*(a + b*x^3)^p])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\log\left(c\left(a+bx^3\right)^p\right)dx = \frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{2}(3bp)\int\frac{x^4}{a+bx^3}\,dx$$
$$= -\frac{3px^2}{4}+\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)+\frac{1}{2}(3ap)\int\frac{x}{a+bx^3}\,dx$$
$$= -\frac{3px^2}{4}+\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)-\frac{\left(a^{2/3}p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{2\sqrt[3]{b}}+\frac{\left(a^{2/3}p\right)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}$$
$$= -\frac{3px^2}{4}-\frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}}+\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)+\frac{\left(a^{2/3}p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}}{4b^{2/3}}$$
$$= -\frac{3px^2}{4}-\frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}}+\frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}}+\frac{1}{2}x^2\log\left(c\left(\right.\right.$$
$$= -\frac{3px^2}{4}-\frac{\sqrt{3}a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}}-\frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}}+\frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\right.}{4b^{2/3}}$$

Mathematica [C] time = 0.00, size = 53, normalized size = 0.36

$$\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)+\frac{3}{4}px^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right)-\frac{3px^2}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (-3*p*x^2)/4 + (3*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/4 + (
x^2*Log[c*(a + b*x^3)^p])/2
```

fricas [A] time = 0.48, size = 150, normalized size = 1.02

$$\frac{1}{2}px^2\log\left(bx^3+a\right)-\frac{3}{4}px^2+\frac{1}{2}x^2\log(c)+\frac{1}{2}\sqrt{3}p\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}a}{3a}\right)-\frac{1}{4}p\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] 1/2*p*x^2*log(b*x^3 + a) - 3/4*p*x^2 + 1/2*x^2*log(c) + 1/2*sqrt(3)*p*(-a^2
/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) - 1/
4*p*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/b^2)^(1/3))
 + 1/2*p*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2/b^2)^(2/3))
```

giac [A] time = 0.20, size = 150, normalized size = 1.02

$$-\frac{1}{4}ab^2p\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab^2}+\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^4}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(\right.\right.}{ab^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/4*a*b^2*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 2*sqrt(3)
*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^
4) - (-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4)) + 1/2
*p*x^2*log(b*x^3 + a) - 1/4*(3*p - 2*log(c))*x^2
```

maple [C] time = 0.45, size = 184, normalized size = 1.25

$$-\frac{i\pi x^2\mathrm{csgn}(ic)\,\mathrm{csgn}\left(i\left(bx^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(bx^3+a\right)^p\right)}{4}+\frac{i\pi x^2\mathrm{csgn}(ic)\,\mathrm{csgn}\left(ic\left(bx^3+a\right)^p\right)^2}{4}+\frac{i\pi x^2\mathrm{csgn}\left(i\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^3+a)^p),x)
```

```
[Out] 1/2*x^2*ln((b*x^3+a)^p)+1/4*I*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*(b*x^3+a)^p)*x
^2*Pi-1/4*I*Pi*x^2*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-1/4*
I*Pi*x^2*csgn(I*c*(b*x^3+a)^p)^3+1/4*I*csgn(I*c)*csgn(I*c*(b*x^3+a)^p)^2*x^
2*Pi+1/2*ln(c)*x^2-3/4*p*x^2+1/2*a/b*p*sum(1/_R*ln(-_R+x),_R=RootOf(_Z^3*b+
a))
```

maxima [A] time = 1.33, size = 131, normalized size = 0.89

$$-\frac{1}{4}bp\left(\frac{3x^2}{b}-\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+\frac{1}{2}x^2\log\left(\left(bx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] -1/4*b*p*(3*x^2/b - 2*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(b^2*(a/b)^(1/3)) - a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*
(a/b)^(1/3)) + 2*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(1/3))) + 1/2*x^2*log((b
*x^3 + a)^p*c)
```

mupad [B] time = 2.38, size = 121, normalized size = 0.82

$$\frac{x^2 \ln\left(c\left(b x^3+a\right)^p\right)}{2}-\frac{3 p x^2}{4}-\frac{a^{2/3} p \ln\left(b^{1/3} x+a^{1/3}\right)}{2 b^{2/3}}-\frac{a^{2/3} p \ln\left(4 b^{1/3} x-2 a^{1/3}-\sqrt{3}\, a^{1/3} 2\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2 b^{2/3}}+\frac{a^{2/3}\,p}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(a + b*x^3)^p),x)
```

```
[Out] (x^2*log(c*(a + b*x^3)^p))/2 - (3*p*x^2)/4 - (a^(2/3)*p*log(b^(1/3)*x + a^(
1/3)))/(2*b^(2/3)) - (a^(2/3)*p*log(4*b^(1/3)*x - 3^(1/2)*a^(1/3)*2i - 2*a^
(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*b^(2/3)) + (a^(2/3)*p*log(3^(1/2)*a^(1/3)
*2i + 4*b^(1/3)*x - 2*a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(2*b^(2/3))
```

sympy [A] time = 142.41, size = 260, normalized size = 1.77

$$\begin{cases} \frac{x^2 \log\left(0^p c\right)}{2} \\ \frac{x^2 \log\left(a^p c\right)}{2} \\ \frac{p x^2 \log\left(b\right)}{2}+\frac{3 p x^2 \log\left(x\right)}{2}-\frac{3 p x^2}{4}+\frac{x^2 \log\left(c\right)}{2} \\ -\frac{(-1)^{\frac{2}{3}} a^{\frac{2}{3}} p\left(\frac{1}{b}\right)^{\frac{2}{3}} \log\left(a+b x^3\right)}{2}+\frac{3(-1)^{\frac{2}{3}} a^{\frac{2}{3}} p\left(\frac{1}{b}\right)^{\frac{2}{3}} \log\left(4(-1)^{\frac{2}{3}} a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,x\sqrt[3]{\frac{1}{b}}+4x^2\right)}{4}-\frac{(-1)^{\frac{2}{3}}\sqrt{3}\,a^{\frac{2}{3}} p\left(\frac{1}{b}\right)^{\frac{2}{3}} \operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\,x}{3\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}}\right)}{2}+\cdots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Piecewise((x**2*log(0**p*c)/2, Eq(a, 0) & Eq(b, 0)), (x**2*log(a**p*c)/2, E
q(b, 0)), (p*x**2*log(b)/2 + 3*p*x**2*log(x)/2 - 3*p*x**2/4 + x**2*log(c)/2
, Eq(a, 0)), (-(-1)**(2/3)*a**(2/3)*p*(1/b)**(2/3)*log(a + b*x**3)/2 + 3*(-
1)**(2/3)*a**(2/3)*p*(1/b)**(2/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) +
 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/4 - (-1)**(2/3)*sqrt(3)*a*
*(2/3)*p*(1/b)**(2/3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x/(3*a**(1/3)*
(1/b)**(1/3)))/2 + p*x**2*log(a + b*x**3)/2 - 3*p*x**2/4 + x**2*log(c)/2, T
rue))
```

3.18 $\int \log \left(c \left(a+b x^3\right)^p\right) \, dx$

Optimal. Leaf size=133

$$-\frac{\sqrt[3]{a} p \log \left(a^{2/3}-\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3} x^2\right)}{2 \sqrt[3]{b}}+x \log \left(c \left(a+b x^3\right)^p\right)+\frac{\sqrt[3]{a} p \log \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\sqrt[3]{b}}-\frac{\sqrt{3} \sqrt[3]{a} p \tan ^{-1}\left(\frac{\sqrt[3]{a}-2 \sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt[3]{b}}$$

```
[Out] -3*p*x+a^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)-1/2*a^(1/3)*p*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(1/3)+x*ln(c*(b*x^3+a)^p)-a^(1/3)*p*arctan(1/
3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)
```

Rubi [A] time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2448, 321, 200, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{a} p \log \left(a^{2/3}-\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3} x^2\right)}{2 \sqrt[3]{b}}+x \log \left(c \left(a+b x^3\right)^p\right)+\frac{\sqrt[3]{a} p \log \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\sqrt[3]{b}}-\frac{\sqrt{3} \sqrt[3]{a} p \tan ^{-1}\left(\frac{\sqrt[3]{a}-2 \sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/b^(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x
^3)^p]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \log\left(c\left(a+bx^3\right)^p\right)dx &= x\log\left(c\left(a+bx^3\right)^p\right)-(3bp)\int\frac{x^3}{a+bx^3}\,dx\\
&= -3px+x\log\left(c\left(a+bx^3\right)^p\right)+(3ap)\int\frac{1}{a+bx^3}\,dx\\
&= -3px+x\log\left(c\left(a+bx^3\right)^p\right)+\left(\sqrt[3]{a}\,p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx+\left(\sqrt[3]{a}\,p\right)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+}\\
&= -3px+\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{1}{2}\left(3a^{2/3}p\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+}\\
&= -3px+\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-\frac{\sqrt[3]{a}\,p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+x\log\left(c\left(a+b\right.\right.\\
&= -3px-\frac{\sqrt{3}\sqrt[3]{a}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}}+\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-\frac{\sqrt[3]{a}\,p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\right.}{2\sqrt[3]{b}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 129, normalized size = 0.97

$$-\frac{\sqrt[3]{a}\,p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-\frac{\sqrt{3}\sqrt[3]{a}\,p\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}-3px$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^
(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x^3)^p]
```

fricas [A] time = 0.47, size = 110, normalized size = 0.83

$$px\log\left(bx^3+a\right)+\sqrt{3}\,p\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)-\frac{1}{2}\,p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] p*x*log(b*x^3 + a) + sqrt(3)*p*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^
(2/3) - sqrt(3)*a)/a) - 1/2*p*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(
2/3)) + p*(a/b)^(1/3)*log(x + (a/b)^(1/3)) - 3*p*x + x*log(c)
```

giac [A] time = 0.19, size = 143, normalized size = 1.08

$$-\frac{1}{2}\,abp\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\right.\right.}{ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/2*a*b*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 2*sqrt(3)*(-a
*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) -
 (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)) + p*x*log
(b*x^3 + a) - (3*p - log(c))*x
```

maple [A] time = 0.05, size = 113, normalized size = 0.85

$$\frac{\sqrt{3}\,ap\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{ap\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{ap\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-3px+x\ln\left(c\left(b\,x^3+a\right)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p),x)
```

```
[Out] x*ln(c*(b*x^3+a)^p)-3*p*x+1/b*p*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/2/b*p*a/(
a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/b*p*a/(a/b)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 1.36, size = 125, normalized size = 0.94

$$-\frac{1}{2}\,bp\left(\frac{6\,x}{b}-\frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\,a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+x\log\left(\left(bx^3+a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] -1/2*b*p*(6*x/b - 2*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^
(1/3))/(b^2*(a/b)^(2/3)) + a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a
/b)^(2/3)) - 2*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))) + x*log((b*x^3 + a
)^p*c)
```

mupad [B] time = 0.46, size = 134, normalized size = 1.01

$$x\,\ln\left(c\left(b\,x^3+a\right)^p\right)-3\,p\,x-\frac{(-a)^{1/3}\,p\,\ln\left((-a)^{4/3}+a\,b^{1/3}\,x\right)}{b^{1/3}}+\frac{(-a)^{1/3}\,p\,\ln\left(2\,a\,b^{1/3}\,x-(-a)^{4/3}-\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\right.}{b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p),x)
```

```
[Out] x*log(c*(a + b*x^3)^p) - 3*p*x - ((-a)^(1/3)*p*log((-a)^(4/3) + a*b^(1/3)*x
))/b^(1/3) + ((-a)^(1/3)*p*log(2*a*b^(1/3)*x - 3^(1/2)*(-a)^(4/3)*1i - (-a)
^(4/3))*((3^(1/2)*1i)/2 + 1/2))/b^(1/3) - ((-a)^(1/3)*p*log(3^(1/2)*(-a)^(4
/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2 - 1/2))/b^(1/3)
```

sympy [A] time = 63.41, size = 231, normalized size = 1.74

$$\begin{cases} x\log\left(0^p c\right) \\ x\log\left(a^p c\right) \\ px\log(b)+3px\log(x)-3px+x\log(c) \\ -\sqrt[3]{-1}\,\sqrt[3]{a}\,bp\left(\frac{1}{b}\right)^{\frac{4}{3}}\log\left(a+bx^3\right)+\frac{3\sqrt[3]{-1}\,\sqrt[3]{a}\,bp\left(\frac{1}{b}\right)^{\frac{4}{3}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,x\sqrt[3]{\frac{1}{b}}+4x^2\right)}{2}+\sqrt[3]{-1}\,\sqrt{3}\,\sqrt[3]{a}\,bp\left(\frac{1}{b}\right)^{\frac{4}{3}}\operatorname{atan}\left(\right. \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p),x)
```

```
[Out] Piecewise((x*log(0**p*c), Eq(a, 0) & Eq(b, 0)), (x*log(a**p*c), Eq(b, 0)),
(p*x*log(b) + 3*p*x*log(x) - 3*p*x + x*log(c), Eq(a, 0)), (-(-1)**(1/3)*a**
(1/3)*b*p*(1/b)**(4/3)*log(a + b*x**3) + 3*(-1)**(1/3)*a**(1/3)*b*p*(1/b)**
(4/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1
/b)**(1/3) + 4*x**2)/2 + (-1)**(1/3)*sqrt(3)*a**(1/3)*b*p*(1/b)**(4/3)*atan
(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x/(3*a**(1/3)*(1/b)**(1/3))) + p*x*log(a
 + b*x**3) - 3*p*x + x*log(c), True))
```

3.19 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x} \, dx$

Optimal. Leaf size=44

$$\frac{1}{3}\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)+\frac{1}{3}p\text{Li}_2\left(\frac{bx^3}{a}+1\right)$$

[Out] 1/3*ln(-b*x^3/a)*ln(c*(b*x^3+a)^p)+1/3*p*polylog(2,1+b*x^3/a)

Rubi [A] time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2394, 2315}

$$\frac{1}{3}p\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)+\frac{1}{3}\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)$$

Antiderivative was successfully verified.

[In] Int[Log[c*(a + b*x^3)^p]/x,x]

[Out] (Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p])/3 + (p*PolyLog[2, 1 + (b*x^3)/a])/3

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x}\,dx,x,x^3\right)\\
&= \frac{1}{3}\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{3}(bp)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx,x,x^3\right)\\
&= \frac{1}{3}\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)+\frac{1}{3}p\text{Li}_2\left(1+\frac{bx^3}{a}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.98

$$\frac{1}{3}\left(\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)+p\text{Li}_2\left(\frac{bx^3+a}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x,x]
[Out] (Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p] + p*PolyLog[2, (a + b*x^3)/a])/3
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^3+a\right)^p c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x,x, algorithm="fricas")
[Out] integral(log((b*x^3 + a)^p*c)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x,x, algorithm="giac")
[Out] integrate(log((b*x^3 + a)^p*c)/x, x)
```

maple [C] time = 0.41, size = 180, normalized size = 4.09

$$-\frac{i\pi\,\text{csgn}(ic)\,\text{csgn}\left(i\left(bx^3+a\right)^p\right)\text{csgn}\left(ic\left(bx^3+a\right)^p\right)\ln(x)}{2}+\frac{i\pi\,\text{csgn}(ic)\,\text{csgn}\left(ic\left(bx^3+a\right)^p\right)^2\ln(x)}{2}+\frac{i\pi\,\text{csgn}\left(i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x,x)
[Out] ln(x)*ln((b*x^3+a)^p)-p*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=Ro
otOf(_Z^3*b+a))+1/2*I*ln(x)*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-
1/2*I*ln(x)*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-1/2*I*ln
(x)*Pi*csgn(I*c*(b*x^3+a)^p)^3+1/2*I*ln(x)*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(
I*c)+ln(c)*ln(x)
```

maxima [B] time = 0.66, size = 80, normalized size = 1.82

$$\frac{1}{3}bp\left(\frac{3\log\left(bx^3+a\right)\log(x)}{b}-\frac{3\log\left(\frac{bx^3}{a}+1\right)\log(x)+\text{Li}_2\left(-\frac{bx^3}{a}\right)}{b}\right)-p\log\left(bx^3+a\right)\log(x)+\log\left(\left(bx^3+a\right)^p c\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x,x, algorithm="maxima")
[Out] 1/3*b*p*(3*log(b*x^3 + a)*log(x)/b - (3*log(b*x^3/a + 1)*log(x) + dilog(-b*
x^3/a))/b) - p*log(b*x^3 + a)*log(x) + log((b*x^3 + a)^p*c)*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln \left(c \left(b\,x^3+a\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/x,x)
```

```
[Out] int(log(c*(a + b*x^3)^p)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c \left(a+bx^3\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x,x)
```

```
[Out] Integral(log(c*(a + b*x**3)**p)/x, x)
```

3.20 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=133

$$\frac{\sqrt[3]{b}\,p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{a}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}-\frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

```
[Out] -b^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)+1/2*b^(1/3)*p*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/a^(1/3)-ln(c*(b*x^3+a)^p)/x-b^(1/3)*p*arctan(1/3*(a^(
1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/a^(1/3)
```

Rubi [A] time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2455, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\,p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{a}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}-\frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x^2,x]
```

```
[Out] -((Sqrt[3]*b^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(
1/3)) - (b^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (b^(1/3)*p*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*a^(1/3)) - Log[c*(a + b*x^3)^p]/x
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^2}\,dx = -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{x} + (3bp)\int \frac{x}{a+bx^3}\,dx$$

$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{x} - \frac{\left(b^{2/3}p\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{\sqrt[3]{a}} + \frac{\left(b^{2/3}p\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{\sqrt[3]{a}}$$

$$= -\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x} + \frac{\left(\sqrt[3]{b}p\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}} + \frac{1}{2}$$

$$= -\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} + \frac{\sqrt[3]{b}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}} - \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} + \frac{\sqrt[3]{b}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\right.}{2\sqrt[3]{a}}$$

Mathematica [C] time = 0.00, size = 47, normalized size = 0.35

$$\frac{3bpx^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right)}{2a} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^2,x]
```

```
[Out] (3*b*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/(2*a) - Log[c*(a +
 b*x^3)^p]/x
```

fricas [A] time = 0.47, size = 126, normalized size = 0.95

$$\frac{2\sqrt{3}px\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right) - px\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right) + 2px\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(b\right.}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*p*x*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqr
t(3)) - p*x*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1/3)) + 2
```

```
*p*x*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)) - 2*p*log(b*x^3 + a) - 2*log(c)
)/x
```

giac [A] time = 0.18, size = 137, normalized size = 1.03

$$-\frac{bp\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a}-\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}p\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{p\log\left(bx^{3}+a\right)}{x}+\frac{\left(-ab^{2}\right)^{\frac{2}{3}}p\log\left(x^{2}+x\left(-\frac{a}{b}\right.\right.}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2,x, algorithm="giac")
```

```
[Out] -b*p*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a - sqrt(3)*(-a*b^2)^(2/3)*p*a
rctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) - p*log(b*x^3 +
a)/x + 1/2*(-a*b^2)^(2/3)*p*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
- log(c)/x
```

maple [C] time = 0.34, size = 184, normalized size = 1.38

$$-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)}{x}-\frac{-i\pi\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)+i\pi\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)^{2}+i}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^2,x)
```

```
[Out] -1/x*ln((b*x^3+a)^p)-1/2*(I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-
I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x
^3+a)^p)^3+I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-2*sum(_R*ln((-4*_R^3*a-3*
b*p^3)*x+a*p*_R^2),_R=RootOf(_Z^3*a+b*p^3))*x+2*ln(c))/x
```

maxima [A] time = 1.54, size = 119, normalized size = 0.89

$$\frac{1}{2}\,bp\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-\frac{\log\left(\left(bx^{3}+a\right)^{p}c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(
a/b)^(1/3)) + log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3)) - 2*lo
g(x + (a/b)^(1/3))/(b*(a/b)^(1/3))) - log((b*x^3 + a)^p*c)/x
```

mupad [B] time = 0.81, size = 149, normalized size = 1.12

$$\frac{(-b)^{1/3}\,p\,\ln\left(a^{1/3}\,(-b)^{8/3}+b^{3}\,x\right)}{a^{1/3}}-\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{x}+\frac{(-b)^{1/3}\,p\,\ln\left(9\,b^{3}\,p^{2}\,x+9\,a^{1/3}\,(-b)^{8/3}\,p^{2}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^{2}\right)\left(-\frac{1}{2}\right.}{a^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/x^2,x)
```

```
[Out] ((-b)^(1/3)*p*log(a^(1/3)*(-b)^(8/3) + b^3*x))/a^(1/3) - log(c*(a + b*x^3)^
p)/x + ((-b)^(1/3)*p*log(9*b^3*p^2*x + 9*a^(1/3)*(-b)^(8/3)*p^2*((3^(1/2)*1
i)/2 - 1/2)^2)*((3^(1/2)*1i)/2 - 1/2))/a^(1/3) - ((-b)^(1/3)*p*log(9*b^3*p^
2*x + 9*a^(1/3)*(-b)^(8/3)*p^2*((3^(1/2)*1i)/2 + 1/2)^2)*((3^(1/2)*1i)/2 +
1/2))/a^(1/3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**2,x)
```

```
[Out] Timed out
```

3.21 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3}\,dx$

Optimal. Leaf size=139

$$-\frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}}+\frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}}-\frac{\sqrt{3}\,b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}$$

```
[Out] 1/2*b^(2/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)-1/4*b^(2/3)*p*ln(a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)-1/2*ln(c*(b*x^3+a)^p)/x^2-1/2*b^(2/3)*p*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/a^(2/3)
```

Rubi [A] time = 0.07, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2455, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}}+\frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}}-\frac{\sqrt{3}\,b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x^3,x]
```

```
[Out] -(Sqrt[3]*b^(2/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*a
^(2/3)) + (b^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*a^(2/3)) - (b^(2/3)*p*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*a^(2/3)) - Log[c*(a + b*x^3
)^p]/(2*x^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3}\,dx &= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}+\frac{1}{2}(3bp)\int \frac{1}{a+bx^3}\,dx\\
&= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}+\frac{(bp)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{2a^{2/3}}+\frac{(bp)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{2/3}}\\
&= \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}-\frac{\left(b^{2/3}p\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4a^{2/3}}+\frac{(3b
\\
&= \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}}-\frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}+\\
&= -\frac{\sqrt{3}b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}}+\frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}}-\frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 134, normalized size = 0.96

$$
-\frac{b^{2/3}px^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2a^{2/3}\log\left(c\left(a+bx^3\right)^p\right)-2b^{2/3}px^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}b^{2/3}px^2\text{ta}}{4a^{2/3}x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^3,x]
```

```
[Out] -1/4*(2*Sqrt[3]*b^(2/3)*p*x^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] -
 2*b^(2/3)*p*x^2*Log[a^(1/3) + b^(1/3)*x] + b^(2/3)*p*x^2*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a^(2/3)*Log[c*(a + b*x^3)^p])/(a^(2/3)*x^
2)
```

fricas [A] time = 0.47, size = 150, normalized size = 1.08

$$
\frac{2\sqrt{3}px^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-px^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2px^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx\right.}{4x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*p*x^2*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2
/3) - sqrt(3)*b)/b) - p*x^2*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(
1/3) + a^2*(b^2/a^2)^(2/3)) + 2*p*x^2*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)
^(1/3)) - 2*p*log(b*x^3 + a) - 2*log(c))/x^2
```

giac [A] time = 0.18, size = 138, normalized size = 0.99

$$-\frac{1}{4}\,bp\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*b*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a - 2*sqrt(3)*(-a*b^2)^
(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) - (-a*b^2
)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)) - 1/2*p*log(b*x^3 +
 a)/x^2 - 1/2*log(c)/x^2
```

maple [C] time = 0.34, size = 197, normalized size = 1.42

$$-\frac{\ln\left(\left(b\,x^3+a\right)^p\right)}{2x^2}-\frac{-2x^2\,\mathrm{RootOf}\left(a^2\_Z^3-b^2p^3\right)\ln\left(-\mathrm{RootOf}\left(a^2\_Z^3-b^2p^3\right)ab\,p^2+\left(-4\,\mathrm{RootOf}\left(a^2\_Z^3-b^2p^3\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^3,x)
```

```
[Out] -1/2/x^2*ln((b*x^3+a)^p)-1/4*(I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p
)^2-I*Pi*csgn(I*c)*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)-I*Pi*csgn(I*c*
(b*x^3+a)^p)^3+I*Pi*csgn(I*c)*csgn(I*c*(b*x^3+a)^p)^2-2*sum(_R*ln((-4*_R^3*
a^2+3*b^2*p^3)*x-a*p^2*_R*b),_R=RootOf(_Z^3*a^2-b^2*p^3))*x^2+2*ln(c))/x^2
```

maxima [A] time = 1.45, size = 120, normalized size = 0.86

$$\frac{1}{4}\,bp\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-\frac{\log\left(\left(bx^3+a\right)^p c\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*p*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(
a/b)^(2/3)) - log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 2*lo
g(x + (a/b)^(1/3))/(b*(a/b)^(2/3))) - 1/2*log((b*x^3 + a)^p*c)/x^2
```

mupad [B] time = 2.61, size = 115, normalized size = 0.83

$$\frac{b^{2/3}\,p\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{2\,a^{2/3}}-\frac{\ln\left(c\left(b\,x^3+a\right)^p\right)}{2\,x^2}-\frac{b^{2/3}\,p\,\ln\left(2\,b^{1/3}\,x-a^{1/3}-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,a^{2/3}}+\frac{b^{2/3}\,p\,\ln\left(2\,b^{1/3}\,x-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/x^3,x)
```

```
[Out] (b^(2/3)*p*log(b^(1/3)*x + a^(1/3)))/(2*a^(2/3)) - log(c*(a + b*x^3)^p)/(2*
x^2) - (b^(2/3)*p*log(2*b^(1/3)*x - 3^(1/2)*a^(1/3)*1i - a^(1/3))*((3^(1/2)
*1i)/2 + 1/2))/(2*a^(2/3)) + (b^(2/3)*p*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*
x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*a^(2/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**3,x)
```

```
[Out] Timed out
```

3.22 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=45

$$-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{3x^3}-\frac{bp\log\left(a+bx^3\right)}{3a}+\frac{bp\log(x)}{a}$$

```
[Out] b*p*ln(x)/a-1/3*b*p*ln(b*x^3+a)/a-1/3*ln(c*(b*x^3+a)^p)/x^3
```

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2454, 2395, 36, 29, 31}

$$-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{3x^3}-\frac{bp\log\left(a+bx^3\right)}{3a}+\frac{bp\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x^4,x]
```

```
[Out] (b*p*Log[x])/a - (b*p*Log[a + b*x^3])/(3*a) - Log[c*(a + b*x^3)^p]/(3*x^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log \left(c \left(a+bx^3\right)^p\right)}{x^4} \, dx = \frac{1}{3} \text{Subst}\left(\int \frac{\log (c(a+bx)^p)}{x^2} \, dx, x, x^3\right)$$
$$= -\frac{\log \left(c \left(a+bx^3\right)^p\right)}{3x^3} + \frac{1}{3}(bp) \text{Subst}\left(\int \frac{1}{x(a+bx)} \, dx, x, x^3\right)$$
$$= -\frac{\log \left(c \left(a+bx^3\right)^p\right)}{3x^3} + \frac{(bp) \text{Subst}\left(\int \frac{1}{x} \, dx, x, x^3\right)}{3a} - \frac{\left(b^2p\right) \text{Subst}\left(\int \frac{1}{a+bx} \, dx, x, x^3\right)}{3a}$$
$$= \frac{bp \log(x)}{a} - \frac{bp \log \left(a+bx^3\right)}{3a} - \frac{\log \left(c \left(a+bx^3\right)^p\right)}{3x^3}$$

Mathematica [A] time = 0.00, size = 45, normalized size = 1.00

$$-\frac{\log \left(c \left(a+bx^3\right)^p\right)}{3x^3} - \frac{bp \log \left(a+bx^3\right)}{3a} + \frac{bp \log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^4,x]
[Out] (b*p*Log[x])/a - (b*p*Log[a + b*x^3])/(3*a) - Log[c*(a + b*x^3)^p]/(3*x^3)
```

fricas [A] time = 0.47, size = 43, normalized size = 0.96

$$\frac{3\, bpx^3 \log(x) - \left(bpx^3 + ap\right) \log \left(bx^3 + a\right) - a \log(c)}{3\, ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^4,x, algorithm="fricas")
[Out] 1/3*(3*b*p*x^3*log(x) - (b*p*x^3 + a*p)*log(b*x^3 + a) - a*log(c))/(a*x^3)
```

giac [A] time = 0.16, size = 58, normalized size = 1.29

$$-\frac{\frac{b^2 p \log\left(bx^3+a\right)}{a} - \frac{b^2 p \log\left(bx^3\right)}{a} + \frac{bp \log\left(bx^3+a\right)}{x^3} + \frac{b \log(c)}{x^3}}{3\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^4,x, algorithm="giac")
[Out] -1/3*(b^2*p*log(b*x^3 + a)/a - b^2*p*log(b*x^3)/a + b*p*log(b*x^3 + a)/x^3
+ b*log(c)/x^3)/b
```

maple [C] time = 0.25, size = 173, normalized size = 3.84

$$-\frac{\ln \left(\left(b\, x^3 + a\right)^p\right)}{3x^3} - \frac{-6bp\, x^3 \ln(x) + 2bp\, x^3 \ln \left(b\, x^3 + a\right) - i\pi a \,\mathrm{csgn}\,(ic) \,\mathrm{csgn}\left(i \left(b\, x^3 + a\right)^p\right) \mathrm{csgn}\left(ic \left(b\, x^3 + a\right)^p\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^4,x)
[Out] -1/3/x^3*ln((b*x^3+a)^p)-1/6*(I*Pi*a*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)
^p)^2-I*Pi*a*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-I*Pi*a*csg
```

```
n(I*c*(b*x^3+a)^p)^3+I*Pi*a*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-6*b*p*ln(x)*x
^3+2*b*p*ln(b*x^3+a)*x^3+2*a*ln(c))/a/x^3
```

maxima [A] time = 0.64, size = 44, normalized size = 0.98

$$-\frac{1}{3}\,bp\left(\frac{\log\left(bx^3+a\right)}{a}-\frac{\log\left(x^3\right)}{a}\right)-\frac{\log\left(\left(bx^3+a\right)^p c\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^4,x, algorithm="maxima")
[Out] -1/3*b*p*(log(b*x^3 + a)/a - log(x^3)/a) - 1/3*log((b*x^3 + a)^p*c)/x^3
```

mupad [B] time = 0.26, size = 41, normalized size = 0.91

$$\frac{b\,p\,\ln(x)}{a}-\frac{b\,p\,\ln\left(b\,x^3+a\right)}{3\,a}-\frac{\ln\left(c\,\left(b\,x^3+a\right)^p\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/x^4,x)
[Out] (b*p*log(x))/a - (b*p*log(a + b*x^3))/(3*a) - log(c*(a + b*x^3)^p)/(3*x^3)
```

sympy [A] time = 10.81, size = 82, normalized size = 1.82

$$\begin{cases} -\frac{p\log\left(a+bx^3\right)}{3x^3}-\frac{\log\left(c\right)}{3x^3}+\frac{bp\log\left(x\right)}{a}-\frac{bp\log\left(a+bx^3\right)}{3a} & \text{for } a\neq 0 \\ -\frac{p\log\left(b\right)}{3x^3}-\frac{p\log\left(x\right)}{x^3}-\frac{p}{3x^3}-\frac{\log\left(c\right)}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**4,x)
[Out] Piecewise((-p*log(a + b*x**3)/(3*x**3) - log(c)/(3*x**3) + b*p*log(x)/a - b
*p*log(a + b*x**3)/(3*a), Ne(a, 0)), (-p*log(b)/(3*x**3) - p*log(x)/x**3 -
p/(3*x**3) - log(c)/(3*x**3), True))
```

3.23 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^5}\,dx$

Optimal. Leaf size=151

$$-\frac{b^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8a^{4/3}}+\frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4a^{4/3}}+\frac{\sqrt{3}b^{4/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4a^{4/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}-$$

```
[Out] -3/4*b*p/a/x+1/4*b^(4/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)-1/8*b^(4/3)*p*ln(a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)-1/4*ln(c*(b*x^3+a)^p)/x^4+1/4
*b^(4/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/a^(4/3
)
```

Rubi [A] time = 0.09, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2455, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{b^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8a^{4/3}}+\frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4a^{4/3}}+\frac{\sqrt{3}b^{4/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4a^{4/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}-$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x^5,x]
```

```
[Out] (-3*b*p)/(4*a*x) + (Sqrt[3]*b^(4/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(4*a^(4/3)) + (b^(4/3)*p*Log[a^(1/3) + b^(1/3)*x])/(4*a^(4/3)
) - (b^(4/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(8*a^(4/3))
- Log[c*(a + b*x^3)^p]/(4*x^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^5}\,dx &= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}+\frac{1}{4}(3bp)\int\frac{1}{x^2\left(a+bx^3\right)}\,dx\\
&= -\frac{3bp}{4ax}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}-\frac{\left(3b^2p\right)\int\frac{x}{a+bx^3}\,dx}{4a}\\
&= -\frac{3bp}{4ax}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}+\frac{\left(b^{5/3}p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{4a^{4/3}}-\frac{\left(b^{5/3}p\right)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4a^{4/3}}\\
&= -\frac{3bp}{4ax}+\frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4a^{4/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}-\frac{\left(b^{4/3}p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{8a^{4/3}}-\\
&= -\frac{3bp}{4ax}+\frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4a^{4/3}}-\frac{b^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8a^{4/3}}-\frac{\log\left(c\left(a+bx^3\right.\right.}{4x^4}\\
&= -\frac{3bp}{4ax}+\frac{\sqrt{3}b^{4/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4a^{4/3}}+\frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4a^{4/3}}-\frac{b^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{\right.}}{8a^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 49, normalized size = 0.32

$$-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}-\frac{3bp\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};-\frac{bx^3}{a}\right)}{4ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^5,x]
```

```
[Out] (-3*b*p*Hypergeometric2F1[-1/3, 1, 2/3, -((b*x^3)/a)])/(4*a*x) - Log[c*(a +
 b*x^3)^p]/(4*x^4)
```

fricas [A] time = 0.47, size = 138, normalized size = 0.91

$$-\frac{2\sqrt{3}\,bpx^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+bpx^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2\,bpx^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(b\right.}{8\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(3)*b*p*x^4*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*
sqrt(3)) + b*p*x^4*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3))
 - 2*b*p*x^4*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) + 6*b*p*x^3 + 2*a*p*log(b
*x^3 + a) + 2*a*log(c))/(a*x^4)
```

giac [A] time = 0.21, size = 153, normalized size = 1.01

$$\frac{1}{8}b^2p\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}+\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b^2}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right.\right.}{a^2b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b^2*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 2*sqrt(3)*(-a*b^
2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) -
(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^2)) - 1/4*p*
log(b*x^3 + a)/x^4 - 1/4*(3*b*p*x^3 + a*log(c))/(a*x^4)
```

maple [C] time = 0.36, size = 215, normalized size = 1.42

$$-\frac{\ln\left(\left(b\,x^3+a\right)^p\right)}{4x^4}-\frac{-2a\,x^4\,\mathrm{RootOf}\left(a^4\_Z^3-b^4p^3\right)\ln\left(-\mathrm{RootOf}\left(a^4\_Z^3-b^4p^3\right)^2a^3bp+\left(-4\,\mathrm{RootOf}\left(a^4\_Z^3-\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^5,x)
```

```
[Out] -1/4/x^4*ln((b*x^3+a)^p)-1/8*(I*Pi*a*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)
^p)^2-I*Pi*a*csgn(I*c)*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)-I*Pi*a*csg
n(I*c*(b*x^3+a)^p)^3+I*Pi*a*csgn(I*c)*csgn(I*c*(b*x^3+a)^p)^2-2*sum(_R*ln((
-4*_R^3*a^4+3*b^4*p^3)*x-a^3*b*p*_R^2),_R=RootOf(_Z^3*a^4-b^4*p^3))*a*x^4+6
*b*p*x^3+2*a*ln(c))/a/x^4
```

maxima [A] time = 1.47, size = 127, normalized size = 0.84

$$-\frac{1}{8}bp\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{6}{ax}\right)-\frac{\log\left(\left(bx^3+a\right)^pc\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^5,x, algorithm="maxima")
```

```
[Out] -1/8*b*p*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*
(a/b)^(1/3)) + log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/3)) - 2*l
og(x + (a/b)^(1/3))/(a*(a/b)^(1/3)) + 6/(a*x)) - 1/4*log((b*x^3 + a)^p*c)/x
^4
```

mupad [B] time = 2.36, size = 125, normalized size = 0.83

$$\frac{b^{4/3}\,p\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{4\,a^{4/3}}-\frac{\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{4\,x^4}-\frac{3\,b\,p}{4\,a\,x}+\frac{b^{4/3}\,p\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}-\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{4\,a^{4/3}}-\frac{b^{4/3}\,p\,\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/x^5,x)
```

```
[Out] (b^(4/3)*p*log(b^(1/3)*x + a^(1/3)))/(4*a^(4/3)) - log(c*(a + b*x^3)^p)/(4*
x^4) - (3*b*p)/(4*a*x) + (b^(4/3)*p*log(4*b^(1/3)*x - 3^(1/2)*a^(1/3)*2i -
2*a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(4*a^(4/3)) - (b^(4/3)*p*log(3^(1/2)*a^(
1/3)*2i + 4*b^(1/3)*x - 2*a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(4*a^(4/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**5,x)
```

```
[Out] Timed out
```

3.24 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^6}\,dx$

Optimal. Leaf size=151

$$\frac{b^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{10a^{5/3}}-\frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{5a^{5/3}}+\frac{\sqrt{3}\,b^{5/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{5a^{5/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5}-\cdots$$

```
[Out] -3/10*b*p/a/x^2-1/5*b^(5/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)+1/10*b^(5/3)*p*
ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)-1/5*ln(c*(b*x^3+a)^p)/x^5
+1/5*b^(5/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/a^
(5/3)
```

Rubi [A] time = 0.09, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2455, 325, 200, 31, 634, 617, 204, 628}

$$\frac{b^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{10a^{5/3}}-\frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{5a^{5/3}}+\frac{\sqrt{3}\,b^{5/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{5a^{5/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x^6,x]
```

```
[Out] (-3*b*p)/(10*a*x^2) + (Sqrt[3]*b^(5/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(5*a^(5/3)) - (b^(5/3)*p*Log[a^(1/3) + b^(1/3)*x])/(5*a^(5
/3)) + (b^(5/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(10*a^(5/
3)) - Log[c*(a + b*x^3)^p]/(5*x^5)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^6}\,dx &= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5}+\frac{1}{5}(3bp)\int \frac{1}{x^3\left(a+bx^3\right)}\,dx\\
&= -\frac{3bp}{10ax^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5}-\frac{\left(3b^2p\right)\int \frac{1}{a+bx^3}\,dx}{5a}\\
&= -\frac{3bp}{10ax^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5}-\frac{\left(b^2p\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{5a^{5/3}}-\frac{\left(b^2p\right)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{5a^{5/3}}\\
&= -\frac{3bp}{10ax^2}-\frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5a^{5/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5}+\frac{\left(b^{5/3}p\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{10a^{5/3}}\\
&= -\frac{3bp}{10ax^2}-\frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5a^{5/3}}+\frac{b^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{10a^{5/3}}-\frac{\log\left(c\left(a+b\right.\right.}{5x^5}\\
&= -\frac{3bp}{10ax^2}+\frac{\sqrt{3}\,b^{5/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{5a^{5/3}}-\frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5a^{5/3}}+\frac{b^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\right.}{10a^{5/3}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 49, normalized size = 0.32

$$-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5}-\frac{3bp\,{}_2F_1\left(-\frac{2}{3},1;\frac{1}{3};-\frac{bx^3}{a}\right)}{10ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^6,x]
```

```
[Out] (-3*b*p*Hypergeometric2F1[-2/3, 1, 1/3, -((b*x^3)/a)])/(10*a*x^2) - Log[c*(
a + b*x^3)^p]/(5*x^5)
```

fricas [A] time = 0.48, size = 172, normalized size = 1.14

$$\frac{2\sqrt{3}\,bpx^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-bpx^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2\,bpx^5\left(-\frac{b}{a}\right.}{10\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^6,x, algorithm="fricas")
```

```
[Out] 1/10*(2*sqrt(3)*b*p*x^5*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^
2)^(2/3) - sqrt(3)*b)/b) - b*p*x^5*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b
^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*b*p*x^5*(-b^2/a^2)^(1/3)*log(b*x
- a*(-b^2/a^2)^(1/3)) - 3*b*p*x^3 - 2*a*p*log(b*x^3 + a) - 2*a*log(c))/(a*x
^5)
```

giac [A] time = 0.21, size = 149, normalized size = 0.99

$$\frac{b^2p\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{5\,a^2}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}bp\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{5\,a^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}bp\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{10\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^6,x, algorithm="giac")
```

```
[Out] 1/5*b^2*p*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - 1/5*sqrt(3)*(-a*b^2
)^(1/3)*b*p*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - 1/1
0*(-a*b^2)^(1/3)*b*p*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 1/5*p*l
og(b*x^3 + a)/x^5 - 1/10*(3*b*p*x^3 + 2*a*log(c))/(a*x^5)
```

maple [C] time = 0.36, size = 216, normalized size = 1.43

$$-\frac{\ln\left(\left(b\,x^3+a\right)^p\right)}{5x^5}-\frac{-2a\,x^5\,\mathrm{RootOf}\left(a^5\_Z^3+b^5p^3\right)\ln\left(-\mathrm{RootOf}\left(a^5\_Z^3+b^5p^3\right)a^2b^3p^2+\left(-4\,\mathrm{RootOf}\left(a^5\_Z^3\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^6,x)
```

```
[Out] -1/5/x^5*ln((b*x^3+a)^p)-1/10*(-2*sum(_R*ln((-4*_R^3*a^5-3*b^5*p^3)*x-a^2*b
^3*p^2*_R),_R=RootOf(_Z^3*a^5+b^5*p^3))*a*x^5+I*Pi*a*csgn(I*(b*x^3+a)^p)*cs
gn(I*c*(b*x^3+a)^p)^2-I*Pi*a*csgn(I*c)*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+
a)^p)-I*Pi*a*csgn(I*c*(b*x^3+a)^p)^3+I*Pi*a*csgn(I*c)*csgn(I*c*(b*x^3+a)^p)
^2+3*b*p*x^3+2*a*ln(c))/a/x^5
```

maxima [A] time = 1.27, size = 128, normalized size = 0.85

$$-\frac{1}{10}\,bp\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{3}{ax^2}\right)-\frac{\log\left(\left(bx^3+a\right)^pc\right.}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^6,x, algorithm="maxima")
```

```
[Out] -1/10*b*p*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a
*(a/b)^(2/3)) - log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) + 2*
log(x + (a/b)^(1/3))/(a*(a/b)^(2/3)) + 3/(a*x^2)) - 1/5*log((b*x^3 + a)^p*c
)/x^5
```

mupad [B] time = 2.45, size = 156, normalized size = 1.03

$$\frac{(-b)^{5/3}\,p\,\ln\left(a^{1/3}\,(-b)^{11/3}-b^4\,x\right)}{5\,a^{5/3}}-\frac{\ln\left(c\,\left(b\,x^3+a\right)^p\right)}{5\,x^5}-\frac{3\,b\,p}{10\,a\,x^2}+\frac{(-b)^{5/3}\,p\,\ln\left(225\,a^2\,b^4\,p\,x-225\,a^{7/3}\,(-b)^{11/3}\,p\,\left(-\frac{1}{2}\right.\right.}{5\,a^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/x^6,x)
```

```
[Out] ((-b)^(5/3)*p*log(a^(1/3)*(-b)^(11/3) - b^4*x))/(5*a^(5/3)) - log(c*(a + b*
x^3)^p)/(5*x^5) - (3*b*p)/(10*a*x^2) + ((-b)^(5/3)*p*log(225*a^2*b^4*p*x -
225*a^(7/3)*(-b)^(11/3)*p*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2))/(
5*a^(5/3)) - ((-b)^(5/3)*p*log(225*a^2*b^4*p*x + 225*a^(7/3)*(-b)^(11/3)*p*
((3^(1/2)*1i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/(5*a^(5/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**6,x)
```

```
[Out] Timed out
```

3.25 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^7}\,dx$

Optimal. Leaf size=64

$$\frac{b^2p\log\left(a+bx^3\right)}{6a^2}-\frac{b^2p\log(x)}{2a^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{6x^6}-\frac{bp}{6ax^3}$$

```
[Out] -1/6*b*p/a/x^3-1/2*b^2*p*ln(x)/a^2+1/6*b^2*p*ln(b*x^3+a)/a^2-1/6*ln(c*(b*x^
3+a)^p)/x^6
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2395, 44}

$$\frac{b^2p\log\left(a+bx^3\right)}{6a^2}-\frac{b^2p\log(x)}{2a^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{6x^6}-\frac{bp}{6ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x^7,x]
```

```
[Out] -(b*p)/(6*a*x^3) - (b^2*p*Log[x])/(2*a^2) + (b^2*p*Log[a + b*x^3])/(6*a^2)
- Log[c*(a + b*x^3)^p]/(6*x^6)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^7}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^3}\,dx, x, x^3\right)$$
$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{6x^6} + \frac{1}{6}(bp)\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, x^3\right)$$
$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{6x^6} + \frac{1}{6}(bp)\,\text{Subst}\left(\int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right)dx, x, x^3\right)$$
$$= -\frac{bp}{6ax^3} - \frac{b^2p\log(x)}{2a^2} + \frac{b^2p\log\left(a+bx^3\right)}{6a^2} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{6x^6}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.88

$$\frac{1}{6}bp\left(\frac{b\log\left(a+bx^3\right)}{a^2} - \frac{3b\log(x)}{a^2} - \frac{1}{ax^3}\right) - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^7,x]
```

```
[Out] (b*p*(-(1/(a*x^3)) - (3*b*Log[x])/a^2 + (b*Log[a + b*x^3])/a^2))/6 - Log[c*
(a + b*x^3)^p]/(6*x^6)
```

fricas [A] time = 0.48, size = 58, normalized size = 0.91

$$-\frac{3\,b^2px^6\log(x) + abpx^3 + a^2\log(c) - \left(b^2px^6 - a^2p\right)\log\left(bx^3+a\right)}{6\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b^2*p*x^6*log(x) + a*b*p*x^3 + a^2*log(c) - (b^2*p*x^6 - a^2*p)*log
(b*x^3 + a))/(a^2*x^6)
```

giac [B] time = 0.18, size = 132, normalized size = 2.06

$$-\frac{\frac{b^3p\log\left(bx^3+a\right)}{\left(bx^3+a\right)^2-2\left(bx^3+a\right)a+a^2} - \frac{b^3p\log\left(bx^3+a\right)}{a^2} + \frac{b^3p\log\left(bx^3\right)}{a^2} + \frac{\left(bx^3+a\right)b^3p-ab^3p+ab^3\log(c)}{\left(bx^3+a\right)^2a-2\left(bx^3+a\right)a^2+a^3}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^7,x, algorithm="giac")
```

```
[Out] -1/6*(b^3*p*log(b*x^3 + a)/((b*x^3 + a)^2 - 2*(b*x^3 + a)*a + a^2) - b^3*p*
log(b*x^3 + a)/a^2 + b^3*p*log(b*x^3)/a^2 + ((b*x^3 + a)*b^3*p - a*b^3*p +
a*b^3*log(c))/((b*x^3 + a)^2*a - 2*(b*x^3 + a)*a^2 + a^3))/b
```

maple [C] time = 0.26, size = 198, normalized size = 3.09

$$-\frac{\ln\left(\left(b\,x^3+a\right)^p\right)}{6x^6} - \frac{6b^2p\,x^6\ln(x) - 2b^2p\,x^6\ln\left(-b\,x^3-a\right) + 2abp\,x^3 - i\pi\,a^2\text{csgn}\,(ic)\,\text{csgn}\left(i\left(b\,x^3+a\right)^p\right)\text{csgn}\left(ic\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^7,x)
```

```
[Out] -1/6/x^6*ln((b*x^3+a)^p)-1/12*(6*b^2*p*ln(x)*x^6-2*b^2*p*ln(-b*x^3-a)*x^6+I
*Pi*a^2*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-I*Pi*a^2*csgn(I*(b*x^3+
a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-I*Pi*a^2*csgn(I*c*(b*x^3+a)^p)^3+I*Pi
*a^2*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+2*a*b*p*x^3+2*a^2*ln(c))/a^2/x^6
```

maxima [A] time = 0.69, size = 54, normalized size = 0.84

$$\frac{1}{6}\,bp\left(\frac{b\log\left(bx^3+a\right)}{a^2}-\frac{b\log\left(x^3\right)}{a^2}-\frac{1}{ax^3}\right)-\frac{\log\left(\left(bx^3+a\right)^p c\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*b*p*(b*log(b*x^3 + a)/a^2 - b*log(x^3)/a^2 - 1/(a*x^3)) - 1/6*log((b*x^
3 + a)^p*c)/x^6
```

mupad [B] time = 0.28, size = 56, normalized size = 0.88

$$\frac{b^2\,p\,\ln\left(b\,x^3+a\right)}{6\,a^2}-\frac{\ln\left(c\,\left(b\,x^3+a\right)^p\right)}{6\,x^6}-\frac{b^2\,p\,\ln(x)}{2\,a^2}-\frac{b\,p}{6\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/x^7,x)
```

```
[Out] (b^2*p*log(a + b*x^3))/(6*a^2) - log(c*(a + b*x^3)^p)/(6*x^6) - (b^2*p*log(
x))/(2*a^2) - (b*p)/(6*a*x^3)
```

sympy [A] time = 45.92, size = 102, normalized size = 1.59

$$\begin{cases} -\frac{p\log\left(a+bx^3\right)}{6x^6}-\frac{\log\left(c\right)}{6x^6}-\frac{bp}{6ax^3}-\frac{b^2p\log\left(x\right)}{2a^2}+\frac{b^2p\log\left(a+bx^3\right)}{6a^2} & \text{for } a\neq 0 \\ -\frac{p\log\left(b\right)}{6x^6}-\frac{p\log\left(x\right)}{2x^6}-\frac{p}{12x^6}-\frac{\log\left(c\right)}{6x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**7,x)
```

```
[Out] Piecewise((-p*log(a + b*x**3)/(6*x**6) - log(c)/(6*x**6) - b*p/(6*a*x**3) -
 b**2*p*log(x)/(2*a**2) + b**2*p*log(a + b*x**3)/(6*a**2), Ne(a, 0)), (-p*l
og(b)/(6*x**6) - p*log(x)/(2*x**6) - p/(12*x**6) - log(c)/(6*x**6), True))
```

3.26 $\int x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$

Optimal. Leaf size=89

$$\frac{b^5 p\log(ax+b)}{5a^5}-\frac{b^4 px}{5a^4}+\frac{b^3 px^2}{10a^3}-\frac{b^2 px^3}{15a^2}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bpx^4}{20a}$$

```
[Out] -1/5*b^4*p*x/a^4+1/10*b^3*p*x^2/a^3-1/15*b^2*p*x^3/a^2+1/20*b*p*x^4/a+1/5*x
^5*ln(c*(a+b/x)^p)+1/5*b^5*p*ln(a*x+b)/a^5
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2455, 263, 43}

$$\frac{b^3 px^2}{10a^3}-\frac{b^2 px^3}{15a^2}-\frac{b^4 px}{5a^4}+\frac{b^5 p\log(ax+b)}{5a^5}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bpx^4}{20a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[c*(a + b/x)^p],x]
```

```
[Out] -(b^4*p*x)/(5*a^4) + (b^3*p*x^2)/(10*a^3) - (b^2*p*x^3)/(15*a^2) + (b*p*x^4
)/(20*a) + (x^5*Log[c*(a + b/x)^p])/5 + (b^5*p*Log[b + a*x])/(5*a^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx &= \frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{5}(bp)\int\frac{x^3}{a+\frac{b}{x}}\,dx\\
&= \frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{5}(bp)\int\frac{x^4}{b+ax}\,dx\\
&= \frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{5}(bp)\int\left(-\frac{b^3}{a^4}+\frac{b^2x}{a^3}-\frac{bx^2}{a^2}+\frac{x^3}{a}+\frac{b^4}{a^4(b+ax)}\right)dx\\
&= -\frac{b^4 px}{5a^4}+\frac{b^3px^2}{10a^3}-\frac{b^2px^3}{15a^2}+\frac{bpx^4}{20a}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b^5p\log(b+ax)}{5a^5}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 85, normalized size = 0.96

$$\frac{12a^5x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+abpx\left(3a^3x^3-4a^2bx^2+6ab^2x-12b^3\right)+12b^5p\log\left(a+\frac{b}{x}\right)+12b^5p\log(x)}{60a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[c*(a + b/x)^p],x]
```

```
[Out] (a*b*p*x*(-12*b^3 + 6*a*b^2*x - 4*a^2*b*x^2 + 3*a^3*x^3) + 12*b^5*p*Log[a +
 b/x] + 12*a^5*x^5*Log[c*(a + b/x)^p] + 12*b^5*p*Log[x])/(60*a^5)
```

fricas [A] time = 0.45, size = 89, normalized size = 1.00

$$\frac{12\,a^5px^5\log\left(\frac{ax+b}{x}\right)+12\,a^5x^5\log(c)+3\,a^4bpx^4-4\,a^3b^2px^3+6\,a^2b^3px^2-12\,ab^4px+12\,b^5p\log\left(ax+b\right)}{60\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/60*(12*a^5*p*x^5*log((a*x + b)/x) + 12*a^5*x^5*log(c) + 3*a^4*b*p*x^4 - 4
*a^3*b^2*p*x^3 + 6*a^2*b^3*p*x^2 - 12*a*b^4*p*x + 12*b^5*p*log(a*x + b))/a^
5
```

giac [B] time = 0.19, size = 308, normalized size = 3.46

$$-\frac{\frac{12\,b^6p\log\left(\frac{ax+b}{x}\right)}{a^5-\frac{5\,(ax+b)a^4}{x}+\frac{10\,(ax+b)^2a^3}{x^2}-\frac{10\,(ax+b)^3a^2}{x^3}+\frac{5\,(ax+b)^4a}{x^4}-\frac{(ax+b)^5}{x^5}}+\frac{12\,b^6p\log\left(-a+\frac{ax+b}{x}\right)}{a^5}-\frac{12\,b^6p\log\left(\frac{ax+b}{x}\right)}{a^5}-\frac{25\,a^4b^6p-12\,a^4b^6\log(c)-\frac{77\,(ax+b)\cdots}{x}}{a^9-\frac{5\,(ax+b)a^8}{x}+\frac{10\,(ax+b\cdots}{x^2}}}{60\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] -1/60*(12*b^6*p*log((a*x + b)/x)/(a^5 - 5*(a*x + b)*a^4/x + 10*(a*x + b)^2*
a^3/x^2 - 10*(a*x + b)^3*a^2/x^3 + 5*(a*x + b)^4*a/x^4 - (a*x + b)^5/x^5) +
 12*b^6*p*log(-a + (a*x + b)/x)/a^5 - 12*b^6*p*log((a*x + b)/x)/a^5 - (25*a
^4*b^6*p - 12*a^4*b^6*log(c) - 77*(a*x + b)*a^3*b^6*p/x + 94*(a*x + b)^2*a^
2*b^6*p/x^2 - 54*(a*x + b)^3*a*b^6*p/x^3 + 12*(a*x + b)^4*b^6*p/x^4)/(a^9 -
 5*(a*x + b)*a^8/x + 10*(a*x + b)^2*a^7/x^2 - 10*(a*x + b)^3*a^6/x^3 + 5*(a
*x + b)^4*a^5/x^4 - (a*x + b)^5*a^4/x^5))/b
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int x^4\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(a+b/x)^p),x)
```

```
[Out] int(x^4*ln(c*(a+b/x)^p),x)
```

maxima [A] time = 0.67, size = 74, normalized size = 0.83

$$\frac{1}{5}\,x^5\log\left(\left(a+\frac{b}{x}\right)^pc\right)+\frac{1}{60}\,bp\left(\frac{12\,b^4\log\left(ax+b\right)}{a^5}+\frac{3\,a^3x^4-4\,a^2bx^3+6\,ab^2x^2-12\,b^3x}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*log((a + b/x)^p*c) + 1/60*b*p*(12*b^4*log(a*x + b)/a^5 + (3*a^3*x^4
 - 4*a^2*b*x^3 + 6*a*b^2*x^2 - 12*b^3*x)/a^4)
```

mupad [B] time = 0.24, size = 77, normalized size = 0.87

$$\frac{x^5\,\ln\left(c\,\left(a+\frac{b}{x}\right)^p\right)}{5}-\frac{b^2\,p\,x^3}{15\,a^2}+\frac{b^3\,p\,x^2}{10\,a^3}+\frac{b^5\,p\,\ln\left(b+a\,x\right)}{5\,a^5}+\frac{b\,p\,x^4}{20\,a}-\frac{b^4\,p\,x}{5\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*log(c*(a + b/x)^p),x)
```

```
[Out] (x^5*log(c*(a + b/x)^p))/5 - (b^2*p*x^3)/(15*a^2) + (b^3*p*x^2)/(10*a^3) +
(b^5*p*log(b + a*x))/(5*a^5) + (b*p*x^4)/(20*a) - (b^4*p*x)/(5*a^4)
```

sympy [A] time = 12.38, size = 122, normalized size = 1.37

$$\begin{cases} \frac{px^5\log\left(a+\frac{b}{x}\right)}{5}+\frac{x^5\log\left(c\right)}{5}+\frac{bpx^4}{20a}-\frac{b^2px^3}{15a^2}+\frac{b^3px^2}{10a^3}-\frac{b^4px}{5a^4}+\frac{b^5p\log\left(ax+b\right)}{5a^5} & \text{for}\ a\neq 0\\ \frac{px^5\log\left(b\right)}{5}-\frac{px^5\log\left(x\right)}{5}+\frac{px^5}{25}+\frac{x^5\log\left(c\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((p*x**5*log(a + b/x)/5 + x**5*log(c)/5 + b*p*x**4/(20*a) - b**2*p
*x**3/(15*a**2) + b**3*p*x**2/(10*a**3) - b**4*p*x/(5*a**4) + b**5*p*log(a*
x + b)/(5*a**5), Ne(a, 0)), (p*x**5*log(b)/5 - p*x**5*log(x)/5 + p*x**5/25
+ x**5*log(c)/5, True))
```

3.27 $\int x^3 \log\left(c \left(a+\frac{b}{x}\right)^p\right) \, dx$

Optimal. Leaf size=75

$$-\frac{b^4 p \log(ax+b)}{4a^4}+\frac{b^3 p x}{4a^3}-\frac{b^2 p x^2}{8a^2}+\frac{1}{4}x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b p x^3}{12a}$$

```
[Out] 1/4*b^3*p*x/a^3-1/8*b^2*p*x^2/a^2+1/12*b*p*x^3/a+1/4*x^4*ln(c*(a+b/x)^p)-1/
4*b^4*p*ln(a*x+b)/a^4
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2455, 263, 43}

$$-\frac{b^2 p x^2}{8a^2}+\frac{b^3 p x}{4a^3}-\frac{b^4 p \log(ax+b)}{4a^4}+\frac{1}{4}x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b p x^3}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b/x)^p],x]
```

```
[Out] (b^3*p*x)/(4*a^3) - (b^2*p*x^2)/(8*a^2) + (b*p*x^3)/(12*a) + (x^4*Log[c*(a
+ b/x)^p])/4 - (b^4*p*Log[b + a*x])/(4*a^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx &= \frac{1}{4}x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{4}(bp)\int \frac{x^2}{a+\frac{b}{x}} \, dx\\
&= \frac{1}{4}x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{4}(bp)\int \frac{x^3}{b+ax} \, dx\\
&= \frac{1}{4}x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{4}(bp)\int \left(\frac{b^2}{a^3}-\frac{bx}{a^2}+\frac{x^2}{a}-\frac{b^3}{a^3(b+ax)}\right) dx\\
&= \frac{b^3 p x}{4a^3}-\frac{b^2 p x^2}{8a^2}+\frac{b p x^3}{12a}+\frac{1}{4}x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)-\frac{b^4 p \log(b+ax)}{4a^4}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 0.99

$$\frac{6a^4x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)+abpx\left(2a^2x^2-3abx+6b^2\right)-6b^4p\log\left(a+\frac{b}{x}\right)-6b^4p\log(x)}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b/x)^p],x]
```

```
[Out] (a*b*p*x*(6*b^2 - 3*a*b*x + 2*a^2*x^2) - 6*b^4*p*Log[a + b/x] + 6*a^4*x^4*L
og[c*(a + b/x)^p] - 6*b^4*p*Log[x])/(24*a^4)
```

fricas [A] time = 0.46, size = 77, normalized size = 1.03

$$\frac{6\,a^4px^4\log\left(\frac{ax+b}{x}\right)+6\,a^4x^4\log(c)+2\,a^3bpx^3-3\,a^2b^2px^2+6\,ab^3px-6\,b^4p\log\left(ax+b\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a^4*p*x^4*log((a*x + b)/x) + 6*a^4*x^4*log(c) + 2*a^3*b*p*x^3 - 3*a
^2*b^2*p*x^2 + 6*a*b^3*p*x - 6*b^4*p*log(a*x + b))/a^4
```

giac [B] time = 0.18, size = 257, normalized size = 3.43

$$\frac{\frac{6\,b^5p\log\left(\frac{ax+b}{x}\right)}{a^4-\frac{4\,(ax+b)a^3}{x}+\frac{6\,(ax+b)^2a^2}{x^2}-\frac{4\,(ax+b)^3a}{x^3}+\frac{(ax+b)^4}{x^4}}+\frac{6\,b^5p\log\left(-a+\frac{ax+b}{x}\right)}{a^4}-\frac{6\,b^5p\log\left(\frac{ax+b}{x}\right)}{a^4}-\frac{11\,a^3b^5p-6\,a^3b^5\log(c)-\frac{26\,(ax+b)a^2b^5p}{x}+\frac{21\,(ax+b)^2ab^5p}{x^2}-6}{a^7-\frac{4\,(ax+b)a^6}{x}+\frac{6\,(ax+b)^2a^5}{x^2}-\frac{4\,(ax+b)^3a^4}{x^3}+\frac{(ax+b)^4a}{x^4}}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] 1/24*(6*b^5*p*log((a*x + b)/x)/(a^4 - 4*(a*x + b)*a^3/x + 6*(a*x + b)^2*a^2
/x^2 - 4*(a*x + b)^3*a/x^3 + (a*x + b)^4/x^4) + 6*b^5*p*log(-a + (a*x + b)/
x)/a^4 - 6*b^5*p*log((a*x + b)/x)/a^4 - (11*a^3*b^5*p - 6*a^3*b^5*log(c) -
26*(a*x + b)*a^2*b^5*p/x + 21*(a*x + b)^2*a*b^5*p/x^2 - 6*(a*x + b)^3*b^5*p
/x^3)/(a^7 - 4*(a*x + b)*a^6/x + 6*(a*x + b)^2*a^5/x^2 - 4*(a*x + b)^3*a^4/
x^3 + (a*x + b)^4*a^3/x^4))/b
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^3\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(a+b/x)^p),x)
```

```
[Out] int(x^3*ln(c*(a+b/x)^p),x)
```

maxima [A] time = 0.66, size = 64, normalized size = 0.85

$$\frac{1}{4}\,x^4\log\left(\left(a+\frac{b}{x}\right)^pc\right)-\frac{1}{24}\,bp\left(\frac{6\,b^3\log\left(ax+b\right)}{a^4}-\frac{2\,a^2x^3-3\,abx^2+6\,b^2x}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((a + b/x)^p*c) - 1/24*b*p*(6*b^3*log(a*x + b)/a^4 - (2*a^2*x^3
- 3*a*b*x^2 + 6*b^2*x)/a^3)
```

mupad [B] time = 0.21, size = 65, normalized size = 0.87

$$\frac{x^4\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{4}-\frac{b^2\,p\,x^2}{8\,a^2}-\frac{b^4\,p\,\ln\left(b+a\,x\right)}{4\,a^4}+\frac{b\,p\,x^3}{12\,a}+\frac{b^3\,p\,x}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(c*(a + b/x)^p),x)
```

```
[Out] (x^4*log(c*(a + b/x)^p))/4 - (b^2*p*x^2)/(8*a^2) - (b^4*p*log(b + a*x))/(4*
a^4) + (b*p*x^3)/(12*a) + (b^3*p*x)/(4*a^3)
```

sympy [A] time = 7.25, size = 109, normalized size = 1.45

$$\begin{cases} \frac{px^4\log\left(a+\frac{b}{x}\right)}{4}+\frac{x^4\log\left(c\right)}{4}+\frac{bpx^3}{12a}-\frac{b^2px^2}{8a^2}+\frac{b^3px}{4a^3}-\frac{b^4p\log\left(ax+b\right)}{4a^4} & \text{for } a\neq 0 \\ \frac{px^4\log\left(b\right)}{4}-\frac{px^4\log\left(x\right)}{4}+\frac{px^4}{16}+\frac{x^4\log\left(c\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((p*x**4*log(a + b/x)/4 + x**4*log(c)/4 + b*p*x**3/(12*a) - b**2*p
*x**2/(8*a**2) + b**3*p*x/(4*a**3) - b**4*p*log(a*x + b)/(4*a**4), Ne(a, 0)
), (p*x**4*log(b)/4 - p*x**4*log(x)/4 + p*x**4/16 + x**4*log(c)/4, True))
```

3.28 $\int x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx$

Optimal. Leaf size=61

$$\frac{b^3 p \log(ax+b)}{3a^3} - \frac{b^2 px}{3a^2} + \frac{1}{3}x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{bpx^2}{6a}$$

```
[Out] -1/3*b^2*p*x/a^2+1/6*b*p*x^2/a+1/3*x^3*ln(c*(a+b/x)^p)+1/3*b^3*p*ln(a*x+b)/
a^3
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2455, 263, 43}

$$-\frac{b^2 px}{3a^2} + \frac{b^3 p \log(ax+b)}{3a^3} + \frac{1}{3}x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{bpx^2}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(a + b/x)^p],x]
```

```
[Out] -(b^2*p*x)/(3*a^2) + (b*p*x^2)/(6*a) + (x^3*Log[c*(a + b/x)^p])/3 + (b^3*p*
Log[b + a*x])/(3*a^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx &= \frac{1}{3}x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{1}{3}(bp)\int \frac{x}{a+\frac{b}{x}}\,dx \\
&= \frac{1}{3}x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{1}{3}(bp)\int \frac{x^2}{b+ax}\,dx \\
&= \frac{1}{3}x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{1}{3}(bp)\int \left(-\frac{b}{a^2}+\frac{x}{a}+\frac{b^2}{a^2(b+ax)}\right) dx \\
&= -\frac{b^2 px}{3a^2} + \frac{bpx^2}{6a} + \frac{1}{3}x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{b^3 p \log(b+ax)}{3a^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 1.02

$$\frac{2a^3x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+2b^3p\log\left(a+\frac{b}{x}\right)+abpx(ax-2b)+2b^3p\log(x)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b/x)^p],x]
```

```
[Out] (a*b*p*x*(-2*b + a*x) + 2*b^3*p*Log[a + b/x] + 2*a^3*x^3*Log[c*(a + b/x)^p]
 + 2*b^3*p*Log[x])/(6*a^3)
```

fricas [A] time = 0.48, size = 64, normalized size = 1.05

$$\frac{2\,a^3px^3\log\left(\frac{ax+b}{x}\right)+2\,a^3x^3\log(c)+a^2bpx^2-2\,ab^2px+2\,b^3p\log\left(ax+b\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*p*x^3*log((a*x + b)/x) + 2*a^3*x^3*log(c) + a^2*b*p*x^2 - 2*a*b^
2*p*x + 2*b^3*p*log(a*x + b))/a^3
```

giac [B] time = 0.20, size = 210, normalized size = 3.44

$$-\frac{\frac{2\,b^4p\log\left(\frac{ax+b}{x}\right)}{a^3-\frac{3\,(ax+b)a^2}{x}+\frac{3\,(ax+b)^2a}{x^2}-\frac{(ax+b)^3}{x^3}}+\frac{2\,b^4p\log\left(-a+\frac{ax+b}{x}\right)}{a^3}-\frac{2\,b^4p\log\left(\frac{ax+b}{x}\right)}{a^3}-\frac{3\,a^2b^4p-2\,a^2b^4\log(c)-\frac{5\,(ax+b)ab^4p}{x}+\frac{2\,(ax+b)^2b^4p}{x^2}}{a^5-\frac{3\,(ax+b)a^4}{x}+\frac{3\,(ax+b)^2a^3}{x^2}-\frac{(ax+b)^3a^2}{x^3}}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] -1/6*(2*b^4*p*log((a*x + b)/x)/(a^3 - 3*(a*x + b)*a^2/x + 3*(a*x + b)^2*a/x
^2 - (a*x + b)^3/x^3) + 2*b^4*p*log(-a + (a*x + b)/x)/a^3 - 2*b^4*p*log((a*
x + b)/x)/a^3 - (3*a^2*b^4*p - 2*a^2*b^4*log(c) - 5*(a*x + b)*a*b^4*p/x + 2
*(a*x + b)^2*b^4*p/x^2)/(a^5 - 3*(a*x + b)*a^4/x + 3*(a*x + b)^2*a^3/x^2 -
(a*x + b)^3*a^2/x^3))/b
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int x^2\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(a+b/x)^p),x)
```

```
[Out] int(x^2*ln(c*(a+b/x)^p),x)
```

maxima [A] time = 0.71, size = 51, normalized size = 0.84

$$\frac{1}{3}\,x^3\log\left(\left(a+\frac{b}{x}\right)^pc\right)+\frac{1}{6}\,bp\left(\frac{2\,b^2\log\left(ax+b\right)}{a^3}+\frac{ax^2-2\,bx}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((a + b/x)^p*c) + 1/6*b*p*(2*b^2*log(a*x + b)/a^3 + (a*x^2 - 2*b
*x)/a^2)
```

mupad [B]   time = 0.23, size = 53, normalized size = 0.87

$$\frac{x^3\,\ln\left(c\,{\left(a+\frac{b}{x}\right)}^p\right)}{3}+\frac{b^3\,p\,\ln\left(b+a\,x\right)}{3\,a^3}+\frac{b\,p\,x^2}{6\,a}-\frac{b^2\,p\,x}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(a + b/x)^p),x)
```

```
[Out] (x^3*log(c*(a + b/x)^p))/3 + (b^3*p*log(b + a*x))/(3*a^3) + (b*p*x^2)/(6*a)
 - (b^2*p*x)/(3*a^2)
```

sympy [A]   time = 4.12, size = 95, normalized size = 1.56

$$\begin{cases} \frac{px^3\log\left(a+\frac{b}{x}\right)}{3}+\frac{x^3\log\left(c\right)}{3}+\frac{bpx^2}{6a}-\frac{b^2px}{3a^2}+\frac{b^3p\log\left(ax+b\right)}{3a^3} & \text{for}\ a\neq 0 \\ \frac{px^3\log\left(b\right)}{3}-\frac{px^3\log\left(x\right)}{3}+\frac{px^3}{9}+\frac{x^3\log\left(c\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((p*x**3*log(a + b/x)/3 + x**3*log(c)/3 + b*p*x**2/(6*a) - b**2*p*
x/(3*a**2) + b**3*p*log(a*x + b)/(3*a**3), Ne(a, 0)), (p*x**3*log(b)/3 - p*
x**3*log(x)/3 + p*x**3/9 + x**3*log(c)/3, True))
```

3.29 $\int x \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx$

Optimal. Leaf size=47

$$-\frac{b^2 p \log(ax+b)}{2a^2}+\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bpx}{2a}$$

```
[Out] 1/2*b*p*x/a+1/2*x^2*ln(c*(a+b/x)^p)-1/2*b^2*p*ln(a*x+b)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2455, 193, 43}

$$-\frac{b^2 p \log(ax+b)}{2a^2}+\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bpx}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b/x)^p],x]
```

```
[Out] (b*p*x)/(2*a) + (x^2*Log[c*(a + b/x)^p])/2 - (b^2*p*Log[b + a*x])/(2*a^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx &= \frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{2}(bp)\int \frac{1}{a+\frac{b}{x}} dx\\
&= \frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{2}(bp)\int \frac{x}{b+ax} dx\\
&= \frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{2}(bp)\int \left(\frac{1}{a}-\frac{b}{a(b+ax)}\right) dx\\
&= \frac{bpx}{2a}+\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)-\frac{b^2 p \log(b+ax)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.85

$$\frac{1}{2}\left(\frac{bp(ax-b\log(ax+b))}{a^2}+x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b/x)^p],x]
```

```
[Out] (x^2*Log[c*(a + b/x)^p] + (b*p*(a*x - b*Log[b + a*x]))/a^2)/2
```

fricas [A] time = 0.42, size = 50, normalized size = 1.06

$$\frac{a^2px^2\log\left(\frac{ax+b}{x}\right)+a^2x^2\log(c)+abpx-b^2p\log(ax+b)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*p*x^2*log((a*x + b)/x) + a^2*x^2*log(c) + a*b*p*x - b^2*p*log(a*x
+ b))/a^2
```

giac [B] time = 0.18, size = 152, normalized size = 3.23

$$\frac{\frac{b^3p\log\left(\frac{ax+b}{x}\right)}{a^2-\frac{2\,(ax+b)a}{x}+\frac{(ax+b)^2}{x^2}}+\frac{b^3p\log\left(-a+\frac{ax+b}{x}\right)}{a^2}-\frac{b^3p\log\left(\frac{ax+b}{x}\right)}{a^2}-\frac{ab^3p-ab^3\log(c)-\frac{(ax+b)b^3p}{x}}{a^3-\frac{2\,(ax+b)a^2}{x}+\frac{(ax+b)^2a}{x^2}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] 1/2*(b^3*p*log((a*x + b)/x)/(a^2 - 2*(a*x + b)*a/x + (a*x + b)^2/x^2) + b^3
*p*log(-a + (a*x + b)/x)/a^2 - b^3*p*log((a*x + b)/x)/a^2 - (a*b^3*p - a*b^
3*log(c) - (a*x + b)*b^3*p/x)/(a^3 - 2*(a*x + b)*a^2/x + (a*x + b)^2*a/x^2)
)/b
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(a+b/x)^p),x)
```

```
[Out] int(x*ln(c*(a+b/x)^p),x)
```

maxima [A] time = 0.64, size = 40, normalized size = 0.85

$$\frac{1}{2}\,bp\left(\frac{x}{a}-\frac{b\log(ax+b)}{a^2}\right)+\frac{1}{2}\,x^2\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(x/a - b*log(a*x + b)/a^2) + 1/2*x^2*log((a + b/x)^p*c)
```

mupad [B] time = 0.25, size = 41, normalized size = 0.87

$$\frac{x^2\,\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{2}+\frac{b\,p\,x}{2\,a}-\frac{b^2\,p\,\ln(b+a\,x)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(a + b/x)^p),x)
```

```
[Out] (x^2*log(c*(a + b/x)^p))/2 + (b*p*x)/(2*a) - (b^2*p*log(b + a*x))/(2*a^2)
```

sympy [**A**] time = 2.19, size = 82, normalized size = 1.74

$$\begin{cases} \frac{px^2\log\left(a+\frac{b}{x}\right)}{2}+\frac{x^2\log(c)}{2}+\frac{bpx}{2a}-\frac{b^2p\log(ax+b)}{2a^2} & \text{for } a\neq 0 \\ \frac{px^2\log(b)}{2}-\frac{px^2\log(x)}{2}+\frac{px^2}{4}+\frac{x^2\log(c)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((p*x**2*log(a + b/x)/2 + x**2*log(c)/2 + b*p*x/(2*a) - b**2*p*log
(a*x + b)/(2*a**2), Ne(a, 0)), (p*x**2*log(b)/2 - p*x**2*log(x)/2 + p*x**2/
4 + x**2*log(c)/2, True))
```

3.30 $\int \log \left(c \left(a+\frac{b}{x}\right)^p\right) \, dx$

Optimal. Leaf size=27

$$x \log \left(c \left(a+\frac{b}{x}\right)^p\right)+\frac{b p \log (a x+b)}{a}$$

```
[Out] x*ln(c*(a+b/x)^p)+b*p*ln(a*x+b)/a
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2448, 263, 31}

$$x \log \left(c \left(a+\frac{b}{x}\right)^p\right)+\frac{b p \log (a x+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p],x]
```

```
[Out] x*Log[c*(a + b/x)^p] + (b*p*Log[b + a*x])/a
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned} \int \log \left(c \left(a+\frac{b}{x}\right)^p\right) \, dx &= x \log \left(c \left(a+\frac{b}{x}\right)^p\right)+(b p) \int \frac{1}{\left(a+\frac{b}{x}\right) x} \, dx\\ &= x \log \left(c \left(a+\frac{b}{x}\right)^p\right)+(b p) \int \frac{1}{b+a x} \, dx\\ &= x \log \left(c \left(a+\frac{b}{x}\right)^p\right)+\frac{b p \log (b+a x)}{a} \end{aligned}$$

Mathematica [A] time = 0.00, size = 37, normalized size = 1.37

$$x \log \left(c \left(a+\frac{b}{x}\right)^p\right)+\frac{b p \log \left(a+\frac{b}{x}\right)}{a}+\frac{b p \log (x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p],x]
[Out] (b*p*Log[a + b/x])/a + x*Log[c*(a + b/x)^p] + (b*p*Log[x])/a
```

fricas [A] time = 0.45, size = 33, normalized size = 1.22

$$\frac{apx\log\left(\frac{ax+b}{x}\right)+bp\log\left(ax+b\right)+ax\log(c)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p),x, algorithm="fricas")
[Out] (a*p*x*log((a*x + b)/x) + b*p*log(a*x + b) + a*x*log(c))/a
```

giac [B] time = 0.16, size = 96, normalized size = 3.56

$$-\frac{\frac{b^{2}p\log\left(-a+\frac{ax+b}{x}\right)}{a}+\frac{b^{2}p\log\left(\frac{ax+b}{x}\right)}{a-\frac{ax+b}{x}}-\frac{b^{2}p\log\left(\frac{ax+b}{x}\right)}{a}+\frac{b^{2}\log(c)}{a-\frac{ax+b}{x}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p),x, algorithm="giac")
[Out] -(b^2*p*log(-a + (a*x + b)/x)/a + b^2*p*log((a*x + b)/x)/(a - (a*x + b)/x)
- b^2*p*log((a*x + b)/x)/a + b^2*log(c)/(a - (a*x + b)/x))/b
```

maple [A] time = 0.05, size = 30, normalized size = 1.11

$$\frac{bp\ln\left(ax+b\right)}{a}+x\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p),x)
[Out] x*ln(c*((a*x+b)/x)^p)+b*p*ln(a*x+b)/a
```

maxima [A] time = 0.71, size = 27, normalized size = 1.00

$$x\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)+\frac{bp\log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p),x, algorithm="maxima")
[Out] x*log((a + b/x)^p*c) + b*p*log(a*x + b)/a
```

mupad [B] time = 0.20, size = 27, normalized size = 1.00

$$x\,\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)+\frac{b\,p\,\ln\left(b+a\,x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p),x)
[Out] x*log(c*(a + b/x)^p) + (b*p*log(b + a*x))/a
```

sympy [A] time = 1.12, size = 48, normalized size = 1.78

$$\begin{cases} px\log\left(a+\frac{b}{x}\right)+x\log\left(c\right)+\frac{bp\log\left(ax+b\right)}{a} & \text{for } a\neq 0 \\ px\log\left(b\right)-px\log\left(x\right)+px+x\log\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((p*x*log(a + b/x) + x*log(c) + b*p*log(a*x + b)/a, Ne(a, 0)), (p*
x*log(b) - p*x*log(x) + p*x + x*log(c), True))
```

3.31 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx$

Optimal. Leaf size=40

$$\log\left(-\frac{b}{ax}\right)\left(-\log\left(c\left(a+\frac{b}{x}\right)^p\right)\right)-p\text{Li}_2\left(\frac{b}{ax}+1\right)$$

```
[Out] -ln(c*(a+b/x)^p)*ln(-b/a/x)-p*polylog(2,1+b/a/x)
```

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2394, 2315}

$$\log\left(-\frac{b}{ax}\right)\left(-\log\left(c\left(a+\frac{b}{x}\right)^p\right)\right)-p\text{PolyLog}\left(2,\frac{b}{ax}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/x,x]
```

```
[Out] -(Log[c*(a + b/x)^p]*Log[-(b/(a*x))]) - p*PolyLog[2, 1 + b/(a*x)]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx &= -\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x}\,dx,x,\frac{1}{x}\right)\\
&= -\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)+(bp)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx,x,\frac{1}{x}\right)\\
&= -\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)-p\text{Li}_2\left(1+\frac{b}{ax}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 41, normalized size = 1.02

$$\log\left(-\frac{b}{ax}\right)\left(-\log\left(c\left(a+\frac{b}{x}\right)^p\right)\right)-p\text{Li}_2\left(\frac{a+\frac{b}{x}}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/x,x]
```

```
[Out] -(Log[c*(a + b/x)^p]*Log[-(b/(a*x))]) - p*PolyLog[2, (a + b/x)/a]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax+b}{x}\right)^p\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x,x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/x, x)
```

giac [B] time = 0.52, size = 152, normalized size = 3.80

$$-\frac{\frac{b^3p\log\left(\frac{ax+b}{x}\right)}{a^2-\frac{2(ax+b)a}{x}+\frac{(ax+b)^2}{x^2}}+\frac{b^3p\log\left(-a+\frac{ax+b}{x}\right)}{a^2}-\frac{b^3p\log\left(\frac{ax+b}{x}\right)}{a^2}-\frac{ab^3p-ab^3\log(c)-\frac{(ax+b)b^3p}{x}}{a^3-\frac{2(ax+b)a^2}{x}+\frac{(ax+b)^2a}{x^2}}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x,x, algorithm="giac")
```

```
[Out] -1/2*(b^3*p*log((a*x + b)/x)/(a^2 - 2*(a*x + b)*a/x + (a*x + b)^2/x^2) + b^
3*p*log(-a + (a*x + b)/x)/a^2 - b^3*p*log((a*x + b)/x)/a^2 - (a*b^3*p - a*b
^3*log(c) - (a*x + b)*b^3*p/x)/(a^3 - 2*(a*x + b)*a^2/x + (a*x + b)^2*a/x^2
))/b^2
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int\frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x,x)
```

```
[Out] int(ln(c*(a+b/x)^p)/x,x)
```

maxima [B] time = 0.68, size = 83, normalized size = 2.08

$$\frac{1}{2}bp\left(\frac{2\log\left(a+\frac{b}{x}\right)\log(x)}{b}+\frac{\log(x)^2}{b}-\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\text{Li}_2\left(-\frac{ax}{b}\right)\right)}{b}\right)-p\log\left(a+\frac{b}{x}\right)\log(x)+\log\left(\left(a+\frac{b}{x}\right)^p\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*log(a + b/x)*log(x)/b + log(x)^2/b - 2*(log(a*x/b + 1)*log(x) +
dilog(-a*x/b))/b) - p*log(a + b/x)*log(x) + log((a + b/x)^p*c)*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/x,x)
```

```
[Out] int(log(c*(a + b/x)^p)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x,x)
```

```
[Out] Integral(log(c*(a + b/x)**p)/x, x)
```

3.32 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=30

$$\frac{p}{x}-\frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}$$

```
[Out] p/x-(a+b/x)*ln(c*(a+b/x)^p)/b
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2389, 2295}

$$\frac{p}{x}-\frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/x^2,x]
```

```
[Out] p/x - ((a + b/x)*Log[c*(a + b/x)^p])/b
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx &= -\text{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, a+\frac{b}{x}\right)}{b}\\ &= \frac{p}{x}-\frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{p}{x}-\frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/x^2,x]
[Out] p/x - ((a + b/x)*Log[c*(a + b/x)^p])/b
```

fricas [A] time = 0.48, size = 36, normalized size = 1.20

$$\frac{bp-b\log(c)-\left(apx+bp\right)\log\left(\frac{ax+b}{x}\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^2,x, algorithm="fricas")
[Out] (b*p - b*log(c) - (a*p*x + b*p)*log((a*x + b)/x))/(b*x)
```

giac [B] time = 0.18, size = 63, normalized size = 2.10

$$-\frac{\frac{(ax+b)p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{x}-\frac{(ax+b)p}{x}+\frac{(ax+b)\log(c)}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^2,x, algorithm="giac")
[Out] -((a*x + b)*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/x - (a*x + b)*p/x + (a*x
+ b)*log(c)/x)/b
```

maple [A] time = 0.05, size = 48, normalized size = 1.60

$$\frac{ap}{b}-\frac{a\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}+\frac{p}{x}-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x^2,x)
[Out] -1/b*ln(c*(a+b/x)^p)*a-1/x*ln(c*(a+b/x)^p)+a/b*p+p/x
```

maxima [A] time = 0.65, size = 50, normalized size = 1.67

$$-bp\left(\frac{a\log(ax+b)}{b^2}-\frac{a\log(x)}{b^2}-\frac{1}{bx}\right)-\frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^2,x, algorithm="maxima")
[Out] -b*p*(a*log(a*x + b)/b^2 - a*log(x)/b^2 - 1/(b*x)) - log((a + b/x)^p*c)/x
```

mupad [B] time = 0.68, size = 40, normalized size = 1.33

$$\frac{p}{x}-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}-\frac{2\,a\,p\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/x^2,x)
```

```
[Out] p/x - log(c*(a + b/x)^p)/x - (2*a*p*atanh((2*a*x)/b + 1))/b
```

sympy [A] time = 2.22, size = 39, normalized size = 1.30

$$\begin{cases} -\frac{ap\log\left(a+\frac{b}{x}\right)}{b} - \frac{p\log\left(a+\frac{b}{x}\right)}{x} + \frac{p}{x} - \frac{\log(c)}{x} & \text{for } b \neq 0 \\ -\frac{\log(a^p c)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x**2,x)
```

```
[Out] Piecewise((-a*p*log(a + b/x)/b - p*log(a + b/x)/x + p/x - log(c)/x, Ne(b, 0
)), (-log(a**p*c)/x, True))
```

3.33 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3} dx$

Optimal. Leaf size=59

$$\frac{a^2 p \log\left(a+\frac{b}{x}\right)}{2b^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}-\frac{ap}{2bx}+\frac{p}{4x^2}$$

```
[Out] 1/4*p/x^2-1/2*a*p/b/x+1/2*a^2*p*ln(a+b/x)/b^2-1/2*ln(c*(a+b/x)^p)/x^2
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2395, 43}

$$\frac{a^2 p \log\left(a+\frac{b}{x}\right)}{2b^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}-\frac{ap}{2bx}+\frac{p}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/x^3,x]
```

```
[Out] p/(4*x^2) - (a*p)/(2*b*x) + (a^2*p*Log[a + b/x])/(2*b^2) - Log[c*(a + b/x)^
p]/(2*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx = -\text{Subst}\left(\int x\log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2} + \frac{1}{2}(bp)\,\text{Subst}\left(\int \frac{x^2}{a+bx}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2} + \frac{1}{2}(bp)\,\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx, x, \frac{1}{x}\right)$$
$$= \frac{p}{4x^2} - \frac{ap}{2bx} + \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 1.00

$$\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2} - \frac{ap}{2bx} + \frac{p}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/x^3,x]
```

```
[Out] p/(4*x^2) - (a*p)/(2*b*x) + (a^2*p*Log[a + b/x])/(2*b^2) - Log[c*(a + b/x)^
p]/(2*x^2)
```

fricas [A] time = 0.47, size = 55, normalized size = 0.93

$$-\frac{2\,abpx - b^2p + 2\,b^2\log(c) - 2\left(a^2px^2 - b^2p\right)\log\left(\frac{ax+b}{x}\right)}{4\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*b*p*x - b^2*p + 2*b^2*log(c) - 2*(a^2*p*x^2 - b^2*p)*log((a*x + b
)/x))/(b^2*x^2)
```

giac [B] time = 0.19, size = 150, normalized size = 2.54

$$\frac{\frac{4\,(ax+b)ap\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{bx} - \frac{4\,(ax+b)ap}{bx} - \frac{2\,(ax+b)^2p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{bx^2} + \frac{4\,(ax+b)a\log(c)}{bx} + \frac{(ax+b)^2p}{bx^2} - \frac{2\,(ax+b)^2\log(c)}{bx^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(4*(a*x + b)*a*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b*x) - 4*(a*x + b
)*a*p/(b*x) - 2*(a*x + b)^2*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b*x^2) +
 4*(a*x + b)*a*log(c)/(b*x) + (a*x + b)^2*p/(b*x^2) - 2*(a*x + b)^2*log(c)/
(b*x^2))/b
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x^3,x)
```

```
[Out] int(ln(c*(a+b/x)^p)/x^3,x)
```

maxima [A] time = 0.65, size = 63, normalized size = 1.07

$$\frac{1}{4}\,bp\left(\frac{2\,a^2\log\left(ax+b\right)}{b^3}-\frac{2\,a^2\log(x)}{b^3}-\frac{2\,ax-b}{b^2x^2}\right)-\frac{\log\left(\left(a+\frac{b}{x}\right)^pc\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*p*(2*a^2*log(a*x + b)/b^3 - 2*a^2*log(x)/b^3 - (2*a*x - b)/(b^2*x^2))
 - 1/2*log((a + b/x)^p*c)/x^2
```

mupad [B] time = 0.34, size = 53, normalized size = 0.90

$$\frac{\frac{p}{2}-\frac{a\,p\,x}{b}}{2\,x^2}-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{2\,x^2}+\frac{a^2\,p\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/x^3,x)
```

```
[Out] (p/2 - (a*p*x)/b)/(2*x^2) - log(c*(a + b/x)^p)/(2*x^2) + (a^2*p*atanh((2*a*
x)/b + 1))/b^2
```

sympy [A] time = 3.93, size = 66, normalized size = 1.12

$$\begin{cases} \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2}-\frac{ap}{2bx}-\frac{p\log\left(a+\frac{b}{x}\right)}{2x^2}+\frac{p}{4x^2}-\frac{\log\left(c\right)}{2x^2} & \text{for}\ b\neq 0 \\ -\frac{\log\left(a^pc\right)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x**3,x)
```

```
[Out] Piecewise((a**2*p*log(a + b/x)/(2*b**2) - a*p/(2*b*x) - p*log(a + b/x)/(2*x
**2) + p/(4*x**2) - log(c)/(2*x**2), Ne(b, 0)), (-log(a**p*c)/(2*x**2), Tru
e))
```

3.34 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=73

$$-\frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3}+\frac{a^2p}{3b^2x}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}-\frac{ap}{6bx^2}+\frac{p}{9x^3}$$

```
[Out] 1/9*p/x^3-1/6*a*p/b/x^2+1/3*a^2*p/b^2/x-1/3*a^3*p*ln(a+b/x)/b^3-1/3*ln(c*(a
+b/x)^p)/x^3
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2395, 43}

$$\frac{a^2p}{3b^2x}-\frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}-\frac{ap}{6bx^2}+\frac{p}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/x^4,x]
```

```
[Out] p/(9*x^3) - (a*p)/(6*b*x^2) + (a^2*p)/(3*b^2*x) - (a^3*p*Log[a + b/x])/(3*b
^3) - Log[c*(a + b/x)^p]/(3*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx = -\text{Subst}\left(\int x^2\log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3} + \frac{1}{3}(bp)\,\text{Subst}\left(\int \frac{x^3}{a+bx}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3} + \frac{1}{3}(bp)\,\text{Subst}\left(\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx, x, \frac{1}{x}\right)$$
$$= \frac{p}{9x^3}-\frac{ap}{6bx^2}+\frac{a^2p}{3b^2x}-\frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}$$

Mathematica [A] time = 0.02, size = 73, normalized size = 1.00

$$-\frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3}+\frac{a^2p}{3b^2x}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}-\frac{ap}{6bx^2}+\frac{p}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/x^4,x]
```

```
[Out] p/(9*x^3) - (a*p)/(6*b*x^2) + (a^2*p)/(3*b^2*x) - (a^3*p*Log[a + b/x])/(3*b
^3) - Log[c*(a + b/x)^p]/(3*x^3)
```

fricas [A] time = 0.48, size = 66, normalized size = 0.90

$$\frac{6\,a^2bpx^2-3\,ab^2px+2\,b^3p-6\,b^3\log(c)-6\left(a^3px^3+b^3p\right)\log\left(\frac{ax+b}{x}\right)}{18\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^4,x, algorithm="fricas")
```

```
[Out] 1/18*(6*a^2*b*p*x^2 - 3*a*b^2*p*x + 2*b^3*p - 6*b^3*log(c) - 6*(a^3*p*x^3 +
 b^3*p)*log((a*x + b)/x))/(b^3*x^3)
```

giac [B] time = 0.22, size = 234, normalized size = 3.21

$$-\frac{\frac{18\,(ax+b)a^2p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^2x}-\frac{18\,(ax+b)a^2p}{b^2x}-\frac{18\,(ax+b)^2ap\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^2x^2}+\frac{18\,(ax+b)a^2\log(c)}{b^2x}+\frac{9\,(ax+b)^2ap}{b^2x^2}+\frac{6\,(ax+b)^3p}{}}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^4,x, algorithm="giac")
```

```
[Out] -1/18*(18*(a*x + b)*a^2*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^2*x) - 18*
(a*x + b)*a^2*p/(b^2*x) - 18*(a*x + b)^2*a*p*log(-b*(a/b - (a*x + b)/(b*x))
 + a)/(b^2*x^2) + 18*(a*x + b)*a^2*log(c)/(b^2*x) + 9*(a*x + b)^2*a*p/(b^2*
x^2) + 6*(a*x + b)^3*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^2*x^3) - 18*(
a*x + b)^2*a*log(c)/(b^2*x^2) - 2*(a*x + b)^3*p/(b^2*x^3) + 6*(a*x + b)^3*l
og(c)/(b^2*x^3))/b
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x^4,x)
```

```
[Out] int(ln(c*(a+b/x)^p)/x^4,x)
```

maxima [A] time = 0.65, size = 74, normalized size = 1.01

$$-\frac{1}{18}\, bp\left(\frac{6\, a^{3} \log\left(ax+b\right)}{b^{4}} - \frac{6\, a^{3} \log\left(x\right)}{b^{4}} - \frac{6\, a^{2} x^{2} - 3\, abx + 2\, b^{2}}{b^{3} x^{3}}\right) - \frac{\log\left(\left(a+\frac{b}{x}\right)^{p} c\right)}{3\, x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -1/18*b*p*(6*a^3*log(a*x + b)/b^4 - 6*a^3*log(x)/b^4 - (6*a^2*x^2 - 3*a*b*x
 + 2*b^2)/(b^3*x^3)) - 1/3*log((a + b/x)^p*c)/x^3
```

mupad [B] time = 0.32, size = 65, normalized size = 0.89

$$\frac{\frac{p}{3} + \frac{a^{2}\, p\, x^{2}}{b^{2}} - \frac{a\, p\, x}{2\, b}}{3\, x^{3}} - \frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{3\, x^{3}} - \frac{2\, a^{3}\, p\, \mathrm{atanh}\left(\frac{2\, a\, x}{b} + 1\right)}{3\, b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/x^4,x)
```

```
[Out] (p/3 + (a^2*p*x^2)/b^2 - (a*p*x)/(2*b))/(3*x^3) - log(c*(a + b/x)^p)/(3*x^3
) - (2*a^3*p*atanh((2*a*x)/b + 1))/(3*b^3)
```

sympy [A] time = 6.43, size = 80, normalized size = 1.10

$$\begin{cases} -\frac{a^{3} p \log\left(a+\frac{b}{x}\right)}{3b^{3}} + \frac{a^{2} p}{3b^{2} x} - \frac{ap}{6bx^{2}} - \frac{p \log\left(a+\frac{b}{x}\right)}{3x^{3}} + \frac{p}{9x^{3}} - \frac{\log\left(c\right)}{3x^{3}} & \text{for}\: b \neq 0 \\ -\frac{\log\left(a^{p} c\right)}{3x^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x**4,x)
```

```
[Out] Piecewise((-a**3*p*log(a + b/x)/(3*b**3) + a**2*p/(3*b**2*x) - a*p/(6*b*x**
2) - p*log(a + b/x)/(3*x**3) + p/(9*x**3) - log(c)/(3*x**3), Ne(b, 0)), (-l
og(a**p*c)/(3*x**3), True))
```

3.35 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5}\,dx$

Optimal. Leaf size=87

$$\frac{a^4p\log\left(a+\frac{b}{x}\right)}{4b^4}-\frac{a^3p}{4b^3x}+\frac{a^2p}{8b^2x^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}-\frac{ap}{12bx^3}+\frac{p}{16x^4}$$

```
[Out] 1/16*p/x^4-1/12*a*p/b/x^3+1/8*a^2*p/b^2/x^2-1/4*a^3*p/b^3/x+1/4*a^4*p*ln(a+
b/x)/b^4-1/4*ln(c*(a+b/x)^p)/x^4
```

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2395, 43}

$$\frac{a^2p}{8b^2x^2}-\frac{a^3p}{4b^3x}+\frac{a^4p\log\left(a+\frac{b}{x}\right)}{4b^4}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}-\frac{ap}{12bx^3}+\frac{p}{16x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/x^5,x]
```

```
[Out] p/(16*x^4) - (a*p)/(12*b*x^3) + (a^2*p)/(8*b^2*x^2) - (a^3*p)/(4*b^3*x) + (
a^4*p*Log[a + b/x])/(4*b^4) - Log[c*(a + b/x)^p]/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5}\,dx = -\text{Subst}\left(\int x^3\log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4} + \frac{1}{4}(bp)\,\text{Subst}\left(\int \frac{x^4}{a+bx}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4} + \frac{1}{4}(bp)\,\text{Subst}\left(\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx, x, \frac{1}{x}\right)$$
$$= \frac{p}{16x^4} - \frac{ap}{12bx^3} + \frac{a^2p}{8b^2x^2} - \frac{a^3p}{4b^3x} + \frac{a^4p\log\left(a+\frac{b}{x}\right)}{4b^4} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}$$

Mathematica [A] time = 0.02, size = 87, normalized size = 1.00

$$\frac{a^4p\log\left(a+\frac{b}{x}\right)}{4b^4} - \frac{a^3p}{4b^3x} + \frac{a^2p}{8b^2x^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4} - \frac{ap}{12bx^3} + \frac{p}{16x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/x^5,x]
```

```
[Out] p/(16*x^4) - (a*p)/(12*b*x^3) + (a^2*p)/(8*b^2*x^2) - (a^3*p)/(4*b^3*x) + (
a^4*p*Log[a + b/x])/(4*b^4) - Log[c*(a + b/x)^p]/(4*x^4)
```

fricas [A] time = 0.47, size = 79, normalized size = 0.91

$$-\frac{12\,a^3bpx^3 - 6\,a^2b^2px^2 + 4\,ab^3px - 3\,b^4p + 12\,b^4\log(c) - 12\left(a^4px^4 - b^4p\right)\log\left(\frac{ax+b}{x}\right)}{48\,b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(12*a^3*b*p*x^3 - 6*a^2*b^2*p*x^2 + 4*a*b^3*p*x - 3*b^4*p + 12*b^4*lo
g(c) - 12*(a^4*p*x^4 - b^4*p)*log((a*x + b)/x))/(b^4*x^4)
```

giac [B] time = 0.20, size = 317, normalized size = 3.64

$$\frac{\frac{48\,(ax+b)a^3p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^3x} - \frac{48\,(ax+b)a^3p}{b^3x} - \frac{72\,(ax+b)^2a^2p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^3x^2} + \frac{48\,(ax+b)a^3\log(c)}{b^3x} + \frac{36\,(ax+b)^2a^2p}{b^3x^2} + \frac{48\,(ax+b)^3ap}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^5,x, algorithm="giac")
```

```
[Out] 1/48*(48*(a*x + b)*a^3*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^3*x) - 48*(
a*x + b)*a^3*p/(b^3*x) - 72*(a*x + b)^2*a^2*p*log(-b*(a/b - (a*x + b)/(b*x)
) + a)/(b^3*x^2) + 48*(a*x + b)*a^3*log(c)/(b^3*x) + 36*(a*x + b)^2*a^2*p/(
b^3*x^2) + 48*(a*x + b)^3*a*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^3*x^3)
 - 72*(a*x + b)^2*a^2*log(c)/(b^3*x^2) - 16*(a*x + b)^3*a*p/(b^3*x^3) - 12*
(a*x + b)^4*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^3*x^4) + 48*(a*x + b)^
3*a*log(c)/(b^3*x^3) + 3*(a*x + b)^4*p/(b^3*x^4) - 12*(a*x + b)^4*log(c)/(b
^3*x^4))/b
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x^5,x)
```

```
[Out] int(ln(c*(a+b/x)^p)/x^5,x)
```

maxima [A] time = 0.70, size = 85, normalized size = 0.98

$$\frac{1}{48}\, bp\left(\frac{12\, a^{4} \log\left(ax+b\right)}{b^{5}} - \frac{12\, a^{4} \log(x)}{b^{5}} - \frac{12\, a^{3} x^{3} - 6\, a^{2} b x^{2} + 4\, a b^{2} x - 3\, b^{3}}{b^{4} x^{4}}\right) - \frac{\log\left(\left(a+\frac{b}{x}\right)^{p} c\right)}{4\, x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^5,x, algorithm="maxima")
```

```
[Out] 1/48*b*p*(12*a^4*log(a*x + b)/b^5 - 12*a^4*log(x)/b^5 - (12*a^3*x^3 - 6*a^2
*b*x^2 + 4*a*b^2*x - 3*b^3)/(b^4*x^4)) - 1/4*log((a + b/x)^p*c)/x^4
```

mupad [B] time = 0.39, size = 78, normalized size = 0.90

$$\frac{\frac{p}{4} + \frac{a^{2}\, p\, x^{2}}{2\, b^{2}} - \frac{a^{3}\, p\, x^{3}}{b^{3}} - \frac{a\, p\, x}{3\, b}}{4\, x^{4}} - \frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{4\, x^{4}} + \frac{a^{4}\, p\, \mathrm{atanh}\left(\frac{2\, a\, x}{b}+1\right)}{2\, b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/x^5,x)
```

```
[Out] (p/4 + (a^2*p*x^2)/(2*b^2) - (a^3*p*x^3)/b^3 - (a*p*x)/(3*b))/(4*x^4) - log
(c*(a + b/x)^p)/(4*x^4) + (a^4*p*atanh((2*a*x)/b + 1))/(2*b^4)
```

sympy [A] time = 10.47, size = 94, normalized size = 1.08

$$\begin{cases} \frac{a^{4} p \log\left(a+\frac{b}{x}\right)}{4 b^{4}} - \frac{a^{3} p}{4 b^{3} x} + \frac{a^{2} p}{8 b^{2} x^{2}} - \frac{a p}{12 b x^{3}} - \frac{p \log\left(a+\frac{b}{x}\right)}{4 x^{4}} + \frac{p}{16 x^{4}} - \frac{\log\left(c\right)}{4 x^{4}} & \text{for}\: b \neq 0 \\ - \frac{\log\left(a^{p} c\right)}{4 x^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x**5,x)
```

```
[Out] Piecewise((a**4*p*log(a + b/x)/(4*b**4) - a**3*p/(4*b**3*x) + a**2*p/(8*b**
2*x**2) - a*p/(12*b*x**3) - p*log(a + b/x)/(4*x**4) + p/(16*x**4) - log(c)/
(4*x**4), Ne(b, 0)), (-log(a**p*c)/(4*x**4), True))
```

3.36 $\int x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx$

Optimal. Leaf size=72

$$\frac{2b^{5/2}p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{5a^{5/2}}-\frac{2b^2px}{5a^2}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{2bpx^3}{15a}$$

```
[Out] -2/5*b^2*p*x/a^2+2/15*b*p*x^3/a+2/5*b^(5/2)*p*arctan(x*a^(1/2)/b^(1/2))/a^(
5/2)+1/5*x^5*ln(c*(a+b/x^2)^p)
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2455, 263, 302, 205}

$$-\frac{2b^2px}{5a^2}+\frac{2b^{5/2}p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{5a^{5/2}}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{2bpx^3}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (-2*b^2*p*x)/(5*a^2) + (2*b*p*x^3)/(15*a) + (2*b^(5/2)*p*ArcTan[(Sqrt[a]*x)
/Sqrt[b]])/(5*a^(5/2)) + (x^5*Log[c*(a + b/x^2)^p])/5
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{1}{5}x^5 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{1}{5}(2bp)\int \frac{x^2}{a+\frac{b}{x^2}} dx$$
$$= \frac{1}{5}x^5 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{1}{5}(2bp)\int \frac{x^4}{b+ax^2} dx$$
$$= \frac{1}{5}x^5 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{1}{5}(2bp)\int \left(-\frac{b}{a^2}+\frac{x^2}{a}+\frac{b^2}{a^2\left(b+ax^2\right)}\right) dx$$
$$= -\frac{2b^2px}{5a^2}+\frac{2bpx^3}{15a}+\frac{1}{5}x^5 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{\left(2b^3p\right)\int \frac{1}{b+ax^2} dx}{5a^2}$$
$$= -\frac{2b^2px}{5a^2}+\frac{2bpx^3}{15a}+\frac{2b^{5/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{5a^{5/2}}+\frac{1}{5}x^5 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.68

$$\frac{1}{5}x^5 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{2bpx^3\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{b}{ax^2}\right)}{15a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (2*b*p*x^3*Hypergeometric2F1[-3/2, 1, -1/2, -(b/(a*x^2))])/(15*a) + (x^5*Lo
g[c*(a + b/x^2)^p])/5
```

fricas [A] time = 0.51, size = 178, normalized size = 2.47

$$\left[\frac{3\,a^2px^5\log\left(\frac{ax^2+b}{x^2}\right)+3\,a^2x^5\log(c)+2\,abpx^3+3\,b^2p\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)-6\,b^2px}{15\,a^2}, \frac{3\,a^2px^5\log\left(\frac{ax^2+b}{x^2}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] [1/15*(3*a^2*p*x^5*log((a*x^2 + b)/x^2) + 3*a^2*x^5*log(c) + 2*a*b*p*x^3 +
3*b^2*p*sqrt(-b/a)*log((a*x^2 + 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)) - 6*b^2*
p*x)/a^2, 1/15*(3*a^2*p*x^5*log((a*x^2 + b)/x^2) + 3*a^2*x^5*log(c) + 2*a*b
*p*x^3 + 6*b^2*p*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b) - 6*b^2*p*x)/a^2]
```

giac [A] time = 0.21, size = 75, normalized size = 1.04

$$\frac{1}{5}\,px^5\log\left(ax^2+b\right)-\frac{1}{5}\,px^5\log\left(x^2\right)+\frac{1}{5}\,x^5\log(c)+\frac{2\,bpx^3}{15\,a}+\frac{2\,b^3p\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{5\,\sqrt{ab}\,a^2}-\frac{2\,b^2px}{5\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] 1/5*p*x^5*log(a*x^2 + b) - 1/5*p*x^5*log(x^2) + 1/5*x^5*log(c) + 2/15*b*p*x
^3/a + 2/5*b^3*p*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 2/5*b^2*p*x/a^2
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int x^4 \ln \left(c \left(a+\frac{b}{x^2}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(a+b/x^2)^p),x)
```

```
[Out] int(x^4*ln(c*(a+b/x^2)^p),x)
```

maxima [A] time = 1.47, size = 59, normalized size = 0.82

$$\frac{1}{5}\,x^5\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)+\frac{2}{15}\,bp\left(\frac{3\,b^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{ax^3-3\,bx}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*log((a + b/x^2)^p*c) + 2/15*b*p*(3*b^2*arctan(a*x/sqrt(a*b))/(sqrt(
a*b)*a^2) + (a*x^3 - 3*b*x)/a^2)
```

mupad [B] time = 0.26, size = 56, normalized size = 0.78

$$\frac{x^5\,\ln\left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)}{5}+\frac{2\,b^{5/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{5\,a^{5/2}}+\frac{2\,b\,p\,x^3}{15\,a}-\frac{2\,b^2\,p\,x}{5\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*log(c*(a + b/x^2)^p),x)
```

```
[Out] (x^5*log(c*(a + b/x^2)^p))/5 + (2*b^(5/2)*p*atan((a^(1/2)*x)/b^(1/2)))/(5*a
^(5/2)) + (2*b*p*x^3)/(15*a) - (2*b^2*p*x)/(5*a^2)
```

sympy [A] time = 92.91, size = 162, normalized size = 2.25

$$\begin{cases} \frac{px^5\log\left(a+\frac{b}{x^2}\right)}{5}+\frac{x^5\log(c)}{5}+\frac{2bpx^3}{15a}-\frac{2b^2px}{5a^2}-\frac{ib^{\frac{5}{2}}p\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+x\right)}{5a^3\sqrt{\frac{1}{a}}}+\frac{ib^{\frac{5}{2}}p\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+x\right)}{5a^3\sqrt{\frac{1}{a}}} & \text{for } a\neq 0 \\ \frac{px^5\log(b)}{5}-\frac{2px^5\log(x)}{5}+\frac{2px^5}{25}+\frac{x^5\log(c)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(a+b/x**2)**p),x)
```

```
[Out] Piecewise((p*x**5*log(a + b/x**2)/5 + x**5*log(c)/5 + 2*b*p*x**3/(15*a) - 2
*b**2*p*x/(5*a**2) - I*b**(5/2)*p*log(-I*sqrt(b)*sqrt(1/a) + x)/(5*a**3*sqr
t(1/a)) + I*b**(5/2)*p*log(I*sqrt(b)*sqrt(1/a) + x)/(5*a**3*sqrt(1/a)), Ne(
a, 0)), (p*x**5*log(b)/5 - 2*p*x**5*log(x)/5 + 2*p*x**5/25 + x**5*log(c)/5,
 True))
```

3.37 $\int x^3 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right) \, dx$

Optimal. Leaf size=51

$$-\frac{b^2 p \log \left(a x^2+b\right)}{4 a^2}+\frac{1}{4} x^4 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)+\frac{b p x^2}{4 a}$$

```
[Out] 1/4*b*p*x^2/a+1/4*x^4*ln(c*(a+b/x^2)^p)-1/4*b^2*p*ln(a*x^2+b)/a^2
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2455, 263, 266, 43}

$$-\frac{b^2 p \log \left(a x^2+b\right)}{4 a^2}+\frac{1}{4} x^4 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)+\frac{b p x^2}{4 a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (b*p*x^2)/(4*a) + (x^4*Log[c*(a + b/x^2)^p])/4 - (b^2*p*Log[b + a*x^2])/(4*
a^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx &= \frac{1}{4}x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{1}{2}(bp)\int \frac{x}{a+\frac{b}{x^2}}\, dx \\
&= \frac{1}{4}x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{1}{2}(bp)\int \frac{x^3}{b+ax^2}\, dx \\
&= \frac{1}{4}x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{1}{4}(bp)\,\text{Subst}\left(\int \frac{x}{b+ax}\, dx, x, x^2\right) \\
&= \frac{1}{4}x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{1}{4}(bp)\,\text{Subst}\left(\int \left(\frac{1}{a}-\frac{b}{a(b+ax)}\right) dx, x, x^2\right) \\
&= \frac{bpx^2}{4a} + \frac{1}{4}x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) - \frac{b^2 p \log\left(b+ax^2\right)}{4a^2}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 56, normalized size = 1.10

$$
\frac{1}{4}bp\left(-\frac{b\log\left(a+\frac{b}{x^2}\right)}{a^2}-\frac{2b\log(x)}{a^2}+\frac{x^2}{a}\right)+\frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (x^4*Log[c*(a + b/x^2)^p])/4 + (b*p*(x^2/a - (b*Log[a + b/x^2])/a^2 - (2*b*
Log[x])/a^2))/4
```

fricas [A] time = 0.45, size = 56, normalized size = 1.10

$$
\frac{a^2 p x^4 \log\left(\frac{ax^2+b}{x^2}\right) + a^2 x^4 \log(c) + abpx^2 - b^2 p \log\left(ax^2+b\right)}{4\,a^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*p*x^4*log((a*x^2 + b)/x^2) + a^2*x^4*log(c) + a*b*p*x^2 - b^2*p*lo
g(a*x^2 + b))/a^2
```

giac [A] time = 0.17, size = 59, normalized size = 1.16

$$
\frac{1}{4}\,px^4\log\left(ax^2+b\right) - \frac{1}{4}\,px^4\log\left(x^2\right) + \frac{1}{4}\,x^4\log(c) + \frac{bpx^2}{4\,a} - \frac{b^2 p \log\left(ax^2+b\right)}{4\,a^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] 1/4*p*x^4*log(a*x^2 + b) - 1/4*p*x^4*log(x^2) + 1/4*x^4*log(c) + 1/4*b*p*x^
2/a - 1/4*b^2*p*log(a*x^2 + b)/a^2
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$
\int x^3 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(a+b/x^2)^p),x)
```

```
[Out] int(x^3*ln(c*(a+b/x^2)^p),x)
```

maxima [A] time = 0.52, size = 44, normalized size = 0.86

$$\frac{1}{4}\,x^4\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)+\frac{1}{4}\,bp\left(\frac{x^2}{a}-\frac{b\log\left(ax^2+b\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((a + b/x^2)^p*c) + 1/4*b*p*(x^2/a - b*log(a*x^2 + b)/a^2)
```

mupad [B] time = 0.24, size = 45, normalized size = 0.88

$$\frac{x^4\,\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{4}-\frac{b^2\,p\,\ln\left(a\,x^2+b\right)}{4\,a^2}+\frac{b\,p\,x^2}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(c*(a + b/x^2)^p),x)
```

```
[Out] (x^4*log(c*(a + b/x^2)^p))/4 - (b^2*p*log(b + a*x^2))/(4*a^2) + (b*p*x^2)/(
4*a)
```

sympy [A] time = 9.55, size = 87, normalized size = 1.71

$$\begin{cases} \frac{px^4\log\left(a+\frac{b}{x^2}\right)}{4}+\frac{x^4\log\left(c\right)}{4}+\frac{bpx^2}{4a}-\frac{b^2p\log\left(ax^2+b\right)}{4a^2} & \text{for}\ a\neq 0 \\ \frac{px^4\log\left(b\right)}{4}-\frac{px^4\log\left(x\right)}{2}+\frac{px^4}{8}+\frac{x^4\log\left(c\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(a+b/x**2)**p),x)
```

```
[Out] Piecewise((p*x**4*log(a + b/x**2)/4 + x**4*log(c)/4 + b*p*x**2/(4*a) - b**2
*p*log(a*x**2 + b)/(4*a**2), Ne(a, 0)), (p*x**4*log(b)/4 - p*x**4*log(x)/2
+ p*x**4/8 + x**4*log(c)/4, True))
```

3.38 $\int x^2 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right) \, dx$

Optimal. Leaf size=58

$$-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3a^{3/2}}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{2bpx}{3a}$$

```
[Out] 2/3*b*p*x/a-2/3*b^(3/2)*p*arctan(x*a^(1/2)/b^(1/2))/a^(3/2)+1/3*x^3*ln(c*(a
+b/x^2)^p)
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2455, 193, 321, 205}

$$-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3a^{3/2}}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{2bpx}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (2*b*p*x)/(3*a) - (2*b^(3/2)*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(3*a^(3/2)) + (
x^3*Log[c*(a + b/x^2)^p])/3
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx &= \frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{3}(2bp)\int\frac{1}{a+\frac{b}{x^2}}\,dx\\ &= \frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{3}(2bp)\int\frac{x^2}{b+ax^2}\,dx\\ &= \frac{2bpx}{3a}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)-\frac{\left(2b^2p\right)\int\frac{1}{b+ax^2}\,dx}{3a}\\ &= \frac{2bpx}{3a}-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3a^{3/2}}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 47, normalized size = 0.81

$$\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{2bpx\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{b}{ax^2}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (2*b*p*x*Hypergeometric2F1[-1/2, 1, 1/2, -(b/(a*x^2))])/(3*a) + (x^3*Log[c*
(a + b/x^2)^p])/3
```

fricas [A] time = 0.47, size = 141, normalized size = 2.43

$$\left[\frac{apx^3\log\left(\frac{ax^2+b}{x^2}\right)+ax^3\log(c)+bp\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2-2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)+2\,bpx}{3\,a},\ \frac{apx^3\log\left(\frac{ax^2+b}{x^2}\right)+ax^3\log(c)-2\,bp\sqrt{\frac{b}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] [1/3*(a*p*x^3*log((a*x^2 + b)/x^2) + a*x^3*log(c) + b*p*sqrt(-b/a)*log((a*x
^2 - 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)) + 2*b*p*x)/a, 1/3*(a*p*x^3*log((a*x
^2 + b)/x^2) + a*x^3*log(c) - 2*b*p*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b) + 2*b
*p*x)/a]
```

giac [A] time = 0.17, size = 63, normalized size = 1.09

$$\frac{1}{3}\,px^3\log\left(ax^2+b\right)-\frac{1}{3}\,px^3\log\left(x^2\right)+\frac{1}{3}\,x^3\log(c)-\frac{2\,b^2p\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,a}+\frac{2\,bpx}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] 1/3*p*x^3*log(a*x^2 + b) - 1/3*p*x^3*log(x^2) + 1/3*x^3*log(c) - 2/3*b^2*p*
arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a) + 2/3*b*p*x/a
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x^2\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(a+b/x^2)^p),x)
```

```
[Out] int(x^2*ln(c*(a+b/x^2)^p),x)
```

maxima [A] time = 1.50, size = 48, normalized size = 0.83

$$\frac{1}{3}\,x^3\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)-\frac{2}{3}\,bp\left(\frac{b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((a + b/x^2)^p*c) - 2/3*b*p*(b*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*
a) - x/a)
```

mupad [B] time = 0.24, size = 44, normalized size = 0.76

$$\frac{x^3\,\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3}-\frac{2\,b^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3\,a^{3/2}}+\frac{2\,b\,p\,x}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(a + b/x^2)^p),x)
```

```
[Out] (x^3*log(c*(a + b/x^2)^p))/3 - (2*b^(3/2)*p*atan((a^(1/2)*x)/b^(1/2)))/(3*a
^(3/2)) + (2*b*p*x)/(3*a)
```

sympy [A] time = 33.14, size = 146, normalized size = 2.52

$$\begin{cases}\frac{px^3\log\left(a+\frac{b}{x^2}\right)}{3}+\frac{x^3\log(c)}{3}+\frac{2bpx}{3a}+\frac{ib^{\frac{3}{2}}p\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+x\right)}{3a^2\sqrt{\frac{1}{a}}}-\frac{ib^{\frac{3}{2}}p\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+x\right)}{3a^2\sqrt{\frac{1}{a}}} & \text{for } a\neq 0\\ \frac{px^3\log(b)}{3}-\frac{2px^3\log(x)}{3}+\frac{2px^3}{9}+\frac{x^3\log(c)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(a+b/x**2)**p),x)
```

```
[Out] Piecewise((p*x**3*log(a + b/x**2)/3 + x**3*log(c)/3 + 2*b*p*x/(3*a) + I*b**
(3/2)*p*log(-I*sqrt(b)*sqrt(1/a) + x)/(3*a**2*sqrt(1/a)) - I*b**(3/2)*p*log
(I*sqrt(b)*sqrt(1/a) + x)/(3*a**2*sqrt(1/a)), Ne(a, 0)), (p*x**3*log(b)/3 -
 2*p*x**3*log(x)/3 + 2*p*x**3/9 + x**3*log(c)/3, True))
```

3.39 $\int x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$

Optimal. Leaf size=37

$$\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bp\log\left(ax^2+b\right)}{2a}$$

```
[Out] 1/2*x^2*ln(c*(a+b/x^2)^p)+1/2*b*p*ln(a*x^2+b)/a
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2455, 263, 260}

$$\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bp\log\left(ax^2+b\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (x^2*Log[c*(a + b/x^2)^p])/2 + (b*p*Log[b + a*x^2])/(2*a)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx &= \frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+(bp)\int\frac{1}{\left(a+\frac{b}{x^2}\right)x}\,dx\\
&= \frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+(bp)\int\frac{x}{b+ax^2}\,dx\\
&= \frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bp\log\left(b+ax^2\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 45, normalized size = 1.22

$$\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bp\log\left(a+\frac{b}{x^2}\right)}{2a}+\frac{bp\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (b*p*Log[a + b/x^2])/(2*a) + (x^2*Log[c*(a + b/x^2)^p])/2 + (b*p*Log[x])/a
```

fricas [A] time = 0.44, size = 42, normalized size = 1.14

$$\frac{apx^2\log\left(\frac{ax^2+b}{x^2}\right) + ax^2\log(c) + bp\log\left(ax^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] 1/2*(a*p*x^2*log((a*x^2 + b)/x^2) + a*x^2*log(c) + b*p*log(a*x^2 + b))/a
```

giac [A] time = 0.17, size = 47, normalized size = 1.27

$$\frac{1}{2}\,px^2\log\left(ax^2+b\right) - \frac{1}{2}\,px^2\log\left(x^2\right) + \frac{1}{2}\,x^2\log(c) + \frac{bp\log\left(ax^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] 1/2*p*x^2*log(a*x^2 + b) - 1/2*p*x^2*log(x^2) + 1/2*x^2*log(c) + 1/2*b*p*lo
g(a*x^2 + b)/a
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(a+b/x^2)^p),x)
```

```
[Out] int(x*ln(c*(a+b/x^2)^p),x)
```

maxima [A] time = 0.59, size = 33, normalized size = 0.89

$$\frac{1}{2}\,x^2\log\left(\left(a+\frac{b}{x^2}\right)^p c\right) + \frac{bp\log\left(ax^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log((a + b/x^2)^p*c) + 1/2*b*p*log(a*x^2 + b)/a
```

mupad [B] time = 0.21, size = 33, normalized size = 0.89

$$\frac{x^2\,\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2} + \frac{b\,p\,\ln\left(a\,x^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(a + b/x^2)^p),x)
```

```
[Out] (x^2*log(c*(a + b/x^2)^p))/2 + (b*p*log(b + a*x^2))/(2*a)
```

<u>**sympy**</u> [**A**] time = 3.35, size = 71, normalized size = 1.92

$$\begin{cases} \frac{px^2\log\left(a+\frac{b}{x^2}\right)}{2} + \frac{x^2\log(c)}{2} + \frac{bp\log(ax^2+b)}{2a} & \text{for } a \neq 0 \\ \frac{px^2\log(b)}{2} - px^2\log(x) + \frac{px^2}{2} + \frac{x^2\log(c)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(a+b/x**2)**p),x)
```

```
[Out] Piecewise((p*x**2*log(a + b/x**2)/2 + x**2*log(c)/2 + b*p*log(a*x**2 + b)/(
2*a), Ne(a, 0)), (p*x**2*log(b)/2 - p*x**2*log(x) + p*x**2/2 + x**2*log(c)/
2, True))
```

3.40 $\int \log \left(c \left(a+\frac{b}{x^2}\right)^p\right) \, dx$

Optimal. Leaf size=41

$$x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)+\frac{2 \sqrt{b} p \tan ^{-1}\left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{\sqrt{a}}$$

```
[Out] x*ln(c*(a+b/x^2)^p)+2*p*arctan(x*a^(1/2)/b^(1/2))*b^(1/2)/a^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2448, 263, 205}

$$x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)+\frac{2 \sqrt{b} p \tan ^{-1}\left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p],x]
```

```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/Sqrt[a] + x*Log[c*(a + b/x^2)^p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \log \left(c \left(a+\frac{b}{x^2}\right)^p\right) \, dx &= x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)+(2 b p) \int \frac{1}{\left(a+\frac{b}{x^2}\right) x^2} \, dx\\
&= x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)+(2 b p) \int \frac{1}{b+a x^2} \, dx\\
&= \frac{2 \sqrt{b} p \tan ^{-1}\left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{\sqrt{a}}+x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.05

$$x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)-\frac{2 \sqrt{b} p \tan ^{-1}\left(\frac{\sqrt{b}}{\sqrt{a} x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p],x]
```

```
[Out] (-2*Sqrt[b]*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[a] + x*Log[c*(a + b/x^2)^p]
```

fricas [A] time = 0.46, size = 107, normalized size = 2.61

$$\left[px\log\left(\frac{ax^2+b}{x^2}\right)+p\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)+x\log(c),px\log\left(\frac{ax^2+b}{x^2}\right)+2\,p\sqrt{\frac{b}{a}}\arctan\left(\frac{ax\sqrt{\frac{b}{a}}}{b}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] [p*x*log((a*x^2 + b)/x^2) + p*sqrt(-b/a)*log((a*x^2 + 2*a*x*sqrt(-b/a) - b)
/(a*x^2 + b)) + x*log(c), p*x*log((a*x^2 + b)/x^2) + 2*p*sqrt(b/a)*arctan(a
*x*sqrt(b/a)/b) + x*log(c)]
```

giac [A] time = 0.19, size = 42, normalized size = 1.02

$$px\log\left(ax^2+b\right)-px\log\left(x^2\right)+\frac{2\,bp\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}+x\log(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] p*x*log(a*x^2 + b) - p*x*log(x^2) + 2*b*p*arctan(a*x/sqrt(a*b))/sqrt(a*b) +
 x*log(c)
```

maple [A] time = 0.05, size = 38, normalized size = 0.93

$$\frac{2bp\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}+x\ln\left(c\left(\frac{a\,x^2+b}{x^2}\right)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p),x)
```

```
[Out] x*ln(c*((a*x^2+b)/x^2)^p)+2*b*p/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2))
```

maxima [A] time = 1.85, size = 33, normalized size = 0.80

$$\frac{2\,bp\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}+x\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] 2*b*p*arctan(a*x/sqrt(a*b))/sqrt(a*b) + x*log((a + b/x^2)^p*c)
```

mupad [B] time = 0.11, size = 33, normalized size = 0.80

$$x\,\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{2\,\sqrt{b}\,p\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p),x)
```

```
[Out] x*log(c*(a + b/x^2)^p) + (2*b^(1/2)*p*atan((a^(1/2)*x)/b^(1/2)))/a^(1/2)
```

sympy [A] time = 11.32, size = 109, normalized size = 2.66

$$\begin{cases} px\log\left(a+\frac{b}{x^2}\right)+x\log(c)-\frac{i\sqrt{b}p\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+x\right)}{a\sqrt{\frac{1}{a}}}+\frac{i\sqrt{b}p\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+x\right)}{a\sqrt{\frac{1}{a}}} & \text{for } a\neq 0 \\ px\log(b)-2px\log(x)+2px+x\log(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p),x)
```

```
[Out] Piecewise((p*x*log(a + b/x**2) + x*log(c) - I*sqrt(b)*p*log(-I*sqrt(b)*sqrt
(1/a) + x)/(a*sqrt(1/a)) + I*sqrt(b)*p*log(I*sqrt(b)*sqrt(1/a) + x)/(a*sqrt
(1/a)), Ne(a, 0)), (p*x*log(b) - 2*p*x*log(x) + 2*p*x + x*log(c), True))
```

3.41 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx$

Optimal. Leaf size=44

$$-\frac{1}{2}\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)-\frac{1}{2}p\text{Li}_2\left(\frac{b}{ax^2}+1\right)$$

```
[Out] -1/2*ln(c*(a+b/x^2)^p)*ln(-b/a/x^2)-1/2*p*polylog(2,1+b/a/x^2)
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2394, 2315}

$$-\frac{1}{2}p\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)-\frac{1}{2}\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/x,x]
```

```
[Out] -(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/2 - (p*PolyLog[2, 1 + b/(a*x^2)])
/2
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x}\,dx,x,\frac{1}{x^2}\right)\right)\\
&= -\frac{1}{2}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)+\frac{1}{2}(bp)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx,x,\frac{1}{x^2}\right)\\
&= -\frac{1}{2}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)-\frac{1}{2}p\text{Li}_2\left(1+\frac{b}{ax^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 45, normalized size = 1.02

$$-\frac{1}{2}\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)-\frac{1}{2}p\text{Li}_2\left(\frac{a+\frac{b}{x^2}}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/x,x]
[Out] -1/2*(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))]) - (p*PolyLog[2, (a + b/x^2)/a
])/2
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax^2+b}{x^2}\right)^p\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x,x, algorithm="fricas")
[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x,x, algorithm="giac")
[Out] integrate(log((a + b/x^2)^p*c)/x, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x,x)
[Out] int(ln(c*(a+b/x^2)^p)/x,x)
```

maxima [B] time = 0.63, size = 89, normalized size = 2.02

$$\frac{1}{2}bp\left(\frac{2\log\left(a+\frac{b}{x^2}\right)\log(x)}{b}+\frac{2\log(x)^2}{b}-\frac{2\log\left(\frac{ax^2}{b}+1\right)\log(x)+\text{Li}_2\left(-\frac{ax^2}{b}\right)}{b}\right)-p\log\left(a+\frac{b}{x^2}\right)\log(x)+\log\left(\left(a+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x,x, algorithm="maxima")
[Out] 1/2*b*p*(2*log(a + b/x^2)*log(x)/b + 2*log(x)^2/b - (2*log(a*x^2/b + 1)*log
(x) + dilog(-a*x^2/b))/b) - p*log(a + b/x^2)*log(x) + log((a + b/x^2)^p*c)*
log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p)/x,x)
```

```
[Out] int(log(c*(a + b/x^2)^p)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x,x)
```

```
[Out] Integral(log(c*(a + b/x**2)**p)/x, x)
```

3.42 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=50

$$-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}+\frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{2p}{x}$$

```
[Out] 2*p/x-ln(c*(a+b/x^2)^p)/x+2*p*arctan(x*a^(1/2)/b^(1/2))*a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2455, 263, 325, 205}

$$-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}+\frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{2p}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/x^2,x]
```

```
[Out] (2*p)/x + (2*Sqrt[a]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/Sqrt[b] - Log[c*(a + b/
x^2)^p]/x
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx = -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x} - (2bp)\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^4}\,dx$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x} - (2bp)\int \frac{1}{x^2\left(b+ax^2\right)}\,dx$$
$$= \frac{2p}{x} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x} + (2ap)\int \frac{1}{b+ax^2}\,dx$$
$$= \frac{2p}{x} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 1.04

$$-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x} - \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{\sqrt{b}} + \frac{2p}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/x^2,x]
```

```
[Out] (2*p)/x - (2*Sqrt[a]*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[b] - Log[c*(a + b/
x^2)^p]/x
```

fricas [A] time = 0.44, size = 119, normalized size = 2.38

$$\left[\frac{px\sqrt{-\frac{a}{b}}\log\left(\frac{ax^2+2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right) - p\log\left(\frac{ax^2+b}{x^2}\right) + 2\,p - \log(c)}{x}, \frac{2\,px\sqrt{\frac{a}{b}}\arctan\left(x\sqrt{\frac{a}{b}}\right) - p\log\left(\frac{ax^2+b}{x^2}\right) + 2\,p - \log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2,x, algorithm="fricas")
```

```
[Out] [(p*x*sqrt(-a/b)*log((a*x^2 + 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b)) - p*log((a
*x^2 + b)/x^2) + 2*p - log(c))/x, (2*p*x*sqrt(a/b)*arctan(x*sqrt(a/b)) - p*
log((a*x^2 + b)/x^2) + 2*p - log(c))/x]
```

giac [A] time = 0.17, size = 54, normalized size = 1.08

$$\frac{2\,ap\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}} - \frac{p\log\left(ax^2+b\right)}{x} + \frac{p\log\left(x^2\right)}{x} + \frac{2\,p - \log(c)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2,x, algorithm="giac")
```

```
[Out] 2*a*p*arctan(a*x/sqrt(a*b))/sqrt(a*b) - p*log(a*x^2 + b)/x + p*log(x^2)/x +
 (2*p - log(c))/x
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x^2,x)
```

```
[Out] int(ln(c*(a+b/x^2)^p)/x^2,x)
```

maxima [A] time = 1.41, size = 49, normalized size = 0.98

$$2\,bp\left(\frac{a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{1}{bx}\right)-\frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2,x, algorithm="maxima")
```

```
[Out] 2*b*p*(a*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/(b*x)) - log((a + b/x^2)^p
*c)/x
```

mupad [B] time = 0.24, size = 42, normalized size = 0.84

$$\frac{2\,p}{x}-\frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}+\frac{2\,\sqrt{a}\,p\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p)/x^2,x)
```

```
[Out] (2*p)/x - log(c*(a + b/x^2)^p)/x + (2*a^(1/2)*p*atan((a^(1/2)*x)/b^(1/2)))/
b^(1/2)
```

sympy [A] time = 25.45, size = 129, normalized size = 2.58

$$\begin{cases} -\frac{\log\left(0^p c\right)}{x} & \text{for}\ a=0 \wedge b=0 \\ -\frac{\log\left(a^p c\right)}{x} & \text{for}\ b=0 \\ -\frac{p\log\left(b\right)}{x}+\frac{2p\log\left(x\right)}{x}+\frac{2p}{x}-\frac{\log\left(c\right)}{x} & \text{for}\ a=0 \\ -\frac{p\log\left(a+\frac{b}{x^2}\right)}{x}+\frac{2p}{x}-\frac{\log\left(c\right)}{x}-\frac{ip\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+x\right)}{\sqrt{b}\sqrt{\frac{1}{a}}}+\frac{ip\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+x\right)}{\sqrt{b}\sqrt{\frac{1}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**2,x)
```

```
[Out] Piecewise((-log(0**p*c)/x, Eq(a, 0) & Eq(b, 0)), (-log(a**p*c)/x, Eq(b, 0))
, (-p*log(b)/x + 2*p*log(x)/x + 2*p/x - log(c)/x, Eq(a, 0)), (-p*log(a + b/
x**2)/x + 2*p/x - log(c)/x - I*p*log(-I*sqrt(b)*sqrt(1/a) + x)/(sqrt(b)*sqr
t(1/a)) + I*p*log(I*sqrt(b)*sqrt(1/a) + x)/(sqrt(b)*sqrt(1/a)), True))
```

3.43 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx$

Optimal. Leaf size=35

$$\frac{p}{2x^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2b}$$

```
[Out] 1/2*p/x^2-1/2*(a+b/x^2)*ln(c*(a+b/x^2)^p)/b
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2389, 2295}

$$\frac{p}{2x^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/x^3,x]
```

```
[Out] p/(2*x^2) - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/(2*b)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx,x,\frac{1}{x^2}\right)\right)\\
&= -\frac{\text{Subst}\left(\int \log\left(cx^p\right)\,dx,x,a+\frac{b}{x^2}\right)}{2b}\\
&= \frac{p}{2x^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.97

$$\frac{1}{2}\left(\frac{p}{x^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/x^3,x]
```

```
[Out] (p/x^2 - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/b)/2
```

fricas [A] time = 0.43, size = 41, normalized size = 1.17

$$\frac{bp-b\log(c)-\left(apx^2+bp\right)\log\left(\frac{ax^2+b}{x^2}\right)}{2\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*p - b*log(c) - (a*p*x^2 + b*p)*log((a*x^2 + b)/x^2))/(b*x^2)
```

giac [B] time = 0.17, size = 65, normalized size = 1.86

$$-\frac{ap\log\left(ax^2+b\right)}{2\,b}+\frac{ap\log(x)}{b}-\frac{p\log\left(ax^2+b\right)}{2\,x^2}+\frac{p\log\left(x^2\right)}{2\,x^2}+\frac{bp-b\log(c)}{2\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*a*p*log(a*x^2 + b)/b + a*p*log(x)/b - 1/2*p*log(a*x^2 + b)/x^2 + 1/2*p
*log(x^2)/x^2 + 1/2*(b*p - b*log(c))/(b*x^2)
```

maple [A] time = 0.05, size = 50, normalized size = 1.43

$$\frac{ap}{2b}-\frac{a\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2b}+\frac{p}{2x^2}-\frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x^3,x)
```

```
[Out] -1/2/b*ln(c*(a+b/x^2)^p)*a-1/2/x^2*ln(c*(a+b/x^2)^p)+1/2*a/b*p+1/2*p/x^2
```

maxima [A] time = 0.65, size = 54, normalized size = 1.54

$$-\frac{1}{2}\,bp\left(\frac{a\log\left(ax^2+b\right)}{b^2}-\frac{a\log\left(x^2\right)}{b^2}-\frac{1}{bx^2}\right)-\frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*p*(a*log(a*x^2 + b)/b^2 - a*log(x^2)/b^2 - 1/(b*x^2)) - 1/2*log((a +
 b/x^2)^p*c)/x^2
```

mupad [B] time = 0.24, size = 47, normalized size = 1.34

$$\frac{p}{2\,x^2}-\frac{\ln\left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)}{2\,x^2}-\frac{a\,p\,\ln\left(a\,x^2+b\right)}{2\,b}+\frac{a\,p\,\ln(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p)/x^3,x)
```

```
[Out] p/(2*x^2) - log(c*(a + b/x^2)^p)/(2*x^2) - (a*p*log(b + a*x^2))/(2*b) + (a*
p*log(x))/b
```

sympy [A] time = 5.65, size = 58, normalized size = 1.66

$$\begin{cases} -\frac{ap\log\left(a+\frac{b}{x^2}\right)}{2b}-\frac{p\log\left(a+\frac{b}{x^2}\right)}{2x^2}+\frac{p}{2x^2}-\frac{\log(c)}{2x^2} & \text{for } b\neq 0 \\ -\frac{\log\left(a^pc\right)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**3,x)
```

```
[Out] Piecewise((-a*p*log(a + b/x**2)/(2*b) - p*log(a + b/x**2)/(2*x**2) + p/(2*x
**2) - log(c)/(2*x**2), Ne(b, 0)), (-log(a**p*c)/(2*x**2), True))
```

3.44 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=68

$$-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3b^{3/2}}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}-\frac{2ap}{3bx}+\frac{2p}{9x^3}$$

```
[Out] 2/9*p/x^3-2/3*a*p/b/x-2/3*a^(3/2)*p*arctan(x*a^(1/2)/b^(1/2))/b^(3/2)-1/3*l
n(c*(a+b/x^2)^p)/x^3
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2455, 263, 325, 205}

$$-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3b^{3/2}}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}-\frac{2ap}{3bx}+\frac{2p}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/x^4,x]
```

```
[Out] (2*p)/(9*x^3) - (2*a*p)/(3*b*x) - (2*a^(3/2)*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])
/(3*b^(3/2)) - Log[c*(a + b/x^2)^p]/(3*x^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx = -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3} - \frac{1}{3}(2bp)\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^6}\,dx$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3} - \frac{1}{3}(2bp)\int \frac{1}{x^4\left(b+ax^2\right)}\,dx$$
$$= \frac{2p}{9x^3} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3} + \frac{1}{3}(2ap)\int \frac{1}{x^2\left(b+ax^2\right)}\,dx$$
$$= \frac{2p}{9x^3} - \frac{2ap}{3bx} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3} - \frac{\left(2a^2p\right)\int \frac{1}{b+ax^2}\,dx}{3b}$$
$$= \frac{2p}{9x^3} - \frac{2ap}{3bx} - \frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3b^{3/2}} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.03

$$\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{3b^{3/2}} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3} - \frac{2ap}{3bx} + \frac{2p}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/x^4,x]
```

```
[Out] (2*p)/(9*x^3) - (2*a*p)/(3*b*x) + (2*a^(3/2)*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])
/(3*b^(3/2)) - Log[c*(a + b/x^2)^p]/(3*x^3)
```

fricas [A] time = 0.47, size = 154, normalized size = 2.26

$$\left[\frac{3\,apx^3\sqrt{-\frac{a}{b}}\,\log\left(\frac{ax^2-2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right) - 6\,apx^2 - 3\,bp\log\left(\frac{ax^2+b}{x^2}\right) + 2\,bp - 3\,b\log(c)}{9\,bx^3}, -\frac{6\,apx^3\sqrt{\frac{a}{b}}\,\arctan\left(x\sqrt{\frac{a}{b}}\right) + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^4,x, algorithm="fricas")
```

```
[Out] [1/9*(3*a*p*x^3*sqrt(-a/b)*log((a*x^2 - 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b))
- 6*a*p*x^2 - 3*b*p*log((a*x^2 + b)/x^2) + 2*b*p - 3*b*log(c))/(b*x^3), -1/
9*(6*a*p*x^3*sqrt(a/b)*arctan(x*sqrt(a/b)) + 6*a*p*x^2 + 3*b*p*log((a*x^2 +
 b)/x^2) - 2*b*p + 3*b*log(c))/(b*x^3)]
```

giac [A] time = 0.19, size = 73, normalized size = 1.07

$$-\frac{2\,a^2p\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b} - \frac{p\log\left(ax^2+b\right)}{3\,x^3} + \frac{p\log\left(x^2\right)}{3\,x^3} - \frac{6\,apx^2 - 2\,bp + 3\,b\log(c)}{9\,bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^4,x, algorithm="giac")
```

```
[Out] -2/3*a^2*p*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/3*p*log(a*x^2 + b)/x^3 +
 1/3*p*log(x^2)/x^3 - 1/9*(6*a*p*x^2 - 2*b*p + 3*b*log(c))/(b*x^3)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x^4,x)
```

```
[Out] int(ln(c*(a+b/x^2)^p)/x^4,x)
```

maxima [A] time = 1.58, size = 62, normalized size = 0.91

$$-\frac{2}{9}\,bp\left(\frac{3\,a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{3\,ax^2-b}{b^2x^3}\right)-\frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -2/9*b*p*(3*a^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^2) + (3*a*x^2 - b)/(b^2*
x^3)) - 1/3*log((a + b/x^2)^p*c)/x^3
```

mupad [B] time = 0.25, size = 55, normalized size = 0.81

$$\frac{\frac{2\,p}{3}-\frac{2\,a\,p\,x^2}{b}}{3\,x^3}-\frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3\,x^3}-\frac{2\,a^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p)/x^4,x)
```

```
[Out] ((2*p)/3 - (2*a*p*x^2)/b)/(3*x^3) - log(c*(a + b/x^2)^p)/(3*x^3) - (2*a^(3/
2)*p*atan((a^(1/2)*x)/b^(1/2)))/(3*b^(3/2))
```

sympy [A] time = 72.06, size = 177, normalized size = 2.60

$$\begin{cases} -\frac{\log\left(0^p c\right)}{3x^3} & \text{for}\: a=0 \wedge b=0 \\ -\frac{\log\left(a^p c\right)}{3x^3} & \text{for}\: b=0 \\ -\frac{p\log\left(b\right)}{3x^3}+\frac{2p\log\left(x\right)}{3x^3}+\frac{2p}{9x^3}-\frac{\log\left(c\right)}{3x^3} & \text{for}\: a=0 \\ -\frac{2ap}{3bx}+\frac{iap\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+x\right)}{3b^{\frac{3}{2}}\sqrt{\frac{1}{a}}}-\frac{iap\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+x\right)}{3b^{\frac{3}{2}}\sqrt{\frac{1}{a}}}-\frac{p\log\left(a+\frac{b}{x^2}\right)}{3x^3}+\frac{2p}{9x^3}-\frac{\log\left(c\right)}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**4,x)
```

```
[Out] Piecewise((-log(0**p*c)/(3*x**3), Eq(a, 0) & Eq(b, 0)), (-log(a**p*c)/(3*x*
*3), Eq(b, 0)), (-p*log(b)/(3*x**3) + 2*p*log(x)/(3*x**3) + 2*p/(9*x**3) -
log(c)/(3*x**3), Eq(a, 0)), (-2*a*p/(3*b*x) + I*a*p*log(-I*sqrt(b)*sqrt(1/a
) + x)/(3*b**(3/2)*sqrt(1/a)) - I*a*p*log(I*sqrt(b)*sqrt(1/a) + x)/(3*b**(3
/2)*sqrt(1/a)) - p*log(a + b/x**2)/(3*x**3) + 2*p/(9*x**3) - log(c)/(3*x**3
), True))
```

3.45 $\int \frac{\log\left(1+\frac{b}{x}\right)}{x}\,dx$

Optimal. Leaf size=8

$$\mathrm{Li}_2\left(-\frac{b}{x}\right)$$

```
[Out] polylog(2,-b/x)
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2391}

$$\mathrm{PolyLog}\left(2,-\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + b/x]/x,x]
[Out] PolyLog[2, -(b/x)]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(1+\frac{b}{x}\right)}{x}\,dx = \mathrm{Li}_2\left(-\frac{b}{x}\right)$$

Mathematica [B] time = 0.00, size = 34, normalized size = 4.25

$$-\mathrm{Li}_2\left(-\frac{-b-x}{x}\right)-\log\left(-\frac{b}{x}\right)\log\left(\frac{b+x}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + b/x]/x,x]
[Out] -(Log[-(b/x)]*Log[(b + x)/x]) - PolyLog[2, -((-b - x)/x)]
```

fricas [A] time = 0.45, size = 11, normalized size = 1.38

$$\mathrm{Li}_2\left(-\frac{b+x}{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+b/x)/x,x, algorithm="fricas")
[Out] dilog(-(b + x)/x + 1)
```

giac [B] time = 0.35, size = 110, normalized size = 13.75

$$-\frac{b^3\left(\frac{1}{\frac{b+x}{x}-1}-\log\left(\frac{|b+x|}{|x|}\right)+\log\left(\left|\frac{b+x}{x}-1\right|\right)\right)+\frac{b^3\log\left(-b\left(\frac{\left(b-\frac{1}{\frac{1}{b}-\frac{b+x}{bx}}\right)\left(\frac{1}{b}-\frac{b+x}{bx}\right)}{b}+\frac{1}{b}\right)+1\right)}{\left(\frac{b+x}{x}-1\right)^2}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+b/x)/x,x, algorithm="giac")
```

```
[Out] -1/2*(b^3*(1/((b + x)/x - 1) - log(abs(b + x)/abs(x)) + log(abs((b + x)/x -
 1))) + b^3*log(-b*((b - 1/(1/b - (b + x)/(b*x)))*(1/b - (b + x)/(b*x))/b +
 1/b) + 1)/((b + x)/x - 1)^2)/b^2
```

maple [A] time = 0.05, size = 9, normalized size = 1.12

$$\operatorname{dilog}\left(\frac{b}{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+b/x)/x,x)
```

```
[Out] dilog(1+b/x)
```

maxima [B] time = 0.71, size = 35, normalized size = 4.38

$$\log\left(b+x\right)\log(x)-\frac{1}{2}\log(x)^2-\log(x)\log\left(\frac{x}{b}+1\right)-\operatorname{Li}_2\left(-\frac{x}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+b/x)/x,x, algorithm="maxima")
```

```
[Out] log(b + x)*log(x) - 1/2*log(x)^2 - log(x)*log(x/b + 1) - dilog(-x/b)
```

mupad [B] time = 0.26, size = 8, normalized size = 1.00

$$\operatorname{polylog}\left(2,-\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(b/x + 1)/x,x)
```

```
[Out] polylog(2, -b/x)
```

sympy [C] time = 2.78, size = 8, normalized size = 1.00

$$\operatorname{Li}_2\left(\frac{be^{i\pi}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+b/x)/x,x)
```

```
[Out] polylog(2, b*exp_polar(I*pi)/x)
```

3.46 $\int x^3 \log\left(c\left(a+b\sqrt{x}\right)^p\right)dx$

Optimal. Leaf size=153

$$-\frac{a^8p\log\left(a+b\sqrt{x}\right)}{4b^8}+\frac{a^7p\sqrt{x}}{4b^7}-\frac{a^6px}{8b^6}+\frac{a^5px^{3/2}}{12b^5}-\frac{a^4px^2}{16b^4}+\frac{a^3px^{5/2}}{20b^3}-\frac{a^2px^3}{24b^2}+\frac{1}{4}x^4\log\left(c\left(a+b\sqrt{x}\right)^p\right)+\frac{apx^{7/2}}{28b}-\frac{px^4}{32}$$

```
[Out] -1/8*a^6*p*x/b^6+1/12*a^5*p*x^(3/2)/b^5-1/16*a^4*p*x^2/b^4+1/20*a^3*p*x^(5/
2)/b^3-1/24*a^2*p*x^3/b^2+1/28*a*p*x^(7/2)/b-1/32*p*x^4-1/4*a^8*p*ln(a+b*x^
(1/2))/b^8+1/4*x^4*ln(c*(a+b*x^(1/2))^p)+1/4*a^7*p*x^(1/2)/b^7
```

Rubi [A] time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2454, 2395, 43}

$$\frac{a^5px^{3/2}}{12b^5}-\frac{a^4px^2}{16b^4}+\frac{a^3px^{5/2}}{20b^3}-\frac{a^2px^3}{24b^2}+\frac{a^7p\sqrt{x}}{4b^7}-\frac{a^6px}{8b^6}-\frac{a^8p\log\left(a+b\sqrt{x}\right)}{4b^8}+\frac{1}{4}x^4\log\left(c\left(a+b\sqrt{x}\right)^p\right)+\frac{apx^{7/2}}{28b}-\frac{px^4}{32}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (a^7*p*Sqrt[x])/(4*b^7) - (a^6*p*x)/(8*b^6) + (a^5*p*x^(3/2))/(12*b^5) - (a
^4*p*x^2)/(16*b^4) + (a^3*p*x^(5/2))/(20*b^3) - (a^2*p*x^3)/(24*b^2) + (a*p
*x^(7/2))/(28*b) - (p*x^4)/32 - (a^8*p*Log[a + b*Sqrt[x]])/(4*b^8) + (x^4*L
og[c*(a + b*Sqrt[x])^p])/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3 \log\left(c\left(a+b\sqrt{x}\right)^p\right) dx = 2\,\text{Subst}\left(\int x^7 \log\left(c(a+bx)^p\right) dx, x, \sqrt{x}\right)$$
$$= \frac{1}{4}x^4 \log\left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{1}{4}(bp)\,\text{Subst}\left(\int \frac{x^8}{a+bx}\,dx, x, \sqrt{x}\right)$$
$$= \frac{1}{4}x^4 \log\left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{1}{4}(bp)\,\text{Subst}\left(\int \left(-\frac{a^7}{b^8}+\frac{a^6x}{b^7}-\frac{a^5x^2}{b^6}+\frac{a^4x^3}{b^5}-\frac{a^3x^4}{b^4}+ \cdots\right.\right.$$
$$= \frac{a^7p\sqrt{x}}{4b^7} - \frac{a^6px}{8b^6} + \frac{a^5px^{3/2}}{12b^5} - \frac{a^4px^2}{16b^4} + \frac{a^3px^{5/2}}{20b^3} - \frac{a^2px^3}{24b^2} + \frac{apx^{7/2}}{28b} - \frac{px^4}{32} - \frac{a^8p\log(\cdots}{4\cdots}$$

Mathematica [A] time = 0.14, size = 134, normalized size = 0.88

$$\frac{1}{4}\left(x^4\log\left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{p\left(840a^8\log\left(a+b\sqrt{x}\right) - 840a^7b\sqrt{x} + 420a^6b^2x - 280a^5b^3x^{3/2} + 210a^4b^4x^2 - 168a^3\cdots\right.}{840b^8}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (-1/840*(p*(-840*a^7*b*Sqrt[x] + 420*a^6*b^2*x - 280*a^5*b^3*x^(3/2) + 210*
a^4*b^4*x^2 - 168*a^3*b^5*x^(5/2) + 140*a^2*b^6*x^3 - 120*a*b^7*x^(7/2) + 1
05*b^8*x^4 + 840*a^8*Log[a + b*Sqrt[x]]))/b^8 + x^4*Log[c*(a + b*Sqrt[x])^p
])/4
```

fricas [A] time = 0.46, size = 129, normalized size = 0.84

$$-\frac{105\,b^8px^4 - 840\,b^8x^4\log(c) + 140\,a^2b^6px^3 + 210\,a^4b^4px^2 + 420\,a^6b^2px - 840\left(b^8px^4 - a^8p\right)\log\left(b\sqrt{x}+a\right) - 8\left(\cdots\right.}{3360\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] -1/3360*(105*b^8*p*x^4 - 840*b^8*x^4*log(c) + 140*a^2*b^6*p*x^3 + 210*a^4*b
^4*p*x^2 + 420*a^6*b^2*p*x - 840*(b^8*p*x^4 - a^8*p)*log(b*sqrt(x) + a) - 8
*(15*a*b^7*p*x^3 + 21*a^3*b^5*p*x^2 + 35*a^5*b^3*p*x + 105*a^7*b*p)*sqrt(x)
)/b^8
```

giac [B] time = 0.22, size = 339, normalized size = 2.22

$$840\,bx^4\log(c) + \left(\frac{840\left(b\sqrt{x}+a\right)^8\log\left(b\sqrt{x}+a\right)}{b^7} - \frac{6720\left(b\sqrt{x}+a\right)^7a\log\left(b\sqrt{x}+a\right)}{b^7} + \frac{23520\left(b\sqrt{x}+a\right)^6a^2\log\left(b\sqrt{x}+a\right)}{b^7} - \frac{47040\left(b\sqrt{x}+a\right)^5a^3\log\cdots}{b^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] 1/3360*(840*b*x^4*log(c) + (840*(b*sqrt(x) + a)^8*log(b*sqrt(x) + a)/b^7 -
6720*(b*sqrt(x) + a)^7*a*log(b*sqrt(x) + a)/b^7 + 23520*(b*sqrt(x) + a)^6*a
^2*log(b*sqrt(x) + a)/b^7 - 47040*(b*sqrt(x) + a)^5*a^3*log(b*sqrt(x) + a)/
b^7 + 58800*(b*sqrt(x) + a)^4*a^4*log(b*sqrt(x) + a)/b^7 - 47040*(b*sqrt(x)
 + a)^3*a^5*log(b*sqrt(x) + a)/b^7 + 23520*(b*sqrt(x) + a)^2*a^6*log(b*sqrt
(x) + a)/b^7 - 6720*(b*sqrt(x) + a)*a^7*log(b*sqrt(x) + a)/b^7 - 105*(b*sqr
t(x) + a)^8/b^7 + 960*(b*sqrt(x) + a)^7*a/b^7 - 3920*(b*sqrt(x) + a)^6*a^2/
b^7 + 9408*(b*sqrt(x) + a)^5*a^3/b^7 - 14700*(b*sqrt(x) + a)^4*a^4/b^7 + 15
680*(b*sqrt(x) + a)^3*a^5/b^7 - 11760*(b*sqrt(x) + a)^2*a^6/b^7 + 6720*(b*s
qrt(x) + a)*a^7/b^7)*p)/b
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int x^3 \ln\left(c\left(b\sqrt{x}+a\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(a+b*x^(1/2))^p),x)
```

```
[Out] int(x^3*ln(c*(a+b*x^(1/2))^p),x)
```

maxima [A] time = 0.63, size = 120, normalized size = 0.78

$$\frac{1}{4}x^4\log\left(\left(b\sqrt{x}+a\right)^p c\right)-\frac{1}{3360}bp\left(\frac{840\,a^8\log\left(b\sqrt{x}+a\right)}{b^9}+\frac{105\,b^7x^4-120\,ab^6x^{\frac{7}{2}}+140\,a^2b^5x^3-168\,a^3b^4x^{\frac{5}{2}}+\dots}{b^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((b*sqrt(x) + a)^p*c) - 1/3360*b*p*(840*a^8*log(b*sqrt(x) + a)/b
^9 + (105*b^7*x^4 - 120*a*b^6*x^(7/2) + 140*a^2*b^5*x^3 - 168*a^3*b^4*x^(5/
2) + 210*a^4*b^3*x^2 - 280*a^5*b^2*x^(3/2) + 420*a^6*b*x - 840*a^7*sqrt(x))
/b^8)
```

mupad [B] time = 0.33, size = 121, normalized size = 0.79

$$\frac{x^4\ln\left(c\left(a+b\sqrt{x}\right)^p\right)}{4}-\frac{p\,x^4}{32}-\frac{a^8\,p\,\ln\left(a+b\sqrt{x}\right)}{4\,b^8}-\frac{a^2\,p\,x^3}{24\,b^2}-\frac{a^4\,p\,x^2}{16\,b^4}+\frac{a^3\,p\,x^{5/2}}{20\,b^3}+\frac{a^5\,p\,x^{3/2}}{12\,b^5}+\frac{a^7\,p\,\sqrt{x}}{4\,b^7}+\frac{a\,p\,x^{7/2}}{28\,b}-\frac{a^6\,p\dots}{8\,b\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(c*(a + b*x^(1/2))^p),x)
```

```
[Out] (x^4*log(c*(a + b*x^(1/2))^p))/4 - (p*x^4)/32 - (a^8*p*log(a + b*x^(1/2)))/
(4*b^8) - (a^2*p*x^3)/(24*b^2) - (a^4*p*x^2)/(16*b^4) + (a^3*p*x^(5/2))/(20
*b^3) + (a^5*p*x^(3/2))/(12*b^5) + (a^7*p*x^(1/2))/(4*b^7) + (a*p*x^(7/2))/
(28*b) - (a^6*p*x)/(8*b^6)
```

sympy [A] time = 27.23, size = 146, normalized size = 0.95

$$-\frac{bp\left(\frac{2a^8\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^8}-\frac{2a^7\sqrt{x}}{b^8}+\frac{a^6x}{b^7}-\frac{2a^5x^{\frac{3}{2}}}{3b^6}+\frac{a^4x^2}{2b^5}-\frac{2a^3x^{\frac{5}{2}}}{5b^4}+\frac{a^2x^3}{3b^3}-\frac{2ax^{\frac{7}{2}}}{7b^2}+\frac{x^4}{4b}\right)}{8}+\frac{x^4\log\left(c\left(a+b\dots\right.\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(a+b*x**(1/2))**p),x)
```

```
[Out] -b*p*(2*a**8*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True))
/b**8 - 2*a**7*sqrt(x)/b**8 + a**6*x/b**7 - 2*a**5*x**(3/2)/(3*b**6) + a**4
*x**2/(2*b**5) - 2*a**3*x**(5/2)/(5*b**4) + a**2*x**3/(3*b**3) - 2*a*x**(7/
2)/(7*b**2) + x**4/(4*b))/8 + x**4*log(c*(a + b*sqrt(x))**p)/4
```

3.47 $\int x^2 \log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx$

Optimal. Leaf size=123

$$-\frac{a^6p\log\left(a+b\sqrt{x}\right)}{3b^6}+\frac{a^5p\sqrt{x}}{3b^5}-\frac{a^4px}{6b^4}+\frac{a^3px^{3/2}}{9b^3}-\frac{a^2px^2}{12b^2}+\frac{1}{3}x^3\log\left(c\left(a+b\sqrt{x}\right)^p\right)+\frac{apx^{5/2}}{15b}-\frac{px^3}{18}$$

```
[Out] -1/6*a^4*p*x/b^4+1/9*a^3*p*x^(3/2)/b^3-1/12*a^2*p*x^2/b^2+1/15*a*p*x^(5/2)/
b-1/18*p*x^3-1/3*a^6*p*ln(a+b*x^(1/2))/b^6+1/3*x^3*ln(c*(a+b*x^(1/2))^p)+1/
3*a^5*p*x^(1/2)/b^5
```

Rubi [A] time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2454, 2395, 43}

$$\frac{a^3px^{3/2}}{9b^3}-\frac{a^2px^2}{12b^2}+\frac{a^5p\sqrt{x}}{3b^5}-\frac{a^4px}{6b^4}-\frac{a^6p\log\left(a+b\sqrt{x}\right)}{3b^6}+\frac{1}{3}x^3\log\left(c\left(a+b\sqrt{x}\right)^p\right)+\frac{apx^{5/2}}{15b}-\frac{px^3}{18}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (a^5*p*Sqrt[x])/(3*b^5) - (a^4*p*x)/(6*b^4) + (a^3*p*x^(3/2))/(9*b^3) - (a^
2*p*x^2)/(12*b^2) + (a*p*x^(5/2))/(15*b) - (p*x^3)/18 - (a^6*p*Log[a + b*Sq
rt[x]])/(3*b^6) + (x^3*Log[c*(a + b*Sqrt[x])^p])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2 \log\left(c\left(a+b\sqrt{x}\right)^p\right) dx = 2\,\text{Subst}\left(\int x^5 \log\left(c(a+bx)^p\right) dx, x, \sqrt{x}\right)$$
$$= \frac{1}{3}x^3 \log\left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{1}{3}(bp)\,\text{Subst}\left(\int \frac{x^6}{a+bx} dx, x, \sqrt{x}\right)$$
$$= \frac{1}{3}x^3 \log\left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{1}{3}(bp)\,\text{Subst}\left(\int \left(-\frac{a^5}{b^6} + \frac{a^4x}{b^5} - \frac{a^3x^2}{b^4} + \frac{a^2x^3}{b^3} - \frac{ax^4}{b^2} + \right.\right.$$
$$= \frac{a^5p\sqrt{x}}{3b^5} - \frac{a^4px}{6b^4} + \frac{a^3px^{3/2}}{9b^3} - \frac{a^2px^2}{12b^2} + \frac{apx^{5/2}}{15b} - \frac{px^3}{18} - \frac{a^6p\log\left(a+b\sqrt{x}\right)}{3b^6} + \frac{1}{3}x^3$$

Mathematica [A] time = 0.06, size = 112, normalized size = 0.91

$$\frac{-60a^6p\log\left(a+b\sqrt{x}\right) + bp\sqrt{x}\left(60a^5 - 30a^4b\sqrt{x} + 20a^3b^2x - 15a^2b^3x^{3/2} + 12ab^4x^2 - 10b^5x^{5/2}\right) + 60b^6x^3\log}{180b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (b*p*Sqrt[x]*(60*a^5 - 30*a^4*b*Sqrt[x] + 20*a^3*b^2*x - 15*a^2*b^3*x^(3/2)
 + 12*a*b^4*x^2 - 10*b^5*x^(5/2)) - 60*a^6*p*Log[a + b*Sqrt[x]] + 60*b^6*x^
3*Log[c*(a + b*Sqrt[x])^p])/(180*b^6)
```

fricas [A] time = 0.47, size = 105, normalized size = 0.85

$$-\frac{10\,b^6px^3 - 60\,b^6x^3\log(c) + 15\,a^2b^4px^2 + 30\,a^4b^2px - 60\left(b^6px^3 - a^6p\right)\log\left(b\sqrt{x}+a\right) - 4\left(3\,ab^5px^2 + 5\,a^3b^3\right.}{180\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] -1/180*(10*b^6*p*x^3 - 60*b^6*x^3*log(c) + 15*a^2*b^4*p*x^2 + 30*a^4*b^2*p*
x - 60*(b^6*p*x^3 - a^6*p)*log(b*sqrt(x) + a) - 4*(3*a*b^5*p*x^2 + 5*a^3*b^
3*p*x + 15*a^5*b*p)*sqrt(x))/b^6
```

giac [B] time = 0.18, size = 255, normalized size = 2.07

$$\frac{60\,bx^3\log(c) + \left(\frac{60\left(b\sqrt{x}+a\right)^6\log\left(b\sqrt{x}+a\right)}{b^5} - \frac{360\left(b\sqrt{x}+a\right)^5a\log\left(b\sqrt{x}+a\right)}{b^5} + \frac{900\left(b\sqrt{x}+a\right)^4a^2\log\left(b\sqrt{x}+a\right)}{b^5} - \frac{1200\left(b\sqrt{x}+a\right)^3a^3\log\left(b\right.}{b^5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] 1/180*(60*b*x^3*log(c) + (60*(b*sqrt(x) + a)^6*log(b*sqrt(x) + a)/b^5 - 360
*(b*sqrt(x) + a)^5*a*log(b*sqrt(x) + a)/b^5 + 900*(b*sqrt(x) + a)^4*a^2*log
(b*sqrt(x) + a)/b^5 - 1200*(b*sqrt(x) + a)^3*a^3*log(b*sqrt(x) + a)/b^5 + 9
00*(b*sqrt(x) + a)^2*a^4*log(b*sqrt(x) + a)/b^5 - 360*(b*sqrt(x) + a)*a^5*l
og(b*sqrt(x) + a)/b^5 - 10*(b*sqrt(x) + a)^6/b^5 + 72*(b*sqrt(x) + a)^5*a/b
^5 - 225*(b*sqrt(x) + a)^4*a^2/b^5 + 400*(b*sqrt(x) + a)^3*a^3/b^5 - 450*(b
*sqrt(x) + a)^2*a^4/b^5 + 360*(b*sqrt(x) + a)*a^5/b^5)*p)/b
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x^2 \ln\left(c\left(b\sqrt{x}+a\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^(1/2)+a)^p),x)
```

```
[Out] int(x^2*ln(c*(b*x^(1/2)+a)^p),x)
```

maxima [A] time = 0.68, size = 98, normalized size = 0.80

$$\frac{1}{3}x^3\log\left(\left(b\sqrt{x}+a\right)^p c\right)-\frac{1}{180}bp\left(\frac{60\,a^6\log\left(b\sqrt{x}+a\right)}{b^7}+\frac{10\,b^5x^3-12\,ab^4x^{\frac{5}{2}}+15\,a^2b^3x^2-20\,a^3b^2x^{\frac{3}{2}}+30\,a^4bx-}{b^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((b*sqrt(x) + a)^p*c) - 1/180*b*p*(60*a^6*log(b*sqrt(x) + a)/b^7
 + (10*b^5*x^3 - 12*a*b^4*x^(5/2) + 15*a^2*b^3*x^2 - 20*a^3*b^2*x^(3/2) + 3
0*a^4*b*x - 60*a^5*sqrt(x))/b^6)
```

mupad [B] time = 0.28, size = 97, normalized size = 0.79

$$\frac{x^3\,\ln\left(c\,{\left(a+b\,\sqrt{x}\right)}^p\right)}{3}-\frac{p\,x^3}{18}-\frac{a^6\,p\,\ln\left(a+b\,\sqrt{x}\right)}{3\,b^6}-\frac{a^2\,p\,x^2}{12\,b^2}+\frac{a^3\,p\,x^{3/2}}{9\,b^3}+\frac{a^5\,p\,\sqrt{x}}{3\,b^5}+\frac{a\,p\,x^{5/2}}{15\,b}-\frac{a^4\,p\,x}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(a + b*x^(1/2))^p),x)
```

```
[Out] (x^3*log(c*(a + b*x^(1/2))^p))/3 - (p*x^3)/18 - (a^6*p*log(a + b*x^(1/2)))/
(3*b^6) - (a^2*p*x^2)/(12*b^2) + (a^3*p*x^(3/2))/(9*b^3) + (a^5*p*x^(1/2))/
(3*b^5) + (a*p*x^(5/2))/(15*b) - (a^4*p*x)/(6*b^4)
```

sympy [A] time = 8.45, size = 119, normalized size = 0.97

$$-\frac{bp\left(\frac{2a^6\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for}\ b=0\\ \frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^6}-\frac{2a^5\sqrt{x}}{b^6}+\frac{a^4x}{b^5}-\frac{2a^3x^{\frac{3}{2}}}{3b^4}+\frac{a^2x^2}{2b^3}-\frac{2ax^{\frac{5}{2}}}{5b^2}+\frac{x^3}{3b}\right)}{6}+\frac{x^3\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(a+b*x**(1/2))**p),x)
```

```
[Out] -b*p*(2*a**6*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True))
/b**6 - 2*a**5*sqrt(x)/b**6 + a**4*x/b**5 - 2*a**3*x**(3/2)/(3*b**4) + a**2
*x**2/(2*b**3) - 2*a*x**(5/2)/(5*b**2) + x**3/(3*b))/6 + x**3*log(c*(a + b*
sqrt(x))**p)/3
```

3.48 $\int x \log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx$

Optimal. Leaf size=93

$$-\frac{a^4 p \log\left(a+b\sqrt{x}\right)}{2b^4}+\frac{a^3 p\sqrt{x}}{2b^3}-\frac{a^2 p x}{4b^2}+\frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)+\frac{a p x^{3/2}}{6b}-\frac{p x^2}{8}$$

```
[Out] -1/4*a^2*p*x/b^2+1/6*a*p*x^(3/2)/b-1/8*p*x^2-1/2*a^4*p*ln(a+b*x^(1/2))/b^4+
1/2*x^2*ln(c*(a+b*x^(1/2))^p)+1/2*a^3*p*x^(1/2)/b^3
```

Rubi [A] time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2395, 43}

$$\frac{a^3 p\sqrt{x}}{2b^3}-\frac{a^2 p x}{4b^2}-\frac{a^4 p \log\left(a+b\sqrt{x}\right)}{2b^4}+\frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)+\frac{a p x^{3/2}}{6b}-\frac{p x^2}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (a^3*p*Sqrt[x])/(2*b^3) - (a^2*p*x)/(4*b^2) + (a*p*x^(3/2))/(6*b) - (p*x^2)
/8 - (a^4*p*Log[a + b*Sqrt[x]])/(2*b^4) + (x^2*Log[c*(a + b*Sqrt[x])^p])/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int x\log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx &= 2\,\text{Subst}\left(\int x^3\log\left(c(a+bx)^p\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{2}(bp)\,\text{Subst}\left(\int\frac{x^4}{a+bx}\,dx, x, \sqrt{x}\right)\\
&= \frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{2}(bp)\,\text{Subst}\Bigg(\int\Bigg(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\\
&= \frac{a^3p\sqrt{x}}{2b^3}-\frac{a^2px}{4b^2}+\frac{apx^{3/2}}{6b}-\frac{px^2}{8}-\frac{a^4p\log\left(a+b\sqrt{x}\right)}{2b^4}+\frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 0.95

$$\frac{-12a^4p\log\left(a+b\sqrt{x}\right)+bp\sqrt{x}\left(12a^3-6a^2b\sqrt{x}+4ab^2x-3b^3x^{3/2}\right)+12b^4x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{24b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (b*p*Sqrt[x]*(12*a^3 - 6*a^2*b*Sqrt[x] + 4*a*b^2*x - 3*b^3*x^(3/2)) - 12*a^
4*p*Log[a + b*Sqrt[x]] + 12*b^4*x^2*Log[c*(a + b*Sqrt[x])^p])/(24*b^4)
```

fricas [A] time = 0.48, size = 80, normalized size = 0.86

$$-\frac{3\,b^4px^2-12\,b^4x^2\log(c)+6\,a^2b^2px-12\left(b^4px^2-a^4p\right)\log\left(b\sqrt{x}+a\right)-4\left(ab^3px+3\,a^3bp\right)\sqrt{x}}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] -1/24*(3*b^4*p*x^2 - 12*b^4*x^2*log(c) + 6*a^2*b^2*p*x - 12*(b^4*p*x^2 - a^
4*p)*log(b*sqrt(x) + a) - 4*(a*b^3*p*x + 3*a^3*b*p)*sqrt(x))/b^4
```

giac [B] time = 0.22, size = 171, normalized size = 1.84

$$\frac{12\,bx^2\log(c)+\left(\frac{12\left(b\sqrt{x}+a\right)^4\log\left(b\sqrt{x}+a\right)}{b^3}-\frac{48\left(b\sqrt{x}+a\right)^3a\log\left(b\sqrt{x}+a\right)}{b^3}+\frac{72\left(b\sqrt{x}+a\right)^2a^2\log\left(b\sqrt{x}+a\right)}{b^3}-\frac{48\left(b\sqrt{x}+a\right)a^3\log\left(b\sqrt{x}+a\right)}{b^3}-\right.}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] 1/24*(12*b*x^2*log(c) + (12*(b*sqrt(x) + a)^4*log(b*sqrt(x) + a)/b^3 - 48*(
b*sqrt(x) + a)^3*a*log(b*sqrt(x) + a)/b^3 + 72*(b*sqrt(x) + a)^2*a^2*log(b*
sqrt(x) + a)/b^3 - 48*(b*sqrt(x) + a)*a^3*log(b*sqrt(x) + a)/b^3 - 3*(b*sqr
t(x) + a)^4/b^3 + 16*(b*sqrt(x) + a)^3*a/b^3 - 36*(b*sqrt(x) + a)^2*a^2/b^3
 + 48*(b*sqrt(x) + a)*a^3/b^3)*p)/b
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x\ln\left(c\left(b\sqrt{x}+a\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^(1/2)+a)^p),x)
```

```
[Out] int(x*ln(c*(b*x^(1/2)+a)^p),x)
```

maxima [A] time = 0.65, size = 76, normalized size = 0.82

$$-\frac{1}{24}\,bp\left(\frac{12\,a^4\log\left(b\sqrt{x}+a\right)}{b^5}+\frac{3\,b^3x^2-4\,ab^2x^{\frac{3}{2}}+6\,a^2bx-12\,a^3\sqrt{x}}{b^4}\right)+\frac{1}{2}\,x^2\log\left(\left(b\sqrt{x}+a\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] -1/24*b*p*(12*a^4*log(b*sqrt(x) + a)/b^5 + (3*b^3*x^2 - 4*a*b^2*x^(3/2) + 6
*a^2*b*x - 12*a^3*sqrt(x))/b^4) + 1/2*x^2*log((b*sqrt(x) + a)^p*c)
```

mupad [B] time = 0.27, size = 73, normalized size = 0.78

$$\frac{x^2 \ln\left(c\left(a+b\sqrt{x}\right)^p\right)}{2}-\frac{p\,x^2}{8}-\frac{a^4\,p\,\ln\left(a+b\sqrt{x}\right)}{2\,b^4}+\frac{a^3\,p\sqrt{x}}{2\,b^3}-\frac{a^2\,p\,x}{4\,b^2}+\frac{a\,p\,x^{3/2}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(a + b*x^(1/2))^p),x)
```

```
[Out] (x^2*log(c*(a + b*x^(1/2))^p))/2 - (p*x^2)/8 - (a^4*p*log(a + b*x^(1/2)))/(
2*b^4) + (a^3*p*x^(1/2))/(2*b^3) - (a^2*p*x)/(4*b^2) + (a*p*x^(3/2))/(6*b)
```

sympy [A] time = 2.91, size = 92, normalized size = 0.99

$$-\frac{bp\left(\frac{2a^4\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^4}-\frac{2a^3\sqrt{x}}{b^4}+\frac{a^2x}{b^3}-\frac{2ax^{\frac{3}{2}}}{3b^2}+\frac{x^2}{2b}\right)}{4}+\frac{x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(a+b*x**(1/2))**p),x)
```

```
[Out] -b*p*(2*a**4*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True))
/b**4 - 2*a**3*sqrt(x)/b**4 + a**2*x/b**3 - 2*a*x**(3/2)/(3*b**2) + x**2/(2
*b))/4 + x**2*log(c*(a + b*sqrt(x))**p)/2
```

3.49 $\int \log \left(c \left(a+b \sqrt{x}\right)^p\right) \, dx$

Optimal. Leaf size=53

$$-\frac{a^2 p \log \left(a+b \sqrt{x}\right)}{b^2}+x \log \left(c \left(a+b \sqrt{x}\right)^p\right)+\frac{a p \sqrt{x}}{b}-\frac{p x}{2}$$

```
[Out] -1/2*p*x-a^2*p*ln(a+b*x^(1/2))/b^2+x*ln(c*(a+b*x^(1/2))^p)+a*p*x^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2448, 266, 43}

$$-\frac{a^2 p \log \left(a+b \sqrt{x}\right)}{b^2}+x \log \left(c \left(a+b \sqrt{x}\right)^p\right)+\frac{a p \sqrt{x}}{b}-\frac{p x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (a*p*Sqrt[x])/b - (p*x)/2 - (a^2*p*Log[a + b*Sqrt[x]])/b^2 + x*Log[c*(a + b
*Sqrt[x])^p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \log \left(c \left(a+b \sqrt{x}\right)^p\right) \, dx &= x \log \left(c \left(a+b \sqrt{x}\right)^p\right)-\frac{1}{2} (b p) \int \frac{\sqrt{x}}{a+b \sqrt{x}} \, dx\\
&= x \log \left(c \left(a+b \sqrt{x}\right)^p\right)-(b p) \operatorname{Subst}\left(\int \frac{x^2}{a+b x} \, dx,x,\sqrt{x}\right)\\
&= x \log \left(c \left(a+b \sqrt{x}\right)^p\right)-(b p) \operatorname{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2 (a+b x)}\right) \, dx,x,\sqrt{x}\right)\\
&= \frac{a p \sqrt{x}}{b}-\frac{p x}{2}-\frac{a^2 p \log \left(a+b \sqrt{x}\right)}{b^2}+x \log \left(c \left(a+b \sqrt{x}\right)^p\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.00

$$-\frac{a^2 p \log \left(a+b \sqrt{x}\right)}{b^2}+x \log \left(c \left(a+b \sqrt{x}\right)^p\right)+\frac{a p \sqrt{x}}{b}-\frac{p x}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (a*p*Sqrt[x])/b - (p*x)/2 - (a^2*p*Log[a + b*Sqrt[x]])/b^2 + x*Log[c*(a + b
*Sqrt[x])^p]
```

fricas [A] time = 0.46, size = 51, normalized size = 0.96

$$-\frac{b^2px - 2\,b^2x\log(c) - 2\,abp\sqrt{x} - 2\left(b^2px - a^2p\right)\log\left(b\sqrt{x} + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*p*x - 2*b^2*x*log(c) - 2*a*b*p*sqrt(x) - 2*(b^2*p*x - a^2*p)*log(
b*sqrt(x) + a))/b^2
```

giac [B] time = 0.18, size = 97, normalized size = 1.83

$$\frac{\frac{\left(2\left(b\sqrt{x}+a\right)^2\log\left(b\sqrt{x}+a\right)-4\left(b\sqrt{x}+a\right)a\log\left(b\sqrt{x}+a\right)-\left(b\sqrt{x}+a\right)^2+4\left(b\sqrt{x}+a\right)a\right)p}{b} + \frac{2\left(\left(b\sqrt{x}+a\right)^2-2\left(b\sqrt{x}+a\right)a\right)\log(c)}{b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] 1/2*((2*(b*sqrt(x) + a)^2*log(b*sqrt(x) + a) - 4*(b*sqrt(x) + a)*a*log(b*sq
rt(x) + a) - (b*sqrt(x) + a)^2 + 4*(b*sqrt(x) + a)*a)*p/b + 2*((b*sqrt(x) +
 a)^2 - 2*(b*sqrt(x) + a)*a)*log(c)/b)/b
```

maple [A] time = 0.05, size = 46, normalized size = 0.87

$$-\frac{a^2p\ln\left(b\sqrt{x} + a\right)}{b^2} - \frac{px}{2} + x\ln\left(c\left(b\sqrt{x} + a\right)^p\right) + \frac{ap\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^(1/2)+a)^p),x)
```

```
[Out] -1/2*p*x-a^2*p*ln(b*x^(1/2)+a)/b^2+x*ln(c*(b*x^(1/2)+a)^p)+a*p*x^(1/2)/b
```

maxima [A] time = 0.59, size = 50, normalized size = 0.94

$$-\frac{1}{2}\,bp\left(\frac{2\,a^2\log\left(b\sqrt{x} + a\right)}{b^3} + \frac{bx - 2\,a\sqrt{x}}{b^2}\right) + x\log\left(\left(b\sqrt{x} + a\right)^p c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] -1/2*b*p*(2*a^2*log(b*sqrt(x) + a)/b^3 + (b*x - 2*a*sqrt(x))/b^2) + x*log((
b*sqrt(x) + a)^p*c)
```

mupad [B] time = 0.12, size = 47, normalized size = 0.89

$$x\,\ln\left(c\left(a + b\,\sqrt{x}\right)^p\right) - \frac{p\,\left(b^2\,x + 2\,a^2\,\ln\left(a + b\,\sqrt{x}\right) - 2\,a\,b\,\sqrt{x}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^(1/2))^p),x)
```

```
[Out] x*log(c*(a + b*x^(1/2))^p) - (p*(b^2*x + 2*a^2*log(a + b*x^(1/2)) - 2*a*b*x
^(1/2)))/(2*b^2)
```

sympy [A] time = 1.50, size = 61, normalized size = 1.15

$$-\frac{bp\left(\frac{2a^2\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^2}-\frac{2a\sqrt{x}}{b^2}+\frac{x}{b}\right)}{2}+x\log\left(c\left(a+b\sqrt{x}\right)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b*x**(1/2))**p),x)
```

```
[Out] -b*p*(2*a**2*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True))
/b**2 - 2*a*sqrt(x)/b**2 + x/b)/2 + x*log(c*(a + b*sqrt(x))**p)
```

3.50 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx$

Optimal. Leaf size=46

$$2\log\left(-\frac{b\sqrt{x}}{a}\right)\log\left(c\left(a+b\sqrt{x}\right)^p\right)+2p\text{Li}_2\left(\frac{\sqrt{x}\,b}{a}+1\right)$$

```
[Out] 2*ln(-b*x^(1/2)/a)*ln(c*(a+b*x^(1/2))^p)+2*p*polylog(2,1+b*x^(1/2)/a)
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2454, 2394, 2315}

$$2p\text{PolyLog}\left(2,\frac{b\sqrt{x}}{a}+1\right)+2\log\left(-\frac{b\sqrt{x}}{a}\right)\log\left(c\left(a+b\sqrt{x}\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*Sqrt[x])^p]/x,x]
```

```
[Out] 2*Log[c*(a + b*Sqrt[x])^p]*Log[-((b*Sqrt[x])/a)] + 2*p*PolyLog[2, 1 + (b*Sq
rt[x])/a]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx &= 2\,\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x}\,dx,x,\sqrt{x}\right)\\
&= 2\log\left(c\left(a+b\sqrt{x}\right)^p\right)\log\left(-\frac{b\sqrt{x}}{a}\right)-(2bp)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx,x,\sqrt{x}\right)\\
&= 2\log\left(c\left(a+b\sqrt{x}\right)^p\right)\log\left(-\frac{b\sqrt{x}}{a}\right)+2p\text{Li}_2\left(1+\frac{b\sqrt{x}}{a}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 47, normalized size = 1.02

$$2\log\left(-\frac{b\sqrt{x}}{a}\right)\log\left(c\left(a+b\sqrt{x}\right)^{p}\right)+2p\text{Li}_2\left(\frac{a+b\sqrt{x}}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p]/x,x]
```

```
[Out] 2*Log[c*(a + b*Sqrt[x])^p]*Log[-((b*Sqrt[x])/a)] + 2*p*PolyLog[2, (a + b*Sq
rt[x])/a]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(b\sqrt{x}+a\right)^{p}c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x,x, algorithm="fricas")
```

```
[Out] integral(log((b*sqrt(x) + a)^p*c)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(\left(b\sqrt{x}+a\right)^{p}c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*sqrt(x) + a)^p*c)/x, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{\ln\left(c\left(b\sqrt{x}+a\right)^{p}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^(1/2)+a)^p)/x,x)
```

```
[Out] int(ln(c*(b*x^(1/2)+a)^p)/x,x)
```

maxima [B] time = 0.64, size = 79, normalized size = 1.72

$$bp\left(\frac{\log\left(b\sqrt{x}+a\right)\log(x)}{b}-\frac{\log(x)\log\left(\frac{b\sqrt{x}}{a}+1\right)+2\,\text{Li}_2\left(-\frac{b\sqrt{x}}{a}\right)}{b}\right)-p\log\left(b\sqrt{x}+a\right)\log(x)+\log\left(\left(b\sqrt{x}+a\right)^{p}c\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x,x, algorithm="maxima")
```

```
[Out] b*p*(log(b*sqrt(x) + a)*log(x)/b - (log(x)*log(b*sqrt(x)/a + 1) + 2*dilog(-
b*sqrt(x)/a))/b) - p*log(b*sqrt(x) + a)*log(x) + log((b*sqrt(x) + a)^p*c)*l
og(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\left(a+b\sqrt{x}\right)^{p}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^(1/2))^p)/x,x)
```

```
[Out] int(log(c*(a + b*x^(1/2))^p)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^{p}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b*x**(1/2))**p)/x,x)
```

```
[Out] Integral(log(c*(a + b*sqrt(x))**p)/x, x)
```

3.51 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=63

$$\frac{b^2p\log\left(a+b\sqrt{x}\right)}{a^2}-\frac{b^2p\log(x)}{2a^2}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}-\frac{bp}{a\sqrt{x}}$$

```
[Out] -1/2*b^2*p*ln(x)/a^2+b^2*p*ln(a+b*x^(1/2))/a^2-ln(c*(a+b*x^(1/2))^p)/x-b*p/
a/x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2454, 2395, 44}

$$\frac{b^2p\log\left(a+b\sqrt{x}\right)}{a^2}-\frac{b^2p\log(x)}{2a^2}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}-\frac{bp}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*Sqrt[x])^p]/x^2,x]
```

```
[Out] -((b*p)/(a*Sqrt[x])) + (b^2*p*Log[a + b*Sqrt[x]])/a^2 - Log[c*(a + b*Sqrt[x
])^p]/x - (b^2*p*Log[x])/(2*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx = 2\,\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^3}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x} + (bp)\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x} + (bp)\,\text{Subst}\left(\int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right)dx, x, \sqrt{x}\right)$$
$$= -\frac{bp}{a\sqrt{x}} + \frac{b^2p\log\left(a+b\sqrt{x}\right)}{a^2} - \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x} - \frac{b^2p\log(x)}{2a^2}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.87

$$-\frac{bp\left(-2b\log\left(a+b\sqrt{x}\right)+\frac{2a}{\sqrt{x}}+b\log(x)\right)}{2a^2} - \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p]/x^2,x]
```

```
[Out] -(Log[c*(a + b*Sqrt[x])^p]/x) - (b*p*((2*a)/Sqrt[x] - 2*b*Log[a + b*Sqrt[x]
] + b*Log[x]))/(2*a^2)
```

fricas [A] time = 0.48, size = 55, normalized size = 0.87

$$-\frac{b^2px\log\left(\sqrt{x}\right) + abp\sqrt{x} + a^2\log(c) - \left(b^2px - a^2p\right)\log\left(b\sqrt{x}+a\right)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^2,x, algorithm="fricas")
```

```
[Out] -(b^2*p*x*log(sqrt(x)) + a*b*p*sqrt(x) + a^2*log(c) - (b^2*p*x - a^2*p)*log
(b*sqrt(x) + a))/(a^2*x)
```

giac [B] time = 0.18, size = 132, normalized size = 2.10

$$-\frac{\frac{b^3p\log\left(b\sqrt{x}+a\right)}{\left(b\sqrt{x}+a\right)^2-2\left(b\sqrt{x}+a\right)a+a^2} - \frac{b^3p\log\left(b\sqrt{x}+a\right)}{a^2} + \frac{b^3p\log\left(b\sqrt{x}\right)}{a^2} + \frac{\left(b\sqrt{x}+a\right)b^3p-ab^3p+ab^3\log(c)}{\left(b\sqrt{x}+a\right)^2a-2\left(b\sqrt{x}+a\right)a^2+a^3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^2,x, algorithm="giac")
```

```
[Out] -(b^3*p*log(b*sqrt(x) + a)/((b*sqrt(x) + a)^2 - 2*(b*sqrt(x) + a)*a + a^2)
- b^3*p*log(b*sqrt(x) + a)/a^2 + b^3*p*log(b*sqrt(x))/a^2 + ((b*sqrt(x) + a
)*b^3*p - a*b^3*p + a*b^3*log(c))/((b*sqrt(x) + a)^2*a - 2*(b*sqrt(x) + a)*
a^2 + a^3))/b
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(b\sqrt{x}+a\right)^p\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^(1/2)+a)^p)/x^2,x)
```

```
[Out] int(ln(c*(b*x^(1/2)+a)^p)/x^2,x)
```

maxima [A] time = 0.62, size = 53, normalized size = 0.84

$$\frac{1}{2}\,bp\left(\frac{2\,b\log\left(b\sqrt{x}+a\right)}{a^2}-\frac{b\log(x)}{a^2}-\frac{2}{a\sqrt{x}}\right)-\frac{\log\left(\left(b\sqrt{x}+a\right)^p c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*b*log(b*sqrt(x) + a)/a^2 - b*log(x)/a^2 - 2/(a*sqrt(x))) - log((
b*sqrt(x) + a)^p*c)/x
```

mupad [B] time = 0.66, size = 49, normalized size = 0.78

$$\frac{2\,b^2\,p\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^2}-\frac{\ln\left(c\,\left(a+b\,\sqrt{x}\right)^p\right)}{x}-\frac{b\,p}{a\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^(1/2))^p)/x^2,x)
```

```
[Out] (2*b^2*p*atanh((2*b*x^(1/2))/a + 1))/a^2 - log(c*(a + b*x^(1/2))^p)/x - (b*
p)/(a*x^(1/2))
```

sympy [A] time = 40.89, size = 461, normalized size = 7.32

$$\begin{cases} -\frac{2a^3p\sqrt{x}\log\left(a+b\sqrt{x}\right)}{2a^3x^{\frac{3}{2}}+2a^2bx^2}-\frac{2a^3\sqrt{x}\log\left(c\right)}{2a^3x^{\frac{3}{2}}+2a^2bx^2}-\frac{2a^2bpx\log\left(a+b\sqrt{x}\right)}{2a^3x^{\frac{3}{2}}+2a^2bx^2}-\frac{2a^2bpx}{2a^3x^{\frac{3}{2}}+2a^2bx^2}-\frac{2a^2bx\log\left(c\right)}{2a^3x^{\frac{3}{2}}+2a^2bx^2}-\frac{ab^2px^{\frac{3}{2}}\log\left(x\right)}{2a^3x^{\frac{3}{2}}+2a^2bx^2}+\frac{2ab^2px^{\frac{3}{2}}\log\left(a+b\cdot\right.}{2a^3x^{\frac{3}{2}}+2a^2bx^2} \\ -\frac{p\log\left(b\right)}{x}-\frac{p\log\left(x\right)}{2x}-\frac{p}{2x}-\frac{\log\left(c\right)}{x} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b*x**(1/2))**p)/x**2,x)
```

```
[Out] Piecewise((-2*a**3*p*sqrt(x)*log(a + b*sqrt(x))/(2*a**3*x**(3/2) + 2*a**2*b
*x**2) - 2*a**3*sqrt(x)*log(c)/(2*a**3*x**(3/2) + 2*a**2*b*x**2) - 2*a**2*b
*p*x*log(a + b*sqrt(x))/(2*a**3*x**(3/2) + 2*a**2*b*x**2) - 2*a**2*b*p*x/(2
*a**3*x**(3/2) + 2*a**2*b*x**2) - 2*a**2*b*x*log(c)/(2*a**3*x**(3/2) + 2*a*
*2*b*x**2) - a*b**2*p*x**(3/2)*log(x)/(2*a**3*x**(3/2) + 2*a**2*b*x**2) + 2
*a*b**2*p*x**(3/2)*log(a + b*sqrt(x))/(2*a**3*x**(3/2) + 2*a**2*b*x**2) - 2
*a*b**2*p*x**(3/2)/(2*a**3*x**(3/2) + 2*a**2*b*x**2) + 2*a*b**2*x**(3/2)*lo
g(c)/(2*a**3*x**(3/2) + 2*a**2*b*x**2) - b**3*p*x**2*log(x)/(2*a**3*x**(3/2
) + 2*a**2*b*x**2) + 2*b**3*p*x**2*log(a + b*sqrt(x))/(2*a**3*x**(3/2) + 2*
a**2*b*x**2) + 2*b**3*x**2*log(c)/(2*a**3*x**(3/2) + 2*a**2*b*x**2), Ne(a,
0)), (-p*log(b)/x - p*log(x)/(2*x) - p/(2*x) - log(c)/x, True))
```

3.52 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3} dx$

Optimal. Leaf size=100

$$\frac{b^4 p \log\left(a+b\sqrt{x}\right)}{2a^4}-\frac{b^4 p \log(x)}{4a^4}-\frac{b^3 p}{2a^3\sqrt{x}}+\frac{b^2 p}{4a^2 x}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2}-\frac{bp}{6ax^{3/2}}$$

```
[Out] -1/6*b*p/a/x^(3/2)+1/4*b^2*p/a^2/x-1/4*b^4*p*ln(x)/a^4+1/2*b^4*p*ln(a+b*x^(
1/2))/a^4-1/2*ln(c*(a+b*x^(1/2))^p)/x^2-1/2*b^3*p/a^3/x^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2454, 2395, 44}

$$-\frac{b^3 p}{2a^3\sqrt{x}}+\frac{b^2 p}{4a^2 x}+\frac{b^4 p \log\left(a+b\sqrt{x}\right)}{2a^4}-\frac{b^4 p \log(x)}{4a^4}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2}-\frac{bp}{6ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*Sqrt[x])^p]/x^3,x]
```

```
[Out] -(b*p)/(6*a*x^(3/2)) + (b^2*p)/(4*a^2*x) - (b^3*p)/(2*a^3*Sqrt[x]) + (b^4*p
*Log[a + b*Sqrt[x]])/(2*a^4) - Log[c*(a + b*Sqrt[x])^p]/(2*x^2) - (b^4*p*Lo
g[x])/(4*a^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx = 2\,\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^5}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2} + \frac{1}{2}(bp)\,\text{Subst}\left(\int \frac{1}{x^4(a+bx)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2} + \frac{1}{2}(bp)\,\text{Subst}\left(\int \left(\frac{1}{ax^4} - \frac{b}{a^2x^3} + \frac{b^2}{a^3x^2} - \frac{b^3}{a^4x} + \frac{b^4}{a^4(a+bx)}\right) d$$
$$= -\frac{bp}{6ax^{3/2}} + \frac{b^2p}{4a^2x} - \frac{b^3p}{2a^3\sqrt{x}} + \frac{b^4p\log\left(a+b\sqrt{x}\right)}{2a^4} - \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2} - \frac{b^4p\log(x)}{4a^4}$$

Mathematica [A] time = 0.05, size = 90, normalized size = 0.90

$$\frac{-6a^4\log\left(c\left(a+b\sqrt{x}\right)^p\right) + abp\sqrt{x}\left(-2a^2+3ab\sqrt{x}-6b^2x\right) + 6b^4px^2\log\left(a+b\sqrt{x}\right) - 3b^4px^2\log(x)}{12a^4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p]/x^3,x]
```

```
[Out] (a*b*p*Sqrt[x]*(-2*a^2 + 3*a*b*Sqrt[x] - 6*b^2*x) + 6*b^4*p*x^2*Log[a + b*S
qrt[x]] - 6*a^4*Log[c*(a + b*Sqrt[x])^p] - 3*b^4*p*x^2*Log[x])/(12*a^4*x^2)
```

fricas [A] time = 0.47, size = 84, normalized size = 0.84

$$-\frac{6\,b^4px^2\log\left(\sqrt{x}\right) - 3\,a^2b^2px + 6\,a^4\log(c) - 6\left(b^4px^2 - a^4p\right)\log\left(b\sqrt{x}+a\right) + 2\left(3\,ab^3px + a^3bp\right)\sqrt{x}}{12\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^4*p*x^2*log(sqrt(x)) - 3*a^2*b^2*p*x + 6*a^4*log(c) - 6*(b^4*p*x
^2 - a^4*p)*log(b*sqrt(x) + a) + 2*(3*a*b^3*p*x + a^3*b*p)*sqrt(x))/(a^4*x^
2)
```

giac [B] time = 0.18, size = 232, normalized size = 2.32

$$-\frac{\frac{6\,b^5p\log\left(b\sqrt{x}+a\right)}{\left(b\sqrt{x}+a\right)^4 - 4\left(b\sqrt{x}+a\right)^3a + 6\left(b\sqrt{x}+a\right)^2a^2 - 4\left(b\sqrt{x}+a\right)a^3 + a^4} - \frac{6\,b^5p\log\left(b\sqrt{x}+a\right)}{a^4} + \frac{6\,b^5p\log\left(b\sqrt{x}\right)}{a^4} + \frac{6\left(b\sqrt{x}+a\right)^3b^5p - 21\left(b\sqrt{x}+a\right)^2ab^5p + 26\,(}{\left(b\sqrt{x}+a\right)^4a^3 - 4\left(b\sqrt{x}+a\right)^3a^4 + 6\,(}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^3,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^5*p*log(b*sqrt(x) + a)/((b*sqrt(x) + a)^4 - 4*(b*sqrt(x) + a)^3*
a + 6*(b*sqrt(x) + a)^2*a^2 - 4*(b*sqrt(x) + a)*a^3 + a^4) - 6*b^5*p*log(b*
sqrt(x) + a)/a^4 + 6*b^5*p*log(b*sqrt(x))/a^4 + (6*(b*sqrt(x) + a)^3*b^5*p
- 21*(b*sqrt(x) + a)^2*a*b^5*p + 26*(b*sqrt(x) + a)*a^2*b^5*p - 11*a^3*b^5*
p + 6*a^3*b^5*log(c))/((b*sqrt(x) + a)^4*a^3 - 4*(b*sqrt(x) + a)^3*a^4 + 6*
(b*sqrt(x) + a)^2*a^5 - 4*(b*sqrt(x) + a)*a^6 + a^7))/b
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(b\sqrt{x}+a\right)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^(1/2)+a)^p)/x^3,x)
```

```
[Out] int(ln(c*(b*x^(1/2)+a)^p)/x^3,x)
```

maxima [A] time = 0.65, size = 76, normalized size = 0.76

$$\frac{1}{12}\,bp\left(\frac{6\,b^3\log\left(b\sqrt{x}+a\right)}{a^4}-\frac{3\,b^3\log(x)}{a^4}-\frac{6\,b^2x-3\,ab\sqrt{x}+2\,a^2}{a^3x^{\frac{3}{2}}}\right)-\frac{\log\left(\left(b\sqrt{x}+a\right)^pc\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*b*p*(6*b^3*log(b*sqrt(x) + a)/a^4 - 3*b^3*log(x)/a^4 - (6*b^2*x - 3*a*
b*sqrt(x) + 2*a^2)/(a^3*x^(3/2))) - 1/2*log((b*sqrt(x) + a)^p*c)/x^2
```

mupad [B] time = 0.46, size = 72, normalized size = 0.72

$$\frac{b^4\,p\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^4}-\frac{\ln\left(c\,\left(a+b\,\sqrt{x}\right)^p\right)}{2\,x^2}-\frac{\frac{b\,p}{3\,a}-\frac{b^2\,p\,\sqrt{x}}{2\,a^2}+\frac{b^3\,p\,x}{a^3}}{2\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^(1/2))^p)/x^3,x)
```

```
[Out] (b^4*p*atanh((2*b*x^(1/2))/a + 1))/a^4 - log(c*(a + b*x^(1/2))^p)/(2*x^2) -
 ((b*p)/(3*a) - (b^2*p*x^(1/2))/(2*a^2) + (b^3*p*x)/a^3)/(2*x^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b*x**(1/2))**p)/x**3,x)
```

```
[Out] Timed out
```

3.53 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=130

$$\frac{b^6p\log\left(a+b\sqrt{x}\right)}{3a^6}-\frac{b^6p\log(x)}{6a^6}-\frac{b^5p}{3a^5\sqrt{x}}+\frac{b^4p}{6a^4x}-\frac{b^3p}{9a^3x^{3/2}}+\frac{b^2p}{12a^2x^2}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3}-\frac{bp}{15ax^{5/2}}$$

```
[Out] -1/15*b*p/a/x^(5/2)+1/12*b^2*p/a^2/x^2-1/9*b^3*p/a^3/x^(3/2)+1/6*b^4*p/a^4/
x-1/6*b^6*p*ln(x)/a^6+1/3*b^6*p*ln(a+b*x^(1/2))/a^6-1/3*ln(c*(a+b*x^(1/2))^
p)/x^3-1/3*b^5*p/a^5/x^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2454, 2395, 44}

$$-\frac{b^3p}{9a^3x^{3/2}}+\frac{b^2p}{12a^2x^2}-\frac{b^5p}{3a^5\sqrt{x}}+\frac{b^4p}{6a^4x}+\frac{b^6p\log\left(a+b\sqrt{x}\right)}{3a^6}-\frac{b^6p\log(x)}{6a^6}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3}-\frac{bp}{15ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*Sqrt[x])^p]/x^4,x]
```

```
[Out] -(b*p)/(15*a*x^(5/2)) + (b^2*p)/(12*a^2*x^2) - (b^3*p)/(9*a^3*x^(3/2)) + (b
^4*p)/(6*a^4*x) - (b^5*p)/(3*a^5*Sqrt[x]) + (b^6*p*Log[a + b*Sqrt[x]])/(3*a
^6) - Log[c*(a + b*Sqrt[x])^p]/(3*x^3) - (b^6*p*Log[x])/(6*a^6)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx = 2\,\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^7}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3} + \frac{1}{3}(bp)\,\text{Subst}\left(\int \frac{1}{x^6(a+bx)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3} + \frac{1}{3}(bp)\,\text{Subst}\left(\int \left(\frac{1}{ax^6} - \frac{b}{a^2x^5} + \frac{b^2}{a^3x^4} - \frac{b^3}{a^4x^3} + \frac{b^4}{a^5x^2} - \frac{b^5}{a^6 \cdots}\right.\right.$$
$$= -\frac{bp}{15ax^{5/2}} + \frac{b^2p}{12a^2x^2} - \frac{b^3p}{9a^3x^{3/2}} + \frac{b^4p}{6a^4x} - \frac{b^5p}{3a^5\sqrt{x}} + \frac{b^6p\log\left(a+b\sqrt{x}\right)}{3a^6} - \frac{\log\left(c\left(a \cdots\right.\right.}{3 \cdots}$$

Mathematica [A] time = 0.07, size = 114, normalized size = 0.88

$$\frac{-60a^6\log\left(c\left(a+b\sqrt{x}\right)^p\right) + abp\sqrt{x}\left(-12a^4+15a^3b\sqrt{x}-20a^2b^2x+30ab^3x^{3/2}-60b^4x^2\right) + 60b^6px^3\log\left(a+b \cdots\right.}{180a^6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p]/x^4,x]
```

```
[Out] (a*b*p*Sqrt[x]*(-12*a^4 + 15*a^3*b*Sqrt[x] - 20*a^2*b^2*x + 30*a*b^3*x^(3/2
) - 60*b^4*x^2) + 60*b^6*p*x^3*Log[a + b*Sqrt[x]] - 60*a^6*Log[c*(a + b*Sqr
t[x])^p] - 30*b^6*p*x^3*Log[x])/(180*a^6*x^3)
```

fricas [A] time = 0.50, size = 109, normalized size = 0.84

$$-\frac{60\,b^6px^3\log\left(\sqrt{x}\right) - 30\,a^2b^4px^2 - 15\,a^4b^2px + 60\,a^6\log(c) - 60\left(b^6px^3 - a^6p\right)\log\left(b\sqrt{x}+a\right) + 4\left(15\,ab^5px^2 \cdots\right.}{180\,a^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^4,x, algorithm="fricas")
```

```
[Out] -1/180*(60*b^6*p*x^3*log(sqrt(x)) - 30*a^2*b^4*p*x^2 - 15*a^4*b^2*p*x + 60*
a^6*log(c) - 60*(b^6*p*x^3 - a^6*p)*log(b*sqrt(x) + a) + 4*(15*a*b^5*p*x^2
+ 5*a^3*b^3*p*x + 3*a^5*b*p)*sqrt(x))/(a^6*x^3)
```

giac [B] time = 0.19, size = 324, normalized size = 2.49

$$-\frac{\frac{60\,b^7p\log\left(b\sqrt{x}+a\right)}{\left(b\sqrt{x}+a\right)^6 - 6\left(b\sqrt{x}+a\right)^5a + 15\left(b\sqrt{x}+a\right)^4a^2 - 20\left(b\sqrt{x}+a\right)^3a^3 + 15\left(b\sqrt{x}+a\right)^2a^4 - 6\left(b\sqrt{x}+a\right)a^5 + a^6} - \frac{60\,b^7p\log\left(b\sqrt{x}+a\right)}{a^6} + \frac{60\,b^7p\log\left(b\sqrt{x}\right)}{a^6} + \cdots}{180\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^4,x, algorithm="giac")
```

```
[Out] -1/180*(60*b^7*p*log(b*sqrt(x) + a)/((b*sqrt(x) + a)^6 - 6*(b*sqrt(x) + a)^
5*a + 15*(b*sqrt(x) + a)^4*a^2 - 20*(b*sqrt(x) + a)^3*a^3 + 15*(b*sqrt(x) +
 a)^2*a^4 - 6*(b*sqrt(x) + a)*a^5 + a^6) - 60*b^7*p*log(b*sqrt(x) + a)/a^6
+ 60*b^7*p*log(b*sqrt(x))/a^6 + (60*(b*sqrt(x) + a)^5*b^7*p - 330*(b*sqrt(x
) + a)^4*a*b^7*p + 740*(b*sqrt(x) + a)^3*a^2*b^7*p - 855*(b*sqrt(x) + a)^2*
a^3*b^7*p + 522*(b*sqrt(x) + a)*a^4*b^7*p - 137*a^5*b^7*p + 60*a^5*b^7*log(
c))/((b*sqrt(x) + a)^6*a^5 - 6*(b*sqrt(x) + a)^5*a^6 + 15*(b*sqrt(x) + a)^4
*a^7 - 20*(b*sqrt(x) + a)^3*a^8 + 15*(b*sqrt(x) + a)^2*a^9 - 6*(b*sqrt(x) +
 a)*a^10 + a^11))/b
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c \left(b \sqrt{x}+a\right)^{p}\right)}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^(1/2)+a)^p)/x^4,x)
```

```
[Out] int(ln(c*(b*x^(1/2)+a)^p)/x^4,x)
```

maxima [A] time = 0.58, size = 98, normalized size = 0.75

$$\frac{1}{180}\, bp\left(\frac{60\, b^{5} \log \left(b\sqrt{x}+a\right)}{a^{6}}-\frac{30\, b^{5} \log(x)}{a^{6}}-\frac{60\, b^{4} x^{2}-30\, ab^{3} x^{\frac{3}{2}}+20\, a^{2} b^{2} x-15\, a^{3} b\sqrt{x}+12\, a^{4}}{a^{5} x^{\frac{5}{2}}}\right)-\frac{\log \left(\left(b\sqrt{x}+\right.\right.}{3\, x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^4,x, algorithm="maxima")
```

```
[Out] 1/180*b*p*(60*b^5*log(b*sqrt(x) + a)/a^6 - 30*b^5*log(x)/a^6 - (60*b^4*x^2
- 30*a*b^3*x^(3/2) + 20*a^2*b^2*x - 15*a^3*b*sqrt(x) + 12*a^4)/(a^5*x^(5/2)
)) - 1/3*log((b*sqrt(x) + a)^p*c)/x^3
```

mupad [B] time = 0.56, size = 97, normalized size = 0.75

$$\frac{2\, b^{6}\, p\, \mathrm{atanh}\left(\frac{2\, b\, \sqrt{x}}{a}+1\right)}{3\, a^{6}}-\frac{\frac{b\, p}{5\, a}-\frac{b^{2}\, p\, \sqrt{x}}{4\, a^{2}}+\frac{b^{5}\, p\, x^{2}}{a^{5}}-\frac{b^{4}\, p\, x^{3/2}}{2\, a^{4}}+\frac{b^{3}\, p\, x}{3\, a^{3}}}{3\, x^{5/2}}-\frac{\ln\left(c\left(a+b\, \sqrt{x}\right)^{p}\right)}{3\, x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^(1/2))^p)/x^4,x)
```

```
[Out] (2*b^6*p*atanh((2*b*x^(1/2))/a + 1))/(3*a^6) - ((b*p)/(5*a) - (b^2*p*x^(1/2
))/(4*a^2) + (b^5*p*x^2)/a^5 - (b^4*p*x^(3/2))/(2*a^4) + (b^3*p*x)/(3*a^3))
/(3*x^(5/2)) - log(c*(a + b*x^(1/2))^p)/(3*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b*x**(1/2))**p)/x**4,x)
```

```
[Out] Timed out
```

3.54 $\int \frac{\log\left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=32

$$\frac{2\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b}-2\sqrt{x}$$

```
[Out] -2*x^(1/2)+2*ln(a+b*x^(1/2))*(a+b*x^(1/2))/b
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2389, 2295}

$$\frac{2\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b}-2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[a + b*Sqrt[x]]/Sqrt[x],x]
```

```
[Out] -2*Sqrt[x] + (2*(a + b*Sqrt[x])*Log[a + b*Sqrt[x]])/b
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \log(a+bx)\,dx, x, \sqrt{x}\right)\\ &= \frac{2\,\text{Subst}\left(\int \log(x)\,dx, x, a+b\sqrt{x}\right)}{b}\\ &= -2\sqrt{x}+\frac{2\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.03

$$2\left(\frac{\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b}-\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a + b*Sqrt[x]]/Sqrt[x],x]
[Out] 2*(-Sqrt[x] + ((a + b*Sqrt[x])*Log[a + b*Sqrt[x]])/b)
```

fricas [A] time = 1.13, size = 28, normalized size = 0.88

$$\frac{2\left(\left(b\sqrt{x}+a\right)\log\left(b\sqrt{x}+a\right)-b\sqrt{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 2*((b*sqrt(x) + a)*log(b*sqrt(x) + a) - b*sqrt(x))/b
```

giac [A] time = 0.17, size = 31, normalized size = 0.97

$$\frac{2\left(\left(b\sqrt{x}+a\right)\log\left(b\sqrt{x}+a\right)-b\sqrt{x}-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 2*((b*sqrt(x) + a)*log(b*sqrt(x) + a) - b*sqrt(x) - a)/b
```

maple [A] time = 0.04, size = 40, normalized size = 1.25

$$2\sqrt{x}\,\ln\left(b\sqrt{x}+a\right)+\frac{2a\ln\left(b\sqrt{x}+a\right)}{b}-2\sqrt{x}-\frac{2a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(b*x^(1/2)+a)/x^(1/2),x)
[Out] 2*x^(1/2)*ln(b*x^(1/2)+a)-2*x^(1/2)+2/b*ln(b*x^(1/2)+a)*a-2*a/b
```

maxima [A] time = 0.58, size = 31, normalized size = 0.97

$$\frac{2\left(\left(b\sqrt{x}+a\right)\log\left(b\sqrt{x}+a\right)-b\sqrt{x}-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 2*((b*sqrt(x) + a)*log(b*sqrt(x) + a) - b*sqrt(x) - a)/b
```

mupad [B] time = 0.28, size = 33, normalized size = 1.03

$$2\sqrt{x}\,\ln\left(a+b\sqrt{x}\right)-2\sqrt{x}+\frac{2\,a\,\ln\left(a+b\sqrt{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b*x^(1/2))/x^(1/2),x)
[Out] 2*x^(1/2)*log(a + b*x^(1/2)) - 2*x^(1/2) + (2*a*log(a + b*x^(1/2)))/b
```

sympy [A] time = 0.71, size = 133, normalized size = 4.16

$$\begin{cases} \frac{2a^2\log\left(a+b\sqrt{x}\right)}{ab+b^2\sqrt{x}} + \frac{2a^2}{ab+b^2\sqrt{x}} + \frac{4ab\sqrt{x}\log\left(a+b\sqrt{x}\right)}{ab+b^2\sqrt{x}} + \frac{2b^2x\log\left(a+b\sqrt{x}\right)}{ab+b^2\sqrt{x}} - \frac{2b^2x}{ab+b^2\sqrt{x}} & \text{for } b \neq 0 \\ 2\sqrt{x}\log(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a+b*x**(1/2))/x**(1/2),x)
```

```
[Out] Piecewise((2*a**2*log(a + b*sqrt(x))/(a*b + b**2*sqrt(x)) + 2*a**2/(a*b + b
**2*sqrt(x)) + 4*a*b*sqrt(x)*log(a + b*sqrt(x))/(a*b + b**2*sqrt(x)) + 2*b*
*2*x*log(a + b*sqrt(x))/(a*b + b**2*sqrt(x)) - 2*b**2*x/(a*b + b**2*sqrt(x)
), Ne(b, 0)), (2*sqrt(x)*log(a), True))
```

3.55 $\int (fx)^m \log\left(c\left(d+ex^3\right)^p\right)\,dx$

Optimal. Leaf size=81

$$\frac{(fx)^{m+1}\log\left(c\left(d+ex^3\right)^p\right)}{f(m+1)}-\frac{3ep(fx)^{m+4}\,{}_2F_1\left(1,\frac{m+4}{3};\frac{m+7}{3};-\frac{ex^3}{d}\right)}{df^4(m+1)(m+4)}$$

```
[Out] -3*e*p*(f*x)^(4+m)*hypergeom([1, 4/3+1/3*m],[7/3+1/3*m],-e*x^3/d)/d/f^4/(1+
m)/(4+m)+(f*x)^(1+m)*ln(c*(e*x^3+d)^p)/f/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2455, 16, 364}

$$\frac{(fx)^{m+1}\log\left(c\left(d+ex^3\right)^p\right)}{f(m+1)}-\frac{3ep(fx)^{m+4}\,{}_2F_1\left(1,\frac{m+4}{3};\frac{m+7}{3};-\frac{ex^3}{d}\right)}{df^4(m+1)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e*x^3)^p],x]
```

```
[Out] (-3*e*p*(f*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, -((e*x^3)/
d)])/(d*f^4*(1 + m)*(4 + m)) + ((f*x)^(1 + m)*Log[c*(d + e*x^3)^p])/(f*(1 +
 m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (fx)^m\log\left(c\left(d+ex^3\right)^p\right)\,dx &= \frac{(fx)^{1+m}\log\left(c\left(d+ex^3\right)^p\right)}{f(1+m)}-\frac{(3ep)\int\frac{x^2(fx)^{1+m}}{d+ex^3}\,dx}{f(1+m)}\\
&= \frac{(fx)^{1+m}\log\left(c\left(d+ex^3\right)^p\right)}{f(1+m)}-\frac{(3ep)\int\frac{(fx)^{3+m}}{d+ex^3}\,dx}{f^3(1+m)}\\
&= -\frac{3ep(fx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{3};\frac{7+m}{3};-\frac{ex^3}{d}\right)}{df^4(1+m)(4+m)}+\frac{(fx)^{1+m}\log\left(c\left(d+ex^3\right)^p\right)}{f(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.86

$$\frac{x(fx)^m \left(d(m+4)\log\left(c\left(d+ex^3\right)^p\right)-3epx^3\,{}_2F_1\left(1,\frac{m+4}{3};\frac{m+7}{3};-\frac{ex^3}{d}\right)\right)}{d(m+1)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^3)^p],x]
```

```
[Out] (x*(f*x)^m*(-3*e*p*x^3*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, -((e*x^3)
/d)] + d*(4 + m)*Log[c*(d + e*x^3)^p]))/(d*(1 + m)*(4 + m))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fx\right)^m\log\left(\left(ex^3+d\right)^p c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((e*x^3 + d)^p*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m\log\left(\left(ex^3+d\right)^p c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*x^3 + d)^p*c), x)
```

maple [F] time = 1.11, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m\ln\left(c\left(e\,x^3+d\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(e*x^3+d)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(e*x^3+d)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{f^m x x^m\log\left(\left(ex^3+d\right)^p\right)}{m+1}+\int\frac{\left(\left(ef^m(m+1)\log(c)-3\,ef^m p\right)x^3+d\,f^m(m+1)\log(c)\right)x^m}{e(m+1)x^3+d(m+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] f^m*x*x^m*log((e*x^3 + d)^p)/(m + 1) + integrate(((e*f^m*(m + 1)*log(c) - 3
*e*f^m*p)*x^3 + d*f^m*(m + 1)*log(c))*x^m/(e*(m + 1)*x^3 + d*(m + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(e\,x^3+d\right)^p\right)\left(f\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^3)^p)*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e*x^3)^p)*(f*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(e*x**3+d)**p),x)
```

```
[Out] Timed out
```

3.56 $\int (fx)^m \log \left(c\left(d+ex^2\right)^p\right) \, dx$

Optimal. Leaf size=81

$$\frac{(fx)^{m+1}\log\left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{2ep(fx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};-\frac{ex^2}{d}\right)}{df^3(m+1)(m+3)}$$

```
[Out] -2*e*p*(f*x)^(3+m)*hypergeom([1, 3/2+1/2*m],[5/2+1/2*m],-e*x^2/d)/d/f^3/(1+
m)/(3+m)+(f*x)^(1+m)*ln(c*(e*x^2+d)^p)/f/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2455, 16, 364}

$$\frac{(fx)^{m+1}\log\left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{2ep(fx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};-\frac{ex^2}{d}\right)}{df^3(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-2*e*p*(f*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/
d)])/(d*f^3*(1 + m)*(3 + m)) + ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p])/(f*(1 +
 m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (fx)^m \log\left(c\left(d+ex^2\right)^p\right)\,dx &= \frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}-\frac{(2ep)\int\frac{x(fx)^{1+m}}{d+ex^2}\,dx}{f(1+m)}\\
&= \frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}-\frac{(2ep)\int\frac{(fx)^{2+m}}{d+ex^2}\,dx}{f^2(1+m)}\\
&= -\frac{2ep(fx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};-\frac{ex^2}{d}\right)}{df^3(1+m)(3+m)}+\frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.86

$$\frac{x(fx)^m\left(d(m+3)\log\left(c\left(d+ex^2\right)^p\right)-2epx^2\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};-\frac{ex^2}{d}\right)\right)}{d(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (x*(f*x)^m*(-2*e*p*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)
/d)] + d*(3 + m)*Log[c*(d + e*x^2)^p]))/(d*(1 + m)*(3 + m))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fx\right)^m\log\left(\left(ex^2+d\right)^pc\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((e*x^2 + d)^p*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(fx\right)^m\log\left(\left(ex^2+d\right)^pc\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*x^2 + d)^p*c), x)
```

maple [F] time = 1.12, size = 0, normalized size = 0.00

$$\int\left(fx\right)^m\ln\left(c\left(e\,x^2+d\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(e*x^2+d)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(e*x^2+d)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{f^mpxx^m\log\left(ex^2+d\right)}{m+1}+\int\frac{\left(df^m(m+1)\log(c)+\left(ef^m(m+1)\log(c)-2\,ef^mp\right)x^2\right)x^m}{e(m+1)x^2+d(m+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] f^m*p*x*x^m*log(e*x^2 + d)/(m + 1) + integrate((d*f^m*(m + 1)*log(c) + (e*f
^m*(m + 1)*log(c) - 2*e*f^m*p)*x^2)*x^m/(e*(m + 1)*x^2 + d*(m + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\ln\left(c\left(e\,x^2+d\right)^p\right)\left(f\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)*(f*x)^m, x)
```

sympy [A] time = 95.99, size = 359, normalized size = 4.43

$$-2ep\left(\begin{cases}\frac{0^m\sqrt{-\frac{d}{e^3}}\log\left(-e\sqrt{-\frac{d}{e^3}}+x\right)}{2}-\frac{0^m\sqrt{-\frac{d}{e^3}}\log\left(e\sqrt{-\frac{d}{e^3}}+x\right)}{2}+\frac{0^m x}{e} \\ \frac{ff^m m x^3 x^m \Phi\left(\frac{ex^2e^{i\pi}}{d},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4dfm\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+4df\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{3ff^m x^3 x^m \Phi\left(\frac{ex^2e^{i\pi}}{d},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4dfm\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+4df\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} \\ -\frac{\begin{cases}\log(d)\log(x)-\frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } |x|<1 \\ -\log(d)\log\left(\frac{1}{x}\right)-\frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(d)-\frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{otherwise}\end{cases}}{2ef}+\frac{\log(fx)\log(d+ex^2)}{2ef}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(e*x**2+d)**p),x)
```

```
[Out] -2*e*p*Piecewise((0**m*sqrt(-d/e**3)*log(-e*sqrt(-d/e**3) + x)/2 - 0**m*sqr
t(-d/e**3)*log(e*sqrt(-d/e**3) + x)/2 + 0**m*x/e, Eq(f, 0) | (Eq(f, 0) & Ne
(m, -1))), (f*f**m*m*x**3*x**m*lerchphi(e*x**2*exp_polar(I*pi)/d, 1, m/2 +
3/2)*gamma(m/2 + 3/2)/(4*d*f*m*gamma(m/2 + 5/2) + 4*d*f*gamma(m/2 + 5/2)) +
 3*f*f**m*x**3*x**m*lerchphi(e*x**2*exp_polar(I*pi)/d, 1, m/2 + 3/2)*gamma(
m/2 + 3/2)/(4*d*f*m*gamma(m/2 + 5/2) + 4*d*f*gamma(m/2 + 5/2)), (m > -oo) &
 (m < oo) & Ne(m, -1)), (-Piecewise((log(d)*log(x) - polylog(2, e*x**2*exp_
polar(I*pi)/d)/2, Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x**2*exp_po
lar(I*pi)/d)/2, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log
(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x**2*exp
_polar(I*pi)/d)/2, True))/(2*e*f) + log(f*x)*log(d + e*x**2)/(2*e*f), True)
) + Piecewise((0**m*x, Eq(f, 0)), (Piecewise(((f*x)**(m + 1)/(m + 1), Ne(m,
 -1)), (log(f*x), True))/f, True))*log(c*(d + e*x**2)**p)
```

3.57 $\int (fx)^m \log(c(d+ex)^p)\,dx$

Optimal. Leaf size=69

$$\frac{(fx)^{m+1}\log(c(d+ex)^p)}{f(m+1)}-\frac{ep(fx)^{m+2}\,{}_2F_1\left(1,m+2;m+3;-\frac{ex}{d}\right)}{df^2(m+1)(m+2)}$$

```
[Out] -e*p*(f*x)^(2+m)*hypergeom([1, 2+m],[3+m],-e*x/d)/d/f^2/(1+m)/(2+m)+(f*x)^(
1+m)*ln(c*(e*x+d)^p)/f/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2395, 64}

$$\frac{(fx)^{m+1}\log(c(d+ex)^p)}{f(m+1)}-\frac{ep(fx)^{m+2}\,{}_2F_1\left(1,m+2;m+3;-\frac{ex}{d}\right)}{df^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e*x)^p],x]
```

```
[Out] -((e*p*(f*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, -((e*x)/d)])/(d*f^2
*(1 + m)*(2 + m))) + ((f*x)^(1 + m)*Log[c*(d + e*x)^p])/(f*(1 + m))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int (fx)^m\log(c(d+ex)^p)\,dx&=\frac{(fx)^{1+m}\log(c(d+ex)^p)}{f(1+m)}-\frac{(ep)\int\frac{(fx)^{1+m}}{d+ex}\,dx}{f(1+m)}\\&=-\frac{ep(fx)^{2+m}\,{}_2F_1\left(1,2+m;3+m;-\frac{ex}{d}\right)}{df^2(1+m)(2+m)}+\frac{(fx)^{1+m}\log(c(d+ex)^p)}{f(1+m)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.81

$$\frac{x(fx)^m\left(d(m+2)\log(c(d+ex)^p)-epx\,{}_2F_1\left(1,m+2;m+3;-\frac{ex}{d}\right)\right)}{d(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x)^p],x]
```

```
[Out] (x*(f*x)^m*(-(e*p*x*Hypergeometric2F1[1, 2 + m, 3 + m, -((e*x)/d)]) + d*(2
+ m)*Log[c*(d + e*x)^p]))/(d*(1 + m)*(2 + m))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fx\right)^{m} \log\left(\left(ex+d\right)^{p} c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x+d)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((e*x + d)^p*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \log\left(\left(ex+d\right)^{p} c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x+d)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*x + d)^p*c), x)
```

maple [F] time = 0.96, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \ln\left(c\left(ex+d\right)^{p}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(e*x+d)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(e*x+d)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{f^{m} x x^{m} \log\left(\left(ex+d\right)^{p}\right)}{m+1} + \int \frac{\left(df^{m}(m+1)\log(c) + \left(ef^{m}(m+1)\log(c) - ef^{m}p\right)x\right)x^{m}}{e(m+1)x + d(m+1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x+d)^p),x, algorithm="maxima")
```

```
[Out] f^m*x*x^m*log((e*x + d)^p)/(m + 1) + integrate((d*f^m*(m + 1)*log(c) + (e*f
^m*(m + 1)*log(c) - e*f^m*p)*x)*x^m/(e*(m + 1)*x + d*(m + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(d+e\,x\right)^{p}\right) \left(f\,x\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x)^p)*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e*x)^p)*(f*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \log\left(c\left(d+ex\right)^{p}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(e*x+d)**p),x)
```

```
[Out] Integral((f*x)**m*log(c*(d + e*x)**p), x)
```

3.58 $\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right)\,dx$

Optimal. Leaf size=67

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(m+1)}+\frac{ep(fx)^m\,{}_2F_1\left(1,-m;1-m;-\frac{e}{dx}\right)}{dm(m+1)}$$

```
[Out] e*p*(f*x)^m*hypergeom([1, -m],[1-m],-e/d/x)/d/m/(1+m)+(f*x)^(1+m)*ln(c*(d+e
/x)^p)/f/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2455, 16, 339, 64}

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(m+1)}+\frac{ep(fx)^m\,{}_2F_1\left(1,-m;1-m;-\frac{e}{dx}\right)}{dm(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e/x)^p],x]
```

```
[Out] (e*p*(f*x)^m*Hypergeometric2F1[1, -m, 1 - m, -(e/(d*x))])/(d*m*(1 + m)) + (
(f*x)^(1 + m)*Log[c*(d + e/x)^p])/(f*(1 + m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] && !RationalQ[m]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right)dx = \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)} + \frac{(ep)\int \frac{(fx)^{1+m}}{\left(d+\frac{e}{x}\right)x^2}dx}{f(1+m)}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)} + \frac{(efp)\int \frac{(fx)^{-1+m}}{d+\frac{e}{x}}dx}{1+m}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)} - \frac{\left(ep\left(\frac{1}{x}\right)^m (fx)^m\right)\text{Subst}\left(\int \frac{x^{-1-m}}{d+ex}dx, x, \frac{1}{x}\right)}{1+m}$$
$$= \frac{ep(fx)^m\,{}_2F_1\left(1,-m;1-m;-\frac{e}{dx}\right)}{dm(1+m)} + \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 0.84

$$\frac{(fx)^m\left(dmx\log\left(c\left(d+\frac{e}{x}\right)^p\right)+ep\,{}_2F_1\left(1,-m;1-m;-\frac{e}{dx}\right)\right)}{dm(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e/x)^p],x]
[Out] ((f*x)^m*(e*p*Hypergeometric2F1[1, -m, 1 - m, -(e/(d*x))] + d*m*x*Log[c*(d
+ e/x)^p]))/(d*m*(1 + m))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fx\right)^m \log\left(c\left(\frac{dx+e}{x}\right)^p\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x)^p),x, algorithm="fricas")
[Out] integral((f*x)^m*log(c*((d*x + e)/x)^p), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x)^p),x, algorithm="giac")
[Out] integrate((f*x)^m*log(c*(d + e/x)^p), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \ln\left(c\left(d+\frac{e}{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(d+e/x)^p),x)
[Out] int((f*x)^m*ln(c*(d+e/x)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{f^m x x^m \log\left((dx+e)^p\right) - f^m x x^m \log\left(x^p\right)}{m+1} + \int \frac{\left(d f^m (m+1) x \log(c) + e f^m (m+1) \log(c) + e f^m p\right) x^m}{d(m+1)x + e(m+1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x)^p),x, algorithm="maxima")
```

```
[Out] (f^m*x*x^m*log((d*x + e)^p) - f^m*x*x^m*log(x^p))/(m + 1) + integrate((d*f^
m*(m + 1)*x*log(c) + e*f^m*(m + 1)*log(c) + e*f^m*p)*x^m/(d*(m + 1)*x + e*(
m + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(d+\frac{e}{x}\right)^p\right)\left(f\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/x)^p)*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e/x)^p)*(f*x)^m, x)
```

sympy [A] time = 21.40, size = 201, normalized size = 3.00

$$ep\left(\begin{cases} \frac{0^m \log(dx+e)}{d} \\ \frac{f^m m x^m \Phi\left(\frac{e e^{i\pi}}{dx}, 1, m e^{i\pi}\right)\Gamma(-m)}{dm\Gamma(1-m)+d\Gamma(1-m)} \\ \frac{\begin{cases} -\frac{1}{dx} & \text{for } e=0 \\ \frac{\begin{cases} \log(d)\log(x) + \operatorname{Li}_2\left(\frac{e e^{i\pi}}{dx}\right) & \text{for } |x|<1 \\ -\log(d)\log\left(\frac{1}{x}\right) + \operatorname{Li}_2\left(\frac{e e^{i\pi}}{dx}\right) & \text{for } \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(d) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(d) + \operatorname{Li}_2\left(\frac{e e^{i\pi}}{dx}\right) & \text{otherwise} \end{cases}}{e} & \text{otherwise} \end{cases}}{f} - \frac{\begin{cases} \frac{1}{dx} \\ \frac{\log\left(d+\frac{e}{x}\right)}{e} \end{cases}}{} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(d+e/x)**p),x)
```

```
[Out] e*p*Piecewise((0**m*log(d*x + e)/d, Eq(f, 0) | (Eq(f, 0) & Ne(m, -1))), (f*
*m*m*x**m*lerchphi(e*exp_polar(I*pi)/(d*x), 1, m*exp_polar(I*pi))*gamma(-m)
/(d*m*gamma(1 - m) + d*gamma(1 - m)), (m > -oo) & (m < oo) & Ne(m, -1)), (P
iecewise((-1/(d*x), Eq(e, 0)), (Piecewise((log(d)*log(x) + polylog(2, e*exp
_polar(I*pi)/(d*x)), Abs(x) < 1), (-log(d)*log(1/x) + polylog(2, e*exp_pola
r(I*pi)/(d*x)), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log
(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) + polylog(2, e*exp_pola
r(I*pi)/(d*x)), True))/e, True))/f - Piecewise((1/(d*x), Eq(e, 0)), (log(d
+ e/x)/e, True))*log(f*x)/f, True)) + Piecewise((0**m*x, Eq(f, 0)), (Piecew
ise(((f*x)**(m + 1)/(m + 1), Ne(m, -1)), (log(f*x), True))/f, True))*log(c*
(d + e/x)**p)
```

3.59 $\int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\,dx$

Optimal. Leaf size=82

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(m+1)}-\frac{2efp(fx)^{m-1}\,{}_2F_1\left(1,\frac{1-m}{2};\frac{3-m}{2};-\frac{e}{dx^2}\right)}{d\left(1-m^2\right)}$$

```
[Out] -2*e*f*p*(f*x)^(-1+m)*hypergeom([1, 1/2-1/2*m],[3/2-1/2*m],-e/d/x^2)/d/(-m^
2+1)+(f*x)^(1+m)*ln(c*(d+e/x^2)^p)/f/(1+m)
```

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2455, 16, 339, 364}

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(m+1)}-\frac{2efp(fx)^{m-1}\,{}_2F_1\left(1,\frac{1-m}{2};\frac{3-m}{2};-\frac{e}{dx^2}\right)}{d\left(1-m^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e/x^2)^p],x]
```

```
[Out] (-2*e*f*p*(f*x)^(-1 + m)*Hypergeometric2F1[1, (1 - m)/2, (3 - m)/2, -(e/(d*
x^2))])/(d*(1 - m^2)) + ((f*x)^(1 + m)*Log[c*(d + e/x^2)^p])/(f*(1 + m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right)dx = \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)} + \frac{(2ep)\int \frac{(fx)^{1+m}}{\left(d+\frac{e}{x^2}\right)x^3}dx}{f(1+m)}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)} + \frac{\left(2ef^2p\right)\int \frac{(fx)^{-2+m}}{d+\frac{e}{x^2}}dx}{1+m}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)} - \frac{\left(2efp\left(\frac{1}{x}\right)^{-1+m}(fx)^{-1+m}\right)\text{Subst}\left(\int \frac{x^{-m}}{d+ex^2}dx, x, \frac{1}{x}\right)}{1+m}$$
$$= -\frac{2efp(fx)^{-1+m}\,{}_2F_1\left(1,\frac{1-m}{2};\frac{3-m}{2};-\frac{e}{dx^2}\right)}{d\left(1-m^2\right)} + \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)}$$

Mathematica [A] time = 0.03, size = 76, normalized size = 0.93

$$\frac{(fx)^m\left(d(m-1)x^2\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)+2ep\,{}_2F_1\left(1,\frac{1}{2}-\frac{m}{2};\frac{3}{2}-\frac{m}{2};-\frac{e}{dx^2}\right)\right)}{d(m-1)(m+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e/x^2)^p],x]
[Out] ((f*x)^m*(2*e*p*Hypergeometric2F1[1, 1/2 - m/2, 3/2 - m/2, -(e/(d*x^2))] + d*(-1 + m)*x^2*Log[c*(d + e/x^2)^p]))/(d*(-1 + m)*(1 + m)*x)
```

fricas [F] time = 1.36, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fx\right)^m \log\left(c\left(\frac{dx^2+e}{x^2}\right)^p\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^2)^p),x, algorithm="fricas")
[Out] integral((f*x)^m*log(c*((d*x^2 + e)/x^2)^p), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^2)^p),x, algorithm="giac")
[Out] integrate((f*x)^m*log(c*(d + e/x^2)^p), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \ln\left(c\left(d+\frac{e}{x^2}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(d+e/x^2)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(d+e/x^2)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{f^{m}pxx^{m}\log\left(dx^{2}+e\right)-2\,f^{m}xx^{m}\log\left(x^{p}\right)}{m+1}+\int\frac{\left(df^{m}(m+1)x^{2}\log(c)+ef^{m}(m+1)\log(c)+2\,ef^{m}p\right)x^{m}}{d(m+1)x^{2}+e(m+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^2)^p),x, algorithm="maxima")
```

```
[Out] (f^m*p*x*x^m*log(d*x^2 + e) - 2*f^m*x*x^m*log(x^p))/(m + 1) + integrate((d*
f^m*(m + 1)*x^2*log(c) + e*f^m*(m + 1)*log(c) + 2*e*f^m*p)*x^m/(d*(m + 1)*x
^2 + e*(m + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\ln\left(c\left(d+\frac{e}{x^{2}}\right)^{p}\right)\left(f\,x\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/x^2)^p)*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e/x^2)^p)*(f*x)^m, x)
```

sympy [A] time = 77.64, size = 348, normalized size = 4.24

$$2ep\begin{cases}-\frac{0^{m}\sqrt{-\frac{1}{de}}\log\left(-e\sqrt{-\frac{1}{de}}+x\right)}{2}+\frac{0^{m}\sqrt{-\frac{1}{de}}\log\left(e\sqrt{-\frac{1}{de}}+x\right)}{2} & \text{for} \\ \frac{ff^{m}mx^{m}\Phi\left(\frac{ee^{i\pi}}{dx^{2}},1,\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4dfmx\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)+4dfx\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)}-\frac{ff^{m}x^{m}\Phi\left(\frac{ee^{i\pi}}{dx^{2}},1,\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4dfmx\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)+4dfx\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} & \text{for} \\ \frac{\begin{cases}\log(d)\log(x)+\frac{\operatorname{Li}_{2}\left(\frac{ee^{i\pi}}{dx^{2}}\right)}{2} & \text{for}\ |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right)+\frac{\operatorname{Li}_{2}\left(\frac{ee^{i\pi}}{dx^{2}}\right)}{2} & \text{for}\ \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log(d)+\frac{\operatorname{Li}_{2}\left(\frac{ee^{i\pi}}{dx^{2}}\right)}{2} & \text{otherwise}\end{cases}}{2ef}-\frac{\log(fx)\log\left(d+\frac{e}{x^{2}}\right)}{2ef} & \text{oth}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(d+e/x**2)**p),x)
```

```
[Out] 2*e*p*Piecewise((-0**m*sqrt(-1/(d*e))*log(-e*sqrt(-1/(d*e)) + x)/2 + 0**m*s
qrt(-1/(d*e))*log(e*sqrt(-1/(d*e)) + x)/2, Eq(f, 0) | (Eq(f, 0) & Ne(m, -1)
)), (f*f**m*m*x**m*lerchphi(e*exp_polar(I*pi)/(d*x**2), 1, 1/2 - m/2)*gamma
(1/2 - m/2)/(4*d*f*m*x*gamma(3/2 - m/2) + 4*d*f*x*gamma(3/2 - m/2)) - f*f**
m*x**m*lerchphi(e*exp_polar(I*pi)/(d*x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)/
(4*d*f*m*x*gamma(3/2 - m/2) + 4*d*f*x*gamma(3/2 - m/2)), (m > -oo) & (m < o
o) & Ne(m, -1)), (Piecewise((log(d)*log(x) + polylog(2, e*exp_polar(I*pi)/(
d*x**2))/2, Abs(x) < 1), (-log(d)*log(1/x) + polylog(2, e*exp_polar(I*pi)/(
d*x**2))/2, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(d)
+ meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) + polylog(2, e*exp_polar(I*
pi)/(d*x**2))/2, True))/(2*e*f) - log(f*x)*log(d + e/x**2)/(2*e*f), True))
+ Piecewise((0**m*x, Eq(f, 0)), (Piecewise(((f*x)**(m + 1)/(m + 1), Ne(m, -
1)), (log(f*x), True))/f, True))*log(c*(d + e/x**2)**p)
```

3.60 $\int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\,dx$

Optimal. Leaf size=85

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(m+1)}-\frac{3ef^2p(fx)^{m-2}\,{}_2F_1\left(1,\frac{2-m}{3};\frac{5-m}{3};-\frac{e}{dx^3}\right)}{d\left(-m^2+m+2\right)}$$

```
[Out] -3*e*f^2*p*(f*x)^(-2+m)*hypergeom([1, 2/3-1/3*m],[5/3-1/3*m],-e/d/x^3)/d/(-
m^2+m+2)+(f*x)^(1+m)*ln(c*(d+e/x^3)^p)/f/(1+m)
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2455, 16, 339, 364}

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(m+1)}-\frac{3ef^2p(fx)^{m-2}\,{}_2F_1\left(1,\frac{2-m}{3};\frac{5-m}{3};-\frac{e}{dx^3}\right)}{d\left(-m^2+m+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e/x^3)^p],x]
```

```
[Out] (-3*e*f^2*p*(f*x)^(-2 + m)*Hypergeometric2F1[1, (2 - m)/3, (5 - m)/3, -(e/(
d*x^3))])/(d*(2 + m - m^2)) + ((f*x)^(1 + m)*Log[c*(d + e/x^3)^p])/(f*(1 +
m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)dx = \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)} + \frac{(3ep)\int \frac{(fx)^{1+m}}{\left(d+\frac{e}{x^3}\right)x^4}dx}{f(1+m)}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)} + \frac{(3ef^3p)\int \frac{(fx)^{-3+m}}{d+\frac{e}{x^3}}dx}{1+m}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)} - \frac{\left(3ef^2p\left(\frac{1}{x}\right)^{-2+m}(fx)^{-2+m}\right)\text{Subst}\left(\int \frac{x^{1-m}}{d+ex^3}dx, x, \frac{1}{x}\right.}{1+m}$$
$$= -\frac{3ef^2p(fx)^{-2+m}\,{}_2F_1\left(1,\frac{2-m}{3};\frac{5-m}{3};-\frac{e}{dx^3}\right)}{d(2+m-m^2)} + \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)}$$

Mathematica [A] time = 0.03, size = 76, normalized size = 0.89

$$\frac{(fx)^m\left(d(m-2)x^3\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)+3ep\,{}_2F_1\left(1,\frac{2}{3}-\frac{m}{3};\frac{5}{3}-\frac{m}{3};-\frac{e}{dx^3}\right)\right)}{d(m-2)(m+1)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e/x^3)^p],x]
[Out] ((f*x)^m*(3*e*p*Hypergeometric2F1[1, 2/3 - m/3, 5/3 - m/3, -(e/(d*x^3))] +
d*(-2 + m)*x^3*Log[c*(d + e/x^3)^p]))/(d*(-2 + m)*(1 + m)*x^2)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fx\right)^m \log\left(c\left(\frac{dx^3+e}{x^3}\right)^p\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^3)^p),x, algorithm="fricas")
[Out] integral((f*x)^m*log(c*((d*x^3 + e)/x^3)^p), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^3)^p),x, algorithm="giac")
[Out] integrate((f*x)^m*log(c*(d + e/x^3)^p), x)
```

maple [F] time = 8.85, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \ln\left(c\left(d+\frac{e}{x^3}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(d+e/x^3)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(d+e/x^3)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{f^{m}xx^{m}\log\left(\left(dx^{3}+e\right)^{p}\right)-3\,f^{m}xx^{m}\log\left(x^{p}\right)}{m+1}+\int\frac{\left(df^{m}(m+1)x^{3}\log(c)+ef^{m}(m+1)\log(c)+3\,ef^{m}p\right)x^{m}}{d(m+1)x^{3}+e(m+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^3)^p),x, algorithm="maxima")
```

```
[Out] (f^m*x*x^m*log((d*x^3 + e)^p) - 3*f^m*x*x^m*log(x^p))/(m + 1) + integrate((
d*f^m*(m + 1)*x^3*log(c) + e*f^m*(m + 1)*log(c) + 3*e*f^m*p)*x^m/(d*(m + 1)
*x^3 + e*(m + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\ln\left(c\left(d+\frac{e}{x^{3}}\right)^{p}\right)\left(f\,x\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/x^3)^p)*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e/x^3)^p)*(f*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(d+e/x**3)**p),x)
```

```
[Out] Timed out
```

3.61 $\int (fx)^m \log\left(c\left(d+e\sqrt{x}\right)^p\right)\,dx$

Optimal. Leaf size=83

$$\frac{(fx)^{m+1}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(m+1)}-\frac{epx^{3/2}(fx)^m\,{}_2F_1\left(1,2m+3;2(m+2);-\frac{e\sqrt{x}}{d}\right)}{d\left(2m^2+5m+3\right)}$$

```
[Out] -e*p*x^(3/2)*(f*x)^m*hypergeom([1, 3+2*m],[4+2*m],-e*x^(1/2)/d)/d/(1+m)/(3+
2*m)+(f*x)^(1+m)*ln(c*(d+e*x^(1/2))^p)/f/(1+m)
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2455, 20, 341, 64}

$$\frac{(fx)^{m+1}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(m+1)}-\frac{epx^{3/2}(fx)^m\,{}_2F_1\left(1,2m+3;2(m+2);-\frac{e\sqrt{x}}{d}\right)}{d\left(2m^2+5m+3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e*Sqrt[x])^p],x]
```

```
[Out] -((e*p*x^(3/2)*(f*x)^m*Hypergeometric2F1[1, 3 + 2*m, 2*(2 + m), -((e*Sqrt[x
])/d)])/(d*(3 + 5*m + 2*m^2))) + ((f*x)^(1 + m)*Log[c*(d + e*Sqrt[x])^p])/(
f*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^m \log\left(c\left(d+e\sqrt{x}\right)^p\right)dx = \frac{(fx)^{1+m}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(1+m)} - \frac{(ep)\int \frac{(fx)^{1+m}}{\left(d+e\sqrt{x}\right)\sqrt{x}}dx}{2f(1+m)}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(1+m)} - \frac{\left(epx^{-m}(fx)^m\right)\int \frac{x^{\frac{1}{2}+m}}{d+e\sqrt{x}}dx}{2(1+m)}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(1+m)} - \frac{\left(epx^{-m}(fx)^m\right)\text{Subst}\left(\int \frac{x^{-1+2\left(\frac{3}{2}+m\right)}}{d+ex}dx, x, \sqrt{x}\right)}{1+m}$$
$$= -\frac{epx^{3/2}(fx)^m\,{}_2F_1\left(1,3+2m;2(2+m);-\frac{e\sqrt{x}}{d}\right)}{d\left(3+5m+2m^2\right)} + \frac{(fx)^{1+m}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(1+m)}$$

Mathematica [A] time = 0.04, size = 76, normalized size = 0.92

$$\frac{x(fx)^m\left(d(2m+3)\log\left(c\left(d+e\sqrt{x}\right)^p\right) - ep\sqrt{x}\,{}_2F_1\left(1,2m+3;2m+4;-\frac{e\sqrt{x}}{d}\right)\right)}{d(m+1)(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*Sqrt[x])^p],x]
```

```
[Out] (x*(f*x)^m*(-(e*p*Sqrt[x]*Hypergeometric2F1[1, 3 + 2*m, 4 + 2*m, -((e*Sqrt[
x])/d)]) + d*(3 + 2*m)*Log[c*(d + e*Sqrt[x])^p]))/(d*(1 + m)*(3 + 2*m))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fx\right)^m \log\left(\left(e\sqrt{x}+d\right)^p c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((e*sqrt(x) + d)^p*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \log\left(\left(e\sqrt{x}+d\right)^p c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*sqrt(x) + d)^p*c), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \ln\left(c\left(e\sqrt{x}+d\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(d+e*x^(1/2))^p),x)
```

```
[Out] int((f*x)^m*ln(c*(d+e*x^(1/2))^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2 f^m p \int \frac{x x^m}{2\left(de(m+1)\sqrt{x} + d^2(m+1)\right)}\,dx + \frac{d f^m (2m+3) p x x^m \log\left(e\sqrt{x} + d\right) + d f^m (2m+3) x x^m \log(c) - ef}{\left(2m^2 + 5m + 3\right) d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] e^2*f^m*p*integrate(1/2*x*x^m/(d*e*(m + 1)*sqrt(x) + d^2*(m + 1)), x) + (d*
f^m*(2*m + 3)*p*x*x^m*log(e*sqrt(x) + d) + d*f^m*(2*m + 3)*x*x^m*log(c) - e
*f^m*p*x^(3/2)*x^m)/((2*m^2 + 5*m + 3)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(d + e\sqrt{x}\right)^p\right)\left(f\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^(1/2))^p)*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e*x^(1/2))^p)*(f*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \log\left(c\left(d + e\sqrt{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(d+e*x**(1/2))**p),x)
```

```
[Out] Integral((f*x)**m*log(c*(d + e*sqrt(x))**p), x)
```

3.62 $\int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)dx$

Optimal. Leaf size=70

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(m+1)}+\frac{px(fx)^m\,{}_2F_1\left(1,2(m+1);2m+3;-\frac{d\sqrt{x}}{e}\right)}{2(m+1)^2}$$

```
[Out] 1/2*p*x*(f*x)^m*hypergeom([1, 2+2*m],[3+2*m],-d*x^(1/2)/e)/(1+m)^2+(f*x)^(1
+m)*ln(c*(d+e/x^(1/2))^p)/f/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2455, 20, 263, 341, 64}

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(m+1)}+\frac{px(fx)^m\,{}_2F_1\left(1,2(m+1);2m+3;-\frac{d\sqrt{x}}{e}\right)}{2(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e/Sqrt[x])^p],x]
```

```
[Out] (p*x*(f*x)^m*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -((d*Sqrt[x])/e)])/(2
*(1 + m)^2) + ((f*x)^(1 + m)*Log[c*(d + e/Sqrt[x])^p])/(f*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx = \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)} + \frac{(ep)\int \frac{(fx)^{1+m}}{\left(d+\frac{e}{\sqrt{x}}\right)x^{3/2}}dx}{2f(1+m)}$$

$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)} + \frac{\left(epx^{-m}(fx)^m\right)\int \frac{x^{-\frac{1}{2}+m}}{d+\frac{e}{\sqrt{x}}}dx}{2(1+m)}$$

$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)} + \frac{\left(epx^{-m}(fx)^m\right)\int \frac{x^m}{e+d\sqrt{x}}dx}{2(1+m)}$$

$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)} + \frac{\left(epx^{-m}(fx)^m\right)\text{Subst}\left(\int \frac{x^{-1+2(1+m)}}{e+dx}dx, x, \sqrt{x}\right)}{1+m}$$

$$= \frac{px(fx)^m\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{d\sqrt{x}}{e}\right)}{2(1+m)^2} + \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 1.10

$$\frac{\sqrt{x}\,(fx)^m\left(d(2m+1)\sqrt{x}\,\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)+ep\,{}_2F_1\left(1,-2m-1;-2m;-\frac{e}{d\sqrt{x}}\right)\right)}{d(m+1)(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e/Sqrt[x])^p],x]
```

```
[Out] (Sqrt[x]*(f*x)^m*(e*p*Hypergeometric2F1[1, -1 - 2*m, -2*m, -(e/(d*Sqrt[x]))
] + d*(1 + 2*m)*Sqrt[x]*Log[c*(d + e/Sqrt[x])^p]))/(d*(1 + m)*(1 + 2*m))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fx\right)^m \log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^p\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log(c*((d*x + e*sqrt(x))/x)^p), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^(1/2))^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log(c*(d + e/sqrt(x))^p), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(d+e/x^(1/2))^p),x)
```

```
[Out] int((f*x)^m*ln(c*(d+e/x^(1/2))^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2 f^m p \int \frac{x x^m}{2\left(de(m+1)\sqrt{x} + e^2(m+1)\right)}\,dx + \frac{2\left(2\,m^2+5\,m+3\right) e f^m p x x^m \log\left(d\sqrt{x}+e\right) - 2\left(2\,m^2+5\,m+3\right) e f^m x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] d^2*f^m*p*integrate(1/2*x*x^m/(d*e*(m + 1)*sqrt(x) + e^2*(m + 1)), x) + 1/2
*(2*(2*m^2 + 5*m + 3)*e*f^m*p*x*x^m*log(d*sqrt(x) + e) - 2*(2*m^2 + 5*m + 3
)*e*f^m*x*x^m*log(x^(1/2*p)) - 2*(m*p + p)*d*f^m*x^(3/2)*x^m + (2*(2*m^2 +
5*m + 3)*e*f^m*log(c) + (2*m*p + 3*p)*e*f^m)*x*x^m)/((2*m^3 + 7*m^2 + 8*m +
 3)*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\left(f\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/x^(1/2))^p)*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e/x^(1/2))^p)*(f*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(d+e/x**(1/2))**p),x)
```

```
[Out] Integral((f*x)**m*log(c*(d + e/sqrt(x))**p), x)
```

3.63 $\int (fx)^m \log \left(c \left(d+ex^n\right)^p\right) \, dx$

Optimal. Leaf size=87

$$\frac{(fx)^{m+1} \log \left(c \left(d+ex^n\right)^p\right)}{f(m+1)}-\frac{enpx^{n+1}(fx)^m {}_2F_1\left(1,\frac{m+n+1}{n};\frac{m+2n+1}{n};-\frac{ex^n}{d}\right)}{d(m+1)(m+n+1)}$$

```
[Out] -e*n*p*x^(1+n)*(f*x)^m*hypergeom([1, (1+m+n)/n],[(1+m+2*n)/n],-e*x^n/d)/d/(
1+m)/(1+m+n)+(f*x)^(1+m)*ln(c*(d+e*x^n)^p)/f/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2455, 20, 364}

$$\frac{(fx)^{m+1} \log \left(c \left(d+ex^n\right)^p\right)}{f(m+1)}-\frac{enpx^{n+1}(fx)^m {}_2F_1\left(1,\frac{m+n+1}{n};\frac{m+2n+1}{n};-\frac{ex^n}{d}\right)}{d(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*n*p*x^(1 + n)*(f*x)^m*Hypergeometric2F1[1, (1 + m + n)/n, (1 + m + 2*n
)/n, -((e*x^n)/d)])/(d*(1 + m)*(1 + m + n))) + ((f*x)^(1 + m)*Log[c*(d + e*
x^n)^p])/(f*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (fx)^m \log \left(c \left(d+ex^n\right)^p\right) \, dx &= \frac{(fx)^{1+m} \log \left(c \left(d+ex^n\right)^p\right)}{f(1+m)}-\frac{(enp) \int \frac{x^{-1+n}(fx)^{1+m}}{d+ex^n} \, dx}{f(1+m)}\\
&= \frac{(fx)^{1+m} \log \left(c \left(d+ex^n\right)^p\right)}{f(1+m)}-\frac{\left(enpx^{-m}(fx)^m\right) \int \frac{x^{m+n}}{d+ex^n} \, dx}{1+m}\\
&= -\frac{enpx^{1+n}(fx)^m {}_2F_1\left(1,\frac{1+m+n}{n};\frac{1+m+2n}{n};-\frac{ex^n}{d}\right)}{d(1+m)(1+m+n)}+\frac{(fx)^{1+m} \log \left(c \left(d+ex^n\right)^p\right)}{f(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.89

$$\frac{x(fx)^m \left(d(m+n+1)\log\left(c\left(d+ex^n\right)^p\right)-enpx^n\,{}_2F_1\left(1,\frac{m+n+1}{n};\frac{m+2n+1}{n};-\frac{ex^n}{d}\right)\right)}{d(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (x*(f*x)^m*(-(e*n*p*x^n*Hypergeometric2F1[1, (1 + m + n)/n, (1 + m + 2*n)/n
, -((e*x^n)/d)]) + d*(1 + m + n)*Log[c*(d + e*x^n)^p]))/(d*(1 + m)*(1 + m +
 n))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fx\right)^m \log\left(\left(ex^n+d\right)^p c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((e*x^n + d)^p*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \log\left(\left(ex^n+d\right)^p c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*x^n + d)^p*c), x)
```

maple [F] time = 1.54, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \ln\left(c\left(e\,x^n+d\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(d+e*x^n)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$df^m np \int \frac{x^m}{e(m+1)x^n+d(m+1)}\, dx + \frac{f^m(m+1)xx^m \log\left(\left(ex^n+d\right)^p\right)-\left(f^m np-f^m(m+1)\log(c)\right)xx^m}{m^2+2\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] d*f^m*n*p*integrate(x^m/(e*(m + 1)*x^n + d*(m + 1)), x) + (f^m*(m + 1)*x*x^
m*log((e*x^n + d)^p) - (f^m*n*p - f^m*(m + 1)*log(c))*x*x^m)/(m^2 + 2*m + 1
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(d+e\,x^n\right)^p\right)\left(f\,x\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)*(f*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \log \left(c\left(d+ex^n\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Integral((f*x)**m*log(c*(d + e*x**n)**p), x)
```

3.64 $\int (fx)^{-1+3n}\log\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=141

$$\frac{(fx)^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3fn}+\frac{d^3px^{-3n}(fx)^{3n}\log\left(d+ex^n\right)}{3e^3fn}-\frac{d^2px^{-2n}(fx)^{3n}}{3e^2fn}+\frac{dpx^{-n}(fx)^{3n}}{6efn}-\frac{p(fx)^{3n}}{9fn}$$

```
[Out] -1/9*p*(f*x)^(3*n)/f/n-1/3*d^2*p*(f*x)^(3*n)/e^2/f/n/(x^(2*n))+1/6*d*p*(f*x
)^(3*n)/e/f/n/(x^n)+1/3*d^3*p*(f*x)^(3*n)*ln(d+e*x^n)/e^3/f/n/(x^(3*n))+1/3
*(f*x)^(3*n)*ln(c*(d+e*x^n)^p)/f/n
```

Rubi [A] time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2455, 20, 266, 43}

$$\frac{(fx)^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3fn}-\frac{d^2px^{-2n}(fx)^{3n}}{3e^2fn}+\frac{d^3px^{-3n}(fx)^{3n}\log\left(d+ex^n\right)}{3e^3fn}+\frac{dpx^{-n}(fx)^{3n}}{6efn}-\frac{p(fx)^{3n}}{9fn}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -(p*(f*x)^(3*n))/(9*f*n) - (d^2*p*(f*x)^(3*n))/(3*e^2*f*n*x^(2*n)) + (d*p*(
f*x)^(3*n))/(6*e*f*n*x^n) + (d^3*p*(f*x)^(3*n)*Log[d + e*x^n])/(3*e^3*f*n*x
^(3*n)) + ((f*x)^(3*n)*Log[c*(d + e*x^n)^p])/(3*f*n)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^{-1+3n}\log\left(c\left(d+ex^n\right)^p\right)dx = \frac{(fx)^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3fn} - \frac{(ep)\int \frac{x^{-1+n}(fx)^{3n}}{d+ex^n}dx}{3f}$$
$$= \frac{(fx)^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3fn} - \frac{\left(epx^{-3n}(fx)^{3n}\right)\int \frac{x^{-1+4n}}{d+ex^n}dx}{3f}$$
$$= \frac{(fx)^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3fn} - \frac{\left(epx^{-3n}(fx)^{3n}\right)\text{Subst}\left(\int \frac{x^3}{d+ex}dx, x, x^n\right)}{3fn}$$
$$= \frac{(fx)^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3fn} - \frac{\left(epx^{-3n}(fx)^{3n}\right)\text{Subst}\left(\int \left(\frac{d^2}{e^3} - \frac{dx}{e^2} + \frac{x^2}{e} - \frac{d^3}{e^3(d+e}\right.\right.}{3fn}$$
$$= -\frac{p(fx)^{3n}}{9fn} - \frac{d^2px^{-2n}(fx)^{3n}}{3e^2fn} + \frac{dpx^{-n}(fx)^{3n}}{6efn} + \frac{d^3px^{-3n}(fx)^{3n}\log\left(d+ex^n\right)}{3e^3fn}$$

Mathematica [A] time = 0.08, size = 92, normalized size = 0.65

$$\frac{x^{-3n}(fx)^{3n}\left(6e^3x^{3n}\log\left(c\left(d+ex^n\right)^p\right)+6d^3p\log\left(d+ex^n\right)-epx^n\left(6d^2-3dex^n+2e^2x^{2n}\right)\right)}{18e^3fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] ((f*x)^(3*n)*(-(e*p*x^n*(6*d^2 - 3*d*e*x^n + 2*e^2*x^(2*n))) + 6*d^3*p*Log[
d + e*x^n] + 6*e^3*x^(3*n)*Log[c*(d + e*x^n)^p]))/(18*e^3*f*n*x^(3*n))
```

fricas [A] time = 0.48, size = 112, normalized size = 0.79

$$\frac{3\,de^2f^{3\,n-1}px^{2\,n}-6\,d^2ef^{3\,n-1}px^n-2\left(e^3p-3\,e^3\log(c)\right)f^{3\,n-1}x^{3\,n}+6\left(e^3f^{3\,n-1}px^{3\,n}+d^3f^{3\,n-1}p\right)\log\left(ex^n+d\right)}{18\,e^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] 1/18*(3*d*e^2*f^(3*n - 1)*p*x^(2*n) - 6*d^2*e*f^(3*n - 1)*p*x^n - 2*(e^3*p
- 3*e^3*log(c))*f^(3*n - 1)*x^(3*n) + 6*(e^3*f^(3*n - 1)*p*x^(3*n) + d^3*f^
(3*n - 1)*p)*log(e*x^n + d))/(e^3*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{3\,n-1}\log\left(\left(ex^n+d\right)^pc\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^(3*n - 1)*log((e*x^n + d)^p*c), x)
```

maple [F] time = 1.57, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{3n-1}\ln\left(c\left(e\,x^n+d\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1+3*n)*ln(c*(e*x^n+d)^p),x)
```

```
[Out] int((f*x)^(-1+3*n)*ln(c*(e*x^n+d)^p),x)
```

maxima [A] time = 0.77, size = 115, normalized size = 0.82

$$\frac{ep\left(\frac{6\,d^3f^{3\,n}\log\left(\frac{ex^n+d}{e}\right)}{e^4n}-\frac{2\,e^2f^{3\,n}x^{3\,n}-3\,def^{3\,n}x^{2\,n}+6\,d^2f^{3\,n}x^n}{e^3n}\right)}{18\,f}+\frac{\left(fx\right)^{3\,n}\log\left((ex^n+d)^pc\right)}{3\,fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] 1/18*e*p*(6*d^3*f^(3*n)*log((e*x^n + d)/e)/(e^4*n) - (2*e^2*f^(3*n)*x^(3*n)
 - 3*d*e*f^(3*n)*x^(2*n) + 6*d^2*f^(3*n)*x^n)/(e^3*n))/f + 1/3*(f*x)^(3*n)*
log((e*x^n + d)^p*c)/(f*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,{\left(f\,x\right)}^{3\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)*(f*x)^(3*n - 1),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)*(f*x)^(3*n - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1+3*n)*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Timed out
```

3.65 $\int (fx)^{-1+2n} \log \left(c\left(d+ex^n\right)^p\right) dx$

Optimal. Leaf size=112

$$\frac{(fx)^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn}-\frac{d^2px^{-2n}(fx)^{2n}\log\left(d+ex^n\right)}{2e^2fn}+\frac{dpx^{-n}(fx)^{2n}}{2efn}-\frac{p(fx)^{2n}}{4fn}$$

```
[Out] -1/4*p*(f*x)^(2*n)/f/n+1/2*d*p*(f*x)^(2*n)/e/f/n/(x^n)-1/2*d^2*p*(f*x)^(2*n
)*ln(d+e*x^n)/e^2/f/n/(x^(2*n))+1/2*(f*x)^(2*n)*ln(c*(d+e*x^n)^p)/f/n
```

Rubi [A] time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2455, 20, 266, 43}

$$\frac{(fx)^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn}-\frac{d^2px^{-2n}(fx)^{2n}\log\left(d+ex^n\right)}{2e^2fn}+\frac{dpx^{-n}(fx)^{2n}}{2efn}-\frac{p(fx)^{2n}}{4fn}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -(p*(f*x)^(2*n))/(4*f*n) + (d*p*(f*x)^(2*n))/(2*e*f*n*x^n) - (d^2*p*(f*x)^(
2*n)*Log[d + e*x^n])/(2*e^2*f*n*x^(2*n)) + ((f*x)^(2*n)*Log[c*(d + e*x^n)^p
])/(2*f*n)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^{-1+2n}\log\left(c\left(d+ex^n\right)^p\right)dx = \frac{(fx)^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn} - \frac{(ep)\int \frac{x^{-1+n}(fx)^{2n}}{d+ex^n}dx}{2f}$$
$$= \frac{(fx)^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn} - \frac{\left(epx^{-2n}(fx)^{2n}\right)\int \frac{x^{-1+3n}}{d+ex^n}dx}{2f}$$
$$= \frac{(fx)^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn} - \frac{\left(epx^{-2n}(fx)^{2n}\right)\text{Subst}\left(\int \frac{x^2}{d+ex}dx, x, x^n\right)}{2fn}$$
$$= \frac{(fx)^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn} - \frac{\left(epx^{-2n}(fx)^{2n}\right)\text{Subst}\left(\int \left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx,\right.}{2fn}$$
$$= -\frac{p(fx)^{2n}}{4fn} + \frac{dpx^{-n}(fx)^{2n}}{2efn} - \frac{d^2px^{-2n}(fx)^{2n}\log\left(d+ex^n\right)}{2e^2fn} + \frac{(fx)^{2n}\log\left(c\left(d+\right.\right.}{2fn}$$

Mathematica [A] time = 0.04, size = 74, normalized size = 0.66

$$-\frac{x^{-2n}(fx)^{2n}\left(ex^n\left(-2ex^n\log\left(c\left(d+ex^n\right)^p\right)-2dp+epx^n\right)+2d^2p\log\left(d+ex^n\right)\right)}{4e^2fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -1/4*((f*x)^(2*n)*(2*d^2*p*Log[d + e*x^n] + e*x^n*(-2*d*p + e*p*x^n - 2*e*x
^n*Log[c*(d + e*x^n)^p])))/(e^2*f*n*x^(2*n))
```

fricas [A] time = 0.49, size = 92, normalized size = 0.82

$$\frac{2\,def^{2\,n-1}px^n-\left(e^2p-2\,e^2\log(c)\right)f^{2\,n-1}x^{2\,n}+2\left(e^2f^{2\,n-1}px^{2\,n}-d^2f^{2\,n-1}p\right)\log\left(ex^n+d\right)}{4\,e^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*e*f^(2*n - 1)*p*x^n - (e^2*p - 2*e^2*log(c))*f^(2*n - 1)*x^(2*n) +
 2*(e^2*f^(2*n - 1)*p*x^(2*n) - d^2*f^(2*n - 1)*p)*log(e*x^n + d))/(e^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{2\,n-1}\log\left(\left(ex^n+d\right)^pc\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^(2*n - 1)*log((e*x^n + d)^p*c), x)
```

maple [F] time = 1.50, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{2n-1}\ln\left(c\left(e\,x^n+d\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1+2*n)*ln(c*(e*x^n+d)^p),x)
```

```
[Out] int((f*x)^(-1+2*n)*ln(c*(e*x^n+d)^p),x)
```

maxima [A] time = 0.69, size = 95, normalized size = 0.85

$$-\frac{ep\left(\frac{2\,d^2 f^{2\,n}\log\left(\frac{ex^n+d}{e}\right)}{e^3 n}+\frac{ef^{2\,n}x^{2\,n}-2\,df^{2\,n}x^n}{e^2 n}\right)}{4\,f}+\frac{\left(fx\right)^{2\,n}\log\left(\left(ex^n+d\right)^p c\right)}{2\,fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/4*e*p*(2*d^2*f^(2*n)*log((e*x^n + d)/e)/(e^3*n) + (e*f^(2*n)*x^(2*n) - 2
*d*f^(2*n)*x^n)/(e^2*n))/f + 1/2*(f*x)^(2*n)*log((e*x^n + d)^p*c)/(f*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,{\left(f\,x\right)}^{2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)*(f*x)^(2*n - 1),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)*(f*x)^(2*n - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1+2*n)*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Timed out
```

3.66 $\int (fx)^{-1+n} \log \left(c\left(d+ex^n\right)^p\right) dx$

Optimal. Leaf size=69

$$\frac{(fx)^n \log \left(c\left(d+ex^n\right)^p\right)}{fn}+\frac{dpx^{-n}(fx)^n \log \left(d+ex^n\right)}{efn}-\frac{p(fx)^n}{fn}$$

```
[Out] -p*(f*x)^n/f/n+d*p*(f*x)^n*ln(d+e*x^n)/e/f/n/(x^n)+(f*x)^n*ln(c*(d+e*x^n)^p
)/f/n
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2455, 20, 266, 43}

$$\frac{(fx)^n \log \left(c\left(d+ex^n\right)^p\right)}{fn}+\frac{dpx^{-n}(fx)^n \log \left(d+ex^n\right)}{efn}-\frac{p(fx)^n}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((p*(f*x)^n)/(f*n)) + (d*p*(f*x)^n*Log[d + e*x^n])/(e*f*n*x^n) + ((f*x)^n*
Log[c*(d + e*x^n)^p])/(f*n)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^{-1+n}\log\left(c\left(d+ex^n\right)^p\right)dx = \frac{(fx)^n\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{(ep)\int\frac{x^{-1+n}(fx)^n}{d+ex^n}dx}{f}$$
$$= \frac{(fx)^n\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{\left(epx^{-n}(fx)^n\right)\int\frac{x^{-1+2n}}{d+ex^n}dx}{f}$$
$$= \frac{(fx)^n\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{\left(epx^{-n}(fx)^n\right)\text{Subst}\left(\int\frac{x}{d+ex}dx,x,x^n\right)}{fn}$$
$$= \frac{(fx)^n\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{\left(epx^{-n}(fx)^n\right)\text{Subst}\left(\int\left(\frac{1}{e}-\frac{d}{e(d+ex)}\right)dx,x,x^n\right)}{fn}$$
$$= -\frac{p(fx)^n}{fn} + \frac{dpx^{-n}(fx)^n\log\left(d+ex^n\right)}{efn} + \frac{(fx)^n\log\left(c\left(d+ex^n\right)^p\right)}{fn}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.70

$$\frac{x^{1-n}(fx)^{n-1}\left(\frac{(d+ex^n)\log\left(c\left(d+ex^n\right)^p\right)}{e}-px^n\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (x^(1 - n)*(f*x)^(-1 + n)*(-(p*x^n) + ((d + e*x^n)*Log[c*(d + e*x^n)^p])/e)
)/n
```

fricas [A] time = 0.47, size = 57, normalized size = 0.83

$$-\frac{\left(ep-e\log(c)\right)f^{n-1}x^n-\left(ef^{n-1}px^n+df^{n-1}p\right)\log\left(ex^n+d\right)}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] -((e*p - e*log(c))*f^(n - 1)*x^n - (e*f^(n - 1)*p*x^n + d*f^(n - 1)*p)*log(
e*x^n + d))/(e*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(fx\right)^{n-1}\log\left(\left(ex^n+d\right)^pc\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^(n - 1)*log((e*x^n + d)^p*c), x)
```

maple [F] time = 1.44, size = 0, normalized size = 0.00

$$\int\left(fx\right)^{n-1}\ln\left(c\left(e\,x^n+d\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(n-1)*ln(c*(e*x^n+d)^p),x)
```

```
[Out] int((f*x)^(n-1)*ln(c*(e*x^n+d)^p),x)
```

maxima [A] time = 0.75, size = 70, normalized size = 1.01

$$-\frac{ep\left(\frac{f^n x^n}{en} - \frac{d f^n \log\left(\frac{ex^n+d}{e}\right)}{e^2 n}\right)}{f} + \frac{\left(fx\right)^n \log\left(\left(ex^n + d\right)^p c\right)}{fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
[Out] -e*p*(f^n*x^n/(e*n) - d*f^n*log((e*x^n + d)/e)/(e^2*n))/f + (f*x)^n*log((e*
x^n + d)^p*c)/(f*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,{\left(f\,x\right)}^{n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)*(f*x)^(n - 1),x)
[Out] int(log(c*(d + e*x^n)^p)*(f*x)^(n - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1+n)*ln(c*(d+e*x**n)**p),x)
[Out] Timed out
```

3.67 $\int \frac{\log\left(c(d+ex^n)^p\right)}{fx}\,dx$

Optimal. Leaf size=50

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn}+\frac{p\mathrm{Li}_2\left(\frac{ex^n}{d}+1\right)}{fn}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/f/n+p*polylog(2,1+e*x^n/d)/f/n
```

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {12, 2454, 2394, 2315}

$$\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{fn}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(f*x),x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f*n) + (p*PolyLog[2, 1 + (e*x^n)/
d])/(f*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{fx}\,dx = \frac{\int \frac{\log\left(c(d+ex^n)^p\right)}{x}dx}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}dx, x, x^n\right)}{fn}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{(ep)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}dx, x, x^n\right)}{fn}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn}$$

Mathematica [**A**] time = 0.01, size = 46, normalized size = 0.92

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)+p\text{Li}_2\left(\frac{ex^n+d}{d}\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(f*x),x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, (d + e*x^n)/d])/(f*n
)
```

fricas [**A**] time = 0.47, size = 63, normalized size = 1.26

$$\frac{np\log\left(ex^n+d\right)\log(x)-np\log(x)\log\left(\frac{ex^n+d}{d}\right)+n\log(c)\log(x)-p\text{Li}_2\left(-\frac{ex^n+d}{d}+1\right)}{fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/f/x,x, algorithm="fricas")
```

```
[Out] (n*p*log(e*x^n + d)*log(x) - n*p*log(x)*log((e*x^n + d)/d) + n*log(c)*log(x
) - p*dilog(-(e*x^n + d)/d + 1))/(f*n)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^n+d\right)^p c\right)}{fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/f/x,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/(f*x), x)
```

maple [**C**] time = 1.79, size = 201, normalized size = 4.02

$$-\frac{i\pi\,\text{csgn}\left(ic\right)\text{csgn}\left(i\left(e\,x^n+d\right)^p\right)\text{csgn}\left(ic\left(e\,x^n+d\right)^p\right)\ln(x)}{2f}+\frac{i\pi\,\text{csgn}\left(ic\right)\text{csgn}\left(ic\left(e\,x^n+d\right)^p\right)^2\ln(x)}{2f}+\frac{i\pi\,\text{csgn}\left(i\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)/f/x,x)
```

```
[Out] 1/f*ln(x)*ln((e*x^n+d)^p)+1/2*I/f*ln(x)*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*
x^n+d)^p)^2-1/2*I/f*ln(x)*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn
(I*c)-1/2*I/f*ln(x)*Pi*csgn(I*c*(e*x^n+d)^p)^3+1/2*I/f*ln(x)*Pi*csgn(I*c*(e
*x^n+d)^p)^2*csgn(I*c)+1/f*ln(c)*ln(x)-1/f*p/n*dilog((e*x^n+d)/d)-1/f*p*ln(
x)*ln((e*x^n+d)/d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,dnp\int\frac{\log(x)}{exx^{n}+dx}\,dx-np\log(x)^{2}+2\,\log\left((ex^{n}+d)^{p}\right)\log(x)+2\,\log(c)\log(x)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/f/x,x, algorithm="maxima")
```

```
[Out] 1/2*(2*d*n*p*integrate(log(x)/(e*x*x^n + d*x), x) - n*p*log(x)^2 + 2*log((e
*x^n + d)^p)*log(x) + 2*log(c)*log(x))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\ln\left(c\,{\left(d+e\,x^{n}\right)}^{p}\right)}{f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(f*x),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\log\left(c(d+ex^{n})^{p}\right)}{x}\,dx}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/f/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)/x, x)/f
```

3.68 $\int (fx)^{-1-n}\log\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=80

$$-\frac{(fx)^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{fn}+\frac{epx^n\log(x)(fx)^{-n}}{df}-\frac{epx^n(fx)^{-n}\log\left(d+ex^n\right)}{dfn}$$

```
[Out] e*p*x^n*ln(x)/d/f/((f*x)^n)-e*p*x^n*ln(d+e*x^n)/d/f/n/((f*x)^n)-ln(c*(d+e*x
^n)^p)/f/n/((f*x)^n)
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2455, 19, 266, 36, 29, 31}

$$-\frac{(fx)^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{fn}+\frac{epx^n\log(x)(fx)^{-n}}{df}-\frac{epx^n(fx)^{-n}\log\left(d+ex^n\right)}{dfn}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (e*p*x^n*Log[x])/(d*f*(f*x)^n) - (e*p*x^n*Log[d + e*x^n])/(d*f*n*(f*x)^n) -
 Log[c*(d + e*x^n)^p]/(f*n*(f*x)^n)
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + n)
*(b*v)^n)/(a*v)^n, Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int (fx)^{-1-n}\log\left(c\left(d+ex^n\right)^p\right)\,dx &= -\frac{(fx)^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{fn}+\frac{(ep)\int \frac{x^{-1+n}(fx)^{-n}}{d+ex^n}\,dx}{f}\\
&= -\frac{(fx)^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{fn}+\frac{\left(epx^n(fx)^{-n}\right)\int \frac{1}{x(d+ex^n)}\,dx}{f}\\
&= -\frac{(fx)^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{fn}+\frac{\left(epx^n(fx)^{-n}\right)\text{Subst}\left(\int \frac{1}{x(d+ex)}\,dx,x,x^n\right)}{fn}\\
&= -\frac{(fx)^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{fn}+\frac{\left(epx^n(fx)^{-n}\right)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^n\right)}{dfn}-\frac{\left(e^2px^n(\right.}{}\\
&= \frac{epx^n(fx)^{-n}\log(x)}{df}-\frac{epx^n(fx)^{-n}\log\left(d+ex^n\right)}{dfn}-\frac{(fx)^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{fn}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 57, normalized size = 0.71

$$
-\frac{(fx)^{-n}\left(d\log\left(c\left(d+ex^n\right)^p\right)+epx^n\log\left(d+ex^n\right)-enpx^n\log(x)\right)}{dfn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p],x]
[Out] -((-(e*n*p*x^n*Log[x]) + e*p*x^n*Log[d + e*x^n] + d*Log[c*(d + e*x^n)^p])/(
d*f*n*(f*x)^n))
```

fricas [A] time = 0.50, size = 75, normalized size = 0.94

$$
\frac{ef^{-n-1}npx^n\log(x)-df^{-n-1}\log(c)-\left(ef^{-n-1}px^n+df^{-n-1}p\right)\log\left(ex^n+d\right)}{dnx^n}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
[Out] (e*f^(-n - 1)*n*p*x^n*log(x) - d*f^(-n - 1)*log(c) - (e*f^(-n - 1)*p*x^n +
d*f^(-n - 1)*p)*log(e*x^n + d))/(d*n*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \left(fx\right)^{-n-1}\log\left(\left(ex^n+d\right)^p c\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
[Out] integrate((f*x)^(-n - 1)*log((e*x^n + d)^p*c), x)
```

maple [F] time = 1.53, size = 0, normalized size = 0.00

$$
\int \left(fx\right)^{-n-1}\ln\left(c\left(e\,x^n+d\right)^p\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1-n)*ln(c*(e*x^n+d)^p),x)
```

[Out] int((f*x)^(-1-n)*ln(c*(e*x^n+d)^p),x)

maxima [A] time = 0.73, size = 71, normalized size = 0.89

$$\frac{ep\left(\frac{\log(x)}{df^n} - \frac{\log\left(\frac{ex^n+d}{e}\right)}{df^n n}\right)}{f} - \frac{\log\left((ex^n + d)^p c\right)}{\left(fx\right)^n fn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")

[Out] e*p*(log(x)/(d*f^n) - log((e*x^n + d)/e)/(d*f^n*n))/f - log((e*x^n + d)^p*c
)/((f*x)^n*f*n)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)}{{\left(f\,x\right)}^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(c*(d + e*x^n)^p)/(f*x)^(n + 1),x)

[Out] int(log(c*(d + e*x^n)^p)/(f*x)^(n + 1), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**(-1-n)*ln(c*(d+e*x**n)**p),x)

[Out] Timed out

3.69 $\int (fx)^{-1-2n}\log\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=120

$$-\frac{(fx)^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn}-\frac{e^2px^{2n}\log(x)(fx)^{-2n}}{2d^2f}+\frac{e^2px^{2n}(fx)^{-2n}\log\left(d+ex^n\right)}{2d^2fn}-\frac{epx^n(fx)^{-2n}}{2dfn}$$

```
[Out] -1/2*e*p*x^n/d/f/n/((f*x)^(2*n))-1/2*e^2*p*x^(2*n)*ln(x)/d^2/f/((f*x)^(2*n)
)+1/2*e^2*p*x^(2*n)*ln(d+e*x^n)/d^2/f/n/((f*x)^(2*n))-1/2*ln(c*(d+e*x^n)^p)
/f/n/((f*x)^(2*n))
```

Rubi [A] time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2455, 20, 266, 44}

$$-\frac{(fx)^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn}-\frac{e^2px^{2n}\log(x)(fx)^{-2n}}{2d^2f}+\frac{e^2px^{2n}(fx)^{-2n}\log\left(d+ex^n\right)}{2d^2fn}-\frac{epx^n(fx)^{-2n}}{2dfn}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -(e*p*x^n)/(2*d*f*n*(f*x)^(2*n)) - (e^2*p*x^(2*n)*Log[x])/(2*d^2*f*(f*x)^(2
*n)) + (e^2*p*x^(2*n)*Log[d + e*x^n])/(2*d^2*f*n*(f*x)^(2*n)) - Log[c*(d +
e*x^n)^p]/(2*f*n*(f*x)^(2*n))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^{-1-2n}\log\left(c\left(d+ex^n\right)^p\right)\,dx = -\frac{(fx)^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn}+\frac{(ep)\int\frac{x^{-1+n}(fx)^{-2n}}{d+ex^n}\,dx}{2f}$$
$$= -\frac{(fx)^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn}+\frac{\left(epx^{2n}(fx)^{-2n}\right)\int\frac{x^{-1-n}}{d+ex^n}\,dx}{2f}$$
$$= -\frac{(fx)^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn}+\frac{\left(epx^{2n}(fx)^{-2n}\right)\text{Subst}\left(\int\frac{1}{x^2(d+ex)}\,dx,x,x^n\right)}{2fn}$$
$$= -\frac{(fx)^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2fn}+\frac{\left(epx^{2n}(fx)^{-2n}\right)\text{Subst}\left(\int\left(\frac{1}{dx^2}-\frac{e}{d^2x}+\frac{e^2}{d^2(d+ex)}\right)\right.}{2fn}$$
$$= -\frac{epx^n(fx)^{-2n}}{2dfn}-\frac{e^2px^{2n}(fx)^{-2n}\log(x)}{2d^2f}+\frac{e^2px^{2n}(fx)^{-2n}\log\left(d+ex^n\right)}{2d^2fn}-\frac{(fx)^{-}}{}$$

Mathematica [A] time = 0.05, size = 76, normalized size = 0.63

$$-\frac{(fx)^{-2n}\left(d\left(d\log\left(c\left(d+ex^n\right)^p\right)+epx^n\right)-e^2px^{2n}\log\left(d+ex^n\right)+e^2npx^{2n}\log(x)\right)}{2d^2fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -1/2*(e^2*n*p*x^(2*n)*Log[x] - e^2*p*x^(2*n)*Log[d + e*x^n] + d*(e*p*x^n +
d*Log[c*(d + e*x^n)^p]))/(d^2*f*n*(f*x)^(2*n))
```

fricas [A] time = 0.49, size = 104, normalized size = 0.87

$$-\frac{e^2f^{-2n-1}npx^{2n}\log(x)+def^{-2n-1}px^n+d^2f^{-2n-1}\log(c)-\left(e^2f^{-2n-1}px^{2n}-d^2f^{-2n-1}p\right)\log\left(ex^n+d\right)}{2\,d^2nx^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] -1/2*(e^2*f^(-2*n - 1)*n*p*x^(2*n)*log(x) + d*e*f^(-2*n - 1)*p*x^n + d^2*f^
(-2*n - 1)*log(c) - (e^2*f^(-2*n - 1)*p*x^(2*n) - d^2*f^(-2*n - 1)*p)*log(e
*x^n + d))/(d^2*n*x^(2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{-2n-1}\log\left(\left(ex^n+d\right)^pc\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^(-2*n - 1)*log((e*x^n + d)^p*c), x)
```

maple [F] time = 1.54, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{-2n-1}\ln\left(c\left(e\,x^n+d\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1-2*n)*ln(c*(e*x^n+d)^p),x)
```

```
[Out] int((f*x)^(-1-2*n)*ln(c*(e*x^n+d)^p),x)
```

maxima [A] time = 0.74, size = 99, normalized size = 0.82

$$-\frac{ep\left(\frac{e\log(x)}{d^2f^{2n}}-\frac{e\log\left(\frac{ex^n+d}{e}\right)}{d^2f^{2n}n}+\frac{1}{df^{2n}nx^n}\right)}{2f}-\frac{\log\left((ex^n+d)^pc\right)}{2\left(fx\right)^{2n}fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/2*e*p*(e*log(x)/(d^2*f^(2*n)) - e*log((e*x^n + d)/e)/(d^2*f^(2*n)*n) + 1
/(d*f^(2*n)*n*x^n))/f - 1/2*log((e*x^n + d)^p*c)/((f*x)^(2*n)*f*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)}{{\left(f\,x\right)}^{2\,n+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(f*x)^(2*n + 1),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(f*x)^(2*n + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1-2*n)*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Timed out
```

3.70 $\int x^2 \log\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=65

$$\frac{1}{3}x^3\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^{n+3}\,{}_2F_1\left(1,\frac{n+3}{n};2+\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(n+3)}$$

```
[Out] -1/3*e*n*p*x^(3+n)*hypergeom([1, (3+n)/n],[2+3/n],-e*x^n/d)/d/(3+n)+1/3*x^3
*ln(c*(d+e*x^n)^p)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2455, 364}

$$\frac{1}{3}x^3\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^{n+3}\,{}_2F_1\left(1,\frac{n+3}{n};2+\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -(e*n*p*x^(3 + n)*Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(
3*d*(3 + n)) + (x^3*Log[c*(d + e*x^n)^p])/3
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\log\left(c\left(d+ex^n\right)^p\right)dx&=\frac{1}{3}x^3\log\left(c\left(d+ex^n\right)^p\right)-\frac{1}{3}(enp)\int\frac{x^{2+n}}{d+ex^n}\,dx\\&=-\frac{enpx^{3+n}\,{}_2F_1\left(1,\frac{3+n}{n};2+\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(3+n)}+\frac{1}{3}x^3\log\left(c\left(d+ex^n\right)^p\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.94

$$\frac{1}{3}x^3\left(\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^n\,{}_2F_1\left(1,\frac{n+3}{n};2+\frac{3}{n};-\frac{ex^n}{d}\right)}{d(n+3)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (x^3*(-((e*n*p*x^n*Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/
(d*(3 + n))) + Log[c*(d + e*x^n)^p]))/3
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\log\left((ex^n+d)^pc\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral(x^2*log((e*x^n + d)^p*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\log\left((ex^n+d)^pc\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((e*x^n + d)^p*c), x)
```

maple [F] time = 1.14, size = 0, normalized size = 0.00

$$\int x^2\ln\left(c\left(e\,x^n+d\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(e*x^n+d)^p),x)
```

```
[Out] int(x^2*ln(c*(e*x^n+d)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{9}\left(np-3\,\log(c)\right)x^3+dnp\int\frac{x^2}{3\,(ex^n+d)}\,dx+\frac{1}{3}\,x^3\log\left((ex^n+d)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/9*(n*p - 3*log(c))*x^3 + d*n*p*integrate(1/3*x^2/(e*x^n + d), x) + 1/3*x
^3*log((e*x^n + d)^p)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(d + e*x^n)^p),x)
```

```
[Out] int(x^2*log(c*(d + e*x^n)^p), x)
```

sympy [C] time = 13.63, size = 104, normalized size = 1.60

$$\frac{x^3\log\left(c\left(d+ex^n\right)^p\right)}{3}-\frac{epx^3x^n\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{3d\Gamma\left(2+\frac{3}{n}\right)}-\frac{epx^3x^n\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{dn\Gamma\left(2+\frac{3}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(d+e*x**n)**p),x)
```

```
[Out] x**3*log(c*(d + e*x**n)**p)/3 - e*p*x**3*x**n*lerchphi(e*x**n*exp_polar(I*p
i)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(3*d*gamma(2 + 3/n)) - e*p*x**3*x**n*lerch
phi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(d*n*gamma(2 + 3/n
))
```

3.71 $\int x \log\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=65

$$\frac{1}{2}x^2\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(n+2)}$$

```
[Out] -1/2*e*n*p*x^(2+n)*hypergeom([1, (2+n)/n],[2+2/n],-e*x^n/d)/d/(2+n)+1/2*x^2
*ln(c*(d+e*x^n)^p)
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2455, 364}

$$\frac{1}{2}x^2\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -(e*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n, 2*(1 + n^(-1)), -((e*x^n)
/d)])/(2*d*(2 + n)) + (x^2*Log[c*(d + e*x^n)^p])/2
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\log\left(c\left(d+ex^n\right)^p\right)\,dx&=\frac{1}{2}x^2\log\left(c\left(d+ex^n\right)^p\right)-\frac{1}{2}(enp)\int\frac{x^{1+n}}{d+ex^n}\,dx\\&=-\frac{enpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2+n)}+\frac{1}{2}x^2\log\left(c\left(d+ex^n\right)^p\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.94

$$\frac{1}{2}x^2\left(\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^n\,{}_2F_1\left(1,\frac{n+2}{n};2+\frac{2}{n};-\frac{ex^n}{d}\right)}{d(n+2)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (x^2*(-((e*n*p*x^n*Hypergeometric2F1[1, (2 + n)/n, 2 + 2/n, -((e*x^n)/d)])/
(d*(2 + n))) + Log[c*(d + e*x^n)^p]))/2
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x\log\left((ex^n+d)^pc\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral(x*log((e*x^n + d)^p*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\log\left((ex^n+d)^pc\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate(x*log((e*x^n + d)^p*c), x)
```

maple [F] time = 1.29, size = 0, normalized size = 0.00

$$\int x\ln\left(c\,(e\,x^n+d)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(e*x^n+d)^p),x)
```

```
[Out] int(x*ln(c*(e*x^n+d)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$dnp\int\frac{x}{2\,(ex^n+d)}\,dx-\frac{1}{4}\left(np-2\,\log(c)\right)x^2+\frac{1}{2}\,x^2\log\left((ex^n+d)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] d*n*p*integrate(1/2*x/(e*x^n + d), x) - 1/4*(n*p - 2*log(c))*x^2 + 1/2*x^2*
log((e*x^n + d)^p)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\ln\left(c\,(d+e\,x^n)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(d + e*x^n)^p),x)
```

```
[Out] int(x*log(c*(d + e*x^n)^p), x)
```

sympy [C] time = 6.63, size = 104, normalized size = 1.60

$$\frac{x^2\log\left(c\,(d+ex^n)^p\right)}{2}-\frac{epx^2x^n\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{2d\Gamma\left(2+\frac{2}{n}\right)}-\frac{epx^2x^n\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{dn\Gamma\left(2+\frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(d+e*x**n)**p),x)
```

```
[Out] x**2*log(c*(d + e*x**n)**p)/2 - e*p*x**2*x**n*lerchphi(e*x**n*exp_polar(I*p
i)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(2*d*gamma(2 + 2/n)) - e*p*x**2*x**n*lerch
phi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(d*n*gamma(2 + 2/n
))
```

3.72 $\int \log\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=54

$$x\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}$$

```
[Out] -e*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)+x*ln(c*(d+e*x
^n)^p)
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2448, 364}

$$x\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d
)])/(d*(1 + n))) + x*Log[c*(d + e*x^n)^p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(c\left(d+ex^n\right)^p\right)\,dx &= x\log\left(c\left(d+ex^n\right)^p\right)-(enp)\int \frac{x^n}{d+ex^n}\,dx\\ &= -\frac{enpx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)}+x\log\left(c\left(d+ex^n\right)^p\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 0.96

$$x\left(\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^n\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p],x]
```

```
[Out] x*(-((e*n*p*x^n*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d)])
/(d*(1 + n))) + Log[c*(d + e*x^n)^p])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\log\left((ex^n + d)^p c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="fricas")
[Out] integral(log((e*x^n + d)^p*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left((ex^n + d)^p c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="giac")
[Out] integrate(log((e*x^n + d)^p*c), x)
```

maple [F] time = 1.04, size = 0, normalized size = 0.00

$$\int \ln\left(c\,(e\,x^n + d)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p),x)
[Out] int(ln(c*(e*x^n+d)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$dnp\int \frac{1}{ex^n + d}\,dx - \left(np - \log(c)\right)x + x\log\left((ex^n + d)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="maxima")
[Out] d*n*p*integrate(1/(e*x^n + d), x) - (n*p - log(c))*x + x*log((e*x^n + d)^p)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(c\,(d + e\,x^n)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p),x)
[Out] int(log(c*(d + e*x^n)^p), x)
```

sympy [C] time = 3.43, size = 48, normalized size = 0.89

$$x\log\left(c\,(d + ex^n)^p\right) + \frac{px\Phi\left(\frac{dx^{-n}e^{i\pi}}{e}, 1, \frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n\Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p),x)
[Out] x*log(c*(d + e*x**n)**p) + p*x*lerchphi(d*x**(-n)*exp_polar(I*pi)/e, 1, exp
_polar(I*pi)/n)*gamma(1/n)/(n*gamma(1 + 1/n))
```

3.73 $\int \frac{\log(c(d+ex^n)^p)}{x}\,dx$

Optimal. Leaf size=44

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{p\text{Li}_2\left(\frac{ex^n}{d}+1\right)}{n}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+p*polylog(2,1+e*x^n/d)/n
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2394, 2315}

$$\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (p*PolyLog[2, 1 + (e*x^n)/d])/
n
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{(ep)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,x^n\right)}{n}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 0.98

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)+p\text{Li}_2\left(\frac{ex^n+d}{d}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, (d + e*x^n)/d])/n
```

fricas [A] time = 0.48, size = 60, normalized size = 1.36

$$\frac{np\log\left(ex^n+d\right)\log(x)-np\log(x)\log\left(\frac{ex^n+d}{d}\right)+n\log(c)\log(x)-p\text{Li}_2\left(-\frac{ex^n+d}{d}+1\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] (n*p*log(e*x^n + d)*log(x) - n*p*log(x)*log((e*x^n + d)/d) + n*log(c)*log(x
) - p*dilog(-(e*x^n + d)/d + 1))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(\left(ex^n+d\right)^pc\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/x, x)
```

maple [C] time = 1.71, size = 177, normalized size = 4.02

$$-\frac{i\pi\,\text{csgn}\left(ic\right)\text{csgn}\left(i\left(e\,x^n+d\right)^p\right)\text{csgn}\left(ic\left(e\,x^n+d\right)^p\right)\ln(x)}{2}+\frac{i\pi\,\text{csgn}\left(ic\right)\text{csgn}\left(ic\left(e\,x^n+d\right)^p\right)^2\ln(x)}{2}+\frac{i\pi\,\text{csgn}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)/x,x)
```

```
[Out] ln(x)*ln((e*x^n+d)^p)+1/2*I*ln(x)*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)
^p)^2-1/2*I*ln(x)*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)-1/
2*I*ln(x)*Pi*csgn(I*c*(e*x^n+d)^p)^3+1/2*I*ln(x)*Pi*csgn(I*c*(e*x^n+d)^p)^2
*csgn(I*c)+ln(c)*ln(x)-p/n*dilog((e*x^n+d)/d)-p*ln(x)*ln((e*x^n+d)/d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$dnp\int\frac{\log(x)}{exx^n+dx}\,dx-\frac{1}{2}np\log(x)^2+\log\left(\left(ex^n+d\right)^p\right)\log(x)+\log(c)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] d*n*p*integrate(log(x)/(e*x*x^n + d*x), x) - 1/2*n*p*log(x)^2 + log((e*x^n
+ d)^p)*log(x) + log(c)*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/x,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\,(d+ex^n)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)/x, x)
```

3.74 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x^2}\,dx$

Optimal. Leaf size=66

$$-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{x}-\frac{enpx^{n-1}\,{}_2F_1\left(1,-\frac{1-n}{n};2-\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1-n)}$$

```
[Out] -e*n*p*x^(-1+n)*hypergeom([1, (-1+n)/n],[2-1/n],-e*x^n/d)/d/(1-n)-ln(c*(d+e
*x^n)^p)/x
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2455, 364}

$$-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{x}-\frac{enpx^{n-1}\,{}_2F_1\left(1,-\frac{1-n}{n};2-\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/x^2,x]
```

```
[Out] -((e*n*p*x^(-1 + n)*Hypergeometric2F1[1, -((1 - n)/n), 2 - n^(-1), -((e*x^n
)/d)])/(d*(1 - n))) - Log[c*(d + e*x^n)^p]/x
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x^2}\,dx&=-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{x}+(enp)\int \frac{x^{-2+n}}{d+ex^n}\,dx\\&=-\frac{enpx^{-1+n}\,{}_2F_1\left(1,-\frac{1-n}{n};2-\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1-n)}-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.03, size = 59, normalized size = 0.89

$$\frac{\frac{enpx^n\,{}_2F_1\left(1,\frac{n-1}{n};2-\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n-1)}-\log\left(c\left(d+ex^n\right)^p\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x^2,x]
```

```
[Out] ((e*n*p*x^n*Hypergeometric2F1[1, (-1 + n)/n, 2 - n^(-1), -((e*x^n)/d)])/(d*
(-1 + n)) - Log[c*(d + e*x^n)^p])/x
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left((ex^n+d)^p c\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left((ex^n+d)^p c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/x^2, x)
```

maple [F] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\,(e\,x^n+d)^p\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)/x^2,x)
```

```
[Out] int(ln(c*(e*x^n+d)^p)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-dnp\int \frac{1}{ex^2x^n+dx^2}\,dx - \frac{np+\log\left((ex^n+d)^p\right)+\log(c)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^2,x, algorithm="maxima")
```

```
[Out] -d*n*p*integrate(1/(e*x^2*x^n + d*x^2), x) - (n*p + log((e*x^n + d)^p) + lo
g(c))/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/x^2,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/x^2, x)
```

sympy [C] time = 7.71, size = 46, normalized size = 0.70

$$-\frac{\log\left(c\,(d+ex^n)^p\right)}{x} + \frac{p\Phi\left(\frac{dx^{-n}e^{i\pi}}{e}, 1, \frac{1}{n}\right)\Gamma\left(-\frac{1}{n}\right)}{nx\Gamma\left(1-\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x**2,x)
```

```
[Out] -log(c*(d + e*x**n)**p)/x + p*lerchphi(d*x**(-n)*exp_polar(I*pi)/e, 1, 1/n)
*gamma(-1/n)/(n*x*gamma(1 - 1/n))
```

3.75 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x^3}\,dx$

Optimal. Leaf size=72

$$-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{2x^2}-\frac{enpx^{n-2}\,{}_2F_1\left(1,-\frac{2-n}{n};2\left(1-\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2-n)}$$

```
[Out] -1/2*e*n*p*x^(-2+n)*hypergeom([1, (-2+n)/n],[2-2/n],-e*x^n/d)/d/(2-n)-1/2*l
n(c*(d+e*x^n)^p)/x^2
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2455, 364}

$$-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{2x^2}-\frac{enpx^{n-2}\,{}_2F_1\left(1,-\frac{2-n}{n};2\left(1-\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/x^3,x]
```

```
[Out] -(e*n*p*x^(-2 + n)*Hypergeometric2F1[1, -((2 - n)/n), 2*(1 - n^(-1)), -((e*
x^n)/d)])/(2*d*(2 - n)) - Log[c*(d + e*x^n)^p]/(2*x^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x^3}\,dx &= -\frac{\log\left(c\left(d+ex^n\right)^p\right)}{2x^2}+\frac{1}{2}(enp)\int \frac{x^{-3+n}}{d+ex^n}\,dx\\ &= -\frac{enpx^{-2+n}\,{}_2F_1\left(1,-\frac{2-n}{n};2\left(1-\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2-n)}-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.86

$$\frac{\frac{enpx^n\,{}_2F_1\left(1,\frac{n-2}{n};2-\frac{2}{n};-\frac{ex^n}{d}\right)}{d(n-2)}-\log\left(c\left(d+ex^n\right)^p\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x^3,x]
```

```
[Out] ((e*n*p*x^n*Hypergeometric2F1[1, (-2 + n)/n, 2 - 2/n, -((e*x^n)/d)])/(d*(-2
 + n)) - Log[c*(d + e*x^n)^p])/(2*x^2)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left((ex^n+d)^p c\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^3,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left((ex^n+d)^p c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^3,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/x^3, x)
```

maple [F] time = 1.00, size = 0, normalized size = 0.00

$$\int\frac{\ln\left(c\,(e\,x^n+d)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)/x^3,x)
```

```
[Out] int(ln(c*(e*x^n+d)^p)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-dnp\int\frac{1}{2\left(ex^3x^n+dx^3\right)}\,dx-\frac{np+2\,\log\left((ex^n+d)^p\right)+2\,\log(c)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^3,x, algorithm="maxima")
```

```
[Out] -d*n*p*integrate(1/2/(e*x^3*x^n + d*x^3), x) - 1/4*(n*p + 2*log((e*x^n + d)
^p) + 2*log(c))/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\ln\left(c\,(d+e\,x^n)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/x^3,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/x^3, x)
```

sympy [C] time = 15.29, size = 51, normalized size = 0.71

$$-\frac{\log\left(c\,(d+ex^n)^p\right)}{2x^2}+\frac{p\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{2}{n}\right)\Gamma\left(-\frac{2}{n}\right)}{nx^2\Gamma\left(1-\frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x**3,x)
```

```
[Out] -log(c*(d + e*x**n)**p)/(2*x**2) + p*lerchphi(d*x**(-n)*exp_polar(I*pi)/e,
1, 2/n)*gamma(-2/n)/(n*x**2*gamma(1 - 2/n))
```

3.76 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x^4} dx$

Optimal. Leaf size=70

$$-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{3x^3}-\frac{enpx^{n-3}\,{}_2F_1\left(1,-\frac{3-n}{n};2-\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(3-n)}$$

```
[Out] -1/3*e*n*p*x^(-3+n)*hypergeom([1, (-3+n)/n],[2-3/n],-e*x^n/d)/d/(3-n)-1/3*l
n(c*(d+e*x^n)^p)/x^3
```

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2455, 364}

$$-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{3x^3}-\frac{enpx^{n-3}\,{}_2F_1\left(1,-\frac{3-n}{n};2-\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/x^4,x]
```

```
[Out] -(e*n*p*x^(-3 + n)*Hypergeometric2F1[1, -((3 - n)/n), 2 - 3/n, -((e*x^n)/d)
])/(3*d*(3 - n)) - Log[c*(d + e*x^n)^p]/(3*x^3)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x^4}\,dx &= -\frac{\log\left(c\left(d+ex^n\right)^p\right)}{3x^3}+\frac{1}{3}(enp)\int \frac{x^{-4+n}}{d+ex^n}\,dx\\ &= -\frac{enpx^{-3+n}\,{}_2F_1\left(1,-\frac{3-n}{n};2-\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(3-n)}-\frac{\log\left(c\left(d+ex^n\right)^p\right)}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.89

$$\frac{\frac{enpx^n\,{}_2F_1\left(1,\frac{n-3}{n};2-\frac{3}{n};-\frac{ex^n}{d}\right)}{d(n-3)}-\log\left(c\left(d+ex^n\right)^p\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x^4,x]
```

```
[Out] ((e*n*p*x^n*Hypergeometric2F1[1, (-3 + n)/n, 2 - 3/n, -((e*x^n)/d)])/(d*(-3
 + n)) - Log[c*(d + e*x^n)^p])/(3*x^3)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left((ex^n + d)^p c\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^4,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left((ex^n + d)^p c\right)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^4,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/x^4, x)
```

maple [F] time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^n + d\right)^p\right)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)/x^4,x)
```

```
[Out] int(ln(c*(e*x^n+d)^p)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-dnp \int \frac{1}{3\left(ex^4x^n + dx^4\right)}\, dx - \frac{np + 3\,\log\left((ex^n + d)^p\right) + 3\,\log(c)}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -d*n*p*integrate(1/3/(e*x^4*x^n + d*x^4), x) - 1/9*(n*p + 3*log((e*x^n + d)
^p) + 3*log(c))/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(d + e\,x^n\right)^p\right)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/x^4,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/x^4, x)
```

sympy [C] time = 32.22, size = 51, normalized size = 0.73

$$-\frac{\log\left(c\left(d + ex^n\right)^p\right)}{3x^3} + \frac{p\Phi\left(\frac{dx^{-n}e^{i\pi}}{e}, 1, \frac{3}{n}\right)\Gamma\left(-\frac{3}{n}\right)}{nx^3\Gamma\left(1 - \frac{3}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x**4,x)
```

```
[Out] -log(c*(d + e*x**n)**p)/(3*x**3) + p*lerchphi(d*x**(-n)*exp_polar(I*pi)/e,
1, 3/n)*gamma(-3/n)/(n*x**3*gamma(1 - 3/n))
```

3.77 $\int x^5 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=215

$$\frac{a^3 p \log\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^3}-\frac{a^3p^2\log^2\left(a+bx^2\right)}{6b^3}-\frac{a^2p\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^3}+\frac{a^2p^2x^2}{b^2}-\frac{p\left(a+bx^2\right)^3}{}$$

```
[Out] a^2*p^2*x^2/b^2-1/4*a*p^2*(b*x^2+a)^2/b^3+1/27*p^2*(b*x^2+a)^3/b^3-1/6*a^3*
p^2*ln(b*x^2+a)^2/b^3-a^2*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b^3+1/2*a*p*(b*x^2+
a)^2*ln(c*(b*x^2+a)^p)/b^3-1/9*p*(b*x^2+a)^3*ln(c*(b*x^2+a)^p)/b^3+1/3*a^3*
p*ln(b*x^2+a)*ln(c*(b*x^2+a)^p)/b^3+1/6*x^6*ln(c*(b*x^2+a)^p)^2
```

Rubi [A] time = 0.30, antiderivative size = 175, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$-\frac{1}{18}p\left(\frac{18a^2\left(a+bx^2\right)}{b^3}-\frac{6a^3\log\left(a+bx^2\right)}{b^3}-\frac{9a\left(a+bx^2\right)^2}{b^3}+\frac{2\left(a+bx^2\right)^3}{b^3}\right)\log\left(c\left(a+bx^2\right)^p\right)+\frac{a^2p^2x^2}{b^2}-\frac{a^3p^2\log^2}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (a^2*p^2*x^2)/b^2 - (a*p^2*(a + b*x^2)^2)/(4*b^3) + (p^2*(a + b*x^2)^3)/(27
*b^3) - (a^3*p^2*Log[a + b*x^2]^2)/(6*b^3) - (p*((18*a^2*(a + b*x^2))/b^3 -
 (9*a*(a + b*x^2)^2)/b^3 + (2*(a + b*x^2)^3)/b^3 - (6*a^3*Log[a + b*x^2])/b
^3)*Log[c*(a + b*x^2)^p])/18 + (x^6*Log[c*(a + b*x^2)^p]^2)/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
```

```
] && EqQ[m, -1])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^5 \log^2\left(c\left(a+bx^2\right)^p\right) dx = \frac{1}{2}\text{Subst}\left(\int x^2 \log^2\left(c(a+bx)^p\right) dx, x, x^2\right)$$

$$= \frac{1}{6}x^6 \log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{3}(bp)\,\text{Subst}\left(\int \frac{x^3 \log\left(c(a+bx)^p\right)}{a+bx} dx, x, x^2\right)$$

$$= \frac{1}{6}x^6 \log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{3}p\,\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3 \log\left(cx^p\right)}{x} dx, x, a+bx^2\right)$$

$$= -\frac{1}{18}p\left(\frac{18a^2\left(a+bx^2\right)}{b^3} - \frac{9a\left(a+bx^2\right)^2}{b^3} + \frac{2\left(a+bx^2\right)^3}{b^3} - \frac{6a^3\log\left(a+bx^2\right)}{b^3}\right)\log$$

$$= -\frac{1}{18}p\left(\frac{18a^2\left(a+bx^2\right)}{b^3} - \frac{9a\left(a+bx^2\right)^2}{b^3} + \frac{2\left(a+bx^2\right)^3}{b^3} - \frac{6a^3\log\left(a+bx^2\right)}{b^3}\right)\log$$

$$= -\frac{1}{18}p\left(\frac{18a^2\left(a+bx^2\right)}{b^3} - \frac{9a\left(a+bx^2\right)^2}{b^3} + \frac{2\left(a+bx^2\right)^3}{b^3} - \frac{6a^3\log\left(a+bx^2\right)}{b^3}\right)\log$$

$$= \frac{a^2p^2x^2}{b^2} - \frac{ap^2\left(a+bx^2\right)^2}{4b^3} + \frac{p^2\left(a+bx^2\right)^3}{27b^3} - \frac{1}{18}p\left(\frac{18a^2\left(a+bx^2\right)}{b^3} - \frac{9a\left(a+bx^2\right)}{b^3}\right.$$

$$= \frac{a^2p^2x^2}{b^2} - \frac{ap^2\left(a+bx^2\right)^2}{4b^3} + \frac{p^2\left(a+bx^2\right)^3}{27b^3} - \frac{a^3p^2\log^2\left(a+bx^2\right)}{6b^3} - \frac{1}{18}p\left(\frac{18a^2\left(a\right.}{b}\right.$$

Mathematica [A] time = 0.07, size = 200, normalized size = 0.93

$$\frac{a^3\log^2\left(c\left(a+bx^2\right)^p\right)}{6b^3}-\frac{a^3p\log\left(c\left(a+bx^2\right)^p\right)}{3b^3}-\frac{5a^3p^2\log\left(a+bx^2\right)}{18b^3}-\frac{a^2px^2\log\left(c\left(a+bx^2\right)^p\right)}{3b^2}+\frac{11a^2p^2x^2}{18b^2}+\frac{1}{6}x^6\log$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (11*a^2*p^2*x^2)/(18*b^2) - (5*a*p^2*x^4)/(36*b) + (p^2*x^6)/27 - (5*a^3*p^
2*Log[a + b*x^2])/(18*b^3) - (a^3*p*Log[c*(a + b*x^2)^p])/(3*b^3) - (a^2*p*
x^2*Log[c*(a + b*x^2)^p])/(3*b^2) + (a*p*x^4*Log[c*(a + b*x^2)^p])/(6*b) -
(p*x^6*Log[c*(a + b*x^2)^p])/9 + (a^3*Log[c*(a + b*x^2)^p]^2)/(6*b^3) + (x^
6*Log[c*(a + b*x^2)^p]^2)/6
```

fricas [A] time = 0.46, size = 189, normalized size = 0.88

$$\frac{4\,b^3p^2x^6+18\,b^3x^6\log(c)^2-15\,ab^2p^2x^4+66\,a^2bp^2x^2+18\left(b^3p^2x^6+a^3p^2\right)\log\left(bx^2+a\right)^2-6\left(2\,b^3p^2x^6-3\,ab^2p^2\right.}{108\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/108*(4*b^3*p^2*x^6 + 18*b^3*x^6*log(c)^2 - 15*a*b^2*p^2*x^4 + 66*a^2*b*p^
2*x^2 + 18*(b^3*p^2*x^6 + a^3*p^2)*log(b*x^2 + a)^2 - 6*(2*b^3*p^2*x^6 - 3*
a*b^2*p^2*x^4 + 6*a^2*b*p^2*x^2 + 11*a^3*p^2 - 6*(b^3*p*x^6 + a^3*p)*log(c)
)*log(b*x^2 + a) - 6*(2*b^3*p*x^6 - 3*a*b^2*p*x^4 + 6*a^2*b*p*x^2)*log(c))/
b^3
```

giac [A] time = 0.19, size = 325, normalized size = 1.51

$$18\,bx^6\log(c)^2+\left(\frac{18\left(bx^2+a\right)^3\log\left(bx^2+a\right)^2}{b^2}-\frac{54\left(bx^2+a\right)^2a\log\left(bx^2+a\right)^2}{b^2}+\frac{54\left(bx^2+a\right)a^2\log\left(bx^2+a\right)^2}{b^2}-\frac{12\left(bx^2+a\right)^3\log\left(bx^2+a\right)}{b^2}+\frac{54\left(bx^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] 1/108*(18*b*x^6*log(c)^2 + (18*(b*x^2 + a)^3*log(b*x^2 + a)^2/b^2 - 54*(b*x
^2 + a)^2*a*log(b*x^2 + a)^2/b^2 + 54*(b*x^2 + a)*a^2*log(b*x^2 + a)^2/b^2
- 12*(b*x^2 + a)^3*log(b*x^2 + a)/b^2 + 54*(b*x^2 + a)^2*a*log(b*x^2 + a)/b
^2 - 108*(b*x^2 + a)*a^2*log(b*x^2 + a)/b^2 + 4*(b*x^2 + a)^3/b^2 - 27*(b*x
^2 + a)^2*a/b^2 + 108*(b*x^2 + a)*a^2/b^2)*p^2 + 6*(6*(b*x^2 + a)^3*log(b*x
^2 + a)/b^2 - 18*(b*x^2 + a)^2*a*log(b*x^2 + a)/b^2 + 18*(b*x^2 + a)*a^2*lo
g(b*x^2 + a)/b^2 - 2*(b*x^2 + a)^3/b^2 + 9*(b*x^2 + a)^2*a/b^2 - 18*(b*x^2
+ a)*a^2/b^2)*p*log(c))/b
```

maple [C] time = 0.52, size = 1436, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] 1/18*(3*I*Pi*b^3*x^6*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-3*I*Pi*b^3
*x^6*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-3*I*Pi*b^3*x^6*csg
n(I*c*(b*x^2+a)^p)^3+3*I*Pi*b^3*x^6*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+6*ln(
c)*b^3*x^6-2*b^3*p*x^6+3*a*b^2*p*x^4-6*a^2*b*p*x^2+6*a^3*p*ln(b*x^2+a))/b^3
```

```
*ln((b*x^2+a)^p)-1/6*a^3*p^2*ln(b*x^2+a)^2/b^3-1/9*ln(c)*p*x^6-1/24*Pi^2*x^
6*csgn(I*c*(b*x^2+a)^p)^6-11/18*a^3*p^2/b^3*ln(b*x^2+a)+1/27*p^2*x^6+11/18*
a^2*p^2*x^2/b^2+1/6*ln(c)^2*x^6-5/36/b*a*p^2*x^4-1/24*Pi^2*x^6*csgn(I*c*(b*
x^2+a)^p)^4*csgn(I*c)^2+1/12*Pi^2*x^6*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)-1/2
4*Pi^2*x^6*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4+1/12*Pi^2*x^6*csgn
(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5+1/6*x^6*ln((b*x^2+a)^p)^2+1/6/b*ln(
c)*a*p*x^4-1/24*Pi^2*x^6*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn
(I*c)^2+1/12*Pi^2*x^6*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)
^2+1/12*Pi^2*x^6*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)-1/
6*Pi^2*x^6*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)-1/3/b^2*ln
(c)*a^2*p*x^2+1/3/b^3*ln(c)*ln(b*x^2+a)*a^3*p-1/6*I*ln(c)*Pi*x^6*csgn(I*c*(
b*x^2+a)^p)^3+1/18*I*Pi*p*x^6*csgn(I*c*(b*x^2+a)^p)^3+1/6*I*ln(c)*Pi*x^6*cs
gn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/6*I*ln(c)*Pi*x^6*csgn(I*(b*x^2+a)^p)*csgn
(I*c*(b*x^2+a)^p)^2-1/18*I*Pi*p*x^6*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/18*
I*Pi*p*x^6*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/12*I/b*Pi*a*p*x^4*
csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/6*I/b^2*Pi*a^2*p*x^2*
csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/6*I/b^3*Pi*ln(b*x^2+a
)*a^3*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/12*I/b*Pi*a*p
*x^4*csgn(I*c*(b*x^2+a)^p)^3+1/6*I/b^2*Pi*a^2*p*x^2*csgn(I*c*(b*x^2+a)^p)^3
-1/6*I/b^3*Pi*ln(b*x^2+a)*a^3*p*csgn(I*c*(b*x^2+a)^p)^3-1/6*I*ln(c)*Pi*x^6*
csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/18*I*Pi*p*x^6*csgn(I*
(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/12*I/b*Pi*a*p*x^4*csgn(I*c*(
b*x^2+a)^p)^2*csgn(I*c)+1/12*I/b*Pi*a*p*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b
*x^2+a)^p)^2-1/6*I/b^2*Pi*a^2*p*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/6*I
/b^2*Pi*a^2*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/6*I/b^3*Pi*
ln(b*x^2+a)*a^3*p*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/6*I/b^3*Pi*ln(b*x^2+a
)*a^3*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2
```

maxima [A] time = 0.70, size = 145, normalized size = 0.67

$$\frac{1}{6}\,x^6\log\left(\left(bx^2+a\right)^p c\right)^2+\frac{1}{18}\,bp\left(\frac{6\,a^3\log\left(bx^2+a\right)}{b^4}-\frac{2\,b^2x^6-3\,abx^4+6\,a^2x^2}{b^3}\right)\log\left(\left(bx^2+a\right)^p c\right)+\frac{\left(4\,b^3x^6-15\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log((b*x^2 + a)^p*c)^2 + 1/18*b*p*(6*a^3*log(b*x^2 + a)/b^4 - (2*b^
2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)/b^3)*log((b*x^2 + a)^p*c) + 1/108*(4*b^3*x^6
 - 15*a*b^2*x^4 + 66*a^2*b*x^2 - 18*a^3*log(b*x^2 + a)^2 - 66*a^3*log(b*x^2
 + a))*p^2/b^3
```

mupad [B] time = 0.31, size = 126, normalized size = 0.59

$$\frac{p^2\,x^6}{27}+\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2\,\left(\frac{x^6}{6}+\frac{a^3}{6\,b^3}\right)-\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)\,\left(\frac{p\,x^6}{9}+\frac{a^2\,p\,x^2}{3\,b^2}-\frac{a\,p\,x^4}{6\,b}\right)-\frac{5\,a\,p^2\,x^4}{36\,b}-\frac{11\,a^3\,p^2\,\ln\left(b\,x^2\right.}{18\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] (p^2*x^6)/27 + log(c*(a + b*x^2)^p)^2*(x^6/6 + a^3/(6*b^3)) - log(c*(a + b*
x^2)^p)*((p*x^6)/9 + (a^2*p*x^2)/(3*b^2) - (a*p*x^4)/(6*b)) - (5*a*p^2*x^4)
/(36*b) - (11*a^3*p^2*log(a + b*x^2))/(18*b^3) + (11*a^2*p^2*x^2)/(18*b^2)
```

sympy [A] time = 22.72, size = 267, normalized size = 1.24

$$\begin{cases}\frac{a^3p^2\log\left(a+bx^2\right)^2}{6b^3}-\frac{11a^3p^2\log\left(a+bx^2\right)}{18b^3}+\frac{a^3p\log\left(c\right)\log\left(a+bx^2\right)}{3b^3}-\frac{a^2p^2x^2\log\left(a+bx^2\right)}{3b^2}+\frac{11a^2p^2x^2}{18b^2}-\frac{a^2px^2\log\left(c\right)}{3b^2}+\frac{ap^2x^4\log\left(a+bx^2\right)}{6b} \\ \frac{x^6\log\left(a^pc\right)^2}{6}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Piecewise((a**3*p**2*log(a + b*x**2)**2/(6*b**3) - 11*a**3*p**2*log(a + b*x
**2)/(18*b**3) + a**3*p*log(c)*log(a + b*x**2)/(3*b**3) - a**2*p**2*x**2*lo
g(a + b*x**2)/(3*b**2) + 11*a**2*p**2*x**2/(18*b**2) - a**2*p*x**2*log(c)/(
3*b**2) + a*p**2*x**4*log(a + b*x**2)/(6*b) - 5*a*p**2*x**4/(36*b) + a*p*x*
*4*log(c)/(6*b) + p**2*x**6*log(a + b*x**2)**2/6 - p**2*x**6*log(a + b*x**2
)/9 + p**2*x**6/27 + p*x**6*log(c)*log(a + b*x**2)/3 - p*x**6*log(c)/9 + x*
*6*log(c)**2/6, Ne(b, 0)), (x**6*log(a**p*c)**2/6, True))
```

3.78 $\int x^3 \log^2\left(c\left(a+bx^2\right)^p\right) dx$

Optimal. Leaf size=145

$$\frac{\left(a+bx^2\right)^2\log^2\left(c\left(a+bx^2\right)^p\right)}{4b^2}-\frac{a\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2}-\frac{p\left(a+bx^2\right)^2\log\left(c\left(a+bx^2\right)^p\right)}{4b^2}+\frac{ap\left(a+bx^2\right)\mathrm{l}}{}$$

```
[Out] -a*p^2*x^2/b+1/8*p^2*(b*x^2+a)^2/b^2+a*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b^2-1/
4*p*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)/b^2-1/2*a*(b*x^2+a)*ln(c*(b*x^2+a)^p)^2/b
^2+1/4*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^2/b^2
```

Rubi [A] time = 0.15, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{\left(a+bx^2\right)^2\log^2\left(c\left(a+bx^2\right)^p\right)}{4b^2}-\frac{a\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2}-\frac{p\left(a+bx^2\right)^2\log\left(c\left(a+bx^2\right)^p\right)}{4b^2}+\frac{ap\left(a+bx^2\right)\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] -((a*p^2*x^2)/b) + (p^2*(a + b*x^2)^2)/(8*b^2) + (a*p*(a + b*x^2)*Log[c*(a
+ b*x^2)^p])/b^2 - (p*(a + b*x^2)^2*Log[c*(a + b*x^2)^p])/(4*b^2) - (a*(a +
 b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*b^2) + ((a + b*x^2)^2*Log[c*(a + b*x^2)^
p]^2)/(4*b^2)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x\log^2\left(c(a+bx)^p\right)\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a\log^2\left(c(a+bx)^p\right)}{b}+\frac{(a+bx)\log^2\left(c(a+bx)^p\right)}{b}\right)dx,x,x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int(a+bx)\log^2\left(c(a+bx)^p\right)\,dx,x,x^2\right)}{2b}-\frac{a\operatorname{Subst}\left(\int\log^2\left(c(a+bx)^p\right)\,dx,x,\right.}{2b}\\
&= \frac{\operatorname{Subst}\left(\int x\log^2\left(cx^p\right)\,dx,x,a+bx^2\right)}{2b^2}-\frac{a\operatorname{Subst}\left(\int\log^2\left(cx^p\right)\,dx,x,a+bx^2\right)}{2b^2}\\
&= -\frac{a\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2}+\frac{\left(a+bx^2\right)^2\log^2\left(c\left(a+bx^2\right)^p\right)}{4b^2}-\frac{p\operatorname{Subst}\left(\int x\,l\right.}{}\\
&= -\frac{ap^2x^2}{b}+\frac{p^2\left(a+bx^2\right)^2}{8b^2}+\frac{ap\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^2}-\frac{p\left(a+bx^2\right)^2\log\left(c\left(a\right.\right.}{4b^2}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 105, normalized size = 0.72

$$\frac{-2\left(a^2-b^2x^4\right)\log^2\left(c\left(a+bx^2\right)^p\right)+2p\left(2a^2+2abx^2-b^2x^4\right)\log\left(c\left(a+bx^2\right)^p\right)+2a^2p^2\log\left(a+bx^2\right)+bp^2x^2\left(b\right.}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (b*p^2*x^2*(-6*a + b*x^2) + 2*a^2*p^2*Log[a + b*x^2] + 2*p*(2*a^2 + 2*a*b*x
^2 - b^2*x^4)*Log[c*(a + b*x^2)^p] - 2*(a^2 - b^2*x^4)*Log[c*(a + b*x^2)^p]
^2)/(8*b^2)
```

fricas [A] time = 0.48, size = 148, normalized size = 1.02

$$\frac{b^2p^2x^4+2\,b^2x^4\log(c)^2-6\,abp^2x^2+2\left(b^2p^2x^4-a^2p^2\right)\log\left(bx^2+a\right)^2-2\left(b^2p^2x^4-2\,abp^2x^2-3\,a^2p^2-2\left(b^2px^4\right.\right.}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*p^2*x^4 + 2*b^2*x^4*log(c)^2 - 6*a*b*p^2*x^2 + 2*(b^2*p^2*x^4 - a^
2*p^2)*log(b*x^2 + a)^2 - 2*(b^2*p^2*x^4 - 2*a*b*p^2*x^2 - 3*a^2*p^2 - 2*(b
^2*p*x^4 - a^2*p)*log(c))*log(b*x^2 + a) - 2*(b^2*p*x^4 - 2*a*b*p*x^2)*log(
c))/b^2
```

giac [A] time = 0.19, size = 207, normalized size = 1.43

$$\frac{\frac{\left(2\left(bx^2+a\right)^2\log\left(bx^2+a\right)^2-4\left(bx^2+a\right)a\log\left(bx^2+a\right)^2-2\left(bx^2+a\right)^2\log\left(bx^2+a\right)+8\left(bx^2+a\right)a\log\left(bx^2+a\right)+\left(bx^2+a\right)^2-8\left(bx^2+a\right)a\right)p^2}{b}+\frac{2\left(2\left(bx^2+a\right)^2\mathrm{lo}\right.}{}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] 1/8*((2*(b*x^2 + a)^2*log(b*x^2 + a)^2 - 4*(b*x^2 + a)*a*log(b*x^2 + a)^2 -
 2*(b*x^2 + a)^2*log(b*x^2 + a) + 8*(b*x^2 + a)*a*log(b*x^2 + a) + (b*x^2 +
 a)^2 - 8*(b*x^2 + a)*a)*p^2/b + 2*(2*(b*x^2 + a)^2*log(b*x^2 + a) - 4*(b*x
^2 + a)*a*log(b*x^2 + a) - (b*x^2 + a)^2 + 4*(b*x^2 + a)*a)*p*log(c)/b + 2*
((b*x^2 + a)^2 - 2*(b*x^2 + a)*a)*log(c)^2/b)/b
```

maple [C] time = 0.47, size = 1242, normalized size = 8.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] -1/4*ln(c)*p*x^4-1/16*Pi^2*x^4*csgn(I*c*(b*x^2+a)^p)^6+1/4*(I*Pi*b^2*x^4*cs
gn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*b^2*x^4*csgn(I*(b*x^2+a)^p)*
csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*b^2*x^4*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*b
^2*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)*b^2*x^4-b^2*p*x^4+2*a*b*p*
x^2-2*a^2*p*ln(b*x^2+a))/b^2*ln((b*x^2+a)^p)+1/8*x^4*p^2+3/4*a^2*p^2/b^2*ln
(b*x^2+a)-3/4*a*p^2*x^2/b+1/4/b^2*a^2*p^2*ln(b*x^2+a)^2-1/16*Pi^2*x^4*csgn(
I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4+1/8*Pi^2*x^4*csgn(I*(b*x^2+a)^p)*c
sgn(I*c*(b*x^2+a)^p)^5+1/8*Pi^2*x^4*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)-1/16*
Pi^2*x^4*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2+1/4*x^4*ln((b*x^2+a)^p)^2+1/4*
I/b^2*Pi*ln(b*x^2+a)*a^2*p*csgn(I*c*(b*x^2+a)^p)^3-1/4*I*ln(c)*Pi*x^4*csgn(
I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/8*I*Pi*p*x^4*csgn(I*(b*x^2
+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/4*I/b*Pi*a*p*x^2*csgn(I*c*(b*x^2+a
)^p)^3+1/4*ln(c)^2*x^4+1/8*Pi^2*x^4*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a
)^p)^3*csgn(I*c)-1/16*Pi^2*x^4*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^
2*csgn(I*c)^2-1/4*Pi^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn
(I*c)+1/8*Pi^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2+
1/2/b*ln(c)*a*p*x^2-1/2/b^2*ln(c)*ln(b*x^2+a)*a^2*p-1/4*I*ln(c)*Pi*x^4*csgn
(I*c*(b*x^2+a)^p)^3+1/8*I*Pi*p*x^4*csgn(I*c*(b*x^2+a)^p)^3+1/4*I*ln(c)*Pi*x
^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/8*I*Pi*p*x^4*csgn(I*(b*x^2
+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/4*I*ln(c)*Pi*x^4*csgn(I*c*(b*x^2+a)^p)^2*c
sgn(I*c)-1/8*I*Pi*p*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/4*I/b*Pi*a*p*x^
2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/4*I/b*Pi*a*p*x^2*csgn(I*c*(
b*x^2+a)^p)^2*csgn(I*c)-1/4*I/b^2*Pi*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)*
csgn(I*c*(b*x^2+a)^p)^2-1/4*I/b^2*Pi*ln(b*x^2+a)*a^2*p*csgn(I*c*(b*x^2+a)^p
)^2*csgn(I*c)-1/4*I/b*Pi*a*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*
csgn(I*c)+1/4*I/b^2*Pi*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^
2+a)^p)*csgn(I*c)
```

maxima [A] time = 0.77, size = 120, normalized size = 0.83

$$\frac{1}{4}x^4\log\left(\left(bx^2+a\right)^pc\right)^2-\frac{1}{4}bp\left(\frac{2a^2\log\left(bx^2+a\right)}{b^3}+\frac{bx^4-2ax^2}{b^2}\right)\log\left(\left(bx^2+a\right)^pc\right)+\frac{\left(b^2x^4-6abx^2+2a^2\log\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((b*x^2 + a)^p*c)^2 - 1/4*b*p*(2*a^2*log(b*x^2 + a)/b^3 + (b*x^4
 - 2*a*x^2)/b^2)*log((b*x^2 + a)^p*c) + 1/8*(b^2*x^4 - 6*a*b*x^2 + 2*a^2*lo
g(b*x^2 + a)^2 + 6*a^2*log(b*x^2 + a))*p^2/b^2
```

mupad [B] time = 0.26, size = 100, normalized size = 0.69

$$\frac{p^2\,x^4}{8}-\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)\,\left(\frac{p\,x^4}{4}-\frac{a\,p\,x^2}{2\,b}\right)+{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}^2\,\left(\frac{x^4}{4}-\frac{a^2}{4\,b^2}\right)-\frac{3\,a\,p^2\,x^2}{4\,b}+\frac{3\,a^2\,p^2\,\ln\left(b\,x^2+a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] (p^2*x^4)/8 - log(c*(a + b*x^2)^p)*((p*x^4)/4 - (a*p*x^2)/(2*b)) + log(c*(a
 + b*x^2)^p)^2*(x^4/4 - a^2/(4*b^2)) - (3*a*p^2*x^2)/(4*b) + (3*a^2*p^2*log
(a + b*x^2))/(4*b^2)
```

sympy [A] time = 8.64, size = 209, normalized size = 1.44

$$\begin{cases} - \frac{a^{2} p^{2} \log{\left(a + b x^{2} \right)}^{2}}{4 b^{2}} + \frac{3 a^{2} p^{2} \log{\left(a + b x^{2} \right)}}{4 b^{2}} - \frac{a^{2} p \log{\left(c \right)} \log{\left(a + b x^{2} \right)}}{2 b^{2}} + \frac{a p^{2} x^{2} \log{\left(a + b x^{2} \right)}}{2 b} - \frac{3 a p^{2} x^{2}}{4 b} + \frac{a p x^{2} \log{\left(c \right)}}{2 b} + \frac{p^{2} x^{4} \log{\left(a + b x^{2} \right)}^{2}}{4} - p^{2} \\ \frac{x^{4} \log{\left(a^{p} c \right)}^{2}}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Piecewise((-a**2*p**2*log(a + b*x**2)**2/(4*b**2) + 3*a**2*p**2*log(a + b*x
**2)/(4*b**2) - a**2*p*log(c)*log(a + b*x**2)/(2*b**2) + a*p**2*x**2*log(a
+ b*x**2)/(2*b) - 3*a*p**2*x**2/(4*b) + a*p*x**2*log(c)/(2*b) + p**2*x**4*l
og(a + b*x**2)**2/4 - p**2*x**4*log(a + b*x**2)/4 + p**2*x**4/8 + p*x**4*lo
g(c)*log(a + b*x**2)/2 - p*x**4*log(c)/4 + x**4*log(c)**2/4, Ne(b, 0)), (x*
*4*log(a**p*c)**2/4, True))
```

3.79 $\int x \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=61

$$\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b}-\frac{p\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}+p^2x^2$$

```
[Out] p^2*x^2-p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b+1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^2/b
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2454, 2389, 2296, 2295}

$$\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b}-\frac{p\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}+p^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] p^2*x^2 - (p*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b + ((a + b*x^2)*Log[c*(a + 
b*x^2)^p]^2)/(2*b)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\log^2\left(c\left(a+bx^2\right)^p\right)dx=\frac{1}{2}\text{Subst}\left(\int\log^2\left(c(a+bx)^p\right)dx,x,x^2\right)$$
$$=\frac{\text{Subst}\left(\int\log^2\left(cx^p\right)dx,x,a+bx^2\right)}{2b}$$
$$=\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b}-\frac{p\,\text{Subst}\left(\int\log\left(cx^p\right)dx,x,a+bx^2\right)}{b}$$
$$=p^2x^2-\frac{p\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}+\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b}$$

Mathematica [A] time = 0.01, size = 63, normalized size = 1.03

$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{b}-2p\left(\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}-px^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (((a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/b - 2*p*(-(p*x^2) + ((a + b*x^2)*Log[
c*(a + b*x^2)^p])/b))/2
```

fricas [A] time = 0.48, size = 96, normalized size = 1.57

$$\frac{2\,bp^2x^2-2\,bpx^2\log(c)+bx^2\log(c)^2+\left(bp^2x^2+ap^2\right)\log\left(bx^2+a\right)^2-2\left(bp^2x^2+ap^2-\left(bpx^2+ap\right)\log(c)\right)\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*p^2*x^2 - 2*b*p*x^2*log(c) + b*x^2*log(c)^2 + (b*p^2*x^2 + a*p^2)*
log(b*x^2 + a)^2 - 2*(b*p^2*x^2 + a*p^2 - (b*p*x^2 + a*p)*log(c))*log(b*x^2
 + a))/b
```

giac [A] time = 0.19, size = 96, normalized size = 1.57

$$\frac{\left(2\,bx^2+\left(bx^2+a\right)\log\left(bx^2+a\right)^2-2\left(bx^2+a\right)\log\left(bx^2+a\right)+2\,a\right)p^2-2\left(bx^2-\left(bx^2+a\right)\log\left(bx^2+a\right)+a\right)p\log(c)+\left(bx^2+a\right)\log(c)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*b*x^2 + (b*x^2 + a)*log(b*x^2 + a)^2 - 2*(b*x^2 + a)*log(b*x^2 + a)
 + 2*a)*p^2 - 2*(b*x^2 - (b*x^2 + a)*log(b*x^2 + a) + a)*p*log(c) + (b*x^2
+ a)*log(c)^2)/b
```

maple [C] time = 0.53, size = 1034, normalized size = 16.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] -1/8*Pi^2*x^2*csgn(I*c*(b*x^2+a)^p)^6-ln(c)*p*x^2+1/2*(I*Pi*b*x^2*csgn(I*(b
*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)*csgn(I*c)-I*Pi*b*x^2*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*b*x^2*csgn(I
*c*(b*x^2+a)^p)^2*csgn(I*c)+2*b*x^2*ln(c)-2*b*p*x^2+2*a*p*ln(b*x^2+a))/b*ln
((b*x^2+a)^p)-1/2/b*a*p^2*ln(b*x^2+a)^2-1/8*Pi^2*x^2*csgn(I*(b*x^2+a)^p)^2*
csgn(I*c*(b*x^2+a)^p)^4+1/4*Pi^2*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)^5+1/4*Pi^2*x^2*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)+1/2*x^2*ln((b*x^2+a)^p
)^2-a*p^2/b*ln(b*x^2+a)-1/8*Pi^2*x^2*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2+p^
2*x^2+1/2*I/b*Pi*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^
2+1/2*I/b*Pi*ln(b*x^2+a)*a*p*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/4*Pi^2*x^2
*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)+1/2*ln(c)^2*x^2+1/
2*I*ln(c)*Pi*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/2*I*Pi*p*x^2
*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/2*I*ln(c)*Pi*x^2*csgn(I*c*(b
*x^2+a)^p)^2*csgn(I*c)-1/2*I*Pi*p*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/8
*Pi^2*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2+1/b*ln(
c)*ln(b*x^2+a)*a*p-1/2*Pi^2*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4
*csgn(I*c)+1/4*Pi^2*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*
c)^2-1/2*I*ln(c)*Pi*x^2*csgn(I*c*(b*x^2+a)^p)^3+1/2*I*Pi*p*x^2*csgn(I*c*(b*
x^2+a)^p)^3-1/2*I/b*Pi*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+
a)^p)*csgn(I*c)+1/2*I*Pi*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*cs
gn(I*c)-1/2*I*ln(c)*Pi*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I
*c)-1/2*I/b*Pi*ln(b*x^2+a)*a*p*csgn(I*c*(b*x^2+a)^p)^3
```

maxima [A] time = 0.73, size = 97, normalized size = 1.59

$$-bp\left(\frac{x^2}{b} - \frac{a\log\left(bx^2+a\right)}{b^2}\right)\log\left(\left(bx^2+a\right)^p c\right) + \frac{1}{2}x^2\log\left(\left(bx^2+a\right)^p c\right)^2 + \frac{\left(2\,bx^2 - a\log\left(bx^2+a\right)^2 - 2\,a\log\left(bx^2\right.\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -b*p*(x^2/b - a*log(b*x^2 + a)/b^2)*log((b*x^2 + a)^p*c) + 1/2*x^2*log((b*x
^2 + a)^p*c)^2 + 1/2*(2*b*x^2 - a*log(b*x^2 + a)^2 - 2*a*log(b*x^2 + a))*p^
2/b
```

mupad [B] time = 0.23, size = 70, normalized size = 1.15

$$p^2\,x^2 + \ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2\,\left(\frac{a}{2\,b} + \frac{x^2}{2}\right) - p\,x^2\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right) - \frac{a\,p^2\,\ln\left(b\,x^2+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] p^2*x^2 + log(c*(a + b*x^2)^p)^2*(a/(2*b) + x^2/2) - p*x^2*log(c*(a + b*x^2
)^p) - (a*p^2*log(a + b*x^2))/b
```

sympy [A] time = 3.13, size = 139, normalized size = 2.28

$$\begin{cases} \frac{ap^2\log\left(a+bx^2\right)^2}{2b} - \frac{ap^2\log\left(a+bx^2\right)}{b} + \frac{ap\log\left(c\right)\log\left(a+bx^2\right)}{b} + \frac{p^2x^2\log\left(a+bx^2\right)^2}{2} - p^2x^2\log\left(a+bx^2\right) + p^2x^2 + px^2\log\left(c\right)\mathrm{lo} \\ \frac{x^2\log\left(a^pc\right)^2}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Piecewise((a*p**2*log(a + b*x**2)**2/(2*b) - a*p**2*log(a + b*x**2)/b + a*p
*log(c)*log(a + b*x**2)/b + p**2*x**2*log(a + b*x**2)**2/2 - p**2*x**2*log(
a + b*x**2) + p**2*x**2 + p*x**2*log(c)*log(a + b*x**2) - p*x**2*log(c) + x
**2*log(c)**2/2, Ne(b, 0)), (x**2*log(a**p*c)**2/2, True))
```

3.80 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x} \, dx$

Optimal. Leaf size=72

$$p\text{Li}_2\left(\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)+\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)+p^2\left(-\text{Li}_3\left(\frac{bx^2}{a}+1\right)\right)$$

```
[Out] 1/2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)^2+p*ln(c*(b*x^2+a)^p)*polylog(2,1+b*x^2/
a)-p^2*polylog(3,1+b*x^2/a)
```

Rubi [A] time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2454, 2396, 2433, 2374, 6589}

$$p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)+p^2\left(-\text{PolyLog}\left(3,\frac{bx^2}{a}+1\right)\right)+\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x,x]
```

```
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p]^2)/2 + p*Log[c*(a + b*x^2)^p]*PolyL
og[2, 1 + (b*x^2)/a] - p^2*PolyLog[3, 1 + (b*x^2)/a]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right) - (bp)\operatorname{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{a+bx}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right) - p\operatorname{Subst}\left(\int \frac{\log\left(cx^p\right)\log\left(-\frac{b\left(-\frac{a}{b}+\frac{x}{b}\right)}{a}\right)}{x}\,dx, x, a+bx^2\right)$$
$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right) + p\log\left(c\left(a+bx^2\right)^p\right)\operatorname{Li}_2\left(1+\frac{bx^2}{a}\right) - p^2\operatorname{Subst}\left(\int$$
$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right) + p\log\left(c\left(a+bx^2\right)^p\right)\operatorname{Li}_2\left(1+\frac{bx^2}{a}\right) - p^2\operatorname{Li}_3\left(1+\frac{b}{}\right.$$

Mathematica [B] time = 0.07, size = 163, normalized size = 2.26

$$2p\left(\log(x)\left(\log\left(a+bx^2\right)-\log\left(\frac{bx^2}{a}+1\right)\right)-\frac{1}{2}\operatorname{Li}_2\left(-\frac{bx^2}{a}\right)\right)\left(\log\left(c\left(a+bx^2\right)^p\right)-p\log\left(a+bx^2\right)\right)+\log(x)\left(\log\left(c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x,x]
```

```
[Out] Log[x]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2 + 2*p*(-(p*Log[a + b*
x^2]) + Log[c*(a + b*x^2)^p])*(Log[x]*(Log[a + b*x^2] - Log[1 + (b*x^2)/a])
 - PolyLog[2, -((b*x^2)/a)]/2) + (p^2*(Log[-((b*x^2)/a)]*Log[a + b*x^2]^2 +
 2*Log[a + b*x^2]*PolyLog[2, 1 + (b*x^2)/a] - 2*PolyLog[3, 1 + (b*x^2)/a]))
/2
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x, x)
```

maple [F] time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x,x)
[Out] int(ln(c*(b*x^2+a)^p)^2/x,x)
```

maxima [A] time = 0.75, size = 118, normalized size = 1.64

$$\frac{1}{2}\left(\log\left(bx^{2}+a\right)^{2}\log\left(-\frac{bx^{2}+a}{a}+1\right)+2\,\mathrm{Li}_{2}\left(\frac{bx^{2}+a}{a}\right)\log\left(bx^{2}+a\right)-2\,\mathrm{Li}_{3}(\frac{bx^{2}+a}{a})\right)p^{2}+\left(\log\left(bx^{2}+a\right)\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x,x, algorithm="maxima")
[Out] 1/2*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog((b*x^2 + a)/a)*log(
b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*p^2 + (log(b*x^2 + a)*log(-(b*x^2
 + a)/a + 1) + dilog((b*x^2 + a)/a))*p*log(c) + log(c)^2*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^2/x,x)
[Out] int(log(c*(a + b*x^2)^p)^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^{2}\right)^{p}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x,x)
[Out] Integral(log(c*(a + b*x**2)**p)**2/x, x)
```

3.81 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^3}\,dx$

Optimal. Leaf size=80

$$-\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2ax^2}+\frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a}+\frac{bp^2\text{Li}_2\left(\frac{bx^2}{a}+1\right)}{a}$$

```
[Out] b*p*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/a-1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^2/a/x^
2+b*p^2*polylog(2,1+b*x^2/a)/a
```

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2454, 2397, 2394, 2315}

$$\frac{bp^2\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{a}-\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2ax^2}+\frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^3,x]
```

```
[Out] (b*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/a - ((a + b*x^2)*Log[c*(a + b*
x^2)^p]^2)/(2*a*x^2) + (b*p^2*PolyLog[2, 1 + (b*x^2)/a])/a
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2ax^2} + \frac{(bp)\operatorname{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, x^2\right)}{a}$$

$$= \frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a} - \frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2ax^2} - \frac{\left(b^2p^2\right)\operatorname{Subst}\left(\right.}{}$$

$$= \frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a} - \frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2ax^2} + \frac{bp^2\operatorname{Li}_2\left(1+\frac{bx^2}{a}\right.}{a}$$

Mathematica [A] time = 0.03, size = 93, normalized size = 1.16

$$-\frac{b\log^2\left(c\left(a+bx^2\right)^p\right)}{2a} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{2x^2} + \frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a} + \frac{bp^2\operatorname{Li}_2\left(\frac{bx^2+a}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^3,x]
```

```
[Out] (b*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/a - (b*Log[c*(a + b*x^2)^p]^2)
/(2*a) - Log[c*(a + b*x^2)^p]^2/(2*x^2) + (b*p^2*PolyLog[2, (a + b*x^2)/a])
/a
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^3, x)
```

maple [C] time = 0.38, size = 841, normalized size = 10.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^3,x)
```

```
[Out] -1/2/x^2*ln((b*x^2+a)^p)^2-b*p*ln((b*x^2+a)^p)/a*ln(b*x^2+a)+2*b*p*ln((b*x^
2+a)^p)/a*ln(x)-2*b*p^2/a*ln(x)*ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-2*b*p^
2/a*ln(x)*ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-2*b*p^2/a*dilog((-b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))-2*b*p^2/a*dilog((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+1/2*b
*p^2/a*ln(b*x^2+a)^2+1/2*I/x^2*ln((b*x^2+a)^p)*Pi*csgn(I*c*(b*x^2+a)^p)^3-1
/2*I/x^2*ln((b*x^2+a)^p)*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/2*I*b*p/a*l
n(b*x^2+a)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/2*I*b*p/a*ln(b*
x^2+a)*Pi*csgn(I*c*(b*x^2+a)^p)^3-1/x^2*ln((b*x^2+a)^p)*ln(c)-1/2*I*b*p/a*l
n(b*x^2+a)*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+I*b*p/a*ln(x)*Pi*csgn(I*c*(
b*x^2+a)^p)^2*csgn(I*c)+1/2*I/x^2*ln((b*x^2+a)^p)*Pi*csgn(I*(b*x^2+a)^p)*cs
gn(I*c*(b*x^2+a)^p)*csgn(I*c)+I*b*p/a*ln(x)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c
*(b*x^2+a)^p)^2-b*p/a*ln(b*x^2+a)*ln(c)-1/2*I/x^2*ln((b*x^2+a)^p)*Pi*csgn(I
*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*b*p/a*ln(x)*Pi*csgn(I*c*(b*x^2+a)^p
)^3+1/2*I*b*p/a*ln(b*x^2+a)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*cs
gn(I*c)-I*b*p/a*ln(x)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c
)+2*b*p/a*ln(x)*ln(c)-1/8*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2
-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*
x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c))^2/x^2
```

maxima [A] time = 0.69, size = 118, normalized size = 1.48

$$\frac{1}{2}\,b^2p^2\left(\frac{\log\left(bx^2+a\right)^2}{ab}-\frac{2\left(2\,\log\left(\frac{bx^2}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx^2}{a}\right)\right)}{ab}\right)-bp\left(\frac{\log\left(bx^2+a\right)}{a}-\frac{\log\left(x^2\right)}{a}\right)\log\left(\left(bx^2+a\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*p^2*(log(b*x^2 + a)^2/(a*b) - 2*(2*log(b*x^2/a + 1)*log(x) + dilog(
-b*x^2/a))/(a*b)) - b*p*(log(b*x^2 + a)/a - log(x^2)/a)*log((b*x^2 + a)^p*c
) - 1/2*log((b*x^2 + a)^p*c)^2/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^2/x^3,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^2/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**3,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**3, x)
```

3.82 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx$

Optimal. Leaf size=129

$$-\frac{b^2 p \log\left(1-\frac{a}{a+bx^2}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2}+\frac{b^2p^2\text{Li}_2\left(\frac{a}{bx^2+a}\right)}{2a^2}+\frac{b^2p^2\log(x)}{a^2}-\frac{bp\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2x^2}-\frac{\log^2\left(\right.}{}$$

```
[Out] b^2*p^2*ln(x)/a^2-1/2*b*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/a^2/x^2-1/4*ln(c*(b*x
^2+a)^p)^2/x^4-1/2*b^2*p*ln(c*(b*x^2+a)^p)*ln(1-a/(b*x^2+a))/a^2+1/2*b^2*p^
2*polylog(2,a/(b*x^2+a))/a^2
```

Rubi [A] time = 0.27, antiderivative size = 147, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31}

$$-\frac{b^2p^2\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2a^2}+\frac{b^2\log^2\left(c\left(a+bx^2\right)^p\right)}{4a^2}-\frac{b^2p\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2}+\frac{b^2p^2\log(x)}{a^2}-\frac{bp\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^5,x]
```

```
[Out] (b^2*p^2*Log[x])/a^2 - (b*p*(a + b*x^2)*Log[c*(a + b*x^2)^p])/(2*a^2*x^2) -
 (b^2*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*a^2) + (b^2*Log[c*(a + b
*x^2)^p]^2)/(4*a^2) - Log[c*(a + b*x^2)^p]^2/(4*x^4) - (b^2*p^2*PolyLog[2,
1 + (b*x^2)/a])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{4x^4} + \frac{1}{2}(bp)\,\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^2(a+bx)}\,dx, x, x^2\right)$$
$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{4x^4} + \frac{1}{2}p\,\text{Subst}\left(\int \frac{\log\left(cx^p\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right)$$
$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{4x^4} + \frac{p\,\text{Subst}\left(\int \frac{\log(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right)}{2a} - \frac{(bp)\,\text{Subst}\left(\int \frac{\log(cx^p)}{x\left(-\frac{a}{b}+\frac{x}{b}\right.}\right.}{2a}$$
$$= -\frac{bp\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2x^2} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{4x^4} - \frac{(bp)\,\text{Subst}\left(\int \frac{\log(cx^p)}{-\frac{a}{b}+\frac{x}{b}}\,dx,\right.}{2a^2}$$
$$= \frac{b^2p^2\log(x)}{a^2} - \frac{bp\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2x^2} - \frac{b^2p\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2}$$
$$= \frac{b^2p^2\log(x)}{a^2} - \frac{bp\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2x^2} - \frac{b^2p\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2}$$

Mathematica [A] time = 0.08, size = 137, normalized size = 1.06

$$\frac{\frac{bx^2\left(-2bpx^2\left(\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)+p\text{Li}_2\left(\frac{bx^2}{a}+1\right)\right)+bx^2\log^2\left(c\left(a+bx^2\right)^p\right)-2ap\log\left(c\left(a+bx^2\right)^p\right)+2bp^2x^2\left(2\log(x)-\log\left(a+bx^2\right)\right)\right)}{a^2} - \log^2\left(c\right.}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^5,x]
```

```
[Out] (-Log[c*(a + b*x^2)^p]^2 + (b*x^2*(2*b*p^2*x^2*(2*Log[x] - Log[a + b*x^2])
- 2*a*p*Log[c*(a + b*x^2)^p] + b*x^2*Log[c*(a + b*x^2)^p]^2 - 2*b*p*x^2*(Lo
g[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2, 1 + (b*x^2)/a])))/a^2)/
(4*x^4)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^5,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^5, x)
```

maple [C] time = 0.39, size = 1080, normalized size = 8.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^5,x)
```

```
[Out] -1/2/x^4*ln((b*x^2+a)^p)*ln(c)-1/16*(-I*Pi*csgn(I*c)*csgn(I*(b*x^2+a)^p)*cs
gn(I*c*(b*x^2+a)^p)+I*Pi*csgn(I*c)*csgn(I*c*(b*x^2+a)^p)^2+I*Pi*csgn(I*(b*x
^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+2*ln(c))^2/x^
4+1/4*I*b^2*p/a^2*ln(b*x^2+a)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^
2-1/4*b^2*p^2/a^2*ln(b*x^2+a)^2-1/2*b^2*p^2/a^2*ln(b*x^2+a)+b^2*p^2/a^2*dil
og((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+b^2*p^2/a^2*dilog((b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))+b^2*p^2*ln(x)/a^2-1/4*I*b*p/a/x^2*Pi*csgn(I*(b*x^2+a)^p)*csgn(
I*c*(b*x^2+a)^p)^2-1/4*I/x^4*ln((b*x^2+a)^p)*Pi*csgn(I*c*(b*x^2+a)^p)^2*csg
n(I*c)-1/4*I/x^4*ln((b*x^2+a)^p)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^
p)^2-1/4/x^4*ln((b*x^2+a)^p)^2-1/2*I*b^2*p/a^2*ln(x)*Pi*csgn(I*(b*x^2+a)^p)
*csgn(I*c*(b*x^2+a)^p)^2+1/4*I*b^2*p/a^2*ln(b*x^2+a)*Pi*csgn(I*c*(b*x^2+a)^
p)^2*csgn(I*c)-1/4*I*b*p/a/x^2*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/2*I*b
^2*p/a^2*ln(x)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/2*I
*b^2*p/a^2*ln(x)*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/4*I*b*p/a/x^2*Pi*cs
gn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/4*I*b^2*p/a^2*ln(b*x^2+
a)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/4*I/x^4*ln((b*x
^2+a)^p)*Pi*csgn(I*c*(b*x^2+a)^p)^3+1/2*b^2*p*ln((b*x^2+a)^p)/a^2*ln(b*x^2+
a)-1/2*b*p*ln((b*x^2+a)^p)/a/x^2-b^2*p*ln((b*x^2+a)^p)/a^2*ln(x)-1/4*I*b^2*
p/a^2*ln(b*x^2+a)*Pi*csgn(I*c*(b*x^2+a)^p)^3+1/4*I*b*p/a/x^2*Pi*csgn(I*c*(b
*x^2+a)^p)^3+1/2*I*b^2*p/a^2*ln(x)*Pi*csgn(I*c*(b*x^2+a)^p)^3+1/4*I/x^4*ln(
(b*x^2+a)^p)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+b^2*p^2
/a^2*ln(x)*ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+b^2*p^2/a^2*ln(x)*ln((b*x+(
-a*b)^(1/2))/(-a*b)^(1/2))+1/2*b^2*p/a^2*ln(b*x^2+a)*ln(c)-1/2*b*p/a/x^2*ln
(c)-b^2*p/a^2*ln(x)*ln(c)
```

maxima [A] time = 0.98, size = 142, normalized size = 1.10

$$-\frac{1}{4}\,b^2p^2\left(\frac{\log\left(bx^2+a\right)^2}{a^2}-\frac{2\left(2\,\log\left(\frac{bx^2}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx^2}{a}\right)\right)}{a^2}+\frac{2\,\log\left(bx^2+a\right)}{a^2}-\frac{4\,\log(x)}{a^2}\right)+\frac{1}{2}\,bp\left(\frac{b\log\left(b\right.}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*p^2*(log(b*x^2 + a)^2/a^2 - 2*(2*log(b*x^2/a + 1)*log(x) + dilog(-
b*x^2/a))/a^2 + 2*log(b*x^2 + a)/a^2 - 4*log(x)/a^2) + 1/2*b*p*(b*log(b*x^2
 + a)/a^2 - b*log(x^2)/a^2 - 1/(a*x^2))*log((b*x^2 + a)^p*c) - 1/4*log((b*x
^2 + a)^p*c)^2/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^2/x^5,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^2/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**5,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**5, x)
```

3.83 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^7} dx$

Optimal. Leaf size=193

$$\frac{b^3 p \log\left(1-\frac{a}{a+bx^2}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^3}-\frac{b^3p^2\text{Li}_2\left(\frac{a}{bx^2+a}\right)}{3a^3}+\frac{b^3p^2\log\left(a+bx^2\right)}{6a^3}-\frac{b^3p^2\log(x)}{a^3}+\frac{b^2p\left(a+bx^2\right)\log\left(c\left(a\right.\right.}{3a^3x^2}$$

```
[Out] -1/6*b^2*p^2/a^2/x^2-b^3*p^2*ln(x)/a^3+1/6*b^3*p^2*ln(b*x^2+a)/a^3-1/6*b*p*
ln(c*(b*x^2+a)^p)/a/x^4+1/3*b^2*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/a^3/x^2-1/6*l
n(c*(b*x^2+a)^p)^2/x^6+1/3*b^3*p*ln(c*(b*x^2+a)^p)*ln(1-a/(b*x^2+a))/a^3-1/
3*b^3*p^2*polylog(2,a/(b*x^2+a))/a^3
```

Rubi [A] time = 0.41, antiderivative size = 211, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$\frac{b^3p^2\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{3a^3}-\frac{b^3\log^2\left(c\left(a+bx^2\right)^p\right)}{6a^3}+\frac{b^3p\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^3}+\frac{b^2p\left(a+bx^2\right)\log\left(c\left(a+b\right.\right.}{3a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^7,x]
```

```
[Out] -(b^2*p^2)/(6*a^2*x^2) - (b^3*p^2*Log[x])/a^3 + (b^3*p^2*Log[a + b*x^2])/(6
*a^3) - (b*p*Log[c*(a + b*x^2)^p])/(6*a*x^4) + (b^2*p*(a + b*x^2)*Log[c*(a
+ b*x^2)^p])/(3*a^3*x^2) + (b^3*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(
3*a^3) - (b^3*Log[c*(a + b*x^2)^p]^2)/(6*a^3) - Log[c*(a + b*x^2)^p]^2/(6*x
^6) + (b^3*p^2*PolyLog[2, 1 + (b*x^2)/a])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
```

```
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{6x^6} + \frac{1}{3}(bp)\,\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^3(a+bx)}\,dx, x, x^2\right)$$
$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{6x^6} + \frac{1}{3}p\,\text{Subst}\left(\int \frac{\log\left(cx^p\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx^2\right)$$
$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{6x^6} + \frac{p\,\text{Subst}\left(\int \frac{\log(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx^2\right)}{3a} - \frac{(bp)\,\text{Subst}\left(\int \frac{\log(cx^p)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,d\right.}{3a}$$
$$= -\frac{bp\log\left(c\left(a+bx^2\right)^p\right)}{6ax^4} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{6x^6} - \frac{(bp)\,\text{Subst}\left(\int \frac{\log(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right)}{3a^2}$$
$$= -\frac{bp\log\left(c\left(a+bx^2\right)^p\right)}{6ax^4} + \frac{b^2p\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^3x^2} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{6x^6} + \frac{(b}{}$$
$$= -\frac{b^2p^2}{6a^2x^2} - \frac{b^3p^2\log(x)}{a^3} + \frac{b^3p^2\log\left(a+bx^2\right)}{6a^3} - \frac{bp\log\left(c\left(a+bx^2\right)^p\right)}{6ax^4} + \frac{b^2p\left(a+bx^2\right)\mathrm{l}}{3}$$
$$= -\frac{b^2p^2}{6a^2x^2} - \frac{b^3p^2\log(x)}{a^3} + \frac{b^3p^2\log\left(a+bx^2\right)}{6a^3} - \frac{bp\log\left(c\left(a+bx^2\right)^p\right)}{6ax^4} + \frac{b^2p\left(a+bx^2\right)\mathrm{l}}{3}$$

Mathematica [A] time = 0.06, size = 205, normalized size = 1.06

$$-\frac{b^3\log^2\left(c\left(a+bx^2\right)^p\right)}{6a^3}+\frac{b^3p\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^3}+\frac{b^3p^2\text{Li}_2\left(\frac{bx^2+a}{a}\right)}{3a^3}+\frac{b^3p^2\log\left(a+bx^2\right)}{2a^3}-\frac{b^3p^2\log(x)}{a^3}+\frac{b^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^7,x]
```

```
[Out] -1/6*(b^2*p^2)/(a^2*x^2) - (b^3*p^2*Log[x])/a^3 + (b^3*p^2*Log[a + b*x^2])/
(2*a^3) - (b*p*Log[c*(a + b*x^2)^p])/(6*a*x^4) + (b^2*p*Log[c*(a + b*x^2)^p
])/(3*a^2*x^2) + (b^3*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(3*a^3) - (
b^3*Log[c*(a + b*x^2)^p]^2)/(6*a^3) - Log[c*(a + b*x^2)^p]^2/(6*x^6) + (b^3
*p^2*PolyLog[2, (a + b*x^2)/a])/(3*a^3)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^7,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^7, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^7,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^7, x)
```

maple [C] time = 0.44, size = 1289, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^7,x)
```

```
[Out] -2/3*b^3*p^2/a^3*dilog((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-2/3*b^3*p^2/a^3*di
log((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+1/6*b^3*p^2/a^3*ln(b*x^2+a)^2-1/3/x^6*
ln((b*x^2+a)^p)*ln(c)-b^3*p^2*ln(x)/a^3+1/2*b^3*p^2*ln(b*x^2+a)/a^3+1/6*I*b
^2*p/a^2/x^2*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/3*I*b^3*p/a^3
*ln(x)*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/6*I*b^2*p/a^2/x^2*Pi*csgn(I*c
*(b*x^2+a)^p)^2*csgn(I*c)-1/6*I*b^3*p/a^3*ln(b*x^2+a)*Pi*csgn(I*(b*x^2+a)^p
)*csgn(I*c*(b*x^2+a)^p)^2+1/3*I*b^3*p/a^3*ln(x)*Pi*csgn(I*(b*x^2+a)^p)*csgn
(I*c*(b*x^2+a)^p)^2-1/12*I*b*p/a/x^4*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2
+a)^p)^2-1/12*I*b*p/a/x^4*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/6*I*b^3*p/
a^3*ln(b*x^2+a)*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/6*b^2*p^2/a^2/x^2+1/
6*I*b^3*p/a^3*ln(b*x^2+a)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn
(I*c)+1/6*I/x^6*ln((b*x^2+a)^p)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p
)*csgn(I*c)-1/3*I*b^3*p/a^3*ln(x)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)*csgn(I*c)-1/24*(-I*Pi*csgn(I*c)*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^
p)+I*Pi*csgn(I*c)*csgn(I*c*(b*x^2+a)^p)^2+I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c
*(b*x^2+a)^p)^2-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+2*ln(c))^2/x^6-1/6*I/x^6*ln((b
*x^2+a)^p)*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/6/x^6*ln((b*x^2+a)^p)^2-1
/6*I*b^2*p/a^2/x^2*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1
/12*I*b*p/a/x^4*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/6*
I/x^6*ln((b*x^2+a)^p)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/6*b*
p/a/x^4*ln(c)+1/3*b^2*p/a^2/x^2*ln(c)-1/3*b^3*p/a^3*ln(b*x^2+a)*ln(c)+2/3*b
^3*p/a^3*ln(x)*ln(c)+1/6*I*b^3*p/a^3*ln(b*x^2+a)*Pi*csgn(I*c*(b*x^2+a)^p)^3
-1/3*I*b^3*p/a^3*ln(x)*Pi*csgn(I*c*(b*x^2+a)^p)^3-2/3*b^3*p^2/a^3*ln(x)*ln(
(-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-2/3*b^3*p^2/a^3*ln(x)*ln((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))-1/6*b*p*ln((b*x^2+a)^p)/a/x^4+2/3*b^3*p*ln((b*x^2+a)^p)/a^
3*ln(x)+1/3*b^2*p*ln((b*x^2+a)^p)/a^2/x^2+1/6*I/x^6*ln((b*x^2+a)^p)*Pi*csgn
(I*c*(b*x^2+a)^p)^3-1/3*b^3*p*ln((b*x^2+a)^p)/a^3*ln(b*x^2+a)+1/12*I*b*p/a/
x^4*Pi*csgn(I*c*(b*x^2+a)^p)^3-1/6*I*b^2*p/a^2/x^2*Pi*csgn(I*c*(b*x^2+a)^p)
^3
```

maxima [A] time = 0.81, size = 173, normalized size = 0.90

$$-\frac{1}{6}\,b^2p^2\left(\frac{2\left(2\,\log\left(\frac{bx^2}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx^2}{a}\right)\right)b}{a^3}-\frac{3\,b\log\left(bx^2+a\right)}{a^3}-\frac{bx^2\log\left(bx^2+a\right)^2-6\,bx^2\log(x)-a}{a^3x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*b^2*p^2*(2*(2*log(b*x^2/a + 1)*log(x) + dilog(-b*x^2/a))*b/a^3 - 3*b*l
og(b*x^2 + a)/a^3 - (b*x^2*log(b*x^2 + a)^2 - 6*b*x^2*log(x) - a)/(a^3*x^2)
) - 1/6*b*p*(2*b^2*log(b*x^2 + a)/a^3 - 2*b^2*log(x^2)/a^3 - (2*b*x^2 - a)/
(a^2*x^4))*log((b*x^2 + a)^p*c) - 1/6*log((b*x^2 + a)^p*c)^2/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^2/x^7,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^2/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**7,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**7, x)
```

3.84 $\int x^4 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=336

$$\frac{4a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{5b^{5/2}}+\frac{4ia^{5/2}p^2\text{Li}_2\left(1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{5b^{5/2}}+\frac{4ia^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5b^{5/2}}-\frac{184a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{75b^{5/2}}$$

```
[Out] 184/75*a^2*p^2*x/b^2-64/225*a*p^2*x^3/b+8/125*p^2*x^5-184/75*a^(5/2)*p^2*ar
ctan(x*b^(1/2)/a^(1/2))/b^(5/2)+4/5*I*a^(5/2)*p^2*arctan(x*b^(1/2)/a^(1/2))
^2/b^(5/2)-4/5*a^2*p*x*ln(c*(b*x^2+a)^p)/b^2+4/15*a*p*x^3*ln(c*(b*x^2+a)^p)
/b-4/25*p*x^5*ln(c*(b*x^2+a)^p)+4/5*a^(5/2)*p*arctan(x*b^(1/2)/a^(1/2))*ln(
c*(b*x^2+a)^p)/b^(5/2)+1/5*x^5*ln(c*(b*x^2+a)^p)^2+8/5*a^(5/2)*p^2*arctan(x
*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/b^(5/2)+4/5*I*a^(5/2)
*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/b^(5/2)
```

Rubi [A] time = 0.41, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {2457, 2476, 2448, 321, 205, 2455, 302, 2470, 12, 4920, 4854, 2402, 2315}

$$\frac{4ia^{5/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5b^{5/2}}-\frac{4a^2px\log\left(c\left(a+bx^2\right)^p\right)}{5b^2}+\frac{4a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{5b^{5/2}}+\frac{184a^2p^2}{75b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (184*a^2*p^2*x)/(75*b^2) - (64*a*p^2*x^3)/(225*b) + (8*p^2*x^5)/125 - (184*
a^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(75*b^(5/2)) + (((4*I)/5)*a^(5/2)*
p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/b^(5/2) + (8*a^(5/2)*p^2*ArcTan[(Sqrt[b]
*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(5*b^(5/2)) - (4*a^2
*p*x*Log[c*(a + b*x^2)^p])/(5*b^2) + (4*a*p*x^3*Log[c*(a + b*x^2)^p])/(15*b
) - (4*p*x^5*Log[c*(a + b*x^2)^p])/25 + (4*a^(5/2)*p*ArcTan[(Sqrt[b]*x)/Sqr
t[a]]*Log[c*(a + b*x^2)^p])/(5*b^(5/2)) + (x^5*Log[c*(a + b*x^2)^p]^2)/5 +
(((4*I)/5)*a^(5/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])
/b^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
+ e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
+ 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
:> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^4 \log^2\left(c\left(a+bx^2\right)^p\right)dx = \frac{1}{5}x^5\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{5}(4bp)\int \frac{x^6\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}dx$$

$$= \frac{1}{5}x^5\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{5}(4bp)\int\left(\frac{a^2\log\left(c\left(a+bx^2\right)^p\right)}{b^3} - \frac{ax^2\log\left(c\left(a+bx\right.\right.}{b^2}\right.$$

$$= \frac{1}{5}x^5\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{5}(4p)\int x^4\log\left(c\left(a+bx^2\right)^p\right)dx - \frac{\left(4a^2p\right)\int\log\left(c\left(\right.\right.}{5b^2}$$

$$= -\frac{4a^2px\log\left(c\left(a+bx^2\right)^p\right)}{5b^2} + \frac{4apx^3\log\left(c\left(a+bx^2\right)^p\right)}{15b} - \frac{4}{25}px^5\log\left(c\left(a+bx^2\right)\right.$$

$$= \frac{8a^2p^2x}{5b^2} - \frac{4a^2px\log\left(c\left(a+bx^2\right)^p\right)}{5b^2} + \frac{4apx^3\log\left(c\left(a+bx^2\right)^p\right)}{15b} - \frac{4}{25}px^5\log\left(c\left(\right.\right.$$

$$= \frac{184a^2p^2x}{75b^2} - \frac{64ap^2x^3}{225b} + \frac{8p^2x^5}{125} - \frac{8a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5b^{5/2}} + \frac{4ia^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5b^{5/2}} -$$

$$= \frac{184a^2p^2x}{75b^2} - \frac{64ap^2x^3}{225b} + \frac{8p^2x^5}{125} - \frac{184a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{75b^{5/2}} + \frac{4ia^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5b^{5/2}}$$

$$= \frac{184a^2p^2x}{75b^2} - \frac{64ap^2x^3}{225b} + \frac{8p^2x^5}{125} - \frac{184a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{75b^{5/2}} + \frac{4ia^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5b^{5/2}}$$

$$= \frac{184a^2p^2x}{75b^2} - \frac{64ap^2x^3}{225b} + \frac{8p^2x^5}{125} - \frac{184a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{75b^{5/2}} + \frac{4ia^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5b^{5/2}}$$

Mathematica [A] time = 0.21, size = 248, normalized size = 0.74

$$60a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(15\log\left(c\left(a+bx^2\right)^p\right)+30p\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)-46p\right)+900ia^{5/2}p^2\text{Li}_2\left(\frac{\sqrt{b}x+i\sqrt{a}}{\sqrt{b}x-i\sqrt{a}}\right)+900ia^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] ((900*I)*a^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + 60*a^(5/2)*p*ArcTan[(S
qrt[b]*x)/Sqrt[a]]*(-46*p + 30*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)] +
 15*Log[c*(a + b*x^2)^p]) + Sqrt[b]*x*(8*p^2*(345*a^2 - 40*a*b*x^2 + 9*b^2*
x^4) - 60*p*(15*a^2 - 5*a*b*x^2 + 3*b^2*x^4)*Log[c*(a + b*x^2)^p] + 225*b^2
```

```
*x^4*Log[c*(a + b*x^2)^p]^2) + (900*I)*a^(5/2)*p^2*PolyLog[2, (I*Sqrt[a] +
Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(1125*b^(5/2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(x^4 \log\left(\left(bx^2+a\right)^p c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
[Out] integral(x^4*log((b*x^2 + a)^p*c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \log\left(\left(bx^2+a\right)^p c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
[Out] integrate(x^4*log((b*x^2 + a)^p*c)^2, x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int x^4 \ln\left(c\left(b\,x^2+a\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(b*x^2+a)^p)^2,x)
[Out] int(x^4*ln(c*(b*x^2+a)^p)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{5}\,p^2x^5\log\left(bx^2+a\right)^2+\int \frac{5\,bx^6\log(c)^2+5\,ax^4\log(c)^2-2\left(\left(2\,p^2-5\,p\log(c)\right)bx^6-5\,apx^4\log(c)\right)\log\left(bx^2+a\right)}{5\left(bx^2+a\right)}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
[Out] 1/5*p^2*x^5*log(b*x^2 + a)^2 + integrate(1/5*(5*b*x^6*log(c)^2 + 5*a*x^4*lo
g(c)^2 - 2*((2*p^2 - 5*p*log(c))*b*x^6 - 5*a*p*x^4*log(c))*log(b*x^2 + a))/
(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \ln\left(c\left(b\,x^2+a\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*log(c*(a + b*x^2)^p)^2,x)
[Out] int(x^4*log(c*(a + b*x^2)^p)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \log\left(c\left(a+bx^2\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(x**4*log(c*(a + b*x**2)**p)**2, x)
```

3.85 $\int x^2 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=294

$$-\frac{4a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^{3/2}}-\frac{4ia^{3/2}p^2\text{Li}_2\left(1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{3b^{3/2}}-\frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}}+\frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}}$$

```
[Out] -32/9*a*p^2*x/b+8/27*p^2*x^3+32/9*a^(3/2)*p^2*arctan(x*b^(1/2)/a^(1/2))/b^(
3/2)-4/3*I*a^(3/2)*p^2*arctan(x*b^(1/2)/a^(1/2))^2/b^(3/2)+4/3*a*p*x*ln(c*(
b*x^2+a)^p)/b-4/9*p*x^3*ln(c*(b*x^2+a)^p)-4/3*a^(3/2)*p*arctan(x*b^(1/2)/a^
(1/2))*ln(c*(b*x^2+a)^p)/b^(3/2)+1/3*x^3*ln(c*(b*x^2+a)^p)^2-8/3*a^(3/2)*p^
2*arctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/b^(3/2)-4/3
*I*a^(3/2)*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/b^(3/2)
```

Rubi [A] time = 0.32, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {2457, 2476, 2448, 321, 205, 2455, 302, 2470, 12, 4920, 4854, 2402, 2315}

$$-\frac{4ia^{3/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3b^{3/2}}-\frac{4a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^{3/2}}-\frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}}+\frac{32a^{3/2}p^2\tan}{9b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (-32*a*p^2*x)/(9*b) + (8*p^2*x^3)/27 + (32*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(9*b^(3/2)) - (((4*I)/3)*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2
)/b^(3/2) - (8*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqr
t[a] + I*Sqrt[b]*x)])/(3*b^(3/2)) + (4*a*p*x*Log[c*(a + b*x^2)^p])/(3*b) -
(4*p*x^3*Log[c*(a + b*x^2)^p])/9 - (4*a^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
*Log[c*(a + b*x^2)^p])/(3*b^(3/2)) + (x^3*Log[c*(a + b*x^2)^p]^2)/3 - (((4*
I)/3)*a^(3/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/b^(3
/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2\log^2\left(c\left(a+bx^2\right)^p\right)dx = \frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{1}{3}(4bp)\int\frac{x^4\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}dx$$

$$=\frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{1}{3}(4bp)\int\left(-\frac{a\log\left(c\left(a+bx^2\right)^p\right)}{b^2}+\frac{x^2\log\left(c\left(a+bx^2\right)^p\right)}{b}\right.$$

$$=\frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{1}{3}(4p)\int x^2\log\left(c\left(a+bx^2\right)^p\right)dx+\frac{(4ap)\int\log\left(c\left(a+b\right.\right.}{3b}$$

$$=\frac{4apx\log\left(c\left(a+bx^2\right)^p\right)}{3b}-\frac{4}{9}px^3\log\left(c\left(a+bx^2\right)^p\right)-\frac{4a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+\right.\right.}{3b^{3/2}}$$

$$=-\frac{8ap^2x}{3b}+\frac{4apx\log\left(c\left(a+bx^2\right)^p\right)}{3b}-\frac{4}{9}px^3\log\left(c\left(a+bx^2\right)^p\right)-\frac{4a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b}$$

$$=-\frac{32ap^2x}{9b}+\frac{8p^2x^3}{27}+\frac{8a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}}-\frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}}+\frac{4apx\log\left(c\left(\right.\right.}{3b}$$

$$=-\frac{32ap^2x}{9b}+\frac{8p^2x^3}{27}+\frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}}-\frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}}-\frac{8a^{3/2}p^2\tan^{-}}{}$$

$$=-\frac{32ap^2x}{9b}+\frac{8p^2x^3}{27}+\frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}}-\frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}}-\frac{8a^{3/2}p^2\tan^{-}}{}$$

$$=-\frac{32ap^2x}{9b}+\frac{8p^2x^3}{27}+\frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}}-\frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}}-\frac{8a^{3/2}p^2\tan^{-}}{}$$

Mathematica [A] time = 0.14, size = 223, normalized size = 0.76

$$\frac{-12a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3\log\left(c\left(a+bx^2\right)^p\right)+6p\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)-8p\right)-36ia^{3/2}p^2\text{Li}_2\left(\frac{\sqrt{b}x+i\sqrt{a}}{\sqrt{b}x-i\sqrt{a}}\right)-36ia^{3/2}p^2\tan^{-1}}{27b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] ((-36*I)*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 - 12*a^(3/2)*p*ArcTan[(S
qrt[b]*x)/Sqrt[a]]*(-8*p + 6*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)] + 3
*Log[c*(a + b*x^2)^p]) + Sqrt[b]*x*(8*p^2*(-12*a + b*x^2) + 12*p*(3*a - b*x
^2)*Log[c*(a + b*x^2)^p] + 9*b*x^2*Log[c*(a + b*x^2)^p]^2) - (36*I)*a^(3/2)
```

```
*p^2*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(27*b^
(3/2))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\log\left(\left(bx^2+a\right)^p c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*log((b*x^2 + a)^p*c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\log\left(\left(bx^2+a\right)^p c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*log((b*x^2 + a)^p*c)^2, x)
```

maple [F] time = 0.96, size = 0, normalized size = 0.00

$$\int x^2\ln\left(c\left(b\,x^2+a\right)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(x^2*ln(c*(b*x^2+a)^p)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}p^2x^3\log\left(bx^2+a\right)^2+\int\frac{3\,bx^4\log(c)^2+3\,ax^2\log(c)^2-2\left(\left(2\,p^2-3\,p\log(c)\right)bx^4-3\,apx^2\log(c)\right)\log\left(bx^2+a\right)}{3\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/3*p^2*x^3*log(b*x^2 + a)^2 + integrate(1/3*(3*b*x^4*log(c)^2 + 3*a*x^2*lo
g(c)^2 - 2*((2*p^2 - 3*p*log(c))*b*x^4 - 3*a*p*x^2*log(c))*log(b*x^2 + a))/
(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\ln\left(c\left(b\,x^2+a\right)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] int(x^2*log(c*(a + b*x^2)^p)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\log\left(c\left(a+bx^2\right)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(x**2*log(c*(a + b*x**2)**p)**2, x)
```

3.86 $\int \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=237

$$x\log^2\left(c\left(a+bx^2\right)^p\right)-4px\log\left(c\left(a+bx^2\right)^p\right)+\frac{4\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}}+\frac{4i\sqrt{a}\,p^2\text{Li}_2\left(1-\frac{2\sqrt{a}}{i\sqrt{b}\,x+\sqrt{a}}\right.}{\sqrt{b}}$$

```
[Out] 8*p^2*x-4*p*x*ln(c*(b*x^2+a)^p)+x*ln(c*(b*x^2+a)^p)^2-8*p^2*arctan(x*b^(1/2
)/a^(1/2))*a^(1/2)/b^(1/2)+4*I*p^2*arctan(x*b^(1/2)/a^(1/2))^2*a^(1/2)/b^(1
/2)+4*p*arctan(x*b^(1/2)/a^(1/2))*ln(c*(b*x^2+a)^p)*a^(1/2)/b^(1/2)+8*p^2*a
rctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))*a^(1/2)/b^(1/2
)+4*I*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))*a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {2450, 2476, 2448, 321, 205, 2470, 12, 4920, 4854, 2402, 2315}

$$\frac{4i\sqrt{a}\,p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{b}}+x\log^2\left(c\left(a+bx^2\right)^p\right)-4px\log\left(c\left(a+bx^2\right)^p\right)+\frac{4\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a\right.\right.}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] 8*p^2*x - (8*Sqrt[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + ((4*I)*Sqrt
[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/Sqrt[b] + (8*Sqrt[a]*p^2*ArcTan[(Sqr
t[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[b] - 4*p*x*
Log[c*(a + b*x^2)^p] + (4*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a +
b*x^2)^p])/Sqrt[b] + x*Log[c*(a + b*x^2)^p]^2 + ((4*I)*Sqrt[a]*p^2*PolyLog[
2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \log^2\left(c\left(a+bx^2\right)^p\right) dx = x\log^2\left(c\left(a+bx^2\right)^p\right) - (4bp)\int \frac{x^2\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2} dx$$

$$= x\log^2\left(c\left(a+bx^2\right)^p\right) - (4bp)\int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{b} - \frac{a\log\left(c\left(a+bx^2\right)^p\right)}{b\left(a+bx^2\right)}\right) dx$$

$$= x\log^2\left(c\left(a+bx^2\right)^p\right) - (4p)\int \log\left(c\left(a+bx^2\right)^p\right) dx + (4ap)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}$$

$$= -4px\log\left(c\left(a+bx^2\right)^p\right) + \frac{4\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}} + x\log^2\left(c\left(a+b\right.\right.$$

$$= 8p^2x - 4px\log\left(c\left(a+bx^2\right)^p\right) + \frac{4\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}} + x\log^2\left(c\right.$$

$$= 8p^2x - \frac{8\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{4i\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}} - 4px\log\left(c\left(a+bx^2\right)^p\right) + \frac{4}{}$$

$$= 8p^2x - \frac{8\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{4i\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}} + \frac{8\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\right.}{\sqrt{b}}$$

$$= 8p^2x - \frac{8\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{4i\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}} + \frac{8\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\right.}{\sqrt{b}}$$

$$= 8p^2x - \frac{8\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{4i\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}} + \frac{8\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\right.}{\sqrt{b}}$$

Mathematica [A] time = 0.08, size = 193, normalized size = 0.81

$$\frac{\sqrt{b}\,x\left(\log^2\left(c\left(a+bx^2\right)^p\right) - 4p\log\left(c\left(a+bx^2\right)^p\right) + 8p^2\right) + 4\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(\log\left(c\left(a+bx^2\right)^p\right) + 2p\log\left(\frac{}{\sqrt{a}}\right.\right.}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] ((4*I)*Sqrt[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + 4*Sqrt[a]*p*ArcTan[(Sqrt
[b]*x)/Sqrt[a]]*(-2*p + 2*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)] + Log[
c*(a + b*x^2)^p]) + Sqrt[b]*x*(8*p^2 - 4*p*Log[c*(a + b*x^2)^p] + Log[c*(a
+ b*x^2)^p]^2) + (4*I)*Sqrt[a]*p^2*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)
*Sqrt[a] + Sqrt[b]*x)])/Sqrt[b]
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\log\left(\left(bx^2+a\right)^p c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^p*c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\left(bx^2+a\right)^p c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)^2, x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \ln\left(c\left(b\,x^2+a\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2,x)
[Out] int(ln(c*(b*x^2+a)^p)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$p^2 x \log\left(bx^2+a\right)^2+\int \frac{bx^2\log(c)^2+a\log(c)^2-2\left(\left(2\,p^2-p\log(c)\right)bx^2-ap\log(c)\right)\log\left(bx^2+a\right)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
[Out] p^2*x*log(b*x^2 + a)^2 + integrate((b*x^2*log(c)^2 + a*log(c)^2 - 2*((2*p^2
 - p*log(c))*b*x^2 - a*p*log(c))*log(b*x^2 + a))/(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(b\,x^2+a\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^2,x)
[Out] int(log(c*(a + b*x^2)^p)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(c\left(a+bx^2\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2,x)
[Out] Integral(log(c*(a + b*x**2)**p)**2, x)
```

3.87 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=190

$$-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x}+\frac{4\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{a}}+\frac{4i\sqrt{b}\,p^2\text{Li}_2\left(1-\frac{2\sqrt{a}}{i\sqrt{b}\,x+\sqrt{a}}\right)}{\sqrt{a}}+\frac{4i\sqrt{b}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -ln(c*(b*x^2+a)^p)^2/x+4*I*p^2*arctan(x*b^(1/2)/a^(1/2))^2*b^(1/2)/a^(1/2)+
4*p*arctan(x*b^(1/2)/a^(1/2))*ln(c*(b*x^2+a)^p)*b^(1/2)/a^(1/2)+8*p^2*arcta
n(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))*b^(1/2)/a^(1/2)+4*
I*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))*b^(1/2)/a^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2457, 205, 2470, 12, 4920, 4854, 2402, 2315}

$$\frac{4i\sqrt{b}\,p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{a}}-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x}+\frac{4\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{a}}+\frac{4i\sqrt{b}\,p^2\tan^{-1}}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^2,x]
```

```
[Out] ((4*I)*Sqrt[b]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/Sqrt[a] + (8*Sqrt[b]*p^2*
ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[
a] + (4*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/Sqrt[a]
 - Log[c*(a + b*x^2)^p]^2/x + ((4*I)*Sqrt[b]*p^2*PolyLog[2, 1 - (2*Sqrt[a])
/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[a]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
```

```
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx = -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x} + (4bp)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx$$

$$= \frac{4\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{a}} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x} - \left(8b^2p^2\right)\int \frac{x\tan^{-1}\left(\cdots\right.}{\sqrt{a}\sqrt{b}\left(a+\cdots\right.}$$

$$= \frac{4\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{a}} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x} - \frac{\left(8b^{3/2}p^2\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a+bx^2}\cdots}{\sqrt{a}}$$

$$= \frac{4i\sqrt{b}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{4\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{a}} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x}$$

$$= \frac{4i\sqrt{b}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{8\sqrt{b}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{a}} + \frac{4\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\cdots}{\sqrt{a}}$$

$$= \frac{4i\sqrt{b}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{8\sqrt{b}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{a}} + \frac{4\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\cdots}{\sqrt{a}}$$

$$= \frac{4i\sqrt{b}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{8\sqrt{b}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{a}} + \frac{4\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\cdots}{\sqrt{a}}$$

Mathematica [A] time = 0.05, size = 173, normalized size = 0.91

$$\frac{-\sqrt{a}\log^2\left(c\left(a+bx^2\right)^p\right)+4\sqrt{b}px\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(\log\left(c\left(a+bx^2\right)^p\right)+2p\log\left(\frac{2i}{-\frac{\sqrt{b}x}{\sqrt{a}}+i}\right)\right)+4i\sqrt{b}p^2x\mathrm{Li}_2\left(\frac{\sqrt{b}x+i\sqrt{a}}{\sqrt{b}x-i\sqrt{a}}\right)}{\sqrt{a}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^2,x]
```

```
[Out] ((4*I)*Sqrt[b]*p^2*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 - Sqrt[a]*Log[c*(a + b*x
^2)^p]^2 + 4*Sqrt[b]*p*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(2*p*Log[(2*I)/(I - (S
qrt[b]*x)/Sqrt[a])] + Log[c*(a + b*x^2)^p]) + (4*I)*Sqrt[b]*p^2*x*PolyLog[2
, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(Sqrt[a]*x)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^2, x)
```

maple [F] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^2,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^2/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{p^2\log\left(bx^2+a\right)^2}{x}+\int \frac{bx^2\log(c)^2+a\log(c)^2+2\left(\left(2\,p^2+p\log(c)\right)bx^2+ap\log(c)\right)\log\left(bx^2+a\right)}{bx^4+ax^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^2,x, algorithm="maxima")
```

```
[Out] -p^2*log(b*x^2 + a)^2/x + integrate((b*x^2*log(c)^2 + a*log(c)^2 + 2*((2*p^
2 + p*log(c))*b*x^2 + a*p*log(c))*log(b*x^2 + a))/(b*x^4 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^2/x^2,x)
[Out] int(log(c*(a + b*x^2)^p)^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**2,x)
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**2, x)
```

3.88 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=254

$$-\frac{4b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^{3/2}}-\frac{4ib^{3/2}p^2\text{Li}_2\left(1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{3a^{3/2}}-\frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3a^{3/2}}+\frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.}{3a^{3/2}}$$

```
[Out] 8/3*b^(3/2)*p^2*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)-4/3*I*b^(3/2)*p^2*arctan(
x*b^(1/2)/a^(1/2))^2/a^(3/2)-4/3*b*p*ln(c*(b*x^2+a)^p)/a/x-4/3*b^(3/2)*p*ar
ctan(x*b^(1/2)/a^(1/2))*ln(c*(b*x^2+a)^p)/a^(3/2)-1/3*ln(c*(b*x^2+a)^p)^2/x
^3-8/3*b^(3/2)*p^2*arctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1
/2)))/a^(3/2)-4/3*I*b^(3/2)*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2))
)/a^(3/2)
```

Rubi [A] time = 0.29, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {2457, 2476, 2455, 205, 2470, 12, 4920, 4854, 2402, 2315}

$$-\frac{4ib^{3/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3a^{3/2}}-\frac{4b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^{3/2}}-\frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3a^{3/2}}+\frac{8b^{3/2}p^2\text{ta}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^4,x]
```

```
[Out] (8*b^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*a^(3/2)) - (((4*I)/3)*b^(3/2
)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/a^(3/2) - (8*b^(3/2)*p^2*ArcTan[(Sqrt[
b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(3*a^(3/2)) - (4*b
*p*Log[c*(a + b*x^2)^p])/(3*a*x) - (4*b^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
*Log[c*(a + b*x^2)^p])/(3*a^(3/2)) - Log[c*(a + b*x^2)^p]^2/(3*x^3) - (((4*
I)/3)*b^(3/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^(3
/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx = -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{3x^3} + \frac{1}{3}(4bp)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2\left(a+bx^2\right)}\,dx$$

$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{3x^3} + \frac{1}{3}(4bp)\int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{ax^2} - \frac{b\log\left(c\left(a+bx^2\right)^p\right)}{a\left(a+bx^2\right)}\right)dx$$

$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{3x^3} + \frac{(4bp)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx}{3a} - \frac{\left(4b^2p\right)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx}{3a}$$

$$= -\frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{3ax} - \frac{4b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^{3/2}} - \frac{\log^2\left(c\left(a+bx\right.\right.}{3x^3}$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{3ax} - \frac{4b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right.\right.}{3a^{3/2}}$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3a^{3/2}} - \frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{3ax} - \frac{4b^{3/2}p\tan}{}$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3a^{3/2}} - \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right.}{3a^{3/2}}$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3a^{3/2}} - \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right.}{3a^{3/2}}$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3a^{3/2}} - \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right.}{3a^{3/2}}$$

Mathematica [A] time = 0.10, size = 207, normalized size = 0.81

$$\frac{-4b^{3/2}px^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(\log\left(c\left(a+bx^2\right)^p\right)+2p\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)-2p\right)-4ib^{3/2}p^2x^3\text{Li}_2\left(\frac{\sqrt{b}\,x+i\sqrt{a}}{\sqrt{b}\,x-i\sqrt{a}}\right)-4ib^{3/2}p^2x^3\tan}{3a^{3/2}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^4,x]
```

```
[Out] ((-4*I)*b^(3/2)*p^2*x^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 - 4*b^(3/2)*p*x^3*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]]*(-2*p + 2*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x
)] + Log[c*(a + b*x^2)^p]) - Sqrt[a]*Log[c*(a + b*x^2)^p]*(4*b*p*x^2 + a*Lo
g[c*(a + b*x^2)^p]) - (4*I)*b^(3/2)*p^2*x^3*PolyLog[2, (I*Sqrt[a] + Sqrt[b]
*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(3*a^(3/2)*x^3)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^4, x)
```

maple [F] time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^4,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^2/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{p^2\log\left(bx^2+a\right)^2}{3\,x^3}+\int \frac{3\,bx^2\log(c)^2+3\,a\log(c)^2+2\left(\left(2\,p^2+3\,p\log(c)\right)bx^2+3\,ap\log(c)\right)\log\left(bx^2+a\right)}{3\left(bx^6+ax^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*p^2*log(b*x^2 + a)^2/x^3 + integrate(1/3*(3*b*x^2*log(c)^2 + 3*a*log(c
)^2 + 2*((2*p^2 + 3*p*log(c))*b*x^2 + 3*a*p*log(c))*log(b*x^2 + a))/(b*x^6
+ a*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^2/x^4,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^2/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**4,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**4, x)
```

3.89 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^6} dx$

Optimal. Leaf size=296

$$\frac{4b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{5a^{5/2}}+\frac{4ib^{5/2}p^2\text{Li}_2\left(1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{5a^{5/2}}+\frac{4ib^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5a^{5/2}}-\frac{32b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\cdot}{\sqrt{a}}\right.}{15a^{5/2}}$$

```
[Out] -8/15*b^2*p^2/a^2/x-32/15*b^(5/2)*p^2*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)+4/5
*I*b^(5/2)*p^2*arctan(x*b^(1/2)/a^(1/2))^2/a^(5/2)-4/15*b*p*ln(c*(b*x^2+a)^
p)/a/x^3+4/5*b^2*p*ln(c*(b*x^2+a)^p)/a^2/x+4/5*b^(5/2)*p*arctan(x*b^(1/2)/a
^(1/2))*ln(c*(b*x^2+a)^p)/a^(5/2)-1/5*ln(c*(b*x^2+a)^p)^2/x^5+8/5*b^(5/2)*p
^2*arctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/a^(5/2)+4/
5*I*b^(5/2)*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/a^(5/2)
```

Rubi [A] time = 0.32, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {2457, 2476, 2455, 325, 205, 2470, 12, 4920, 4854, 2402, 2315}

$$\frac{4ib^{5/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5a^{5/2}}+\frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{5a^2x}+\frac{4b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{5a^{5/2}}-\frac{8b^2p^2}{15a^2x}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^6,x]
```

```
[Out] (-8*b^2*p^2)/(15*a^2*x) - (32*b^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(15*
a^(5/2)) + (((4*I)/5)*b^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/a^(5/2) +
(8*b^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqr
t[b]*x)])/(5*a^(5/2)) - (4*b*p*Log[c*(a + b*x^2)^p])/(15*a*x^3) + (4*b^2*p*
Log[c*(a + b*x^2)^p])/(5*a^2*x) + (4*b^(5/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*
Log[c*(a + b*x^2)^p])/(5*a^(5/2)) - Log[c*(a + b*x^2)^p]^2/(5*x^5) + (((4*I
)/5)*b^(5/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^(5/
2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^6}\,dx = -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{5x^5} + \frac{1}{5}(4bp)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^4\left(a+bx^2\right)}\,dx$$

$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{5x^5} + \frac{1}{5}(4bp)\int\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{ax^4} - \frac{b\log\left(c\left(a+bx^2\right)^p\right)}{a^2x^2} + \frac{b^2}{\ldots}\right.$$

$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{5x^5} + \frac{(4bp)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx}{5a} - \frac{\left(4b^2p\right)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx}{5a^2} + \frac{\left(4b^{\ldots}\right.}{\ldots}$$

$$= -\frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{15ax^3} + \frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{5a^2x} + \frac{4b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a\ldots\right.\right.}{5a^{5/2}}$$

$$= -\frac{8b^2p^2}{15a^2x} - \frac{8b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5a^{5/2}} - \frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{15ax^3} + \frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{5a^2x}$$

$$= -\frac{8b^2p^2}{15a^2x} - \frac{32b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{15a^{5/2}} + \frac{4ib^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{5a^{5/2}} - \frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{15ax^3}$$

$$= -\frac{8b^2p^2}{15a^2x} - \frac{32b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{15a^{5/2}} + \frac{4ib^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{5a^{5/2}} + \frac{8b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\mathrm{lo}\ldots}{5a^{5/2}}$$

$$= -\frac{8b^2p^2}{15a^2x} - \frac{32b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{15a^{5/2}} + \frac{4ib^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{5a^{5/2}} + \frac{8b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\mathrm{lo}\ldots}{5a^{5/2}}$$

$$= -\frac{8b^2p^2}{15a^2x} - \frac{32b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{15a^{5/2}} + \frac{4ib^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{5a^{5/2}} + \frac{8b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\mathrm{lo}\ldots}{5a^{5/2}}$$

Mathematica [C] time = 0.29, size = 277, normalized size = 0.94

$$-\frac{3\log^2\left(c\left(a+bx^2\right)^p\right) + \frac{4bpx^2\left(a^{3/2}\log\left(c\left(a+bx^2\right)^p\right) - 3b^{3/2}x^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right) - 3ib^{3/2}px^3\left(\mathrm{Li}_2\left(\frac{\sqrt{b}\,x+i\sqrt{a}}{\sqrt{b}\,x-i\sqrt{a}}\right) + \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(\tan^{-1}\ldots\right.\right.\right.}{a^{5/2}}}{15x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^6,x]
```

```
[Out] -1/15*(3*Log[c*(a + b*x^2)^p]^2 + (4*b*p*x^2*(6*b^(3/2)*p*x^3*ArcTan[(Sqrt[
b]*x)/Sqrt[a]] + 2*Sqrt[a]*b*p*x^2*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^2
)/a)] + a^(3/2)*Log[c*(a + b*x^2)^p] - 3*Sqrt[a]*b*x^2*Log[c*(a + b*x^2)^p]
 - 3*b^(3/2)*x^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p] - (3*I)*b
^(3/2)*p*x^3*(ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(ArcTan[(Sqrt[b]*x)/Sqrt[a]] - (2
*I)*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)]) + PolyLog[2, (I*Sqrt[a] + Sqr
t[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])))/a^(5/2))/x^5
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^6,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^6, x)
```

maple [F] time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^6,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^2/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{p^2\log\left(bx^2+a\right)^2}{5\,x^5}+\int \frac{5\,bx^2\log(c)^2+5\,a\log(c)^2+2\left(\left(2\,p^2+5\,p\log(c)\right)bx^2+5\,ap\log(c)\right)\log\left(bx^2+a\right)}{5\left(bx^8+ax^6\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*p^2*log(b*x^2 + a)^2/x^5 + integrate(1/5*(5*b*x^2*log(c)^2 + 5*a*log(c
)^2 + 2*((2*p^2 + 5*p*log(c))*b*x^2 + 5*a*p*log(c))*log(b*x^2 + a))/(b*x^8
+ a*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^2/x^6,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^2/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**6,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**6, x)
```

3.90 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^8} dx$

Optimal. Leaf size=338

$$-\frac{4b^{7/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{7a^{7/2}}-\frac{4ib^{7/2}p^2\text{Li}_2\left(1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{7a^{7/2}}-\frac{4ib^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{7a^{7/2}}+\frac{184b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.}{105a^{7/2}}$$

```
[Out] -8/105*b^2*p^2/a^2/x^3+64/105*b^3*p^2/a^3/x+184/105*b^(7/2)*p^2*arctan(x*b^
(1/2)/a^(1/2))/a^(7/2)-4/7*I*b^(7/2)*p^2*arctan(x*b^(1/2)/a^(1/2))^2/a^(7/2
)-4/35*b*p*ln(c*(b*x^2+a)^p)/a/x^5+4/21*b^2*p*ln(c*(b*x^2+a)^p)/a^2/x^3-4/7
*b^3*p*ln(c*(b*x^2+a)^p)/a^3/x-4/7*b^(7/2)*p*arctan(x*b^(1/2)/a^(1/2))*ln(c
*(b*x^2+a)^p)/a^(7/2)-1/7*ln(c*(b*x^2+a)^p)^2/x^7-8/7*b^(7/2)*p^2*arctan(x*
b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/a^(7/2)-4/7*I*b^(7/2)*
p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/a^(7/2)
```

Rubi [A] time = 0.38, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {2457, 2476, 2455, 325, 205, 2470, 12, 4920, 4854, 2402, 2315}

$$-\frac{4ib^{7/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{7a^{7/2}}-\frac{4b^3p\log\left(c\left(a+bx^2\right)^p\right)}{7a^3x}+\frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{21a^2x^3}-\frac{4b^{7/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\right.}{7a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^8,x]
```

```
[Out] (-8*b^2*p^2)/(105*a^2*x^3) + (64*b^3*p^2)/(105*a^3*x) + (184*b^(7/2)*p^2*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(105*a^(7/2)) - (((4*I)/7)*b^(7/2)*p^2*ArcTan[(S
qrt[b]*x)/Sqrt[a]]^2)/a^(7/2) - (8*b^(7/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*
Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(7*a^(7/2)) - (4*b*p*Log[c*(a + b
*x^2)^p])/(35*a*x^5) + (4*b^2*p*Log[c*(a + b*x^2)^p])/(21*a^2*x^3) - (4*b^3
*p*Log[c*(a + b*x^2)^p])/(7*a^3*x) - (4*b^(7/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a
]]*Log[c*(a + b*x^2)^p])/(7*a^(7/2)) - Log[c*(a + b*x^2)^p]^2/(7*x^7) - (((
4*I)/7)*b^(7/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^
(7/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^8}\,dx = -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{7x^7} + \frac{1}{7}(4bp)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^6\left(a+bx^2\right)}\,dx$$

$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{7x^7} + \frac{1}{7}(4bp)\int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{ax^6} - \frac{b\log\left(c\left(a+bx^2\right)^p\right)}{a^2x^4} + \frac{b^2\log\right.$$

$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{7x^7} + \frac{(4bp)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^6}\,dx}{7a} - \frac{\left(4b^2p\right)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx}{7a^2} + \frac{\left(4b^3p\right)}{}$$

$$= -\frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{35ax^5} + \frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{21a^2x^3} - \frac{4b^3p\log\left(c\left(a+bx^2\right)^p\right)}{7a^3x} - \frac{4b^{7/2}p}{}$$

$$= -\frac{8b^2p^2}{105a^2x^3} + \frac{8b^3p^2}{21a^3x} + \frac{8b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{7a^{7/2}} - \frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{35ax^5} + \frac{4b^2p\log\left(c\left(a\right.\right.}{21a^2x^3}$$

$$= -\frac{8b^2p^2}{105a^2x^3} + \frac{64b^3p^2}{105a^3x} + \frac{32b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{21a^{7/2}} - \frac{4ib^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{7a^{7/2}} - \frac{4bp\log\left(c\left(a\right.\right.}{35ax}$$

$$= -\frac{8b^2p^2}{105a^2x^3} + \frac{64b^3p^2}{105a^3x} + \frac{184b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{105a^{7/2}} - \frac{4ib^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{7a^{7/2}} - \frac{8b^{7/2}p^2\tan}{}$$

$$= -\frac{8b^2p^2}{105a^2x^3} + \frac{64b^3p^2}{105a^3x} + \frac{184b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{105a^{7/2}} - \frac{4ib^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{7a^{7/2}} - \frac{8b^{7/2}p^2\tan}{}$$

$$= -\frac{8b^2p^2}{105a^2x^3} + \frac{64b^3p^2}{105a^3x} + \frac{184b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{105a^{7/2}} - \frac{4ib^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{7a^{7/2}} - \frac{8b^{7/2}p^2\tan}{}$$

Mathematica [C] time = 0.24, size = 334, normalized size = 0.99

$$-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{7x^7} + \frac{4bp\left(5a^{3/2}bx^2\log\left(c\left(a+bx^2\right)^p\right) - 3a^{5/2}\log\left(c\left(a+bx^2\right)^p\right) - 2a^{3/2}bpx^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{bx^2}{a}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^8,x]
```

```
[Out] -1/7*Log[c*(a + b*x^2)^p]^2/x^7 + (4*b*p*(30*b^(5/2)*p*x^5*ArcTan[(Sqrt[b]*
x)/Sqrt[a]] - 2*a^(3/2)*b*p*x^2*Hypergeometric2F1[-3/2, 1, -1/2, -((b*x^2)/
a)] + 10*Sqrt[a]*b^2*p*x^4*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^2)/a)] -
3*a^(5/2)*Log[c*(a + b*x^2)^p] + 5*a^(3/2)*b*x^2*Log[c*(a + b*x^2)^p] - 15*
Sqrt[a]*b^2*x^4*Log[c*(a + b*x^2)^p] - 15*b^(5/2)*x^5*ArcTan[(Sqrt[b]*x)/Sq
rt[a]]*Log[c*(a + b*x^2)^p] - (15*I)*b^(5/2)*p*x^5*(ArcTan[(Sqrt[b]*x)/Sqrt
[a]]*(ArcTan[(Sqrt[b]*x)/Sqrt[a]] - (2*I)*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt
[b]*x)]) + PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)]))
)/(105*a^(7/2)*x^5)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^8,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^8,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^8, x)
```

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^8,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^2/x^8,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{p^2\log\left(bx^2+a\right)^2}{7\,x^7}+\int \frac{7\,bx^2\log(c)^2+7\,a\log(c)^2+2\left(\left(2\,p^2+7\,p\log(c)\right)bx^2+7\,ap\log(c)\right)\log\left(bx^2+a\right)}{7\left(bx^{10}+ax^8\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^8,x, algorithm="maxima")
```

```
[Out] -1/7*p^2*log(b*x^2 + a)^2/x^7 + integrate(1/7*(7*b*x^2*log(c)^2 + 7*a*log(c
)^2 + 2*((2*p^2 + 7*p*log(c))*b*x^2 + 7*a*p*log(c))*log(b*x^2 + a))/(b*x^10
 + a*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^2/x^8,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^2/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**8,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**8, x)
```

3.91 $\int x^5 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=334

$$\frac{3a^2p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^3}+\frac{a^2\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b^3}-\frac{3a^2p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^3}-\frac{3a^2p}{b^2}$$

```
[Out] -3*a^2*p^3*x^2/b^2+3/8*a*p^3*(b*x^2+a)^2/b^3-1/27*p^3*(b*x^2+a)^3/b^3+3*a^2
*p^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b^3-3/4*a*p^2*(b*x^2+a)^2*ln(c*(b*x^2+a)^p
)/b^3+1/9*p^2*(b*x^2+a)^3*ln(c*(b*x^2+a)^p)/b^3-3/2*a^2*p*(b*x^2+a)*ln(c*(b
*x^2+a)^p)^2/b^3+3/4*a*p*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^2/b^3-1/6*p*(b*x^2+a
)^3*ln(c*(b*x^2+a)^p)^2/b^3+1/2*a^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^3/b^3-1/2*a
*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^3/b^3+1/6*(b*x^2+a)^3*ln(c*(b*x^2+a)^p)^3/b^
3
```

Rubi [A] time = 0.36, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{3a^2p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^3}-\frac{3a^2p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^3}+\frac{a^2\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b^3}-\frac{3a^2p}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (-3*a^2*p^3*x^2)/b^2 + (3*a*p^3*(a + b*x^2)^2)/(8*b^3) - (p^3*(a + b*x^2)^3
)/(27*b^3) + (3*a^2*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b^3 - (3*a*p^2*(a
 + b*x^2)^2*Log[c*(a + b*x^2)^p])/(4*b^3) + (p^2*(a + b*x^2)^3*Log[c*(a + b
*x^2)^p])/(9*b^3) - (3*a^2*p*(a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*b^3) +
(3*a*p*(a + b*x^2)^2*Log[c*(a + b*x^2)^p]^2)/(4*b^3) - (p*(a + b*x^2)^3*Log
[c*(a + b*x^2)^p]^2)/(6*b^3) + (a^2*(a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2*
b^3) - (a*(a + b*x^2)^2*Log[c*(a + b*x^2)^p]^3)/(2*b^3) + ((a + b*x^2)^3*Lo
g[c*(a + b*x^2)^p]^3)/(6*b^3)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^5 \log^3\left(c\left(a+bx^2\right)^p\right)dx = \frac{1}{2}\operatorname{Subst}\left(\int x^2\log^3\left(c(a+bx)^p\right)dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{a^2\log^3\left(c(a+bx)^p\right)}{b^2} - \frac{2a(a+bx)\log^3\left(c(a+bx)^p\right)}{b^2} + \frac{(a+bx)^2\log^3\left(\right.}{b^2}\right.\right.$$
$$= \frac{\operatorname{Subst}\left(\int(a+bx)^2\log^3\left(c(a+bx)^p\right)dx, x, x^2\right)}{2b^2} - \frac{a\operatorname{Subst}\left(\int(a+bx)\log^3\left(c(a+bx\right.\right.}{b^2}$$
$$= \frac{\operatorname{Subst}\left(\int x^2\log^3\left(cx^p\right)dx, x, a+bx^2\right)}{2b^3} - \frac{a\operatorname{Subst}\left(\int x\log^3\left(cx^p\right)dx, x, a+bx^2\right)}{b^3} + $$
$$= \frac{a^2\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b^3} - \frac{a\left(a+bx^2\right)^2\log^3\left(c\left(a+bx^2\right)^p\right)}{2b^3} + \frac{\left(a+bx^2\right)^3\log}{}$$
$$= -\frac{3a^2p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^3} + \frac{3ap\left(a+bx^2\right)^2\log^2\left(c\left(a+bx^2\right)^p\right)}{4b^3} - \frac{p\left(a+\right.}{}$$
$$= -\frac{3a^2p^3x^2}{b^2} + \frac{3ap^3\left(a+bx^2\right)^2}{8b^3} - \frac{p^3\left(a+bx^2\right)^3}{27b^3} + \frac{3a^2p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^3}$$

Mathematica [A] time = 0.20, size = 309, normalized size = 0.93

$$\frac{11a^3p^2\log\left(c\left(a+bx^2\right)^p\right)}{6b^3} + \frac{a^3\log^3\left(c\left(a+bx^2\right)^p\right)}{6b^3} - \frac{11a^3p\log^2\left(c\left(a+bx^2\right)^p\right)}{12b^3} + \frac{19a^3p^3\log\left(a+bx^2\right)}{36b^3} + \frac{11a^2p^2x^2\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (-85*a^2*p^3*x^2)/(36*b^2) + (19*a*p^3*x^4)/(72*b) - (p^3*x^6)/27 + (19*a^3
*p^3*Log[a + b*x^2])/(36*b^3) + (11*a^3*p^2*Log[c*(a + b*x^2)^p])/(6*b^3) +
 (11*a^2*p^2*x^2*Log[c*(a + b*x^2)^p])/(6*b^2) - (5*a*p^2*x^4*Log[c*(a + b*
x^2)^p])/(12*b) + (p^2*x^6*Log[c*(a + b*x^2)^p])/9 - (11*a^3*p*Log[c*(a + b
*x^2)^p]^2)/(12*b^3) - (a^2*p*x^2*Log[c*(a + b*x^2)^p]^2)/(2*b^2) + (a*p*x^
4*Log[c*(a + b*x^2)^p]^2)/(4*b) - (p*x^6*Log[c*(a + b*x^2)^p]^2)/6 + (a^3*L
og[c*(a + b*x^2)^p]^3)/(6*b^3) + (x^6*Log[c*(a + b*x^2)^p]^3)/6
```

fricas [A] time = 0.47, size = 359, normalized size = 1.07

$$-\frac{8\,b^3p^3x^6 - 36\,b^3x^6\log(c)^3 - 57\,ab^2p^3x^4 + 510\,a^2bp^3x^2 - 36\left(b^3p^3x^6 + a^3p^3\right)\log\left(bx^2+a\right)^3 + 18\left(2\,b^3p^3x^6 - \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] -1/216*(8*b^3*p^3*x^6 - 36*b^3*x^6*log(c)^3 - 57*a*b^2*p^3*x^4 + 510*a^2*b*
p^3*x^2 - 36*(b^3*p^3*x^6 + a^3*p^3)*log(b*x^2 + a)^3 + 18*(2*b^3*p^3*x^6 -
 3*a*b^2*p^3*x^4 + 6*a^2*b*p^3*x^2 + 11*a^3*p^3 - 6*(b^3*p^2*x^6 + a^3*p^2)
*log(c))*log(b*x^2 + a)^2 + 18*(2*b^3*p*x^6 - 3*a*b^2*p*x^4 + 6*a^2*b*p*x^2
)*log(c)^2 - 6*(4*b^3*p^3*x^6 - 15*a*b^2*p^3*x^4 + 66*a^2*b*p^3*x^2 + 85*a^
3*p^3 + 18*(b^3*p*x^6 + a^3*p)*log(c)^2 - 6*(2*b^3*p^2*x^6 - 3*a*b^2*p^2*x^
4 + 6*a^2*b*p^2*x^2 + 11*a^3*p^2)*log(c))*log(b*x^2 + a) - 6*(4*b^3*p^2*x^6
 - 15*a*b^2*p^2*x^4 + 66*a^2*b*p^2*x^2)*log(c))/b^3
```

giac [A] time = 0.20, size = 595, normalized size = 1.78

$$\frac{36\,bx^6\log(c)^3 + \left(\frac{36\left(bx^2+a\right)^3\log\left(bx^2+a\right)^3}{b^2} - \frac{108\left(bx^2+a\right)^2a\log\left(bx^2+a\right)^3}{b^2} + \frac{108\left(bx^2+a\right)a^2\log\left(bx^2+a\right)^3}{b^2} - \frac{36\left(bx^2+a\right)^3\log\left(bx^2+a\right)^2}{b^2} + \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] 1/216*(36*b*x^6*log(c)^3 + (36*(b*x^2 + a)^3*log(b*x^2 + a)^3/b^2 - 108*(b*
x^2 + a)^2*a*log(b*x^2 + a)^3/b^2 + 108*(b*x^2 + a)*a^2*log(b*x^2 + a)^3/b^
2 - 36*(b*x^2 + a)^3*log(b*x^2 + a)^2/b^2 + 162*(b*x^2 + a)^2*a*log(b*x^2 +
 a)^2/b^2 - 324*(b*x^2 + a)*a^2*log(b*x^2 + a)^2/b^2 + 24*(b*x^2 + a)^3*log
(b*x^2 + a)/b^2 - 162*(b*x^2 + a)^2*a*log(b*x^2 + a)/b^2 + 648*(b*x^2 + a)*
a^2*log(b*x^2 + a)/b^2 - 8*(b*x^2 + a)^3/b^2 + 81*(b*x^2 + a)^2*a/b^2 - 648
*(b*x^2 + a)*a^2/b^2)*p^3 + 6*(18*(b*x^2 + a)^3*log(b*x^2 + a)^2/b^2 - 54*(
b*x^2 + a)^2*a*log(b*x^2 + a)^2/b^2 + 54*(b*x^2 + a)*a^2*log(b*x^2 + a)^2/b
^2 - 12*(b*x^2 + a)^3*log(b*x^2 + a)/b^2 + 54*(b*x^2 + a)^2*a*log(b*x^2 + a
)/b^2 - 108*(b*x^2 + a)*a^2*log(b*x^2 + a)/b^2 + 4*(b*x^2 + a)^3/b^2 - 27*(
b*x^2 + a)^2*a/b^2 + 108*(b*x^2 + a)*a^2/b^2)*p^2*log(c) + 18*(6*(b*x^2 + a
)^3*log(b*x^2 + a)/b^2 - 18*(b*x^2 + a)^2*a*log(b*x^2 + a)/b^2 + 18*(b*x^2
+ a)*a^2*log(b*x^2 + a)/b^2 - 2*(b*x^2 + a)^3/b^2 + 9*(b*x^2 + a)^2*a/b^2 -
 18*(b*x^2 + a)*a^2/b^2)*p*log(c)^2)/b
```

maple [C] time = 1.13, size = 5905, normalized size = 17.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.72, size = 239, normalized size = 0.72

$$\frac{1}{6}x^6\log\left(\left(bx^2+a\right)^pc\right)^3+\frac{1}{12}bp\left(\frac{6a^3\log\left(bx^2+a\right)}{b^4}-\frac{2b^2x^6-3abx^4+6a^2x^2}{b^3}\right)\log\left(\left(bx^2+a\right)^pc\right)^2-\frac{1}{216}bp\left(\frac{\left(8b^3x^6\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log((b*x^2 + a)^p*c)^3 + 1/12*b*p*(6*a^3*log(b*x^2 + a)/b^4 - (2*b^
2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)/b^3)*log((b*x^2 + a)^p*c)^2 - 1/216*b*p*((8*
b^3*x^6 - 57*a*b^2*x^4 - 36*a^3*log(b*x^2 + a)^3 + 510*a^2*b*x^2 - 198*a^3*
log(b*x^2 + a)^2 - 510*a^3*log(b*x^2 + a))*p^2/b^4 - 6*(4*b^3*x^6 - 15*a*b^
2*x^4 + 66*a^2*b*x^2 - 18*a^3*log(b*x^2 + a)^2 - 66*a^3*log(b*x^2 + a))*p*l
og((b*x^2 + a)^p*c)/b^4)
```

mupad [B] time = 0.36, size = 187, normalized size = 0.56

$$\ln\left(c\left(b\,x^2+a\right)^p\right)^3\left(\frac{x^6}{6}+\frac{a^3}{6\,b^3}\right)-\ln\left(c\left(b\,x^2+a\right)^p\right)^2\left(\frac{p\,x^6}{6}+\frac{11\,a^3\,p}{12\,b^3}+\frac{a^2\,p\,x^2}{2\,b^2}-\frac{a\,p\,x^4}{4\,b}\right)-\frac{p^3\,x^6}{27}+\frac{\ln\left(c\left(b\,x^2+a\right)^p\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] log(c*(a + b*x^2)^p)^3*(x^6/6 + a^3/(6*b^3)) - log(c*(a + b*x^2)^p)^2*((p*x
^6)/6 + (11*a^3*p)/(12*b^3) + (a^2*p*x^2)/(2*b^2) - (a*p*x^4)/(4*b)) - (p^3
*x^6)/27 + (log(c*(a + b*x^2)^p)*((b*p^2*x^6)/3 - (5*a*p^2*x^4)/4 + (11*a^2
*p^2*x^2)/(2*b)))/(3*b) + (19*a*p^3*x^4)/(72*b) + (85*a^3*p^3*log(a + b*x^2
))/(36*b^3) - (85*a^2*p^3*x^2)/(36*b^2)
```

sympy [A] time = 37.19, size = 561, normalized size = 1.68

$$\begin{cases}\frac{a^3p^3\log\left(a+bx^2\right)^3}{6b^3}-\frac{11a^3p^3\log\left(a+bx^2\right)^2}{12b^3}+\frac{85a^3p^3\log\left(a+bx^2\right)}{36b^3}+\frac{a^3p^2\log\left(c\right)\log\left(a+bx^2\right)^2}{2b^3}-\frac{11a^3p^2\log\left(c\right)\log\left(a+bx^2\right)}{6b^3}+\frac{a^3p\log\left(c\right)^2\log\left(a\right.}{2b^3} \\ \frac{x^6\log\left(a^pc\right)^3}{6}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Piecewise((a**3*p**3*log(a + b*x**2)**3/(6*b**3) - 11*a**3*p**3*log(a + b*x
**2)**2/(12*b**3) + 85*a**3*p**3*log(a + b*x**2)/(36*b**3) + a**3*p**2*log(
c)*log(a + b*x**2)**2/(2*b**3) - 11*a**3*p**2*log(c)*log(a + b*x**2)/(6*b**
3) + a**3*p*log(c)**2*log(a + b*x**2)/(2*b**3) - a**2*p**3*x**2*log(a + b*x
**2)**2/(2*b**2) + 11*a**2*p**3*x**2*log(a + b*x**2)/(6*b**2) - 85*a**2*p**
3*x**2/(36*b**2) - a**2*p**2*x**2*log(c)*log(a + b*x**2)/b**2 + 11*a**2*p**
2*x**2*log(c)/(6*b**2) - a**2*p*x**2*log(c)**2/(2*b**2) + a*p**3*x**4*log(a
 + b*x**2)**2/(4*b) - 5*a*p**3*x**4*log(a + b*x**2)/(12*b) + 19*a*p**3*x**4
/(72*b) + a*p**2*x**4*log(c)*log(a + b*x**2)/(2*b) - 5*a*p**2*x**4*log(c)/(
12*b) + a*p*x**4*log(c)**2/(4*b) + p**3*x**6*log(a + b*x**2)**3/6 - p**3*x*
*6*log(a + b*x**2)**2/6 + p**3*x**6*log(a + b*x**2)/9 - p**3*x**6/27 + p**2
*x**6*log(c)*log(a + b*x**2)**2/2 - p**2*x**6*log(c)*log(a + b*x**2)/3 + p*
*2*x**6*log(c)/9 + p*x**6*log(c)**2*log(a + b*x**2)/2 - p*x**6*log(c)**2/6
+ x**6*log(c)**3/6, Ne(b, 0)), (x**6*log(a**p*c)**3/6, True))
```

3.92 $\int x^3 \log^3\left(c\left(a+bx^2\right)^p\right) dx$

Optimal. Leaf size=211

$$\frac{3p^2\left(a+bx^2\right)^2\log\left(c\left(a+bx^2\right)^p\right)}{8b^2}-\frac{3ap^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^2}+\frac{\left(a+bx^2\right)^2\log^3\left(c\left(a+bx^2\right)^p\right)}{4b^2}-\frac{a\left(a+bx\right.}{}$$

```
[Out] 3*a*p^3*x^2/b-3/16*p^3*(b*x^2+a)^2/b^2-3*a*p^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/
b^2+3/8*p^2*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)/b^2+3/2*a*p*(b*x^2+a)*ln(c*(b*x^2
+a)^p)^2/b^2-3/8*p*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^2/b^2-1/2*a*(b*x^2+a)*ln(c
*(b*x^2+a)^p)^3/b^2+1/4*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^3/b^2
```

Rubi [A] time = 0.21, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{3p^2\left(a+bx^2\right)^2\log\left(c\left(a+bx^2\right)^p\right)}{8b^2}-\frac{3ap^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^2}-\frac{3p\left(a+bx^2\right)^2\log^2\left(c\left(a+bx^2\right)^p\right)}{8b^2}+\frac{3ap\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (3*a*p^3*x^2)/b - (3*p^3*(a + b*x^2)^2)/(16*b^2) - (3*a*p^2*(a + b*x^2)*Log
[c*(a + b*x^2)^p])/b^2 + (3*p^2*(a + b*x^2)^2*Log[c*(a + b*x^2)^p])/(8*b^2)
 + (3*a*p*(a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*b^2) - (3*p*(a + b*x^2)^2*
Log[c*(a + b*x^2)^p]^2)/(8*b^2) - (a*(a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2
*b^2) + ((a + b*x^2)^2*Log[c*(a + b*x^2)^p]^3)/(4*b^2)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\log^3\left(c\left(a+bx^2\right)^p\right)dx &= \frac{1}{2}\text{Subst}\left(\int x\log^3\left(c(a+bx)^p\right)dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a\log^3\left(c(a+bx)^p\right)}{b}+\frac{(a+bx)\log^3\left(c(a+bx)^p\right)}{b}\right)dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int(a+bx)\log^3\left(c(a+bx)^p\right)dx,x,x^2\right)}{2b}-\frac{a\,\text{Subst}\left(\int\log^3\left(c(a+bx)^p\right)dx,x,\right.}{2b}\\
&= \frac{\text{Subst}\left(\int x\log^3\left(cx^p\right)dx,x,a+bx^2\right)}{2b^2}-\frac{a\,\text{Subst}\left(\int\log^3\left(cx^p\right)dx,x,a+bx^2\right)}{2b^2}\\
&= -\frac{a\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b^2}+\frac{\left(a+bx^2\right)^2\log^3\left(c\left(a+bx^2\right)^p\right)}{4b^2}-\frac{(3p)\,\text{Subst}\left(\int\right.}{}\\
&= \frac{3ap\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2}-\frac{3p\left(a+bx^2\right)^2\log^2\left(c\left(a+bx^2\right)^p\right)}{8b^2}-\frac{a\left(a+bx^2\right.}{}\\
&= \frac{3ap^3x^2}{b}-\frac{3p^3\left(a+bx^2\right)^2}{16b^2}-\frac{3ap^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^2}+\frac{3p^2\left(a+bx^2\right)^2\log}{8b^2}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 237, normalized size = 1.12

$$-\frac{9a^2p^2\log\left(c\left(a+bx^2\right)^p\right)}{4b^2}-\frac{a^2\log^3\left(c\left(a+bx^2\right)^p\right)}{4b^2}+\frac{9a^2p\log^2\left(c\left(a+bx^2\right)^p\right)}{8b^2}-\frac{3a^2p^3\log\left(a+bx^2\right)}{8b^2}-\frac{9ap^2x^2\log\left(c\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (21*a*p^3*x^2)/(8*b) - (3*p^3*x^4)/16 - (3*a^2*p^3*Log[a + b*x^2])/(8*b^2)
- (9*a^2*p^2*Log[c*(a + b*x^2)^p])/(4*b^2) - (9*a*p^2*x^2*Log[c*(a + b*x^2)
^p])/(4*b) + (3*p^2*x^4*Log[c*(a + b*x^2)^p])/8 + (9*a^2*p*Log[c*(a + b*x^2
)^p]^2)/(8*b^2) + (3*a*p*x^2*Log[c*(a + b*x^2)^p]^2)/(4*b) - (3*p*x^4*Log[c
```

```
*(a + b*x^2)^p]^2)/8 - (a^2*Log[c*(a + b*x^2)^p]^3)/(4*b^2) + (x^4*Log[c*(a
 + b*x^2)^p]^3)/4
```

fricas [A] time = 0.47, size = 275, normalized size = 1.30

$$-\frac{3\,b^2p^3x^4 - 4\,b^2x^4\log(c)^3 - 42\,abp^3x^2 - 4\left(b^2p^3x^4 - a^2p^3\right)\log\left(bx^2+a\right)^3 + 6\left(b^2p^3x^4 - 2\,abp^3x^2 - 3\,a^2p^3 - 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(3*b^2*p^3*x^4 - 4*b^2*x^4*log(c)^3 - 42*a*b*p^3*x^2 - 4*(b^2*p^3*x^4
 - a^2*p^3)*log(b*x^2 + a)^3 + 6*(b^2*p^3*x^4 - 2*a*b*p^3*x^2 - 3*a^2*p^3 -
 2*(b^2*p^2*x^4 - a^2*p^2)*log(c))*log(b*x^2 + a)^2 + 6*(b^2*p*x^4 - 2*a*b*
p*x^2)*log(c)^2 - 6*(b^2*p^3*x^4 - 6*a*b*p^3*x^2 - 7*a^2*p^3 + 2*(b^2*p*x^4
 - a^2*p)*log(c)^2 - 2*(b^2*p^2*x^4 - 2*a*b*p^2*x^2 - 3*a^2*p^2)*log(c))*lo
g(b*x^2 + a) - 6*(b^2*p^2*x^4 - 6*a*b*p^2*x^2)*log(c))/b^2
```

giac [A] time = 0.22, size = 360, normalized size = 1.71

$$\frac{\left(4\left(bx^2+a\right)^2\log\left(bx^2+a\right)^3 - 8\left(bx^2+a\right)a\log\left(bx^2+a\right)^3 - 6\left(bx^2+a\right)^2\log\left(bx^2+a\right)^2 + 24\left(bx^2+a\right)a\log\left(bx^2+a\right)^2 + 6\left(bx^2+a\right)^2\log\left(bx^2+a\right) - 48\left(bx^2+a\right)a\log\left(\right.\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] 1/16*((4*(b*x^2 + a)^2*log(b*x^2 + a)^3 - 8*(b*x^2 + a)*a*log(b*x^2 + a)^3
- 6*(b*x^2 + a)^2*log(b*x^2 + a)^2 + 24*(b*x^2 + a)*a*log(b*x^2 + a)^2 + 6*
(b*x^2 + a)^2*log(b*x^2 + a) - 48*(b*x^2 + a)*a*log(b*x^2 + a) - 3*(b*x^2 +
 a)^2 + 48*(b*x^2 + a)*a)*p^3/b + 6*(2*(b*x^2 + a)^2*log(b*x^2 + a)^2 - 4*(
b*x^2 + a)*a*log(b*x^2 + a)^2 - 2*(b*x^2 + a)^2*log(b*x^2 + a) + 8*(b*x^2 +
 a)*a*log(b*x^2 + a) + (b*x^2 + a)^2 - 8*(b*x^2 + a)*a)*p^2*log(c)/b + 6*(2
*(b*x^2 + a)^2*log(b*x^2 + a) - 4*(b*x^2 + a)*a*log(b*x^2 + a) - (b*x^2 + a
)^2 + 4*(b*x^2 + a)*a)*p*log(c)^2/b + 4*((b*x^2 + a)^2 - 2*(b*x^2 + a)*a)*l
og(c)^3/b)/b
```

maple [C] time = 1.06, size = 4942, normalized size = 23.42

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] -3/16*x^4*p^3-3/8*ln(c)^2*p*x^4+3/8*ln(c)*p^2*x^4+21/8*a*p^3*x^2/b+3/8*(-I*
Pi*b^2*x^4*csgn(I*c)*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)+I*Pi*b^2*x^4
*csgn(I*c)*csgn(I*c*(b*x^2+a)^p)^2+I*Pi*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*
c*(b*x^2+a)^p)^2-I*Pi*b^2*x^4*csgn(I*c*(b*x^2+a)^p)^3-b^2*p*x^4+2*b^2*x^4*l
n(c)+2*a*b*p*x^2-2*a^2*p*ln(b*x^2+a))/b^2*ln((b*x^2+a)^p)^2-3/16*ln(c)*Pi^2
*x^4*csgn(I*c*(b*x^2+a)^p)^6+3/32*Pi^2*p*x^4*csgn(I*c*(b*x^2+a)^p)^6+1/32*I
*Pi^3*x^4*csgn(I*c*(b*x^2+a)^p)^9-21/8*a^2*p^3/b^2*ln(b*x^2+a)-3/16/b*Pi^2*
a*p*x^2*csgn(I*c*(b*x^2+a)^p)^6+3/16/b^2*Pi^2*ln(b*x^2+a)*a^2*p*csgn(I*c*(b
*x^2+a)^p)^6-3/16*ln(c)*Pi^2*x^4*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p
)^2*csgn(I*c)^2+3/16*(4*ln(c)^2*b^2*x^4+4*a^2*p^2*ln(b*x^2+a)^2+12*ln(b*x^2
+a)*a^2*p^2-12*x^2*b*a*p^2+2*x^4*b^2*p^2-8*ln(c)*ln(b*x^2+a)*a^2*p-4*ln(c)*
b^2*p*x^4-Pi^2*b^2*x^4*csgn(I*c*(b*x^2+a)^p)^6-Pi^2*b^2*x^4*csgn(I*c*(b*x^2
+a)^p)^4*csgn(I*c)^2-4*I*ln(c)*Pi*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x
^2+a)^p)*csgn(I*c)-4*I*Pi*a*b*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^
p)*csgn(I*c)+4*I*ln(c)*Pi*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)
```

```
^2+4*I*ln(c)*Pi*b^2*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-4*I*Pi*a*b*p*x^2*
csgn(I*c*(b*x^2+a)^p)^3+2*Pi^2*b^2*x^4*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^
2+a)^p)^3*csgn(I*c)-Pi^2*b^2*x^4*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p
)^2*csgn(I*c)^2+8*ln(c)*a*b*p*x^2-4*I*Pi*ln(b*x^2+a)*a^2*p*csgn(I*c*(b*x^2+
a)^p)^2*csgn(I*c)-2*I*Pi*b^2*p*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p
)^2-2*I*Pi*b^2*p*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*I*Pi*b^2*p*x^4*csg
n(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+4*I*Pi*ln(b*x^2+a)*a^2*p*c
sgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+4*I*Pi*a*b*p*x^2*csgn(I*
(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+4*I*Pi*a*b*p*x^2*csgn(I*c*(b*x^2+a)^p)
^2*csgn(I*c)-Pi^2*b^2*x^4*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4+2*P
i^2*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5+2*Pi^2*b^2*x^4*csgn
(I*c*(b*x^2+a)^p)^5*csgn(I*c)-4*Pi^2*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(
b*x^2+a)^p)^4*csgn(I*c)+2*Pi^2*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+
a)^p)^3*csgn(I*c)^2+2*I*Pi*b^2*p*x^4*csgn(I*c*(b*x^2+a)^p)^3-4*I*ln(c)*Pi*b
^2*x^4*csgn(I*c*(b*x^2+a)^p)^3+4*I*Pi*ln(b*x^2+a)*a^2*p*csgn(I*c*(b*x^2+a)^
p)^3-4*I*Pi*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2)/
b^2*ln((b*x^2+a)^p)-1/4/b^2*a^2*p^3*ln(b*x^2+a)^3-9/8/b^2*a^2*p^3*ln(b*x^2+
a)^2-3/16*ln(c)*Pi^2*x^4*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2+3/8*ln(c)*Pi^2
*x^4*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)-3/16*ln(c)*Pi^2*x^4*csgn(I*(b*x^2+a)
^p)^2*csgn(I*c*(b*x^2+a)^p)^4+3/8*ln(c)*Pi^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I
*c*(b*x^2+a)^p)^5+3/32*Pi^2*p*x^4*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2-3/16*
Pi^2*p*x^4*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)+3/32*Pi^2*p*x^4*csgn(I*(b*x^2+
a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4-3/16*Pi^2*p*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*
c*(b*x^2+a)^p)^5+3/4/b*ln(c)^2*a*p*x^2+3/4/b^2*ln(c)*a^2*p^2*ln(b*x^2+a)^2-
9/4/b*ln(c)*a*p^2*x^2-3/4/b^2*ln(c)^2*ln(b*x^2+a)*a^2*p+9/4/b^2*ln(c)*ln(b*
x^2+a)*a^2*p^2-1/32*I*Pi^3*x^4*csgn(I*c*(b*x^2+a)^p)^6*csgn(I*c)^3+3/32*I*P
i^3*x^4*csgn(I*c*(b*x^2+a)^p)^7*csgn(I*c)^2-3/32*I*Pi^3*x^4*csgn(I*c*(b*x^2
+a)^p)^8*csgn(I*c)-1/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)^p)^3*csgn(I*c*(b*x^2+a)
^p)^6+3/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^7-3/32*I*
Pi^3*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^8-3/8*I*ln(c)^2*Pi*x^4*c
sgn(I*c*(b*x^2+a)^p)^3-3/16*I*Pi*p^2*x^4*csgn(I*c*(b*x^2+a)^p)^3-3/16/b*Pi^
2*a*p*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2+3/8/b*P
i^2*a*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2+3/8/b*P
i^2*a*p*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)-3/4/b*P
i^2*a*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)+3/16/b^2*
Pi^2*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I
*c)^2-3/8/b^2*Pi^2*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)^3*csgn(I*c)^2-3/8/b^2*Pi^2*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)^2*csgn
(I*c*(b*x^2+a)^p)^3*csgn(I*c)+3/4/b^2*Pi^2*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+
a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)+3/8*I*ln(c)*Pi*p*x^4*csgn(I*(b*x^2+
a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-3/4*I/b*ln(c)*Pi*a*p*x^2*csgn(I*c*(b*
x^2+a)^p)^3+3/8*I/b^2*Pi*a^2*p^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*ln(b*x^2
+a)^2+3/8*I/b^2*Pi*a^2*p^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*ln(b
*x^2+a)^2-9/8*I/b*Pi*a*p^2*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-9/8*I/b*Pi
*a*p^2*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+3/4*I/b^2*ln(c)*Pi*l
n(b*x^2+a)*a^2*p*csgn(I*c*(b*x^2+a)^p)^3+9/8*I/b^2*Pi*ln(b*x^2+a)*a^2*p^2*c
sgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+9/8*I/b^2*Pi*ln(b*x^2+a)*a^2*p^2*csgn(I*(b
*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/4*x^4*ln((b*x^2+a)^p)^3+3/8*ln(c)*Pi^2
*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2+3/8*ln(c)*Pi^2
*x^4*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)-3/4*ln(c)*Pi^2
*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)+3/32*Pi^2*p*x^4*
csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2-3/16*Pi^2*p*x^4*c
sgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2-3/16*Pi^2*p*x^4*csgn
(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)+3/8*Pi^2*p*x^4*csgn(I*(
b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)+1/32*I*Pi^3*x^4*csgn(I*(b*x^2
+a)^p)^3*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^3-3/32*I*Pi^3*x^4*csgn(I*(b*x^2+
a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^3+3/32*I*Pi^3*x^4*csgn(I*(b*x^2+a
)^p)*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)^3-3/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)^p
)^3*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2+9/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)^p)
```

```
^2*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)^2-9/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)^p)*
csgn(I*c*(b*x^2+a)^p)^6*csgn(I*c)^2+3/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)^p)^3*c
sgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)-9/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)^p)^2*csgn
(I*c*(b*x^2+a)^p)^6*csgn(I*c)+9/32*I*Pi^3*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)^7*csgn(I*c)+3/8*I*ln(c)^2*Pi*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(
I*c)+3/8*I*ln(c)^2*Pi*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+3/8*I
*ln(c)*Pi*p*x^4*csgn(I*c*(b*x^2+a)^p)^3+3/16*I*Pi*p^2*x^4*csgn(I*c*(b*x^2+a
)^p)^2*csgn(I*c)+3/16*I*Pi*p^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p
)^2+3/4*I/b*ln(c)*Pi*a*p*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+3/4*I/b*ln(c
)*Pi*a*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-3/8*I/b^2*Pi*a^2*p
^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*ln(b*x^2+a)^2+9/8*I/
b*Pi*a*p^2*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-3/4*I/b^
2*ln(c)*Pi*ln(b*x^2+a)*a^2*p*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-3/4*I/b^2*ln
(c)*Pi*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-9/8*I/
b^2*Pi*ln(b*x^2+a)*a^2*p^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I
*c)-3/8*I*ln(c)^2*Pi*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c
)-3/8*I*ln(c)*Pi*p*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-3/8*I*ln(c)*Pi*p*x
^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-3/16*I*Pi*p^2*x^4*csgn(I*(b*
x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-3/8*I/b^2*Pi*a^2*p^2*csgn(I*c*(b*
x^2+a)^p)^3*ln(b*x^2+a)^2+9/8*I/b*Pi*a*p^2*x^2*csgn(I*c*(b*x^2+a)^p)^3-9/8*
I/b^2*Pi*ln(b*x^2+a)*a^2*p^2*csgn(I*c*(b*x^2+a)^p)^3+1/4*ln(c)^3*x^4-3/4*I/
b*ln(c)*Pi*a*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+3/4*
I/b^2*ln(c)*Pi*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*
csgn(I*c)-3/16/b*Pi^2*a*p*x^2*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2+3/8/b*Pi^
2*a*p*x^2*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)-3/16/b*Pi^2*a*p*x^2*csgn(I*(b*x
^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4+3/8/b*Pi^2*a*p*x^2*csgn(I*(b*x^2+a)^p)*c
sgn(I*c*(b*x^2+a)^p)^5+3/16/b^2*Pi^2*ln(b*x^2+a)*a^2*p*csgn(I*c*(b*x^2+a)^p
)^4*csgn(I*c)^2-3/8/b^2*Pi^2*ln(b*x^2+a)*a^2*p*csgn(I*c*(b*x^2+a)^p)^5*csgn
(I*c)+3/16/b^2*Pi^2*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2
+a)^p)^4-3/8/b^2*Pi^2*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2
+a)^p)^5
```

maxima [A] time = 0.76, size = 203, normalized size = 0.96

$$\frac{1}{4}x^4\log\left(\left(bx^2+a\right)^pc\right)^3-\frac{3}{8}bp\left(\frac{2a^2\log\left(bx^2+a\right)}{b^3}+\frac{bx^4-2ax^2}{b^2}\right)\log\left(\left(bx^2+a\right)^pc\right)^2-\frac{1}{16}bp\left(\frac{\left(3b^2x^4+4a^2\log\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((b*x^2 + a)^p*c)^3 - 3/8*b*p*(2*a^2*log(b*x^2 + a)/b^3 + (b*x^4
 - 2*a*x^2)/b^2)*log((b*x^2 + a)^p*c)^2 - 1/16*b*p*((3*b^2*x^4 + 4*a^2*log(
b*x^2 + a)^3 - 42*a*b*x^2 + 18*a^2*log(b*x^2 + a)^2 + 42*a^2*log(b*x^2 + a)
)*p^2/b^3 - 6*(b^2*x^4 - 6*a*b*x^2 + 2*a^2*log(b*x^2 + a)^2 + 6*a^2*log(b*x
^2 + a))*p*log((b*x^2 + a)^p*c)/b^3)
```

mupad [B] time = 0.30, size = 144, normalized size = 0.68

$$\ln\left(c\left(b\,x^2+a\right)^p\right)^2\,\left(\frac{9\,a^2\,p}{8\,b^2}-\frac{3\,p\,x^4}{8}+\frac{3\,a\,p\,x^2}{4\,b}\right)-\frac{3\,p^3\,x^4}{16}+\ln\left(c\left(b\,x^2+a\right)^p\right)\,\left(\frac{3\,p^2\,x^4}{8}-\frac{9\,a\,p^2\,x^2}{4\,b}\right)+\ln\left(c\left(b\,x^2+a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] log(c*(a + b*x^2)^p)^2*((9*a^2*p)/(8*b^2) - (3*p*x^4)/8 + (3*a*p*x^2)/(4*b)
) - (3*p^3*x^4)/16 + log(c*(a + b*x^2)^p)*((3*p^2*x^4)/8 - (9*a*p^2*x^2)/(4
*b)) + log(c*(a + b*x^2)^p)^3*(x^4/4 - a^2/(4*b^2)) + (21*a*p^3*x^2)/(8*b)
- (21*a^2*p^3*log(a + b*x^2))/(8*b^2)
```

sympy [A] time = 15.11, size = 450, normalized size = 2.13

$$\begin{cases} -\frac{a^2p^3\log\left(a+bx^2\right)^3}{4b^2} + \frac{9a^2p^3\log\left(a+bx^2\right)^2}{8b^2} - \frac{21a^2p^3\log\left(a+bx^2\right)}{8b^2} - \frac{3a^2p^2\log\left(c\right)\log\left(a+bx^2\right)^2}{4b^2} + \frac{9a^2p^2\log\left(c\right)\log\left(a+bx^2\right)}{4b^2} - \frac{3a^2p\log\left(c\right)^2\log(}{4b^2} \\ \frac{x^4\log\left(a^pc\right)^3}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Piecewise((-a**2*p**3*log(a + b*x**2)**3/(4*b**2) + 9*a**2*p**3*log(a + b*x
**2)**2/(8*b**2) - 21*a**2*p**3*log(a + b*x**2)/(8*b**2) - 3*a**2*p**2*log(
c)*log(a + b*x**2)**2/(4*b**2) + 9*a**2*p**2*log(c)*log(a + b*x**2)/(4*b**2
) - 3*a**2*p*log(c)**2*log(a + b*x**2)/(4*b**2) + 3*a*p**3*x**2*log(a + b*x
**2)**2/(4*b) - 9*a*p**3*x**2*log(a + b*x**2)/(4*b) + 21*a*p**3*x**2/(8*b)
+ 3*a*p**2*x**2*log(c)*log(a + b*x**2)/(2*b) - 9*a*p**2*x**2*log(c)/(4*b) +
 3*a*p*x**2*log(c)**2/(4*b) + p**3*x**4*log(a + b*x**2)**3/4 - 3*p**3*x**4*
log(a + b*x**2)**2/8 + 3*p**3*x**4*log(a + b*x**2)/8 - 3*p**3*x**4/16 + 3*p
**2*x**4*log(c)*log(a + b*x**2)**2/4 - 3*p**2*x**4*log(c)*log(a + b*x**2)/4
 + 3*p**2*x**4*log(c)/8 + 3*p*x**4*log(c)**2*log(a + b*x**2)/4 - 3*p*x**4*l
og(c)**2/8 + x**4*log(c)**3/4, Ne(b, 0)), (x**4*log(a**p*c)**3/4, True))
```

3.93 $\int x \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=93

$$\frac{3p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}+\frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b}-\frac{3p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b}-3p^3x^2$$

```
[Out] -3*p^3*x^2+3*p^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b-3/2*p*(b*x^2+a)*ln(c*(b*x^2+
a)^p)^2/b+1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^3/b
```

Rubi [A] time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2454, 2389, 2296, 2295}

$$\frac{3p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}-\frac{3p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b}+\frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b}-3p^3x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -3*p^3*x^2 + (3*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b - (3*p*(a + b*x^2)*
Log[c*(a + b*x^2)^p]^2)/(2*b) + ((a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2*b)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x \log^3\left(c\left(a+bx^2\right)^p\right) dx = \frac{1}{2}\operatorname{Subst}\left(\int \log^3\left(c(a+bx)^p\right) dx, x, x^2\right)$$
$$= \frac{\operatorname{Subst}\left(\int \log^3\left(cx^p\right) dx, x, a+bx^2\right)}{2b}$$
$$= \frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b} - \frac{(3p)\operatorname{Subst}\left(\int \log^2\left(cx^p\right) dx, x, a+bx^2\right)}{2b}$$
$$= -\frac{3p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b} + \frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b} + \frac{\left(3p^2\right)\operatorname{Subst}\left(\int\right.}{}$$
$$= -3p^3x^2 + \frac{3p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b} - \frac{3p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b} + \frac{(a}{}$$

Mathematica [A] time = 0.01, size = 87, normalized size = 0.94

$$\frac{6p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right) + \left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right) - 3p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right) - 6bp^3x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (-6*b*p^3*x^2 + 6*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p] - 3*p*(a + b*x^2)*Lo
g[c*(a + b*x^2)^p]^2 + (a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2*b)
```

fricas [A] time = 0.49, size = 176, normalized size = 1.89

$$-\frac{6\,bp^3x^2 - 6\,bp^2x^2\log(c) + 3\,bpx^2\log(c)^2 - bx^2\log(c)^3 - \left(bp^3x^2 + ap^3\right)\log\left(bx^2+a\right)^3 + 3\left(bp^3x^2 + ap^3 - \left(bp^2x\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*b*p^3*x^2 - 6*b*p^2*x^2*log(c) + 3*b*p*x^2*log(c)^2 - b*x^2*log(c)^
3 - (b*p^3*x^2 + a*p^3)*log(b*x^2 + a)^3 + 3*(b*p^3*x^2 + a*p^3 - (b*p^2*x^
2 + a*p^2)*log(c))*log(b*x^2 + a)^2 - 3*(2*b*p^3*x^2 + 2*a*p^3 + (b*p*x^2 +
 a*p)*log(c)^2 - 2*(b*p^2*x^2 + a*p^2)*log(c))*log(b*x^2 + a))/b
```

giac [A] time = 0.17, size = 169, normalized size = 1.82

$$\frac{\left(\left(bx^2+a\right)\log\left(bx^2+a\right)^3 - 6\,bx^2 - 3\left(bx^2+a\right)\log\left(bx^2+a\right)^2 + 6\left(bx^2+a\right)\log\left(bx^2+a\right) - 6\,a\right)p^3 + 3\left(2\,bx^2 + (b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] 1/2*(((b*x^2 + a)*log(b*x^2 + a)^3 - 6*b*x^2 - 3*(b*x^2 + a)*log(b*x^2 + a)
^2 + 6*(b*x^2 + a)*log(b*x^2 + a) - 6*a)*p^3 + 3*(2*b*x^2 + (b*x^2 + a)*log
(b*x^2 + a)^2 - 2*(b*x^2 + a)*log(b*x^2 + a) + 2*a)*p^2*log(c) - 3*(b*x^2 -
 (b*x^2 + a)*log(b*x^2 + a) + a)*p*log(c)^2 + (b*x^2 + a)*log(c)^3)/b
```

maple [C] time = 1.03, size = 3925, normalized size = 42.20

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] -3/8/b*Pi^2*ln(b*x^2+a)*a*p*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2+3/4/b*Pi^2*
ln(b*x^2+a)*a*p*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)-3/8/b*Pi^2*ln(b*x^2+a)*a*
p*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4+3/4/b*Pi^2*ln(b*x^2+a)*a*p*
csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5+3/4*I/b*Pi*a*p^2*csgn(I*c*(b*x^
2+a)^p)^3*ln(b*x^2+a)^2+3/2*I/b*Pi*ln(b*x^2+a)*a*p^2*csgn(I*c*(b*x^2+a)^p)^
3-3/4*I*ln(c)^2*Pi*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-
3/2*I*ln(c)*Pi*p*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-3/2*I*ln(c)*Pi*p*x^2
*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-3/2*I*Pi*p^2*x^2*csgn(I*(b*x^2
+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+3/8*(4*ln(c)^2*b*x^2-4*a*p^2*ln(b*x^
2+a)^2-8*ln(b*x^2+a)*a*p^2+8*x^2*b*p^2-8*ln(c)*b*p*x^2+8*ln(c)*ln(b*x^2+a)*
a*p-Pi^2*b*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2-Pi
^2*b*x^2*csgn(I*c*(b*x^2+a)^p)^6+2*Pi^2*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)^5+2*Pi^2*b*x^2*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)-Pi^2*b*x^2*cs
gn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2-4*I*ln(c)*Pi*b*x^2*csgn(I*c*(b*x^2+a)^p)^
3+2*Pi^2*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2+2*Pi
^2*b*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)-4*I*Pi*b*p
*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-4*I*Pi*b*p*x^2*csgn(I*c*(b
*x^2+a)^p)^2*csgn(I*c)+4*I*Pi*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)^2+4*I*Pi*b*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csg
n(I*c)-4*I*Pi*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csg
n(I*c)-4*Pi^2*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)+4
*I*Pi*b*p*x^2*csgn(I*c*(b*x^2+a)^p)^3-4*I*Pi*ln(b*x^2+a)*a*p*csgn(I*c*(b*x^
2+a)^p)^3+4*I*Pi*ln(b*x^2+a)*a*p*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-4*I*ln(c
)*Pi*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-Pi^2*b*x^2*c
sgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4+4*I*ln(c)*Pi*b*x^2*csgn(I*(b*x
^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+4*I*ln(c)*Pi*b*x^2*csgn(I*c*(b*x^2+a)^p)^2
*csgn(I*c))/b*ln((b*x^2+a)^p)-3/2*ln(c)^2*p*x^2+3*ln(c)*p^2*x^2+3/4*(I*Pi*b
*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*b*x^2*csgn(I*(b*x^2+a
)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*b*x^2*csgn(I*c*(b*x^2+a)^p)^3+I*P
i*b*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*b*x^2*ln(c)-2*b*p*x^2+2*a*p*ln(
b*x^2+a))/b*ln((b*x^2+a)^p)^2+3/16*I*Pi^3*x^2*csgn(I*c*(b*x^2+a)^p)^7*csgn(
I*c)^2-3/8*ln(c)*Pi^2*x^2*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2+3/4*ln(c)*Pi^
2*x^2*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)-3/8*ln(c)*Pi^2*x^2*csgn(I*(b*x^2+a)
^p)^2*csgn(I*c*(b*x^2+a)^p)^4+3/4*ln(c)*Pi^2*x^2*csgn(I*(b*x^2+a)^p)*csgn(I
*c*(b*x^2+a)^p)^5+3/8*Pi^2*p*x^2*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2-3/4*Pi
^2*p*x^2*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)+3/8*Pi^2*p*x^2*csgn(I*(b*x^2+a)^
p)^2*csgn(I*c*(b*x^2+a)^p)^4-3/4*Pi^2*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b
*x^2+a)^p)^5-3/2/b*ln(c)*a*p^2*ln(b*x^2+a)^2+3/2/b*ln(c)^2*ln(b*x^2+a)*a*p-
3/b*ln(c)*ln(b*x^2+a)*a*p^2-1/16*I*Pi^3*x^2*csgn(I*c*(b*x^2+a)^p)^6*csgn(I*
c)^3-3/16*I*Pi^3*x^2*csgn(I*c*(b*x^2+a)^p)^8*csgn(I*c)-1/16*I*Pi^3*x^2*csgn
(I*(b*x^2+a)^p)^3*csgn(I*c*(b*x^2+a)^p)^6+3/16*I*Pi^3*x^2*csgn(I*(b*x^2+a)^
p)^2*csgn(I*c*(b*x^2+a)^p)^7-3/16*I*Pi^3*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(
b*x^2+a)^p)^8-3/4*I*ln(c)^2*Pi*x^2*csgn(I*c*(b*x^2+a)^p)^3-3/2*I*Pi*p^2*x^2
*csgn(I*c*(b*x^2+a)^p)^3+3*a*p^3/b*ln(b*x^2+a)+1/16*I*Pi^3*x^2*csgn(I*c*(b*
x^2+a)^p)^9+1/2/b*a*p^3*ln(b*x^2+a)^3+3/2/b*a*p^3*ln(b*x^2+a)^2-3/8*ln(c)*P
i^2*x^2*csgn(I*c*(b*x^2+a)^p)^6+3/8*Pi^2*p*x^2*csgn(I*c*(b*x^2+a)^p)^6+3/4/
b*Pi^2*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c
)^2+3/4/b*Pi^2*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^
3*csgn(I*c)-3/2/b*Pi^2*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+
a)^p)^4*csgn(I*c)+3/2*I*ln(c)*Pi*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+
a)^p)*csgn(I*c)-3/4*I/b*Pi*a*p^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*ln(b*x^2
+a)^2-3/4*I/b*Pi*a*p^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*ln(b*x^2
+a)^2-3/2*I/b*ln(c)*Pi*ln(b*x^2+a)*a*p*csgn(I*c*(b*x^2+a)^p)^3-3/2*I/b*Pi*l
n(b*x^2+a)*a*p^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-3/2*I/b*Pi*ln(b*x^2+a)*a
*p^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-3/8/b*Pi^2*ln(b*x^2+a)*a*p
*csgn(I*c*(b*x^2+a)^p)^6-3/8*ln(c)*Pi^2*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*
```

```
(b*x^2+a)^p)^2*csgn(I*c)^2+3/4*ln(c)*Pi^2*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)^3*csgn(I*c)^2-3*p^3*x^2+3/4*I/b*Pi*a*p^2*csgn(I*(b*x^2+a)^p)*c
sgn(I*c*(b*x^2+a)^p)*csgn(I*c)*ln(b*x^2+a)^2+3/2*I/b*ln(c)*Pi*ln(b*x^2+a)*a
*p*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+3/2*I/b*ln(c)*Pi*ln(b*x^2+a)*a*p*csgn(
I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+3/2*I/b*Pi*ln(b*x^2+a)*a*p^2*csgn(I*
(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-3/8/b*Pi^2*ln(b*x^2+a)*a*p*csg
n(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2+1/2*x^2*ln((b*x^2+a)
^p)^3+1/2*ln(c)^3*x^2-3/2*I/b*ln(c)*Pi*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*
csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+3/4*ln(c)*Pi^2*x^2*csgn(I*(b*x^2+a)^p)^2*cs
gn(I*c*(b*x^2+a)^p)^3*csgn(I*c)-3/2*ln(c)*Pi^2*x^2*csgn(I*(b*x^2+a)^p)*csgn
(I*c*(b*x^2+a)^p)^4*csgn(I*c)+3/8*Pi^2*p*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c
*(b*x^2+a)^p)^2*csgn(I*c)^2-3/4*Pi^2*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*
x^2+a)^p)^3*csgn(I*c)^2-3/4*Pi^2*p*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^
2+a)^p)^3*csgn(I*c)+3/2*Pi^2*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p
)^4*csgn(I*c)+3/2*I*Pi*p^2*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+3/2*I*Pi*p
^2*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/16*I*Pi^3*x^2*csgn(I*(
b*x^2+a)^p)^3*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^3-3/16*I*Pi^3*x^2*csgn(I*(b
*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^3+3/16*I*Pi^3*x^2*csgn(I*(b*
x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)^3-3/16*I*Pi^3*x^2*csgn(I*(b*x^2
+a)^p)^3*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2+9/16*I*Pi^3*x^2*csgn(I*(b*x^2+
a)^p)^2*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)^2-9/16*I*Pi^3*x^2*csgn(I*(b*x^2+a
)^p)*csgn(I*c*(b*x^2+a)^p)^6*csgn(I*c)^2+3/16*I*Pi^3*x^2*csgn(I*(b*x^2+a)^p
)^3*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)-9/16*I*Pi^3*x^2*csgn(I*(b*x^2+a)^p)^2
*csgn(I*c*(b*x^2+a)^p)^6*csgn(I*c)+9/16*I*Pi^3*x^2*csgn(I*(b*x^2+a)^p)*csgn
(I*c*(b*x^2+a)^p)^7*csgn(I*c)+3/4*I*ln(c)^2*Pi*x^2*csgn(I*c*(b*x^2+a)^p)^2*
csgn(I*c)+3/4*I*ln(c)^2*Pi*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+
3/2*I*ln(c)*Pi*p*x^2*csgn(I*c*(b*x^2+a)^p)^3
```

maxima [A] time = 0.75, size = 164, normalized size = 1.76

$$-\frac{3}{2}\,bp\left(\frac{x^2}{b}-\frac{a\log\left(bx^2+a\right)}{b^2}\right)\log\left(\left(bx^2+a\right)^p c\right)^2+\frac{1}{2}\,x^2\log\left(\left(bx^2+a\right)^p c\right)^3+\frac{1}{2}\,bp\left(\frac{\left(a\log\left(bx^2+a\right)^3-6\,bx^2+3\,a\log\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -3/2*b*p*(x^2/b - a*log(b*x^2 + a)/b^2)*log((b*x^2 + a)^p*c)^2 + 1/2*x^2*lo
g((b*x^2 + a)^p*c)^3 + 1/2*b*p*((a*log(b*x^2 + a)^3 - 6*b*x^2 + 3*a*log(b*x
^2 + a)^2 + 6*a*log(b*x^2 + a))*p^2/b^2 + 3*(2*b*x^2 - a*log(b*x^2 + a)^2 -
 2*a*log(b*x^2 + a))*p*log((b*x^2 + a)^p*c)/b^2)
```

mupad [B] time = 0.24, size = 103, normalized size = 1.11

$$\ln\left(c\left(b\,x^2+a\right)^p\right)^3\,\left(\frac{a}{2\,b}+\frac{x^2}{2}\right)-\ln\left(c\left(b\,x^2+a\right)^p\right)^2\,\left(\frac{3\,p\,x^2}{2}+\frac{3\,a\,p}{2\,b}\right)-3\,p^3\,x^2+3\,p^2\,x^2\,\ln\left(c\left(b\,x^2+a\right)^p\right)+\frac{3\,a\,p^3\,\ln\left(\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] log(c*(a + b*x^2)^p)^3*(a/(2*b) + x^2/2) - log(c*(a + b*x^2)^p)^2*((3*p*x^2
)/2 + (3*a*p)/(2*b)) - 3*p^3*x^2 + 3*p^2*x^2*log(c*(a + b*x^2)^p) + (3*a*p^
3*log(a + b*x^2))/b
```

sympy [A] time = 5.33, size = 301, normalized size = 3.24

$$\begin{cases}\frac{ap^3\log\left(a+bx^2\right)^3}{2b}-\frac{3ap^3\log\left(a+bx^2\right)^2}{2b}+\frac{3ap^3\log\left(a+bx^2\right)}{b}+\frac{3ap^2\log\left(c\right)\log\left(a+bx^2\right)^2}{2b}-\frac{3ap^2\log\left(c\right)\log\left(a+bx^2\right)}{b}+\frac{3ap\log\left(c\right)^2\log\left(a+bx^2\right)}{2b} \\ \frac{x^2\log\left(a^pc\right)^3}{2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Piecewise((a*p**3*log(a + b*x**2)**3/(2*b) - 3*a*p**3*log(a + b*x**2)**2/(2
*b) + 3*a*p**3*log(a + b*x**2)/b + 3*a*p**2*log(c)*log(a + b*x**2)**2/(2*b)
 - 3*a*p**2*log(c)*log(a + b*x**2)/b + 3*a*p*log(c)**2*log(a + b*x**2)/(2*b
) + p**3*x**2*log(a + b*x**2)**3/2 - 3*p**3*x**2*log(a + b*x**2)**2/2 + 3*p
**3*x**2*log(a + b*x**2) - 3*p**3*x**2 + 3*p**2*x**2*log(c)*log(a + b*x**2)
**2/2 - 3*p**2*x**2*log(c)*log(a + b*x**2) + 3*p**2*x**2*log(c) + 3*p*x**2*
log(c)**2*log(a + b*x**2)/2 - 3*p*x**2*log(c)**2/2 + x**2*log(c)**3/2, Ne(b
, 0)), (x**2*log(a**p*c)**3/2, True))
```

3.94 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x}\,dx$

Optimal. Leaf size=106

$$-3p^2\mathrm{Li}_3\left(\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)+\frac{3}{2}p\mathrm{Li}_2\left(\frac{bx^2}{a}+1\right)\log^2\left(c\left(a+bx^2\right)^p\right)+\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right)+3p$$

```
[Out] 1/2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)^3+3/2*p*ln(c*(b*x^2+a)^p)^2*polylog(2,1+
b*x^2/a)-3*p^2*ln(c*(b*x^2+a)^p)*polylog(3,1+b*x^2/a)+3*p^3*polylog(4,1+b*x
^2/a)
```

Rubi [A] time = 0.16, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2396, 2433, 2374, 2383, 6589}

$$-3p^2\mathrm{PolyLog}\left(3,\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)+\frac{3}{2}p\mathrm{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log^2\left(c\left(a+bx^2\right)^p\right)+3p^3\mathrm{PolyLog}\left(4,\frac{bx^2}{a}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x,x]
```

```
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p]^3)/2 + (3*p*Log[c*(a + b*x^2)^p]^2*
PolyLog[2, 1 + (b*x^2)/a])/2 - 3*p^2*Log[c*(a + b*x^2)^p]*PolyLog[3, 1 + (b
*x^2)/a] + 3*p^3*PolyLog[4, 1 + (b*x^2)/a]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log^3\left(c(a+bx)^p\right)}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) - \frac{1}{2}(3bp)\operatorname{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)\log^2\left(c(a+bx)^p\right)}{a+bx}\right.$$

$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) - \frac{1}{2}(3p)\operatorname{Subst}\left(\int \frac{\log^2\left(cx^p\right)\log\left(-\frac{b\left(-\frac{a}{b}+\frac{x}{b}\right)}{a}\right)}{x}\,dx,\right.$$

$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) + \frac{3}{2}p\log^2\left(c\left(a+bx^2\right)^p\right)\operatorname{Li}_2\left(1+\frac{bx^2}{a}\right) - \left(3p^2\right)\mathrm{S}$$

$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) + \frac{3}{2}p\log^2\left(c\left(a+bx^2\right)^p\right)\operatorname{Li}_2\left(1+\frac{bx^2}{a}\right) - 3p^2\log$$

$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) + \frac{3}{2}p\log^2\left(c\left(a+bx^2\right)^p\right)\operatorname{Li}_2\left(1+\frac{bx^2}{a}\right) - 3p^2\log$$

Mathematica [B] time = 0.10, size = 279, normalized size = 2.63

$$-\frac{3}{2}p^2\left(-2\operatorname{Li}_3\left(\frac{bx^2}{a}+1\right)+2\operatorname{Li}_2\left(\frac{bx^2}{a}+1\right)\log\left(a+bx^2\right)+\log\left(-\frac{bx^2}{a}\right)\log^2\left(a+bx^2\right)\right)\left(p\log\left(a+bx^2\right)-\log\left(c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x,x]
```

```
[Out] Log[x]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^3 + 3*p*(-(p*Log[a + b*
x^2]) + Log[c*(a + b*x^2)^p])^2*(Log[x]*(Log[a + b*x^2] - Log[1 + (b*x^2)/a
]) - PolyLog[2, -((b*x^2)/a)]/2) - (3*p^2*(p*Log[a + b*x^2] - Log[c*(a + b*
x^2)^p])*(Log[-((b*x^2)/a)]*Log[a + b*x^2]^2 + 2*Log[a + b*x^2]*PolyLog[2,
1 + (b*x^2)/a] - 2*PolyLog[3, 1 + (b*x^2)/a]))/2 + (p^3*(Log[-((b*x^2)/a)]*
Log[a + b*x^2]^3 + 3*Log[a + b*x^2]^2*PolyLog[2, 1 + (b*x^2)/a] - 6*Log[a +
 b*x^2]*PolyLog[3, 1 + (b*x^2)/a] + 6*PolyLog[4, 1 + (b*x^2)/a]))/2
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x, x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3/x,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x,x)
```

maxima [B] time = 0.65, size = 217, normalized size = 2.05

$$\frac{1}{2}\left(\log\left(bx^2+a\right)^3\log\left(-\frac{bx^2+a}{a}+1\right)+3\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)^2-6\,\log\left(bx^2+a\right)\mathrm{Li}_3\left(\frac{bx^2+a}{a}\right)+6\,\mathrm{Li}_4\left(\frac{bx^2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x,x, algorithm="maxima")
```

```
[Out] 1/2*(log(b*x^2 + a)^3*log(-(b*x^2 + a)/a + 1) + 3*dilog((b*x^2 + a)/a)*log(
b*x^2 + a)^2 - 6*log(b*x^2 + a)*polylog(3, (b*x^2 + a)/a) + 6*polylog(4, (b
*x^2 + a)/a))*p^3 + 3/2*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog
((b*x^2 + a)/a)*log(b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*p^2*log(c) +
3/2*(log(b*x^2 + a)*log(-(b*x^2 + a)/a + 1) + dilog((b*x^2 + a)/a))*p*log(c
)^2 + log(c)^3*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^3/x,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**3/x, x)
```

3.95 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^3} dx$

Optimal. Leaf size=119

$$\frac{3bp^2\text{Li}_2\left(\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)}{a}-\frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2ax^2}+\frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2a}-\frac{3bp^3\text{l}}{}$$

```
[Out] 3/2*b*p*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)^2/a-1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^
3/a/x^2+3*b*p^2*ln(c*(b*x^2+a)^p)*polylog(2,1+b*x^2/a)/a-3*b*p^3*polylog(3,
1+b*x^2/a)/a
```

Rubi [A] time = 0.15, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2397, 2396, 2433, 2374, 6589}

$$\frac{3bp^2\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)}{a}-\frac{3bp^3\text{PolyLog}\left(3,\frac{bx^2}{a}+1\right)}{a}+\frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2a}-$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^3,x]
```

```
[Out] (3*b*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p]^2)/(2*a) - ((a + b*x^2)*Log[c
*(a + b*x^2)^p]^3)/(2*a*x^2) + (3*b*p^2*Log[c*(a + b*x^2)^p]*PolyLog[2, 1 +
 (b*x^2)/a])/a - (3*b*p^3*PolyLog[3, 1 + (b*x^2)/a])/a
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\log^3\left(c(a+bx)^p\right)}{x^2}\,dx,x,x^2\right)$$

$$= -\frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2ax^2}+\frac{(3bp)\,\text{Subst}\left(\int \frac{\log^2(c(a+bx)^p)}{x}\,dx,x,x^2\right)}{2a}$$

$$= \frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2a}-\frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2ax^2}-\frac{\left(3b^2p^2\right)\text{Subst}\left(\right.}{}$$

$$= \frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2a}-\frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2ax^2}-\frac{\left(3bp^2\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2a}-\frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2ax^2}+\frac{3bp^2\log\left(c\left(a+\right.\right.}{}$$

$$= \frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2a}-\frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2ax^2}+\frac{3bp^2\log\left(c\left(a+\right.\right.}{}$$

Mathematica [B] time = 0.30, size = 302, normalized size = 2.54

$$-\frac{-6bp^2x^2\text{Li}_2\left(\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)-3bp^2x^2\log^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)+12bp^2x^2\log(x)\log\left(a+bx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^3,x]
```

```
[Out] -1/2*(-6*b*p^3*x^2*Log[x]*Log[a + b*x^2]^2 + 3*b*p^3*x^2*Log[-((b*x^2)/a)]*
Log[a + b*x^2]^2 + b*p^3*x^2*Log[a + b*x^2]^3 + 12*b*p^2*x^2*Log[x]*Log[a +
 b*x^2]*Log[c*(a + b*x^2)^p] - 6*b*p^2*x^2*Log[-((b*x^2)/a)]*Log[a + b*x^2]
*Log[c*(a + b*x^2)^p] - 3*b*p^2*x^2*Log[a + b*x^2]^2*Log[c*(a + b*x^2)^p] -
 6*b*p*x^2*Log[x]*Log[c*(a + b*x^2)^p]^2 + 3*b*p*x^2*Log[a + b*x^2]*Log[c*(
a + b*x^2)^p]^2 + a*Log[c*(a + b*x^2)^p]^3 - 6*b*p^2*x^2*Log[c*(a + b*x^2)^
p]*PolyLog[2, 1 + (b*x^2)/a] + 6*b*p^3*x^2*PolyLog[3, 1 + (b*x^2)/a])/(a*x^
2)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^3, x)
```

maple [F] time = 1.19, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^3,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x^3,x)
```

maxima [A] time = 1.13, size = 202, normalized size = 1.70

$$\frac{1}{2}\left(\frac{3\left(\log\left(bx^2+a\right)^2\log\left(-\frac{bx^2+a}{a}+1\right)+2\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)-2\,\mathrm{Li}_3\left(\frac{bx^2+a}{a}\right)\right)p^2}{a}+\frac{6\left(\log\left(bx^2+a\right)\log\left(-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog((b*x^2 + a)/a)*l
og(b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*p^2/a + 6*(log(b*x^2 + a)*log(
-(b*x^2 + a)/a + 1) + dilog((b*x^2 + a)/a))*p*log(c)/a + 6*log(c)^2*log(x)/
a - (p^2*log(b*x^2 + a)^3 + 3*p*log(b*x^2 + a)^2*log(c) + 3*log(b*x^2 + a)*
log(c)^2)/a)*b*p - 1/2*log((b*x^2 + a)^p*c)^3/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^3/x^3,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^3/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x**3,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**3/x**3, x)
```

3.96 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^5} \, dx$

Optimal. Leaf size=219

$$\frac{3b^2p^2\text{Li}_2\left(\frac{a}{bx^2+a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2}+\frac{3b^2p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2}-\frac{3b^2p\log\left(1-\frac{a}{a+bx^2}\right)\log^2\left(c\left(a+bx^2\right.\right.}{4a^2}$$

```
[Out] 3/2*b^2*p^2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/a^2-3/4*b*p*(b*x^2+a)*ln(c*(b*x^
2+a)^p)^2/a^2/x^2-1/4*ln(c*(b*x^2+a)^p)^3/x^4-3/4*b^2*p*ln(c*(b*x^2+a)^p)^2
*ln(1-a/(b*x^2+a))/a^2+3/2*b^2*p^2*ln(c*(b*x^2+a)^p)*polylog(2,a/(b*x^2+a))
/a^2+3/2*b^2*p^3*polylog(2,1+b*x^2/a)/a^2+3/2*b^2*p^3*polylog(3,a/(b*x^2+a)
)/a^2
```

Rubi [A] time = 0.43, antiderivative size = 236, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391}

$$-\frac{3b^2p^2\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2}+\frac{3b^2p^3\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2a^2}+\frac{3b^2p^3\text{PolyLog}\left(3,\frac{bx^2}{a}+1\right)}{2a^2}+\frac{3b^2\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^5,x]
```

```
[Out] (3*b^2*p^2*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*a^2) - (3*b*p*(a + b*
x^2)*Log[c*(a + b*x^2)^p]^2)/(4*a^2*x^2) - (3*b^2*p*Log[-((b*x^2)/a)]*Log[c
*(a + b*x^2)^p]^2)/(4*a^2) + (b^2*Log[c*(a + b*x^2)^p]^3)/(4*a^2) - Log[c*(
a + b*x^2)^p]^3/(4*x^4) + (3*b^2*p^3*PolyLog[2, 1 + (b*x^2)/a])/(2*a^2) - (
3*b^2*p^2*Log[c*(a + b*x^2)^p]*PolyLog[2, 1 + (b*x^2)/a])/(2*a^2) + (3*b^2*
p^3*PolyLog[3, 1 + (b*x^2)/a])/(2*a^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\log^3\left(c(a+bx)^p\right)}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{4x^4} + \frac{1}{4}(3bp)\,\text{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^2(a+bx)}\,dx, x, x^2\right)$$

$$= -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{4x^4} + \frac{1}{4}(3p)\,\text{Subst}\left(\int \frac{\log^2\left(cx^p\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right)$$

$$= -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{4x^4} + \frac{(3p)\,\text{Subst}\left(\int \frac{\log^2(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right)}{4a} - \frac{(3bp)\,\text{Subst}\left(\int \frac{\log}{x(-}\right.}{4}$$

$$= -\frac{3bp\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{4a^2x^2} - \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{4x^4} - \frac{(3bp)\,\text{Subst}\left(\int \frac{\log^2(cx^p}{-\frac{a}{b}+\frac{x}{b}}\right.}{4a^2}$$

$$= \frac{3b^2p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2} - \frac{3bp\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{4a^2x^2} - \frac{3b^2p\log}{}$$

$$= \frac{3b^2p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2} - \frac{3bp\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{4a^2x^2} - \frac{3b^2p\log}{}$$

$$= \frac{3b^2p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2} - \frac{3bp\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{4a^2x^2} - \frac{3b^2p\log}{}$$

Mathematica [B] time = 0.37, size = 478, normalized size = 2.18

$$\frac{-a^2\log^3\left(c\left(a+bx^2\right)^p\right) + 6b^2p^2x^4\text{Li}_2\left(\frac{bx^2}{a}+1\right)\left(p-\log\left(c\left(a+bx^2\right)^p\right)\right) - 3b^2p^2x^4\log^2\left(a+bx^2\right)\log\left(c\left(a+bx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^5,x]
```

```
[Out] (-12*b^2*p^3*x^4*Log[x]*Log[a + b*x^2] + 6*b^2*p^3*x^4*Log[-((b*x^2)/a)]*Lo
g[a + b*x^2] + 3*b^2*p^3*x^4*Log[a + b*x^2]^2 - 6*b^2*p^3*x^4*Log[x]*Log[a
+ b*x^2]^2 + 3*b^2*p^3*x^4*Log[-((b*x^2)/a)]*Log[a + b*x^2]^2 + b^2*p^3*x^4
*Log[a + b*x^2]^3 + 12*b^2*p^2*x^4*Log[x]*Log[c*(a + b*x^2)^p] - 6*b^2*p^2*
x^4*Log[a + b*x^2]*Log[c*(a + b*x^2)^p] + 12*b^2*p^2*x^4*Log[x]*Log[a + b*x
^2]*Log[c*(a + b*x^2)^p] - 6*b^2*p^2*x^4*Log[-((b*x^2)/a)]*Log[a + b*x^2]*L
og[c*(a + b*x^2)^p] - 3*b^2*p^2*x^4*Log[a + b*x^2]^2*Log[c*(a + b*x^2)^p] -
 3*a*b*p*x^2*Log[c*(a + b*x^2)^p]^2 - 6*b^2*p*x^4*Log[x]*Log[c*(a + b*x^2)^
p]^2 + 3*b^2*p*x^4*Log[a + b*x^2]*Log[c*(a + b*x^2)^p]^2 - a^2*Log[c*(a + b
*x^2)^p]^3 + 6*b^2*p^2*x^4*(p - Log[c*(a + b*x^2)^p])*PolyLog[2, 1 + (b*x^2
)/a] + 6*b^2*p^3*x^4*PolyLog[3, 1 + (b*x^2)/a])/(4*a^2*x^4)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^5,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^5,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^5, x)
```

maple [F] time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^5,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x^5,x)
```

maxima [A] time = 1.36, size = 270, normalized size = 1.23

$$-\frac{1}{4}\left(\frac{3\left(\log\left(bx^2+a\right)^2\log\left(-\frac{bx^2+a}{a}+1\right)+2\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)-2\,\mathrm{Li}_3\left(\frac{bx^2+a}{a}\right)\right)bp^2}{a^2}-\frac{6\left(p^2-p\log(c)\right)\left(\log\left(b\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*(3*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog((b*x^2 + a)/a)*
log(b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*b*p^2/a^2 - 6*(p^2 - p*log(c)
)*(log(b*x^2 + a)*log(-(b*x^2 + a)/a + 1) + dilog((b*x^2 + a)/a))*b/a^2 - 6
*(2*p*log(c) - log(c)^2)*b*log(x)/a^2 - (b*p^2*x^2*log(b*x^2 + a)^3 - 3*((p
^2 - p*log(c))*b*x^2 + a*p^2)*log(b*x^2 + a)^2 - 3*a*log(c)^2 - 3*((2*p*log
(c) - log(c)^2)*b*x^2 + 2*a*p*log(c))*log(b*x^2 + a))/(a^2*x^2))*b*p - 1/4*
log((b*x^2 + a)^p*c)^3/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^3/x^5,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^3/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x**5,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**3/x**5, x)
```

3.97 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx$

Optimal. Leaf size=352

$$-\frac{b^3p^2\text{Li}_2\left(\frac{a}{bx^2+a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^3}-\frac{b^3p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^3}-\frac{b^3p^2\log\left(1-\frac{a}{a+bx^2}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^3}+$$

```
[Out] b^3*p^3*ln(x)/a^3-1/2*b^2*p^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/a^3/x^2-b^3*p^2*l
n(-b*x^2/a)*ln(c*(b*x^2+a)^p)/a^3-1/4*b*p*ln(c*(b*x^2+a)^p)^2/a/x^4+1/2*b^2
*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)^2/a^3/x^2-1/6*ln(c*(b*x^2+a)^p)^3/x^6-1/2*b^
3*p^2*ln(c*(b*x^2+a)^p)*ln(1-a/(b*x^2+a))/a^3+1/2*b^3*p*ln(c*(b*x^2+a)^p)^2
*ln(1-a/(b*x^2+a))/a^3+1/2*b^3*p^3*polylog(2,a/(b*x^2+a))/a^3-b^3*p^2*ln(c*
(b*x^2+a)^p)*polylog(2,a/(b*x^2+a))/a^3-b^3*p^3*polylog(2,1+b*x^2/a)/a^3-b^
3*p^3*polylog(3,a/(b*x^2+a))/a^3
```

Rubi [A] time = 0.74, antiderivative size = 331, normalized size of antiderivative = 0.94, number of steps used = 22, number of rules used = 16, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31}

$$\frac{b^3p^2\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^3}-\frac{3b^3p^3\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2a^3}-\frac{b^3p^3\text{PolyLog}\left(3,\frac{bx^2}{a}+1\right)}{a^3}-\frac{3b^3p^2\log(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^7,x]
```

```
[Out] (b^3*p^3*Log[x])/a^3 - (b^2*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/(2*a^3*x^
2) - (3*b^3*p^2*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*a^3) + (b^3*p*Lo
g[c*(a + b*x^2)^p]^2)/(4*a^3) - (b*p*Log[c*(a + b*x^2)^p]^2)/(4*a*x^4) + (b
^2*p*(a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*a^3*x^2) + (b^3*p*Log[-((b*x^2)
/a)]*Log[c*(a + b*x^2)^p]^2)/(2*a^3) - (b^3*Log[c*(a + b*x^2)^p]^3)/(6*a^3)
 - Log[c*(a + b*x^2)^p]^3/(6*x^6) - (3*b^3*p^3*PolyLog[2, 1 + (b*x^2)/a])/(
2*a^3) + (b^3*p^2*Log[c*(a + b*x^2)^p]*PolyLog[2, 1 + (b*x^2)/a])/a^3 - (b^
3*p^3*PolyLog[3, 1 + (b*x^2)/a])/a^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
```

```
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log^3\left(c(a+bx)^p\right)}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{6x^6} + \frac{1}{2}(bp)\operatorname{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^3(a+bx)}\,dx, x, x^2\right)$$

$$= -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{6x^6} + \frac{1}{2}p\operatorname{Subst}\left(\int \frac{\log^2\left(cx^p\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx^2\right)$$

$$= -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{6x^6} + \frac{p\operatorname{Subst}\left(\int \frac{\log^2(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx^2\right)}{2a} - \frac{(bp)\operatorname{Subst}\left(\int \frac{\log^2(cx^p)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)\ldots}\right.}{2a}$$

$$= -\frac{bp\log^2\left(c\left(a+bx^2\right)^p\right)}{4ax^4} - \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{6x^6} - \frac{(bp)\operatorname{Subst}\left(\int \frac{\log^2(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+b\ldots\right.}{2a^2}$$

$$= -\frac{bp\log^2\left(c\left(a+bx^2\right)^p\right)}{4ax^4} + \frac{b^2p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2a^3x^2} - \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{6x^6}$$

$$= -\frac{b^2p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^3x^2} - \frac{b^3p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^3} - \frac{bp\log^2\ldots}{}$$

$$= \frac{b^3p^3\log(x)}{a^3} - \frac{b^2p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^3x^2} - \frac{3b^3p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\ldots\right.\right.}{2a^3}$$

$$= \frac{b^3p^3\log(x)}{a^3} - \frac{b^2p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^3x^2} - \frac{3b^3p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\ldots\right.\right.}{2a^3}$$

Mathematica [**A**] time = 0.42, size = 571, normalized size = 1.62

$$-\frac{2a^3\log^3\left(c\left(a+bx^2\right)^p\right) + 3a^2bpx^2\log^2\left(c\left(a+bx^2\right)^p\right) + 6b^3p^2x^6\operatorname{Li}_2\left(\frac{bx^2}{a}+1\right)\left(3p-2\log\left(c\left(a+bx^2\right)^p\right)\right) - 6b\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^7,x]
```

```
[Out] -1/12*(-6*b^3*p^3*x^6*Log[-((b*x^2)/a)] + 6*b^3*p^3*x^6*Log[a + b*x^2] - 36
*b^3*p^3*x^6*Log[x]*Log[a + b*x^2] + 18*b^3*p^3*x^6*Log[-((b*x^2)/a)]*Log[a
 + b*x^2] + 9*b^3*p^3*x^6*Log[a + b*x^2]^2 - 12*b^3*p^3*x^6*Log[x]*Log[a +
b*x^2]^2 + 6*b^3*p^3*x^6*Log[-((b*x^2)/a)]*Log[a + b*x^2]^2 + 2*b^3*p^3*x^6
*Log[a + b*x^2]^3 + 6*a*b^2*p^2*x^4*Log[c*(a + b*x^2)^p] + 36*b^3*p^2*x^6*L
og[x]*Log[c*(a + b*x^2)^p] - 18*b^3*p^2*x^6*Log[a + b*x^2]*Log[c*(a + b*x^2
)^p] + 24*b^3*p^2*x^6*Log[x]*Log[a + b*x^2]*Log[c*(a + b*x^2)^p] - 12*b^3*p
^2*x^6*Log[-((b*x^2)/a)]*Log[a + b*x^2]*Log[c*(a + b*x^2)^p] - 6*b^3*p^2*x^
6*Log[a + b*x^2]^2*Log[c*(a + b*x^2)^p] + 3*a^2*b*p*x^2*Log[c*(a + b*x^2)^p
]^2 - 6*a*b^2*p*x^4*Log[c*(a + b*x^2)^p]^2 - 12*b^3*p*x^6*Log[x]*Log[c*(a +
 b*x^2)^p]^2 + 6*b^3*p*x^6*Log[a + b*x^2]*Log[c*(a + b*x^2)^p]^2 + 2*a^3*Lo
g[c*(a + b*x^2)^p]^3 + 6*b^3*p^2*x^6*(3*p - 2*Log[c*(a + b*x^2)^p])*PolyLog
[2, 1 + (b*x^2)/a] + 12*b^3*p^3*x^6*PolyLog[3, 1 + (b*x^2)/a])/(a^3*x^6)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^7,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x^7, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^7,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^7, x)
```

maple [F] time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^7,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x^7,x)
```

maxima [A] time = 1.27, size = 338, normalized size = 0.96

$$\frac{1}{12}\left(\frac{6\left(\log\left(bx^2+a\right)^2\log\left(-\frac{bx^2+a}{a}+1\right)+2\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)-2\,\mathrm{Li}_3\left(\frac{bx^2+a}{a}\right)\right)b^2p^2}{a^3}-\frac{6\left(3\,p^2-2\,p\log(c)\right)\left(\mathrm{lo}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/12*(6*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog((b*x^2 + a)/a)*
log(b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*b^2*p^2/a^3 - 6*(3*p^2 - 2*p*
log(c))*(log(b*x^2 + a)*log(-(b*x^2 + a)/a + 1) + dilog((b*x^2 + a)/a))*b^2
/a^3 + 12*(p^2 - 3*p*log(c) + log(c)^2)*b^2*log(x)/a^3 - (2*b^2*p^2*x^4*log
(b*x^2 + a)^3 + 6*(p*log(c) - log(c)^2)*a*b*x^2 + 3*a^2*log(c)^2 - 3*((3*p^
2 - 2*p*log(c))*b^2*x^4 + 2*a*b*p^2*x^2 - a^2*p^2)*log(b*x^2 + a)^2 + 6*((p
^2 - 3*p*log(c) + log(c)^2)*b^2*x^4 + (p^2 - 2*p*log(c))*a*b*x^2 + a^2*p*lo
g(c))*log(b*x^2 + a))/(a^3*x^4))*b*p - 1/6*log((b*x^2 + a)^p*c)^3/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^3/x^7,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^3/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x**7,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**3/x**7, x)
```

3.98 $\int x^2 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=380

$$-\frac{2a^2p\mathrm{Int}\left(\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2},x\right)}{b}+\frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^{3/2}}+\frac{32ia^{3/2}p^3\mathrm{Li}_2\left(1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{3b^{3/2}}+\frac{32ia^{3/2}p^3\tan}{3b^{3/}}$$

```
[Out] 208/9*a*p^3*x/b-16/27*p^3*x^3-208/9*a^(3/2)*p^3*arctan(x*b^(1/2)/a^(1/2))/b
^(3/2)+32/3*I*a^(3/2)*p^3*arctan(x*b^(1/2)/a^(1/2))^2/b^(3/2)-32/3*a*p^2*x*
ln(c*(b*x^2+a)^p)/b+8/9*p^2*x^3*ln(c*(b*x^2+a)^p)+32/3*a^(3/2)*p^2*arctan(x
*b^(1/2)/a^(1/2))*ln(c*(b*x^2+a)^p)/b^(3/2)+2*a*p*x*ln(c*(b*x^2+a)^p)^2/b-2
/3*p*x^3*ln(c*(b*x^2+a)^p)^2+1/3*x^3*ln(c*(b*x^2+a)^p)^3+64/3*a^(3/2)*p^3*a
rctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/b^(3/2)+32/3*I
*a^(3/2)*p^3*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/b^(3/2)-2*a^2*p*U
nintegrable(ln(c*(b*x^2+a)^p)^2/(b*x^2+a),x)/b
```

Rubi [A] time = 0.83, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (208*a*p^3*x)/(9*b) - (16*p^3*x^3)/27 - (208*a^(3/2)*p^3*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(9*b^(3/2)) + (((32*I)/3)*a^(3/2)*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]
]^2)/b^(3/2) + (64*a^(3/2)*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/
(Sqrt[a] + I*Sqrt[b]*x)])/(3*b^(3/2)) - (32*a*p^2*x*Log[c*(a + b*x^2)^p])/(
3*b) + (8*p^2*x^3*Log[c*(a + b*x^2)^p])/9 + (32*a^(3/2)*p^2*ArcTan[(Sqrt[b]
*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/(3*b^(3/2)) + (2*a*p*x*Log[c*(a + b*x^2)
^p]^2)/b - (2*p*x^3*Log[c*(a + b*x^2)^p]^2)/3 + (x^3*Log[c*(a + b*x^2)^p]^3
)/3 + (((32*I)/3)*a^(3/2)*p^3*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[
b]*x)])/b^(3/2) - (2*a^2*p*Defer[Int][Log[c*(a + b*x^2)^p]^2/(a + b*x^2), x
])/b
```

Rubi steps

$$\int x^2\log^3\left(c\left(a+bx^2\right)^p\right)dx=\frac{1}{3}x^3\log^3\left(c\left(a+bx^2\right)^p\right)-(2bp)\int\frac{x^4\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}dx$$

$$=\frac{1}{3}x^3\log^3\left(c\left(a+bx^2\right)^p\right)-(2bp)\int\left(-\frac{a\log^2\left(c\left(a+bx^2\right)^p\right)}{b^2}+\frac{x^2\log^2\left(c\left(a+bx^2\right.\right.}{b}\right.$$

$$=\frac{1}{3}x^3\log^3\left(c\left(a+bx^2\right)^p\right)-(2p)\int x^2\log^2\left(c\left(a+bx^2\right)^p\right)dx+\frac{(2ap)\int\log^2\left(c\left(a\right.\right.}{b}$$

$$=\frac{2apx\log^2\left(c\left(a+bx^2\right)^p\right)}{b}-\frac{2}{3}px^3\log^2\left(c\left(a+bx^2\right)^p\right)+\frac{1}{3}x^3\log^3\left(c\left(a+bx^2\right)^p\right)$$

$$=\frac{2apx\log^2\left(c\left(a+bx^2\right)^p\right)}{b}-\frac{2}{3}px^3\log^2\left(c\left(a+bx^2\right)^p\right)+\frac{1}{3}x^3\log^3\left(c\left(a+bx^2\right)^p\right)$$

$$=\frac{2apx\log^2\left(c\left(a+bx^2\right)^p\right)}{b}-\frac{2}{3}px^3\log^2\left(c\left(a+bx^2\right)^p\right)+\frac{1}{3}x^3\log^3\left(c\left(a+bx^2\right)^p\right)$$

$$=-\frac{32ap^2x\log\left(c\left(a+bx^2\right)^p\right)}{3b}+\frac{8}{9}p^2x^3\log\left(c\left(a+bx^2\right)^p\right)+\frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}}$$

$$=\frac{64ap^3x}{3b}-\frac{32ap^2x\log\left(c\left(a+bx^2\right)^p\right)}{3b}+\frac{8}{9}p^2x^3\log\left(c\left(a+bx^2\right)^p\right)+\frac{32a^{3/2}p^2\tan}{}$$

$$=\frac{208ap^3x}{9b}-\frac{16p^3x^3}{27}-\frac{64a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}}+\frac{32ia^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}}-\frac{32ap^2x}{}$$

$$=\frac{208ap^3x}{9b}-\frac{16p^3x^3}{27}-\frac{208a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}}+\frac{32ia^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}}+\frac{64a^{3/2}}{}$$

$$=\frac{208ap^3x}{9b}-\frac{16p^3x^3}{27}-\frac{208a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}}+\frac{32ia^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}}+\frac{64a^{3/2}}{}$$

$$=\frac{208ap^3x}{9b}-\frac{16p^3x^3}{27}-\frac{208a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}}+\frac{32ia^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}}+\frac{64a^{3/2}}{}$$

Mathematica [A] time = 3.89, size = 909, normalized size = 2.39

$$\frac{\left(-48\left(4\sqrt{bx^2}\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{-a}}\right)\left(\log\left(bx^2+a\right)-\log\left(\frac{bx^2}{a}+1\right)\right)-\sqrt{-a}\sqrt{-\frac{bx^2}{a}}\left(\log^2\left(\frac{bx^2}{a}+1\right)-4\log\left(\frac{1}{2}\left(\sqrt{-\frac{bx^2}{a}}\right.\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (2*a*p*x*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/b - (2*a^(3/2)*p*A
rcTan[(Sqrt[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/
b^(3/2) + p*x^3*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])
^2 + (x^3*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2*(-2*p - p*Log[a +
b*x^2] + Log[c*(a + b*x^2)^p]))/3 + 3*p^2*(-(p*Log[a + b*x^2]) + Log[c*(a +
 b*x^2)^p])*((x^3*Log[a + b*x^2]^2)/3 - (4*((9*I)*a^(3/2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]]^2 + 3*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-8 + 6*Log[(2*Sqrt[a]
)/(Sqrt[a] + I*Sqrt[b]*x)] + 3*Log[a + b*x^2]) + Sqrt[b]*x*(24*a - 2*b*x^2
+ (-9*a + 3*b*x^2)*Log[a + b*x^2]) + (9*I)*a^(3/2)*PolyLog[2, (I*Sqrt[a] +
Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)]))/(27*b^(3/2))) + (p^3*(416*Sqrt[-a]
*a^(3/2)*Sqrt[(b*x^2)/(a + b*x^2)]*Sqrt[a + b*x^2]*ArcSin[Sqrt[a]/Sqrt[a +
b*x^2]] + (2*Sqrt[-a]*b*x^2*(624*a - 16*b*x^2 + (-288*a + 24*b*x^2)*Log[a +
 b*x^2] + 18*(3*a - b*x^2)*Log[a + b*x^2]^2 + 9*b*x^2*Log[a + b*x^2]^3))/3
+ 36*Sqrt[-a]*a^(3/2)*Sqrt[(b*x^2)/(a + b*x^2)]*(8*Sqrt[a]*HypergeometricPF
Q[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, a/(a + b*x^2)] + Log[a + b*x^2]*(4
*Sqrt[a]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, a/(a + b*x^2)] + Sq
rt[a + b*x^2]*ArcSin[Sqrt[a]/Sqrt[a + b*x^2]]*Log[a + b*x^2])) - 48*a^2*(4*
Sqrt[b*x^2]*ArcTanh[Sqrt[b*x^2]/Sqrt[-a]]*(Log[a + b*x^2] - Log[1 + (b*x^2)
/a]) - Sqrt[-a]*Sqrt[-((b*x^2)/a)]*(Log[1 + (b*x^2)/a]^2 - 4*Log[1 + (b*x^2
)/a]*Log[(1 + Sqrt[-((b*x^2)/a)])/2] + 2*Log[(1 + Sqrt[-((b*x^2)/a)])/2]^2
- 4*PolyLog[2, 1/2 - Sqrt[-((b*x^2)/a)]/2]))))/(18*Sqrt[-a]*b^2*x)
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2 \log\left(\left(bx^2 + a\right)^p c\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2*log((b*x^2 + a)^p*c)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \log\left(\left(bx^2 + a\right)^p c\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*log((b*x^2 + a)^p*c)^3, x)
```

maple [A] time = 47.76, size = 0, normalized size = 0.00

$$\int x^2 \ln\left(c\left(b\,x^2 + a\right)^p\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(x^2*ln(c*(b*x^2+a)^p)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}p^3x^3\log\left(bx^2 + a\right)^3 + \int \frac{bx^4\log(c)^3 + ax^2\log(c)^3 - \left(\left(2p^3 - 3p^2\log(c)\right)bx^4 - 3ap^2x^2\log(c)\right)\log\left(bx^2 + a\right)^2 + 3}{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] 1/3*p^3*x^3*log(b*x^2 + a)^3 + integrate((b*x^4*log(c)^3 + a*x^2*log(c)^3 -
 ((2*p^3 - 3*p^2*log(c))*b*x^4 - 3*a*p^2*x^2*log(c))*log(b*x^2 + a)^2 + 3*(
b*p*x^4*log(c)^2 + a*p*x^2*log(c)^2)*log(b*x^2 + a))/(b*x^2 + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] int(x^2*log(c*(a + b*x^2)^p)^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \log\left(c\left(a+bx^2\right)^p\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(x**2*log(c*(a + b*x**2)**p)**3, x)
```

3.99 $\int \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=290

$$6ap\text{Int}\left(\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2},x\right)+24p^2x\log\left(c\left(a+bx^2\right)^p\right)-\frac{24\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}}+x\log^3\left(c\left(a+\right.\right.$$

```
[Out] -48*p^3*x+24*p^2*x*ln(c*(b*x^2+a)^p)-6*p*x*ln(c*(b*x^2+a)^p)^2+x*ln(c*(b*x^
2+a)^p)^3+48*p^3*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(1/2)-24*I*p^3*arctan(
x*b^(1/2)/a^(1/2))^2*a^(1/2)/b^(1/2)-24*p^2*arctan(x*b^(1/2)/a^(1/2))*ln(c*
(b*x^2+a)^p)*a^(1/2)/b^(1/2)-48*p^3*arctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/
(a^(1/2)+I*x*b^(1/2)))*a^(1/2)/b^(1/2)-24*I*p^3*polylog(2,1-2*a^(1/2)/(a^(1
/2)+I*x*b^(1/2)))*a^(1/2)/b^(1/2)+6*a*p*Unintegrable(ln(c*(b*x^2+a)^p)^2/(b
*x^2+a),x)
```

Rubi [A] time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -48*p^3*x + (48*Sqrt[a]*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] - ((24*I)*
Sqrt[a]*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/Sqrt[b] - (48*Sqrt[a]*p^3*ArcTan
[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[b] + 2
4*p^2*x*Log[c*(a + b*x^2)^p] - (24*Sqrt[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*
Log[c*(a + b*x^2)^p])/Sqrt[b] - 6*p*x*Log[c*(a + b*x^2)^p]^2 + x*Log[c*(a +
 b*x^2)^p]^3 - ((24*I)*Sqrt[a]*p^3*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*
Sqrt[b]*x)])/Sqrt[b] + 6*a*p*Defer[Int][Log[c*(a + b*x^2)^p]^2/(a + b*x^2),
 x]
```

Rubi steps

$$\int \log^3\left(c\left(a+bx^2\right)^p\right)dx = x\log^3\left(c\left(a+bx^2\right)^p\right) - (6bp)\int \frac{x^2\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}dx$$

$$= x\log^3\left(c\left(a+bx^2\right)^p\right) - (6bp)\int \left(\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{b} - \frac{a\log^2\left(c\left(a+bx^2\right)^p\right)}{b\left(a+bx^2\right)}\right)dx$$

$$= x\log^3\left(c\left(a+bx^2\right)^p\right) - (6p)\int \log^2\left(c\left(a+bx^2\right)^p\right)dx + (6ap)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}$$

$$= -6px\log^2\left(c\left(a+bx^2\right)^p\right) + x\log^3\left(c\left(a+bx^2\right)^p\right) + (6ap)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}dx$$

$$= -6px\log^2\left(c\left(a+bx^2\right)^p\right) + x\log^3\left(c\left(a+bx^2\right)^p\right) + (6ap)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}dx$$

$$= -6px\log^2\left(c\left(a+bx^2\right)^p\right) + x\log^3\left(c\left(a+bx^2\right)^p\right) + (6ap)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}dx$$

$$= 24p^2x\log\left(c\left(a+bx^2\right)^p\right) - \frac{24\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}} - 6px\log^2\left(c\left(\right.\right.$$

$$= -48p^3x + 24p^2x\log\left(c\left(a+bx^2\right)^p\right) - \frac{24\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}} - 6p$$

$$= -48p^3x + \frac{48\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{24i\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}} + 24p^2x\log\left(c\left(a+bx^2\right.\right.$$

$$= -48p^3x + \frac{48\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{24i\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}} - \frac{48\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

$$= -48p^3x + \frac{48\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{24i\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}} - \frac{48\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

$$= -48p^3x + \frac{48\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{24i\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}} - \frac{48\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 3.53, size = 789, normalized size = 2.72

$$p^3\left(-6\sqrt{-a^2}\sqrt{\frac{bx^2}{a+bx^2}}\left(8\sqrt{a}\ {}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{a}{bx^2+a}\right) + \log\left(a+bx^2\right)\left(4\sqrt{a}\ {}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{a}{bx^2+a}\right) + \sqrt{a+\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (6*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a +
b*x^2)^p])^2)/Sqrt[b] + 3*p*x*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(
```

```
a + b*x^2)^p])^2 + x*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2*(-6*p -
 p*Log[a + b*x^2] + Log[c*(a + b*x^2)^p]) - (3*p^2*(p*Log[a + b*x^2] - Log[
c*(a + b*x^2)^p])*((4*I)*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + 4*Sqrt[a]*
ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-2 + 2*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)
] + Log[a + b*x^2]) + Sqrt[b]*x*(8 - 4*Log[a + b*x^2] + Log[a + b*x^2]^2) +
 (4*I)*Sqrt[a]*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x
)]))/Sqrt[b] + (p^3*(-48*Sqrt[-a^2]*Sqrt[(b*x^2)/(a + b*x^2)]*Sqrt[a + b*x^
2]*ArcSin[Sqrt[a]/Sqrt[a + b*x^2]] + Sqrt[-a]*b*x^2*(-48 + 24*Log[a + b*x^2
] - 6*Log[a + b*x^2]^2 + Log[a + b*x^2]^3) - 6*Sqrt[-a^2]*Sqrt[(b*x^2)/(a +
 b*x^2)]*(8*Sqrt[a]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}
, a/(a + b*x^2)] + Log[a + b*x^2]*(4*Sqrt[a]*HypergeometricPFQ[{1/2, 1/2, 1
/2}, {3/2, 3/2}, a/(a + b*x^2)] + Sqrt[a + b*x^2]*ArcSin[Sqrt[a]/Sqrt[a + b
*x^2]]*Log[a + b*x^2])) + 24*a*Sqrt[b*x^2]*ArcTanh[Sqrt[b*x^2]/Sqrt[-a]]*(L
og[a + b*x^2] - Log[1 + (b*x^2)/a]) + 6*(-a)^(3/2)*Sqrt[-((b*x^2)/a)]*(Log[
1 + (b*x^2)/a]^2 - 4*Log[1 + (b*x^2)/a]*Log[(1 + Sqrt[-((b*x^2)/a)])/2] + 2
*Log[(1 + Sqrt[-((b*x^2)/a)])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[-((b*x^2)/a)]/
2])))/(Sqrt[-a]*b*x)
```

fricas [**A**] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\log\left(\left(bx^2+a\right)^p c\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3, x)
```

giac [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\left(bx^2+a\right)^p c\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3, x)
```

maple [**A**] time = 0.73, size = 0, normalized size = 0.00

$$\int \ln\left(c\left(b\,x^2+a\right)^p\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3,x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$p^3x\log\left(bx^2+a\right)^3+\int \frac{bx^2\log(c)^3+a\log(c)^3-3\left(\left(2p^3-p^2\log(c)\right)bx^2-ap^2\log(c)\right)\log\left(bx^2+a\right)^2+3\left(bpx^2\log\right.}{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] p^3*x*log(b*x^2 + a)^3 + integrate((b*x^2*log(c)^3 + a*log(c)^3 - 3*((2*p^3
 - p^2*log(c))*b*x^2 - a*p^2*log(c))*log(b*x^2 + a)^2 + 3*(b*p*x^2*log(c)^2
 + a*p*log(c)^2)*log(b*x^2 + a))/(b*x^2 + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(b\,x^2+a\right)^p\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(c\left(a+bx^2\right)^p\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**3, x)
```

3.100 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=51

$$6bp\text{Int}\left(\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2},x\right)-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x}$$

```
[Out] -ln(c*(b*x^2+a)^p)^3/x+6*b*p*Unintegrable(ln(c*(b*x^2+a)^p)^2/(b*x^2+a),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^2,x]
```

```
[Out] -(Log[c*(a + b*x^2)^p]^3/x) + 6*b*p*Defer[Int][Log[c*(a + b*x^2)^p]^2/(a + b*x^2), x]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx=-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x}+(6bp)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx$$

Mathematica [A] time = 0.83, size = 505, normalized size = 9.90

$$\frac{p^3\left(-96\sqrt{a}\sqrt{1-\frac{a}{a+bx^2}}\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{a}{bx^2+a}\right)-48\sqrt{a}\sqrt{1-\frac{a}{a+bx^2}}\log\left(a+bx^2\right)\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{a}{bx^2+a}\right)-2\right.}{2\sqrt{a}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^2,x]
```

```
[Out] (p^3*(-96*Sqrt[a]*Sqrt[1 - a/(a + b*x^2)]*HypergeometricPFQ[{1/2, 1/2, 1/2,
 1/2}, {3/2, 3/2, 3/2}, a/(a + b*x^2)] - 48*Sqrt[a]*Sqrt[1 - a/(a + b*x^2)]
*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, a/(a + b*x^2)]*Log[a + b*x^
2] - 2*Log[a + b*x^2]^2*(6*Sqrt[a + b*x^2]*Sqrt[1 - a/(a + b*x^2)]*ArcSin[S
qrt[a]/Sqrt[a + b*x^2]] + Sqrt[a]*Log[a + b*x^2])))/(2*Sqrt[a]*x) + (6*Sqrt
[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^
p])^2)/Sqrt[a] - (3*p*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^
2)^p])^2)/x - (-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^3/x + 3*p^2*(-(p
*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])*(-(Log[a + b*x^2]^2/x) + (4*Sqrt[b
]*(ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(I*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*Log[(2*I)
/(I - (Sqrt[b]*x)/Sqrt[a])] + Log[a + b*x^2]) + I*PolyLog[2, (I*Sqrt[a] + S
qrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)]))/Sqrt[a])
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^2, x)
```

maple [A] time = 7.52, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^2,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^3/x^2,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^3/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^3}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x**2,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**3/x**2, x)
```

3.101 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=254

$$-\frac{2b^2p\text{Int}\left(\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2},x\right)}{a}+\frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^{3/2}}+\frac{8ib^{3/2}p^3\text{Li}_2\left(1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{a^{3/2}}+\frac{8ib^{3/2}p^3\tan^{-1}}{a^{3/2}}$$

```
[Out] 8*I*b^(3/2)*p^3*arctan(x*b^(1/2)/a^(1/2))^2/a^(3/2)+8*b^(3/2)*p^2*arctan(x*
b^(1/2)/a^(1/2))*ln(c*(b*x^2+a)^p)/a^(3/2)-2*b*p*ln(c*(b*x^2+a)^p)^2/a/x-1/
3*ln(c*(b*x^2+a)^p)^3/x^3+16*b^(3/2)*p^3*arctan(x*b^(1/2)/a^(1/2))*ln(2*a^(
1/2)/(a^(1/2)+I*x*b^(1/2)))/a^(3/2)+8*I*b^(3/2)*p^3*polylog(2,1-2*a^(1/2)/(
a^(1/2)+I*x*b^(1/2)))/a^(3/2)-2*b^2*p*Unintegrable(ln(c*(b*x^2+a)^p)^2/(b*x
^2+a),x)/a
```

Rubi [A] time = 0.35, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^4,x]
```

```
[Out] ((8*I)*b^(3/2)*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/a^(3/2) + (16*b^(3/2)*p^3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^(3
/2) + (8*b^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/a^(3
/2) - (2*b*p*Log[c*(a + b*x^2)^p]^2)/(a*x) - Log[c*(a + b*x^2)^p]^3/(3*x^3)
 + ((8*I)*b^(3/2)*p^3*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/
a^(3/2) - (2*b^2*p*Defer[Int][Log[c*(a + b*x^2)^p]^2/(a + b*x^2), x])/a
```

Rubi steps

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx = -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{3x^3} + (2bp)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^2\left(a+bx^2\right)}\,dx$$

$$= -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{3x^3} + (2bp)\int \left(\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{ax^2} - \frac{b\log^2\left(c\left(a+bx^2\right)^p\right)}{a\left(a+bx^2\right)}\right)dx$$

$$= -\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{3x^3} + \frac{(2bp)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx}{a} - \frac{\left(2b^2p\right)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx}{a}$$

$$= -\frac{2bp\log^2\left(c\left(a+bx^2\right)^p\right)}{ax} - \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{3x^3} - \frac{\left(2b^2p\right)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx}{a} + \frac{\left(8b^2p^2\right.}{}$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^{3/2}} - \frac{2bp\log^2\left(c\left(a+bx^2\right)^p\right)}{ax} - \frac{\log^3\left(c\left(a+bx^2\right)^{p}\right)}{3x^3}$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^{3/2}} - \frac{2bp\log^2\left(c\left(a+bx^2\right)^p\right)}{ax} - \frac{\log^3\left(c\left(a+bx^2\right)^{p}\right)}{3x^3}$$

$$= \frac{8ib^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{a^{3/2}} + \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^{3/2}} - \frac{2bp\log^2\left(c\left(a+bx\right.\right.}{ax}$$

$$= \frac{8ib^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{a^{3/2}} + \frac{16b^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{a^{3/2}} + \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

$$= \frac{8ib^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{a^{3/2}} + \frac{16b^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{a^{3/2}} + \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

$$= \frac{8ib^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{a^{3/2}} + \frac{16b^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{a^{3/2}} + \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A] time = 2.71, size = 851, normalized size = 3.35

$$\left(-a^2\log^3\left(bx^2+a\right) - 6abx^2\log^2\left(bx^2+a\right) + 6\sqrt{a}\left(\frac{bx^2}{bx^2+a}\right)^{3/2}\left(bx^2+a\right)^{3/2}\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{bx^2+a}}\right)\log^2\left(bx^2+a\right) + 24\sqrt{-a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^4,x]
```

```
[Out] (a^2*(p*Log[a + b*x^2] - Log[c*(a + b*x^2)^p])^3 - 6*a*b*p*x^2*(-(p*Log[a +
 b*x^2]) + Log[c*(a + b*x^2)^p])^2 - 6*Sqrt[a]*b^(3/2)*p*x^3*ArcTan[(Sqrt[b
]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2 - 3*a^2*p*Log[
a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2 + 3*Sqrt[a]*p^2*(
p*Log[a + b*x^2] - Log[c*(a + b*x^2)^p])*(a^(3/2)*Log[a + b*x^2]^2 + 4*b*x^
2*(I*Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + Sqrt[a]*Log[a + b*x^2] + Sqr
t[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-2 + 2*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqr
t[b]*x)] + Log[a + b*x^2]) + I*Sqrt[b]*x*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)
```

```
/((-I)*Sqrt[a] + Sqrt[b]*x)])) + p^3*(48*a*b*x^2*Sqrt[(b*x^2)/(a + b*x^2)]*
HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, a/(a + b*x^2)] + 2
4*Sqrt[-a]*(b*x^2)^(3/2)*ArcTanh[Sqrt[b*x^2]/Sqrt[-a]]*Log[a + b*x^2] + 24*
a*b*x^2*Sqrt[(b*x^2)/(a + b*x^2)]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2,
3/2}, a/(a + b*x^2)]*Log[a + b*x^2] - 6*a*b*x^2*Log[a + b*x^2]^2 + 6*Sqrt[a
]*((b*x^2)/(a + b*x^2))^(3/2)*(a + b*x^2)^(3/2)*ArcSin[Sqrt[a]/Sqrt[a + b*x
^2]]*Log[a + b*x^2]^2 - a^2*Log[a + b*x^2]^3 - 24*Sqrt[-a]*(b*x^2)^(3/2)*Ar
cTanh[Sqrt[b*x^2]/Sqrt[-a]]*Log[1 + (b*x^2)/a] - 6*a^2*(-((b*x^2)/a))^(3/2)
*Log[1 + (b*x^2)/a]^2 + 24*a^2*(-((b*x^2)/a))^(3/2)*Log[1 + (b*x^2)/a]*Log[
(1 + Sqrt[-((b*x^2)/a)])/2] - 12*a^2*(-((b*x^2)/a))^(3/2)*Log[(1 + Sqrt[-((
b*x^2)/a)])/2]^2 + 24*a^2*(-((b*x^2)/a))^(3/2)*PolyLog[2, 1/2 - Sqrt[-((b*x
^2)/a)]/2]))/(3*a^2*x^3)
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x^4, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^4, x)
```

maple [A] time = 22.77, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^4,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{p^3\log\left(bx^2+a\right)^3}{3\,x^3}+\int \frac{bx^2\log(c)^3+a\log(c)^3+\left(\left(2\,p^3+3\,p^2\log(c)\right)bx^2+3\,ap^2\log(c)\right)\log\left(bx^2+a\right)^2+3\left(b\right.}{bx^6+ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*p^3*log(b*x^2 + a)^3/x^3 + integrate((b*x^2*log(c)^3 + a*log(c)^3 + ((
2*p^3 + 3*p^2*log(c))*b*x^2 + 3*a*p^2*log(c))*log(b*x^2 + a)^2 + 3*(b*p*x^2
*log(c)^2 + a*p*log(c)^2)*log(b*x^2 + a))/(b*x^6 + a*x^4), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^3/x^4,x)
[Out] int(log(c*(a + b*x^2)^p)^3/x^4, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x**4,x)
[Out] Integral(log(c*(a + b*x**2)**p)**3/x**4, x)
```

3.102 $\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal. Leaf size=107

$$\frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2b^2p}-\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2b^2p}$$

```
[Out] -1/2*a*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b^2/p/((c*(b*x^2+a)^p)^(1/p))+1/2*
(b*x^2+a)^2*Ei(2*ln(c*(b*x^2+a)^p)/p)/b^2/p/((c*(b*x^2+a)^p)^(2/p))
```

Rubi [A] time = 0.15, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2454, 2399, 2389, 2300, 2178, 2390, 2310}

$$\frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2b^2p}-\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2b^2p}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Log[c*(a + b*x^2)^p],x]
```

```
[Out] -(a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b^2*p*(c*(a + b*x
^2)^p)^p^(-1)) + ((a + b*x^2)^2*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p])/
(2*b^2*p*(c*(a + b*x^2)^p)^(2/p))
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\log\left(c(a+bx)^p\right)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{b\log\left(c(a+bx)^p\right)}+\frac{a+bx}{b\log\left(c(a+bx)^p\right)}\right)dx, x, x^2\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{a+bx}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{2b} - \frac{a\operatorname{Subst}\left(\int \frac{1}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{2b}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{x}{\log(cx^p)}\,dx, x, a+bx^2\right)}{2b^2} - \frac{a\operatorname{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, a+bx^2\right)}{2b^2}$$
$$= \frac{\left(\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\right)\operatorname{Subst}\left(\int \frac{e^{\frac{2x}{p}}}{x}\,dx, x, \log\left(c\left(a+bx^2\right)^p\right)\right)}{2b^2p} - \frac{\left(a\left(a+bx^2\right)\left(\cdots\right.\right.}{\cdots}$$
$$= -\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{2b^2p} + \frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\cdots}{\cdots}\right.}{2b^2p}$$

Mathematica [A] time = 0.15, size = 96, normalized size = 0.90

$$-\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-2/p}\left(a\left(c\left(a+bx^2\right)^p\right)^{\frac{1}{p}}\operatorname{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)-\left(a+bx^2\right)\operatorname{Ei}\left(\frac{2\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)\right)}{2b^2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Log[c*(a + b*x^2)^p],x]
```

```
[Out] -1/2*((a + b*x^2)*(a*(c*(a + b*x^2)^p)^p^(-1)*ExpIntegralEi[Log[c*(a + b*x^
2)^p]/p] - (a + b*x^2)*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p]))/(b^2*p*(
c*(a + b*x^2)^p)^(2/p))
```

fricas [A] time = 0.43, size = 68, normalized size = 0.64

$$-\frac{ac^{\left(\frac{1}{p}\right)}\,\mathrm{log\_integral}\left(\left(bx^2+a\right)c^{\left(\frac{1}{p}\right)}\right)-\mathrm{log\_integral}\left(\left(b^2x^4+2\,abx^2+a^2\right)c^{\frac{2}{p}}\right)}{2\,b^2c^{\frac{2}{p}}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] -1/2*(a*c^(1/p)*log_integral((b*x^2 + a)*c^(1/p)) - log_integral((b^2*x^4 +
 2*a*b*x^2 + a^2)*c^(2/p)))/(b^2*c^(2/p)*p)
```

giac [A] time = 0.17, size = 73, normalized size = 0.68

$$-\frac{\frac{a\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)}{bc^{\left(\frac{1}{p}\right)}p}-\frac{\mathrm{Ei}\left(\frac{2\log(c)}{p}+2\log\left(bx^2+a\right)\right)}{bc^{\frac{2}{p}}p}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] -1/2*(a*Ei(log(c)/p + log(b*x^2 + a))/(b*c^(1/p)*p) - Ei(2*log(c)/p + 2*log
(b*x^2 + a))/(b*c^(2/p)*p))/b
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\ln\left(c\left(b\,x^2+a\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(b*x^2+a)^p),x)
```

```
[Out] int(x^3/ln(c*(b*x^2+a)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] integrate(x^3/log((b*x^2 + a)^p*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\ln\left(c\left(b\,x^2+a\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/log(c*(a + b*x^2)^p),x)
```

```
[Out] int(x^3/log(c*(a + b*x^2)^p), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(b*x**2+a)**p),x)
```

```
[Out] Integral(x**3/log(c*(a + b*x**2)**p), x)
```

3.103 $\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=51

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2bp}$$

```
[Out] 1/2*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b/p/((c*(b*x^2+a)^p)^(1/p))
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2454, 2389, 2300, 2178}

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2bp}$$

Antiderivative was successfully verified.

```
[In] Int[x/Log[c*(a + b*x^2)^p],x]
```

```
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b*p*(c*(a + b*x^2)^p
)^p^(-1))
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\log\left(c(a+bx)^p\right)}\,dx, x, x^2\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, a+bx^2\right)}{2b}$$
$$= \frac{\left(\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\right)\operatorname{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c\left(a+bx^2\right)^p\right)\right)}{2bp}$$
$$= \frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{2bp}$$

Mathematica [**A**] time = 0.04, size = 51, normalized size = 1.00

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2bp}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Log[c*(a + b*x^2)^p],x]
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b*p*(c*(a + b*x^2)^p
)^p^(-1))
```

fricas [**A**] time = 0.44, size = 29, normalized size = 0.57

$$\frac{\operatorname{log\_integral}\left(\left(bx^2+a\right)c^{\left(\frac{1}{p}\right)}\right)}{2\,bc^{\left(\frac{1}{p}\right)}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] 1/2*log_integral((b*x^2 + a)*c^(1/p))/(b*c^(1/p)*p)
```

giac [**A**] time = 0.16, size = 31, normalized size = 0.61

$$\frac{\operatorname{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)}{2\,bc^{\left(\frac{1}{p}\right)}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] 1/2*Ei(log(c)/p + log(b*x^2 + a))/(b*c^(1/p)*p)
```

maple [**C**] time = 1.26, size = 272, normalized size = 5.33

$$-\frac{\left(b\,x^2+a\right)c^{-\frac{1}{p}}\left(\left(b\,x^2+a\right)^p\right)^{-\frac{1}{p}}\operatorname{Ei}\left(1,-\ln\left(b\,x^2+a\right)-\frac{-i\pi\operatorname{csgn}(ic)\operatorname{csgn}\left(i\left(b\,x^2+a\right)^p\right)\operatorname{csgn}\left(ic\left(b\,x^2+a\right)^p\right)+i\pi\operatorname{csgn}(ic)\operatorname{csgn}\left(ic\left(b\,x^2+\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(b*x^2+a)^p),x)
```

```
[Out] -1/2/b/p*(b*x^2+a)*c^(-1/p)*((b*x^2+a)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(b*x
^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*
(b*x^2+a)^p))/p)*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(
b*x^2+a)^p)^2-I*Pi*csgn(I*c)*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)-I*Pi
*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c)*csgn(I*c*(b*x^2+a)^p)^2+2*ln(c)+2*l
n((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] integrate(x/log((b*x^2 + a)^p*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\ln\left(c\left(b\,x^2+a\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/log(c*(a + b*x^2)^p),x)
```

```
[Out] int(x/log(c*(a + b*x^2)^p), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(b*x**2+a)**p),x)
```

```
[Out] Integral(x/log(c*(a + b*x**2)**p), x)
```

3.104 $\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x/ln(c*(b*x^2+a)^p),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Log[c*(a + b*x^2)^p]),x]
[Out] Defer[Int][1/(x*Log[c*(a + b*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Log[c*(a + b*x^2)^p]),x]
[Out] Integrate[1/(x*Log[c*(a + b*x^2)^p]), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{x\log\left(\left(bx^2+a\right)^p c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] integral(1/(x*log((b*x^2 + a)^p*c)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] integrate(1/(x*log((b*x^2 + a)^p*c)), x)
```

maple [A] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{1}{x \ln \left(c \left(b\,x^{2}+a\right)^{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(c*(b*x^2+a)^p),x)
```

```
[Out] int(1/x/ln(c*(b*x^2+a)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \log \left(\left(bx^{2}+a\right)^{p} c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*log((b*x^2 + a)^p*c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\ \ln \left(c \left(b\,x^{2}+a\right)^{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(c*(a + b*x^2)^p)),x)
```

```
[Out] int(1/(x*log(c*(a + b*x^2)^p)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \log \left(c \left(a+bx^{2}\right)^{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(c*(b*x**2+a)**p),x)
```

```
[Out] Integral(1/(x*log(c*(a + b*x**2)**p)), x)
```

3.105 $\int \frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^3/ln(c*(b*x^2+a)^p),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*Log[c*(a + b*x^2)^p]),x]
[Out] Defer[Int][1/(x^3*Log[c*(a + b*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)} dx = \int \frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)} dx$$

Mathematica [A] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]),x]
[Out] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]), x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^3 \log\left(\left(bx^2+a\right)^p c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] integral(1/(x^3*log((b*x^2 + a)^p*c)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log\left(\left(bx^2+a\right)^p c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] integrate(1/(x^3*log((b*x^2 + a)^p*c)), x)
```

maple [A] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \ln \left(c \left(b\,x^{2}+a\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/ln(c*(b*x^2+a)^p),x)
[Out] int(1/x^3/ln(c*(b*x^2+a)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \log \left(\left(bx^{2}+a\right)^{p} c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] integrate(1/(x^3*log((b*x^2 + a)^p*c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^{3}\,\ln \left(c\,\left(b\,x^{2}+a\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*log(c*(a + b*x^2)^p)),x)
[Out] int(1/(x^3*log(c*(a + b*x^2)^p)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \log \left(c \left(a+bx^{2}\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(c*(b*x**2+a)**p),x)
[Out] Integral(1/(x**3*log(c*(a + b*x**2)**p)), x)
```

3.106 $\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(x^2/ln(c*(b*x^2+a)^p),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/Log[c*(a + b*x^2)^p],x]
[Out] Defer[Int][x^2/Log[c*(a + b*x^2)^p], x]
```

Rubi steps

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)} dx = \int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)} dx$$

Mathematica [A] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/Log[c*(a + b*x^2)^p],x]
[Out] Integrate[x^2/Log[c*(a + b*x^2)^p], x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] integral(x^2/log((b*x^2 + a)^p*c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] integrate(x^2/log((b*x^2 + a)^p*c), x)
```

maple [A] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/ln(c*(b*x^2+a)^p),x)
[Out] int(x^2/ln(c*(b*x^2+a)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] integrate(x^2/log((b*x^2 + a)^p*c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/log(c*(a + b*x^2)^p),x)
[Out] int(x^2/log(c*(a + b*x^2)^p), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/ln(c*(b*x**2+a)**p),x)
[Out] Integral(x**2/log(c*(a + b*x**2)**p), x)
```

3.107 $\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left(\frac{1}{\log\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/ln(c*(b*x^2+a)^p),x)
```

Rubi [A] time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(a + b*x^2)^p]^(-1),x]
[Out] Defer[Int][Log[c*(a + b*x^2)^p]^(-1), x]
```

Rubi steps

$$\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^(-1),x]
[Out] Integrate[Log[c*(a + b*x^2)^p]^(-1), x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{\log\left(\left(bx^2+a\right)^p c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] integral(1/log((b*x^2 + a)^p*c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] integrate(1/log((b*x^2 + a)^p*c), x)
```

maple [A] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{1}{\ln \left(c \left(b\,x^{2}+a\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(c*(b*x^2+a)^p),x)
[Out] int(1/ln(c*(b*x^2+a)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log \left(\left(bx^{2}+a\right)^{p} c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] integrate(1/log((b*x^2 + a)^p*c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\ln \left(c \left(b\,x^{2}+a\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(c*(a + b*x^2)^p),x)
[Out] int(1/log(c*(a + b*x^2)^p), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log \left(c \left(a+bx^{2}\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(c*(b*x**2+a)**p),x)
[Out] Integral(1/log(c*(a + b*x**2)**p), x)
```

3.108 $\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/ln(c*(b*x^2+a)^p),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Log[c*(a + b*x^2)^p]),x]
[Out] Defer[Int][1/(x^2*Log[c*(a + b*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]),x]
[Out] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^2 \log\left(\left(bx^2+a\right)^p c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] integral(1/(x^2*log((b*x^2 + a)^p*c)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] integrate(1/(x^2*log((b*x^2 + a)^p*c)), x)
```

maple [A] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \ln \left(c \left(b \,x^{2}+a \right)^{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/ln(c*(b*x^2+a)^p),x)
[Out] int(1/x^2/ln(c*(b*x^2+a)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \log\left({\left(b x^{2} + a\right)}^{p} c\right)}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] integrate(1/(x^2*log((b*x^2 + a)^p*c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*log(c*(a + b*x^2)^p)),x)
[Out] int(1/(x^2*log(c*(a + b*x^2)^p)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \log{\left(c \left(a + b x^{2}\right)^{p} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(c*(b*x**2+a)**p),x)
[Out] Integral(1/(x**2*log(c*(a + b*x**2)**p)), x)
```

3.109 $\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=138

$$\frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{b^2p^2}-\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2b^2p^2}-\frac{x^2\left(a+\right.}{2bp\log\left(c\left(a\right.\right.}$$

```
[Out] -1/2*a*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b^2/p^2/((c*(b*x^2+a)^p)^(1/p))+(b
*x^2+a)^2*Ei(2*ln(c*(b*x^2+a)^p)/p)/b^2/p^2/((c*(b*x^2+a)^p)^(2/p))-1/2*x^2
*(b*x^2+a)/b/p/ln(c*(b*x^2+a)^p)
```

Rubi [A] time = 0.21, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2454, 2400, 2399, 2389, 2300, 2178, 2390, 2310}

$$\frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{b^2p^2}-\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2b^2p^2}-\frac{x^2\left(a+\right.}{2bp\log\left(c\left(a\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] -(a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b^2*p^2*(c*(a + b
*x^2)^p)^p^(-1)) + ((a + b*x^2)^2*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p]
)/(b^2*p^2*(c*(a + b*x^2)^p)^(2/p)) - (x^2*(a + b*x^2))/(2*b*p*Log[c*(a + b
*x^2)^p])
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2400

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((d + e*x)*(f + g*x)^q*(a + b*Log[c*(d + e
*x)^n])^(p + 1))/(b*e*n*(p + 1)), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[(q*(e*f - d*g))
/(b*e*n*(p + 1)), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\log^2\left(c(a+bx)^p\right)}\,dx, x, x^2\right)$$

$$= -\frac{x^2\left(a+bx^2\right)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{x}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{p} + \frac{a\,\text{Subst}\left(\int \frac{1}{\log(c(a+bx)^p}\right.}{2bp}$$

$$= -\frac{x^2\left(a+bx^2\right)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int\left(-\frac{a}{b\log(c(a+bx)^p)}+\frac{a+bx}{b\log(c(a+bx)^p)}\right)dx, x, x^2\right)}{p} + -\frac{a}{}$$

$$= -\frac{x^2\left(a+bx^2\right)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{a+bx}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{bp} - \frac{a\,\text{Subst}\left(\int \frac{1}{\log(c(a+bx)^p}\right.}{bp}$$

$$= \frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{2b^2p^2} - \frac{x^2\left(a+bx^2\right)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}}{}$$

$$= \frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{2b^2p^2} - \frac{x^2\left(a+bx^2\right)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\left(\left(a+b\right.\right.}{}$$

$$= -\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{2b^2p^2} + \frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\,\text{E}}{b^2p^2}$$

Mathematica [A] time = 0.16, size = 157, normalized size = 1.14

$$-\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-2/p}\left(a\left(c\left(a+bx^2\right)^p\right)^{\frac{1}{p}}\log\left(c\left(a+bx^2\right)^p\right)\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)-2\left(a+bx^2\right)\log\left(c\left(a+b\right.\right.\right.}{2b^2p^2\log\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] -1/2*((a + b*x^2)*(b*p*x^2*(c*(a + b*x^2)^p)^(2/p) + a*(c*(a + b*x^2)^p)^p^
(-1)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p]*Log[c*(a + b*x^2)^p] - 2*(a + b*
x^2)*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p]*Log[c*(a + b*x^2)^p]))/(b^2*
p^2*(c*(a + b*x^2)^p)^(2/p)*Log[c*(a + b*x^2)^p])
```

fricas [A] time = 0.46, size = 141, normalized size = 1.02

$$-\frac{\left(ap\log\left(bx^2+a\right)+a\log(c)\right)c^{\left(\frac{1}{p}\right)}\text{log\_integral}\left(\left(bx^2+a\right)c^{\left(\frac{1}{p}\right)}\right)+\left(b^2px^4+abpx^2\right)c^{\frac{2}{p}}-2\left(p\log\left(bx^2+a\right)+\text{lo}\right.}{2\left(b^2p^3\log\left(bx^2+a\right)+b^2p^2\log(c)\right)c^{\frac{2}{p}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a*p*log(b*x^2 + a) + a*log(c))*c^(1/p)*log_integral((b*x^2 + a)*c^(1
/p)) + (b^2*p*x^4 + a*b*p*x^2)*c^(2/p) - 2*(p*log(b*x^2 + a) + log(c))*log_
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*c^(2/p)))/((b^2*p^3*log(b*x^2 + a) + b
^2*p^2*log(c))*c^(2/p))
```

giac [B] time = 0.20, size = 298, normalized size = 2.16

$$-\frac{\frac{(bx^2+a)^2 p}{bp^3\log(bx^2+a)+bp^2\log(c)} - \frac{(bx^2+a)ap}{bp^3\log(bx^2+a)+bp^2\log(c)} + \frac{ap\mathrm{Ei}\left(\frac{\log(c)}{p}+\log(bx^2+a)\right)\log(bx^2+a)}{\left(bp^3\log(bx^2+a)+bp^2\log(c)\right)c^{\left(\frac{1}{p}\right)}} - \frac{2p\mathrm{Ei}\left(\frac{2\log(c)}{p}+2\log(bx^2+a)\right)\log(bx^2+a)}{\left(bp^3\log(bx^2+a)+bp^2\log(c)\right)c^{\frac{2}{p}}} +}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] -1/2*((b*x^2 + a)^2*p/(b*p^3*log(b*x^2 + a) + b*p^2*log(c)) - (b*x^2 + a)*a
*p/(b*p^3*log(b*x^2 + a) + b*p^2*log(c)) + a*p*Ei(log(c)/p + log(b*x^2 + a)
)*log(b*x^2 + a)/((b*p^3*log(b*x^2 + a) + b*p^2*log(c))*c^(1/p)) - 2*p*Ei(2
*log(c)/p + 2*log(b*x^2 + a))*log(b*x^2 + a)/((b*p^3*log(b*x^2 + a) + b*p^2
*log(c))*c^(2/p)) + a*Ei(log(c)/p + log(b*x^2 + a))*log(c)/((b*p^3*log(b*x^
2 + a) + b*p^2*log(c))*c^(1/p)) - 2*Ei(2*log(c)/p + 2*log(b*x^2 + a))*log(c
)/((b*p^3*log(b*x^2 + a) + b*p^2*log(c))*c^(2/p)))/b
```

maple [F] time = 6.55, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\ln\left(c\left(bx^2+a\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(x^3/ln(c*(b*x^2+a)^p)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bx^4+ax^2}{2\left(bp^2\log\left(bx^2+a\right)+bp\log(c)\right)}+\int \frac{2bx^3+ax}{bp^2\log\left(bx^2+a\right)+bp\log(c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^4 + a*x^2)/(b*p^2*log(b*x^2 + a) + b*p*log(c)) + integrate((2*b*x
^3 + a*x)/(b*p^2*log(b*x^2 + a) + b*p*log(c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\ln\left(c\left(bx^2+a\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] int(x^3/log(c*(a + b*x^2)^p)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(x**3/log(c*(a + b*x**2)**p)**2, x)
```

3.110 $\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal. Leaf size=83

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2bp^2}-\frac{a+bx^2}{2bp\log\left(c\left(a+bx^2\right)^p\right)}$$

```
[Out] 1/2*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b/p^2/((c*(b*x^2+a)^p)^(1/p))+1/2*(-b
*x^2-a)/b/p/ln(c*(b*x^2+a)^p)
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2454, 2389, 2297, 2300, 2178}

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2bp^2}-\frac{a+bx^2}{2bp\log\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b*p^2*(c*(a + b*x^2)
^p)^p^(-1)) - (a + b*x^2)/(2*b*p*Log[c*(a + b*x^2)^p])
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\log^2\left(c(a+bx)^p\right)}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx^p)}\,dx, x, a+bx^2\right)}{2b}$$
$$= -\frac{a+bx^2}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, a+bx^2\right)}{2bp}$$
$$= -\frac{a+bx^2}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\left(\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c\right.\right.}{2bp^2}$$
$$= \frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{2bp^2} - \frac{a+bx^2}{2bp\log\left(c\left(a+bx^2\right)^p\right)}$$

Mathematica [A] time = 0.05, size = 97, normalized size = 1.17

$$-\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\left(p\left(c\left(a+bx^2\right)^p\right)^{\frac{1}{p}} - \log\left(c\left(a+bx^2\right)^p\right)\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)\right)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] -1/2*((a + b*x^2)*(p*(c*(a + b*x^2)^p)^p^(-1) - ExpIntegralEi[Log[c*(a + b*
x^2)^p]/p]*Log[c*(a + b*x^2)^p]))/(b*p^2*(c*(a + b*x^2)^p)^p^(-1)*Log[c*(a
+ b*x^2)^p])
```

fricas [A] time = 0.46, size = 78, normalized size = 0.94

$$-\frac{\left(bpx^2+ap\right)c^{\left(\frac{1}{p}\right)} - \left(p\log\left(bx^2+a\right)+\log(c)\right)\text{log\_integral}\left(\left(bx^2+a\right)c^{\left(\frac{1}{p}\right)}\right)}{2\left(bp^3\log\left(bx^2+a\right)+bp^2\log(c)\right)c^{\left(\frac{1}{p}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((b*p*x^2 + a*p)*c^(1/p) - (p*log(b*x^2 + a) + log(c))*log_integral((b
*x^2 + a)*c^(1/p)))/((b*p^3*log(b*x^2 + a) + b*p^2*log(c))*c^(1/p))
```

giac [A] time = 0.18, size = 141, normalized size = 1.70

$$-\frac{\left(bx^2+a\right)p}{2\left(bp^3\log\left(bx^2+a\right)+bp^2\log(c)\right)} + \frac{p\text{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)}{2\left(bp^3\log\left(bx^2+a\right)+bp^2\log(c)\right)c^{\left(\frac{1}{p}\right)}} + \frac{\text{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\text{l}}{2\left(bp^3\log\left(bx^2+a\right)+bp^2\text{log}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b*x^2 + a)*p/(b*p^3*log(b*x^2 + a) + b*p^2*log(c)) + 1/2*p*Ei(log(c)/
p + log(b*x^2 + a))*log(b*x^2 + a)/((b*p^3*log(b*x^2 + a) + b*p^2*log(c))*c
^(1/p)) + 1/2*Ei(log(c)/p + log(b*x^2 + a))*log(c)/((b*p^3*log(b*x^2 + a) +
 b*p^2*log(c))*c^(1/p))
```

maple [C] time = 1.22, size = 421, normalized size = 5.07

$$-\frac{\left(b\,x^{2}+a\right)c^{-\frac{1}{p}}\left(\left(b\,x^{2}+a\right)^{p}\right)^{-\frac{1}{p}}\operatorname{Ei}\left(1,-\ln\left(b\,x^{2}+a\right)-\frac{-i\pi\operatorname{csgn}(ic)\operatorname{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\operatorname{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)+i\pi\operatorname{csgn}(ic)\operatorname{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right.}{\quad}\right.}{\quad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] -1/(2*ln(c)+2*ln((b*x^2+a)^p)+I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p
)^2-I*Pi*csgn(I*c)*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)-I*Pi*csgn(I*c*
(b*x^2+a)^p)^3+I*Pi*csgn(I*c)*csgn(I*c*(b*x^2+a)^p)^2)/p/b*(b*x^2+a)-1/2/p^
2/b*(b*x^2+a)*c^(-1/p)*((b*x^2+a)^p)^(-1/p)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*
c*(b*x^2+a)^p))*(csgn(I*(b*x^2+a)^p)-csgn(I*c*(b*x^2+a)^p))/p*csgn(I*c*(b*x
^2+a)^p))*Ei(1,-ln(b*x^2+a)-1/2*(-I*Pi*csgn(I*c)*csgn(I*(b*x^2+a)^p)*csgn(I
*c*(b*x^2+a)^p)+I*Pi*csgn(I*c)*csgn(I*c*(b*x^2+a)^p)^2+I*Pi*csgn(I*(b*x^2+a
)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*c*(b*x^2+a)^p)^3-2*p*ln(b*x^2+a)+2
*ln(c)+2*ln((b*x^2+a)^p))/p)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bx^{2}+a}{2\left(bp^{2}\log\left(bx^{2}+a\right)+bp\log(c)\right)}+\int\frac{x}{p^{2}\log\left(bx^{2}+a\right)+p\log(c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)/(b*p^2*log(b*x^2 + a) + b*p*log(c)) + integrate(x/(p^2*log
(b*x^2 + a) + p*log(c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x}{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] int(x/log(c*(a + b*x^2)^p)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\log\left(c\left(a+bx^{2}\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(x/log(c*(a + b*x**2)**p)**2, x)
```

3.111 $\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)},x\right)$$

[Out] Unintegrable(1/x/ln(c*(b*x^2+a)^p)^2,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/(x*Log[c*(a + b*x^2)^p]^2),x]

[Out] Defer[Int][1/(x*Log[c*(a + b*x^2)^p]^2), x]

Rubi steps

$$\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x*Log[c*(a + b*x^2)^p]^2),x]

[Out] Integrate[1/(x*Log[c*(a + b*x^2)^p]^2), x]

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x\log\left(\left(bx^2+a\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")

[Out] integral(1/(x*log((b*x^2 + a)^p*c)^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*log((b*x^2 + a)^p*c)^2), x)
```

maple [A] time = 1.84, size = 0, normalized size = 0.00

$$\int \frac{1}{x \ln \left(c \left(b \,x^{2}+a \right)^{p}\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(1/x/ln(c*(b*x^2+a)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-a \int \frac{1}{bp^{2}x^{3} \log \left(bx^{2} + a\right) + bpx^{3} \log(c)}\,dx - \frac{bx^{2} + a}{2 \left(bp^{2}x^{2} \log \left(bx^{2} + a\right) + bpx^{2} \log(c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -a*integrate(1/(b*p^2*x^3*log(b*x^2 + a) + b*p*x^3*log(c)), x) - 1/2*(b*x^2
 + a)/(b*p^2*x^2*log(b*x^2 + a) + b*p*x^2*log(c))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x \ln \left(c \left(b \,x^{2}+a \right)^{p}\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(c*(a + b*x^2)^p)^2),x)
```

```
[Out] int(1/(x*log(c*(a + b*x^2)^p)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \log \left(c \left(a + bx^{2}\right)^{p}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(1/(x*log(c*(a + b*x**2)**p)**2), x)
```

3.112 $\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

```
[Out] Unintegrable(1/x^3/ln(c*(b*x^2+a)^p)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*Log[c*(a + b*x^2)^p]^2),x]
```

```
[Out] Defer[Int][1/(x^3*Log[c*(a + b*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} dx = \int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} dx$$

Mathematica [A] time = 1.55, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^2),x]
```

```
[Out] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^2), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^3 \log\left(\left(bx^2+a\right)^p c\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x^3*log((b*x^2 + a)^p*c)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log\left(\left(bx^2+a\right)^p c\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*log((b*x^2 + a)^p*c)^2), x)
```

maple [A] time = 3.76, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \ln \left(c \left(b\,x^{2}+a\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(1/x^3/ln(c*(b*x^2+a)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bx^{2}+a}{2\left(bp^{2}x^{4}\log\left(bx^{2}+a\right)+bpx^{4}\log(c)\right)}-\int \frac{bx^{2}+2\,a}{bp^{2}x^{5}\log\left(bx^{2}+a\right)+bpx^{5}\log(c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)/(b*p^2*x^4*log(b*x^2 + a) + b*p*x^4*log(c)) - integrate((b
*x^2 + 2*a)/(b*p^2*x^5*log(b*x^2 + a) + b*p*x^5*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^{3} \ln \left(c \left(b\,x^{2}+a\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*log(c*(a + b*x^2)^p)^2),x)
```

```
[Out] int(1/(x^3*log(c*(a + b*x^2)^p)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \log \left(c \left(a+bx^{2}\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(1/(x**3*log(c*(a + b*x**2)**p)**2), x)
```

3.113 $\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(x^2/ln(c*(b*x^2+a)^p)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/Log[c*(a + b*x^2)^p]^2,x]
[Out] Defer[Int][x^2/Log[c*(a + b*x^2)^p]^2, x]
```

Rubi steps

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/Log[c*(a + b*x^2)^p]^2,x]
[Out] Integrate[x^2/Log[c*(a + b*x^2)^p]^2, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
[Out] integral(x^2/log((b*x^2 + a)^p*c)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/log((b*x^2 + a)^p*c)^2, x)
```

maple [A] time = 3.52, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\ln \left(c \left(b\,x^{2}+a\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(x^2/ln(c*(b*x^2+a)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bx^{3}+ax}{2\left(bp^{2}\log\left(bx^{2}+a\right)+bp\log\left(c\right)\right)}+\int \frac{3\,bx^{2}+a}{2\left(bp^{2}\log\left(bx^{2}+a\right)+bp\log\left(c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^3 + a*x)/(b*p^2*log(b*x^2 + a) + b*p*log(c)) + integrate(1/2*(3*b
*x^2 + a)/(b*p^2*log(b*x^2 + a) + b*p*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^{2}}{\ln \left(c \left(b\,x^{2}+a\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] int(x^2/log(c*(a + b*x^2)^p)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\log \left(c \left(a+bx^{2}\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(x**2/log(c*(a + b*x**2)**p)**2, x)
```

3.114 $\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/ln(c*(b*x^2+a)^p)^2,x)
```

Rubi [A] time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(a + b*x^2)^p]^(-2),x]
[Out] Defer[Int][Log[c*(a + b*x^2)^p]^(-2), x]
```

Rubi steps

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^(-2),x]
[Out] Integrate[Log[c*(a + b*x^2)^p]^(-2), x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\log\left(\left(bx^2+a\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^p*c)^(-2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^(-2), x)
```

maple [A] time = 3.49, size = 0, normalized size = 0.00

$$\int \frac{1}{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(1/ln(c*(b*x^2+a)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bx^{2}+a}{2\left(bp^{2}x\log\left(bx^{2}+a\right)+bpx\log(c)\right)}+\int\frac{bx^{2}-a}{2\left(bp^{2}x^{2}\log\left(bx^{2}+a\right)+bpx^{2}\log(c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)/(b*p^2*x*log(b*x^2 + a) + b*p*x*log(c)) + integrate(1/2*(b
*x^2 - a)/(b*p^2*x^2*log(b*x^2 + a) + b*p*x^2*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] int(1/log(c*(a + b*x^2)^p)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log\left(c\left(a+bx^{2}\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**(-2), x)
```

3.115 $\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/ln(c*(b*x^2+a)^p)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Log[c*(a + b*x^2)^p]^2),x]
[Out] Defer[Int][1/(x^2*Log[c*(a + b*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A] time = 1.25, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^2),x]
[Out] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^2), x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{x^2 \log\left(\left(bx^2+a\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
[Out] integral(1/(x^2*log((b*x^2 + a)^p*c)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*log((b*x^2 + a)^p*c)^2), x)
```

maple [A] time = 3.52, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(1/x^2/ln(c*(b*x^2+a)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bx^2+a}{2\left(bp^2x^3\log\left(bx^2+a\right)+bpx^3\log(c)\right)}-\int \frac{bx^2+3\,a}{2\left(bp^2x^4\log\left(bx^2+a\right)+bpx^4\log(c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)/(b*p^2*x^3*log(b*x^2 + a) + b*p*x^3*log(c)) - integrate(1/
2*(b*x^2 + 3*a)/(b*p^2*x^4*log(b*x^2 + a) + b*p*x^4*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*log(c*(a + b*x^2)^p)^2),x)
```

```
[Out] int(1/(x^2*log(c*(a + b*x^2)^p)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(1/(x**2*log(c*(a + b*x**2)**p)**2), x)
```

3.116 $\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=204

$$\frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{b^2p^3}-\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{4b^2p^3}-\frac{a\left(a+\right.}{4b^2p^2\log\left(c\right.}$$

```
[Out] -1/4*a*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b^2/p^3/((c*(b*x^2+a)^p)^(1/p))+(b
*x^2+a)^2*Ei(2*ln(c*(b*x^2+a)^p)/p)/b^2/p^3/((c*(b*x^2+a)^p)^(2/p))-1/4*x^2
*(b*x^2+a)/b/p/ln(c*(b*x^2+a)^p)^2-1/4*a*(b*x^2+a)/b^2/p^2/ln(c*(b*x^2+a)^p
)-1/2*x^2*(b*x^2+a)/b/p^2/ln(c*(b*x^2+a)^p)
```

Rubi [A] time = 0.29, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2454, 2400, 2399, 2389, 2300, 2178, 2390, 2310, 2297}

$$\frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{b^2p^3}-\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{4b^2p^3}-\frac{a\left(a+\right.}{4b^2p^2\log\left(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -(a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(4*b^2*p^3*(c*(a + b
*x^2)^p)^p^(-1)) + ((a + b*x^2)^2*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p]
)/(b^2*p^3*(c*(a + b*x^2)^p)^(2/p)) - (x^2*(a + b*x^2))/(4*b*p*Log[c*(a + b
*x^2)^p]^2) - (a*(a + b*x^2))/(4*b^2*p^2*Log[c*(a + b*x^2)^p]) - (x^2*(a +
b*x^2))/(2*b*p^2*Log[c*(a + b*x^2)^p])
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2400

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((d + e*x)*(f + g*x)^q*(a + b*Log[c*(d + e
*x)^n])^(p + 1))/(b*e*n*(p + 1)), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[(q*(e*f - d*g))
/(b*e*n*(p + 1)), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{\log^3\left(c(a+bx)^p\right)}\,dx, x, x^2\right)$$

$$= -\frac{x^2\left(a+bx^2\right)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{x}{\log^2(c(a+bx)^p)}\,dx, x, x^2\right)}{2p} + \frac{a\,\text{Subst}\left(\int \frac{1}{\log^2(c(a+b}\right.}{4bp}$$

$$= -\frac{x^2\left(a+bx^2\right)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2\left(a+bx^2\right)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{x}{\log(c(a+bx)^p)}\,dx, x, x\right.}{p^2}$$

$$= -\frac{x^2\left(a+bx^2\right)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a\left(a+bx^2\right)}{4b^2p^2\log\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2\left(a+bx^2\right)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)} +$$

$$= -\frac{x^2\left(a+bx^2\right)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a\left(a+bx^2\right)}{4b^2p^2\log\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2\left(a+bx^2\right)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)} +$$

$$= \frac{3a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{4b^2p^3} - \frac{x^2\left(a+bx^2\right)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{}{4b^2p}$$

$$= \frac{3a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{4b^2p^3} - \frac{x^2\left(a+bx^2\right)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{}{4b^2p}$$

$$= -\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{4b^2p^3} + \frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\text{E}}{b^2p^3}$$

Mathematica [A] time = 0.21, size = 185, normalized size = 0.91

$$-\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-2/p}\left(a\left(c\left(a+bx^2\right)^p\right)^{\frac{1}{p}}\log^2\left(c\left(a+bx^2\right)^p\right)\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right) - 4\left(a+bx^2\right)\log^2\left(c\left(a+\right.\right.\right.}{4b^2p^3\log^2\left(c\left(a+bx^2\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -1/4*((a + b*x^2)*(a*(c*(a + b*x^2)^p)^p^(-1)*ExpIntegralEi[Log[c*(a + b*x^
2)^p]/p]*Log[c*(a + b*x^2)^p]^2 - 4*(a + b*x^2)*ExpIntegralEi[(2*Log[c*(a +
 b*x^2)^p])/p]*Log[c*(a + b*x^2)^p]^2 + p*(c*(a + b*x^2)^p)^(2/p)*(b*p*x^2
+ (a + 2*b*x^2)*Log[c*(a + b*x^2)^p])))/(b^2*p^3*(c*(a + b*x^2)^p)^(2/p)*Lo
g[c*(a + b*x^2)^p]^2)
```

fricas [A] time = 0.44, size = 270, normalized size = 1.32

$$-\frac{\left(ap^2\log\left(bx^2+a\right)^2 + 2\,ap\log\left(bx^2+a\right)\log(c) + a\log(c)^2\right)c^{\left(\frac{1}{p}\right)}\text{log\_integral}\left(\left(bx^2+a\right)c^{\left(\frac{1}{p}\right)}\right) + \left(b^2p^2x^4 + ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((a*p^2*log(b*x^2 + a)^2 + 2*a*p*log(b*x^2 + a)*log(c) + a*log(c)^2)*c
^(1/p)*log_integral((b*x^2 + a)*c^(1/p)) + (b^2*p^2*x^4 + a*b*p^2*x^2 + (2*
b^2*p^2*x^4 + 3*a*b*p^2*x^2 + a^2*p^2)*log(b*x^2 + a) + (2*b^2*p*x^4 + 3*a*
b*p*x^2 + a^2*p)*log(c))*c^(2/p) - 4*(p^2*log(b*x^2 + a)^2 + 2*p*log(b*x^2
+ a)*log(c) + log(c)^2)*log_integral((b^2*x^4 + 2*a*b*x^2 + a^2)*c^(2/p)))/
((b^2*p^5*log(b*x^2 + a)^2 + 2*b^2*p^4*log(b*x^2 + a)*log(c) + b^2*p^3*log(
c)^2)*c^(2/p))
```

giac [B] time = 0.24, size = 840, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*(b*x^2 + a)^2*p^2*log(b*x^2 + a)/(b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*
log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) - (b*x^2 + a)*a*p^2*log(b*x^2 + a)/
(b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) +
 a*p^2*Ei(log(c)/p + log(b*x^2 + a))*log(b*x^2 + a)^2/((b*p^5*log(b*x^2 + a
)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p)) + (b*x^2 + a
)^2*p^2/(b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log
(c)^2) - (b*x^2 + a)*a*p^2/(b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)
*log(c) + b*p^3*log(c)^2) - 4*p^2*Ei(2*log(c)/p + 2*log(b*x^2 + a))*log(b*x
^2 + a)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*
log(c)^2)*c^(2/p)) + 2*(b*x^2 + a)^2*p*log(c)/(b*p^5*log(b*x^2 + a)^2 + 2*b
*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) - (b*x^2 + a)*a*p*log(c)/(b*p^
5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) + 2*a*
p*Ei(log(c)/p + log(b*x^2 + a))*log(b*x^2 + a)*log(c)/((b*p^5*log(b*x^2 + a
)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p)) - 8*p*Ei(2*l
og(c)/p + 2*log(b*x^2 + a))*log(b*x^2 + a)*log(c)/((b*p^5*log(b*x^2 + a)^2
+ 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(2/p)) + a*Ei(log(c)/p
+ log(b*x^2 + a))*log(c)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a
)*log(c) + b*p^3*log(c)^2)*c^(1/p)) - 4*Ei(2*log(c)/p + 2*log(b*x^2 + a))*l
og(c)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*lo
g(c)^2)*c^(2/p)))/b
```

maple [F] time = 6.59, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(x^3/ln(c*(b*x^2+a)^p)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^{2}\left(p+2\,\log(c)\right)x^{4}+ab\left(p+3\,\log(c)\right)x^{2}+a^{2}\log(c)+\left(2\,b^{2}px^{4}+3\,abpx^{2}+a^{2}p\right)\log\left(bx^{2}+a\right)}{4\left(b^{2}p^{4}\log\left(bx^{2}+a\right)^{2}+2\,b^{2}p^{3}\log\left(bx^{2}+a\right)\log(c)+b^{2}p^{2}\log(c)^{2}\right)}+\int\frac{4\cdots}{2\left(bp^{3}\log\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(p + 2*log(c))*x^4 + a*b*(p + 3*log(c))*x^2 + a^2*log(c) + (2*b^2
*p*x^4 + 3*a*b*p*x^2 + a^2*p)*log(b*x^2 + a))/(b^2*p^4*log(b*x^2 + a)^2 + 2
*b^2*p^3*log(b*x^2 + a)*log(c) + b^2*p^2*log(c)^2) + integrate(1/2*(4*b*x^3
 + 3*a*x)/(b*p^3*log(b*x^2 + a) + b*p^2*log(c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] int(x^3/log(c*(a + b*x^2)^p)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(x**3/log(c*(a + b*x**2)**p)**3, x)
```

3.117 $\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal. Leaf size=114

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{4bp^3}-\frac{a+bx^2}{4bp^2\log\left(c\left(a+bx^2\right)^p\right)}-\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)}$$

```
[Out] 1/4*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b/p^3/((c*(b*x^2+a)^p)^(1/p))+1/4*(-b
*x^2-a)/b/p/ln(c*(b*x^2+a)^p)^2+1/4*(-b*x^2-a)/b/p^2/ln(c*(b*x^2+a)^p)
```

Rubi [A] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2454, 2389, 2297, 2300, 2178}

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{4bp^3}-\frac{a+bx^2}{4bp^2\log\left(c\left(a+bx^2\right)^p\right)}-\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(4*b*p^3*(c*(a + b*x^2)
^p)^p^(-1)) - (a + b*x^2)/(4*b*p*Log[c*(a + b*x^2)^p]^2) - (a + b*x^2)/(4*b
*p^2*Log[c*(a + b*x^2)^p])
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\log^3\left(c(a+bx)^p\right)}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\log^3(cx^p)}\,dx, x, a+bx^2\right)}{2b}$$
$$= -\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx^p)}\,dx, x, a+bx^2\right)}{4bp}$$
$$= -\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a+bx^2}{4bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, a+b\right.}{4bp^2}$$
$$= -\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a+bx^2}{4bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\left(\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/}\right.}{}$$
$$= \frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{4bp^3} - \frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{}{4bp^2\log}$$

Mathematica [A] time = 0.06, size = 113, normalized size = 0.99

$$-\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\left(p\left(c\left(a+bx^2\right)^p\right)^{\frac{1}{p}}\left(\log\left(c\left(a+bx^2\right)^p\right)+p\right)-\log^2\left(c\left(a+bx^2\right)^p\right)\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right.\right.}{4bp^3\log^2\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -1/4*((a + b*x^2)*(-(ExpIntegralEi[Log[c*(a + b*x^2)^p]/p]*Log[c*(a + b*x^2
)^p]^2) + p*(c*(a + b*x^2)^p)^p^(-1)*(p + Log[c*(a + b*x^2)^p])))/(b*p^3*(c
*(a + b*x^2)^p)^p^(-1)*Log[c*(a + b*x^2)^p]^2)
```

fricas [A] time = 0.45, size = 157, normalized size = 1.38

$$-\frac{\left(bp^2x^2+ap^2+\left(bp^2x^2+ap^2\right)\log\left(bx^2+a\right)+\left(bpx^2+ap\right)\log(c)\right)c^{\left(\frac{1}{p}\right)}-\left(p^2\log\left(bx^2+a\right)^2+2\,p\log\left(bx^2+a\right.\right.}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log(c)+bp^3\log(c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((b*p^2*x^2 + a*p^2 + (b*p^2*x^2 + a*p^2)*log(b*x^2 + a) + (b*p*x^2 +
a*p)*log(c))*c^(1/p) - (p^2*log(b*x^2 + a)^2 + 2*p*log(b*x^2 + a)*log(c) +
log(c)^2)*log_integral((b*x^2 + a)*c^(1/p)))/((b*p^5*log(b*x^2 + a)^2 + 2*b
*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p))
```

giac [B] time = 0.18, size = 406, normalized size = 3.56

$$-\frac{(bx^2+a)p^2\log(bx^2+a)}{4\left(bp^5\log(bx^2+a)^2+2\,bp^4\log(bx^2+a)\log(c)+bp^3\log(c)^2\right)}+\frac{p^2\mathrm{Ei}\left(\frac{\log(c)}{p}+\log(bx^2+a)\right)\log}{4\left(bp^5\log(bx^2+a)^2+2\,bp^4\log(bx^2+a)\log\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] -1/4*(b*x^2 + a)*p^2*log(b*x^2 + a)/(b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b
*x^2 + a)*log(c) + b*p^3*log(c)^2) + 1/4*p^2*Ei(log(c)/p + log(b*x^2 + a))*
log(b*x^2 + a)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) +
 b*p^3*log(c)^2)*c^(1/p)) - 1/4*(b*x^2 + a)*p^2/(b*p^5*log(b*x^2 + a)^2 + 2
*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) - 1/4*(b*x^2 + a)*p*log(c)/(
b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) +
1/2*p*Ei(log(c)/p + log(b*x^2 + a))*log(b*x^2 + a)*log(c)/((b*p^5*log(b*x^2
 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p)) + 1/4*Ei
(log(c)/p + log(b*x^2 + a))*log(c)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log
(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p))
```

maple [C] time = 1.22, size = 716, normalized size = 6.28

$$-\frac{(b\,x^2+a)\,c^{-\frac{1}{p}}\left((b\,x^2+a)^p\right)^{-\frac{1}{p}}\,\mathrm{Ei}\left(1,-\ln(b\,x^2+a)-\frac{-i\pi\,\mathrm{csgn}(ic)\mathrm{csgn}\left(i(b\,x^2+a)^p\right)\mathrm{csgn}\left(ic(b\,x^2+a)^p\right)+i\pi\,\mathrm{csgn}(ic)\mathrm{csgn}\left(ic(b\,x^2+a)^p\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] -1/2*(I*Pi*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*b*x^2*csg
n(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*b*x^2*csgn(I*c*(b*x^2
+a)^p)^3+I*Pi*b*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+I*Pi*a*csgn(I*(b*x^2+
a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)*csgn(I*c)-I*Pi*a*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*a*csgn(I*c*(b*x^2+a)^p)^2
*csgn(I*c)+2*b*x^2*ln(c)+2*b*x^2*ln((b*x^2+a)^p)+2*a*ln(c)+2*a*ln((b*x^2+a)
^p)+2*b*p*x^2+2*a*p)/p^2/(-I*Pi*csgn(I*c)*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x
^2+a)^p)+I*Pi*csgn(I*c)*csgn(I*c*(b*x^2+a)^p)^2+I*Pi*csgn(I*(b*x^2+a)^p)*cs
gn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+2*ln(c)+2*ln((b*x^2+a)^p
))^2/b-1/4/p^3/b*(b*x^2+a)*((b*x^2+a)^p)^(-1/p)*c^(-1/p)*exp(1/2*I*Pi*(csgn
(I*c)-csgn(I*c*(b*x^2+a)^p))*(csgn(I*(b*x^2+a)^p)-csgn(I*c*(b*x^2+a)^p))/p*
csgn(I*c*(b*x^2+a)^p))*Ei(1,-ln(b*x^2+a)-1/2*(-I*Pi*csgn(I*c)*csgn(I*(b*x^2
+a)^p)*csgn(I*c*(b*x^2+a)^p)+I*Pi*csgn(I*c)*csgn(I*c*(b*x^2+a)^p)^2+I*Pi*cs
gn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*c*(b*x^2+a)^p)^3-2*p*
ln(b*x^2+a)+2*ln(c)+2*ln((b*x^2+a)^p))/p)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b(p+\log(c))x^2+a(p+\log(c))+(bpx^2+ap)\log(bx^2+a)}{4\left(bp^4\log(bx^2+a)^2+2\,bp^3\log(bx^2+a)\log(c)+bp^2\log(c)^2\right)}+\int\frac{x}{2\left(p^3\log(bx^2+a)+p^2\log(c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*(p + log(c))*x^2 + a*(p + log(c)) + (b*p*x^2 + a*p)*log(b*x^2 + a))
/(b*p^4*log(b*x^2 + a)^2 + 2*b*p^3*log(b*x^2 + a)*log(c) + b*p^2*log(c)^2)
+ integrate(1/2*x/(p^3*log(b*x^2 + a) + p^2*log(c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] int(x/log(c*(a + b*x^2)^p)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(x/log(c*(a + b*x**2)**p)**3, x)
```

3.118 $\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)},x\right)$$

[Out] Unintegrable(1/x/ln(c*(b*x^2+a)^p)^3,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/(x*Log[c*(a + b*x^2)^p]^3),x]

[Out] Defer[Int][1/(x*Log[c*(a + b*x^2)^p]^3), x]

Rubi steps

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x*Log[c*(a + b*x^2)^p]^3),x]

[Out] Integrate[1/(x*Log[c*(a + b*x^2)^p]^3), x]

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x\log\left(\left(bx^2+a\right)^pc\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")

[Out] integral(1/(x*log((b*x^2 + a)^p*c)^3), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\log\left(\left(bx^2+a\right)^pc\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x*log((b*x^2 + a)^p*c)^3), x)
```

maple [A] time = 3.74, size = 0, normalized size = 0.00

$$\int \frac{1}{x \ln \left(c \left(b\,x^{2}+a\right)^{p}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(1/x/ln(c*(b*x^2+a)^p)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2px^4 + ab\left(p - \log(c)\right)x^2 - a^2\log(c) - \left(abpx^2 + a^2p\right)\log\left(bx^2+a\right)}{4\left(b^2p^4x^4\log\left(bx^2+a\right)^2 + 2\,b^2p^3x^4\log\left(bx^2+a\right)\log(c) + b^2p^2x^4\log(c)^2\right)} + \int \frac{abx^2 + 2\,a^2}{2\left(b^2p^3x^5\log\left(bx^2+a\right) + b^2p\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*p*x^4 + a*b*(p - log(c))*x^2 - a^2*log(c) - (a*b*p*x^2 + a^2*p)*l
og(b*x^2 + a))/(b^2*p^4*x^4*log(b*x^2 + a)^2 + 2*b^2*p^3*x^4*log(b*x^2 + a)
*log(c) + b^2*p^2*x^4*log(c)^2) + integrate(1/2*(a*b*x^2 + 2*a^2)/(b^2*p^3*
x^5*log(b*x^2 + a) + b^2*p^2*x^5*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x \ln \left(c \left(b\,x^{2}+a\right)^{p}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(c*(a + b*x^2)^p)^3),x)
```

```
[Out] int(1/(x*log(c*(a + b*x^2)^p)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \log \left(c \left(a + bx^{2}\right)^{p}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(1/(x*log(c*(a + b*x**2)**p)**3), x)
```

3.119 $\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^3/ln(c*(b*x^2+a)^p)^3,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*Log[c*(a + b*x^2)^p]^3),x]
[Out] Defer[Int][1/(x^3*Log[c*(a + b*x^2)^p]^3), x]
```

Rubi steps

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A] time = 3.20, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^3),x]
[Out] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^3), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{x^3 \log\left(\left(bx^2+a\right)^p c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
[Out] integral(1/(x^3*log((b*x^2 + a)^p*c)^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*log((b*x^2 + a)^p*c)^3), x)
```

maple [A] time = 3.92, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \ln \left(c \left(b\,x^{2}+a\right)^{p}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(1/x^3/ln(c*(b*x^2+a)^p)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^{2}\left(p-\log(c)\right)x^{4}+ab\left(p-3\,\log(c)\right)x^{2}-2\,a^{2}\log(c)-\left(b^{2}px^{4}+3\,abpx^{2}+2\,a^{2}p\right)\log\left(bx^{2}+a\right)}{4\left(b^{2}p^{4}x^{6}\log\left(bx^{2}+a\right)^{2}+2\,b^{2}p^{3}x^{6}\log\left(bx^{2}+a\right)\log(c)+b^{2}p^{2}x^{6}\log(c)^{2}\right)}+\int\frac{}{2\left(b^{2}p^{3}x^{7}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(p - log(c))*x^4 + a*b*(p - 3*log(c))*x^2 - 2*a^2*log(c) - (b^2*p
*x^4 + 3*a*b*p*x^2 + 2*a^2*p)*log(b*x^2 + a))/(b^2*p^4*x^6*log(b*x^2 + a)^2
 + 2*b^2*p^3*x^6*log(b*x^2 + a)*log(c) + b^2*p^2*x^6*log(c)^2) + integrate(
1/2*(b^2*x^4 + 6*a*b*x^2 + 6*a^2)/(b^2*p^3*x^7*log(b*x^2 + a) + b^2*p^2*x^7
*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^{3} \ln \left(c \left(b\,x^{2}+a\right)^{p}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*log(c*(a + b*x^2)^p)^3),x)
```

```
[Out] int(1/(x^3*log(c*(a + b*x^2)^p)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \log \left(c \left(a+bx^{2}\right)^{p}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(1/(x**3*log(c*(a + b*x**2)**p)**3), x)
```

3.120 $\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(x^2/ln(c*(b*x^2+a)^p)^3,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/Log[c*(a + b*x^2)^p]^3,x]
[Out] Defer[Int][x^2/Log[c*(a + b*x^2)^p]^3, x]
```

Rubi steps

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Mathematica [A] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/Log[c*(a + b*x^2)^p]^3,x]
[Out] Integrate[x^2/Log[c*(a + b*x^2)^p]^3, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
[Out] integral(x^2/log((b*x^2 + a)^p*c)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
[Out] integrate(x^2/log((b*x^2 + a)^p*c)^3, x)
```

maple [A] time = 3.65, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/ln(c*(b*x^2+a)^p)^3,x)
[Out] int(x^2/ln(c*(b*x^2+a)^p)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2\left(2\,p+3\,\log(c)\right)x^4+2\,ab\left(p+2\,\log(c)\right)x^2+a^2\log(c)+\left(3\,b^2px^4+4\,abpx^2+a^2p\right)\log\left(bx^2+a\right)}{8\left(b^2p^4x\log\left(bx^2+a\right)^2+2\,b^2p^3x\log\left(bx^2+a\right)\log(c)+b^2p^2x\log(c)^2\right)}+\int \frac{}{8\left(b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
[Out] -1/8*(b^2*(2*p + 3*log(c))*x^4 + 2*a*b*(p + 2*log(c))*x^2 + a^2*log(c) + (3
*b^2*p*x^4 + 4*a*b*p*x^2 + a^2*p)*log(b*x^2 + a))/(b^2*p^4*x*log(b*x^2 + a)
^2 + 2*b^2*p^3*x*log(b*x^2 + a)*log(c) + b^2*p^2*x*log(c)^2) + integrate(1/
8*(9*b^2*x^4 + 4*a*b*x^2 - a^2)/(b^2*p^3*x^2*log(b*x^2 + a) + b^2*p^2*x^2*l
og(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/log(c*(a + b*x^2)^p)^3,x)
[Out] int(x^2/log(c*(a + b*x^2)^p)^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/ln(c*(b*x**2+a)**p)**3,x)
[Out] Integral(x**2/log(c*(a + b*x**2)**p)**3, x)
```

3.121 $\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/ln(c*(b*x^2+a)^p)^3,x)
```

Rubi [A] time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(a + b*x^2)^p]^(-3),x]
```

```
[Out] Defer[Int][Log[c*(a + b*x^2)^p]^(-3), x]
```

Rubi steps

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx = \int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Mathematica [A] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^(-3),x]
```

```
[Out] Integrate[Log[c*(a + b*x^2)^p]^(-3), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\log\left(\left(bx^2+a\right)^p c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^(-3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log\left(\left(bx^2+a\right)^p c\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)^(-3), x)
```

maple [A] time = 3.73, size = 0, normalized size = 0.00

$$\int \frac{1}{\ln \left(c \left(b\,x^{2}+a\right)^{p}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(c*(b*x^2+a)^p)^3,x)
[Out] int(1/ln(c*(b*x^2+a)^p)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^{2}\left(2\,p+\log(c)\right)x^{4}+2\,abpx^{2}-a^{2}\log(c)+\left(b^{2}px^{4}-a^{2}p\right)\log\left(bx^{2}+a\right)}{8\left(b^{2}p^{4}x^{3}\log\left(bx^{2}+a\right)^{2}+2\,b^{2}p^{3}x^{3}\log\left(bx^{2}+a\right)\log(c)+b^{2}p^{2}x^{3}\log(c)^{2}\right)}+\int\frac{b^{2}x^{4}+3\,a^{2}}{8\left(b^{2}p^{3}x^{4}\log\left(bx^{2}+a\right)+b^{2}p\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
[Out] -1/8*(b^2*(2*p + log(c))*x^4 + 2*a*b*p*x^2 - a^2*log(c) + (b^2*p*x^4 - a^2*
p)*log(b*x^2 + a))/(b^2*p^4*x^3*log(b*x^2 + a)^2 + 2*b^2*p^3*x^3*log(b*x^2
+ a)*log(c) + b^2*p^2*x^3*log(c)^2) + integrate(1/8*(b^2*x^4 + 3*a^2)/(b^2*
p^3*x^4*log(b*x^2 + a) + b^2*p^2*x^4*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\ln \left(c \left(b\,x^{2}+a\right)^{p}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(c*(a + b*x^2)^p)^3,x)
[Out] int(1/log(c*(a + b*x^2)^p)^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log \left(c \left(a+bx^{2}\right)^{p}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(c*(b*x**2+a)**p)**3,x)
[Out] Integral(log(c*(a + b*x**2)**p)**(-3), x)
```

3.122 $\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/ln(c*(b*x^2+a)^p)^3,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Log[c*(a + b*x^2)^p]^3),x]
```

```
[Out] Defer[Int][1/(x^2*Log[c*(a + b*x^2)^p]^3), x]
```

Rubi steps

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A] time = 2.26, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^3),x]
```

```
[Out] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^3), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{x^2 \log\left(\left(bx^2+a\right)^p c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*log((b*x^2 + a)^p*c)^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*log((b*x^2 + a)^p*c)^3), x)
```

maple [A] time = 4.75, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \ln \left(c \left(b\,x^{2}+a\right)^{p}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(1/x^2/ln(c*(b*x^2+a)^p)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^{2}\left(2\,p-\log(c)\right)x^{4}+2\,ab\left(p-2\,\log(c)\right)x^{2}-3\,a^{2}\log(c)-\left(b^{2}px^{4}+4\,abpx^{2}+3\,a^{2}p\right)\log\left(bx^{2}+a\right)}{8\left(b^{2}p^{4}x^{5}\log\left(bx^{2}+a\right)^{2}+2\,b^{2}p^{3}x^{5}\log\left(bx^{2}+a\right)\log(c)+b^{2}p^{2}x^{5}\log(c)^{2}\right)}+\int \frac{}{8\left(b^{2}p^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(b^2*(2*p - log(c))*x^4 + 2*a*b*(p - 2*log(c))*x^2 - 3*a^2*log(c) - (b
^2*p*x^4 + 4*a*b*p*x^2 + 3*a^2*p)*log(b*x^2 + a))/(b^2*p^4*x^5*log(b*x^2 +
a)^2 + 2*b^2*p^3*x^5*log(b*x^2 + a)*log(c) + b^2*p^2*x^5*log(c)^2) + integr
ate(1/8*(b^2*x^4 + 12*a*b*x^2 + 15*a^2)/(b^2*p^3*x^6*log(b*x^2 + a) + b^2*p
^2*x^6*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^{2} \ln \left(c \left(b\,x^{2}+a\right)^{p}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*log(c*(a + b*x^2)^p)^3),x)
```

```
[Out] int(1/(x^2*log(c*(a + b*x^2)^p)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \log \left(c \left(a+bx^{2}\right)^{p}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(1/(x**2*log(c*(a + b*x**2)**p)**3), x)
```

3.123 $\int \frac{x^3}{\log\left(c\left(a+bx^2\right)\right)}\,dx$

Optimal. Leaf size=45

$$\frac{\text{Ei}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{2b^2c^2}-\frac{a\text{li}\left(c\left(bx^2+a\right)\right)}{2b^2c}$$

```
[Out] 1/2*Ei(2*ln(c*(b*x^2+a)))/b^2/c^2-1/2*a*Li(c*(b*x^2+a))/b^2/c
```

Rubi [A] time = 0.10, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2454, 2399, 2389, 2298, 2390, 2309, 2178}

$$\frac{\text{Ei}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{2b^2c^2}-\frac{a\text{li}\left(c\left(bx^2+a\right)\right)}{2b^2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Log[c*(a + b*x^2)],x]
```

```
[Out] ExpIntegralEi[2*Log[c*(a + b*x^2)]]/(2*b^2*c^2) - (a*LogIntegral[c*(a + b*x
^2)])/(2*b^2*c)
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\log\left(c\left(a+bx^2\right)\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\log(c(a+bx))}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{b\log(c(a+bx))}+\frac{a+bx}{b\log(c(a+bx))}\right)dx, x, x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{a+bx}{\log(c(a+bx))}\,dx, x, x^2\right)}{2b} - \frac{a\operatorname{Subst}\left(\int \frac{1}{\log(c(a+bx))}\,dx, x, x^2\right)}{2b}\\
&= \frac{\operatorname{Subst}\left(\int \frac{x}{\log(cx)}\,dx, x, a+bx^2\right)}{2b^2} - \frac{a\operatorname{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{2b^2}\\
&= -\frac{a\operatorname{li}\left(c\left(a+bx^2\right)\right)}{2b^2c} + \frac{\operatorname{Subst}\left(\int \frac{e^{2x}}{x}\,dx, x, \log\left(c\left(a+bx^2\right)\right)\right)}{2b^2c^2}\\
&= \frac{\operatorname{Ei}\left(2\log\left(c\left(a+bx^2\right)\right)\right)}{2b^2c^2} - \frac{a\operatorname{li}\left(c\left(a+bx^2\right)\right)}{2b^2c}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 41, normalized size = 0.91

$$\frac{\operatorname{Ei}\left(2\log\left(bcx^2+ac\right)\right) - ac\operatorname{Ei}\left(\log\left(bcx^2+ac\right)\right)}{2b^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Log[c*(a + b*x^2)],x]
```

```
[Out] (-(a*c*ExpIntegralEi[Log[a*c + b*c*x^2]]) + ExpIntegralEi[2*Log[a*c + b*c*x
^2]])/(2*b^2*c^2)
```

fricas [A] time = 0.43, size = 54, normalized size = 1.20

$$-\frac{ac\operatorname{log\_integral}\left(bcx^2+ac\right) - \operatorname{log\_integral}\left(b^2c^2x^4+2\,abc^2x^2+a^2c^2\right)}{2\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*c*log_integral(b*c*x^2 + a*c) - log_integral(b^2*c^2*x^4 + 2*a*b*c^
2*x^2 + a^2*c^2))/(b^2*c^2)
```

giac [A] time = 0.16, size = 38, normalized size = 0.84

$$-\frac{ac\operatorname{Ei}\left(\log\left(\left(bx^2+a\right)c\right)\right) - \operatorname{Ei}\left(2\,\log\left(\left(bx^2+a\right)c\right)\right)}{2\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)),x, algorithm="giac")
```

```
[Out] -1/2*(a*c*Ei(log((b*x^2 + a)*c)) - Ei(2*log((b*x^2 + a)*c)))/(b^2*c^2)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\ln\left(\left(b\,x^2+a\right)c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(b*x^2+a)),x)
```

```
[Out] int(x^3/ln(c*(b*x^2+a)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log\left(\left(bx^2+a\right)c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/log((b*x^2 + a)*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\ln\left(c\,\left(b\,x^2+a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/log(c*(a + b*x^2)),x)
```

```
[Out] int(x^3/log(c*(a + b*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log\left(ac+bcx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(b*x**2+a)),x)
```

```
[Out] Integral(x**3/log(a*c + b*c*x**2), x)
```

3.124 $\int \frac{x}{\log(c(a+bx^2))} dx$

Optimal. Leaf size=20

$$\frac{\text{li}\left(c\left(bx^2+a\right)\right)}{2bc}$$

```
[Out] 1/2*Li(c*(b*x^2+a))/b/c
```

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2454, 2389, 2298}

$$\frac{\text{li}\left(c\left(bx^2+a\right)\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[x/Log[c*(a + b*x^2)],x]
```

```
[Out] LogIntegral[c*(a + b*x^2)]/(2*b*c)
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\log\left(c\left(a+bx^2\right)\right)} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\log(c(a+bx))} dx, x, x^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{\log(cx)} dx, x, a+bx^2\right)}{2b} \\
&= \frac{\text{li}\left(c\left(a+bx^2\right)\right)}{2bc}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$\frac{\text{li}\left(c\left(bx^2+a\right)\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Log[c*(a + b*x^2)],x]
[Out] LogIntegral[c*(a + b*x^2)]/(2*b*c)
```

fricas [A] time = 0.41, size = 19, normalized size = 0.95

$$\frac{\log\_integral\left(bcx^2 + ac\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)),x, algorithm="fricas")
[Out] 1/2*log_integral(b*c*x^2 + a*c)/(b*c)
```

giac [A] time = 0.16, size = 19, normalized size = 0.95

$$\frac{\mathrm{Ei}\left(\log\left(\left(bx^2 + a\right)c\right)\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)),x, algorithm="giac")
[Out] 1/2*Ei(log((b*x^2 + a)*c))/(b*c)
```

maple [A] time = 0.05, size = 23, normalized size = 1.15

$$-\frac{\mathrm{Ei}\left(1, -\ln\left(\left(b\,x^2 + a\right)c\right)\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln((b*x^2+a)*c),x)
[Out] -1/2/b/c*Ei(1,-ln((b*x^2+a)*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\log\left(\left(bx^2 + a\right)c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)),x, algorithm="maxima")
[Out] integrate(x/log((b*x^2 + a)*c), x)
```

mupad [B] time = 0.35, size = 18, normalized size = 0.90

$$\frac{\mathrm{logint}\left(c\,\left(b\,x^2 + a\right)\right)}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/log(c*(a + b*x^2)),x)
[Out] logint(c*(a + b*x^2))/(2*b*c)
```

sympy [A] time = 2.16, size = 27, normalized size = 1.35

$$\begin{cases} \frac{x^2}{2\log\left(ac\right)} & \text{for}\ b = 0 \\ 0 & \text{for}\ c = 0 \\ \frac{\mathrm{Ei}\left(\log\left(ac + bcx^2\right)\right)}{2bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(b*x**2+a)),x)
```

```
[Out] Piecewise((x**2/(2*log(a*c)), Eq(b, 0)), (0, Eq(c, 0)), (Ei(log(a*c + b*c*x
**2))/(2*b*c), True))
```

3.125 $\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)\right)} dx$

Optimal. Leaf size=71

$$\frac{\operatorname{Ei}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{b^2c^2}-\frac{a\operatorname{li}\left(c\left(bx^2+a\right)\right)}{2b^2c}-\frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)}$$

```
[Out] Ei(2*ln(c*(b*x^2+a)))/b^2/c^2-1/2*a*Li(c*(b*x^2+a))/b^2/c-1/2*x^2*(b*x^2+a)
/b/ln(c*(b*x^2+a))
```

Rubi [A] time = 0.13, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2454, 2400, 2399, 2389, 2298, 2390, 2309, 2178}

$$\frac{\operatorname{Ei}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{b^2c^2}-\frac{a\operatorname{li}\left(c\left(bx^2+a\right)\right)}{2b^2c}-\frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Log[c*(a + b*x^2)]^2,x]
```

```
[Out] ExpIntegralEi[2*Log[c*(a + b*x^2)]]/(b^2*c^2) - (x^2*(a + b*x^2))/(2*b*Log[
c*(a + b*x^2)]) - (a*LogIntegral[c*(a + b*x^2)])/(2*b^2*c)
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
```

```
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2400

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((d + e*x)*(f + g*x)^q*(a + b*Log[c*(d + e
*x)^n])^(p + 1))/(b*e*n*(p + 1)), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[(q*(e*f - d*g))
/(b*e*n*(p + 1)), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\log^2(c(a+bx))}\,dx, x, x^2\right)$$

$$= -\frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\,\text{Subst}\left(\int \frac{1}{\log(c(a+bx))}\,dx, x, x^2\right)}{2b} + \text{Subst}\left(\int \frac{x}{\log(c(a+bx)}\right.$$

$$= -\frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\,\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{2b^2} + \text{Subst}\left(\int \left(-\frac{a}{b\log(c(a}\right.\right.$$

$$= -\frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\text{li}\left(c\left(a+bx^2\right)\right)}{2b^2c} + \frac{\text{Subst}\left(\int \frac{a+bx}{\log(c(a+bx))}\,dx, x, x^2\right)}{b} - \frac{a\,\text{Subs}}{}$$

$$= -\frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\text{li}\left(c\left(a+bx^2\right)\right)}{2b^2c} + \frac{\text{Subst}\left(\int \frac{x}{\log(cx)}\,dx, x, a+bx^2\right)}{b^2} - \frac{a\,\text{Sub}}{}$$

$$= -\frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} - \frac{a\text{li}\left(c\left(a+bx^2\right)\right)}{2b^2c} + \frac{\text{Subst}\left(\int \frac{e^{2x}}{x}\,dx, x, \log\left(c\left(a+bx^2\right)\right)\right)}{b^2c^2}$$

$$= \frac{\text{Ei}\left(2\log\left(c\left(a+bx^2\right)\right)\right)}{b^2c^2} - \frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} - \frac{a\text{li}\left(c\left(a+bx^2\right)\right)}{2b^2c}$$

Mathematica [A] time = 0.12, size = 66, normalized size = 0.93

$$-\frac{-\frac{2\text{Ei}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{c^2} + \frac{a\text{Ei}\left(\log\left(c\left(bx^2+a\right)\right)\right)}{c} + \frac{bx^2\left(a+bx^2\right)}{\log\left(c\left(a+bx^2\right)\right)}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Log[c*(a + b*x^2)]^2,x]
```

```
[Out] -1/2*((a*ExpIntegralEi[Log[c*(a + b*x^2)]])/c - (2*ExpIntegralEi[2*Log[c*(a
 + b*x^2)]])/c^2 + (b*x^2*(a + b*x^2))/Log[c*(a + b*x^2)])/b^2
```

fricas [A] time = 0.43, size = 99, normalized size = 1.39

$$-\frac{b^2c^2x^4 + abc^2x^2 + \left(ac\operatorname{log\_integral}\left(bcx^2 + ac\right) - 2\operatorname{log\_integral}\left(b^2c^2x^4 + 2abc^2x^2 + a^2c^2\right)\right)\log\left(bcx^2 + ac\right)}{2b^2c^2\log\left(bcx^2 + ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2*x^4 + a*b*c^2*x^2 + (a*c*log_integral(b*c*x^2 + a*c) - 2*log_
integral(b^2*c^2*x^4 + 2*a*b*c^2*x^2 + a^2*c^2))*log(b*c*x^2 + a*c))/(b^2*c
^2*log(b*c*x^2 + a*c))
```

giac [A] time = 0.17, size = 89, normalized size = 1.25

$$-\frac{ac\operatorname{Ei}\left(\log\left(\left(bx^2 + a\right)c\right)\right) - \frac{\left(bcx^2+ac\right)ac}{\log\left(\left(bx^2+a\right)c\right)} + \frac{\left(bcx^2+ac\right)^2}{\log\left(\left(bx^2+a\right)c\right)} - 2\operatorname{Ei}\left(2\log\left(\left(bx^2 + a\right)c\right)\right)}{2b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a))^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*c*Ei(log((b*x^2 + a)*c)) - (b*c*x^2 + a*c)*a*c/log((b*x^2 + a)*c) +
 (b*c*x^2 + a*c)^2/log((b*x^2 + a)*c) - 2*Ei(2*log((b*x^2 + a)*c)))/(b^2*c^
2)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\ln\left(\left(bx^2 + a\right)c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln((b*x^2+a)*c)^2,x)
```

```
[Out] int(x^3/ln((b*x^2+a)*c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bx^4 + ax^2}{2\left(b\log\left(bx^2 + a\right) + b\log(c)\right)} + \int \frac{2bx^3 + ax}{b\log\left(bx^2 + a\right) + b\log(c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^4 + a*x^2)/(b*log(b*x^2 + a) + b*log(c)) + integrate((2*b*x^3 + a
*x)/(b*log(b*x^2 + a) + b*log(c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\ln\left(c\left(bx^2 + a\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/log(c*(a + b*x^2))^2,x)
```

```
[Out] int(x^3/log(c*(a + b*x^2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-ax^2 - bx^4}{2b\log\left(c\left(a + bx^2\right)\right)} + \frac{\int \frac{ax}{\log\left(ac+bcx^2\right)}\,dx + \int \frac{2bx^3}{\log\left(ac+bcx^2\right)}\,dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(b*x**2+a))**2,x)
```

```
[Out] (-a*x**2 - b*x**4)/(2*b*log(c*(a + b*x**2))) + (Integral(a*x/log(a*c + b*c*
x**2), x) + Integral(2*b*x**3/log(a*c + b*c*x**2), x))/b
```

3.126 $\int \frac{x}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx$

Optimal. Leaf size=47

$$\frac{\text{li}\left(c\left(bx^2+a\right)\right)}{2bc}-\frac{a+bx^2}{2b\log\left(c\left(a+bx^2\right)\right)}$$

```
[Out] 1/2*Li(c*(b*x^2+a))/b/c+1/2*(-b*x^2-a)/b/ln(c*(b*x^2+a))
```

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2454, 2389, 2297, 2298}

$$\frac{\text{li}\left(c\left(bx^2+a\right)\right)}{2bc}-\frac{a+bx^2}{2b\log\left(c\left(a+bx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Log[c*(a + b*x^2)]^2,x]
```

```
[Out] -(a + b*x^2)/(2*b*Log[c*(a + b*x^2)]) + LogIntegral[c*(a + b*x^2)]/(2*b*c)
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\log^2(c(a+bx))}\,dx, x, x^2\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{\log^2(cx)}\,dx, x, a+bx^2\right)}{2b}$$
$$= -\frac{a+bx^2}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{2b}$$
$$= -\frac{a+bx^2}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{\operatorname{li}\left(c\left(a+bx^2\right)\right)}{2bc}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.91

$$\frac{\frac{\operatorname{li}\left(c\left(bx^2+a\right)\right)}{c} - \frac{a+bx^2}{\log\left(c\left(a+bx^2\right)\right)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Log[c*(a + b*x^2)]^2,x]
[Out] (-((a + b*x^2)/Log[c*(a + b*x^2)]) + LogIntegral[c*(a + b*x^2)]/c)/(2*b)
```

fricas [A] time = 0.41, size = 55, normalized size = 1.17

$$-\frac{bcx^2 + ac - \log\left(bcx^2+ac\right)\operatorname{log\_integral}\left(bcx^2+ac\right)}{2\,bc\log\left(bcx^2+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a))^2,x, algorithm="fricas")
[Out] -1/2*(b*c*x^2 + a*c - log(b*c*x^2 + a*c)*log_integral(b*c*x^2 + a*c))/(b*c*
log(b*c*x^2 + a*c))
```

giac [A] time = 0.16, size = 45, normalized size = 0.96

$$-\frac{\frac{bcx^2+ac}{\log\left(\left(bx^2+a\right)c\right)} - \operatorname{Ei}\left(\log\left(\left(bx^2+a\right)c\right)\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a))^2,x, algorithm="giac")
[Out] -1/2*((b*c*x^2 + a*c)/log((b*x^2 + a)*c) - Ei(log((b*x^2 + a)*c)))/(b*c)
```

maple [A] time = 0.05, size = 59, normalized size = 1.26

$$-\frac{x^2}{2\ln\left(\left(b\,x^2+a\right)c\right)} - \frac{a}{2b\ln\left(\left(b\,x^2+a\right)c\right)} - \frac{\operatorname{Ei}\left(1, -\ln\left(\left(b\,x^2+a\right)c\right)\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln((b*x^2+a)*c)^2,x)
```

```
[Out] -1/2/ln((b*x^2+a)*c)*x^2-1/2/b/ln((b*x^2+a)*c)*a-1/2/b/c*Ei(1,-ln((b*x^2+a)
*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bx^2+a}{2\left(b\log\left(bx^2+a\right)+b\log(c)\right)}+\int\frac{x}{\log\left(bx^2+a\right)+\log(c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)/(b*log(b*x^2 + a) + b*log(c)) + integrate(x/(log(b*x^2 + a
) + log(c)), x)
```

mupad [B] time = 0.36, size = 46, normalized size = 0.98

$$\frac{\mathrm{logint}\left(c\,\left(b\,x^2+a\right)\right)}{2\,b\,c}-\frac{\frac{b\,x^2}{2}+\frac{a}{2}}{b\,\ln\left(c\,\left(b\,x^2+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/log(c*(a + b*x^2))^2,x)
```

```
[Out] logint(c*(a + b*x^2))/(2*b*c) - (a/2 + (b*x^2)/2)/(b*log(c*(a + b*x^2)))
```

sympy [A] time = 2.16, size = 49, normalized size = 1.04

$$\begin{cases}\frac{x^2}{2\log(ac)} & \text{for}\ b=0\\ 0 & \text{for}\ c=0\\ \frac{\mathrm{Ei}\left(\log\left(ac+bcx^2\right)\right)}{2bc} & \text{otherwise}\end{cases}+\frac{-a-bx^2}{2b\log\left(c\left(a+bx^2\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(b*x**2+a))**2,x)
```

```
[Out] Piecewise((x**2/(2*log(a*c)), Eq(b, 0)), (0, Eq(c, 0)), (Ei(log(a*c + b*c*x
**2))/(2*b*c), True)) + (-a - b*x**2)/(2*b*log(c*(a + b*x**2)))
```

3.127 $\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx$

Optimal. Leaf size=127

$$\frac{\text{Ei}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{b^2c^2}-\frac{a\text{li}\left(c\left(bx^2+a\right)\right)}{4b^2c}-\frac{a\left(a+bx^2\right)}{4b^2\log\left(c\left(a+bx^2\right)\right)}-\frac{x^2\left(a+bx^2\right)}{4b\log^2\left(c\left(a+bx^2\right)\right)}-\frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)}$$

```
[Out] Ei(2*ln(c*(b*x^2+a)))/b^2/c^2-1/4*a*Li(c*(b*x^2+a))/b^2/c-1/4*x^2*(b*x^2+a)
/b/ln(c*(b*x^2+a))^2-1/4*a*(b*x^2+a)/b^2/ln(c*(b*x^2+a))-1/2*x^2*(b*x^2+a)/
b/ln(c*(b*x^2+a))
```

Rubi [A] time = 0.17, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {2454, 2400, 2399, 2389, 2298, 2390, 2309, 2178, 2297}

$$\frac{\text{Ei}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{b^2c^2}-\frac{a\text{li}\left(c\left(bx^2+a\right)\right)}{4b^2c}-\frac{a\left(a+bx^2\right)}{4b^2\log\left(c\left(a+bx^2\right)\right)}-\frac{x^2\left(a+bx^2\right)}{4b\log^2\left(c\left(a+bx^2\right)\right)}-\frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Log[c*(a + b*x^2)]^3,x]
```

```
[Out] ExpIntegralEi[2*Log[c*(a + b*x^2)]]/(b^2*c^2) - (x^2*(a + b*x^2))/(4*b*Log[
c*(a + b*x^2)]^2) - (a*(a + b*x^2))/(4*b^2*Log[c*(a + b*x^2)]) - (x^2*(a +
b*x^2))/(2*b*Log[c*(a + b*x^2)]) - (a*LogIntegral[c*(a + b*x^2)])/(4*b^2*c)
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2400

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((d + e*x)*(f + g*x)^q*(a + b*Log[c*(d + e
*x)^n])^(p + 1))/(b*e*n*(p + 1)), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[(q*(e*f - d*g))
/(b*e*n*(p + 1)), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\log^3(c(a+bx))}\,dx, x, x^2\right)$$

$$= -\frac{x^2\left(a+bx^2\right)}{4b\log^2\left(c\left(a+bx^2\right)\right)} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\log^2(c(a+bx))}\,dx, x, x^2\right) + \frac{a\operatorname{Subst}\left(\int \frac{1}{\log^2(c(\ldots}\right)}{4b\ldots}$$

$$= -\frac{x^2\left(a+bx^2\right)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\operatorname{Subst}\left(\int \frac{1}{\log^2(cx)}\,dx, x, a+bx^2\right)}{4b^2} + \ldots$$

$$= -\frac{x^2\left(a+bx^2\right)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a\left(a+bx^2\right)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\operatorname{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{x^2\left(a+bx^2\right)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a\left(a+bx^2\right)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{3a\operatorname{li}\left(c\left(a+\ldots\right.\right.}{4b^2c\ldots}$$

$$= -\frac{x^2\left(a+bx^2\right)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a\left(a+bx^2\right)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{3a\operatorname{li}\left(c\left(a+\ldots\right.\right.}{4b^2c\ldots}$$

$$= -\frac{x^2\left(a+bx^2\right)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a\left(a+bx^2\right)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2\left(a+bx^2\right)}{2b\log\left(c\left(a+bx^2\right)\right)} - \frac{a\operatorname{li}\left(c\left(a+b\ldots\right.\right.}{4b^2c\ldots}$$

$$= \frac{\operatorname{Ei}\left(2\log\left(c\left(a+bx^2\right)\right)\right)}{b^2c^2} - \frac{x^2\left(a+bx^2\right)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a\left(a+bx^2\right)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2\left(a\ldots\right.}{2b\log\left(c\ldots\right.}$$

Mathematica [A] time = 0.13, size = 87, normalized size = 0.69

$$-\frac{-\frac{4\operatorname{Ei}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{c^2} + \frac{a\operatorname{Ei}\left(\log\left(c\left(bx^2+a\right)\right)\right)}{c} + \frac{\left(a+bx^2\right)\left(\left(a+2bx^2\right)\log\left(c\left(a+bx^2\right)\right)+bx^2\right)}{\log^2\left(c\left(a+bx^2\right)\right)}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Log[c*(a + b*x^2)]^3,x]
```

```
[Out] -1/4*((a*ExpIntegralEi[Log[c*(a + b*x^2)]])/c - (4*ExpIntegralEi[2*Log[c*(a
 + b*x^2)]])/c^2 + ((a + b*x^2)*(b*x^2 + (a + 2*b*x^2)*Log[c*(a + b*x^2)]))
/Log[c*(a + b*x^2)]^2)/b^2
```

fricas [A] time = 0.43, size = 142, normalized size = 1.12

$$-\frac{b^2c^2x^4 + abc^2x^2 + \left(ac\operatorname{log\_integral}\left(bcx^2+ac\right) - 4\operatorname{log\_integral}\left(b^2c^2x^4 + 2abc^2x^2 + a^2c^2\right)\right)\log\left(bcx^2+ac\ldots\right.}{4b^2c^2\log\left(bcx^2+ac\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*c^2*x^4 + a*b*c^2*x^2 + (a*c*log_integral(b*c*x^2 + a*c) - 4*log_
integral(b^2*c^2*x^4 + 2*a*b*c^2*x^2 + a^2*c^2))*log(b*c*x^2 + a*c)^2 + (2*
b^2*c^2*x^4 + 3*a*b*c^2*x^2 + a^2*c^2)*log(b*c*x^2 + a*c))/(b^2*c^2*log(b*c
*x^2 + a*c)^2)
```

giac [A] time = 0.17, size = 141, normalized size = 1.11

$$-\frac{ac\operatorname{Ei}\left(\log\left(\left(bx^2+a\right)c\right)\right) - \frac{\left(bcx^2+ac\right)ac}{\log\left(\left(bx^2+a\right)c\right)} - \frac{\left(bcx^2+ac\right)ac}{\log\left(\left(bx^2+a\right)c\right)^2} + \frac{2\left(bcx^2+ac\right)^2}{\log\left(\left(bx^2+a\right)c\right)} + \frac{\left(bcx^2+ac\right)^2}{\log\left(\left(bx^2+a\right)c\right)^2} - 4\operatorname{Ei}\left(2\log\left(\left(bx^2+a\right)c\right)\right)}{4b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a))^3,x, algorithm="giac")
```

```
[Out] -1/4*(a*c*Ei(log((b*x^2 + a)*c)) - (b*c*x^2 + a*c)*a*c/log((b*x^2 + a)*c) -
 (b*c*x^2 + a*c)*a*c/log((b*x^2 + a)*c)^2 + 2*(b*c*x^2 + a*c)^2/log((b*x^2
+ a)*c) + (b*c*x^2 + a*c)^2/log((b*x^2 + a)*c)^2 - 4*Ei(2*log((b*x^2 + a)*c
)))/(b^2*c^2)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\ln\left(\left(b\,x^2+a\right)c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln((b*x^2+a)*c)^3,x)
```

```
[Out] int(x^3/ln((b*x^2+a)*c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2x^4\left(2\,\log(c)+1\right)+abx^2\left(3\,\log(c)+1\right)+a^2\log(c)+\left(2\,b^2x^4+3\,abx^2+a^2\right)\log\left(bx^2+a\right)}{4\left(b^2\log\left(bx^2+a\right)^2+2\,b^2\log\left(bx^2+a\right)\log(c)+b^2\log(c)^2\right)}+\int\frac{4\,bx^3+}{2\left(b\log\left(bx^2+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*x^4*(2*log(c) + 1) + a*b*x^2*(3*log(c) + 1) + a^2*log(c) + (2*b^2
*x^4 + 3*a*b*x^2 + a^2)*log(b*x^2 + a))/(b^2*log(b*x^2 + a)^2 + 2*b^2*log(b
*x^2 + a)*log(c) + b^2*log(c)^2) + integrate(1/2*(4*b*x^3 + 3*a*x)/(b*log(b
*x^2 + a) + b*log(c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\ln\left(c\,\left(b\,x^2+a\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/log(c*(a + b*x^2))^3,x)
```

```
[Out] int(x^3/log(c*(a + b*x^2))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{3ax}{\log\left(ac+bcx^2\right)}\,dx+\int \frac{4bx^3}{\log\left(ac+bcx^2\right)}\,dx}{2b}+\frac{-abx^2-b^2x^4+\left(-a^2-3abx^2-2b^2x^4\right)\log\left(c\left(a+bx^2\right)\right)}{4b^2\log\left(c\left(a+bx^2\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(b*x**2+a))**3,x)
```

```
[Out] (Integral(3*a*x/log(a*c + b*c*x**2), x) + Integral(4*b*x**3/log(a*c + b*c*x
**2), x))/(2*b) + (-a*b*x**2 - b**2*x**4 + (-a**2 - 3*a*b*x**2 - 2*b**2*x**
4)*log(c*(a + b*x**2)))/(4*b**2*log(c*(a + b*x**2))**2)
```

3.128 $\int \frac{x}{\log^3(c(a+bx^2))} dx$

Optimal. Leaf size=73

$$\frac{\mathrm{li}\left(c\left(bx^2+a\right)\right)}{4bc}-\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)}-\frac{a+bx^2}{4b\log\left(c\left(a+bx^2\right)\right)}$$

```
[Out] 1/4*Li(c*(b*x^2+a))/b/c+1/4*(-b*x^2-a)/b/ln(c*(b*x^2+a))^2+1/4*(-b*x^2-a)/b
/ln(c*(b*x^2+a))
```

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2454, 2389, 2297, 2298}

$$\frac{\mathrm{li}\left(c\left(bx^2+a\right)\right)}{4bc}-\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)}-\frac{a+bx^2}{4b\log\left(c\left(a+bx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Log[c*(a + b*x^2)]^3,x]
```

```
[Out] -(a + b*x^2)/(4*b*Log[c*(a + b*x^2)]^2) - (a + b*x^2)/(4*b*Log[c*(a + b*x^2
)]) + LogIntegral[c*(a + b*x^2)]/(4*b*c)
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\log^3(c(a+bx))}\,dx, x, x^2\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{\log^3(cx)}\,dx, x, a+bx^2\right)}{2b}$$
$$= -\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{\log^2(cx)}\,dx, x, a+bx^2\right)}{4b}$$
$$= -\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a+bx^2}{4b\log\left(c\left(a+bx^2\right)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{4b}$$
$$= -\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a+bx^2}{4b\log\left(c\left(a+bx^2\right)\right)} + \frac{\operatorname{li}\left(c\left(a+bx^2\right)\right)}{4bc}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.75

$$\frac{\frac{\operatorname{li}\left(c\left(bx^2+a\right)\right)}{c} - \frac{\left(a+bx^2\right)\left(\log\left(c\left(a+bx^2\right)\right)+1\right)}{\log^2\left(c\left(a+bx^2\right)\right)}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Log[c*(a + b*x^2)]^3,x]
```

```
[Out] (-(((a + b*x^2)*(1 + Log[c*(a + b*x^2)]))/Log[c*(a + b*x^2)]^2) + LogIntegr
al[c*(a + b*x^2)]/c)/(4*b)
```

fricas [A] time = 0.45, size = 79, normalized size = 1.08

$$-\frac{bcx^2 - \log\left(bcx^2+ac\right)^2 \operatorname{log\_integral}\left(bcx^2+ac\right) + ac + \left(bcx^2+ac\right)\log\left(bcx^2+ac\right)}{4\,bc\log\left(bcx^2+ac\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*c*x^2 - log(b*c*x^2 + a*c)^2*log_integral(b*c*x^2 + a*c) + a*c + (b
*c*x^2 + a*c)*log(b*c*x^2 + a*c))/(b*c*log(b*c*x^2 + a*c)^2)
```

giac [A] time = 0.18, size = 68, normalized size = 0.93

$$-\frac{\frac{bcx^2+ac}{\log\left(\left(bx^2+a\right)c\right)} + \frac{bcx^2+ac}{\log\left(\left(bx^2+a\right)c\right)^2} - \operatorname{Ei}\left(\log\left(\left(bx^2+a\right)c\right)\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a))^3,x, algorithm="giac")
```

```
[Out] -1/4*((b*c*x^2 + a*c)/log((b*x^2 + a)*c) + (b*c*x^2 + a*c)/log((b*x^2 + a)*
c)^2 - Ei(log((b*x^2 + a)*c)))/(b*c)
```

maple [A] time = 0.05, size = 94, normalized size = 1.29

$$-\frac{x^2}{4\ln\left(\left(b\,x^2+a\right)c\right)} - \frac{x^2}{4\ln\left(\left(b\,x^2+a\right)c\right)^2} - \frac{a}{4b\ln\left(\left(b\,x^2+a\right)c\right)} - \frac{\operatorname{Ei}\left(1,-\ln\left(\left(b\,x^2+a\right)c\right)\right)}{4bc} - \frac{a}{4b\ln\left(\left(b\,x^2+a\right)c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln((b*x^2+a)*c)^3,x)
```

```
[Out] -1/4/ln((b*x^2+a)*c)^2*x^2-1/4/b/ln((b*x^2+a)*c)^2*a-1/4*x^2/ln((b*x^2+a)*c
)-1/4*a/b/ln((b*x^2+a)*c)-1/4/b/c*Ei(1,-ln((b*x^2+a)*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bx^2\left(\log(c)+1\right)+a\left(\log(c)+1\right)+\left(bx^2+a\right)\log\left(bx^2+a\right)}{4\left(b\log\left(bx^2+a\right)^2+2\,b\log\left(bx^2+a\right)\log(c)+b\log(c)^2\right)}+\int\frac{x}{2\left(\log\left(bx^2+a\right)+\log(c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*x^2*(log(c) + 1) + a*(log(c) + 1) + (b*x^2 + a)*log(b*x^2 + a))/(b*
log(b*x^2 + a)^2 + 2*b*log(b*x^2 + a)*log(c) + b*log(c)^2) + integrate(1/2*
x/(log(b*x^2 + a) + log(c)), x)
```

mupad [B] time = 0.45, size = 74, normalized size = 1.01

$$\frac{\mathrm{logint}\left(c\,\left(b\,x^2+a\right)\right)}{4\,b\,c}-\frac{\frac{a\,c}{4}+\ln\left(c\,\left(b\,x^2+a\right)\right)\,\left(\frac{b\,c\,x^2}{4}+\frac{a\,c}{4}\right)+\frac{b\,c\,x^2}{4}}{b\,c\,\ln\left(c\,\left(b\,x^2+a\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/log(c*(a + b*x^2))^3,x)
```

```
[Out] logint(c*(a + b*x^2))/(4*b*c) - ((a*c)/4 + log(c*(a + b*x^2))*((a*c)/4 + (b
*c*x^2)/4) + (b*c*x^2)/4)/(b*c*log(c*(a + b*x^2))^2)
```

sympy [A] time = 2.19, size = 70, normalized size = 0.96

$$\frac{\begin{cases}\frac{x^2}{2\log\left(ac\right)} & \text{for}\: b=0\\0 & \text{for}\: c=0\\\frac{\operatorname{Ei}\left(\log\left(ac+bcx^2\right)\right)}{2bc} & \text{otherwise}\end{cases}}{2}+\frac{-a-bx^2+\left(-a-bx^2\right)\log\left(c\left(a+bx^2\right)\right)}{4b\log\left(c\left(a+bx^2\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(b*x**2+a))**3,x)
```

```
[Out] Piecewise((x**2/(2*log(a*c)), Eq(b, 0)), (0, Eq(c, 0)), (Ei(log(a*c + b*c*x
**2))/(2*b*c), True))/2 + (-a - b*x**2 + (-a - b*x**2)*log(c*(a + b*x**2)))
/(4*b*log(c*(a + b*x**2))**2)
```

3.129 $\int x^5 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx$

Optimal. Leaf size=150

$$\frac{\left(d+ex^3\right)^2\log^2\left(c\left(d+ex^3\right)^p\right)}{6e^2}-\frac{d\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e^2}-\frac{p\left(d+ex^3\right)^2\log\left(c\left(d+ex^3\right)^p\right)}{6e^2}+\frac{2dp\left(d+ex^3\right)\log}{3e^2}$$

```
[Out] -2/3*d*p^2*x^3/e+1/12*p^2*(e*x^3+d)^2/e^2+2/3*d*p*(e*x^3+d)*ln(c*(e*x^3+d)^
p)/e^2-1/6*p*(e*x^3+d)^2*ln(c*(e*x^3+d)^p)/e^2-1/3*d*(e*x^3+d)*ln(c*(e*x^3+
d)^p)^2/e^2+1/6*(e*x^3+d)^2*ln(c*(e*x^3+d)^p)^2/e^2
```

Rubi [A] time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{\left(d+ex^3\right)^2\log^2\left(c\left(d+ex^3\right)^p\right)}{6e^2}-\frac{d\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e^2}-\frac{p\left(d+ex^3\right)^2\log\left(c\left(d+ex^3\right)^p\right)}{6e^2}+\frac{2dp\left(d+ex^3\right)\log}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (-2*d*p^2*x^3)/(3*e) + (p^2*(d + e*x^3)^2)/(12*e^2) + (2*d*p*(d + e*x^3)*Lo
g[c*(d + e*x^3)^p])/(3*e^2) - (p*(d + e*x^3)^2*Log[c*(d + e*x^3)^p])/(6*e^2
) - (d*(d + e*x^3)*Log[c*(d + e*x^3)^p]^2)/(3*e^2) + ((d + e*x^3)^2*Log[c*(
d + e*x^3)^p]^2)/(6*e^2)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int x^5\log^2\left(c\left(d+ex^3\right)^p\right)dx &= \frac{1}{3}\text{Subst}\left(\int x\log^2\left(c(d+ex)^p\right)dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(-\frac{d\log^2\left(c(d+ex)^p\right)}{e}+\frac{(d+ex)\log^2\left(c(d+ex)^p\right)}{e}\right)dx,x,x^3\right)\\ &= \frac{\text{Subst}\left(\int(d+ex)\log^2\left(c(d+ex)^p\right)dx,x,x^3\right)}{3e}-\frac{d\,\text{Subst}\left(\int\log^2\left(c(d+ex)^p\right)dx,\right.}{3e}\\ &= \frac{\text{Subst}\left(\int x\log^2\left(cx^p\right)dx,x,d+ex^3\right)}{3e^2}-\frac{d\,\text{Subst}\left(\int\log^2\left(cx^p\right)dx,x,d+ex^3\right)}{3e^2}\\ &= -\frac{d\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e^2}+\frac{\left(d+ex^3\right)^2\log^2\left(c\left(d+ex^3\right)^p\right)}{6e^2}-\frac{p\,\text{Subst}\left(\int\right.}{}\\ &= -\frac{2dp^2x^3}{3e}+\frac{p^2\left(d+ex^3\right)^2}{12e^2}+\frac{2dp\left(d+ex^3\right)\log\left(c\left(d+ex^3\right)^p\right)}{3e^2}-\frac{p\left(d+ex^3\right)^2\log}{6e^2}\end{aligned}$$

Mathematica [A] time = 0.07, size = 105, normalized size = 0.70

$$\frac{-2\left(d^2-e^2x^6\right)\log^2\left(c\left(d+ex^3\right)^p\right)+2p\left(2d^2+2dex^3-e^2x^6\right)\log\left(c\left(d+ex^3\right)^p\right)+2d^2p^2\log\left(d+ex^3\right)+ep^2x^3\left(\right.}{12e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (e*p^2*x^3*(-6*d + e*x^3) + 2*d^2*p^2*Log[d + e*x^3] + 2*p*(2*d^2 + 2*d*e*x
^3 - e^2*x^6)*Log[c*(d + e*x^3)^p] - 2*(d^2 - e^2*x^6)*Log[c*(d + e*x^3)^p]
^2)/(12*e^2)
```

fricas [A] time = 0.49, size = 148, normalized size = 0.99

$$\frac{e^2p^2x^6+2\,e^2x^6\log(c)^2-6\,dep^2x^3+2\left(e^2p^2x^6-d^2p^2\right)\log\left(ex^3+d\right)^2-2\left(e^2p^2x^6-2\,dep^2x^3-3\,d^2p^2-2\left(e^2px\right.\right.}{12\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(e^2*p^2*x^6 + 2*e^2*x^6*log(c)^2 - 6*d*e*p^2*x^3 + 2*(e^2*p^2*x^6 - d
^2*p^2)*log(e*x^3 + d)^2 - 2*(e^2*p^2*x^6 - 2*d*e*p^2*x^3 - 3*d^2*p^2 - 2*(
e^2*p*x^6 - d^2*p)*log(c))*log(e*x^3 + d) - 2*(e^2*p*x^6 - 2*d*e*p*x^3)*log
(c))/e^2
```

giac [A] time = 0.17, size = 221, normalized size = 1.47

$$\frac{1}{12}\left(\left(2\left(x^3e+d\right)^2\log\left(x^3e+d\right)^2-4\left(x^3e+d\right)d\log\left(x^3e+d\right)^2-2\left(x^3e+d\right)^2\log\left(x^3e+d\right)+8\left(x^3e+d\right)d\log\left(x^3e\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] 1/12*((2*(x^3*e + d)^2*log(x^3*e + d)^2 - 4*(x^3*e + d)*d*log(x^3*e + d)^2
- 2*(x^3*e + d)^2*log(x^3*e + d) + 8*(x^3*e + d)*d*log(x^3*e + d) + (x^3*e
+ d)^2 - 8*(x^3*e + d)*d)*p^2*e^(-1) + 2*(2*(x^3*e + d)^2*log(x^3*e + d) -
4*(x^3*e + d)*d*log(x^3*e + d) - (x^3*e + d)^2 + 4*(x^3*e + d)*d)*p*e^(-1)*
log(c) + 2*((x^3*e + d)^2 - 2*(x^3*e + d)*d)*e^(-1)*log(c)^2)*e^(-1)
```

maple [C] time = 0.67, size = 1242, normalized size = 8.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] -1/2*d*p^2*x^3/e-1/6*p*x^6*ln(c)+1/6*(I*Pi*e^2*x^6*csgn(I*(e*x^3+d)^p)*csgn
(I*c*(e*x^3+d)^p)^2-I*Pi*e^2*x^6*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*
csgn(I*c)-I*Pi*e^2*x^6*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*e^2*x^6*csgn(I*c*(e*x^3
+d)^p)^2*csgn(I*c)+2*ln(c)*e^2*x^6-e^2*p*x^6+2*d*e*p*x^3-2*d^2*p*ln(e*x^3+d
))/e^2*ln((e*x^3+d)^p)+1/6*ln(c)^2*x^6+1/2*d^2*p^2/e^2*ln(e*x^3+d)-1/24*Pi^
2*x^6*csgn(I*c*(e*x^3+d)^p)^6+1/6/e^2*d^2*p^2*ln(e*x^3+d)^2-1/24*Pi^2*x^6*c
sgn(I*(e*x^3+d)^p)^2*csgn(I*c*(e*x^3+d)^p)^4+1/12*Pi^2*x^6*csgn(I*(e*x^3+d)
^p)*csgn(I*c*(e*x^3+d)^p)^5+1/12*p^2*x^6+1/12*Pi^2*x^6*csgn(I*c*(e*x^3+d)^p
)^5*csgn(I*c)-1/24*Pi^2*x^6*csgn(I*c*(e*x^3+d)^p)^4*csgn(I*c)^2-1/6*I/e*Pi*
d*p*x^3*csgn(I*c*(e*x^3+d)^p)^3+1/6*I/e^2*Pi*ln(e*x^3+d)*d^2*p*csgn(I*c*(e*
x^3+d)^p)^3-1/6*I*ln(c)*Pi*x^6*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*cs
gn(I*c)+1/12*I*Pi*p*x^6*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)
-1/6*I/e*Pi*d*p*x^3*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)+1/6
*I/e^2*Pi*ln(e*x^3+d)*d^2*p*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(
I*c)+1/6*x^6*ln((e*x^3+d)^p)^2+1/6*I/e*Pi*d*p*x^3*csgn(I*(e*x^3+d)^p)*csgn(
I*c*(e*x^3+d)^p)^2+1/6*I/e*Pi*d*p*x^3*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)-1/6
*I/e^2*Pi*ln(e*x^3+d)*d^2*p*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-1/6
*I/e^2*Pi*ln(e*x^3+d)*d^2*p*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+1/6*I*ln(c)*P
i*x^6*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2+1/6*I*ln(c)*Pi*x^6*csgn(I
*c*(e*x^3+d)^p)^2*csgn(I*c)-1/12*I*Pi*p*x^6*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e
*x^3+d)^p)^2-1/12*I*Pi*p*x^6*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+1/12*Pi^2*x^
6*csgn(I*(e*x^3+d)^p)^2*csgn(I*c*(e*x^3+d)^p)^3*csgn(I*c)-1/24*Pi^2*x^6*csg
n(I*(e*x^3+d)^p)^2*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)^2-1/6*Pi^2*x^6*csgn(I*
(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^4*csgn(I*c)+1/12*Pi^2*x^6*csgn(I*(e*x^3+
d)^p)*csgn(I*c*(e*x^3+d)^p)^3*csgn(I*c)^2+1/3/e*ln(c)*d*p*x^3-1/3/e^2*ln(c)
*ln(e*x^3+d)*d^2*p-1/6*I*ln(c)*Pi*x^6*csgn(I*c*(e*x^3+d)^p)^3+1/12*I*Pi*p*x
^6*csgn(I*c*(e*x^3+d)^p)^3
```

maxima [A] time = 0.66, size = 120, normalized size = 0.80

$$\frac{1}{6}x^6\log\left(\left(ex^3+d\right)^pc\right)^2-\frac{1}{6}ep\left(\frac{2d^2\log\left(ex^3+d\right)}{e^3}+\frac{ex^6-2dx^3}{e^2}\right)\log\left(\left(ex^3+d\right)^pc\right)+\frac{\left(e^2x^6-6dex^3+2d^2\log\left(ex^3\right.\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log((e*x^3 + d)^p*c)^2 - 1/6*e*p*(2*d^2*log(e*x^3 + d)/e^3 + (e*x^6
 - 2*d*x^3)/e^2)*log((e*x^3 + d)^p*c) + 1/12*(e^2*x^6 - 6*d*e*x^3 + 2*d^2*l
og(e*x^3 + d)^2 + 6*d^2*log(e*x^3 + d))*p^2/e^2
```

mupad [B] time = 0.28, size = 100, normalized size = 0.67

$$\frac{p^2\,x^6}{12}-\ln\left(c\,{\left(e\,x^3+d\right)}^p\right)\,\left(\frac{p\,x^6}{6}-\frac{d\,p\,x^3}{3\,e}\right)+{\ln\left(c\,{\left(e\,x^3+d\right)}^p\right)}^2\,\left(\frac{x^6}{6}-\frac{d^2}{6\,e^2}\right)-\frac{d\,p^2\,x^3}{2\,e}+\frac{d^2\,p^2\,\ln\left(e\,x^3+d\right)}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] (p^2*x^6)/12 - log(c*(d + e*x^3)^p)*((p*x^6)/6 - (d*p*x^3)/(3*e)) + log(c*(
d + e*x^3)^p)^2*(x^6/6 - d^2/(6*e^2)) - (d*p^2*x^3)/(2*e) + (d^2*p^2*log(d
+ e*x^3))/(2*e^2)
```

sympy [A] time = 31.92, size = 206, normalized size = 1.37

$$\begin{cases} -\frac{d^2p^2\log\left(d+ex^3\right)^2}{6e^2}+\frac{d^2p^2\log\left(d+ex^3\right)}{2e^2}-\frac{d^2p\log\left(c\right)\log\left(d+ex^3\right)}{3e^2}+\frac{dp^2x^3\log\left(d+ex^3\right)}{3e}-\frac{dp^2x^3}{2e}+\frac{dpx^3\log\left(c\right)}{3e}+\frac{p^2x^6\log\left(d+ex^3\right)^2}{6}-\frac{p \\ \frac{x^6\log\left(cd^p\right)^2}{6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Piecewise((-d**2*p**2*log(d + e*x**3)**2/(6*e**2) + d**2*p**2*log(d + e*x**
3)/(2*e**2) - d**2*p*log(c)*log(d + e*x**3)/(3*e**2) + d*p**2*x**3*log(d +
e*x**3)/(3*e) - d*p**2*x**3/(2*e) + d*p*x**3*log(c)/(3*e) + p**2*x**6*log(d
 + e*x**3)**2/6 - p**2*x**6*log(d + e*x**3)/6 + p**2*x**6/12 + p*x**6*log(c
)*log(d + e*x**3)/3 - p*x**6*log(c)/6 + x**6*log(c)**2/6, Ne(e, 0)), (x**6*
log(c*d**p)**2/6, True))
```

3.130 $\int x^2 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx$

Optimal. Leaf size=66

$$\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e}-\frac{2p\left(d+ex^3\right)\log\left(c\left(d+ex^3\right)^p\right)}{3e}+\frac{2p^2x^3}{3}$$

```
[Out] 2/3*p^2*x^3-2/3*p*(e*x^3+d)*ln(c*(e*x^3+d)^p)/e+1/3*(e*x^3+d)*ln(c*(e*x^3+d
)^p)^2/e
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2454, 2389, 2296, 2295}

$$\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e}-\frac{2p\left(d+ex^3\right)\log\left(c\left(d+ex^3\right)^p\right)}{3e}+\frac{2p^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (2*p^2*x^3)/3 - (2*p*(d + e*x^3)*Log[c*(d + e*x^3)^p])/(3*e) + ((d + e*x^3)
*Log[c*(d + e*x^3)^p]^2)/(3*e)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2 \log^2\left(c\left(d+ex^3\right)^p\right) dx &= \frac{1}{3}\operatorname{Subst}\left(\int \log^2\left(c(d+ex)^p\right) dx, x, x^3\right) \\
&= \frac{\operatorname{Subst}\left(\int \log^2\left(cx^p\right) dx, x, d+ex^3\right)}{3e} \\
&= \frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e} - \frac{(2p)\operatorname{Subst}\left(\int \log\left(cx^p\right) dx, x, d+ex^3\right)}{3e} \\
&= \frac{2p^2x^3}{3} - \frac{2p\left(d+ex^3\right)\log\left(c\left(d+ex^3\right)^p\right)}{3e} + \frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 63, normalized size = 0.95

$$\frac{1}{3}\left(\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{e} - 2p\left(\frac{\left(d+ex^3\right)\log\left(c\left(d+ex^3\right)^p\right)}{e} - px^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (((d + e*x^3)*Log[c*(d + e*x^3)^p]^2)/e - 2*p*(-(p*x^3) + ((d + e*x^3)*Log[
c*(d + e*x^3)^p])/e))/3
```

fricas [A] time = 0.44, size = 96, normalized size = 1.45

$$\frac{2\,ep^2x^3 - 2\,epx^3\log(c) + ex^3\log(c)^2 + \left(ep^2x^3 + dp^2\right)\log\left(ex^3+d\right)^2 - 2\left(ep^2x^3 + dp^2 - \left(epx^3 + dp\right)\log(c)\right)\log}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*e*p^2*x^3 - 2*e*p*x^3*log(c) + e*x^3*log(c)^2 + (e*p^2*x^3 + d*p^2)*
log(e*x^3 + d)^2 - 2*(e*p^2*x^3 + d*p^2 - (e*p*x^3 + d*p)*log(c))*log(e*x^3
 + d))/e
```

giac [A] time = 0.17, size = 104, normalized size = 1.58

$$\frac{1}{3}\left(\left(2\,x^3e + \left(x^3e+d\right)\log\left(x^3e+d\right)^2 - 2\left(x^3e+d\right)\log\left(x^3e+d\right) + 2\,d\right)p^2 - 2\left(x^3e - \left(x^3e+d\right)\log\left(x^3e+d\right) + d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] 1/3*((2*x^3*e + (x^3*e + d)*log(x^3*e + d)^2 - 2*(x^3*e + d)*log(x^3*e + d)
 + 2*d)*p^2 - 2*(x^3*e - (x^3*e + d)*log(x^3*e + d) + d)*p*log(c) + (x^3*e
+ d)*log(c)^2)*e^(-1)
```

maple [C] time = 0.56, size = 1036, normalized size = 15.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] 1/3*(I*Pi*e*x^3*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*e*x^3*csgn
(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*e*x^3*csgn(I*c*(e*x^3+
d)^p)^3+I*Pi*e*x^3*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)*e*x^3-2*x^3*p*
e+2*d*p*ln(e*x^3+d))/e*ln((e*x^3+d)^p)-2/3*ln(c)*p*x^3-1/12*Pi^2*x^3*csgn(I
*c*(e*x^3+d)^p)^6-2/3*d*p^2/e*ln(e*x^3+d)+1/6*Pi^2*x^3*csgn(I*c*(e*x^3+d)^p
)^5*csgn(I*c)-1/12*Pi^2*x^3*csgn(I*c*(e*x^3+d)^p)^4*csgn(I*c)^2-1/3/e*d*p^2
*ln(e*x^3+d)^2-1/12*Pi^2*x^3*csgn(I*(e*x^3+d)^p)^2*csgn(I*c*(e*x^3+d)^p)^4+
1/6*Pi^2*x^3*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^5-1/3*I/e*Pi*ln(e*x^
3+d)*d*p*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)+1/3*ln(c)^2*x^
3+1/3*x^3*ln((e*x^3+d)^p)^2+2/3/e*ln(c)*ln(e*x^3+d)*d*p+1/6*Pi^2*x^3*csgn(I
*(e*x^3+d)^p)^2*csgn(I*c*(e*x^3+d)^p)^3*csgn(I*c)-1/12*Pi^2*x^3*csgn(I*(e*x
^3+d)^p)^2*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)^2-1/3*Pi^2*x^3*csgn(I*(e*x^3+d
)^p)*csgn(I*c*(e*x^3+d)^p)^4*csgn(I*c)+1/6*Pi^2*x^3*csgn(I*(e*x^3+d)^p)*csg
n(I*c*(e*x^3+d)^p)^3*csgn(I*c)^2-1/3*I*ln(c)*Pi*x^3*csgn(I*c*(e*x^3+d)^p)^3
+1/3*I*Pi*p*x^3*csgn(I*c*(e*x^3+d)^p)^3+2/3*p^2*x^3+1/3*I*ln(c)*Pi*x^3*csgn
(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2+1/3*I*ln(c)*Pi*x^3*csgn(I*c*(e*x^3+
d)^p)^2*csgn(I*c)-1/3*I*Pi*p*x^3*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^
2-1/3*I*Pi*p*x^3*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+1/3*I/e*Pi*ln(e*x^3+d)*d
*p*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2+1/3*I/e*Pi*ln(e*x^3+d)*d*p*c
sgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)-1/3*I*ln(c)*Pi*x^3*csgn(I*(e*x^3+d)^p)*csg
n(I*c*(e*x^3+d)^p)*csgn(I*c)+1/3*I*Pi*p*x^3*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e
*x^3+d)^p)*csgn(I*c)-1/3*I/e*Pi*ln(e*x^3+d)*d*p*csgn(I*c*(e*x^3+d)^p)^3
```

maxima [A] time = 0.66, size = 97, normalized size = 1.47

$$\frac{1}{3}\,x^3\log\left(\left(ex^3+d\right)^pc\right)^2-\frac{2}{3}\left(\frac{x^3}{e}-\frac{d\log\left(ex^3+d\right)}{e^2}\right)ep\log\left(\left(ex^3+d\right)^pc\right)+\frac{\left(2\,ex^3-d\log\left(ex^3+d\right)^2-2\,d\log\left(ex^3+\right.\right.}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((e*x^3 + d)^p*c)^2 - 2/3*(x^3/e - d*log(e*x^3 + d)/e^2)*e*p*log
((e*x^3 + d)^p*c) + 1/3*(2*e*x^3 - d*log(e*x^3 + d)^2 - 2*d*log(e*x^3 + d))
*p^2/e
```

mupad [B] time = 0.23, size = 71, normalized size = 1.08

$$\frac{2\,p^2\,x^3}{3}+\ln\left(c\,\left(e\,x^3+d\right)^p\right)^2\,\left(\frac{d}{3\,e}+\frac{x^3}{3}\right)-\frac{2\,p\,x^3\,\ln\left(c\,\left(e\,x^3+d\right)^p\right)}{3}-\frac{2\,d\,p^2\,\ln\left(e\,x^3+d\right)}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] (2*p^2*x^3)/3 + log(c*(d + e*x^3)^p)^2*(d/(3*e) + x^3/3) - (2*p*x^3*log(c*(
d + e*x^3)^p))/3 - (2*d*p^2*log(d + e*x^3))/(3*e)
```

sympy [A] time = 7.88, size = 160, normalized size = 2.42

$$\begin{cases} \frac{dp^2\log\left(d+ex^3\right)^2}{3e}-\frac{2dp^2\log\left(d+ex^3\right)}{3e}+\frac{2dp\log\left(c\right)\log\left(d+ex^3\right)}{3e}+\frac{p^2x^3\log\left(d+ex^3\right)^2}{3}-\frac{2p^2x^3\log\left(d+ex^3\right)}{3}+\frac{2p^2x^3}{3}+\frac{2px^3\log\left(c\right)\log\left(d+ex^3\right)}{3} \\ \frac{x^3\log\left(cd^p\right)^2}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Piecewise((d*p**2*log(d + e*x**3)**2/(3*e) - 2*d*p**2*log(d + e*x**3)/(3*e)
 + 2*d*p*log(c)*log(d + e*x**3)/(3*e) + p**2*x**3*log(d + e*x**3)**2/3 - 2*
p**2*x**3*log(d + e*x**3)/3 + 2*p**2*x**3/3 + 2*p*x**3*log(c)*log(d + e*x**
3)/3 - 2*p*x**3*log(c)/3 + x**3*log(c)**2/3, Ne(e, 0)), (x**3*log(c*d**p)**
2/3, True))
```

3.131 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x}\,dx$

Optimal. Leaf size=77

$$\frac{2}{3}p\text{Li}_2\left(\frac{ex^3}{d}+1\right)\log\left(c\left(d+ex^3\right)^p\right)+\frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right)-\frac{2}{3}p^2\text{Li}_3\left(\frac{ex^3}{d}+1\right)$$

```
[Out] 1/3*ln(-e*x^3/d)*ln(c*(e*x^3+d)^p)^2+2/3*p*ln(c*(e*x^3+d)^p)*polylog(2,1+e*
x^3/d)-2/3*p^2*polylog(3,1+e*x^3/d)
```

Rubi [A] time = 0.11, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2454, 2396, 2433, 2374, 6589}

$$\frac{2}{3}p\text{PolyLog}\left(2,\frac{ex^3}{d}+1\right)\log\left(c\left(d+ex^3\right)^p\right)-\frac{2}{3}p^2\text{PolyLog}\left(3,\frac{ex^3}{d}+1\right)+\frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x,x]
```

```
[Out] (Log[-((e*x^3)/d)]*Log[c*(d + e*x^3)^p]^2)/3 + (2*p*Log[c*(d + e*x^3)^p]*Po
lyLog[2, 1 + (e*x^3)/d])/3 - (2*p^2*PolyLog[3, 1 + (e*x^3)/d])/3
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\log^2\left(c(d+ex)^p\right)}{x}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right)-\frac{1}{3}(2ep)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\log\left(c(d+ex)^p\right)}{d+ex}\,dx\right.$$
$$= \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right)-\frac{1}{3}(2p)\,\text{Subst}\left(\int \frac{\log\left(cx^p\right)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx,x\right.$$
$$= \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right)+\frac{2}{3}p\log\left(c\left(d+ex^3\right)^p\right)\text{Li}_2\left(1+\frac{ex^3}{d}\right)-\frac{1}{3}\left(2p^2\right)\text{S}$$
$$= \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right)+\frac{2}{3}p\log\left(c\left(d+ex^3\right)^p\right)\text{Li}_2\left(1+\frac{ex^3}{d}\right)-\frac{2}{3}p^2\text{Li}_3\left(\right.$$

Mathematica [B] time = 0.10, size = 163, normalized size = 2.12

$$2p\left(\log(x)\left(\log\left(d+ex^3\right)-\log\left(\frac{ex^3}{d}+1\right)\right)-\frac{1}{3}\text{Li}_2\left(-\frac{ex^3}{d}\right)\right)\left(\log\left(c\left(d+ex^3\right)^p\right)-p\log\left(d+ex^3\right)\right)+\log(x)\left(\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x,x]
```

```
[Out] Log[x]*(-(p*Log[d + e*x^3]) + Log[c*(d + e*x^3)^p])^2 + 2*p*(-(p*Log[d + e*
x^3]) + Log[c*(d + e*x^3)^p])*(Log[x]*(Log[d + e*x^3] - Log[1 + (e*x^3)/d])
 - PolyLog[2, -((e*x^3)/d)]/3) + (p^2*(Log[-((e*x^3)/d)]*Log[d + e*x^3]^2 +
 2*Log[d + e*x^3]*PolyLog[2, 1 + (e*x^3)/d] - 2*PolyLog[3, 1 + (e*x^3)/d]))
/3
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^3+d\right)^p c\right)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^3 + d)^p*c)^2/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^3+d\right)^p c\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)^2/x, x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^{3}+d\right)^{p}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)^2/x,x)
[Out] int(ln(c*(e*x^3+d)^p)^2/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^{3}+d\right)^{p}c\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x,x, algorithm="maxima")
[Out] integrate(log((e*x^3 + d)^p*c)^2/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(e\,x^{3}+d\right)^{p}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^3)^p)^2/x,x)
[Out] int(log(c*(d + e*x^3)^p)^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+ex^{3}\right)^{p}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x,x)
[Out] Integral(log(c*(d + e*x**3)**p)**2/x, x)
```

3.132 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=86

$$-\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3dx^3}+\frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c\left(d+ex^3\right)^p\right)}{3d}+\frac{2ep^2\text{Li}_2\left(\frac{ex^3}{d}+1\right)}{3d}$$

```
[Out] 2/3*e*p*ln(-e*x^3/d)*ln(c*(e*x^3+d)^p)/d-1/3*(e*x^3+d)*ln(c*(e*x^3+d)^p)^2/
d/x^3+2/3*e*p^2*polylog(2,1+e*x^3/d)/d
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2454, 2397, 2394, 2315}

$$\frac{2ep^2\text{PolyLog}\left(2,\frac{ex^3}{d}+1\right)}{3d}-\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3dx^3}+\frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c\left(d+ex^3\right)^p\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x^4,x]
```

```
[Out] (2*e*p*Log[-((e*x^3)/d)]*Log[c*(d + e*x^3)^p])/(3*d) - ((d + e*x^3)*Log[c*(
d + e*x^3)^p]^2)/(3*d*x^3) + (2*e*p^2*PolyLog[2, 1 + (e*x^3)/d])/(3*d)
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^4}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\log^2\left(c(d+ex)^p\right)}{x^2}\,dx, x, x^3\right)$$
$$= -\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3dx^3} + \frac{(2ep)\operatorname{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^3\right)}{3d}$$
$$= \frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c\left(d+ex^3\right)^p\right)}{3d} - \frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3dx^3} - \frac{\left(2e^2p^2\right)\operatorname{Subst}\left(\int\right.}{3}$$
$$= \frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c\left(d+ex^3\right)^p\right)}{3d} - \frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3dx^3} + \frac{2ep^2\operatorname{Li}_2\left(1+\frac{ex^3}{d}\right)}{3d}$$

Mathematica [A] time = 0.04, size = 84, normalized size = 0.98

$$\frac{-\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right) + 2epx^3\log\left(-\frac{ex^3}{d}\right)\log\left(c\left(d+ex^3\right)^p\right) + 2ep^2x^3\operatorname{Li}_2\left(\frac{ex^3}{d}+1\right)}{3dx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x^4,x]
[Out] (2*e*p*x^3*Log[-((e*x^3)/d)]*Log[c*(d + e*x^3)^p] - (d + e*x^3)*Log[c*(d +
e*x^3)^p]^2 + 2*e*p^2*x^3*PolyLog[2, 1 + (e*x^3)/d])/(3*d*x^3)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\log\left(\left(ex^3+d\right)^p c\right)^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^4,x, algorithm="fricas")
[Out] integral(log((e*x^3 + d)^p*c)^2/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^3+d\right)^p c\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^4,x, algorithm="giac")
[Out] integrate(log((e*x^3 + d)^p*c)^2/x^4, x)
```

maple [C] time = 0.53, size = 771, normalized size = 8.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)^2/x^4,x)
[Out] -1/3/x^3*ln((e*x^3+d)^p)^2+2*p*e*ln((e*x^3+d)^p)/d*ln(x)-2/3*p*e*ln((e*x^3+
d)^p)/d*ln(e*x^3+d)-2*p^2*e/d*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),
```

```
_R1=RootOf(_Z^3*e+d))+1/3*p^2*e/d*ln(e*x^3+d)^2-1/3*I*p*e/d*ln(e*x^3+d)*Pi*
csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+I*p*e/d*ln(x)*Pi*csgn(I*(e*x^3+d)^p)*csgn
(I*c*(e*x^3+d)^p)^2+1/3*I*p*e/d*ln(e*x^3+d)*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c
*(e*x^3+d)^p)*csgn(I*c)+1/3*I/x^3*ln((e*x^3+d)^p)*Pi*csgn(I*(e*x^3+d)^p)*cs
gn(I*c*(e*x^3+d)^p)*csgn(I*c)-2/3/x^3*ln((e*x^3+d)^p)*ln(c)-1/3*I/x^3*ln((e
*x^3+d)^p)*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+1/3*I*p*e/d*ln(e*x^3+d)*Pi*
csgn(I*c*(e*x^3+d)^p)^3+I*p*e/d*ln(x)*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)-
I*p*e/d*ln(x)*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)+2*p*e/
d*ln(x)*ln(c)-I*p*e/d*ln(x)*Pi*csgn(I*c*(e*x^3+d)^p)^3+1/3*I/x^3*ln((e*x^3+
d)^p)*Pi*csgn(I*c*(e*x^3+d)^p)^3-1/3*I*p*e/d*ln(e*x^3+d)*Pi*csgn(I*(e*x^3+d
)^p)*csgn(I*c*(e*x^3+d)^p)^2-1/3*I/x^3*ln((e*x^3+d)^p)*Pi*csgn(I*(e*x^3+d)^
p)*csgn(I*c*(e*x^3+d)^p)^2-2/3*p*e/d*ln(e*x^3+d)*ln(c)-1/12*(I*Pi*csgn(I*(e
*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3
+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*
csgn(I*c)+2*ln(c))^2/x^3
```

maxima [A] time = 0.82, size = 118, normalized size = 1.37

$$\frac{1}{3}e^{2}p^{2}\left(\frac{\log\left(ex^{3}+d\right)^{2}}{de}-\frac{2\left(3\log\left(\frac{ex^{3}}{d}+1\right)\log(x)+\mathrm{Li}_{2}\left(-\frac{ex^{3}}{d}\right)\right)}{de}\right)-\frac{2}{3}ep\left(\frac{\log\left(ex^{3}+d\right)}{d}-\frac{\log\left(x^{3}\right)}{d}\right)\log\left(\left(ex^{3}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*e^2*p^2*(log(e*x^3 + d)^2/(d*e) - 2*(3*log(e*x^3/d + 1)*log(x) + dilog(
-e*x^3/d))/(d*e)) - 2/3*e*p*(log(e*x^3 + d)/d - log(x^3)/d)*log((e*x^3 + d)
^p*c) - 1/3*log((e*x^3 + d)^p*c)^2/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\ln\left(c\left(e\,x^{3}+d\right)^{p}\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^3)^p)^2/x^4,x)
```

```
[Out] int(log(c*(d + e*x^3)^p)^2/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(c\left(d+ex^{3}\right)^{p}\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x**4,x)
```

```
[Out] Integral(log(c*(d + e*x**3)**p)**2/x**4, x)
```

3.133 $\int x \log^2\left(c\left(d+ex^3\right)^p\right)\,dx$

Optimal. Leaf size=1294

$$\frac{9x^2p^2}{4}+\frac{d^{2/3}\log^2\left(\sqrt[3]{e}\,x+\sqrt[3]{d}\right)p^2}{2e^{2/3}}-\frac{\sqrt[3]{-1}\,d^{2/3}\log^2\left(\sqrt[3]{d}-\sqrt[3]{-1}\,\sqrt[3]{e}\,x\right)p^2}{2e^{2/3}}+\frac{(-1)^{2/3}d^{2/3}\log^2\left((-1)^{2/3}\sqrt[3]{e}\,x+\sqrt[3]{d}\right)p^2}{2e^{2/3}}+-$$

```
[Out] -1/2*(-1)^(1/3)*d^(2/3)*p^2*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)^2/e^(2/3)+1/2*
(-1)^(2/3)*d^(2/3)*p^2*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)^2/e^(2/3)+3/2*d^(2/
3)*p^2*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))*3^(1/2)/e^(2/3)+d^
(2/3)*p^2*ln(d^(1/3)+e^(1/3)*x)*ln((-(-1)^(2/3)*d^(1/3)-e^(1/3)*x)/(1-(-1)^
(2/3))/d^(1/3))/e^(2/3)+d^(2/3)*p^2*ln(d^(1/3)+e^(1/3)*x)*ln((-1)^(1/3)*(d^
(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(2/3)-d^(2/3)*p*ln(d^
(1/3)+e^(1/3)*x)*ln(c*(e*x^3+d)^p)/e^(2/3)-3/2*p*x^2*ln(c*(e*x^3+d)^p)-(-1)
^(1/3)*d^(2/3)*p^2*ln((-1)^(1/3)*(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3)
)*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/e^(2/3)+(-1)^(2/3)*d^(2/3)*p^2*ln((-1)^(
1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(d^(1/3)+(-1)
^(2/3)*e^(1/3)*x)/e^(2/3)+(-1)^(2/3)*d^(2/3)*p^2*ln(-(-1)^(2/3)*(d^(1/3)+e^
(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/e^(2/3)-(
-1)^(2/3)*d^(2/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^
(1/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e
^(2/3)-(-1)^(2/3)*d^(2/3)*p^2*ln(-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2
/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/e^(
2/3)+(-1)^(1/3)*d^(2/3)*p^2*ln(-(-1)^(1/3)*((-1)^(2/3)*d^(1/3)+e^(1/3)*x)/(
1-(-1)^(2/3))/d^(1/3))*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1
)^(2/3))/d^(1/3))/e^(2/3)-(-1)^(1/3)*d^(2/3)*p^2*ln(d^(1/3)-(-1)^(1/3)*e^(1
/3)*x)*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3)
)/e^(2/3)+(-1)^(1/3)*d^(2/3)*p*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)*ln(c*(e*x^3
+d)^p)/e^(2/3)-(-1)^(2/3)*d^(2/3)*p*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)*ln(c*(
e*x^3+d)^p)/e^(2/3)+d^(2/3)*p^2*polylog(2,(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3)
)/d^(1/3))/e^(2/3)+1/2*x^2*ln(c*(e*x^3+d)^p)^2+9/4*p^2*x^2+3/2*d^(2/3)*p^2*
ln(d^(1/3)+e^(1/3)*x)/e^(2/3)+1/2*d^(2/3)*p^2*ln(d^(1/3)+e^(1/3)*x)^2/e^(2/
3)-3/4*d^(2/3)*p^2*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/e^(2/3)+d^(2/3
)*p^2*polylog(2,2*(d^(1/3)+e^(1/3)*x)/d^(1/3)/(3-I*3^(1/2)))/e^(2/3)-(-1)^(
2/3)*d^(2/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d
^(1/3))/e^(2/3)-(-1)^(1/3)*d^(2/3)*p^2*polylog(2,(d^(1/3)-(-1)^(1/3)*e^(1/3
)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(2/3)-(-1)^(2/3)*d^(2/3)*p^2*polylog(2,(-1)^
(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(2/3)+(-1)^(
1/3)*d^(2/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-
1)^(2/3))/d^(1/3))/e^(2/3)
```

Rubi [A] time = 1.92, antiderivative size = 1300, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.187, Rules used = {2457, 2476, 2455, 321, 292, 31, 634, 617, 204, 628, 2462, 260, 2416, 2390, 2301, 2394, 2393, 2391, 12}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (9*p^2*x^2)/4 + (3*Sqrt[3]*d^(2/3)*p^2*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt
[3]*d^(1/3))])/(2*e^(2/3)) + (3*d^(2/3)*p^2*Log[d^(1/3) + e^(1/3)*x])/(2*e^
(2/3)) + (d^(2/3)*p^2*Log[d^(1/3) + e^(1/3)*x]^2)/(2*e^(2/3)) + (d^(2/3)*p^
2*Log[d^(1/3) + e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1
)^(2/3))*d^(1/3)))])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*Log[((-1)^(1/3)*(d^(
```

```
1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) - (-1)^(1/3)*e^(
1/3)*x])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)
*x]^2)/(2*e^(2/3)) + ((-1)^(2/3)*d^(2/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e
^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]
)/e^(2/3) + ((-1)^(2/3)*d^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e
^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])
/e^(2/3) + ((-1)^(2/3)*d^(2/3)*p^2*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]^2)/(
2*e^(2/3)) - ((-1)^(2/3)*d^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*
e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x
)/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3) + (d^(2/3)*p^2*Log[d^(1/3) + e^(1/3)
*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(
1/3))])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*
x]*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^
(1/3)))])/e^(2/3) - (3*d^(2/3)*p^2*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3
)*x^2])/(4*e^(2/3)) - (3*p*x^2*Log[c*(d + e*x^3)^p])/2 - (d^(2/3)*p*Log[d^(
1/3) + e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(2/3) + ((-1)^(1/3)*d^(2/3)*p*Log
[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(2/3) - ((-1)^(2/3
)*d^(2/3)*p*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(2/
3) + (x^2*Log[c*(d + e*x^3)^p]^2)/2 + (d^(2/3)*p^2*PolyLog[2, (d^(1/3) + e^
(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3) + (d^(2/3)*p^2*PolyLog[2, (2*
(d^(1/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*d^(1/3))])/e^(2/3) - ((-1)^(1/3)*d^
(2/3)*p^2*PolyLog[2, -(((-1)^(1/3)*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 -
(-1)^(2/3))*d^(1/3)))])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*PolyLog[2, (d^(1/
3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3) - ((-1)^(2/
3)*d^(2/3)*p^2*PolyLog[2, ((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1
 + (-1)^(1/3))*d^(1/3))])/e^(2/3) + ((-1)^(2/3)*d^(2/3)*p^2*PolyLog[2, (d^(
1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/e^(2/3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int x\log^2\left(c\left(d+ex^3\right)^p\right)dx = \frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)-(3ep)\int\frac{x^4\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}dx$$

$$= \frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)-(3ep)\int\left(\frac{x\log\left(c\left(d+ex^3\right)^p\right)}{e}-\frac{dx\log\left(c\left(d+ex^3\right)^p\right)}{e\left(d+ex^3\right)}\right)dx$$

$$= \frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)-(3p)\int x\log\left(c\left(d+ex^3\right)^p\right)dx+(3dp)\int\frac{x\log\left(c\left(d+ex\right.\right.}{d+ex^3}$$

$$= -\frac{3}{2}px^2\log\left(c\left(d+ex^3\right)^p\right)+\frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)+(3dp)\int\left(-\frac{\log\left(c\left(d+ex^3\right)\right.}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{d}+\sqrt[3]{}\right.}\right.$$

$$= \frac{9p^2x^2}{4}-\frac{3}{2}px^2\log\left(c\left(d+ex^3\right)^p\right)+\frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)-\frac{\left(d^{2/3}p\right)\int\frac{\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}}{\sqrt[3]{e}}$$

$$= \frac{9p^2x^2}{4}-\frac{3}{2}px^2\log\left(c\left(d+ex^3\right)^p\right)-\frac{d^{2/3}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{e^{2/3}}+\frac{\sqrt[3]{-1}}{}$$

$$= \frac{9p^2x^2}{4}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2e^{2/3}}-\frac{3}{2}px^2\log\left(c\left(d+ex^3\right)^p\right)-\frac{d^{2/3}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}\right.}{e^{2}}$$

$$= \frac{9p^2x^2}{4}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2e^{2/3}}-\frac{3d^{2/3}p^2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{4e^{2/3}}-\frac{3}{2}px^2\,\mathrm{lo}$$

$$= \frac{9p^2x^2}{4}+\frac{3\sqrt{3}d^{2/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2e^{2/3}}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2e^{2/3}}+\frac{d^{2/3}p^2\log\left(\sqrt[3]{d}\right.}{}$$

$$= \frac{9p^2x^2}{4}+\frac{3\sqrt{3}d^{2/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2e^{2/3}}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2e^{2/3}}+\frac{d^{2/3}p^2\log^2\left(\sqrt[3]{}\right.}{2e^{2/3}}$$

$$= \frac{9p^2x^2}{4}+\frac{3\sqrt{3}d^{2/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2e^{2/3}}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2e^{2/3}}+\frac{d^{2/3}p^2\log^2\left(\sqrt[3]{}\right.}{2e^{2/3}}$$

$$= \frac{9p^2x^2}{4}+\frac{3\sqrt{3}d^{2/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2e^{2/3}}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2e^{2/3}}+\frac{d^{2/3}p^2\log^2\left(\sqrt[3]{}\right.}{2e^{2/3}}$$

Mathematica [C] time = 1.23, size = 823, normalized size = 0.64

$$\frac{1}{4}\left(2x^2\log^2\left(c\left(ex^3+d\right)^p\right)+\frac{p\left(-9e^{2/3}p\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{ex^3}{d}\right)-1\right)x^2-6e^{2/3}\log\left(c\left(ex^3+d\right)^p\right)x^2-4d^{2/3}\log\left(-\sqrt[3]{e}x\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (2*x^2*Log[c*(d + e*x^3)^p]^2 + (p*(-9*e^(2/3)*p*x^2*(-1 + Hypergeometric2F
1[2/3, 1, 5/3, -((e*x^3)/d)]) - 6*e^(2/3)*x^2*Log[c*(d + e*x^3)^p] - 4*d^(2
/3)*Log[-d^(1/3) - e^(1/3)*x]*Log[c*(d + e*x^3)^p] + 4*(-1)^(1/3)*d^(2/3)*L
og[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p] - 4*(-1)^(2/3)*d^(
2/3)*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p] - 2*(-1)^(1/
3)*d^(2/3)*p*(Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*(2*Log[((-1)^(1/3)*(d^(1
/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/3) + (-1)^(1/3)*e
^(1/3)*x] + 2*Log[((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((-1 + (-1)
^(2/3))*d^(1/3))]) + 2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-
1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (-d^(1/3) + (-1)^(1/3)*e^(1/3)*x)/((-1 +
 (-1)^(2/3))*d^(1/3))]) + 2*(-1)^(2/3)*d^(2/3)*p*(Log[-d^(1/3) - (-1)^(2/3)
*e^(1/3)*x]*(2*Log[((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((-1 + (-1)^(2/3))*d^
(1/3))] + 2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1
/3))*d^(1/3))] + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]) + 2*PolyLog[2, (d^(1
/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (d^(
1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))]) + 2*d^(2/3)*p*(Lo
g[-d^(1/3) - e^(1/3)*x]*(Log[-d^(1/3) - e^(1/3)*x] + 2*(Log[((-1)^(1/3)*d^(
1/3) - e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + Log[(I + Sqrt[3] - ((2*I)*e
^(1/3)*x)/d^(1/3))/(3*I + Sqrt[3])])) + 2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/
((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, ((2*I)*(1 + (e^(1/3)*x)/d^(1/3))
)/(3*I + Sqrt[3])])))/e^(2/3))/4
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(x\log\left(\left(ex^3+d\right)^p c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(x*log((e*x^3 + d)^p*c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\log\left(\left(ex^3+d\right)^p c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x*log((e*x^3 + d)^p*c)^2, x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int x\ln\left(c\left(e\,x^3+d\right)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(x*ln(c*(e*x^3+d)^p)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2\log\left(\left(ex^3+d\right)^p\right)^2+\int\frac{ex^4\log(c)^2+dx\log(c)^2-\left(\left(3\,ep-2\,e\log(c)\right)x^4-2\,dx\log(c)\right)\log\left(\left(ex^3+d\right)^p\right)}{ex^3+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log((e*x^3 + d)^p)^2 + integrate((e*x^4*log(c)^2 + d*x*log(c)^2 - (
(3*e*p - 2*e*log(c))*x^4 - 2*d*x*log(c))*log((e*x^3 + d)^p))/(e*x^3 + d), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \ln\left(c\left(e x^3+d\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(x*log(c*(d + e*x^3)^p)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \log\left(c\left(d+ex^3\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(x*log(c*(d + e*x**3)**p)**2, x)
```

3.134 $\int \log^2\left(c\left(d+ex^3\right)^p\right)\,dx$

Optimal. Leaf size=1304

$$-\frac{\sqrt[3]{d}\log^2\left(-\sqrt[3]{e}x-\sqrt[3]{d}\right)p^2}{\sqrt[3]{e}}-\frac{(-1)^{2/3}\sqrt[3]{d}\log^2\left(\sqrt[3]{-1}\sqrt[3]{e}x-\sqrt[3]{d}\right)p^2}{\sqrt[3]{e}}+\frac{\sqrt[3]{-1}\sqrt[3]{d}\log^2\left(-(-1)^{2/3}\sqrt[3]{e}x-\sqrt[3]{d}\right)p^2}{\sqrt[3]{e}}+18$$

```
[Out] 6*d^(1/3)*p^2*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))*3^(1/2)/e^(
1/3)-2*d^(1/3)*p^2*ln(-d^(1/3)-e^(1/3)*x)*ln((-(-1)^(2/3)*d^(1/3)-e^(1/3)*x
)/(1-(-1)^(2/3))/d^(1/3))/e^(1/3)-2*d^(1/3)*p^2*ln(-d^(1/3)-e^(1/3)*x)*ln((
-1)^(1/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(1/3)+2*
d^(1/3)*p*ln(-d^(1/3)-e^(1/3)*x)*ln(c*(e*x^3+d)^p)/e^(1/3)+(-1)^(1/3)*d^(1/
3)*p^2*ln(-d^(1/3)-(-1)^(2/3)*e^(1/3)*x)^2/e^(1/3)-(-1)^(2/3)*d^(1/3)*p^2*l
n(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)^2/e^(1/3)-2*d^(1/3)*p^2*polylog(2,(d^(1/3)
+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(1/3)-6*p*x*ln(c*(e*x^3+d)^p)-2*(-1)^
(2/3)*d^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))
*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)/e^(1/3)+2*(-1)^(1/3)*d^(1/3)*p^2*ln(-(-1
)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-d^(1/3)-(-1)^(2/3)*
e^(1/3)*x)/e^(1/3)+2*(-1)^(1/3)*d^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/
3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(-d^(1/3)-(-1)^(2/3)*e^(1/3)*x)/e^(
1/3)-2*(-1)^(1/3)*d^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/
(1+(-1)^(1/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^
(1/3))/e^(1/3)-2*(-1)^(1/3)*d^(1/3)*p^2*ln(-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/
(1-(-1)^(2/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^
(1/3))/e^(1/3)+2*(-1)^(2/3)*d^(1/3)*p^2*ln(-(-1)^(1/3)*((-1)^(2/3)*d^(1/3)+
e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/
3)*x)/(1-(-1)^(2/3))/d^(1/3))/e^(1/3)-2*(-1)^(2/3)*d^(1/3)*p^2*ln(-d^(1/3)+
(-1)^(1/3)*e^(1/3)*x)*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)
^(2/3))/d^(1/3))/e^(1/3)+2*(-1)^(2/3)*d^(1/3)*p*ln(-d^(1/3)+(-1)^(1/3)*e^(1
/3)*x)*ln(c*(e*x^3+d)^p)/e^(1/3)-2*(-1)^(1/3)*d^(1/3)*p*ln(-d^(1/3)-(-1)^(2
/3)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/e^(1/3)+2*(-1)^(2/3)*d^(1/3)*p^2*polylog(2
,-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/e^(1/3)
-2*(-1)^(1/3)*d^(1/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)
^(2/3))/d^(1/3))/e^(1/3)-2*(-1)^(2/3)*d^(1/3)*p^2*polylog(2,(d^(1/3)-(-1)^(
1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(1/3)-2*(-1)^(1/3)*d^(1/3)*p^2*po
lylog(2,(-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e
^(1/3)+x*ln(c*(e*x^3+d)^p)^2-6*d^(1/3)*p^2*ln(d^(1/3)+e^(1/3)*x)/e^(1/3)+3*
d^(1/3)*p^2*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/e^(1/3)-2*d^(1/3)*p^2
*polylog(2,2*(d^(1/3)+e^(1/3)*x)/d^(1/3)/(3-I*3^(1/2)))/e^(1/3)-d^(1/3)*p^2
*ln(-d^(1/3)-e^(1/3)*x)^2/e^(1/3)+18*p^2*x
```

Rubi [A] time = 1.79, antiderivative size = 1310, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 20, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.429, Rules used = {2450, 2476, 2448, 321, 200, 31, 634, 617, 204, 628, 2471, 2462, 260, 2416, 2390, 2301, 2394, 2393, 2391, 12}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] 18*p^2*x + (6*Sqrt[3]*d^(1/3)*p^2*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d
^(1/3))])/e^(1/3) - (d^(1/3)*p^2*Log[-d^(1/3) - e^(1/3)*x]^2)/e^(1/3) - (6*
d^(1/3)*p^2*Log[d^(1/3) + e^(1/3)*x])/e^(1/3) - (2*d^(1/3)*p^2*Log[-d^(1/3)
 - e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1
/3)))])/e^(1/3) - (2*(-1)^(2/3)*d^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) + e^(1
```

```
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x])/e
^(1/3) - ((-1)^(2/3)*d^(1/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]^2)/e^
(1/3) + (2*(-1)^(1/3)*d^(1/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/
((1 - (-1)^(2/3))*d^(1/3)))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/e^(1/3)
+ (2*(-1)^(1/3)*d^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x
))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/e^(1/3
) + ((-1)^(1/3)*d^(1/3)*p^2*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2)/e^(1/3)
 - (2*(-1)^(1/3)*d^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*
x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 +
(-1)^(1/3))*d^(1/3))])/e^(1/3) - (2*d^(1/3)*p^2*Log[-d^(1/3) - e^(1/3)*x]*L
og[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))
])/e^(1/3) - (2*(-1)^(2/3)*d^(1/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]
*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1
/3)))])/e^(1/3) + (3*d^(1/3)*p^2*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*
x^2])/e^(1/3) - 6*p*x*Log[c*(d + e*x^3)^p] + (2*d^(1/3)*p*Log[-d^(1/3) - e^
(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(1/3) + (2*(-1)^(2/3)*d^(1/3)*p*Log[-d^(1/
3) + (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(1/3) - (2*(-1)^(1/3)*d^
(1/3)*p*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(1/3)
+ x*Log[c*(d + e*x^3)^p]^2 - (2*d^(1/3)*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x
)/((1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) - (2*d^(1/3)*p^2*PolyLog[2, (2*(d^(1
/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*d^(1/3))])/e^(1/3) - (2*(-1)^(2/3)*d^(1/
3)*p^2*PolyLog[2, -(((-1)^(1/3)*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1
)^(2/3))*d^(1/3)))])/e^(1/3) - (2*(-1)^(2/3)*d^(1/3)*p^2*PolyLog[2, (d^(1/3
) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) - (2*(-1)^(1
/3)*d^(1/3)*p^2*PolyLog[2, ((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((
1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) + (2*(-1)^(1/3)*d^(1/3)*p^2*PolyLog[2, (
d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/e^(1/3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = x\log^2\left(c\left(d+ex^3\right)^p\right) - (6ep)\int \frac{x^3\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}\,dx$$

$$= x\log^2\left(c\left(d+ex^3\right)^p\right) - (6ep)\int \left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{e} - \frac{d\log\left(c\left(d+ex^3\right)^p\right)}{e\left(d+ex^3\right)}\right)dx$$

$$= x\log^2\left(c\left(d+ex^3\right)^p\right) - (6p)\int \log\left(c\left(d+ex^3\right)^p\right)dx + (6dp)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}$$

$$= -6px\log\left(c\left(d+ex^3\right)^p\right) + x\log^2\left(c\left(d+ex^3\right)^p\right) + (6dp)\int\left(-\frac{\log\left(c\left(d+ex^3\right)^p\right)}{3d^{2/3}\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}\right.$$

$$= 18p^2x - 6px\log\left(c\left(d+ex^3\right)^p\right) + x\log^2\left(c\left(d+ex^3\right)^p\right) - \left(2\sqrt[3]{d}\,p\right)\int \frac{\log\left(c\left(d+ex\right.\right.}{-\sqrt[3]{d}-\sqrt[3]{e}}$$

$$= 18p^2x - 6px\log\left(c\left(d+ex^3\right)^p\right) + \frac{2\sqrt[3]{d}\,p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{e}} + \frac{2(}{}$$

$$= 18p^2x - \frac{6\sqrt[3]{d}\,p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}} - 6px\log\left(c\left(d+ex^3\right)^p\right) + \frac{2\sqrt[3]{d}\,p\log\left(-\sqrt[3]{d}-\right.}{}$$

$$= 18p^2x - \frac{6\sqrt[3]{d}\,p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}} + \frac{3\sqrt[3]{d}\,p^2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2\right)}{\sqrt[3]{e}} - 6px\log$$

$$= 18p^2x + \frac{6\sqrt{3}\sqrt[3]{d}\,p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt[3]{e}} - \frac{6\sqrt[3]{d}\,p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}} - \frac{2\sqrt[3]{d}\,p^2\log\left(-\right.}{}$$

$$= 18p^2x + \frac{6\sqrt{3}\sqrt[3]{d}\,p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt[3]{e}} - \frac{\sqrt[3]{d}\,p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}} - \frac{6\sqrt[3]{d}\,p^2\log\left(\right.}{\sqrt[3]{e}}$$

$$= 18p^2x + \frac{6\sqrt{3}\sqrt[3]{d}\,p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt[3]{e}} - \frac{\sqrt[3]{d}\,p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}} - \frac{6\sqrt[3]{d}\,p^2\log\left(\right.}{\sqrt[3]{e}}$$

$$= 18p^2x + \frac{6\sqrt{3}\sqrt[3]{d}\,p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt[3]{e}} - \frac{\sqrt[3]{d}\,p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}} - \frac{6\sqrt[3]{d}\,p^2\log\left(\right.}{\sqrt[3]{e}}$$

Mathematica [A] time = 0.73, size = 1090, normalized size = 0.84

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] x*Log[c*(d + e*x^3)^p]^2 - 6*e*p*(-1/2*(p*((6*x)/e - ((2*d^(1/3)*Log[d^(1/3
) + e^(1/3)*x])/e^(1/3) - d^(1/3)*((2*Sqrt[3]*ArcTan[(d^(1/3) - 2*e^(1/3)*x
)/(Sqrt[3]*d^(1/3))])/e^(1/3) + Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x
```

```
^2]/e^(1/3)))/e)) + (x*Log[c*(d + e*x^3)^p])/e - (d^(1/3)*Log[-d^(1/3) - e^
(1/3)*x]*Log[c*(d + e*x^3)^p])/(3*e^(4/3)) - ((-1)^(2/3)*d^(1/3)*Log[-d^(1/
3) + (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(3*e^(4/3)) + ((-1)^(1/3)*
d^(1/3)*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(3*e^(4/
3)) + (d^(1/3)*p*(Log[-d^(1/3) - e^(1/3)*x]^2/e^(1/3) + (2*Log[-d^(1/3) - e
^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))
)])/e^(1/3) + (2*Log[-d^(1/3) - e^(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^
(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]])/e^(1/3) + (2*PolyLog[2, (d^(
1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))]])/e^(1/3) + (2*PolyLog[2, (d^(
1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))]])/e^(1/3)))/(6*e) + ((-1)^(2/3
)*d^(1/3)*p*((2*Log[((-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^
(1/3))]*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]])/e^(1/3) + Log[-d^(1/3) + (-1)
^(1/3)*e^(1/3)*x]^2/e^(1/3) + (2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[-
(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]
)/e^(1/3) + (2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1)^(1/3)
)*d^(1/3))]])/e^(1/3) + (2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 -
 (-1)^(2/3))*d^(1/3))]])/e^(1/3)))/(6*e) - ((-1)^(1/3)*d^(1/3)*p*((2*Log[-((
(-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[-d^(1/3)
 - (-1)^(2/3)*e^(1/3)*x]])/e^(1/3) + (2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3
)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)
*x]])/e^(1/3) + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2/e^(1/3) + (2*PolyLog[
2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))]])/e^(1/3) +
(2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))]])
/e^(1/3)))/(6*e))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\log\left(\left(ex^3+d\right)^p c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^3 + d)^p*c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\left(ex^3+d\right)^p c\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)^2, x)
```

maple [F] time = 0.91, size = 0, normalized size = 0.00

$$\int \ln\left(c\left(e\,x^3+d\right)^p\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(ln(c*(e*x^3+d)^p)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x\log\left(\left(ex^3+d\right)^p\right)^2+\int \frac{ex^3\log(c)^2+d\log(c)^2-2\left(\left(3\,ep-e\log(c)\right)x^3-d\log(c)\right)\log\left(\left(ex^3+d\right)^p\right)}{ex^3+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] x*log((e*x^3 + d)^p)^2 + integrate((e*x^3*log(c)^2 + d*log(c)^2 - 2*((3*e*p
 - e*log(c))*x^3 - d*log(c))*log((e*x^3 + d)^p))/(e*x^3 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(e\,x^3+d\right)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(log(c*(d + e*x^3)^p)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(c\left(d+ex^3\right)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(log(c*(d + e*x**3)**p)**2, x)
```

3.135 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=1137

$$\frac{\sqrt[3]{e}\log^2\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{\sqrt[3]{d}}-\frac{\sqrt[3]{-1}\sqrt[3]{e}\log^2\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)p^2}{\sqrt[3]{d}}+\frac{(-1)^{2/3}\sqrt[3]{e}\log^2\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{\sqrt[3]{d}}+\frac{2\sqrt[3]{e}\log\left(\right.}{}$$

```
[Out] e^(1/3)*p^2*ln(d^(1/3)+e^(1/3)*x)^2/d^(1/3)+2*e^(1/3)*p^2*ln(d^(1/3)+e^(1/3
)*x)*ln((-(-1)^(2/3)*d^(1/3)-e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(1/3)-2*(
-1)^(1/3)*e^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1
/3))*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/d^(1/3)-(-1)^(1/3)*e^(1/3)*p^2*ln(d^(
1/3)-(-1)^(1/3)*e^(1/3)*x)^2/d^(1/3)+2*(-1)^(2/3)*e^(1/3)*p^2*ln(-(-1)^(2/3
)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)
*x)/d^(1/3)+2*(-1)^(2/3)*e^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1
/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/d^(1/3)+(-1
)^(2/3)*e^(1/3)*p^2*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)^2/d^(1/3)-2*(-1)^(2/3)
*e^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^
(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(1/3)+2*
e^(1/3)*p^2*ln(d^(1/3)+e^(1/3)*x)*ln((-1)^(1/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)
*x)/(1+(-1)^(1/3))/d^(1/3))/d^(1/3)-2*(-1)^(2/3)*e^(1/3)*p^2*ln(-(-1)^(2/3)
*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)
*x)/(1-(-1)^(2/3))/d^(1/3))/d^(1/3)+2*(-1)^(1/3)*e^(1/3)*p^2*ln(-(-1)^(1/3)
*((-1)^(2/3)*d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-(-1)^(2/3)*(d^(
1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(1/3)-2*(-1)^(1/3)*e^(
1/3)*p^2*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3
)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(1/3)-2*e^(1/3)*p*ln(d^(1/3)+e^(1/3)
*x)*ln(c*(e*x^3+d)^p)/d^(1/3)+2*(-1)^(1/3)*e^(1/3)*p*ln(d^(1/3)-(-1)^(1/3)*
e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(1/3)-2*(-1)^(2/3)*e^(1/3)*p*ln(d^(1/3)+(-1)
^(2/3)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(1/3)-ln(c*(e*x^3+d)^p)^2/x+2*e^(1/3)
*p^2*polylog(2,(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(1/3)-2*(-1)^(
2/3)*e^(1/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d
^(1/3))/d^(1/3)-2*(-1)^(1/3)*e^(1/3)*p^2*polylog(2,(d^(1/3)-(-1)^(1/3)*e^(1
/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(1/3)-2*(-1)^(2/3)*e^(1/3)*p^2*polylog(2,(
-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(1/3)+2*
(-1)^(1/3)*e^(1/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)
/(1-(-1)^(2/3))/d^(1/3))/d^(1/3)+2*e^(1/3)*p^2*polylog(2,2*(d^(1/3)+e^(1/3)
*x)/d^(1/3)/(3-I*3^(1/2)))/d^(1/3)
```

Rubi [A] time = 1.33, antiderivative size = 1143, normalized size of antiderivative = 1.01, number of steps used = 39, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {2457, 2476, 2462, 260, 2416, 2390, 2301, 2394, 2393, 2391, 12}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x^2,x]
```

```
[Out] (e^(1/3)*p^2*Log[d^(1/3) + e^(1/3)*x]^2)/d^(1/3) + (2*e^(1/3)*p^2*Log[d^(1/
3) + e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^
(1/3)))])/d^(1/3) - (2*(-1)^(1/3)*e^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) + e^
(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x])/
d^(1/3) - ((-1)^(1/3)*e^(1/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]^2)/d^
(1/3) + (2*(-1)^(2/3)*e^(1/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/
((1 - (-1)^(2/3))*d^(1/3)))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/d^(1/3) +
 (2*(-1)^(2/3)*e^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x)
)/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/d^(1/3)
```

```
+ ((-1)^(2/3)*e^(1/3)*p^2*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]^2)/d^(1/3) -
(2*(-1)^(2/3)*e^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))
/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1
)^(1/3))*d^(1/3))])/d^(1/3) + (2*e^(1/3)*p^2*Log[d^(1/3) + e^(1/3)*x]*Log[(
(-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/d
^(1/3) - (2*(-1)^(1/3)*e^(1/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[
-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))
])/d^(1/3) - (2*e^(1/3)*p*Log[d^(1/3) + e^(1/3)*x]*Log[c*(d + e*x^3)^p])/d^
(1/3) + (2*(-1)^(1/3)*e^(1/3)*p*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[c*(
d + e*x^3)^p])/d^(1/3) - (2*(-1)^(2/3)*e^(1/3)*p*Log[d^(1/3) + (-1)^(2/3)*e
^(1/3)*x]*Log[c*(d + e*x^3)^p])/d^(1/3) - Log[c*(d + e*x^3)^p]^2/x + (2*e^(
1/3)*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/d^(1
/3) + (2*e^(1/3)*p^2*PolyLog[2, (2*(d^(1/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*
d^(1/3))])/d^(1/3) - (2*(-1)^(1/3)*e^(1/3)*p^2*PolyLog[2, -(((-1)^(1/3)*((-
1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/d^(1/3) - (2*(
-1)^(1/3)*e^(1/3)*p^2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1
)^(1/3))*d^(1/3))])/d^(1/3) - (2*(-1)^(2/3)*e^(1/3)*p^2*PolyLog[2, ((-1)^(1
/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/d^(1/3)
+ (2*(-1)^(2/3)*e^(1/3)*p^2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1
 - (-1)^(2/3))*d^(1/3))])/d^(1/3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^2}\,dx = -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x} + (6ep)\int \frac{x\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}\,dx$$

$$= -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x} + (6ep)\int\left(-\frac{\log\left(c\left(d+ex^3\right)^p\right)}{3\sqrt[3]{d}\,\sqrt[3]{e}\,\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)} - \frac{(-1)^{2/3}\log\left(c\left(d+ex^3 \cdots\right.\right.}{3\sqrt[3]{d}\,\sqrt[3]{e}\,\left(\sqrt[3]{d}-\sqrt[3]{-1}\,\cdots\right.}\right.$$

$$= -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x} - \frac{\left(2e^{2/3}p\right)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}+\sqrt[3]{e}\,x}\,dx}{\sqrt[3]{d}} + \frac{\left(2\sqrt[3]{-1}\,e^{2/3}p\right)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}\,x}\cdots}{\sqrt[3]{d}}$$

$$= -\frac{2\sqrt[3]{e}\,p\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{-1}\,\sqrt[3]{e}\,p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\,\sqrt[3]{e}\,x\right)\log\cdots}{\sqrt[3]{d}}$$

$$= -\frac{2\sqrt[3]{e}\,p\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{-1}\,\sqrt[3]{e}\,p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\,\sqrt[3]{e}\,x\right)\log\cdots}{\sqrt[3]{d}}$$

$$= -\frac{2\sqrt[3]{e}\,p\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{-1}\,\sqrt[3]{e}\,p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\,\sqrt[3]{e}\,x\right)\log\cdots}{\sqrt[3]{d}}$$

$$= \frac{2\sqrt[3]{e}\,p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{\sqrt[3]{d}} - \frac{2\sqrt[3]{-1}\,\sqrt[3]{e}\,p^2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\cdots}{\sqrt[3]{d}}$$

$$= \frac{\sqrt[3]{e}\,p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{e}\,p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{\sqrt[3]{d}} - \frac{2\sqrt[3]{-1}\cdots}{\cdots}$$

$$= \frac{\sqrt[3]{e}\,p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{e}\,p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{\sqrt[3]{d}} - \frac{2\sqrt[3]{-1}\cdots}{\cdots}$$

$$= \frac{\sqrt[3]{e}\,p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{e}\,p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{\sqrt[3]{d}} - \frac{2\sqrt[3]{-1}\cdots}{\cdots}$$

Mathematica [A] time = 0.87, size = 742, normalized size = 0.65

$$-\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x} - \frac{\sqrt[3]{e}\,p\left(2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(c\left(d+ex^3\right)^p\right) - 2\sqrt[3]{-1}\,\log\left(\sqrt[3]{-1}\,\sqrt[3]{e}\,x-\sqrt[3]{d}\right)\log\left(c\left(d+\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x^2,x]
```

```
[Out] -(Log[c*(d + e*x^3)^p]^2/x) - (e^(1/3)*p*(2*Log[-d^(1/3) - e^(1/3)*x]*Log[c
*(d + e*x^3)^p] - 2*(-1)^(1/3)*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[c*(
d + e*x^3)^p] + 2*(-1)^(2/3)*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d
+ e*x^3)^p] + (-1)^(1/3)*p*(Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*(2*Log[((-
```

```
1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/3)
+ (-1)^(1/3)*e^(1/3)*x] + 2*Log[((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x
))/((-1 + (-1)^(2/3))*d^(1/3))]) + 2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/
3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (-d^(1/3) + (-1)^(1/3)*e^(
1/3)*x)/((-1 + (-1)^(2/3))*d^(1/3))]) - (-1)^(2/3)*p*(Log[-d^(1/3) - (-1)^(
2/3)*e^(1/3)*x]*(2*Log[((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((-1 + (-1)^(2/3)
)*d^(1/3))] + 2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1
)^(1/3))*d^(1/3))] + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]) + 2*PolyLog[2, (
d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2,
(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))]) - p*(Log[-d^(
1/3) - e^(1/3)*x]*(Log[-d^(1/3) - e^(1/3)*x] + 2*(Log[((-1)^(1/3)*d^(1/3) -
 e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + Log[(I + Sqrt[3] - ((2*I)*e^(1/3)
*x)/d^(1/3))/(3*I + Sqrt[3])])) + 2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 +
(-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, ((2*I)*(1 + (e^(1/3)*x)/d^(1/3)))/(3*I
 + Sqrt[3])])))/d^(1/3)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^3+d\right)^p c\right)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^3 + d)^p*c)^2/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^3+d\right)^p c\right)^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)^2/x^2, x)
```

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)^2/x^2,x)
```

```
[Out] int(ln(c*(e*x^3+d)^p)^2/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(\left(ex^3+d\right)^p\right)^2}{x}+\int \frac{ex^3\log(c)^2+d\log(c)^2+2\left(\left(3\,ep+e\log(c)\right)x^3+d\log(c)\right)\log\left(\left(ex^3+d\right)^p\right)}{ex^5+dx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^2,x, algorithm="maxima")
```

```
[Out] -log((e*x^3 + d)^p)^2/x + integrate((e*x^3*log(c)^2 + d*log(c)^2 + 2*((3*e*
p + e*log(c))*x^3 + d*log(c))*log((e*x^3 + d)^p))/(e*x^5 + d*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^3)^p)^2/x^2,x)
```

```
[Out] int(log(c*(d + e*x^3)^p)^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+ex^3\right)^p\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x**2,x)
```

```
[Out] Integral(log(c*(d + e*x**3)**p)**2/x**2, x)
```

3.136 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^3} dx$

Optimal. Leaf size=1170

$$-\frac{e^{2/3}\log^2\left(-\sqrt[3]{e}x-\sqrt[3]{d}\right)p^2}{2d^{2/3}}-\frac{(-1)^{2/3}e^{2/3}\log^2\left(\sqrt[3]{-1}\sqrt[3]{e}x-\sqrt[3]{d}\right)p^2}{2d^{2/3}}+\frac{\sqrt[3]{-1}e^{2/3}\log^2\left(-(-1)^{2/3}\sqrt[3]{e}x-\sqrt[3]{d}\right)p^2}{2d^{2/3}}-\frac{e^{2/3}\log}{}$$

```
[Out] -1/2*e^(2/3)*p^2*ln(-d^(1/3)-e^(1/3)*x)^2/d^(2/3)-e^(2/3)*p^2*ln(-d^(1/3)-e
^(1/3)*x)*ln((-(-1)^(2/3)*d^(1/3)-e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(2/3
)-(-1)^(2/3)*e^(2/3)*p^2*ln((-1)^(1/3)*(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d
^(1/3))*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)/d^(2/3)-1/2*(-1)^(2/3)*e^(2/3)*p^
2*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)^2/d^(2/3)+(-1)^(1/3)*e^(2/3)*p^2*ln(-(-
1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-d^(1/3)-(-1)^(2/3)
*e^(1/3)*x)/d^(2/3)+(-1)^(1/3)*e^(2/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3
)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(-d^(1/3)-(-1)^(2/3)*e^(1/3)*x)/d^(2
/3)+1/2*(-1)^(1/3)*e^(2/3)*p^2*ln(-d^(1/3)-(-1)^(2/3)*e^(1/3)*x)^2/d^(2/3)-
(-1)^(1/3)*e^(2/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)
^(1/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/
d^(2/3)-e^(2/3)*p^2*ln(-d^(1/3)-e^(1/3)*x)*ln((-1)^(1/3)*(d^(1/3)+(-1)^(2/3
)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(2/3)-(-1)^(1/3)*e^(2/3)*p^2*ln(-(-1
)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*
e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(2/3)+(-1)^(2/3)*e^(2/3)*p^2*ln(-(-1)^
(1/3)*((-1)^(2/3)*d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-(-1)^(2/3)
*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(2/3)-(-1)^(2/3)*
e^(2/3)*p^2*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^
(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(2/3)+e^(2/3)*p*ln(-d^(1/3)-e^(1
/3)*x)*ln(c*(e*x^3+d)^p)/d^(2/3)+(-1)^(2/3)*e^(2/3)*p*ln(-d^(1/3)+(-1)^(1/3
)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(2/3)-(-1)^(1/3)*e^(2/3)*p*ln(-d^(1/3)-(-1
)^(2/3)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(2/3)-1/2*ln(c*(e*x^3+d)^p)^2/x^2-e^
(2/3)*p^2*polylog(2,(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(2/3)-(-1
)^(1/3)*e^(2/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3)
)/d^(1/3))/d^(2/3)-(-1)^(2/3)*e^(2/3)*p^2*polylog(2,(d^(1/3)-(-1)^(1/3)*e^(
1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(2/3)-(-1)^(1/3)*e^(2/3)*p^2*polylog(2,(-
1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(2/3)+(-1
)^(2/3)*e^(2/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1
-(-1)^(2/3))/d^(1/3))/d^(2/3)-e^(2/3)*p^2*polylog(2,2*(d^(1/3)+e^(1/3)*x)/d
^(1/3)/(3-I*3^(1/2)))/d^(2/3)
```

Rubi [A] time = 1.34, antiderivative size = 1176, normalized size of antiderivative = 1.01, number of steps used = 39, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {2457, 2471, 2462, 260, 2416, 2390, 2301, 2394, 2393, 2391, 12}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x^3,x]
```

```
[Out] -(e^(2/3)*p^2*Log[-d^(1/3) - e^(1/3)*x]^2)/(2*d^(2/3)) - (e^(2/3)*p^2*Log[-
d^(1/3) - e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3
))*d^(1/3)))])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) +
 e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*
x])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]^
2)/(2*d^(2/3)) + ((-1)^(1/3)*e^(2/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/
3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/d
^(2/3) + ((-1)^(1/3)*e^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/d
```

```
^(2/3) + ((-1)^(1/3)*e^(2/3)*p^2*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2)/(2
*d^(2/3)) - ((-1)^(1/3)*e^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e
^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)
/((1 + (-1)^(1/3))*d^(1/3))])/d^(2/3) - (e^(2/3)*p^2*Log[-d^(1/3) - e^(1/3)
*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(
1/3))])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)
*x]*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d
^(1/3)))])/d^(2/3) + (e^(2/3)*p*Log[-d^(1/3) - e^(1/3)*x]*Log[c*(d + e*x^3)
^p])/d^(2/3) + ((-1)^(2/3)*e^(2/3)*p*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*L
og[c*(d + e*x^3)^p])/d^(2/3) - ((-1)^(1/3)*e^(2/3)*p*Log[-d^(1/3) - (-1)^(2
/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/d^(2/3) - Log[c*(d + e*x^3)^p]^2/(2*x^
2) - (e^(2/3)*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3
))])/d^(2/3) - (e^(2/3)*p^2*PolyLog[2, (2*(d^(1/3) + e^(1/3)*x))/((3 - I*Sq
rt[3])*d^(1/3))])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*PolyLog[2, -(((-1)^(1/3
)*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/d^(2/3) -
 ((-1)^(2/3)*e^(2/3)*p^2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 +
(-1)^(1/3))*d^(1/3))])/d^(2/3) - ((-1)^(1/3)*e^(2/3)*p^2*PolyLog[2, ((-1)^(
1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/d^(2/3)
 + ((-1)^(1/3)*e^(2/3)*p^2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1
- (-1)^(2/3))*d^(1/3))])/d^(2/3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^3}\,dx &= -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{2x^2} + (3ep)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}\,dx \\
&= -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{2x^2} + (3ep)\int\left(-\frac{\log\left(c\left(d+ex^3\right)^p\right)}{3d^{2/3}\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)} - \frac{\log\left(c\left(d+ex^3\right)^p\right)}{3d^{2/3}\left(-\sqrt[3]{d}+\sqrt[3]{-1}\,\sqrt[3]{e}\,x\right.}\right. \\
&= -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{2x^2} - \frac{(ep)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{-\sqrt[3]{d}-\sqrt[3]{e}\,x}\,dx}{d^{2/3}} - \frac{(ep)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{-\sqrt[3]{d}+\sqrt[3]{-1}\,\sqrt[3]{e}\,x}\,dx}{d^{2/3}} - \frac{(ep)\int -}{} \\
&= \frac{e^{2/3}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(c\left(d+ex^3\right)^p\right)}{d^{2/3}} + \frac{(-1)^{2/3}e^{2/3}p\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\,\sqrt[3]{e}\,x\right)\log}{d^{2/3}} \\
&= \frac{e^{2/3}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(c\left(d+ex^3\right)^p\right)}{d^{2/3}} + \frac{(-1)^{2/3}e^{2/3}p\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\,\sqrt[3]{e}\,x\right)\log}{d^{2/3}} \\
&= \frac{e^{2/3}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(c\left(d+ex^3\right)^p\right)}{d^{2/3}} + \frac{(-1)^{2/3}e^{2/3}p\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\,\sqrt[3]{e}\,x\right)\log}{d^{2/3}} \\
&= -\frac{e^{2/3}p^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{d^{2/3}} - \frac{(-1)^{2/3}e^{2/3}p^2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right.}{d^{2/3}} \\
&= -\frac{e^{2/3}p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{2d^{2/3}} - \frac{e^{2/3}p^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{d^{2/3}} - \frac{(-}{} \\
&= -\frac{e^{2/3}p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{2d^{2/3}} - \frac{e^{2/3}p^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{d^{2/3}} - \frac{(-}{} \\
&= -\frac{e^{2/3}p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{2d^{2/3}} - \frac{e^{2/3}p^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{d^{2/3}} - \frac{(-}{}
\end{aligned}
$$

Mathematica [A] time = 0.87, size = 745, normalized size = 0.64

$$
\frac{1}{2}\left(-\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^2} + \frac{e^{2/3}p\left(2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(c\left(d+ex^3\right)^p\right) + 2(-1)^{2/3}\log\left(\sqrt[3]{-1}\,\sqrt[3]{e}\,x-\sqrt[3]{d}\right)\log\left(c\right.\right.}{}\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x^3,x]
```

```
[Out] (-(Log[c*(d + e*x^3)^p]^2/x^2) + (e^(2/3)*p*(2*Log[-d^(1/3) - e^(1/3)*x]*Lo
g[c*(d + e*x^3)^p] + 2*(-1)^(2/3)*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[
c*(d + e*x^3)^p] - 2*(-1)^(1/3)*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*
(d + e*x^3)^p] - (-1)^(2/3)*p*(Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*(2*Log[
```

```
((-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/
3) + (-1)^(1/3)*e^(1/3)*x] + 2*Log[((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3
)*x))/((-1 + (-1)^(2/3))*d^(1/3))]) + 2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^
(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (-d^(1/3) + (-1)^(1/3)*
e^(1/3)*x)/((-1 + (-1)^(2/3))*d^(1/3))]) + (-1)^(1/3)*p*(Log[-d^(1/3) - (-1
)^(2/3)*e^(1/3)*x]*(2*Log[((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((-1 + (-1)^(2
/3))*d^(1/3))] + 2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 +
(-1)^(1/3))*d^(1/3))] + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]) + 2*PolyLog[2
, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[
2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))]) - p*(Log[-
d^(1/3) - e^(1/3)*x]*(Log[-d^(1/3) - e^(1/3)*x] + 2*(Log[((-1)^(1/3)*d^(1/3
) - e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + Log[(I + Sqrt[3] - ((2*I)*e^(1
/3)*x)/d^(1/3))/(3*I + Sqrt[3])])) + 2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1
 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, ((2*I)*(1 + (e^(1/3)*x)/d^(1/3)))/(
3*I + Sqrt[3])])))/d^(2/3))/2
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^3+d\right)^p c\right)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^3 + d)^p*c)^2/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^3+d\right)^p c\right)^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)^2/x^3, x)
```

maple [F] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)^2/x^3,x)
```

```
[Out] int(ln(c*(e*x^3+d)^p)^2/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(\left(ex^3+d\right)^p\right)^2}{2\,x^2}+\int \frac{ex^3\log(c)^2+d\log(c)^2+\left(\left(3\,ep+2\,e\log(c)\right)x^3+2\,d\log(c)\right)\log\left(\left(ex^3+d\right)^p\right)}{ex^6+dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log((e*x^3 + d)^p)^2/x^2 + integrate((e*x^3*log(c)^2 + d*log(c)^2 + ((
3*e*p + 2*e*log(c))*x^3 + 2*d*log(c))*log((e*x^3 + d)^p))/(e*x^6 + d*x^3),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^3)^p)^2/x^3,x)
```

```
[Out] int(log(c*(d + e*x^3)^p)^2/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+ex^3\right)^p\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x**3,x)
```

```
[Out] Integral(log(c*(d + e*x**3)**p)**2/x**3, x)
```

3.137 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^5}\,dx$

Optimal. Leaf size=1328

$$-\frac{e^{4/3}\log^2\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{4d^{4/3}}+\frac{\sqrt[3]{-1}e^{4/3}\log^2\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)p^2}{4d^{4/3}}-\frac{(-1)^{2/3}e^{4/3}\log^2\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{4d^{4/3}}-\frac{3\sqrt{3}e^{4/3}}{}$$

```
[Out] -1/2*e^(4/3)*p^2*ln(d^(1/3)+e^(1/3)*x)*ln((-(-1)^(2/3)*d^(1/3)-e^(1/3)*x)/(
1-(-1)^(2/3))/d^(1/3))/d^(4/3)+1/4*(-1)^(1/3)*e^(4/3)*p^2*ln(d^(1/3)-(-1)^(
1/3)*e^(1/3)*x)^2/d^(4/3)-1/4*(-1)^(2/3)*e^(4/3)*p^2*ln(d^(1/3)+(-1)^(2/3)*
e^(1/3)*x)^2/d^(4/3)-1/2*e^(4/3)*p^2*ln(d^(1/3)+e^(1/3)*x)*ln((-1)^(1/3)*(d
^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(4/3)-3/2*e*p*ln(c*(
e*x^3+d)^p)/d/x+1/2*e^(4/3)*p*ln(d^(1/3)+e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(4/
3)+1/2*(-1)^(2/3)*e^(4/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-
(-1)^(2/3))/d^(1/3))/d^(4/3)+1/2*(-1)^(1/3)*e^(4/3)*p^2*polylog(2,(d^(1/3)-
(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(4/3)+1/2*(-1)^(2/3)*e^(4/3
)*p^2*polylog(2,(-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^
(1/3))/d^(4/3)-1/2*(-1)^(1/3)*e^(4/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+(-
1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(4/3)+1/2*(-1)^(1/3)*e^(4/3)*
p^2*ln((-1)^(1/3)*(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(d^(1/3)-(-
1)^(1/3)*e^(1/3)*x)/d^(4/3)-1/2*(-1)^(2/3)*e^(4/3)*p^2*ln(-(-1)^(2/3)*(d^(1
/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/d^(
4/3)-1/2*(-1)^(2/3)*e^(4/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x
)/(1+(-1)^(1/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/d^(4/3)+1/2*(-1)
^(2/3)*e^(4/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/
3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(4
/3)+1/2*(-1)^(2/3)*e^(4/3)*p^2*ln(-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(
2/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^
(4/3)-1/2*(-1)^(1/3)*e^(4/3)*p^2*ln(-(-1)^(1/3)*((-1)^(2/3)*d^(1/3)+e^(1/3)
*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(
1-(-1)^(2/3))/d^(1/3))/d^(4/3)+1/2*(-1)^(1/3)*e^(4/3)*p^2*ln(d^(1/3)-(-1)^(
1/3)*e^(1/3)*x)*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3)
)/d^(1/3))/d^(4/3)-1/2*(-1)^(1/3)*e^(4/3)*p*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x
)*ln(c*(e*x^3+d)^p)/d^(4/3)+1/2*(-1)^(2/3)*e^(4/3)*p*ln(d^(1/3)+(-1)^(2/3)*
e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(4/3)-1/4*ln(c*(e*x^3+d)^p)^2/x^4-3/2*e^(4/3
)*p^2*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))*3^(1/2)/d^(4/3)-3/2
*e^(4/3)*p^2*ln(d^(1/3)+e^(1/3)*x)/d^(4/3)-1/4*e^(4/3)*p^2*ln(d^(1/3)+e^(1/
3)*x)^2/d^(4/3)+3/4*e^(4/3)*p^2*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d
^(4/3)-1/2*e^(4/3)*p^2*polylog(2,2*(d^(1/3)+e^(1/3)*x)/d^(1/3)/(3-I*3^(1/2)
))/d^(4/3)-1/2*e^(4/3)*p^2*polylog(2,(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(
1/3))/d^(4/3)
```

Rubi [A] time = 1.72, antiderivative size = 1334, normalized size of antiderivative = 1.00, number of steps used = 48, number of rules used = 18, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2457, 2476, 2455, 292, 31, 634, 617, 204, 628, 2462, 260, 2416, 2390, 2301, 2394, 2393, 2391, 12}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x^5,x]
```

```
[Out] (-3*Sqrt[3]*e^(4/3)*p^2*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/
(2*d^(4/3)) - (3*e^(4/3)*p^2*Log[d^(1/3) + e^(1/3)*x])/(2*d^(4/3)) - (e^(4/
3)*p^2*Log[d^(1/3) + e^(1/3)*x]^2)/(4*d^(4/3)) - (e^(4/3)*p^2*Log[d^(1/3) +
 e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3
```

```
)))])/(2*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) + e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x])/(2
*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]^2)/
(4*d^(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*
x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/(2*d^
(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/(2
*d^(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]^2)/
(4*d^(4/3)) + ((-1)^(2/3)*e^(4/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)
*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*
x)/((1 + (-1)^(1/3))*d^(1/3))]])/(2*d^(4/3)) - (e^(4/3)*p^2*Log[d^(1/3) + e^
(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3)
)*d^(1/3))]])/(2*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*Log[d^(1/3) - (-1)^(1/3)
*e^(1/3)*x]*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^
(2/3))*d^(1/3)))]])/(2*d^(4/3)) + (3*e^(4/3)*p^2*Log[d^(2/3) - d^(1/3)*e^(1/
3)*x + e^(2/3)*x^2])/(4*d^(4/3)) - (3*e*p*Log[c*(d + e*x^3)^p])/(2*d*x) + (
e^(4/3)*p*Log[d^(1/3) + e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(2*d^(4/3)) - ((-1
)^(1/3)*e^(4/3)*p*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])
/(2*d^(4/3)) + ((-1)^(2/3)*e^(4/3)*p*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]*Lo
g[c*(d + e*x^3)^p])/(2*d^(4/3)) - Log[c*(d + e*x^3)^p]^2/(4*x^4) - (e^(4/3)
*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))]])/(2*d^(4/
3)) - (e^(4/3)*p^2*PolyLog[2, (2*(d^(1/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*d^
(1/3))]])/(2*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*PolyLog[2, -(((-1)^(1/3)*((-
1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]])/(2*d^(4/3)) +
((-1)^(1/3)*e^(4/3)*p^2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (
-1)^(1/3))*d^(1/3))]])/(2*d^(4/3)) + ((-1)^(2/3)*e^(4/3)*p^2*PolyLog[2, ((-1
)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]])/(2*d
^(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*
x)/((1 - (-1)^(2/3))*d^(1/3))]])/(2*d^(4/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^5}\,dx = -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{4x^4} + \frac{1}{2}(3ep)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{x^2\left(d+ex^3\right)}\,dx$$

$$= -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{4x^4} + \frac{1}{2}(3ep)\int \left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{dx^2} - \frac{ex\log\left(c\left(d+ex^3\right)^p\right)}{d\left(d+ex^3\right)}\right)dx$$

$$= -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{4x^4} + \frac{(3ep)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{x^2}\,dx}{2d} - \frac{\left(3e^2p\right)\int \frac{x\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}\,dx}{2d}$$

$$= -\frac{3ep\log\left(c\left(d+ex^3\right)^p\right)}{2dx} - \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{4x^4} - \frac{\left(3e^2p\right)\int\left(-\frac{\log\left(c\left(d+ex^3\right)^p\right)}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} - \frac{(-1)^{2/3}\cdots}{3\sqrt[3]{d}\sqrt[3]{e}\cdots}\right.}{}$$

$$= -\frac{3ep\log\left(c\left(d+ex^3\right)^p\right)}{2dx} - \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{4x^4} + \frac{\left(e^{5/3}p\right)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{2d^{4/3}} - \frac{\left(\sqrt[3]{-1}\,e^{5/\cdots}\right.}{}$$

$$= -\frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} - \frac{3ep\log\left(c\left(d+ex^3\right)^p\right)}{2dx} + \frac{e^{4/3}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c\left(d\cdots\right.\right.}{2d^{4/3}}$$

$$= -\frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} + \frac{3e^{4/3}p^2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{4d^{4/3}} - \frac{3ep\log\left(c\left(d+e\cdots\right.\right.}{2dx}$$

$$= -\frac{3\sqrt{3}\,e^{4/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2d^{4/3}} - \frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} + \frac{3e^{4/3}p^2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{\cdots}\right.}{4d^{4/3}}$$

$$= -\frac{3\sqrt{3}\,e^{4/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2d^{4/3}} - \frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} - \frac{e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\text{l}\cdots}{2d^{4/3}}$$

$$= -\frac{3\sqrt{3}\,e^{4/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2d^{4/3}} - \frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} - \frac{e^{4/3}p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{4d^{4/3}}$$

$$= -\frac{3\sqrt{3}\,e^{4/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2d^{4/3}} - \frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} - \frac{e^{4/3}p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{4d^{4/3}}$$

$$= -\frac{3\sqrt{3}\,e^{4/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2d^{4/3}} - \frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} - \frac{e^{4/3}p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{4d^{4/3}}$$

Mathematica [C] time = 1.64, size = 847, normalized size = 0.64

$$\frac{epx^3\left(9ep\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{ex^3}{d}\right)x^3+2d^{2/3}\sqrt[3]{e}\log\left(-\sqrt[3]{e}x-\sqrt[3]{d}\right)\log\left(c\left(ex^3+d\right)^p\right)x-2\sqrt[3]{-1}\,d^{2/3}\sqrt[3]{e}\log\left(\sqrt[3]{-1}\sqrt[3]{e}x-\sqrt[3]{d}\right)\log\left(c\left(ex^3+d\right)^p\right)x+2(-1)^{2/3}d^{2/3}\sqrt[3]{e}\log\left(\cdots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x^5,x]
```

```
[Out] (-Log[c*(d + e*x^3)^p]^2 + (e*p*x^3*(9*e*p*x^3*Hypergeometric2F1[2/3, 1, 5/
3, -((e*x^3)/d)] - 6*d*Log[c*(d + e*x^3)^p] + 2*d^(2/3)*e^(1/3)*x*Log[-d^(1
/3) - e^(1/3)*x]*Log[c*(d + e*x^3)^p] - 2*(-1)^(1/3)*d^(2/3)*e^(1/3)*x*Log[
-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p] + 2*(-1)^(2/3)*d^(2/3
)*e^(1/3)*x*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p] + (-1
)^(1/3)*d^(2/3)*e^(1/3)*p*x*(Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*(2*Log[((
-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/3)
 + (-1)^(1/3)*e^(1/3)*x] + 2*Log[((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*
x))/((-1 + (-1)^(2/3))*d^(1/3))]) + 2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1
/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (-d^(1/3) + (-1)^(1/3)*e^
(1/3)*x)/((-1 + (-1)^(2/3))*d^(1/3))]) - (-1)^(2/3)*d^(2/3)*e^(1/3)*p*x*(Lo
g[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*(2*Log[((-1)^(2/3)*(d^(1/3) + e^(1/3)*x)
)/((-1 + (-1)^(2/3))*d^(1/3))] + 2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^
(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x
]) + 2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3
))] + 2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/
3))]) - d^(2/3)*e^(1/3)*p*x*(Log[-d^(1/3) - e^(1/3)*x]*(Log[-d^(1/3) - e^(1
/3)*x] + 2*(Log[((-1)^(1/3)*d^(1/3) - e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))
] + Log[(I + Sqrt[3] - ((2*I)*e^(1/3)*x)/d^(1/3))/(3*I + Sqrt[3])])) + 2*Po
lyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (
(2*I)*(1 + (e^(1/3)*x)/d^(1/3)))/(3*I + Sqrt[3])]))))/d^2)/(4*x^4)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^3+d\right)^p c\right)^2}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^5,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^3 + d)^p*c)^2/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^3+d\right)^p c\right)^2}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)^2/x^5, x)
```

maple [F] time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)^2/x^5,x)
```

```
[Out] int(ln(c*(e*x^3+d)^p)^2/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(\left(ex^3+d\right)^p\right)^2}{4\,x^4}+\int \frac{2\,ex^3\log(c)^2+2\,d\log(c)^2+\left(\left(3\,ep+4\,e\log(c)\right)x^3+4\,d\log(c)\right)\log\left(\left(ex^3+d\right)^p\right)}{2\left(ex^8+dx^5\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*log((e*x^3 + d)^p)^2/x^4 + integrate(1/2*(2*e*x^3*log(c)^2 + 2*d*log(c
)^2 + ((3*e*p + 4*e*log(c))*x^3 + 4*d*log(c))*log((e*x^3 + d)^p))/(e*x^8 +
d*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^3)^p)^2/x^5,x)
```

```
[Out] int(log(c*(d + e*x^3)^p)^2/x^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x**5,x)
```

```
[Out] Timed out
```

3.138 $\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal. Leaf size=164

$$\frac{d^2\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^3p}+\frac{\left(d+ex^3\right)^3\left(c\left(d+ex^3\right)^p\right)^{-3/p}\text{Ei}\left(\frac{3\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^3p}-\frac{2d\left(d+ex^3\right)\ldots}{}$$

```
[Out] 1/3*d^2*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e^3/p/((c*(e*x^3+d)^p)^(1/p))-2/3
*d*(e*x^3+d)^2*Ei(2*ln(c*(e*x^3+d)^p)/p)/e^3/p/((c*(e*x^3+d)^p)^(2/p))+1/3*
(e*x^3+d)^3*Ei(3*ln(c*(e*x^3+d)^p)/p)/e^3/p/((c*(e*x^3+d)^p)^(3/p))
```

Rubi [A] time = 0.24, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2454, 2399, 2389, 2300, 2178, 2390, 2310}

$$\frac{d^2\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^3p}+\frac{\left(d+ex^3\right)^3\left(c\left(d+ex^3\right)^p\right)^{-3/p}\text{Ei}\left(\frac{3\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^3p}-\frac{2d\left(d+ex^3\right)\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Log[c*(d + e*x^3)^p],x]
```

```
[Out] (d^2*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e^3*p*(c*(d + e*
x^3)^p)^p^(-1)) - (2*d*(d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])
/p])/(3*e^3*p*(c*(d + e*x^3)^p)^(2/p)) + ((d + e*x^3)^3*ExpIntegralEi[(3*Lo
g[c*(d + e*x^3)^p])/p])/(3*e^3*p*(c*(d + e*x^3)^p)^(3/p))
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
```

```
qQ[e*f - d*g, 0]
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\log\left(c(d+ex)^p\right)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{d^2}{e^2\log\left(c(d+ex)^p\right)} - \frac{2d(d+ex)}{e^2\log\left(c(d+ex)^p\right)} + \frac{(d+ex)^2}{e^2\log\left(c(d+ex)^p\right)}\right) dx, x, x^3\right)$$

$$= \frac{\text{Subst}\left(\int \frac{(d+ex)^2}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3e^2} - \frac{(2d)\,\text{Subst}\left(\int \frac{d+ex}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3e^2} + \frac{d^2\,\text{Subst}\left(\int \frac{\ldots}{\log\ldots}\right.}{\ldots}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e^3} - \frac{(2d)\,\text{Subst}\left(\int \frac{x}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e^3} + \frac{d^2\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{\left(\left(d+ex^3\right)^3\left(c\left(d+ex^3\right)^p\right)^{-3/p}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{p}}}{x}\,dx, x, \log\left(c\left(d+ex^3\right)^p\right)\right)}{3e^3p} - \frac{\left(2d\left(d+ex^3\right)^2\ldots\right.}{\ldots}$$

$$= \frac{d^2\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^3p} - \frac{2d\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\text{Ei}\left(\ldots\right.}{3e^3p}$$

Mathematica [A] time = 0.25, size = 146, normalized size = 0.89

$$\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-3/p}\left(d^2\left(c\left(d+ex^3\right)^p\right)^{2/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right) - \left(d+ex^3\right)\left(2d\left(c\left(d+ex^3\right)^p\right)^{\frac{1}{p}}\text{Ei}\left(\frac{2\log\left(c\left(ex^3+d\right)^p\ldots\right.}{p}\right.\right.\right.}{3e^3p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Log[c*(d + e*x^3)^p],x]
```

```
[Out] ((d + e*x^3)*(d^2*(c*(d + e*x^3)^p)^(2/p)*ExpIntegralEi[Log[c*(d + e*x^3)^p
]/p] - (d + e*x^3)*(2*d*(c*(d + e*x^3)^p)^p^(-1)*ExpIntegralEi[(2*Log[c*(d
+ e*x^3)^p])/p] - (d + e*x^3)*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p])))/
(3*e^3*p*(c*(d + e*x^3)^p)^(3/p))
```

fricas [A] time = 0.47, size = 116, normalized size = 0.71

$$\frac{c^{\frac{2}{p}}d^2\,\mathrm{log\_integral}\left(\left(ex^3+d\right)c^{\left(\frac{1}{p}\right)}\right)-2\,c^{\left(\frac{1}{p}\right)}d\,\mathrm{log\_integral}\left(\left(e^2x^6+2\,dex^3+d^2\right)c^{\frac{2}{p}}\right)+\mathrm{log\_integral}\left(\left(e^3x^9+3\right.\right.}{3\,c^{\frac{3}{p}}e^3p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] 1/3*(c^(2/p)*d^2*log_integral((e*x^3 + d)*c^(1/p)) - 2*c^(1/p)*d*log_integr
al((e^2*x^6 + 2*d*e*x^3 + d^2)*c^(2/p)) + log_integral((e^3*x^9 + 3*d*e^2*x
^6 + 3*d^2*e*x^3 + d^3)*c^(3/p)))/(c^(3/p)*e^3*p)
```

giac [A] time = 0.18, size = 108, normalized size = 0.66

$$\frac{d^2\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(x^3e+d\right)\right)e^{(-3)}}{3\,c^{\left(\frac{1}{p}\right)}p}-\frac{2\,d\mathrm{Ei}\left(\frac{2\,\log(c)}{p}+2\,\log\left(x^3e+d\right)\right)e^{(-3)}}{3\,c^{\frac{2}{p}}p}+\frac{\mathrm{Ei}\left(\frac{3\,\log(c)}{p}+3\,\log\left(x^3e+d\right)\right)e^{(-3)}}{3\,c^{\frac{3}{p}}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] 1/3*d^2*Ei(log(c)/p + log(x^3*e + d))*e^(-3)/(c^(1/p)*p) - 2/3*d*Ei(2*log(c
)/p + 2*log(x^3*e + d))*e^(-3)/(c^(2/p)*p) + 1/3*Ei(3*log(c)/p + 3*log(x^3*
e + d))*e^(-3)/(c^(3/p)*p)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/ln(c*(e*x^3+d)^p),x)
```

```
[Out] int(x^8/ln(c*(e*x^3+d)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(x^8/log((e*x^3 + d)^p*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/log(c*(d + e*x^3)^p),x)
```

```
[Out] int(x^8/log(c*(d + e*x^3)^p), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/ln(c*(e*x**3+d)**p),x)
[Out] Integral(x**8/log(c*(d + e*x**3)**p), x)
```

3.139 $\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=107

$$\frac{\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^2p}-\frac{d\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^2p}$$

```
[Out] -1/3*d*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e^2/p/((c*(e*x^3+d)^p)^(1/p))+1/3*
(e*x^3+d)^2*Ei(2*ln(c*(e*x^3+d)^p)/p)/e^2/p/((c*(e*x^3+d)^p)^(2/p))
```

Rubi [A] time = 0.15, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2454, 2399, 2389, 2300, 2178, 2390, 2310}

$$\frac{\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^2p}-\frac{d\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^2p}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Log[c*(d + e*x^3)^p],x]
```

```
[Out] -(d*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e^2*p*(c*(d + e*x
^3)^p)^p^(-1)) + ((d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p])/
(3*e^2*p*(c*(d + e*x^3)^p)^(2/p))
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{\log\left(c(d+ex)^p\right)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\operatorname{Subst}\left(\int\left(-\frac{d}{e\log\left(c(d+ex)^p\right)}+\frac{d+ex}{e\log\left(c(d+ex)^p\right)}\right)dx, x, x^3\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{d+ex}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3e} - \frac{d\operatorname{Subst}\left(\int \frac{1}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3e}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e^2} - \frac{d\operatorname{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e^2}$$

$$= \frac{\left(\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\right)\operatorname{Subst}\left(\int \frac{e^{\frac{2x}{p}}}{x}\,dx, x, \log\left(c\left(d+ex^3\right)^p\right)\right)}{3e^2p} - \frac{\left(d\left(d+ex^3\right)\left(\right.\right.}{}$$

$$= -\frac{d\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^2p} + \frac{\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log\ldots}{}\right.}{3e^2p}$$

Mathematica [A] time = 0.12, size = 96, normalized size = 0.90

$$-\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-2/p}\left(d\left(c\left(d+ex^3\right)^p\right)^{\frac{1}{p}}\operatorname{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)-\left(d+ex^3\right)\operatorname{Ei}\left(\frac{2\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)\right)}{3e^2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Log[c*(d + e*x^3)^p],x]
```

```
[Out] -1/3*((d + e*x^3)*(d*(c*(d + e*x^3)^p)^p^(-1)*ExpIntegralEi[Log[c*(d + e*x^
3)^p]/p] - (d + e*x^3)*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p]))/(e^2*p*(
c*(d + e*x^3)^p)^(2/p))
```

fricas [A] time = 0.46, size = 68, normalized size = 0.64

$$-\frac{c^{\left(\frac{1}{p}\right)}d\,\mathrm{log\_integral}\left(\left(ex^3+d\right)c^{\left(\frac{1}{p}\right)}\right)-\mathrm{log\_integral}\left(\left(e^2x^6+2\,dex^3+d^2\right)c^{\frac{2}{p}}\right)}{3\,c^{\frac{2}{p}}e^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] -1/3*(c^(1/p)*d*log_integral((e*x^3 + d)*c^(1/p)) - log_integral((e^2*x^6 +
 2*d*e*x^3 + d^2)*c^(2/p)))/(c^(2/p)*e^2*p)
```

giac [A] time = 0.18, size = 72, normalized size = 0.67

$$-\frac{1}{3}\left(\frac{d\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(x^{3}e+d\right)\right)e^{(-1)}}{c^{\left(\frac{1}{p}\right)}p}-\frac{\mathrm{Ei}\left(\frac{2\log(c)}{p}+2\log\left(x^{3}e+d\right)\right)e^{(-1)}}{c^{\frac{2}{p}}p}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] -1/3*(d*Ei(log(c)/p + log(x^3*e + d))*e^(-1)/(c^(1/p)*p) - Ei(2*log(c)/p +
2*log(x^3*e + d))*e^(-1)/(c^(2/p)*p))*e^(-1)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\ln\left(c\left(e\,x^{3}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/ln(c*(e*x^3+d)^p),x)
```

```
[Out] int(x^5/ln(c*(e*x^3+d)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\log\left(\left(ex^{3}+d\right)^{p}c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(x^5/log((e*x^3 + d)^p*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5}}{\ln\left(c\left(e\,x^{3}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/log(c*(d + e*x^3)^p),x)
```

```
[Out] int(x^5/log(c*(d + e*x^3)^p), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\log\left(c\left(d+ex^{3}\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Integral(x**5/log(c*(d + e*x**3)**p), x)
```

3.140 $\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal. Leaf size=51

$$\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3ep}$$

```
[Out] 1/3*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e/p/((c*(e*x^3+d)^p)^(1/p))
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2454, 2389, 2300, 2178}

$$\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3ep}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Log[c*(d + e*x^3)^p],x]
```

```
[Out] ((d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e*p*(c*(d + e*x^3)^p
)^p^(-1))
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\log\left(c(d+ex)^p\right)}\,dx, x, x^3\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e}$$
$$= \frac{\left(\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\right)\operatorname{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c\left(d+ex^3\right)^p\right)\right)}{3ep}$$
$$= \frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3ep}$$

Mathematica [**A**] time = 0.04, size = 51, normalized size = 1.00

$$\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3ep}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Log[c*(d + e*x^3)^p],x]
[Out] ((d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e*p*(c*(d + e*x^3)^p
)^p^(-1))
```

fricas [**A**] time = 0.46, size = 29, normalized size = 0.57

$$\frac{\text{log\_integral}\left(\left(ex^3+d\right)c^{\left(\frac{1}{p}\right)}\right)}{3\,c^{\left(\frac{1}{p}\right)}ep}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] 1/3*log_integral((e*x^3 + d)*c^(1/p))/(c^(1/p)*e*p)
```

giac [**A**] time = 0.18, size = 31, normalized size = 0.61

$$\frac{\operatorname{Ei}\left(\frac{\log(c)}{p}+\log\left(x^3e+d\right)\right)e^{(-1)}}{3\,c^{\left(\frac{1}{p}\right)}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] 1/3*Ei(log(c)/p + log(x^3*e + d))*e^(-1)/(c^(1/p)*p)
```

maple [**C**] time = 1.28, size = 272, normalized size = 5.33

$$-\frac{\left(e\,x^3+d\right)c^{-\frac{1}{p}}\left(\left(e\,x^3+d\right)^p\right)^{-\frac{1}{p}}\operatorname{Ei}\left(1,-\ln\left(e\,x^3+d\right)-\frac{-i\pi\operatorname{csgn}(ic)\operatorname{csgn}\left(i\left(e\,x^3+d\right)^p\right)\operatorname{csgn}\left(ic\left(e\,x^3+d\right)^p\right)+i\pi\operatorname{csgn}(ic)\operatorname{csgn}\left(ic\left(e\,x^3+\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/ln(c*(e*x^3+d)^p),x)
```

```
[Out] -1/3/e/p*(e*x^3+d)*c^(-1/p)*((e*x^3+d)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*x
^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*
(e*x^3+d)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(
e*x^3+d)^p)^2-I*Pi*csgn(I*c)*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)-I*Pi
*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c)*csgn(I*c*(e*x^3+d)^p)^2+2*ln(c)+2*l
n((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(x^2/log((e*x^3 + d)^p*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/log(c*(d + e*x^3)^p),x)
```

```
[Out] int(x^2/log(c*(d + e*x^3)^p), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Integral(x**2/log(c*(d + e*x**3)**p), x)
```

3.141 $\int \frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

[Out] Unintegrable(1/x/ln(c*(e*x^3+d)^p),x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/(x*Log[c*(d + e*x^3)^p]),x]

[Out] Defer[Int][1/(x*Log[c*(d + e*x^3)^p]), x]

Rubi steps

$$\int \frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [A] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x*Log[c*(d + e*x^3)^p]),x]

[Out] Integrate[1/(x*Log[c*(d + e*x^3)^p]), x]

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x\log\left(\left(ex^3+d\right)^p c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(c*(e*x^3+d)^p),x, algorithm="fricas")

[Out] integral(1/(x*log((e*x^3 + d)^p*c)), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate(1/(x*log((e*x^3 + d)^p*c)), x)
```

maple [A]   time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{1}{x \ln \left(c \left(e\,x^{3}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(c*(e*x^3+d)^p),x)
[Out] int(1/x/ln(c*(e*x^3+d)^p),x)
```

maxima [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \log \left(\left(e x^{3}+d\right)^{p} c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(1/(x*log((e*x^3 + d)^p*c)), x)
```

mupad [A]   time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\,\ln \left(c\,\left(e\,x^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(c*(d + e*x^3)^p)),x)
[Out] int(1/(x*log(c*(d + e*x^3)^p)), x)
```

sympy [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \log \left(c \left(d + e x^{3}\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(c*(e*x**3+d)**p),x)
[Out] Integral(1/(x*log(c*(d + e*x**3)**p)), x)
```

3.142 $\int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^4/ln(c*(e*x^3+d)^p),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*Log[c*(d + e*x^3)^p]),x]
[Out] Defer[Int][1/(x^4*Log[c*(d + e*x^3)^p]), x]
```

Rubi steps

$$\int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)} dx$$

Mathematica [A] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*Log[c*(d + e*x^3)^p]),x]
[Out] Integrate[1/(x^4*Log[c*(d + e*x^3)^p]), x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^4 \log\left(\left(ex^3+d\right)^p c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral(1/(x^4*log((e*x^3 + d)^p*c)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \log\left(\left(ex^3+d\right)^p c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] integrate(1/(x^4*log((e*x^3 + d)^p*c)), x)
```

maple [A] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/ln(c*(e*x^3+d)^p),x)
```

```
[Out] int(1/x^4/ln(c*(e*x^3+d)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*log((e*x^3 + d)^p*c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^4\,\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*log(c*(d + e*x^3)^p)),x)
```

```
[Out] int(1/(x^4*log(c*(d + e*x^3)^p)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Integral(1/(x**4*log(c*(d + e*x**3)**p)), x)
```

3.143 $\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(x^3/ln(c*(e*x^3+d)^p),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/Log[c*(d + e*x^3)^p],x]
[Out] Defer[Int][x^3/Log[c*(d + e*x^3)^p], x]
```

Rubi steps

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/Log[c*(d + e*x^3)^p],x]
[Out] Integrate[x^3/Log[c*(d + e*x^3)^p], x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\log\left(\left(ex^3+d\right)^p c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral(x^3/log((e*x^3 + d)^p*c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate(x^3/log((e*x^3 + d)^p*c), x)
```

maple [A] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\ln \left(c \left(e\,x^{3}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(e*x^3+d)^p),x)
[Out] int(x^3/ln(c*(e*x^3+d)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\log \left(\left(ex^{3}+d\right)^{p} c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(x^3/log((e*x^3 + d)^p*c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^{3}}{\ln \left(c \left(e\,x^{3}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/log(c*(d + e*x^3)^p),x)
[Out] int(x^3/log(c*(d + e*x^3)^p), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\log \left(c \left(d+ex^{3}\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(e*x**3+d)**p),x)
[Out] Integral(x**3/log(c*(d + e*x**3)**p), x)
```

3.144 $\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{x}{\log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

[Out] Unintegrable(x/ln(c*(e*x^3+d)^p),x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[x/Log[c*(d + e*x^3)^p],x]

[Out] Defer[Int][x/Log[c*(d + e*x^3)^p], x]

Rubi steps

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[x/Log[c*(d + e*x^3)^p],x]

[Out] Integrate[x/Log[c*(d + e*x^3)^p], x]

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{x}{\log\left(\left(ex^3+d\right)^p c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(c*(e*x^3+d)^p),x, algorithm="fricas")

[Out] integral(x/log((e*x^3 + d)^p*c), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate(x/log((e*x^3 + d)^p*c), x)
```

maple [A] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{x}{\ln\left(c\left(e\,x^{3}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(e*x^3+d)^p),x)
[Out] int(x/ln(c*(e*x^3+d)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\log\left(\left(ex^{3}+d\right)^{p}c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(x/log((e*x^3 + d)^p*c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x}{\ln\left(c\left(e\,x^{3}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/log(c*(d + e*x^3)^p),x)
[Out] int(x/log(c*(d + e*x^3)^p), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\log\left(c\left(d+ex^{3}\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(e*x**3+d)**p),x)
[Out] Integral(x/log(c*(d + e*x**3)**p), x)
```

3.145 $\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)} dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{\log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/ln(c*(e*x^3+d)^p),x)
```

Rubi [A] time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^3)^p]^(-1),x]
[Out] Defer[Int][Log[c*(d + e*x^3)^p]^(-1), x]
```

Rubi steps

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)} dx$$

Mathematica [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^(-1),x]
[Out] Integrate[Log[c*(d + e*x^3)^p]^(-1), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\log\left(\left(ex^3+d\right)^p c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral(1/log((e*x^3 + d)^p*c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log\left(\left(ex^3+d\right)^p c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] integrate(1/log((e*x^3 + d)^p*c), x)
```

maple [A] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{1}{\ln\left(c\left(e\,x^{3}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(c*(e*x^3+d)^p),x)
```

```
[Out] int(1/ln(c*(e*x^3+d)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log\left(\left(ex^{3}+d\right)^{p}c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/log((e*x^3 + d)^p*c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\ln\left(c\left(e\,x^{3}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(c*(d + e*x^3)^p),x)
```

```
[Out] int(1/log(c*(d + e*x^3)^p), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log\left(c\left(d+ex^{3}\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Integral(1/log(c*(d + e*x**3)**p), x)
```

3.146 $\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/ln(c*(e*x^3+d)^p),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Log[c*(d + e*x^3)^p]),x]
[Out] Defer[Int][1/(x^2*Log[c*(d + e*x^3)^p]), x]
```

Rubi steps

$$\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Mathematica [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Log[c*(d + e*x^3)^p]),x]
[Out] Integrate[1/(x^2*Log[c*(d + e*x^3)^p]), x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^2 \log\left(\left(ex^3+d\right)^p c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral(1/(x^2*log((e*x^3 + d)^p*c)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log\left(\left(ex^3+d\right)^p c\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate(1/(x^2*log((e*x^3 + d)^p*c)), x)
```

maple [A] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \ln \left(c \left(e\,x^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/ln(c*(e*x^3+d)^p),x)
[Out] int(1/x^2/ln(c*(e*x^3+d)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log \left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(1/(x^2*log((e*x^3 + d)^p*c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2\,\ln \left(c\,{\left(e\,x^3+d\right)}^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*log(c*(d + e*x^3)^p)),x)
[Out] int(1/(x^2*log(c*(d + e*x^3)^p)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log \left(c \left(d+ex^3\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(c*(e*x**3+d)**p),x)
[Out] Integral(1/(x**2*log(c*(d + e*x**3)**p)), x)
```

3.147 $\int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)}, x\right)$$

```
[Out] Unintegrable(1/x^3/ln(c*(e*x^3+d)^p),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*Log[c*(d + e*x^3)^p]),x]
```

```
[Out] Defer[Int][1/(x^3*Log[c*(d + e*x^3)^p]), x]
```

Rubi steps

$$\int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Mathematica [A] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*Log[c*(d + e*x^3)^p]),x]
```

```
[Out] Integrate[1/(x^3*Log[c*(d + e*x^3)^p]), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^3 \log\left(\left(ex^3+d\right)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] integral(1/(x^3*log((e*x^3 + d)^p*c)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log\left(\left(ex^3+d\right)^p c\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate(1/(x^3*log((e*x^3 + d)^p*c)), x)
```

maple [A] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/ln(c*(e*x^3+d)^p),x)
[Out] int(1/x^3/ln(c*(e*x^3+d)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(1/(x^3*log((e*x^3 + d)^p*c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^3\,\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*log(c*(d + e*x^3)^p)),x)
[Out] int(1/(x^3*log(c*(d + e*x^3)^p)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(c*(e*x**3+d)**p),x)
[Out] Integral(1/(x**3*log(c*(d + e*x**3)**p)), x)
```

3.148 $\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=195

$$\frac{d^2\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^3p^2}+\frac{\left(d+ex^3\right)^3\left(c\left(d+ex^3\right)^p\right)^{-3/p}\operatorname{Ei}\left(\frac{3\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{e^3p^2}-\frac{4d\left(d+ex^3\right)}{}$$

```
[Out] 1/3*d^2*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e^3/p^2/((c*(e*x^3+d)^p)^(1/p))-4
/3*d*(e*x^3+d)^2*Ei(2*ln(c*(e*x^3+d)^p)/p)/e^3/p^2/((c*(e*x^3+d)^p)^(2/p))+
(e*x^3+d)^3*Ei(3*ln(c*(e*x^3+d)^p)/p)/e^3/p^2/((c*(e*x^3+d)^p)^(3/p))-1/3*x
^6*(e*x^3+d)/e/p/ln(c*(e*x^3+d)^p)
```

Rubi [A] time = 0.38, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2454, 2400, 2399, 2389, 2300, 2178, 2390, 2310}

$$\frac{d^2\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^3p^2}+\frac{\left(d+ex^3\right)^3\left(c\left(d+ex^3\right)^p\right)^{-3/p}\operatorname{Ei}\left(\frac{3\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{e^3p^2}-\frac{4d\left(d+ex^3\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (d^2*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e^3*p^2*(c*(d +
e*x^3)^p)^p^(-1)) - (4*d*(d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p
])/p])/(3*e^3*p^2*(c*(d + e*x^3)^p)^(2/p)) + ((d + e*x^3)^3*ExpIntegralEi[(
3*Log[c*(d + e*x^3)^p])/p])/(e^3*p^2*(c*(d + e*x^3)^p)^(3/p)) - (x^6*(d + e
*x^3))/(3*e*p*Log[c*(d + e*x^3)^p])
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2400

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((d + e*x)*(f + g*x)^q*(a + b*Log[c*(d + e
*x)^n])^(p + 1))/(b*e*n*(p + 1)), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[(q*(e*f - d*g))
/(b*e*n*(p + 1)), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\log^2\left(c(d+ex)^p\right)}\,dx, x, x^3\right)$$

$$= -\frac{x^6\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{x^2}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{p} + \frac{(2d)\,\text{Subst}\left(\int \frac{x}{\log(c(d+e}\right.}{3ep}$$

$$= -\frac{x^6\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\text{Subst}\left(\int \left(\frac{d^2}{e^2\log(c(d+ex)^p)} - \frac{2d(d+ex)}{e^2\log(c(d+ex)^p)} + \frac{(d+ex)^2}{e^2\log(c(d+ex)^p)}\right)\right.}{p}$$

$$= -\frac{x^6\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{(d+ex)^2}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{e^2p} + \frac{(2d)\,\text{Subst}\left(\int \frac{d+ex}{\log(c(d+e}\right.}{3e^2p}$$

$$= -\frac{x^6\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{x^2}{\log(cx^p)}\,dx, x, d+ex^3\right)}{e^3p} + \frac{(2d)\,\text{Subst}\left(\int \frac{x}{\log(cx^p)}\right.}{3e^3p}$$

$$= -\frac{x^6\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\left(\left(d+ex^3\right)^3\left(c\left(d+ex^3\right)^p\right)^{-3/p}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{p}}}{x}\,dx, x, \log\left(\right.\right.}{e^3p^2}$$

$$= \frac{d^2\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^3p^2} - \frac{4d\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}}{3e^3p^2}$$

Mathematica [A] time = 0.28, size = 290, normalized size = 1.49

$$\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-3/p}\left(d^2\left(c\left(d+ex^3\right)^p\right)^{2/p}\log\left(c\left(d+ex^3\right)^p\right)\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right) + 3d^2\log\left(c\left(d+ex^3\right)^p\right)\text{Ei}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] ((d + e*x^3)*(-(e^2*p*x^6*(c*(d + e*x^3)^p)^(3/p)) + d^2*(c*(d + e*x^3)^p)^
(2/p)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p]*Log[c*(d + e*x^3)^p] - 4*d*(d +
 e*x^3)*(c*(d + e*x^3)^p)^p^(-1)*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p]*
Log[c*(d + e*x^3)^p] + 3*d^2*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p]*Log[
c*(d + e*x^3)^p] + 6*d*e*x^3*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p]*Log[
c*(d + e*x^3)^p] + 3*e^2*x^6*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p]*Log[
c*(d + e*x^3)^p]))/(3*e^3*p^2*(c*(d + e*x^3)^p)^(3/p)*Log[c*(d + e*x^3)^p])
```

fricas [A] time = 0.45, size = 211, normalized size = 1.08

$$-\frac{4\left(dp\log\left(ex^3+d\right)+d\log(c)\right)c^{\left(\frac{1}{p}\right)}\,\text{log\_integral}\left(\left(e^2x^6+2\,dex^3+d^2\right)c^{\frac{2}{p}}\right) - \left(d^2p\log\left(ex^3+d\right)+d^2\log(c)\right)c^{\frac{2}{p}}}{3\left(e^3p^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*(d*p*log(e*x^3 + d) + d*log(c))*c^(1/p)*log_integral((e^2*x^6 + 2*d
*e*x^3 + d^2)*c^(2/p)) - (d^2*p*log(e*x^3 + d) + d^2*log(c))*c^(2/p)*log_in
tegral((e*x^3 + d)*c^(1/p)) + (e^3*p*x^9 + d*e^2*p*x^6)*c^(3/p) - 3*(p*log(
e*x^3 + d) + log(c))*log_integral((e^3*x^9 + 3*d*e^2*x^6 + 3*d^2*e*x^3 + d^
3)*c^(3/p)))/((e^3*p^3*log(e*x^3 + d) + e^3*p^2*log(c))*c^(3/p))
```

giac [B] time = 0.23, size = 494, normalized size = 2.53

$$-\frac{\left(x^3e+d\right)^3p}{3\left(p^3e^3\log\left(x^3e+d\right)+p^2e^3\log(c)\right)}+\frac{2\left(x^3e+d\right)^2dp}{3\left(p^3e^3\log\left(x^3e+d\right)+p^2e^3\log(c)\right)}-\frac{\left(x^3e+d\right)d^2p}{3\left(p^3e^3\log\left(x^3e+d\right)+p^2e^3\log(c)\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] -1/3*(x^3*e + d)^3*p/(p^3*e^3*log(x^3*e + d) + p^2*e^3*log(c)) + 2/3*(x^3*e
 + d)^2*d*p/(p^3*e^3*log(x^3*e + d) + p^2*e^3*log(c)) - 1/3*(x^3*e + d)*d^2
*p/(p^3*e^3*log(x^3*e + d) + p^2*e^3*log(c)) + 1/3*d^2*p*Ei(log(c)/p + log(
x^3*e + d))*log(x^3*e + d)/((p^3*e^3*log(x^3*e + d) + p^2*e^3*log(c))*c^(1/
p)) - 4/3*d*p*Ei(2*log(c)/p + 2*log(x^3*e + d))*log(x^3*e + d)/((p^3*e^3*lo
g(x^3*e + d) + p^2*e^3*log(c))*c^(2/p)) + 1/3*d^2*Ei(log(c)/p + log(x^3*e +
 d))*log(c)/((p^3*e^3*log(x^3*e + d) + p^2*e^3*log(c))*c^(1/p)) + p*Ei(3*lo
g(c)/p + 3*log(x^3*e + d))*log(x^3*e + d)/((p^3*e^3*log(x^3*e + d) + p^2*e^
3*log(c))*c^(3/p)) - 4/3*d*Ei(2*log(c)/p + 2*log(x^3*e + d))*log(c)/((p^3*e
^3*log(x^3*e + d) + p^2*e^3*log(c))*c^(2/p)) + Ei(3*log(c)/p + 3*log(x^3*e
+ d))*log(c)/((p^3*e^3*log(x^3*e + d) + p^2*e^3*log(c))*c^(3/p))
```

maple [F] time = 3.98, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(x^8/ln(c*(e*x^3+d)^p)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ex^9+dx^6}{3\left(ep\log\left(\left(ex^3+d\right)^p\right)+ep\log(c)\right)}+\int\frac{3\,ex^8+2\,dx^5}{ep\log\left(\left(ex^3+d\right)^p\right)+ep\log(c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^9 + d*x^6)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate((3*e
*x^8 + 2*d*x^5)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(x^8/log(c*(d + e*x^3)^p)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(x**8/log(c*(d + e*x**3)**p)**2, x)
```

3.149 $\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=141

$$\frac{2\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^2p^2}-\frac{d\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^2p^2}-\frac{x^3\left(d+ex^3\right)}{3ep\log\left(c\left(d+\right.\right.}$$

```
[Out] -1/3*d*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e^2/p^2/((c*(e*x^3+d)^p)^(1/p))+2/
3*(e*x^3+d)^2*Ei(2*ln(c*(e*x^3+d)^p)/p)/e^2/p^2/((c*(e*x^3+d)^p)^(2/p))-1/3
*x^3*(e*x^3+d)/e/p/ln(c*(e*x^3+d)^p)
```

Rubi [A] time = 0.21, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2454, 2400, 2399, 2389, 2300, 2178, 2390, 2310}

$$\frac{2\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^2p^2}-\frac{d\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^2p^2}-\frac{x^3\left(d+ex^3\right)}{3ep\log\left(c\left(d+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] -(d*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e^2*p^2*(c*(d + e
*x^3)^p)^p^(-1)) + (2*(d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/
p])/(3*e^2*p^2*(c*(d + e*x^3)^p)^(2/p)) - (x^3*(d + e*x^3))/(3*e*p*Log[c*(d
 + e*x^3)^p])
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2399

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2400

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((d + e*x)*(f + g*x)^q*(a + b*Log[c*(d + e
*x)^n])^(p + 1))/(b*e*n*(p + 1)), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[(q*(e*f - d*g))
/(b*e*n*(p + 1)), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{\log^2\left(c(d+ex)^p\right)}\,dx, x, x^3\right)$$

$$= -\frac{x^3\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{2\,\text{Subst}\left(\int \frac{x}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3p} + \frac{d\,\text{Subst}\left(\int \frac{1}{\log(c(d+ex)^p)}\right.}{3ep}$$

$$= -\frac{x^3\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{2\,\text{Subst}\left(\int\left(-\frac{d}{e\log(c(d+ex)^p)} + \frac{d+ex}{e\log(c(d+ex)^p)}\right)dx, x, x^3\right)}{3p} + \frac{d\,\text{Su}}{}$$

$$= -\frac{x^3\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{2\,\text{Subst}\left(\int \frac{d+ex}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3ep} - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{\log(c(d+ex)}\right.}{3ep}$$

$$= \frac{d\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^2p^2} - \frac{x^3\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{2\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{d\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^2p^2} - \frac{x^3\left(d+ex^3\right)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\left(2\left(d+ex^3\right.\right.}{}$$

$$= -\frac{d\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^2p^2} + \frac{2\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\text{Ei}\left(\right.}{3e^2p^2}$$

Mathematica [A] time = 0.14, size = 157, normalized size = 1.11

$$-\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-2/p}\left(d\left(c\left(d+ex^3\right)^p\right)^{\frac{1}{p}}\log\left(c\left(d+ex^3\right)^p\right)\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right) - 2\left(d+ex^3\right)\log\left(c\left(d+ex^3\right)^p\right.\right.}{3e^2p^2\log\left(c\left(d+ex^3\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] -1/3*((d + e*x^3)*(e*p*x^3*(c*(d + e*x^3)^p)^(2/p) + d*(c*(d + e*x^3)^p)^p^
(-1)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p]*Log[c*(d + e*x^3)^p] - 2*(d + e*
x^3)*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p]*Log[c*(d + e*x^3)^p]))/(e^2*
p^2*(c*(d + e*x^3)^p)^(2/p)*Log[c*(d + e*x^3)^p])
```

fricas [A] time = 0.46, size = 141, normalized size = 1.00

$$-\frac{\left(dp\log\left(ex^3+d\right) + d\log(c)\right)c^{\left(\frac{1}{p}\right)}\text{log\_integral}\left(\left(ex^3+d\right)c^{\left(\frac{1}{p}\right)}\right) + \left(e^2px^6 + depx^3\right)c^{\frac{2}{p}} - 2\left(p\log\left(ex^3+d\right) + \log(c\right.}{3\left(e^2p^3\log\left(ex^3+d\right) + e^2p^2\log(c)\right)c^{\frac{2}{p}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] -1/3*((d*p*log(e*x^3 + d) + d*log(c))*c^(1/p)*log_integral((e*x^3 + d)*c^(1
/p)) + (e^2*p*x^6 + d*e*p*x^3)*c^(2/p) - 2*(p*log(e*x^3 + d) + log(c))*log_
integral((e^2*x^6 + 2*d*e*x^3 + d^2)*c^(2/p)))/((e^2*p^3*log(e*x^3 + d) + e
^2*p^2*log(c))*c^(2/p))
```

giac [B] time = 0.22, size = 323, normalized size = 2.29

$$-\frac{1}{3}\left(\frac{\left(x^3e+d\right)^2p}{p^3e\log\left(x^3e+d\right)+p^2e\log(c)}-\frac{\left(x^3e+d\right)dp}{p^3e\log\left(x^3e+d\right)+p^2e\log(c)}+\frac{dp\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(x^3e+d\right)\right)\log\left(x^3e+\right.}{\left(p^3e\log\left(x^3e+d\right)+p^2e\log(c)\right)c^{\left(\frac{1}{p}\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] -1/3*((x^3*e + d)^2*p/(p^3*e*log(x^3*e + d) + p^2*e*log(c)) - (x^3*e + d)*d
*p/(p^3*e*log(x^3*e + d) + p^2*e*log(c)) + d*p*Ei(log(c)/p + log(x^3*e + d)
)*log(x^3*e + d)/((p^3*e*log(x^3*e + d) + p^2*e*log(c))*c^(1/p)) - 2*p*Ei(2
*log(c)/p + 2*log(x^3*e + d))*log(x^3*e + d)/((p^3*e*log(x^3*e + d) + p^2*e
*log(c))*c^(2/p)) + d*Ei(log(c)/p + log(x^3*e + d))*log(c)/((p^3*e*log(x^3*
e + d) + p^2*e*log(c))*c^(1/p)) - 2*Ei(2*log(c)/p + 2*log(x^3*e + d))*log(c
)/((p^3*e*log(x^3*e + d) + p^2*e*log(c))*c^(2/p)))*e^(-1)
```

maple [F] time = 7.07, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(x^5/ln(c*(e*x^3+d)^p)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ex^6+dx^3}{3\left(ep\log\left(\left(ex^3+d\right)^p\right)+ep\log(c)\right)}+\int \frac{2\,ex^5+dx^2}{ep\log\left(\left(ex^3+d\right)^p\right)+ep\log(c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^6 + d*x^3)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate((2*e
*x^5 + d*x^2)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(x^5/log(c*(d + e*x^3)^p)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(x**5/log(c*(d + e*x**3)**p)**2, x)
```

3.150 $\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal. Leaf size=83

$$\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3ep^2}-\frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

```
[Out] 1/3*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e/p^2/((c*(e*x^3+d)^p)^(1/p))+1/3*(-e
*x^3-d)/e/p/ln(c*(e*x^3+d)^p)
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2454, 2389, 2297, 2300, 2178}

$$\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3ep^2}-\frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] ((d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e*p^2*(c*(d + e*x^3)
^p)^p^(-1)) - (d + e*x^3)/(3*e*p*Log[c*(d + e*x^3)^p])
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{\log^2\left(c(d+ex)^p\right)}\,dx,x,x^3\right)$$
$$=\frac{\text{Subst}\left(\int \frac{1}{\log^2(cx^p)}\,dx,x,d+ex^3\right)}{3e}$$
$$=-\frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}+\frac{\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx,x,d+ex^3\right)}{3ep}$$
$$=-\frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}+\frac{\left(\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx,x,\log\left(c\left(d+\right.\right.\right.}{3ep^2}$$
$$=\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3ep^2}-\frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

Mathematica [A] time = 0.05, size = 97, normalized size = 1.17

$$-\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\left(p\left(c\left(d+ex^3\right)^p\right)^{\frac{1}{p}}-\log\left(c\left(d+ex^3\right)^p\right)\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)\right)}{3ep^2\log\left(c\left(d+ex^3\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] -1/3*((d + e*x^3)*(p*(c*(d + e*x^3)^p)^p^(-1) - ExpIntegralEi[Log[c*(d + e*
x^3)^p]/p]*Log[c*(d + e*x^3)^p]))/(e*p^2*(c*(d + e*x^3)^p)^p^(-1)*Log[c*(d
+ e*x^3)^p])
```

fricas [A] time = 0.43, size = 78, normalized size = 0.94

$$-\frac{\left(epx^3+dp\right)c^{\left(\frac{1}{p}\right)}-\left(p\log\left(ex^3+d\right)+\log(c)\right)\text{log\_integral}\left(\left(ex^3+d\right)c^{\left(\frac{1}{p}\right)}\right)}{3\left(ep^3\log\left(ex^3+d\right)+ep^2\log(c)\right)c^{\left(\frac{1}{p}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] -1/3*((e*p*x^3 + d*p)*c^(1/p) - (p*log(e*x^3 + d) + log(c))*log_integral((e
*x^3 + d)*c^(1/p)))/((e*p^3*log(e*x^3 + d) + e*p^2*log(c))*c^(1/p))
```

giac [A] time = 0.18, size = 154, normalized size = 1.86

$$-\frac{\left(x^3e+d\right)p}{3\left(p^3e\log\left(x^3e+d\right)+p^2e\log(c)\right)}+\frac{p\text{Ei}\left(\frac{\log(c)}{p}+\log\left(x^3e+d\right)\right)\log\left(x^3e+d\right)}{3\left(p^3e\log\left(x^3e+d\right)+p^2e\log(c)\right)c^{\left(\frac{1}{p}\right)}}+\frac{\text{Ei}\left(\frac{\log(c)}{p}+\log\left(x^3e+d\right)\right)\log(\ldots}{3\left(p^3e\log\left(x^3e+d\right)+p^2e\log(c)\right)\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] -1/3*(x^3*e + d)*p/(p^3*e*log(x^3*e + d) + p^2*e*log(c)) + 1/3*p*Ei(log(c)/
p + log(x^3*e + d))*log(x^3*e + d)/((p^3*e*log(x^3*e + d) + p^2*e*log(c))*c
^(1/p)) + 1/3*Ei(log(c)/p + log(x^3*e + d))*log(c)/((p^3*e*log(x^3*e + d) +
 p^2*e*log(c))*c^(1/p))
```

maple [C] time = 1.33, size = 421, normalized size = 5.07

$$-\frac{\left(e\,x^{3}+d\right)c^{-\frac{1}{p}}\left(\left(e\,x^{3}+d\right)^{p}\right)^{-\frac{1}{p}}\mathrm{Ei}\left(1,-\ln\left(e\,x^{3}+d\right)-\frac{-i\pi\,\mathrm{csgn}(ic)\mathrm{csgn}\left(i\left(e\,x^{3}+d\right)^{p}\right)\mathrm{csgn}\left(ic\left(e\,x^{3}+d\right)^{p}\right)+i\pi\,\mathrm{csgn}(ic)\mathrm{csgn}\left(ic\left(e\,x^{3}+\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] -2/3/(2*ln(c)+2*ln((e*x^3+d)^p)+I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)
^p)^2-I*Pi*csgn(I*c)*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)-I*Pi*csgn(I*
c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c)*csgn(I*c*(e*x^3+d)^p)^2)/p/e*(e*x^3+d)-1/3/
p^2/e*(e*x^3+d)*((e*x^3+d)^p)^(-1/p)*c^(-1/p)*exp(1/2*I*Pi*(csgn(I*c)-csgn(
I*c*(e*x^3+d)^p))*(csgn(I*(e*x^3+d)^p)-csgn(I*c*(e*x^3+d)^p))/p*csgn(I*c*(e
*x^3+d)^p))*Ei(1,-ln(e*x^3+d)-1/2*(-I*Pi*csgn(I*c)*csgn(I*(e*x^3+d)^p)*csgn
(I*c*(e*x^3+d)^p)+I*Pi*csgn(I*c)*csgn(I*c*(e*x^3+d)^p)^2+I*Pi*csgn(I*(e*x^3
+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*c*(e*x^3+d)^p)^3-2*p*ln(e*x^3+d)
+2*ln(c)+2*ln((e*x^3+d)^p))/p)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ex^{3}+d}{3\left(ep\log\left(\left(ex^{3}+d\right)^{p}\right)+ep\log(c)\right)}+\int\frac{x^{2}}{p\log\left(\left(ex^{3}+d\right)^{p}\right)+p\log(c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate(x^2/(p*l
og((e*x^3 + d)^p) + p*log(c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{2}}{\ln\left(c\left(e\,x^{3}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(x^2/log(c*(d + e*x^3)^p)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2}}{\log\left(c\left(d+ex^{3}\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/ln(c*(e*x**3+d)**p)**2,x)
[Out] Integral(x**2/log(c*(d + e*x**3)**p)**2, x)
```

3.151 $\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Log[c*(d + e*x^3)^p]^2),x]
[Out] Defer[Int][1/(x*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [A] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Log[c*(d + e*x^3)^p]^2),x]
[Out] Integrate[1/(x*Log[c*(d + e*x^3)^p]^2), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x\log\left(\left(ex^3+d\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] integral(1/(x*log((e*x^3 + d)^p*c)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*log((e*x^3 + d)^p*c)^2), x)
```

maple [A] time = 2.06, size = 0, normalized size = 0.00

$$\int \frac{1}{x \ln\left(c\left(e\,x^{3}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(1/x/ln(c*(e*x^3+d)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-d\int \frac{1}{epx^{4}\log\left(\left(ex^{3}+d\right)^{p}\right)+epx^{4}\log(c)}\,dx-\frac{ex^{3}+d}{3\left(epx^{3}\log\left(\left(ex^{3}+d\right)^{p}\right)+epx^{3}\log(c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -d*integrate(1/(e*p*x^4*log((e*x^3 + d)^p) + e*p*x^4*log(c)), x) - 1/3*(e*x
^3 + d)/(e*p*x^3*log((e*x^3 + d)^p) + e*p*x^3*log(c))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x \ln\left(c\left(e\,x^{3}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(c*(d + e*x^3)^p)^2),x)
```

```
[Out] int(1/(x*log(c*(d + e*x^3)^p)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\log\left(c\left(d+ex^{3}\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(1/(x*log(c*(d + e*x**3)**p)**2), x)
```

3.152 $\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^4/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*Log[c*(d + e*x^3)^p]^2),x]
[Out] Defer[Int][1/(x^4*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Mathematica [A] time = 1.53, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*Log[c*(d + e*x^3)^p]^2),x]
[Out] Integrate[1/(x^4*Log[c*(d + e*x^3)^p]^2), x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^4 \log\left(\left(ex^3+d\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] integral(1/(x^4*log((e*x^3 + d)^p*c)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \log\left(\left(ex^3+d\right)^p c\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^4*log((e*x^3 + d)^p*c)^2), x)
```

maple [A] time = 4.34, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \ln \left(c \left(e\,x^{3}+d\right)^{p}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(1/x^4/ln(c*(e*x^3+d)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ex^{3}+d}{3\left(epx^{6}\log\left(\left(ex^{3}+d\right)^{p}\right)+epx^{6}\log\left(c\right)\right)}-\int \frac{ex^{3}+2\,d}{epx^{7}\log\left(\left(ex^{3}+d\right)^{p}\right)+epx^{7}\log\left(c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*x^6*log((e*x^3 + d)^p) + e*p*x^6*log(c)) - integrate(
(e*x^3 + 2*d)/(e*p*x^7*log((e*x^3 + d)^p) + e*p*x^7*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^{4} \ln \left(c \left(e\,x^{3}+d\right)^{p}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*log(c*(d + e*x^3)^p)^2),x)
```

```
[Out] int(1/(x^4*log(c*(d + e*x^3)^p)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \log \left(c \left(d+ex^{3}\right)^{p}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(1/(x**4*log(c*(d + e*x**3)**p)**2), x)
```

3.153 $\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(x^3/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/Log[c*(d + e*x^3)^p]^2,x]
[Out] Defer[Int][x^3/Log[c*(d + e*x^3)^p]^2, x]
```

Rubi steps

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Mathematica [A] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/Log[c*(d + e*x^3)^p]^2,x]
[Out] Integrate[x^3/Log[c*(d + e*x^3)^p]^2, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\log\left(\left(ex^3+d\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] integral(x^3/log((e*x^3 + d)^p*c)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log\left(\left(ex^3+d\right)^p c\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/log((e*x^3 + d)^p*c)^2, x)
```

maple [A] time = 3.77, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\ln \left(c \left(e\,x^{3}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(x^3/ln(c*(e*x^3+d)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ex^{4}+dx}{3\left(ep\log\left(\left(ex^{3}+d\right)^{p}\right)+ep\log\left(c\right)\right)}+\int \frac{4\,ex^{3}+d}{3\left(ep\log\left(\left(ex^{3}+d\right)^{p}\right)+ep\log\left(c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^4 + d*x)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate(1/3*(4
*e*x^3 + d)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^{3}}{\ln \left(c \left(e\,x^{3}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(x^3/log(c*(d + e*x^3)^p)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\log \left(c \left(d+ex^{3}\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(x**3/log(c*(d + e*x**3)**p)**2, x)
```

3.154 $\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(x/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/Log[c*(d + e*x^3)^p]^2,x]
[Out] Defer[Int][x/Log[c*(d + e*x^3)^p]^2, x]
```

Rubi steps

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [A] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/Log[c*(d + e*x^3)^p]^2,x]
[Out] Integrate[x/Log[c*(d + e*x^3)^p]^2, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\log\left(\left(ex^3+d\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] integral(x/log((e*x^3 + d)^p*c)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x/log((e*x^3 + d)^p*c)^2, x)
```

maple [A] time = 4.05, size = 0, normalized size = 0.00

$$\int \frac{x}{\ln\left(c\left(e\,x^{3}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(x/ln(c*(e*x^3+d)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ex^{3}+d}{3\left(epx\log\left(\left(ex^{3}+d\right)^{p}\right)+epx\log\left(c\right)\right)}+\int \frac{2\,ex^{3}-d}{3\left(epx^{2}\log\left(\left(ex^{3}+d\right)^{p}\right)+epx^{2}\log\left(c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*x*log((e*x^3 + d)^p) + e*p*x*log(c)) + integrate(1/3*
(2*e*x^3 - d)/(e*p*x^2*log((e*x^3 + d)^p) + e*p*x^2*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x}{\ln\left(c\left(e\,x^{3}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(x/log(c*(d + e*x^3)^p)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\log\left(c\left(d+ex^{3}\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(x/log(c*(d + e*x**3)**p)**2, x)
```

3.155 $\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [A] time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^3)^p]^(-2),x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^3)^p]^(-2), x]
```

Rubi steps

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^(-2),x]
```

```
[Out] Integrate[Log[c*(d + e*x^3)^p]^(-2), x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\log\left(\left(ex^3+d\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^3 + d)^p*c)^(-2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)^(-2), x)
```

maple [A] time = 3.95, size = 0, normalized size = 0.00

$$\int \frac{1}{\ln\left(c\left(e\,x^{3}+d\right)^{p}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(1/ln(c*(e*x^3+d)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ex^{3}+d}{3\left(epx^{2}\log\left(\left(ex^{3}+d\right)^{p}\right)+epx^{2}\log\left(c\right)\right)}+\int \frac{ex^{3}-2\,d}{3\left(epx^{3}\log\left(\left(ex^{3}+d\right)^{p}\right)+epx^{3}\log\left(c\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*x^2*log((e*x^3 + d)^p) + e*p*x^2*log(c)) + integrate(
1/3*(e*x^3 - 2*d)/(e*p*x^3*log((e*x^3 + d)^p) + e*p*x^3*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\ln\left(c\left(e\,x^{3}+d\right)^{p}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(1/log(c*(d + e*x^3)^p)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log\left(c\left(d+ex^{3}\right)^{p}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(log(c*(d + e*x**3)**p)**(-2), x)
```

3.156 $\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Log[c*(d + e*x^3)^p]^2),x]
[Out] Defer[Int][1/(x^2*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Mathematica [A] time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Log[c*(d + e*x^3)^p]^2),x]
[Out] Integrate[1/(x^2*Log[c*(d + e*x^3)^p]^2), x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^2 \log\left(\left(ex^3+d\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] integral(1/(x^2*log((e*x^3 + d)^p*c)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log\left(\left(ex^3+d\right)^p c\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*log((e*x^3 + d)^p*c)^2), x)
```

maple [A] time = 4.24, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \ln \left(c \left(e \,x^{3}+d \right)^{p}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(1/x^2/ln(c*(e*x^3+d)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ex^{3}+d}{3 \left(epx^{4} \log \left(\left(ex^{3}+d\right)^{p}\right)+epx^{4} \log(c)\right)}-\int \frac{ex^{3}+4\,d}{3 \left(epx^{5} \log \left(\left(ex^{3}+d\right)^{p}\right)+epx^{5} \log(c)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*x^4*log((e*x^3 + d)^p) + e*p*x^4*log(c)) - integrate(
1/3*(e*x^3 + 4*d)/(e*p*x^5*log((e*x^3 + d)^p) + e*p*x^5*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^{2} \ln \left(c \left(e \,x^{3}+d \right)^{p}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*log(c*(d + e*x^3)^p)^2),x)
```

```
[Out] int(1/(x^2*log(c*(d + e*x^3)^p)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \log \left(c \left(d+ex^{3}\right)^{p}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(1/(x**2*log(c*(d + e*x**3)**p)**2), x)
```

3.157 $\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^3/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*Log[c*(d + e*x^3)^p]^2),x]
[Out] Defer[Int][1/(x^3*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Mathematica [A] time = 1.24, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*Log[c*(d + e*x^3)^p]^2),x]
[Out] Integrate[1/(x^3*Log[c*(d + e*x^3)^p]^2), x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^3 \log\left(\left(ex^3+d\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] integral(1/(x^3*log((e*x^3 + d)^p*c)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log\left(\left(ex^3+d\right)^p c\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*log((e*x^3 + d)^p*c)^2), x)
```

maple [A] time = 4.25, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \ln \left(c \left(e\,x^{3}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(1/x^3/ln(c*(e*x^3+d)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ex^{3}+d}{3\left(epx^{5}\log\left(\left(ex^{3}+d\right)^{p}\right)+epx^{5}\log\left(c\right)\right)}-\int \frac{2\,ex^{3}+5\,d}{3\left(epx^{6}\log\left(\left(ex^{3}+d\right)^{p}\right)+epx^{6}\log\left(c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*x^5*log((e*x^3 + d)^p) + e*p*x^5*log(c)) - integrate(
1/3*(2*e*x^3 + 5*d)/(e*p*x^6*log((e*x^3 + d)^p) + e*p*x^6*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^{3} \ln \left(c \left(e\,x^{3}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*log(c*(d + e*x^3)^p)^2),x)
```

```
[Out] int(1/(x^3*log(c*(d + e*x^3)^p)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \log \left(c \left(d+ex^{3}\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(1/(x**3*log(c*(d + e*x**3)**p)**2), x)
```

3.158 $\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=77

$$\frac{(fx)^{m+1}\log^3\left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{6ep\text{Int}\left(\frac{(fx)^{m+2}\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2},x\right)}{f^2(m+1)}$$

```
[Out] (f*x)^(1+m)*ln(c*(e*x^2+d)^p)^3/f/(1+m)-6*e*p*Unintegrable((f*x)^(2+m)*ln(c
*(e*x^2+d)^p)^2/(e*x^2+d),x)/f^2/(1+m)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p]^3)/(f*(1 + m)) - (6*e*p*Defer[Int][((f*
x)^(2 + m)*Log[c*(d + e*x^2)^p]^2)/(d + e*x^2), x])/(f^2*(1 + m))
```

Rubi steps

$$\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\frac{(fx)^{1+m}\log^3\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}-\frac{(6ep)\int \frac{(fx)^{2+m}\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx}{f^2(1+m)}$$

Mathematica [A] time = 2.20, size = 994, normalized size = 12.91

$$(fx)^m\left(\frac{6p^3\left(d\left(\left(-\frac{ex^2}{d}\right)^{\frac{m+1}{2}}-1\right)\log^2\left(ex^2+d\right)+(m+1)\left(ex^2+d\right){}_3F_2\left(1,1,\frac{1}{2}-\frac{m}{2};2,2;\frac{ex^2}{d}+1\right)\log\left(ex^2+d\right)-(m+1)\left(ex^2+d\right){}_4F_3\left(1,1,1,\frac{1}{2}-\frac{m}{2};2,2,2;\frac{ex^2}{d}+1\right)\right)\left(\right.}{e}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] ((f*x)^m*((1 + m)*p^3*x^2*Log[d + e*x^2]^3 + (6*p^3*(-((e*x^2)/d))^(1/2 - m
/2)*(-((1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1, 1/2 - m/2}, {2, 2, 2
}, 1 + (e*x^2)/d]) + (1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1/2 - m/2
}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2] + d*(-1 + (-((e*x^2)/d))^((1 + m)/
2))*Log[d + e*x^2]^2))/e + (6*d*(1 + m)*p^3*((e*x^2)/(d + e*x^2))^(1/2 - m/
2)*(8*HypergeometricPFQ[{1/2 - m/2, 1/2 - m/2, 1/2 - m/2, 1/2 - m/2}, {3/2
- m/2, 3/2 - m/2, 3/2 - m/2}, d/(d + e*x^2)] + (-1 + m)*Log[d + e*x^2]*(-4*
HypergeometricPFQ[{1/2 - m/2, 1/2 - m/2, 1/2 - m/2}, {3/2 - m/2, 3/2 - m/2}
, d/(d + e*x^2)] + (-1 + m)*Hypergeometric2F1[1/2 - m/2, 1/2 - m/2, 3/2 - m
/2, d/(d + e*x^2)]*Log[d + e*x^2])))/(e*(-1 + m)^3) - (3*p^2*(-((e*x^2)/d))
^(1/2 - m/2)*(-((1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1, 1/2 - m/2},
 {2, 2, 2}, 1 + (e*x^2)/d]) + (1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1,
1/2 - m/2}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2] + d*(-1 + (-((e*x^2)/d))^
```

```
((1 + m)/2))*Log[d + e*x^2]^2)*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])
)/e - (3*m*p^2*(-((e*x^2)/d))^(1/2 - m/2)*(-((1 + m)*(d + e*x^2)*Hypergeome
tricPFQ[{1, 1, 1, 1/2 - m/2}, {2, 2, 2}, 1 + (e*x^2)/d]) + (1 + m)*(d + e*x
^2)*HypergeometricPFQ[{1, 1, 1/2 - m/2}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x
^2] + d*(-1 + (-((e*x^2)/d))^((1 + m)/2))*Log[d + e*x^2]^2)*(-(p*Log[d + e*
x^2]) + Log[c*(d + e*x^2)^p]))/e + (3*p*x^2*(-2*e*x^2*Hypergeometric2F1[1,
(3 + m)/2, (5 + m)/2, -((e*x^2)/d)] + d*(3 + m)*Log[d + e*x^2])*(-(p*Log[d
+ e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(d*(3 + m)) + (3*m*p*x^2*(-2*e*x^2*Hyp
ergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/d)] + d*(3 + m)*Log[d + e
*x^2])*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(d*(3 + m)) + x^2*(-
(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^3 + m*x^2*(-(p*Log[d + e*x^2]) +
 Log[c*(d + e*x^2)^p])^3))/((1 + m)^2*x)
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(fx\right)^{m}\log\left(\left(ex^{2}+d\right)^{p}c\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((e*x^2 + d)^p*c)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m}\log\left(\left(ex^{2}+d\right)^{p}c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*x^2 + d)^p*c)^3, x)
```

maple [A] time = 0.87, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m}\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(e*x^2+d)^p)^3,x)
```

```
[Out] int((f*x)^m*ln(c*(e*x^2+d)^p)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{f^{m}p^{3}xx^{m}\log\left(ex^{2}+d\right)^{3}}{m+1}+\int \frac{3\left(\left(mp^{2}+p^{2}\right)df^{m}\log(c)-\left(2\,ef^{m}p^{3}-\left(mp^{2}+p^{2}\right)ef^{m}\log(c)\right)x^{2}\right)x^{m}\log\left(ex^{2}+d\right)^{2}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
```

```
[Out] f^m*p^3*x*x^m*log(e*x^2 + d)^3/(m + 1) + integrate((3*((m*p^2 + p^2)*d*f^m*
log(c) - (2*e*f^m*p^3 - (m*p^2 + p^2)*e*f^m*log(c))*x^2)*x^m*log(e*x^2 + d)
^2 + 3*((m*p + p)*e*f^m*x^2*log(c)^2 + (m*p + p)*d*f^m*log(c)^2)*x^m*log(e*
x^2 + d) + (e*f^m*(m + 1)*x^2*log(c)^3 + d*f^m*(m + 1)*log(c)^3)*x^m)/(e*(m
 + 1)*x^2 + d*(m + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{3}\left(f\,x\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^3*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^3*(f*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(e*x**2+d)**p)**3,x)
```

```
[Out] Timed out
```

3.159 $\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right) dx$

Optimal. Leaf size=75

$$\frac{(fx)^{m+1}\log^2\left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{4ep\text{Int}\left(\frac{(fx)^{m+2}\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2},x\right)}{f^2(m+1)}$$

```
[Out] (f*x)^(1+m)*ln(c*(e*x^2+d)^p)^2/f/(1+m)-4*e*p*Unintegrable((f*x)^(2+m)*ln(c
*(e*x^2+d)^p)/(e*x^2+d),x)/f^2/(1+m)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p]^2)/(f*(1 + m)) - (4*e*p*Defer[Int][((f*
x)^(2 + m)*Log[c*(d + e*x^2)^p])/(d + e*x^2), x])/(f^2*(1 + m))
```

Rubi steps

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right) dx=\frac{(fx)^{1+m}\log^2\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}-\frac{(4ep)\int \frac{(fx)^{2+m}\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}dx}{f^2(1+m)}$$

Mathematica [A] time = 1.08, size = 466, normalized size = 6.21

$$(fx)^m\left(\frac{4d(m+1)p^2\left(\frac{ex^2}{d+ex^2}\right)^{\frac{1}{2}-\frac{m}{2}}\left((m-1)\log\left(d+ex^2\right){}_2F_1\left(\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2};\frac{3}{2}-\frac{m}{2};\frac{d}{ex^2+d}\right)-2\,{}_3F_2\left(\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2};\frac{3}{2}-\frac{m}{2},\frac{3}{2}-\frac{m}{2};\frac{d}{ex^2+d}\right)\right)}{e(m-1)^2x}+\frac{2p\left(p\log\left(d+ex^2\right)-\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((f*x)^m*(4*p^2*x*((2*e*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e
*x^2)/d)])/(d*(3 + m)) - Log[d + e*x^2]) + (1 + m)*p^2*x*Log[d + e*x^2]^2 +
 (4*d*(1 + m)*p^2*((e*x^2)/(d + e*x^2))^(1/2 - m/2)*(-2*HypergeometricPFQ[{
1/2 - m/2, 1/2 - m/2, 1/2 - m/2}, {3/2 - m/2, 3/2 - m/2}, d/(d + e*x^2)] +
(-1 + m)*Hypergeometric2F1[1/2 - m/2, 1/2 - m/2, 3/2 - m/2, d/(d + e*x^2)]*
Log[d + e*x^2]))/(e*(-1 + m)^2*x) + (2*p*(2*e*x^3*Hypergeometric2F1[1, (3 +
 m)/2, (5 + m)/2, -((e*x^2)/d)] - d*(3 + m)*x*Log[d + e*x^2])*(p*Log[d + e*
x^2] - Log[c*(d + e*x^2)^p]))/(d*(3 + m)) - (2*m*p*(-2*e*x^3*Hypergeometric
2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/d)] + d*(3 + m)*x*Log[d + e*x^2])*(p
*Log[d + e*x^2] - Log[c*(d + e*x^2)^p]))/(d*(3 + m)) + x*(-(p*Log[d + e*x^2
]) + Log[c*(d + e*x^2)^p])^2 + m*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)
^p])^2))/(1 + m)^2
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fx\right)^m\log\left(\left(ex^2+d\right)^pc\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((e*x^2 + d)^p*c)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \log\left(\left(ex^2+d\right)^p c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*x^2 + d)^p*c)^2, x)
```

maple [A] time = 1.27, size = 0, normalized size = 0.00

$$\int (fx)^m \ln\left(c\left(e\,x^2+d\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((f*x)^m*ln(c*(e*x^2+d)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{f^m p^2 x x^m \log\left(ex^2+d\right)^2}{m+1} + \int \frac{2\left(\left(mp+p\right)df^m \log(c) - \left(2\,ef^m p^2 - \left(mp+p\right)ef^m \log(c)\right)x^2\right)x^m \log\left(ex^2+d\right) + (}{e(m+1)x^2 + d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] f^m*p^2*x*x^m*log(e*x^2 + d)^2/(m + 1) + integrate((2*((m*p + p)*d*f^m*log(
c) - (2*e*f^m*p^2 - (m*p + p)*e*f^m*log(c))*x^2)*x^m*log(e*x^2 + d) + (e*f^
m*(m + 1)*x^2*log(c)^2 + d*f^m*(m + 1)*log(c)^2)*x^m)/(e*(m + 1)*x^2 + d*(m
 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(e\,x^2+d\right)^p\right)^2 \left(f\,x\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^2*(f*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \log\left(c\left(d+ex^2\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f*x)**m*log(c*(d + e*x**2)**p)**2, x)
```

3.160 $\int (fx)^m \log\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=81

$$\frac{(fx)^{m+1}\log\left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{2ep(fx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};-\frac{ex^2}{d}\right)}{df^3(m+1)(m+3)}$$

```
[Out] -2*e*p*(f*x)^(3+m)*hypergeom([1, 3/2+1/2*m],[5/2+1/2*m],-e*x^2/d)/d/f^3/(1+
m)/(3+m)+(f*x)^(1+m)*ln(c*(e*x^2+d)^p)/f/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2455, 16, 364}

$$\frac{(fx)^{m+1}\log\left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{2ep(fx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};-\frac{ex^2}{d}\right)}{df^3(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-2*e*p*(f*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/
d)])/(d*f^3*(1 + m)*(3 + m)) + ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p])/(f*(1 +
 m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (fx)^m\log\left(c\left(d+ex^2\right)^p\right)\,dx&=\frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}-\frac{(2ep)\int\frac{x(fx)^{1+m}}{d+ex^2}\,dx}{f(1+m)}\\&=\frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}-\frac{(2ep)\int\frac{(fx)^{2+m}}{d+ex^2}\,dx}{f^2(1+m)}\\&=-\frac{2ep(fx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};-\frac{ex^2}{d}\right)}{df^3(1+m)(3+m)}+\frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 0.86

$$\frac{x(fx)^m \left(d(m+3)\log\left(c\left(d+ex^2\right)^p\right) - 2epx^2\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};-\frac{ex^2}{d}\right)\right)}{d(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (x*(f*x)^m*(-2*e*p*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)
/d)] + d*(3 + m)*Log[c*(d + e*x^2)^p]))/(d*(1 + m)*(3 + m))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fx\right)^m \log\left(\left(ex^2+d\right)^p c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((e*x^2 + d)^p*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \log\left(\left(ex^2+d\right)^p c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*x^2 + d)^p*c), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \ln\left(c\left(e\,x^2+d\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(e*x^2+d)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(e*x^2+d)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{f^m p x x^m \log\left(ex^2+d\right)}{m+1} + \int \frac{\left(df^m(m+1)\log(c) + \left(ef^m(m+1)\log(c) - 2\,ef^m p\right)x^2\right)x^m}{e(m+1)x^2 + d(m+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] f^m*p*x*x^m*log(e*x^2 + d)/(m + 1) + integrate((d*f^m*(m + 1)*log(c) + (e*f
^m*(m + 1)*log(c) - 2*e*f^m*p)*x^2)*x^m/(e*(m + 1)*x^2 + d*(m + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(e\,x^2+d\right)^p\right)\left(f\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)*(f*x)^m, x)
```

sympy [A] time = 94.77, size = 359, normalized size = 4.43

$$-2ep\begin{cases}\begin{cases}\frac{0^m\sqrt{-\frac{d}{e^3}}\log\left(-e\sqrt{-\frac{d}{e^3}}+x\right)}{2}-\frac{0^m\sqrt{-\frac{d}{e^3}}\log\left(e\sqrt{-\frac{d}{e^3}}+x\right)}{2}+\frac{0^m x}{e} & \text{fo}\\ \frac{ff^m m x^3 x^m \Phi\left(\frac{ex^2e^{i\pi}}{d},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4dfm\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+4df\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{3ff^m x^3 x^m \Phi\left(\frac{ex^2e^{i\pi}}{d},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4dfm\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+4df\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} & \text{fo}\\ -\frac{\begin{cases}\log(d)\log(x)-\frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } |x|<1\\ -\log(d)\log\left(\frac{1}{x}\right)-\frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(d)-\frac{\mathrm{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{otherwise}\end{cases}}{2ef}+\frac{\log(fx)\log(d+ex^2)}{2ef} & \text{o}\end{cases}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(e*x**2+d)**p),x)
```

```
[Out] -2*e*p*Piecewise((0**m*sqrt(-d/e**3)*log(-e*sqrt(-d/e**3) + x)/2 - 0**m*sqr
t(-d/e**3)*log(e*sqrt(-d/e**3) + x)/2 + 0**m*x/e, Eq(f, 0) | (Eq(f, 0) & Ne
(m, -1))), (f*f**m*m*x**3*x**m*lerchphi(e*x**2*exp_polar(I*pi)/d, 1, m/2 +
3/2)*gamma(m/2 + 3/2)/(4*d*f*m*gamma(m/2 + 5/2) + 4*d*f*gamma(m/2 + 5/2)) +
 3*f*f**m*x**3*x**m*lerchphi(e*x**2*exp_polar(I*pi)/d, 1, m/2 + 3/2)*gamma(
m/2 + 3/2)/(4*d*f*m*gamma(m/2 + 5/2) + 4*d*f*gamma(m/2 + 5/2)), (m > -oo) &
 (m < oo) & Ne(m, -1)), (-Piecewise((log(d)*log(x) - polylog(2, e*x**2*exp_
polar(I*pi)/d)/2, Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x**2*exp_po
lar(I*pi)/d)/2, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log
(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x**2*exp
_polar(I*pi)/d)/2, True))/(2*e*f) + log(f*x)*log(d + e*x**2)/(2*e*f), True)
) + Piecewise((0**m*x, Eq(f, 0)), (Piecewise(((f*x)**(m + 1)/(m + 1), Ne(m,
 -1)), (log(f*x), True))/f, True))*log(c*(d + e*x**2)**p)
```

3.161 $\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((f*x)^m/ln(c*(e*x^2+d)^p),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m/Log[c*(d + e*x^2)^p],x]
[Out] Defer[Int][(f*x)^m/Log[c*(d + e*x^2)^p], x]
```

Rubi steps

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m/Log[c*(d + e*x^2)^p],x]
[Out] Integrate[(f*x)^m/Log[c*(d + e*x^2)^p], x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx\right)^m}{\log\left(\left(ex^2+d\right)^p c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral((f*x)^m/log((e*x^2 + d)^p*c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^m}{\log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate((f*x)^m/log((e*x^2 + d)^p*c), x)
```

maple [A] time = 3.06, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{\ln\left(c\left(e\,x^2+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/ln(c*(e*x^2+d)^p),x)
[Out] int((f*x)^m/ln(c*(e*x^2+d)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{\log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate((f*x)^m/log((e*x^2 + d)^p*c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(f\,x)^m}{\ln\left(c\left(e\,x^2+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/log(c*(d + e*x^2)^p),x)
[Out] int((f*x)^m/log(c*(d + e*x^2)^p), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m/ln(c*(e*x**2+d)**p),x)
[Out] Integral((f*x)**m/log(c*(d + e*x**2)**p), x)
```

3.162 $\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((f*x)^m/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m/Log[c*(d + e*x^2)^p]^2,x]
[Out] Defer[Int][(f*x)^m/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi steps

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m/Log[c*(d + e*x^2)^p]^2,x]
[Out] Integrate[(f*x)^m/Log[c*(d + e*x^2)^p]^2, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(fx\right)^m}{\log\left(\left(ex^2+d\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral((f*x)^m/log((e*x^2 + d)^p*c)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^m}{\log\left(\left(ex^2+d\right)^p c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^m/log((e*x^2 + d)^p*c)^2, x)
```

maple [A] time = 6.75, size = 0, normalized size = 0.00

$$\int \frac{(fx)^{m}}{\ln \left(c \left(e\,x^{2}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((f*x)^m/ln(c*(e*x^2+d)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(ef^{m}x^{2}+df^{m}\right)x^{m}}{2\left(ep^{2}x\log\left(ex^{2}+d\right)+epx\log(c)\right)}+\int \frac{\left(ef^{m}(m+1)x^{2}+df^{m}(m-1)\right)x^{m}}{2\left(ep^{2}x^{2}\log\left(ex^{2}+d\right)+epx^{2}\log(c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*f^m*x^2 + d*f^m)*x^m/(e*p^2*x*log(e*x^2 + d) + e*p*x*log(c)) + inte
grate(1/2*(e*f^m*(m + 1)*x^2 + d*f^m*(m - 1))*x^m/(e*p^2*x^2*log(e*x^2 + d)
 + e*p*x^2*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(fx)^{m}}{\ln \left(c \left(e\,x^{2}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/log(c*(d + e*x^2)^p)^2,x)
```

```
[Out] int((f*x)^m/log(c*(d + e*x^2)^p)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^{m}}{\log \left(c \left(d+ex^{2}\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f*x)**m/log(c*(d + e*x**2)**p)**2, x)
```

3.163 $\int (fx)^{-1+3n} \log^2\left(c\left(d+ex^n\right)^p\right) dx$

Optimal. Leaf size=372

$$\frac{2d^3px^{1-3n}(fx)^{3n-1}\log\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{3e^3n}-\frac{2d^2px^{1-3n}(fx)^{3n-1}\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{e^3n}-\frac{2px^{1-3n}(f\cdots}{}$$

[Out] 2*d^2*p^2*x^(1-2*n)*(f*x)^(-1+3*n)/e^2/n-1/2*d*p^2*x^(1-3*n)*(f*x)^(-1+3*n)*(d+e*x^n)^2/e^3/n+2/27*p^2*x^(1-3*n)*(f*x)^(-1+3*n)*(d+e*x^n)^3/e^3/n-1/3*d^3*p^2*x^(1-3*n)*(f*x)^(-1+3*n)*ln(d+e*x^n)^2/e^3/n-2*d^2*p*x^(1-3*n)*(f*x)^(-1+3*n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)/e^3/n+d*p*x^(1-3*n)*(f*x)^(-1+3*n)*(d+e*x^n)^2*ln(c*(d+e*x^n)^p)/e^3/n-2/9*p*x^(1-3*n)*(f*x)^(-1+3*n)*(d+e*x^n)^3*ln(c*(d+e*x^n)^p)/e^3/n+2/3*d^3*p*x^(1-3*n)*(f*x)^(-1+3*n)*ln(d+e*x^n)*ln(c*(d+e*x^n)^p)/e^3/n+1/3*x*(f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p)^2/n

Rubi [A] time = 0.32, antiderivative size = 278, normalized size of antiderivative = 0.75, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2456, 2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$-\frac{px^{1-3n}(fx)^{3n-1}\left(\frac{18d^2(d+ex^n)}{e^3}-\frac{6d^3\log(d+ex^n)}{e^3}-\frac{9d(d+ex^n)^2}{e^3}+\frac{2(d+ex^n)^3}{e^3}\right)\log\left(c\left(d+ex^n\right)^p\right)}{9n}+\frac{x(fx)^{3n-1}\log^2\left(c\left(d+e\cdots\right.\right.}{3n}$$

Antiderivative was successfully verified.

[In] Int[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p]^2,x]

[Out] (2*d^2*p^2*x^(1 - 2*n)*(f*x)^(-1 + 3*n))/(e^2*n) - (d*p^2*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*(d + e*x^n)^2)/(2*e^3*n) + (2*p^2*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*(d + e*x^n)^3)/(27*e^3*n) - (d^3*p^2*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*Log[d + e*x^n]^2)/(3*e^3*n) - (p*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*((18*d^2*(d + e*x^n))/e^3 - (9*d*(d + e*x^n)^2)/e^3 + (2*(d + e*x^n)^3)/e^3 - (6*d^3*Log[d + e*x^n])/e^3)*Log[c*(d + e*x^n)^p])/(9*n) + (x*(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p]^2)/(3*n)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rubi steps

$$\int (fx)^{-1+3n}\log^2\left(c\left(d+ex^n\right)^p\right)dx = \left(x^{1-3n}(fx)^{-1+3n}\right)\int x^{-1+3n}\log^2\left(c\left(d+ex^n\right)^p\right)dx$$
$$= \frac{\left(x^{1-3n}(fx)^{-1+3n}\right)\text{Subst}\left(\int x^2\log^2\left(c(d+ex)^p\right)dx,x,x^n\right)}{n}$$
$$= \frac{x(fx)^{-1+3n}\log^2\left(c\left(d+ex^n\right)^p\right)}{3n} - \frac{\left(2epx^{1-3n}(fx)^{-1+3n}\right)\text{Subst}\left(\int \frac{x^3\log(c(d+\cdots}{d+ex}\right.}{3n}$$
$$= \frac{x(fx)^{-1+3n}\log^2\left(c\left(d+ex^n\right)^p\right)}{3n} - \frac{\left(2px^{1-3n}(fx)^{-1+3n}\right)\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3\log\cdots}{x}\right.}{3n}$$
$$= -\frac{px^{1-3n}(fx)^{-1+3n}\left(\frac{18d^2(d+ex^n)}{e^3}-\frac{9d(d+ex^n)^2}{e^3}+\frac{2(d+ex^n)^3}{e^3}-\frac{6d^3\log(d+ex^n)}{e^3}\right)\log\left(\cdots\right.}{9n}$$
$$= -\frac{px^{1-3n}(fx)^{-1+3n}\left(\frac{18d^2(d+ex^n)}{e^3}-\frac{9d(d+ex^n)^2}{e^3}+\frac{2(d+ex^n)^3}{e^3}-\frac{6d^3\log(d+ex^n)}{e^3}\right)\log\left(\cdots\right.}{9n}$$
$$= -\frac{px^{1-3n}(fx)^{-1+3n}\left(\frac{18d^2(d+ex^n)}{e^3}-\frac{9d(d+ex^n)^2}{e^3}+\frac{2(d+ex^n)^3}{e^3}-\frac{6d^3\log(d+ex^n)}{e^3}\right)\log\left(\cdots\right.}{9n}$$
$$= \frac{2d^2p^2x^{1-2n}(fx)^{-1+3n}}{e^2n} - \frac{dp^2x^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^2}{2e^3n} + \frac{2p^2x^{1-3n}(fx)^{-1+\cdots}}{27e^3\cdots}$$
$$= \frac{2d^2p^2x^{1-2n}(fx)^{-1+3n}}{e^2n} - \frac{dp^2x^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^2}{2e^3n} + \frac{2p^2x^{1-3n}(fx)^{-1+\cdots}}{27e^3\cdots}$$

Mathematica [A] time = 0.17, size = 171, normalized size = 0.46

$$\frac{x^{-3n}(fx)^{3n}\left(6d^3p\log\left(d+ex^n\right)\left(6\log\left(c\left(d+ex^n\right)^p\right)-11p\right)+ex^n\left(-6p\left(6d^2-3dex^n+2e^2x^{2n}\right)\log\left(c\left(d+ex^n\right)^p\cdots\right.\right.\right.}{54e^3fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] ((f*x)^(3*n)*(-18*d^3*p^2*Log[d + e*x^n]^2 + 6*d^3*p*Log[d + e*x^n]*(-11*p
+ 6*Log[c*(d + e*x^n)^p]) + e*x^n*(p^2*(66*d^2 - 15*d*e*x^n + 4*e^2*x^(2*n)
) - 6*p*(6*d^2 - 3*d*e*x^n + 2*e^2*x^(2*n))*Log[c*(d + e*x^n)^p] + 18*e^2*x
^(2*n)*Log[c*(d + e*x^n)^p]^2)))/(54*e^3*f*n*x^(3*n))
```

fricas [A] time = 0.45, size = 266, normalized size = 0.72

$$\frac{2\left(2e^3p^2-6e^3p\log(c)+9e^3\log(c)^2\right)f^{3n-1}x^{3n}-3\left(5de^2p^2-6de^2p\log(c)\right)f^{3n-1}x^{2n}+6\left(11d^2ep^2-6d^2ep\log\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/54*(2*(2*e^3*p^2 - 6*e^3*p*log(c) + 9*e^3*log(c)^2)*f^(3*n - 1)*x^(3*n) -
 3*(5*d*e^2*p^2 - 6*d*e^2*p*log(c))*f^(3*n - 1)*x^(2*n) + 6*(11*d^2*e*p^2 -
 6*d^2*e*p*log(c))*f^(3*n - 1)*x^n + 18*(e^3*f^(3*n - 1)*p^2*x^(3*n) + d^3*
f^(3*n - 1)*p^2)*log(e*x^n + d)^2 + 6*(3*d*e^2*f^(3*n - 1)*p^2*x^(2*n) - 6*
d^2*e*f^(3*n - 1)*p^2*x^n - 2*(e^3*p^2 - 3*e^3*p*log(c))*f^(3*n - 1)*x^(3*n
) - (11*d^3*p^2 - 6*d^3*p*log(c))*f^(3*n - 1))*log(e*x^n + d))/(e^3*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{3n-1} \log \left(\left(ex^{n}+d\right)^{p} c\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^(3*n - 1)*log((e*x^n + d)^p*c)^2, x)
```

maple [F] time = 1.82, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{3n-1} \ln \left(c \left(e\,x^{n}+d\right)^{p}\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3*n-1)*ln(c*(e*x^n+d)^p)^2,x)
```

```
[Out] int((f*x)^(3*n-1)*ln(c*(e*x^n+d)^p)^2,x)
```

maxima [A] time = 0.56, size = 239, normalized size = 0.64

$$\frac{ep\left(\frac{6\,d^{3} f^{3\,n} \log\left(\frac{ex^{n}+d}{e}\right)}{e^{4} n} - \frac{2\,e^{2} f^{3\,n} x^{3\,n} - 3\,def^{3\,n} x^{2\,n} + 6\,d^{2} f^{3\,n} x^{n}}{e^{3} n}\right) \log\left(\left(ex^{n}+d\right)^{p} c\right)}{9\,f} + \frac{\left(fx\right)^{3\,n} \log\left(\left(ex^{n}+d\right)^{p} c\right)^{2}}{3\,fn} - \frac{\left(18\,d^{3} f^{3\,n} \log\left(e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/9*e*p*(6*d^3*f^(3*n)*log((e*x^n + d)/e)/(e^4*n) - (2*e^2*f^(3*n)*x^(3*n)
- 3*d*e*f^(3*n)*x^(2*n) + 6*d^2*f^(3*n)*x^n)/(e^3*n))*log((e*x^n + d)^p*c)/
f + 1/3*(f*x)^(3*n)*log((e*x^n + d)^p*c)^2/(f*n) - 1/54*(18*d^3*f^(3*n)*log
(e*x^n + d)^2 - 4*e^3*f^(3*n)*x^(3*n) + 15*d*e^2*f^(3*n)*x^(2*n) - 66*d^2*e
*f^(3*n)*x^n - 6*(6*f^(3*n)*log(e) - 11*f^(3*n))*d^3*log(e*x^n + d))*p^2/(e
^3*f*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln \left(c \left(d+e\,x^{n}\right)^{p}\right)^{2} \left(f\,x\right)^{3\,n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^2*(f*x)^(3*n - 1),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^2*(f*x)^(3*n - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1+3*n)*ln(c*(d+e*x**n)**p)**2,x)
```

```
[Out] Timed out
```

3.164 $\int (fx)^{-1+2n} \log^2\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=255

$$\frac{x^{1-2n}(fx)^{2n-1}\left(d+ex^n\right)^2\log^2\left(c\left(d+ex^n\right)^p\right)}{2e^2n}-\frac{dx^{1-2n}(fx)^{2n-1}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{e^2n}-\frac{px^{1-2n}(fx)^{2n-1}\left(d+\right.}{2}$$

```
[Out] -2*d*p^2*x^(1-n)*(f*x)^(-1+2*n)/e/n+1/4*p^2*x^(1-2*n)*(f*x)^(-1+2*n)*(d+e*x
^n)^2/e^2/n+2*d*p*x^(1-2*n)*(f*x)^(-1+2*n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)/e^2/
n-1/2*p*x^(1-2*n)*(f*x)^(-1+2*n)*(d+e*x^n)^2*ln(c*(d+e*x^n)^p)/e^2/n-d*x^(1
-2*n)*(f*x)^(-1+2*n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)^2/e^2/n+1/2*x^(1-2*n)*(f*x
)^(-1+2*n)*(d+e*x^n)^2*ln(c*(d+e*x^n)^p)^2/e^2/n
```

Rubi [A] time = 0.19, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2456, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{x^{1-2n}(fx)^{2n-1}\left(d+ex^n\right)^2\log^2\left(c\left(d+ex^n\right)^p\right)}{2e^2n}-\frac{dx^{1-2n}(fx)^{2n-1}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{e^2n}-\frac{px^{1-2n}(fx)^{2n-1}\left(d+\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (-2*d*p^2*x^(1 - n)*(f*x)^(-1 + 2*n))/(e*n) + (p^2*x^(1 - 2*n)*(f*x)^(-1 +
2*n)*(d + e*x^n)^2)/(4*e^2*n) + (2*d*p*x^(1 - 2*n)*(f*x)^(-1 + 2*n)*(d + e*
x^n)*Log[c*(d + e*x^n)^p])/(e^2*n) - (p*x^(1 - 2*n)*(f*x)^(-1 + 2*n)*(d + e
*x^n)^2*Log[c*(d + e*x^n)^p])/(2*e^2*n) - (d*x^(1 - 2*n)*(f*x)^(-1 + 2*n)*(
d + e*x^n)*Log[c*(d + e*x^n)^p]^2)/(e^2*n) + (x^(1 - 2*n)*(f*x)^(-1 + 2*n)*
(d + e*x^n)^2*Log[c*(d + e*x^n)^p]^2)/(2*e^2*n)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int (fx)^{-1+2n}\log^2\left(c\left(d+ex^n\right)^p\right)\,dx &= \left(x^{1-2n}(fx)^{-1+2n}\right)\int x^{-1+2n}\log^2\left(c\left(d+ex^n\right)^p\right)\,dx\\
&= \frac{\left(x^{1-2n}(fx)^{-1+2n}\right)\text{Subst}\left(\int x\log^2\left(c(d+ex)^p\right)\,dx,x,x^n\right)}{n}\\
&= \frac{\left(x^{1-2n}(fx)^{-1+2n}\right)\text{Subst}\left(\int\left(-\frac{d\log^2(c(d+ex)^p)}{e}+\frac{(d+ex)\log^2(c(d+ex)^p)}{e}\right)\,dx,x,x^n\right)}{n}\\
&= \frac{\left(x^{1-2n}(fx)^{-1+2n}\right)\text{Subst}\left(\int(d+ex)\log^2\left(c(d+ex)^p\right)\,dx,x,x^n\right)}{en}-\frac{\left(dx^{1-2n}(fx\right.}{}\\
&= \frac{\left(x^{1-2n}(fx)^{-1+2n}\right)\text{Subst}\left(\int x\log^2\left(cx^p\right)\,dx,x,d+ex^n\right)}{e^2n}-\frac{\left(dx^{1-2n}(fx)^{-1+2n}\right)}{}\\
&= -\frac{dx^{1-2n}(fx)^{-1+2n}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{e^2n}+\frac{x^{1-2n}(fx)^{-1+2n}\left(d+ex^n\right)^2}{2e^2n}\\
&= -\frac{2dp^2x^{1-n}(fx)^{-1+2n}}{en}+\frac{p^2x^{1-2n}(fx)^{-1+2n}\left(d+ex^n\right)^2}{4e^2n}+\frac{2dpx^{1-2n}(fx)^{-1+2n}(d}{}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 140, normalized size = 0.55

$$\frac{x^{-2n}(fx)^{2n}\left(2d^2p\log\left(d+ex^n\right)\left(3p-2\log\left(c\left(d+ex^n\right)^p\right)\right)+ex^n\left(2ex^n\log^2\left(c\left(d+ex^n\right)^p\right)+2p\left(2d-ex^n\right)\log\left(c\left(d\right.\right.\right.\right.}{4e^2fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] ((f*x)^(2*n)*(2*d^2*p^2*Log[d + e*x^n]^2 + 2*d^2*p*Log[d + e*x^n]*(3*p - 2*
Log[c*(d + e*x^n)^p]) + e*x^n*(p^2*(-6*d + e*x^n) + 2*p*(2*d - e*x^n)*Log[c
*(d + e*x^n)^p] + 2*e*x^n*Log[c*(d + e*x^n)^p]^2)))/(4*e^2*f*n*x^(2*n))
```

fricas [A] time = 0.45, size = 204, normalized size = 0.80

$$\frac{\left(e^2p^2 - 2\,e^2p\log(c) + 2\,e^2\log(c)^2\right)f^{2\,n-1}x^{2\,n} - 2\left(3\,dep^2 - 2\,dep\log(c)\right)f^{2\,n-1}x^n + 2\left(e^2f^{2\,n-1}p^2x^{2\,n} - d^2f^{2\,n-1}p\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((e^2*p^2 - 2*e^2*p*log(c) + 2*e^2*log(c)^2)*f^(2*n - 1)*x^(2*n) - 2*(3
*d*e*p^2 - 2*d*e*p*log(c))*f^(2*n - 1)*x^n + 2*(e^2*f^(2*n - 1)*p^2*x^(2*n)
 - d^2*f^(2*n - 1)*p^2)*log(e*x^n + d)^2 + 2*(2*d*e*f^(2*n - 1)*p^2*x^n - (
e^2*p^2 - 2*e^2*p*log(c))*f^(2*n - 1)*x^(2*n) + (3*d^2*p^2 - 2*d^2*p*log(c)
)*f^(2*n - 1))*log(e*x^n + d))/(e^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{2\,n-1}\log\left(\left(ex^n + d\right)^p c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^(2*n - 1)*log((e*x^n + d)^p*c)^2, x)
```

maple [F] time = 1.72, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{2n-1}\ln\left(c\left(e\,x^n + d\right)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(2*n-1)*ln(c*(e*x^n+d)^p)^2,x)
```

```
[Out] int((f*x)^(2*n-1)*ln(c*(e*x^n+d)^p)^2,x)
```

maxima [A] time = 0.57, size = 200, normalized size = 0.78

$$-\frac{ep\left(\frac{2\,d^2f^{2\,n}\log\left(\frac{ex^n+d}{e}\right)}{e^3n} + \frac{ef^{2\,n}x^{2\,n}-2\,df^{2\,n}x^n}{e^2n}\right)\log\left(\left(ex^n + d\right)^p c\right)}{2\,f} + \frac{\left(fx\right)^{2\,n}\log\left(\left(ex^n + d\right)^p c\right)^2}{2\,fn} + \frac{\left(2\,d^2f^{2\,n}\log\left(ex^n + d\right)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*e*p*(2*d^2*f^(2*n)*log((e*x^n + d)/e)/(e^3*n) + (e*f^(2*n)*x^(2*n) - 2
*d*f^(2*n)*x^n)/(e^2*n))*log((e*x^n + d)^p*c)/f + 1/2*(f*x)^(2*n)*log((e*x^
n + d)^p*c)^2/(f*n) + 1/4*(2*d^2*f^(2*n)*log(e*x^n + d)^2 + e^2*f^(2*n)*x^(
2*n) - 6*d*e*f^(2*n)*x^n - 2*(2*f^(2*n)*log(e) - 3*f^(2*n))*d^2*log(e*x^n +
 d))*p^2/(e^2*f*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(d + e\,x^n\right)^p\right)^2\left(f\,x\right)^{2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^2*(f*x)^(2*n - 1),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^2*(f*x)^(2*n - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1+2*n)*ln(c*(d+e*x**n)**p)**2,x)
```

```
[Out] Timed out
```

3.165 $\int (fx)^{-1+n} \log^2\left(c\left(d+ex^n\right)^p\right) dx$

Optimal. Leaf size=101

$$\frac{x^{1-n}(fx)^{n-1}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{en}-\frac{2px^{1-n}(fx)^{n-1}\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{en}+\frac{2p^2x(fx)^{n-1}}{n}$$

```
[Out] 2*p^2*x*(f*x)^(-1+n)/n-2*p*x^(1-n)*(f*x)^(-1+n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)
/e/n+x^(1-n)*(f*x)^(-1+n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)^2/e/n
```

Rubi [A] time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2456, 2454, 2389, 2296, 2295}

$$\frac{x^{1-n}(fx)^{n-1}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{en}-\frac{2px^{1-n}(fx)^{n-1}\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{en}+\frac{2p^2x(fx)^{n-1}}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (2*p^2*x*(f*x)^(-1 + n))/n - (2*p*x^(1 - n)*(f*x)^(-1 + n)*(d + e*x^n)*Log[
c*(d + e*x^n)^p])/(e*n) + (x^(1 - n)*(f*x)^(-1 + n)*(d + e*x^n)*Log[c*(d +
e*x^n)^p]^2)/(e*n)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rubi steps

$$\int (fx)^{-1+n}\log^2\left(c\,(d+ex^n)^p\right)dx = \left(x^{1-n}(fx)^{-1+n}\right)\int x^{-1+n}\log^2\left(c\,(d+ex^n)^p\right)dx$$
$$= \frac{\left(x^{1-n}(fx)^{-1+n}\right)\text{Subst}\left(\int\log^2\left(c(d+ex)^p\right)dx,x,x^n\right)}{n}$$
$$= \frac{\left(x^{1-n}(fx)^{-1+n}\right)\text{Subst}\left(\int\log^2\left(cx^p\right)dx,x,d+ex^n\right)}{en}$$
$$= \frac{x^{1-n}(fx)^{-1+n}\,(d+ex^n)\log^2\left(c\,(d+ex^n)^p\right)}{en} - \frac{\left(2px^{1-n}(fx)^{-1+n}\right)\text{Subst}\left(\int\log\right.}{en}$$
$$= \frac{2p^2x(fx)^{-1+n}}{n} - \frac{2px^{1-n}(fx)^{-1+n}\,(d+ex^n)\log\left(c\,(d+ex^n)^p\right)}{en} + \frac{x^{1-n}(fx)^{-1+n}}{}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 0.73

$$\frac{x^{-n}(fx)^n\left((d+ex^n)\log^2\left(c\,(d+ex^n)^p\right)-2p\,(d+ex^n)\log\left(c\,(d+ex^n)^p\right)+2ep^2x^n\right)}{efn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] ((f*x)^n*(2*e*p^2*x^n - 2*p*(d + e*x^n)*Log[c*(d + e*x^n)^p] + (d + e*x^n)*
Log[c*(d + e*x^n)^p]^2))/(e*f*n*x^n)
```

fricas [A] time = 0.43, size = 121, normalized size = 1.20

$$\frac{\left(2\,ep^2-2\,ep\log(c)+e\log(c)^2\right)f^{n-1}x^n+\left(ef^{n-1}p^2x^n+df^{n-1}p^2\right)\log\left(ex^n+d\right)^2-2\left(\left(ep^2-ep\log(c)\right)f^{n-1}x^n+\left(d\right.\right.}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] ((2*e*p^2 - 2*e*p*log(c) + e*log(c)^2)*f^(n - 1)*x^n + (e*f^(n - 1)*p^2*x^n
 + d*f^(n - 1)*p^2)*log(e*x^n + d)^2 - 2*((e*p^2 - e*p*log(c))*f^(n - 1)*x^
n + (d*p^2 - d*p*log(c))*f^(n - 1))*log(e*x^n + d))/(e*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^{n-1}\log\left((ex^n+d)^pc\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^(n - 1)*log((e*x^n + d)^p*c)^2, x)
```

maple [F] time = 1.64, size = 0, normalized size = 0.00

$$\int (fx)^{n-1}\ln\left(c\,(e\,x^n+d)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(n-1)*ln(c*(e*x^n+d)^p)^2,x)
```

```
[Out] int((f*x)^(n-1)*ln(c*(e*x^n+d)^p)^2,x)
```

maxima [A] time = 0.54, size = 146, normalized size = 1.45

$$-\frac{2\,ep\left(\frac{f^n x^n}{en}-\frac{df^n\log\left(\frac{ex^n+d}{e}\right)}{e^2n}\right)\log\left((ex^n+d)^p c\right)}{f}+\frac{\left(fx\right)^n\log\left((ex^n+d)^p c\right)^2}{fn}-\frac{\left(df^n\log\left(ex^n+d\right)^2-2\,ef^nx^n-2\left(f^n\right.\right.}{efn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

```
[Out] -2*e*p*(f^n*x^n/(e*n) - d*f^n*log((e*x^n + d)/e)/(e^2*n))*log((e*x^n + d)^p
*c)/f + (f*x)^n*log((e*x^n + d)^p*c)^2/(f*n) - (d*f^n*log(e*x^n + d)^2 - 2*
e*f^n*x^n - 2*(f^n*log(e) - f^n)*d*log(e*x^n + d))*p^2/(e*f*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\,(d+e\,x^n)^p\right)^2\,\left(f\,x\right)^{n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^2*(f*x)^(n - 1),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^2*(f*x)^(n - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1+n)*ln(c*(d+e*x**n)**p)**2,x)
```

```
[Out] Timed out
```

3.166 $\int \frac{\log^2\left(c(d+ex^n)^p\right)}{fx}\,dx$

Optimal. Leaf size=88

$$\frac{2p\text{Li}_2\left(\frac{ex^n}{d}+1\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn}+\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{fn}-\frac{2p^2\text{Li}_3\left(\frac{ex^n}{d}+1\right)}{fn}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)^2/f/n+2*p*ln(c*(d+e*x^n)^p)*polylog(2,1+e*x^
n/d)/f/n-2*p^2*polylog(3,1+e*x^n/d)/f/n
```

Rubi [A] time = 0.11, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {12, 2454, 2396, 2433, 2374, 6589}

$$\frac{2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn}-\frac{2p^2\text{PolyLog}\left(3,\frac{ex^n}{d}+1\right)}{fn}+\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]^2/(f*x),x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]^2)/(f*n) + (2*p*Log[c*(d + e*x^n)^p
]*PolyLog[2, 1 + (e*x^n)/d])/(f*n) - (2*p^2*PolyLog[3, 1 + (e*x^n)/d])/(f*n
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^n\right)^p\right)}{fx}\,dx = \frac{\int \frac{\log^2\left(c\left(d+ex^n\right)^p\right)}{x}\,dx}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{\log^2\left(c(d+ex)^p\right)}{x}\,dx, x, x^n\right)}{fn}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{(2ep)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\log\left(c(d+ex)^p\right)}{d+ex}\,dx, x, x^n\right)}{fn}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{(2p)\,\text{Subst}\left(\int \frac{\log\left(cx^p\right)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+ex^n\right)}{fn}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{fn} + \frac{2p\log\left(c\left(d+ex^n\right)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn} - \frac{\left(2p^2\right)\text{Subst}\left(\right.}{}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{fn} + \frac{2p\log\left(c\left(d+ex^n\right)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn} - \frac{2p^2\text{Li}_3\left(1+\right.}{fn}$$

Mathematica [A] time = 0.11, size = 168, normalized size = 1.91

$$\frac{2p\left(\log(x)\left(\log\left(d+ex^n\right)-\log\left(\frac{ex^n}{d}+1\right)\right)-\frac{\text{Li}_2\left(-\frac{ex^n}{d}\right)}{n}\right)\left(\log\left(c\left(d+ex^n\right)^p\right)-p\log\left(d+ex^n\right)\right)+\log(x)\left(\log\left(c\left(d\right.\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^2/(f*x),x]
```

```
[Out] (Log[x]*(-(p*Log[d + e*x^n]) + Log[c*(d + e*x^n)^p])^2 + 2*p*(-(p*Log[d + e
*x^n]) + Log[c*(d + e*x^n)^p])*(Log[x]*(Log[d + e*x^n] - Log[1 + (e*x^n)/d]
) - PolyLog[2, -((e*x^n)/d)]/n) + (p^2*(Log[-((e*x^n)/d)]*Log[d + e*x^n]^2
+ 2*Log[d + e*x^n]*PolyLog[2, 1 + (e*x^n)/d] - 2*PolyLog[3, 1 + (e*x^n)/d])
)/n)/f
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^n+d\right)^p c\right)^2}{fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^2/f/x,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)^2/(f*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^n + d\right)^p c\right)^2}{fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^2/f/x,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^2/(f*x), x)
```

maple [C] time = 3.52, size = 1473, normalized size = 16.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)^2/f/x,x)
```

```
[Out] -I/f/n*ln((e*x^n+d)^p)*Pi*ln(x^n)*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)
*csgn(I*c)-I/f/n*Pi*ln((e*x^n+d)/d)*ln(x^n)*p*csgn(I*c*(e*x^n+d)^p)^2*csgn(
I*c)+1/f/n*ln((e*x^n+d)^p)^2*ln(e*x^n)-2/f/n*polylog(3,(e*x^n+d)/d)*p^2-1/4
/f*ln(x)*Pi^2*csgn(I*c*(e*x^n+d)^p)^6+1/f*ln(x)*ln(c)^2+I/f/n*Pi*ln((e*x^n+
d)/d)*ln(x^n)*p*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)-I/f/n*P
i*dilog((e*x^n+d)/d)*p*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2-I/f/n*Pi
*dilog((e*x^n+d)/d)*p*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)+I/f/n*ln((e*x^n+d)^
p)*Pi*ln(x^n)*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)+I/f/n*Pi*ln((e*x^n+d)/d)*ln
(x^n)*p*csgn(I*c*(e*x^n+d)^p)^3-I/f*ln(x)*ln(c)*Pi*csgn(I*(e*x^n+d)^p)*csgn
(I*c*(e*x^n+d)^p)*csgn(I*c)+I/f/n*ln((e*x^n+d)^p)*Pi*ln(x^n)*csgn(I*(e*x^n+
d)^p)*csgn(I*c*(e*x^n+d)^p)^2+1/2/f*ln(x)*Pi^2*csgn(I*c*(e*x^n+d)^p)^5*csgn
(I*c)+I/f/n*Pi*dilog((e*x^n+d)/d)*p*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^
p)*csgn(I*c)-I/f/n*Pi*ln((e*x^n+d)/d)*ln(x^n)*p*csgn(I*(e*x^n+d)^p)*csgn(I*
c*(e*x^n+d)^p)^2+2/f/n*polylog(2,(e*x^n+d)/d)*ln(e*x^n+d)*p^2-2/f/n*ln(e*x^
n+d)*dilog(-1/d*e*x^n)*p^2+2/f/n*ln((e*x^n+d)^p)*dilog(-1/d*e*x^n)*p+1/f/n*
ln(e*x^n+d)^2*ln(e*x^n)*p^2+1/f/n*ln(1-(e*x^n+d)/d)*ln(e*x^n+d)^2*p^2-2/f/n
*ln(e*x^n+d)^2*ln(-1/d*e*x^n)*p^2-2/f/n*ln(c)*dilog((e*x^n+d)/d)*p+I/f*ln(x
)*ln(c)*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)+I/f/n*Pi*dilog((e*x^n+d)/d)*p*
csgn(I*c*(e*x^n+d)^p)^3+I/f*ln(x)*ln(c)*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*
x^n+d)^p)^2-I/f/n*ln((e*x^n+d)^p)*Pi*ln(x^n)*csgn(I*c*(e*x^n+d)^p)^3-2/f/n*
ln(c)*ln((e*x^n+d)/d)*ln(x^n)*p-1/4/f*ln(x)*Pi^2*csgn(I*c*(e*x^n+d)^p)^4*cs
gn(I*c)^2-1/4/f*ln(x)*Pi^2*csgn(I*(e*x^n+d)^p)^2*csgn(I*c*(e*x^n+d)^p)^4+1/
2/f*ln(x)*Pi^2*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^5+2/f/n*ln((e*x^n+
d)^p)*ln(c)*ln(x^n)+2/f/n*ln((e*x^n+d)^p)*ln(e*x^n+d)*ln(-1/d*e*x^n)*p-2/f/
n*ln((e*x^n+d)^p)*ln(e*x^n+d)*ln(e*x^n)*p-1/f*ln(x)*Pi^2*csgn(I*(e*x^n+d)^p
)*csgn(I*c*(e*x^n+d)^p)^4*csgn(I*c)-1/4/f*ln(x)*Pi^2*csgn(I*(e*x^n+d)^p)^2*
csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)^2+1/2/f*ln(x)*Pi^2*csgn(I*(e*x^n+d)^p)^2*
csgn(I*c*(e*x^n+d)^p)^3*csgn(I*c)+1/2/f*ln(x)*Pi^2*csgn(I*(e*x^n+d)^p)*csgn
(I*c*(e*x^n+d)^p)^3*csgn(I*c)^2-I/f*ln(x)*ln(c)*Pi*csgn(I*c*(e*x^n+d)^p)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{d\left(\frac{\log(x)}{d} - \frac{\log\left(\frac{ex^n+d}{e}\right)}{dn}\right)\log(c)^2 + \log\left(\left(ex^n + d\right)^p\right)^2 \log(x) + \frac{\log(c)^2 \log\left(\frac{ex^n+d}{e}\right)}{n} - \int \frac{2\left(\left(enp\log(x) - e\log(c)\right)x^n - d\log(c)\right)\log\left(\left(ex^n+d\right.\right.}{exx^n+dx}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^2/f/x,x, algorithm="maxima")
```

```
[Out] (log((e*x^n + d)^p)^2*log(x) - integrate(-(e*x^n*log(c)^2 + d*log(c)^2 - 2*
((e*n*p*log(x) - e*log(c))*x^n - d*log(c))*log((e*x^n + d)^p))/(e*x*x^n + d
*x), x))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^2}{f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^2/(f*x),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^2/(f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\log\left(c(d+ex^n)^p\right)^2}{x}\,dx}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**2/f/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)**2/x, x)/f
```

3.167 $\int (fx)^{-1-n} \log^2\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=124

$$-\frac{x(fx)^{-n-1}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{dn}+\frac{2epx^{n+1}(fx)^{-n-1}\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{dn}+\frac{2ep^2x^{n+1}(fx)^{-n-1}\text{Li}_2\left(\frac{e\text{…}}{\text{…}}\right)}{dn}$$

```
[Out] 2*e*p*x^(1+n)*(f*x)^(-1-n)*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/d/n-x*(f*x)^(-1-n
)*(d+e*x^n)*ln(c*(d+e*x^n)^p)^2/d/n+2*e*p^2*x^(1+n)*(f*x)^(-1-n)*polylog(2,
1+e*x^n/d)/d/n
```

Rubi [A] time = 0.11, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2456, 2454, 2397, 2394, 2315}

$$\frac{2ep^2x^{n+1}(fx)^{-n-1}\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{dn}-\frac{x(fx)^{-n-1}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{dn}+\frac{2epx^{n+1}(fx)^{-n-1}\log\left(-\frac{ex^n}{d}\right)\log\text{…}}{dn}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (2*e*p*x^(1 + n)*(f*x)^(-1 - n)*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(d*
n) - (x*(f*x)^(-1 - n)*(d + e*x^n)*Log[c*(d + e*x^n)^p]^2)/(d*n) + (2*e*p^2
*x^(1 + n)*(f*x)^(-1 - n)*PolyLog[2, 1 + (e*x^n)/d])/(d*n)
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
```

```
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rubi steps

$$\int (fx)^{-1-n}\log^2\left(c\left(d+ex^n\right)^p\right)dx = \left(x^{1+n}(fx)^{-1-n}\right)\int x^{-1-n}\log^2\left(c\left(d+ex^n\right)^p\right)dx$$
$$= \frac{\left(x^{1+n}(fx)^{-1-n}\right)\text{Subst}\left(\int \frac{\log^2(c(d+ex)^p)}{x^2}dx, x, x^n\right)}{n}$$
$$= -\frac{x(fx)^{-1-n}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{dn} + \frac{\left(2epx^{1+n}(fx)^{-1-n}\right)\text{Subst}\left(\int \text{lo}\right.}{dn}$$
$$= \frac{2epx^{1+n}(fx)^{-1-n}\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{dn} - \frac{x(fx)^{-1-n}\left(d+ex^n\right)\log^2\left(\right.}{dn}$$
$$= \frac{2epx^{1+n}(fx)^{-1-n}\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{dn} - \frac{x(fx)^{-1-n}\left(d+ex^n\right)\log^2\left(\right.}{dn}$$

Mathematica [A] time = 0.09, size = 148, normalized size = 1.19

$$-\frac{(fx)^{-n}\left(d\log^2\left(c\left(d+ex^n\right)^p\right)+2epx^n\log\left(-dx^{-n}-e\right)\log\left(c\left(d+ex^n\right)^p\right)+2ep^2x^n\text{Li}_2\left(\frac{dx^{-n}}{e}+1\right)-ep^2x^n\log^2\left(\right.\right.}{dfn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] -((2*e*p^2*x^n*Log[-(d/(e*x^n))]*Log[-e - d/x^n] - e*p^2*x^n*Log[-e - d/x^n
]^2 + 2*e*p*x^n*Log[-e - d/x^n]*Log[c*(d + e*x^n)^p] + d*Log[c*(d + e*x^n)^
p]^2 + 2*e*p^2*x^n*PolyLog[2, 1 + d/(e*x^n)])/(d*f*n*(f*x)^n))
```

fricas [A] time = 0.45, size = 197, normalized size = 1.59

$$-\frac{2\,ef^{-n-1}np^2x^n\log(x)\log\left(\frac{ex^n+d}{d}\right)-2\,ef^{-n-1}npx^n\log(c)\log(x)+2\,ef^{-n-1}p^2x^n\text{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+df^{-n-1}\log(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] -(2*e*f^(-n - 1)*n*p^2*x^n*log(x)*log((e*x^n + d)/d) - 2*e*f^(-n - 1)*n*p*x
^n*log(c)*log(x) + 2*e*f^(-n - 1)*p^2*x^n*dilog(-(e*x^n + d)/d + 1) + d*f^(
-n - 1)*log(c)^2 + (e*f^(-n - 1)*p^2*x^n + d*f^(-n - 1)*p^2)*log(e*x^n + d)
^2 + 2*(d*f^(-n - 1)*p*log(c) - (e*n*p^2*log(x) - e*p*log(c))*f^(-n - 1)*x^
n)*log(e*x^n + d))/(d*n*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^{-n-1}\log\left(\left(ex^n+d\right)^pc\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^(-n - 1)*log((e*x^n + d)^p*c)^2, x)
```

maple [F] time = 1.81, size = 0, normalized size = 0.00

$$\int \left(f x\right)^{-n-1} \ln \left(c \left(e \,x^{n}+d\right)^{p}\right)^{2}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-n-1)*ln(c*(e*x^n+d)^p)^2,x)
```

```
[Out] int((f*x)^(-n-1)*ln(c*(e*x^n+d)^p)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(en^2p^2x^n \log(x)^2 - ep^2x^n \log\left(ex^n + d\right)^2 + d \log\left(\left(ex^n + d\right)^p\right)^2 + d \log(c)^2 - 2\left(enpx^n \log(x) - epx^n \log\left(ex^n + d\right)\right.\right.}{dnx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

```
[Out] -(e*n^2*p^2*x^n*log(x)^2 - e*p^2*x^n*log(e*x^n + d)^2 + d*log((e*x^n + d)^p
)^2 + d*log(c)^2 - 2*(e*n*p*x^n*log(x) - e*p*x^n*log(e*x^n + d) - d*log(c))
*log((e*x^n + d)^p))*f^(-n - 1)/(d*n*x^n) + integrate(2*(e*n*p^2*log(x) + e
*p*log(c))/(e*f^(n + 1)*x*x^n + d*f^(n + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)}^2}{{\left(f\,x\right)}^{n+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^2/(f*x)^(n + 1),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^2/(f*x)^(n + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1-n)*ln(c*(d+e*x**n)**p)**2,x)
```

```
[Out] Timed out
```

3.168 $\int (fx)^{-1-2n}\log^2\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=200

$$-\frac{e^2px^{2n+1}(fx)^{-2n-1}\log\left(1-\frac{d}{d+ex^n}\right)\log\left(c\left(d+ex^n\right)^p\right)}{d^2n}-\frac{epx^{n+1}(fx)^{-2n-1}\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{d^2n}-\frac{x(fx)^{-2n-\cdots}}{\cdots}$$

```
[Out] e^2*p^2*x^(1+2*n)*(f*x)^(-1-2*n)*ln(x)/d^2-e*p*x^(1+n)*(f*x)^(-1-2*n)*(d+e*
x^n)*ln(c*(d+e*x^n)^p)/d^2/n-1/2*x*(f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p)^2/n-e^2
*p*x^(1+2*n)*(f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p)*ln(1-d/(d+e*x^n))/d^2/n+e^2*p
^2*x^(1+2*n)*(f*x)^(-1-2*n)*polylog(2,d/(d+e*x^n))/d^2/n
```

Rubi [A] time = 0.32, antiderivative size = 238, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {2456, 2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31}

$$-\frac{e^2p^2x^{2n+1}(fx)^{-2n-1}\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{d^2n}+\frac{e^2x^{2n+1}(fx)^{-2n-1}\log^2\left(c\left(d+ex^n\right)^p\right)}{2d^2n}-\frac{e^2px^{2n+1}(fx)^{-2n-1}\log\left(-\frac{ex^n}{d}\cdots\right)}{d^2n}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (e^2*p^2*x^(1 + 2*n)*(f*x)^(-1 - 2*n)*Log[x])/d^2 - (e*p*x^(1 + n)*(f*x)^(-
1 - 2*n)*(d + e*x^n)*Log[c*(d + e*x^n)^p])/(d^2*n) - (e^2*p*x^(1 + 2*n)*(f*
x)^(-1 - 2*n)*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(d^2*n) - (x*(f*x)^(-
1 - 2*n)*Log[c*(d + e*x^n)^p]^2)/(2*n) + (e^2*x^(1 + 2*n)*(f*x)^(-1 - 2*n)*
Log[c*(d + e*x^n)^p]^2)/(2*d^2*n) - (e^2*p^2*x^(1 + 2*n)*(f*x)^(-1 - 2*n)*P
olyLog[2, 1 + (e*x^n)/d])/(d^2*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
```

```
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rubi steps

$$\int (fx)^{-1-2n}\log^2\left(c\left(d+ex^n\right)^p\right)dx = \left(x^{1+2n}(fx)^{-1-2n}\right)\int x^{-1-2n}\log^2\left(c\left(d+ex^n\right)^p\right)dx$$
$$= \frac{\left(x^{1+2n}(fx)^{-1-2n}\right)\text{Subst}\left(\int \frac{\log^2(c(d+ex)^p)}{x^3}dx, x, x^n\right)}{n}$$
$$= -\frac{x(fx)^{-1-2n}\log^2\left(c\left(d+ex^n\right)^p\right)}{2n} + \frac{\left(epx^{1+2n}(fx)^{-1-2n}\right)\text{Subst}\left(\int \frac{\log(c(d+ex)}{x^2(d+ex)}\right.}{n}$$
$$= -\frac{x(fx)^{-1-2n}\log^2\left(c\left(d+ex^n\right)^p\right)}{2n} + \frac{\left(px^{1+2n}(fx)^{-1-2n}\right)\text{Subst}\left(\int \frac{\log(cx^p)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}d\right.}{n}$$
$$= -\frac{x(fx)^{-1-2n}\log^2\left(c\left(d+ex^n\right)^p\right)}{2n} + \frac{\left(px^{1+2n}(fx)^{-1-2n}\right)\text{Subst}\left(\int \frac{\log(cx^p)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}dx\right.}{dn}$$
$$= -\frac{epx^{1+n}(fx)^{-1-2n}\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{d^2n} - \frac{x(fx)^{-1-2n}\log^2\left(c\left(d+e\right.\right.}{2n}$$
$$= \frac{e^2p^2x^{1+2n}(fx)^{-1-2n}\log(x)}{d^2} - \frac{epx^{1+n}(fx)^{-1-2n}\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{d^2n}$$
$$= \frac{e^2p^2x^{1+2n}(fx)^{-1-2n}\log(x)}{d^2} - \frac{epx^{1+n}(fx)^{-1-2n}\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{d^2n}$$

Mathematica [A] time = 0.32, size = 288, normalized size = 1.44

$$\frac{(fx)^{-2n}\left(-d^2\log^2\left(c\left(d+ex^n\right)^p\right)+2e^2px^{2n}\log\left(e-ex^{-n}\right)\log\left(c\left(d+ex^n\right)^p\right)+2e^2npx^{2n}\log(x)\left(-\log\left(c\left(d+ex\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (e^2*n^2*p^2*x^(2*n)*Log[x]^2 + e^2*p^2*x^(2*n)*Log[e + d/x^n]^2 - 2*e^2*p^
2*x^(2*n)*Log[e - e/x^n] - 2*e^2*p^2*x^(2*n)*Log[e + d/x^n]*Log[e - e/x^n]
- 2*d*e*p*x^n*Log[c*(d + e*x^n)^p] + 2*e^2*p*x^(2*n)*Log[e - e/x^n]*Log[c*(
d + e*x^n)^p] - d^2*Log[c*(d + e*x^n)^p]^2 + 2*e^2*n*p*x^(2*n)*Log[x]*(p +
p*Log[e + d/x^n] - p*Log[e - e/x^n] - Log[c*(d + e*x^n)^p] + p*Log[1 + (e*x
^n)/d]) + 2*e^2*p^2*x^(2*n)*PolyLog[2, -((e*x^n)/d)])/(2*d^2*f*n*(f*x)^(2*n
))
```

fricas [A] time = 0.49, size = 279, normalized size = 1.40

$$\frac{2e^2f^{-2n-1}np^2x^{2n}\log(x)\log\left(\frac{ex^n+d}{d}\right)+2e^2f^{-2n-1}p^2x^{2n}\text{Li}_2\left(-\frac{ex^n+d}{d}+1\right)-2def^{-2n-1}px^n\log(c)-d^2f^{-2n-1}\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*e^2*f^(-2*n - 1)*n*p^2*x^(2*n)*log(x)*log((e*x^n + d)/d) + 2*e^2*f^(
-2*n - 1)*p^2*x^(2*n)*dilog(-(e*x^n + d)/d + 1) - 2*d*e*f^(-2*n - 1)*p*x^n*
log(c) - d^2*f^(-2*n - 1)*log(c)^2 + 2*(e^2*n*p^2 - e^2*n*p*log(c))*f^(-2*n
 - 1)*x^(2*n)*log(x) + (e^2*f^(-2*n - 1)*p^2*x^(2*n) - d^2*f^(-2*n - 1)*p^2
```

```
)*log(e*x^n + d)^2 - 2*(d*e*f^(-2*n - 1)*p^2*x^n + d^2*f^(-2*n - 1)*p*log(c
) + (e^2*n*p^2*log(x) + e^2*p^2 - e^2*p*log(c))*f^(-2*n - 1)*x^(2*n))*log(e
*x^n + d))/(d^2*n*x^(2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{-2n-1} \log\left(\left(ex^n + d\right)^p c\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^(-2*n - 1)*log((e*x^n + d)^p*c)^2, x)
```

maple [F] time = 1.68, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{-2n-1} \ln\left(c\left(e\,x^n + d\right)^p\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-2*n-1)*ln(c*(e*x^n+d)^p)^2,x)
```

```
[Out] int((f*x)^(-2*n-1)*ln(c*(e*x^n+d)^p)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(e^2n^2p^2x^{2n}\log(x)^2 - e^2p^2x^{2n}\log\left(ex^n + d\right)^2 - 2\,depx^n\log(c) - d^2\log\left(\left(ex^n + d\right)^p\right)^2 - d^2\log(c)^2 - 2\left(e^2npx^{2n}\log\right.\right.}{2\,d^2nx^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(e^2*n^2*p^2*x^(2*n)*log(x)^2 - e^2*p^2*x^(2*n)*log(e*x^n + d)^2 - 2*d*
e*p*x^n*log(c) - d^2*log((e*x^n + d)^p)^2 - d^2*log(c)^2 - 2*(e^2*n*p*x^(2*
n)*log(x) - e^2*p*x^(2*n)*log(e*x^n + d) + d*e*p*x^n + d^2*log(c))*log((e*x
^n + d)^p))*f^(-2*n - 1)/(d^2*n*x^(2*n)) - integrate((e^2*n*p^2*log(x) - e^
2*p^2 + e^2*p*log(c))/(d*e*f^(2*n + 1)*x*x^n + d^2*f^(2*n + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(d + e\,x^n\right)^p\right)^2}{\left(f\,x\right)^{2\,n+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^2/(f*x)^(2*n + 1),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^2/(f*x)^(2*n + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1-2*n)*ln(c*(d+e*x**n)**p)**2,x)
```

```
[Out] Timed out
```

3.169 $\int \frac{\log(1+ex^n)}{x} dx$

Optimal. Leaf size=13

$$-\frac{\text{Li}_2(-ex^n)}{n}$$

[Out] -polylog(2,-e*x^n)/n

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2391}

$$-\frac{\text{PolyLog}(2,-ex^n)}{n}$$

Antiderivative was successfully verified.

[In] Int[Log[1 + e*x^n]/x,x]

[Out] -(PolyLog[2, -(e*x^n)]/n)

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log(1+ex^n)}{x} dx = -\frac{\text{Li}_2(-ex^n)}{n}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{\text{Li}_2(-ex^n)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[Log[1 + e*x^n]/x,x]

[Out] -(PolyLog[2, -(e*x^n)]/n)

fricas [A] time = 0.47, size = 12, normalized size = 0.92

$$-\frac{\text{Li}_2(-ex^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(1+e*x^n)/x,x, algorithm="fricas")

[Out] -dilog(-e*x^n)/n

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(ex^n+1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(log(e*x^n + 1)/x, x)
```

maple [A] time = 0.06, size = 14, normalized size = 1.08

$$-\frac{\operatorname{dilog}(e\,x^{n}+1)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+e*x^n)/x,x)
```

```
[Out] -1/n*dilog(1+e*x^n)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,n\log(x)^{2}+n\int\frac{\log(x)}{exx^{n}+x}\,dx+\log\left(ex^{n}+1\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*x^n)/x,x, algorithm="maxima")
```

```
[Out] -1/2*n*log(x)^2 + n*integrate(log(x)/(e*x*x^n + x), x) + log(e*x^n + 1)*log
(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int\frac{\ln\left(e\,x^{n}+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*x^n + 1)/x,x)
```

```
[Out] int(log(e*x^n + 1)/x, x)
```

sympy [C] time = 3.12, size = 14, normalized size = 1.08

$$-\frac{\operatorname{Li}_{2}\left(ex^{n}e^{i\pi}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+e*x**n)/x,x)
```

```
[Out] -polylog(2, e*x**n*exp_polar(I*pi))/n
```

3.170 $\int \frac{\log(2+ex^n)}{x} \, dx$

Optimal. Leaf size=21

$$\log(2)\log(x)-\frac{\text{Li}_2\left(-\frac{ex^n}{2}\right)}{n}$$

```
[Out] ln(2)*ln(x)-polylog(2,-1/2*e*x^n)/n
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2454, 2392, 2391}

$$\log(2)\log(x)-\frac{\text{PolyLog}\left(2,-\frac{ex^n}{2}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[2 + e*x^n]/x,x]
```

```
[Out] Log[2]*Log[x] - PolyLog[2, -(e*x^n)/2]/n
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(2+ex^n\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{\log(2+ex)}{x}\,dx,x,x^n\right)}{n}\\ &= \log(2)\log(x)+\frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{2}\right)}{x}\,dx,x,x^n\right)}{n}\\ &= \log(2)\log(x)-\frac{\text{Li}_2\left(-\frac{ex^n}{2}\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\log(2)\log(x)-\frac{\text{Li}_2\left(-\frac{ex^n}{2}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[2 + e*x^n]/x,x]
```

```
[Out] Log[2]*Log[x] - PolyLog[2, -1/2*(e*x^n)]/n
```

fricas [B] time = 0.44, size = 40, normalized size = 1.90

$$\frac{n\log\left(ex^{n}+2\right)\log(x)-n\log\left(\frac{1}{2}\,ex^{n}+1\right)\log(x)-\mathrm{Li}_{2}\left(-\frac{1}{2}\,ex^{n}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2+e*x^n)/x,x, algorithm="fricas")
```

```
[Out] (n*log(e*x^n + 2)*log(x) - n*log(1/2*e*x^n + 1)*log(x) - dilog(-1/2*e*x^n))
/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(ex^{n}+2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2+e*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(log(e*x^n + 2)/x, x)
```

maple [B] time = 0.06, size = 56, normalized size = 2.67

$$\frac{\ln\left(-\frac{e\,x^{n}}{2}\right)\ln\left(e\,x^{n}+2\right)}{n}-\frac{\ln\left(-\frac{e\,x^{n}}{2}\right)\ln\left(\frac{e\,x^{n}}{2}+1\right)}{n}-\frac{\mathrm{dilog}\left(\frac{e\,x^{n}}{2}+1\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(2+e*x^n)/x,x)
```

```
[Out] -1/n*ln(-1/2*e*x^n)*ln(1/2*e*x^n+1)+1/n*ln(-1/2*e*x^n)*ln(2+e*x^n)-1/n*dilo
g(1/2*e*x^n+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,n\log(x)^{2}+2\,n\int\frac{\log(x)}{exx^{n}+2\,x}\,dx+\log\left(ex^{n}+2\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2+e*x^n)/x,x, algorithm="maxima")
```

```
[Out] -1/2*n*log(x)^2 + 2*n*integrate(log(x)/(e*x*x^n + 2*x), x) + log(e*x^n + 2)
*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int\frac{\ln\left(e\,x^{n}+2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*x^n + 2)/x,x)
```

```
[Out] int(log(e*x^n + 2)/x, x)
```

sympy [C] time = 3.72, size = 78, normalized size = 3.71

$$\begin{cases} \log(2)\log(x) - \frac{\operatorname{Li}_2\left(\frac{ex^n e^{i\pi}}{2}\right)}{n} & \text{for } |x| < 1 \\ -\log(2)\log\left(\frac{1}{x}\right) - \frac{\operatorname{Li}_2\left(\frac{ex^n e^{i\pi}}{2}\right)}{n} & \text{for } \frac{1}{|x|} < 1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| x\right)\log(2) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| x\right)\log(2) - \frac{\operatorname{Li}_2\left(\frac{ex^n e^{i\pi}}{2}\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(2+e*x**n)/x,x)
```

```
[Out] Piecewise((log(2)*log(x) - polylog(2, e*x**n*exp_polar(I*pi)/2)/n, Abs(x) <
 1), (-log(2)*log(1/x) - polylog(2, e*x**n*exp_polar(I*pi)/2)/n, 1/Abs(x) <
 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(2) + meijerg(((1, 1), ())
, ((), (0, 0)), x)*log(2) - polylog(2, e*x**n*exp_polar(I*pi)/2)/n, True))
```

3.171 $\int \frac{\log(2(3+ex^n))}{x} dx$

Optimal. Leaf size=21

$$\log(6)\log(x) - \frac{\text{Li}_2\left(-\frac{ex^n}{3}\right)}{n}$$

```
[Out] ln(6)*ln(x)-polylog(2,-1/3*e*x^n)/n
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2454, 2392, 2391}

$$\log(6)\log(x) - \frac{\text{PolyLog}\left(2, -\frac{ex^n}{3}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[2*(3 + e*x^n)]/x,x]
```

```
[Out] Log[6]*Log[x] - PolyLog[2, -(e*x^n)/3]/n
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\log(2(3+ex^n))}{x} dx &= \frac{\text{Subst}\left(\int \frac{\log(2(3+ex))}{x} dx, x, x^n\right)}{n}\\ &= \log(6)\log(x) + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{3}\right)}{x} dx, x, x^n\right)}{n}\\ &= \log(6)\log(x) - \frac{\text{Li}_2\left(-\frac{ex^n}{3}\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\log(6)\log(x) - \frac{\text{Li}_2\left(-\frac{ex^n}{3}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[2*(3 + e*x^n)]/x,x]
[Out] Log[6]*Log[x] - PolyLog[2, -1/3*(e*x^n)]/n
```

fricas [B] time = 0.43, size = 41, normalized size = 1.95

$$\frac{n\log\left(2\,ex^n+6\right)\log(x)-n\log\left(\frac{1}{3}\,ex^n+1\right)\log(x)-\mathrm{Li}_2\left(-\frac{1}{3}\,ex^n\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(6+2*e*x^n)/x,x, algorithm="fricas")
[Out] (n*log(2*e*x^n + 6)*log(x) - n*log(1/3*e*x^n + 1)*log(x) - dilog(-1/3*e*x^n
))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(2\,ex^n+6\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(6+2*e*x^n)/x,x, algorithm="giac")
[Out] integrate(log(2*e*x^n + 6)/x, x)
```

maple [B] time = 0.07, size = 57, normalized size = 2.71

$$\frac{\ln\left(-\frac{e\,x^n}{3}\right)\ln\left(2e\,x^n+6\right)}{n}-\frac{\ln\left(-\frac{e\,x^n}{3}\right)\ln\left(\frac{e\,x^n}{3}+1\right)}{n}-\frac{\mathrm{dilog}\left(\frac{e\,x^n}{3}+1\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(6+2*e*x^n)/x,x)
[Out] -1/n*ln(-1/3*e*x^n)*ln(1/3*e*x^n+1)+1/n*ln(-1/3*e*x^n)*ln(6+2*e*x^n)-1/n*di
log(1/3*e*x^n+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,n\log(x)^2+3\,n\int\frac{\log(x)}{exx^n+3\,x}\,dx+\log(2)\log(x)+\log\left(ex^n+3\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(6+2*e*x^n)/x,x, algorithm="maxima")
[Out] -1/2*n*log(x)^2 + 3*n*integrate(log(x)/(e*x*x^n + 3*x), x) + log(2)*log(x)
+ log(e*x^n + 3)*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int\frac{\ln\left(2\,e\,x^n+6\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(2*e*x^n + 6)/x,x)
[Out] int(log(2*e*x^n + 6)/x, x)
```

sympy [C] time = 3.87, size = 78, normalized size = 3.71

$$\begin{cases} \log(6)\log(x) - \frac{\text{Li}_2\left(\frac{ex^n e^{i\pi}}{3}\right)}{n} & \text{for } |x| < 1 \\ -\log(6)\log\left(\frac{1}{x}\right) - \frac{\text{Li}_2\left(\frac{ex^n e^{i\pi}}{3}\right)}{n} & \text{for } \frac{1}{|x|} < 1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| x\right)\log(6) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| x\right)\log(6) - \frac{\text{Li}_2\left(\frac{ex^n e^{i\pi}}{3}\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(6+2*e*x**n)/x,x)
```

```
[Out] Piecewise((log(6)*log(x) - polylog(2, e*x**n*exp_polar(I*pi)/3)/n, Abs(x) <
 1), (-log(6)*log(1/x) - polylog(2, e*x**n*exp_polar(I*pi)/3)/n, 1/Abs(x) <
 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(6) + meijerg(((1, 1), ())
, ((), (0, 0)), x)*log(6) - polylog(2, e*x**n*exp_polar(I*pi)/3)/n, True))
```

3.172 $\int \frac{\log(c(d+ex^n))}{x} dx$

Optimal. Leaf size=41

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)\right)}{n}+\frac{\mathrm{Li}_2\left(\frac{ex^n}{d}+1\right)}{n}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n))/n+polylog(2,1+e*x^n/d)/n
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2454, 2394, 2315}

$$\frac{\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)])/n + PolyLog[2, 1 + (e*x^n)/d]/n
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(d+ex^n\right)\right)}{x} dx &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{x} dx, x, x^n\right)}{n}\\ &= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)\right)}{n}-\frac{e\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex} dx, x, x^n\right)}{n}\\ &= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)\right)}{n}+\frac{\mathrm{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.95

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)\right)+\mathrm{Li}_2\left(\frac{ex^n+d}{d}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)] + PolyLog[2, (d + e*x^n)/d])/n
```

fricas [A] time = 0.45, size = 54, normalized size = 1.32

$$\frac{n\log\left(cex^{n}+cd\right)\log(x)-n\log(x)\log\left(\frac{ex^{n}+d}{d}\right)-\mathrm{Li}_2\left(-\frac{ex^{n}+d}{d}+1\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n))/x,x, algorithm="fricas")
```

```
[Out] (n*log(c*e*x^n + c*d)*log(x) - n*log(x)*log((e*x^n + d)/d) - dilog(-(e*x^n
+ d)/d + 1))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left((ex^{n}+d)c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n))/x,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)*c)/x, x)
```

maple [A] time = 0.14, size = 41, normalized size = 1.00

$$\frac{\ln\left(-\frac{e\,x^{n}}{d}\right)\ln\left(ce\,x^{n}+cd\right)}{n}+\frac{\mathrm{dilog}\left(-\frac{e\,x^{n}}{d}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d))/x,x)
```

```
[Out] 1/n*ln(c*e*x^n+c*d)*ln(-1/d*e*x^n)+1/n*dilog(-1/d*e*x^n)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$dn\int\frac{\log(x)}{exx^{n}+dx}\,dx-\frac{1}{2}\,n\log(x)^2+\log\left(ex^{n}+d\right)\log(x)+\log(c)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n))/x,x, algorithm="maxima")
```

```
[Out] d*n*integrate(log(x)/(e*x*x^n + d*x), x) - 1/2*n*log(x)^2 + log(e*x^n + d)*
log(x) + log(c)*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\ln\left(c\,\left(d+e\,x^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n))/x,x)
```

```
[Out] int(log(c*(d + e*x^n))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(cd + cex^n)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n))/x,x)
```

```
[Out] Integral(log(c*d + c*e*x**n)/x, x)
```

3.173 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x}\,dx$

Optimal. Leaf size=44

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{p\text{Li}_2\left(\frac{ex^n}{d}+1\right)}{n}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+p*polylog(2,1+e*x^n/d)/n
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2454, 2394, 2315}

$$\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (p*PolyLog[2, 1 + (e*x^n)/d])/
n
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{(ep)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,x^n\right)}{n}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.98

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)+p\text{Li}_2\left(\frac{ex^n+d}{d}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, (d + e*x^n)/d])/n
```

fricas [A] time = 0.44, size = 60, normalized size = 1.36

$$\frac{np\log\left(ex^n+d\right)\log(x)-np\log(x)\log\left(\frac{ex^n+d}{d}\right)+n\log(c)\log(x)-p\text{Li}_2\left(-\frac{ex^n+d}{d}+1\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] (n*p*log(e*x^n + d)*log(x) - n*p*log(x)*log((e*x^n + d)/d) + n*log(c)*log(x
) - p*dilog(-(e*x^n + d)/d + 1))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(\left(ex^n+d\right)^pc\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/x, x)
```

maple [C] time = 0.08, size = 177, normalized size = 4.02

$$-\frac{i\pi\,\text{csgn}\left(ic\right)\text{csgn}\left(i\left(e\,x^n+d\right)^p\right)\text{csgn}\left(ic\left(e\,x^n+d\right)^p\right)\ln(x)}{2}+\frac{i\pi\,\text{csgn}\left(ic\right)\text{csgn}\left(ic\left(e\,x^n+d\right)^p\right)^2\ln(x)}{2}+\frac{i\pi\,\text{csgn}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)/x,x)
```

```
[Out] ln(x)*ln((e*x^n+d)^p)+1/2*I*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2*
ln(x)-1/2*I*Pi*csgn(I*c)*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*ln(x)-1/
2*I*Pi*csgn(I*c*(e*x^n+d)^p)^3*ln(x)+1/2*I*Pi*csgn(I*c)*csgn(I*c*(e*x^n+d)^
p)^2*ln(x)+ln(c)*ln(x)-p/n*dilog((e*x^n+d)/d)-p*ln(x)*ln((e*x^n+d)/d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$dnp\int\frac{\log(x)}{exx^n+dx}\,dx-\frac{1}{2}np\log(x)^2+\log\left(\left(ex^n+d\right)^p\right)\log(x)+\log(c)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] d*n*p*integrate(log(x)/(e*x*x^n + d*x), x) - 1/2*n*p*log(x)^2 + log((e*x^n
+ d)^p)*log(x) + log(c)*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/x,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\,(d+ex^n)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)/x, x)
```

3.174 $\int \frac{\log^2(c(d+ex^n)^p)}{x} dx$

Optimal. Leaf size=79

$$\frac{2p\text{Li}_2\left(\frac{ex^n}{d}+1\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{2p^2\text{Li}_3\left(\frac{ex^n}{d}+1\right)}{n}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)^2/n+2*p*ln(c*(d+e*x^n)^p)*polylog(2,1+e*x^n/
d)/n-2*p^2*polylog(3,1+e*x^n/d)/n
```

Rubi [A] time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2454, 2396, 2433, 2374, 6589}

$$\frac{2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{2p^2\text{PolyLog}\left(3,\frac{ex^n}{d}+1\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]^2/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]^2)/n + (2*p*Log[c*(d + e*x^n)^p]*Po
lyLog[2, 1 + (e*x^n)/d])/n - (2*p^2*PolyLog[3, 1 + (e*x^n)/d])/n
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\log^2(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{n} - \frac{(2ep)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^p)}{d+ex}\,dx, x, x^n\right)}{n}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{n} - \frac{(2p)\,\text{Subst}\left(\int \frac{\log(cx^p)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+ex^n\right)}{n}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{2p\log\left(c\left(d+ex^n\right)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{\left(2p^2\right)\text{Subst}\left(\int\right.}{}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{2p\log\left(c\left(d+ex^n\right)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{2p^2\text{Li}_3\left(1+\frac{ex^n}{d}\right)}{n}$$

Mathematica [B] time = 0.07, size = 164, normalized size = 2.08

$$2p\left(\log(x)\left(\log\left(d+ex^n\right)-\log\left(\frac{ex^n}{d}+1\right)\right)-\frac{\text{Li}_2\left(-\frac{ex^n}{d}\right)}{n}\right)\left(\log\left(c\left(d+ex^n\right)^p\right)-p\log\left(d+ex^n\right)\right)+\log(x)\left(\log\left(c\left(d\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^2/x,x]
```

```
[Out] Log[x]*(-(p*Log[d + e*x^n]) + Log[c*(d + e*x^n)^p])^2 + 2*p*(-(p*Log[d + e*
x^n]) + Log[c*(d + e*x^n)^p])*(Log[x]*(Log[d + e*x^n] - Log[1 + (e*x^n)/d])
 - PolyLog[2, -((e*x^n)/d)]/n) + (p^2*(Log[-((e*x^n)/d)]*Log[d + e*x^n]^2 +
 2*Log[d + e*x^n]*PolyLog[2, 1 + (e*x^n)/d] - 2*PolyLog[3, 1 + (e*x^n)/d]))
/n
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^n+d\right)^p c\right)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^2/x,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)^2/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^n+d\right)^p c\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^2/x,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^2/x, x)
```

maple [C] time = 3.15, size = 1356, normalized size = 17.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)^2/x,x)
```

```
[Out] 1/n*ln((e*x^n+d)^p)^2*ln(e*x^n)-2/n*polylog(3,(e*x^n+d)/d)*p^2-1/4*Pi^2*ln(
x)*csgn(I*c*(e*x^n+d)^p)^6-I/n*Pi*ln((e*x^n+d)/d)*ln(x^n)*p*csgn(I*c*(e*x^n
+d)^p)^2*csgn(I*c)-I/n*Pi*ln((e*x^n+d)/d)*ln(x^n)*p*csgn(I*(e*x^n+d)^p)*csg
n(I*c*(e*x^n+d)^p)^2+I/n*Pi*dilog((e*x^n+d)/d)*p*csgn(I*(e*x^n+d)^p)*csgn(I
*c*(e*x^n+d)^p)*csgn(I*c)-I/n*Pi*ln((e*x^n+d)^p)*ln(x^n)*csgn(I*(e*x^n+d)^p
)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)+2/n*ln(c)*ln((e*x^n+d)^p)*ln(x^n)-2/n*ln(
c)*dilog((e*x^n+d)/d)*p+2/n*polylog(2,(e*x^n+d)/d)*ln(e*x^n+d)*p^2-2/n*ln(e
*x^n+d)*dilog(-1/d*e*x^n)*p^2+2/n*ln((e*x^n+d)^p)*dilog(-1/d*e*x^n)*p+I/n*P
i*ln((e*x^n+d)/d)*ln(x^n)*p*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(
I*c)+1/2*Pi^2*ln(x)*csgn(I*c*(e*x^n+d)^p)^5*csgn(I*c)-1/4*Pi^2*ln(x)*csgn(I
*(e*x^n+d)^p)^2*csgn(I*c*(e*x^n+d)^p)^4+I/n*Pi*ln((e*x^n+d)/d)*ln(x^n)*p*cs
gn(I*c*(e*x^n+d)^p)^3-I/n*Pi*dilog((e*x^n+d)/d)*p*csgn(I*c*(e*x^n+d)^p)^2*c
sgn(I*c)-1/4*Pi^2*ln(x)*csgn(I*c*(e*x^n+d)^p)^4*csgn(I*c)^2+1/2*Pi^2*ln(x)*
csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^5+1/n*ln(e*x^n+d)^2*ln(e*x^n)*p^2
+1/n*ln(-(e*x^n+d)/d+1)*ln(e*x^n+d)^2*p^2-2/n*ln(e*x^n+d)^2*ln(-1/d*e*x^n)*
p^2-I*ln(c)*Pi*ln(x)*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)+ln
(c)^2*ln(x)+I/n*Pi*ln((e*x^n+d)^p)*ln(x^n)*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*
x^n+d)^p)^2+I/n*Pi*ln((e*x^n+d)^p)*ln(x^n)*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c
)+2/n*ln((e*x^n+d)^p)*ln(e*x^n+d)*ln(-1/d*e*x^n)*p-2/n*ln((e*x^n+d)^p)*ln(e
*x^n+d)*ln(e*x^n)*p-2/n*ln(c)*ln((e*x^n+d)/d)*ln(x^n)*p+1/2*Pi^2*ln(x)*csgn
(I*(e*x^n+d)^p)^2*csgn(I*c*(e*x^n+d)^p)^3*csgn(I*c)-1/4*Pi^2*ln(x)*csgn(I*(
e*x^n+d)^p)^2*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)^2-Pi^2*ln(x)*csgn(I*(e*x^n+
d)^p)*csgn(I*c*(e*x^n+d)^p)^4*csgn(I*c)+1/2*Pi^2*ln(x)*csgn(I*(e*x^n+d)^p)*
csgn(I*c*(e*x^n+d)^p)^3*csgn(I*c)^2-I*ln(c)*Pi*ln(x)*csgn(I*c*(e*x^n+d)^p)^
3+I/n*Pi*dilog((e*x^n+d)/d)*p*csgn(I*c*(e*x^n+d)^p)^3+I*ln(c)*Pi*ln(x)*csgn
(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2-I/n*Pi*ln((e*x^n+d)^p)*ln(x^n)*csgn
(I*c*(e*x^n+d)^p)^3-I/n*Pi*dilog((e*x^n+d)/d)*p*csgn(I*(e*x^n+d)^p)*csgn(I*
c*(e*x^n+d)^p)^2+I*ln(c)*Pi*ln(x)*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\log\left(\left(ex^n+d\right)^p\right)^2\log(x) - \int -\frac{ex^n\log(c)^2 + d\log(c)^2 - 2\left(\left(enp\log(x) - e\log(c)\right)x^n - d\log(c)\right)\log\left(\left(ex^n+d\right)^p\right)}{exx^n + dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^2/x,x, algorithm="maxima")
```

```
[Out] log((e*x^n + d)^p)^2*log(x) - integrate(-(e*x^n*log(c)^2 + d*log(c)^2 - 2*(
(e*n*p*log(x) - e*log(c))*x^n - d*log(c))*log((e*x^n + d)^p))/(e*x*x^n + d*
x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^2/x,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+ex^{n}\right)^{p}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**2/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)**2/x, x)
```

3.175 $\int \frac{\log^3(c(d+ex^n)^p)}{x} dx$

Optimal. Leaf size=113

$$-\frac{6p^2\text{Li}_3\left(\frac{ex^n}{d}+1\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{3p\text{Li}_2\left(\frac{ex^n}{d}+1\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{6p^3\text{I}}{}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)^3/n+3*p*ln(c*(d+e*x^n)^p)^2*polylog(2,1+e*x^
n/d)/n-6*p^2*ln(c*(d+e*x^n)^p)*polylog(3,1+e*x^n/d)/n+6*p^3*polylog(4,1+e*x
^n/d)/n
```

Rubi [A] time = 0.15, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2396, 2433, 2374, 2383, 6589}

$$-\frac{6p^2\text{PolyLog}\left(3,\frac{ex^n}{d}+1\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{3p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{6p^3\text{PolyLog}\left(4,\frac{ex^n}{d}\right.}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]^3/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]^3)/n + (3*p*Log[c*(d + e*x^n)^p]^2*
PolyLog[2, 1 + (e*x^n)/d])/n - (6*p^2*Log[c*(d + e*x^n)^p]*PolyLog[3, 1 + (
e*x^n)/d])/n + (6*p^3*PolyLog[4, 1 + (e*x^n)/d])/n
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\log^3(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c\left(d+ex^n\right)^p\right)}{n} - \frac{(3ep)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\log^2(c(d+ex)^p)}{d+ex}\,dx, x, x^n\right)}{n}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c\left(d+ex^n\right)^p\right)}{n} - \frac{(3p)\,\text{Subst}\left(\int \frac{\log^2(cx^p)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+ex^n\right)}{n}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{3p\log^2\left(c\left(d+ex^n\right)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{\left(6p^2\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{3p\log^2\left(c\left(d+ex^n\right)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{6p^2\log\left(c\left(d+\right.\right.}{}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{3p\log^2\left(c\left(d+ex^n\right)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{6p^2\log\left(c\left(d+\right.\right.}{}$$

Mathematica [B] time = 0.11, size = 270, normalized size = 2.39

$$\frac{-6p^2\text{Li}_3\left(\frac{ex^n}{d}+1\right)\log\left(c\left(d+ex^n\right)^p\right) + 3np^2\log(x)\log^2\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right) - 3p^2\log\left(-\frac{ex^n}{d}\right)\log^2\left(d+ex^n\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^3/x,x]
```

```
[Out] (-(n*p^3*Log[x]*Log[d + e*x^n]^3) + p^3*Log[-((e*x^n)/d)]*Log[d + e*x^n]^3
+ 3*n*p^2*Log[x]*Log[d + e*x^n]^2*Log[c*(d + e*x^n)^p] - 3*p^2*Log[-((e*x^n
)/d)]*Log[d + e*x^n]^2*Log[c*(d + e*x^n)^p] - 3*n*p*Log[x]*Log[d + e*x^n]*L
og[c*(d + e*x^n)^p]^2 + 3*p*Log[-((e*x^n)/d)]*Log[d + e*x^n]*Log[c*(d + e*x
^n)^p]^2 + n*Log[x]*Log[c*(d + e*x^n)^p]^3 + 3*p*Log[c*(d + e*x^n)^p]^2*Pol
yLog[2, 1 + (e*x^n)/d] - 6*p^2*Log[c*(d + e*x^n)^p]*PolyLog[3, 1 + (e*x^n)/
d] + 6*p^3*PolyLog[4, 1 + (e*x^n)/d])/n
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^n+d\right)^p c\right)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^3/x,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)^3/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^n + d\right)^p c\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^3/x,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^3/x, x)
```

maple [C] time = 3.94, size = 6131, normalized size = 54.26

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)^3/x,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\log\left(\left(ex^n + d\right)^p\right)^3 \log(x) - \int -\frac{ex^n \log(c)^3 + d\log(c)^3 - 3\left(\left(enp\log(x) - e\log(c)\right)x^n - d\log(c)\right)\log\left(\left(ex^n + d\right)^p\right.}{exx^n + dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^3/x,x, algorithm="maxima")
```

```
[Out] log((e*x^n + d)^p)^3*log(x) - integrate(-(e*x^n*log(c)^3 + d*log(c)^3 - 3*(
(e*n*p*log(x) - e*log(c))*x^n - d*log(c))*log((e*x^n + d)^p)^2 + 3*(e*x^n*l
og(c)^2 + d*log(c)^2)*log((e*x^n + d)^p))/(e*x*x^n + d*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^3/x,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d + ex^n\right)^p\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**3/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)**3/x, x)
```

3.176 $\int (d+ex)^3 \log(c(a+bx)^p)\,dx$

Optimal. Leaf size=140

$$-\frac{p(bd-ae)^4\log(a+bx)}{4b^4e}-\frac{px(bd-ae)^3}{4b^3}-\frac{p(d+ex)^2(bd-ae)^2}{8b^2e}+\frac{(d+ex)^4\log(c(a+bx)^p)}{4e}-\frac{p(d+ex)^3(bd-ae)}{12be}-\frac{p}{}$$

```
[Out] -1/4*(-a*e+b*d)^3*p*x/b^3-1/8*(-a*e+b*d)^2*p*(e*x+d)^2/b^2/e-1/12*(-a*e+b*d
)*p*(e*x+d)^3/b/e-1/16*p*(e*x+d)^4/e-1/4*(-a*e+b*d)^4*p*ln(b*x+a)/b^4/e+1/4
*(e*x+d)^4*ln(c*(b*x+a)^p)/e
```

Rubi [A] time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2395, 43}

$$-\frac{px(bd-ae)^3}{4b^3}-\frac{p(d+ex)^2(bd-ae)^2}{8b^2e}-\frac{p(bd-ae)^4\log(a+bx)}{4b^4e}+\frac{(d+ex)^4\log(c(a+bx)^p)}{4e}-\frac{p(d+ex)^3(bd-ae)}{12be}-\frac{p}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Log[c*(a + b*x)^p],x]
```

```
[Out] -((b*d - a*e)^3*p*x)/(4*b^3) - ((b*d - a*e)^2*p*(d + e*x)^2)/(8*b^2*e) - ((
b*d - a*e)*p*(d + e*x)^3)/(12*b*e) - (p*(d + e*x)^4)/(16*e) - ((b*d - a*e)^
4*p*Log[a + b*x])/(4*b^4*e) + ((d + e*x)^4*Log[c*(a + b*x)^p])/(4*e)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3\log(c(a+bx)^p)\,dx &= \frac{(d+ex)^4\log(c(a+bx)^p)}{4e}-\frac{(bp)\int\frac{(d+ex)^4}{a+bx}\,dx}{4e}\\ &= \frac{(d+ex)^4\log(c(a+bx)^p)}{4e}-\frac{(bp)\int\left(\frac{e(bd-ae)^3}{b^4}+\frac{(bd-ae)^4}{b^4(a+bx)}+\frac{e(bd-ae)^2(d+ex)}{b^3}+\frac{e(bd-ae)(}{b^2}\right.}{4e}\\ &= -\frac{(bd-ae)^3px}{4b^3}-\frac{(bd-ae)^2p(d+ex)^2}{8b^2e}-\frac{(bd-ae)p(d+ex)^3}{12be}-\frac{p(d+ex)^4}{16e}-\frac{(bd}{}\end{aligned}$$

Mathematica [A] time = 0.21, size = 185, normalized size = 1.32

$$-\frac{12a^2ep\left(a^2e^2-4abde+6b^2d^2\right)\log(a+bx)+bpx\left(-12a^3e^3+6a^2be^2(8d+ex)-4ab^2e\left(18d^2+6dex+e^2x^2\right)+b^3\right.}{48b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Log[c*(a + b*x)^p],x]
```

```
[Out] -1/48*(b*p*x*(-12*a^3*e^3 + 6*a^2*b*e^2*(8*d + e*x) - 4*a*b^2*e*(18*d^2 + 6
*d*e*x + e^2*x^2) + b^3*(48*d^3 + 36*d^2*e*x + 16*d*e^2*x^2 + 3*e^3*x^3)) +
 12*a^2*e*(6*b^2*d^2 - 4*a*b*d*e + a^2*e^2)*p*Log[a + b*x] - 12*b^3*(4*a*d^
3 + b*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3))*Log[c*(a + b*x)^p])/b^
4
```

fricas [B] time = 0.45, size = 269, normalized size = 1.92

$$-\frac{3\,b^4e^3px^4 + 4\left(4\,b^4de^2 - ab^3e^3\right)px^3 + 6\left(6\,b^4d^2e - 4\,ab^3de^2 + a^2b^2e^3\right)px^2 + 12\left(4\,b^4d^3 - 6\,ab^3d^2e + 4\,a^2b^2de^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x+a)^p),x, algorithm="fricas")
```

```
[Out] -1/48*(3*b^4*e^3*p*x^4 + 4*(4*b^4*d*e^2 - a*b^3*e^3)*p*x^3 + 6*(6*b^4*d^2*e
 - 4*a*b^3*d*e^2 + a^2*b^2*e^3)*p*x^2 + 12*(4*b^4*d^3 - 6*a*b^3*d^2*e + 4*a
^2*b^2*d*e^2 - a^3*b*e^3)*p*x - 12*(b^4*e^3*p*x^4 + 4*b^4*d*e^2*p*x^3 + 6*b
^4*d^2*e*p*x^2 + 4*b^4*d^3*p*x + (4*a*b^3*d^3 - 6*a^2*b^2*d^2*e + 4*a^3*b*d
*e^2 - a^4*e^3)*p)*log(b*x + a) - 12*(b^4*e^3*x^4 + 4*b^4*d*e^2*x^3 + 6*b^4
*d^2*e*x^2 + 4*b^4*d^3*x)*log(c))/b^4
```

giac [B] time = 0.20, size = 558, normalized size = 3.99

$$\frac{(bx+a)d^3p\log(bx+a)}{b}+\frac{3\,(bx+a)^2d^2pe\log(bx+a)}{2\,b^2}-\frac{3\,(bx+a)ad^2pe\log(bx+a)}{b^2}-\frac{(bx+a)d^3p}{b}-\frac{3\,(bx+a)^2}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x+a)^p),x, algorithm="giac")
```

```
[Out] (b*x + a)*d^3*p*log(b*x + a)/b + 3/2*(b*x + a)^2*d^2*p*e*log(b*x + a)/b^2 -
 3*(b*x + a)*a*d^2*p*e*log(b*x + a)/b^2 - (b*x + a)*d^3*p/b - 3/4*(b*x + a)
^2*d^2*p*e/b^2 + 3*(b*x + a)*a*d^2*p*e/b^2 + (b*x + a)^3*d*p*e^2*log(b*x +
a)/b^3 - 3*(b*x + a)^2*a*d*p*e^2*log(b*x + a)/b^3 + 3*(b*x + a)*a^2*d*p*e^2
*log(b*x + a)/b^3 + (b*x + a)*d^3*log(c)/b + 3/2*(b*x + a)^2*d^2*e*log(c)/b
^2 - 3*(b*x + a)*a*d^2*e*log(c)/b^2 - 1/3*(b*x + a)^3*d*p*e^2/b^3 + 3/2*(b*
x + a)^2*a*d*p*e^2/b^3 - 3*(b*x + a)*a^2*d*p*e^2/b^3 + 1/4*(b*x + a)^4*p*e^
3*log(b*x + a)/b^4 - (b*x + a)^3*a*p*e^3*log(b*x + a)/b^4 + 3/2*(b*x + a)^2
*a^2*p*e^3*log(b*x + a)/b^4 - (b*x + a)*a^3*p*e^3*log(b*x + a)/b^4 + (b*x +
 a)^3*d*e^2*log(c)/b^3 - 3*(b*x + a)^2*a*d*e^2*log(c)/b^3 + 3*(b*x + a)*a^2
*d*e^2*log(c)/b^3 - 1/16*(b*x + a)^4*p*e^3/b^4 + 1/3*(b*x + a)^3*a*p*e^3/b^
4 - 3/4*(b*x + a)^2*a^2*p*e^3/b^4 + (b*x + a)*a^3*p*e^3/b^4 + 1/4*(b*x + a)
^4*e^3*log(c)/b^4 - (b*x + a)^3*a*e^3*log(c)/b^4 + 3/2*(b*x + a)^2*a^2*e^3*
log(c)/b^4 - (b*x + a)*a^3*e^3*log(c)/b^4
```

maple [C] time = 0.54, size = 766, normalized size = 5.47

$$d\,e^2x^3\ln(c)+\frac{3d^2e\,x^2\ln(c)}{2}-\frac{d^4p\ln(bx+a)}{4e}+\frac{e^3x^4\ln(c)}{4}+d^3x\ln(c)+\frac{(ex+d)^4\ln\left((bx+a)^p\right)}{4e}-\frac{e^3p\,x^4}{16}-\frac{3d^2ep\,x^2}{4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*ln(c*(b*x+a)^p),x)
```

```
[Out] e^2*ln(c)*d*x^3+3/2*e*ln(c)*d^2*x^2-1/4/e*ln(b*x+a)*d^4*p+1/4*e^3*ln(c)*x^4
+ln(c)*d^3*x+1/4*(e*x+d)^4/e*ln((b*x+a)^p)-1/16*e^3*p*x^4-1/2*I*Pi*d^3*x*cs
gn(I*c*(b*x+a)^p)^3-3/4*d^2*e*p*x^2+1/12/b*e^3*a*p*x^3-1/8/b^2*e^3*a^2*p*x^
2+1/4/b^3*e^3*a^3*p*x-d^3*p*x-1/4/b^4*e^3*ln(b*x+a)*a^4*p+1/b*ln(b*x+a)*a*d
^3*p-1/8*I*e^3*Pi*x^4*csgn(I*c*(b*x+a)^p)^3-1/3*d*e^2*p*x^3+1/b^3*e^2*ln(b*
```

```
x+a)*a^3*d*p-3/2/b^2*e*ln(b*x+a)*a^2*d^2*p-1/2*I*e^2*Pi*d*x^3*csgn(I*(b*x+a
)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)-3/4*I*e*Pi*d^2*x^2*csgn(I*(b*x+a)^p)*csg
n(I*c*(b*x+a)^p)*csgn(I*c)+1/2/b*e^2*a*d*p*x^2-1/b^2*e^2*a^2*d*p*x+3/2/b*e*
a*d^2*p*x+1/2*I*e^2*Pi*d*x^3*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)+3/4*I*e*Pi*d^2
*x^2*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+3/4*I*e*Pi*d^2*x^2*csgn(I*c*(b
*x+a)^p)^2*csgn(I*c)-1/8*I*e^3*Pi*x^4*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)
*csgn(I*c)+1/2*I*e^2*Pi*d*x^3*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2-1/2*I
*Pi*d^3*x*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)+1/2*I*Pi*d^3*x*cs
gn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+1/2*I*Pi*d^3*x*csgn(I*c*(b*x+a)^p)^2*
csgn(I*c)+1/8*I*e^3*Pi*x^4*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+1/8*I*e^
3*Pi*x^4*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)-1/2*I*e^2*Pi*d*x^3*csgn(I*c*(b*x+a
)^p)^3-3/4*I*e*Pi*d^2*x^2*csgn(I*c*(b*x+a)^p)^3
```

maxima [A] time = 0.45, size = 214, normalized size = 1.53

$$-\frac{1}{48}\,bp\left(\frac{3\,b^3e^3x^4+4\left(4\,b^3de^2-ab^2e^3\right)x^3+6\left(6\,b^3d^2e-4\,ab^2de^2+a^2be^3\right)x^2+12\left(4\,b^3d^3-6\,ab^2d^2e+4\,a^2bde^2-\right.}{b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x+a)^p),x, algorithm="maxima")
```

```
[Out] -1/48*b*p*((3*b^3*e^3*x^4 + 4*(4*b^3*d*e^2 - a*b^2*e^3)*x^3 + 6*(6*b^3*d^2*
e - 4*a*b^2*d*e^2 + a^2*b*e^3)*x^2 + 12*(4*b^3*d^3 - 6*a*b^2*d^2*e + 4*a^2*
b*d*e^2 - a^3*e^3)*x)/b^4 - 12*(4*a*b^3*d^3 - 6*a^2*b^2*d^2*e + 4*a^3*b*d*e
^2 - a^4*e^3)*log(b*x + a)/b^5) + 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2
+ 4*d^3*x)*log((b*x + a)^p*c)
```

mupad [B] time = 0.31, size = 208, normalized size = 1.49

$$\ln\left(c\,(a+b\,x)^p\right)\,\left(d^3\,x+\frac{3\,d^2\,e\,x^2}{2}+d\,e^2\,x^3+\frac{e^3\,x^4}{4}\right)+x^2\,\left(\frac{a\,\left(d\,e^2\,p-\frac{a\,e^3\,p}{4\,b}\right)}{2\,b}-\frac{3\,d^2\,e\,p}{4}\right)-x\,\left(d^3\,p+\frac{a\,\left(\frac{a\,\left(d\,e^2\,p-\frac{a\,e^3\,p}{4\,b}\right)}{b}-\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)*(d + e*x)^3,x)
```

```
[Out] log(c*(a + b*x)^p)*(d^3*x + (e^3*x^4)/4 + (3*d^2*e*x^2)/2 + d*e^2*x^3) + x^
2*((a*(d*e^2*p - (a*e^3*p)/(4*b)))/(2*b) - (3*d^2*e*p)/4) - x*(d^3*p + (a*(
(a*(d*e^2*p - (a*e^3*p)/(4*b)))/b - (3*d^2*e*p)/2))/b) - x^3*((d*e^2*p)/3 -
 (a*e^3*p)/(12*b)) - (e^3*p*x^4)/16 - (log(a + b*x)*(a^4*e^3*p - 4*a*b^3*d^
3*p - 4*a^3*b*d*e^2*p + 6*a^2*b^2*d^2*e*p))/(4*b^4)
```

sympy [A] time = 6.40, size = 369, normalized size = 2.64

$$\begin{cases}-\frac{a^4e^3p\log(a+bx)}{4b^4}+\frac{a^3de^2p\log(a+bx)}{b^3}+\frac{a^3e^3px}{4b^3}-\frac{3a^2d^2ep\log(a+bx)}{2b^2}-\frac{a^2de^2px}{b^2}-\frac{a^2e^3px^2}{8b^2}+\frac{ad^3p\log(a+bx)}{b}+\frac{3ad^2epx}{2b}+\frac{ade^2px^2}{2b}+\\ \left(d^3x+\frac{3d^2ex^2}{2}+de^2x^3+\frac{e^3x^4}{4}\right)\log(a^pc)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*ln(c*(b*x+a)**p),x)
```

```
[Out] Piecewise((-a**4*e**3*p*log(a + b*x)/(4*b**4) + a**3*d*e**2*p*log(a + b*x)/
b**3 + a**3*e**3*p*x/(4*b**3) - 3*a**2*d**2*e*p*log(a + b*x)/(2*b**2) - a**
```

```
2*d*e**2*p*x/b**2 - a**2*e**3*p*x**2/(8*b**2) + a*d**3*p*log(a + b*x)/b + 3
*a*d**2*e*p*x/(2*b) + a*d*e**2*p*x**2/(2*b) + a*e**3*p*x**3/(12*b) + d**3*p
*x*log(a + b*x) - d**3*p*x + d**3*x*log(c) + 3*d**2*e*p*x**2*log(a + b*x)/2
 - 3*d**2*e*p*x**2/4 + 3*d**2*e*x**2*log(c)/2 + d*e**2*p*x**3*log(a + b*x)
- d*e**2*p*x**3/3 + d*e**2*x**3*log(c) + e**3*p*x**4*log(a + b*x)/4 - e**3*
p*x**4/16 + e**3*x**4*log(c)/4, Ne(b, 0)), ((d**3*x + 3*d**2*e*x**2/2 + d*e
**2*x**3 + e**3*x**4/4)*log(a**p*c), True))
```

3.177 $\int (d+ex)^2 \log(c(a+bx)^p)\, dx$

Optimal. Leaf size=112

$$-\frac{p(bd-ae)^3\log(a+bx)}{3b^3e}-\frac{px(bd-ae)^2}{3b^2}+\frac{(d+ex)^3\log(c(a+bx)^p)}{3e}-\frac{p(d+ex)^2(bd-ae)}{6be}-\frac{p(d+ex)^3}{9e}$$

```
[Out] -1/3*(-a*e+b*d)^2*p*x/b^2-1/6*(-a*e+b*d)*p*(e*x+d)^2/b/e-1/9*p*(e*x+d)^3/e-
1/3*(-a*e+b*d)^3*p*ln(b*x+a)/b^3/e+1/3*(e*x+d)^3*ln(c*(b*x+a)^p)/e
```

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2395, 43}

$$-\frac{px(bd-ae)^2}{3b^2}-\frac{p(bd-ae)^3\log(a+bx)}{3b^3e}+\frac{(d+ex)^3\log(c(a+bx)^p)}{3e}-\frac{p(d+ex)^2(bd-ae)}{6be}-\frac{p(d+ex)^3}{9e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Log[c*(a + b*x)^p],x]
```

```
[Out] -((b*d - a*e)^2*p*x)/(3*b^2) - ((b*d - a*e)*p*(d + e*x)^2)/(6*b*e) - (p*(d
+ e*x)^3)/(9*e) - ((b*d - a*e)^3*p*Log[a + b*x])/(3*b^3*e) + ((d + e*x)^3*L
og[c*(a + b*x)^p])/(3*e)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\log(c(a+bx)^p)\,dx &= \frac{(d+ex)^3\log(c(a+bx)^p)}{3e}-\frac{(bp)\int \frac{(d+ex)^3}{a+bx}\,dx}{3e}\\ &= \frac{(d+ex)^3\log(c(a+bx)^p)}{3e}-\frac{(bp)\int \left(\frac{e(bd-ae)^2}{b^3}+\frac{(bd-ae)^3}{b^3(a+bx)}+\frac{e(bd-ae)(d+ex)}{b^2}+\frac{e(d+ex)^2}{b}\right)}{3e}\\ &= -\frac{(bd-ae)^2px}{3b^2}-\frac{(bd-ae)p(d+ex)^2}{6be}-\frac{p(d+ex)^3}{9e}-\frac{(bd-ae)^3p\log(a+bx)}{3b^3e}+\end{aligned}$$

Mathematica [A] time = 0.11, size = 121, normalized size = 1.08

$$\frac{b\left(6b\left(3ad^2+bx\left(3d^2+3dex+e^2x^2\right)\right)\log(c(a+bx)^p)-px\left(6a^2e^2-3abe(6d+ex)+b^2\left(18d^2+9dex+2e^2x^2\right)\right)\right)}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Log[c*(a + b*x)^p],x]
```

```
[Out] (6*a^2*e*(-3*b*d + a*e)*p*Log[a + b*x] + b*(-(p*x*(6*a^2*e^2 - 3*a*b*e*(6*d
 + e*x) + b^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2))) + 6*b*(3*a*d^2 + b*x*(3*d^2
+ 3*d*e*x + e^2*x^2))*Log[c*(a + b*x)^p]))/(18*b^3)
```

fricas [A] time = 0.42, size = 172, normalized size = 1.54

$$-\frac{2\,b^3e^2px^3 + 3\left(3\,b^3de - ab^2e^2\right)px^2 + 6\left(3\,b^3d^2 - 3\,ab^2de + a^2be^2\right)px - 6\left(b^3e^2px^3 + 3\,b^3depx^2 + 3\,b^3d^2px + \left(3\right.\right.}{18\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x+a)^p),x, algorithm="fricas")
```

```
[Out] -1/18*(2*b^3*e^2*p*x^3 + 3*(3*b^3*d*e - a*b^2*e^2)*p*x^2 + 6*(3*b^3*d^2 - 3
*a*b^2*d*e + a^2*b*e^2)*p*x - 6*(b^3*e^2*p*x^3 + 3*b^3*d*e*p*x^2 + 3*b^3*d^
2*p*x + (3*a*b^2*d^2 - 3*a^2*b*d*e + a^3*e^2)*p)*log(b*x + a) - 6*(b^3*e^2*
x^3 + 3*b^3*d*e*x^2 + 3*b^3*d^2*x)*log(c))/b^3
```

giac [B] time = 0.18, size = 313, normalized size = 2.79

$$\frac{(bx+a)d^2p\log(bx+a)}{b} + \frac{(bx+a)^2dpe\log(bx+a)}{b^2} - \frac{2\,(bx+a)adpe\log(bx+a)}{b^2} - \frac{(bx+a)d^2p}{b} - \frac{(bx+a)^2dpe}{2\,b^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x+a)^p),x, algorithm="giac")
```

```
[Out] (b*x + a)*d^2*p*log(b*x + a)/b + (b*x + a)^2*d*p*e*log(b*x + a)/b^2 - 2*(b*
x + a)*a*d*p*e*log(b*x + a)/b^2 - (b*x + a)*d^2*p/b - 1/2*(b*x + a)^2*d*p*e
/b^2 + 2*(b*x + a)*a*d*p*e/b^2 + 1/3*(b*x + a)^3*p*e^2*log(b*x + a)/b^3 - (
b*x + a)^2*a*p*e^2*log(b*x + a)/b^3 + (b*x + a)*a^2*p*e^2*log(b*x + a)/b^3
+ (b*x + a)*d^2*log(c)/b + (b*x + a)^2*d*e*log(c)/b^2 - 2*(b*x + a)*a*d*e*l
og(c)/b^2 - 1/9*(b*x + a)^3*p*e^2/b^3 + 1/2*(b*x + a)^2*a*p*e^2/b^3 - (b*x
+ a)*a^2*p*e^2/b^3 + 1/3*(b*x + a)^3*e^2*log(c)/b^3 - (b*x + a)^2*a*e^2*log
(c)/b^3 + (b*x + a)*a^2*e^2*log(c)/b^3
```

maple [C] time = 0.45, size = 537, normalized size = 4.79

$$de\,x^2\ln(c) - \frac{d^3p\ln(bx+a)}{3e} + \frac{e^2x^3\ln(c)}{3} + d^2x\ln(c) - \frac{e^2p\,x^3}{9} - d^2px + \frac{(ex+d)^3\ln\left((bx+a)^p\right)}{3e} - \frac{dep\,x^2}{2} + \frac{adepx}{b} - \frac{a^2d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*ln(c*(b*x+a)^p),x)
```

```
[Out] e*ln(c)*d*x^2-1/3/e*ln(b*x+a)*d^3*p+1/3*e^2*ln(c)*x^3+ln(c)*d^2*x-1/9*e^2*p
*x^3-d^2*p*x+1/3*(e*x+d)^3/e*ln((b*x+a)^p)-1/2*d*e*p*x^2+1/b*e*a*d*p*x-1/b^
2*e*ln(b*x+a)*a^2*d*p+1/6*I*e^2*Pi*x^3*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p
)^2+1/6*I*e^2*Pi*x^3*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)-1/2*I*e*Pi*d*x^2*csgn(
I*c*(b*x+a)^p)^3+1/2*I*Pi*d^2*x*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+1/2
*I*Pi*d^2*x*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)-1/6*I*e^2*Pi*x^3*csgn(I*(b*x+a)
^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)+1/2*I*e*Pi*d*x^2*csgn(I*(b*x+a)^p)*csgn(I
*c*(b*x+a)^p)^2-1/2*I*e*Pi*d*x^2*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn
(I*c)+1/3/b^3*e^2*ln(b*x+a)*a^3*p-1/6*I*e^2*Pi*x^3*csgn(I*c*(b*x+a)^p)^3-1/
2*I*Pi*d^2*x*csgn(I*c*(b*x+a)^p)^3+1/b*ln(b*x+a)*a*d^2*p-1/3/b^2*e^2*a^2*p*
x+1/6/b*e^2*a*p*x^2+1/2*I*e*Pi*d*x^2*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)-1/2*I*
Pi*d^2*x*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)
```

maxima [A] time = 0.44, size = 136, normalized size = 1.21

$$-\frac{1}{18}\,bp\left(\frac{2\,b^2e^2x^3 + 3\left(3\,b^2de - abe^2\right)x^2 + 6\left(3\,b^2d^2 - 3\,abde + a^2e^2\right)x}{b^3} - \frac{6\left(3\,ab^2d^2 - 3\,a^2bde + a^3e^2\right)\log(bx+a}{b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x+a)^p),x, algorithm="maxima")
```

```
[Out] -1/18*b*p*((2*b^2*e^2*x^3 + 3*(3*b^2*d*e - a*b*e^2)*x^2 + 6*(3*b^2*d^2 - 3*
a*b*d*e + a^2*e^2)*x)/b^3 - 6*(3*a*b^2*d^2 - 3*a^2*b*d*e + a^3*e^2)*log(b*x
 + a)/b^4) + 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)*log((b*x + a)^p*c)
```

mupad [B] time = 0.27, size = 131, normalized size = 1.17

$$\ln\left(c\,(a+b\,x)^p\right)\,\left(d^2\,x+d\,e\,x^2+\frac{e^2\,x^3}{3}\right)-x^2\,\left(\frac{d\,e\,p}{2}-\frac{a\,e^2\,p}{6\,b}\right)-x\,\left(d^2\,p-\frac{a\,\left(d\,e\,p-\frac{a\,e^2\,p}{3\,b}\right)}{b}\right)-\frac{e^2\,p\,x^3}{9}+\frac{\ln\left(a+b\,x\right)\,\left(p\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)*(d + e*x)^2,x)
```

```
[Out] log(c*(a + b*x)^p)*(d^2*x + (e^2*x^3)/3 + d*e*x^2) - x^2*((d*e*p)/2 - (a*e^
2*p)/(6*b)) - x*(d^2*p - (a*(d*e*p - (a*e^2*p)/(3*b)))/b) - (e^2*p*x^3)/9 +
 (log(a + b*x)*(a^3*e^2*p + 3*a*b^2*d^2*p - 3*a^2*b*d*e*p))/(3*b^3)
```

sympy [A] time = 3.02, size = 223, normalized size = 1.99

$$\begin{cases} \frac{a^3e^2p\log\left(a+bx\right)}{3b^3}-\frac{a^2dep\log\left(a+bx\right)}{b^2}-\frac{a^2e^2px}{3b^2}+\frac{ad^2p\log\left(a+bx\right)}{b}+\frac{adepx}{b}+\frac{ae^2px^2}{6b}+d^2px\log\left(a+bx\right)-d^2px+d^2x\log\left(c\right)+\ldots \\ \left(d^2x+dex^2+\frac{e^2x^3}{3}\right)\log\left(a^pc\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*ln(c*(b*x+a)**p),x)
```

```
[Out] Piecewise((a**3*e**2*p*log(a + b*x)/(3*b**3) - a**2*d*e*p*log(a + b*x)/b**2
 - a**2*e**2*p*x/(3*b**2) + a*d**2*p*log(a + b*x)/b + a*d*e*p*x/b + a*e**2*
p*x**2/(6*b) + d**2*p*x*log(a + b*x) - d**2*p*x + d**2*x*log(c) + d*e*p*x**
2*log(a + b*x) - d*e*p*x**2/2 + d*e*x**2*log(c) + e**2*p*x**3*log(a + b*x)/
3 - e**2*p*x**3/9 + e**2*x**3*log(c)/3, Ne(b, 0)), ((d**2*x + d*e*x**2 + e*
*2*x**3/3)*log(a**p*c), True))
```

3.178 $\int (d+ex)\log \left(c(a+bx)^p\right)\,dx$

Optimal. Leaf size=84

$$-\frac{p(bd-ae)^2\log(a+bx)}{2b^2e}+\frac{(d+ex)^2\log\left(c(a+bx)^p\right)}{2e}-\frac{px(bd-ae)}{2b}-\frac{p(d+ex)^2}{4e}$$

```
[Out] -1/2*(-a*e+b*d)*p*x/b-1/4*p*(e*x+d)^2/e-1/2*(-a*e+b*d)^2*p*ln(b*x+a)/b^2/e+
1/2*(e*x+d)^2*ln(c*(b*x+a)^p)/e
```

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2395, 43}

$$-\frac{p(bd-ae)^2\log(a+bx)}{2b^2e}+\frac{(d+ex)^2\log\left(c(a+bx)^p\right)}{2e}-\frac{px(bd-ae)}{2b}-\frac{p(d+ex)^2}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Log[c*(a + b*x)^p],x]
```

```
[Out] -((b*d - a*e)*p*x)/(2*b) - (p*(d + e*x)^2)/(4*e) - ((b*d - a*e)^2*p*Log[a +
 b*x])/(2*b^2*e) + ((d + e*x)^2*Log[c*(a + b*x)^p])/(2*e)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\log\left(c(a+bx)^p\right)\,dx &= \frac{(d+ex)^2\log\left(c(a+bx)^p\right)}{2e}-\frac{(bp)\int\frac{(d+ex)^2}{a+bx}\,dx}{2e}\\ &= \frac{(d+ex)^2\log\left(c(a+bx)^p\right)}{2e}-\frac{(bp)\int\left(\frac{e(bd-ae)}{b^2}+\frac{(bd-ae)^2}{b^2(a+bx)}+\frac{e(d+ex)}{b}\right)dx}{2e}\\ &= -\frac{(bd-ae)px}{2b}-\frac{p(d+ex)^2}{4e}-\frac{(bd-ae)^2p\log(a+bx)}{2b^2e}+\frac{(d+ex)^2\log\left(c(a+b\right.}{2e}\end{aligned}$$

Mathematica [A] time = 0.05, size = 82, normalized size = 0.98

$$-\frac{a^2ep\log(a+bx)}{2b^2}+\frac{d(a+bx)\log\left(c(a+bx)^p\right)}{b}+\frac{1}{2}ex^2\log\left(c(a+bx)^p\right)+\frac{aepx}{2b}-dpx-\frac{1}{4}epx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Log[c*(a + b*x)^p],x]
```

```
[Out] -(d*p*x) + (a*e*p*x)/(2*b) - (e*p*x^2)/4 - (a^2*e*p*Log[a + b*x])/(2*b^2) +
 (e*x^2*Log[c*(a + b*x)^p])/2 + (d*(a + b*x)*Log[c*(a + b*x)^p])/b
```

fricas [A] time = 0.43, size = 91, normalized size = 1.08

$$-\frac{b^2epx^2 + 2\left(2\,b^2d - abe\right)px - 2\left(b^2epx^2 + 2\,b^2dpx + \left(2\,abd - a^2e\right)p\right)\log\left(bx + a\right) - 2\left(b^2ex^2 + 2\,b^2dx\right)\log(c)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x+a)^p),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*e*p*x^2 + 2*(2*b^2*d - a*b*e)*p*x - 2*(b^2*e*p*x^2 + 2*b^2*d*p*x
+ (2*a*b*d - a^2*e)*p)*log(b*x + a) - 2*(b^2*e*x^2 + 2*b^2*d*x)*log(c))/b^2
```

giac [A] time = 0.17, size = 142, normalized size = 1.69

$$\frac{(bx + a)dp\log\left(bx + a\right)}{b} + \frac{(bx + a)^2pe\log\left(bx + a\right)}{2\,b^2} - \frac{(bx + a)ape\log\left(bx + a\right)}{b^2} - \frac{(bx + a)dp}{b} - \frac{(bx + a)^2pe}{4\,b^2} + \frac{(bx + a)ap}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x+a)^p),x, algorithm="giac")
```

```
[Out] (b*x + a)*d*p*log(b*x + a)/b + 1/2*(b*x + a)^2*p*e*log(b*x + a)/b^2 - (b*x
+ a)*a*p*e*log(b*x + a)/b^2 - (b*x + a)*d*p/b - 1/4*(b*x + a)^2*p*e/b^2 + (
b*x + a)*a*p*e/b^2 + (b*x + a)*d*log(c)/b + 1/2*(b*x + a)^2*e*log(c)/b^2 -
(b*x + a)*a*e*log(c)/b^2
```

maple [A] time = 0.09, size = 83, normalized size = 0.99

$$-\frac{ep\,x^2}{4} + \frac{e\,x^2\ln\left(c\,e^{p\ln(bx+a)}\right)}{2} - \frac{a^2ep\ln\left(bx + a\right)}{2b^2} + \frac{adp\ln\left(bx + a\right)}{b} + \frac{aepx}{2b} - dpx + dx\ln\left(c\left(bx + a\right)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*ln(c*(b*x+a)^p),x)
```

```
[Out] d*x*ln(c*(b*x+a)^p)-d*p*x+d/b*p*a*ln(b*x+a)+1/2*e*x^2*ln(c*exp(p*ln(b*x+a))
)-1/4*e*p*x^2-1/2*p*a^2*e/b^2*ln(b*x+a)+1/2*a*p*e/b*x
```

maxima [A] time = 0.44, size = 74, normalized size = 0.88

$$-\frac{1}{4}\,bp\left(\frac{bex^2 + 2\left(2\,bd - ae\right)x}{b^2} - \frac{2\left(2\,abd - a^2e\right)\log\left(bx + a\right)}{b^3}\right) + \frac{1}{2}\left(ex^2 + 2\,dx\right)\log\left((bx + a)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x+a)^p),x, algorithm="maxima")
```

```
[Out] -1/4*b*p*((b*e*x^2 + 2*(2*b*d - a*e)*x)/b^2 - 2*(2*a*b*d - a^2*e)*log(b*x +
 a)/b^3) + 1/2*(e*x^2 + 2*d*x)*log((b*x + a)^p*c)
```

mupad [B] time = 0.25, size = 68, normalized size = 0.81

$$\ln\left(c\,(a + b\,x)^p\right)\,\left(\frac{e\,x^2}{2} + d\,x\right) - x\,\left(d\,p - \frac{a\,e\,p}{2\,b}\right) - \frac{e\,p\,x^2}{4} - \frac{\ln\left(a + b\,x\right)\,\left(a^2\,e\,p - 2\,a\,b\,d\,p\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)*(d + e*x),x)
```

```
[Out] log(c*(a + b*x)^p)*(d*x + (e*x^2)/2) - x*(d*p - (a*e*p)/(2*b)) - (e*p*x^2)/
4 - (log(a + b*x)*(a^2*e*p - 2*a*b*d*p))/(2*b^2)
```

sympy [A] time = 1.38, size = 116, normalized size = 1.38

$$\begin{cases} -\frac{a^2ep\log(a+bx)}{2b^2} + \frac{adp\log(a+bx)}{b} + \frac{aepx}{2b} + dpx\log(a+bx) - dpx + dx\log(c) + \frac{epx^2\log(a+bx)}{2} - \frac{epx^2}{4} + \frac{ex^2\log(c)}{2} & \text{fo} \\ \left(dx + \frac{ex^2}{2}\right)\log(a^pc) & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*ln(c*(b*x+a)**p),x)
```

```
[Out] Piecewise((-a**2*e*p*log(a + b*x)/(2*b**2) + a*d*p*log(a + b*x)/b + a*e*p*x
/(2*b) + d*p*x*log(a + b*x) - d*p*x + d*x*log(c) + e*p*x**2*log(a + b*x)/2
- e*p*x**2/4 + e*x**2*log(c)/2, Ne(b, 0)), ((d*x + e*x**2/2)*log(a**p*c), T
rue))
```

3.179 $\int \log\left(c(a+bx)^p\right)\,dx$

Optimal. Leaf size=24

$$\frac{(a+bx)\log\left(c(a+bx)^p\right)}{b}-px$$

```
[Out] -p*x+(b*x+a)*ln(c*(b*x+a)^p)/b
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2389, 2295}

$$\frac{(a+bx)\log\left(c(a+bx)^p\right)}{b}-px$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p],x]
```

```
[Out] -(p*x) + ((a + b*x)*Log[c*(a + b*x)^p])/b
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(c(a+bx)^p\right)\,dx &= \frac{\text{Subst}\left(\int \log\left(cx^p\right)\,dx,x,a+bx\right)}{b}\\ &= -px+\frac{(a+bx)\log\left(c(a+bx)^p\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{(a+bx)\log\left(c(a+bx)^p\right)}{b}-px$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p],x]
```

```
[Out] -(p*x) + ((a + b*x)*Log[c*(a + b*x)^p])/b
```

fricas [A] time = 0.42, size = 32, normalized size = 1.33

$$-\frac{bpx-bx\log(c)-\left(bpx+ap\right)\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p),x, algorithm="fricas")
```

```
[Out] -(b*p*x - b*x*log(c) - (b*p*x + a*p)*log(b*x + a))/b
```

giac [A] time = 0.16, size = 39, normalized size = 1.62

$$\frac{(bx+a)p\log(bx+a)}{b} - \frac{(bx+a)p}{b} + \frac{(bx+a)\log(c)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p),x, algorithm="giac")
[Out] (b*x + a)*p*log(b*x + a)/b - (b*x + a)*p/b + (b*x + a)*log(c)/b
```

maple [A] time = 0.07, size = 30, normalized size = 1.25

$$\frac{ap\ln(bx+a)}{b} - px + x\ln\left(c\,(bx+a)^{p}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p),x)
[Out] x*ln(c*(b*x+a)^p)-p*x+1/b*p*a*ln(b*x+a)
```

maxima [A] time = 0.43, size = 35, normalized size = 1.46

$$-bp\left(\frac{x}{b} - \frac{a\log(bx+a)}{b^2}\right) + x\log\left((bx+a)^{p}c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p),x, algorithm="maxima")
[Out] -b*p*(x/b - a*log(b*x + a)/b^2) + x*log((b*x + a)^p*c)
```

mupad [B] time = 0.07, size = 29, normalized size = 1.21

$$x\,\ln\left(c\,(a+b\,x)^{p}\right) - p\,x + \frac{a\,p\,\ln(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p),x)
[Out] x*log(c*(a + b*x)^p) - p*x + (a*p*log(a + b*x))/b
```

sympy [A] time = 0.46, size = 37, normalized size = 1.54

$$\begin{cases} \frac{ap\log(a+bx)}{b} + px\log(a+bx) - px + x\log(c) & \text{for}\ b \neq 0 \\ x\log(a^{p}c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p),x)
[Out] Piecewise((a*p*log(a + b*x)/b + p*x*log(a + b*x) - p*x + x*log(c), Ne(b, 0)
), (x*log(a**p*c), True))
```

3.180 $\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx$

Optimal. Leaf size=58

$$\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}+\frac{p\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{e}$$

```
[Out] ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/e+p*polylog(2,-e*(b*x+a)/(-a*e+b*d
))/e
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2394, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e}+\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, -((e*
(a + b*x))/(b*d - a*e))])/e
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx &= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}-\frac{(bp)\int \frac{\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}\,dx}{e}\\ &= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}-\frac{p\,\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{bd-ae}\right)}{x}\,dx,x,a+bx\right)}{e}\\ &= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}+\frac{p\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{e}\end{aligned}$$

Mathematica [A] time = 0.00, size = 57, normalized size = 0.98

$$\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}+\frac{p\text{Li}_2\left(\frac{e(a+bx)}{ae-bd}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, (e*(a
 + b*x))/(-(b*d) + a*e)])/e
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left((bx+a)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)^p*c)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left((bx+a)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^p*c)/(e*x + d), x)
```

maple [C] time = 0.32, size = 242, normalized size = 4.17

$$-\frac{i\pi\,\text{csgn}(ic)\,\text{csgn}\left(i(bx+a)^p\right)\text{csgn}\left(ic(bx+a)^p\right)\ln(ex+d)}{2e}+\frac{i\pi\,\text{csgn}(ic)\,\text{csgn}\left(ic(bx+a)^p\right)^2\ln(ex+d)}{2e}+\frac{i\pi$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/(e*x+d),x)
```

```
[Out] ln(e*x+d)/e*ln((b*x+a)^p)-1/e*p*dilog((a*e-b*d+(e*x+d)*b)/(a*e-b*d))-1/e*p*
ln(e*x+d)*ln((a*e-b*d+(e*x+d)*b)/(a*e-b*d))+1/2*I*ln(e*x+d)/e*Pi*csgn(I*(b*
x+a)^p)*csgn(I*c*(b*x+a)^p)^2-1/2*I*ln(e*x+d)/e*Pi*csgn(I*(b*x+a)^p)*csgn(I
*c*(b*x+a)^p)*csgn(I*c)-1/2*I*ln(e*x+d)/e*Pi*csgn(I*c*(b*x+a)^p)^3+1/2*I*ln
(e*x+d)/e*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)+ln(e*x+d)/e*ln(c)
```

maxima [B] time = 0.47, size = 118, normalized size = 2.03

$$\frac{bp\left(\frac{\log(bx+a)\log(ex+d)}{b}-\frac{\log(ex+d)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\text{Li}_2\left(\frac{bex+bd}{bd-ae}\right)}{b}\right)}{e}-\frac{p\log(bx+a)\log(ex+d)}{e}+\frac{\log\left((bx+a)^p c\right)\log(ex+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*(log(b*x + a)*log(e*x + d)/b - (log(e*x + d)*log(-(b*e*x + b*d)/(b*d -
a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))/b)/e - p*log(b*x + a)*log(e*x
 + d)/e + log((b*x + a)^p*c)*log(e*x + d)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\,(a+b\,x)^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)/(d + e*x),x)
[Out] int(log(c*(a + b*x)^p)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\,(a+bx)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/(e*x+d),x)
[Out] Integral(log(c*(a + b*x)**p)/(d + e*x), x)
```

3.181 $\int \frac{\log(c(a+bx)^p)}{(d+ex)^2} dx$

Optimal. Leaf size=68

$$-\frac{\log\left(c(a+bx)^p\right)}{e(d+ex)}+\frac{bp\log(a+bx)}{e(bd-ae)}-\frac{bp\log(d+ex)}{e(bd-ae)}$$

```
[Out] b*p*ln(b*x+a)/e/(-a*e+b*d)-ln(c*(b*x+a)^p)/e/(e*x+d)-b*p*ln(e*x+d)/e/(-a*e+
b*d)
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2395, 36, 31}

$$-\frac{\log\left(c(a+bx)^p\right)}{e(d+ex)}+\frac{bp\log(a+bx)}{e(bd-ae)}-\frac{bp\log(d+ex)}{e(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x)^2,x]
```

```
[Out] (b*p*Log[a + b*x])/(e*(b*d - a*e)) - Log[c*(a + b*x)^p]/(e*(d + e*x)) - (b*
p*Log[d + e*x])/(e*(b*d - a*e))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^2}\,dx &= -\frac{\log\left(c(a+bx)^p\right)}{e(d+ex)}+\frac{(bp)\int \frac{1}{(a+bx)(d+ex)}\,dx}{e}\\
&= -\frac{\log\left(c(a+bx)^p\right)}{e(d+ex)}-\frac{(bp)\int \frac{1}{d+ex}\,dx}{bd-ae}+\frac{\left(b^2p\right)\int \frac{1}{a+bx}\,dx}{e(bd-ae)}\\
&= \frac{bp\log(a+bx)}{e(bd-ae)}-\frac{\log\left(c(a+bx)^p\right)}{e(d+ex)}-\frac{bp\log(d+ex)}{e(bd-ae)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 52, normalized size = 0.76

$$\frac{\frac{bp(\log(a+bx)-\log(d+ex))}{bd-ae}-\frac{\log(c(a+bx)^p)}{d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x)^2,x]
```

```
[Out] (-(Log[c*(a + b*x)^p]/(d + e*x)) + (b*p*(Log[a + b*x] - Log[d + e*x]))/(b*d
 - a*e))/e
```

fricas [A] time = 0.43, size = 80, normalized size = 1.18

$$\frac{\left(bepx+aep\right)\log\left(bx+a\right)-\left(bepx+bdp\right)\log\left(ex+d\right)-(bd-ae)\log(c)}{bd^2e-ade^2+\left(bde^2-ae^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] ((b*e*p*x + a*e*p)*log(b*x + a) - (b*e*p*x + b*d*p)*log(e*x + d) - (b*d - a
*e)*log(c))/(b*d^2*e - a*d*e^2 + (b*d*e^2 - a*e^3)*x)
```

giac [A] time = 0.16, size = 91, normalized size = 1.34

$$\frac{bpxe\log\left(bx+a\right)-bpxe\log\left(xe+d\right)+ape\log\left(bx+a\right)-bdp\log\left(xe+d\right)-bd\log(c)+ae\log(c)}{bdxe^2+bd^2e-axe^3-ade^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (b*p*x*e*log(b*x + a) - b*p*x*e*log(x*e + d) + a*p*e*log(b*x + a) - b*d*p*l
og(x*e + d) - b*d*log(c) + a*e*log(c))/(b*d*x*e^2 + b*d^2*e - a*x*e^3 - a*d
*e^2)
```

maple [C] time = 0.45, size = 329, normalized size = 4.84

$$-\frac{\ln\left(\left(bx+a\right)^p\right)}{\left(ex+d\right)e}-\frac{-i\pi ae\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(bx+a\right)^p\right)\mathrm{csgn}\left(ic\left(bx+a\right)^p\right)+i\pi ae\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(bx+a\right)^p\right)^2+i\pi a e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/(e*x+d)^2,x)
```

```
[Out] -1/e/(e*x+d)*ln((b*x+a)^p)-1/2*(I*Pi*a*e*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)
^p)^2-I*Pi*a*e*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)-I*Pi*a*e*csg
n(I*c*(b*x+a)^p)^3+I*Pi*a*e*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)-I*Pi*b*d*csgn(I
*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+I*Pi*b*d*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+
a)^p)*csgn(I*c)+I*Pi*b*d*csgn(I*c*(b*x+a)^p)^3-I*Pi*b*d*csgn(I*c*(b*x+a)^p)
^2*csgn(I*c)+2*ln(b*x+a)*b*e*p*x-2*ln(-e*x-d)*b*e*p*x+2*ln(b*x+a)*b*d*p-2*l
n(-e*x-d)*b*d*p+2*ln(c)*a*e-2*b*d*ln(c))/(e*x+d)/e/(a*e-b*d)
```

maxima [A] time = 0.44, size = 65, normalized size = 0.96

$$\frac{bp\left(\frac{\log(bx+a)}{bd-ae}-\frac{\log(ex+d)}{bd-ae}\right)}{e}-\frac{\log\left(\left(bx+a\right)^pc\right)}{(ex+d)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] b*p*(log(b*x + a)/(b*d - a*e) - log(e*x + d)/(b*d - a*e))/e - log((b*x + a)
^p*c)/((e*x + d)*e)
```

mupad [B] time = 1.07, size = 70, normalized size = 1.03

$$-\frac{\ln\left(c\,(a+b\,x)^p\right)}{e\,\left(d+e\,x\right)}+\frac{b\,p\,\mathrm{atan}\left(\frac{a\,e\,1\mathrm{i}+b\,d\,1\mathrm{i}+b\,e\,x\,2\mathrm{i}}{a\,e-b\,d}\right)\,2\mathrm{i}}{a\,e^2-b\,d\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)/(d + e*x)^2,x)
```

```
[Out] (b*p*atan((a*e*1i + b*d*1i + b*e*x*2i)/(a*e - b*d))*2i)/(a*e^2 - b*d*e) - l
og(c*(a + b*x)^p)/(e*(d + e*x))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/(e*x+d)**2,x)
```

```
[Out] Exception raised: NotImplementedError
```

3.182 $\int \frac{\log(c(a+bx)^p)}{(d+ex)^3} dx$

Optimal. Leaf size=105

$$\frac{b^2p\log(a+bx)}{2e(bd-ae)^2}-\frac{b^2p\log(d+ex)}{2e(bd-ae)^2}-\frac{\log(c(a+bx)^p)}{2e(d+ex)^2}+\frac{bp}{2e(d+ex)(bd-ae)}$$

```
[Out] 1/2*b*p/e/(-a*e+b*d)/(e*x+d)+1/2*b^2*p*ln(b*x+a)/e/(-a*e+b*d)^2-1/2*ln(c*(b
*x+a)^p)/e/(e*x+d)^2-1/2*b^2*p*ln(e*x+d)/e/(-a*e+b*d)^2
```

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2395, 44}

$$\frac{b^2p\log(a+bx)}{2e(bd-ae)^2}-\frac{b^2p\log(d+ex)}{2e(bd-ae)^2}-\frac{\log(c(a+bx)^p)}{2e(d+ex)^2}+\frac{bp}{2e(d+ex)(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x)^3,x]
```

```
[Out] (b*p)/(2*e*(b*d - a*e)*(d + e*x)) + (b^2*p*Log[a + b*x])/(2*e*(b*d - a*e)^2
) - Log[c*(a + b*x)^p]/(2*e*(d + e*x)^2) - (b^2*p*Log[d + e*x])/(2*e*(b*d -
 a*e)^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(c(a+bx)^p)}{(d+ex)^3}dx&=-\frac{\log(c(a+bx)^p)}{2e(d+ex)^2}+\frac{(bp)\int \frac{1}{(a+bx)(d+ex)^2}dx}{2e}\\&=-\frac{\log(c(a+bx)^p)}{2e(d+ex)^2}+\frac{(bp)\int \left(\frac{b^2}{(bd-ae)^2(a+bx)}-\frac{e}{(bd-ae)(d+ex)^2}-\frac{be}{(bd-ae)^2(d+ex)}\right)dx}{2e}\\&=\frac{bp}{2e(bd-ae)(d+ex)}+\frac{b^2p\log(a+bx)}{2e(bd-ae)^2}-\frac{\log(c(a+bx)^p)}{2e(d+ex)^2}-\frac{b^2p\log(d+ex)}{2e(bd-ae)^2}\end{aligned}$$

Mathematica [A] time = 0.10, size = 80, normalized size = 0.76

$$\frac{\frac{bp(d+ex)(b(d+ex)\log(a+bx)-ae-b(d+ex)\log(d+ex)+bd)}{(bd-ae)^2}-\log(c(a+bx)^p)}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x)^3,x]
```

```
[Out] (-Log[c*(a + b*x)^p] + (b*p*(d + e*x)*(b*d - a*e + b*(d + e*x)*Log[a + b*x]
 - b*(d + e*x)*Log[d + e*x]))/(b*d - a*e)^2)/(2*e*(d + e*x)^2)
```

fricas [B] time = 0.45, size = 236, normalized size = 2.25

$$\frac{\left(b^2de - abe^2\right)px + \left(b^2d^2 - abde\right)p + \left(b^2e^2px^2 + 2\,b^2depx + \left(2\,abde - a^2e^2\right)p\right)\log\left(bx + a\right) - \left(b^2e^2px^2 + 2\,b^2dep\right.}{2\left(b^2d^4e - 2\,abd^3e^2 + a^2d^2e^3 + \left(b^2d^2e^3 - 2\,abde^4 + a^2e^5\right)x^2 + 2\left(b^2d^3e^2 - 2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*((b^2*d*e - a*b*e^2)*p*x + (b^2*d^2 - a*b*d*e)*p + (b^2*e^2*p*x^2 + 2*b
^2*d*e*p*x + (2*a*b*d*e - a^2*e^2)*p)*log(b*x + a) - (b^2*e^2*p*x^2 + 2*b^2
*d*e*p*x + b^2*d^2*p)*log(e*x + d) - (b^2*d^2 - 2*a*b*d*e + a^2*e^2)*log(c)
)/(b^2*d^4*e - 2*a*b*d^3*e^2 + a^2*d^2*e^3 + (b^2*d^2*e^3 - 2*a*b*d*e^4 + a
^2*e^5)*x^2 + 2*(b^2*d^3*e^2 - 2*a*b*d^2*e^3 + a^2*d*e^4)*x)
```

giac [B] time = 0.17, size = 266, normalized size = 2.53

$$\frac{b^2px^2e^2\log\left(bx + a\right) + 2\,b^2dpxe\log\left(bx + a\right) - b^2px^2e^2\log\left(xe + d\right) - 2\,b^2dpxe\log\left(xe + d\right) + b^2dpxe + 2\,abdpe}{2\left(b^2d^2x^2e^3 + 2\,b^2d^3xe^2 + b^2d^4e - 2\,abdx^2e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*p*x^2*e^2*log(b*x + a) + 2*b^2*d*p*x*e*log(b*x + a) - b^2*p*x^2*e^
2*log(x*e + d) - 2*b^2*d*p*x*e*log(x*e + d) + b^2*d*p*x*e + 2*a*b*d*p*e*log
(b*x + a) - b^2*d^2*p*log(x*e + d) + b^2*d^2*p - a*b*p*x*e^2 - a*b*d*p*e -
a^2*p*e^2*log(b*x + a) - b^2*d^2*log(c) + 2*a*b*d*e*log(c) - a^2*e^2*log(c)
)/(b^2*d^2*x^2*e^3 + 2*b^2*d^3*x*e^2 + b^2*d^4*e - 2*a*b*d*x^2*e^4 - 4*a*b*
d^2*x*e^3 - 2*a*b*d^3*e^2 + a^2*x^2*e^5 + 2*a^2*d*x*e^4 + a^2*d^2*e^3)
```

maple [C] time = 0.57, size = 582, normalized size = 5.54

$$-\frac{\ln\left(\left(bx + a\right)^p\right)}{2\left(ex + d\right)^2 e} - \frac{2b^2d^2p\ln\left(ex + d\right) - 2b^2d^2p\ln\left(-bx - a\right) + 2a^2e^2\ln(c) + 2abdep + 2ab\,e^2px - 2b^2depx - 2b^2d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/(e*x+d)^3,x)
```

```
[Out] -1/2/e/(e*x+d)^2*ln((b*x+a)^p)-1/4*(2*ln(e*x+d)*b^2*d^2*p-2*ln(-b*x-a)*b^2*
d^2*p+I*Pi*b^2*d^2*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)+I*Pi*a^2*e^2*csgn(I*(b*x
+a)^p)*csgn(I*c*(b*x+a)^p)^2+I*Pi*a^2*e^2*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)+2
*ln(c)*a^2*e^2+2*a*b*d*p*e+2*a*b*e^2*p*x-2*b^2*d*e*p*x+2*I*Pi*a*b*d*e*csgn(
I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)-2*b^2*d^2*p-2*I*Pi*a*b*d*e*csgn(
I*c*(b*x+a)^p)^2*csgn(I*c)+2*b^2*d^2*ln(c)-2*I*Pi*a*b*d*e*csgn(I*(b*x+a)^p)
*csgn(I*c*(b*x+a)^p)^2+I*Pi*b^2*d^2*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2
-I*Pi*b^2*d^2*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)+2*I*Pi*a*b*d*
e*csgn(I*c*(b*x+a)^p)^3+4*ln(e*x+d)*b^2*d*e*p*x-4*ln(-b*x-a)*b^2*d*e*p*x+2*
ln(e*x+d)*b^2*e^2*p*x^2-2*ln(-b*x-a)*b^2*e^2*p*x^2-4*ln(c)*a*b*d*e-I*Pi*b^2
*d^2*csgn(I*c*(b*x+a)^p)^3-I*Pi*a^2*e^2*csgn(I*c*(b*x+a)^p)^3-I*Pi*a^2*e^2*
csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c))/(e*x+d)^2/(a*e-b*d)^2/e
```

maxima [A] time = 0.45, size = 120, normalized size = 1.14

$$\frac{bp\left(\frac{b\log(bx+a)}{b^2d^2-2\,abde+a^2e^2} - \frac{b\log(ex+d)}{b^2d^2-2\,abde+a^2e^2} + \frac{1}{bd^2-ade+\left(bde-ae^2\right)x}\right)}{2\,e} - \frac{\log\left(\left(bx + a\right)^p c\right)}{2\left(ex + d\right)^2 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(b*log(b*x + a)/(b^2*d^2 - 2*a*b*d*e + a^2*e^2) - b*log(e*x + d)/(b
^2*d^2 - 2*a*b*d*e + a^2*e^2) + 1/(b*d^2 - a*d*e + (b*d*e - a*e^2)*x))/e -
1/2*log((b*x + a)^p*c)/((e*x + d)^2*e)
```

mupad [B] time = 0.64, size = 96, normalized size = 0.91

$$-\frac{\ln\left(c\,(a+b\,x)^p\right)}{2\,e\,(d+e\,x)^2}-\frac{b\,p}{2\,e\,\left(a\,e-b\,d\right)\,\left(d+e\,x\right)}-\frac{b^2\,p\,\mathrm{atan}\left(\frac{a\,e\,1\mathrm{i}+b\,d\,1\mathrm{i}+b\,e\,x\,2\mathrm{i}}{a\,e-b\,d}\right)\,1\mathrm{i}}{e\,(a\,e-b\,d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)/(d + e*x)^3,x)
```

```
[Out] - log(c*(a + b*x)^p)/(2*e*(d + e*x)^2) - (b*p)/(2*e*(a*e - b*d)*(d + e*x))
- (b^2*p*atan((a*e*1i + b*d*1i + b*e*x*2i)/(a*e - b*d))*1i)/(e*(a*e - b*d)^
2)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/(e*x+d)**3,x)
```

```
[Out] Exception raised: NotImplementedError
```

3.183 $\int \frac{\log(c(a+bx)^p)}{(d+ex)^4} dx$

Optimal. Leaf size=133

$$\frac{b^3p\log(a+bx)}{3e(bd-ae)^3}-\frac{b^3p\log(d+ex)}{3e(bd-ae)^3}+\frac{b^2p}{3e(d+ex)(bd-ae)^2}-\frac{\log\left(c(a+bx)^p\right)}{3e(d+ex)^3}+\frac{bp}{6e(d+ex)^2(bd-ae)}$$

```
[Out] 1/6*b*p/e/(-a*e+b*d)/(e*x+d)^2+1/3*b^2*p/e/(-a*e+b*d)^2/(e*x+d)+1/3*b^3*p*l
n(b*x+a)/e/(-a*e+b*d)^3-1/3*ln(c*(b*x+a)^p)/e/(e*x+d)^3-1/3*b^3*p*ln(e*x+d)
/e/(-a*e+b*d)^3
```

Rubi [A] time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2395, 44}

$$\frac{b^2p}{3e(d+ex)(bd-ae)^2}+\frac{b^3p\log(a+bx)}{3e(bd-ae)^3}-\frac{b^3p\log(d+ex)}{3e(bd-ae)^3}-\frac{\log\left(c(a+bx)^p\right)}{3e(d+ex)^3}+\frac{bp}{6e(d+ex)^2(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x)^4,x]
```

```
[Out] (b*p)/(6*e*(b*d - a*e)*(d + e*x)^2) + (b^2*p)/(3*e*(b*d - a*e)^2*(d + e*x))
 + (b^3*p*Log[a + b*x])/(3*e*(b*d - a*e)^3) - Log[c*(a + b*x)^p]/(3*e*(d +
e*x)^3) - (b^3*p*Log[d + e*x])/(3*e*(b*d - a*e)^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^4}dx &= -\frac{\log\left(c(a+bx)^p\right)}{3e(d+ex)^3}+\frac{(bp)\int \frac{1}{(a+bx)(d+ex)^3}dx}{3e}\\ &= -\frac{\log\left(c(a+bx)^p\right)}{3e(d+ex)^3}+\frac{(bp)\int \left(\frac{b^3}{(bd-ae)^3(a+bx)}-\frac{e}{(bd-ae)(d+ex)^3}-\frac{be}{(bd-ae)^2(d+ex)^2}-\frac{b^2e}{(bd-ae)^3(d+ex)}\right.}{3e}\\ &= \frac{bp}{6e(bd-ae)(d+ex)^2}+\frac{b^2p}{3e(bd-ae)^2(d+ex)}+\frac{b^3p\log(a+bx)}{3e(bd-ae)^3}-\frac{\log\left(c(a+bx)^p\right)}{3e(d+ex)^3}-\frac{b}{}\end{aligned}$$

Mathematica [A] time = 0.15, size = 105, normalized size = 0.79

$$\frac{\frac{bp(d+ex)\left(2b^2(d+ex)^2\log(a+bx)+(bd-ae)(-ae+3bd+2bex)-2b^2(d+ex)^2\log(d+ex)\right)}{(bd-ae)^3}-2\log\left(c(a+bx)^p\right)}{6e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x)^4,x]
```

```
[Out] (-2*Log[c*(a + b*x)^p] + (b*p*(d + e*x)*((b*d - a*e)*(3*b*d - a*e + 2*b*e*x
) + 2*b^2*(d + e*x)^2*Log[a + b*x] - 2*b^2*(d + e*x)^2*Log[d + e*x]))/(b*d
- a*e)^3)/(6*e*(d + e*x)^3)
```

fricas [B] time = 0.47, size = 443, normalized size = 3.33

$$\frac{2\left(b^3de^2 - ab^2e^3\right)px^2 + \left(5\,b^3d^2e - 6\,ab^2de^2 + a^2be^3\right)px + \left(3\,b^3d^3 - 4\,ab^2d^2e + a^2bde^2\right)p + 2\left(b^3e^3px^3 + 3\,b^3de^2px^2\right.}{6\left(b^3d^6e - 3\,ab^2d^5e^2 + 3\,a^2bd^4e^3 - a^3d^3e^4 + \left(b^3d^3e^4 - 3\,ab^2d^2e^5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(b^3*d*e^2 - a*b^2*e^3)*p*x^2 + (5*b^3*d^2*e - 6*a*b^2*d*e^2 + a^2*b
*e^3)*p*x + (3*b^3*d^3 - 4*a*b^2*d^2*e + a^2*b*d*e^2)*p + 2*(b^3*e^3*p*x^3
+ 3*b^3*d*e^2*p*x^2 + 3*b^3*d^2*e*p*x + (3*a*b^2*d^2*e - 3*a^2*b*d*e^2 + a^
3*e^3)*p)*log(b*x + a) - 2*(b^3*e^3*p*x^3 + 3*b^3*d*e^2*p*x^2 + 3*b^3*d^2*e
*p*x + b^3*d^3*p)*log(e*x + d) - 2*(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2
 - a^3*e^3)*log(c))/(b^3*d^6*e - 3*a*b^2*d^5*e^2 + 3*a^2*b*d^4*e^3 - a^3*d^
3*e^4 + (b^3*d^3*e^4 - 3*a*b^2*d^2*e^5 + 3*a^2*b*d*e^6 - a^3*e^7)*x^3 + 3*(
b^3*d^4*e^3 - 3*a*b^2*d^3*e^4 + 3*a^2*b*d^2*e^5 - a^3*d*e^6)*x^2 + 3*(b^3*d
^5*e^2 - 3*a*b^2*d^4*e^3 + 3*a^2*b*d^3*e^4 - a^3*d^2*e^5)*x)
```

giac [B] time = 0.19, size = 495, normalized size = 3.72

$$\frac{2\,b^3px^3e^3\log\left(bx + a\right) + 6\,b^3dpx^2e^2\log\left(bx + a\right) + 6\,b^3d^2pxe\log\left(bx + a\right) - 2\,b^3px^3e^3\log\left(xe + d\right) - 6\,b^3dpx^2e^2\log}{6\left(b^3d^3x^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 1/6*(2*b^3*p*x^3*e^3*log(b*x + a) + 6*b^3*d*p*x^2*e^2*log(b*x + a) + 6*b^3*
d^2*p*x*e*log(b*x + a) - 2*b^3*p*x^3*e^3*log(x*e + d) - 6*b^3*d*p*x^2*e^2*l
og(x*e + d) - 6*b^3*d^2*p*x*e*log(x*e + d) + 2*b^3*d*p*x^2*e^2 + 5*b^3*d^2*
p*x*e + 6*a*b^2*d^2*p*e*log(b*x + a) - 2*b^3*d^3*p*log(x*e + d) + 3*b^3*d^3
*p - 2*a*b^2*p*x^2*e^3 - 6*a*b^2*d*p*x*e^2 - 4*a*b^2*d^2*p*e - 6*a^2*b*d*p*
e^2*log(b*x + a) - 2*b^3*d^3*log(c) + 6*a*b^2*d^2*e*log(c) + a^2*b*p*x*e^3
+ a^2*b*d*p*e^2 + 2*a^3*p*e^3*log(b*x + a) - 6*a^2*b*d*e^2*log(c) + 2*a^3*e
^3*log(c))/(b^3*d^3*x^3*e^4 + 3*b^3*d^4*x^2*e^3 + 3*b^3*d^5*x*e^2 + b^3*d^6
*e - 3*a*b^2*d^2*x^3*e^5 - 9*a*b^2*d^3*x^2*e^4 - 9*a*b^2*d^4*x*e^3 - 3*a*b^
2*d^5*e^2 + 3*a^2*b*d*x^3*e^6 + 9*a^2*b*d^2*x^2*e^5 + 9*a^2*b*d^3*x*e^4 + 3
*a^2*b*d^4*e^3 - a^3*x^3*e^7 - 3*a^3*d*x^2*e^6 - 3*a^3*d^2*x*e^5 - a^3*d^3*
e^4)
```

maple [C] time = 0.66, size = 873, normalized size = 6.56

$$-\frac{\ln\left(\left(bx + a\right)^p\right)}{3\left(ex + d\right)^3 e} + \frac{2b^3d^3\ln(c) - 2a^3e^3\ln(c) + 2a\,b^2e^3p\,x^2 - 2b^3d\,e^2p\,x^2 - a^2b\,e^3px - 5b^3d^2epx - a^2bd\,e^2p + 4a\,b^2d^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/(e*x+d)^4,x)
```

```
[Out] -1/3/e/(e*x+d)^3*ln((b*x+a)^p)+1/6*(2*ln(c)*b^3*d^3-2*ln(c)*a^3*e^3+2*a*b^2
*e^3*p*x^2-2*b^3*d*e^2*p*x^2-a^2*b*e^3*p*x-5*b^3*d^2*e*p*x-a^2*b*d*p*e^2+4*
a*b^2*d^2*p*e-I*Pi*a^3*e^3*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2-3*b^3*d^
```

```
3*p+6*a*b^2*d*e^2*p*x+2*ln(-e*x-d)*b^3*d^3*p-2*ln(b*x+a)*b^3*d^3*p+3*I*Pi*a
*b^2*d^2*e*csgn(I*c*(b*x+a)^p)^3-I*Pi*b^3*d^3*csgn(I*(b*x+a)^p)*csgn(I*c*(b
*x+a)^p)*csgn(I*c)+I*Pi*a^3*e^3*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(
I*c)-I*Pi*a^3*e^3*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)+I*Pi*b^3*d^3*csgn(I*(b*x+
a)^p)*csgn(I*c*(b*x+a)^p)^2+I*Pi*b^3*d^3*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)+3*
I*Pi*a^2*b*d*e^2*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)+3*I*Pi*a^2*b*d*e^2*csgn(I*
(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2-3*I*Pi*a*b^2*d^2*e*csgn(I*c*(b*x+a)^p)^2*c
sgn(I*c)-3*I*Pi*a*b^2*d^2*e*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2-3*I*Pi*
a^2*b*d*e^2*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)+I*Pi*a^3*e^3*cs
gn(I*c*(b*x+a)^p)^3-I*Pi*b^3*d^3*csgn(I*c*(b*x+a)^p)^3+3*I*Pi*a*b^2*d^2*e*c
sgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)-2*ln(b*x+a)*b^3*e^3*p*x^3+2*
ln(-e*x-d)*b^3*e^3*p*x^3+6*ln(c)*a^2*b*d*e^2-6*ln(c)*a*b^2*d^2*e-3*I*Pi*a^2
*b*d*e^2*csgn(I*c*(b*x+a)^p)^3-6*ln(b*x+a)*b^3*d*e^2*p*x^2+6*ln(-e*x-d)*b^3
*d*e^2*p*x^2-6*ln(b*x+a)*b^3*d^2*e*p*x+6*ln(-e*x-d)*b^3*d^2*e*p*x)/(e*x+d)^
3/(a^2*e^2-2*a*b*d*e+b^2*d^2)/(a*e-b*d)/e
```

maxima [A] time = 0.47, size = 232, normalized size = 1.74

$$\frac{\left(\frac{2\,b^2\log(bx+a)}{b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3}-\frac{2\,b^2\log(ex+d)}{b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3}+\frac{2\,bex+3\,bd-ae}{b^2d^4-2\,abd^3e+a^2d^2e^2+\left(b^2d^2e^2-2\,abde^3+a^2e^4\right)x^2+2\,\left(b^2d^3e-2\,abd^2e^2+a^2de^3\right)\right.}{6\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*log(b*x + a)/(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)
 - 2*b^2*log(e*x + d)/(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3) +
 (2*b*e*x + 3*b*d - a*e)/(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^2 + (b^2*d^2*e^
2 - 2*a*b*d*e^3 + a^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^2 + a^2*d*e^3)*
x))*b*p/e - 1/3*log((b*x + a)^p*c)/((e*x + d)^3*e)
```

mupad [B] time = 0.76, size = 145, normalized size = 1.09

$$\frac{b^2\,p\,x}{3\,(a\,e-b\,d)^2\,(d+e\,x)^2}-\frac{\ln\left(c\,(a+b\,x)^p\right)}{3\,e\,(d+e\,x)^3}-\frac{a\,b\,p}{6\,(a\,e-b\,d)^2\,(d+e\,x)^2}+\frac{b^2\,d\,p}{2\,e\,(a\,e-b\,d)^2\,(d+e\,x)^2}+\frac{b^3\,p\,\mathrm{atan}\left(\frac{a\,e\,1\mathrm{i}+b\,d}{a\,e-}\right.}{3\,e\,(a\,e-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)/(d + e*x)^4,x)
```

```
[Out] (b^2*p*x)/(3*(a*e - b*d)^2*(d + e*x)^2) - log(c*(a + b*x)^p)/(3*e*(d + e*x)
^3) + (b^3*p*atan((a*e*1i + b*d*1i + b*e*x*2i)/(a*e - b*d))*2i)/(3*e*(a*e -
 b*d)^3) - (a*b*p)/(6*(a*e - b*d)^2*(d + e*x)^2) + (b^2*d*p)/(2*e*(a*e - b*
d)^2*(d + e*x)^2)
```

sympy [A] time = 39.66, size = 6069, normalized size = 45.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/(e*x+d)**4,x)
```

```
[Out] Piecewise((-3*p*log(b*d/e + b*x)/(9*d**3*e + 27*d**2*e**2*x + 27*d*e**3*x**
2 + 9*e**4*x**3) - p/(9*d**3*e + 27*d**2*e**2*x + 27*d*e**3*x**2 + 9*e**4*x
**3) - 3*log(c)/(9*d**3*e + 27*d**2*e**2*x + 27*d*e**3*x**2 + 9*e**4*x**3),
 Eq(a, b*d/e)), ((a*p*log(a + b*x)/b + p*x*log(a + b*x) - p*x + x*log(c))/d
**4, Eq(e, 0)), (-2*a**3*e**3*p*log(a + b*x)/(6*a**3*d**3*e**4 + 18*a**3*d*
*2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 -
54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 +
```

```
18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18
*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4
*e**3*x**2 - 6*b**3*d**3*e**4*x**3) - 2*a**3*e**3*log(c)/(6*a**3*d**3*e**4
+ 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*
d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*
e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e*
*4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x -
18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) + 6*a**2*b*d*e**2*p*log(a +
 b*x)/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**
3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*
e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e*
*3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e
- 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) - a
**2*b*d*e**2*p/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**
2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**
2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**
2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**
3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*
x**3) + 6*a**2*b*d*e**2*log(c)/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18
*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3
*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5
*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e
**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6
*b**3*d**3*e**4*x**3) - a**2*b*e**3*p*x/(6*a**3*d**3*e**4 + 18*a**3*d**2*e*
*5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a*
*2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*
b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b*
*2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3
*x**2 - 6*b**3*d**3*e**4*x**3) - 6*a*b**2*d**2*e*p*log(a + b*x)/(6*a**3*d**
3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*
a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a*
*2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*
d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e*
*2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) + 4*a*b**2*d**2*e*p/
(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7
*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x
**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x +
 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b
**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) - 6*a*b**
2*d**2*e*log(c)/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x*
*2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a*
*2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b*
*2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b*
*3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4
*x**3) + 6*a*b**2*d*e**2*p*x/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a
**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e
**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e
**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**
5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b
**3*d**3*e**4*x**3) + 2*a*b**2*e**3*p*x**2/(6*a**3*d**3*e**4 + 18*a**3*d**2
*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54
*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18
*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a
*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e
**3*x**2 - 6*b**3*d**3*e**4*x**3) + 2*b**3*d**3*p*log(d/e + x)/(6*a**3*d**3
*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a
**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**
2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d
**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**
```

```
2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) - 3*b**3*d**3*p/(6*a*
*3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3
 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 -
 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a
*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d
**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) - 6*b**3*d**2*
e*p*x*log(a + b*x)/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6
*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54
*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a
*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6
*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e
**4*x**3) + 6*b**3*d**2*e*p*x*log(d/e + x)/(6*a**3*d**3*e**4 + 18*a**3*d**2
*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54
*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18
*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a
*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e
**3*x**2 - 6*b**3*d**3*e**4*x**3) - 5*b**3*d**2*e*p*x/(6*a**3*d**3*e**4 + 1
8*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**
4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**
6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*
x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*
b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) - 6*b**3*d**2*e*x*log(c)/(6*a*
*3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3
 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 -
 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a
*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d
**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) - 6*b**3*d*e**
2*p*x**2*log(a + b*x)/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e
**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x -
 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 5
4*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3
- 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**
3*e**4*x**3) + 6*b**3*d*e**2*p*x**2*log(d/e + x)/(6*a**3*d**3*e**4 + 18*a**
3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**
3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**
3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2
+ 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*
d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) - 2*b**3*d*e**2*p*x**2/(6*a**3*d**3
*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a
**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**
2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d
**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**
2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) - 6*b**3*d*e**2*x**2*
log(c)/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a*
*3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2
*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e
**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e
 - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) -
2*b**3*e**3*p*x**3*log(a + b*x)/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 1
8*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**
3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**
5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*
e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 -
6*b**3*d**3*e**4*x**3) + 2*b**3*e**3*p*x**3*log(d/e + x)/(6*a**3*d**3*e**4
+ 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*
d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*
e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e*
*4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x -
18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) - 2*b**3*e**3*x**3*log(c)/(
```

```
6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*
x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x*
*2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x +
54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b*
*3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3), True))
```

3.184 $\int (d+ex)^3 \log \left(c \left(a+bx^2\right)^p\right) dx$

Optimal. Leaf size=178

$$-\frac{p \left(a^2e^4-6abd^2e^2+b^2d^4\right) \log \left(a+bx^2\right)}{4b^2e}+\frac{2\sqrt{a}\, dp \left(bd^2-ae^2\right) \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}+\frac{(d+ex)^4 \log \left(c \left(a+bx^2\right)^p\right)}{4e}-\frac{epx^2 \ldots}{\ldots}$$

```
[Out] -2*d*(-a*e^2+b*d^2)*p*x/b-1/4*e*(-a*e^2+6*b*d^2)*p*x^2/b-2/3*d*e^2*p*x^3-1/
8*e^3*p*x^4-1/4*(a^2*e^4-6*a*b*d^2*e^2+b^2*d^4)*p*ln(b*x^2+a)/b^2/e+1/4*(e*
x+d)^4*ln(c*(b*x^2+a)^p)/e+2*d*(-a*e^2+b*d^2)*p*arctan(x*b^(1/2)/a^(1/2))*a
^(1/2)/b^(3/2)
```

Rubi [A] time = 0.16, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2463, 801, 635, 205, 260}

$$-\frac{p \left(a^2e^4-6abd^2e^2+b^2d^4\right) \log \left(a+bx^2\right)}{4b^2e}+\frac{2\sqrt{a}\, dp \left(bd^2-ae^2\right) \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}+\frac{(d+ex)^4 \log \left(c \left(a+bx^2\right)^p\right)}{4e}-\frac{epx^2 \ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (-2*d*(b*d^2 - a*e^2)*p*x)/b - (e*(6*b*d^2 - a*e^2)*p*x^2)/(4*b) - (2*d*e^2
*p*x^3)/3 - (e^3*p*x^4)/8 + (2*Sqrt[a]*d*(b*d^2 - a*e^2)*p*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/b^(3/2) - ((b^2*d^4 - 6*a*b*d^2*e^2 + a^2*e^4)*p*Log[a + b*x^2
])/(4*b^2*e) + ((d + e*x)^4*Log[c*(a + b*x^2)^p])/(4*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)^3 \log\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{(d+ex)^4\log\left(c\left(a+bx^2\right)^p\right)}{4e} - \frac{(bp)\int \frac{x(d+ex)^4}{a+bx^2}\,dx}{2e}$$

$$= \frac{(d+ex)^4\log\left(c\left(a+bx^2\right)^p\right)}{4e} - \frac{(bp)\int\left(\frac{4de\left(bd^2-ae^2\right)}{b^2} + \frac{e^2\left(6bd^2-ae^2\right)x}{b^2} + \frac{4de^3x^2}{b} + \frac{e^4x}{b}\right.}{2e}$$

$$= -\frac{2d\left(bd^2-ae^2\right)px}{b} - \frac{e\left(6bd^2-ae^2\right)px^2}{4b} - \frac{2}{3}de^2px^3 - \frac{1}{8}e^3px^4 + \frac{(d+ex)^4\log}{4}$$

$$= -\frac{2d\left(bd^2-ae^2\right)px}{b} - \frac{e\left(6bd^2-ae^2\right)px^2}{4b} - \frac{2}{3}de^2px^3 - \frac{1}{8}e^3px^4 + \frac{(d+ex)^4\log}{4}$$

$$= -\frac{2d\left(bd^2-ae^2\right)px}{b} - \frac{e\left(6bd^2-ae^2\right)px^2}{4b} - \frac{2}{3}de^2px^3 - \frac{1}{8}e^3px^4 + \frac{2\sqrt{a}\,d\left(bd^2-\right.}{}$$

Mathematica [A] time = 0.78, size = 249, normalized size = 1.40

$$\frac{-6p\left(a^2e^4+4\sqrt{-a}\,b^{3/2}d^3e-6abd^2e^2+4(-a)^{3/2}\sqrt{b}\,de^3+b^2d^4\right)\log\left(\sqrt{-a}-\sqrt{b}\,x\right)-6p\left(a^2e^4-4\sqrt{-a}\,b^{3/2}d^3e-6ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (-6*(b^2*d^4 + 4*Sqrt[-a]*b^(3/2)*d^3*e - 6*a*b*d^2*e^2 + 4*(-a)^(3/2)*Sqrt
[b]*d*e^3 + a^2*e^4)*p*Log[Sqrt[-a] - Sqrt[b]*x] - 6*(b^2*d^4 - 4*Sqrt[-a]*
b^(3/2)*d^3*e - 6*a*b*d^2*e^2 + 4*Sqrt[-a]*a*Sqrt[b]*d*e^3 + a^2*e^4)*p*Log
[Sqrt[-a] + Sqrt[b]*x] + b*(6*a*e^3*p*x*(8*d + e*x) - b*e*p*x*(48*d^3 + 36*
d^2*e*x + 16*d*e^2*x^2 + 3*e^3*x^3) + 6*b*(d + e*x)^4*Log[c*(a + b*x^2)^p])
)/(24*b^2*e)
```

fricas [A] time = 0.47, size = 498, normalized size = 2.80

$$\left[-\frac{3\,b^2e^3px^4+16\,b^2de^2px^3+6\left(6\,b^2d^2e-abe^3\right)px^2-24\left(b^2d^3-abde^2\right)p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+48\left(b^2d^3-ab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*b^2*e^3*p*x^4 + 16*b^2*d*e^2*p*x^3 + 6*(6*b^2*d^2*e - a*b*e^3)*p*
x^2 - 24*(b^2*d^3 - a*b*d*e^2)*p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) -
 a)/(b*x^2 + a)) + 48*(b^2*d^3 - a*b*d*e^2)*p*x - 6*(b^2*e^3*p*x^4 + 4*b^2*
d*e^2*p*x^3 + 6*b^2*d^2*e*p*x^2 + 4*b^2*d^3*p*x + (6*a*b*d^2*e - a^2*e^3)*p
)*log(b*x^2 + a) - 6*(b^2*e^3*x^4 + 4*b^2*d*e^2*x^3 + 6*b^2*d^2*e*x^2 + 4*b
^2*d^3*x)*log(c))/b^2, -1/24*(3*b^2*e^3*p*x^4 + 16*b^2*d*e^2*p*x^3 + 6*(6*b
^2*d^2*e - a*b*e^3)*p*x^2 - 48*(b^2*d^3 - a*b*d*e^2)*p*sqrt(a/b)*arctan(b*x
*sqrt(a/b)/a) + 48*(b^2*d^3 - a*b*d*e^2)*p*x - 6*(b^2*e^3*p*x^4 + 4*b^2*d*e
^2*p*x^3 + 6*b^2*d^2*e*p*x^2 + 4*b^2*d^3*p*x + (6*a*b*d^2*e - a^2*e^3)*p)*l
og(b*x^2 + a) - 6*(b^2*e^3*x^4 + 4*b^2*d*e^2*x^3 + 6*b^2*d^2*e*x^2 + 4*b^2*
d^3*x)*log(c))/b^2]
```

giac [A] time = 0.21, size = 273, normalized size = 1.53

$$\frac{2\left(abd^3p - a^2dpe^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b} + \frac{6\,b^2px^4e^3\log\left(bx^2+a\right) + 24\,b^2dpx^3e^2\log\left(bx^2+a\right) + 36\,b^2d^2px^2e\log\left(bx^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 2*(a*b*d^3*p - a^2*d*p*e^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/24*(6*b
^2*p*x^4*e^3*log(b*x^2 + a) + 24*b^2*d*p*x^3*e^2*log(b*x^2 + a) + 36*b^2*d^
2*p*x^2*e*log(b*x^2 + a) - 3*b^2*p*x^4*e^3 - 16*b^2*d*p*x^3*e^2 - 36*b^2*d^
2*p*x^2*e + 24*b^2*d^3*p*x*log(b*x^2 + a) + 6*b^2*x^4*e^3*log(c) + 24*b^2*d
*x^3*e^2*log(c) + 36*b^2*d^2*x^2*e*log(c) - 48*b^2*d^3*p*x + 36*a*b*d^2*p*e
*log(b*x^2 + a) + 24*b^2*d^3*x*log(c) + 6*a*b*p*x^2*e^3 + 48*a*b*d*p*x*e^2
- 6*a^2*p*e^3*log(b*x^2 + a))/b^2
```

maple [C] time = 0.72, size = 1330, normalized size = 7.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*ln(c*(b*x^2+a)^p),x)
```

```
[Out] d*e^2*x^3*ln(c)+3/2*d^2*e*x^2*ln(c)-1/4/e*p*ln(-a^2*d*e^3+a*b*d^3*e+(-a^3*b
*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)*x)*d^4-1/4/e*p*ln(-a^2*d*e^
3+a*b*d^3*e-(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)*x)*d^4+1
/4*(e*x+d)^4/e*ln((b*x^2+a)^p)+1/4*e^3*x^4*ln(c)+d^3*x*ln(c)-1/8*e^3*p*x^4-
3/2*d^2*e*p*x^2-1/4/b^2*e^3*p*ln(-a^2*d*e^3+a*b*d^3*e+(-a^3*b*d^2*e^6+2*a^2
*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)*x)*a^2-1/4/b^2*e^3*p*ln(-a^2*d*e^3+a*b*d^
3*e-(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)*x)*a^2-1/b^2/e*p
*ln(-a^2*d*e^3+a*b*d^3*e+(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(
1/2)*x)*(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)+1/b^2/e*p*ln
(-a^2*d*e^3+a*b*d^3*e-(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2
)*x)*(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)-1/8*I*e^3*Pi*x^
4*csgn(I*c*(b*x^2+a)^p)^3-1/2*I*Pi*d^3*csgn(I*c*(b*x^2+a)^p)^3*x+2/b*a*d*p*
e^2*x-2*d^3*p*x-2/3*d*e^2*p*x^3+3/2/b*e*p*ln(-a^2*d*e^3+a*b*d^3*e+(-a^3*b*d
^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)*x)*a*d^2+3/2/b*e*p*ln(-a^2*d*
e^3+a*b*d^3*e-(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)*x)*a*d
^2+1/2*I*Pi*d^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*x+1/2*I*Pi*d^3*
csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*x+1/8*I*e^3*Pi*x^4*csgn(I*(b*x^2+a)^p)*cs
gn(I*c*(b*x^2+a)^p)^2+1/8*I*e^3*Pi*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/
2*I*e^2*Pi*d*x^3*csgn(I*c*(b*x^2+a)^p)^3-3/4*I*e*Pi*d^2*x^2*csgn(I*c*(b*x^2
+a)^p)^3+1/4/b*a*e^3*p*x^2+1/2*I*e^2*Pi*d*x^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)^2+1/2*I*e^2*Pi*d*x^3*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+3/4*I*e
*Pi*d^2*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+3/4*I*e*Pi*d^2*x^2*
csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/8*I*e^3*Pi*x^4*csgn(I*(b*x^2+a)^p)*csgn
(I*c*(b*x^2+a)^p)*csgn(I*c)-1/2*I*Pi*d^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^
2+a)^p)*csgn(I*c)*x-1/2*I*e^2*Pi*d*x^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+
a)^p)*csgn(I*c)-3/4*I*e*Pi*d^2*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p
)*csgn(I*c)
```

maxima [A] time = 0.99, size = 177, normalized size = 0.99

$$\frac{1}{24}\,bp\left(\frac{48\left(abd^3 - a^2de^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} - \frac{3\,be^3x^4 + 16\,bde^2x^3 + 6\left(6\,bd^2e - ae^3\right)x^2 + 48\left(bd^3 - ade^2\right)x}{b^2} + \frac{6\left(6\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 1/24*b*p*(48*(a*b*d^3 - a^2*d*e^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) -
(3*b*e^3*x^4 + 16*b*d*e^2*x^3 + 6*(6*b*d^2*e - a*e^3)*x^2 + 48*(b*d^3 - a*d
*e^2)*x)/b^2 + 6*(6*a*b*d^2*e - a^2*e^3)*log(b*x^2 + a)/b^3) + 1/4*(e^3*x^4
 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)*log((b*x^2 + a)^p*c)
```

mupad [B] time = 0.46, size = 222, normalized size = 1.25

$$\frac{e^3\,x^4\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{4}-2\,d^3\,p\,x-\frac{e^3\,p\,x^4}{8}+d^3\,x\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)+\frac{3\,d^2\,e\,x^2\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{2}+d\,e^2\,x^3\,\ln\left(c\,\left(b\,x^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)*(d + e*x)^3,x)
```

```
[Out] (e^3*x^4*log(c*(a + b*x^2)^p))/4 - 2*d^3*p*x - (e^3*p*x^4)/8 + d^3*x*log(c*
(a + b*x^2)^p) + (3*d^2*e*x^2*log(c*(a + b*x^2)^p))/2 + d*e^2*x^3*log(c*(a
+ b*x^2)^p) - (3*d^2*e*p*x^2)/2 - (2*d*e^2*p*x^3)/3 + (a*e^3*p*x^2)/(4*b) +
 (2*a^(1/2)*d^3*p*atan((b^(1/2)*x)/a^(1/2)))/b^(1/2) - (a^2*e^3*p*log(a + b
*x^2))/(4*b^2) - (2*a^(3/2)*d*e^2*p*atan((b^(1/2)*x)/a^(1/2)))/b^(3/2) + (2
*a*d*e^2*p*x)/b + (3*a*d^2*e*p*log(a + b*x^2))/(2*b)
```

sympy [A] time = 47.87, size = 422, normalized size = 2.37

$$\begin{cases} -\frac{i a^{\frac{3}{2}} d e^2 p \log\left(a+bx^2\right)}{b^2\sqrt{\frac{1}{b}}}+\frac{2 i a^{\frac{3}{2}} d e^2 p \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{b^2\sqrt{\frac{1}{b}}}+\frac{i\sqrt{a}\, d^3 p \log\left(a+bx^2\right)}{b\sqrt{\frac{1}{b}}}-\frac{2i\sqrt{a}\, d^3 p \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{b\sqrt{\frac{1}{b}}}-\frac{a^2 e^3 p \log\left(a+bx^2\right)}{4b^2}+\frac{3ad^2ep\log}{2} \\ \left(d^3x+\frac{3d^2ex^2}{2}+de^2x^3+\frac{e^3x^4}{4}\right)\log\left(a^pc\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((-I*a**(3/2)*d*e**2*p*log(a + b*x**2)/(b**2*sqrt(1/b)) + 2*I*a**(
3/2)*d*e**2*p*log(-I*sqrt(a)*sqrt(1/b) + x)/(b**2*sqrt(1/b)) + I*sqrt(a)*d*
*3*p*log(a + b*x**2)/(b*sqrt(1/b)) - 2*I*sqrt(a)*d**3*p*log(-I*sqrt(a)*sqrt
(1/b) + x)/(b*sqrt(1/b)) - a**2*e**3*p*log(a + b*x**2)/(4*b**2) + 3*a*d**2*
e*p*log(a + b*x**2)/(2*b) + 2*a*d*e**2*p*x/b + a*e**3*p*x**2/(4*b) + d**3*p
*x*log(a + b*x**2) - 2*d**3*p*x + d**3*x*log(c) + 3*d**2*e*p*x**2*log(a + b
*x**2)/2 - 3*d**2*e*p*x**2/2 + 3*d**2*e*x**2*log(c)/2 + d*e**2*p*x**3*log(a
 + b*x**2) - 2*d*e**2*p*x**3/3 + d*e**2*x**3*log(c) + e**3*p*x**4*log(a + b
*x**2)/4 - e**3*p*x**4/8 + e**3*x**4*log(c)/4, Ne(b, 0)), ((d**3*x + 3*d**2
*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4)*log(a**p*c), True))
```

3.185 $\int (d+ex)^2 \log \left(c \left(a+bx^2\right)^p\right) \, dx$

Optimal. Leaf size=141

$$\frac{2\sqrt{a}\,p\left(3bd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3b^{3/2}}+\frac{(d+ex)^3\log\left(c\left(a+bx^2\right)^p\right)}{3e}-\frac{dp\left(bd^2-3ae^2\right)\log\left(a+bx^2\right)}{3be}-\frac{2px\left(3bd^2-ae^2\right.}{3b}$$

```
[Out] -2/3*(-a*e^2+3*b*d^2)*p*x/b-d*e*p*x^2-2/9*e^2*p*x^3-1/3*d*(-3*a*e^2+b*d^2)*
p*ln(b*x^2+a)/b/e+1/3*(e*x+d)^3*ln(c*(b*x^2+a)^p)/e+2/3*(-a*e^2+3*b*d^2)*p*
arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)
```

Rubi [A] time = 0.13, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2463, 801, 635, 205, 260}

$$\frac{2\sqrt{a}\,p\left(3bd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3b^{3/2}}+\frac{(d+ex)^3\log\left(c\left(a+bx^2\right)^p\right)}{3e}-\frac{dp\left(bd^2-3ae^2\right)\log\left(a+bx^2\right)}{3be}-\frac{2px\left(3bd^2-ae^2\right.}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (-2*(3*b*d^2 - a*e^2)*p*x)/(3*b) - d*e*p*x^2 - (2*e^2*p*x^3)/9 + (2*Sqrt[a]
*(3*b*d^2 - a*e^2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*b^(3/2)) - (d*(b*d^2 -
 3*a*e^2)*p*Log[a + b*x^2])/(3*b*e) + ((d + e*x)^3*Log[c*(a + b*x^2)^p])/(3
*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)^2 \log\left(c\left(a+bx^2\right)^p\right) dx = \frac{(d+ex)^3 \log\left(c\left(a+bx^2\right)^p\right)}{3e} - \frac{(2bp)\int \frac{x(d+ex)^3}{a+bx^2}dx}{3e}$$
$$= \frac{(d+ex)^3 \log\left(c\left(a+bx^2\right)^p\right)}{3e} - \frac{(2bp)\int \left(\frac{e\left(3bd^2-ae^2\right)}{b^2} + \frac{3de^2x}{b} + \frac{e^3x^2}{b} - \frac{ae\left(3bd^2-ae^2\right)\cdots}{b^2(a\cdots}\right.}{3e}$$
$$= -\frac{2\left(3bd^2-ae^2\right)px}{3b} - depx^2 - \frac{2}{9}e^2px^3 + \frac{(d+ex)^3 \log\left(c\left(a+bx^2\right)^p\right)}{3e} + \frac{(2p)\int\cdots}{\cdots}$$
$$= -\frac{2\left(3bd^2-ae^2\right)px}{3b} - depx^2 - \frac{2}{9}e^2px^3 + \frac{(d+ex)^3 \log\left(c\left(a+bx^2\right)^p\right)}{3e} - \frac{\left(2d\left(b\cdots\right.\right.}{\cdots}$$
$$= -\frac{2\left(3bd^2-ae^2\right)px}{3b} - depx^2 - \frac{2}{9}e^2px^3 + \frac{2\sqrt{a}\left(3bd^2-ae^2\right)p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}} - \frac{d\cdots}{\cdots}$$

Mathematica [A] time = 0.48, size = 211, normalized size = 1.50

$$\frac{3p\left(-3\sqrt{-a}bd^2e + 3a\sqrt{b}de^2 + \sqrt{-a}ae^3 - b^{3/2}d^3\right)\log\left(\sqrt{-a} - \sqrt{b}x\right) - 3p\left(-3\sqrt{-a}bd^2e - 3a\sqrt{b}de^2 + \sqrt{-a}ae^3 + b\cdots\right.}{9b^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (3*(-(b^(3/2)*d^3) - 3*Sqrt[-a]*b*d^2*e + 3*a*Sqrt[b]*d*e^2 + Sqrt[-a]*a*e^
3)*p*Log[Sqrt[-a] - Sqrt[b]*x] - 3*(b^(3/2)*d^3 - 3*Sqrt[-a]*b*d^2*e - 3*a*
Sqrt[b]*d*e^2 + Sqrt[-a]*a*e^3)*p*Log[Sqrt[-a] + Sqrt[b]*x] + Sqrt[b]*(6*a*
e^3*p*x - b*e*p*x*(18*d^2 + 9*d*e*x + 2*e^2*x^2) + 3*b*(d + e*x)^3*Log[c*(a
 + b*x^2)^p]))/(9*b^(3/2)*e)
```

fricas [A] time = 0.45, size = 320, normalized size = 2.27

$$\left[-\frac{2\,be^2px^3 + 9\,bdepx^2 - 3\left(3\,bd^2 - ae^2\right)p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) + 6\left(3\,bd^2 - ae^2\right)px - 3\left(be^2px^3 + 3\,bdepx^2 + 3\cdots\right.}{9\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [-1/9*(2*b*e^2*p*x^3 + 9*b*d*e*p*x^2 - 3*(3*b*d^2 - a*e^2)*p*sqrt(-a/b)*log
((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 6*(3*b*d^2 - a*e^2)*p*x - 3*
(b*e^2*p*x^3 + 3*b*d*e*p*x^2 + 3*b*d^2*p*x + 3*a*d*e*p)*log(b*x^2 + a) - 3*
(b*e^2*x^3 + 3*b*d*e*x^2 + 3*b*d^2*x)*log(c))/b, -1/9*(2*b*e^2*p*x^3 + 9*b*
d*e*p*x^2 - 6*(3*b*d^2 - a*e^2)*p*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 6*(3*
b*d^2 - a*e^2)*p*x - 3*(b*e^2*p*x^3 + 3*b*d*e*p*x^2 + 3*b*d^2*p*x + 3*a*d*e
*p)*log(b*x^2 + a) - 3*(b*e^2*x^3 + 3*b*d*e*x^2 + 3*b*d^2*x)*log(c))/b]
```

giac [A] time = 0.20, size = 173, normalized size = 1.23

$$\frac{2\left(3\,abd^2p - a^2pe^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{3\sqrt{ab}\,b} + \frac{3\,bpx^3e^2\log\left(bx^2+a\right) + 9\,bdpx^2e\log\left(bx^2+a\right) - 2\,bpx^3e^2 - 9\,bdpx^2e + 9\,bd^2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 2/3*(3*a*b*d^2*p - a^2*p*e^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/9*(3*
b*p*x^3*e^2*log(b*x^2 + a) + 9*b*d*p*x^2*e*log(b*x^2 + a) - 2*b*p*x^3*e^2 -
 9*b*d*p*x^2*e + 9*b*d^2*p*x*log(b*x^2 + a) + 3*b*x^3*e^2*log(c) + 9*b*d*x^
2*e*log(c) - 18*b*d^2*p*x + 9*a*d*p*e*log(b*x^2 + a) + 9*b*d^2*x*log(c) + 6
*a*p*x*e^2)/b
```

maple [C] time = 0.60, size = 965, normalized size = 6.84

$$\frac{(ex+d)^3 \ln\left(\left(b\,x^2+a\right)^p\right)}{3e}+de\,x^2\ln(c)-\frac{d^3p\ln\left(-a^2e^3+3ab\,d^2e-\sqrt{-a^3b\,e^6+6a^2b^2d^2e^4-9a\,b^3d^4e^2}\,x\right)}{3e}-\frac{d^3p\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*ln(c*(b*x^2+a)^p),x)
```

```
[Out] 1/3*(e*x+d)^3/e*ln((b*x^2+a)^p)+d*e*x^2*ln(c)-1/3/e*p*ln(-a^2*e^3+3*a*b*d^2
*e-(-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*b^3*d^4*e^2)^(1/2)*x)*d^3-1/3/e*p*ln(-
a^2*e^3+3*a*b*d^2*e+(-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*b^3*d^4*e^2)^(1/2)*x)
*d^3+1/3*e^2*x^3*ln(c)+d^2*x*ln(c)-2/9*e^2*p*x^3-2*d^2*p*x-d*e*p*x^2-1/2*I*
e*Pi*d*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+2/3/b*a*p*e^
2*x+1/6*I*e^2*Pi*x^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/6*I*e^2*
Pi*x^3*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/2*I*e*Pi*d*x^2*csgn(I*c*(b*x^2+a
)^p)^3+1/3/b^2/e*p*ln(-a^2*e^3+3*a*b*d^2*e-(-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*
a*b^3*d^4*e^2)^(1/2)*x)*(-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*b^3*d^4*e^2)^(1/2
)-1/3/b^2/e*p*ln(-a^2*e^3+3*a*b*d^2*e+(-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*b^3
*d^4*e^2)^(1/2)*x)*(-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*b^3*d^4*e^2)^(1/2)-1/6
*I*e^2*Pi*x^3*csgn(I*c*(b*x^2+a)^p)^3-1/2*I*Pi*d^2*csgn(I*c*(b*x^2+a)^p)^3*
x-1/6*I*e^2*Pi*x^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/2*
I*e*Pi*d*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/2*I*e*Pi*d*x^2*c
sgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/2*I*Pi*d^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)*csgn(I*c)*x+1/b*e*p*ln(-a^2*e^3+3*a*b*d^2*e-(-a^3*b*e^6+6*a^2*
b^2*d^2*e^4-9*a*b^3*d^4*e^2)^(1/2)*x)*a*d+1/b*e*p*ln(-a^2*e^3+3*a*b*d^2*e+(
-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*b^3*d^4*e^2)^(1/2)*x)*a*d+1/2*I*Pi*d^2*csg
n(I*c*(b*x^2+a)^p)^2*csgn(I*c)*x+1/2*I*Pi*d^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)^2*x
```

maxima [A] time = 0.99, size = 131, normalized size = 0.93

$$\frac{1}{9}\left(\frac{9\,ade\log\left(bx^2+a\right)}{b^2}+\frac{6\left(3\,abd^2-a^2e^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}-\frac{2\,be^2x^3+9\,bdex^2+6\left(3\,bd^2-ae^2\right)x}{b^2}\right)bp+\frac{1}{3}\left(e^2x^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 1/9*(9*a*d*e*log(b*x^2 + a)/b^2 + 6*(3*a*b*d^2 - a^2*e^2)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*b^2) - (2*b*e^2*x^3 + 9*b*d*e*x^2 + 6*(3*b*d^2 - a*e^2)*x)/
b^2)*b*p + 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)*log((b*x^2 + a)^p*c)
```

mupad [B] time = 3.28, size = 263, normalized size = 1.87

$$\frac{e^2\,x^3\,\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{3}-2\,d^2\,p\,x-\frac{2\,e^2\,p\,x^3}{9}+d^2\,x\,\ln\left(c\,\left(b\,x^2+a\right)^p\right)+d\,e\,x^2\,\ln\left(c\,\left(b\,x^2+a\right)^p\right)-d\,e\,p\,x^2+\frac{2\,a\,e^2\,p\,x}{3\,b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)*(d + e*x)^2,x)
```

```
[Out] (e^2*x^3*log(c*(a + b*x^2)^p))/3 - 2*d^2*p*x - (2*e^2*p*x^3)/9 + d^2*x*log(
c*(a + b*x^2)^p) + d*e*x^2*log(c*(a + b*x^2)^p) - d*e*p*x^2 + (2*a*e^2*p*x)
/(3*b) - (2*a^(1/2)*d^2*p*atan((3*a^(1/2)*b^(3/2)*d^2*p*x)/(a^2*e^2*p - 3*a
*b*d^2*p) - (a^(3/2)*b^(1/2)*e^2*p*x)/(a^2*e^2*p - 3*a*b*d^2*p)))/b^(1/2) +
 (2*a^(3/2)*e^2*p*atan((3*a^(1/2)*b^(3/2)*d^2*p*x)/(a^2*e^2*p - 3*a*b*d^2*p
) - (a^(3/2)*b^(1/2)*e^2*p*x)/(a^2*e^2*p - 3*a*b*d^2*p)))/(3*b^(3/2)) + (a*
d*e*p*log(a + b*x^2))/b
```

sympy [A] time = 24.02, size = 309, normalized size = 2.19

$$\begin{cases} -\frac{ia^{\frac{3}{2}}e^2p\log\left(a+bx^2\right)}{3b^2\sqrt{\frac{1}{b}}} + \frac{2ia^{\frac{3}{2}}e^2p\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{3b^2\sqrt{\frac{1}{b}}} + \frac{i\sqrt{a}\,d^2p\log\left(a+bx^2\right)}{b\sqrt{\frac{1}{b}}} - \frac{2i\sqrt{a}\,d^2p\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{b\sqrt{\frac{1}{b}}} + \frac{adep\log\left(a+bx^2\right)}{b} + \frac{2ae^2px}{3b} + d \\ \left(d^2x + dex^2 + \frac{e^2x^3}{3}\right)\log\left(a^pc\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((-I*a**(3/2)*e**2*p*log(a + b*x**2)/(3*b**2*sqrt(1/b)) + 2*I*a**(
3/2)*e**2*p*log(-I*sqrt(a)*sqrt(1/b) + x)/(3*b**2*sqrt(1/b)) + I*sqrt(a)*d*
*2*p*log(a + b*x**2)/(b*sqrt(1/b)) - 2*I*sqrt(a)*d**2*p*log(-I*sqrt(a)*sqrt
(1/b) + x)/(b*sqrt(1/b)) + a*d*e*p*log(a + b*x**2)/b + 2*a*e**2*p*x/(3*b) +
 d**2*p*x*log(a + b*x**2) - 2*d**2*p*x + d**2*x*log(c) + d*e*p*x**2*log(a +
 b*x**2) - d*e*p*x**2 + d*e*x**2*log(c) + e**2*p*x**3*log(a + b*x**2)/3 - 2
*e**2*p*x**3/9 + e**2*x**3*log(c)/3, Ne(b, 0)), ((d**2*x + d*e*x**2 + e**2*
x**3/3)*log(a**p*c), True))
```

3.186 $\int (d+ex)\log \left(c\left(a+bx^2\right)^p\right) \, dx$

Optimal. Leaf size=99

$$\frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}-\frac{p\left(bd^2-ae^2\right)\log\left(a+bx^2\right)}{2be}+\frac{2\sqrt{a}\,dp\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}-2dpx-\frac{1}{2}epx^2$$

```
[Out] -2*d*p*x-1/2*e*p*x^2-1/2*(-a*e^2+b*d^2)*p*ln(b*x^2+a)/b/e+1/2*(e*x+d)^2*ln(
c*(b*x^2+a)^p)/e+2*d*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2463, 801, 635, 205, 260}

$$\frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}-\frac{p\left(bd^2-ae^2\right)\log\left(a+bx^2\right)}{2be}+\frac{2\sqrt{a}\,dp\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}-2dpx-\frac{1}{2}epx^2$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Log[c*(a + b*x^2)^p],x]
```

```
[Out] -2*d*p*x - (e*p*x^2)/2 + (2*Sqrt[a]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b
] - ((b*d^2 - a*e^2)*p*Log[a + b*x^2])/(2*b*e) + ((d + e*x)^2*Log[c*(a + b*
x^2)^p])/(2*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)\log\left(c\left(a+bx^2\right)^p\right)dx = \frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e} - \frac{(bp)\int\frac{x(d+ex)^2}{a+bx^2}dx}{e}$$

$$= \frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e} - \frac{(bp)\int\left(\frac{2de}{b}+\frac{e^2x}{b}-\frac{2ade-\left(bd^2-ae^2\right)x}{b\left(a+bx^2\right)}\right)dx}{e}$$

$$= -2dpx - \frac{1}{2}epx^2 + \frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e} + \frac{p\int\frac{2ade-\left(bd^2-ae^2\right)x}{a+bx^2}dx}{e}$$

$$= -2dpx - \frac{1}{2}epx^2 + \frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e} + (2adp)\int\frac{1}{a+bx^2}dx + \frac{\left(\left(-bd\right.\right.}{}$$

$$= -2dpx - \frac{1}{2}epx^2 + \frac{2\sqrt{a}\,dp\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{\left(bd^2-ae^2\right)p\log\left(a+bx^2\right)}{2be} + \frac{(d+ex}{}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 0.84

$$dx\log\left(c\left(a+bx^2\right)^p\right) + \frac{1}{2}e\left(\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b} - px^2\right) + \frac{2\sqrt{a}\,dp\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} - 2dpx$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Log[c*(a + b*x^2)^p],x]
```

```
[Out] -2*d*p*x + (2*Sqrt[a]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + d*x*Log[c*
(a + b*x^2)^p] + (e*(-(p*x^2) + ((a + b*x^2)*Log[c*(a + b*x^2)^p])/b))/2
```

fricas [A] time = 0.45, size = 198, normalized size = 2.00

$$\left[-\frac{bepx^2 - 2\,bdp\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) + 4\,bdpx - \left(bepx^2+2\,bdpx+aep\right)\log\left(bx^2+a\right) - \left(bex^2+2\,bdx\right)\log(c}{2\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [-1/2*(b*e*p*x^2 - 2*b*d*p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b
*x^2 + a)) + 4*b*d*p*x - (b*e*p*x^2 + 2*b*d*p*x + a*e*p)*log(b*x^2 + a) - (
b*e*x^2 + 2*b*d*x)*log(c))/b, -1/2*(b*e*p*x^2 - 4*b*d*p*sqrt(a/b)*arctan(b*
x*sqrt(a/b)/a) + 4*b*d*p*x - (b*e*p*x^2 + 2*b*d*p*x + a*e*p)*log(b*x^2 + a)
 - (b*e*x^2 + 2*b*d*x)*log(c))/b]
```

giac [A] time = 0.18, size = 100, normalized size = 1.01

$$\frac{2\,adp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{bpx^2e\log\left(bx^2+a\right) - bpx^2e + 2\,bdpx\log\left(bx^2+a\right) + bx^2e\log(c) - 4\,bdpx + ape\log\left(bx^2+\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 2*a*d*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*(b*p*x^2*e*log(b*x^2 + a) - b
*p*x^2*e + 2*b*d*p*x*log(b*x^2 + a) + b*x^2*e*log(c) - 4*b*d*p*x + a*p*e*lo
g(b*x^2 + a) + 2*b*d*x*log(c))/b
```

maple [A] time = 0.07, size = 93, normalized size = 0.94

$$\frac{2adp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{ep\,x^2}{2}+\frac{e\,x^2\ln\left(c\left(b\,x^2+a\right)^p\right)}{2}-2dpx+dx\ln\left(c\left(b\,x^2+a\right)^p\right)-\frac{aep}{2b}+\frac{ae\ln\left(c\left(b\,x^2+a\right)^p\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*ln(c*(b*x^2+a)^p),x)
```

```
[Out] d*x*ln(c*(b*x^2+a)^p)-2*d*p*x+2*d*p*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
+1/2*e*ln(c*(b*x^2+a)^p)*x^2-1/2*e*p*x^2+1/2*e/b*ln(c*(b*x^2+a)^p)*a-1/2*a*
p*e/b
```

maxima [A] time = 0.98, size = 80, normalized size = 0.81

$$\frac{1}{2}\left(\frac{4\,ad\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{ae\log\left(bx^2+a\right)}{b^2}-\frac{ex^2+4\,dx}{b}\right)bp+\frac{1}{2}\left(ex^2+2\,dx\right)\log\left(\left(bx^2+a\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*d*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + a*e*log(b*x^2 + a)/b^2 - (
e*x^2 + 4*d*x)/b)*b*p + 1/2*(e*x^2 + 2*d*x)*log((b*x^2 + a)^p*c)
```

mupad [B] time = 1.13, size = 81, normalized size = 0.82

$$d\,x\,\ln\left(c\left(b\,x^2+a\right)^p\right)-\frac{e\,p\,x^2}{2}-2\,d\,p\,x+\frac{e\,x^2\,\ln\left(c\left(b\,x^2+a\right)^p\right)}{2}+\frac{a\,e\,p\,\ln\left(b\,x^2+a\right)}{2\,b}+\frac{2\,\sqrt{a}\,d\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)*(d + e*x),x)
```

```
[Out] d*x*log(c*(a + b*x^2)^p) - (e*p*x^2)/2 - 2*d*p*x + (e*x^2*log(c*(a + b*x^2)
^p))/2 + (a*e*p*log(a + b*x^2))/(2*b) + (2*a^(1/2)*d*p*atan((b^(1/2)*x)/a^(
1/2)))/b^(1/2)
```

sympy [A] time = 11.68, size = 160, normalized size = 1.62

$$\begin{cases}\frac{i\sqrt{a}dp\log\left(a+bx^2\right)}{b\sqrt{\frac{1}{b}}}-\frac{2i\sqrt{a}dp\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{b\sqrt{\frac{1}{b}}}+\frac{aep\log\left(a+bx^2\right)}{2b}+dpx\log\left(a+bx^2\right)-2dpx+dx\log\left(c\right)+\frac{epx^2\log\left(a+bx^2\right)}{2} \\ \left(dx+\frac{ex^2}{2}\right)\log\left(a^pc\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((I*sqrt(a)*d*p*log(a + b*x**2)/(b*sqrt(1/b)) - 2*I*sqrt(a)*d*p*lo
g(-I*sqrt(a)*sqrt(1/b) + x)/(b*sqrt(1/b)) + a*e*p*log(a + b*x**2)/(2*b) + d
*p*x*log(a + b*x**2) - 2*d*p*x + d*x*log(c) + e*p*x**2*log(a + b*x**2)/2 -
e*p*x**2/2 + e*x**2*log(c)/2, Ne(b, 0)), ((d*x + e*x**2/2)*log(a**p*c), Tru
e))
```

3.187 $\int \log \left(c \left(a+b x^2\right)^p\right) \, dx$

Optimal. Leaf size=45

$$x \log \left(c \left(a+b x^2\right)^p\right)+\frac{2 \sqrt{a} p \tan ^{-1}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{\sqrt{b}}-2 p x$$

```
[Out] -2*p*x+x*ln(c*(b*x^2+a)^p)+2*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2448, 321, 205}

$$x \log \left(c \left(a+b x^2\right)^p\right)+\frac{2 \sqrt{a} p \tan ^{-1}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{\sqrt{b}}-2 p x$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p],x]
```

```
[Out] -2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + b
*x^2)^p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \log \left(c \left(a+b x^2\right)^p\right) \, dx &= x \log \left(c \left(a+b x^2\right)^p\right)-(2 b p) \int \frac{x^2}{a+b x^2} \, dx\\
&= -2 p x+x \log \left(c \left(a+b x^2\right)^p\right)+(2 a p) \int \frac{1}{a+b x^2} \, dx\\
&= -2 p x+\frac{2 \sqrt{a} p \tan ^{-1}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{\sqrt{b}}+x \log \left(c \left(a+b x^2\right)^p\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 45, normalized size = 1.00

$$x \log \left(c \left(a+b x^2\right)^p\right)+\frac{2 \sqrt{a} p \tan ^{-1}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{\sqrt{b}}-2 p x$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p],x]
[Out] -2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + b
*x^2)^p]
```

fricas [A] time = 0.45, size = 107, normalized size = 2.38

$$\left[px\log\left(bx^2+a\right)+p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-2\,px+x\log(c),px\log\left(bx^2+a\right)+2\,p\sqrt{\frac{a}{b}}\arctan\left(\frac{bx}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] [p*x*log(b*x^2 + a) + p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^
2 + a)) - 2*p*x + x*log(c), p*x*log(b*x^2 + a) + 2*p*sqrt(a/b)*arctan(b*x*s
qrt(a/b)/a) - 2*p*x + x*log(c)]
```

giac [A] time = 0.16, size = 41, normalized size = 0.91

$$px\log\left(bx^2+a\right)+\frac{2\,ap\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\left(2\,p-\log(c)\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] p*x*log(b*x^2 + a) + 2*a*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - (2*p - log(c))
*x
```

maple [A] time = 0.07, size = 38, normalized size = 0.84

$$\frac{2ap\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-2px+x\ln\left(c\left(b\,x^2+a\right)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p),x)
[Out] x*ln(c*(b*x^2+a)^p)-2*p*x+2*p*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 0.99, size = 45, normalized size = 1.00

$$2\,bp\left(\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}-\frac{x}{b}\right)+x\log\left(\left(bx^2+a\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] 2*b*p*(a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - x/b) + x*log((b*x^2 + a)^p*c
)
```

mupad [B] time = 0.08, size = 37, normalized size = 0.82

$$x\,\ln\left(c\left(b\,x^2+a\right)^p\right)-2\,p\,x+\frac{2\,\sqrt{a}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p),x)
```

```
[Out] x*log(c*(a + b*x^2)^p) - 2*p*x + (2*a^(1/2)*p*atan((b^(1/2)*x)/a^(1/2)))/b^
(1/2)
```

sympy [A] time = 5.84, size = 90, normalized size = 2.00

$$\begin{cases} \frac{i\sqrt{a}\,p\log\left(a+bx^2\right)}{b\sqrt{\frac{1}{b}}} - \frac{2i\sqrt{a}\,p\log\left(-i\sqrt{a}\,\sqrt{\frac{1}{b}}+x\right)}{b\sqrt{\frac{1}{b}}} + px\log\left(a+bx^2\right) - 2px + x\log\left(c\right) & \text{for}\ b \neq 0 \\ x\log\left(a^p c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((I*sqrt(a)*p*log(a + b*x**2)/(b*sqrt(1/b)) - 2*I*sqrt(a)*p*log(-I
*sqrt(a)*sqrt(1/b) + x)/(b*sqrt(1/b)) + p*x*log(a + b*x**2) - 2*p*x + x*log
(c), Ne(b, 0)), (x*log(a**p*c), True))
```

3.188 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=201

$$\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e}-\frac{p\log(d$$

```
[Out] ln(e*x+d)*ln(c*(b*x^2+a)^p)/e-p*ln(e*x+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-
a)^(1/2)+d*b^(1/2)))/e-p*ln(e*x+d)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1
/2)+d*b^(1/2)))/e-p*polylog(2,(e*x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/e-
p*polylog(2,(e*x+d)*b^(1/2)/(e*(-a)^(1/2)+d*b^(1/2)))/e
```

Rubi [A] time = 0.27, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2462, 260, 2416, 2394, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{e(\sqrt{-}}{\sqrt{-a}}\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])
/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx = \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{(2bp)\int \frac{x\log(d+ex)}{a+bx^2}\,dx}{e}$$

$$= \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{(2bp)\int \left(-\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{e}$$

$$= \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e} + \frac{\left(\sqrt{b}p\right)\int \frac{\log(d+ex)}{\sqrt{-a}-\sqrt{b}x}\,dx}{e} - \frac{\left(\sqrt{b}p\right)\int \frac{\log(d+ex)}{\sqrt{-a}+\sqrt{b}x}\,dx}{e}$$

$$= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log}{e}$$

$$= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log}{e}$$

$$= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log}{e}$$

Mathematica [A] time = 0.08, size = 201, normalized size = 1.00

$$\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{p\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e} - \frac{p\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e} - \frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e} - \frac{p\log(d+e}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])
/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

maple [C] time = 0.38, size = 366, normalized size = 1.82

$$-\frac{i\pi \operatorname{csgn}(ic)\operatorname{csgn}\left(i\left(bx^2+a\right)^p\right)\operatorname{csgn}\left(ic\left(bx^2+a\right)^p\right)\ln(ex+d)}{2e}+\frac{i\pi \operatorname{csgn}(ic)\operatorname{csgn}\left(ic\left(bx^2+a\right)^p\right)^2\ln(ex+d)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/(e*x+d),x)
```

```
[Out] ln(e*x+d)/e*ln((b*x^2+a)^p)-p/e*ln(e*x+d)*ln((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)
/(e*(-a*b)^(1/2)+b*d))-p/e*ln(e*x+d)*ln((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(
-a*b)^(1/2)-b*d))-p/e*dilog((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+
b*d))-p/e*dilog((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d))+1/2*I*
ln(e*x+d)/e*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/2*I*ln(e*x+d)/
e*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/2*I*ln(e*x+d)/e*
Pi*csgn(I*c*(b*x^2+a)^p)^3+1/2*I*ln(e*x+d)/e*Pi*csgn(I*c*(b*x^2+a)^p)^2*csg
n(I*c)+1/e*ln(c)*ln(e*x+d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/(d + e*x),x)
```

```
[Out] int(log(c*(a + b*x^2)^p)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)/(d + e*x), x)
```

3.189 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{(d+ex)^2} dx$

Optimal. Leaf size=119

$$-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}+\frac{bdp\log\left(a+bx^2\right)}{e\left(ae^2+bd^2\right)}-\frac{2bdp\log(d+ex)}{e\left(ae^2+bd^2\right)}+\frac{2\sqrt{a}\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{ae^2+bd^2}$$

```
[Out] -2*b*d*p*ln(e*x+d)/e/(a*e^2+b*d^2)+b*d*p*ln(b*x^2+a)/e/(a*e^2+b*d^2)-ln(c*(
b*x^2+a)^p)/e/(e*x+d)+2*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)*b^(1/2)/(a*e^2+
b*d^2)
```

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2463, 801, 635, 205, 260}

$$-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}+\frac{bdp\log\left(a+bx^2\right)}{e\left(ae^2+bd^2\right)}-\frac{2bdp\log(d+ex)}{e\left(ae^2+bd^2\right)}+\frac{2\sqrt{a}\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{ae^2+bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(d + e*x)^2,x]
```

```
[Out] (2*Sqrt[a]*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b*d^2 + a*e^2) - (2*b*d*
p*Log[d + e*x])/(e*(b*d^2 + a*e^2)) + (b*d*p*Log[a + b*x^2])/(e*(b*d^2 + a*
e^2)) - Log[c*(a + b*x^2)^p]/(e*(d + e*x))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{(d+ex)^2}\,dx = -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)} + \frac{(2bp)\int \frac{x}{(d+ex)\left(a+bx^2\right)}\,dx}{e}$$
$$= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)} + \frac{(2bp)\int \left(-\frac{de}{\left(bd^2+ae^2\right)(d+ex)} + \frac{ae+bdx}{\left(bd^2+ae^2\right)\left(a+bx^2\right)}\right)dx}{e}$$
$$= -\frac{2bdp\log(d+ex)}{e\left(bd^2+ae^2\right)} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)} + \frac{(2bp)\int \frac{ae+bdx}{a+bx^2}\,dx}{e\left(bd^2+ae^2\right)}$$
$$= -\frac{2bdp\log(d+ex)}{e\left(bd^2+ae^2\right)} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)} + \frac{(2abp)\int \frac{1}{a+bx^2}\,dx}{bd^2+ae^2} + \frac{\left(2b^2dp\right)\int \frac{x}{a+bx^2}\,dx}{e\left(bd^2+ae^2\right)}$$
$$= \frac{2\sqrt{a}\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{bd^2+ae^2} - \frac{2bdp\log(d+ex)}{e\left(bd^2+ae^2\right)} + \frac{bdp\log\left(a+bx^2\right)}{e\left(bd^2+ae^2\right)} - \frac{\log\left(c\left(a+bx^2\right)^p\right.}{e(d+ex)}$$

Mathematica [A] time = 0.08, size = 137, normalized size = 1.15

$$\frac{-bd^2\log\left(c\left(a+bx^2\right)^p\right) - ae^2\log\left(c\left(a+bx^2\right)^p\right) + bd^2p\log\left(a+bx^2\right) + bdepx\log\left(a+bx^2\right) + 2\sqrt{a}\sqrt{b}\,ep(d+e}{e(d+ex)\left(ae^2+bd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(d + e*x)^2,x]
```

```
[Out] (2*Sqrt[a]*Sqrt[b]*e*p*(d + e*x)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] - 2*b*d*p*(d +
 e*x)*Log[d + e*x] + b*d^2*p*Log[a + b*x^2] + b*d*e*p*x*Log[a + b*x^2] - b*
d^2*Log[c*(a + b*x^2)^p] - a*e^2*Log[c*(a + b*x^2)^p])/(e*(b*d^2 + a*e^2)*(
d + e*x))
```

fricas [A] time = 0.47, size = 261, normalized size = 2.19

$$\left[\frac{\left(e^2px+dep\right)\sqrt{-ab}\,\log\left(\frac{bx^2+2\sqrt{-ab}\,x-a}{bx^2+a}\right) + \left(bdepx-ae^2p\right)\log\left(bx^2+a\right) - 2\left(bdepx+bd^2p\right)\log\left(ex+d\right) - \left(bd^2\right.}{bd^3e+ade^3+\left(bd^2e^2+ae^4\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [((e^2*p*x + d*e*p)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) + (b*d*e*p*x - a*e^2*p)*log(b*x^2 + a) - 2*(b*d*e*p*x + b*d^2*p)*log(e*x
+ d) - (b*d^2 + a*e^2)*log(c))/(b*d^3*e + a*d*e^3 + (b*d^2*e^2 + a*e^4)*x),
 (2*(e^2*p*x + d*e*p)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (b*d*e*p*x - a*e^2*
p)*log(b*x^2 + a) - 2*(b*d*e*p*x + b*d^2*p)*log(e*x + d) - (b*d^2 + a*e^2)*
log(c))/(b*d^3*e + a*d*e^3 + (b*d^2*e^2 + a*e^4)*x)]
```

giac [A] time = 0.22, size = 158, normalized size = 1.33

$$\frac{bdp\log\left(bx^2+a\right)}{bd^2e+ae^3} + \frac{2\,abp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(bd^2+ae^2\right)\sqrt{ab}} - \frac{2\,bdpxe\log\left(xe+d\right) + bd^2p\log\left(bx^2+a\right) + 2\,bd^2p\log\left(xe+d\right) + ape^2}{bd^2xe^2+bd^3e+axe^4+ade^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] b*d*p*log(b*x^2 + a)/(b*d^2*e + a*e^3) + 2*a*b*p*arctan(b*x/sqrt(a*b))/((b*
d^2 + a*e^2)*sqrt(a*b)) - (2*b*d*p*x*e*log(x*e + d) + b*d^2*p*log(b*x^2 + a
) + 2*b*d^2*p*log(x*e + d) + a*p*e^2*log(b*x^2 + a) + b*d^2*log(c) + a*e^2*
log(c))/(b*d^2*x*e^2 + b*d^3*e + a*x*e^4 + a*d*e^3)
```

maple [C] time = 0.61, size = 1233, normalized size = 10.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/(e*x+d)^2,x)
```

```
[Out] -1/e/(e*x+d)*ln((b*x^2+a)^p)-1/2*b*(I*Pi*b*d^2*csgn(I*(b*x^2+a)^p)*csgn(I*c
*(b*x^2+a)^p)^2+I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*e^2+I*Pi
*b*d^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*
c*(b*x^2+a)^p)*csgn(I*c)*e^2+I*Pi*a*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*e^2-I
*Pi*b*d^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*b*d^2*cs
gn(I*c*(b*x^2+a)^p)^3-I*Pi*a*csgn(I*c*(b*x^2+a)^p)^3*e^2-2*(-a*b)^(1/2)*ln(
(-a^2*b*e^3+7*a*b^2*d^2*e+5*(-a*b)^(1/2)*a*b*d*e^2-3*(-a*b)^(1/2)*b^2*d^3)*
x-5*a^2*b*d*e^2-(-a*b)^(1/2)*a^2*e^3+7*(-a*b)^(1/2)*a*b*d^2*e+3*a*b^2*d^3)*
e^2*p*x-2*ln((-a^2*b*e^3+7*a*b^2*d^2*e+5*(-a*b)^(1/2)*a*b*d*e^2-3*(-a*b)^(1
/2)*b^2*d^3)*x-5*a^2*b*d*e^2-(-a*b)^(1/2)*a^2*e^3+7*(-a*b)^(1/2)*a*b*d^2*e+
3*a*b^2*d^3)*b*d*e*p*x+2*(-a*b)^(1/2)*ln((-a^2*b*e^3+7*a*b^2*d^2*e-5*(-a*b)
^(1/2)*a*b*d*e^2+3*(-a*b)^(1/2)*b^2*d^3)*x-5*a^2*b*d*e^2+(-a*b)^(1/2)*a^2*e
^3-7*(-a*b)^(1/2)*a*b*d^2*e+3*a*b^2*d^3)*e^2*p*x-2*ln((-a^2*b*e^3+7*a*b^2*d
^2*e-5*(-a*b)^(1/2)*a*b*d*e^2+3*(-a*b)^(1/2)*b^2*d^3)*x-5*a^2*b*d*e^2+(-a*b
)^(1/2)*a^2*e^3-7*(-a*b)^(1/2)*a*b*d^2*e+3*a*b^2*d^3)*b*d*e*p*x+4*ln(e*x+d)
*b*d*e*p*x-2*(-a*b)^(1/2)*ln((-a^2*b*e^3+7*a*b^2*d^2*e+5*(-a*b)^(1/2)*a*b*d
*e^2-3*(-a*b)^(1/2)*b^2*d^3)*x-5*a^2*b*d*e^2-(-a*b)^(1/2)*a^2*e^3+7*(-a*b)^
(1/2)*a*b*d^2*e+3*a*b^2*d^3)*d*e*p-2*ln((-a^2*b*e^3+7*a*b^2*d^2*e+5*(-a*b)^
(1/2)*a*b*d*e^2-3*(-a*b)^(1/2)*b^2*d^3)*x-5*a^2*b*d*e^2-(-a*b)^(1/2)*a^2*e^
3+7*(-a*b)^(1/2)*a*b*d^2*e+3*a*b^2*d^3)*b*d^2*p+2*(-a*b)^(1/2)*ln((-a^2*b*e
^3+7*a*b^2*d^2*e-5*(-a*b)^(1/2)*a*b*d*e^2+3*(-a*b)^(1/2)*b^2*d^3)*x-5*a^2*b
*d*e^2+(-a*b)^(1/2)*a^2*e^3-7*(-a*b)^(1/2)*a*b*d^2*e+3*a*b^2*d^3)*d*e*p-2*l
n((-a^2*b*e^3+7*a*b^2*d^2*e-5*(-a*b)^(1/2)*a*b*d*e^2+3*(-a*b)^(1/2)*b^2*d^3
)*x-5*a^2*b*d*e^2+(-a*b)^(1/2)*a^2*e^3-7*(-a*b)^(1/2)*a*b*d^2*e+3*a*b^2*d^3
)*b*d^2*p+4*ln(e*x+d)*b*d^2*p+2*ln(c)*a*e^2+2*b*d^2*ln(c))/(e*x+d)/(b*d-(-a
*b)^(1/2)*e)/(b*d+(-a*b)^(1/2)*e)/e
```

maxima [A] time = 1.51, size = 108, normalized size = 0.91

$$\frac{\left(\frac{2\,ae\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(bd^2+ae^2\right)\sqrt{ab}}+\frac{d\log\left(bx^2+a\right)}{bd^2+ae^2}-\frac{2\,d\log\left(ex+d\right)}{bd^2+ae^2}\right)bp}{e}-\frac{\log\left(\left(bx^2+a\right)^p c\right)}{(ex+d)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] (2*a*e*arctan(b*x/sqrt(a*b))/((b*d^2 + a*e^2)*sqrt(a*b)) + d*log(b*x^2 + a)
/(b*d^2 + a*e^2) - 2*d*log(e*x + d)/(b*d^2 + a*e^2))*b*p/e - log((b*x^2 + a
)^p*c)/((e*x + d)*e)
```

mupad [B] time = 1.26, size = 337, normalized size = 2.83

$$\frac{\ln\left(\frac{4\,b^3\,p^2\,x}{e}-\frac{p\,\left(b\,d+e\,\sqrt{-a\,b}\right)\,\left(2\,a\,b^2\,e\,p+2\,b^3\,d\,p\,x-\frac{2\,b^2\,e\,p\,\left(b\,d+e\,\sqrt{-a\,b}\right)\,\left(-b\,x\,d^2+4\,a\,d\,e+3\,a\,x\,e^2\right)}{b\,d^2\,e+a\,e^3}\right)}{b\,d^2\,e+a\,e^3}\right)\,\left(b\,d\,p+e\,p\,\sqrt{-a\,b}\right)}{b\,d^2\,e+a\,e^3}-\frac{\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{e\,\left(d+e\,x\right)}\cdot$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/(d + e*x)^2,x)
```

```
[Out] (log((4*b^3*p^2*x)/e - (p*(b*d + e*(-a*b)^(1/2))*(2*a*b^2*e*p + 2*b^3*d*p*x
 - (2*b^2*e*p*(b*d + e*(-a*b)^(1/2))*(4*a*d*e + 3*a*e^2*x - b*d^2*x))/(a*e^
3 + b*d^2*e)))/(a*e^3 + b*d^2*e))*(b*d*p + e*p*(-a*b)^(1/2)))/(a*e^3 + b*d^
2*e) - log(c*(a + b*x^2)^p)/(e*(d + e*x)) + (log((4*b^3*p^2*x)/e - (p*(b*d
- e*(-a*b)^(1/2))*(2*a*b^2*e*p + 2*b^3*d*p*x - (2*b^2*e*p*(b*d - e*(-a*b)^(
1/2))*(4*a*d*e + 3*a*e^2*x - b*d^2*x))/(a*e^3 + b*d^2*e)))/(a*e^3 + b*d^2*e
))*(b*d*p - e*p*(-a*b)^(1/2)))/(a*e^3 + b*d^2*e) - (2*b*d*p*log(d + e*x))/(
a*e^3 + b*d^2*e)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.190 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{(d+ex)^3}\,dx$

Optimal. Leaf size=174

$$\frac{2\sqrt{a}\,b^{3/2}dp\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\left(ae^2+bd^2\right)^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2}+\frac{bp\left(bd^2-ae^2\right)\log\left(a+bx^2\right)}{2e\left(ae^2+bd^2\right)^2}+\frac{bdp}{e(d+ex)\left(ae^2+bd^2\right)}-\frac{bp\left(bd^2-ae^2\right)\cdots}{e\left(ae^2+\cdots\right.}$$

```
[Out] b*d*p/e/(a*e^2+b*d^2)/(e*x+d)-b*(-a*e^2+b*d^2)*p*ln(e*x+d)/e/(a*e^2+b*d^2)^
2+1/2*b*(-a*e^2+b*d^2)*p*ln(b*x^2+a)/e/(a*e^2+b*d^2)^2-1/2*ln(c*(b*x^2+a)^p
)/e/(e*x+d)^2+2*b^(3/2)*d*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/(a*e^2+b*d^2)
^2
```

Rubi [A] time = 0.14, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2463, 801, 635, 205, 260}

$$\frac{2\sqrt{a}\,b^{3/2}dp\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\left(ae^2+bd^2\right)^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2}+\frac{bp\left(bd^2-ae^2\right)\log\left(a+bx^2\right)}{2e\left(ae^2+bd^2\right)^2}+\frac{bdp}{e(d+ex)\left(ae^2+bd^2\right)}-\frac{bp\left(bd^2-ae^2\right)\cdots}{e\left(ae^2+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(d + e*x)^3,x]
```

```
[Out] (b*d*p)/(e*(b*d^2 + a*e^2)*(d + e*x)) + (2*Sqrt[a]*b^(3/2)*d*p*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(b*d^2 + a*e^2)^2 - (b*(b*d^2 - a*e^2)*p*Log[d + e*x])/(e*
(b*d^2 + a*e^2)^2) + (b*(b*d^2 - a*e^2)*p*Log[a + b*x^2])/(2*e*(b*d^2 + a*e
^2)^2) - Log[c*(a + b*x^2)^p]/(2*e*(d + e*x)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
```

```
&& (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{(d+ex)^3}\,dx = -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2}+\frac{(bp)\int \frac{x}{(d+ex)^2\left(a+bx^2\right)}\,dx}{e}$$

$$= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2}+\frac{(bp)\int\left(-\frac{de}{\left(bd^2+ae^2\right)(d+ex)^2}+\frac{e\left(-bd^2+ae^2\right)}{\left(bd^2+ae^2\right)^2(d+ex)}+\frac{b\left(2ade+\left(bd^2-ae^2\right)x\right)}{\left(bd^2+ae^2\right)^2\left(a+bx^2\right)}\right)}{e}$$

$$= \frac{bdp}{e\left(bd^2+ae^2\right)(d+ex)}-\frac{b\left(bd^2-ae^2\right)p\log(d+ex)}{e\left(bd^2+ae^2\right)^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2}+\frac{\left(b^2p\right)\int}{e(}$$

$$= \frac{bdp}{e\left(bd^2+ae^2\right)(d+ex)}-\frac{b\left(bd^2-ae^2\right)p\log(d+ex)}{e\left(bd^2+ae^2\right)^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2}+\frac{\left(2ab^2dp\right.}{\left(bd^2\right.}$$

$$= \frac{bdp}{e\left(bd^2+ae^2\right)(d+ex)}+\frac{2\sqrt{a}\,b^{3/2}dp\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\left(bd^2+ae^2\right)^2}-\frac{b\left(bd^2-ae^2\right)p\log(d+ex)}{e\left(bd^2+ae^2\right)^2}+\frac{b\,(}{}$$

Mathematica [A] time = 0.57, size = 217, normalized size = 1.25

$$\frac{\frac{bp(d+ex)\left((d+ex)\left(\sqrt{-a}\,bd^2+2a\sqrt{b}\,de+(-a)^{3/2}e^2\right)\log\left(\sqrt{-a}-\sqrt{b}\,x\right)+(d+ex)\left(\sqrt{-a}\,bd^2-2a\sqrt{b}\,de+(-a)^{3/2}e^2\right)\log\left(\sqrt{-a}+\sqrt{b}\,x\right)+2\sqrt{-a}\left(-(d+ex)\left(bd^2-ae^2\right.\right.\right.}{\sqrt{-a}\left(ae^2+bd^2\right)^2}}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(d + e*x)^3,x]
```

```
[Out] ((b*p*(d + e*x)*((Sqrt[-a]*b*d^2 + 2*a*Sqrt[b]*d*e + (-a)^(3/2)*e^2)*(d + e
*x)*Log[Sqrt[-a] - Sqrt[b]*x] + (Sqrt[-a]*b*d^2 - 2*a*Sqrt[b]*d*e + (-a)^(3
/2)*e^2)*(d + e*x)*Log[Sqrt[-a] + Sqrt[b]*x] + 2*Sqrt[-a]*(b*d^3 + a*d*e^2
- (b*d^2 - a*e^2)*(d + e*x)*Log[d + e*x])))/(Sqrt[-a]*(b*d^2 + a*e^2)^2) -
Log[c*(a + b*x^2)^p])/(2*e*(d + e*x)^2)
```

fricas [B] time = 0.55, size = 744, normalized size = 4.28

$$\left[\frac{2\left(b^2d^3e+abde^3\right)px+2\left(bde^3px^2+2\,bd^2e^2px+bd^3ep\right)\sqrt{-ab}\,\log\left(\frac{bx^2+2\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\left(b^2d^4+abd^2e^2\right)p+\left(\left(b^2\right.\right.}{2\left(b^2d^6\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*d^3*e + a*b*d*e^3)*p*x + 2*(b*d*e^3*p*x^2 + 2*b*d^2*e^2*p*x +
b*d^3*e*p)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(b^
2*d^4 + a*b*d^2*e^2)*p + ((b^2*d^2*e^2 - a*b*e^4)*p*x^2 + 2*(b^2*d^3*e - a*
b*d*e^3)*p*x - (3*a*b*d^2*e^2 + a^2*e^4)*p)*log(b*x^2 + a) - 2*((b^2*d^2*e^
2 - a*b*e^4)*p*x^2 + 2*(b^2*d^3*e - a*b*d*e^3)*p*x + (b^2*d^4 - a*b*d^2*e^2
)*p)*log(e*x + d) - (b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4)*log(c))/(b^2*d^6*e
+ 2*a*b*d^4*e^3 + a^2*d^2*e^5 + (b^2*d^4*e^3 + 2*a*b*d^2*e^5 + a^2*e^7)*x^2
 + 2*(b^2*d^5*e^2 + 2*a*b*d^3*e^4 + a^2*d*e^6)*x), 1/2*(2*(b^2*d^3*e + a*b*
```

```
d*e^3)*p*x + 4*(b*d*e^3*p*x^2 + 2*b*d^2*e^2*p*x + b*d^3*e*p)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a) + 2*(b^2*d^4 + a*b*d^2*e^2)*p + ((b^2*d^2*e^2 - a*b*e^4)*
p*x^2 + 2*(b^2*d^3*e - a*b*d*e^3)*p*x - (3*a*b*d^2*e^2 + a^2*e^4)*p)*log(b*
x^2 + a) - 2*((b^2*d^2*e^2 - a*b*e^4)*p*x^2 + 2*(b^2*d^3*e - a*b*d*e^3)*p*x
 + (b^2*d^4 - a*b*d^2*e^2)*p)*log(e*x + d) - (b^2*d^4 + 2*a*b*d^2*e^2 + a^2
*e^4)*log(c))/(b^2*d^6*e + 2*a*b*d^4*e^3 + a^2*d^2*e^5 + (b^2*d^4*e^3 + 2*a
*b*d^2*e^5 + a^2*e^7)*x^2 + 2*(b^2*d^5*e^2 + 2*a*b*d^3*e^4 + a^2*d*e^6)*x)]
```

giac [B] time = 0.22, size = 420, normalized size = 2.41

$$\frac{2\,ab^2dp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^2d^4+2\,abd^2e^2+a^2e^4\right)\sqrt{ab}}+\frac{\left(b^2d^2p-abpe^2\right)\log\left(bx^2+a\right)}{2\left(b^2d^4e+2\,abd^2e^3+a^2e^5\right)}-\frac{2\,b^2d^2px^2e^2\log\left(xe+d\right)+4\,b^2d^3pxe\log\left(xe+d\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 2*a*b^2*d*p*arctan(b*x/sqrt(a*b))/((b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4)*sqrt
(a*b)) + 1/2*(b^2*d^2*p - a*b*p*e^2)*log(b*x^2 + a)/(b^2*d^4*e + 2*a*b*d^2*
e^3 + a^2*e^5) - 1/2*(2*b^2*d^2*p*x^2*e^2*log(x*e + d) + 4*b^2*d^3*p*x*e*lo
g(x*e + d) - 2*b^2*d^3*p*x*e + b^2*d^4*p*log(b*x^2 + a) + 2*b^2*d^4*p*log(x
*e + d) - 2*b^2*d^4*p + 2*a*b*d^2*p*e^2*log(b*x^2 + a) - 2*a*b*p*x^2*e^4*lo
g(x*e + d) - 4*a*b*d*p*x*e^3*log(x*e + d) - 2*a*b*d^2*p*e^2*log(x*e + d) +
b^2*d^4*log(c) - 2*a*b*d*p*x*e^3 - 2*a*b*d^2*p*e^2 + 2*a*b*d^2*e^2*log(c) +
 a^2*p*e^4*log(b*x^2 + a) + a^2*e^4*log(c))/(b^2*d^4*x^2*e^3 + 2*b^2*d^5*x*
e^2 + b^2*d^6*e + 2*a*b*d^2*x^2*e^5 + 4*a*b*d^3*x*e^4 + 2*a*b*d^4*e^3 + a^2
*x^2*e^7 + 2*a^2*d*x*e^6 + a^2*d^2*e^5)
```

maple [C] time = 0.92, size = 2684, normalized size = 15.43

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/(e*x+d)^3,x)
```

```
[Out] -1/2/e/(e*x+d)^2*ln((b*x^2+a)^p)+1/4*(-2*I*Pi*a*b*d^2*e^2*csgn(I*(b*x^2+a)^
p)*csgn(I*c*(b*x^2+a)^p)^2-2*ln(c)*b^2*d^4-2*ln(c)*a^2*e^4+4*b^2*d^4*p+2*su
m(_R*ln(((3*a^3*e^8+5*a^2*b*d^2*e^6+a*b^2*d^4*e^4-b^3*d^6*e^2)*_R^2+(3*a^2*
b*e^5*p+2*a*b^2*d^2*e^3*p-b^3*d^4*e*p)*_R+2*b^3*d^2*p^2)*x+(4*a^3*d*e^7+8*a
^2*b*d^3*e^5+4*a*b^2*d^5*e^3)*_R^2+2*a*b^2*d*e*p^2),_R=RootOf((a^2*e^6+2*a*
b*d^2*e^4+b^2*d^4*e^2)*_Z^2+(2*a*b*e^3*p-2*b^2*d^2*e*p)*_Z+b^2*p^2))*a^2*e^
7*x^2+2*sum(_R*ln(((3*a^3*e^8+5*a^2*b*d^2*e^6+a*b^2*d^4*e^4-b^3*d^6*e^2)*_R
^2+(3*a^2*b*e^5*p+2*a*b^2*d^2*e^3*p-b^3*d^4*e*p)*_R+2*b^3*d^2*p^2)*x+(4*a^3
*d*e^7+8*a^2*b*d^3*e^5+4*a*b^2*d^5*e^3)*_R^2+2*a*b^2*d*e*p^2),_R=RootOf((a^
2*e^6+2*a*b*d^2*e^4+b^2*d^4*e^2)*_Z^2+(2*a*b*e^3*p-2*b^2*d^2*e*p)*_Z+b^2*p^
2))*a^2*d^2*e^5+2*sum(_R*ln(((3*a^3*e^8+5*a^2*b*d^2*e^6+a*b^2*d^4*e^4-b^3*d
^6*e^2)*_R^2+(3*a^2*b*e^5*p+2*a*b^2*d^2*e^3*p-b^3*d^4*e*p)*_R+2*b^3*d^2*p^2
)*x+(4*a^3*d*e^7+8*a^2*b*d^3*e^5+4*a*b^2*d^5*e^3)*_R^2+2*a*b^2*d*e*p^2),_R=
RootOf((a^2*e^6+2*a*b*d^2*e^4+b^2*d^4*e^2)*_Z^2+(2*a*b*e^3*p-2*b^2*d^2*e*p)
*_Z+b^2*p^2))*b^2*d^6*e-4*ln(e*x+d)*b^2*d^4*p+4*a*d^2*b*p*e^2+I*Pi*a^2*e^4*
csgn(I*c*(b*x^2+a)^p)^3+I*Pi*b^2*d^4*csgn(I*c*(b*x^2+a)^p)^3-2*I*Pi*a*b*d^2
*e^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+4*sum(_R*ln(((3*a^3*e^8+5*a^2*b*d^2*
e^6+a*b^2*d^4*e^4-b^3*d^6*e^2)*_R^2+(3*a^2*b*e^5*p+2*a*b^2*d^2*e^3*p-b^3*d^
4*e*p)*_R+2*b^3*d^2*p^2)*x+(4*a^3*d*e^7+8*a^2*b*d^3*e^5+4*a*b^2*d^5*e^3)*_R
^2+2*a*b^2*d*e*p^2),_R=RootOf((a^2*e^6+2*a*b*d^2*e^4+b^2*d^4*e^2)*_Z^2+(2*a
*b*e^3*p-2*b^2*d^2*e*p)*_Z+b^2*p^2))*a*b*d^2*e^5*x^2+8*sum(_R*ln(((3*a^3*e^
8+5*a^2*b*d^2*e^6+a*b^2*d^4*e^4-b^3*d^6*e^2)*_R^2+(3*a^2*b*e^5*p+2*a*b^2*d^
2*e^3*p-b^3*d^4*e*p)*_R+2*b^3*d^2*p^2)*x+(4*a^3*d*e^7+8*a^2*b*d^3*e^5+4*a*b
^2*d^5*e^3)*_R^2+2*a*b^2*d*e*p^2),_R=RootOf((a^2*e^6+2*a*b*d^2*e^4+b^2*d^4*
```

```
e^2)*_Z^2+(2*a*b*e^3*p-2*b^2*d^2*e*p)*_Z+b^2*p^2))*a*b*d^3*e^4*x+4*ln(e*x+d
)*a*b*e^4*p*x^2-4*ln(e*x+d)*b^2*d^2*e^2*p*x^2-8*ln(e*x+d)*b^2*d^3*e*p*x+4*l
n(e*x+d)*a*b*d^2*e^2*p+4*a*d*p*e^3*x*b+2*I*Pi*a*b*d^2*e^2*csgn(I*(b*x^2+a)^
p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+2*sum(_R*ln(((3*a^3*e^8+5*a^2*b*d^2*e^6+
a*b^2*d^4*e^4-b^3*d^6*e^2)*_R^2+(3*a^2*b*e^5*p+2*a*b^2*d^2*e^3*p-b^3*d^4*e*
p)*_R+2*b^3*d^2*p^2)*x+(4*a^3*d*e^7+8*a^2*b*d^3*e^5+4*a*b^2*d^5*e^3)*_R^2+2
*a*b^2*d*e*p^2),_R=RootOf((a^2*e^6+2*a*b*d^2*e^4+b^2*d^4*e^2)*_Z^2+(2*a*b*e
^3*p-2*b^2*d^2*e*p)*_Z+b^2*p^2))*b^2*d^4*e^3*x^2+4*sum(_R*ln(((3*a^3*e^8+5*
a^2*b*d^2*e^6+a*b^2*d^4*e^4-b^3*d^6*e^2)*_R^2+(3*a^2*b*e^5*p+2*a*b^2*d^2*e^
3*p-b^3*d^4*e*p)*_R+2*b^3*d^2*p^2)*x+(4*a^3*d*e^7+8*a^2*b*d^3*e^5+4*a*b^2*d
^5*e^3)*_R^2+2*a*b^2*d*e*p^2),_R=RootOf((a^2*e^6+2*a*b*d^2*e^4+b^2*d^4*e^2)
*_Z^2+(2*a*b*e^3*p-2*b^2*d^2*e*p)*_Z+b^2*p^2))*a^2*d*e^6*x+4*sum(_R*ln(((3*
a^3*e^8+5*a^2*b*d^2*e^6+a*b^2*d^4*e^4-b^3*d^6*e^2)*_R^2+(3*a^2*b*e^5*p+2*a*
b^2*d^2*e^3*p-b^3*d^4*e*p)*_R+2*b^3*d^2*p^2)*x+(4*a^3*d*e^7+8*a^2*b*d^3*e^5
+4*a*b^2*d^5*e^3)*_R^2+2*a*b^2*d*e*p^2),_R=RootOf((a^2*e^6+2*a*b*d^2*e^4+b^
2*d^4*e^2)*_Z^2+(2*a*b*e^3*p-2*b^2*d^2*e*p)*_Z+b^2*p^2))*b^2*d^5*e^2*x+4*su
m(_R*ln(((3*a^3*e^8+5*a^2*b*d^2*e^6+a*b^2*d^4*e^4-b^3*d^6*e^2)*_R^2+(3*a^2*
b*e^5*p+2*a*b^2*d^2*e^3*p-b^3*d^4*e*p)*_R+2*b^3*d^2*p^2)*x+(4*a^3*d*e^7+8*a
^2*b*d^3*e^5+4*a*b^2*d^5*e^3)*_R^2+2*a*b^2*d*e*p^2),_R=RootOf((a^2*e^6+2*a*
b*d^2*e^4+b^2*d^4*e^2)*_Z^2+(2*a*b*e^3*p-2*b^2*d^2*e*p)*_Z+b^2*p^2))*a*b*d^
4*e^3-4*ln(c)*a*b*d^2*e^2+4*d^3*p*x*b^2*e+I*Pi*b^2*d^4*csgn(I*(b*x^2+a)^p)*
csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+8*ln(e*x+d)*a*b*d*e^3*p*x-I*Pi*b^2*d^4*csgn
(I*c*(b*x^2+a)^p)^2*csgn(I*c)-I*Pi*a^2*e^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*
x^2+a)^p)^2-I*Pi*a^2*e^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-I*Pi*b^2*d^4*csg
n(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+2*I*Pi*a*b*d^2*e^2*csgn(I*c*(b*x^2
+a)^p)^3+I*Pi*a^2*e^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c))/
(e*x+d)^2/e/(a*e^2+b*d^2)^2
```

maxima [A] time = 1.07, size = 206, normalized size = 1.18

$$\frac{\left(\frac{4\,abde\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^2d^4+2\,abd^2e^2+a^2e^4\right)\sqrt{ab}}+\frac{\left(bd^2-ae^2\right)\log\left(bx^2+a\right)}{b^2d^4+2\,abd^2e^2+a^2e^4}-\frac{2\left(bd^2-ae^2\right)\log(ex+d)}{b^2d^4+2\,abd^2e^2+a^2e^4}+\frac{2\,d}{bd^3+ade^2+\left(bd^2e+ae^3\right)x}\right)bp}{2\,e}-\frac{\log\left(\left(bx^2+a\right)^pc\right)}{2\,(ex+d)^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b*d*e*arctan(b*x/sqrt(a*b))/((b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4)*s
qrt(a*b)) + (b*d^2 - a*e^2)*log(b*x^2 + a)/(b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e
^4) - 2*(b*d^2 - a*e^2)*log(e*x + d)/(b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4) +
2*d/(b*d^3 + a*d*e^2 + (b*d^2*e + a*e^3)*x))*b*p/e - 1/2*log((b*x^2 + a)^p*
c)/((e*x + d)^2*e)
```

mupad [B] time = 0.98, size = 272, normalized size = 1.56

$$\frac{\ln\left(b^2\,x+\sqrt{-a\,b^3}\right)\,\left(b^2\,d^2\,p-a\,b\,e^2\,p+2\,d\,e\,p\,\sqrt{-a\,b^3}\right)}{2\,\left(a^2\,e^5+2\,a\,b\,d^2\,e^3+b^2\,d^4\,e\right)}-\frac{\ln\left(d+e\,x\right)\,\left(b^2\,d^2\,p-a\,b\,e^2\,p\right)}{a^2\,e^5+2\,a\,b\,d^2\,e^3+b^2\,d^4\,e}-\frac{\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{2\,e\,\left(d^2+2\,d\,e\,x+e^2\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/(d + e*x)^3,x)
```

```
[Out] (log(b^2*x + (-a*b^3)^(1/2))*(b^2*d^2*p - a*b*e^2*p + 2*d*e*p*(-a*b^3)^(1/2
)))/(2*(a^2*e^5 + b^2*d^4*e + 2*a*b*d^2*e^3)) - (log(d + e*x)*(b^2*d^2*p -
a*b*e^2*p))/(a^2*e^5 + b^2*d^4*e + 2*a*b*d^2*e^3) - log(c*(a + b*x^2)^p)/(2
*e*(d^2 + e^2*x^2 + 2*d*e*x)) - (log(b^2*x - (-a*b^3)^(1/2))*(a*b*e^2*p - b
^2*d^2*p + 2*d*e*p*(-a*b^3)^(1/2)))/(2*(a^2*e^5 + b^2*d^4*e + 2*a*b*d^2*e^3
)) + (b*d*p)/((d*e + e^2*x)*(a*e^2 + b*d^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

3.191 $\int (d+ex)^3 \log\left(c\left(a+bx^3\right)^p\right)\,dx$

Optimal. Leaf size=320

$$-\frac{\sqrt[3]{a}\,p\left(-6\sqrt[3]{a}\,b^{2/3}d^2e-ae^3+4bd^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{8b^{4/3}}+\frac{\sqrt[3]{a}\,p\left(-6\sqrt[3]{a}\,b^{2/3}d^2e-ae^3+4bd^3\right)\log\left(\sqrt[3]{a}\right.}{4b^{4/3}}$$

```
[Out] -3/4*(-a*e^3+4*b*d^3)*p*x/b-9/4*d^2*e*p*x^2-d*e^2*p*x^3-3/16*e^3*p*x^4+1/4*
a^(1/3)*(4*b*d^3-6*a^(1/3)*b^(2/3)*d^2*e-a*e^3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(
4/3)-1/8*a^(1/3)*(4*b*d^3-6*a^(1/3)*b^(2/3)*d^2*e-a*e^3)*p*ln(a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/b^(4/3)-1/4*d*(-4*a*e^3+b*d^3)*p*ln(b*x^3+a)/b/e+
1/4*(e*x+d)^4*ln(c*(b*x^3+a)^p)/e-1/4*a^(1/3)*(4*b*d^3+6*a^(1/3)*b^(2/3)*d^
2*e-a*e^3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(4
/3)
```

Rubi [A] time = 0.74, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {2463, 1836, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\sqrt[3]{a}\,p\left(-6\sqrt[3]{a}\,b^{2/3}d^2e-ae^3+4bd^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{8b^{4/3}}+\frac{\sqrt[3]{a}\,p\left(-6\sqrt[3]{a}\,b^{2/3}d^2e-ae^3+4bd^3\right)\log\left(\sqrt[3]{a}\right.}{4b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (-3*(4*b*d^3 - a*e^3)*p*x)/(4*b) - (9*d^2*e*p*x^2)/4 - d*e^2*p*x^3 - (3*e^3
*p*x^4)/16 - (Sqrt[3]*a^(1/3)*(4*b*d^3 + 6*a^(1/3)*b^(2/3)*d^2*e - a*e^3)*p
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(4*b^(4/3)) + (a^(1/3)*
(4*b*d^3 - 6*a^(1/3)*b^(2/3)*d^2*e - a*e^3)*p*Log[a^(1/3) + b^(1/3)*x])/(4*
b^(4/3)) - (a^(1/3)*(4*b*d^3 - 6*a^(1/3)*b^(2/3)*d^2*e - a*e^3)*p*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(8*b^(4/3)) - (d*(b*d^3 - 4*a*e^3)*p
*Log[a + b*x^3])/(4*b*e) + ((d + e*x)^4*Log[c*(a + b*x^3)^p])/(4*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)^3 \log\left(c\left(a+bx^3\right)^p\right)dx = \frac{(d+ex)^4\log\left(c\left(a+bx^3\right)^p\right)}{4e} - \frac{(3bp)\int \frac{x^2(d+ex)^4}{a+bx^3}dx}{4e}$$

$$= -\frac{3}{16}e^3px^4 + \frac{(d+ex)^4\log\left(c\left(a+bx^3\right)^p\right)}{4e} - \frac{(3p)\int \frac{x^2\left(4bd^4+4e\left(4bd^3-ae^3\right)x+24bd^2 \dots}{a+bx^3}}{16e}$$

$$= -de^2px^3 - \frac{3}{16}e^3px^4 + \frac{(d+ex)^4\log\left(c\left(a+bx^3\right)^p\right)}{4e} - \frac{p\int \frac{x^2\left(12bd\left(bd^3-4ae^3\right)+12 \dots}{\dots}}{1 \dots}$$

$$= -de^2px^3 - \frac{3}{16}e^3px^4 + \frac{(d+ex)^4\log\left(c\left(a+bx^3\right)^p\right)}{4e} - \frac{p\int \left(12e\left(4bd^3-ae^3\right) \dots\right.}{\dots}$$

$$= -\frac{3\left(4bd^3-ae^3\right)px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 + \frac{(d+ex)^4\log\left(c\left(a+ \dots\right.\right.}{4e}$$

$$= -\frac{3\left(4bd^3-ae^3\right)px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 + \frac{(d+ex)^4\log\left(c\left(a+ \dots\right.\right.}{4e}$$

$$= -\frac{3\left(4bd^3-ae^3\right)px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 - \frac{d\left(bd^3-4ae^3\right)p\log\left(\dots\right.}{4be}$$

$$= -\frac{3\left(4bd^3-ae^3\right)px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 + \frac{\sqrt[3]{a}\left(4bd^3-6\sqrt[3]{a}\,b^{2/3} \dots\right.}{\dots}$$

$$= -\frac{3\left(4bd^3-ae^3\right)px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 + \frac{\sqrt[3]{a}\left(4bd^3-6\sqrt[3]{a}\,b^{2/3} \dots\right.}{\dots}$$

$$= -\frac{3\left(4bd^3-ae^3\right)px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 - \frac{\sqrt{3}\sqrt[3]{a}\left(4bd^3+6\sqrt[3]{a} \dots\right.}{\dots}$$

Mathematica [C] time = 0.51, size = 264, normalized size = 0.82

$$\frac{\frac{\sqrt[3]{a}ep\left(ae^3-4bd^3\right)\left(\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{2b^{4/3}} + \frac{\sqrt[3]{a}ep\left(4bd^3-ae^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{4/3}} + (d+ex)^4\log\left(c\left(a+bx^3\right)^p \dots\right.}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Log[c*(a + b*x^3)^p],x]
```

```
[Out] ((3*e*(-4*b*d^3 + a*e^3)*p*x)/b - 9*d^2*e^2*p*x^2 - 4*d*e^3*p*x^3 - (3*e^4*
p*x^4)/4 + 9*d^2*e^2*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)] + (
a^(1/3)*e*(4*b*d^3 - a*e^3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) + (a^(1/3)*
e*(-4*b*d^3 + a*e^3)*p*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2*b^(4/3)) - (d*(b*d
^3 - 4*a*e^3)*p*Log[a + b*x^3])/b + (d + e*x)^4*Log[c*(a + b*x^3)^p])/(4*e)
```

fricas [C] time = 11.38, size = 8840, normalized size = 27.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] -1/16*(3*b*e^3*p*x^4 + 16*b*d*e^2*p*x^3 + 36*b*d^2*e*p*x^2 + 12*(4*b*d^3 -
a*e^3)*p*x - 2*(8*a*d*e^2*p/b - 4*(1/2)^(2/3)*(8*a^2*d^2*e^4*p^2/b^2 - (12*
a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(128*a^3*d^3*e^6*p
^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*
d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3
*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)
^(1/3) - (1/2)^(1/3)*(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^
2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d
^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 +
4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3)*(I*sqrt(3) + 1))*b*log(-3/2*(
8*a*d*e^2*p/b - 4*(1/2)^(2/3)*(8*a^2*d^2*e^4*p^2/b^2 - (12*a*b*d^5*e*p^2 +
5*a^2*d^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*
a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*
d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^
2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3) - (1/2)^(
1/3)*(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a
*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9
)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*
p^3 - a^4*e^9*p^3)/b^4)^(1/3)*(I*sqrt(3) + 1))^2*b^3*d^2*e - (64*b^3*d^9 +
168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*p^2*x - 1/2*(16*b^3*d^6 - 5
6*a*b^2*d^3*e^3 + a^2*b*e^6)*(8*a*d*e^2*p/b - 4*(1/2)^(2/3)*(8*a^2*d^2*e^4*
p^2/b^2 - (12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(128
*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/
b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b
^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4
*e^9*p^3)/b^4)^(1/3) - (1/2)^(1/3)*(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^
5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^
3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*
d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3)*(I*sqrt(3) + 1)
)*p - 4*(56*a*b^2*d^7*e^2 + 14*a^2*b*d^4*e^5 - a^3*d*e^8)*p^2) - (24*a*d*e^
2*p + 2*sqrt(3/2)*sqrt(1/2)*b*sqrt((16*(8*a*d*e^2*p/b - 4*(1/2)^(2/3)*(8*a^
2*d^2*e^4*p^2/b^2 - (12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)/b^2)*(-I*sqrt(3)
 + 1)/(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*
a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^
9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6
*p^3 - a^4*e^9*p^3)/b^4)^(1/3) - (1/2)^(1/3)*(128*a^3*d^3*e^6*p^3/b^3 - 24*
(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*
b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 2
4*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3)*(I*sq
rt(3) + 1))*a*b*d*e^2*p - (8*a*d*e^2*p/b - 4*(1/2)^(2/3)*(8*a^2*d^2*e^4*p^2
/b^2 - (12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(128*a^
3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3
 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4
+ (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^
9*p^3)/b^4)^(1/3) - (1/2)^(1/3)*(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e
*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 +
 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6
*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3)*(I*sqrt(3) + 1))^2
*b^2 - 32*(12*a*b*d^5*e - a^2*d^2*e^4)*p^2)/b^2) - (8*a*d*e^2*p/b - 4*(1/2)
^(2/3)*(8*a^2*d^2*e^4*p^2/b^2 - (12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)/b^2)
*(-I*sqrt(3) + 1)/(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d
^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*
e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a
^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3) - (1/2)^(1/3)*(128*a^3*d^3*e^6*p
^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*
d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3
*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)
^(1/3)*(I*sqrt(3) + 1))*b)*log(3/4*(8*a*d*e^2*p/b - 4*(1/2)^(2/3)*(8*a^2*d^
2*e^4*p^2/b^2 - (12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1
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)/(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*
e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a
*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3
 - a^4*e^9*p^3)/b^4)^(1/3) - (1/2)^(1/3)*(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*
a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*
d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^
2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3)*(I*sqrt(3
) + 1))^2*b^3*d^2*e - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 -
a^3*e^9)*p^2*x + 1/4*(16*b^3*d^6 - 56*a*b^2*d^3*e^3 + a^2*b*e^6)*(8*a*d*e^2
p/b - 4(1/2)^(2/3)*(8*a^2*d^2*e^4*p^2/b^2 - (12*a*b*d^5*e*p^2 + 5*a^2*d^2
*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e
*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 +
 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6
*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3) - (1/2)^(1/3)*(128
*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/
b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b
^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4
*e^9*p^3)/b^4)^(1/3)*(I*sqrt(3) + 1))*p + 2*(56*a*b^2*d^7*e^2 + 14*a^2*b*d^
4*e^5 - a^3*d*e^8)*p^2 + 1/2*sqrt(3/2)*sqrt(1/2)*(3*(8*a*d*e^2*p/b - 4*(1/2
)^(2/3)*(8*a^2*d^2*e^4*p^2/b^2 - (12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)/b^2
)*(-I*sqrt(3) + 1)/(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*
d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3
*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*
a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3) - (1/2)^(1/3)*(128*a^3*d^3*e^6*
p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3
*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^
3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4
)^(1/3)*(I*sqrt(3) + 1))*b^3*d^2*e - (16*b^3*d^6 + 16*a*b^2*d^3*e^3 + a^2*b
*e^6)*p)*sqrt((16*(8*a*d*e^2*p/b - 4*(1/2)^(2/3)*(8*a^2*d^2*e^4*p^2/b^2 - (
12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(128*a^3*d^3*e^
6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b
^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*
b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b
^4)^(1/3) - (1/2)^(1/3)*(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5
*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*
b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3
 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3)*(I*sqrt(3) + 1))*a*b*d*e^2
*p - (8*a*d*e^2*p/b - 4*(1/2)^(2/3)*(8*a^2*d^2*e^4*p^2/b^2 - (12*a*b*d^5*e*
p^2 + 5*a^2*d^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(128*a^3*d^3*e^6*p^3/b^3 - 2
4*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*
a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 +
 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3) - (
1/2)^(1/3)*(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*
p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a
^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^
3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3)*(I*sqrt(3) + 1))^2*b^2 - 32*(12*a*b*d^5
*e - a^2*d^2*e^4)*p^2)/b^2)) - (24*a*d*e^2*p - 2*sqrt(3/2)*sqrt(1/2)*b*sqrt
((16*(8*a*d*e^2*p/b - 4*(1/2)^(2/3)*(8*a^2*d^2*e^4*p^2/b^2 - (12*a*b*d^5*e*
p^2 + 5*a^2*d^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(128*a^3*d^3*e^6*p^3/b^3 - 2
4*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*
a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 +
 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3) - (
1/2)^(1/3)*(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*
p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a
^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^
3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3)*(I*sqrt(3) + 1))*a*b*d*e^2*p - (8*a*d*e
^2*p/b - 4*(1/2)^(2/3)*(8*a^2*d^2*e^4*p^2/b^2 - (12*a*b*d^5*e*p^2 + 5*a^2*d
^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5
*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3

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 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d
^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3) - (1/2)^(1/3)*(1
28*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*
p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3
/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a
^4*e^9*p^3)/b^4)^(1/3)*(I*sqrt(3) + 1))^2*b^2 - 32*(12*a*b*d^5*e - a^2*d^2*
e^4)*p^2)/b^2) - (8*a*d*e^2*p/b - 4*(1/2)^(2/3)*(8*a^2*d^2*e^4*p^2/b^2 - (1
2*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(128*a^3*d^3*e^6
*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^
3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b
^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^
4)^(1/3) - (1/2)^(1/3)*(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*
a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b
*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3
 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3)*(I*sqrt(3) + 1))*b)*log(3/4
*(8*a*d*e^2*p/b - 4*(1/2)^(2/3)*(8*a^2*d^2*e^4*p^2/b^2 - (12*a*b*d^5*e*p^2
 + 5*a^2*d^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(128*a^3*d^3*e^6*p^3/b^3 - 24*(1
2*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^
2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*
a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3) - (1/2)
^(1/3)*(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)
*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e
^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^
6*p^3 - a^4*e^9*p^3)/b^4)^(1/3)*(I*sqrt(3) + 1))^2*b^3*d^2*e - (64*b^3*d^9
 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*p^2*x + 1/4*(16*b^3*d^6 -
 56*a*b^2*d^3*e^3 + a^2*b*e^6)*(8*a*d*e^2*p/b - 4*(1/2)^(2/3)*(8*a^2*d^2*e^
4*p^2/b^2 - (12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(1
28*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*
p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3
/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a
^4*e^9*p^3)/b^4)^(1/3) - (1/2)^(1/3)*(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*
d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*
e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^
2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3)*(I*sqrt(3) +
1))*p + 2*(56*a*b^2*d^7*e^2 + 14*a^2*b*d^4*e^5 - a^3*d*e^8)*p^2 - 1/2*sqrt(
3/2)*sqrt(1/2)*(3*(8*a*d*e^2*p/b - 4*(1/2)^(2/3)*(8*a^2*d^2*e^4*p^2/b^2 - (
12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(128*a^3*d^3*e^
6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b
^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*
b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b
^4)^(1/3) - (1/2)^(1/3)*(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5
*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*
b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3
 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3)*(I*sqrt(3) + 1))*b^3*d^2*e
 - (16*b^3*d^6 + 16*a*b^2*d^3*e^3 + a^2*b*e^6)*p)*sqrt((16*(8*a*d*e^2*p/b -
 4*(1/2)^(2/3)*(8*a^2*d^2*e^4*p^2/b^2 - (12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p
^2)/b^2)*(-I*sqrt(3) + 1)/(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 +
 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^
2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p
^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3) - (1/2)^(1/3)*(128*a^3*d
^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 -
(64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (
64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p
^3)/b^4)^(1/3)*(I*sqrt(3) + 1))*a*b*d*e^2*p - (8*a*d*e^2*p/b - 4*(1/2)^(2/3
)*(8*a^2*d^2*e^4*p^2/b^2 - (12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)/b^2)*(-I*
sqrt(3) + 1)/(128*a^3*d^3*e^6*p^3/b^3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^
4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 -
 a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*
d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3) - (1/2)^(1/3)*(128*a^3*d^3*e^6*p^3/b^
```

```
3 - 24*(12*a*b*d^5*e*p^2 + 5*a^2*d^2*e^4*p^2)*a*d*e^2*p/b^3 - (64*b^3*d^9 +
 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*
p^3 + 24*a^2*b^2*d^6*e^3*p^3 + 4*a^3*b*d^3*e^6*p^3 - a^4*e^9*p^3)/b^4)^(1/3
)*(I*sqrt(3) + 1))^2*b^2 - 32*(12*a*b*d^5*e - a^2*d^2*e^4)*p^2)/b^2)) - 4*(
b*e^3*p*x^4 + 4*b*d*e^2*p*x^3 + 6*b*d^2*e*p*x^2 + 4*b*d^3*p*x)*log(b*x^3 +
a) - 4*(b*e^3*x^4 + 4*b*d*e^2*x^3 + 6*b*d^2*e*x^2 + 4*b*d^3*x)*log(c))/b
```

giac [A] time = 0.24, size = 407, normalized size = 1.27

$$-\frac{\left(6\,abd^2p\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+4\,abd^3p-a^2pe^3\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{4\,ab}+\frac{4\,bpx^4e^3\log\left(bx^3+a\right)+16\,bdpx^3e^2\log\left(bx^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/4*(6*a*b*d^2*p*(-a/b)^(1/3)*e + 4*a*b*d^3*p - a^2*p*e^3)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/(a*b) + 1/16*(4*b*p*x^4*e^3*log(b*x^3 + a) + 16*b*
d*p*x^3*e^2*log(b*x^3 + a) + 24*b*d^2*p*x^2*e*log(b*x^3 + a) - 3*b*p*x^4*e^
3 - 16*b*d*p*x^3*e^2 - 36*b*d^2*p*x^2*e + 16*b*d^3*p*x*log(b*x^3 + a) + 4*b
*x^4*e^3*log(c) + 16*b*d*x^3*e^2*log(c) + 24*b*d^2*x^2*e*log(c) - 48*b*d^3*
p*x + 16*b*d^3*x*log(c) + 16*a*d*p*e^2*log(b*x^3 + a) + 12*a*p*x*e^3)/b + 1
/4*(4*sqrt(3)*(-a*b^2)^(1/3)*b*d^3*p - 6*sqrt(3)*(-a*b^2)^(2/3)*d^2*p*e - s
qrt(3)*(-a*b^2)^(1/3)*a*p*e^3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/b^2 + 1/8*(4*(-a*b^2)^(1/3)*b*d^3*p + 6*(-a*b^2)^(2/3)*d^2*p*e -
(-a*b^2)^(1/3)*a*p*e^3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^2
```

maple [C] time = 0.76, size = 738, normalized size = 2.31

$$\frac{3a\,e^3px}{4b}+d\,e^2x^3\ln(c)+\frac{3d^2e\,x^2\ln(c)}{2}+\frac{e^3x^4\ln(c)}{4}+d^3x\ln(c)+\frac{(ex+d)^4\ln\left(\left(b\,x^3+a\right)^p\right)}{4e}-\frac{3e^3p\,x^4}{16}-\frac{9d^2ep\,x^2}{4}-\frac{i\pi e^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*ln(c*(b*x^3+a)^p),x)
```

```
[Out] 3/4/b*e^3*a*p*x+d*e^2*x^3*ln(c)+3/2*d^2*e*x^2*ln(c)+1/4*e^3*x^4*ln(c)+d^3*x
*ln(c)+1/4/b^2*p/e*sum((b*d*(4*a*e^3-b*d^3)*_R^2+6*a*b*d^2*e^2*_R-a^2*e^4+4
*a*b*d^3*e)/_R^2*ln(-_R+x),_R=RootOf(_Z^3*b+a))-1/8*I*e^3*Pi*x^4*csgn(I*c*(
b*x^3+a)^p)^3+1/4*(e*x+d)^4/e*ln((b*x^3+a)^p)-3/16*e^3*p*x^4-9/4*d^2*e*p*x^
2-3*d^3*p*x-1/2*I*e^2*Pi*d*x^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*cs
gn(I*c)-3/4*I*e*Pi*d^2*x^2*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I
*c)-d*e^2*p*x^3-1/8*I*e^3*Pi*x^4*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*
csgn(I*c)+1/2*I*e^2*Pi*d*x^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-1/
2*I*Pi*d^3*x*csgn(I*c*(b*x^3+a)^p)^3+1/2*I*e^2*Pi*d*x^3*csgn(I*c*(b*x^3+a)^
p)^2*csgn(I*c)+3/4*I*e*Pi*d^2*x^2*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)
^2+3/4*I*e*Pi*d^2*x^2*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-1/2*I*Pi*d^3*x*csgn
(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)+1/2*I*Pi*d^3*x*csgn(I*c*(b*
x^3+a)^p)^2*csgn(I*c)+1/8*I*e^3*Pi*x^4*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+
a)^p)^2+1/8*I*e^3*Pi*x^4*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-1/2*I*e^2*Pi*d*x
^3*csgn(I*c*(b*x^3+a)^p)^3-3/4*I*e*Pi*d^2*x^2*csgn(I*c*(b*x^3+a)^p)^3+1/2*I
*Pi*d^3*x*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2
```

maxima [A] time = 1.02, size = 332, normalized size = 1.04

$$\frac{1}{16}\,bp\left(\frac{4\,\sqrt{3}\left(6\,abd^2e\left(\frac{a}{b}\right)^{\frac{2}{3}}+4\,abd^3\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2e^3\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2}-\frac{3\,be^3x^4+16\,bde^2x^3+36\,bd^2ex^2+1}{b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] 1/16*b*p*(4*sqrt(3)*(6*a*b*d^2*e*(a/b)^(2/3) + 4*a*b*d^3*(a/b)^(1/3) - a^2*
e^3*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2
) - (3*b*e^3*x^4 + 16*b*d*e^2*x^3 + 36*b*d^2*e*x^2 + 12*(4*b*d^3 - a*e^3)*x
)/b^2 + 2*(8*a*b*d*e^2*(a/b)^(2/3) + 6*a*b*d^2*e*(a/b)^(1/3) - 4*a*b*d^3 +
a^2*e^3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 4*(4*a*
b*d*e^2*(a/b)^(2/3) - 6*a*b*d^2*e*(a/b)^(1/3) + 4*a*b*d^3 - a^2*e^3)*log(x
+ (a/b)^(1/3))/(b^3*(a/b)^(2/3))) + 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^
2 + 4*d^3*x)*log((b*x^3 + a)^p*c)
```

mupad [B] time = 0.95, size = 536, normalized size = 1.68

$$\ln\left(c\left(b\,x^3+a\right)^p\right)\left(d^3\,x+\frac{3\,d^2\,e\,x^2}{2}+d\,e^2\,x^3+\frac{e^3\,x^4}{4}\right)-x\left(3\,d^3\,p-\frac{3\,a\,e^3\,p}{4\,b}\right)+\left(\sum_{k=1}^{3}\ln\left(x\left(\frac{9\,a^3\,d\,e^5\,p^2}{4}+\frac{45\,b\,a^2\,d^4\,e^2}{4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)*(d + e*x)^3,x)
```

```
[Out] log(c*(a + b*x^3)^p)*(d^3*x + (e^3*x^4)/4 + (3*d^2*e*x^2)/2 + d*e^2*x^3) -
x*(3*d^3*p - (3*a*e^3*p)/(4*b)) + symsum(log(x*((9*a^3*d*e^5*p^2)/4 + (45*a
^2*b*d^4*e^2*p^2)/4) + root(64*b^4*c^3 - 192*a*b^3*c^2*d*e^2*p + 288*a*b^3*
c*d^5*e*p^2 + 120*a^2*b^2*c*d^2*e^4*p^2 - 4*a^3*b*d^3*e^6*p^3 - 24*a^2*b^2*
d^6*e^3*p^3 - 64*a*b^3*d^9*p^3 + a^4*e^9*p^3, c, k)*(x*(9*a*b^2*d^3*p - (9*
a^2*b*e^3*p)/4) + 9*root(64*b^4*c^3 - 192*a*b^3*c^2*d*e^2*p + 288*a*b^3*c*d
^5*e*p^2 + 120*a^2*b^2*c*d^2*e^4*p^2 - 4*a^3*b*d^3*e^6*p^3 - 24*a^2*b^2*d^6
*e^3*p^3 - 64*a*b^3*d^9*p^3 + a^4*e^9*p^3, c, k)*a*b^2 - 18*a^2*b*d*e^2*p)
+ (45*a^3*d^2*e^4*p^2)/8 + (27*a^2*b*d^5*e*p^2)/2)*root(64*b^4*c^3 - 192*a*
b^3*c^2*d*e^2*p + 288*a*b^3*c*d^5*e*p^2 + 120*a^2*b^2*c*d^2*e^4*p^2 - 4*a^3
*b*d^3*e^6*p^3 - 24*a^2*b^2*d^6*e^3*p^3 - 64*a*b^3*d^9*p^3 + a^4*e^9*p^3, c
, k), k, 1, 3) - (3*e^3*p*x^4)/16 - (9*d^2*e*p*x^2)/4 - d*e^2*p*x^3
```

sympy [A] time = 79.68, size = 265, normalized size = 0.83

$$-\frac{3a^2e^3p\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)}{4b}+3ad^3p\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)+\frac{9a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*ln(c*(b*x**3+a)**p),x)
```

```
[Out] -3*a**2*e**3*p*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
/(4*b) + 3*a*d**3*p*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a +
 x))) + 9*a*d**2*e*p*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2
*a*b + x)))/2 + a*d*e**2*p*Piecewise((x**3/a, Eq(b, 0)), (log(a + b*x**3)/b
```

```
, True)) + 3*a*e**3*p*x/(4*b) - 3*d**3*p*x + d**3*x*log(c*(a + b*x**3)**p)
- 9*d**2*e*p*x**2/4 + 3*d**2*e*x**2*log(c*(a + b*x**3)**p)/2 - d*e**2*p*x**
3 + d*e**2*x**3*log(c*(a + b*x**3)**p) - 3*e**3*p*x**4/16 + e**3*x**4*log(c
*(a + b*x**3)**p)/4
```

3.192 $\int (d+ex)^2 \log\left(c\left(a+bx^3\right)^p\right)\,dx$

Optimal. Leaf size=250

$$-\frac{\sqrt[3]{a}\,dp\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2b^{2/3}}+\frac{\sqrt[3]{a}\,dp\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}-\frac{\sqrt{3}\,\sqrt[3]{a}\,dp\left(\sqrt[3]{a}\,e+\right.}{}$$

```
[Out] -3*d^2*p*x-3/2*d*e*p*x^2-1/3*e^2*p*x^3+a^(1/3)*d*(b^(1/3)*d-a^(1/3)*e)*p*ln
(a^(1/3)+b^(1/3)*x)/b^(2/3)-1/2*a^(1/3)*d*(b^(1/3)*d-a^(1/3)*e)*p*ln(a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)-1/3*(-a*e^3+b*d^3)*p*ln(b*x^3+a)/b
/e+1/3*(e*x+d)^3*ln(c*(b*x^3+a)^p)/e-a^(1/3)*d*(b^(1/3)*d+a^(1/3)*e)*p*arct
an(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)
```

Rubi [A] time = 0.48, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {2463, 1836, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\sqrt[3]{a}\,dp\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2b^{2/3}}+\frac{\sqrt[3]{a}\,dp\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}-\frac{\sqrt{3}\,\sqrt[3]{a}\,dp\left(\sqrt[3]{a}\,e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*d^2*p*x - (3*d*e*p*x^2)/2 - (e^2*p*x^3)/3 - (Sqrt[3]*a^(1/3)*d*(b^(1/3)*
d + a^(1/3)*e)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(2/3)
 + (a^(1/3)*d*(b^(1/3)*d - a^(1/3)*e)*p*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) -
 (a^(1/3)*d*(b^(1/3)*d - a^(1/3)*e)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(2*b^(2/3)) - ((b*d^3 - a*e^3)*p*Log[a + b*x^3])/(3*b*e) + ((d +
 e*x)^3*Log[c*(a + b*x^3)^p])/(3*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)^2 \log\left(c\left(a+bx^3\right)^p\right)\,dx = \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} - \frac{(bp)\int \frac{x^2(d+ex)^3}{a+bx^3}dx}{e}$$
$$= -\frac{1}{3}e^2px^3 + \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} - \frac{p\int \frac{x^2\left(3\left(bd^3-ae^3\right)+9bd^2ex+9bde^2x^2\right)}{a+bx^3}dx}{3e}$$
$$= -\frac{1}{3}e^2px^3 + \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} - \frac{p\int \left(9d^2e+9de^2x-\frac{3\left(3ad^2e+3ade^2x-(\right.}{a+bx^3}\right.}{3e}$$
$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 + \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} + \frac{p\int \frac{3ad^2e+3ade^2x-}{a+bx}}{e}$$
$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 + \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} + \frac{p\int \frac{3ad^2e+3ade^2x}{a+bx^3}}{e}$$
$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 - \frac{\left(bd^3-ae^3\right)p\log\left(a+bx^3\right)}{3be} + \frac{(d+ex)^3\log\left(c\left(\right.\right.}{3e}$$
$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 + \frac{\sqrt[3]{a}\,d\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}} - \frac{\left(bd^3\right.}{}$$
$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 + \frac{\sqrt[3]{a}\,d\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}} - \frac{\sqrt[3]{a}\,d}{}$$
$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 - \frac{\sqrt{3}\sqrt[3]{a}\,d\left(\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}} + $$

Mathematica [C] time = 0.33, size = 218, normalized size = 0.87

$$\frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right) - \frac{p\left(3\sqrt[3]{a}\,b^{2/3}d^2e\left(\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)-6\sqrt[3]{a}\,b^{2/3}d^2e\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+2\left(bd^3-ae^3\right)\right.}{2b}}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (-1/2*(p*(18*b*d^2*e*x + 9*b*d*e^2*x^2 + 2*b*e^3*x^3 - 9*b*d*e^2*x^2*Hyperg
eometric2F1[2/3, 1, 5/3, -((b*x^3)/a)] - 6*a^(1/3)*b^(2/3)*d^2*e*Log[a^(1/3
) + b^(1/3)*x] + 3*a^(1/3)*b^(2/3)*d^2*e*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*
x)/a^(1/3))/Sqrt[3]] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]) + 2*
(b*d^3 - a*e^3)*Log[a + b*x^3]))/b + (d + e*x)^3*Log[c*(a + b*x^3)^p])/(3*e
)
```

fricas [C] time = 2.76, size = 5799, normalized size = 23.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*e^2*p*x^3 + 18*b*d*e*p*x^2 + 36*b*d^2*p*x - 2*(2*a*e^2*p/b - 2*(
1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt
```

```
(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d
^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)
^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2
- 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e
^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))*b*log(1/4*(2*a*e^2*p/b - 2*(1/2)^(2/3)*
(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(2
*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 +
a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) - (1
/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*
d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3
)^(1/3)*(I*sqrt(3) + 1))^2*b^2*e + 9*(b^2*d^5 + a*b*d^2*e^3)*p^2*x + 1/2*(3
*b^2*d^3 - 2*a*b*e^3)*(2*a*e^2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*
b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b
*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3
 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3
/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)
*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))
*p + (15*a*b*d^3*e^2 + a^2*e^5)*p^2) - (6*a*e^2*p - (2*a*e^2*p/b - 2*(1/2)^
(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) +
 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*
p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3
) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(
9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^
3)/b^3)^(1/3)*(I*sqrt(3) + 1))*b - 3*sqrt(1/3)*b*sqrt((4*(2*a*e^2*p/b - 2*(
1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt
(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d
^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)
^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2
- 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e
^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))*a*b*e^2*p - (2*a*e^2*p/b - 2*(1/2)^(2/3
)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/
(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2
+ a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) -
(1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*
b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b
^3)^(1/3)*(I*sqrt(3) + 1))^2*b^2 - 4*(36*a*b*d^3*e + a^2*e^4)*p^2)/b^2))*lo
g(-1/4*(2*a*e^2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a
^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a
*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^
6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^
3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 +
(27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))^2*b^2*e + 18*(
b^2*d^5 + a*b*d^2*e^3)*p^2*x - 1/2*(3*b^2*d^3 - 2*a*b*e^3)*(2*a*e^2*p/b - 2
*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sq
rt(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b
*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^
3)^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^
2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3
*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))*p - (15*a*b*d^3*e^2 + a^2*e^5)*p^2 +
3/4*sqrt(1/3)*((2*a*e^2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e
*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 +
a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*
a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 +
27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*
p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))*b^2*e
- 2*(3*b^2*d^3 + a*b*e^3)*p)*sqrt((4*(2*a*e^2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*
p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*
p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p
^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) - (1/2)^(1/3)
```

```
*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2
 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(
I*sqrt(3) + 1))*a*b*e^2*p - (2*a*e^2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 -
 (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^3 +
 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2
*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^3*e
^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^
4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3)
 + 1))^2*b^2 - 4*(36*a*b*d^3*e + a^2*e^4)*p^2)/b^2)) - (6*a*e^2*p - (2*a*e^
2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^
2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 -
 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^
6*p^3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d
^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*
p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))*b + 3*sqrt(1/3)*b*sqrt((4*(2
*a*e^2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^
2)/b^2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/
b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a
^3*e^6*p^3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3
)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2
*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))*a*b*e^2*p - (2*a*e^2*p/
b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(
-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(
9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^
3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p
^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3
+ a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))^2*b^2 - 4*(36*a*b*d^3*e + a^2*e^
4)*p^2)/b^2))*log(-1/4*(2*a*e^2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a
*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(
b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^
3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^
3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2
)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1)
)^2*b^2*e + 18*(b^2*d^5 + a*b*d^2*e^3)*p^2*x - 1/2*(3*b^2*d^3 - 2*a*b*e^3)*
(2*a*e^2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*
p^2)/b^2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^
3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 +
 a^3*e^6*p^3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e
^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b
^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt(3) + 1))*p - (15*a*b*d^3*e^2 +
 a^2*e^5)*p^2 - 3/4*sqrt(1/3)*((2*a*e^2*p/b - 2*(1/2)^(2/3)*(a^2*e^4*p^2/b^
2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(2*a^3*e^6*p^3/b^
3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*
e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) - (1/2)^(1/3)*(2*a^
3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^2
*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3)*(I*sqrt
(3) + 1))*b^2*e - 2*(3*b^2*d^3 + a*b*e^3)*p)*sqrt((4*(2*a*e^2*p/b - 2*(1/2)
^(2/3)*(a^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3)
+ 1)/(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e
*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/
3) - (1/2)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*
(9*a*b*d^3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p
^3)/b^3)^(1/3)*(I*sqrt(3) + 1))*a*b*e^2*p - (2*a*e^2*p/b - 2*(1/2)^(2/3)*(a
^2*e^4*p^2/b^2 - (9*a*b*d^3*e*p^2 + a^2*e^4*p^2)/b^2)*(-I*sqrt(3) + 1)/(2*a
^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^3*e*p^2 + a^
2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^(1/3) - (1/2
)^(1/3)*(2*a^3*e^6*p^3/b^3 + 27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 - 3*(9*a*b*d^
3*e*p^2 + a^2*e^4*p^2)*a*e^2*p/b^3 + (27*a*b^2*d^6*p^3 + a^3*e^6*p^3)/b^3)^
(1/3)*(I*sqrt(3) + 1))^2*b^2 - 4*(36*a*b*d^3*e + a^2*e^4)*p^2)/b^2)) - 4*(b
```

```
*e^2*p*x^3 + 3*b*d*e*p*x^2 + 3*b*d^2*p*x)*log(b*x^3 + a) - 4*(b*e^2*x^3 + 3
*b*d*e*x^2 + 3*b*d^2*x)*log(c))/b
```

giac [A] time = 0.21, size = 298, normalized size = 1.19

$$-\frac{\left(adp\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+ad^2p\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a}+\frac{2\,bpx^3e^2\log\left(bx^3+a\right)+6\,bdpx^2e\log\left(bx^3+a\right)-2\,bpx^3e^2-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -(a*d*p*(-a/b)^(1/3)*e + a*d^2*p)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a
 + 1/6*(2*b*p*x^3*e^2*log(b*x^3 + a) + 6*b*d*p*x^2*e*log(b*x^3 + a) - 2*b*p
*x^3*e^2 - 9*b*d*p*x^2*e + 6*b*d^2*p*x*log(b*x^3 + a) + 2*b*x^3*e^2*log(c)
+ 6*b*d*x^2*e*log(c) - 18*b*d^2*p*x + 6*b*d^2*x*log(c) + 2*a*p*e^2*log(b*x^
3 + a))/b + (sqrt(3)*(-a*b^2)^(1/3)*b*d^2*p - sqrt(3)*(-a*b^2)^(2/3)*d*p*e)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^2 + 1/2*((-a*b^2)^
(1/3)*b*d^2*p + (-a*b^2)^(2/3)*d*p*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/
3))/b^2
```

maple [C] time = 0.77, size = 537, normalized size = 2.15

$$\frac{i\pi de\,x^2\mathrm{csgn}\left(i\left(b\,x^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^3+a\right)^p\right)^2}{2}+\frac{i\pi\,d^2x\,\mathrm{csgn}\left(i\left(b\,x^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^3+a\right)^p\right)^2}{2}-\frac{i\pi\,e^2x^3\mathrm{cs}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*ln(c*(b*x^3+a)^p),x)
```

```
[Out] 1/3*(e*x+d)^3/e*ln((b*x^3+a)^p)+1/2*I*e*Pi*d*x^2*csgn(I*(b*x^3+a)^p)*csgn(I
*c*(b*x^3+a)^p)^2+1/2*I*Pi*d^2*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2*
x-1/6*I*e^2*Pi*x^3*csgn(I*c*(b*x^3+a)^p)^3+1/2*I*Pi*d^2*csgn(I*c*(b*x^3+a)^
p)^2*csgn(I*c)*x-1/2*I*e*Pi*d*x^2*csgn(I*c*(b*x^3+a)^p)^3-1/2*I*Pi*d^2*csgn
(I*c*(b*x^3+a)^p)^3*x+1/6*I*e^2*Pi*x^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+
a)^p)^2+1/2*I*e*Pi*d*x^2*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+1/6*I*e^2*Pi*x^3
*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-1/2*I*e*Pi*d*x^2*csgn(I*(b*x^3+a)^p)*csg
n(I*c*(b*x^3+a)^p)*csgn(I*c)-1/6*I*e^2*Pi*x^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*
(b*x^3+a)^p)*csgn(I*c)-1/2*I*Pi*d^2*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^
p)*csgn(I*c)*x+1/3*e^2*x^3*ln(c)-1/3*e^2*p*x^3+d*e*x^2*ln(c)-3/2*d*e*p*x^2+
d^2*x*ln(c)-3*d^2*p*x+1/3*p/b/e*sum(((a*e^3-b*d^3)*_R^2+3*a*d*e^2*_R+3*a*d^
2*e)/_R^2*ln(-_R+x),_R=RootOf(_Z^3*b+a))
```

maxima [A] time = 1.02, size = 249, normalized size = 1.00

$$-\frac{1}{6}\,bp\left(\frac{2\,e^2x^3+9\,dex^2+18\,d^2x}{b}-\frac{6\,\sqrt{3}\left(abde\left(\frac{a}{b}\right)^{\frac{2}{3}}+abd^2\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2}-\frac{\left(2\,ae^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+3\,ade\left(\frac{a}{b}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] -1/6*b*p*((2*e^2*x^3 + 9*d*e*x^2 + 18*d^2*x)/b - 6*sqrt(3)*(a*b*d*e*(a/b)^(
2/3) + a*b*d^2*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/(a*b^2) - (2*a*e^2*(a/b)^(2/3) + 3*a*d*e*(a/b)^(1/3) - 3*a*d^2)*log(x^
2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) - 2*(a*e^2*(a/b)^(2/3) -
 3*a*d*e*(a/b)^(1/3) + 3*a*d^2)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))) + 1
/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)*log((b*x^3 + a)^p*c)
```

mupad [B] time = 0.32, size = 358, normalized size = 1.43

$$\left(\sum_{k=1}^{3} \ln\left(\mathrm{root}\left(27\,b^3\,c^3 - 27\,a\,b^2\,c^2\,e^2\,p + 81\,a\,b^2\,c\,d^3\,e\,p^2 + 9\,a^2\,b\,c\,e^4\,p^2 - 27\,a\,b^2\,d^6\,p^3 - a^3\,e^6\,p^3, c, k\right)\,\left(\mathrm{root}\left(27\,b^3\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)*(d + e*x)^2,x)
```

```
[Out] symsum(log(root(27*b^3*c^3 - 27*a*b^2*c^2*e^2*p + 81*a*b^2*c*d^3*e*p^2 + 9*
a^2*b*c*e^4*p^2 - 27*a*b^2*d^6*p^3 - a^3*e^6*p^3, c, k)*(9*root(27*b^3*c^3
- 27*a*b^2*c^2*e^2*p + 81*a*b^2*c*d^3*e*p^2 + 9*a^2*b*c*e^4*p^2 - 27*a*b^2*
d^6*p^3 - a^3*e^6*p^3, c, k)*a*b^2 - 6*a^2*b*e^2*p + 9*a*b^2*d^2*p*x) + a^3
*e^4*p^2 + 9*a^2*b*d^3*e*p^2 + 6*a^2*b*d^2*e^2*p^2*x)*root(27*b^3*c^3 - 27*
a*b^2*c^2*e^2*p + 81*a*b^2*c*d^3*e*p^2 + 9*a^2*b*c*e^4*p^2 - 27*a*b^2*d^6*p
^3 - a^3*e^6*p^3, c, k), k, 1, 3) + log(c*(a + b*x^3)^p)*(d^2*x + (e^2*x^3)
/3 + d*e*x^2) - 3*d^2*p*x - (e^2*p*x^3)/3 - (3*d*e*p*x^2)/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Timed out
```

3.193 $\int (d+ex)\log\left(c\left(a+bx^3\right)^p\right)\,dx$

Optimal. Leaf size=229

$$-\frac{\sqrt[3]{a}\,p\left(2\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{4b^{2/3}}+\frac{\sqrt[3]{a}\,p\left(2\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2b^{2/3}}-\frac{\sqrt{3}\,\sqrt[3]{a}\,p\left(\sqrt[3]{a}\,e+\right.}{}$$

```
[Out] -3*d*p*x-3/4*e*p*x^2+1/2*a^(1/3)*(2*b^(1/3)*d-a^(1/3)*e)*p*ln(a^(1/3)+b^(1/
3)*x)/b^(2/3)-1/4*a^(1/3)*(2*b^(1/3)*d-a^(1/3)*e)*p*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/b^(2/3)-1/2*d^2*p*ln(b*x^3+a)/e+1/2*(e*x+d)^2*ln(c*(b*x^
3+a)^p)/e-1/2*a^(1/3)*(2*b^(1/3)*d+a^(1/3)*e)*p*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)
```

Rubi [A] time = 0.32, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {2463, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\sqrt[3]{a}\,p\left(2\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{4b^{2/3}}+\frac{\sqrt[3]{a}\,p\left(2\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2b^{2/3}}-\frac{\sqrt{3}\,\sqrt[3]{a}\,p\left(\sqrt[3]{a}\,e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*d*p*x - (3*e*p*x^2)/4 - (Sqrt[3]*a^(1/3)*(2*b^(1/3)*d + a^(1/3)*e)*p*Arc
Tan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*b^(2/3)) + (a^(1/3)*(2*b
^(1/3)*d - a^(1/3)*e)*p*Log[a^(1/3) + b^(1/3)*x])/(2*b^(2/3)) - (a^(1/3)*(2
*b^(1/3)*d - a^(1/3)*e)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
4*b^(2/3)) - (d^2*p*Log[a + b*x^3])/(2*e) + ((d + e*x)^2*Log[c*(a + b*x^3)^
p])/(2*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)\log\left(c\left(a+bx^3\right)^p\right)dx = \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} - \frac{(3bp)\int \frac{x^2(d+ex)^2}{a+bx^3}dx}{2e}$$
$$= \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} - \frac{(3bp)\int\left(\frac{2de}{b}+\frac{e^2x}{b}-\frac{2ade+ae^2x-bd^2x^2}{b\left(a+bx^3\right)}\right)dx}{2e}$$
$$= -3dpx - \frac{3}{4}epx^2 + \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} + \frac{(3p)\int\frac{2ade+ae^2x-bd^2x^2}{a+bx^3}dx}{2e}$$
$$= -3dpx - \frac{3}{4}epx^2 + \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} + \frac{(3p)\int\frac{2ade+ae^2x}{a+bx^3}dx}{2e} - \frac{(3bd$$
$$= -3dpx - \frac{3}{4}epx^2 - \frac{d^2p\log\left(a+bx^3\right)}{2e} + \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} + \frac{p\int -$$
$$= -3dpx - \frac{3}{4}epx^2 + \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}} - \frac{d^2p\log\left(a+bx\right.}{2e}$$
$$= -3dpx - \frac{3}{4}epx^2 + \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}} - \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d-\right.}{}$$
$$= -3dpx - \frac{3}{4}epx^2 - \frac{\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{b}d+\sqrt[3]{a}e\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}} + \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d\right.}{}$$

Mathematica [C] time = 0.07, size = 204, normalized size = 0.89

$$-\frac{\sqrt[3]{a}dp\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}} + \frac{\sqrt{3}\sqrt[3]{a}dp\tan^{-1}\left(\frac{2b^{2/3}x-\sqrt[3]{a}\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}\right)}{\sqrt[3]{b}} + dx\log\left(c\left(a+bx^3\right)^p\right) + \frac{1}{2}ex^2\log\left(c\left(a+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*d*p*x - (3*e*p*x^2)/4 + (Sqrt[3]*a^(1/3)*d*p*ArcTan[(-(a^(1/3)*b^(1/3))
+ 2*b^(2/3)*x)/(Sqrt[3]*a^(1/3)*b^(1/3))])/b^(1/3) + (3*e*p*x^2*Hypergeomet
ric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/4 + (a^(1/3)*d*p*Log[a^(1/3) + b^(1/3)*x
])/b^(1/3) - (a^(1/3)*d*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
2*b^(1/3)) + d*x*Log[c*(a + b*x^3)^p] + (e*x^2*Log[c*(a + b*x^3)^p])/2
```

fricas [C] time = 1.25, size = 2284, normalized size = 9.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] -3/4*e*p*x^2 - 3*d*p*x + 1/4*(4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8
*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (
1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2
)^(1/3)*(I*sqrt(3) + 1))*log(4*a*d*e^2*p^2 + 2*(4*(1/2)^(2/3)*a*d*e*p^2*(-I
*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)
/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3
- a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*p + 1/4*(4*(1/2)^(2/3)*a*d
```

```
*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^
2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*
b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))^2*b*e + (8*b*d^3 + a*e
^3)*p^2*x) - 1/8*(4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e
^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*(
(8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*s
qrt(3) + 1) - sqrt(3)*sqrt(-(32*a*d*e*p^2 + (4*(1/2)^(2/3)*a*d*e*p^2*(-I*sq
rt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^
2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a
^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))^2*b)/b))*log(-2*a*d*e^2*p^2 - (4*(1
/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b
*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^
3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*p -
 1/8*(4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^
2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a
*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))
^2*b*e + (8*b*d^3 + a*e^3)*p^2*x + 1/8*sqrt(3)*(8*b*d^2*p - (4*(1/2)^(2/3)*
a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 -
 a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8
*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*e)*sqrt(-(32*a*d*
e*p^2 + (4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3
/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3
+ a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) +
1))^2*b)/b)) - 1/8*(4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a
*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)
*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I
*sqrt(3) + 1) + sqrt(3)*sqrt(-(32*a*d*e*p^2 + (4*(1/2)^(2/3)*a*d*e*p^2*(-I*
sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/
b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 -
 a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))^2*b)/b))*log(-2*a*d*e^2*p^2 - (4*
(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a
*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*
p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*p
 - 1/8*(4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/
b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 +
 a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1
))^2*b*e + (8*b*d^3 + a*e^3)*p^2*x - 1/8*sqrt(3)*(8*b*d^2*p - (4*(1/2)^(2/3
)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3
 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 +
(8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*e)*sqrt(-(32*a*
d*e*p^2 + (4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p
^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^
3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3)
+ 1))^2*b)/b)) + 1/2*(e*p*x^2 + 2*d*p*x)*log(b*x^3 + a) + 1/2*(e*x^2 + 2*d*
x)*log(c)
```

giac [A] time = 0.19, size = 220, normalized size = 0.96

$$\frac{1}{2}\,px^2e\log\left(bx^3+a\right)-\frac{3}{4}\,px^2e+dpx\log\left(bx^3+a\right)+\frac{1}{2}\,x^2e\log(c)-3\,dpx+dx\log(c)-\frac{\left(ap\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+2\,adp\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\right.}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] 1/2*p*x^2*e*log(b*x^3 + a) - 3/4*p*x^2*e + d*p*x*log(b*x^3 + a) + 1/2*x^2*e
*log(c) - 3*d*p*x + d*x*log(c) - 1/2*(a*p*(-a/b)^(1/3)*e + 2*a*d*p)*(-a/b)^
(1/3)*log(abs(x - (-a/b)^(1/3)))/a + 1/2*(2*sqrt(3)*(-a*b^2)^(1/3)*b*d*p -
```

```
sqrt(3)*(-a*b^2)^(2/3)*p*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/b^2 + 1/4*(2*(-a*b^2)^(1/3)*b*d*p + (-a*b^2)^(2/3)*p*e)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/b^2
```

maple [C] time = 0.74, size = 335, normalized size = 1.46

$$-\frac{i\pi e\,x^2\mathrm{csgn}\,(ic)\,\mathrm{csgn}\left(i\left(b\,x^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^3+a\right)^p\right)}{4}+\frac{i\pi e\,x^2\mathrm{csgn}\,(ic)\,\mathrm{csgn}\left(ic\left(b\,x^3+a\right)^p\right)^2}{4}+\frac{i\pi e\,x^2\mathrm{csgn}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*ln(c*(b*x^3+a)^p),x)
```

```
[Out] (1/2*e*x^2+d*x)*ln((b*x^3+a)^p)-1/4*I*Pi*e*x^2*csgn(I*(b*x^3+a)^p)*csgn(I*c
*(b*x^3+a)^p)*csgn(I*c)+1/4*I*csgn(I*c)*csgn(I*c*(b*x^3+a)^p)^2*x^2*e*Pi+1/
4*I*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*(b*x^3+a)^p)*x^2*e*Pi-1/4*I*Pi*e*x^2*csg
n(I*c*(b*x^3+a)^p)^3-1/2*I*Pi*d*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*c
sgn(I*c)*x+1/2*I*Pi*d*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)*x+1/2*I*Pi*d*csgn(I
*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2*x-1/2*I*Pi*d*csgn(I*c*(b*x^3+a)^p)^3*
x+1/2*ln(c)*e*x^2-3/4*e*p*x^2+ln(c)*d*x-3*d*p*x+1/2*a*p/b*sum((_R*e+2*d)/_R
^2*ln(-_R+x),_R=RootOf(_Z^3*b+a))
```

maxima [A] time = 1.01, size = 187, normalized size = 0.82

$$-\frac{1}{4}\,bp\left(\frac{3\left(ex^2+4\,dx\right)}{b}-\frac{2\,\sqrt{3}\left(ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)\right.}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] -1/4*b*p*(3*(e*x^2 + 4*d*x)/b - 2*sqrt(3)*(a*e*(a/b)^(1/3) + 2*a*d)*arctan(
1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) - (a*e*(a/b)
^(1/3) - 2*a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) +
2*(a*e*(a/b)^(1/3) - 2*a*d)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))) + 1/2*(
e*x^2 + 2*d*x)*log((b*x^3 + a)^p*c)
```

mupad [B] time = 0.31, size = 210, normalized size = 0.92

$$\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(8\,b^2\,c^3+12\,a\,b\,c\,d\,e\,p^2-8\,a\,b\,d^3\,p^3+a^2\,e^3\,p^3,c,k\right)\left(\mathrm{root}\left(8\,b^2\,c^3+12\,a\,b\,c\,d\,e\,p^2-8\,a\,b\,d^3\,p^3+a^2\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)*(d + e*x),x)
```

```
[Out] symsum(log(root(8*b^2*c^3 + 12*a*b*c*d*e*p^2 - 8*a*b*d^3*p^3 + a^2*e^3*p^3,
 c, k)*(9*root(8*b^2*c^3 + 12*a*b*c*d*e*p^2 - 8*a*b*d^3*p^3 + a^2*e^3*p^3,
c, k)*a*b^2 + 9*a*b^2*d*p*x) + (9*a^2*b*d*e*p^2)/2 + (9*a^2*b*e^2*p^2*x)/4)
*root(8*b^2*c^3 + 12*a*b*c*d*e*p^2 - 8*a*b*d^3*p^3 + a^2*e^3*p^3, c, k), k,
 1, 3) + log(c*(a + b*x^3)^p)*(d*x + (e*x^2)/2) - (3*e*p*x^2)/4 - 3*d*p*x
```

sympy [A] time = 141.24, size = 520, normalized size = 2.27

$$\begin{cases} \left(dx + \frac{ex^2}{2}\right)\log\left(0^p c\right) \\ \left(dx + \frac{ex^2}{2}\right)\log\left(a^p c\right) \\ dpx\log(b) + 3dpx\log(x) - 3dpx + dx\log(c) + \frac{epx^2\log(b)}{2} + \frac{3epx^2\log(x)}{2} - \frac{3epx^2}{4} + \frac{ex^2\log(c)}{2} \\ -\frac{(-1)^{\frac{2}{3}}a^{\frac{2}{3}}ep\left(\frac{1}{b}\right)^{\frac{2}{3}}\log\left(a+bx^3\right)}{2} + \frac{3(-1)^{\frac{2}{3}}a^{\frac{2}{3}}ep\left(\frac{1}{b}\right)^{\frac{2}{3}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}x\sqrt[3]{\frac{1}{b}}+4x^2\right)}{4} - \frac{(-1)^{\frac{2}{3}}\sqrt{3}a^{\frac{2}{3}}ep\left(\frac{1}{b}\right)^{\frac{2}{3}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}x}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{2} - \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Piecewise(((d*x + e*x**2/2)*log(0**p*c), Eq(a, 0) & Eq(b, 0)), ((d*x + e*x*
*2/2)*log(a**p*c), Eq(b, 0)), (d*p*x*log(b) + 3*d*p*x*log(x) - 3*d*p*x + d*
x*log(c) + e*p*x**2*log(b)/2 + 3*e*p*x**2*log(x)/2 - 3*e*p*x**2/4 + e*x**2*
log(c)/2, Eq(a, 0)), (-(-1)**(2/3)*a**(2/3)*e*p*(1/b)**(2/3)*log(a + b*x**3
)/2 + 3*(-1)**(2/3)*a**(2/3)*e*p*(1/b)**(2/3)*log(4*(-1)**(2/3)*a**(2/3)*(1
/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/4 - (-1)**(2/3
)*sqrt(3)*a**(2/3)*e*p*(1/b)**(2/3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*
x/(3*a**(1/3)*(1/b)**(1/3)))/2 + 3*(-1)**(1/3)*a**(1/3)*b*d*p*(1/b)**(4/3)*
log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(
1/3) + 4*x**2)/2 + (-1)**(1/3)*sqrt(3)*a**(1/3)*b*d*p*(1/b)**(4/3)*atan(sqr
t(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x/(3*a**(1/3)*(1/b)**(1/3))) - (-1)**(1/3)*a
**(1/3)*d*p*(1/b)**(1/3)*log(a + b*x**3) + d*p*x*log(a + b*x**3) - 3*d*p*x
+ d*x*log(c) + e*p*x**2*log(a + b*x**3)/2 - 3*e*p*x**2/4 + e*x**2*log(c)/2,
 True))
```

3.194 $\int \log \left(c \left(a+b x^3\right)^p\right) \, dx$

Optimal. Leaf size=133

$$-\frac{\sqrt[3]{a} p \log \left(a^{2/3}-\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3} x^2\right)}{2 \sqrt[3]{b}}+x \log \left(c \left(a+b x^3\right)^p\right)+\frac{\sqrt[3]{a} p \log \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\sqrt[3]{b}}-\frac{\sqrt{3} \sqrt[3]{a} p \tan^{-1}\left(\frac{\sqrt[3]{a}-2 \sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt[3]{b}}$$

```
[Out] -3*p*x+a^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)-1/2*a^(1/3)*p*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(1/3)+x*ln(c*(b*x^3+a)^p)-a^(1/3)*p*arctan(1/
3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)
```

Rubi [A] time = 0.09, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2448, 321, 200, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{a} p \log \left(a^{2/3}-\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3} x^2\right)}{2 \sqrt[3]{b}}+x \log \left(c \left(a+b x^3\right)^p\right)+\frac{\sqrt[3]{a} p \log \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\sqrt[3]{b}}-\frac{\sqrt{3} \sqrt[3]{a} p \tan^{-1}\left(\frac{\sqrt[3]{a}-2 \sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/b^(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x
^3)^p]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \log\left(c\left(a+bx^3\right)^p\right)dx &= x\log\left(c\left(a+bx^3\right)^p\right)-(3bp)\int\frac{x^3}{a+bx^3}\,dx\\
&= -3px+x\log\left(c\left(a+bx^3\right)^p\right)+(3ap)\int\frac{1}{a+bx^3}\,dx\\
&= -3px+x\log\left(c\left(a+bx^3\right)^p\right)+\left(\sqrt[3]{a}\,p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx+\left(\sqrt[3]{a}\,p\right)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+\ldots}\\
&= -3px+\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{1}{2}\left(3a^{2/3}p\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+\ldots}\\
&= -3px+\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-\frac{\sqrt[3]{a}\,p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+x\log\left(c\left(a+b\ldots\right.\right.\\
&= -3px-\frac{\sqrt{3}\,\sqrt[3]{a}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{b}}+\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-\frac{\sqrt[3]{a}\,p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\ldots\right.}{2\sqrt[3]{b}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 129, normalized size = 0.97

$$-\frac{\sqrt[3]{a}\,p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-\frac{\sqrt{3}\,\sqrt[3]{a}\,p\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}-3px$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^
(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x^3)^p]
```

fricas [A] time = 0.46, size = 110, normalized size = 0.83

$$px\log\left(bx^3+a\right)+\sqrt{3}p\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-\frac{1}{2}p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] p*x*log(b*x^3 + a) + sqrt(3)*p*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^
(2/3) - sqrt(3)*a)/a) - 1/2*p*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(
2/3)) + p*(a/b)^(1/3)*log(x + (a/b)^(1/3)) - 3*p*x + x*log(c)
```

giac [A] time = 0.19, size = 143, normalized size = 1.08

$$-\frac{1}{2}abp\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\right.\right.}{ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/2*a*b*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 2*sqrt(3)*(-a
*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) -
 (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)) + p*x*log
(b*x^3 + a) - (3*p - log(c))*x
```

maple [A] time = 0.06, size = 113, normalized size = 0.85

$$\frac{\sqrt{3}\,ap\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{ap\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{ap\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-3px+x\ln\left(c\left(bx^3+a\right)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p),x)
```

```
[Out] x*ln(c*(b*x^3+a)^p)-3*p*x+1/b*p*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/2/b*p*a/(
a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/b*p*a/(a/b)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 1.01, size = 125, normalized size = 0.94

$$-\frac{1}{2}bp\left(\frac{6x}{b}-\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+x\log\left(\left(bx^3+a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] -1/2*b*p*(6*x/b - 2*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^
(1/3))/(b^2*(a/b)^(2/3)) + a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a
/b)^(2/3)) - 2*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))) + x*log((b*x^3 + a
)^p*c)
```

mupad [B] time = 0.00, size = 134, normalized size = 1.01

$$x\ln\left(c\left(b\,x^3+a\right)^p\right)-3\,p\,x-\frac{(-a)^{1/3}\,p\,\ln\left((-a)^{4/3}+a\,b^{1/3}\,x\right)}{b^{1/3}}+\frac{(-a)^{1/3}\,p\,\ln\left(2\,a\,b^{1/3}\,x-(-a)^{4/3}-\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\right.}{b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p),x)
```

```
[Out] x*log(c*(a + b*x^3)^p) - 3*p*x - ((-a)^(1/3)*p*log((-a)^(4/3) + a*b^(1/3)*x
))/b^(1/3) + ((-a)^(1/3)*p*log(2*a*b^(1/3)*x - 3^(1/2)*(-a)^(4/3)*1i - (-a)
^(4/3))*((3^(1/2)*1i)/2 + 1/2))/b^(1/3) - ((-a)^(1/3)*p*log(3^(1/2)*(-a)^(4
/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2 - 1/2))/b^(1/3)
```

sympy [A] time = 60.15, size = 231, normalized size = 1.74

$$\begin{cases} x\log\left(0^p c\right) \\ x\log\left(a^p c\right) \\ px\log(b)+3px\log(x)-3px+x\log(c) \\ -\sqrt[3]{-1}\,\sqrt[3]{a}\,bp\left(\frac{1}{b}\right)^{\frac{4}{3}}\log\left(a+bx^3\right)+\frac{3\sqrt[3]{-1}\,\sqrt[3]{a}\,bp\left(\frac{1}{b}\right)^{\frac{4}{3}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,x\sqrt[3]{\frac{1}{b}}+4x^2\right)}{2}+\sqrt[3]{-1}\,\sqrt{3}\,\sqrt[3]{a}\,bp\left(\frac{1}{b}\right)^{\frac{4}{3}}\operatorname{atan}\left(\right. \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p),x)
```

```
[Out] Piecewise((x*log(0**p*c), Eq(a, 0) & Eq(b, 0)), (x*log(a**p*c), Eq(b, 0)),
(p*x*log(b) + 3*p*x*log(x) - 3*p*x + x*log(c), Eq(a, 0)), (-(-1)**(1/3)*a**
(1/3)*b*p*(1/b)**(4/3)*log(a + b*x**3) + 3*(-1)**(1/3)*a**(1/3)*b*p*(1/b)**
(4/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1
/b)**(1/3) + 4*x**2)/2 + (-1)**(1/3)*sqrt(3)*a**(1/3)*b*p*(1/b)**(4/3)*atan
(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x/(3*a**(1/3)*(1/b)**(1/3))) + p*x*log(a
 + b*x**3) - 3*p*x + x*log(c), True))
```

3.195 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=308

$$\frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}-\frac{p\log(d+ex)\,\text{lo}}{}$$

```
[Out] -p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e-p*ln(-e*((-
1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e-p
*ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/
3)*e))*ln(e*x+d)/e+ln(e*x+d)*ln(c*(b*x^3+a)^p)/e-p*polylog(2,b^(1/3)*(e*x+d
)/(b^(1/3)*d-a^(1/3)*e))/e-p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3
)*a^(1/3)*e))/e-p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e
))/e
```

Rubi [A] time = 0.52, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2462, 260, 2416, 2394, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a\right.\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x]
)/e) - (p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3
)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)
*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e + (Log[d +
 e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1
/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2
/3)*a^(1/3)*e)])/e
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx = \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{(3bp)\int \frac{x^2\log(d+ex)}{a+bx^3}\,dx}{e}$$

$$= \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{(3bp)\int\left(\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(d+ex)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\text{l}}{3b^{2/3}\left((-\right.}\right.}{e}$$

$$= \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{\left(\sqrt[3]{b}p\right)\int \frac{\log(d+ex)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e} - \frac{\left(\sqrt[3]{b}p\right)\int \frac{\log(d+ex)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e} - \frac{(}{}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-}}{}\right.}{}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-}}{}\right.}{}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-}}{}\right.}{}$$

Mathematica [A] time = 0.16, size = 313, normalized size = 1.02

$$\frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e} - \frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e} - \frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e} - \frac{p\log(d+ex)\log\left(\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x]
)/e) - (p*Log[-(((-1)^(2/3)*e*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*d
- (-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3) +
 (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e
 + (Log[d + e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x)
```

```
)/(b^(1/3)*d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d + (-1)^(1/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - (-1)^(2/3)*a^(1/3)*e)])/e
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^3+a\right)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

maple [C] time = 0.60, size = 261, normalized size = 0.85

$$-\frac{i\pi \operatorname{csgn}(ic)\operatorname{csgn}\left(i\left(b\,x^3+a\right)^p\right)\operatorname{csgn}\left(ic\left(b\,x^3+a\right)^p\right)\ln(ex+d)}{2e}+\frac{i\pi \operatorname{csgn}(ic)\operatorname{csgn}\left(ic\left(b\,x^3+a\right)^p\right)^2\ln(ex+d)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/(e*x+d),x)
[Out] ln(e*x+d)/e*ln((b*x^3+a)^p)-p/e*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-
e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))+1/2*I
*ln(e*x+d)/e*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-1/2*I*ln(e*x+d)
/e*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-1/2*I*ln(e*x+d)/e
*Pi*csgn(I*c*(b*x^3+a)^p)^3+1/2*I*ln(e*x+d)/e*Pi*csgn(I*c*(b*x^3+a)^p)^2*cs
gn(I*c)+1/e*ln(c)*ln(e*x+d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(b\,x^3+a\right)^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/(d + e*x),x)
```

```
[Out] int(log(c*(a + b*x^3)^p)/(d + e*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

3.196 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{(d+ex)^2}\,dx$

Optimal. Leaf size=292

$$-\frac{\sqrt[3]{a}\sqrt[3]{b}\,p\left(\sqrt[3]{a}\,e+\sqrt[3]{b}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\left(bd^3-ae^3\right)}-\frac{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}e^2+\sqrt[3]{a}\sqrt[3]{b}\,de+b^{2/3}d^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)}+\ldots$$

```
[Out] a^(1/3)*b^(1/3)*(b^(1/3)*d+a^(1/3)*e)*p*ln(a^(1/3)+b^(1/3)*x)/(-a*e^3+b*d^3
)-3*b*d^2*p*ln(e*x+d)/e/(-a*e^3+b*d^3)-1/2*a^(1/3)*b^(1/3)*(b^(1/3)*d+a^(1/
3)*e)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-a*e^3+b*d^3)+b*d^2*p*ln
(b*x^3+a)/e/(-a*e^3+b*d^3)-ln(c*(b*x^3+a)^p)/e/(e*x+d)-a^(1/3)*b^(1/3)*p*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/(b^(2/3)*d^2+a^(1/3
)*b^(1/3)*d*e+a^(2/3)*e^2)
```

Rubi [A] time = 0.55, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2463, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\sqrt[3]{a}\sqrt[3]{b}\,p\left(\sqrt[3]{a}\,e+\sqrt[3]{b}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\left(bd^3-ae^3\right)}-\frac{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}e^2+\sqrt[3]{a}\sqrt[3]{b}\,de+b^{2/3}d^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(d + e*x)^2,x]
```

```
[Out] -((Sqrt[3]*a^(1/3)*b^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(b^(2/3)*d^2 + a^(1/3)*b^(1/3)*d*e + a^(2/3)*e^2)) + (a^(1/3)*b^(1/3)*
(b^(1/3)*d + a^(1/3)*e)*p*Log[a^(1/3) + b^(1/3)*x])/(b*d^3 - a*e^3) - (3*b*
d^2*p*Log[d + e*x])/(e*(b*d^3 - a*e^3)) - (a^(1/3)*b^(1/3)*(b^(1/3)*d + a^(
1/3)*e)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*(b*d^3 - a*e^3
)) + (b*d^2*p*Log[a + b*x^3])/(e*(b*d^3 - a*e^3)) - Log[c*(a + b*x^3)^p]/(e
*(d + e*x))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{(d+ex)^2}\,dx = -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)} + \frac{(3bp)\int \frac{x^2}{(d+ex)\left(a+bx^3\right)}\,dx}{e}$$

$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)} + \frac{(3bp)\int \left(-\frac{d^2e}{\left(bd^3-ae^3\right)(d+ex)} + \frac{ade-ae^2x+bd^2x^2}{\left(bd^3-ae^3\right)\left(a+bx^3\right)}\right)dx}{e}$$

$$= -\frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)} + \frac{(3bp)\int \frac{ade-ae^2x+bd^2x^2}{a+bx^3}\,dx}{e\left(bd^3-ae^3\right)}$$

$$= -\frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)} + \frac{(3bp)\int \frac{ade-ae^2x}{a+bx^3}\,dx}{e\left(bd^3-ae^3\right)} + \frac{\left(3b^2d^2p\right)\int \frac{x^2}{a+bx^3}}{e\left(bd^3-ae^3\right)}$$

$$= -\frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} + \frac{bd^2p\log\left(a+bx^3\right)}{e\left(bd^3-ae^3\right)} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)} + \frac{\left(b^{2/3}p\right)\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}\right.}{}}{a^2}$$

$$= \frac{\sqrt[3]{a}\,b^{2/3}\left(d+\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{bd^3-ae^3} - \frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} + \frac{bd^2p\log\left(a+bx^3\right)}{e\left(bd^3-ae^3\right)} -$$

$$= \frac{\sqrt[3]{a}\,b^{2/3}\left(d+\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{bd^3-ae^3} - \frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} - \frac{\sqrt[3]{a}\,b^{2/3}\left(d+\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)p\,\mathrm{lo}}{2\left(b\right.}$$

$$= -\frac{\sqrt{3}\,\sqrt[3]{a}\,\sqrt[3]{b}\,\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{bd^3-ae^3} + \frac{\sqrt[3]{a}\,b^{2/3}\left(d+\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{\right.}}{bd^3-ae^3}$$

Mathematica [C] time = 0.64, size = 202, normalized size = 0.69

$$-\frac{\frac{b^{2/3}dp\left(\sqrt[3]{a}\,e\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)-2\sqrt[3]{b}\,d\log\left(a+bx^3\right)-2\sqrt[3]{a}\,e\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+2\sqrt{3}\,\sqrt[3]{a}\,e\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6\sqrt[3]{b}\,d\log(d+ex)\right)+3be^2px^2\,{}_2F_1\left(\right.}{2bd^3-2ae^3}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(d + e*x)^2,x]
```

```
[Out] -(((3*b*e^2*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)] + b^(2/3)*d*
p*(2*Sqrt[3]*a^(1/3)*e*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*a^(1
/3)*e*Log[a^(1/3) + b^(1/3)*x] + 6*b^(1/3)*d*Log[d + e*x] + a^(1/3)*e*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*b^(1/3)*d*Log[a + b*x^3]))/(2
*b*d^3 - 2*a*e^3) + Log[c*(a + b*x^3)^p]/(d + e*x))/e)
```

fricas [C] time = 1.53, size = 7010, normalized size = 24.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*d^4*e - a*d*e^4 + (b*d^3*e^2 - a*e^5)*x)*(2*b*d^2*p/(b*d^3*e - a*
e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*
```

```
sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2
 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/
(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p
^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6)
+ 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))*log(3/2*(2*b*d^2*p/
(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 -
 a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^
3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) +
 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 -
3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*
e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*
e*p + b*e*p^2*x - 2*b*d*p^2 - 1/4*(b*d^3*e^2 - a*e^5)*(2*b*d^2*p/(b*d^3*e -
 a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(
-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*
e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p
^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^
3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^
6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))^2) - 4*(b*d^3 -
a*e^3)*p*log(b*x^3 + a) + (6*b*d^2*e*p*x + 6*b*d^3*p - (b*d^4*e - a*d*e^4 +
 (b*d^3*e^2 - a*e^5)*x)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*
e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(
b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)
) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) -
(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*
d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^
3)^2)^(1/3)*(I*sqrt(3) + 1)) + sqrt(3)*(b*d^4*e - a*d*e^4 + (b*d^3*e^2 - a*
e^5)*x)*sqrt(-((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(2*b*d^2*p/(b*d^3*e
- a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*
(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3
*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*
p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d
^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e
^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))^2 - 4*(b^2*d^5*
e - a*b*d^2*e^4)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e
^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e
 - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2
*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^
6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e -
 a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(
1/3)*(I*sqrt(3) + 1))*p + 4*(b^2*d^4 - 4*a*b*d*e^3)*p^2)/(b^2*d^6*e^2 - 2*a
*b*d^3*e^5 + a^2*e^8)))*log(-3/2*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^
2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*
d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e
 - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)
^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a
*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d
^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*e*p + 2*b*e*p^2*x + 2*b*d*p^2 +
 1/4*(b*d^3*e^2 - a*e^5)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3
*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/
(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4
)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) -
 (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b
*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e
^3)^2)^(1/3)*(I*sqrt(3) + 1))^2 + 1/4*sqrt(3)*(b*d^3*e^2 - a*e^5)*(2*b*d^2*
p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2
 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*
p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6)
 + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3
- 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^
```

```
3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))*sqrt
(-((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (
b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3)
+ 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5
)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 -
 a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d
^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*
b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))^2 - 4*(b^2*d^5*e - a*b*d^2*
e^4)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*
p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3
- 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^
3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3
*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1
/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt
(3) + 1))*p + 4*(b^2*d^4 - 4*a*b*d*e^3)*p^2)/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 +
 a^2*e^8))) + (6*b*d^2*e*p*x + 6*b*d^3*p - (b*d^4*e - a*d*e^4 + (b*d^3*e^2
- a*e^5)*x)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2
 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*
e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^
3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3
/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^
4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*
(I*sqrt(3) + 1)) - sqrt(3)*(b*d^4*e - a*d*e^4 + (b*d^3*e^2 - a*e^5)*x)*sqrt
(-((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (
b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3)
+ 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5
)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 -
 a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d
^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*
b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))^2 - 4*(b^2*d^5*e - a*b*d^2*
e^4)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*
p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3
- 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^
3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3
*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1
/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt
(3) + 1))*p + 4*(b^2*d^4 - 4*a*b*d*e^3)*p^2)/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 +
 a^2*e^8)))*log(-3/2*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e -
 a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d
^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) +
 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^
3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3
*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^
2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*e*p + 2*b*e*p^2*x + 2*b*d*p^2 + 1/4*(b*d^3*
e^2 - a*e^5)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^
2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a
*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p
^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^
3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e
^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)
*(I*sqrt(3) + 1))^2 - 1/4*sqrt(3)*(b*d^3*e^2 - a*e^5)*(2*b*d^2*p/(b*d^3*e -
 a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(
-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*
e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p
^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^
3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^
6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))*sqrt(-((b^2*d^6*
e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/
(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^
```

```
6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e -
 a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(
1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e
^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3
 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))^2 - 4*(b^2*d^5*e - a*b*d^2*e^4)*(2*b*d^
2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e
^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^
3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^
6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^
3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*
d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))*p
+ 4*(b^2*d^4 - 4*a*b*d*e^3)*p^2)/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)))
- 12*(b*d^2*e*p*x + b*d^3*p)*log(e*x + d) - 4*(b*d^3 - a*e^3)*log(c))/(b*d^
4*e - a*d*e^4 + (b*d^3*e^2 - a*e^5)*x)
```

giac [A] time = 0.32, size = 398, normalized size = 1.36

$$\frac{bd^2p\log\left(\left|bx^3+a\right|\right)}{bd^3e-ae^4}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}bp\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2d^2-\left(-ab^2\right)^{\frac{1}{3}}bde+\left(-ab^2\right)^{\frac{2}{3}}e^2}-\frac{\left(ab^3d^4pe^2-ab^3d^3p\left(-\frac{a}{b}\right)^{\frac{1}{3}}e^3-a^2b^2dpe^5+a^2b^2p\left(-\right.\right.}{ab^3d^6e^2-2a^2b^2d^3e^5+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] b*d^2*p*log(abs(b*x^3 + a))/(b*d^3*e - a*e^4) + sqrt(3)*(-a*b^2)^(1/3)*b*p*
arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(b^2*d^2 - (-a*b^2)^(
1/3)*b*d*e + (-a*b^2)^(2/3)*e^2) - (a*b^3*d^4*p*e^2 - a*b^3*d^3*p*(-a/b)^(1
/3)*e^3 - a^2*b^2*d*p*e^5 + a^2*b^2*p*(-a/b)^(1/3)*e^6)*(-a/b)^(1/3)*log(ab
s(x - (-a/b)^(1/3)))/(a*b^3*d^6*e^2 - 2*a^2*b^2*d^3*e^5 + a^3*b*e^8) + 1/2*
((-a*b^2)^(1/3)*b*d*p - (-a*b^2)^(2/3)*p*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/
b)^(2/3))/(b^2*d^3 - a*b*e^3) - (3*b*d^2*p*x*e*log(x*e + d) + b*d^3*p*log(b
*x^3 + a) + 3*b*d^3*p*log(x*e + d) + b*d^3*log(c) - a*p*e^3*log(b*x^3 + a)
- a*e^3*log(c))/(b*d^3*x*e^2 + b*d^4*e - a*x*e^5 - a*d*e^4)
```

maple [C] time = 0.83, size = 1068, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/(e*x+d)^2,x)
```

```
[Out] -1/e/(e*x+d)*ln((b*x^3+a)^p)+1/2*(-I*Pi*b*d^3*csgn(I*c*(b*x^3+a)^p)^3-I*Pi*
a*e^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2+I*Pi*a*e^3*csgn(I*(b*x^3+
a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)+I*Pi*b*d^3*csgn(I*(b*x^3+a)^p)*csgn(I
*c*(b*x^3+a)^p)^2-I*Pi*b*d^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn
(I*c)+I*Pi*a*e^3*csgn(I*c*(b*x^3+a)^p)^3+I*Pi*b*d^3*csgn(I*c*(b*x^3+a)^p)^2
*csgn(I*c)-I*Pi*a*e^3*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+2*sum(_R*ln(((-4*a*
e^7-2*b*d^3*e^4)*_R^3-3*_R^2*b*d^2*e^3*p+8*_R*b*d*e^2*p^2-3*b*e*p^3)*x+(-5*
a*d*e^6-b*d^4*e^3)*_R^3+(a*e^5*p-b*d^3*e^2*p)*_R^2+5*b*d^2*e*p^2*_R-3*b*d*p
^3),_R=RootOf((a*e^6-b*d^3*e^3)*_Z^3+3*b*d^2*e^2*p*_Z^2-3*b*d*e*p^2*_Z+b*p^
3))*a*e^5*x-2*sum(_R*ln(((-4*a*e^7-2*b*d^3*e^4)*_R^3-3*_R^2*b*d^2*e^3*p+8*_
R*b*d*e^2*p^2-3*b*e*p^3)*x+(-5*a*d*e^6-b*d^4*e^3)*_R^3+(a*e^5*p-b*d^3*e^2*p
)*_R^2+5*b*d^2*e*p^2*_R-3*b*d*p^3),_R=RootOf((a*e^6-b*d^3*e^3)*_Z^3+3*b*d^2
*e^2*p*_Z^2-3*b*d*e*p^2*_Z+b*p^3))*b*d^3*e^2*x+2*sum(_R*ln(((-4*a*e^7-2*b*d
^3*e^4)*_R^3-3*_R^2*b*d^2*e^3*p+8*_R*b*d*e^2*p^2-3*b*e*p^3)*x+(-5*a*d*e^6-b
*d^4*e^3)*_R^3+(a*e^5*p-b*d^3*e^2*p)*_R^2+5*b*d^2*e*p^2*_R-3*b*d*p^3),_R=Ro
```

```
otOf((a*e^6-b*d^3*e^3)*_Z^3+3*b*d^2*e^2*p*_Z^2-3*b*d*e*p^2*_Z+b*p^3))*a*d*e
^4-2*sum(_R*ln(((-4*a*e^7-2*b*d^3*e^4)*_R^3-3*_R^2*b*d^2*e^3*p+8*_R*b*d*e^2
*p^2-3*b*e*p^3)*x+(-5*a*d*e^6-b*d^4*e^3)*_R^3+(a*e^5*p-b*d^3*e^2*p)*_R^2+5*
b*d^2*e*p^2*_R-3*b*d*p^3),_R=RootOf((a*e^6-b*d^3*e^3)*_Z^3+3*b*d^2*e^2*p*_Z
^2-3*b*d*e*p^2*_Z+b*p^3))*b*d^4*e+6*ln(-e*x-d)*b*d^2*e*p*x+6*ln(-e*x-d)*b*d
^3*p-2*ln(c)*a*e^3+2*b*d^3*ln(c))/(e*x+d)/e/(a*e^3-b*d^3)
```

maxima [A] time = 1.01, size = 311, normalized size = 1.07

$$-\frac{\frac{6\,d^2\log(ex+d)}{bd^3-ae^3}+\frac{2\sqrt{3}\left(ae^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-ade\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(b^2d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-abe^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(2\,bd^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-ae^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-ade\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^2d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-abe^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\left(bd^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ae^2 \cdots\right.}{b^2d^3\left(\cdots\right.}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(6*d^2*log(e*x + d)/(b*d^3 - a*e^3) + 2*sqrt(3)*(a*e^2*(a/b)^(2/3) - a
*d*e*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2
*d^3*(a/b)^(2/3) - a*b*e^3*(a/b)^(2/3))*(a/b)^(1/3)) - (2*b*d^2*(a/b)^(2/3)
 - a*e^2*(a/b)^(1/3) - a*d*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*d
^3*(a/b)^(2/3) - a*b*e^3*(a/b)^(2/3)) - 2*(b*d^2*(a/b)^(2/3) + a*e^2*(a/b)^
(1/3) + a*d*e)*log(x + (a/b)^(1/3))/(b^2*d^3*(a/b)^(2/3) - a*b*e^3*(a/b)^(2
/3)))*b*p/e - log((b*x^3 + a)^p*c)/((e*x + d)*e)
```

mupad [B] time = 0.49, size = 736, normalized size = 2.52

$$\left(\sum_{k=1}^{3}\ln\left(-\frac{27\,a\,b^4\,d\,p^3+27\,a\,b^4\,e\,p^3\,x+\mathrm{root}\left(b\,d^3\,e^3\,z^3-a\,e^6\,z^3-3\,b\,d^2\,e^2\,p\,z^2+3\,b\,d\,e\,p^2\,z-b\,p^3,z,k\right)^3\,a\,b^4\,d^4 \cdots}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/(d + e*x)^2,x)
```

```
[Out] symsum(log(-(27*a*b^4*d*p^3 + 27*a*b^4*e*p^3*x + 9*root(b*d^3*e^3*z^3 - a*e
^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^3*a*b^4*d^4*e^3 +
 45*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*
p^3, z, k)^3*a^2*b^3*d*e^6 - 9*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2
*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^2*a^2*b^3*e^5*p + 36*root(b*d^3*e^3*z
^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^3*a^2*b^3
*e^7*x + 9*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2
*z - b*p^3, z, k)^2*a*b^4*d^3*e^2*p + 18*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3
*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^3*a*b^4*d^3*e^4*x - 45*root
(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z,
k)*a*b^4*d^2*e*p^2 - 72*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2
+ 3*b*d*e*p^2*z - b*p^3, z, k)*a*b^4*d*e^2*p^2*x + 27*root(b*d^3*e^3*z^3 -
a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^2*a*b^4*d^2*e^
3*p*x)/e^2)*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^
2*z - b*p^3, z, k), k, 1, 3) - log(c*(a + b*x^3)^p)/(d*e + e^2*x) + (3*b*d^
2*p*log(d + e*x))/(a*e^4 - b*d^3*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.197 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{(d+ex)^3}\,dx$

Optimal. Leaf size=391

$$-\frac{\sqrt[3]{a}\,b^{2/3}p\left(3\sqrt[3]{a}\,b^{2/3}d^2e+ae^3+2bd^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{4\left(bd^3-ae^3\right)^2}+\frac{\sqrt[3]{a}\,b^{2/3}p\left(3\sqrt[3]{a}\,b^{2/3}d^2e+ae^3+2bd^3\right)\log\left(\right.}{2\left(bd^3-ae^3\right)^2}$$

```
[Out] 3/2*b*d^2*p/e/(-a*e^3+b*d^3)/(e*x+d)+1/2*a^(1/3)*b^(2/3)*(2*b*d^3+3*a^(1/3)
*b^(2/3)*d^2*e+a*e^3)*p*ln(a^(1/3)+b^(1/3)*x)/(-a*e^3+b*d^3)^2-3/2*b*d*(2*a
*e^3+b*d^3)*p*ln(e*x+d)/e/(-a*e^3+b*d^3)^2-1/4*a^(1/3)*b^(2/3)*(2*b*d^3+3*a
^(1/3)*b^(2/3)*d^2*e+a*e^3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-a
*e^3+b*d^3)^2+1/2*b*d*(2*a*e^3+b*d^3)*p*ln(b*x^3+a)/e/(-a*e^3+b*d^3)^2-1/2*
ln(c*(b*x^3+a)^p)/e/(e*x+d)^2-1/2*a^(1/3)*b^(2/3)*(2*b*d^3-3*a^(1/3)*b^(2/3
)*d^2*e+a*e^3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/
(-a*e^3+b*d^3)^2
```

Rubi [A] time = 0.71, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2463, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\sqrt[3]{a}\,b^{2/3}p\left(3\sqrt[3]{a}\,b^{2/3}d^2e+ae^3+2bd^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{4\left(bd^3-ae^3\right)^2}+\frac{\sqrt[3]{a}\,b^{2/3}p\left(3\sqrt[3]{a}\,b^{2/3}d^2e+ae^3+2bd^3\right)\log\left(\right.}{2\left(bd^3-ae^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(d + e*x)^3,x]
```

```
[Out] (3*b*d^2*p)/(2*e*(b*d^3 - a*e^3)*(d + e*x)) - (Sqrt[3]*a^(1/3)*b^(2/3)*(2*b
*d^3 - 3*a^(1/3)*b^(2/3)*d^2*e + a*e^3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(2*(b*d^3 - a*e^3)^2) + (a^(1/3)*b^(2/3)*(2*b*d^3 + 3*a^(
1/3)*b^(2/3)*d^2*e + a*e^3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*(b*d^3 - a*e^3)^
2) - (3*b*d*(b*d^3 + 2*a*e^3)*p*Log[d + e*x])/(2*e*(b*d^3 - a*e^3)^2) - (a^
(1/3)*b^(2/3)*(2*b*d^3 + 3*a^(1/3)*b^(2/3)*d^2*e + a*e^3)*p*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*(b*d^3 - a*e^3)^2) + (b*d*(b*d^3 + 2*a*
e^3)*p*Log[a + b*x^3])/(2*e*(b*d^3 - a*e^3)^2) - Log[c*(a + b*x^3)^p]/(2*e*
(d + e*x)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{(d+ex)^3}\,dx = -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2} + \frac{(3bp)\int \frac{x^2}{(d+ex)^2\left(a+bx^3\right)}\,dx}{2e}$$

$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2} + \frac{(3bp)\int\left(-\frac{d^2e}{\left(bd^3-ae^3\right)(d+ex)^2} - \frac{de\left(bd^3+2ae^3\right)}{\left(bd^3-ae^3\right)^2(d+ex)} + \frac{ae\left(2bd^3+ae^3\right)-3abd^2e}{\left(bd^3-ae^3\right)}\right.}{2e}$$

$$= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)} - \frac{3bd\left(bd^3+2ae^3\right)p\log(d+ex)}{2e\left(bd^3-ae^3\right)^2} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2} + \frac{(3b}{}$$

$$= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)} - \frac{3bd\left(bd^3+2ae^3\right)p\log(d+ex)}{2e\left(bd^3-ae^3\right)^2} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2} + \frac{(3b}{}$$

$$= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)} - \frac{3bd\left(bd^3+2ae^3\right)p\log(d+ex)}{2e\left(bd^3-ae^3\right)^2} + \frac{bd\left(bd^3+2ae^3\right)p\log\left(a\right.}{2e\left(bd^3-ae^3\right)^2}$$

$$= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)} + \frac{\sqrt[3]{a}\,b^{2/3}\left(2bd^3+3\sqrt[3]{a}\,b^{2/3}d^2e+ae^3\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2\left(bd^3-ae^3\right)^2} -$$

$$= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)} + \frac{\sqrt[3]{a}\,b^{2/3}\left(2bd^3+3\sqrt[3]{a}\,b^{2/3}d^2e+ae^3\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2\left(bd^3-ae^3\right)^2} -$$

$$= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)} - \frac{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}\left(2bd^3-3\sqrt[3]{a}\,b^{2/3}d^2e+ae^3\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2\left(bd^3-ae^3\right)^2}$$

Mathematica [C] time = 0.69, size = 303, normalized size = 0.77

$$\frac{\frac{b^{2/3}p(d+ex)\left(-\sqrt[3]{a}\,e(d+ex)\left(ae^3+2bd^3\right)\left(\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)-9b^{4/3}d^2e^2x^2(d+ex)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right)+2\sqrt[3]{b}\,d(d+ex)\left(2ae^3\right.}{\left(bd^3-ae^3\right)^2}}{4e(d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(d + e*x)^3,x]
```

```
[Out] ((b^(2/3)*p*(d + e*x)*(6*b^(1/3)*d^2*(b*d^3 - a*e^3) - 9*b^(4/3)*d^2*e^2*x^
2*(d + e*x)*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)] + 2*a^(1/3)*e*(2*b
*d^3 + a*e^3)*(d + e*x)*Log[a^(1/3) + b^(1/3)*x] - 6*b^(1/3)*d*(b*d^3 + 2*a
*e^3)*(d + e*x)*Log[d + e*x] - a^(1/3)*e*(2*b*d^3 + a*e^3)*(d + e*x)*(2*Sqr
t[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2]) + 2*b^(1/3)*d*(b*d^3 + 2*a*e^3)*(d + e*x)*Log[a + b
*x^3]))/(b*d^3 - a*e^3)^2 - 2*Log[c*(a + b*x^3)^p])/(4*e*(d + e*x)^2)
```

fricas [C] time = 7.23, size = 13236, normalized size = 33.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(24*(b^2*d^5*e - a*b*d^2*e^4)*p*x + 2*(b^2*d^8*e - 2*a*b*d^5*e^4 + a^2
*d^2*e^7 + (b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9)*x^2 + 2*(b^2*d^7*e^2 - 2
*a*b*d^4*e^5 + a^2*d*e^8)*x)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a
^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^
7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2
*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))
+ 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*
e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d
^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^
3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*
b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9
) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2
*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) +
 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))*
log((8*b^2*d^3*e + a*b*e^4)*p^2*x - 3/16*(b^2*d^8*e^2 - 2*a*b*d^5*e^5 + a^2
*d^2*e^8)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*
p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) +
 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^
3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2
*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d
^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^
4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((
b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7
)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 +
 a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^
2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p
+ 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))^2 + 1/4*(10*b^2*d^6
*e + 16*a*b*d^3*e^4 + a^2*e^7)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 +
 a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*
e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b
^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)
) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 +
a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2
*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*
e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*
a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e
^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p +
 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3)
 + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)
)*p - (7*b^2*d^4 + 2*a*b*d*e^3)*p^2) - 8*(b^2*d^6 - 2*a*b*d^3*e^3 + a^2*e^6
)*p*log(b*x^3 + a) + 24*(b^2*d^6 - a*b*d^3*e^3)*p + (12*(b^2*d^4*e^2 + 2*a*
b*d*e^5)*p*x^2 + 24*(b^2*d^5*e + 2*a*b*d^2*e^4)*p*x - (b^2*d^8*e - 2*a*b*d^
5*e^4 + a^2*d^2*e^7 + (b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9)*x^2 + 2*(b^2*
d^7*e^2 - 2*a*b*d^4*e^5 + a^2*d*e^8)*x)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*
d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e
^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^
2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 +
 a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8
*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*
p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p +
 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d
^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^
6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^
2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(
I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 +
 a^2*e^7)) + 12*(b^2*d^6 + 2*a*b*d^3*e^3)*p + sqrt(3)*(b^2*d^8*e - 2*a*b*d^
5*e^4 + a^2*d^2*e^7 + (b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9)*x^2 + 2*(b^2*
d^7*e^2 - 2*a*b*d^4*e^5 + a^2*d*e^8)*x)*sqrt(-((b^4*d^12*e^2 - 4*a*b^3*d^9*
```

e^5 + 6*a^2*b^2*d^6*e^8 - 4*a^3*b*d^3*e^11 + a^4*e^14)*((b^2*d^2*p^2/(b^2*d
^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*
e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*
d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e -
2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2
e^9) + 1/16(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p
 + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/
16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 +
a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3
 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e
^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*
e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e -
2*a*b*d^3*e^4 + a^2*e^7))^2 - 8*(b^4*d^10*e - 3*a^2*b^2*d^4*e^7 + 2*a^3*b*d
e^10)((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p +
 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)
/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e
^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^
6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3
- a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 +
 a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2
*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))
+ 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*
e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d
^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2
*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))*p + 16*(b^4*d^8 - 20*a
*b^3*d^5*e^3 - 8*a^2*b^2*d^2*e^6)*p^2)/(b^4*d^12*e^2 - 4*a*b^3*d^9*e^5 + 6*
a^2*b^2*d^6*e^8 - 4*a^3*b*d^3*e^11 + a^4*e^14)))*log(2*(8*b^2*d^3*e + a*b*e
^4)*p^2*x + 3/16*(b^2*d^8*e^2 - 2*a*b*d^5*e^5 + a^2*d^2*e^8)*((b^2*d^2*p^2/
(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^
2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p +
2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^
6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6
 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2
*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) -
4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*
e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d
^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3
 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4
+ a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^
6*e - 2*a*b*d^3*e^4 + a^2*e^7))^2 - 1/4*(10*b^2*d^6*e + 16*a*b*d^3*e^4 + a^
2*e^7)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p +
 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)
/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e
^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^
6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3
- a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 +
 a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2
*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))
+ 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*
e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d
^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2
*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))*p + (7*b^2*d^4 + 2*a*b
*d*e^3)*p^2 + 1/16*sqrt(3)*(3*(b^2*d^8*e^2 - 2*a*b*d^5*e^5 + a^2*d^2*e^8)*(
(b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d
*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*
(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2
e^8)(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 -
2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)
^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7

)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 4*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)*p)*sqrt(-((b^4*d^12*e^2 - 4*a*b^3*d^9*e^5 + 6*a^2*b^2*d^6*e^8 - 4*a^3*b*d^3*e^11 + a^4*e^14)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))^2 - 8*(b^4*d^10*e - 3*a^2*b^2*d^4*e^7 + 2*a^3*b*d*e^10)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))*p + 16*(b^4*d^8 - 20*a*b^3*d^5*e^3 - 8*a^2*b^2*d^2*e^6)*p^2)/(b^4*d^12*e^2 - 4*a*b^3*d^9*e^5 + 6*a^2*b^2*d^6*e^8 - 4*a^3*b*d^3*e^11 + a^4*e^14))) + (12*(b^2*d^4*e^2 + 2*a*b*d*e^5)*p*x^2 + 24*(b^2*d^5*e + 2*a*b*d^2*e^4)*p*x - (b^2*d^8*e - 2*a*b*d^5*e^4 + a^2*d^2*e^7 + (b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9)*x^2 + 2*(b^2*d^7*e^2 - 2*a*b*d^4*e^5 + a^2*d*e^8)*x)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 12*(b^2*d^6 + 2*a*b*d^3*e^3)*p - sqrt(3)*(b^2*d^8*e - 2*a*b*d^5*e^4 + a^2*d^2*e^7 + (b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9)*x^2 + 2*(b^2*d^7*e^2 - 2*a*b*d^4*e^5 + a^2*d*e^8)*x)*sqrt(-((b^4*d^12*e^2 - 4*a*b^3*d^9*e^5 + 6*a^2*b^2*d^6*e^8 - 4*a^3*b*d^3*e^11 + a^4*e^14)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*

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b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9
) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2
*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) +
 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))^
2 - 8*(b^4*d^10*e - 3*a^2*b^2*d^4*e^7 + 2*a^3*b*d*e^10)*((b^2*d^2*p^2/(b^2*
d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6
*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b
*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e -
 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^
2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*
p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3
/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 +
 a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^
3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*
e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2
*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e -
 2*a*b*d^3*e^4 + a^2*e^7))*p + 16*(b^4*d^8 - 20*a*b^3*d^5*e^3 - 8*a^2*b^2*d
^2*e^6)*p^2)/(b^4*d^12*e^2 - 4*a*b^3*d^9*e^5 + 6*a^2*b^2*d^6*e^8 - 4*a^3*b*
d^3*e^11 + a^4*e^14)))*log(2*(8*b^2*d^3*e + a*b*e^4)*p^2*x + 3/16*(b^2*d^8*
e^2 - 2*a*b*d^5*e^5 + a^2*d^2*e^8)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e
^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 +
a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2
/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*
e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^
3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/
(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*
b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e
- 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a
^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4
*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqr
t(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*
e^7))^2 - 1/4*(10*b^2*d^6*e + 16*a*b*d^3*e^4 + a^2*e^7)*((b^2*d^2*p^2/(b^2*
d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6
*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b
*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e -
 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^
2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*
p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3
/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 +
 a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^
3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*
e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2
*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e -
 2*a*b*d^3*e^4 + a^2*e^7))*p + (7*b^2*d^4 + 2*a*b*d*e^3)*p^2 - 1/16*sqrt(3)
*(3*(b^2*d^8*e^2 - 2*a*b*d^5*e^5 + a^2*d^2*e^8)*((b^2*d^2*p^2/(b^2*d^6*e^2
- 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a
*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p
)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d
^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) +
 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*
b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2
*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8
)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*
b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 +
 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)
^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d
^3*e^4 + a^2*e^7)) + 4*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)*p)*sqrt(-((b^4
*d^12*e^2 - 4*a*b^3*d^9*e^5 + 6*a^2*b^2*d^6*e^8 - 4*a^3*b*d^3*e^11 + a^4*e^
14)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*
```

```
a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-
3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5
+ a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e
^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a
*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^
2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b^2*d^
6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1
/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3
)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2*d^6*
e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*
b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))^2 - 8*(b^4*d^10*e - 3*a^2
*b^2*d^4*e^7 + 2*a^3*b*d*e^10)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 +
 a^2*e^8) - (b^2*d^4*p + 2*a*b*d*e^3*p)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*
e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*e^3*p)*b^2*d^2*p^2/((b
^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)
) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 +
a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*e^3*p)^3/(b^2
*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*
e^3*p)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*
a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e
^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p +
 2*a*b*d*e^3*p)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3)
 + 1) + 4*(b^2*d^4*p + 2*a*b*d*e^3*p)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)
)*p + 16*(b^4*d^8 - 20*a*b^3*d^5*e^3 - 8*a^2*b^2*d^2*e^6)*p^2)/(b^4*d^12*e^
2 - 4*a*b^3*d^9*e^5 + 6*a^2*b^2*d^6*e^8 - 4*a^3*b*d^3*e^11 + a^4*e^14))) -
24*((b^2*d^4*e^2 + 2*a*b*d*e^5)*p*x^2 + 2*(b^2*d^5*e + 2*a*b*d^2*e^4)*p*x +
 (b^2*d^6 + 2*a*b*d^3*e^3)*p)*log(e*x + d) - 8*(b^2*d^6 - 2*a*b*d^3*e^3 + a
^2*e^6)*log(c))/(b^2*d^8*e - 2*a*b*d^5*e^4 + a^2*d^2*e^7 + (b^2*d^6*e^3 - 2
*a*b*d^3*e^6 + a^2*e^9)*x^2 + 2*(b^2*d^7*e^2 - 2*a*b*d^4*e^5 + a^2*d*e^8)*x
)
```

giac [B] time = 0.72, size = 790, normalized size = 2.02

$$-\frac{\left(2\,ab^5d^9pe^2 - 3\,ab^5d^8p\left(-\frac{a}{b}\right)^{\frac{1}{3}}e^3 - 3\,a^2b^4d^6pe^5 + 6\,a^2b^4d^5p\left(-\frac{a}{b}\right)^{\frac{1}{3}}e^6 - 3\,a^3b^3d^2p\left(-\frac{a}{b}\right)^{\frac{1}{3}}e^9 + a^4b^2pe^{11}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log}{2\left(ab^5d^{12}e^2 - 4\,a^2b^4d^9e^5 + 6\,a^3b^3d^6e^8 - 4\,a^4b^2d^3e^{11} + a^5be^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*b^5*d^9*p*e^2 - 3*a*b^5*d^8*p*(-a/b)^(1/3)*e^3 - 3*a^2*b^4*d^6*p*
e^5 + 6*a^2*b^4*d^5*p*(-a/b)^(1/3)*e^6 - 3*a^3*b^3*d^2*p*(-a/b)^(1/3)*e^9 +
 a^4*b^2*p*e^11)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5*d^12*e^2 -
4*a^2*b^4*d^9*e^5 + 6*a^3*b^3*d^6*e^8 - 4*a^4*b^2*d^3*e^11 + a^5*b*e^14) +
3/2*(2*(-a*b^2)^(1/3)*b*d*p - (-a*b^2)^(2/3)*p*e)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^2*d^4 - 2*sqrt(3)*(-a*b^2)^(1/3)*b*
d^3*e + 2*sqrt(3)*a*b*d*e^3 + 3*sqrt(3)*(-a*b^2)^(2/3)*d^2*e^2 - sqrt(3)*(-
a*b^2)^(1/3)*a*e^4) + 1/4*(2*(-a*b^2)^(1/3)*b*d^3*p - 3*(-a*b^2)^(2/3)*d^2*
p*e + (-a*b^2)^(1/3)*a*p*e^3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^2
*d^6 - 2*a*b*d^3*e^3 + a^2*e^6) + 1/2*(b^2*d^4*p + 2*a*b*d*p*e^3)*log(abs(b
*x^3 + a))/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7) - 1/2*(3*b^2*d^4*p*x^2*e^2
*log(x*e + d) + 6*b^2*d^5*p*x*e*log(x*e + d) - 3*b^2*d^5*p*x*e + b^2*d^6*p*
log(b*x^3 + a) + 3*b^2*d^6*p*log(x*e + d) - 3*b^2*d^6*p + b^2*d^6*log(c) -
2*a*b*d^3*p*e^3*log(b*x^3 + a) + 6*a*b*d*p*x^2*e^5*log(x*e + d) + 12*a*b*d^
2*p*x*e^4*log(x*e + d) + 6*a*b*d^3*p*e^3*log(x*e + d) + 3*a*b*d^2*p*x*e^4 +
 3*a*b*d^3*p*e^3 - 2*a*b*d^3*e^3*log(c) + a^2*p*e^6*log(b*x^3 + a) + a^2*e^
```

```
6*log(c))/(b^2*d^6*x^2*e^3 + 2*b^2*d^7*x*e^2 + b^2*d^8*e - 2*a*b*d^3*x^2*e^
6 - 4*a*b*d^4*x*e^5 - 2*a*b*d^5*e^4 + a^2*x^2*e^9 + 2*a^2*d*x*e^8 + a^2*d^2
*e^7)
```

maple [C] time = 0.84, size = 4085, normalized size = 10.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/(e*x+d)^3,x)
```

```
[Out] -1/2/e/(e*x+d)^2*ln((b*x^3+a)^p)+1/4*(-2*I*Pi*a*b*d^3*e^3*csgn(I*(b*x^3+a)^
p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-6*a*b*d^2*e^4*p*x+6*b^2*d^6*p-I*Pi*b^2*d
^6*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+6*b^2*d^5*e*p*x-2*ln(c)*a^2*e^6-2*ln(c
)*b^2*d^6+2*sum(_R*ln(((-4*a^3*e^13+6*a^2*b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(1
4*a^2*b*d*e^9*p-10*a*b^2*d^4*e^6*p-4*b^3*d^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*p
^2+6*b^3*d^5*e^2*p^2)*_R+3*a*b^2*e^4*p^3)*x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9-
3*a*b^2*d^7*e^6-b^3*d^10*e^3)*_R^3+(8*a^2*b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^3
*d^8*e^2*p)*_R^2+(-a^2*b*e^7*p^2+5*a*b^2*d^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3*
a*b^2*d*e^3*p^3-3*b^3*d^4*p^3),_R=RootOf((a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^3
)*_Z^3+(-6*a*b*d*e^5*p-3*b^2*d^4*e^2*p)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*a
^2*e^9*x^2+2*sum(_R*ln(((-4*a^3*e^13+6*a^2*b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(
14*a^2*b*d*e^9*p-10*a*b^2*d^4*e^6*p-4*b^3*d^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*
p^2+6*b^3*d^5*e^2*p^2)*_R+3*a*b^2*e^4*p^3)*x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9
-3*a*b^2*d^7*e^6-b^3*d^10*e^3)*_R^3+(8*a^2*b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^
3*d^8*e^2*p)*_R^2+(-a^2*b*e^7*p^2+5*a*b^2*d^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3
*a*b^2*d*e^3*p^3-3*b^3*d^4*p^3),_R=RootOf((a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^
3)*_Z^3+(-6*a*b*d*e^5*p-3*b^2*d^4*e^2*p)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*
a^2*d^2*e^7+2*sum(_R*ln(((-4*a^3*e^13+6*a^2*b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+
(14*a^2*b*d*e^9*p-10*a*b^2*d^4*e^6*p-4*b^3*d^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5
*p^2+6*b^3*d^5*e^2*p^2)*_R+3*a*b^2*e^4*p^3)*x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^
9-3*a*b^2*d^7*e^6-b^3*d^10*e^3)*_R^3+(8*a^2*b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b
^3*d^8*e^2*p)*_R^2+(-a^2*b*e^7*p^2+5*a*b^2*d^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-
3*a*b^2*d*e^3*p^3-3*b^3*d^4*p^3),_R=RootOf((a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e
^3)*_Z^3+(-6*a*b*d*e^5*p-3*b^2*d^4*e^2*p)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))
*b^2*d^8*e-6*ln(e*x+d)*b^2*d^6*p+2*sum(_R*ln(((-4*a^3*e^13+6*a^2*b*d^3*e^10
-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-10*a*b^2*d^4*e^6*p-4*b^3*d^7*e^3*p)*
_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2*p^2)*_R+3*a*b^2*e^4*p^3)*x+(-5*a^3*
d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-b^3*d^10*e^3)*_R^3+(8*a^2*b*d^2*e^8*
p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_R^2+(-a^2*b*e^7*p^2+5*a*b^2*d^3*e^4*p^2
+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p^3-3*b^3*d^4*p^3),_R=RootOf((a^2*e^9-2*
a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*b*d*e^5*p-3*b^2*d^4*e^2*p)*_Z^2+3*b^2*d
^2*e*p^2*_Z-b^2*p^3))*b^2*d^6*e^3*x^2+4*sum(_R*ln(((-4*a^3*e^13+6*a^2*b*d^3
*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-10*a*b^2*d^4*e^6*p-4*b^3*d^7*e^
3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2*p^2)*_R+3*a*b^2*e^4*p^3)*x+(-5
*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-b^3*d^10*e^3)*_R^3+(8*a^2*b*d^2
*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_R^2+(-a^2*b*e^7*p^2+5*a*b^2*d^3*e^
4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p^3-3*b^3*d^4*p^3),_R=RootOf((a^2*e
^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*b*d*e^5*p-3*b^2*d^4*e^2*p)*_Z^2+3*
b^2*d^2*e*p^2*_Z-b^2*p^3))*a^2*d*e^8*x+4*sum(_R*ln(((-4*a^3*e^13+6*a^2*b*d^
3*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-10*a*b^2*d^4*e^6*p-4*b^3*d^7*e
^3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2*p^2)*_R+3*a*b^2*e^4*p^3)*x+(-
5*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-b^3*d^10*e^3)*_R^3+(8*a^2*b*d^
2*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_R^2+(-a^2*b*e^7*p^2+5*a*b^2*d^3*e
^4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p^3-3*b^3*d^4*p^3),_R=RootOf((a^2*
e^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*b*d*e^5*p-3*b^2*d^4*e^2*p)*_Z^2+3
*b^2*d^2*e*p^2*_Z-b^2*p^3))*b^2*d^7*e^2*x-4*sum(_R*ln(((-4*a^3*e^13+6*a^2*b
*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-10*a*b^2*d^4*e^6*p-4*b^3*d^
7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2*p^2)*_R+3*a*b^2*e^4*p^3)*x
+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-b^3*d^10*e^3)*_R^3+(8*a^2*b
```

```
*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_R^2+(-a^2*b*e^7*p^2+5*a*b^2*d^
3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p^3-3*b^3*d^4*p^3),_R=RootOf((a
^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*b*d*e^5*p-3*b^2*d^4*e^2*p)*_Z^
2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*a*b*d^5*e^4+4*ln(c)*a*b*d^3*e^3-6*a*d^3*e^3*
b*p+I*Pi*a^2*e^6*csgn(I*c*(b*x^3+a)^p)^3+I*Pi*b^2*d^6*csgn(I*c*(b*x^3+a)^p)
^3-4*sum(_R*ln(((-4*a^3*e^13+6*a^2*b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b
*d*e^9*p-10*a*b^2*d^4*e^6*p-4*b^3*d^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^
3*d^5*e^2*p^2)*_R+3*a*b^2*e^4*p^3)*x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2
*d^7*e^6-b^3*d^10*e^3)*_R^3+(8*a^2*b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^
2*p)*_R^2+(-a^2*b*e^7*p^2+5*a*b^2*d^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d
*e^3*p^3-3*b^3*d^4*p^3),_R=RootOf((a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+
(-6*a*b*d*e^5*p-3*b^2*d^4*e^2*p)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*a*b*d^3*
e^6*x^2-8*sum(_R*ln(((-4*a^3*e^13+6*a^2*b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(14*
a^2*b*d*e^9*p-10*a*b^2*d^4*e^6*p-4*b^3*d^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2
+6*b^3*d^5*e^2*p^2)*_R+3*a*b^2*e^4*p^3)*x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9-3*
a*b^2*d^7*e^6-b^3*d^10*e^3)*_R^3+(8*a^2*b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d
^8*e^2*p)*_R^2+(-a^2*b*e^7*p^2+5*a*b^2*d^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*
b^2*d*e^3*p^3-3*b^3*d^4*p^3),_R=RootOf((a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*
_Z^3+(-6*a*b*d*e^5*p-3*b^2*d^4*e^2*p)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*a*b
*d^4*e^5*x-6*ln(e*x+d)*b^2*d^4*e^2*p*x^2-12*ln(e*x+d)*b^2*d^5*e*p*x-12*ln(e
*x+d)*a*b*d^3*e^3*p+2*I*Pi*a*b*d^3*e^3*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+2*
I*Pi*a*b*d^3*e^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-I*Pi*a^2*e^6*c
sgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-I*Pi*a^2*e^6*csgn(I*c*(b*x^3+a)^
p)^2*csgn(I*c)-I*Pi*b^2*d^6*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2+I*P
i*a^2*e^6*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)+I*Pi*b^2*d^6*
csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-2*I*Pi*a*b*d^3*e^3*csgn
(I*c*(b*x^3+a)^p)^3-12*ln(e*x+d)*a*b*d*e^5*p*x^2-24*ln(e*x+d)*a*b*d^2*e^4*p
*x)/(e*x+d)^2/(-a*e^3+b*d^3)^2/e
```

maxima [A] time = 1.03, size = 517, normalized size = 1.32

$$-\frac{\left(\frac{2\sqrt{3}\left(3abd^2e^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-2abd^3e\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2e^4\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(b^3d^6\left(\frac{a}{b}\right)^{\frac{2}{3}}-2ab^2d^3e^3\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^2be^6\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{6d^2}{bd^4-ade^3+\left(bd^3e-ae^4\right)x}+\frac{6\left(bd^4+2ade^3\right)\log(ex+d)}{b^2d^6-2abd^3e^3+a^2e^6}-\frac{\left(2b^2d^4\left(\frac{a}{b}\right)^{\frac{2}{3}}+4a\cdots\right.}{\cdots}\right.}{4e\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(3)*(3*a*b*d^2*e^2*(a/b)^(2/3) - 2*a*b*d^3*e*(a/b)^(1/3) - a^2*
e^4*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^3*
d^6*(a/b)^(2/3) - 2*a*b^2*d^3*e^3*(a/b)^(2/3) + a^2*b*e^6*(a/b)^(2/3))*(a/b
)^(1/3)) - 6*d^2/(b*d^4 - a*d*e^3 + (b*d^3*e - a*e^4)*x) + 6*(b*d^4 + 2*a*d
*e^3)*log(e*x + d)/(b^2*d^6 - 2*a*b*d^3*e^3 + a^2*e^6) - (2*b^2*d^4*(a/b)^(
2/3) + 4*a*b*d*e^3*(a/b)^(2/3) - 3*a*b*d^2*e^2*(a/b)^(1/3) - 2*a*b*d^3*e -
a^2*e^4)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*d^6*(a/b)^(2/3) - 2*a*
b^2*d^3*e^3*(a/b)^(2/3) + a^2*b*e^6*(a/b)^(2/3)) - 2*(b^2*d^4*(a/b)^(2/3) +
 2*a*b*d*e^3*(a/b)^(2/3) + 3*a*b*d^2*e^2*(a/b)^(1/3) + 2*a*b*d^3*e + a^2*e^
4)*log(x + (a/b)^(1/3))/(b^3*d^6*(a/b)^(2/3) - 2*a*b^2*d^3*e^3*(a/b)^(2/3)
+ a^2*b*e^6*(a/b)^(2/3)))*b*p/e - 1/2*log((b*x^3 + a)^p*c)/((e*x + d)^2*e)
```

mupad [B] time = 0.88, size = 2227, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/(d + e*x)^3,x)
```

```
[Out] symsum(log(-(27*a*b^6*d^4*p^3 + 216*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3
*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^
2*e*p^2*z + b^2*p^3, z, k)^3*a^2*b^5*d^7*e^6 - 648*root(16*a*b*d^3*e^6*z^3
- 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p
*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^3*a^3*b^4*d^4*e^9 + 72*root(16*a*
b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12
*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^3*a*b^6*d^10*e^3 +
360*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*
e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^3*a^4
*b^3*d*e^12 + 18*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^
3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3
, z, k)*a^3*b^4*e^7*p^2 + 288*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 -
 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^
2*z + b^2*p^3, z, k)^3*a^4*b^3*e^13*x + 27*a^2*b^5*d*e^3*p^3 - 27*a^2*b^5*e
^4*p^3*x + 36*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 +
 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z
, k)^2*a*b^6*d^8*e^2*p + 144*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 -
8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2
*z + b^2*p^3, z, k)^3*a*b^6*d^9*e^4*x - 90*root(16*a*b*d^3*e^6*z^3 - 8*b^2*
d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6
*b^2*d^2*e*p^2*z + b^2*p^3, z, k)*a^2*b^5*d^3*e^4*p^2 + 252*root(16*a*b*d^3
*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*
d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^2*a^2*b^5*d^5*e^5*p - 28
8*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^
5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^2*a^3*b
^4*d^2*e^8*p - 432*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*
z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p
^3, z, k)^3*a^3*b^4*d^3*e^10*x - 90*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3
*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^
2*e*p^2*z + b^2*p^3, z, k)*a*b^6*d^6*e*p^2 - 54*root(16*a*b*d^3*e^6*z^3 - 8
*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^
2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)*a^2*b^5*d^2*e^5*p^2*x + 360*root(16*
a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 +
12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^2*a^2*b^5*d^4*e^6
*p*x - 108*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24
*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k
)*a*b^6*d^5*e^2*p^2*x + 144*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8
*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*
z + b^2*p^3, z, k)^2*a*b^6*d^7*e^3*p*x - 504*root(16*a*b*d^3*e^6*z^3 - 8*b^
2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 -
 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^2*a^3*b^4*d*e^9*p*x)/(8*a^2*e^8 + 8*b^2
*d^6*e^2 - 16*a*b*d^3*e^5))*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8
*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*
z + b^2*p^3, z, k), k, 1, 3) - log(c*(a + b*x^3)^p)/(2*(d^2*e + e^3*x^2 + 2
*d*e^2*x)) - (3*b*d^2*p)/(2*a*d*e^4 - 2*b*d^4*e + 2*a*e^5*x - 2*b*d^3*e^2*x
) - (3*b^2*d^4*p*log(d + e*x))/(2*a^2*e^7 + 2*b^2*d^6*e - 4*a*b*d^3*e^4) -
(6*a*b*d*e^3*p*log(d + e*x))/(2*a^2*e^7 + 2*b^2*d^6*e - 4*a*b*d^3*e^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

3.198 $\int (d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx$

Optimal. Leaf size=139

$$-\frac{p(ad-be)^4\log(ax+b)}{4a^4e}+\frac{be^2px^2(4ad-be)}{8a^2}+\frac{bepx\left(6a^2d^2-4abde+b^2e^2\right)}{4a^3}+\frac{(d+ex)^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e}+\frac{be^3px^3}{12a}+\cdots$$

```
[Out] 1/4*b*e*(6*a^2*d^2-4*a*b*d*e+b^2*e^2)*p*x/a^3+1/8*b*e^2*(4*a*d-b*e)*p*x^2/a
^2+1/12*b*e^3*p*x^3/a+1/4*(e*x+d)^4*ln(c*(a+b/x)^p)/e+1/4*d^4*p*ln(x)/e-1/4
*(a*d-b*e)^4*p*ln(a*x+b)/a^4/e
```

Rubi [A] time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2463, 514, 72}

$$\frac{bepx\left(6a^2d^2-4abde+b^2e^2\right)}{4a^3}+\frac{be^2px^2(4ad-be)}{8a^2}-\frac{p(ad-be)^4\log(ax+b)}{4a^4e}+\frac{(d+ex)^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e}+\frac{be^3px^3}{12a}+\frac{d^4\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Log[c*(a + b/x)^p],x]
```

```
[Out] (b*e*(6*a^2*d^2 - 4*a*b*d*e + b^2*e^2)*p*x)/(4*a^3) + (b*e^2*(4*a*d - b*e)*
p*x^2)/(8*a^2) + (b*e^3*p*x^3)/(12*a) + ((d + e*x)^4*Log[c*(a + b/x)^p])/(4
*e) + (d^4*p*Log[x])/(4*e) - ((a*d - b*e)^4*p*Log[b + a*x])/(4*a^4*e)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{(d+ex)^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e} + \frac{(bp)\int \frac{(d+ex)^4}{\left(a+\frac{b}{x}\right)x^2} dx}{4e}$$
$$= \frac{(d+ex)^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e} + \frac{(bp)\int \frac{(d+ex)^4}{x(b+ax)} dx}{4e}$$
$$= \frac{(d+ex)^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e} + \frac{(bp)\int \left(\frac{e^2(6a^2d^2-4abde+b^2e^2)}{a^3} + \frac{d^4}{bx} + \frac{e^3(4ad-be)x}{a^2} + e\right.}{4e}$$
$$= \frac{be\left(6a^2d^2-4abde+b^2e^2\right)px}{4a^3} + \frac{be^2(4ad-be)px^2}{8a^2} + \frac{be^3px^3}{12a} + \frac{(d+ex)^4 \log}{4e}$$

Mathematica [A] time = 0.15, size = 114, normalized size = 0.82

$$\frac{-\frac{p(ad-be)^4\log(ax+b)}{a^4} + \frac{be^2px\left(2a^2\left(18d^2+6dex+e^2x^2\right)-3abe(8d+ex)+6b^2e^2\right)}{6a^3} + (d+ex)^4\log\left(c\left(a+\frac{b}{x}\right)^p\right) + d^4p\log(x)}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Log[c*(a + b/x)^p],x]
```

```
[Out] ((b*e^2*p*x*(6*b^2*e^2 - 3*a*b*e*(8*d + e*x) + 2*a^2*(18*d^2 + 6*d*e*x + e^
2*x^2)))/(6*a^3) + (d + e*x)^4*Log[c*(a + b/x)^p] + d^4*p*Log[x] - ((a*d -
b*e)^4*p*Log[b + a*x])/a^4)/(4*e)
```

fricas [A] time = 0.41, size = 239, normalized size = 1.72

$$2\,a^3be^3px^3 + 3\left(4\,a^3bde^2 - a^2b^2e^3\right)px^2 + 6\left(6\,a^3bd^2e - 4\,a^2b^2de^2 + ab^3e^3\right)px + 6\left(4\,a^3bd^3 - 6\,a^2b^2d^2e + 4\,ab^3de\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/24*(2*a^3*b*e^3*p*x^3 + 3*(4*a^3*b*d*e^2 - a^2*b^2*e^3)*p*x^2 + 6*(6*a^3*
b*d^2*e - 4*a^2*b^2*d*e^2 + a*b^3*e^3)*p*x + 6*(4*a^3*b*d^3 - 6*a^2*b^2*d^2
*e + 4*a*b^3*d*e^2 - b^4*e^3)*p*log(a*x + b) + 6*(a^4*e^3*x^4 + 4*a^4*d*e^2
*x^3 + 6*a^4*d^2*e*x^2 + 4*a^4*d^3*x)*log(c) + 6*(a^4*e^3*p*x^4 + 4*a^4*d*e
^2*p*x^3 + 6*a^4*d^2*e*p*x^2 + 4*a^4*d^3*p*x)*log((a*x + b)/x))/a^4
```

giac [B] time = 0.30, size = 1659, normalized size = 11.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] -1/24*(24*a^7*b^2*d^3*p*log(-a + (a*x + b)/x) - 36*a^6*b^3*d^2*p*e*log(-a +
 (a*x + b)/x) + 36*a^6*b^3*d^2*p*e - 96*(a*x + b)*a^6*b^2*d^3*p*log(-a + (a
*x + b)/x)/x + 24*a^5*b^4*d*p*e^2*log(-a + (a*x + b)/x) + 144*(a*x + b)*a^5
*b^3*d^2*p*e*log(-a + (a*x + b)/x)/x + 24*a^7*b^2*d^3*log(c) - 36*a^6*b^3*d
^2*e*log(c) + 24*(a*x + b)*a^6*b^2*d^3*p*log((a*x + b)/x)/x - 72*(a*x + b)*
a^5*b^3*d^2*p*e*log((a*x + b)/x)/x - 36*a^5*b^4*d*p*e^2 - 108*(a*x + b)*a^5
*b^3*d^2*p*e/x + 144*(a*x + b)^2*a^5*b^2*d^3*p*log(-a + (a*x + b)/x)/x^2 -
```

```
6*a^4*b^5*p*e^3*log(-a + (a*x + b)/x) - 96*(a*x + b)*a^4*b^4*d*p*e^2*log(-a
 + (a*x + b)/x)/x - 216*(a*x + b)^2*a^4*b^3*d^2*p*e*log(-a + (a*x + b)/x)/x
^2 - 72*(a*x + b)*a^6*b^2*d^3*log(c)/x + 24*a^5*b^4*d*e^2*log(c) + 72*(a*x
+ b)*a^5*b^3*d^2*e*log(c)/x - 72*(a*x + b)^2*a^5*b^2*d^3*p*log((a*x + b)/x)
/x^2 + 72*(a*x + b)*a^4*b^4*d*p*e^2*log((a*x + b)/x)/x + 180*(a*x + b)^2*a^
4*b^3*d^2*p*e*log((a*x + b)/x)/x^2 + 11*a^4*b^5*p*e^3 + 96*(a*x + b)*a^4*b^
4*d*p*e^2/x + 108*(a*x + b)^2*a^4*b^3*d^2*p*e/x^2 - 96*(a*x + b)^3*a^4*b^2*
d^3*p*log(-a + (a*x + b)/x)/x^3 + 24*(a*x + b)*a^3*b^5*p*e^3*log(-a + (a*x
+ b)/x)/x + 144*(a*x + b)^2*a^3*b^4*d*p*e^2*log(-a + (a*x + b)/x)/x^2 + 144
*(a*x + b)^3*a^3*b^3*d^2*p*e*log(-a + (a*x + b)/x)/x^3 + 72*(a*x + b)^2*a^5
*b^2*d^3*log(c)/x^2 - 6*a^4*b^5*e^3*log(c) - 24*(a*x + b)*a^4*b^4*d*e^2*log
(c)/x - 36*(a*x + b)^2*a^4*b^3*d^2*e*log(c)/x^2 + 72*(a*x + b)^3*a^4*b^2*d^
3*p*log((a*x + b)/x)/x^3 - 24*(a*x + b)*a^3*b^5*p*e^3*log((a*x + b)/x)/x -
144*(a*x + b)^2*a^3*b^4*d*p*e^2*log((a*x + b)/x)/x^2 - 144*(a*x + b)^3*a^3*
b^3*d^2*p*e*log((a*x + b)/x)/x^3 - 26*(a*x + b)*a^3*b^5*p*e^3/x - 84*(a*x +
 b)^2*a^3*b^4*d*p*e^2/x^2 - 36*(a*x + b)^3*a^3*b^3*d^2*p*e/x^3 + 24*(a*x +
b)^4*a^3*b^2*d^3*p*log(-a + (a*x + b)/x)/x^4 - 36*(a*x + b)^2*a^2*b^5*p*e^3
*log(-a + (a*x + b)/x)/x^2 - 96*(a*x + b)^3*a^2*b^4*d*p*e^2*log(-a + (a*x +
 b)/x)/x^3 - 36*(a*x + b)^4*a^2*b^3*d^2*p*e*log(-a + (a*x + b)/x)/x^4 - 24*
(a*x + b)^3*a^4*b^2*d^3*log(c)/x^3 - 24*(a*x + b)^4*a^3*b^2*d^3*p*log((a*x
+ b)/x)/x^4 + 36*(a*x + b)^2*a^2*b^5*p*e^3*log((a*x + b)/x)/x^2 + 96*(a*x +
 b)^3*a^2*b^4*d*p*e^2*log((a*x + b)/x)/x^3 + 36*(a*x + b)^4*a^2*b^3*d^2*p*e
*log((a*x + b)/x)/x^4 + 21*(a*x + b)^2*a^2*b^5*p*e^3/x^2 + 24*(a*x + b)^3*a
^2*b^4*d*p*e^2/x^3 + 24*(a*x + b)^3*a*b^5*p*e^3*log(-a + (a*x + b)/x)/x^3 +
 24*(a*x + b)^4*a*b^4*d*p*e^2*log(-a + (a*x + b)/x)/x^4 - 24*(a*x + b)^3*a*
b^5*p*e^3*log((a*x + b)/x)/x^3 - 24*(a*x + b)^4*a*b^4*d*p*e^2*log((a*x + b)
/x)/x^4 - 6*(a*x + b)^3*a*b^5*p*e^3/x^3 - 6*(a*x + b)^4*b^5*p*e^3*log(-a +
(a*x + b)/x)/x^4 + 6*(a*x + b)^4*b^5*p*e^3*log((a*x + b)/x)/x^4)/((a^8 - 4*
(a*x + b)*a^7/x + 6*(a*x + b)^2*a^6/x^2 - 4*(a*x + b)^3*a^5/x^3 + (a*x + b)
^4*a^4/x^4)*b)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int (ex+d)^3 \ln\left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*ln(c*(a+b/x)^p),x)
```

```
[Out] int((e*x+d)^3*ln(c*(a+b/x)^p),x)
```

maxima [A] time = 0.46, size = 166, normalized size = 1.19

$$\frac{1}{24}bp\left(\frac{2a^2e^3x^3+3\left(4a^2de^2-abe^3\right)x^2+6\left(6a^2d^2e-4abde^2+b^2e^3\right)x}{a^3}+\frac{6\left(4a^3d^3-6a^2bd^2e+4ab^2de^2-b^3e^3\right)\mathrm{lo}}{a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/24*b*p*((2*a^2*e^3*x^3 + 3*(4*a^2*d*e^2 - a*b*e^3)*x^2 + 6*(6*a^2*d^2*e -
 4*a*b*d*e^2 + b^2*e^3)*x)/a^3 + 6*(4*a^3*d^3 - 6*a^2*b*d^2*e + 4*a*b^2*d*e
^2 - b^3*e^3)*log(a*x + b)/a^4) + 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2
+ 4*d^3*x)*log((a + b/x)^p*c)
```

mupad [B] time = 0.34, size = 184, normalized size = 1.32

$$x\left(\frac{b\left(\frac{b^2e^3p}{4a^2}-\frac{bde^2p}{a}\right)}{a}+\frac{3bd^2ep}{2a}\right)+\ln\left(c\left(a+\frac{b}{x}\right)^p\right)\left(d^3x+\frac{3d^2ex^2}{2}+de^2x^3+\frac{e^3x^4}{4}\right)-x^2\left(\frac{b^2e^3p}{8a^2}-\frac{bde^2p}{2a}\right)-\frac{\mathrm{ln}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)*(d + e*x)^3,x)
```

```
[Out] x*((b*((b^2*e^3*p)/(4*a^2) - (b*d*e^2*p)/a))/a + (3*b*d^2*e*p)/(2*a)) + log
(c*(a + b/x)^p)*(d^3*x + (e^3*x^4)/4 + (3*d^2*e*x^2)/2 + d*e^2*x^3) - x^2*(
(b^2*e^3*p)/(8*a^2) - (b*d*e^2*p)/(2*a)) - (log(b + a*x)*(b^4*e^3*p - 4*a^3
*b*d^3*p - 4*a*b^3*d*e^2*p + 6*a^2*b^2*d^2*e*p))/(4*a^4) + (b*e^3*p*x^3)/(1
2*a)
```

sympy [A] time = 9.45, size = 484, normalized size = 3.48

$$\begin{cases} d^3px\log\left(a+\frac{b}{x}\right)+d^3x\log(c)+\frac{3d^2epx^2\log\left(a+\frac{b}{x}\right)}{2}+\frac{3d^2ex^2\log(c)}{2}+de^2px^3\log\left(a+\frac{b}{x}\right)+de^2x^3\log(c)+\frac{e^3px^4\log(}{4} \\ d^3px\log(b)-d^3px\log(x)+d^3px+d^3x\log(c)+\frac{3d^2epx^2\log(b)}{2}-\frac{3d^2epx^2\log(x)}{2}+\frac{3d^2epx^2}{4}+\frac{3d^2ex^2\log(c)}{2}+de^2px^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((d**3*p*x*log(a + b/x) + d**3*x*log(c) + 3*d**2*e*p*x**2*log(a +
b/x)/2 + 3*d**2*e*x**2*log(c)/2 + d*e**2*p*x**3*log(a + b/x) + d*e**2*x**3*
log(c) + e**3*p*x**4*log(a + b/x)/4 + e**3*x**4*log(c)/4 + b*d**3*p*log(x +
 b/a)/a + 3*b*d**2*e*p*x/(2*a) + b*d*e**2*p*x**2/(2*a) + b*e**3*p*x**3/(12*
a) - 3*b**2*d**2*e*p*log(x + b/a)/(2*a**2) - b**2*d*e**2*p*x/a**2 - b**2*e*
*3*p*x**2/(8*a**2) + b**3*d*e**2*p*log(x + b/a)/a**3 + b**3*e**3*p*x/(4*a**
3) - b**4*e**3*p*log(x + b/a)/(4*a**4), Ne(a, 0)), (d**3*p*x*log(b) - d**3*
p*x*log(x) + d**3*p*x + d**3*x*log(c) + 3*d**2*e*p*x**2*log(b)/2 - 3*d**2*e
*p*x**2*log(x)/2 + 3*d**2*e*p*x**2/4 + 3*d**2*e*x**2*log(c)/2 + d*e**2*p*x*
*3*log(b) - d*e**2*p*x**3*log(x) + d*e**2*p*x**3/3 + d*e**2*x**3*log(c) + e
**3*p*x**4*log(b)/4 - e**3*p*x**4*log(x)/4 + e**3*p*x**4/16 + e**3*x**4*log
(c)/4, True))
```

3.199 $\int (d+ex)^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$

Optimal. Leaf size=102

$$-\frac{p(ad-be)^3\log(ax+b)}{3a^3e}+\frac{bepx(3ad-be)}{3a^2}+\frac{(d+ex)^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e}+\frac{be^2px^2}{6a}+\frac{d^3p\log(x)}{3e}$$

```
[Out] 1/3*b*e*(3*a*d-b*e)*p*x/a^2+1/6*b*e^2*p*x^2/a+1/3*(e*x+d)^3*ln(c*(a+b/x)^p)
/e+1/3*d^3*p*ln(x)/e-1/3*(a*d-b*e)^3*p*ln(a*x+b)/a^3/e
```

Rubi [A] time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2463, 514, 72}

$$\frac{bepx(3ad-be)}{3a^2}-\frac{p(ad-be)^3\log(ax+b)}{3a^3e}+\frac{(d+ex)^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e}+\frac{be^2px^2}{6a}+\frac{d^3p\log(x)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Log[c*(a + b/x)^p],x]
```

```
[Out] (b*e*(3*a*d - b*e)*p*x)/(3*a^2) + (b*e^2*p*x^2)/(6*a) + ((d + e*x)^3*Log[c*
(a + b/x)^p])/(3*e) + (d^3*p*Log[x])/(3*e) - ((a*d - b*e)^3*p*Log[b + a*x])
/(3*a^3*e)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{(d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{(bp)\int \frac{(d+ex)^3}{\left(a+\frac{b}{x}\right)x^2} dx}{3e}$$
$$= \frac{(d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{(bp)\int \frac{(d+ex)^3}{x(b+ax)} dx}{3e}$$
$$= \frac{(d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{(bp)\int \left(\frac{e^2(3ad-be)}{a^2} + \frac{d^3}{bx} + \frac{e^3x}{a} - \frac{(ad-be)^3}{a^2b(b+ax)}\right) dx}{3e}$$
$$= \frac{be(3ad-be)px}{3a^2} + \frac{be^2px^2}{6a} + \frac{(d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{d^3p\log(x)}{3e} - \frac{(ad-$$

Mathematica [A] time = 0.08, size = 86, normalized size = 0.84

$$\frac{2a^3(d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) + p\left(2a^3d^3\log(x) + abe^2x(6ad+aex-2be) - 2(ad-be)^3\log(ax+b)\right)}{6a^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Log[c*(a + b/x)^p],x]
```

```
[Out] (2*a^3*(d + e*x)^3*Log[c*(a + b/x)^p] + p*(a*b*e^2*x*(6*a*d - 2*b*e + a*e*x
) + 2*a^3*d^3*Log[x] - 2*(a*d - b*e)^3*Log[b + a*x]))/(6*a^3*e)
```

fricas [A] time = 0.43, size = 153, normalized size = 1.50

$$\frac{a^2be^2px^2 + 2\left(3a^2bde - ab^2e^2\right)px + 2\left(3a^2bd^2 - 3ab^2de + b^3e^2\right)p\log(ax+b) + 2\left(a^3e^2x^3 + 3a^3dex^2 + 3a^3d^2x\right.}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/6*(a^2*b*e^2*p*x^2 + 2*(3*a^2*b*d*e - a*b^2*e^2)*p*x + 2*(3*a^2*b*d^2 - 3
*a*b^2*d*e + b^3*e^2)*p*log(a*x + b) + 2*(a^3*e^2*x^3 + 3*a^3*d*e*x^2 + 3*a
^3*d^2*x)*log(c) + 2*(a^3*e^2*p*x^3 + 3*a^3*d*e*p*x^2 + 3*a^3*d^2*p*x)*log(
(a*x + b)/x))/a^3
```

giac [B] time = 0.26, size = 918, normalized size = 9.00

$$-\frac{6a^5b^2d^2p\log\left(-a+\frac{ax+b}{x}\right) - 6a^4b^3dpe\log\left(-a+\frac{ax+b}{x}\right) + 6a^4b^3dpe - \frac{18(ax+b)a^4b^2d^2p\log\left(-a+\frac{ax+b}{x}\right)}{x} + 2a^3b^4pe^2\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] -1/6*(6*a^5*b^2*d^2*p*log(-a + (a*x + b)/x) - 6*a^4*b^3*d*p*e*log(-a + (a*x
 + b)/x) + 6*a^4*b^3*d*p*e - 18*(a*x + b)*a^4*b^2*d^2*p*log(-a + (a*x + b)/
x)/x + 2*a^3*b^4*p*e^2*log(-a + (a*x + b)/x) + 18*(a*x + b)*a^3*b^3*d*p*e*l
og(-a + (a*x + b)/x)/x + 6*a^5*b^2*d^2*log(c) - 6*a^4*b^3*d*e*log(c) + 6*(a
*x + b)*a^4*b^2*d^2*p*log((a*x + b)/x)/x - 12*(a*x + b)*a^3*b^3*d*p*e*log((
```

```
a*x + b)/x)/x - 3*a^3*b^4*p*e^2 - 12*(a*x + b)*a^3*b^3*d*p*e/x + 18*(a*x +
b)^2*a^3*b^2*d^2*p*log(-a + (a*x + b)/x)/x^2 - 6*(a*x + b)*a^2*b^4*p*e^2*lo
g(-a + (a*x + b)/x)/x - 18*(a*x + b)^2*a^2*b^3*d*p*e*log(-a + (a*x + b)/x)/
x^2 - 12*(a*x + b)*a^4*b^2*d^2*log(c)/x + 2*a^3*b^4*e^2*log(c) + 6*(a*x + b
)*a^3*b^3*d*e*log(c)/x - 12*(a*x + b)^2*a^3*b^2*d^2*p*log((a*x + b)/x)/x^2
+ 6*(a*x + b)*a^2*b^4*p*e^2*log((a*x + b)/x)/x + 18*(a*x + b)^2*a^2*b^3*d*p
*e*log((a*x + b)/x)/x^2 + 5*(a*x + b)*a^2*b^4*p*e^2/x + 6*(a*x + b)^2*a^2*b
^3*d*p*e/x^2 - 6*(a*x + b)^3*a^2*b^2*d^2*p*log(-a + (a*x + b)/x)/x^3 + 6*(a
*x + b)^2*a*b^4*p*e^2*log(-a + (a*x + b)/x)/x^2 + 6*(a*x + b)^3*a*b^3*d*p*e
*log(-a + (a*x + b)/x)/x^3 + 6*(a*x + b)^2*a^3*b^2*d^2*log(c)/x^2 + 6*(a*x
+ b)^3*a^2*b^2*d^2*p*log((a*x + b)/x)/x^3 - 6*(a*x + b)^2*a*b^4*p*e^2*log((
a*x + b)/x)/x^2 - 6*(a*x + b)^3*a*b^3*d*p*e*log((a*x + b)/x)/x^3 - 2*(a*x +
 b)^2*a*b^4*p*e^2/x^2 - 2*(a*x + b)^3*b^4*p*e^2*log(-a + (a*x + b)/x)/x^3 +
 2*(a*x + b)^3*b^4*p*e^2*log((a*x + b)/x)/x^3)/((a^6 - 3*(a*x + b)*a^5/x +
3*(a*x + b)^2*a^4/x^2 - (a*x + b)^3*a^3/x^3)*b)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int (ex+d)^2 \ln\left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*ln(c*(a+b/x)^p),x)
```

```
[Out] int((e*x+d)^2*ln(c*(a+b/x)^p),x)
```

maxima [A] time = 0.45, size = 102, normalized size = 1.00

$$\frac{1}{6}bp\left(\frac{ae^2x^2+2\left(3\,ade-be^2\right)x}{a^2}+\frac{2\left(3\,a^2d^2-3\,abde+b^2e^2\right)\log\left(ax+b\right)}{a^3}\right)+\frac{1}{3}\left(e^2x^3+3\,dex^2+3\,d^2x\right)\log\left(\left(a+\frac{b}{x}\right)^{p}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/6*b*p*((a*e^2*x^2 + 2*(3*a*d*e - b*e^2)*x)/a^2 + 2*(3*a^2*d^2 - 3*a*b*d*e
 + b^2*e^2)*log(a*x + b)/a^3) + 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)*log((a
+ b/x)^p*c)
```

mupad [B] time = 0.32, size = 111, normalized size = 1.09

$$\ln\left(c\left(a+\frac{b}{x}\right)^p\right)\left(d^2\,x+d\,e\,x^2+\frac{e^2\,x^3}{3}\right)-x\,\left(\frac{b^2\,e^2\,p}{3\,a^2}-\frac{b\,d\,e\,p}{a}\right)+\frac{\ln\left(b+a\,x\right)\,\left(3\,p\,a^2\,b\,d^2-3\,p\,a\,b^2\,d\,e+p\,b^3\,e^2\right)}{3\,a^3}+\frac{b\,e^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)*(d + e*x)^2,x)
```

```
[Out] log(c*(a + b/x)^p)*(d^2*x + (e^2*x^3)/3 + d*e*x^2) - x*((b^2*e^2*p)/(3*a^2)
 - (b*d*e*p)/a) + (log(b + a*x)*(b^3*e^2*p + 3*a^2*b*d^2*p - 3*a*b^2*d*e*p)
)/(3*a^3) + (b*e^2*p*x^2)/(6*a)
```

sympy [A] time = 4.94, size = 298, normalized size = 2.92

$$\begin{cases} d^2px\log\left(a+\frac{b}{x}\right)+d^2x\log\left(c\right)+depx^2\log\left(a+\frac{b}{x}\right)+dex^2\log\left(c\right)+\frac{e^2px^3\log\left(a+\frac{b}{x}\right)}{3}+\frac{e^2x^3\log\left(c\right)}{3}+\frac{bd^2p\log\left(x+\frac{b}{a}\right)}{a}+\frac{bd}{a} \\ d^2px\log\left(b\right)-d^2px\log\left(x\right)+d^2px+d^2x\log\left(c\right)+depx^2\log\left(b\right)-depx^2\log\left(x\right)+\frac{depx^2}{2}+dex^2\log\left(c\right)+\frac{e^2px^3\log\left(\right.}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((d**2*p*x*log(a + b/x) + d**2*x*log(c) + d*e*p*x**2*log(a + b/x)
+ d*e*x**2*log(c) + e**2*p*x**3*log(a + b/x)/3 + e**2*x**3*log(c)/3 + b*d**
2*p*log(x + b/a)/a + b*d*e*p*x/a + b*e**2*p*x**2/(6*a) - b**2*d*e*p*log(x +
 b/a)/a**2 - b**2*e**2*p*x/(3*a**2) + b**3*e**2*p*log(x + b/a)/(3*a**3), Ne
(a, 0)), (d**2*p*x*log(b) - d**2*p*x*log(x) + d**2*p*x + d**2*x*log(c) + d*
e*p*x**2*log(b) - d*e*p*x**2*log(x) + d*e*p*x**2/2 + d*e*x**2*log(c) + e**2
*p*x**3*log(b)/3 - e**2*p*x**3*log(x)/3 + e**2*p*x**3/9 + e**2*x**3*log(c)/
3, True))
```

3.200 $\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$

Optimal. Leaf size=78

$$-\frac{p(ad-be)^2\log(ax+b)}{2a^2e}+\frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{bepx}{2a}+\frac{d^2p\log(x)}{2e}$$

```
[Out] 1/2*b*e*p*x/a+1/2*(e*x+d)^2*ln(c*(a+b/x)^p)/e+1/2*d^2*p*ln(x)/e-1/2*(a*d-b*
e)^2*p*ln(a*x+b)/a^2/e
```

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2463, 514, 72}

$$-\frac{p(ad-be)^2\log(ax+b)}{2a^2e}+\frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{bepx}{2a}+\frac{d^2p\log(x)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Log[c*(a + b/x)^p],x]
```

```
[Out] (b*e*p*x)/(2*a) + ((d + e*x)^2*Log[c*(a + b/x)^p])/(2*e) + (d^2*p*Log[x])/(
2*e) - ((a*d - b*e)^2*p*Log[b + a*x])/(2*a^2*e)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx = \frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} + \frac{(bp)\int \frac{(d+ex)^2}{\left(a+\frac{b}{x}\right)x^2}dx}{2e}$$
$$= \frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} + \frac{(bp)\int \frac{(d+ex)^2}{x(b+ax)}dx}{2e}$$
$$= \frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} + \frac{(bp)\int \left(\frac{e^2}{a}+\frac{d^2}{bx}-\frac{(ad-be)^2}{ab(b+ax)}\right)dx}{2e}$$
$$= \frac{bepx}{2a} + \frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} + \frac{d^2p\log(x)}{2e} - \frac{(ad-be)^2p\log(b+ax)}{2a^2e}$$

Mathematica [A] time = 0.03, size = 85, normalized size = 1.09

$$\frac{1}{2}bep\left(\frac{x}{a}-\frac{b\log(ax+b)}{a^2}\right)+dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{2}ex^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bdp\log\left(a+\frac{b}{x}\right)}{a}+\frac{bdp\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Log[c*(a + b/x)^p],x]
```

```
[Out] (b*d*p*Log[a + b/x])/a + d*x*Log[c*(a + b/x)^p] + (e*x^2*Log[c*(a + b/x)^p]
)/2 + (b*d*p*Log[x])/a + (b*e*p*(x/a - (b*Log[b + a*x])/a^2))/2
```

fricas [A] time = 0.42, size = 80, normalized size = 1.03

$$\frac{abepx+\left(2\,abd-b^2e\right)p\log\left(ax+b\right)+\left(a^2ex^2+2\,a^2dx\right)\log(c)+\left(a^2epx^2+2\,a^2dpx\right)\log\left(\frac{ax+b}{x}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*e*p*x + (2*a*b*d - b^2*e)*p*log(a*x + b) + (a^2*e*x^2 + 2*a^2*d*x)
*log(c) + (a^2*e*p*x^2 + 2*a^2*d*p*x)*log((a*x + b)/x))/a^2
```

giac [B] time = 0.19, size = 394, normalized size = 5.05

$$-\frac{2\,a^3b^2dp\log\left(-a+\frac{ax+b}{x}\right)-a^2b^3pe\log\left(-a+\frac{ax+b}{x}\right)+a^2b^3pe-\frac{4\,(ax+b)a^2b^2dp\log\left(-a+\frac{ax+b}{x}\right)}{x}+\frac{2\,(ax+b)ab^3pe\log\left(-a+\frac{ax+b}{x}\right)}{x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^3*b^2*d*p*log(-a + (a*x + b)/x) - a^2*b^3*p*e*log(-a + (a*x + b)/
x) + a^2*b^3*p*e - 4*(a*x + b)*a^2*b^2*d*p*log(-a + (a*x + b)/x)/x + 2*(a*x
 + b)*a*b^3*p*e*log(-a + (a*x + b)/x)/x + 2*a^3*b^2*d*log(c) - a^2*b^3*e*lo
g(c) + 2*(a*x + b)*a^2*b^2*d*p*log((a*x + b)/x)/x - 2*(a*x + b)*a*b^3*p*e*l
og((a*x + b)/x)/x - (a*x + b)*a*b^3*p*e/x + 2*(a*x + b)^2*a*b^2*d*p*log(-a
+ (a*x + b)/x)/x^2 - (a*x + b)^2*b^3*p*e*log(-a + (a*x + b)/x)/x^2 - 2*(a*x
 + b)*a^2*b^2*d*log(c)/x - 2*(a*x + b)^2*a*b^2*d*p*log((a*x + b)/x)/x^2 + (
```

```
a*x + b)^2*b^3*p*e*log((a*x + b)/x)/x^2)/((a^4 - 2*(a*x + b)*a^3/x + (a*x +
 b)^2*a^2/x^2)*b)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (ex+d)\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*ln(c*(a+b/x)^p),x)
```

```
[Out] int((e*x+d)*ln(c*(a+b/x)^p),x)
```

maxima [A] time = 0.46, size = 55, normalized size = 0.71

$$\frac{1}{2}\,bp\left(\frac{ex}{a}+\frac{(2\,ad-be)\log(ax+b)}{a^2}\right)+\frac{1}{2}\left(ex^2+2\,dx\right)\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(e*x/a + (2*a*d - b*e)*log(a*x + b)/a^2) + 1/2*(e*x^2 + 2*d*x)*log(
(a + b/x)^p*c)
```

mupad [B] time = 0.30, size = 57, normalized size = 0.73

$$\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)\left(\frac{e\,x^2}{2}+d\,x\right)-\frac{\ln(b+a\,x)\,\left(b^2\,e\,p-2\,a\,b\,d\,p\right)}{2\,a^2}+\frac{b\,e\,p\,x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)*(d + e*x),x)
```

```
[Out] log(c*(a + b/x)^p)*(d*x + (e*x^2)/2) - (log(b + a*x)*(b^2*e*p - 2*a*b*d*p))
/(2*a^2) + (b*e*p*x)/(2*a)
```

sympy [A] time = 2.35, size = 156, normalized size = 2.00

$$\begin{cases} dpx\log\left(a+\frac{b}{x}\right)+dx\log(c)+\frac{epx^2\log\left(a+\frac{b}{x}\right)}{2}+\frac{ex^2\log(c)}{2}+\frac{bdp\log\left(x+\frac{b}{a}\right)}{a}+\frac{bepx}{2a}-\frac{b^2ep\log\left(x+\frac{b}{a}\right)}{2a^2} & \text{for } a\neq 0 \\ dpx\log(b)-dpx\log(x)+dpx+dx\log(c)+\frac{epx^2\log(b)}{2}-\frac{epx^2\log(x)}{2}+\frac{epx^2}{4}+\frac{ex^2\log(c)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((d*p*x*log(a + b/x) + d*x*log(c) + e*p*x**2*log(a + b/x)/2 + e*x*
*2*log(c)/2 + b*d*p*log(x + b/a)/a + b*e*p*x/(2*a) - b**2*e*p*log(x + b/a)/
(2*a**2), Ne(a, 0)), (d*p*x*log(b) - d*p*x*log(x) + d*p*x + d*x*log(c) + e*
p*x**2*log(b)/2 - e*p*x**2*log(x)/2 + e*p*x**2/4 + e*x**2*log(c)/2, True))
```

3.201 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=113

$$\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}-\frac{p\text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e}+\frac{p\text{Li}_2\left(\frac{ex}{d}+1\right)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+$$

```
[Out] ln(c*(a+b/x)^p)*ln(e*x+d)/e+p*ln(-e*x/d)*ln(e*x+d)/e-p*ln(-e*(a*x+b)/(a*d-b
*e))*ln(e*x+d)/e-p*polylog(2,a*(e*x+d)/(a*d-b*e))/e+p*polylog(2,1+e*x/d)/e
```

Rubi [A] time = 0.16, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2462, 260, 2416, 2394, 2315, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e}+\frac{p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e}+\frac{p$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e -
(p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e - (p*PolyLog[2, (a*(d
+ e*x))/(a*d - b*e)])/e + (p*PolyLog[2, 1 + (e*x)/d])/e
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{(bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{(bp)\int \left(\frac{\log(d+ex)}{bx} - \frac{a\log(d+ex)}{b(b+ax)}\right)dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\int \frac{\log(d+ex)}{x}\,dx}{e} - \frac{(ap)\int \frac{\log(d+ex)}{b+ax}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} -$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} +$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} -$$

Mathematica [A] time = 0.03, size = 114, normalized size = 1.01

$$\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} - \frac{p\text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)}{e} - \frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e} + \frac{p\text{Li}_2\left(\frac{d+ex}{d}\right)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e -
(p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e + (p*PolyLog[2, (d + e
*x)/d])/e - (p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/e
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax+b}{x}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a+\frac{b}{x}\right)^{p} c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x)^p*c)/(e*x + d), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x)^p)/(e*x+d),x)
```

maxima [A] time = 0.49, size = 159, normalized size = 1.41

$$\frac{bp\left(\frac{\log(ex+d)\log\left(a+\frac{b}{x}\right)}{b}-\frac{\log(ex+d)\log\left(-\frac{aex+ad}{ad-be}+1\right)+\mathrm{Li}_2\left(\frac{aex+ad}{ad-be}\right)}{b}+\frac{\log(ex+d)\log\left(-\frac{ex+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{ex+d}{d}\right)}{b}\right)}{e}-\frac{p\log(ex+d)\log\left(a+\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*(log(e*x + d)*log(a + b/x)/b - (log(e*x + d)*log(-(a*e*x + a*d)/(a*d -
b*e) + 1) + dilog((a*e*x + a*d)/(a*d - b*e)))/b + (log(e*x + d)*log(-(e*x +
 d)/d + 1) + dilog((e*x + d)/d))/b)/e - p*log(e*x + d)*log(a + b/x)/e + log
((a + b/x)^p*c)*log(e*x + d)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/(d + e*x),x)
```

```
[Out] int(log(c*(a + b/x)^p)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b/x)**p)/(d + e*x), x)
```

3.202 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2} dx$

Optimal. Leaf size=81

$$-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)}+\frac{ap\log(ax+b)}{e(ad-be)}-\frac{bp\log(d+ex)}{d(ad-be)}-\frac{p\log(x)}{de}$$

```
[Out] -ln(c*(a+b/x)^p)/e/(e*x+d)-p*ln(x)/d/e+a*p*ln(a*x+b)/e/(a*d-b*e)-b*p*ln(e*x
+d)/d/(a*d-b*e)
```

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2463, 514, 72}

$$-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)}+\frac{ap\log(ax+b)}{e(ad-be)}-\frac{bp\log(d+ex)}{d(ad-be)}-\frac{p\log(x)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x)^2,x]
```

```
[Out] -(Log[c*(a + b/x)^p]/(e*(d + e*x))) - (p*Log[x])/(d*e) + (a*p*Log[b + a*x])
/(e*(a*d - b*e)) - (b*p*Log[d + e*x])/(d*(a*d - b*e))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx = -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{(bp)\int \frac{1}{\left(a+\frac{b}{x}\right)x^2(d+ex)}\,dx}{e}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{(bp)\int \frac{1}{x(b+ax)(d+ex)}\,dx}{e}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{(bp)\int \left(\frac{1}{bdx}+\frac{a^2}{b(-ad+be)(b+ax)}+\frac{e^2}{d(ad-be)(d+ex)}\right)dx}{e}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{p\log(x)}{de} + \frac{ap\log(b+ax)}{e(ad-be)} - \frac{bp\log(d+ex)}{d(ad-be)}$$

Mathematica [A] time = 0.07, size = 81, normalized size = 1.00

$$-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} + \frac{ap\log(ax+b)}{e(ad-be)} - \frac{bp\log(d+ex)}{d(ad-be)} - \frac{p\log(x)}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x)^2,x]
```

```
[Out] -(Log[c*(a + b/x)^p]/(e*(d + e*x))) - (p*Log[x])/(d*e) + (a*p*Log[b + a*x])
/(e*(a*d - b*e)) - (b*p*Log[d + e*x])/(d*(a*d - b*e))
```

fricas [A] time = 0.52, size = 148, normalized size = 1.83

$$-\frac{\left(ad^2-bde\right)p\log\left(\frac{ax+b}{x}\right)-\left(adepx+ad^2p\right)\log(ax+b)+\left(be^2px+bdep\right)\log(ex+d)+\left(ad^2-bde\right)\log(c)+\left(\right.}{ad^3e-bd^2e^2+\left(ad^2e^2-bde^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -((a*d^2 - b*d*e)*p*log((a*x + b)/x) - (a*d*e*p*x + a*d^2*p)*log(a*x + b) +
 (b*e^2*p*x + b*d*e*p)*log(e*x + d) + (a*d^2 - b*d*e)*log(c) + ((a*d*e - b*
e^2)*p*x + (a*d^2 - b*d*e)*p)*log(x))/(a*d^3*e - b*d^2*e^2 + (a*d^2*e^2 - b
*d*e^3)*x)
```

giac [B] time = 0.21, size = 192, normalized size = 2.37

$$-\frac{ab^2dp\log\left(-ad+be+\frac{(ax+b)d}{x}\right)-b^3pe\log\left(-ad+be+\frac{(ax+b)d}{x}\right)-\frac{(ax+b)b^2dp\log\left(-ad+be+\frac{(ax+b)d}{x}\right)}{x}+ab^2d\log(c)-b}{\left(a^2d^3-2\,abd^2e-\frac{(ax+b)ad^3}{x}+b^2de^2+\frac{(ax+b)bd^2e}{x}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(a*b^2*d*p*log(-a*d + b*e + (a*x + b)*d/x) - b^3*p*e*log(-a*d + b*e + (a*x
 + b)*d/x) - (a*x + b)*b^2*d*p*log(-a*d + b*e + (a*x + b)*d/x)/x + a*b^2*d*
log(c) - b^3*e*log(c) + (a*x + b)*b^2*d*p*log((a*x + b)/x)/x)/((a^2*d^3 - 2
*a*b*d^2*e - (a*x + b)*a*d^3/x + b^2*d*e^2 + (a*x + b)*b*d^2*e/x)*b)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{\left(ex+d\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/(e*x+d)^2,x)
```

```
[Out] int(ln(c*(a+b/x)^p)/(e*x+d)^2,x)
```

maxima [A] time = 0.46, size = 85, normalized size = 1.05

$$\frac{bp\left(\frac{a\log(ax+b)}{abd-b^2e}-\frac{e\log(ex+d)}{ad^2-bde}-\frac{\log(x)}{bd}\right)}{e}-\frac{\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)}{(ex+d)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] b*p*(a*log(a*x + b)/(a*b*d - b^2*e) - e*log(e*x + d)/(a*d^2 - b*d*e) - log(
x)/(b*d))/e - log((a + b/x)^p*c)/((e*x + d)*e)
```

mupad [B] time = 0.53, size = 85, normalized size = 1.05

$$-\frac{\ln\left(c\left(\frac{b+a\,x}{x}\right)^{p}\right)}{x\,e^2+d\,e}-\frac{p\,\ln(x)}{d\,e}-\frac{a\,p\,\ln\left(b+a\,x\right)}{b\,e^2-a\,d\,e}-\frac{b\,p\,\ln\left(d+e\,x\right)}{a\,d^2-b\,d\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/(d + e*x)^2,x)
```

```
[Out] - log(c*((b + a*x)/x)^p)/(d*e + e^2*x) - (p*log(x))/(d*e) - (a*p*log(b + a*
x))/(b*e^2 - a*d*e) - (b*p*log(d + e*x))/(a*d^2 - b*d*e)
```

sympy [A] time = 7.62, size = 585, normalized size = 7.22

$$\begin{cases} \frac{dp\log\left(\frac{d}{e}+x\right)}{d^2e+de^2x}+\frac{epx\log(b)}{d^2e+de^2x}-\frac{epx\log(x)}{d^2e+de^2x}+\frac{epx\log\left(\frac{d}{e}+x\right)}{d^2e+de^2x}+\frac{ex\log(c)}{d^2e+de^2x} \\ -\frac{dp}{d^2e+de^2x}+\frac{epx\log\left(\frac{b}{x}+\frac{be}{d}\right)}{d^2e+de^2x}+\frac{ex\log(c)}{d^2e+de^2x} \\ \frac{-\frac{ap\log\left(a+\frac{b}{x}\right)}{b}-\frac{p\log\left(a+\frac{b}{x}\right)}{x}+\frac{p}{x}-\frac{\log(c)}{x}}{e^2} \\ \tilde{\infty}\left(px\log\left(a+\frac{b}{x}\right)+x\log(c)+\frac{bp\log(ax+b)}{a}\right) \\ \frac{px\log\left(a+\frac{b}{x}\right)+x\log(c)+\frac{bp\log(ax+b)}{a}}{d^2} \\ \frac{adpx\log\left(a+\frac{b}{x}\right)}{ad^3+ad^2ex-bd^2e-bde^2x}+\frac{adx\log(c)}{ad^3+ad^2ex-bd^2e-bde^2x}+\frac{bdp\log\left(x+\frac{b}{a}\right)}{ad^3+ad^2ex-bd^2e-bde^2x}-\frac{bdp\log\left(\frac{d}{e}+x\right)}{ad^3+ad^2ex-bd^2e-bde^2x}-\frac{bepx\log\left(a+\frac{b}{x}\right)}{ad^3+ad^2ex-bd^2e-bde^2x}+\frac{bepx}{ad^3+ad^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d)**2,x)
```

```
[Out] Piecewise((d*p*log(d/e + x)/(d**2*e + d*e**2*x) + e*p*x*log(b)/(d**2*e + d*
e**2*x) - e*p*x*log(x)/(d**2*e + d*e**2*x) + e*p*x*log(d/e + x)/(d**2*e + d
*e**2*x) + e*x*log(c)/(d**2*e + d*e**2*x), Eq(a, 0)), (-d*p/(d**2*e + d*e**
2*x) + e*p*x*log(b/x + b*e/d)/(d**2*e + d*e**2*x) + e*x*log(c)/(d**2*e + d*
e**2*x), Eq(a, b*e/d)), ((-a*p*log(a + b/x)/b - p*log(a + b/x)/x + p/x - lo
g(c)/x)/e**2, Eq(d, 0)), (zoo*(p*x*log(a + b/x) + x*log(c) + b*p*log(a*x +
b)/a), Eq(d, -e*x)), ((p*x*log(a + b/x) + x*log(c) + b*p*log(a*x + b)/a)/d*
*2, Eq(e, 0)), (a*d*p*x*log(a + b/x)/(a*d**3 + a*d**2*e*x - b*d**2*e - b*d*
e**2*x) + a*d*x*log(c)/(a*d**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x) + b*d*
p*log(x + b/a)/(a*d**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x) - b*d*p*log(d/
e + x)/(a*d**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x) - b*e*p*x*log(a + b/x)
/(a*d**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x) + b*e*p*x*log(x + b/a)/(a*d*
*3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x) - b*e*p*x*log(d/e + x)/(a*d**3 + a
*d**2*e*x - b*d**2*e - b*d*e**2*x) - b*e*x*log(c)/(a*d**3 + a*d**2*e*x - b*
d**2*e - b*d*e**2*x), True))
```

3.203 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx$

Optimal. Leaf size=127

$$\frac{a^2p\log(ax+b)}{2e(ad-be)^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2}-\frac{bp(2ad-be)\log(d+ex)}{2d^2(ad-be)^2}+\frac{bp}{2d(d+ex)(ad-be)}-\frac{p\log(x)}{2d^2e}$$

```
[Out] 1/2*b*p/d/(a*d-b*e)/(e*x+d)-1/2*ln(c*(a+b/x)^p)/e/(e*x+d)^2-1/2*p*ln(x)/d^2
/e+1/2*a^2*p*ln(a*x+b)/e/(a*d-b*e)^2-1/2*b*(2*a*d-b*e)*p*ln(e*x+d)/d^2/(a*d
-b*e)^2
```

Rubi [A] time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2463, 514, 72}

$$\frac{a^2p\log(ax+b)}{2e(ad-be)^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2}-\frac{bp(2ad-be)\log(d+ex)}{2d^2(ad-be)^2}+\frac{bp}{2d(d+ex)(ad-be)}-\frac{p\log(x)}{2d^2e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x)^3,x]
```

```
[Out] (b*p)/(2*d*(a*d - b*e)*(d + e*x)) - Log[c*(a + b/x)^p]/(2*e*(d + e*x)^2) -
(p*Log[x])/(2*d^2*e) + (a^2*p*Log[b + a*x])/(2*e*(a*d - b*e)^2) - (b*(2*a*d
 - b*e)*p*Log[d + e*x])/(2*d^2*(a*d - b*e)^2)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx = -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2} - \frac{(bp)\int \frac{1}{\left(a+\frac{b}{x}\right)x^2(d+ex)^2}\,dx}{2e}$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2} - \frac{(bp)\int \frac{1}{x(b+ax)(d+ex)^2}\,dx}{2e}$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2} - \frac{(bp)\int \left(\frac{1}{bd^2x} - \frac{a^3}{b(-ad+be)^2(b+ax)} + \frac{e^2}{d(ad-be)(d+ex)^2} + \frac{e^2(2ad-be)}{d^2(ad-be)^2(d+ex)}\right) d}{2e}$$
$$= \frac{bp}{2d(ad-be)(d+ex)} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2} - \frac{p\log(x)}{2d^2e} + \frac{a^2p\log(b+ax)}{2e(ad-be)^2} - \frac{b(2ad-be)p}{2d^2(ad}$$

Mathematica [A] time = 0.20, size = 113, normalized size = 0.89

$$\frac{\frac{a^2p\log(ax+b)}{(ad-be)^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2} + \frac{bep(be-2ad)\log(d+ex)}{d^2(ad-be)^2} + \frac{bep}{d(d+ex)(ad-be)} - \frac{p\log(x)}{d^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x)^3,x]
```

```
[Out] ((b*e*p)/(d*(a*d - b*e)*(d + e*x)) - Log[c*(a + b/x)^p]/(d + e*x)^2 - (p*Lo
g[x])/d^2 + (a^2*p*Log[b + a*x])/(a*d - b*e)^2 + (b*e*(-2*a*d + b*e)*p*Log[
d + e*x])/(d^2*(a*d - b*e)^2))/(2*e)
```

fricas [B] time = 1.17, size = 428, normalized size = 3.37

$$\frac{\left(abd^2e^2 - b^2de^3\right)px - \left(a^2d^4 - 2\,abd^3e + b^2d^2e^2\right)p\log\left(\frac{ax+b}{x}\right) + \left(abd^3e - b^2d^2e^2\right)p + \left(a^2d^2e^2px^2 + 2\,a^2d^3epx + a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*((a*b*d^2*e^2 - b^2*d*e^3)*p*x - (a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2)*
p*log((a*x + b)/x) + (a*b*d^3*e - b^2*d^2*e^2)*p + (a^2*d^2*e^2*p*x^2 + 2*a
^2*d^3*e*p*x + a^2*d^4*p)*log(a*x + b) - ((2*a*b*d*e^3 - b^2*e^4)*p*x^2 + 2
*(2*a*b*d^2*e^2 - b^2*d*e^3)*p*x + (2*a*b*d^3*e - b^2*d^2*e^2)*p)*log(e*x +
 d) - (a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2)*log(c) - ((a^2*d^2*e^2 - 2*a*b*
d*e^3 + b^2*e^4)*p*x^2 + 2*(a^2*d^3*e - 2*a*b*d^2*e^2 + b^2*d*e^3)*p*x + (a
^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2)*p)*log(x))/(a^2*d^6*e - 2*a*b*d^5*e^2 +
 b^2*d^4*e^3 + (a^2*d^4*e^3 - 2*a*b*d^3*e^4 + b^2*d^2*e^5)*x^2 + 2*(a^2*d^5
*e^2 - 2*a*b*d^4*e^3 + b^2*d^3*e^4)*x)
```

giac [B] time = 0.24, size = 805, normalized size = 6.34

$$-\frac{2\,a^3b^2d^3p\log\left(-ad+be+\frac{(ax+b)d}{x}\right) - 5\,a^2b^3d^2pe\log\left(-ad+be+\frac{(ax+b)d}{x}\right) - a^2b^3d^2pe - \frac{4\,(ax+b)a^2b^2d^3p\log\left(-ad+be\right.}{x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^3*b^2*d^3*p*log(-a*d + b*e + (a*x + b)*d/x) - 5*a^2*b^3*d^2*p*e*l
og(-a*d + b*e + (a*x + b)*d/x) - a^2*b^3*d^2*p*e - 4*(a*x + b)*a^2*b^2*d^3*
p*log(-a*d + b*e + (a*x + b)*d/x)/x + 4*a*b^4*d*p*e^2*log(-a*d + b*e + (a*x
 + b)*d/x) + 6*(a*x + b)*a*b^3*d^2*p*e*log(-a*d + b*e + (a*x + b)*d/x)/x +
2*a^3*b^2*d^3*log(c) - 5*a^2*b^3*d^2*e*log(c) + 2*(a*x + b)*a^2*b^2*d^3*p*l
og((a*x + b)/x)/x - 2*(a*x + b)*a*b^3*d^2*p*e*log((a*x + b)/x)/x + 2*a*b^4*
d*p*e^2 + (a*x + b)*a*b^3*d^2*p*e/x + 2*(a*x + b)^2*a*b^2*d^3*p*log(-a*d +
b*e + (a*x + b)*d/x)/x^2 - b^5*p*e^3*log(-a*d + b*e + (a*x + b)*d/x) - 2*(a
*x + b)*b^4*d*p*e^2*log(-a*d + b*e + (a*x + b)*d/x)/x - (a*x + b)^2*b^3*d^2
*p*e*log(-a*d + b*e + (a*x + b)*d/x)/x^2 - 2*(a*x + b)*a^2*b^2*d^3*log(c)/x
 + 4*a*b^4*d*e^2*log(c) + 4*(a*x + b)*a*b^3*d^2*e*log(c)/x - 2*(a*x + b)^2*
a*b^2*d^3*p*log((a*x + b)/x)/x^2 + (a*x + b)^2*b^3*d^2*p*e*log((a*x + b)/x)
/x^2 - b^5*p*e^3 - (a*x + b)*b^4*d*p*e^2/x - b^5*e^3*log(c) - 2*(a*x + b)*b
^4*d*e^2*log(c)/x)/((a^4*d^6 - 4*a^3*b*d^5*e - 2*(a*x + b)*a^3*d^6/x + 6*a^
2*b^2*d^4*e^2 + 6*(a*x + b)*a^2*b*d^5*e/x + (a*x + b)^2*a^2*d^6/x^2 - 4*a*b
^3*d^3*e^3 - 6*(a*x + b)*a*b^2*d^4*e^2/x - 2*(a*x + b)^2*a*b*d^5*e/x^2 + b^
4*d^2*e^4 + 2*(a*x + b)*b^3*d^3*e^3/x + (a*x + b)^2*b^2*d^4*e^2/x^2)*b)
```

maple [**F**] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{(ex+d)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/(e*x+d)^3,x)
```

```
[Out] int(ln(c*(a+b/x)^p)/(e*x+d)^3,x)
```

maxima [**A**] time = 0.48, size = 160, normalized size = 1.26

$$\frac{\left(\frac{a^{2}\log(ax+b)}{a^{2}bd^{2}-2\,ab^{2}de+b^{3}e^{2}}-\frac{\left(2\,ade-be^{2}\right)\log(ex+d)}{a^{2}d^{4}-2\,abd^{3}e+b^{2}d^{2}e^{2}}+\frac{e}{ad^{3}-bd^{2}e+\left(ad^{2}e-bde^{2}\right)x}-\frac{\log(x)}{bd^{2}}\right)bp}{2\,e}-\frac{\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)}{2\,(ex+d)^{2}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*log(a*x + b)/(a^2*b*d^2 - 2*a*b^2*d*e + b^3*e^2) - (2*a*d*e - b*e^
2)*log(e*x + d)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2) + e/(a*d^3 - b*d^2*e
+ (a*d^2*e - b*d*e^2)*x) - log(x)/(b*d^2))*b*p/e - 1/2*log((a + b/x)^p*c)/(
(e*x + d)^2*e)
```

mupad [**B**] time = 1.08, size = 217, normalized size = 1.71

$$\frac{a^{2}\,p\,\ln(b+a\,x)}{2\,a^{2}\,d^{2}\,e-4\,a\,b\,d\,e^{2}+2\,b^{2}\,e^{3}}-\frac{\ln\left(c\left(\frac{b+a\,x}{x}\right)^{p}\right)}{2\left(d^{2}\,e+2\,d\,e^{2}\,x+e^{3}\,x^{2}\right)}-\frac{p\,\ln(x)}{2\,d^{2}\,e}-\frac{b\,e\,p}{2\,b\,d^{2}\,e^{2}-2\,a\,d^{3}\,e+2\,b\,d\,e^{3}\,x-2\,a\,d^{2}\,e^{2}\,x}+\frac{\cdots}{2\,a^{2}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/(d + e*x)^3,x)
```

```
[Out] (a^2*p*log(b + a*x))/(2*b^2*e^3 + 2*a^2*d^2*e - 4*a*b*d*e^2) - log(c*((b +
a*x)/x)^p)/(2*(d^2*e + e^3*x^2 + 2*d*e^2*x)) - (p*log(x))/(2*d^2*e) - (b*e*
p)/(2*b*d^2*e^2 - 2*a*d^3*e + 2*b*d*e^3*x - 2*a*d^2*e^2*x) + (b^2*e*p*log(d
 + e*x))/(2*a^2*d^4 + 2*b^2*d^2*e^2 - 4*a*b*d^3*e) - (2*a*b*d*p*log(d + e*x
))/(2*a^2*d^4 + 2*b^2*d^2*e^2 - 4*a*b*d^3*e)
```

sympy [A] time = 23.00, size = 4527, normalized size = 35.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d)**3,x)
```

```
[Out] Piecewise((d**2*p*log(d/e + x)/(2*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2
) - d**2*p/(2*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2) + 2*d*e*p*x*log(b)
/(2*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2) - 2*d*e*p*x*log(x)/(2*d**4*e
 + 4*d**3*e**2*x + 2*d**2*e**3*x**2) + 2*d*e*p*x*log(d/e + x)/(2*d**4*e + 4
*d**3*e**2*x + 2*d**2*e**3*x**2) - d*e*p*x/(2*d**4*e + 4*d**3*e**2*x + 2*d*
*2*e**3*x**2) + 2*d*e*x*log(c)/(2*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2
) + e**2*p*x**2*log(b)/(2*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2) - e**2
*p*x**2*log(x)/(2*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2) + e**2*p*x**2*
log(d/e + x)/(2*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2) + e**2*x**2*log(
c)/(2*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2), Eq(a, 0)), ((p*x*log(a +
b/x) + x*log(c) + b*p*log(a*x + b)/a)/d**3, Eq(e, 0)), (-3*d**2*p/(4*d**4*e
 + 8*d**3*e**2*x + 4*d**2*e**3*x**2) + 4*d*e*p*x*log(b/x + b*e/d)/(4*d**4*e
 + 8*d**3*e**2*x + 4*d**2*e**3*x**2) - 2*d*e*p*x/(4*d**4*e + 8*d**3*e**2*x
+ 4*d**2*e**3*x**2) + 4*d*e*x*log(c)/(4*d**4*e + 8*d**3*e**2*x + 4*d**2*e**
3*x**2) + 2*e**2*p*x**2*log(b/x + b*e/d)/(4*d**4*e + 8*d**3*e**2*x + 4*d**2
*e**3*x**2) + 2*e**2*x**2*log(c)/(4*d**4*e + 8*d**3*e**2*x + 4*d**2*e**3*x*
*2), Eq(a, b*e/d)), ((a**2*p*log(a + b/x)/(2*b**2) - a*p/(2*b*x) - p*log(a
+ b/x)/(2*x**2) + p/(4*x**2) - log(c)/(2*x**2))/e**3, Eq(d, 0)), (zoo*(p*x*
log(a + b/x) + x*log(c) + b*p*log(a*x + b)/a), Eq(d, -e*x)), (2*a**2*d**3*p
*x*log(a + b/x)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*
a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 +
4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) + 2*a**2*d**3*x*log(c)/(2*a**2*
d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*
e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b
**2*d**2*e**4*x**2) + a**2*d**2*e*p*x**2*log(a + b/x)/(2*a**2*d**6 + 4*a**2
*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*
b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4
*x**2) + a**2*d**2*e*x**2*log(c)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d*
*4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*
b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) + 2*a*b*d**3*p
*log(x + b/a)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*
b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*
b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) - 2*a*b*d**3*p*log(d/e + x)/(2*a*
*2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d*
*4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x +
2*b**2*d**2*e**4*x**2) + a*b*d**3*p/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2
*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 +
 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) - 4*a*b*d**
2*e*p*x*log(a + b/x)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2
 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e*
*2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) + 4*a*b*d**2*e*p*x*log(x +
 b/a)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e
 - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**
3*e**3*x + 2*b**2*d**2*e**4*x**2) - 4*a*b*d**2*e*p*x*log(d/e + x)/(2*a**2*d
**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e
**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b*
*2*d**2*e**4*x**2) + a*b*d**2*e*p*x/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2
*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 +
 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) - 4*a*b*d**
2*e*x*log(c)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b
*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b
**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) - 2*a*b*d*e**2*p*x**2*log(a + b/x)
```

```
/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*
a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**
3*x + 2*b**2*d**2*e**4*x**2) + 2*a*b*d*e**2*p*x**2*log(x + b/a)/(2*a**2*d**
6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**
2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2
*d**2*e**4*x**2) - 2*a*b*d*e**2*p*x**2*log(d/e + x)/(2*a**2*d**6 + 4*a**2*d
**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*
d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x
**2) - 2*a*b*d*e**2*x**2*log(c)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**
4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b
**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) - b**2*d**2*e*p
*log(x + b/a)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*
b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*
b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) + b**2*d**2*e*p*log(d/e + x)/(2*a
**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d
**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x +
 2*b**2*d**2*e**4*x**2) - b**2*d**2*e*p/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*
a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x*
*2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) + 2*b**
2*d*e**2*p*x*log(a + b/x)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2
*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d*
*4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) - 2*b**2*d*e**2*p*x*l
og(x + b/a)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*
d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b*
*2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) + 2*b**2*d*e**2*p*x*log(d/e + x)/(2
*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b
*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x
 + 2*b**2*d**2*e**4*x**2) - b**2*d*e**2*p*x/(2*a**2*d**6 + 4*a**2*d**5*e*x
+ 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**
3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) + 2
*b**2*d*e**2*x*log(c)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**
2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e
**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) + b**2*e**3*p*x**2*log(a
+ b/x)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*
e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d*
*3*e**3*x + 2*b**2*d**2*e**4*x**2) - b**2*e**3*p*x**2*log(x + b/a)/(2*a**2*
d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*
e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b
**2*d**2*e**4*x**2) + b**2*e**3*p*x**2*log(d/e + x)/(2*a**2*d**6 + 4*a**2*d
**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*
d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x
**2) + b**2*e**3*x**2*log(c)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e
**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2
*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2), True))
```

3.204 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4} \, dx$

Optimal. Leaf size=175

$$\frac{a^3p\log(ax+b)}{3e(ad-be)^3}-\frac{bp\left(3a^2d^2-3abde+b^2e^2\right)\log(d+ex)}{3d^3(ad-be)^3}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}+\frac{bp(2ad-be)}{3d^2(d+ex)(ad-be)^2}+\frac{bp}{6d(d+ex)^2}$$

```
[Out] 1/6*b*p/d/(a*d-b*e)/(e*x+d)^2+1/3*b*(2*a*d-b*e)*p/d^2/(a*d-b*e)^2/(e*x+d)-1
/3*ln(c*(a+b/x)^p)/e/(e*x+d)^3-1/3*p*ln(x)/d^3/e+1/3*a^3*p*ln(a*x+b)/e/(a*d
-b*e)^3-1/3*b*(3*a^2*d^2-3*a*b*d*e+b^2*e^2)*p*ln(e*x+d)/d^3/(a*d-b*e)^3
```

Rubi [A] time = 0.17, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2463, 514, 72}

$$-\frac{bp\left(3a^2d^2-3abde+b^2e^2\right)\log(d+ex)}{3d^3(ad-be)^3}+\frac{a^3p\log(ax+b)}{3e(ad-be)^3}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}+\frac{bp(2ad-be)}{3d^2(d+ex)(ad-be)^2}+\frac{b}{6d(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x)^4,x]
```

```
[Out] (b*p)/(6*d*(a*d - b*e)*(d + e*x)^2) + (b*(2*a*d - b*e)*p)/(3*d^2*(a*d - b*e
)^2*(d + e*x)) - Log[c*(a + b/x)^p]/(3*e*(d + e*x)^3) - (p*Log[x])/(3*d^3*e
) + (a^3*p*Log[b + a*x])/(3*e*(a*d - b*e)^3) - (b*(3*a^2*d^2 - 3*a*b*d*e +
b^2*e^2)*p*Log[d + e*x])/(3*d^3*(a*d - b*e)^3)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx = -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3} - \frac{(bp)\int \frac{1}{\left(a+\frac{b}{x}\right)x^2(d+ex)^3}\,dx}{3e}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3} - \frac{(bp)\int \frac{1}{x(b+ax)(d+ex)^3}\,dx}{3e}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3} - \frac{(bp)\int \left(\frac{1}{bd^3x} + \frac{a^4}{b(-ad+be)^3(b+ax)} + \frac{e^2}{d(ad-be)(d+ex)^3} + \frac{e^2(2ad-be)}{d^2(ad-be)^2(d+ex)^2} + \frac{e^2(\cdots}{\cdots}\right.}{3e}$$

$$= \frac{bp}{6d(ad-be)(d+ex)^2} + \frac{b(2ad-be)p}{3d^2(ad-be)^2(d+ex)} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3} - \frac{p\log(x)}{3d^3e} + \frac{a^3p\log(\cdots}{3e(ad-\cdots}$$

Mathematica [A] time = 0.28, size = 164, normalized size = 0.94

$$\frac{\frac{a^3p\log(ax+b)}{(ad-be)^3} - \frac{bep\left(3a^2d^2-3abde+b^2e^2\right)\log(d+ex)}{d^3(ad-be)^3} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3} + \frac{bep(2ad-be)}{d^2(d+ex)(ad-be)^2} + \frac{bep}{2d(d+ex)^2(ad-be)} - \frac{p\log(x)}{d^3}}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x)^4,x]
```

```
[Out] ((b*e*p)/(2*d*(a*d - b*e)*(d + e*x)^2) + (b*e*(2*a*d - b*e)*p)/(d^2*(a*d -
b*e)^2*(d + e*x)) - Log[c*(a + b/x)^p]/(d + e*x)^3 - (p*Log[x])/d^3 + (a^3*
p*Log[b + a*x])/(a*d - b*e)^3 - (b*e*(3*a^2*d^2 - 3*a*b*d*e + b^2*e^2)*p*Lo
g[d + e*x])/(d^3*(a*d - b*e)^3))/(3*e)
```

fricas [B] time = 6.46, size = 818, normalized size = 4.67

$$\frac{2\left(2a^2bd^3e^3 - 3ab^2d^2e^4 + b^3de^5\right)px^2 + \left(9a^2bd^4e^2 - 14ab^2d^3e^3 + 5b^3d^2e^4\right)px - 2\left(a^3d^6 - 3a^2bd^5e + 3ab^2d^4e^2 - \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(2*a^2*b*d^3*e^3 - 3*a*b^2*d^2*e^4 + b^3*d*e^5)*p*x^2 + (9*a^2*b*d^4
*e^2 - 14*a*b^2*d^3*e^3 + 5*b^3*d^2*e^4)*p*x - 2*(a^3*d^6 - 3*a^2*b*d^5*e +
 3*a*b^2*d^4*e^2 - b^3*d^3*e^3)*p*log((a*x + b)/x) + (5*a^2*b*d^5*e - 8*a*b
^2*d^4*e^2 + 3*b^3*d^3*e^3)*p + 2*(a^3*d^3*e^3*p*x^3 + 3*a^3*d^4*e^2*p*x^2
+ 3*a^3*d^5*e*p*x + a^3*d^6*p)*log(a*x + b) - 2*((3*a^2*b*d^2*e^4 - 3*a*b^2
*d*e^5 + b^3*e^6)*p*x^3 + 3*(3*a^2*b*d^3*e^3 - 3*a*b^2*d^2*e^4 + b^3*d*e^5)
*p*x^2 + 3*(3*a^2*b*d^4*e^2 - 3*a*b^2*d^3*e^3 + b^3*d^2*e^4)*p*x + (3*a^2*b
*d^5*e - 3*a*b^2*d^4*e^2 + b^3*d^3*e^3)*p)*log(e*x + d) - 2*(a^3*d^6 - 3*a^
2*b*d^5*e + 3*a*b^2*d^4*e^2 - b^3*d^3*e^3)*log(c) - 2*((a^3*d^3*e^3 - 3*a^2
*b*d^2*e^4 + 3*a*b^2*d*e^5 - b^3*e^6)*p*x^3 + 3*(a^3*d^4*e^2 - 3*a^2*b*d^3*
e^3 + 3*a*b^2*d^2*e^4 - b^3*d*e^5)*p*x^2 + 3*(a^3*d^5*e - 3*a^2*b*d^4*e^2 +
 3*a*b^2*d^3*e^3 - b^3*d^2*e^4)*p*x + (a^3*d^6 - 3*a^2*b*d^5*e + 3*a*b^2*d^
4*e^2 - b^3*d^3*e^3)*p)*log(x))/(a^3*d^9*e - 3*a^2*b*d^8*e^2 + 3*a*b^2*d^7*
e^3 - b^3*d^6*e^4 + (a^3*d^6*e^4 - 3*a^2*b*d^5*e^5 + 3*a*b^2*d^4*e^6 - b^3*
d^3*e^7)*x^3 + 3*(a^3*d^7*e^3 - 3*a^2*b*d^6*e^4 + 3*a*b^2*d^5*e^5 - b^3*d^4
*e^6)*x^2 + 3*(a^3*d^8*e^2 - 3*a^2*b*d^7*e^3 + 3*a*b^2*d^6*e^4 - b^3*d^5*e^
5)*x)
```

giac [B] time = 0.32, size = 1841, normalized size = 10.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -1/6*(6*a^5*b^2*d^5*p*log(-a*d + b*e + (a*x + b)*d/x) - 24*a^4*b^3*d^4*p*e*
log(-a*d + b*e + (a*x + b)*d/x) - 6*a^4*b^3*d^4*p*e - 18*(a*x + b)*a^4*b^2*
d^5*p*log(-a*d + b*e + (a*x + b)*d/x)/x + 38*a^3*b^4*d^3*p*e^2*log(-a*d + b
*e + (a*x + b)*d/x) + 54*(a*x + b)*a^3*b^3*d^4*p*e*log(-a*d + b*e + (a*x +
b)*d/x)/x + 6*a^5*b^2*d^5*log(c) - 24*a^4*b^3*d^4*e*log(c) + 6*(a*x + b)*a^
4*b^2*d^5*p*log((a*x + b)/x)/x - 12*(a*x + b)*a^3*b^3*d^4*p*e*log((a*x + b)
/x)/x + 21*a^3*b^4*d^3*p*e^2 + 12*(a*x + b)*a^3*b^3*d^4*p*e/x + 18*(a*x + b
)^2*a^3*b^2*d^5*p*log(-a*d + b*e + (a*x + b)*d/x)/x^2 - 30*a^2*b^5*d^2*p*e^
3*log(-a*d + b*e + (a*x + b)*d/x) - 60*(a*x + b)*a^2*b^4*d^3*p*e^2*log(-a*d
 + b*e + (a*x + b)*d/x)/x - 36*(a*x + b)^2*a^2*b^3*d^4*p*e*log(-a*d + b*e +
 (a*x + b)*d/x)/x^2 - 12*(a*x + b)*a^4*b^2*d^5*log(c)/x + 38*a^3*b^4*d^3*e^
2*log(c) + 42*(a*x + b)*a^3*b^3*d^4*e*log(c)/x - 12*(a*x + b)^2*a^3*b^2*d^5
*p*log((a*x + b)/x)/x^2 + 6*(a*x + b)*a^2*b^4*d^3*p*e^2*log((a*x + b)/x)/x
+ 18*(a*x + b)^2*a^2*b^3*d^4*p*e*log((a*x + b)/x)/x^2 - 27*a^2*b^5*d^2*p*e^
3 - 31*(a*x + b)*a^2*b^4*d^3*p*e^2/x - 6*(a*x + b)^2*a^2*b^3*d^4*p*e/x^2 -
6*(a*x + b)^3*a^2*b^2*d^5*p*log(-a*d + b*e + (a*x + b)*d/x)/x^3 + 12*a*b^6*
d*p*e^4*log(-a*d + b*e + (a*x + b)*d/x) + 30*(a*x + b)*a*b^5*d^2*p*e^3*log(
-a*d + b*e + (a*x + b)*d/x)/x + 24*(a*x + b)^2*a*b^4*d^3*p*e^2*log(-a*d + b
*e + (a*x + b)*d/x)/x^2 + 6*(a*x + b)^3*a*b^3*d^4*p*e*log(-a*d + b*e + (a*x
 + b)*d/x)/x^3 + 6*(a*x + b)^2*a^3*b^2*d^5*log(c)/x^2 - 30*a^2*b^5*d^2*e^3*
log(c) - 54*(a*x + b)*a^2*b^4*d^3*e^2*log(c)/x - 18*(a*x + b)^2*a^2*b^3*d^4
*e*log(c)/x^2 + 6*(a*x + b)^3*a^2*b^2*d^5*p*log((a*x + b)/x)/x^3 - 6*(a*x +
 b)^2*a*b^4*d^3*p*e^2*log((a*x + b)/x)/x^2 - 6*(a*x + b)^3*a*b^3*d^4*p*e*lo
g((a*x + b)/x)/x^3 + 15*a*b^6*d*p*e^4 + 26*(a*x + b)*a*b^5*d^2*p*e^3/x + 10
*(a*x + b)^2*a*b^4*d^3*p*e^2/x^2 - 2*b^7*p*e^5*log(-a*d + b*e + (a*x + b)*d
/x) - 6*(a*x + b)*b^6*d*p*e^4*log(-a*d + b*e + (a*x + b)*d/x)/x - 6*(a*x +
b)^2*b^5*d^2*p*e^3*log(-a*d + b*e + (a*x + b)*d/x)/x^2 - 2*(a*x + b)^3*b^4*
d^3*p*e^2*log(-a*d + b*e + (a*x + b)*d/x)/x^3 + 12*a*b^6*d*e^4*log(c) + 30*
(a*x + b)*a*b^5*d^2*e^3*log(c)/x + 18*(a*x + b)^2*a*b^4*d^3*e^2*log(c)/x^2
+ 2*(a*x + b)^3*b^4*d^3*p*e^2*log((a*x + b)/x)/x^3 - 3*b^7*p*e^5 - 7*(a*x +
 b)*b^6*d*p*e^4/x - 4*(a*x + b)^2*b^5*d^2*p*e^3/x^2 - 2*b^7*e^5*log(c) - 6*
(a*x + b)*b^6*d*e^4*log(c)/x - 6*(a*x + b)^2*b^5*d^2*e^3*log(c)/x^2)/((a^6*
d^9 - 6*a^5*b*d^8*e - 3*(a*x + b)*a^5*d^9/x + 15*a^4*b^2*d^7*e^2 + 15*(a*x
+ b)*a^4*b*d^8*e/x + 3*(a*x + b)^2*a^4*d^9/x^2 - 20*a^3*b^3*d^6*e^3 - 30*(a
*x + b)*a^3*b^2*d^7*e^2/x - 12*(a*x + b)^2*a^3*b*d^8*e/x^2 - (a*x + b)^3*a^
3*d^9/x^3 + 15*a^2*b^4*d^5*e^4 + 30*(a*x + b)*a^2*b^3*d^6*e^3/x + 18*(a*x +
 b)^2*a^2*b^2*d^7*e^2/x^2 + 3*(a*x + b)^3*a^2*b*d^8*e/x^3 - 6*a*b^5*d^4*e^5
 - 15*(a*x + b)*a*b^4*d^5*e^4/x - 12*(a*x + b)^2*a*b^3*d^6*e^3/x^2 - 3*(a*x
 + b)^3*a*b^2*d^7*e^2/x^3 + b^6*d^3*e^6 + 3*(a*x + b)*b^5*d^4*e^5/x + 3*(a*
x + b)^2*b^4*d^5*e^4/x^2 + (a*x + b)^3*b^3*d^6*e^3/x^3)*b)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{(ex+d)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/(e*x+d)^4,x)
```

```
[Out] int(ln(c*(a+b/x)^p)/(e*x+d)^4,x)
```

maxima [A] time = 0.52, size = 299, normalized size = 1.71

$$\frac{\left(\frac{2\,a^3 \log(ax+b)}{a^3bd^3-3\,a^2b^2d^2e+3\,ab^3de^2-b^4e^3} - \frac{2\left(3\,a^2d^2e-3\,abde^2+b^2e^3\right)\log(ex+d)}{a^3d^6-3\,a^2bd^5e+3\,ab^2d^4e^2-b^3d^3e^3} + \frac{5\,ad^2e-3\,bde^2+2\left(2\,ade^2-be^3\right)x}{a^2d^6-2\,abd^5e+b^2d^4e^2+\left(a^2d^4e^2-2\,abd^3e^3+b^2d^2e^4\right)x^2+2\left(a^2d^5e-2\,abd^4e^2+\right.}}{6\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^3*log(a*x + b)/(a^3*b*d^3 - 3*a^2*b^2*d^2*e + 3*a*b^3*d*e^2 - b^4*
e^3) - 2*(3*a^2*d^2*e - 3*a*b*d*e^2 + b^2*e^3)*log(e*x + d)/(a^3*d^6 - 3*a^
2*b*d^5*e + 3*a*b^2*d^4*e^2 - b^3*d^3*e^3) + (5*a*d^2*e - 3*b*d*e^2 + 2*(2*
a*d*e^2 - b*e^3)*x)/(a^2*d^6 - 2*a*b*d^5*e + b^2*d^4*e^2 + (a^2*d^4*e^2 - 2
*a*b*d^3*e^3 + b^2*d^2*e^4)*x^2 + 2*(a^2*d^5*e - 2*a*b*d^4*e^2 + b^2*d^3*e^
3)*x) - 2*log(x)/(b*d^3))*b*p/e - 1/3*log((a + b/x)^p*c)/((e*x + d)^3*e)
```

mupad [B] time = 1.85, size = 662, normalized size = 3.78

$$\frac{p\,\ln\left(d+e\,x\right)}{3\,d^3\,e}-\frac{3\,b^2\,e^2\,p}{2\,\left(3\,a^2\,d^5\,e+6\,a^2\,d^4\,e^2\,x+3\,a^2\,d^3\,e^3\,x^2-6\,a\,b\,d^4\,e^2-12\,a\,b\,d^3\,e^3\,x-6\,a\,b\,d^2\,e^4\,x^2+3\,b^2\,d^3\,e^3+6\,b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/(d + e*x)^4,x)
```

```
[Out] (p*log(d + e*x))/(3*d^3*e) - (3*b^2*e^2*p)/(2*(3*a^2*d^5*e + 3*b^2*d^3*e^3
+ 6*a^2*d^4*e^2*x + 6*b^2*d^2*e^4*x + 3*b^2*d*e^5*x^2 + 3*a^2*d^3*e^3*x^2 -
 6*a*b*d^4*e^2 - 12*a*b*d^3*e^3*x - 6*a*b*d^2*e^4*x^2)) - (p*log(x))/(3*d^3
*e) - (a^3*p*log(b + a*x))/(3*b^3*e^4 - 3*a^3*d^3*e + 9*a^2*b*d^2*e^2 - 9*a
*b^2*d*e^3) - log(c*((b + a*x)/x)^p)/(3*(d^3*e + e^4*x^3 + 3*d^2*e^2*x + 3*
d*e^3*x^2)) - (b^2*e^3*p*x)/(3*a^2*d^6*e + 3*b^2*d^4*e^3 + 6*a^2*d^5*e^2*x
+ 6*b^2*d^3*e^4*x + 3*a^2*d^4*e^3*x^2 + 3*b^2*d^2*e^5*x^2 - 6*a*b*d^5*e^2 -
 12*a*b*d^4*e^3*x - 6*a*b*d^3*e^4*x^2) - (a^3*d^3*p*log(d + e*x))/(3*a^3*d^
6*e - 3*b^3*d^3*e^4 + 9*a*b^2*d^4*e^3 - 9*a^2*b*d^5*e^2) + (5*a*b*d*e*p)/(2
*(3*a^2*d^5*e + 3*b^2*d^3*e^3 + 6*a^2*d^4*e^2*x + 6*b^2*d^2*e^4*x + 3*b^2*d
*e^5*x^2 + 3*a^2*d^3*e^3*x^2 - 6*a*b*d^4*e^2 - 12*a*b*d^3*e^3*x - 6*a*b*d^2
*e^4*x^2)) + (2*a*b*d*e^2*p*x)/(3*a^2*d^6*e + 3*b^2*d^4*e^3 + 6*a^2*d^5*e^2
*x + 6*b^2*d^3*e^4*x + 3*a^2*d^4*e^3*x^2 + 3*b^2*d^2*e^5*x^2 - 6*a*b*d^5*e^
2 - 12*a*b*d^4*e^3*x - 6*a*b*d^3*e^4*x^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d)**4,x)
```

```
[Out] Timed out
```

3.205 $\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx$

Optimal. Leaf size=105

$$-\frac{\text{Li}_2\left(\frac{a(c+dx)}{ac-bd}\right)}{d}+\frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}-\frac{\log(c+dx)\log\left(-\frac{d(ax+b)}{ac-bd}\right)}{d}+\frac{\text{Li}_2\left(\frac{dx}{c}+1\right)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}$$

```
[Out] ln(a+b/x)*ln(d*x+c)/d+ln(-d*x/c)*ln(d*x+c)/d-ln(-d*(a*x+b)/(a*c-b*d))*ln(d*
x+c)/d-polylog(2,a*(d*x+c)/(a*c-b*d))/d+polylog(2,1+d*x/c)/d
```

Rubi [A] time = 0.17, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2462, 260, 2416, 2394, 2315, 2393, 2391}

$$-\frac{\text{PolyLog}\left(2,\frac{a(c+dx)}{ac-bd}\right)}{d}+\frac{\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{d}+\frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}-\frac{\log(c+dx)\log\left(-\frac{d(ax+b)}{ac-bd}\right)}{d}+\frac{\log\left(-\frac{dx}{c}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[a + b/x]/(c + d*x),x]
```

```
[Out] (Log[a + b/x]*Log[c + d*x])/d + (Log[-((d*x)/c)]*Log[c + d*x])/d - (Log[-((
d*(b + a*x))/(a*c - b*d))]*Log[c + d*x])/d - PolyLog[2, (a*(c + d*x))/(a*c
- b*d)]/d + PolyLog[2, 1 + (d*x)/c]/d
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx &= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{b\int\frac{\log(c+dx)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{d}\\ &= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{b\int\left(\frac{\log(c+dx)}{bx}-\frac{a\log(c+dx)}{b(b+ax)}\right)dx}{d}\\ &= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{\int\frac{\log(c+dx)}{x}\,dx}{d}-\frac{a\int\frac{\log(c+dx)}{b+ax}\,dx}{d}\\ &= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}-\frac{\log\left(-\frac{d(b+ax)}{ac-bd}\right)\log(c+dx)}{d}-\int\frac{\log\left(-\frac{dx}{c}\right)}{c+dx}\\ &= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}-\frac{\log\left(-\frac{d(b+ax)}{ac-bd}\right)\log(c+dx)}{d}+\frac{\mathrm{Li}_2\left(1+\frac{dx}{c}\right)}{d}\\ &= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}-\frac{\log\left(-\frac{d(b+ax)}{ac-bd}\right)\log(c+dx)}{d}-\frac{\mathrm{Li}_2\left(\frac{a(c+dx)}{ac-bd}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 80, normalized size = 0.76

$$\frac{-\mathrm{Li}_2\left(\frac{a(c+dx)}{ac-bd}\right)+\log(c+dx)\left(-\log\left(\frac{d(ax+b)}{bd-ac}\right)+\log\left(a+\frac{b}{x}\right)+\log\left(-\frac{dx}{c}\right)\right)+\mathrm{Li}_2\left(\frac{dx}{c}+1\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a + b/x]/(c + d*x),x]
```

```
[Out] ((Log[a + b/x] + Log[-((d*x)/c)] - Log[(d*(b + a*x))/(-(a*c) + b*d)])*Log[c
 + d*x] - PolyLog[2, (a*(c + d*x))/(a*c - b*d)] + PolyLog[2, 1 + (d*x)/c])/
d
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log\left(\frac{ax+b}{x}\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b/x)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log((a*x + b)/x)/(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(a + \frac{b}{x}\right)}{dx + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b/x)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log(a + b/x)/(d*x + c), x)
```

maple [A] time = 0.14, size = 114, normalized size = 1.09

$$\frac{\ln\left(\frac{-ac+bd+\left(a+\frac{b}{x}\right)c}{-ac+bd}\right)\ln\left(a+\frac{b}{x}\right)}{d} - \frac{\ln\left(-\frac{b}{ax}\right)\ln\left(a+\frac{b}{x}\right)}{d} + \frac{\mathrm{dilog}\left(\frac{-ac+bd+\left(a+\frac{b}{x}\right)c}{-ac+bd}\right)}{d} - \frac{\mathrm{dilog}\left(-\frac{b}{ax}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a+b/x)/(d*x+c),x)
```

```
[Out] -1/d*ln(a+b/x)*ln(-b/a/x)-1/d*dilog(-b/a/x)+1/d*dilog((c*(a+b/x)-a*c+b*d)/(
-a*c+b*d))+1/d*ln(a+b/x)*ln((c*(a+b/x)-a*c+b*d)/(-a*c+b*d))
```

maxima [A] time = 0.48, size = 82, normalized size = 0.78

$$-\frac{\log\left(\frac{dx}{c}+1\right)\log(x) + \mathrm{Li}_2\left(-\frac{dx}{c}\right)}{d} + \frac{\log\left(ax+b\right)\log\left(\frac{adx+bd}{ac-bd}+1\right) + \mathrm{Li}_2\left(-\frac{adx+bd}{ac-bd}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b/x)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -(log(d*x/c + 1)*log(x) + dilog(-d*x/c))/d + (log(a*x + b)*log((a*d*x + b*d
)/(a*c - b*d) + 1) + dilog(-(a*d*x + b*d)/(a*c - b*d)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(a + \frac{b}{x}\right)}{c + d\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b/x)/(c + d*x),x)
```

```
[Out] int(log(a + b/x)/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(a + \frac{b}{x}\right)}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a+b/x)/(d*x+c),x)
```

```
[Out] Integral(log(a + b/x)/(c + d*x), x)
```

3.206 $\int (d+ex)^m \log \left(c \left(a+bx^3\right)^p\right) dx$

Optimal. Leaf size=301

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+bx^3\right)^p\right)}{e(m+1)}+\frac{\sqrt[3]{b}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)}{e(m+1)(m+2)\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)}+\frac{\sqrt[3]{b}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;\right.}{e(m+1)(m+2)\left(\sqrt[3]{-1}\right.}$$

```
[Out] b^(1/3)*p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],b^(1/3)*(e*x+d)/(b^(1/3)*d
-a^(1/3)*e))/e/(b^(1/3)*d-a^(1/3)*e)/(1+m)/(2+m)+b^(1/3)*p*(e*x+d)^(2+m)*hy
pergeom([1, 2+m],[3+m],b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/e/
(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e)/(1+m)/(2+m)+b^(1/3)*p*(e*x+d)^(2+m)*hyperg
eom([1, 2+m],[3+m],b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/e/(b^(
1/3)*d-(-1)^(2/3)*a^(1/3)*e)/(1+m)/(2+m)+(e*x+d)^(1+m)*ln(c*(b*x^3+a)^p)/e/
(1+m)
```

Rubi [A] time = 0.76, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2463, 6725, 68}

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+bx^3\right)^p\right)}{e(m+1)}+\frac{\sqrt[3]{b}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)}{e(m+1)(m+2)\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)}+\frac{\sqrt[3]{b}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;\right.}{e(m+1)(m+2)\left(\sqrt[3]{-1}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (b^(1/3)*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (b^(1/3)*(d
 + e*x))/(b^(1/3)*d - a^(1/3)*e)])/(e*(b^(1/3)*d - a^(1/3)*e)*(1 + m)*(2 +
m)) + (b^(1/3)*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (b^(1
/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/(e*(b^(1/3)*d + (-1)^(1
/3)*a^(1/3)*e)*(1 + m)*(2 + m)) + (b^(1/3)*p*(d + e*x)^(2 + m)*Hypergeometr
ic2F1[1, 2 + m, 3 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*
e)])/(e*(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)*(1 + m)*(2 + m)) + ((d + e*x)^(1
 + m)*Log[c*(a + b*x^3)^p])/(e*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int (d+ex)^m \log\left(c\left(a+bx^3\right)^p\right) dx = \frac{(d+ex)^{1+m}\log\left(c\left(a+bx^3\right)^p\right)}{e(1+m)} - \frac{(3bp)\int \frac{x^2(d+ex)^{1+m}}{a+bx^3}\,dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+bx^3\right)^p\right)}{e(1+m)} - \frac{(3bp)\int\left(\frac{(d+ex)^{1+m}}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(d+ex)^{1+m}}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\right.}\right.}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+bx^3\right)^p\right)}{e(1+m)} - \frac{\left(\sqrt[3]{b}p\right)\int \frac{(d+ex)^{1+m}}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e(1+m)} - \frac{\left(\sqrt[3]{b}p\right)\int \frac{(d+ex}{-\sqrt[3]{-1}\sqrt[3]{}}}{e(1+m}$$

$$= \frac{\sqrt[3]{b}p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)(1+m)(2+m)} + \frac{\sqrt[3]{b}p(d+ex)^{2+m}\,{}_2F_1\left(\right.}{e\left(\sqrt[3]{b}d+\sqrt[3]{-1}\right.}$$

Mathematica [A] time = 0.68, size = 239, normalized size = 0.79

$$\frac{(d+ex)^{m+1}\left(\log\left(c\left(a+bx^3\right)^p\right) - \frac{\sqrt[3]{b}p(d+ex)\left(-\frac{{}_2F_1\left(1,m+2;m+3;\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e} - \frac{{}_2F_1\left(1,m+2;m+3;\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d} - \frac{{}_2F_1\left(1,m+2;m+3;\frac{\sqrt[3]{b}(d+ex}{\sqrt[3]{b}d-(-1)^{2/3}}\right.}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right.}{m+2}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b*x^3)^p],x]
```

```
[Out] ((d + e*x)^(1 + m)*(-((b^(1/3)*p*(d + e*x)*(-(Hypergeometric2F1[1, 2 + m, 3
 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)]/(b^(1/3)*d - a^(1/3)*e))
 - Hypergeometric2F1[1, 2 + m, 3 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)
^(1/3)*a^(1/3)*e)]/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e) - Hypergeometric2F1[1
, 2 + m, 3 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)]/(b^
(1/3)*d - (-1)^(2/3)*a^(1/3)*e)))/(2 + m)) + Log[c*(a + b*x^3)^p]))/(e*(1 +
 m))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left((ex+d)^m \log\left(\left(bx^3+a\right)^p c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m*log((b*x^3 + a)^p*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^m \log\left(\left(bx^3+a\right)^p c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m*log((b*x^3 + a)^p*c), x)
```

maple [F] time = 1.17, size = 0, normalized size = 0.00

$$\int (ex+d)^{m} \ln \left(c \left(b\,x^{3}+a\right)^{p}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*ln(c*(b*x^3+a)^p),x)
```

```
[Out] int((e*x+d)^m*ln(c*(b*x^3+a)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(ex+d)(ex+d)^{m}\log\left(\left(bx^{3}+a\right)^{p}\right)}{e(m+1)}+\int -\frac{\left(3\,bdpx^{2}-\left(e(m+1)\log(c)-3\,ep\right)bx^{3}-ae(m+1)\log(c)\right)(ex+d)^{m}}{be(m+1)x^{3}+ae(m+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] (e*x + d)*(e*x + d)^m*log((b*x^3 + a)^p)/(e*(m + 1)) + integrate(-(3*b*d*p*
x^2 - (e*(m + 1)*log(c) - 3*e*p)*b*x^3 - a*e*(m + 1)*log(c))*(e*x + d)^m/(b
*e*(m + 1)*x^3 + a*e*(m + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,{\left(b\,x^3+a\right)}^p\right)\,{\left(d+e\,x\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)*(d + e*x)^m,x)
```

```
[Out] int(log(c*(a + b*x^3)^p)*(d + e*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Timed out
```

3.207 $\int (d+ex)^m \log\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=205

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+bx^2\right)^p\right)}{e(m+1)}+\frac{\sqrt{b}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{e(m+1)(m+2)\left(\sqrt{b}\,d-\sqrt{-a}\,e\right)}+\frac{\sqrt{b}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m\right.}{e(m+1)(m+2)\left(\sqrt{\right.}}$$

```
[Out] (e*x+d)^(1+m)*ln(c*(b*x^2+a)^p)/e/(1+m)+p*(e*x+d)^(2+m)*hypergeom([1, 2+m],
[3+m],(e*x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))*b^(1/2)/e/(1+m)/(2+m)/(-e*
(-a)^(1/2)+d*b^(1/2))+p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],(e*x+d)*b^(1
/2)/(e*(-a)^(1/2)+d*b^(1/2)))*b^(1/2)/e/(1+m)/(2+m)/(e*(-a)^(1/2)+d*b^(1/2)
)
```

Rubi [A] time = 0.25, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2463, 831, 68}

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+bx^2\right)^p\right)}{e(m+1)}+\frac{\sqrt{b}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{e(m+1)(m+2)\left(\sqrt{b}\,d-\sqrt{-a}\,e\right)}+\frac{\sqrt{b}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m\right.}{e(m+1)(m+2)\left(\sqrt{\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (Sqrt[b]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[b]*(d
 + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/(e*(Sqrt[b]*d - Sqrt[-a]*e)*(1 + m)*(2
+ m)) + (Sqrt[b]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sq
rt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/(e*(Sqrt[b]*d + Sqrt[-a]*e)*(1
+ m)*(2 + m)) + ((d + e*x)^(1 + m)*Log[c*(a + b*x^2)^p])/(e*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)^m \log\left(c\left(a+bx^2\right)^p\right)dx = \frac{(d+ex)^{1+m}\log\left(c\left(a+bx^2\right)^p\right)}{e(1+m)} - \frac{(2bp)\int \frac{x(d+ex)^{1+m}}{a+bx^2}dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+bx^2\right)^p\right)}{e(1+m)} - \frac{(2bp)\int\left(-\frac{(d+ex)^{1+m}}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{(d+ex)^{1+m}}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+bx^2\right)^p\right)}{e(1+m)} + \frac{\left(\sqrt{b}p\right)\int\frac{(d+ex)^{1+m}}{\sqrt{-a}-\sqrt{b}x}dx}{e(1+m)} - \frac{\left(\sqrt{b}p\right)\int\frac{(d+ex)^{1+m}}{\sqrt{-a}+\sqrt{b}x}}{e(1+m)}$$

$$= \frac{\sqrt{b}p(d+ex)^{2+m}{}_2F_1\left(1,2+m;3+m;\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e\left(\sqrt{b}d-\sqrt{-a}e\right)(1+m)(2+m)} + \frac{\sqrt{b}p(d+ex)^{2+m}{}_2F_1\left(1,2\right.}{e\left(\sqrt{b}d+\sqrt{-a}e\right.}$$

Mathematica [A] time = 0.25, size = 176, normalized size = 0.86

$$\frac{(d+ex)^{m+1}\left(\log\left(c\left(a+bx^2\right)^p\right)+\frac{\sqrt{b}p(d+ex)\left(\left(\sqrt{-a}e+\sqrt{b}d\right){}_2F_1\left(1,m+2;m+3;\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)+\left(\sqrt{b}d-\sqrt{-a}e\right){}_2F_1\left(1,m+2;m+3;\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)\right)}{(m+2)\left(ae^2+bd^2\right)}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b*x^2)^p],x]
```

```
[Out] ((d + e*x)^(1 + m)*((Sqrt[b]*p*(d + e*x)*((Sqrt[b]*d + Sqrt[-a]*e)*Hypergeo
metric2F1[1, 2 + m, 3 + m, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] +
(Sqrt[b]*d - Sqrt[-a]*e)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[b]*(d + e
*x))/(Sqrt[b]*d + Sqrt[-a]*e)]))/((b*d^2 + a*e^2)*(2 + m)) + Log[c*(a + b*x
^2)^p]))/(e*(1 + m))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((ex+d)^m\log\left(\left(bx^2+a\right)^pc\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m*log((b*x^2 + a)^p*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^m\log\left(\left(bx^2+a\right)^pc\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m*log((b*x^2 + a)^p*c), x)
```

maple [F] time = 1.36, size = 0, normalized size = 0.00

$$\int (ex+d)^m\ln\left(c\left(b\,x^2+a\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*ln(c*(b*x^2+a)^p),x)
```

```
[Out] int((e*x+d)^m*ln(c*(b*x^2+a)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(epx+dp\right)(ex+d)^m\log\left(bx^2+a\right)}{e(m+1)}+\int-\frac{\left(2\,bdpx-\left(e(m+1)\log(c)-2\,ep\right)bx^2-ae(m+1)\log(c)\right)(ex+d)^m}{be(m+1)x^2+ae(m+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] (e*p*x + d*p)*(e*x + d)^m*log(b*x^2 + a)/(e*(m + 1)) + integrate(-(2*b*d*p*
x - (e*(m + 1)*log(c) - 2*e*p)*b*x^2 - a*e*(m + 1)*log(c))*(e*x + d)^m/(b*e
*(m + 1)*x^2 + a*e*(m + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(b\,x^2+a\right)^p\right)\,(d+e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)*(d + e*x)^m,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)*(d + e*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Timed out
```

3.208 $\int (d+ex)^m \log\left(c(a+bx)^p\right) dx$

Optimal. Leaf size=89

$$\frac{(d+ex)^{m+1}\log\left(c(a+bx)^p\right)}{e(m+1)}+\frac{bp(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{b(d+ex)}{bd-ae}\right)}{e(m+1)(m+2)(bd-ae)}$$

```
[Out] b*p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],b*(e*x+d)/(-a*e+b*d))/e/(-a*e+b*
d)/(1+m)/(2+m)+(e*x+d)^(1+m)*ln(c*(b*x+a)^p)/e/(1+m)
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2395, 68}

$$\frac{(d+ex)^{m+1}\log\left(c(a+bx)^p\right)}{e(m+1)}+\frac{bp(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{b(d+ex)}{bd-ae}\right)}{e(m+1)(m+2)(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Log[c*(a + b*x)^p],x]
```

```
[Out] (b*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (b*(d + e*x))/(b*
d - a*e)])/(e*(b*d - a*e)*(1 + m)*(2 + m)) + ((d + e*x)^(1 + m)*Log[c*(a +
b*x)^p])/(e*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m \log\left(c(a+bx)^p\right) dx &= \frac{(d+ex)^{1+m}\log\left(c(a+bx)^p\right)}{e(1+m)}-\frac{(bp)\int \frac{(d+ex)^{1+m}}{a+bx}\,dx}{e(1+m)}\\ &= \frac{bp(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{b(d+ex)}{bd-ae}\right)}{e(bd-ae)(1+m)(2+m)}+\frac{(d+ex)^{1+m}\log\left(c(a+bx)^p\right)}{e(1+m)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 77, normalized size = 0.87

$$\frac{(d+ex)^{m+1}\left(\log\left(c(a+bx)^p\right)+\frac{bp(d+ex)\,{}_2F_1\left(1,m+2;m+3;\frac{b(d+ex)}{bd-ae}\right)}{(m+2)(bd-ae)}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b*x)^p],x]
```

```
[Out] ((d + e*x)^(1 + m)*((b*p*(d + e*x)*Hypergeometric2F1[1, 2 + m, 3 + m, (b*(d
 + e*x))/(b*d - a*e)])/((b*d - a*e)*(2 + m)) + Log[c*(a + b*x)^p]))/(e*(1 +
 m))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left((ex+d)^m \log\left((bx+a)^p c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x+a)^p),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m*log((b*x + a)^p*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^m \log\left((bx+a)^p c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x+a)^p),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m*log((b*x + a)^p*c), x)
```

maple [F] time = 1.23, size = 0, normalized size = 0.00

$$\int (ex+d)^m \ln\left(c\,(bx+a)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*ln(c*(b*x+a)^p),x)
```

```
[Out] int((e*x+d)^m*ln(c*(b*x+a)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(ex+d)(ex+d)^m \log\left((bx+a)^p\right)}{e(m+1)} + \int \frac{\left(ae(m+1)\log(c) - bdp + \left(e(m+1)\log(c) - ep\right)bx\right)(ex+d)^m}{be(m+1)x + ae(m+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x+a)^p),x, algorithm="maxima")
```

```
[Out] (e*x + d)*(e*x + d)^m*log((b*x + a)^p)/(e*(m + 1)) + integrate((a*e*(m + 1)
*log(c) - b*d*p + (e*(m + 1)*log(c) - e*p)*b*x)*(e*x + d)^m/(b*e*(m + 1)*x
+ a*e*(m + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\,(a+b\,x)^p\right)\,(d+e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)*(d + e*x)^m,x)
```

```
[Out] int(log(c*(a + b*x)^p)*(d + e*x)^m, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*ln(c*(b*x+a)**p),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.209 $\int (d+ex)^m \log \left(c \left(a+\frac{b}{x}\right)^p\right) \, dx$

Optimal. Leaf size=135

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(m+1)}+\frac{ap(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{a(d+ex)}{ad-be}\right)}{e(m+1)(m+2)(ad-be)}-\frac{p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{ex}{d}\right.}{de\left(m^2+3m+2\right)}$$

```
[Out] a*p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],a*(e*x+d)/(a*d-b*e))/e/(a*d-b*e)
/(1+m)/(2+m)-p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],1+e*x/d)/d/e/(m^2+3*m
+2)+(e*x+d)^(1+m)*ln(c*(a+b/x)^p)/e/(1+m)
```

Rubi [A] time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2463, 514, 86, 65, 68}

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(m+1)}+\frac{ap(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{a(d+ex)}{ad-be}\right)}{e(m+1)(m+2)(ad-be)}-\frac{p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{ex}{d}\right.}{de\left(m^2+3m+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Log[c*(a + b/x)^p],x]
```

```
[Out] (a*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (a*(d + e*x))/(a*
d - b*e)])/(e*(a*d - b*e)*(1 + m)*(2 + m)) - (p*(d + e*x)^(2 + m)*Hypergeom
etric2F1[1, 2 + m, 3 + m, 1 + (e*x)/d])/(d*e*(2 + 3*m + m^2)) + ((d + e*x)^
(1 + m)*Log[c*(a + b/x)^p])/(e*(1 + m))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m \log\left(c\left(a+\frac{b}{x}\right)^p\right)dx &= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(1+m)}+\frac{(bp)\int \frac{(d+ex)^{1+m}}{\left(a+\frac{b}{x}\right)x^2}dx}{e(1+m)}\\ &= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(1+m)}+\frac{(bp)\int \frac{(d+ex)^{1+m}}{x(b+ax)}dx}{e(1+m)}\\ &= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(1+m)}+\frac{p\int \frac{(d+ex)^{1+m}}{x}dx}{e(1+m)}-\frac{(ap)\int \frac{(d+ex)^{1+m}}{b+ax}dx}{e(1+m)}\\ &= \frac{ap(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{a(d+ex)}{ad-be}\right)}{e(ad-be)(1+m)(2+m)}-\frac{p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3\right.}{de\left(2+3m+m^2\right.}\end{aligned}$$

Mathematica [A] time = 0.08, size = 123, normalized size = 0.91

$$\frac{(d+ex)^{m+1}\left((ad-be)\left(p(d+ex)\,{}_2F_1\left(1,m+2;m+3;\frac{ex}{d}+1\right)-d(m+2)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\right)-adp(d+ex)\,{}_2F_1\left(\right.}{de(m+1)(m+2)(be-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b/x)^p],x]
```

```
[Out] ((d + e*x)^(1 + m)*(-(a*d*p*(d + e*x)*Hypergeometric2F1[1, 2 + m, 3 + m, (a
*(d + e*x))/(a*d - b*e)]) + (a*d - b*e)*(p*(d + e*x)*Hypergeometric2F1[1, 2
 + m, 3 + m, 1 + (e*x)/d] - d*(2 + m)*Log[c*(a + b/x)^p])))/(d*e*(-(a*d) +
b*e)*(1 + m)*(2 + m))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((ex+d)^m \log\left(c\left(\frac{ax+b}{x}\right)^p\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m*log(c*((a*x + b)/x)^p), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^m \log\left(\left(a+\frac{b}{x}\right)^p c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m*log((a + b/x)^p*c), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int (ex+d)^m \ln\left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*ln(c*(a+b/x)^p),x)
```

```
[Out] int((e*x+d)^m*ln(c*(a+b/x)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(ex+d)(ex+d)^m \log\left((ax+b)^p\right)}{e(m+1)} - \int -\frac{\left(be(m+1)\log(c) - adp + \left(e(m+1)\log(c) - ep\right)ax - (ae(m+1)x + be(m$$

$$ae(m+1)x + be(m+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] (e*x + d)*(e*x + d)^m*log((a*x + b)^p)/(e*(m + 1)) - integrate(-(b*e*(m + 1
)*log(c) - a*d*p + (e*(m + 1)*log(c) - e*p)*a*x - (a*e*(m + 1)*x + b*e*(m +
 1))*log(x^p))*(e*x + d)^m/(a*e*(m + 1)*x + b*e*(m + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(a+\frac{b}{x}\right)^p\right) (d+e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)*(d + e*x)^m,x)
```

```
[Out] int(log(c*(a + b/x)^p)*(d + e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*ln(c*(a+b/x)**p),x)
```

```
[Out] Integral((d + e*x)**m*log(c*(a + b/x)**p), x)
```

3.210 $\int (d+ex)^m \log \left(c \left(a+\frac{b}{x^2}\right)^p\right) dx$

Optimal. Leaf size=257

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(m+1)}+\frac{\sqrt{-a}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{e(m+1)(m+2)\left(\sqrt{-a}\,d-\sqrt{b}\,e\right)}+\frac{\sqrt{-a}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m\right.}{e(m+1)(m+2)\left(\right.}$$

```
[Out] -2*p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],1+e*x/d)/d/e/(m^2+3*m+2)+(e*x+d
)^(1+m)*ln(c*(a+b/x^2)^p)/e/(1+m)+p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],
(e*x+d)*(-a)^(1/2)/(d*(-a)^(1/2)-e*b^(1/2)))*(-a)^(1/2)/e/(1+m)/(2+m)/(d*(-
a)^(1/2)-e*b^(1/2))+p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],(e*x+d)*(-a)^(
1/2)/(d*(-a)^(1/2)+e*b^(1/2)))*(-a)^(1/2)/e/(1+m)/(2+m)/(d*(-a)^(1/2)+e*b^(
1/2))
```

Rubi [A] time = 0.53, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2463, 1570, 961, 65, 831, 68}

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(m+1)}+\frac{\sqrt{-a}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{e(m+1)(m+2)\left(\sqrt{-a}\,d-\sqrt{b}\,e\right)}+\frac{\sqrt{-a}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m\right.}{e(m+1)(m+2)\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (Sqrt[-a]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[-a]*
(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/(e*(Sqrt[-a]*d - Sqrt[b]*e)*(1 + m)*(
2 + m)) + (Sqrt[-a]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m,
(Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/(e*(Sqrt[-a]*d + Sqrt[b]*e)
*(1 + m)*(2 + m)) - (2*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 +
m, 1 + (e*x)/d])/(d*e*(2 + 3*m + m^2)) + ((d + e*x)^(1 + m)*Log[c*(a + b/x^
2)^p])/(e*(1 + m))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && !RationalQ[m
]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 1570

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)} + \frac{(2bp)\int \frac{(d+ex)^{1+m}}{\left(a+\frac{b}{x^2}\right)x^3}\,dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)} + \frac{(2bp)\int \frac{(d+ex)^{1+m}}{x\left(b+ax^2\right)}\,dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)} + \frac{(2bp)\int \left(\frac{(d+ex)^{1+m}}{bx} - \frac{ax(d+ex)^{1+m}}{b\left(b+ax^2\right)}\right) dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)} + \frac{(2p)\int \frac{(d+ex)^{1+m}}{x}\,dx}{e(1+m)} - \frac{(2ap)\int \frac{x(d+ex)^{1+m}}{b+ax^2}\,dx}{e(1+m)}$$

$$= -\frac{2p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;1+\frac{ex}{d}\right)}{de\left(2+3m+m^2\right)} + \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)}$$

$$= -\frac{2p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;1+\frac{ex}{d}\right)}{de\left(2+3m+m^2\right)} + \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)}$$

$$= \frac{\sqrt{-a}\,p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{e\left(\sqrt{-a}\,d-\sqrt{b}\,e\right)(1+m)(2+m)} + \frac{\sqrt{-a}\,p(d+ex)^{2+m}\,{}_2F_1\left(\right.}{e\left(\sqrt{-a}\,d+\sqrt{b}\right.}$$

Mathematica [A] time = 0.47, size = 211, normalized size = 0.82

$$\frac{(d+ex)^{m+1}\left(\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{p(d+ex)\left(-2\left(ad^2+be^2\right){}_2F_1\left(1,m+2;m+3;\frac{ex}{d}+1\right)+d\left(ad-\sqrt{-a}\,\sqrt{b}\,e\right){}_2F_1\left(1,m+2;m+3;\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)+d\left(\sqrt{-a}\,\sqrt{b}\,e\right.}{d(m+2)\left(ad^2+be^2\right)}\right.}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b/x^2)^p],x]
```

```
[Out] ((d + e*x)^(1 + m)*((p*(d + e*x)*(d*(a*d - Sqrt[-a]*Sqrt[b]*e)*Hypergeometr
ic2F1[1, 2 + m, 3 + m, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)] + d*(
a*d + Sqrt[-a]*Sqrt[b]*e)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[-a]*(d +
 e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] - 2*(a*d^2 + b*e^2)*Hypergeometric2F1[1, 2
 + m, 3 + m, 1 + (e*x)/d]))/(d*(a*d^2 + b*e^2)*(2 + m)) + Log[c*(a + b/x^2)
^p]))/(e*(1 + m))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left((ex+d)^m \log\left(c\left(\frac{ax^2+b}{x^2}\right)^p\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m*log(c*((a*x^2 + b)/x^2)^p), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^m \log\left(\left(a+\frac{b}{x^2}\right)^p c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m*log((a + b/x^2)^p*c), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int (ex+d)^m \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*ln(c*(a+b/x^2)^p),x)
```

```
[Out] int((e*x+d)^m*ln(c*(a+b/x^2)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(epx+dp)(ex+d)^m \log\left(ax^2+b\right)}{e(m+1)} - \int \frac{\left(2\,adpx - \left(e(m+1)\log(c) - 2\,ep\right)ax^2 - be(m+1)\log(c) + 2\left(ae(m+1)\right.}{ae(m+1)x^2 + be(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] (e*p*x + d*p)*(e*x + d)^m*log(a*x^2 + b)/(e*(m + 1)) - integrate((2*a*d*p*x
 - (e*(m + 1)*log(c) - 2*e*p)*a*x^2 - b*e*(m + 1)*log(c) + 2*(a*e*(m + 1)*x
^2 + b*e*(m + 1))*log(x^p))*(e*x + d)^m/(a*e*(m + 1)*x^2 + b*e*(m + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)(d+e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p)*(d + e*x)^m,x)
```

```
[Out] int(log(c*(a + b/x^2)^p)*(d + e*x)^m, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*ln(c*(a+b/x**2)**p),x)
```

```
[Out] Timed out
```

3.211 $\int (f+gx)^m \log\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=23

$$\text{Int}\left((f+gx)^m \log\left(c\left(d+ex^n\right)^p\right),x\right)$$

```
[Out] Unintegrable((g*x+f)^m*ln(c*(d+e*x^n)^p),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (f+gx)^m \log\left(c\left(d+ex^n\right)^p\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x)^m*Log[c*(d + e*x^n)^p],x]
[Out] Defer[Int][(f + g*x)^m*Log[c*(d + e*x^n)^p], x]
```

Rubi steps

$$\int (f+gx)^m \log\left(c\left(d+ex^n\right)^p\right)\,dx=\int (f+gx)^m \log\left(c\left(d+ex^n\right)^p\right)\,dx$$

Mathematica [A] time = 0.56, size = 0, normalized size = 0.00

$$\int (f+gx)^m \log\left(c\left(d+ex^n\right)^p\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x)^m*Log[c*(d + e*x^n)^p],x]
[Out] Integrate[(f + g*x)^m*Log[c*(d + e*x^n)^p], x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(gx+f\right)^m \log\left(\left(ex^n+d\right)^p c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^m*log(c*(d+e*x^n)^p),x, algorithm="fricas")
[Out] integral((g*x + f)^m*log((e*x^n + d)^p*c), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^m*log(c*(d+e*x^n)^p),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to check sign: (4/(sign(t_nostep)-1)/2)>(-4/(sign(
t_nostep)-1)/2)Unable to check sign: (4/(sign(x)-1)/2)>(-4/(sign(x)-1)/2)Si
mplification assuming f near 0Unable to divide, perhaps due to rounding err
or%%%{-1,[0,0,6,3,6,0,2,2,0,1,0]%%%}+%%%{1,[0,0,6,2,6,1,2,2,0,0,1]%%%}+%%%{
1,[0,0,6,2,6,0,2,2,0,0,1]%%%}+%%%{-6,[0,0,5,3,5,0,2,2,1,1,0]%%%}+%%%{5,[0,0
```

,5,2,5,1,2,2,1,0,1]%%%}+%%%{5,[0,0,5,2,5,0,2,2,1,0,1]%%%}+%%%{-15,[0,0,4,3,
4,0,2,2,2,1,0]%%%}+%%%{10,[0,0,4,2,4,1,2,2,2,0,1]%%%}+%%%{10,[0,0,4,2,4,0,2
,2,2,0,1]%%%}+%%%{-20,[0,0,3,3,3,0,2,2,3,1,0]%%%}+%%%{10,[0,0,3,2,3,1,2,2,3
,0,1]%%%}+%%%{10,[0,0,3,2,3,0,2,2,3,0,1]%%%}+%%%{-15,[0,0,2,3,2,0,2,2,4,1,0
]%%%}+%%%{5,[0,0,2,2,2,1,2,2,4,0,1]%%%}+%%%{5,[0,0,2,2,2,0,2,2,4,0,1]%%%}+%
%%{-6,[0,0,1,3,1,0,2,2,5,1,0]%%%}+%%%{1,[0,0,1,2,1,1,2,2,5,0,1]%%%}+%%%{1,[
0,0,1,2,1,0,2,2,5,0,1]%%%}+%%%{-1,[0,0,0,3,0,0,2,2,6,1,0]%%%} / %%%{1,[0,0,
6,2,5,0,1,2,0,0,0]%%%}+%%%{5,[0,0,5,2,4,0,1,2,1,0,0]%%%}+%%%{10,[0,0,4,2,3,
0,1,2,2,0,0]%%%}+%%%{10,[0,0,3,2,2,0,1,2,3,0,0]%%%}+%%%{5,[0,0,2,2,1,0,1,2,
4,0,0]%%%}+%%%{1,[0,0,1,2,0,0,1,2,5,0,0]%%%} Error: Bad Argument Value

maple [A] time = 1.77, size = 0, normalized size = 0.00

$$\int \left(gx+f\right)^{m} \ln\left(c\left(e\,x^{n}+d\right)^{p}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^m*ln(c*(e*x^n+d)^p),x)
```

```
[Out] int((g*x+f)^m*ln(c*(e*x^n+d)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(gx+f\right)\left(gx+f\right)^{m}\log\left(\left(ex^{n}+d\right)^{p}\right)}{g(m+1)}+\int\frac{\left(dg(m+1)x\log(c)-\left(efnp+\left(egnp-eg(m+1)\log(c)\right)x\right)x^{n}\right)\left(gx+f\right)^{m}}{eg(m+1)xx^{n}+dg(m+1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^m*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] (g*x + f)*(g*x + f)^m*log((e*x^n + d)^p)/(g*(m + 1)) + integrate((d*g*(m +
1)*x*log(c) - (e*f*n*p + (e*g*n*p - e*g*(m + 1)*log(c))*x)*x^n)*(g*x + f)^m
/(e*g*(m + 1)*x*x^n + d*g*(m + 1)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \ln\left(c\left(d+e\,x^{n}\right)^{p}\right)\,\left(f+g\,x\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)*(f + g*x)^m,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)*(f + g*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**m*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Timed out
```

3.212 $\int (f+gx)^3 \log\left(c\left(d+ex^n\right)^p\right) dx$

Optimal. Leaf size=234

$$\frac{(f+gx)^4\log\left(c\left(d+ex^n\right)^p\right)}{4g}-\frac{f^4p\log\left(d+ex^n\right)}{4g}-\frac{ef^3npx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}-\frac{3ef^2gnpx^{n+2}\,{}_2F_1\left(1,\frac{n+}{n}\right.}{2d(n+}$$

```
[Out] -e*f^3*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)-3/2*e*f^2
*g*n*p*x^(2+n)*hypergeom([1, (2+n)/n],[2+2/n],-e*x^n/d)/d/(2+n)-e*f*g^2*n*p
*x^(3+n)*hypergeom([1, (3+n)/n],[2+3/n],-e*x^n/d)/d/(3+n)-1/4*e*g^3*n*p*x^(
4+n)*hypergeom([1, (4+n)/n],[2+4/n],-e*x^n/d)/d/(4+n)-1/4*f^4*p*ln(d+e*x^n)
/g+1/4*(g*x+f)^4*ln(c*(d+e*x^n)^p)/g
```

Rubi [A] time = 0.23, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2463, 1844, 260, 364}

$$\frac{(f+gx)^4\log\left(c\left(d+ex^n\right)^p\right)}{4g}-\frac{3ef^2gnpx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(n+2)}-\frac{f^4p\log\left(d+ex^n\right)}{4g}-\frac{ef^3npx^{n+1}\,{}_2F_1\left(1,\right.}{d(n}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*f^3*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^
n)/d)])/(d*(1 + n))) - (3*e*f^2*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n
)/n, 2*(1 + n^(-1)), -((e*x^n)/d)])/(2*d*(2 + n)) - (e*f*g^2*n*p*x^(3 + n)*
Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(d*(3 + n)) - (e*g^
3*n*p*x^(4 + n)*Hypergeometric2F1[1, (4 + n)/n, 2*(1 + 2/n), -((e*x^n)/d)])
/(4*d*(4 + n)) - (f^4*p*Log[d + e*x^n])/(4*g) + ((f + g*x)^4*Log[c*(d + e*x
^n)^p])/(4*g)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (f+gx)^3 \log\left(c(d+ex^n)^p\right) dx = \frac{(f+gx)^4 \log\left(c(d+ex^n)^p\right)}{4g} - \frac{(enp)\int \frac{x^{-1+n}(f+gx)^4}{d+ex^n} dx}{4g}$$
$$= \frac{(f+gx)^4 \log\left(c(d+ex^n)^p\right)}{4g} - \frac{(enp)\int \left(\frac{f^4x^{-1+n}}{d+ex^n} + \frac{4f^3gx^n}{d+ex^n} + \frac{6f^2g^2x^{1+n}}{d+ex^n} + \frac{4fg^3x^{2+n}}{d+ex^n}\right.}{4g}$$
$$= \frac{(f+gx)^4 \log\left(c(d+ex^n)^p\right)}{4g} - \left(ef^3np\right)\int \frac{x^n}{d+ex^n} dx - \frac{\left(ef^4np\right)\int \frac{x^{-1+n}}{d+ex^n} dx}{4g} -$$
$$= -\frac{ef^3npx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)} - \frac{3ef^2gnpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right);\right.}{2d(2+n)}$$

Mathematica [A] time = 0.49, size = 224, normalized size = 0.96

$$\frac{(f+gx)^4 \log\left(c(d+ex^n)^p\right) - enp\left(\frac{f^4\log(d+ex^n)}{en} + \frac{4f^3gx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)} + \frac{6f^2g^2x^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{d(n+2)} + \frac{4fg^3x\right.}{4g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (-(e*n*p*((4*f^3*g*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -
((e*x^n)/d)])/(d*(1 + n)) + (6*f^2*g^2*x^(2 + n)*Hypergeometric2F1[1, (2 +
n)/n, 2*(1 + n^(-1)), -((e*x^n)/d)])/(d*(2 + n)) + (4*f*g^3*x^(3 + n)*Hyper
geometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(d*(3 + n)) + (g^4*x^(4
+ n)*Hypergeometric2F1[1, (4 + n)/n, 2 + 4/n, -((e*x^n)/d)])/(d*(4 + n)) +
(f^4*Log[d + e*x^n])/(e*n))) + (f + g*x)^4*Log[c*(d + e*x^n)^p])/(4*g)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(g^3x^3 + 3\,fg^2x^2 + 3\,f^2gx + f^3\right)\log\left((ex^n+d)^pc\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)*log((e*x^n + d)^p*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (gx+f)^3 \log\left((ex^n+d)^pc\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^3*log((e*x^n + d)^p*c), x)
```

maple [F] time = 1.80, size = 0, normalized size = 0.00

$$\int (gx+f)^3 \ln\left(c\,(e\,x^n+d)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*ln(c*(e*x^n+d)^p),x)
```

```
[Out] int((g*x+f)^3*ln(c*(e*x^n+d)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{16}\left(g^3np-4\,g^3\log(c)\right)x^4-\frac{1}{3}\left(fg^2np-3\,fg^2\log(c)\right)x^3-\frac{3}{4}\left(f^2gnp-2\,f^2g\log(c)\right)x^2-\left(f^3np-f^3\log(c)\right)x+\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/16*(g^3*n*p - 4*g^3*log(c))*x^4 - 1/3*(f*g^2*n*p - 3*f*g^2*log(c))*x^3 -
 3/4*(f^2*g*n*p - 2*f^2*g*log(c))*x^2 - (f^3*n*p - f^3*log(c))*x + 1/4*(g^3
*x^4 + 4*f*g^2*x^3 + 6*f^2*g*x^2 + 4*f^3*x)*log((e*x^n + d)^p) + integrate(
1/4*(d*g^3*n*p*x^3 + 4*d*f*g^2*n*p*x^2 + 6*d*f^2*g*n*p*x + 4*d*f^3*n*p)/(e*
x^n + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,{\left(f+g\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)*(f + g*x)^3,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)*(f + g*x)^3, x)
```

sympy [C] time = 32.59, size = 415, normalized size = 1.77

$$f^3x\log\left(c\left(d+ex^n\right)^p\right)+\frac{f^3px\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}+\frac{3f^2gx^2\log\left(c\left(d+ex^n\right)^p\right)}{2}+fg^2x^3\log\left(c\left(d+ex^n\right)^p\right)+\frac{g^3x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*ln(c*(d+e*x**n)**p),x)
```

```
[Out] f**3*x*log(c*(d + e*x**n)**p) + f**3*p*x*lerchphi(d*x**(-n)*exp_polar(I*pi)
/e, 1, exp_polar(I*pi)/n)*gamma(1/n)/(n*gamma(1 + 1/n)) + 3*f**2*g*x**2*log
(c*(d + e*x**n)**p)/2 + f*g**2*x**3*log(c*(d + e*x**n)**p) + g**3*x**4*log(
c*(d + e*x**n)**p)/4 - 3*e*f**2*g*p*x**2*x**n*lerchphi(e*x**n*exp_polar(I*p
i)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(2*d*gamma(2 + 2/n)) - 3*e*f**2*g*p*x**2*x
**n*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(d*n*gamm
a(2 + 2/n)) - e*f*g**2*p*x**3*x**n*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1
+ 3/n)*gamma(1 + 3/n)/(d*gamma(2 + 3/n)) - 3*e*f*g**2*p*x**3*x**n*lerchphi(
e*x**n*exp_polar(I*pi)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(d*n*gamma(2 + 3/n)) -
 e*g**3*p*x**4*x**n*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 4/n)*gamma(1
+ 4/n)/(4*d*gamma(2 + 4/n)) - e*g**3*p*x**4*x**n*lerchphi(e*x**n*exp_polar(
I*pi)/d, 1, 1 + 4/n)*gamma(1 + 4/n)/(d*n*gamma(2 + 4/n))
```

3.213 $\int (f+gx)^2 \log \left(c \left(d+ex^n\right)^p\right) dx$

Optimal. Leaf size=181

$$\frac{(f+gx)^3 \log \left(c \left(d+ex^n\right)^p\right)}{3g}-\frac{f^3 p \log \left(d+ex^n\right)}{3g}-\frac{ef^2 npx^{n+1} {}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}-\frac{efgnpx^{n+2} {}_2F_1\left(1,\frac{n+2}{n};2\left(\right.\right.}{d(n+2)}$$

```
[Out] -e*f^2*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)-e*f*g*n*p
*x^(2+n)*hypergeom([1, (2+n)/n],[2+2/n],-e*x^n/d)/d/(2+n)-1/3*e*g^2*n*p*x^(
3+n)*hypergeom([1, (3+n)/n],[2+3/n],-e*x^n/d)/d/(3+n)-1/3*f^3*p*ln(d+e*x^n)
/g+1/3*(g*x+f)^3*ln(c*(d+e*x^n)^p)/g
```

Rubi [A] time = 0.18, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2463, 1844, 260, 364}

$$\frac{(f+gx)^3 \log \left(c \left(d+ex^n\right)^p\right)}{3g}-\frac{f^3 p \log \left(d+ex^n\right)}{3g}-\frac{ef^2 npx^{n+1} {}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}-\frac{efgnpx^{n+2} {}_2F_1\left(1,\frac{n+2}{n};2\left(\right.\right.}{d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*f^2*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^
n)/d)])/(d*(1 + n))) - (e*f*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n,
 2*(1 + n^(-1)), -((e*x^n)/d)])/(d*(2 + n)) - (e*g^2*n*p*x^(3 + n)*Hypergeo
metric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(3*d*(3 + n)) - (f^3*p*Log[
d + e*x^n])/(3*g) + ((f + g*x)^3*Log[c*(d + e*x^n)^p])/(3*g)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (f+gx)^2 \log\left(c\left(d+ex^n\right)^p\right)\,dx = \frac{(f+gx)^3\log\left(c\left(d+ex^n\right)^p\right)}{3g} - \frac{(enp)\int\frac{x^{-1+n}(f+gx)^3}{d+ex^n}\,dx}{3g}$$
$$= \frac{(f+gx)^3\log\left(c\left(d+ex^n\right)^p\right)}{3g} - \frac{(enp)\int\left(\frac{f^3x^{-1+n}}{d+ex^n}+\frac{3f^2gx^n}{d+ex^n}+\frac{3fg^2x^{1+n}}{d+ex^n}+\frac{g^3x^{2+n}}{d+ex^n}\right.}{3g}$$
$$= \frac{(f+gx)^3\log\left(c\left(d+ex^n\right)^p\right)}{3g} - \left(ef^2np\right)\int\frac{x^n}{d+ex^n}\,dx - \frac{\left(ef^3np\right)\int\frac{x^{-1+n}}{d+ex^n}\,dx}{3g}$$
$$= -\frac{ef^2npx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)} - \frac{efgnpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right)\right.}{d(2+n)}$$

Mathematica [A] time = 0.25, size = 178, normalized size = 0.98

$$\frac{(f+gx)^3\log\left(c\left(d+ex^n\right)^p\right)-enp\left(\frac{f^3\log(d+ex^n)}{en}+\frac{3f^2gx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}+\frac{3fg^2x^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{d(n+2)}+\frac{g^3x}{}\right.}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (-(e*n*p*((3*f^2*g*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -
((e*x^n)/d)])/(d*(1 + n)) + (3*f*g^2*x^(2 + n)*Hypergeometric2F1[1, (2 + n)
/n, 2*(1 + n^(-1)), -((e*x^n)/d)])/(d*(2 + n)) + (g^3*x^(3 + n)*Hypergeomet
ric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(d*(3 + n)) + (f^3*Log[d + e*x
^n])/(e*n))) + (f + g*x)^3*Log[c*(d + e*x^n)^p])/(3*g)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(g^2x^2+2\,fgx+f^2\right)\log\left(\left(ex^n+d\right)^pc\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)*log((e*x^n + d)^p*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(gx+f\right)^2\log\left(\left(ex^n+d\right)^pc\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^2*log((e*x^n + d)^p*c), x)
```

maple [F] time = 1.88, size = 0, normalized size = 0.00

$$\int\left(gx+f\right)^2\ln\left(c\left(e\,x^n+d\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*ln(c*(e*x^n+d)^p),x)
```

```
[Out] int((g*x+f)^2*ln(c*(e*x^n+d)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{9}\left(g^2np-3\,g^2\log(c)\right)x^3-\frac{1}{2}\left(fgnp-2\,fg\log(c)\right)x^2-\left(f^2np-f^2\log(c)\right)x+\frac{1}{3}\left(g^2x^3+3\,fgx^2+3\,f^2x\right)\log\left((ex^n+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/9*(g^2*n*p - 3*g^2*log(c))*x^3 - 1/2*(f*g*n*p - 2*f*g*log(c))*x^2 - (f^2
*n*p - f^2*log(c))*x + 1/3*(g^2*x^3 + 3*f*g*x^2 + 3*f^2*x)*log((e*x^n + d)^
p) + integrate(1/3*(d*g^2*n*p*x^2 + 3*d*f*g*n*p*x + 3*d*f^2*n*p)/(e*x^n + d
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,{\left(f+g\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)*(f + g*x)^2,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)*(f + g*x)^2, x)
```

sympy [C] time = 20.06, size = 284, normalized size = 1.57

$$f^2x\log\left(c\,(d+ex^n)^p\right)+\frac{f^2px\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}+fgx^2\log\left(c\,(d+ex^n)^p\right)+\frac{g^2x^3\log\left(c\,(d+ex^n)^p\right)}{3}-\frac{efgpx^2x^n\Phi}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*ln(c*(d+e*x**n)**p),x)
```

```
[Out] f**2*x*log(c*(d + e*x**n)**p) + f**2*p*x*lerchphi(d*x**(-n)*exp_polar(I*pi)
/e, 1, exp_polar(I*pi)/n)*gamma(1/n)/(n*gamma(1 + 1/n)) + f*g*x**2*log(c*(d
 + e*x**n)**p) + g**2*x**3*log(c*(d + e*x**n)**p)/3 - e*f*g*p*x**2*x**n*ler
chphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(d*gamma(2 + 2/n
)) - 2*e*f*g*p*x**2*x**n*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gam
ma(1 + 2/n)/(d*n*gamma(2 + 2/n)) - e*g**2*p*x**3*x**n*lerchphi(e*x**n*exp_p
olar(I*pi)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(3*d*gamma(2 + 3/n)) - e*g**2*p*x*
*3*x**n*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(d*n*
gamma(2 + 3/n))
```

3.214 $\int (f+gx)\log\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=132

$$\frac{(f+gx)^2\log\left(c\left(d+ex^n\right)^p\right)}{2g}-\frac{f^2p\log\left(d+ex^n\right)}{2g}-\frac{efnpx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}-\frac{egnpx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(\right.\right.}{2d(n+2)}$$

```
[Out] -e*f*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)-1/2*e*g*n*p
*x^(2+n)*hypergeom([1, (2+n)/n],[2+2/n],-e*x^n/d)/d/(2+n)-1/2*f^2*p*ln(d+e*
x^n)/g+1/2*(g*x+f)^2*ln(c*(d+e*x^n)^p)/g
```

Rubi [A] time = 0.14, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2463, 1844, 260, 364}

$$\frac{(f+gx)^2\log\left(c\left(d+ex^n\right)^p\right)}{2g}-\frac{f^2p\log\left(d+ex^n\right)}{2g}-\frac{efnpx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}-\frac{egnpx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(\right.\right.}{2d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*f*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)
/d)])/(d*(1 + n))) - (e*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n, 2*(
1 + n^(-1)), -((e*x^n)/d)])/(2*d*(2 + n)) - (f^2*p*Log[d + e*x^n])/(2*g) +
((f + g*x)^2*Log[c*(d + e*x^n)^p])/(2*g)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !IGtQ[m, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (f+gx)\log\left(c\left(d+ex^n\right)^p\right)dx = \frac{(f+gx)^2\log\left(c\left(d+ex^n\right)^p\right)}{2g} - \frac{(enp)\int\frac{x^{-1+n}(f+gx)^2}{d+ex^n}dx}{2g}$$

$$= \frac{(f+gx)^2\log\left(c\left(d+ex^n\right)^p\right)}{2g} - \frac{(enp)\int\left(\frac{f^2x^{-1+n}}{d+ex^n}+\frac{2fgx^n}{d+ex^n}+\frac{g^2x^{1+n}}{d+ex^n}\right)dx}{2g}$$

$$= \frac{(f+gx)^2\log\left(c\left(d+ex^n\right)^p\right)}{2g} - (efnp)\int\frac{x^n}{d+ex^n}dx - \frac{\left(ef^2np\right)\int\frac{x^{-1+n}}{d+ex^n}dx}{2g} - \frac{1}{2}($$

$$= -\frac{efnpx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)} - \frac{egnpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2+n)}$$

Mathematica [A] time = 0.13, size = 130, normalized size = 0.98

$$fx\log\left(c\left(d+ex^n\right)^p\right)+\frac{1}{2}gx^2\log\left(c\left(d+ex^n\right)^p\right)-\frac{efnpx^{n+1}\,{}_2F_1\left(1,\frac{n+1}{n};\frac{n+1}{n}+1;-\frac{ex^n}{d}\right)}{d(n+1)}-\frac{egnpx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};\frac{n+2}{n}+\right.}{2d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*Log[c*(d + e*x^n)^p],x]
[Out] -((e*f*n*p*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/n, 1 + (1 + n)/n, -((e*x^
n)/d)])/(d*(1 + n))) - (e*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n, 1
 + (2 + n)/n, -((e*x^n)/d)])/(2*d*(2 + n)) + f*x*Log[c*(d + e*x^n)^p] + (g*
x^2*Log[c*(d + e*x^n)^p])/2
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(gx+f\right)\log\left(\left(ex^n+d\right)^pc\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
[Out] integral((g*x + f)*log((e*x^n + d)^p*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(gx+f\right)\log\left(\left(ex^n+d\right)^pc\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*log(c*(d+e*x^n)^p),x, algorithm="giac")
[Out] integrate((g*x + f)*log((e*x^n + d)^p*c), x)
```

maple [F] time = 2.19, size = 0, normalized size = 0.00

$$\int\left(gx+f\right)\ln\left(c\left(e\,x^n+d\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*ln(c*(e*x^n+d)^p),x)
[Out] int((g*x+f)*ln(c*(e*x^n+d)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}\left(gnp-2\,g\log(c)\right)x^2-\left(fnp-f\log(c)\right)x+\frac{1}{2}\left(gx^2+2\,fx\right)\log\left((ex^n+d)^p\right)+\int\frac{dgnpx+2\,dfnp}{2\,(ex^n+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/4*(g*n*p - 2*g*log(c))*x^2 - (f*n*p - f*log(c))*x + 1/2*(g*x^2 + 2*f*x)*
log((e*x^n + d)^p) + integrate(1/2*(d*g*n*p*x + 2*d*f*n*p)/(e*x^n + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\ln\left(c\,(d+e\,x^n)^p\right)\,\left(f+g\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)*(f + g*x),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)*(f + g*x), x)
```

sympy [C] time = 12.86, size = 162, normalized size = 1.23

$$fx\log\left(c\,(d+ex^n)^p\right)+\frac{fpx\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}+\frac{gx^2\log\left(c\,(d+ex^n)^p\right)}{2}-\frac{egpx^2x^n\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{2d\Gamma\left(2+\frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*ln(c*(d+e*x**n)**p),x)
```

```
[Out] f*x*log(c*(d + e*x**n)**p) + f*p*x*lerchphi(d*x**(-n)*exp_polar(I*pi)/e, 1,
 exp_polar(I*pi)/n)*gamma(1/n)/(n*gamma(1 + 1/n)) + g*x**2*log(c*(d + e*x**
n)**p)/2 - e*g*p*x**2*x**n*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*g
amma(1 + 2/n)/(2*d*gamma(2 + 2/n)) - e*g*p*x**2*x**n*lerchphi(e*x**n*exp_po
lar(I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(d*n*gamma(2 + 2/n))
```

3.215 $\int \log\left(c\left(d+ex^n\right)^p\right)\,dx$

Optimal. Leaf size=54

$$x\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}$$

```
[Out] -e*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)+x*ln(c*(d+e*x
^n)^p)
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2448, 364}

$$x\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d
)])/(d*(1 + n))) + x*Log[c*(d + e*x^n)^p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(c\left(d+ex^n\right)^p\right)\,dx&=x\log\left(c\left(d+ex^n\right)^p\right)-(enp)\int\frac{x^n}{d+ex^n}\,dx\\&=-\frac{enpx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)}+x\log\left(c\left(d+ex^n\right)^p\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 0.96

$$x\left(\log\left(c\left(d+ex^n\right)^p\right)-\frac{enpx^n\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p],x]
```

```
[Out] x*(-((e*n*p*x^n*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d)])
/(d*(1 + n))) + Log[c*(d + e*x^n)^p])
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\log\left((ex^n + d)^p c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="fricas")
[Out] integral(log((e*x^n + d)^p*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left((ex^n + d)^p c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="giac")
[Out] integrate(log((e*x^n + d)^p*c), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \ln\left(c\,(e\,x^n + d)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p),x)
[Out] int(ln(c*(e*x^n+d)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$dnp\int \frac{1}{ex^n + d}\,dx - \left(np - \log(c)\right)x + x\log\left((ex^n + d)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="maxima")
[Out] d*n*p*integrate(1/(e*x^n + d), x) - (n*p - log(c))*x + x*log((e*x^n + d)^p)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(c\,(d + e\,x^n)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p),x)
[Out] int(log(c*(d + e*x^n)^p), x)
```

sympy [C] time = 3.43, size = 48, normalized size = 0.89

$$x\log\left(c\,(d + ex^n)^p\right) + \frac{px\Phi\left(\frac{dx^{-n}e^{i\pi}}{e}, 1, \frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n\Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p),x)
[Out] x*log(c*(d + e*x**n)**p) + p*x*lerchphi(d*x**(-n)*exp_polar(I*pi)/e, 1, exp
_polar(I*pi)/n)*gamma(1/n)/(n*gamma(1 + 1/n))
```

3.216 $\int \frac{\log\left(c(d+ex^n)^p\right)}{f+gx}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{\log\left(c\left(d+ex^n\right)^p\right)}{f+gx},x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)/(g*x+f),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{f+gx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]/(f + g*x),x]
[Out] Defer[Int][Log[c*(d + e*x^n)^p]/(f + g*x), x]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{f+gx}\,dx=\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{f+gx}\,dx$$

Mathematica [A] time = 1.71, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{f+gx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\log\left(\left(ex^n+d\right)^p c\right)}{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f),x, algorithm="fricas")
[Out] integral(log((e*x^n + d)^p*c)/(g*x + f), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^n+d\right)^p c\right)}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/(g*x + f), x)
```

maple [A] time = 1.64, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\,(e\,x^{n}+d)^{p}\right)}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)/(g*x+f),x)
[Out] int(ln(c*(e*x^n+d)^p)/(g*x+f),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left((ex^{n}+d)^{p}c\right)}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f),x, algorithm="maxima")
[Out] integrate(log((e*x^n + d)^p*c)/(g*x + f), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\,(d+e\,x^{n})^{p}\right)}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(f + g*x),x)
[Out] int(log(c*(d + e*x^n)^p)/(f + g*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\,(d+ex^{n})^{p}\right)}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/(g*x+f),x)
[Out] Integral(log(c*(d + e*x**n)**p)/(f + g*x), x)
```

3.217 $\int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^2} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^2},x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)/(g*x+f)^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]/(f + g*x)^2,x]
[Out] Defer[Int][Log[c*(d + e*x^n)^p]/(f + g*x)^2, x]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^2} dx = \int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^2} dx$$

Mathematica [A] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^2,x]
[Out] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^2, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^n+d\right)^p c\right)}{g^2x^2+2\,fgx+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^2,x, algorithm="fricas")
[Out] integral(log((e*x^n + d)^p*c)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^n+d\right)^p c\right)}{\left(gx+f\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/(g*x + f)^2, x)
```

maple [A] time = 1.37, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^{n}+d\right)^{p}\right)}{\left(gx+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)/(g*x+f)^2,x)
```

```
[Out] int(ln(c*(e*x^n+d)^p)/(g*x+f)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-dnp\int \frac{1}{dg^{2}x^{2}+dfgx+\left(eg^{2}x^{2}+efgx\right)x^{n}}\,dx-\frac{np\log\left(gx+f\right)}{fg}-\frac{f\log\left(\left(ex^{n}+d\right)^{p}\right)+f\log(c)-\left(gnpx+fnp\right.}{fg^{2}x+f^{2}g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] -d*n*p*integrate(1/(d*g^2*x^2 + d*f*g*x + (e*g^2*x^2 + e*f*g*x)*x^n), x) -
n*p*log(g*x + f)/(f*g) - (f*log((e*x^n + d)^p) + f*log(c) - (g*n*p*x + f*n*
p)*log(x))/(f*g^2*x + f^2*g)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\left(d+e\,x^{n}\right)^{p}\right)}{\left(f+g\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(f + g*x)^2,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(f + g*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+ex^{n}\right)^{p}\right)}{\left(f+gx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/(g*x+f)**2,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)/(f + g*x)**2, x)
```

3.218 $\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^3}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^3},x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)/(g*x+f)^3,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]/(f + g*x)^3,x]
[Out] Defer[Int][Log[c*(d + e*x^n)^p]/(f + g*x)^3, x]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^3}\,dx=\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^3}\,dx$$

Mathematica [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{(f+gx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^3,x]
[Out] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^3, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\log\left(\left(ex^n+d\right)^p c\right)}{g^3x^3+3\,fg^2x^2+3\,f^2gx+f^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^3,x, algorithm="fricas")
[Out] integral(log((e*x^n + d)^p*c)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^n+d\right)^p c\right)}{\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/(g*x + f)^3, x)
```

maple [A] time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\,(e\,x^{n}+d)^{p}\right)}{\left(gx+f\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)/(g*x+f)^3,x)
```

```
[Out] int(ln(c*(e*x^n+d)^p)/(g*x+f)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-dnp\int \frac{1}{2\left(dg^{3}x^{3}+2\,dfg^{2}x^{2}+df^{2}gx+\left(eg^{3}x^{3}+2\,efg^{2}x^{2}+ef^{2}gx\right)x^{n}\right)}\,dx+\frac{fgnpx+f^{2}np-f^{2}\log\left((ex^{n}+d\right.}{2\left(f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] -d*n*p*integrate(1/2/(d*g^3*x^3 + 2*d*f*g^2*x^2 + d*f^2*g*x + (e*g^3*x^3 +
2*e*f*g^2*x^2 + e*f^2*g*x)*x^n), x) + 1/2*(f*g*n*p*x + f^2*n*p - f^2*log((e
*x^n + d)^p) - f^2*log(c) + (g^2*n*p*x^2 + 2*f*g*n*p*x + f^2*n*p)*log(x))/(
f^2*g^3*x^2 + 2*f^3*g^2*x + f^4*g) - 1/2*n*p*log(g*x + f)/(f^2*g)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\,(d+e\,x^{n})^{p}\right)}{\left(f+g\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(f + g*x)^3,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(f + g*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/(g*x+f)**3,x)
```

```
[Out] Timed out
```

3.219 $\int \frac{x^3 \log(c(a+bx)^p)}{d+ex} dx$

Optimal. Leaf size=250

$$\frac{a^3p\log(a+bx)}{3b^3e}+\frac{a^2dp\log(a+bx)}{2b^2e^2}-\frac{a^2px}{3b^2e}-\frac{d^3\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^4}+\frac{d^2(a+bx)\log\left(c(a+bx)^p\right)}{be^3}-\frac{dx^2\log(}{}$$

```
[Out] -d^2*p*x/e^3-1/2*a*d*p*x/b/e^2-1/3*a^2*p*x/b^2/e+1/4*d*p*x^2/e^2+1/6*a*p*x^
2/b/e-1/9*p*x^3/e+1/2*a^2*d*p*ln(b*x+a)/b^2/e^2+1/3*a^3*p*ln(b*x+a)/b^3/e-1
/2*d*x^2*ln(c*(b*x+a)^p)/e^2+1/3*x^3*ln(c*(b*x+a)^p)/e+d^2*(b*x+a)*ln(c*(b*
x+a)^p)/b/e^3-d^3*ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/e^4-d^3*p*polylo
g(2,-e*(b*x+a)/(-a*e+b*d))/e^4
```

Rubi [A] time = 0.25, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {43, 2416, 2389, 2295, 2395, 2394, 2393, 2391}

$$-\frac{d^3p\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^4}+\frac{a^2dp\log(a+bx)}{2b^2e^2}-\frac{a^2px}{3b^2e}+\frac{a^3p\log(a+bx)}{3b^3e}+\frac{d^2(a+bx)\log\left(c(a+bx)^p\right)}{be^3}-\frac{d^3\log\left(c(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] -((d^2*p*x)/e^3) - (a*d*p*x)/(2*b*e^2) - (a^2*p*x)/(3*b^2*e) + (d*p*x^2)/(4
*e^2) + (a*p*x^2)/(6*b*e) - (p*x^3)/(9*e) + (a^2*d*p*Log[a + b*x])/(2*b^2*e
^2) + (a^3*p*Log[a + b*x])/(3*b^3*e) - (d*x^2*Log[c*(a + b*x)^p])/(2*e^2) +
 (x^3*Log[c*(a + b*x)^p])/(3*e) + (d^2*(a + b*x)*Log[c*(a + b*x)^p])/(b*e^3
) - (d^3*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e^4 - (d^3*p*Po
lyLog[2, -((e*(a + b*x))/(b*d - a*e))])/e^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\int \frac{x^3 \log (c(a+bx)^p)}{d+ex}\,dx = \int \left(\frac{d^2 \log (c(a+bx)^p)}{e^3} - \frac{dx \log (c(a+bx)^p)}{e^2} + \frac{x^2 \log (c(a+bx)^p)}{e} - \frac{d^3 \log (c(a+}{e^3(d+ex}\right.$$

$$= \frac{d^2 \int \log (c(a+bx)^p)\,dx}{e^3} - \frac{d^3 \int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{e^3} - \frac{d \int x \log (c(a+bx)^p)\,dx}{e^2} + \frac{\int x^2 \mathrm{l}}{}$$

$$= -\frac{dx^2 \log (c(a+bx)^p)}{2e^2} + \frac{x^3 \log (c(a+bx)^p)}{3e} - \frac{d^3 \log (c(a+bx)^p) \log \left(\frac{b(d+ex)}{bd-ae}\right)}{e^4} + \frac{d^2}{}$$

$$= -\frac{d^2 px}{e^3} - \frac{dx^2 \log (c(a+bx)^p)}{2e^2} + \frac{x^3 \log (c(a+bx)^p)}{3e} + \frac{d^2(a+bx) \log (c(a+bx)^p)}{be^3} -$$

$$= -\frac{d^2 px}{e^3} - \frac{adpx}{2be^2} - \frac{a^2 px}{3b^2 e} + \frac{dpx^2}{4e^2} + \frac{apx^2}{6be} - \frac{px^3}{9e} + \frac{a^2 dp \log(a+bx)}{2b^2 e^2} + \frac{a^3 p \log(a+bx}{3b^3 e}$$

Mathematica [A] time = 0.20, size = 183, normalized size = 0.73

$$\frac{b\left(6b \log (c(a+bx)^p)\left(-6bd^3 \log \left(\frac{b(d+ex)}{bd-ae}\right) + 6ad^2 e + bex\left(6d^2 - 3dex + 2e^2 x^2\right)\right) - epx\left(12a^2 e^2 - 6abe(ex-3d)\right.}{36b^3 e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] (6*a^2*e^2*(3*b*d + 2*a*e)*p*Log[a + b*x] + b*(-(e*p*x*(12*a^2*e^2 - 6*a*b*
e*(-3*d + e*x) + b^2*(36*d^2 - 9*d*e*x + 4*e^2*x^2))) + 6*b*Log[c*(a + b*x)
^p]*(6*a*d^2*e + b*e*x*(6*d^2 - 3*d*e*x + 2*e^2*x^2) - 6*b*d^3*Log[(b*(d +
e*x))/(b*d - a*e)])) - 36*b^3*d^3*p*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)
])/(36*b^3*e^4)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3\log\left((bx+a)^pc\right)}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x^3*log((b*x + a)^p*c)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\log\left((bx+a)^pc\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^3*log((b*x + a)^p*c)/(e*x + d), x)
```

maple [C] time = 0.33, size = 919, normalized size = 3.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x+a)^p)/(e*x+d),x)
[Out] 1/4*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)/e^2*x^2*d+1/2*I*Pi
*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)*d^3/e^4*ln(e*x+d)-1/2*I*Pi
*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)/e^3*x*d^2-1/9*p*x^3/e+1/3*
ln((b*x+a)^p)/e*x^3-1/4*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2/e^2*x^
2*d+1/3*ln(c)/e*x^3-1/6*I*Pi*csgn(I*c*(b*x+a)^p)^3/e*x^3+1/6*I*Pi*csgn(I*c*
(b*x+a)^p)^2*csgn(I*c)/e*x^3+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*d^3/e^4*ln(e*x+
d)+1/2/b^2*p/e^2*a^2*ln(a*e-b*d+(e*x+d)*b)*d+1/b*p/e^3*a*ln(a*e-b*d+(e*x+d)
*b)*d^2-1/4*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)/e^2*x^2*d-1/6*I*Pi*csgn(I*
(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)/e*x^3-1/3/b^2*p/e^2*a^2*d+1/3/b^3*
p/e*a^3*ln(a*e-b*d+(e*x+d)*b)+p/e^4*d^3*ln(e*x+d)*ln((a*e-b*d+(e*x+d)*b)/(a
*e-b*d))-2/3/b*p/e^3*a*d^2-d^2*p*x/e^3+1/6*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*
(b*x+a)^p)^2/e*x^3+1/4*I*Pi*csgn(I*c*(b*x+a)^p)^3/e^2*x^2*d-1/2*I*Pi*csgn(I
*c*(b*x+a)^p)^3/e^3*x*d^2-1/2*ln((b*x+a)^p)/e^2*x^2*d+1/2*I*Pi*csgn(I*(b*x+
a)^p)*csgn(I*c*(b*x+a)^p)^2/e^3*x*d^2+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I
*c)/e^3*x*d^2-1/2*ln(c)/e^2*x^2*d+ln(c)/e^3*x*d^2-1/2*I*Pi*csgn(I*c*(b*x+a)
^p)^2*csgn(I*c)*d^3/e^4*ln(e*x+d)-ln(c)*d^3/e^4*ln(e*x+d)+ln((b*x+a)^p)/e^3
*x*d^2-ln((b*x+a)^p)*d^3/e^4*ln(e*x+d)-49/36*p/e^4*d^3+p/e^4*d^3*dilog((a*e
-b*d+(e*x+d)*b)/(a*e-b*d))-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2
*d^3/e^4*ln(e*x+d)-1/3*a^2*p*x/b^2/e+1/6*a*p*x^2/b/e+1/4*d*p*x^2/e^2-1/2*a*
d*p*x/b/e^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\log\left((bx+a)^pc\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^3*log((b*x + a)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\ln\left(c\,{\left(a+b\,x\right)}^p\right)}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(c*(a + b*x)^p))/(d + e*x),x)
[Out] int((x^3*log(c*(a + b*x)^p))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \log \left(c \left(a + bx\right)^{p}\right)}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x+a)**p)/(e*x+d),x)
[Out] Integral(x**3*log(c*(a + b*x)**p)/(d + e*x), x)
```

3.220 $\int \frac{x^2 \log(c(a+bx)^p)}{d+ex} \, dx$

Optimal. Leaf size=159

$$-\frac{a^2 p \log(a+bx)}{2b^2 e}+\frac{d^2 \log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^3}-\frac{d(a+bx)\log\left(c(a+bx)^p\right)}{be^2}+\frac{x^2\log\left(c(a+bx)^p\right)}{2e}+\frac{d^2 p \text{Li}_2\left(-\frac{e(a\cdot}{bd}\right.}{e^3}$$

[Out] d*p*x/e^2+1/2*a*p*x/b/e-1/4*p*x^2/e-1/2*a^2*p*ln(b*x+a)/b^2/e+1/2*x^2*ln(c*(b*x+a)^p)/e-d*(b*x+a)*ln(c*(b*x+a)^p)/b/e^2+d^2*ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/e^3+d^2*p*polylog(2,-e*(b*x+a)/(-a*e+b*d))/e^3

Rubi [A] time = 0.17, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {43, 2416, 2389, 2295, 2395, 2394, 2393, 2391}

$$\frac{d^2 p \text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^3}-\frac{a^2 p \log(a+bx)}{2b^2 e}+\frac{d^2\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^3}-\frac{d(a+bx)\log\left(c(a+bx)^p\right)}{be^2}+\frac{x^2\log(\cdots}{}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Log[c*(a + b*x)^p])/(d + e*x),x]

[Out] (d*p*x)/e^2 + (a*p*x)/(2*b*e) - (p*x^2)/(4*e) - (a^2*p*Log[a + b*x])/(2*b^2*e) + (x^2*Log[c*(a + b*x)^p])/(2*e) - (d*(a + b*x)*Log[c*(a + b*x)^p])/(b*e^2) + (d^2*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e^3 + (d^2*p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/e^3

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \log (c(a+bx)^p)}{d+ex}\,dx &= \int \left(-\frac{d\log (c(a+bx)^p)}{e^2}+\frac{x\log (c(a+bx)^p)}{e}+\frac{d^2\log (c(a+bx)^p)}{e^2(d+ex)}\right)dx\\
&= -\frac{d\int \log (c(a+bx)^p)\,dx}{e^2}+\frac{d^2\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{e^2}+\frac{\int x\log (c(a+bx)^p)\,dx}{e}\\
&= \frac{x^2\log (c(a+bx)^p)}{2e}+\frac{d^2\log (c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^3}-\frac{d\,\mathrm{Subst}\left(\int \log (cx^p)\,dx,x,\right.}{be^2}\\
&= \frac{dpx}{e^2}+\frac{x^2\log (c(a+bx)^p)}{2e}-\frac{d(a+bx)\log (c(a+bx)^p)}{be^2}+\frac{d^2\log (c(a+bx)^p)\log\left(\frac{b}{b}\right.}{e^3}\\
&= \frac{dpx}{e^2}+\frac{apx}{2be}-\frac{px^2}{4e}-\frac{a^2p\log(a+bx)}{2b^2e}+\frac{x^2\log (c(a+bx)^p)}{2e}-\frac{d(a+bx)\log (c(a+b}{be^2}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 127, normalized size = 0.80

$$\frac{-2a^2e^2p\log(a+bx)+4b^2d^2p\mathrm{Li}_2\left(\frac{e(a+bx)}{ae-bd}\right)+b\log (c(a+bx)^p)\left(4bd^2\log\left(\frac{b(d+ex)}{bd-ae}\right)-4ade+2bex(ex-2d)\right)+b}{4b^2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] (b*e*p*x*(4*b*d + 2*a*e - b*e*x) - 2*a^2*e^2*p*Log[a + b*x] + b*Log[c*(a +
b*x)^p]*(-4*a*d*e + 2*b*e*x*(-2*d + e*x) + 4*b*d^2*Log[(b*(d + e*x))/(b*d -
 a*e)]) + 4*b^2*d^2*p*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)])/(4*b^2*e^3)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^2\log\left((bx+a)^pc\right)}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^2*log((b*x + a)^p*c)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log\left((bx+a)^p c\right)}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((b*x + a)^p*c)/(e*x + d), x)
```

maple [C] time = 0.32, size = 666, normalized size = 4.19

$$-\frac{p\,x^2}{4e}-\frac{i\pi\,x^2\mathrm{csgn}\left(ic\,(bx+a)^p\right)^3}{4e}+\frac{x^2\ln(c)}{2e}-\frac{adp\ln\left(ae-bd+(ex+d)\,b\right)}{b\,e^2}-\frac{a^2p\ln\left(ae-bd+(ex+d)\,b\right)}{2b^2e}+\frac{adp}{2b\,e^2}-\frac{d^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x+a)^p)/(e*x+d),x)
```

```
[Out] -1/4*I*Pi*csgn(I*c*(b*x+a)^p)^3/e*x^2-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(
b*x+a)^p)*csgn(I*c)*d^2/e^3*ln(e*x+d)+1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(
b*x+a)^p)*csgn(I*c)/e^2*x*d-1/4*p*x^2/e+1/2*ln(c)/e*x^2+1/4*I*Pi*csgn(I*c*(
b*x+a)^p)^2*csgn(I*c)/e*x^2-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*d^2/e^3*ln(e*x+d
)+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3/e^2*x*d+1/4*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*
c*(b*x+a)^p)^2/e*x^2-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)/e^2*x*d-1/b*p
/e^2*a*ln(a*e-b*d+(e*x+d)*b)*d-1/2/b^2*p/e*a^2*ln(a*e-b*d+(e*x+d)*b)+1/2/b*
p/e^2*a*d-p/e^3*d^2*ln(e*x+d)*ln((a*e-b*d+(e*x+d)*b)/(a*e-b*d))+5/4*p/e^3*d
^2+d*p*x/e^2-ln(c)/e^2*x*d+ln(c)*d^2/e^3*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x+a)^
p)*csgn(I*c*(b*x+a)^p)^2/e^2*x*d+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)*d
^2/e^3*ln(e*x+d)+1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*d^2/e^3*l
n(e*x+d)+1/2*ln((b*x+a)^p)/e*x^2+ln((b*x+a)^p)*d^2/e^3*ln(e*x+d)-ln((b*x+a)
^p)/e^2*x*d-1/4*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)/e*x^2-
p/e^3*d^2*dilog((a*e-b*d+(e*x+d)*b)/(a*e-b*d))+1/2*a*p*x/b/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log\left((bx+a)^p c\right)}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^2*log((b*x + a)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\ln\left(c\,{\left(a+b\,x\right)}^p\right)}{d+e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(c*(a + b*x)^p))/(d + e*x),x)
```

```
[Out] int((x^2*log(c*(a + b*x)^p))/(d + e*x), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log\left(c\,(a+bx)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x+a)**p)/(e*x+d),x)
```

```
[Out] Integral(x**2*log(c*(a + b*x)**p)/(d + e*x), x)
```

3.221 $\int \frac{x\log(c(a+bx)^p)}{d+ex}\,dx$

Optimal. Leaf size=91

$$-\frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2}+\frac{(a+bx)\log\left(c(a+bx)^p\right)}{be}-\frac{dp\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{e^2}-\frac{px}{e}$$

```
[Out] -p*x/e+(b*x+a)*ln(c*(b*x+a)^p)/b/e-d*ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d
))/e^2-d*p*polylog(2,-e*(b*x+a)/(-a*e+b*d))/e^2
```

Rubi [A] time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {43, 2416, 2389, 2295, 2394, 2393, 2391}

$$-\frac{dp\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^2}-\frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2}+\frac{(a+bx)\log\left(c(a+bx)^p\right)}{be}-\frac{px}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] -((p*x)/e) + ((a + b*x)*Log[c*(a + b*x)^p])/(b*e) - (d*Log[c*(a + b*x)^p]*L
og[(b*(d + e*x))/(b*d - a*e)])/e^2 - (d*p*PolyLog[2, -((e*(a + b*x))/(b*d -
 a*e))])/e^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\int \frac{x\log(c(a+bx)^p)}{d+ex}\,dx = \int\left(\frac{\log(c(a+bx)^p)}{e}-\frac{d\log(c(a+bx)^p)}{e(d+ex)}\right)dx$$
$$= \frac{\int \log(c(a+bx)^p)\,dx}{e} - \frac{d\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{e}$$
$$= -\frac{d\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2} + \frac{\text{Subst}\left(\int \log(cx^p)\,dx, x, a+bx\right)}{be} + \frac{(bdp)\int \frac{\log\left(\frac{b}{\cdot}\right.}{a+}}{e^2}$$
$$= -\frac{px}{e} + \frac{(a+bx)\log(c(a+bx)^p)}{be} - \frac{d\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2} + \frac{(dp)\,\text{Subst}\left(\int \frac{\mathrm{l}}{\cdot}\right.}{\cdot}$$
$$= -\frac{px}{e} + \frac{(a+bx)\log(c(a+bx)^p)}{be} - \frac{d\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2} - \frac{dp\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right.}{e^2}$$

Mathematica [A] time = 0.03, size = 79, normalized size = 0.87

$$\frac{\log(c(a+bx)^p)\left(-bd\log\left(\frac{b(d+ex)}{bd-ae}\right)+ae+bex\right)-bdp\text{Li}_2\left(\frac{e(a+bx)}{ae-bd}\right)-bepx}{be^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(a + b*x)^p])/(d + e*x),x]
[Out] (-(b*e*p*x) + Log[c*(a + b*x)^p]*(a*e + b*e*x - b*d*Log[(b*(d + e*x))/(b*d
- a*e)]) - b*d*p*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)])/(b*e^2)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x\log\left((bx+a)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x*log((b*x + a)^p*c)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\log\left((bx+a)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")

[Out] integrate(x*log((b*x + a)^p*c)/(e*x + d), x)

maple [C] time = 0.32, size = 427, normalized size = 4.69

$$\frac{i\pi d\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(bx+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(bx+a\right)^{p}\right)\ln\left(ex+d\right)}{2e^{2}}-\frac{i\pi d\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(bx+a\right)^{p}\right)^{2}\ln\left(ex+d\right)}{2e^{2}}-\frac{i\pi d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*ln(c*(b*x+a)^p)/(e*x+d),x)

[Out] ln((b*x+a)^p)/e*x-ln((b*x+a)^p)*d/e^2*ln(e*x+d)-p*x/e-p/e^2*d+1/b*p/e*a*ln(a*e-b*d+(e*x+d)*b)+p/e^2*d*dilog((a*e-b*d+(e*x+d)*b)/(a*e-b*d))+p/e^2*d*ln(e*x+d)*ln((a*e-b*d+(e*x+d)*b)/(a*e-b*d))+1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)*d/e^2*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*d/e^2*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*d/e^2*ln(e*x+d)+1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2/e*x-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)/e*x-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)*d/e^2*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)/e*x-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3/e*x+ln(c)/e*x-ln(c)*d/e^2*ln(e*x+d)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\log\left(\left(bx+a\right)^{p}c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")

[Out] integrate(x*log((b*x + a)^p*c)/(e*x + d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\ln\left(c\,{\left(a+b\,x\right)}^{p}\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*log(c*(a + b*x)^p))/(d + e*x),x)

[Out] int((x*log(c*(a + b*x)^p))/(d + e*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\log\left(c\left(a+bx\right)^{p}\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*ln(c*(b*x+a)**p)/(e*x+d),x)

[Out] Integral(x*log(c*(a + b*x)**p)/(d + e*x), x)

3.222 $\int \frac{\log(c(a+bx)^p)}{d+ex} dx$

Optimal. Leaf size=58

$$\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}+\frac{p\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{e}$$

```
[Out] ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/e+p*polylog(2,-e*(b*x+a)/(-a*e+b*d
))/e
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2394, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e}+\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, -((e*
(a + b*x))/(b*d - a*e))])/e
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx &= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}-\frac{(bp)\int \frac{\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}\,dx}{e}\\
&= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}-\frac{p\,\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{bd-ae}\right)}{x}\,dx,x,a+bx\right)}{e}\\
&= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}+\frac{p\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{e}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 57, normalized size = 0.98

$$\frac{\log \left(c(a+bx)^p\right) \log \left(\frac{b(d+ex)}{bd-ae}\right)}{e}+\frac{p\text{Li}_2\left(\frac{e(a+bx)}{ae-bd}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, (e*(a
 + b*x))/(-(b*d) + a*e)])/e
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log \left((bx+a)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)^p*c)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left((bx+a)^p c\right)}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^p*c)/(e*x + d), x)
```

maple [C] time = 0.10, size = 242, normalized size = 4.17

$$-\frac{i\pi \operatorname{csgn}(ic) \operatorname{csgn}\left(i(bx+a)^p\right) \operatorname{csgn}\left(ic(bx+a)^p\right) \ln(ex+d)}{2e}+\frac{i\pi \operatorname{csgn}(ic) \operatorname{csgn}\left(ic(bx+a)^p\right)^2 \ln(ex+d)}{2e}+\frac{i\pi \operatorname{cs}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/(e*x+d),x)
```

```
[Out] 1/e*ln((b*x+a)^p)*ln(e*x+d)-1/e*p*dilog((a*e-b*d+(e*x+d)*b)/(a*e-b*d))-1/e*
p*ln((a*e-b*d+(e*x+d)*b)/(a*e-b*d))*ln(e*x+d)+1/2*I*Pi/e*csgn(I*(b*x+a)^p)*
csgn(I*c*(b*x+a)^p)^2*ln(e*x+d)-1/2*I*Pi/e*csgn(I*c)*csgn(I*(b*x+a)^p)*csgn
(I*c*(b*x+a)^p)*ln(e*x+d)-1/2*I*Pi/e*csgn(I*c*(b*x+a)^p)^3*ln(e*x+d)+1/2*I*
Pi/e*csgn(I*c)*csgn(I*c*(b*x+a)^p)^2*ln(e*x+d)+1/e*ln(c)*ln(e*x+d)
```

maxima [B] time = 0.47, size = 118, normalized size = 2.03

$$\frac{bp\left(\frac{\log(bx+a)\log(ex+d)}{b}-\frac{\log(ex+d)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\text{Li}_2\left(\frac{bex+bd}{bd-ae}\right)}{b}\right)}{e}-\frac{p\log(bx+a)\log(ex+d)}{e}+\frac{\log\left((bx+a)^p c\right)\log(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*(log(b*x + a)*log(e*x + d)/b - (log(e*x + d)*log(-(b*e*x + b*d)/(b*d -
a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))/b)/e - p*log(b*x + a)*log(e*x
 + d)/e + log((b*x + a)^p*c)*log(e*x + d)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\,{\left(a+b\,x\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)/(d + e*x),x)
[Out] int(log(c*(a + b*x)^p)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx\right)^{p}\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/(e*x+d),x)
[Out] Integral(log(c*(a + b*x)**p)/(d + e*x), x)
```

3.223 $\int \frac{\log(c(a+bx)^p)}{x(d+ex)}\,dx$

Optimal. Leaf size=97

$$-\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d}+\frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d}-\frac{p\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{d}+\frac{p\text{Li}_2\left(\frac{bx}{a}+1\right)}{d}$$

```
[Out] ln(-b*x/a)*ln(c*(b*x+a)^p)/d-ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/d-p*p
olylog(2,-e*(b*x+a)/(-a*e+b*d))/d+p*polylog(2,1+b*x/a)/d
```

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {36, 29, 31, 2416, 2394, 2315, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d}-\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d}+\frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(x*(d + e*x)),x]
```

```
[Out] (Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d - (Log[c*(a + b*x)^p]*Log[(b*(d + e*
x))/(b*d - a*e)])/d - (p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/d + (p*P
olyLog[2, 1 + (b*x)/a])/d
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(c(a+bx)^p)}{x(d+ex)}\,dx &= \int \left(\frac{\log(c(a+bx)^p)}{dx} - \frac{e\log(c(a+bx)^p)}{d(d+ex)}\right)dx \\
&= \frac{\int \frac{\log(c(a+bx)^p)}{x}\,dx}{d} - \frac{e\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{d} \\
&= \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d} - \frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{(bp)\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx}{d} + \frac{(bp)
\end{aligned}$$

$$\begin{aligned}
&= \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d} - \frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} + \frac{p\mathrm{Li}_2\left(1+\frac{bx}{a}\right)}{d} + \frac{p\,\mathrm{Subst}
\end{aligned}$$

$$\begin{aligned}
&= \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d} - \frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{p\mathrm{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{p\mathrm{Li}_2\left(
\end{aligned}$$

Mathematica [A] time = 0.02, size = 98, normalized size = 1.01

$$-\frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} + \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d} - \frac{p\mathrm{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{p\mathrm{Li}_2\left(\frac{a+bx}{a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(x*(d + e*x)),x]
```

```
[Out] (Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d - (Log[c*(a + b*x)^p]*Log[(b*(d + e*
x))/(b*d - a*e)])/d + (p*PolyLog[2, (a + b*x)/a])/d - (p*PolyLog[2, -((e*(a
 + b*x))/(b*d - a*e))])/d
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log\left((bx+a)^p c\right)}{ex^2+dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)^p*c)/(e*x^2 + d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left((bx+a)^p c\right)}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^p*c)/((e*x + d)*x), x)
```

maple [C] time = 0.27, size = 420, normalized size = 4.33

$$-\frac{i\pi \operatorname{csgn}(ic)\operatorname{csgn}\left(i(bx+a)^p\right)\operatorname{csgn}\left(ic(bx+a)^p\right)\ln(x)}{2d}+\frac{i\pi \operatorname{csgn}(ic)\operatorname{csgn}\left(i(bx+a)^p\right)\operatorname{csgn}\left(ic(bx+a)^p\right)\ln(ex}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/x/(e*x+d),x)
```

```
[Out] ln((b*x+a)^p)/d*ln(x)-ln((b*x+a)^p)/d*ln(e*x+d)-p/d*dilog((b*x+a)/a)-p/d*ln
(x)*ln((b*x+a)/a)+p/d*dilog((a*e-b*d+(e*x+d)*b)/(a*e-b*d))+p/d*ln(e*x+d)*ln
((a*e-b*d+(e*x+d)*b)/(a*e-b*d))+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)/d*
ln(x)-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)/d*ln(x)-1/2*
I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2/d*ln(e*x+d)-1/2*I*Pi*csgn(I*c*
(b*x+a)^p)^3/d*ln(x)+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3/d*ln(e*x+d)+1/2*I*Pi*cs
gn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)/d*ln(e*x+d)+1/2*I*Pi*csgn(I*(
b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2/d*ln(x)-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn
(I*c)/d*ln(e*x+d)+1/d*ln(c)*ln(x)-1/d*ln(c)*ln(e*x+d)
```

maxima [A] time = 0.65, size = 123, normalized size = 1.27

$$-bp\left(\frac{\log\left(\frac{bx}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)}{bd}-\frac{\log(ex+d)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\mathrm{Li}_2\left(\frac{bex+bd}{bd-ae}\right)}{bd}\right)-\left(\frac{\log(ex+d)}{d}-\frac{\log(x)}{d}\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -b*p*((log(b*x/a + 1)*log(x) + dilog(-b*x/a))/(b*d) - (log(e*x + d)*log(-(b
*e*x + b*d)/(b*d - a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))/(b*d)) - (
log(e*x + d)/d - log(x)/d)*log((b*x + a)^p*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,(a+b\,x)^p\right)}{x\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)/(x*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b*x)^p)/(x*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(a+bx)^p\right)}{x(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/x/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b*x)**p)/(x*(d + e*x)), x)
```

3.224 $\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=146

$$-\frac{e\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^2}+\frac{e\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2}-\frac{\log\left(c(a+bx)^p\right)}{dx}+\frac{ep\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{d^2}-\frac{ep\text{Li}_2\left(\frac{bx}{a}+\right.}{d^2}$$

```
[Out] b*p*ln(x)/a/d-b*p*ln(b*x+a)/a/d-ln(c*(b*x+a)^p)/d/x-e*ln(-b*x/a)*ln(c*(b*x+
a)^p)/d^2+e*ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/d^2+e*p*polylog(2,-e*(
b*x+a)/(-a*e+b*d))/d^2-e*p*polylog(2,1+b*x/a)/d^2
```

Rubi [A] time = 0.17, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {44, 2416, 2395, 36, 29, 31, 2394, 2315, 2393, 2391}

$$\frac{ep\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d^2}-\frac{e\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^2}+\frac{e\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (b*p*Log[x])/(a*d) - (b*p*Log[a + b*x])/(a*d) - Log[c*(a + b*x)^p]/(d*x) -
(e*Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d^2 + (e*Log[c*(a + b*x)^p]*Log[(b*(
d + e*x))/(b*d - a*e)])/d^2 + (e*p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))]
)/d^2 - (e*p*PolyLog[2, 1 + (b*x)/a])/d^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\int \frac{\log\left(c(a+bx)^p\right)}{x^2(d+ex)}\,dx = \int \left(\frac{\log\left(c(a+bx)^p\right)}{dx^2} - \frac{e\log\left(c(a+bx)^p\right)}{d^2x} + \frac{e^2\log\left(c(a+bx)^p\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log(c(a+bx)^p)}{x^2}\,dx}{d} - \frac{e\int \frac{\log(c(a+bx)^p)}{x}\,dx}{d^2} + \frac{e^2\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{d^2}$$

$$= -\frac{\log\left(c(a+bx)^p\right)}{dx} - \frac{e\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^2} + \frac{e\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2} + \frac{(bp)}{}$$

$$= -\frac{\log\left(c(a+bx)^p\right)}{dx} - \frac{e\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^2} + \frac{e\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2} - \frac{ep\mathrm{Li}}{}$$

$$= \frac{bp\log(x)}{ad} - \frac{bp\log(a+bx)}{ad} - \frac{\log\left(c(a+bx)^p\right)}{dx} - \frac{e\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^2} + \frac{e\log(c(}{}$$

Mathematica [A] time = 0.05, size = 139, normalized size = 0.95

$$\frac{aex\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right) - ad\log\left(c(a+bx)^p\right) - aex\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right) + aepx\mathrm{Li}_2\left(\frac{e(a+bx)}{ae-bd}\right) - bdpx}{ad^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (b*d*p*x*Log[x] - b*d*p*x*Log[a + b*x] - a*d*Log[c*(a + b*x)^p] - a*e*x*Log
[-((b*x)/a)]*Log[c*(a + b*x)^p] + a*e*x*Log[c*(a + b*x)^p]*Log[(b*(d + e*x)
```

```
)/(b*d - a*e)] + a*e*p*x*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)] - a*e*p*x
*PolyLog[2, 1 + (b*x)/a])/(a*d^2*x)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left((bx+a)^p c\right)}{ex^3+dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)^p*c)/(e*x^3 + d*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left((bx+a)^p c\right)}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^p*c)/((e*x + d)*x^2), x)
```

maple [C] time = 0.25, size = 615, normalized size = 4.21

$$\frac{i\pi e \operatorname{csgn}(ic)\operatorname{csgn}\left(i(bx+a)^p\right)\operatorname{csgn}\left(ic(bx+a)^p\right)\ln(x)}{2d^2} - \frac{i\pi e \operatorname{csgn}(ic)\operatorname{csgn}\left(i(bx+a)^p\right)\operatorname{csgn}\left(ic(bx+a)^p\right)\ln}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/x^2/(e*x+d),x)
```

```
[Out] -ln((b*x+a)^p)/d/x-ln((b*x+a)^p)*e/d^2*ln(x)+ln((b*x+a)^p)*e/d^2*ln(e*x+d)-
p*e/d^2*dilog((a*e-b*d+(e*x+d)*b)/(a*e-b*d))-p*e/d^2*ln(e*x+d)*ln((a*e-b*d+
(e*x+d)*b)/(a*e-b*d))+b*p*ln(x)/a/d-b*p*ln(b*x+a)/a/d+p*e/d^2*dilog((b*x+a)
/a)+p*e/d^2*ln(x)*ln((b*x+a)/a)+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3/d/x-1/2*I*Pi
*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)/d/x-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*e/d^2*l
n(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*e/d^2*ln(x)-1/2*I*Pi*csgn(I*(b*x+a)
^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)*e/d^2*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x+a)^p
)*csgn(I*c*(b*x+a)^p)^2/d/x+1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^
2*e/d^2*ln(e*x+d)+1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)*
e/d^2*ln(x)+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)*e/d^2*ln(e*x+d)-1/2*I*
Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)*e/d^2*ln(x)+1/2*I*Pi*csgn(I*(b*x+a)^p)*c
sgn(I*c*(b*x+a)^p)*csgn(I*c)/d/x-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a
)^p)^2*e/d^2*ln(x)-ln(c)/d/x-ln(c)*e/d^2*ln(x)+ln(c)*e/d^2*ln(e*x+d)
```

maxima [A] time = 0.65, size = 156, normalized size = 1.07

$$bp\left(\frac{\left(\log\left(\frac{bx}{a}+1\right)\log(x)+\operatorname{Li}_2\left(-\frac{bx}{a}\right)\right)e}{bd^2} - \frac{\left(\log(ex+d)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\operatorname{Li}_2\left(\frac{bex+bd}{bd-ae}\right)\right)e}{bd^2} - \frac{\log(bx+a)}{ad} + \frac{\log}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*((log(b*x/a + 1)*log(x) + dilog(-b*x/a))*e/(b*d^2) - (log(e*x + d)*log(
-(b*e*x + b*d)/(b*d - a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))*e/(b*d^
```

```
2) - log(b*x + a)/(a*d) + log(x)/(a*d)) + (e*log(e*x + d)/d^2 - e*log(x)/d^
2 - 1/(d*x))*log((b*x + a)^p*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,(a+b\,x)^p\right)}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)/(x^2*(d + e*x)),x)
[Out] int(log(c*(a + b*x)^p)/(x^2*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/x**2/(e*x+d),x)
[Out] Timed out
```

3.225 $\int \frac{\log(c(a+bx)^p)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=227

$$-\frac{b^2p\log(x)}{2a^2d}+\frac{b^2p\log(a+bx)}{2a^2d}+\frac{e^2\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^3}-\frac{e^2\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^3}+\frac{e\log\left(c(a+bx)\right.}{d^2x}$$

```
[Out] -1/2*b*p/a/d/x-1/2*b^2*p*ln(x)/a^2/d-b*e*p*ln(x)/a/d^2+1/2*b^2*p*ln(b*x+a)/
a^2/d+b*e*p*ln(b*x+a)/a/d^2-1/2*ln(c*(b*x+a)^p)/d/x^2+e*ln(c*(b*x+a)^p)/d^2
/x+e^2*ln(-b*x/a)*ln(c*(b*x+a)^p)/d^3-e^2*ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*
e+b*d))/d^3-e^2*p*polylog(2,-e*(b*x+a)/(-a*e+b*d))/d^3+e^2*p*polylog(2,1+b*
x/a)/d^3
```

Rubi [A] time = 0.22, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {44, 2416, 2395, 36, 29, 31, 2394, 2315, 2393, 2391}

$$-\frac{e^2p\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d^3}-\frac{b^2p\log(x)}{2a^2d}+\frac{b^2p\log(a+bx)}{2a^2d}+\frac{e^2\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx\right.}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(x^3*(d + e*x)),x]
```

```
[Out] -(b*p)/(2*a*d*x) - (b^2*p*Log[x])/(2*a^2*d) - (b*e*p*Log[x])/(a*d^2) + (b^2
*p*Log[a + b*x])/(2*a^2*d) + (b*e*p*Log[a + b*x])/(a*d^2) - Log[c*(a + b*x)
^p]/(2*d*x^2) + (e*Log[c*(a + b*x)^p])/(d^2*x) + (e^2*Log[-((b*x)/a)]*Log[c
*(a + b*x)^p])/d^3 - (e^2*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)]
)/d^3 - (e^2*p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/d^3 + (e^2*p*PolyL
og[2, 1 + (b*x)/a])/d^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\int \frac{\log(c(a+bx)^p)}{x^3(d+ex)}\,dx = \int \left(\frac{\log(c(a+bx)^p)}{dx^3} - \frac{e\log(c(a+bx)^p)}{d^2x^2} + \frac{e^2\log(c(a+bx)^p)}{d^3x} - \frac{e^3\log(c(a+bx)^p)}{d^3(d+ex)}\right) dx$$

$$= \frac{\int \frac{\log(c(a+bx)^p)}{x^3}\,dx}{d} - \frac{e\int \frac{\log(c(a+bx)^p)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\log(c(a+bx)^p)}{x}\,dx}{d^3} - \frac{e^3\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{d^3}$$

$$= -\frac{\log(c(a+bx)^p)}{2dx^2} + \frac{e\log(c(a+bx)^p)}{d^2x} + \frac{e^2\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d^3} - \frac{e^2\log(c(a+bx)}{d}$$

$$= -\frac{\log(c(a+bx)^p)}{2dx^2} + \frac{e\log(c(a+bx)^p)}{d^2x} + \frac{e^2\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d^3} - \frac{e^2\log(c(a+bx)}{d}$$

$$= -\frac{bp}{2adx} - \frac{b^2p\log(x)}{2a^2d} - \frac{bep\log(x)}{ad^2} + \frac{b^2p\log(a+bx)}{2a^2d} + \frac{bep\log(a+bx)}{ad^2} - \frac{\log(c(a+bx)^p}{2dx^2}$$

Mathematica [A] time = 0.18, size = 188, normalized size = 0.83

$$-\frac{\frac{bd^2p(-bx\log(a+bx)+a+bx\log(x))}{a^2x} + \frac{d^2\log(c(a+bx)^p)}{x^2} + 2e^2\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right) - \frac{2de\log(c(a+bx)^p)}{x} - 2e^2\log\left(-\frac{bx}{a}\right)\log}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*((2*b*d*e*p*(Log[x] - Log[a + b*x]))/a + (b*d^2*p*(a + b*x*Log[x] - b*
x*Log[a + b*x]))/(a^2*x) + (d^2*Log[c*(a + b*x)^p])/x^2 - (2*d*e*Log[c*(a +
 b*x)^p])/x - 2*e^2*Log[-((b*x)/a)]*Log[c*(a + b*x)^p] + 2*e^2*Log[c*(a + b
*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)] + 2*e^2*p*PolyLog[2, (e*(a + b*x))/(-
(b*d) + a*e)] - 2*e^2*p*PolyLog[2, 1 + (b*x)/a])/d^3
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left((bx+a)^p c\right)}{ex^4+dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)^p*c)/(e*x^4 + d*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left((bx+a)^p c\right)}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^p*c)/((e*x + d)*x^3), x)
```

maple [C] time = 0.25, size = 850, normalized size = 3.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/x^3/(e*x+d),x)
```

```
[Out] 1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)*e^2/d^3*ln(e*x+d)-
1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)*e^2/d^3*ln(x)+ln((
b*x+a)^p)*e/d^2/x-ln((b*x+a)^p)*e^2/d^3*ln(e*x+d)+ln((b*x+a)^p)*e^2/d^3*ln(
x)-1/2*ln(c)/d/x^2-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)
*e/d^2/x-1/2*ln((b*x+a)^p)/d/x^2+ln(c)*e^2/d^3*ln(x)+ln(c)*e/d^2/x-ln(c)*e^
2/d^3*ln(e*x+d)-p*e^2/d^3*dilog((b*x+a)/a)+p*e^2/d^3*dilog((a*e-b*d+(e*x+d)
*b)/(a*e-b*d))-1/4*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2/d/x^2-1/2*I
*Pi*csgn(I*c*(b*x+a)^p)^3*e^2/d^3*ln(x)-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*e/d^
2/x+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*e^2/d^3*ln(e*x+d)-1/4*I*Pi*csgn(I*c*(b*x
+a)^p)^2*csgn(I*c)/d/x^2-1/2*b*p/a/d/x-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(
I*c)*e^2/d^3*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)*e/d^2/x+1/2
*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*e^2/d^3*ln(x)+1/2*I*Pi*csgn(I
*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*e/d^2/x-p*e^2/d^3*ln(x)*ln((b*x+a)/a)+p*e
^2/d^3*ln(e*x+d)*ln((a*e-b*d+(e*x+d)*b)/(a*e-b*d))+1/2*I*Pi*csgn(I*c*(b*x+a
)^p)^2*csgn(I*c)*e^2/d^3*ln(x)-b*e*p*ln(x)/a/d^2+b*e*p*ln(b*x+a)/a/d^2+1/4*
I*Pi*csgn(I*c*(b*x+a)^p)^3/d/x^2-1/2*b^2*p*ln(x)/a^2/d+1/2*b^2*p*ln(b*x+a)/
a^2/d+1/4*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)/d/x^2-1/2*I*
Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*e^2/d^3*ln(e*x+d)
```

maxima [A] time = 1.01, size = 216, normalized size = 0.95

$$\frac{1}{2}\left(2\,e\left(\frac{\log\left(bx+a\right)}{ad^2}-\frac{\log(x)}{ad^2}\right)-\frac{2\left(\log\left(\frac{bx}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)\right)e^2}{bd^3}+\frac{2\left(\log\left(ex+d\right)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\mathrm{L}\right.}{bd^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*(2*e*(log(b*x + a)/(a*d^2) - log(x)/(a*d^2)) - 2*(log(b*x/a + 1)*log(x)
 + dilog(-b*x/a))*e^2/(b*d^3) + 2*(log(e*x + d)*log(-(b*e*x + b*d)/(b*d - a
*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))*e^2/(b*d^3) + b*log(b*x + a)/(
a^2*d) - b*log(x)/(a^2*d) - 1/(a*d*x))*b*p - 1/2*(2*e^2*log(e*x + d)/d^3 -
2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2))*log((b*x + a)^p*c)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,(a+b\,x)^p\right)}{x^3\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)/(x^3*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b*x)^p)/(x^3*(d + e*x)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

3.226 $\int \frac{x^3 \log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=394

$$-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3b^{3/2}e}-\frac{d^3\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^4}+\frac{d^2x\log\left(c\left(a+bx^2\right)^p\right)}{e^3}-\frac{d\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2be^2}$$

```
[Out] -2*d^2*p*x/e^3+2/3*a*p*x/b/e+1/2*d*p*x^2/e^2-2/9*p*x^3/e-2/3*a^(3/2)*p*arct
an(x*b^(1/2)/a^(1/2))/b^(3/2)/e+d^2*x*ln(c*(b*x^2+a)^p)/e^3+1/3*x^3*ln(c*(b
*x^2+a)^p)/e-1/2*d*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b/e^2-d^3*ln(e*x+d)*ln(c*(b*
x^2+a)^p)/e^4+d^3*p*ln(e*x+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-a)^(1/2)+d*b
^(1/2)))/e^4+d^3*p*ln(e*x+d)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1/2)+d*
b^(1/2)))/e^4+d^3*p*polylog(2,(e*x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/e^
4+d^3*p*polylog(2,(e*x+d)*b^(1/2)/(e*(-a)^(1/2)+d*b^(1/2)))/e^4+2*d^2*p*arc
tan(x*b^(1/2)/a^(1/2))*a^(1/2)/e^3/b^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {2466, 2448, 321, 205, 2454, 2389, 2295, 2455, 302, 2462, 260, 2416, 2394, 2393, 2391}

$$\frac{d^3p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^4}+\frac{d^3p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e^4}-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3b^{3/2}e}-\frac{d^3\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (-2*d^2*p*x)/e^3 + (2*a*p*x)/(3*b*e) + (d*p*x^2)/(2*e^2) - (2*p*x^3)/(9*e)
+ (2*Sqrt[a]*d^2*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*e^3) - (2*a^(3/2)*
p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*b^(3/2)*e) + (d^3*p*Log[(e*(Sqrt[-a] - Sq
rt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(Sq
rt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e*x])/e^4 + (d^2*x*
Log[c*(a + b*x^2)^p])/e^3 + (x^3*Log[c*(a + b*x^2)^p])/(3*e) - (d*(a + b*x^
2)*Log[c*(a + b*x^2)^p])/(2*b*e^2) - (d^3*Log[d + e*x]*Log[c*(a + b*x^2)^p]
)/e^4 + (d^3*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e^
4 + (d^3*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^4
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^3 \log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx = \int \left(\frac{d^2 \log\left(c\left(a+bx^2\right)^p\right)}{e^3} - \frac{dx \log\left(c\left(a+bx^2\right)^p\right)}{e^2} + \frac{x^2 \log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{d^3 \mathrm{lo}\right.$$

$$= \frac{d^2 \int \log\left(c\left(a+bx^2\right)^p\right)dx}{e^3} - \frac{d^3 \int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}dx}{e^3} - \frac{d \int x \log\left(c\left(a+bx^2\right)^p\right)dx}{e^2}$$

$$= \frac{d^2 x \log\left(c\left(a+bx^2\right)^p\right)}{e^3} + \frac{x^3 \log\left(c\left(a+bx^2\right)^p\right)}{3e} - \frac{d^3 \log(d+ex) \log\left(c\left(a+bx^2\right)^p\right.}{e^4}$$

$$= -\frac{2d^2px}{e^3} + \frac{d^2 x \log\left(c\left(a+bx^2\right)^p\right)}{e^3} + \frac{x^3 \log\left(c\left(a+bx^2\right)^p\right)}{3e} - \frac{d^3 \log(d+ex) \log\left(c\right.}{e^4}$$

$$= -\frac{2d^2px}{e^3} + \frac{2apx}{3be} + \frac{dpx^2}{2e^2} - \frac{2px^3}{9e} + \frac{2\sqrt{a}\,d^2p \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e^3} + \frac{d^2 x \log\left(c\left(a+bx^2\right)^p\right.}{e^3}$$

$$= -\frac{2d^2px}{e^3} + \frac{2apx}{3be} + \frac{dpx^2}{2e^2} - \frac{2px^3}{9e} + \frac{2\sqrt{a}\,d^2p \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e^3} - \frac{2a^{3/2}p \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3b^{3/2}e} +$$

$$= -\frac{2d^2px}{e^3} + \frac{2apx}{3be} + \frac{dpx^2}{2e^2} - \frac{2px^3}{9e} + \frac{2\sqrt{a}\,d^2p \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e^3} - \frac{2a^{3/2}p \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3b^{3/2}e} +$$

$$= -\frac{2d^2px}{e^3} + \frac{2apx}{3be} + \frac{dpx^2}{2e^2} - \frac{2px^3}{9e} + \frac{2\sqrt{a}\,d^2p \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e^3} - \frac{2a^{3/2}p \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3b^{3/2}e} +$$

Mathematica [A] time = 0.35, size = 338, normalized size = 0.86

$$-4e^3p\left(\frac{3a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}} - \frac{3ax}{b} + x^3\right) - 18d^3 \log(d+ex) \log\left(c\left(a+bx^2\right)^p\right) + 18d^2 ex \log\left(c\left(a+bx^2\right)^p\right) + 9de^2\left(px\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (-4*e^3*p*((-3*a*x)/b + x^3 + (3*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/
2)) - 36*d^2*e*p*(x - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b]) + 18*d
^2*e*x*Log[c*(a + b*x^2)^p] + 6*e^3*x^3*Log[c*(a + b*x^2)^p] - 18*d^3*Log[d
 + e*x]*Log[c*(a + b*x^2)^p] + 9*d*e^2*(p*x^2 - ((a + b*x^2)*Log[c*(a + b*x
^2)^p])/b) + 18*d^3*p*((Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a
]*e)] + Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqrt[b]*d) + Sqrt[-a]*e)])*Log[d
+ e*x] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] + PolyLog
[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)]))/(18*e^4)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3 \log\left(\left(bx^2+a\right)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^3*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \log\left(\left(bx^2+a\right)^p c\right)}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^3*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

maple [C] time = 0.33, size = 1083, normalized size = 2.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x^2+a)^p)/(e*x+d),x)
```

```
[Out] -2/9/e*p*x^3+1/3/e*x^3*ln(c)+p/e^4*d^3*ln(e*x+d)*ln((b*d-(e*x+d)*b+(-a*b)^(
1/2)*e)/(b*d+(-a*b)^(1/2)*e))+p/e^4*d^3*ln(e*x+d)*ln((-b*d+(e*x+d)*b+(-a*b)
^(1/2)*e)/(-b*d+(-a*b)^(1/2)*e))-1/2*ln((b*x^2+a)^p)/e^2*x^2*d+ln((b*x^2+a)
^p)/e^3*x*d^2-ln((b*x^2+a)^p)*d^3/e^4*ln(e*x+d)+2/3*a/b*d/e^2*p+p/e^4*d^3*d
ilog((-b*d+(e*x+d)*b+(-a*b)^(1/2)*e)/(-b*d+(-a*b)^(1/2)*e))+p/e^4*d^3*dilog
((b*d-(e*x+d)*b+(-a*b)^(1/2)*e)/(b*d+(-a*b)^(1/2)*e))-1/2*I*Pi*csgn(I*(b*x^
2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/e^3*x*d^2-2*d^2/e^3*p*x+1/3*ln((b*x
^2+a)^p)/e*x^3+1/6*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2/e*x^3+1
/4*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/e^2*x^2*d-1/2*d
/e^2*x^2*ln(c)+d^2/e^3*x*ln(c)+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3*d^3/e^4*ln(
e*x+d)-1/2/b*p/e^2*a*d*ln(b*(e*x+d)^2-2*(e*x+d)*b*d+a*e^2+b*d^2)-2/3/b*p/e*
a^2/(a*b)^(1/2)*arctan(1/2*(2*(e*x+d)*b-2*b*d)/e/(a*b)^(1/2))+2*p/e^3*a/(a*
b)^(1/2)*arctan(1/2*(2*(e*x+d)*b-2*b*d)/e/(a*b)^(1/2))*d^2-1/2*I*Pi*csgn(I*
c*(b*x^2+a)^p)^3/e^3*x*d^2+1/6*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/e*x^3
+1/4*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/e^2*x^2*d-d^3/e^4*ln(c)*ln(e*x+d)-49/18*d
^3/e^4*p+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*d^3/e
^4*ln(e*x+d)+2/3*a/b/e*p*x-1/6*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^
```

```
p)*csgn(I*c)/e*x^3-1/4*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2/e^2
*x^2*d+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2/e^3*x*d^2-1/6*I
*Pi*csgn(I*c*(b*x^2+a)^p)^3/e*x^3+1/2*d/e^2*p*x^2+1/2*I*Pi*csgn(I*c*(b*x^2+
a)^p)^2*csgn(I*c)/e^3*x*d^2-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)^2*d^3/e^4*ln(e*x+d)-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*d^3/e^4*
ln(e*x+d)-1/4*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/e^2*x^2*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \log\left(\left(bx^2+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^3*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(c*(a + b*x^2)^p))/(d + e*x),x)
```

```
[Out] int((x^3*log(c*(a + b*x^2)^p))/(d + e*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x**2+a)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

3.227 $\int \frac{x^2 \log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=313

$$\frac{d^2 \log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^3}-\frac{dx\log\left(c\left(a+bx^2\right)^p\right)}{e^2}+\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2be}-\frac{d^2p\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^3}-\frac{d^2p\text{L}}{}$$

```
[Out] 2*d*p*x/e^2-1/2*p*x^2/e-d*x*ln(c*(b*x^2+a)^p)/e^2+1/2*(b*x^2+a)*ln(c*(b*x^2
+a)^p)/b/e+d^2*ln(e*x+d)*ln(c*(b*x^2+a)^p)/e^3-d^2*p*ln(e*x+d)*ln(e*((-a)^(
1/2)-x*b^(1/2))/(e*(-a)^(1/2)+d*b^(1/2)))/e^3-d^2*p*ln(e*x+d)*ln(-e*((-a)^(
1/2)+x*b^(1/2))/(-e*(-a)^(1/2)+d*b^(1/2)))/e^3-d^2*p*polylog(2,(e*x+d)*b^(1
/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/e^3-d^2*p*polylog(2,(e*x+d)*b^(1/2)/(e*(-a)^
(1/2)+d*b^(1/2)))/e^3-2*d*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/e^2/b^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2466, 2448, 321, 205, 2454, 2389, 2295, 2462, 260, 2416, 2394, 2393, 2391}

$$-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^3}-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e^3}+\frac{d^2\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^3}-\frac{dx\log\left(c\left(a+bx^2\right)\right.}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (2*d*p*x)/e^2 - (p*x^2)/(2*e) - (2*Sqrt[a]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(Sqrt[b]*e^2) - (d^2*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a
]*e)]*Log[d + e*x])/e^3 - (d^2*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*
d - Sqrt[-a]*e))]*Log[d + e*x])/e^3 - (d*x*Log[c*(a + b*x^2)^p])/e^2 + ((a
+ b*x^2)*Log[c*(a + b*x^2)^p])/(2*b*e) + (d^2*Log[d + e*x]*Log[c*(a + b*x^2
)^p])/e^3 - (d^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)]
)/e^3 - (d^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^
3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^2 \log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx = \int \left(-\frac{d\log\left(c\left(a+bx^2\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+bx^2\right)^p\right)}{e}+\frac{d^2\log\left(c\left(a+bx^2\right)^p\right)}{e^2(d+ex)}\right)dx$$

$$= -\frac{d\int \log\left(c\left(a+bx^2\right)^p\right)dx}{e^2}+\frac{d^2\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}dx}{e^2}+\frac{\int x\log\left(c\left(a+bx^2\right)^p\right)dx}{e}$$

$$= -\frac{dx\log\left(c\left(a+bx^2\right)^p\right)}{e^2}+\frac{d^2\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^3}+\frac{\text{Subst}\left(\int \log\left(c(a+b\right.\right.}{2e}$$

$$= \frac{2dpx}{e^2}-\frac{dx\log\left(c\left(a+bx^2\right)^p\right)}{e^2}+\frac{d^2\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^3}+\frac{\text{Subst}\left(\int \log\left(c\right.\right.}{}$$

$$= \frac{2dpx}{e^2}-\frac{px^2}{2e}-\frac{2\sqrt{a}\,dp\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e^2}-\frac{dx\log\left(c\left(a+bx^2\right)^p\right)}{e^2}+\frac{\left(a+bx^2\right)\log\left(c\left(a+\right.\right.}{2be}$$

$$= \frac{2dpx}{e^2}-\frac{px^2}{2e}-\frac{2\sqrt{a}\,dp\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e^2}-\frac{d^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{e^3}-\frac{d^2p\log\left(\right.}{}$$

$$= \frac{2dpx}{e^2}-\frac{px^2}{2e}-\frac{2\sqrt{a}\,dp\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e^2}-\frac{d^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{e^3}-\frac{d^2p\log\left(\right.}{}$$

$$= \frac{2dpx}{e^2}-\frac{px^2}{2e}-\frac{2\sqrt{a}\,dp\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e^2}-\frac{d^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{e^3}-\frac{d^2p\log\left(\right.}{}$$

Mathematica [A] time = 0.17, size = 271, normalized size = 0.87

$$\frac{2d^2\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)-2dex\log\left(c\left(a+bx^2\right)^p\right)+\frac{e^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}-2d^2p\left(\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)+\text{Li}_2\right.}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (-(e^2*p*x^2) + 4*d*e*p*(x - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])
 - 2*d*e*x*Log[c*(a + b*x^2)^p] + (e^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b
+ 2*d^2*Log[d + e*x]*Log[c*(a + b*x^2)^p] - 2*d^2*p*((Log[(e*(Sqrt[-a] - Sq
rt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)] + Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqr
t[b]*d) + Sqrt[-a]*e)])*Log[d + e*x] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt
[b]*d - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]
*e)]))/(2*e^3)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2\log\left(\left(bx^2+a\right)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x^2*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log\left(\left(bx^2 + a\right)^p c\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^2*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

maple [C] time = 0.31, size = 825, normalized size = 2.64

$$-\frac{d^2 p \ln\left(\frac{bd-(ex+d)b+\sqrt{-ab}\, e}{bd+\sqrt{-ab}\, e}\right) \ln(ex+d)}{e^3} - \frac{d^2 p \ln\left(\frac{-bd+(ex+d)b+\sqrt{-ab}\, e}{-bd+\sqrt{-ab}\, e}\right) \ln(ex+d)}{e^3} - \frac{p x^2}{2e} + \frac{x^2 \ln(c)}{2e} - \frac{i\pi d^2 \mathrm{csgn}(ic)\, \mathrm{cs}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^2+a)^p)/(e*x+d),x)
[Out] -p/e^3*d^2*ln(e*x+d)*ln((b*d-(e*x+d)*b+(-a*b)^(1/2)*e)/(b*d+(-a*b)^(1/2)*e)
)-p/e^3*d^2*ln(e*x+d)*ln((-b*d+(e*x+d)*b+(-a*b)^(1/2)*e)/(-b*d+(-a*b)^(1/2)
*e))-1/2/e*p*x^2-1/4*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/e*x^2+1/2/e*x^2*ln(c)+1/2
/b*p/e*a*ln(a*e^2+b*d^2-2*(e*x+d)*b*d+(e*x+d)^2*b)+ln((b*x^2+a)^p)*d^2/e^3*
ln(e*x+d)-ln((b*x^2+a)^p)/e^2*x*d+5/2*d^2/e^3*p+1/2*ln((b*x^2+a)^p)/e*x^2+2
*d/e^2*p*x-2*p/e^2*a*d/(a*b)^(1/2)*arctan(1/2*(-2*b*d+2*(e*x+d)*b)/(a*b)^(1
/2)/e)-d/e^2*x*ln(c)+d^2/e^3*ln(c)*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*c
sgn(I*c*(b*x^2+a)^p)^2/e^2*x*d+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*d
^2/e^3*ln(e*x+d)-p/e^3*d^2*dilog((b*d-(e*x+d)*b+(-a*b)^(1/2)*e)/(b*d+(-a*b)
^(1/2)*e))-p/e^3*d^2*dilog((-b*d+(e*x+d)*b+(-a*b)^(1/2)*e)/(-b*d+(-a*b)^(1/
2)*e))-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*d^2/e^3
*ln(e*x+d)+1/4*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2/e*x^2-1/2*I
*Pi*csgn(I*c*(b*x^2+a)^p)^3*d^2/e^3*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p
)^3/e^2*x*d+1/4*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/e*x^2+1/2*I*Pi*csgn(
I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/e^2*x*d-1/2*I*Pi*csgn(I*c*(b
*x^2+a)^p)^2*csgn(I*c)/e^2*x*d-1/4*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2
+a)^p)*csgn(I*c)/e*x^2+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2
*d^2/e^3*ln(e*x+d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log\left(\left(bx^2 + a\right)^p c\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^2*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\, \ln\left(c\left(b\, x^2 + a\right)^p\right)}{d + e\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(c*(a + b*x^2)^p))/(d + e*x),x)
[Out] int((x^2*log(c*(a + b*x^2)^p))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log \left(c \left(a+bx^2\right)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**2+a)**p)/(e*x+d),x)
[Out] Integral(x**2*log(c*(a + b*x**2)**p)/(d + e*x), x)
```

3.228 $\int \frac{x\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=256

$$-\frac{d\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+bx^2\right)^p\right)}{e}+\frac{dp\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^2}+\frac{dp\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e^2}+\frac{dp\log(d+e\cdots}{\cdots}$$

```
[Out] -2*p*x/e+x*ln(c*(b*x^2+a)^p)/e-d*ln(e*x+d)*ln(c*(b*x^2+a)^p)/e^2+d*p*ln(e*x
+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-a)^(1/2)+d*b^(1/2)))/e^2+d*p*ln(e*x+d)
*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1/2)+d*b^(1/2)))/e^2+d*p*polylog(2,
(e*x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/e^2+d*p*polylog(2,(e*x+d)*b^(1/2
)/(e*(-a)^(1/2)+d*b^(1/2)))/e^2+2*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/e/b^(
1/2)
```

Rubi [A] time = 0.27, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2466, 2448, 321, 205, 2462, 260, 2416, 2394, 2393, 2391}

$$\frac{dp\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^2}+\frac{dp\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e^2}-\frac{d\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+bx^2\right)^p\right)}{e}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (-2*p*x)/e + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*e) + (d*p*L
og[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/e^2 +
 (d*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e
*x])/e^2 + (x*Log[c*(a + b*x^2)^p])/e - (d*Log[d + e*x]*Log[c*(a + b*x^2)^p
])/e^2 + (d*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e^2
 + (d*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx = \int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{d\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}\right)dx$$

$$= \frac{\int \log\left(c\left(a+bx^2\right)^p\right)dx}{e} - \frac{d\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}dx}{e}$$

$$= \frac{x\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^2} + \frac{(2bdp)\int \frac{x\log(d+ex)}{a+bx^2}dx}{e^2} -$$

$$= -\frac{2px}{e} + \frac{x\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^2} + \frac{(2bdp)\int\left(-\frac{}{2\sqrt{}}\right.}{}$$

$$= -\frac{2px}{e} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e} + \frac{x\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c\left(a+bx^2\right.\right.}{e^2}$$

$$= -\frac{2px}{e} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e} + \frac{dp\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left(\sqrt{-a}\right.}{\sqrt{b}d-}\right.}{}$$

$$= -\frac{2px}{e} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e} + \frac{dp\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left(\sqrt{-a}\right.}{\sqrt{b}d-}\right.}{}$$

$$= -\frac{2px}{e} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e} + \frac{dp\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left(\sqrt{-a}\right.}{\sqrt{b}d-}\right.}{}$$

Mathematica [A] time = 0.12, size = 225, normalized size = 0.88

$$\frac{-d\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right) + ex\log\left(c\left(a+bx^2\right)^p\right) + dp\left(\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right) + \text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right) + \log(d+ex\right.}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (-2*e*p*(x - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b]) + e*x*Log[c*(a
+ b*x^2)^p] - d*Log[d + e*x]*Log[c*(a + b*x^2)^p] + d*p*((Log[(e*(Sqrt[-a]
- Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)] + Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-
(Sqrt[b]*d) + Sqrt[-a]*e)])*Log[d + e*x] + PolyLog[2, (Sqrt[b]*(d + e*x))/(
Sqrt[b]*d - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt
[-a]*e)]))/e^2
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x\log\left(\left(bx^2+a\right)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\log\left(\left(bx^2+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

maple [C] time = 0.31, size = 576, normalized size = 2.25

$$\frac{i\pi d\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)\ln\left(ex+d\right)}{2e^2}-\frac{i\pi d\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2\ln\left(ex+d\right)}{2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^2+a)^p)/(e*x+d),x)
```

```
[Out] ln((b*x^2+a)^p)/e*x-ln((b*x^2+a)^p)*d/e^2*ln(e*x+d)-2/e*p*x-2*d/e^2*p+2*p/e
*a/(a*b)^(1/2)*arctan(1/2*(-2*b*d+2*(e*x+d)*b)/(a*b)^(1/2)/e)+p/e^2*d*ln(e*
x+d)*ln((b*d-(e*x+d)*b+(-a*b)^(1/2)*e)/(b*d+(-a*b)^(1/2)*e))+p/e^2*d*ln(e*x
+d)*ln((-b*d+(e*x+d)*b+(-a*b)^(1/2)*e)/(-b*d+(-a*b)^(1/2)*e))+p/e^2*d*dilog
((b*d-(e*x+d)*b+(-a*b)^(1/2)*e)/(b*d+(-a*b)^(1/2)*e))+p/e^2*d*dilog((-b*d+(
e*x+d)*b+(-a*b)^(1/2)*e)/(-b*d+(-a*b)^(1/2)*e))+1/2*I*Pi*csgn(I*(b*x^2+a)^p
)*csgn(I*c*(b*x^2+a)^p)^2/e*x-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+
a)^p)^2*d/e^2*ln(e*x+d)-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*d/e^2*ln
(e*x+d)+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*d/e^2*
ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3*d/e^2*ln(e*x+d)-1/2*I*Pi*csgn(I*
(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/e*x+1/2*I*Pi*csgn(I*c*(b*x^2+a
)^p)^2*csgn(I*c)/e*x-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/e*x+1/e*x*ln(c)-d/e^2
*ln(c)*ln(e*x+d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\log\left(\left(bx^2+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\ln\left(c\left(b\,x^2+a\right)^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(c*(a + b*x^2)^p))/(d + e*x),x)
```

```
[Out] int((x*log(c*(a + b*x^2)^p))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**2+a)**p)/(e*x+d),x)
```

```
[Out] Integral(x*log(c*(a + b*x**2)**p)/(d + e*x), x)
```

3.229 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=201

$$\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e}-\frac{p\log(d+e}{}$$

```
[Out] ln(e*x+d)*ln(c*(b*x^2+a)^p)/e-p*ln(e*x+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-
a)^(1/2)+d*b^(1/2)))/e-p*ln(e*x+d)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1
/2)+d*b^(1/2)))/e-p*polylog(2,(e*x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/e-
p*polylog(2,(e*x+d)*b^(1/2)/(e*(-a)^(1/2)+d*b^(1/2)))/e
```

Rubi [A] time = 0.18, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2462, 260, 2416, 2394, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\right.}{\sqrt{-a}e+}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])
/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx = \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{(2bp)\int \frac{x\log(d+ex)}{a+bx^2}\,dx}{e}$$
$$= \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{(2bp)\int\left(-\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{e}$$
$$= \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e} + \frac{\left(\sqrt{b}p\right)\int \frac{\log(d+ex)}{\sqrt{-a}-\sqrt{b}x}\,dx}{e} - \frac{\left(\sqrt{b}p\right)\int \frac{\log(d+ex)}{\sqrt{-a}+\sqrt{b}x}\,dx}{e}$$
$$= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)}{}$$
$$= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)}{}$$
$$= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)}{}$$

Mathematica [A] time = 0.03, size = 201, normalized size = 1.00

$$\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{p\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e} - \frac{p\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e} - \frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e} - \frac{p\log(d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])
/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2 + a\right)^p c\right)}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

maple [C] time = 0.10, size = 366, normalized size = 1.82

$$-\frac{i\pi \,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)\ln\left(ex+d\right)}{2e}+\frac{i\pi \,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2\ln\left(ex+d\right)}{2e}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/(e*x+d),x)
[Out] ln(e*x+d)/e*ln((b*x^2+a)^p)-1/e*p*ln((b*d-(e*x+d)*b+(-a*b)^(1/2)*e)/(b*d+(-
a*b)^(1/2)*e))*ln(e*x+d)-1/e*p*ln((-b*d+(e*x+d)*b+(-a*b)^(1/2)*e)/(-b*d+(-a
*b)^(1/2)*e))*ln(e*x+d)-p/e*dilog((b*d-(e*x+d)*b+(-a*b)^(1/2)*e)/(b*d+(-a*b
)^(1/2)*e))-p/e*dilog((-b*d+(e*x+d)*b+(-a*b)^(1/2)*e)/(-b*d+(-a*b)^(1/2)*e)
)+1/2*I*ln(e*x+d)/e*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/2*I*ln
(e*x+d)/e*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/2*I*ln(e
*x+d)/e*Pi*csgn(I*c*(b*x^2+a)^p)^3+1/2*I*ln(e*x+d)/e*Pi*csgn(I*c*(b*x^2+a)^
p)^2*csgn(I*c)+1/e*ln(c)*ln(e*x+d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2 + a\right)^p c\right)}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/(d + e*x),x)
[Out] int(log(c*(a + b*x^2)^p)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a + bx^2\right)^p\right)}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/(e*x+d),x)
[Out] Integral(log(c*(a + b*x**2)**p)/(d + e*x), x)
```

3.230 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x(d+ex)}\,dx$

Optimal. Leaf size=247

$$-\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d}+\frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d}+\frac{p\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d}+\frac{p\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d}+\frac{p\log(d}{}$$

```
[Out] 1/2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/d-ln(e*x+d)*ln(c*(b*x^2+a)^p)/d+p*ln(e*x
+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-a)^(1/2)+d*b^(1/2)))/d+p*ln(e*x+d)*ln(
-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1/2)+d*b^(1/2)))/d+1/2*p*polylog(2,1+b*
x^2/a)/d+p*polylog(2,(e*x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/d+p*polylog
(2,(e*x+d)*b^(1/2)/(e*(-a)^(1/2)+d*b^(1/2)))/d
```

Rubi [A] time = 0.30, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2466, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2d}-\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(x*(d + e*x)),x]
```

```
[Out] (p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/d
 + (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e
*x])/d + (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*d) - (Log[d + e*x]*Log
[c*(a + b*x^2)^p])/d + (p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[
-a]*e)])/d + (p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/d
 + (p*PolyLog[2, 1 + (b*x^2)/a])/(2*d)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{dx} - \frac{e\log\left(c\left(a+bx^2\right)^p\right)}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}dx}{d} - \frac{e\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}dx}{d}$$

$$= -\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d} + \frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}dx, x, x^2\right)}{2d} + \frac{(2bp)\int \frac{x\log(d+}{a+bx^2}}{d}$$

$$= \frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d} - \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d} - \frac{(bp)\,\text{Subst}\left(\int \frac{\log(}{a+}\right.}{2d}$$

$$= \frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d} - \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d} + \frac{p\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{2d} - $$

$$= \frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d} + \frac{\log\left(-\frac{bx^2}{a}\right)\text{lo}}{}$$

$$= \frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d} + \frac{\log\left(-\frac{bx^2}{a}\right)\text{lo}}{}$$

$$= \frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d} + \frac{\log\left(-\frac{bx^2}{a}\right)\text{lo}}{}$$

Mathematica [A] time = 0.07, size = 232, normalized size = 0.94

$$-\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d} + \frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right) + p\text{Li}_2\left(\frac{bx^2+a}{a}\right)}{2d} + \frac{p\left(\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right) + \text{Li}_2\left(\frac{\sqrt{b}(}{\sqrt{b}d+}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(x*(d + e*x)),x]
```

```
[Out] -((Log[d + e*x]*Log[c*(a + b*x^2)^p])/d) + (p*(Log[(e*(Sqrt[-a] - Sqrt[b]*x
))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x] + Log[-((e*(Sqrt[-a] + Sqrt[b]*x)
)/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e*x] + PolyLog[2, (Sqrt[b]*(d + e*x))/
(Sqrt[b]*d - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqr
t[-a]*e)]))/d + (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2, (a +
 b*x^2)/a])/(2*d)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)}{ex^2+dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)/(e*x^2 + d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x), x)
```

maple [C] time = 0.25, size = 624, normalized size = 2.53

$$-\frac{i\pi\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)\ln(x)}{2d}+\frac{i\pi\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x/(e*x+d),x)
```

```
[Out] ln((b*x^2+a)^p)/d*ln(x)-ln((b*x^2+a)^p)/d*ln(e*x+d)-p/d*ln(x)*ln((-b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))-p/d*ln(x)*ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-p/d*d
ilog((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-p/d*dilog((b*x+(-a*b)^(1/2))/(-a*b)^
(1/2))+p/d*ln(e*x+d)*ln((b*d-(e*x+d)*b+(-a*b)^(1/2)*e)/(b*d+(-a*b)^(1/2)*e)
)+p/d*ln(e*x+d)*ln((-b*d+(e*x+d)*b+(-a*b)^(1/2)*e)/(-b*d+(-a*b)^(1/2)*e))+p
/d*dilog((b*d-(e*x+d)*b+(-a*b)^(1/2)*e)/(b*d+(-a*b)^(1/2)*e))+p/d*dilog((-b
*d+(e*x+d)*b+(-a*b)^(1/2)*e)/(-b*d+(-a*b)^(1/2)*e))-1/2*I*Pi*csgn(I*c*(b*x^
2+a)^p)^3/d*ln(x)+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/d*ln(x)+1/2*I*
Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/d*ln(e*x+d)-1/2*I*Pi
*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/d*ln(x)-1/2*I*Pi*csgn(
I*c*(b*x^2+a)^p)^2*csgn(I*c)/d*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(
I*c*(b*x^2+a)^p)^2/d*ln(e*x+d)+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2
+a)^p)^2/d*ln(x)+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/d*ln(e*x+d)+1/d*ln(c)*ln(
x)-1/d*ln(c)*ln(e*x+d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/(x*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b*x^2)^p)/(x*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)/(x*(d + e*x)), x)
```

3.231 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=306

$$-\frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d^2}+\frac{e\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{dx}-\frac{ep\text{Li}_2\left(\frac{bx^2}{a}+1\right)}{2d^2}-\frac{ep\text{Li}_2\left(\frac{\sqrt{\ }}{\sqrt{\ }}\right.}{d}$$

```
[Out] -ln(c*(b*x^2+a)^p)/d/x-1/2*e*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/d^2+e*ln(e*x+d)
*ln(c*(b*x^2+a)^p)/d^2-e*p*ln(e*x+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-a)^(1
/2)+d*b^(1/2)))/d^2-e*p*ln(e*x+d)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1/
2)+d*b^(1/2)))/d^2-1/2*e*p*polylog(2,1+b*x^2/a)/d^2-e*p*polylog(2,(e*x+d)*b
^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/d^2-e*p*polylog(2,(e*x+d)*b^(1/2)/(e*(-a)
^(1/2)+d*b^(1/2)))/d^2+2*p*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/d/a^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2466, 2455, 205, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$-\frac{ep\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{d^2}-\frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*d) - (e*p*Log[(e*(Sqrt[-
a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/d^2 - (e*p*Log[-((
e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e*x])/d^2 - Lo
g[c*(a + b*x^2)^p]/(d*x) - (e*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*d^
2) + (e*Log[d + e*x]*Log[c*(a + b*x^2)^p])/d^2 - (e*p*PolyLog[2, (Sqrt[b]*(
d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/d^2 - (e*p*PolyLog[2, (Sqrt[b]*(d + e*
x))/(Sqrt[b]*d + Sqrt[-a]*e)])/d^2 - (e*p*PolyLog[2, 1 + (b*x^2)/a])/(2*d^2
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{dx^2} - \frac{e\log\left(c\left(a+bx^2\right)^p\right)}{d^2x} + \frac{e^2\log\left(c\left(a+bx^2\right)^p\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2}dx}{d} - \frac{e\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}dx}{d^2}$$

$$= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{dx} + \frac{e\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d^2} - \frac{e\,\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}dx, x, \right.}{2d^2}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d^2} + \frac{e\log(d+}{}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d^2} + \frac{e\log(d+}{}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{ep\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+}{d^2}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{ep\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+}{d^2}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{ep\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+}{d^2}$$

Mathematica [A] time = 0.21, size = 268, normalized size = 0.88

$$-\frac{-2e\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right) + \frac{2d\log\left(c\left(a+bx^2\right)^p\right)}{x} + e\left(\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right) + p\text{Li}_2\left(\frac{bx^2}{a}+1\right)\right) + 2ep\left(\right.}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(x^2*(d + e*x)),x]
```

```
[Out] -1/2*((-4*Sqrt[b]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (2*d*Log[c*(a
+ b*x^2)^p])/x - 2*e*Log[d + e*x]*Log[c*(a + b*x^2)^p] + 2*e*p*((Log[(e*(Sq
rt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)] + Log[(e*(Sqrt[-a] + Sqrt[b]
*x))/(-(Sqrt[b]*d) + Sqrt[-a]*e)])*Log[d + e*x] + PolyLog[2, (Sqrt[b]*(d +
e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d
 + Sqrt[-a]*e)]) + e*(Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2,
 1 + (b*x^2)/a]))/d^2
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)}{ex^3+dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)/(e*x^3 + d*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x^2), x)
```

maple [C] time = 0.28, size = 831, normalized size = 2.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^2/(e*x+d),x)
```

```
[Out] 1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/d/x-1/d/x*ln(c)-ln((b*x^2+a)^p)/d/x+ln((b*
x^2+a)^p)*e/d^2*ln(e*x+d)-ln((b*x^2+a)^p)*e/d^2*ln(x)+2*b*p/d/(a*b)^(1/2)*a
rctan(1/(a*b)^(1/2)*b*x)+p*e/d^2*ln(x)*ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
+p*e/d^2*ln(x)*ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+1/2*I*Pi*csgn(I*(b*x^2+a
)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*e/d^2*ln(x)+1/d^2*e*ln(c)*ln(e*x+d)-1/
d^2*e*ln(c)*ln(x)-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/d/x-1/2*I*Pi*c
sgn(I*c*(b*x^2+a)^p)^3*e/d^2*ln(e*x+d)-p*e/d^2*dilog((b*d-(e*x+d)*b+(-a*b)^
(1/2)*e)/(b*d+(-a*b)^(1/2)*e))-p*e/d^2*dilog((-b*d+(e*x+d)*b+(-a*b)^(1/2)*e
)/(-b*d+(-a*b)^(1/2)*e))+p*e/d^2*dilog((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+p*
e/d^2*dilog((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*c
sgn(I*c*(b*x^2+a)^p)^2/d/x+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3*e/d^2*ln(x)-1/2
*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*e/d^2*ln(e*x+d)+1
/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*e/d^2*ln(e*x+d)+1/2*I*P
i*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/d/x-1/2*I*Pi*csgn(I*(
b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*e/d^2*ln(x)-1/2*I*Pi*csgn(I*c*(b*x^2+a)
^p)^2*csgn(I*c)*e/d^2*ln(x)-p*e/d^2*ln(e*x+d)*ln((b*d-(e*x+d)*b+(-a*b)^(1/2
)*e)/(b*d+(-a*b)^(1/2)*e))-p*e/d^2*ln(e*x+d)*ln((-b*d+(e*x+d)*b+(-a*b)^(1/2
)*e)/(-b*d+(-a*b)^(1/2)*e))+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*e/d^
2*ln(e*x+d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/(x^2*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b*x^2)^p)/(x^2*(d + e*x)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

3.232 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=371

$$\frac{e^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d^3}-\frac{e^2\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d^3}+\frac{e\log\left(c\left(a+bx^2\right)^p\right)}{d^2x}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2dx^2}+e$$

```
[Out] b*p*ln(x)/a/d-1/2*b*p*ln(b*x^2+a)/a/d-1/2*ln(c*(b*x^2+a)^p)/d/x^2+e*ln(c*(b
*x^2+a)^p)/d^2/x+1/2*e^2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/d^3-e^2*ln(e*x+d)*l
n(c*(b*x^2+a)^p)/d^3+e^2*p*ln(e*x+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-a)^(1
/2)+d*b^(1/2)))/d^3+e^2*p*ln(e*x+d)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(
1/2)+d*b^(1/2)))/d^3+1/2*e^2*p*polylog(2,1+b*x^2/a)/d^3+e^2*p*polylog(2,(e*
x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/d^3+e^2*p*polylog(2,(e*x+d)*b^(1/2)
/(e*(-a)^(1/2)+d*b^(1/2)))/d^3-2*e*p*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/d^2/
a^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {2466, 2454, 2395, 36, 29, 31, 2455, 205, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{e^2p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}e+\sqrt{b}d}\right)}{d^3}+\frac{e^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+b\right.\right.}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(x^3*(d + e*x)),x]
```

```
[Out] (-2*Sqrt[b]*e*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*d^2) + (b*p*Log[x])/(
a*d) + (e^2*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[
d + e*x])/d^3 + (e^2*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-
a]*e))]*Log[d + e*x])/d^3 - (b*p*Log[a + b*x^2])/(2*a*d) - Log[c*(a + b*x^2
)^p]/(2*d*x^2) + (e*Log[c*(a + b*x^2)^p])/(d^2*x) + (e^2*Log[-((b*x^2)/a)]*
Log[c*(a + b*x^2)^p])/(2*d^3) - (e^2*Log[d + e*x]*Log[c*(a + b*x^2)^p])/d^3
 + (e^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/d^3 + (
e^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/d^3 + (e^2*
p*PolyLog[2, 1 + (b*x^2)/a])/(2*d^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^3(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{dx^3} - \frac{e\log\left(c\left(a+bx^2\right)^p\right)}{d^2x^2} + \frac{e^2\log\left(c\left(a+bx^2\right)^p\right)}{d^3x} - \frac{e^3\log\left(c\left(a\right.\right.}{d^3(d+}\right.$$

$$= \frac{\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^3}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}\,dx}{d^3} - \frac{e^3\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}}{d^3}$$

$$= \frac{e\log\left(c\left(a+bx^2\right)^p\right)}{d^2x} - \frac{e^2\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d^3} + \frac{\mathrm{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x^2}\,dx,\right.}{2d}$$

$$= -\frac{2\sqrt{b}\,ep\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d^2} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+bx^2\right)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx^2}{a}\right)\mathrm{l}}{2}$$

$$= -\frac{2\sqrt{b}\,ep\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d^2} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+bx^2\right)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx^2}{a}\right)\mathrm{l}}{2}$$

$$= -\frac{2\sqrt{b}\,ep\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d^2} + \frac{bp\log(x)}{ad} + \frac{e^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{d^3} + \frac{e^2p\log\left(-\right.}{}$$

$$= -\frac{2\sqrt{b}\,ep\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d^2} + \frac{bp\log(x)}{ad} + \frac{e^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{d^3} + \frac{e^2p\log\left(-\right.}{}$$

$$= -\frac{2\sqrt{b}\,ep\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d^2} + \frac{bp\log(x)}{ad} + \frac{e^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{d^3} + \frac{e^2p\log\left(-\right.}{}$$

Mathematica [A] time = 0.22, size = 320, normalized size = 0.86

$$\frac{-\frac{d^2\log\left(c\left(a+bx^2\right)^p\right)}{x^2} - 2e^2\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right) + \frac{2de\log\left(c\left(a+bx^2\right)^p\right)}{x} + e^2\left(\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right) + p\mathrm{I}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(x^3*(d + e*x)),x]
```

```
[Out] ((-4*Sqrt[b]*d*e*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (b*d^2*p*(2*Log[x
] - Log[a + b*x^2]))/a - (d^2*Log[c*(a + b*x^2)^p])/x^2 + (2*d*e*Log[c*(a +
 b*x^2)^p])/x - 2*e^2*Log[d + e*x]*Log[c*(a + b*x^2)^p] + 2*e^2*p*((Log[(e*
(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)] + Log[(e*(Sqrt[-a] + Sqrt
[b]*x))/(-(Sqrt[b]*d) + Sqrt[-a]*e)])*Log[d + e*x] + PolyLog[2, (Sqrt[b]*(d
 + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b
]*d + Sqrt[-a]*e)]) + e^2*(Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyL
og[2, 1 + (b*x^2)/a]))/(2*d^3)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^p c\right)}{ex^4+dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)/(e*x^4 + d*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x^3), x)
```

maple [C] time = 0.28, size = 1071, normalized size = 2.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^3/(e*x+d),x)
```

```
[Out] ln((b*x^2+a)^p)*e^2/d^3*ln(x)+ln((b*x^2+a)^p)*e/d^2/x-ln((b*x^2+a)^p)*e^2/d
^3*ln(e*x+d)-p*e^2/d^3*dilog((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+p*e^2/d^3*dil
og((b*d-(e*x+d)*b+(-a*b)^(1/2)*e)/(b*d+(-a*b)^(1/2)*e))+p*e^2/d^3*dilog((-b
*d+(e*x+d)*b+(-a*b)^(1/2)*e)/(-b*d+(-a*b)^(1/2)*e))-p*e^2/d^3*dilog((-b*x+(
-a*b)^(1/2))/(-a*b)^(1/2))-1/2*ln((b*x^2+a)^p)/d/x^2-1/2/d/x^2*ln(c)-2*b*p/
d^2*e/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)-1/4*I*Pi*csgn(I*(b*x^2+a)^p)*cs
gn(I*c*(b*x^2+a)^p)^2/d/x^2+1/d^3*e^2*ln(c)*ln(x)+1/d^2*e/x*ln(c)-1/d^3*e^2
*ln(c)*ln(e*x+d)-1/4*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/d/x^2-1/2*I*Pi*
csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*e^2/d^3*ln(x)-1/2*I*Pi*
csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*e/d^2/x-1/2*I*Pi*csgn(I
*c*(b*x^2+a)^p)^3*e^2/d^3*ln(x)+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3*e^2/d^3*ln
(e*x+d)-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3*e/d^2/x+1/2*I*Pi*csgn(I*(b*x^2+a)^
p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*e^2/d^3*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x^2
+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*e^2/d^3*ln(e*x+d)+1/4*I*Pi*csgn(I*(b*x^2+a)^
p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/d/x^2-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*c
sgn(I*c)*e^2/d^3*ln(e*x+d)-p*e^2/d^3*ln(x)*ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1
/2))-p*e^2/d^3*ln(x)*ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+p*e^2/d^3*ln(e*x+d
)*ln((b*d-(e*x+d)*b+(-a*b)^(1/2)*e)/(b*d+(-a*b)^(1/2)*e))+p*e^2/d^3*ln(e*x+
d)*ln((-b*d+(e*x+d)*b+(-a*b)^(1/2)*e)/(-b*d+(-a*b)^(1/2)*e))+1/2*I*Pi*csgn(
I*c*(b*x^2+a)^p)^2*csgn(I*c)*e^2/d^3*ln(x)+1/4*I*Pi*csgn(I*c*(b*x^2+a)^p)^3
/d/x^2+1/a*b/d*p*ln(x)-1/2*b*p*ln(b*x^2+a)/a/d+1/2*I*Pi*csgn(I*c*(b*x^2+a)^
```

```
p)^2*csgn(I*c)*e/d^2/x+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2
*e^2/d^3*ln(x)+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*e/d^2/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^p c\right)}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)}{x^3\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/(x^3*(d + e*x)),x)
[Out] int(log(c*(a + b*x^2)^p)/(x^3*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**3/(e*x+d),x)
[Out] Timed out
```

3.233 $\int \frac{x^3 \log\left(c\left(a+bx^3\right)^p\right)}{d+ex} dx$

Optimal. Leaf size=692

$$-\frac{\sqrt[3]{a}\, d^2 p \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\, x+b^{2/3}x^2\right)}{2\sqrt[3]{b}\, e^3}-\frac{a^{2/3} d p \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\, x+b^{2/3}x^2\right)}{4b^{2/3}e^2}+\frac{a^{2/3} d p \log\left(\sqrt[3]{a}+\sqrt[3]{b}\, x\right)}{2b^{2/3}e^2}+\frac{\sqrt{3}\, a^{2/3} d p}{}$$

```
[Out] -3*d^2*p*x/e^3+3/4*d*p*x^2/e^2-1/3*p*x^3/e+a^(1/3)*d^2*p*ln(a^(1/3)+b^(1/3)
*x)/b^(1/3)/e^3+1/2*a^(2/3)*d*p*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)/e^2+d^3*p*ln(
-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e^4+d^3*p*ln(-e*((-
1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e^4
+d^3*p*ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)
*a^(1/3)*e))*ln(e*x+d)/e^4-1/2*a^(1/3)*d^2*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/b^(1/3)/e^3-1/4*a^(2/3)*d*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/b^(2/3)/e^2+d^2*x*ln(c*(b*x^3+a)^p)/e^3-1/2*d*x^2*ln(c*(b*x^3+a)^p)/
e^2+1/3*(b*x^3+a)*ln(c*(b*x^3+a)^p)/b/e-d^3*ln(e*x+d)*ln(c*(b*x^3+a)^p)/e^4
+d^3*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1/3)*e))/e^4+d^3*p*polylog(2
,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/e^4+d^3*p*polylog(2,b^(1
/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/e^4-a^(1/3)*d^2*p*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)/e^3+1/2*a^(2/3)*d*p*
arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)/e^2
```

Rubi [A] time = 0.89, antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 20, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.870, Rules used = {2466, 2448, 321, 200, 31, 634, 617, 204, 628, 2455, 292, 2454, 2389, 2295, 2462, 260, 2416, 2394, 2393, 2391}

$$\frac{d^3 p \text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^4}+\frac{d^3 p \text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e^4}+\frac{d^3 p \text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^4}-\frac{\sqrt[3]{a}\, d^2 p \log\left(a^{2/3}-\right.}{2\sqrt[3]{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] (-3*d^2*p*x)/e^3 + (3*d*p*x^2)/(4*e^2) - (p*x^3)/(3*e) - (Sqrt[3]*a^(1/3)*d
^2*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(b^(1/3)*e^3) + (Sq
rt[3]*a^(2/3)*d*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*b^(
2/3)*e^2) + (a^(1/3)*d^2*p*Log[a^(1/3) + b^(1/3)*x])/(b^(1/3)*e^3) + (a^(2/
3)*d*p*Log[a^(1/3) + b^(1/3)*x])/(2*b^(2/3)*e^2) + (d^3*p*Log[-((e*(a^(1/3)
 + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/e^4 + (d^3*p*Log[-((
e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e))]*Lo
g[d + e*x])/e^4 + (d^3*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)
)/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e^4 - (a^(1/3)*d^2*p*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)*e^3) - (a^(2/3)*d*
p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*b^(2/3)*e^2) + (d^2*x*
Log[c*(a + b*x^3)^p])/e^3 - (d*x^2*Log[c*(a + b*x^3)^p])/(2*e^2) + ((a + b*
x^3)*Log[c*(a + b*x^3)^p])/(3*b*e) - (d^3*Log[d + e*x]*Log[c*(a + b*x^3)^p]
)/e^4 + (d^3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/e^4
 + (d^3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)
])/e^4 + (d^3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1
/3)*e)])/e^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^3 \log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx = \int\left(\frac{d^2\log\left(c\left(a+bx^3\right)^p\right)}{e^3} - \frac{dx\log\left(c\left(a+bx^3\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{d^3 \mathrm{lo}\ldots\right.$$

$$= \frac{d^2\int\log\left(c\left(a+bx^3\right)^p\right)dx}{e^3} - \frac{d^3\int\frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}dx}{e^3} - \frac{d\int x\log\left(c\left(a+bx^3\right)^p\right)dx}{e^2}\ldots$$

$$= \frac{d^2x\log\left(c\left(a+bx^3\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+bx^3\right)^p\right)}{2e^2} - \frac{d^3\log(d+ex)\log\left(c\left(a+bx^3\right)\ldots\right.}{e^4}$$

$$= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} + \frac{d^2x\log\left(c\left(a+bx^3\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+bx^3\right)^p\right)}{2e^2} - \frac{d^3\log(d+\ldots}{}$$

$$= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} - \frac{px^3}{3e} + \frac{d^2x\log\left(c\left(a+bx^3\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+bx^3\right)^p\right)}{2e^2} + \frac{(a\ldots}{}$$

$$= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} - \frac{px^3}{3e} + \frac{\sqrt[3]{a}\,d^2p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}\,e^3} + \frac{a^{2/3}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e^2} + \ldots$$

$$= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} - \frac{px^3}{3e} + \frac{\sqrt[3]{a}\,d^2p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}\,e^3} + \frac{a^{2/3}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e^2} + \ldots$$

$$= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} - \frac{px^3}{3e} - \frac{\sqrt{3}\sqrt[3]{a}\,d^2p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}\,e^3} + \frac{\sqrt{3}a^{2/3}dp\tan^{-1}\left(\frac{\sqrt[3]{a}-\ldots}{\sqrt{3}\ldots}\right.}{2b^{2/3}e^2}$$

Mathematica [C] time = 0.65, size = 497, normalized size = 0.72

$$-\frac{\frac{6d^2ep\left(\sqrt[3]{a}\left(\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)-2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+6\sqrt[3]{b}x\right)}{\sqrt[3]{b}} + 12d^3\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] -1/12*(9*d*e^2*p*x^2*(-1 + Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)]) +
(6*d^2*e*p*(6*b^(1/3)*x - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*(2*S
qrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])))/b^(1/3) - 12*d^2*e*x*Log[c*(a + b*x^3)^p] + 6*d
*e^2*x^2*Log[c*(a + b*x^3)^p] + 12*d^3*Log[d + e*x]*Log[c*(a + b*x^3)^p] +
(4*e^3*(b*p*x^3 - (a + b*x^3)*Log[c*(a + b*x^3)^p]))/b - 12*d^3*p*(Log[(e*(
```

```
(-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d
+ e*x] + Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a^(1/3)*e)]*Log[d +
e*x] + Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + (-1)^(2/3)*
a^(1/3)*e)]*Log[d + e*x] + PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1
/3)*e)] + PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)
] + PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)]))/e^
4
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3 \log\left(\left(bx^3+a\right)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x^3*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \log\left(\left(bx^3+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^3*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

maple [C] time = 0.56, size = 912, normalized size = 1.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x^3+a)^p)/(e*x+d),x)
[Out] -1/3/e*p*x^3+1/3/e*x^3*ln(c)+1/6/b*p/e*a*sum((2*_R^2-7*_R*d+11*d^2)/(_R^2-2
*_R*d+d^2)*ln(e*x-_R+d),_R=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3)
)-ln((b*x^3+a)^p)*d^3/e^4*ln(e*x+d)-1/2*ln((b*x^3+a)^p)/e^2*x^2*d+ln((b*x^3
+a)^p)/e^3*x*d^2+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3*d^3/e^4*ln(e*x+d)-3*d^2/e
^3*p*x+p/e^4*d^3*sum(ln((-e*x+_R1-d)/_R1)*ln(e*x+d)+dilog((-e*x+_R1-d)/_R1)
,_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))+1/3*ln((b*x^3+a)^p)/
e*x^3-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/e^3*x*d^2+1/4*I*Pi*csgn(I*c*(b*x^3+a
)^p)^3/e^2*x^2*d+1/6*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)/e*x^3+1/6*I*Pi*
csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/e*x^3+1/4*I*Pi*csgn(I*(b*x^3+a)
^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/e^2*x^2*d-1/2*d/e^2*x^2*ln(c)+d^2/e^3*x
*ln(c)+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*d^3/e^4
*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/e^3
*x*d^2-d^3/e^4*ln(c)*ln(e*x+d)-49/12*d^3/e^4*p-1/6*I*Pi*csgn(I*(b*x^3+a)^p)
*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/e*x^3-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csg
n(I*c)*d^3/e^4*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)
^2*d^3/e^4*ln(e*x+d)-1/6*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/e*x^3+3/4*d/e^2*p*x^2
-1/4*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)/e^2*x^2*d+1/2*I*Pi*csgn(I*c*(b*
x^3+a)^p)^2*csgn(I*c)/e^3*x*d^2+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^
3+a)^p)^2/e^3*x*d^2-1/4*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/e^
2*x^2*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \log\left(\left(bx^3 + a\right)^p c\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^3*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \ln\left(c\left(b\,x^3 + a\right)^p\right)}{d + e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(c*(a + b*x^3)^p))/(d + e*x),x)
[Out] int((x^3*log(c*(a + b*x^3)^p))/(d + e*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x**3+a)**p)/(e*x+d),x)
[Out] Timed out
```

3.234 $\int \frac{x^2 \log\left(c\left(a+bx^3\right)^p\right)}{d+ex} dx$

Optimal. Leaf size=643

$$\frac{\sqrt[3]{a}\, dp \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\, x+b^{2/3}x^2\right)}{2\sqrt[3]{b}\, e^2}+\frac{a^{2/3}p \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\, x+b^{2/3}x^2\right)}{4b^{2/3}e}-\frac{a^{2/3}p \log\left(\sqrt[3]{a}+\sqrt[3]{b}\, x\right)}{2b^{2/3}e}-\frac{\sqrt{3}\, a^{2/3}p \tan^{-1}}{2b^{2/}}$$

```
[Out] 3*d*p*x/e^2-3/4*p*x^2/e-a^(1/3)*d*p*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)/e^2-1/2*a
^(2/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)/e-d^2*p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b
^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e^3-d^2*p*ln(-e*((-1)^(2/3)*a^(1/3)+b^(1/3)*
x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e^3-d^2*p*ln((-1)^(1/3)*e*(a
^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))*ln(e*x+d)/e^
3+1/2*a^(1/3)*d*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(1/3)/e^2+1/4
*a^(2/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)/e-d*x*ln(c*(b*
x^3+a)^p)/e^2+1/2*x^2*ln(c*(b*x^3+a)^p)/e+d^2*ln(e*x+d)*ln(c*(b*x^3+a)^p)/e
^3-d^2*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1/3)*e))/e^3-d^2*p*polylog
(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/e^3-d^2*p*polylog(2,b^
(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/e^3+a^(1/3)*d*p*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)/e^2-1/2*a^(2/3)*p*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)/e
```

Rubi [A] time = 0.77, antiderivative size = 643, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 17, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.739, Rules used = {2466, 2448, 321, 200, 31, 634, 617, 204, 628, 2455, 292, 2462, 260, 2416, 2394, 2393, 2391}

$$-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^3}-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e^3}-\frac{d^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^3}+\frac{\sqrt[3]{a}\, dp \log\left(a^{2/3}-\right.}{2\sqrt[}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] (3*d*p*x)/e^2 - (3*p*x^2)/(4*e) + (Sqrt[3]*a^(1/3)*d*p*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(b^(1/3)*e^2) - (Sqrt[3]*a^(2/3)*p*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*b^(2/3)*e) - (a^(1/3)*d*p*Log[
a^(1/3) + b^(1/3)*x])/(b^(1/3)*e^2) - (a^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/
(2*b^(2/3)*e) - (d^2*p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)
*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))
/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[((-1)^
(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e
)]*Log[d + e*x])/e^3 + (a^(1/3)*d*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(2*b^(1/3)*e^2) + (a^(2/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(4*b^(2/3)*e) - (d*x*Log[c*(a + b*x^3)^p])/e^2 + (x^2*Log[c*(a +
 b*x^3)^p])/(2*e) + (d^2*Log[d + e*x]*Log[c*(a + b*x^3)^p])/e^3 - (d^2*p*Po
lyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/e^3 - (d^2*p*PolyLog
[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/e^3 - (d^2*p*P
olyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/e^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^2 \log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx = \int \left(-\frac{d\log\left(c\left(a+bx^3\right)^p\right)}{e^2} + \frac{x\log\left(c\left(a+bx^3\right)^p\right)}{e} + \frac{d^2\log\left(c\left(a+bx^3\right)^p\right)}{e^2(d+ex)}\right)dx$$

$$= -\frac{d\int \log\left(c\left(a+bx^3\right)^p\right)dx}{e^2} + \frac{d^2\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}dx}{e^2} + \frac{\int x\log\left(c\left(a+bx^3\right)^p\right)dx}{e}$$

$$= -\frac{dx\log\left(c\left(a+bx^3\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} + \frac{d^2\log(d+ex)\log\left(c\left(a+bx^3\right)\right.}{e^3}$$

$$= \frac{3dpx}{e^2} - \frac{3px^2}{4e} - \frac{dx\log\left(c\left(a+bx^3\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} + \frac{d^2\log(d+ex)\,\mathrm{lo}}{e}$$

$$= \frac{3dpx}{e^2} - \frac{3px^2}{4e} - \frac{dx\log\left(c\left(a+bx^3\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} + \frac{d^2\log(d+ex)\,\mathrm{lo}}{e}$$

$$= \frac{3dpx}{e^2} - \frac{3px^2}{4e} - \frac{\sqrt[3]{a}\,dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}\,e^2} - \frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e} - \frac{d^2p\log\left(-\frac{e(}{}\right.}{}$$

$$= \frac{3dpx}{e^2} - \frac{3px^2}{4e} - \frac{\sqrt[3]{a}\,dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}\,e^2} - \frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e} - \frac{d^2p\log\left(-\frac{e(}{}\right.}{}$$

$$= \frac{3dpx}{e^2} - \frac{3px^2}{4e} + \frac{\sqrt{3}\sqrt[3]{a}\,dp\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}\,e^2} - \frac{\sqrt{3}\,a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}e} - \frac{\sqrt[3]{a}\,\cdots}{}$$

Mathematica [C] time = 0.41, size = 504, normalized size = 0.78

$$-\frac{-\frac{2\sqrt[3]{a}\,dep\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}} - 4d^2\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right) + 4dex\log\left(c\left(a+bx^3\right)^p\right) - 2e^2x^2\log\left(c\left(a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] -1/4*(-12*d*e*p*x + 3*e^2*p*x^2 - (4*Sqrt[3]*a^(1/3)*d*e*p*ArcTan[(1 - (2*b
^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) - 3*e^2*p*x^2*Hypergeometric2F1[2/3, 1
, 5/3, -((b*x^3)/a)] + (4*a^(1/3)*d*e*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) +
 4*d^2*p*Log[(e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a
^(1/3)*e)]*Log[d + e*x] + 4*d^2*p*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*
d) + a^(1/3)*e)]*Log[d + e*x] + 4*d^2*p*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3
)*x))/(-(b^(1/3)*d) + (-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] - (2*a^(1/3)*d*e*
p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 4*d*e*x*Log[c*(
a + b*x^3)^p] - 2*e^2*x^2*Log[c*(a + b*x^3)^p] - 4*d^2*Log[d + e*x]*Log[c*(
a + b*x^3)^p] + 4*d^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)
*e)] + 4*d^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/
3)*e)] + 4*d^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(
1/3)*e)])/e^3
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2\log\left(\left(bx^3+a\right)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x^2*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\log\left(\left(bx^3+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^2*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

maple [C] time = 0.54, size = 704, normalized size = 1.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^3+a)^p)/(e*x+d),x)
[Out] 1/2*ln((b*x^3+a)^p)/e*x^2-ln((b*x^3+a)^p)/e^2*x*d+ln((b*x^3+a)^p)*d^2/e^3*l
n(e*x+d)-p/e^3*d^2*sum(ln((-e*x+_R1-d)/_R1)*ln(e*x+d)+dilog((-e*x+_R1-d)/_R
1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))-3/4/e*p*x^2+3*d/e^
2*p*x+15/4*d^2/e^3*p+1/2/b*p*a*sum((_R-3*d)/(_R^2-2*_R*d+d^2)*ln(e*x-_R+d),
_R=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))+1/4*I*Pi*csgn(I*(b*x^3
+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/e*x^2-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*
(b*x^3+a)^p)*csgn(I*c)*d^2/e^3*ln(e*x+d)-1/4*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(
I*c*(b*x^3+a)^p)*csgn(I*c)/e*x^2-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x
^3+a)^p)^2/e^2*x*d+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2*d^2
/e^3*ln(e*x+d)-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3*d^2/e^3*ln(e*x+d)+1/4*I*Pi*
csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)/e*x^2-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*cs
gn(I*c)/e^2*x*d+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)*d^2/e^3*ln(e*x+d
)-1/4*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/e*x^2+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/e
^2*x*d+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/e^2*x*d
+1/2/e*x^2*ln(c)-d/e^2*x*ln(c)+d^2/e^3*ln(c)*ln(e*x+d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\log\left(\left(bx^3+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^2*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(c*(a + b*x^3)^p))/(d + e*x),x)
[Out] int((x^2*log(c*(a + b*x^3)^p))/(d + e*x), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**3+a)**p)/(e*x+d),x)
[Out] Timed out
```

3.235 $\int \frac{x \log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=457

$$-\frac{\sqrt[3]{a}\,p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}e}-\frac{d\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+bx^3\right)^p\right)}{e}+\frac{dp\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^2}+$$

```
[Out] -3*p*x/e+a^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)/e+d*p*ln(-e*(a^(1/3)+b^(1/
3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e^2+d*p*ln(-e*((-1)^(2/3)*a^(1/3)+b^
(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e^2+d*p*ln((-1)^(1/3)*
e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))*ln(e*x+d
)/e^2-1/2*a^(1/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(1/3)/e+x*l
n(c*(b*x^3+a)^p)/e-d*ln(e*x+d)*ln(c*(b*x^3+a)^p)/e^2+d*p*polylog(2,b^(1/3)*
(e*x+d)/(b^(1/3)*d-a^(1/3)*e))/e^2+d*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d
+(-1)^(1/3)*a^(1/3)*e))/e^2+d*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(
2/3)*a^(1/3)*e))/e^2-a^(1/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(
1/2))*3^(1/2)/b^(1/3)/e
```

Rubi [A] time = 0.63, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {2466, 2448, 321, 200, 31, 634, 617, 204, 628, 2462, 260, 2416, 2394, 2393, 2391}

$$\frac{dp\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^2}+\frac{dp\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e^2}+\frac{dp\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^2}-\frac{\sqrt[3]{a}\,p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\right.}{2\sqrt[3]{b}e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] (-3*p*x)/e - (Sqrt[3]*a^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(b^(1/3)*e) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/(b^(1/3)*e) + (d
*p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/
e^2 + (d*p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/
3)*a^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^
(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e^2 - (
a^(1/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)*e) + (
x*Log[c*(a + b*x^3)^p])/e - (d*Log[d + e*x]*Log[c*(a + b*x^3)^p])/e^2 + (d*
p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/e^2 + (d*p*PolyL
og[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/e^2 + (d*p*P
olyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/e^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx = \int \left(\frac{\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{d\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)}\right)dx$$

$$= \frac{\int \log\left(c\left(a+bx^3\right)^p\right)dx}{e} - \frac{d\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx}{e}$$

$$= \frac{x\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^2} + \frac{(3bdp)\int \frac{x^2\log(d+ex)}{a+bx^3}\,dx}{e^2} - \frac{(3\ldots}{}$$

$$= -\frac{3px}{e} + \frac{x\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^2} + \frac{(3bdp)\int\left(\frac{\log(d+\ldots}{3b^{2/3}\left(\sqrt[3]{a}\ldots\right.}\ldots\right.}{}$$

$$= -\frac{3px}{e} + \frac{x\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^2} + \frac{\left(\sqrt[3]{b}\,dp\right)\int \frac{\log(d+ex\ldots}{\sqrt[3]{a}+\sqrt[3]{b}\ldots}}{e^2}$$

$$= -\frac{3px}{e} + \frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,e} + \frac{dp\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left((-1\ldots}{\sqrt[3]{b}\,d\ldots}\right.}{}$$

$$= -\frac{3px}{e} + \frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,e} + \frac{dp\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left((-1\ldots}{\sqrt[3]{b}\ldots}\right.}{}$$

$$= -\frac{3px}{e} - \frac{\sqrt{3}\,\sqrt[3]{a}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{b}\,e} + \frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,e} + \frac{dp\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\ldots}{e^2}$$

Mathematica [A] time = 0.21, size = 430, normalized size = 0.94

$$-\frac{\sqrt[3]{a}ep\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}-2d\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)+2ex\log\left(c\left(a+bx^3\right)^p\right)+2dp\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] (-6*e*p*x - (2*Sqrt[3]*a^(1/3)*e*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]])/b^(1/3) + (2*a^(1/3)*e*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + 2*d*p*Log
[(e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*L
og[d + e*x] + 2*d*p*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a^(1/3)*e
)]*Log[d + e*x] + 2*d*p*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(-(b^(1/3)
*d) + (-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] - (a^(1/3)*e*p*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 2*e*x*Log[c*(a + b*x^3)^p] - 2*d*Lo
g[d + e*x]*Log[c*(a + b*x^3)^p] + 2*d*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(
1/3)*d - a^(1/3)*e)] + 2*d*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-
1)^(1/3)*a^(1/3)*e)] + 2*d*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-
1)^(2/3)*a^(1/3)*e)])/(2*e^2)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x\log\left(\left(bx^3+a\right)^pc\right)}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\log\left(\left(bx^3+a\right)^pc\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

maple [C] time = 0.54, size = 500, normalized size = 1.09

$$\frac{aep\ln\left(ex-\text{RootOf}\left(b\_Z^3-3\_Z^2bd+3\_Zbd^2+ae^3-bd^3\right)+d\right)}{b\left(\text{RootOf}\left(b\_Z^3-3\_Z^2bd+3\_Zbd^2+ae^3-bd^3\right)^2-2\,\text{RootOf}\left(b\_Z^3-3\_Z^2bd+3\_Zbd^2+ae^3-bd^3\right)d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^3+a)^p)/(e*x+d),x)
```

```
[Out] ln((b*x^3+a)^p)/e*x-ln((b*x^3+a)^p)*d/e^2*ln(e*x+d)-3/e*p*x-3*d/e^2*p+1/b*p
*e*a*sum(1/(_R^2-2*_R*d+d^2)*ln(e*x-_R+d),_R=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*
b*d^2+a*e^3-b*d^3))+p/e^2*d*sum(ln((-e*x+_R1-d)/_R1)*ln(e*x+d)+dilog((-e*x+
```

```
_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))-1/2*I*Pi*
csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/e*x+1/2*I*Pi*csgn(I*(b*
x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*d/e^2*ln(e*x+d)+1/2*I*Pi*csgn(I*c
*(b*x^3+a)^p)^2*csgn(I*c)/e*x-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+
a)^p)^2*d/e^2*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3*d/e^2*ln(e*x+d)-1/
2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)*d/e^2*ln(e*x+d)+1/2*I*Pi*csgn(I*(b
*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/e*x-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/e*x
+1/e*x*ln(c)-d/e^2*ln(c)*ln(e*x+d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\log\left(\left(bx^3+a\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(c*(a + b*x^3)^p))/(d + e*x),x)
[Out] int((x*log(c*(a + b*x^3)^p))/(d + e*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**3+a)**p)/(e*x+d),x)
[Out] Timed out
```

3.236 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=308

$$\frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}-\frac{p\log(d+ex)\log}{}$$

```
[Out] -p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e-p*ln(-e*((-
1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e-p
*ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/
3)*e))*ln(e*x+d)/e+ln(e*x+d)*ln(c*(b*x^3+a)^p)/e-p*polylog(2,b^(1/3)*(e*x+d
)/(b^(1/3)*d-a^(1/3)*e))/e-p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3
)*a^(1/3)*e))/e-p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e
))/e
```

Rubi [A] time = 0.39, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2462, 260, 2416, 2394, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a\right.\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x]
)/e) - (p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3
)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)
*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e + (Log[d +
 e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1
/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2
/3)*a^(1/3)*e)])/e
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx = \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{(3bp)\int \frac{x^2\log(d+ex)}{a+bx^3}\,dx}{e}$$

$$= \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{(3bp)\int \left(\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(d+ex)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\mathrm{l}\ldots}{3b^{2/3}\left((\ldots\right.}\right.}{e}$$

$$= \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{\left(\sqrt[3]{b}p\right)\int \frac{\log(d+ex)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e} - \frac{\left(\sqrt[3]{b}p\right)\int \frac{\log(d+ex)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e} - \ldots$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-}\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-}\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-}\ldots}{\ldots}\right.}{\ldots}$$

Mathematica [A] time = 0.06, size = 313, normalized size = 1.02

$$\frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{p\,\mathrm{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e} - \frac{p\,\mathrm{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e} - \frac{p\,\mathrm{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e} - \frac{p\log(d+ex)\log\left(\ldots\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x]
)/e) - (p*Log[-(((-1)^(2/3)*e*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*d
- (-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3) +
 (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e
 + (Log[d + e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x)
```

```
)/(b^(1/3)*d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d + (-1)^(1/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - (-1)^(2/3)*a^(1/3)*e)])/e
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^3+a\right)^p c\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

maple [C] time = 0.09, size = 261, normalized size = 0.85

$$-\frac{i\pi \operatorname{csgn}(ic)\operatorname{csgn}\left(i\left(bx^3+a\right)^p\right)\operatorname{csgn}\left(ic\left(bx^3+a\right)^p\right)\ln(ex+d)}{2e}+\frac{i\pi \operatorname{csgn}(ic)\operatorname{csgn}\left(ic\left(bx^3+a\right)^p\right)^2\ln(ex+d)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/(e*x+d),x)
[Out] 1/e*ln((b*x^3+a)^p)*ln(e*x+d)-p/e*sum(ln((-e*x+_R1-d)/_R1)*ln(e*x+d)+dilog(
(-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))+1/2
*I*Pi/e*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2*ln(e*x+d)-1/2*I*Pi/e*cs
gn(I*c)*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*ln(e*x+d)-1/2*I*Pi/e*csgn
(I*c*(b*x^3+a)^p)^3*ln(e*x+d)+1/2*I*Pi/e*csgn(I*c)*csgn(I*c*(b*x^3+a)^p)^2*
ln(e*x+d)+1/e*ln(c)*ln(e*x+d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(bx^3+a\right)^p\right)}{d+ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/(d + e*x),x)
[Out] int(log(c*(a + b*x^3)^p)/(d + e*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/(e*x+d),x)
[Out] Timed out
```

3.237 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x(d+ex)}\,dx$

Optimal. Leaf size=352

$$-\frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d}+\frac{\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d}+\frac{p\mathrm{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d}+\frac{p\mathrm{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d}+\frac{p\mathrm{Li}_2}{}$$

```
[Out] p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/d+p*ln(-e*((-1
)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/d+p*
ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3
)*e))*ln(e*x+d)/d+1/3*ln(-b*x^3/a)*ln(c*(b*x^3+a)^p)/d-ln(e*x+d)*ln(c*(b*x^
3+a)^p)/d+p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1/3)*e))/d+p*polylog(2,
b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/d+p*polylog(2,b^(1/3)*(e*
x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/d+1/3*p*polylog(2,1+b*x^3/a)/d
```

Rubi [A] time = 0.56, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2466, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(x*(d + e*x)),x]
```

```
[Out] (p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/
d + (p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a
^(1/3)*e))]*Log[d + e*x])/d + (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^
(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d + (Log[-((b*x
^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d) - (Log[d + e*x]*Log[c*(a + b*x^3)^p])/d
 + (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/d + (p*PolyL
og[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/d + (p*PolyL
og[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/d + (p*PolyL
og[2, 1 + (b*x^3)/a])/(3*d)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+bx^3\right)^p\right)}{dx} - \frac{e\log\left(c\left(a+bx^3\right)^p\right)}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}dx}{d} - \frac{e\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}dx}{d}$$

$$= -\frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d} + \frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}dx, x, x^3\right)}{3d} + \frac{(3bp)\int \frac{x^2\log(d}{a+bx}}{d}$$

$$= \frac{\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d} - \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d} - \frac{(bp)\,\text{Subst}\left(\int \frac{\log(}{a+}\right.}{3d}$$

$$= \frac{\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d} - \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d} + \frac{p\text{Li}_2\left(1+\frac{bx^3}{a}\right)}{3d} +$$

$$= \frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\right.}{}$$

$$= \frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\right.}{}$$

$$= \frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\right.}{}$$

Mathematica [A] time = 0.06, size = 358, normalized size = 1.02

$$-\frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d} + \frac{\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d} + \frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d} + \frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d} + \frac{p\text{Li}_2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(x*(d + e*x)),x]
```

```
[Out] (p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/
d + (p*Log[-(((-1)^(2/3)*e*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*d - (
-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/d + (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-
1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d +
(Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d) - (Log[d + e*x]*Log[c*(a + b
*x^3)^p])/d + (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/d
 + (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/d
 + (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/d
 + (p*PolyLog[2, (a + b*x^3)/a])/(3*d)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^3+a\right)^p c\right)}{ex^2+dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^3 + a)^p*c)/(e*x^2 + d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x), x)
```

maple [C] time = 0.49, size = 461, normalized size = 1.31

$$-\frac{i\pi\,\mathrm{csgn}\,(ic)\,\mathrm{csgn}\left(i\left(b\,x^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^3+a\right)^p\right)\ln(x)}{2d}+\frac{i\pi\,\mathrm{csgn}\,(ic)\,\mathrm{csgn}\left(i\left(b\,x^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^3+a\right)^p\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x/(e*x+d),x)
```

```
[Out] ln((b*x^3+a)^p)/d*ln(x)-ln((b*x^3+a)^p)/d*ln(e*x+d)-p/d*sum(ln(x)*ln((_R1-x
)/_R1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3*b+a))+p/d*sum(ln((-e*x+_R1-d)/_R1
)*ln(e*x+d)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2
+a*e^3-b*d^3))+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/d*ln(x)
-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/d*ln(e*x+d)-1/2*I*Pi*
csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/d*ln(x)+1/2*I*Pi*csgn(I
*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/d*ln(e*x+d)-1/2*I*Pi*csgn(I*c
*(b*x^3+a)^p)^3/d*ln(x)+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/d*ln(e*x+d)+1/2*I*
Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)/d*ln(x)-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)
^2*csgn(I*c)/d*ln(e*x+d)+1/d*ln(c)*ln(x)-1/d*ln(c)*ln(e*x+d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(b\,x^3+a\right)^p\right)}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/(x*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b*x^3)^p)/(x*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x/(e*x+d),x)
```

```
[Out] Timed out
```

3.238 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=510

$$\frac{\sqrt[3]{b}\,p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{a}\,d}-\frac{e\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d^2}+\frac{e\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d^2}-\frac{\log\left(c\left(a+\right.\right.}{dx}$$

```
[Out] -b^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/d-e*p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b
^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/d^2-e*p*ln(-e*((-1)^(2/3)*a^(1/3)+b^(1/3)*x)
/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/d^2-e*p*ln((-1)^(1/3)*e*(a^(1/
3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))*ln(e*x+d)/d^2+1/
2*b^(1/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/d-ln(c*(b*x^3
+a)^p)/d/x-1/3*e*ln(-b*x^3/a)*ln(c*(b*x^3+a)^p)/d^2+e*ln(e*x+d)*ln(c*(b*x^3
+a)^p)/d^2-e*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1/3)*e))/d^2-e*p*pol
ylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/d^2-e*p*polylog(2,
b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/d^2-1/3*e*p*polylog(2,1+b
*x^3/a)/d^2-b^(1/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(
1/2)/a^(1/3)/d
```

Rubi [A] time = 0.68, antiderivative size = 510, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 16, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.696, Rules used = {2466, 2455, 292, 31, 634, 617, 204, 628, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$-\frac{ep\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)}{3d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(x^2*(d + e*x)),x]
```

```
[Out] -((Sqrt[3]*b^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(a^
(1/3)*d)) - (b^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/(a^(1/3)*d) - (e*p*Log[-((
e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/d^2 - (e*p
*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)
*e))]*Log[d + e*x])/d^2 - (e*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(1
/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d^2 + (b^(1/3)*p*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*a^(1/3)*d) - Log[c*(a +
b*x^3)^p]/(d*x) - (e*Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d^2) + (e*L
og[d + e*x]*Log[c*(a + b*x^3)^p])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d + e*x))
/(b^(1/3)*d - a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/
3)*d + (-1)^(1/3)*a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b
^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, 1 + (b*x^3)/a])/(3
*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^2(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+bx^3\right)^p\right)}{dx^2} - \frac{e\log\left(c\left(a+bx^3\right)^p\right)}{d^2x} + \frac{e^2\log\left(c\left(a+bx^3\right)^p\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^2}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx}{d^2}$$

$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{dx} + \frac{e\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d^2} - \frac{e\,\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx,\right.}{3d^2}$$

$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d^2} + \frac{e\log(d+ex)\log\left(c\left(a+b\right.\right.}{d^2}$$

$$= -\frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}\,d} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d^2} + \frac{e}{}$$

$$= -\frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}\,d} - \frac{ep\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\right.}{\sqrt[3]{b}\,d-(-1)^{2/3}}\right.}{d^2}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,d} - \frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}\,d} - \frac{ep\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d}{d^2}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,d} - \frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}\,d} - \frac{ep\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d}{d^2}$$

Mathematica [C] time = 0.07, size = 424, normalized size = 0.83

$$-\frac{e\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d^2} + \frac{e\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d^2} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{dx} - \frac{ep\text{Li}_2\left(\frac{bx^3+a}{a}\right)}{3d^2} - \frac{ep\text{Li}_2\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (3*b*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/(2*a*d) - (e*p*Log
[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/d^2 -
(e*p*Log[-(((-1)^(2/3)*e*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*d - (-1
)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/d^2 - (e*p*Log[((-1)^(1/3)*e*(a^(1/3) +
(-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d^
2 - Log[c*(a + b*x^3)^p]/(d*x) - (e*Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p])
/(3*d^2) + (e*Log[d + e*x]*Log[c*(a + b*x^3)^p])/d^2 - (e*p*PolyLog[2, (b^(
1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d
 + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (b^(1/3
)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (a
+ b*x^3)/a]))/(3*d^2)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^3+a\right)^p c\right)}{ex^3+dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^3 + a)^p*c)/(e*x^3 + d*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x^2), x)
```

maple [C] time = 0.49, size = 732, normalized size = 1.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^2/(e*x+d),x)
```

```
[Out] -ln((b*x^3+a)^p)/d/x-ln((b*x^3+a)^p)*e/d^2*ln(x)+ln((b*x^3+a)^p)*e/d^2*ln(e
*x+d)-p*e/d^2*sum(ln((-e*x+_R1-d)/_R1)*ln(e*x+d)+dilog((-e*x+_R1-d)/_R1),_R
1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))-p/d/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))+1/2*p/d/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+p/d*3^(1/2
)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+p*e/d^2*sum(ln(x)*ln(
(_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3*b+a))-1/2*I*Pi*csgn(I*c*(b*
x^3+a)^p)^2*csgn(I*c)/d/x+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3*e/d^2*ln(x)-1/2*
I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/d/x-1/2*I*Pi*csgn(I*c*(b*x
^3+a)^p)^3*e/d^2*ln(e*x+d)-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)*e/d^2
*ln(x)-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2*e/d^2*ln(x)+1/2
*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/d/x-1/2*I*Pi*csgn
(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*e/d^2*ln(e*x+d)+1/2*I*Pi*cs
gn(I*c*(b*x^3+a)^p)^2*csgn(I*c)*e/d^2*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^3+a)
^p)^3/d/x+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*e/d^
2*ln(x)+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2*e/d^2*ln(e*x+d
)-1/d/x*ln(c)-1/d^2*e*ln(c)*ln(x)+1/d^2*e*ln(c)*ln(e*x+d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(b\,x^3+a\right)^p\right)}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/(x^2*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b*x^3)^p)/(x^2*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

3.239 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=674

$$-\frac{\sqrt[3]{b}\,ep\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{a}\,d^2}-\frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{4a^{2/3}d}+\frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2a^{2/3}d}-\frac{\sqrt{3}\,b^{2/3}p\tan^{-1}}{2a^{2}}$$

```
[Out] 1/2*b^(2/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/d+b^(1/3)*e*p*ln(a^(1/3)+b^(1/3
)*x)/a^(1/3)/d^2+e^2*p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(
e*x+d)/d^3+e^2*p*ln(-e*((-1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)
*a^(1/3)*e))*ln(e*x+d)/d^3+e^2*p*ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3
)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))*ln(e*x+d)/d^3-1/4*b^(2/3)*p*ln(a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/d-1/2*b^(1/3)*e*p*ln(a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/d^2-1/2*ln(c*(b*x^3+a)^p)/d/x^2+e*ln(c*(
b*x^3+a)^p)/d^2/x+1/3*e^2*ln(-b*x^3/a)*ln(c*(b*x^3+a)^p)/d^3-e^2*ln(e*x+d)*
ln(c*(b*x^3+a)^p)/d^3+e^2*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1/3)*e)
)/d^3+e^2*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/d^3
+e^2*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/d^3+1/3*
e^2*p*polylog(2,1+b*x^3/a)/d^3-1/2*b^(2/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*
x)/a^(1/3)*3^(1/2))*3^(1/2)/a^(2/3)/d+b^(1/3)*e*p*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/a^(1/3)/d^2
```

Rubi [A] time = 0.79, antiderivative size = 674, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 17, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.739, Rules used = {2466, 2455, 200, 31, 634, 617, 204, 628, 292, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{e^2p\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)}{3d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(x^3*(d + e*x)),x]
```

```
[Out] -(Sqrt[3]*b^(2/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*a
^(2/3)*d) + (Sqrt[3]*b^(1/3)*e*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(a^(1/3)*d^2) + (b^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*a^(2/3)*d)
 + (b^(1/3)*e*p*Log[a^(1/3) + b^(1/3)*x])/(a^(1/3)*d^2) + (e^2*p*Log[-((e*(
a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/d^3 + (e^2*p*
Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*
e))]*Log[d + e*x])/d^3 + (e^2*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(
1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d^3 - (b^(2/3)*p
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*a^(2/3)*d) - (b^(1/3)*e
*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*a^(1/3)*d^2) - Log[c*
(a + b*x^3)^p]/(2*d*x^2) + (e*Log[c*(a + b*x^3)^p])/(d^2*x) + (e^2*Log[-((b
*x^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d^3) - (e^2*Log[d + e*x]*Log[c*(a + b*x^
3)^p])/d^3 + (e^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)]
)/d^3 + (e^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/
3)*e)])/d^3 + (e^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)
*a^(1/3)*e)])/d^3 + (e^2*p*PolyLog[2, 1 + (b*x^3)/a])/(3*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+bx^3\right)^p\right)}{dx^3} - \frac{e\log\left(c\left(a+bx^3\right)^p\right)}{d^2x^2} + \frac{e^2\log\left(c\left(a+bx^3\right)^p\right)}{d^3x} - \frac{e^3\log\left(c\left(a\right.\right.}{d^3(d+}\right.$$

$$= \frac{\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3}dx}{d} - \frac{e\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^2}dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}dx}{d^3} - \frac{e^3\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}}{d^3}$$

$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+bx^3\right)^p\right)}{d^2x} - \frac{e^2\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d^3} + \frac{e^2}{}$$

$$= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+bx^3\right)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d^3} - \frac{e^2}{}$$

$$= \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}d} + \frac{\sqrt[3]{b}ep\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d^2} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a\right.\right.}{d^2}$$

$$= \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}d} + \frac{\sqrt[3]{b}ep\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d^2} + \frac{e^2p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+}{d^3}$$

$$= -\frac{\sqrt{3}b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}d} + \frac{\sqrt{3}\sqrt[3]{b}ep\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d^2} + \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{2a^{2/3}d}$$

$$= -\frac{\sqrt{3}b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}d} + \frac{\sqrt{3}\sqrt[3]{b}ep\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d^2} + \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{2a^{2/3}d}$$

Mathematica [C] time = 0.39, size = 542, normalized size = 0.80

$$-\frac{3b^{2/3}d^2p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}} + \frac{6b^{2/3}d^2p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} - \frac{6\sqrt{3}b^{2/3}d^2p\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{6d^2\log\left(c\left(a+bx^3\right)^p\right)}{x^2} - 12e^2\log(d+$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(x^3*(d + e*x)),x]
```

```
[Out] ((-6*Sqrt[3]*b^(2/3)*d^2*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(
2/3) - (18*b*d*e*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/a + (6
*b^(2/3)*d^2*p*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + 12*e^2*p*Log[(e*((-1)^(1
/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x]
+ 12*e^2*p*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a^(1/3)*e)]*Log[d
+ e*x] + 12*e^2*p*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) +
(-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] - (3*b^(2/3)*d^2*p*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3) - (6*d^2*Log[c*(a + b*x^3)^p])/x^2 + (1
2*d*e*Log[c*(a + b*x^3)^p])/x + 4*e^2*Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p
] - 12*e^2*Log[d + e*x]*Log[c*(a + b*x^3)^p] + 12*e^2*p*PolyLog[2, (b^(1/3)
*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)] + 12*e^2*p*PolyLog[2, (b^(1/3)*(d + e*
x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)] + 12*e^2*p*PolyLog[2, (b^(1/3)*(d +
```

```
e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)] + 4*e^2*p*PolyLog[2, 1 + (b*x^3)
/a])/(12*d^3)
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^3+a\right)^p c\right)}{ex^4+dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^3 + a)^p*c)/(e*x^4 + d*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x^3), x)
```

maple [C] time = 0.50, size = 1025, normalized size = 1.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^3/(e*x+d),x)
```

```
[Out] -1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3*e^2/d^3*ln(x)-1/2*I*Pi*csgn(I*c*(b*x^3+a)
^p)^3*e/d^2/x-ln((b*x^3+a)^p)*e^2/d^3*ln(e*x+d)+ln((b*x^3+a)^p)*e^2/d^3*ln(
x)+ln((b*x^3+a)^p)*e/d^2/x-1/4*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)/d/x^2
-1/4*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/d/x^2+1/2*I*Pi*csgn(I
*c*(b*x^3+a)^p)^3*e^2/d^3*ln(e*x+d)+1/2*p/d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1
/4*p/d/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-p*e^2/d^3*sum(ln(x)*ln
((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3*b+a))+p*e^2/d^3*sum(ln((-e
*x+_R1-d)/_R1)*ln(e*x+d)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b
*d+3*_Z*b*d^2+a*e^3-b*d^3))-1/2/d/x^2*ln(c)-1/2*ln((b*x^3+a)^p)/d/x^2+1/d^3
*e^2*ln(c)*ln(x)+1/d^2*e/x*ln(c)-1/d^3*e^2*ln(c)*ln(e*x+d)-p/d^2*e*3^(1/2)/
(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/2*I*Pi*csgn(I*(b*x^3+
a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*e^2/d^3*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*
x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*e^2/d^3*ln(x)-1/2*I*Pi*csgn(I*(b*
x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*e/d^2/x-1/2*I*Pi*csgn(I*(b*x^3+a)
^p)*csgn(I*c*(b*x^3+a)^p)^2*e^2/d^3*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p
)^2*csgn(I*c)*e/d^2/x+1/4*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/d/x^2+1/2*I*Pi*csgn(
I*c*(b*x^3+a)^p)^2*csgn(I*c)*e^2/d^3*ln(x)-1/2*p/d^2*e/(a/b)^(1/3)*ln(x^2-(
a/b)^(1/3)*x+(a/b)^(2/3))+1/2*p/d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))+p/d^2*e/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/2*I*Pi*csgn(I*c*
(b*x^3+a)^p)^2*csgn(I*c)*e^2/d^3*ln(e*x+d)+1/4*I*Pi*csgn(I*(b*x^3+a)^p)*csg
n(I*c*(b*x^3+a)^p)*csgn(I*c)/d/x^2+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b
*x^3+a)^p)^2*e^2/d^3*ln(x)+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^
p)^2*e/d^2/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(b\,x^3+a\right)^p\right)}{x^3\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/(x^3*(d + e*x)),x)
[Out] int(log(c*(a + b*x^3)^p)/(x^3*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**3/(e*x+d),x)
[Out] Timed out
```

3.240 $\int \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=297

$$\frac{b^3 p \log(ax+b)}{3a^3 e}+\frac{b^2 dp \log(ax+b)}{2a^2 e^2}-\frac{b^2 px}{3a^2 e}-\frac{d^3 \log(d+ex) \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^4}+\frac{d^2 x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3}-\frac{dx^2 \log\left(c\left(a+\right.\right.}{2e^2}$$

```
[Out] -1/2*b*d*p*x/a/e^2-1/3*b^2*p*x/a^2/e+1/6*b*p*x^2/a/e+d^2*x*ln(c*(a+b/x)^p)/
e^3-1/2*d*x^2*ln(c*(a+b/x)^p)/e^2+1/3*x^3*ln(c*(a+b/x)^p)/e+b*d^2*p*ln(a*x+
b)/a/e^3+1/2*b^2*d*p*ln(a*x+b)/a^2/e^2+1/3*b^3*p*ln(a*x+b)/a^3/e-d^3*ln(c*(
a+b/x)^p)*ln(e*x+d)/e^4-d^3*p*ln(-e*x/d)*ln(e*x+d)/e^4+d^3*p*ln(-e*(a*x+b)/
(a*d-b*e))*ln(e*x+d)/e^4+d^3*p*polylog(2,a*(e*x+d)/(a*d-b*e))/e^4-d^3*p*pol
ylog(2,1+e*x/d)/e^4
```

Rubi [A] time = 0.32, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {2466, 2448, 263, 31, 2455, 193, 43, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$\frac{d^3 p \text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^4}-\frac{d^3 p \text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^4}+\frac{b^2 dp \log(ax+b)}{2a^2 e^2}-\frac{b^2 px}{3a^2 e}+\frac{b^3 p \log(ax+b)}{3a^3 e}+\frac{d^2 x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] -(b*d*p*x)/(2*a*e^2) - (b^2*p*x)/(3*a^2*e) + (b*p*x^2)/(6*a*e) + (d^2*x*Log
[c*(a + b/x)^p])/e^3 - (d*x^2*Log[c*(a + b/x)^p])/(2*e^2) + (x^3*Log[c*(a +
 b/x)^p])/(3*e) + (b*d^2*p*Log[b + a*x])/(a*e^3) + (b^2*d*p*Log[b + a*x])/(
2*a^2*e^2) + (b^3*p*Log[b + a*x])/(3*a^3*e) - (d^3*Log[c*(a + b/x)^p]*Log[d
 + e*x])/e^4 - (d^3*p*Log[-((e*x)/d)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(
b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^4 + (d^3*p*PolyLog[2, (a*(d + e*x))
/(a*d - b*e)])/e^4 - (d^3*p*PolyLog[2, 1 + (e*x)/d])/e^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
```

```
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \left(\frac{d^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} - \frac{d^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3(d+ex)}\right.$$

$$= \frac{d^2 \int \log\left(c\left(a+\frac{b}{x}\right)^p\right)dx}{e^3} - \frac{d^3 \int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}dx}{e^3} - \frac{d \int x \log\left(c\left(a+\frac{b}{x}\right)^p\right)dx}{e^2} + \frac{\int x^2 \log\ldots}{}$$

$$= \frac{d^2 x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} - \frac{d^3 \log\left(c\left(a+\frac{b}{x}\right)\ldots\right.}{e^4}$$

$$= \frac{d^2 x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} - \frac{d^3 \log\left(c\left(a+\frac{b}{x}\right)\ldots\right.}{e^4}$$

$$= \frac{d^2 x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{bd^2 p \log(b+ax\ldots}{ae^3}$$

$$= -\frac{bdpx}{2ae^2} - \frac{b^2px}{3a^2e} + \frac{bpx^2}{6ae} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a\ldots\right.\right.}{3e}$$

$$= -\frac{bdpx}{2ae^2} - \frac{b^2px}{3a^2e} + \frac{bpx^2}{6ae} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a\ldots\right.\right.}{3e}$$

$$= -\frac{bdpx}{2ae^2} - \frac{b^2px}{3a^2e} + \frac{bpx^2}{6ae} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a\ldots\right.\right.}{3e}$$

Mathematica [A] time = 0.23, size = 251, normalized size = 0.85

$$\frac{\frac{be^3p\left(2b^2\log\left(a+\frac{b}{x}\right)+ax(ax-2b)+2b^2\log(x)\right)}{a^3} + \frac{3bde^2p(b\log(ax+b)-ax)}{a^2} - 6d^3\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right) + 6d^2ex\log\left(c\left(a+\frac{b}{x}\right)^p\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] (6*d^2*e*x*Log[c*(a + b/x)^p] - 3*d*e^2*x^2*Log[c*(a + b/x)^p] + 2*e^3*x^3*
Log[c*(a + b/x)^p] + (6*b*d^2*e*p*(Log[a + b/x] + Log[x]))/a + (b*e^3*p*(a*
x*(-2*b + a*x) + 2*b^2*Log[a + b/x] + 2*b^2*Log[x]))/a^3 + (3*b*d*e^2*p*(-(
a*x) + b*Log[b + a*x]))/a^2 - 6*d^3*Log[c*(a + b/x)^p]*Log[d + e*x] - 6*d^3
*p*((Log[-((e*x)/d)] - Log[(e*(b + a*x))/(-(a*d) + b*e)])*Log[d + e*x] - Po
lyLog[2, (a*(d + e*x))/(a*d - b*e)] + PolyLog[2, 1 + (e*x)/d]))/(6*e^4)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{x^3 \log\left(c\left(\frac{ax+b}{x}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^3*log(c*((a*x + b)/x)^p)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \log\left(\left(a + \frac{b}{x}\right)^p c\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^3*log((a + b/x)^p*c)/(e*x + d), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^3 \ln\left(c\left(a + \frac{b}{x}\right)^p\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(a+b/x)^p)/(e*x+d),x)
```

```
[Out] int(x^3*ln(c*(a+b/x)^p)/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \log\left(\left(a + \frac{b}{x}\right)^p c\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^3*log((a + b/x)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \ln\left(c\left(a + \frac{b}{x}\right)^p\right)}{d + e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(c*(a + b/x)^p))/(d + e*x),x)
```

```
[Out] int((x^3*log(c*(a + b/x)^p))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \log\left(c\left(a + \frac{b}{x}\right)^p\right)}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(a+b/x)**p)/(e*x+d),x)
```

```
[Out] Integral(x**3*log(c*(a + b/x)**p)/(d + e*x), x)
```

3.241 $\int \frac{x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex} dx$

Optimal. Leaf size=219

$$-\frac{b^2 p \log(ax+b)}{2a^2 e}+\frac{d^2 \log(d+ex) \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3}-\frac{dx \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}-\frac{d^2 p \text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)}{e^3}-\frac{d^2}{}$$

```
[Out] 1/2*b*p*x/a/e-d*x*ln(c*(a+b/x)^p)/e^2+1/2*x^2*ln(c*(a+b/x)^p)/e-b*d*p*ln(a*
x+b)/a/e^2-1/2*b^2*p*ln(a*x+b)/a^2/e+d^2*ln(c*(a+b/x)^p)*ln(e*x+d)/e^3+d^2*
p*ln(-e*x/d)*ln(e*x+d)/e^3-d^2*p*ln(-e*(a*x+b)/(a*d-b*e))*ln(e*x+d)/e^3-d^2
*p*polylog(2,a*(e*x+d)/(a*d-b*e))/e^3+d^2*p*polylog(2,1+e*x/d)/e^3
```

Rubi [A] time = 0.27, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {2466, 2448, 263, 31, 2455, 193, 43, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$-\frac{d^2 p \text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^3}+\frac{d^2 p \text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^3}-\frac{b^2 p \log(ax+b)}{2a^2 e}+\frac{d^2 \log(d+ex) \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3}-\frac{dx \log\left(c\left(\right.\right.}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] (b*p*x)/(2*a*e) - (d*x*Log[c*(a + b/x)^p])/e^2 + (x^2*Log[c*(a + b/x)^p])/(
2*e) - (b*d*p*Log[b + a*x])/(a*e^2) - (b^2*p*Log[b + a*x])/(2*a^2*e) + (d^2
*Log[c*(a + b/x)^p]*Log[d + e*x])/e^3 + (d^2*p*Log[-((e*x)/d)]*Log[d + e*x]
)/e^3 - (d^2*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^3 - (d^2*p
*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/e^3 + (d^2*p*PolyLog[2, 1 + (e*x)/d
])/e^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \left(-\frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} + \frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2(d+ex)}\right)dx$$

$$= -\frac{d\int \log\left(c\left(a+\frac{b}{x}\right)^p\right)dx}{e^2} + \frac{d^2\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}dx}{e^2} + \frac{\int x\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx}{e}$$

$$= -\frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^3} + \frac{(bd^2p}{}$$

$$= -\frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^3} + \frac{(bd^2p}{}$$

$$= -\frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} - \frac{bdp\log(b+ax)}{ae^2} + \frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log}{e^3}$$

$$= \frac{bpx}{2ae} - \frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} - \frac{bdp\log(b+ax)}{ae^2} - \frac{b^2p\log(b+ax)}{2a^2e}$$

$$= \frac{bpx}{2ae} - \frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} - \frac{bdp\log(b+ax)}{ae^2} - \frac{b^2p\log(b+ax)}{2a^2e}$$

$$= \frac{bpx}{2ae} - \frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} - \frac{bdp\log(b+ax)}{ae^2} - \frac{b^2p\log(b+ax)}{2a^2e}$$

Mathematica [A] time = 0.13, size = 183, normalized size = 0.84

$$\frac{\frac{be^2p(ax-b\log(ax+b))}{a^2} + 2d^2\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right) - 2dex\log\left(c\left(a+\frac{b}{x}\right)^p\right) + e^2x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right) + 2d^2p\left(-\text{Li}_2\right.}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] (-2*d*e*x*Log[c*(a + b/x)^p] + e^2*x^2*Log[c*(a + b/x)^p] - (2*b*d*e*p*(Log
[a + b/x] + Log[x]))/a + (b*e^2*p*(a*x - b*Log[b + a*x]))/a^2 + 2*d^2*Log[c
*(a + b/x)^p]*Log[d + e*x] + 2*d^2*p*((Log[-((e*x)/d)] - Log[(e*(b + a*x))/
(-(a*d) + b*e)])*Log[d + e*x] - PolyLog[2, (a*(d + e*x))/(a*d - b*e)] + Pol
yLog[2, 1 + (e*x)/d]))/(2*e^3)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2\log\left(c\left(\frac{ax+b}{x}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^2*log(c*((a*x + b)/x)^p)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log\left(\left(a + \frac{b}{x}\right)^p c\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^2*log((a + b/x)^p*c)/(e*x + d), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{x^2 \ln\left(c\left(a + \frac{b}{x}\right)^p\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(a+b/x)^p)/(e*x+d),x)
[Out] int(x^2*ln(c*(a+b/x)^p)/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log\left(\left(a + \frac{b}{x}\right)^p c\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^2*log((a + b/x)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\, \ln\left(c\left(a + \frac{b}{x}\right)^p\right)}{d + e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(c*(a + b/x)^p))/(d + e*x),x)
[Out] int((x^2*log(c*(a + b/x)^p))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log\left(c\left(a + \frac{b}{x}\right)^p\right)}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(a+b/x)**p)/(e*x+d),x)
[Out] Integral(x**2*log(c*(a + b/x)**p)/(d + e*x), x)
```

3.242 $\int \frac{x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=151

$$-\frac{d\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}+\frac{dp\text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)}{e^2}+\frac{dp\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e^2}+\frac{bp\log(ax+}{ae}$$

```
[Out] x*ln(c*(a+b/x)^p)/e+b*p*ln(a*x+b)/a/e-d*ln(c*(a+b/x)^p)*ln(e*x+d)/e^2-d*p*l
n(-e*x/d)*ln(e*x+d)/e^2+d*p*ln(-e*(a*x+b)/(a*d-b*e))*ln(e*x+d)/e^2+d*p*poly
log(2,a*(e*x+d)/(a*d-b*e))/e^2-d*p*polylog(2,1+e*x/d)/e^2
```

Rubi [A] time = 0.21, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {2466, 2448, 263, 31, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$\frac{dp\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^2}-\frac{dp\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^2}-\frac{d\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}+\frac{dp\log(d+e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] (x*Log[c*(a + b/x)^p])/e + (b*p*Log[b + a*x])/(a*e) - (d*Log[c*(a + b/x)^p]
*Log[d + e*x])/e^2 - (d*p*Log[-((e*x)/d)]*Log[d + e*x])/e^2 + (d*p*Log[-((e
*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^2 + (d*p*PolyLog[2, (a*(d + e*x))
/(a*d - b*e)])/e^2 - (d*p*PolyLog[2, 1 + (e*x)/d])/e^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)}\right)dx$$

$$= \frac{\int \log\left(c\left(a+\frac{b}{x}\right)^p\right)dx}{e} - \frac{d\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}dx}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{(bdp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}dx}{e^2} + \frac{(bp)\int \frac{1}{\left(a+\right.}}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{(bdp)\int\left(\frac{\log(d+ex)}{bx} - \frac{a\log(d+ex)}{b(b+ax)}\right)d}{e^2}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} + \frac{bp\log(b+ax)}{ae} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{(dp)\int \frac{\log(d+ex)}{x}}{e^2}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} + \frac{bp\log(b+ax)}{ae} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{dp\log\left(-\frac{ex}{d}\right)\text{lo}}{e^2}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} + \frac{bp\log(b+ax)}{ae} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{dp\log\left(-\frac{ex}{d}\right)\text{lo}}{e^2}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} + \frac{bp\log(b+ax)}{ae} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{dp\log\left(-\frac{ex}{d}\right)\text{lo}}{e^2}$$

Mathematica [A] time = 0.06, size = 149, normalized size = 0.99

$$-\frac{d\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} - \frac{dp\left(-\text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right) - \log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right) + \text{Li}_2\left(\frac{d+ex}{d}\right)\right.}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] (x*Log[c*(a + b/x)^p])/e + (b*p*(Log[a + b/x]/a + Log[x]/a))/e - (d*Log[c*(
a + b/x)^p]*Log[d + e*x])/e^2 - (d*p*(Log[-((e*x)/d)]*Log[d + e*x] - Log[-(
(e*(b + a*x))/(a*d - b*e))]*Log[d + e*x] + PolyLog[2, (d + e*x)/d] - PolyLo
g[2, (a*(d + e*x))/(a*d - b*e)]))/e^2
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x\log\left(c\left(\frac{ax+b}{x}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x*log(c*((a*x + b)/x)^p)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \log\left(\left(a + \frac{b}{x}\right)^{p} c\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x*log((a + b/x)^p*c)/(e*x + d), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x \ln\left(c \left(a + \frac{b}{x}\right)^{p}\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(a+b/x)^p)/(e*x+d),x)
[Out] int(x*ln(c*(a+b/x)^p)/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \log\left(\left(a + \frac{b}{x}\right)^{p} c\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x*log((a + b/x)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \ln\left(c \left(a + \frac{b}{x}\right)^{p}\right)}{d + e\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(c*(a + b/x)^p))/(d + e*x),x)
[Out] int((x*log(c*(a + b/x)^p))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \log\left(c \left(a + \frac{b}{x}\right)^{p}\right)}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(a+b/x)**p)/(e*x+d),x)
[Out] Integral(x*log(c*(a + b/x)**p)/(d + e*x), x)
```

3.243 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=113

$$\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}-\frac{p\text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e}+\frac{p\text{Li}_2\left(\frac{ex}{d}+1\right)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}$$

```
[Out] ln(c*(a+b/x)^p)*ln(e*x+d)/e+p*ln(-e*x/d)*ln(e*x+d)/e-p*ln(-e*(a*x+b)/(a*d-b
*e))*ln(e*x+d)/e-p*polylog(2,a*(e*x+d)/(a*d-b*e))/e+p*polylog(2,1+e*x/d)/e
```

Rubi [A] time = 0.15, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2462, 260, 2416, 2394, 2315, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e}+\frac{p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e}+\frac{p\log$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e -
(p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e - (p*PolyLog[2, (a*(d
+ e*x))/(a*d - b*e)])/e + (p*PolyLog[2, 1 + (e*x)/d])/e
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx &= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{(bp)\int\frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{e}\\
&= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{(bp)\int\left(\frac{\log(d+ex)}{bx}-\frac{a\log(d+ex)}{b(b+ax)}\right)dx}{e}\\
&= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{p\int\frac{\log(d+ex)}{x}\,dx}{e}-\frac{(ap)\int\frac{\log(d+ex)}{b+ax}\,dx}{e}\\
&= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e}\\
&= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e}\\
&= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 114, normalized size = 1.01

$$
\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}-\frac{p\text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e}+\frac{p\text{Li}_2\left(\frac{d+ex}{d}\right)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+e}{e}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e -
(p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e + (p*PolyLog[2, (d + e
*x)/d])/e - (p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/e
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\log\left(c\left(\frac{ax+b}{x}\right)^p\right)}{ex+d}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(log(c*((a*x + b)/x)^p)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a + \frac{b}{x}\right)^{p} c\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(log((a + b/x)^p*c)/(e*x + d), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a + \frac{b}{x}\right)^{p}\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)*ln(c*(a+b/x)^p),x)
[Out] int(1/(e*x+d)*ln(c*(a+b/x)^p),x)
```

maxima [A] time = 0.74, size = 159, normalized size = 1.41

$$\frac{bp\left(\frac{\log(ex+d)\log\left(a+\frac{b}{x}\right)}{b} - \frac{\log(ex+d)\log\left(-\frac{aex+ad}{ad-be}+1\right)+\mathrm{Li}_2\left(\frac{aex+ad}{ad-be}\right)}{b} + \frac{\log(ex+d)\log\left(-\frac{ex+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{ex+d}{d}\right)}{b}\right)}{e} - \frac{p\log(ex+d)\log\left(a+\frac{b}{x}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
[Out] b*p*(log(e*x + d)*log(a + b/x)/b - (log(e*x + d)*log(-(a*e*x + a*d)/(a*d -
b*e) + 1) + dilog((a*e*x + a*d)/(a*d - b*e)))/b + (log(e*x + d)*log(-(e*x +
 d)/d + 1) + dilog((e*x + d)/d))/b)/e - p*log(e*x + d)*log(a + b/x)/e + log
((a + b/x)^p*c)*log(e*x + d)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(a + \frac{b}{x}\right)^{p}\right)}{d + e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/(d + e*x),x)
[Out] int(log(c*(a + b/x)^p)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a + \frac{b}{x}\right)^{p}\right)}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d),x)
[Out] Integral(log(c*(a + b/x)**p)/(d + e*x), x)
```

3.244 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)}\,dx$

Optimal. Leaf size=159

$$-\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d}-\frac{\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d}+\frac{p\text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)}{d}+\frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{d}-\frac{p\text{Li}_2}{}$$

```
[Out] -ln(c*(a+b/x)^p)*ln(-b/a/x)/d-ln(c*(a+b/x)^p)*ln(e*x+d)/d-p*ln(-e*x/d)*ln(e
*x+d)/d+p*ln(-e*(a*x+b)/(a*d-b*e))*ln(e*x+d)/d-p*polylog(2,1+b/a/x)/d+p*pol
ylog(2,a*(e*x+d)/(a*d-b*e))/d-p*polylog(2,1+e*x/d)/d
```

Rubi [A] time = 0.25, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2466, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d}-\frac{p\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{d}-\frac{p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d}-\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d}-\frac{\log\left(-\frac{b}{ax}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(x*(d + e*x)),x]
```

```
[Out] -((Log[c*(a + b/x)^p]*Log[-(b/(a*x))])/d) - (Log[c*(a + b/x)^p]*Log[d + e*x
])/d - (p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b + a*x))/(a*d - b
*e))]*Log[d + e*x])/d - (p*PolyLog[2, 1 + b/(a*x)])/d + (p*PolyLog[2, (a*(d
 + e*x))/(a*d - b*e)])/d - (p*PolyLog[2, 1 + (e*x)/d])/d
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{dx} - \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, \frac{1}{x}\right)}{d} - \frac{(bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{(bp)\int\left(\frac{\log(d+ex)}{bx} - \frac{a\log}{b(b}\right.}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\text{Li}_2\left(1+\frac{b}{ax}\right)}{d} - p\int$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex}{d}$$

Mathematica [A] time = 0.06, size = 139, normalized size = 0.87

$$-\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right) + \log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right) - p\text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right) - p\log(d+ex)\log\left(\frac{e(ax+b)}{be-ad}\right) + p\text{Li}_2}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(x*(d + e*x)),x]
```

```
[Out] -((Log[c*(a + b/x)^p]*Log[-(b/(a*x))] + Log[c*(a + b/x)^p]*Log[d + e*x] + p
*Log[-((e*x)/d)]*Log[d + e*x] - p*Log[(e*(b + a*x))/(-(a*d) + b*e)]*Log[d +
 e*x] + p*PolyLog[2, 1 + b/(a*x)] - p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)]
 + p*PolyLog[2, 1 + (e*x)/d])/d)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax+b}{x}\right)^p\right)}{ex^2+dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/(e*x^2 + d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x/(e*x+d),x, algorithm="giac")
[Out] integrate(log((a + b/x)^p*c)/((e*x + d)*x), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{(ex+d)\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x/(e*x+d),x)
[Out] int(ln(c*(a+b/x)^p)/x/(e*x+d),x)
```

maxima [A] time = 1.05, size = 179, normalized size = 1.13

$$-\frac{1}{2}bp\left(\frac{2\log(ex+d)\log(x)-\log(x)^{2}}{bd}+\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)}{bd}-\frac{2\left(\log\left(\frac{ex}{d}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ex}{d}\right)\right)}{bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x/(e*x+d),x, algorithm="maxima")
[Out] -1/2*b*p*((2*log(e*x + d)*log(x) - log(x)^2)/(b*d) + 2*(log(a*x/b + 1)*log(
x) + dilog(-a*x/b))/(b*d) - 2*(log(e*x/d + 1)*log(x) + dilog(-e*x/d))/(b*d)
 - 2*(log(e*x + d)*log(-(a*e*x + a*d)/(a*d - b*e) + 1) + dilog((a*e*x + a*d
)/(a*d - b*e)))/(b*d)) - (log(e*x + d)/d - log(x)/d)*log((a + b/x)^p*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/(x*(d + e*x)),x)
[Out] int(log(c*(a + b/x)^p)/(x*(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x/(e*x+d),x)
[Out] Integral(log(c*(a + b/x)**p)/(x*(d + e*x)), x)
```

3.245 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)} \, dx$

Optimal. Leaf size=198

$$\frac{e\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2}+\frac{e\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2}-\frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd}+\frac{ep\text{Li}_2\left(\frac{b}{ax}+1\right)}{d^2}-\frac{ep\text{Li}_2\left(\right.}{}$$

```
[Out] p/d/x-(a+b/x)*ln(c*(a+b/x)^p)/b/d+e*ln(c*(a+b/x)^p)*ln(-b/a/x)/d^2+e*ln(c*(
a+b/x)^p)*ln(e*x+d)/d^2+e*p*ln(-e*x/d)*ln(e*x+d)/d^2-e*p*ln(-e*(a*x+b)/(a*d
-b*e))*ln(e*x+d)/d^2+e*p*polylog(2,1+b/a/x)/d^2-e*p*polylog(2,a*(e*x+d)/(a*
d-b*e))/d^2+e*p*polylog(2,1+e*x/d)/d^2
```

Rubi [A] time = 0.28, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {2466, 2454, 2389, 2295, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{ep\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d^2}+\frac{ep\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^2}+\frac{e\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2}+\frac{e\log(d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(x^2*(d + e*x)),x]
```

```
[Out] p/(d*x) - ((a + b/x)*Log[c*(a + b/x)^p])/(b*d) + (e*Log[c*(a + b/x)^p]*Log[
-(b/(a*x))])/d^2 + (e*Log[c*(a + b/x)^p]*Log[d + e*x])/d^2 + (e*p*Log[-((e*
x)/d)]*Log[d + e*x])/d^2 - (e*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e
*x])/d^2 + (e*p*PolyLog[2, 1 + b/(a*x)])/d^2 - (e*p*PolyLog[2, (a*(d + e*x)
)/(a*d - b*e)])/d^2 + (e*p*PolyLog[2, 1 + (e*x)/d])/d^2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{dx^2} - \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2x} + \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}dx}{d^2}$$

$$= \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} - \frac{\text{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\,\text{Subst}\left(\int \frac{\log(}{}\right.}{}$$

$$= \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} - \frac{\text{Subst}\left(\int \log\left(cx^p\right)\,d\right.}{bd}$$

$$= \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log}{d^2}$$

$$= \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log}{d^2}$$

$$= \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log}{d^2}$$

$$= \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log}{d^2}$$

Mathematica [A] time = 0.10, size = 166, normalized size = 0.84

$$\frac{e\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right) - \frac{d\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b} + e\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right) + ep\left(-\text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right) + \log(d+\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(x^2*(d + e*x)),x]
```

```
[Out] ((d*p)/x - (d*(a + b/x)*Log[c*(a + b/x)^p])/b + e*Log[c*(a + b/x)^p]*Log[-(
b/(a*x))] + e*Log[c*(a + b/x)^p]*Log[d + e*x] + e*p*PolyLog[2, 1 + b/(a*x)]
 + e*p*((Log[-((e*x)/d)] - Log[(e*(b + a*x))/(-(a*d) + b*e)])*Log[d + e*x]
- PolyLog[2, (a*(d + e*x))/(a*d - b*e)] + PolyLog[2, 1 + (e*x)/d]))/d^2
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax+b}{x}\right)^p\right)}{ex^3+dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/(e*x^3 + d*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a+\frac{b}{x}\right)^{p} c\right)}{(ex+d)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x)^p*c)/((e*x + d)*x^2), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{(ex+d)\,x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x^2/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x)^p)/x^2/(e*x+d),x)
```

maxima [A] time = 0.88, size = 230, normalized size = 1.16

$$\frac{1}{2}bp\left(\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)e}{bd^2}-\frac{2\left(\log\left(\frac{ex}{d}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ex}{d}\right)\right)e}{bd^2}-\frac{2\left(\log(ex+d)\log\left(-\frac{aex+ad}{ad-be}+\right.\right.}{bd^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*e/(b*d^2) - 2*(log(e*x/d
 + 1)*log(x) + dilog(-e*x/d))*e/(b*d^2) - 2*(log(e*x + d)*log(-(a*e*x + a*d
)/(a*d - b*e) + 1) + dilog((a*e*x + a*d)/(a*d - b*e)))*e/(b*d^2) - 2*a*log(
a*x + b)/(b^2*d) + 2*a*log(x)/(b^2*d) + (2*e*log(e*x + d)*log(x) - e*log(x)
^2)/(b*d^2) + 2/(b*d*x)) + (e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x))*lo
g((a + b/x)^p*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x^{2}\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/(x^2*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b/x)^p)/(x^2*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

3.246 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=287

$$\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d}-\frac{e^2\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^3}-\frac{e^2\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^3}+\frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2}-\cdots$$

```
[Out] 1/4*p/d/x^2-1/2*a*p/b/d/x-e*p/d^2/x+1/2*a^2*p*ln(a+b/x)/b^2/d+e*(a+b/x)*ln(
c*(a+b/x)^p)/b/d^2-1/2*ln(c*(a+b/x)^p)/d/x^2-e^2*ln(c*(a+b/x)^p)*ln(-b/a/x)
/d^3-e^2*ln(c*(a+b/x)^p)*ln(e*x+d)/d^3-e^2*p*ln(-e*x/d)*ln(e*x+d)/d^3+e^2*p
*ln(-e*(a*x+b)/(a*d-b*e))*ln(e*x+d)/d^3-e^2*p*polylog(2,1+b/a/x)/d^3+e^2*p*
polylog(2,a*(e*x+d)/(a*d-b*e))/d^3-e^2*p*polylog(2,1+e*x/d)/d^3
```

Rubi [A] time = 0.33, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2466, 2454, 2395, 43, 2389, 2295, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$-\frac{e^2p\,\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{d^3}+\frac{e^2p\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d^3}-\frac{e^2p\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^3}+\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d}-\frac{e^2\log\left(-\frac{b}{ax}\right)\log\cdots}{d^3}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(x^3*(d + e*x)),x]
```

```
[Out] p/(4*d*x^2) - (a*p)/(2*b*d*x) - (e*p)/(d^2*x) + (a^2*p*Log[a + b/x])/(2*b^2
*d) + (e*(a + b/x)*Log[c*(a + b/x)^p])/(b*d^2) - Log[c*(a + b/x)^p]/(2*d*x^
2) - (e^2*Log[c*(a + b/x)^p]*Log[-(b/(a*x))])/d^3 - (e^2*Log[c*(a + b/x)^p]
*Log[d + e*x])/d^3 - (e^2*p*Log[-((e*x)/d)]*Log[d + e*x])/d^3 + (e^2*p*Log[
-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/d^3 - (e^2*p*PolyLog[2, 1 + b/(
a*x)])/d^3 + (e^2*p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/d^3 - (e^2*p*Pol
yLog[2, 1 + (e*x)/d])/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
```

```
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{dx^3} - \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2x^2} + \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^3x} - \frac{e^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^3(d+ex)}\right.$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx}{d^3} - \frac{e^3\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx}{d^3}$$

$$= -\frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^3} - \frac{\text{Subst}\left(\int x\log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2} - \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^3} - \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^3} -$$

$$= -\frac{ep}{d^2x} + \frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2} - \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^3}$$

$$= \frac{p}{4dx^2} - \frac{ap}{2bdx} - \frac{ep}{d^2x} + \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d} + \frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2}$$

$$= \frac{p}{4dx^2} - \frac{ap}{2bdx} - \frac{ep}{d^2x} + \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d} + \frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2}$$

$$= \frac{p}{4dx^2} - \frac{ap}{2bdx} - \frac{ep}{d^2x} + \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d} + \frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2}$$

Mathematica [A] time = 0.24, size = 241, normalized size = 0.84

$$-\frac{-\frac{d^2p\left(2a^2x^2\log\left(a+\frac{b}{x}\right)+b(b-2ax)\right)}{b^2x^2} + \frac{2d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2} + 4e^2\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right) - \frac{4de\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b} + 4e^2\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(x^3*(d + e*x)),x]
```

```
[Out] -1/4*((4*d*e*p)/x - (d^2*p*(b*(b - 2*a*x) + 2*a^2*x^2*Log[a + b/x]))/(b^2*x
^2) - (4*d*e*(a + b/x)*Log[c*(a + b/x)^p])/b + (2*d^2*Log[c*(a + b/x)^p])/x
^2 + 4*e^2*Log[c*(a + b/x)^p]*Log[-(b/(a*x))] + 4*e^2*Log[c*(a + b/x)^p]*Lo
g[d + e*x] + 4*e^2*p*PolyLog[2, 1 + b/(a*x)] + 4*e^2*p*((Log[-((e*x)/d)] -
Log[(e*(b + a*x))/(-(a*d) + b*e)])*Log[d + e*x] - PolyLog[2, (a*(d + e*x))/
(a*d - b*e)] + PolyLog[2, 1 + (e*x)/d]))/d^3
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax+b}{x}\right)^p\right)}{ex^4+dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/(e*x^4 + d*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a+\frac{b}{x}\right)^{p} c\right)}{(ex+d)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x)^p*c)/((e*x + d)*x^3), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{(ex+d)\,x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x^3/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x)^p)/x^3/(e*x+d),x)
```

maxima [A] time = 1.01, size = 307, normalized size = 1.07

$$\frac{1}{4}\left(4\,e\left(\frac{a\log\left(ax+b\right)}{b^{2}d^{2}}-\frac{a\log(x)}{b^{2}d^{2}}-\frac{1}{bd^{2}x}\right)-\frac{4\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)e^{2}}{bd^{3}}+\frac{4\left(\log\left(\frac{ex}{d}+1\right)\log(x)+\mathrm{Li}_2\left(\right.\right.}{bd^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*(4*e*(a*log(a*x + b)/(b^2*d^2) - a*log(x)/(b^2*d^2) - 1/(b*d^2*x)) - 4*
(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*e^2/(b*d^3) + 4*(log(e*x/d + 1)*log
(x) + dilog(-e*x/d))*e^2/(b*d^3) + 4*(log(e*x + d)*log(-(a*e*x + a*d)/(a*d
- b*e) + 1) + dilog((a*e*x + a*d)/(a*d - b*e)))*e^2/(b*d^3) + 2*a^2*log(a*x
 + b)/(b^3*d) - 2*a^2*log(x)/(b^3*d) - 2*(2*e^2*log(e*x + d)*log(x) - e^2*l
og(x)^2)/(b*d^3) - (2*a*x - b)/(b^2*d*x^2))*b*p - 1/2*(2*e^2*log(e*x + d)/d
^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2))*log((a + b/x)^p*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x^{3}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/(x^3*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b/x)^p)/(x^3*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

3.247 $\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} dx$

Optimal. Leaf size=421

$$-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}e}-\frac{d^3\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^4}+\frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3}-\frac{dx^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2}+\frac{x^3\log\left(c\left(a\right.\right.}{3e}$$

```
[Out] 2/3*b*p*x/a/e-2/3*b^(3/2)*p*arctan(x*a^(1/2)/b^(1/2))/a^(3/2)/e+d^2*x*ln(c*
(a+b/x^2)^p)/e^3-1/2*d*x^2*ln(c*(a+b/x^2)^p)/e^2+1/3*x^3*ln(c*(a+b/x^2)^p)/
e-d^3*ln(c*(a+b/x^2)^p)*ln(e*x+d)/e^4-2*d^3*p*ln(-e*x/d)*ln(e*x+d)/e^4-1/2*
b*d*p*ln(a*x^2+b)/a/e^2+d^3*p*ln(e*x+d)*ln(-e*(x*(-a)^(1/2)+b^(1/2))/(d*(-a
)^(1/2)-e*b^(1/2)))/e^4+d^3*p*ln(e*x+d)*ln(e*(-x*(-a)^(1/2)+b^(1/2))/(d*(-a
)^(1/2)+e*b^(1/2)))/e^4-2*d^3*p*polylog(2,1+e*x/d)/e^4+d^3*p*polylog(2,(e*x
+d)*(-a)^(1/2)/(d*(-a)^(1/2)-e*b^(1/2)))/e^4+d^3*p*polylog(2,(e*x+d)*(-a)^(
1/2)/(d*(-a)^(1/2)+e*b^(1/2)))/e^4+2*d^2*p*arctan(x*a^(1/2)/b^(1/2))*b^(1/2
)/e^3/a^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {2466, 2448, 263, 205, 2455, 260, 193, 321, 2462, 2416, 2394, 2315, 2393, 2391}

$$\frac{d^3p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^4}+\frac{d^3p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^4}-\frac{2d^3p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^4}-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}e}-\frac{d^3\log(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] (2*b*p*x)/(3*a*e) + (2*Sqrt[b]*d^2*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[a]*
e^3) - (2*b^(3/2)*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(3*a^(3/2)*e) + (d^2*x*Log
[c*(a + b/x^2)^p])/e^3 - (d*x^2*Log[c*(a + b/x^2)^p])/(2*e^2) + (x^3*Log[c*
(a + b/x^2)^p])/(3*e) - (d^3*Log[c*(a + b/x^2)^p]*Log[d + e*x])/e^4 - (2*d^
3*p*Log[-((e*x)/d)]*Log[d + e*x])/e^4 + (d^3*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x
))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(Sqrt[b] +
 Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d + e*x])/e^4 - (b*d*p*Log[b +
 a*x^2])/(2*a*e^2) + (d^3*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - S
qrt[b]*e)])/e^4 + (d^3*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt
[b]*e)])/e^4 - (2*d^3*p*PolyLog[2, 1 + (e*x)/d])/e^4
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
```

```
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \left(\frac{d^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d^3 \log\left(c\left(a\right.\right.}{e^3(d+}\right.$$

$$= \frac{d^2 \int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx}{e^3} - \frac{d^3 \int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}dx}{e^3} - \frac{d \int x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx}{e^2} + \frac{\int x^2}{}$$

$$= \frac{d^2 x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e} - \frac{d^3 \log\left(c\left(a+\right.\right.}{}$$

$$= \frac{d^2 x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e} - \frac{d^3 \log\left(c\left(a+\right.\right.}{}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}\,d^2 p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e^3} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} + \frac{x^3 \mathrm{l}}{}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}\,d^2 p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e^3} - \frac{2b^{3/2} p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3a^{3/2} e} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2 \mathrm{lo}}{}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}\,d^2 p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e^3} - \frac{2b^{3/2} p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3a^{3/2} e} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2 \mathrm{lo}}{}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}\,d^2 p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e^3} - \frac{2b^{3/2} p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3a^{3/2} e} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2 \mathrm{lo}}{}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}\,d^2 p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e^3} - \frac{2b^{3/2} p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3a^{3/2} e} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2 \mathrm{lo}}{}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}\,d^2 p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e^3} - \frac{2b^{3/2} p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3a^{3/2} e} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2 \mathrm{lo}}{}$$

Mathematica [C] time = 0.44, size = 375, normalized size = 0.89

$$-\frac{6d^3 \log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right) - 6d^2 ex \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + 3de^2x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) - 2e^3x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] -1/6*((12*Sqrt[b]*d^2*e*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[a] - (4*b*e^3*p
*x*Hypergeometric2F1[-1/2, 1, 1/2, -(b/(a*x^2))])/a - 6*d^2*e*x*Log[c*(a +
b/x^2)^p] + 3*d*e^2*x^2*Log[c*(a + b/x^2)^p] - 2*e^3*x^3*Log[c*(a + b/x^2)^
p] + (3*b*d*e^2*p*(Log[a + b/x^2] + 2*Log[x]))/a + 6*d^3*Log[c*(a + b/x^2)^
p]*Log[d + e*x] + 6*d^3*p*(2*Log[-((e*x)/d)]*Log[d + e*x] - Log[(e*(Sqrt[b]
 - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x] - Log[(e*(Sqrt[b] +
Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Log[d + e*x] - PolyLog[2, (Sqrt[-
a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)] - PolyLog[2, (Sqrt[-a]*(d + e*x))/(
Sqrt[-a]*d + Sqrt[b]*e)] + 2*PolyLog[2, 1 + (e*x)/d]))/e^4
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3\log\left(c\left(\frac{ax^2+b}{x^2}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^3*log(c*((a*x^2 + b)/x^2)^p)/(e*x + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming a near 0Simplificati
on assuming d near 0Evaluation time: 0.75Not invertible Error: Bad Argument
 Value
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x^3\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

```
[Out] int(x^3*ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^3*log((a + b/x^2)^p*c)/(e*x + d), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\ln\left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(c*(a + b/x^2)^p))/(d + e*x),x)
[Out] int((x^3*log(c*(a + b/x^2)^p))/(d + e*x), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \log \left(c \left(a + \frac{b}{x^{2}}\right)^{p}\right)}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(a+b/x**2)**p)/(e*x+d),x)
[Out] Integral(x**3*log(c*(a + b/x**2)**p)/(d + e*x), x)
```

3.248 $\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} dx$

Optimal. Leaf size=353

$$\frac{d^2 \log(d+ex) \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} - \frac{d^2 p \text{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^3} - \frac{d^2 p \text{Li}_2\left(\frac{\sqrt{-}}{\sqrt{-}}\right.}{e^3}$$

```
[Out] -d*x*ln(c*(a+b/x^2)^p)/e^2+1/2*x^2*ln(c*(a+b/x^2)^p)/e+d^2*ln(c*(a+b/x^2)^p
)*ln(e*x+d)/e^3+2*d^2*p*ln(-e*x/d)*ln(e*x+d)/e^3+1/2*b*p*ln(a*x^2+b)/a/e-d^
2*p*ln(e*x+d)*ln(-e*(x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)-e*b^(1/2)))/e^3-d^
2*p*ln(e*x+d)*ln(e*(-x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)+e*b^(1/2)))/e^3+2*
d^2*p*polylog(2,1+e*x/d)/e^3-d^2*p*polylog(2,(e*x+d)*(-a)^(1/2)/(d*(-a)^(1/
2)-e*b^(1/2)))/e^3-d^2*p*polylog(2,(e*x+d)*(-a)^(1/2)/(d*(-a)^(1/2)+e*b^(1/
2)))/e^3-2*d*p*arctan(x*a^(1/2)/b^(1/2))*b^(1/2)/e^2/a^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {2466, 2448, 263, 205, 2455, 260, 2462, 2416, 2394, 2315, 2393, 2391}

$$-\frac{d^2 p \text{PolyLog}\left(2, \frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^3} - \frac{d^2 p \text{PolyLog}\left(2, \frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^3} + \frac{2d^2 p \text{PolyLog}\left(2, \frac{ex}{d}+1\right)}{e^3} + \frac{d^2 \log(d+ex) \log\left(c\left(a\right.\right.}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] (-2*Sqrt[b]*d*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[a]*e^2) - (d*x*Log[c*(a
+ b/x^2)^p])/e^2 + (x^2*Log[c*(a + b/x^2)^p])/(2*e) + (d^2*Log[c*(a + b/x^2
)^p]*Log[d + e*x])/e^3 + (2*d^2*p*Log[-((e*x)/d)]*Log[d + e*x])/e^3 - (d^2*
p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/e^
3 - (d^2*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[
d + e*x])/e^3 + (b*p*Log[b + a*x^2])/(2*a*e) - (d^2*p*PolyLog[2, (Sqrt[-a]*
(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/e^3 - (d^2*p*PolyLog[2, (Sqrt[-a]*(d
+ e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/e^3 + (2*d^2*p*PolyLog[2, 1 + (e*x)/d])/
e^3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}dx = \int \left(-\frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}+\frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2(d+ex)}\right)dx$$

$$= -\frac{d\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx}{e^2}+\frac{d^2\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}dx}{e^2}+\frac{\int x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx}{e}$$

$$= -\frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e}+\frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^3}+$$

$$= -\frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e}+\frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^3}+$$

$$= -\frac{2\sqrt{b}dp\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2}-\frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e}+\frac{d^2\log\left(c\left(\right.\right.}{}$$

$$= -\frac{2\sqrt{b}dp\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2}-\frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e}+\frac{d^2\log\left(c\left(\right.\right.}{}$$

$$= -\frac{2\sqrt{b}dp\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2}-\frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e}+\frac{d^2\log\left(c\left(\right.\right.}{}$$

$$= -\frac{2\sqrt{b}dp\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2}-\frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e}+\frac{d^2\log\left(c\left(\right.\right.}{}$$

$$= -\frac{2\sqrt{b}dp\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2}-\frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e}+\frac{d^2\log\left(c\left(\right.\right.}{}$$

$$= -\frac{2\sqrt{b}dp\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2}-\frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e}+\frac{d^2\log\left(c\left(\right.\right.}{}$$

Mathematica [A] time = 0.23, size = 319, normalized size = 0.90

$$\frac{2d^2\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)-2dex\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+e^2x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+2d^2p\left(-\text{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] ((4*Sqrt[b]*d*e*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[a] - 2*d*e*x*Log[c*(a +
 b/x^2)^p] + e^2*x^2*Log[c*(a + b/x^2)^p] + (b*e^2*p*(Log[a + b/x^2] + 2*Lo
g[x]))/a + 2*d^2*Log[c*(a + b/x^2)^p]*Log[d + e*x] + 2*d^2*p*(2*Log[-((e*x)
/d)]*Log[d + e*x] - Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)
]*Log[d + e*x] - Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)
]*Log[d + e*x] - PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)]
- PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] + 2*PolyLog[2,
1 + (e*x)/d]))/(2*e^3)
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2 \log\left(c\left(\frac{ax^2+b}{x^2}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x^2*log(c*((a*x^2 + b)/x^2)^p)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^2*log((a + b/x^2)^p*c)/(e*x + d), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^2 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(a+b/x^2)^p)/(e*x+d),x)
[Out] int(x^2*ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^2*log((a + b/x^2)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(c*(a + b/x^2)^p))/(d + e*x),x)
[Out] int((x^2*log(c*(a + b/x^2)^p))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log \left(c \left(a + \frac{b}{x^2}\right)^p\right)}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(a+b/x**2)**p)/(e*x+d),x)
[Out] Integral(x**2*log(c*(a + b/x**2)**p)/(d + e*x), x)
```

3.249 $\int \frac{x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} dx$

Optimal. Leaf size=291

$$-\frac{d \log(d+ex) \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}+\frac{dp\text{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^2}+\frac{dp\text{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^2}+\frac{dp \log(d+ex) \log}{e^2}$$

```
[Out] x*ln(c*(a+b/x^2)^p)/e-d*ln(c*(a+b/x^2)^p)*ln(e*x+d)/e^2-2*d*p*ln(-e*x/d)*ln
(e*x+d)/e^2+d*p*ln(e*x+d)*ln(-e*(x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)-e*b^(1
/2)))/e^2+d*p*ln(e*x+d)*ln(e*(-x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)+e*b^(1/2
)))/e^2-2*d*p*polylog(2,1+e*x/d)/e^2+d*p*polylog(2,(e*x+d)*(-a)^(1/2)/(d*(-
a)^(1/2)-e*b^(1/2)))/e^2+d*p*polylog(2,(e*x+d)*(-a)^(1/2)/(d*(-a)^(1/2)+e*b
^(1/2)))/e^2+2*p*arctan(x*a^(1/2)/b^(1/2))*b^(1/2)/e/a^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {2466, 2448, 263, 205, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$\frac{dp\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^2}+\frac{dp\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^2}-\frac{2dp\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^2}-\frac{d \log(d+ex) \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[a]*e) + (x*Log[c*(a + b/x^2
)^p])/e - (d*Log[c*(a + b/x^2)^p]*Log[d + e*x])/e^2 - (2*d*p*Log[-((e*x)/d)
]*Log[d + e*x])/e^2 + (d*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqr
t[b]*e)]*Log[d + e*x])/e^2 + (d*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a
]*d - Sqrt[b]*e))]*Log[d + e*x])/e^2 + (d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))
/(Sqrt[-a]*d - Sqrt[b]*e)])/e^2 + (d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqr
t[-a]*d + Sqrt[b]*e)])/e^2 - (2*d*p*PolyLog[2, 1 + (e*x)/d])/e^2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(d+ex)}\right)dx$$

$$= \frac{\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx}{e} - \frac{d\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}dx}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{(2bdp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^2}\right)x^3}dx}{e^2} + \frac{(2bp)\,\dots}{\dots}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{(2bdp)\int \left(\frac{\log(d+ex)}{bx} - \frac{ax\log(d+e\dots}{b\left(b+ax^2\right)}\dots\right.}{e^2}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{(2dp)\int\dots}{\dots}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2dp\log\dots}{\dots}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2dp\log\dots}{\dots}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2dp\log\dots}{\dots}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2dp\log\dots}{\dots}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2dp\log\dots}{\dots}$$

Mathematica [A] time = 0.17, size = 271, normalized size = 0.93

$$\frac{-d\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + ex\log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + dp\text{Li}_2\left(\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right) + dp\text{Li}_2\left(\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right) + dp\log(d+ex)\dots}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] ((-2*Sqrt[b]*e*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[a] + e*x*Log[c*(a + b/x^
2)^p] - d*Log[c*(a + b/x^2)^p]*Log[d + e*x] - 2*d*p*Log[-((e*x)/d)]*Log[d +
 e*x] + d*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d
+ e*x] + d*p*Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Lo
g[d + e*x] + d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)]
+ d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] - 2*d*p*Pol
yLog[2, 1 + (e*x)/d])/e^2
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x\log\left(c\left(\frac{ax^2+b}{x^2}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x*log(c*((a*x^2 + b)/x^2)^p)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x*log((a + b/x^2)^p*c)/(e*x + d), x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int\frac{x\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(a+b/x^2)^p)/(e*x+d),x)
[Out] int(x*ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x*log((a + b/x^2)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x\,\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(c*(a + b/x^2)^p))/(d + e*x),x)
[Out] int((x*log(c*(a + b/x^2)^p))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \log \left(c \left(a + \frac{b}{x^2}\right)^p\right)}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(a+b/x**2)**p)/(e*x+d),x)
[Out] Integral(x*log(c*(a + b/x**2)**p)/(d + e*x), x)
```

3.250 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} dx$

Optimal. Leaf size=241

$$\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e}-\frac{p\log(d+$$

```
[Out] ln(c*(a+b/x^2)^p)*ln(e*x+d)/e+2*p*ln(-e*x/d)*ln(e*x+d)/e-p*ln(e*x+d)*ln(-e*
(x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)-e*b^(1/2)))/e-p*ln(e*x+d)*ln(e*(-x*(-a
)^(1/2)+b^(1/2))/(d*(-a)^(1/2)+e*b^(1/2)))/e+2*p*polylog(2,1+e*x/d)/e-p*pol
ylog(2,(e*x+d)*(-a)^(1/2)/(d*(-a)^(1/2)-e*b^(1/2)))/e-p*polylog(2,(e*x+d)*(
-a)^(1/2)/(d*(-a)^(1/2)+e*b^(1/2)))/e
```

Rubi [A] time = 0.33, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2462, 260, 2416, 2394, 2315, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e}+\frac{2p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}-$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x^2)^p]*Log[d + e*x])/e + (2*p*Log[-((e*x)/d)]*Log[d + e*x])/
e - (p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x
])/e - (p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d
 + e*x])/e - (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/
e - (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/e + (2*p*
PolyLog[2, 1 + (e*x)/d])/e
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{(2bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^2}\right)x^3}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{(2bp)\int \left(\frac{\log(d+ex)}{bx} - \frac{ax\log(d+ex)}{b\left(b+ax^2\right)}\right)dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{(2p)\int \frac{\log(d+ex)}{x}\,dx}{e} - \frac{(2ap)\int \frac{x\log(d+ex)}{b+ax^2}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - (2p)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx - \frac{(2ap)\ldots}{\ldots}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} + \frac{2p\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{e} + \frac{\left(\sqrt{-a}\,p\right)\ldots}{\ldots}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)\log(d+\ldots}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)\log(d+\ldots}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)\log(d+\ldots}{e}$$

Mathematica [A] time = 0.06, size = 242, normalized size = 1.00

$$\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{p\mathrm{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{e} - \frac{p\mathrm{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)}{e} - \frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)}{e} - \frac{p\log(d+e\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x^2)^p]*Log[d + e*x])/e + (2*p*Log[-((e*x)/d)]*Log[d + e*x])/
e - (p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x
])/e - (p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d
 + e*x])/e + (2*p*PolyLog[2, (d + e*x)/d])/e - (p*PolyLog[2, (Sqrt[-a]*(d +
 e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/e - (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(S
qrt[-a]*d + Sqrt[b]*e)])/e
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax^2+b}{x^2}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/(e*x + d), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p)/(d + e*x),x)
[Out] int(log(c*(a + b/x^2)^p)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/(e*x+d),x)
[Out] Integral(log(c*(a + b/x**2)**p)/(d + e*x), x)
```

3.251 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)} dx$

Optimal. Leaf size=287

$$-\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d}-\frac{\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2d}+\frac{p\text{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d}+\frac{p\text{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d}+\frac{p\log(d+$$

```
[Out] -1/2*ln(c*(a+b/x^2)^p)*ln(-b/a/x^2)/d-ln(c*(a+b/x^2)^p)*ln(e*x+d)/d-2*p*ln(
-e*x/d)*ln(e*x+d)/d+p*ln(e*x+d)*ln(-e*(x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)-
e*b^(1/2)))/d+p*ln(e*x+d)*ln(e*(-x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)+e*b^(1
/2)))/d-1/2*p*polylog(2,1+b/a/x^2)/d-2*p*polylog(2,1+e*x/d)/d+p*polylog(2,(
e*x+d)*(-a)^(1/2)/(d*(-a)^(1/2)-e*b^(1/2)))/d+p*polylog(2,(e*x+d)*(-a)^(1/2
)/(d*(-a)^(1/2)+e*b^(1/2)))/d
```

Rubi [A] time = 0.46, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2466, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d}-\frac{p\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)}{2d}-\frac{2p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d}-\frac{\log(d+$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/(x*(d + e*x)),x]
```

```
[Out] -(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/(2*d) - (Log[c*(a + b/x^2)^p]*Log
[d + e*x])/d - (2*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[(e*(Sqrt[b] -
Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/d + (p*Log[-((e*(Sqrt[
b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d + e*x])/d - (p*PolyLog[2
, 1 + b/(a*x^2)])/(2*d) + (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d -
Sqrt[b]*e)])/d + (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e
)])/d - (2*p*PolyLog[2, 1 + (e*x)/d])/d
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} - \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}dx}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}dx, x, \frac{1}{x^2}\right)}{2d} - \frac{(2bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^2}\right)x^3}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} + \frac{(bp)\,\text{Subst}\left(\int \frac{\log(-}{a+b}\right.}{2d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{p\text{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d} - (2$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

Mathematica [A] time = 0.12, size = 264, normalized size = 0.92

$$-\frac{2\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right) - 2p\text{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right) - 2p\text{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right) - 2p}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/(x*(d + e*x)),x]
```

```
[Out] -1/2*(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))] + 2*Log[c*(a + b/x^2)^p]*Log[d
 + e*x] + 4*p*Log[-((e*x)/d)]*Log[d + e*x] - 2*p*Log[(e*(Sqrt[b] - Sqrt[-a]
*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x] - 2*p*Log[(e*(Sqrt[b] + Sqrt[-a
]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Log[d + e*x] + p*PolyLog[2, 1 + b/(a*x^2
)] - 2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)] - 2*p*Po
lyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] + 4*p*PolyLog[2, 1
+ (e*x)/d])/d
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax^2+b}{x^2}\right)^p\right)}{ex^2+dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x/(e*x+d),x, algorithm="fricas")
[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/(e*x^2 + d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x/(e*x+d),x, algorithm="giac")
[Out] integrate(log((a + b/x^2)^p*c)/((e*x + d)*x), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{(ex+d)\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x/(e*x+d),x)
[Out] int(ln(c*(a+b/x^2)^p)/x/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((a + b/x^2)^p*c)/((e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p)/(x*(d + e*x)),x)
[Out] int(log(c*(a + b/x^2)^p)/(x*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x/(e*x+d),x)
```

```
[Out] Timed out
```

3.252 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=357

$$\frac{e\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2d^2}+\frac{e\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx}+\frac{ep\text{Li}_2\left(\frac{b}{ax^2}+1\right)}{2d^2}-\frac{ep\text{Li}_2\left(\frac{\sqrt{-a}(d\cdot}{\sqrt{-a}d-}\right.}{d^2}$$

```
[Out] 2*p/d/x-ln(c*(a+b/x^2)^p)/d/x+1/2*e*ln(c*(a+b/x^2)^p)*ln(-b/a/x^2)/d^2+e*ln
(c*(a+b/x^2)^p)*ln(e*x+d)/d^2+2*e*p*ln(-e*x/d)*ln(e*x+d)/d^2-e*p*ln(e*x+d)*
ln(-e*(x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)-e*b^(1/2)))/d^2-e*p*ln(e*x+d)*ln
(e*(-x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)+e*b^(1/2)))/d^2+1/2*e*p*polylog(2,
1+b/a/x^2)/d^2+2*e*p*polylog(2,1+e*x/d)/d^2-e*p*polylog(2,(e*x+d)*(-a)^(1/2
)/(d*(-a)^(1/2)-e*b^(1/2)))/d^2-e*p*polylog(2,(e*x+d)*(-a)^(1/2)/(d*(-a)^(1
/2)+e*b^(1/2)))/d^2+2*p*arctan(x*a^(1/2)/b^(1/2))*a^(1/2)/d/b^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2466, 2455, 263, 325, 205, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{ep\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)}{2d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^2}+\frac{2ep\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^2}+\frac{e\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (2*p)/(d*x) + (2*Sqrt[a]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[b]*d) - Log[c
*(a + b/x^2)^p]/(d*x) + (e*Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/(2*d^2)
+ (e*Log[c*(a + b/x^2)^p]*Log[d + e*x])/d^2 + (2*e*p*Log[-((e*x)/d)]*Log[d
+ e*x])/d^2 - (e*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]
*Log[d + e*x])/d^2 - (e*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sq
rt[b]*e))]*Log[d + e*x])/d^2 + (e*p*PolyLog[2, 1 + b/(a*x^2)])/(2*d^2) - (e
*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/d^2 - (e*p*Po
lyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/d^2 + (2*e*p*PolyL
og[2, 1 + (e*x)/d])/d^2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx^2} - \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x} + \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx}{d^2}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^2} + \frac{e\,\mathrm{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, \frac{1}{x^2}\right)}{2d^2}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^2} -$$

$$= \frac{2p}{dx} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+e}{d^2}$$

$$= \frac{2p}{dx} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} + \frac{e\log(}{}$$

$$= \frac{2p}{dx} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} + \frac{e\log(}{}$$

$$= \frac{2p}{dx} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} + \frac{e\log(}{}$$

$$= \frac{2p}{dx} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} + \frac{e\log(}{}$$

$$= \frac{2p}{dx} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} + \frac{e\log(}{}$$

Mathematica [A] time = 0.21, size = 320, normalized size = 0.90

$$2e\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right) - \frac{2d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x} + e\left(\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + p\mathrm{Li}_2\left(\frac{b}{ax^2}+1\right)\right) + 2ep\left(-\mathrm{Li}_2\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (4*d*p*(x^(-1) - (Sqrt[a]*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[b]) - (2*d*Log[
c*(a + b/x^2)^p])/x + 2*e*Log[c*(a + b/x^2)^p]*Log[d + e*x] + e*(Log[c*(a +
 b/x^2)^p]*Log[-(b/(a*x^2))] + p*PolyLog[2, 1 + b/(a*x^2)]) + 2*e*p*(2*Log[
-((e*x)/d)]*Log[d + e*x] - Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqr
t[b]*e)]*Log[d + e*x] - Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqr
t[b]*e)]*Log[d + e*x] - PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[
b]*e)] - PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] + 2*Poly
Log[2, 1 + (e*x)/d]))/(2*d^2)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax^2+b}{x^2}\right)^p\right)}{ex^3+dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/(e*x^3 + d*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/((e*x + d)*x^2), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{(ex+d)\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x^2/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x^2)^p)/x^2/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/((e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2\,(d+e\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p)/(x^2*(d + e*x)),x)
[Out] int(log(c*(a + b/x^2)^p)/(x^2*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**2/(e*x+d),x)
[Out] Timed out
```

3.253 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=414

$$-\frac{e^2\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2d^3}-\frac{e^2\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^3}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right.\right.}{2bd}$$

```
[Out] 1/2*p/d/x^2-2*e*p/d^2/x-1/2*(a+b/x^2)*ln(c*(a+b/x^2)^p)/b/d+e*ln(c*(a+b/x^2
)^p)/d^2/x-1/2*e^2*ln(c*(a+b/x^2)^p)*ln(-b/a/x^2)/d^3-e^2*ln(c*(a+b/x^2)^p)
*ln(e*x+d)/d^3-2*e^2*p*ln(-e*x/d)*ln(e*x+d)/d^3+e^2*p*ln(e*x+d)*ln(-e*(x*(-
a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)-e*b^(1/2)))/d^3+e^2*p*ln(e*x+d)*ln(e*(-x*(-
a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)+e*b^(1/2)))/d^3-1/2*e^2*p*polylog(2,1+b/a/x
^2)/d^3-2*e^2*p*polylog(2,1+e*x/d)/d^3+e^2*p*polylog(2,(e*x+d)*(-a)^(1/2)/(
d*(-a)^(1/2)-e*b^(1/2)))/d^3+e^2*p*polylog(2,(e*x+d)*(-a)^(1/2)/(d*(-a)^(1/
2)+e*b^(1/2)))/d^3-2*e*p*arctan(x*a^(1/2)/b^(1/2))*a^(1/2)/d^2/b^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {2466, 2454, 2389, 2295, 2455, 263, 325, 205, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$-\frac{e^2p\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)}{2d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^3}-\frac{2e^2p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/(x^3*(d + e*x)),x]
```

```
[Out] p/(2*d*x^2) - (2*e*p)/(d^2*x) - (2*Sqrt[a]*e*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])
/(Sqrt[b]*d^2) - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/(2*b*d) + (e*Log[c*(a +
 b/x^2)^p])/(d^2*x) - (e^2*Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/(2*d^3)
- (e^2*Log[c*(a + b/x^2)^p]*Log[d + e*x])/d^3 - (2*e^2*p*Log[-((e*x)/d)]*Lo
g[d + e*x])/d^3 + (e^2*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[
b]*e)]*Log[d + e*x])/d^3 + (e^2*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a
]*d - Sqrt[b]*e))]*Log[d + e*x])/d^3 - (e^2*p*PolyLog[2, 1 + b/(a*x^2)])/(2
*d^3) + (e^2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/d
^3 + (e^2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/d^3
- (2*e^2*p*PolyLog[2, 1 + (e*x)/d])/d^3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx=\int\left(\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx^3}-\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x^2}+\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^3x}-\frac{e^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^3(d+ex)}\right.$$

$$=\frac{\int\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}dx}{d}-\frac{e\int\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}dx}{d^2}+\frac{e^2\int\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}dx}{d^3}-\frac{e^3\int\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}dx}{d^3}$$

$$=\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^3}-\frac{\text{Subst}\left(\int\log\left(c(a+bx)^p\right)dx,x\right.}{2d}$$

$$=\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^3}-\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^3}$$

$$=\frac{p}{2dx^2}-\frac{2ep}{d^2x}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\text{lo}}{2d^3}$$

$$=\frac{p}{2dx^2}-\frac{2ep}{d^2x}-\frac{2\sqrt{a}\,ep\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}-$$

$$=\frac{p}{2dx^2}-\frac{2ep}{d^2x}-\frac{2\sqrt{a}\,ep\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}-$$

$$=\frac{p}{2dx^2}-\frac{2ep}{d^2x}-\frac{2\sqrt{a}\,ep\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}-$$

$$=\frac{p}{2dx^2}-\frac{2ep}{d^2x}-\frac{2\sqrt{a}\,ep\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}-$$

$$=\frac{p}{2dx^2}-\frac{2ep}{d^2x}-\frac{2\sqrt{a}\,ep\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}-$$

Mathematica [A] time = 0.30, size = 364, normalized size = 0.88

$$d^2\left(\frac{p}{x^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{b}\right)-2e^2\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{2de\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}-e^2\left(\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/(x^3*(d + e*x)),x]
```

```
[Out] (4*d*e*p*(-x^(-1) + (Sqrt[a]*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[b]) + (2*d*e
*Log[c*(a + b/x^2)^p])/x + d^2*(p/x^2 - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/
b) - 2*e^2*Log[c*(a + b/x^2)^p]*Log[d + e*x] - e^2*(Log[c*(a + b/x^2)^p]*Lo
g[-(b/(a*x^2))] + p*PolyLog[2, 1 + b/(a*x^2)]) - 2*e^2*p*(2*Log[-((e*x)/d)]
*Log[d + e*x] - Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Lo
g[d + e*x] - Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Lo
g[d + e*x] - PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)] - Po
```

```
lyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] + 2*PolyLog[2, 1 +
(e*x)/d]))/(2*d^3)
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax^2+b}{x^2}\right)^p\right)}{ex^4+dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3/(e*x+d),x, algorithm="fricas")
[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/(e*x^4 + d*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3/(e*x+d),x, algorithm="giac")
[Out] integrate(log((a + b/x^2)^p*c)/((e*x + d)*x^3), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{(ex+d)\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x^3/(e*x+d),x)
[Out] int(ln(c*(a+b/x^2)^p)/x^3/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((a + b/x^2)^p*c)/((e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p)/(x^3*(d + e*x)),x)
[Out] int(log(c*(a + b/x^2)^p)/(x^3*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**3/(e*x+d),x)
[Out] Timed out
```

3.254 $\int \frac{x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=714

$$-\frac{\sqrt[3]{b}\,d^2p\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}\right)}{2\sqrt[3]{a}\,e^3}-\frac{b^{2/3}dp\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}\right)}{4a^{2/3}e^2}+\frac{b^{2/3}dp\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{2a^{2/3}e^2}+\frac{\sqrt{3}\,b^{2/}}{}$$

```
[Out] d^2*x*ln(c*(a+b/x^3)^p)/e^3-1/2*d*x^2*ln(c*(a+b/x^3)^p)/e^2+1/3*x^3*ln(c*(a
+b/x^3)^p)/e+b^(1/3)*d^2*p*ln(b^(1/3)+a^(1/3)*x)/a^(1/3)/e^3+1/2*b^(2/3)*d*
p*ln(b^(1/3)+a^(1/3)*x)/a^(2/3)/e^2-d^3*ln(c*(a+b/x^3)^p)*ln(e*x+d)/e^4-3*d
^3*p*ln(-e*x/d)*ln(e*x+d)/e^4+d^3*p*ln(-e*(b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-b^
(1/3)*e))*ln(e*x+d)/e^4+d^3*p*ln(-e*((-1)^(2/3)*b^(1/3)+a^(1/3)*x)/(a^(1/3)
*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/e^4+d^3*p*ln((-1)^(1/3)*e*(b^(1/3)+(-1)
^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*x+d)/e^4-1/2*b^(1/
3)*d^2*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(1/3)/e^3-1/4*b^(2/3)*
d*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(2/3)/e^2+1/3*b*p*ln(a*x^3+
b)/a/e+d^3*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/e^4+d^3*p*pol
ylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/e^4+d^3*p*polylog(
2,a^(1/3)*(e*x+d)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))/e^4-3*d^3*p*polylog(2,1
+e*x/d)/e^4-b^(1/3)*d^2*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))
*3^(1/2)/a^(1/3)/e^3+1/2*b^(2/3)*d*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/
3)*3^(1/2))*3^(1/2)/a^(2/3)/e^2
```

Rubi [A] time = 0.92, antiderivative size = 714, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 18, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.783, Rules used = {2466, 2448, 263, 200, 31, 634, 617, 204, 628, 2455, 292, 260, 2462, 2416, 2394, 2315, 2393, 2391}

$$\frac{d^3p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^4}+\frac{d^3p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^4}+\frac{d^3p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^4}-\frac{3d^3p\text{PolyLog}(}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] -((Sqrt[3]*b^(1/3)*d^2*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])
/(a^(1/3)*e^3)) + (Sqrt[3]*b^(2/3)*d*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt
[3]*b^(1/3))])/(2*a^(2/3)*e^2) + (d^2*x*Log[c*(a + b/x^3)^p])/e^3 - (d*x^2*
Log[c*(a + b/x^3)^p])/(2*e^2) + (x^3*Log[c*(a + b/x^3)^p])/(3*e) + (b^(1/3)
*d^2*p*Log[b^(1/3) + a^(1/3)*x])/(a^(1/3)*e^3) + (b^(2/3)*d*p*Log[b^(1/3) +
 a^(1/3)*x])/(2*a^(2/3)*e^2) - (d^3*Log[c*(a + b/x^3)^p]*Log[d + e*x])/e^4
- (3*d^3*p*Log[-((e*x)/d)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(b^(1/3) + a
^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*((
-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d
+ e*x])/e^4 + (d^3*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a
^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e^4 - (b^(1/3)*d^2*p*Log[b^
(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(2*a^(1/3)*e^3) - (b^(2/3)*d*p*Lo
g[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(4*a^(2/3)*e^2) + (b*p*Log[b
+ a*x^3])/(3*a*e) + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1
/3)*e)])/e^4 + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3
)*b^(1/3)*e)])/e^4 + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1
)^(2/3)*b^(1/3)*e)])/e^4 - (3*d^3*p*PolyLog[2, 1 + (e*x)/d])/e^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \left(\frac{d^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d^3 \log\left(c\left(a\right.\right.}{e^3(d+}\right.$$

$$= \frac{d^2 \int \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)dx}{e^3} - \frac{d^3 \int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}dx}{e^3} - \frac{d \int x \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)dx}{e^2} + \frac{\int x^2}{}$$

$$= \frac{d^2 x \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} - \frac{d^3 \log\left(c\left(a+\right.\right.}{}$$

$$= \frac{d^2 x \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} - \frac{d^3 \log\left(c\left(a+\right.\right.}{}$$

$$= \frac{d^2 x \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} - \frac{d^3 \log\left(c\left(a+\right.\right.}{}$$

$$= \frac{d^2 x \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} + \frac{\sqrt[3]{b}\, d^2 p \log(}{\sqrt[3]{a}}$$

$$= \frac{d^2 x \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} + \frac{\sqrt[3]{b}\, d^2 p \log(}{\sqrt[3]{a}}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}\, d^2 p \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}\, e^3} + \frac{\sqrt{3}\, b^{2/3} d p \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3} e^2} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x^3}\right)^p\right.}{e^3}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}\, d^2 p \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}\, e^3} + \frac{\sqrt{3}\, b^{2/3} d p \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3} e^2} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x^3}\right)^p\right.}{e^3}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}\, d^2 p \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}\, e^3} + \frac{\sqrt{3}\, b^{2/3} d p \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3} e^2} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x^3}\right)^p\right.}{e^3}$$

Mathematica [C] time = 0.42, size = 505, normalized size = 0.71

$$\frac{x^2\left(-6ad^3 \log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right) + 6ad^2 ex \log\left(c\left(a+\frac{b}{x^3}\right)^p\right) - 3ade^2x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right) + 2ae^3x^3 \log\left(c\left(a\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] (9*b*d*e^2*p*x*Hypergeometric2F1[1/3, 1, 4/3, -(b/(a*x^3))] - 9*b*d^2*e*p*H
ypergeometric2F1[2/3, 1, 5/3, -(b/(a*x^3))] + x^2*(2*b*e^3*p*Log[a + b/x^3]
 + 6*a*d^2*e*x*Log[c*(a + b/x^3)^p] - 3*a*d*e^2*x^2*Log[c*(a + b/x^3)^p] +
2*a*e^3*x^3*Log[c*(a + b/x^3)^p] + 6*b*e^3*p*Log[x] - 6*a*d^3*Log[c*(a + b/
x^3)^p]*Log[d + e*x] - 18*a*d^3*p*Log[-((e*x)/d)]*Log[d + e*x] + 6*a*d^3*p*
Log[(e*((-1)^(1/3)*b^(1/3) - a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)
```

```
]*Log[d + e*x] + 6*a*d^3*p*Log[(e*(b^(1/3) + a^(1/3)*x))/(-(a^(1/3)*d) + b^
(1/3)*e)]*Log[d + e*x] + 6*a*d^3*p*Log[(e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))
/(-(a^(1/3)*d) + (-1)^(2/3)*b^(1/3)*e)]*Log[d + e*x] + 6*a*d^3*p*PolyLog[2,
 (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)] + 6*a*d^3*p*PolyLog[2, (a^(1/
3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)] + 6*a*d^3*p*PolyLog[2, (a
^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)] - 18*a*d^3*p*PolyLog[
2, 1 + (e*x)/d]))/(6*a*e^4*x^2)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3\log\left(c\left(\frac{ax^3+b}{x^3}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^3*log(c*((a*x^3 + b)/x^3)^p)/(e*x + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming a near 0Simplificati
on assuming d near 0Evaluation time: 0.86Not invertible Error: Bad Argument
 Value
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x^3\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

```
[Out] int(x^3*ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^3*log((a + b/x^3)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\ln\left(c\,{\left(a+\frac{b}{x^3}\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(c*(a + b/x^3)^p))/(d + e*x),x)
[Out] int((x^3*log(c*(a + b/x^3)^p))/(d + e*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(a+b/x**3)**p)/(e*x+d),x)
[Out] Timed out
```

3.255 $\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=666

$$\frac{\sqrt[3]{b}\,dp\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}\right)}{2\sqrt[3]{a}\,e^2}+\frac{b^{2/3}p\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}\right)}{4a^{2/3}e}-\frac{b^{2/3}p\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{2a^{2/3}e}-\frac{\sqrt{3}\,b^{2/3}p\tan}{2}$$

```
[Out] -d*x*ln(c*(a+b/x^3)^p)/e^2+1/2*x^2*ln(c*(a+b/x^3)^p)/e-b^(1/3)*d*p*ln(b^(1/
3)+a^(1/3)*x)/a^(1/3)/e^2-1/2*b^(2/3)*p*ln(b^(1/3)+a^(1/3)*x)/a^(2/3)/e+d^2
*ln(c*(a+b/x^3)^p)*ln(e*x+d)/e^3+3*d^2*p*ln(-e*x/d)*ln(e*x+d)/e^3-d^2*p*ln(
-e*(b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e*x+d)/e^3-d^2*p*ln(-e*((-
1)^(2/3)*b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/e^3
-d^2*p*ln((-1)^(1/3)*e*(b^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)
*b^(1/3)*e))*ln(e*x+d)/e^3+1/2*b^(1/3)*d*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(
2/3)*x^2)/a^(1/3)/e^2+1/4*b^(2/3)*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^
2)/a^(2/3)/e-d^2*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/e^3-d^2
*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/e^3-d^2*p*po
lylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))/e^3+3*d^2*p*polyl
og(2,1+e*x/d)/e^3+b^(1/3)*d*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1
/2))*3^(1/2)/a^(1/3)/e^2-1/2*b^(2/3)*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(
1/3)*3^(1/2))*3^(1/2)/a^(2/3)/e
```

Rubi [A] time = 0.73, antiderivative size = 666, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 18, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.783, Rules used = {2466, 2448, 263, 200, 31, 634, 617, 204, 628, 2455, 292, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$-\frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^3}-\frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^3}-\frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^3}+\frac{3d^2p\,\text{PolyLog}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] (Sqrt[3]*b^(1/3)*d*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(a^
(1/3)*e^2) - (Sqrt[3]*b^(2/3)*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(
1/3))])/(2*a^(2/3)*e) - (d*x*Log[c*(a + b/x^3)^p])/e^2 + (x^2*Log[c*(a + b/
x^3)^p])/(2*e) - (b^(1/3)*d*p*Log[b^(1/3) + a^(1/3)*x])/(a^(1/3)*e^2) - (b^
(2/3)*p*Log[b^(1/3) + a^(1/3)*x])/(2*a^(2/3)*e) + (d^2*Log[c*(a + b/x^3)^p]
*Log[d + e*x])/e^3 + (3*d^2*p*Log[-((e*x)/d)]*Log[d + e*x])/e^3 - (d^2*p*Lo
g[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/e^3 -
 (d^2*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*
b^(1/3)*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(
2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e^3 + (b
^(1/3)*d*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(2*a^(1/3)*e^2)
+ (b^(2/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(4*a^(2/3)*e)
- (d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/e^3 - (d^
2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/e^3
 - (d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)
])/e^3 + (3*d^2*p*PolyLog[2, 1 + (e*x)/d])/e^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \left(-\frac{d \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{d^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2(d+ex)}\right) dx$$

$$= -\frac{d\int \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)dx}{e^2} + \frac{d^2 \int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}dx}{e^2} + \frac{\int x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)dx}{e}$$

$$= -\frac{dx \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e} + \frac{d^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3} + \cdots$$

$$= -\frac{dx \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e} + \frac{d^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3} + \cdots$$

$$= -\frac{dx \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e} + \frac{d^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3} + \cdots$$

$$= -\frac{dx \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e} - \frac{\sqrt[3]{b}\,dp \log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,e^2} - \frac{b^{2/3}p\log(\cdots}{2\cdots}$$

$$= -\frac{dx \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e} - \frac{\sqrt[3]{b}\,dp \log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,e^2} - \frac{b^{2/3}p\log(\cdots}{2\cdots}$$

$$= \frac{\sqrt{3}\sqrt[3]{b}\,dp\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e^2} - \frac{\sqrt{3}\,b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e} - \frac{dx \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \cdots$$

$$= \frac{\sqrt{3}\sqrt[3]{b}\,dp\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e^2} - \frac{\sqrt{3}\,b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e} - \frac{dx \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \cdots$$

$$= \frac{\sqrt{3}\sqrt[3]{b}\,dp\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e^2} - \frac{\sqrt{3}\,b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e} - \frac{dx \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \cdots$$

Mathematica [C] time = 0.23, size = 443, normalized size = 0.67

$$\frac{ax^2\left(2d^2\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right) - 2dex\log\left(c\left(a+\frac{b}{x^3}\right)^p\right) + e^2x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right) - 2d^2p\mathrm{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right) - 2\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] (-3*b*e^2*p*x*Hypergeometric2F1[1/3, 1, 4/3, -(b/(a*x^3))] + 3*b*d*e*p*Hype
rgeometric2F1[2/3, 1, 5/3, -(b/(a*x^3))] + a*x^2*(-2*d*e*x*Log[c*(a + b/x^3
)^p] + e^2*x^2*Log[c*(a + b/x^3)^p] + 2*d^2*Log[c*(a + b/x^3)^p]*Log[d + e*
x] + 6*d^2*p*Log[-((e*x)/d)]*Log[d + e*x] - 2*d^2*p*Log[(e*((-1)^(1/3)*b^(1
/3) - a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x] - 2*d^2*
p*Log[(e*(b^(1/3) + a^(1/3)*x))/(-(a^(1/3)*d) + b^(1/3)*e)]*Log[d + e*x] -
```

```
2*d^2*p*Log[(e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(-(a^(1/3)*d) + (-1)^(2/3)
*b^(1/3)*e)]*Log[d + e*x] - 2*d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)
*d - b^(1/3)*e)] - 2*d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)
^(1/3)*b^(1/3)*e)] - 2*d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-
1)^(2/3)*b^(1/3)*e)] + 6*d^2*p*PolyLog[2, 1 + (e*x)/d]))/(2*a*e^3*x^2)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2\log\left(c\left(\frac{ax^3+b}{x^3}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^2*log(c*((a*x^3 + b)/x^3)^p)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((a + b/x^3)^p*c)/(e*x + d), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x^2\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

```
[Out] int(x^2*ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^2*log((a + b/x^3)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\ln\left(c\,{\left(a+\frac{b}{x^3}\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(c*(a + b/x^3)^p))/(d + e*x),x)
[Out] int((x^2*log(c*(a + b/x^3)^p))/(d + e*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(a+b/x**3)**p)/(e*x+d),x)
[Out] Timed out
```

3.256 $\int \frac{x \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex} dx$

Optimal. Leaf size=488

$$-\frac{\sqrt[3]{b}\, p \log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}\, x+b^{2/3}\right)}{2\sqrt[3]{a}\, e}-\frac{d\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}+\frac{dp\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^2}+$$

```
[Out] x*ln(c*(a+b/x^3)^p)/e+b^(1/3)*p*ln(b^(1/3)+a^(1/3)*x)/a^(1/3)/e-d*ln(c*(a+b
/x^3)^p)*ln(e*x+d)/e^2-3*d*p*ln(-e*x/d)*ln(e*x+d)/e^2+d*p*ln(-e*(b^(1/3)+a^
(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e*x+d)/e^2+d*p*ln(-e*((-1)^(2/3)*b^(1/3)
+a^(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/e^2+d*p*ln((-1)^(1/
3)*e*(b^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*
x+d)/e^2-1/2*b^(1/3)*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(1/3)/e+
d*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/e^2+d*p*polylog(2,a^(1
/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/e^2+d*p*polylog(2,a^(1/3)*(e*
x+d)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))/e^2-3*d*p*polylog(2,1+e*x/d)/e^2-b^(
1/3)*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))*3^(1/2)/a^(1/3)/e
```

Rubi [A] time = 0.60, antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 16, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.762, Rules used = {2466, 2448, 263, 200, 31, 634, 617, 204, 628, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$\frac{dp\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^2}+\frac{dp\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^2}+\frac{dp\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^2}-\frac{3dp\text{PolyLog}\left(2,\frac{ex}{d}\right.}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] -((Sqrt[3]*b^(1/3)*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(a^
(1/3)*e)) + (x*Log[c*(a + b/x^3)^p])/e + (b^(1/3)*p*Log[b^(1/3) + a^(1/3)*x
])/(a^(1/3)*e) - (d*Log[c*(a + b/x^3)^p]*Log[d + e*x])/e^2 - (3*d*p*Log[-((
e*x)/d)]*Log[d + e*x])/e^2 + (d*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*
d - b^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^
(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log
[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^
(1/3)*e)]*Log[d + e*x])/e^2 - (b^(1/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x +
a^(2/3)*x^2])/(2*a^(1/3)*e) + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*
d - b^(1/3)*e)])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1
)^(1/3)*b^(1/3)*e)])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d -
 (-1)^(2/3)*b^(1/3)*e)])/e^2 - (3*d*p*PolyLog[2, 1 + (e*x)/d])/e^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e(d+ex)}\right)dx$$

$$= \frac{\int \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)dx}{e} - \frac{d\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}dx}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{(3bdp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^3}\right)x^4}dx}{e^2} + \frac{(3bp)\ldots}{}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{(3bdp)\int\left(\frac{\log(d+ex)}{bx} - \frac{ax^2\log(d+\ldots}{b\left(b+ax^3\right)}\ldots\right)}{e^2}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{(3dp)\int\frac{\log(d+ex)}{x}dx}{e^2} + \frac{(3adp)\ldots}{}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{3dp\ldots}{}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{3dp\ldots}{}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e} + \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,e} - \frac{d\log\left(\ldots\right.}{}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e} + \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,e} - \frac{d\log\left(\ldots\right.}{}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e} + \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,e} - \frac{d\log\left(\ldots\right.}{}$$

Mathematica [C] time = 0.12, size = 403, normalized size = 0.83

$$-\frac{d\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{dp\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^2} + \frac{dp\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^2} + \frac{dp\text{Li}_2\left(\frac{\sqrt[3]{a}(d\ldots}{\sqrt[3]{a}d-(-1)\ldots}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] (-3*b*p*Hypergeometric2F1[2/3, 1, 5/3, -(b/(a*x^3))])/(2*a*e*x^2) + (x*Log[
c*(a + b/x^3)^p])/e - (d*Log[c*(a + b/x^3)^p]*Log[d + e*x])/e^2 - (3*d*p*Lo
g[-((e*x)/d)]*Log[d + e*x])/e^2 + (d*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(
1/3)*d - b^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log[-(((-1)^(2/3)*e*(b^(1/3)
 - (-1)^(1/3)*a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x]
)/e^2 + (d*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d
 + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e^2 - (3*d*p*PolyLog[2, (d + e*x)/d
```

```
])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/e^2
+ (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/
e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e
)])/e^2
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x \log\left(c \left(\frac{ax^3+b}{x^3}\right)^p\right)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x*log(c*((a*x^3 + b)/x^3)^p)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \log\left(\left(a + \frac{b}{x^3}\right)^p c\right)}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x*log((a + b/x^3)^p*c)/(e*x + d), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x \ln\left(c \left(a + \frac{b}{x^3}\right)^p\right)}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

```
[Out] int(x*ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \log\left(\left(a + \frac{b}{x^3}\right)^p c\right)}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x*log((a + b/x^3)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\, \ln\left(c \left(a + \frac{b}{x^3}\right)^p\right)}{d + e\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(c*(a + b/x^3)^p))/(d + e*x),x)
```

```
[Out] int((x*log(c*(a + b/x^3)^p))/(d + e*x), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(a+b/x**3)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

3.257 $\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=344

$$\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e}-\frac{p\log(d+ex)\log}{e}$$

```
[Out] ln(c*(a+b/x^3)^p)*ln(e*x+d)/e+3*p*ln(-e*x/d)*ln(e*x+d)/e-p*ln(-e*(b^(1/3)+a
^(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e*x+d)/e-p*ln(-e*((-1)^(2/3)*b^(1/3)+a^
(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/e-p*ln((-1)^(1/3)*e*(b
^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*x+d)/e-
p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/e-p*polylog(2,a^(1/3)*(e
*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/e-p*polylog(2,a^(1/3)*(e*x+d)/(a^(1
/3)*d-(-1)^(2/3)*b^(1/3)*e))/e+3*p*polylog(2,1+e*x/d)/e
```

Rubi [A] time = 0.41, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2462, 260, 2416, 2394, 2315, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e}+\frac{3p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^3)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x^3)^p]*Log[d + e*x])/e + (3*p*Log[-((e*x)/d)]*Log[d + e*x])/
e - (p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*
x])/e - (p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/
3)*b^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3
)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e - (p*Poly
Log[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/e - (p*PolyLog[2, (a^(
1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/e - (p*PolyLog[2, (a^(
1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/e + (3*p*PolyLog[2, 1
+ (e*x)/d])/e
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{(3bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^3}\right)x^4}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{(3bp)\int \left(\frac{\log(d+ex)}{bx} - \frac{ax^2\log(d+ex)}{b\left(b+ax^3\right)}\right)dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{(3p)\int \frac{\log(d+ex)}{x}\,dx}{e} - \frac{(3ap)\int \frac{x^2\log(d+ex)}{b+ax^3}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - (3p)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx - \frac{(3ap}{}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} + \frac{3p\text{Li}_2\left(1+\frac{ex}{d}\right)}{e} - \frac{\left(\sqrt[3]{a}\,p\right)\int}{}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right)\log(d+}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right)\log(d+}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right)\log(d+}{e}$$

Mathematica [A] time = 0.09, size = 350, normalized size = 1.02

$$\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e}-\frac{p\log(d+ex)\log}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^3)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x^3)^p]*Log[d + e*x])/e + (3*p*Log[-((e*x)/d)]*Log[d + e*x])/
e - (p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*
x])/e - (p*Log[-(((-1)^(2/3)*e*(b^(1/3) - (-1)^(1/3)*a^(1/3)*x))/(a^(1/3)*d
 - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(b^(1/3)
+ (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/
e + (3*p*PolyLog[2, (d + e*x)/d])/e - (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^
(1/3)*d - b^(1/3)*e)])/e - (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (
-1)^(1/3)*b^(1/3)*e)])/e - (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (
-1)^(2/3)*b^(1/3)*e)])/e
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax^3+b}{x^3}\right)^p\right)}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^3 + b)/x^3)^p)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^3)^p*c)/(e*x + d), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int\frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((a + b/x^3)^p*c)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^3)^p)/(d + e*x),x)
```

```
[Out] int(log(c*(a + b/x^3)^p)/(d + e*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**3)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

3.258 $\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx$

Optimal. Leaf size=388

$$-\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d}-\frac{\log\left(-\frac{b}{ax^3}\right)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3d}+\frac{p\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d}+\frac{p\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d}+\frac{p\text{Li}_2\left(\right.}{}$$

```
[Out] -1/3*ln(c*(a+b/x^3)^p)*ln(-b/a/x^3)/d-ln(c*(a+b/x^3)^p)*ln(e*x+d)/d-3*p*ln(
-e*x/d)*ln(e*x+d)/d+p*ln(-e*(b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e
*x+d)/d+p*ln(-e*((-1)^(2/3)*b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3
)*e))*ln(e*x+d)/d+p*ln((-1)^(1/3)*e*(b^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3)
*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*x+d)/d-1/3*p*polylog(2,1+b/a/x^3)/d+p*polylo
g(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/d+p*polylog(2,a^(1/3)*(e*x+d)/(a
^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/d+p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-(-
1)^(2/3)*b^(1/3)*e))/d-3*p*polylog(2,1+e*x/d)/d
```

Rubi [A] time = 0.54, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2466, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d}-\frac{p\text{PolyLog}\left(2,\frac{b}{ax^3}+1\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^3)^p]/(x*(d + e*x)),x]
```

```
[Out] -(Log[c*(a + b/x^3)^p]*Log[-(b/(a*x^3))])/(3*d) - (Log[c*(a + b/x^3)^p]*Log
[d + e*x])/d - (3*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b^(1/3)
+ a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/d + (p*Log[-((e*((-1)
^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e
*x])/d + (p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d
+ (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/d - (p*PolyLog[2, 1 + b/(a*x^3)])/(3
*d) + (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d + (p*Po
lyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d + (p*Po
lyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d - (3*p*
PolyLog[2, 1 + (e*x)/d])/d
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} - \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d(d+ex)}\right) dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x}dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}dx}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}dx, x, \frac{1}{x^3}\right)}{3d} - \frac{(3bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^3}\right)x^4}\, d}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} + \frac{(bp)\,\text{Subst}\left(\int \frac{\log(-}{a+b}\right.}{3d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{p\text{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d} - \frac{(3}{}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{3p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{3p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{3p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{3p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{3p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

Mathematica [A] time = 0.10, size = 395, normalized size = 1.02

$$-\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d} - \frac{\log\left(-\frac{b}{ax^3}\right)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3d} + \frac{p\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d} + \frac{p\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d} + \frac{p\text{Li}_2\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^3)^p]/(x*(d + e*x)),x]
```

```
[Out] -1/3*(Log[c*(a + b/x^3)^p]*Log[-(b/(a*x^3))])/d - (Log[c*(a + b/x^3)^p]*Log
[d + e*x])/d - (3*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b^(1/3)
+ a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/d + (p*Log[-(((-1)^(2
/3)*e*(b^(1/3) - (-1)^(1/3)*a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))
]*Log[d + e*x])/d + (p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/
(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/d - (p*PolyLog[2, (a + b/
x^3)/a])/(3*d) - (3*p*PolyLog[2, (d + e*x)/d])/d + (p*PolyLog[2, (a^(1/3)*(
```

```
d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d + (p*PolyLog[2, (a^(1/3)*(d + e*x))/(
a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d + (p*PolyLog[2, (a^(1/3)*(d + e*x))/(
a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax^3+b}{x^3}\right)^p\right)}{ex^2+dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^3 + b)/x^3)^p)/(e*x^2 + d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^3)^p*c)/((e*x + d)*x), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{(ex+d)\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^3)^p)/x/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x^3)^p)/x/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((a + b/x^3)^p*c)/((e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^3)^p)/(x*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b/x^3)^p)/(x*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**3)**p)/x/(e*x+d),x)
```

```
[Out] Timed out
```

3.259 $\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=557

$$\frac{\sqrt[3]{a}\,p\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}\right)}{2\sqrt[3]{b}\,d}+\frac{e\log\left(-\frac{b}{ax^3}\right)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3d^2}+\frac{e\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right.\right.}{dx}$$

```
[Out] 3*p/d/x-ln(c*(a+b/x^3)^p)/d/x+1/3*e*ln(c*(a+b/x^3)^p)*ln(-b/a/x^3)/d^2-a^(1
/3)*p*ln(b^(1/3)+a^(1/3)*x)/b^(1/3)/d+e*ln(c*(a+b/x^3)^p)*ln(e*x+d)/d^2+3*e
*p*ln(-e*x/d)*ln(e*x+d)/d^2-e*p*ln(-e*(b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-b^(1/3
)*e))*ln(e*x+d)/d^2-e*p*ln(-e*((-1)^(2/3)*b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-(-1
)^(2/3)*b^(1/3)*e))*ln(e*x+d)/d^2-e*p*ln((-1)^(1/3)*e*(b^(1/3)+(-1)^(2/3)*a
^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*x+d)/d^2+1/2*a^(1/3)*p*ln(
b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/b^(1/3)/d+1/3*e*p*polylog(2,1+b/a/x^
3)/d^2-e*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/d^2-e*p*polylog
(2,a^(1/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/d^2-e*p*polylog(2,a^(1
/3)*(e*x+d)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))/d^2+3*e*p*polylog(2,1+e*x/d)/
d^2-a^(1/3)*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(
1/3)/d
```

Rubi [A] time = 0.67, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {2466, 2455, 263, 325, 292, 31, 634, 617, 204, 628, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$\frac{ep\text{PolyLog}\left(2,\frac{b}{ax^3}+1\right)}{3d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^3)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (3*p)/(d*x) - (Sqrt[3]*a^(1/3)*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^
(1/3))])/(b^(1/3)*d) - Log[c*(a + b/x^3)^p]/(d*x) + (e*Log[c*(a + b/x^3)^p]
*Log[-(b/(a*x^3))])/(3*d^2) - (a^(1/3)*p*Log[b^(1/3) + a^(1/3)*x])/(b^(1/3)
*d) + (e*Log[c*(a + b/x^3)^p]*Log[d + e*x])/d^2 + (3*e*p*Log[-((e*x)/d)]*Lo
g[d + e*x])/d^2 - (e*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)
*e))]*Log[d + e*x])/d^2 - (e*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(
a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/d^2 - (e*p*Log[((-1)^(1/3
)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*L
og[d + e*x])/d^2 + (a^(1/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2
])/(2*b^(1/3)*d) + (e*p*PolyLog[2, 1 + b/(a*x^3)])/(3*d^2) - (e*p*PolyLog[2
, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (a^(1
/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (
a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d^2 + (3*e*p*PolyLo
g[2, 1 + (e*x)/d])/d^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx^2} - \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} + \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2}dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x}dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}dx}{d^2}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^2} + \frac{e\,\mathrm{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}dx, x,\right.}{3d^2}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex}{d^2}$$

$$= \frac{3p}{dx} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d}{d^2}$$

$$= \frac{3p}{dx} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d}{d^2}$$

$$= \frac{3p}{dx} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} - \frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{b}\,d} +$$

$$= \frac{3p}{dx} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} - \frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{b}\,d} +$$

$$= \frac{3p}{dx} - \frac{\sqrt{3}\sqrt[3]{a}\,p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}\,d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right.}{3d^2}$$

$$= \frac{3p}{dx} - \frac{\sqrt{3}\sqrt[3]{a}\,p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}\,d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right.}{3d^2}$$

Mathematica [C] time = 0.22, size = 429, normalized size = 0.77

$$4ax^3\left(3ex\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right) - 3d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right) + ex\log\left(-\frac{b}{ax^3}\right)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right) - 3epx\mathrm{Li}_2\left(\frac{\sqrt[3]{a}}{\sqrt[3]{a}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^3)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (9*b*d*p*Hypergeometric2F1[1, 4/3, 7/3, -(b/(a*x^3))] + 4*a*x^3*(-3*d*Log[c
*(a + b/x^3)^p] + e*x*Log[c*(a + b/x^3)^p]*Log[-(b/(a*x^3))] + 3*e*x*Log[c*
(a + b/x^3)^p]*Log[d + e*x] + 9*e*p*x*Log[-((e*x)/d)]*Log[d + e*x] - 3*e*p*
x*Log[(e*((-1)^(1/3)*b^(1/3) - a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*
e)]*Log[d + e*x] - 3*e*p*x*Log[(e*(b^(1/3) + a^(1/3)*x))/(-(a^(1/3)*d) + b^
(1/3)*e)]*Log[d + e*x] - 3*e*p*x*Log[(e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(
```

```
-(a^(1/3)*d) + (-1)^(2/3)*b^(1/3)*e)]*Log[d + e*x] + e*p*x*PolyLog[2, 1 + b
/(a*x^3)] - 3*e*p*x*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)]
 - 3*e*p*x*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e
)] - 3*e*p*x*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)
*e)] + 9*e*p*x*PolyLog[2, 1 + (e*x)/d]))/(12*a*d^2*x^4)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax^3+b}{x^3}\right)^p\right)}{ex^3+dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x^2/(e*x+d),x, algorithm="fricas")
[Out] integral(log(c*((a*x^3 + b)/x^3)^p)/(e*x^3 + d*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x^2/(e*x+d),x, algorithm="giac")
[Out] integrate(log((a + b/x^3)^p*c)/((e*x + d)*x^2), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{(ex+d)\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^3)^p)/x^2/(e*x+d),x)
[Out] int(ln(c*(a+b/x^3)^p)/x^2/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x^2/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((a + b/x^3)^p*c)/((e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^3)^p)/(x^2*(d + e*x)),x)
[Out] int(log(c*(a + b/x^3)^p)/(x^2*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**3)**p)/x**2/(e*x+d),x)
[Out] Timed out
```

3.260 $\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=737

$$-\frac{\sqrt[3]{a}\,ep\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{2\sqrt[3]{b}\,d^2}-\frac{a^{2/3}p\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{4b^{2/3}d}+\frac{a^{2/3}p\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{2b^{2/3}d}-\frac{\sqrt{3}\,a^{2/3}p\tan}{2b}$$

```
[Out] 3/4*p/d/x^2-3*e*p/d^2/x-1/2*ln(c*(a+b/x^3)^p)/d/x^2+e*ln(c*(a+b/x^3)^p)/d^2
/x-1/3*e^2*ln(c*(a+b/x^3)^p)*ln(-b/a/x^3)/d^3+1/2*a^(2/3)*p*ln(b^(1/3)+a^(1
/3)*x)/b^(2/3)/d+a^(1/3)*e*p*ln(b^(1/3)+a^(1/3)*x)/b^(1/3)/d^2-e^2*ln(c*(a+
b/x^3)^p)*ln(e*x+d)/d^3-3*e^2*p*ln(-e*x/d)*ln(e*x+d)/d^3+e^2*p*ln(-e*(b^(1/
3)+a^(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e*x+d)/d^3+e^2*p*ln(-e*((-1)^(2/3)*
b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/d^3+e^2*p*ln
((-1)^(1/3)*e*(b^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*
e))*ln(e*x+d)/d^3-1/4*a^(2/3)*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/b
^(2/3)/d-1/2*a^(1/3)*e*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/b^(1/3)/
d^2-1/3*e^2*p*polylog(2,1+b/a/x^3)/d^3+e^2*p*polylog(2,a^(1/3)*(e*x+d)/(a^(
1/3)*d-b^(1/3)*e))/d^3+e^2*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3
)*b^(1/3)*e))/d^3+e^2*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-(-1)^(2/3)*b^(
1/3)*e))/d^3-3*e^2*p*polylog(2,1+e*x/d)/d^3-1/2*a^(2/3)*p*arctan(1/3*(b^(1/
3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)/d+a^(1/3)*e*p*arctan(1/3*(
b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)/d^2
```

Rubi [A] time = 0.80, antiderivative size = 737, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 19, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.826, Rules used = {2466, 2455, 263, 325, 200, 31, 634, 617, 204, 628, 292, 2454, 2394, 2315, 2462, 260, 2416, 2393, 2391}

$$-\frac{e^2p\,\text{PolyLog}\left(2,\frac{b}{ax^3}+1\right)}{3d^3}+\frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^3}+\frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^3}+\frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+}{\sqrt[3]{a}d-(-1)}\right.}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^3)^p]/(x^3*(d + e*x)),x]
```

```
[Out] (3*p)/(4*d*x^2) - (3*e*p)/(d^2*x) - (Sqrt[3]*a^(2/3)*p*ArcTan[(b^(1/3) - 2*
a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(2*b^(2/3)*d) + (Sqrt[3]*a^(1/3)*e*p*ArcTan[
(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(b^(1/3)*d^2) - Log[c*(a + b/x^
3)^p]/(2*d*x^2) + (e*Log[c*(a + b/x^3)^p])/(d^2*x) - (e^2*Log[c*(a + b/x^3)
^p]*Log[-(b/(a*x^3))])/(3*d^3) + (a^(2/3)*p*Log[b^(1/3) + a^(1/3)*x])/(2*b^
(2/3)*d) + (a^(1/3)*e*p*Log[b^(1/3) + a^(1/3)*x])/(b^(1/3)*d^2) - (e^2*Log[
c*(a + b/x^3)^p]*Log[d + e*x])/d^3 - (3*e^2*p*Log[-((e*x)/d)]*Log[d + e*x])
/d^3 + (e^2*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log
[d + e*x])/d^3 + (e^2*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)
*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/d^3 + (e^2*p*Log[((-1)^(1/3)*e*(
b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d
+ e*x])/d^3 - (a^(2/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(4
*b^(2/3)*d) - (a^(1/3)*e*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/
(2*b^(1/3)*d^2) - (e^2*p*PolyLog[2, 1 + b/(a*x^3)])/(3*d^3) + (e^2*p*PolyLo
g[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d^3 + (e^2*p*PolyLog[2,
(a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d^3 + (e^2*p*PolyL
og[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d^3 - (3*e^2
*p*PolyLog[2, 1 + (e*x)/d])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
```

```
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx^3} - \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x^2} + \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^3x} - \frac{e^3\log\left(c\left(a+\frac{b}{x^3}\right.\right.}{d^3(d+ex)}\right.$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x}\,dx}{d^3} - \frac{e^3\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx}{d^3}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^3} - \frac{e^2\,\mathrm{Sub}}{}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^3} - \frac{e^2\log\left(\right.}{}$$

$$= \frac{3p}{4dx^2} - \frac{3ep}{d^2x} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{}{a}\right.}{3d^3}$$

$$= \frac{3p}{4dx^2} - \frac{3ep}{d^2x} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{}{a}\right.}{3d^3}$$

$$= \frac{3p}{4dx^2} - \frac{3ep}{d^2x} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{}{a}\right.}{3d^3}$$

$$= \frac{3p}{4dx^2} - \frac{3ep}{d^2x} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{}{a}\right.}{3d^3}$$

$$= \frac{3p}{4dx^2} - \frac{3ep}{d^2x} - \frac{\sqrt{3}\,a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{2b^{2/3}d} + \frac{\sqrt{3}\,\sqrt[3]{a}\,ep\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{b}\,d^2} - \frac{\log\left(c\left(a\right.\right.}{2dx}$$

$$= \frac{3p}{4dx^2} - \frac{3ep}{d^2x} - \frac{\sqrt{3}\,a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{2b^{2/3}d} + \frac{\sqrt{3}\,\sqrt[3]{a}\,ep\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{b}\,d^2} - \frac{\log\left(c\left(a\right.\right.}{2dx}$$

Mathematica [C] time = 0.30, size = 520, normalized size = 0.71

$$\frac{-10ax^3\left(3d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right) + 6e^2x^2\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right) - 6dex\log\left(c\left(a+\frac{b}{x^3}\right)^p\right) + 2e^2x^2\log\left(-\frac{b}{ax^3}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^3)^p]/(x^3*(d + e*x)),x]
```

```
[Out] (-45*b*d*e*p*x*Hypergeometric2F1[1, 4/3, 7/3, -(b/(a*x^3))] + 18*b*d^2*p*Hy
pergeometric2F1[1, 5/3, 8/3, -(b/(a*x^3))] - 10*a*x^3*(3*d^2*Log[c*(a + b/x
```

```
^3)^p] - 6*d*e*x*Log[c*(a + b/x^3)^p] + 2*e^2*x^2*Log[c*(a + b/x^3)^p]*Log[
-(b/(a*x^3))] + 6*e^2*x^2*Log[c*(a + b/x^3)^p]*Log[d + e*x] + 18*e^2*p*x^2*
Log[-((e*x)/d)]*Log[d + e*x] - 6*e^2*p*x^2*Log[(e*((-1)^(1/3)*b^(1/3) - a^(
1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x] - 6*e^2*p*x^2*Log
[(e*(b^(1/3) + a^(1/3)*x))/(-(a^(1/3)*d) + b^(1/3)*e)]*Log[d + e*x] - 6*e^2
*p*x^2*Log[(e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(-(a^(1/3)*d) + (-1)^(2/3)*
b^(1/3)*e)]*Log[d + e*x] + 2*e^2*p*x^2*PolyLog[2, 1 + b/(a*x^3)] - 6*e^2*p*
x^2*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)] - 6*e^2*p*x^2*P
olyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)] - 6*e^2*p
*x^2*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)] + 1
8*e^2*p*x^2*PolyLog[2, 1 + (e*x)/d]))/(60*a*d^3*x^5)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(c\left(\frac{ax^3+b}{x^3}\right)^p\right)}{ex^4+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^3 + b)/x^3)^p)/(e*x^4 + d*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^3)^p*c)/((e*x + d)*x^3), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{(ex+d)\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^3)^p)/x^3/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x^3)^p)/x^3/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((a + b/x^3)^p*c)/((e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^3)^p)/(x^3*(d + e*x)),x)
[Out] int(log(c*(a + b/x^3)^p)/(x^3*(d + e*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**3)**p)/x**3/(e*x+d),x)
[Out] Timed out
```

3.261 $\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=749

$$\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt{f}\sqrt{g}}+\frac{ip\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(i\sqrt[3]{e}\sqrt{f}+\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{Li}_2\left(\frac{2i\sqrt{f}\sqrt{g}\left(\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}\right)}{\left(\sqrt[3]{e}\sqrt{f}+\sqrt[6]{-1}\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}+1\right)}{2\sqrt{f}\sqrt{g}}+$$

```
[Out] arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^3+d)^p)/f^(1/2)/g^(1/2)+3*p*arctan(x*g^
(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan
(x*g^(1/2)/f^(1/2))*ln(2*(d^(1/3)+e^(1/3)*x)*f^(1/2)*g^(1/2)/(I*e^(1/3)*f^(
1/2)+d^(1/3)*g^(1/2))/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan(x*g^(
1/2)/f^(1/2))*ln(-2*I*((-1)^(2/3)*d^(1/3)+e^(1/3)*x)*f^(1/2)*g^(1/2)/(e^(1/
3)*f^(1/2)+(-1)^(1/6)*d^(1/3)*g^(1/2))/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/
2)-p*arctan(x*g^(1/2)/f^(1/2))*ln(2*(-1)^(5/6)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*
x)*f^(1/2)*g^(1/2)/(e^(1/3)*f^(1/2)+(-1)^(5/6)*d^(1/3)*g^(1/2))/(f^(1/2)-I*
x*g^(1/2)))/f^(1/2)/g^(1/2)-3/2*I*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1
/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1-2*(d^(1/3)+e^(1/3)*x)*f^(1/2)*g^(
1/2)/(I*e^(1/3)*f^(1/2)+d^(1/3)*g^(1/2))/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(
1/2)+1/2*I*p*polylog(2,1+2*I*((-1)^(2/3)*d^(1/3)+e^(1/3)*x)*f^(1/2)*g^(1/2)
/(e^(1/3)*f^(1/2)+(-1)^(1/6)*d^(1/3)*g^(1/2))/(f^(1/2)-I*x*g^(1/2)))/f^(1/2
)/g^(1/2)+1/2*I*p*polylog(2,1-2*(-1)^(5/6)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)*f
^(1/2)*g^(1/2)/(e^(1/3)*f^(1/2)+(-1)^(5/6)*d^(1/3)*g^(1/2))/(f^(1/2)-I*x*g^
(1/2)))/f^(1/2)/g^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 749, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {205, 2470, 12, 260, 6725, 4856, 2402, 2315, 2447}

$$\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt[3]{d}\sqrt{g}+i\sqrt[3]{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1+\frac{2i\sqrt{f}\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt[6]{-1}\sqrt[3]{d}\sqrt{g}+\sqrt[3]{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]/(f + g*x^2),x]
```

```
[Out] (3*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(
d^(1/3) + e^(1/3)*x))/((I*e^(1/3)*Sqrt[f] + d^(1/3)*Sqrt[g])*(Sqrt[f] - I*S
qrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[((-2*I)
*Sqrt[f]*Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((e^(1/3)*Sqrt[f] + (-1)
^(1/6)*d^(1/3)*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*A
rcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*(-1)^(5/6)*Sqrt[f]*Sqrt[g]*(d^(1/3) + (-1
)^(2/3)*e^(1/3)*x))/((e^(1/3)*Sqrt[f] + (-1)^(5/6)*d^(1/3)*Sqrt[g])*(Sqrt[f
] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*
(d + e*x^3)^p])/(Sqrt[f]*Sqrt[g]) - (((3*I)/2)*p*PolyLog[2, 1 - (2*Sqrt[f])
/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 - (2*S
qrt[f]*Sqrt[g]*(d^(1/3) + e^(1/3)*x))/((I*e^(1/3)*Sqrt[f] + d^(1/3)*Sqrt[g]
)*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + ((
2*I)*Sqrt[f]*Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((e^(1/3)*Sqrt[f] +
(-1)^(1/6)*d^(1/3)*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) +
((I/2)*p*PolyLog[2, 1 - (2*(-1)^(5/6)*Sqrt[f]*Sqrt[g]*(d^(1/3) + (-1)^(2/3)
*e^(1/3)*x))/((e^(1/3)*Sqrt[f] + (-1)^(5/6)*d^(1/3)*Sqrt[g])*(Sqrt[f] - I*S
qrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{f+gx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt{f}\,\sqrt{g}} - (3ep)\int \frac{x^2\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}\,\sqrt{g}\,\left(d+ex^3\right)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt{f}\,\sqrt{g}} - \frac{(3ep)\int \frac{x^2\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{d+ex^3}\,dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt{f}\,\sqrt{g}} - \frac{(3ep)\int \left(\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{3e^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{3e^{2/3}\left(-\sqrt[3]{-1}\,\sqrt[3]{d}+\sqrt[3]{e}\,x\right)} + \frac{\text{ta}}{3e^{2/3}(-}\right.}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt{f}\,\sqrt{g}} - \frac{\left(\sqrt[3]{e}\,p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt[3]{d}+\sqrt[3]{e}\,x}\,dx}{\sqrt{f}\,\sqrt{g}} - \frac{\left(\sqrt[3]{e}\,p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{-\sqrt[3]{-1}\,\sqrt[3]{d}+\sqrt[3]{e}\,x}\,dx}{\sqrt{f}\,\sqrt{g}} - $$

$$= \frac{3p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(i\sqrt[3]{e}\,\sqrt{f}+\sqrt[3]{d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan}{}$$

$$= \frac{3p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(i\sqrt[3]{e}\,\sqrt{f}+\sqrt[3]{d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan}{}$$

$$= \frac{3p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(i\sqrt[3]{e}\,\sqrt{f}+\sqrt[3]{d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan}{}$$

Mathematica [A] time = 0.62, size = 867, normalized size = 1.16

$$\frac{-p\log\left(\frac{\sqrt{g}\left(\sqrt[3]{e}\,x+\sqrt[3]{d}\right)}{\sqrt[3]{e}\,\sqrt{-f}+\sqrt[3]{d}\,\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}\,x\right) - p\log\left(\frac{\sqrt{g}\left(\sqrt[3]{e}\,x-\sqrt[3]{-1}\,\sqrt[3]{d}\right)}{\sqrt[3]{e}\,\sqrt{-f}-\sqrt[3]{-1}\,\sqrt[3]{d}\,\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}\,x\right) - p\log\left(\frac{\sqrt{g}\left(\sqrt[3]{e}\,x+(-1)^{2/3}\right.}{\sqrt[3]{e}\,\sqrt{-f}+(-1)^{2/3}\,\sqrt[3]{}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]/(f + g*x^2),x]
```

```
[Out] (-(p*Log[(Sqrt[g]*(d^(1/3) + e^(1/3)*x))/(e^(1/3)*Sqrt[-f] + d^(1/3)*Sqrt[g
])]*Log[Sqrt[-f] - Sqrt[g]*x]) - p*Log[(Sqrt[g]*(-((-1)^(1/3)*d^(1/3)) + e^
(1/3)*x))/(e^(1/3)*Sqrt[-f] - (-1)^(1/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] - S
qrt[g]*x] - p*Log[(Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/(e^(1/3)*Sqrt[
-f] + (-1)^(2/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*x] + p*Log[-((Sqr
t[g]*(d^(1/3) + e^(1/3)*x))/(e^(1/3)*Sqrt[-f] - d^(1/3)*Sqrt[g]))]*Log[Sqrt
[-f] + Sqrt[g]*x] + p*Log[(Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/(-(e^(
1/3)*Sqrt[-f]) + (-1)^(2/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*x] + p
*Log[((-1)^(1/3)*Sqrt[g]*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/(e^(1/3)*Sqrt[-f
] + (-1)^(1/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*x] + Log[Sqrt[-f] -
 Sqrt[g]*x]*Log[c*(d + e*x^3)^p] - Log[Sqrt[-f] + Sqrt[g]*x]*Log[c*(d + e*x
^3)^p] - p*PolyLog[2, (e^(1/3)*(Sqrt[-f] - Sqrt[g]*x))/(e^(1/3)*Sqrt[-f] +
d^(1/3)*Sqrt[g])] - p*PolyLog[2, (e^(1/3)*(Sqrt[-f] - Sqrt[g]*x))/(e^(1/3)*
```

```
Sqrt[-f] - (-1)^(1/3)*d^(1/3)*Sqrt[g])] - p*PolyLog[2, (e^(1/3)*(Sqrt[-f] -
 Sqrt[g]*x))/(e^(1/3)*Sqrt[-f] + (-1)^(2/3)*d^(1/3)*Sqrt[g])] + p*PolyLog[2
, (e^(1/3)*(Sqrt[-f] + Sqrt[g]*x))/(e^(1/3)*Sqrt[-f] - d^(1/3)*Sqrt[g])] +
p*PolyLog[2, (e^(1/3)*(Sqrt[-f] + Sqrt[g]*x))/(e^(1/3)*Sqrt[-f] + (-1)^(1/3
)*d^(1/3)*Sqrt[g])] + p*PolyLog[2, (e^(1/3)*(Sqrt[-f] + Sqrt[g]*x))/(e^(1/3
)*Sqrt[-f] - (-1)^(2/3)*d^(1/3)*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^3+d\right)^p c\right)}{gx^2+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^3 + d)^p*c)/(g*x^2 + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^3+d\right)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)/(g*x^2 + f), x)
```

maple [C] time = 0.78, size = 327, normalized size = 0.44

$$-\frac{i\pi \arctan\left(\frac{gx}{\sqrt{fg}}\right)\operatorname{csgn}(ic)\operatorname{csgn}\left(i\left(e\,x^3+d\right)^p\right)\operatorname{csgn}\left(ic\left(e\,x^3+d\right)^p\right)}{2\sqrt{fg}}+\frac{i\pi \arctan\left(\frac{gx}{\sqrt{fg}}\right)\operatorname{csgn}(ic)\operatorname{csgn}\left(ic\left(e\,x^3\right.\right.}{2\sqrt{fg}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)/(g*x^2+f),x)
```

```
[Out] (ln((e*x^3+d)^p)-p*ln(e*x^3+d))/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)+1/2*p
/g*sum(1/_alpha*(ln(x-_alpha)*ln(e*x^3+d)-sum(ln(x-_alpha)*ln((_R1-x+_alpha
)/_R1)+dilog((_R1-x+_alpha)/_R1),_R1=RootOf(_Z^3*e*g+3*_Z^2*_alpha*e*g-3*_Z
*e*f-_alpha*e*f+d*g))),_alpha=RootOf(_Z^2*g+f))+1/2*I/(f*g)^(1/2)*arctan(1/
(f*g)^(1/2)*g*x)*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-1/2*I/(f*g)
^(1/2)*arctan(1/(f*g)^(1/2)*g*x)*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^
p)*csgn(I*c)-1/2*I/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)*Pi*csgn(I*c*(e*x^3
+d)^p)^3+1/2*I/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)*Pi*csgn(I*c*(e*x^3+d)^
p)^2*csgn(I*c)+1/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)*ln(c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^3+d\right)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)/(g*x^2 + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(e\,x^3+d\right)^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^3)^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e*x^3)^p)/(f + g*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)/(g*x**2+f),x)
[Out] Timed out
```

3.262 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=533

$$\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}+\frac{ip\text{Li}_2\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}+1\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{2\sqrt{f}\sqrt{g}}-$$

```
[Out] arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(1/2)/g^(1/2)+2*p*arctan(x*g^
(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan
(x*g^(1/2)/f^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-p*arcta
n(x*g^(1/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-I*p*pol
ylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2
,1+2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2
)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1-2*((-d)^
(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(
-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {205, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,\right.}{\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] (2*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*S
qrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[
g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt
[f]]*Log[c*(d + e*x^2)^p])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (
2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sq
rt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1
 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d
]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\,\sqrt{g}} - (2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}\,\sqrt{g}\,\left(d+ex^2\right)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\,\sqrt{g}} - \frac{(2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\,\sqrt{g}} - \frac{(2ep)\int \left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{\left(\sqrt{e}\,p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{\sqrt{f}\,\sqrt{g}} - \frac{\left(\sqrt{e}\,p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{f}\,\sqrt{g}} -$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{f}\,\sqrt{g}} -$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{f}\,\sqrt{g}} -$$

Mathematica [A] time = 0.37, size = 564, normalized size = 1.06

$$-\frac{i\left(2i\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right) + p\,\mathrm{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}-i\sqrt{-d}\,\sqrt{g}}\right) + p\,\mathrm{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}+i\sqrt{-d}\,\sqrt{g}}\right) - p\,\mathrm{Li}_2\left(\frac{\sqrt{e}\left(i\sqrt{g}\,x+\sqrt{f}\right)}{\sqrt{e}\,\sqrt{f}-i\sqrt{-d}\,\sqrt{g}}\right) -}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] ((-1/2*I)*(p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt
[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + p*Log[(Sqrt[g]*(Sqrt[-d] +
Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x
)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] +
 Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-
d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]
*x)/Sqrt[f]] + (2*I)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + p*P
olyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*S
qrt[g])] + p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f]
+ I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sq
rt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*S
qrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/(Sqrt[f]*Sqrt[g])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

maple [C] time = 1.48, size = 504, normalized size = 0.95

$$-\frac{i\pi \arctan\left(\frac{gx}{\sqrt{fg}}\right)\operatorname{csgn}(ic)\operatorname{csgn}\left(i\left(ex^2+d\right)^p\right)\operatorname{csgn}\left(ic\left(ex^2+d\right)^p\right)}{2\sqrt{fg}}+\frac{i\pi \arctan\left(\frac{gx}{\sqrt{fg}}\right)\operatorname{csgn}(ic)\operatorname{csgn}\left(ic\left(ex^2+c\right.\right.}{2\sqrt{fg}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

```
[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)+1/2*p
/g*sum(1/_alpha*(ln(-_alpha+x)*ln(e*x^2+d)-ln(-_alpha+x)*(ln((RootOf(_Z^2*e
*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e
*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x
+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))-dilog((RootOf(_
Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_al
pha*e*g+d*g-e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,in
dex=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2))),_alpha=
RootOf(_Z^2*g+f))+1/2*I/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)*Pi*csgn(I*(e*
x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*
x)*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I/(f*g)^(1/2)
*arctan(1/(f*g)^(1/2)*g*x)*Pi*csgn(I*c*(e*x^2+d)^p)^3+1/2*I/(f*g)^(1/2)*arc
tan(1/(f*g)^(1/2)*g*x)*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/(f*g)^(1/2)*a
rctan(1/(f*g)^(1/2)*g*x)*ln(c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^2),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Integral(log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

3.263 $\int \frac{\log(c(d+ex)^p)}{f+gx^2}\,dx$

Optimal. Leaf size=229

$$\frac{\log(c(d+ex)^p)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log(c(d+ex)^p)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{p\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2\sqrt{-f}\sqrt{g}}$$

```
[Out] 1/2*ln(c*(e*x+d)^p)*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/(
-f)^(1/2)/g^(1/2)-1/2*ln(c*(e*x+d)^p)*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(
1/2)-d*g^(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*p*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f
)^(1/2)-d*g^(1/2)))/(-f)^(1/2)/g^(1/2)+1/2*p*polylog(2,(e*x+d)*g^(1/2)/(e*(
-f)^(1/2)+d*g^(1/2)))/(-f)^(1/2)/g^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2409, 2394, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\log(c(d+ex)^p)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log(c(d+ex)^p)\log}{2\sqrt{-f}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x)^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e*x)^p]*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])
])/(2*Sqrt[-f]*Sqrt[g]) - (Log[c*(d + e*x)^p]*Log[(e*(Sqrt[-f] + Sqrt[g]*x)
)/(e*Sqrt[-f] - d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, -((Sqrt[g
]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[
2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g])
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule steps

$$\int \frac{\log(c(d+ex)^p)}{f+gx^2}\,dx = \int \left(\frac{\sqrt{-f}\,\log(c(d+ex)^p)}{2f\left(\sqrt{-f}-\sqrt{g}\,x\right)} + \frac{\sqrt{-f}\,\log(c(d+ex)^p)}{2f\left(\sqrt{-f}+\sqrt{g}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{\log(c(d+ex)^p)}{\sqrt{-f}-\sqrt{g}\,x}\,dx}{2\sqrt{-f}} - \frac{\int \frac{\log(c(d+ex)^p)}{\sqrt{-f}+\sqrt{g}\,x}\,dx}{2\sqrt{-f}}$$

$$= \frac{\log(c(d+ex)^p)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\,\sqrt{g}} - \frac{\log(c(d+ex)^p)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}\,x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\,\sqrt{g}} - \frac{(ep)\int \frac{\log\left(\frac{e(}{e}\right.}{}}{2\sqrt{-}}$$

$$= \frac{\log(c(d+ex)^p)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\,\sqrt{g}} - \frac{\log(c(d+ex)^p)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}\,x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\,\sqrt{g}} + \frac{p\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{\log(c(d+ex)^p)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\,\sqrt{g}} - \frac{\log(c(d+ex)^p)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}\,x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\,\sqrt{g}} - \frac{p\text{Li}_2\left(-\frac{\sqrt{g}(}{e\sqrt{-f}}\right.}{2\sqrt{-f}\,\sqrt{}}$$

Mathematica [A] time = 0.13, size = 178, normalized size = 0.78

$$\frac{\log(c(d+ex)^p)\left(\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x\right)}{d\sqrt{g}+e\sqrt{-f}}\right)-\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}\,x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)\right)-p\text{Li}_2\left(-\frac{\sqrt{g}\,(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)+p\text{Li}_2\left(\frac{\sqrt{g}\,(d+ex)}{\sqrt{g}\,d+e\sqrt{-f}}\right)}{2\sqrt{-f}\,\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x)^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e*x)^p]*(Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g]
)] - Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])]) - p*PolyLog[
2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + p*PolyLog[2, (Sqrt[g]
*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left((ex+d)^p c\right)}{gx^2+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x + d)^p*c)/(g*x^2 + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left((ex+d)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x + d)^p*c)/(g*x^2 + f), x)
```

maple [C] time = 0.67, size = 419, normalized size = 1.83

$$-\frac{i\pi \arctan\left(\frac{gx}{\sqrt{fg}}\right) \mathrm{csgn}\left(ic\right) \mathrm{csgn}\left(i\left(ex+d\right)^{p}\right) \mathrm{csgn}\left(ic\left(ex+d\right)^{p}\right)}{2\sqrt{fg}}+\frac{i\pi \arctan\left(\frac{gx}{\sqrt{fg}}\right) \mathrm{csgn}\left(ic\right) \mathrm{csgn}\left(ic\left(ex+d\right)^{p}\right)^{2}}{2\sqrt{fg}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x+d)^p)/(g*x^2+f),x)
```

```
[Out] (ln((e*x+d)^p)-p*ln(e*x+d))/(f*g)^(1/2)*arctan(1/2*(-2*d*g+2*(e*x+d)*g)/(f*
g)^(1/2)/e)+1/2*p*ln(e*x+d)/(-f*g)^(1/2)*ln((d*g+(-f*g)^(1/2)*e-(e*x+d)*g)/
(d*g+(-f*g)^(1/2)*e))-1/2*p*ln(e*x+d)/(-f*g)^(1/2)*ln((-d*g+(-f*g)^(1/2)*e+
(e*x+d)*g)/(-d*g+(-f*g)^(1/2)*e))+1/2*p/(-f*g)^(1/2)*dilog((d*g+(-f*g)^(1/2
)*e-(e*x+d)*g)/(d*g+(-f*g)^(1/2)*e))-1/2*p/(-f*g)^(1/2)*dilog((-d*g+(-f*g)^
(1/2)*e+(e*x+d)*g)/(-d*g+(-f*g)^(1/2)*e))+1/2*I/(f*g)^(1/2)*arctan(1/(f*g)^
(1/2)*g*x)*Pi*csgn(I*(e*x+d)^p)*csgn(I*c*(e*x+d)^p)^2-1/2*I/(f*g)^(1/2)*arc
tan(1/(f*g)^(1/2)*g*x)*Pi*csgn(I*(e*x+d)^p)*csgn(I*c*(e*x+d)^p)*csgn(I*c)-1
/2*I/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)*Pi*csgn(I*c*(e*x+d)^p)^3+1/2*I/(
f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)*Pi*csgn(I*c*(e*x+d)^p)^2*csgn(I*c)+1/(
f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)*ln(c)
```

maxima [C] time = 1.29, size = 309, normalized size = 1.35

$$\frac{ep\left(\frac{2\arctan\left(\frac{gx}{\sqrt{fg}}\right)\log(ex+d)}{e}+\frac{\arctan\left(\frac{(e^2x+de)\sqrt{f}\sqrt{g}}{e^2f+d^2g},\frac{degx+d^2g}{e^2f+d^2g}\right)\log\left(gx^2+f\right)-\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{e^2gx^2+2degx+d^2g}{e^2f+d^2g}\right)-i\,\mathrm{Li}_2\left(\frac{degx+e^2f-(ie^2x-ide)\sqrt{f}\sqrt{g}}{e^2f+2ide\sqrt{f}\sqrt{g}-d^2g}\right.}{e}\right.}{2\sqrt{fg}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*e*p*(2*arctan(g*x/sqrt(f*g))*log(e*x + d)/e + (arctan2((e^2*x + d*e)*sq
rt(f)*sqrt(g)/(e^2*f + d^2*g), (d*e*g*x + d^2*g)/(e^2*f + d^2*g))*log(g*x^2
 + f) - arctan(sqrt(g)*x/sqrt(f))*log((e^2*g*x^2 + 2*d*e*g*x + d^2*g)/(e^2*
f + d^2*g)) - I*dilog((d*e*g*x + e^2*f - (I*e^2*x - I*d*e)*sqrt(f)*sqrt(g))
/(e^2*f + 2*I*d*e*sqrt(f)*sqrt(g) - d^2*g)) + I*dilog((d*e*g*x + e^2*f + (I
*e^2*x - I*d*e)*sqrt(f)*sqrt(g))/(e^2*f - 2*I*d*e*sqrt(f)*sqrt(g) - d^2*g))
)/e)/sqrt(f*g) - p*arctan(g*x/sqrt(f*g))*log(e*x + d)/sqrt(f*g) + arctan(g*
x/sqrt(f*g))*log((e*x + d)^p*c)/sqrt(f*g)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(d+e\,x\right)}^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x)^p)/(f + g*x^2),x)
```

```
[Out] int(log(c*(d + e*x)^p)/(f + g*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+ex\right)^{p}\right)}{f+gx^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x+d)**p)/(g*x**2+f),x)
```

```
[Out] Integral(log(c*(d + e*x)**p)/(f + g*x**2), x)
```

3.264 $\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=360

$$\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}}+\frac{ip\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(e+dx)}{(i\sqrt{f}d+e\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}}-\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(dx+e)}{(\sqrt{f}-i\sqrt{g}x)(e\sqrt{g}+id\sqrt{f})}\right)}{\sqrt{f}\sqrt{g}}+\frac{ip}{}$$

```
[Out] arctan(x*g^(1/2)/f^(1/2))*ln(c*(d+e/x)^p)/f^(1/2)/g^(1/2)+p*arctan(x*g^(1/2
)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan(x*g
^(1/2)/f^(1/2))*ln(2*(d*x+e)*f^(1/2)*g^(1/2)/(I*d*f^(1/2)+e*g^(1/2))/(f^(1/
2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,-I*x*g^(1/2)/f^(1/2))/f^
(1/2)/g^(1/2)-1/2*I*p*polylog(2,I*x*g^(1/2)/f^(1/2))/f^(1/2)/g^(1/2)-1/2*I*
p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*poly
log(2,1-2*(d*x+e)*f^(1/2)*g^(1/2)/(I*d*f^(1/2)+e*g^(1/2))/(f^(1/2)-I*x*g^(1
/2)))/f^(1/2)/g^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {205, 2470, 12, 260, 6688, 4876, 4848, 2391, 4856, 2402, 2315, 2447}

$$\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(dx+e)}{(\sqrt{f}-i\sqrt{g}x)(e\sqrt{g}+id\sqrt{f})}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e/x)^p]/(f + g*x^2),x]
```

```
[Out] (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e/x)^p])/(Sqrt[f]*Sqrt[g]) + (p*Arc
Tan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]
*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(e + d*x)
)/((I*d*Sqrt[f] + e*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) +
 ((I/2)*p*PolyLog[2, ((-I)*Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - ((I/2)*
p*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - ((I/2)*p*PolyLog[2
, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*Po
lyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(e + d*x))/((I*d*Sqrt[f] + e*Sqrt[g])*(Sqrt
[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + (ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(d+\frac{e}{x}\right)x^2}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\left(d+\frac{e}{x}\right)x^2}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{x(e+dx)}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(ep)\int \left(\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{ex} - \frac{d\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{e(e+dx)}\right)dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{p\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{x}\,dx}{\sqrt{f}\sqrt{g}} - \frac{(dp)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{e+dx}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\cdots\right.}{\sqrt{f}\cdots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\cdots\right.}{\sqrt{f}\cdots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\cdots\right.}{\sqrt{f}\cdots}$$

Mathematica [A] time = 0.26, size = 373, normalized size = 1.04

$$\frac{\log\left(\sqrt{-f}-\sqrt{g}\,x\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)-\log\left(\sqrt{-f}+\sqrt{g}\,x\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)-p\,\text{Li}_2\left(\frac{d\left(\sqrt{-f}-\sqrt{g}\,x\right)}{\sqrt{-f}\,d+e\sqrt{g}}\right)+p\,\text{Li}_2\left(\frac{d\left(\sqrt{g}\,x+\cdots\right.}{d\sqrt{-f}-\cdots}\right.\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/x)^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e/x)^p]*Log[Sqrt[-f] - Sqrt[g]*x] + p*Log[(Sqrt[g]*x)/Sqrt[-f]]
*Log[Sqrt[-f] - Sqrt[g]*x] - p*Log[(Sqrt[g]*(e + d*x))/(d*Sqrt[-f] + e*Sqrt
[g])]*Log[Sqrt[-f] - Sqrt[g]*x] - Log[c*(d + e/x)^p]*Log[Sqrt[-f] + Sqrt[g]
*x] - p*Log[(f*Sqrt[g]*x)/(-f)^(3/2)]*Log[Sqrt[-f] + Sqrt[g]*x] + p*Log[-((
Sqrt[g]*(e + d*x))/(d*Sqrt[-f] - e*Sqrt[g]))]*Log[Sqrt[-f] + Sqrt[g]*x] - p
*PolyLog[2, (d*(Sqrt[-f] - Sqrt[g]*x))/(d*Sqrt[-f] + e*Sqrt[g])] + p*PolyLo
g[2, (d*(Sqrt[-f] + Sqrt[g]*x))/(d*Sqrt[-f] - e*Sqrt[g])] - p*PolyLog[2, 1
+ (Sqrt[g]*x)/Sqrt[-f]] + p*PolyLog[2, 1 + (f*Sqrt[g]*x)/(-f)^(3/2)])/(2*Sq
rt[-f]*Sqrt[g])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(c\left(\frac{dx+e}{x}\right)^{p}\right)}{gx^{2}+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log(c*((d*x + e)/x)^p)/(g*x^2 + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^{p}\right)}{gx^{2}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log(c*(d + e/x)^p)/(g*x^2 + f), x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(d+\frac{e}{x}\right)^{p}\right)}{g\,x^{2}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/x)^p)/(g*x^2+f),x)
```

```
[Out] int(ln(c*(d+e/x)^p)/(g*x^2+f),x)
```

maxima [A] time = 1.27, size = 377, normalized size = 1.05

$$\frac{ep\left(\frac{4\arctan\left(\frac{gx}{\sqrt{fg}}\right)\log\left(d+\frac{e}{x}\right)}{e}-\frac{\left(\pi-2\arctan\left(\frac{\left(d^{2}x+de\right)\sqrt{f}\sqrt{g}}{d^{2}f+e^{2}g},\frac{degx+e^{2}g}{d^{2}f+e^{2}g}\right)\right)\log\left(gx^{2}+f\right)-4\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)+2\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{d^{2}gx^{2}+2degx+}{d^{2}f+e^{2}g}\right.}{}\right.}{4\sqrt{fg}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/4*e*p*(4*arctan(g*x/sqrt(f*g))*log(d + e/x)/e - ((pi - 2*arctan2((d^2*x +
 d*e)*sqrt(f)*sqrt(g)/(d^2*f + e^2*g), (d*e*g*x + e^2*g)/(d^2*f + e^2*g)))*
log(g*x^2 + f) - 4*arctan(sqrt(g)*x/sqrt(f))*log(sqrt(g)*x/sqrt(f)) + 2*arc
tan(sqrt(g)*x/sqrt(f))*log((d^2*g*x^2 + 2*d*e*g*x + e^2*g)/(d^2*f + e^2*g))
 + 2*I*dilog((I*sqrt(g)*x + sqrt(f))/sqrt(f)) - 2*I*dilog(-(I*sqrt(g)*x - s
qrt(f))/sqrt(f)) + 2*I*dilog((d*e*g*x + d^2*f - (I*d^2*x - I*d*e)*sqrt(f)*s
qrt(g))/(d^2*f + 2*I*d*e*sqrt(f)*sqrt(g) - e^2*g)) - 2*I*dilog((d*e*g*x + d
^2*f + (I*d^2*x - I*d*e)*sqrt(f)*sqrt(g))/(d^2*f - 2*I*d*e*sqrt(f)*sqrt(g)
- e^2*g)))/e)/sqrt(f*g) - p*arctan(g*x/sqrt(f*g))*log(d + e/x)/sqrt(f*g) +
arctan(g*x/sqrt(f*g))*log(c*(d + e/x)^p)/sqrt(f*g)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(d+\frac{e}{x}\right)^{p}\right)}{g\,x^{2}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/x)^p)/(f + g*x^2),x)
```

```
[Out] int(log(c*(d + e/x)^p)/(f + g*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^{p}\right)}{f+gx^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/x)**p)/(g*x**2+f),x)
```

```
[Out] Integral(log(c*(d + e/x)**p)/(f + g*x**2), x)
```

3.265 $\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2} dx$

Optimal. Leaf size=597

$$\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}}+\frac{ip\text{Li}_2\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{e}-\sqrt{-d}x\right)}{\left(i\sqrt{-d}\sqrt{f}-\sqrt{e}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}+1\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}x+\sqrt{e}\right)}{\left(i\sqrt{-d}\sqrt{f}+\sqrt{e}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}}-\frac{p\tan}{}$$

```
[Out] arctan(x*g^(1/2)/f^(1/2))*ln(c*(d+e/x^2)^p)/f^(1/2)/g^(1/2)+2*p*arctan(x*g^
(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan
(x*g^(1/2)/f^(1/2))*ln(-2*(-x*(-d)^(1/2)+e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-
I*x*g^(1/2))/(I*(-d)^(1/2)*f^(1/2)-e^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-p*arct
an(x*g^(1/2)/f^(1/2))*ln(2*(x*(-d)^(1/2)+e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-
I*x*g^(1/2))/(I*(-d)^(1/2)*f^(1/2)+e^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)+I*p*po
lylog(2,-I*x*g^(1/2)/f^(1/2))/f^(1/2)/g^(1/2)-I*p*polylog(2,I*x*g^(1/2)/f^(
1/2))/f^(1/2)/g^(1/2)-I*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1
/2)/g^(1/2)+1/2*I*p*polylog(2,1+2*(-x*(-d)^(1/2)+e^(1/2))*f^(1/2)*g^(1/2)/(
f^(1/2)-I*x*g^(1/2))/(I*(-d)^(1/2)*f^(1/2)-e^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2
)+1/2*I*p*polylog(2,1-2*(x*(-d)^(1/2)+e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x
*g^(1/2))/(I*(-d)^(1/2)*f^(1/2)+e^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 597, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {205, 2470, 12, 260, 6688, 4928, 4848, 2391, 4856, 2402, 2315, 2447}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{e}-\sqrt{-d}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{e}\sqrt{g}+i\sqrt{-d}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}x+\sqrt{e}\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{e}\sqrt{g}+i\sqrt{-d}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,-\frac{i}{}\right.}{\sqrt{f}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e/x^2)^p]/(f + g*x^2),x]
```

```
[Out] (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e/x^2)^p])/(Sqrt[f]*Sqrt[g]) + (2*p
*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqr
t[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqr
t[e] - Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] - Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sq
rt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[
f]*Sqrt[g]*(Sqrt[e] + Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] + Sqrt[e]*Sqrt[g])*
(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (I*p*PolyLog[2, ((-I)*Sqrt[g
]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]])/
(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)
])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[e]
- Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] - Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]
*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt
[e] + Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] + Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sqr
t[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + (2ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(d+\frac{e}{x^2}\right)x^3}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(2ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\left(d+\frac{e}{x^2}\right)x^3}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(2ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{x\left(e+dx^2\right)}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(2ep)\int \left(\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{ex} - \frac{dx\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{e\left(e+dx^2\right)}\right)dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(2p)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{x}\,dx}{\sqrt{f}\sqrt{g}} - \frac{(2dp)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{e+dx^2}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(ip)\int \frac{\log\left(1-\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)}{x}\,dx}{\sqrt{f}\sqrt{g}} - \frac{(ip)\int \frac{\log\left(1+\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)}{x}\,dx}{\sqrt{f}\sqrt{g}} - \frac{(2dp)\int \cdots}{\cdots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{ip\mathrm{Li}_2\left(-\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}} - \frac{ip\mathrm{Li}_2\left(\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}} + \frac{\left(\sqrt{-d}\,p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{e}-\sqrt{-d}\,x}\cdots}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\cdots\right.}{\cdots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\cdots\right.}{\cdots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.39, size = 706, normalized size = 1.18

$$\frac{\log\left(\sqrt{-f}-\sqrt{g}\,x\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right) - \log\left(\sqrt{-f}+\sqrt{g}\,x\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right) - p\mathrm{Li}_2\left(\frac{\sqrt{-d}\left(\sqrt{-f}-\sqrt{g}\,x\right)}{\sqrt{-d}\sqrt{-f}-\sqrt{e}\sqrt{g}}\right) - p\mathrm{Li}_2\left(\frac{\sqrt{-d}\left(\cdots\right.}{\sqrt{-d}\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/x^2)^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e/x^2)^p]*Log[Sqrt[-f] - Sqrt[g]*x] + 2*p*Log[(Sqrt[g]*x)/Sqrt[
-f]]*Log[Sqrt[-f] - Sqrt[g]*x] - p*Log[(Sqrt[g]*(-Sqrt[e] + Sqrt[-d]*x))/(S
qrt[-d]*Sqrt[-f] - Sqrt[e]*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*x] - p*Log[(Sqr
t[g]*(Sqrt[e] + Sqrt[-d]*x))/(Sqrt[-d]*Sqrt[-f] + Sqrt[e]*Sqrt[g])]*Log[Sqr
t[-f] - Sqrt[g]*x] - Log[c*(d + e/x^2)^p]*Log[Sqrt[-f] + Sqrt[g]*x] - 2*p*L
og[(f*Sqrt[g]*x)/(-f)^(3/2)]*Log[Sqrt[-f] + Sqrt[g]*x] + p*Log[(Sqrt[g]*(Sq
rt[e] - Sqrt[-d]*x))/(Sqrt[-d]*Sqrt[-f] + Sqrt[e]*Sqrt[g])]*Log[Sqrt[-f] +
Sqrt[g]*x] + p*Log[-((Sqrt[g]*(Sqrt[e] + Sqrt[-d]*x))/(Sqrt[-d]*Sqrt[-f] -
Sqrt[e]*Sqrt[g]))]*Log[Sqrt[-f] + Sqrt[g]*x] - p*PolyLog[2, (Sqrt[-d]*(Sqrt
[-f] - Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f] - Sqrt[e]*Sqrt[g])] - p*PolyLog[2, (S
qrt[-d]*(Sqrt[-f] - Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f] + Sqrt[e]*Sqrt[g])] + p*
PolyLog[2, (Sqrt[-d]*(Sqrt[-f] + Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f] - Sqrt[e]*S
qrt[g])] + p*PolyLog[2, (Sqrt[-d]*(Sqrt[-f] + Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f
] + Sqrt[e]*Sqrt[g])] - 2*p*PolyLog[2, 1 + (Sqrt[g]*x)/Sqrt[-f]] + 2*p*Poly
Log[2, 1 + (f*Sqrt[g]*x)/(-f)^(3/2)])/(2*Sqrt[-f]*Sqrt[g])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(c\left(\frac{dx^2+e}{x^2}\right)^p\right)}{gx^2+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x^2)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log(c*((d*x^2 + e)/x^2)^p)/(g*x^2 + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x^2)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log(c*(d + e/x^2)^p)/(g*x^2 + f), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/x^2)^p)/(g*x^2+f),x)
```

```
[Out] int(ln(c*(d+e/x^2)^p)/(g*x^2+f),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x^2)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] integrate(log(c*(d + e/x^2)^p)/(g*x^2 + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/x^2)^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e/x^2)^p)/(f + g*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/x**2)**p)/(g*x**2+f),x)
[Out] Timed out
```

3.266 $\int \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=541

$$-\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{d\sqrt[4]{g}+e\sqrt{-\sqrt{-f}}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)}{d\sqrt[4]{g}+e\sqrt[4]{-f}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log}{2\sqrt{-f}}$$

```
[Out] 1/2*ln(c*(d+e*x^(1/2))^p)*ln(e*((-f)^(1/4)-g^(1/4)*x^(1/2))/(e*(-f)^(1/4)+d
*g^(1/4)))/(-f)^(1/2)/g^(1/2)+1/2*ln(c*(d+e*x^(1/2))^p)*ln(e*((-f)^(1/4)+g^
(1/4)*x^(1/2))/(e*(-f)^(1/4)-d*g^(1/4)))/(-f)^(1/2)/g^(1/2)-1/2*ln(c*(d+e*x
^(1/2))^p)*ln(e*(g^(1/4)*x^(1/2)+(-(-f)^(1/2))^(1/2))/(-d*g^(1/4)+e*(-(-f)^
(1/2))^(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*ln(c*(d+e*x^(1/2))^p)*ln(e*(-g^(1/4)*
x^(1/2)+(-(-f)^(1/2))^(1/2))/(d*g^(1/4)+e*(-(-f)^(1/2))^(1/2)))/(-f)^(1/2)/
g^(1/2)+1/2*p*polylog(2,-g^(1/4)*(d+e*x^(1/2))/(e*(-f)^(1/4)-d*g^(1/4)))/(-
f)^(1/2)/g^(1/2)+1/2*p*polylog(2,g^(1/4)*(d+e*x^(1/2))/(e*(-f)^(1/4)+d*g^(1
/4)))/(-f)^(1/2)/g^(1/2)-1/2*p*polylog(2,-g^(1/4)*(d+e*x^(1/2))/(-d*g^(1/4)
+e*(-(-f)^(1/2))^(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*p*polylog(2,g^(1/4)*(d+e*x^
(1/2))/(d*g^(1/4)+e*(-(-f)^(1/2))^(1/2)))/(-f)^(1/2)/g^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 541, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2472, 275, 205, 2416, 260, 2394, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,-\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\text{PolyLog}\left(2,-\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{d\sqrt[4]{g}+e\sqrt{-\sqrt{-f}}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[4]{g}}{d\sqrt[4]{}}\right)}{2\sqrt{-f}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*Sqrt[x])^p]/(f + g*x^2),x]
```

```
[Out] -(Log[c*(d + e*Sqrt[x])^p]*Log[(e*(Sqrt[-Sqrt[-f]] - g^(1/4)*Sqrt[x]))/(e*S
qrt[-Sqrt[-f]] + d*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) + (Log[c*(d + e*Sqrt[x])
^p]*Log[(e*((-f)^(1/4) - g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4) + d*g^(1/4))])/(2*
Sqrt[-f]*Sqrt[g]) - (Log[c*(d + e*Sqrt[x])^p]*Log[(e*(Sqrt[-Sqrt[-f]] + g^(
1/4)*Sqrt[x]))/(e*Sqrt[-Sqrt[-f]] - d*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) + (Lo
g[c*(d + e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) + g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4)
 - d*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, -((g^(1/4)*(d + e*Sqrt
[x]))/(e*Sqrt[-Sqrt[-f]] - d*g^(1/4)))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[
2, -((g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) - d*g^(1/4)))])/(2*Sqrt[-f]*Sq
rt[g]) - (p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(e*Sqrt[-Sqrt[-f]] + d*g^(
1/4))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(e*(
-f)^(1/4) + d*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2472

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Su
bst[Int[x^(k - 1)*(f + g*x^(k*s))^r*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x],
 x, x^(1/k)], x] /; IntegerQ[k*s]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q,
r, s}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x\log\left(c(d+ex)^p\right)}{f+gx^4}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\mathrm{Subst}\left(\int\left(-\frac{\sqrt{g}\,x\log\left(c(d+ex)^p\right)}{2\sqrt{-f}\left(\sqrt{-f}\sqrt{g}-gx^2\right)} - \frac{\sqrt{g}\,x\log\left(c(d+ex)^p\right)}{2\sqrt{-f}\left(\sqrt{-f}\sqrt{g}+gx^2\right)}\right)dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{g}\,\mathrm{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{\sqrt{-f}\sqrt{g}-gx^2}\,dx, x, \sqrt{x}\right)}{\sqrt{-f}} - \frac{\sqrt{g}\,\mathrm{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{\sqrt{-f}\sqrt{g}+gx^2}\,dx, x, \sqrt{x}\right)}{\sqrt{-f}}$$

$$= -\frac{\sqrt{g}\,\mathrm{Subst}\left(\int\left(-\frac{\log(c(d+ex)^p)}{2g^{3/4}\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\,x\right)} + \frac{\log(c(d+ex)^p)}{2g^{3/4}\left(\sqrt{-\sqrt{-f}}+\sqrt[4]{g}\,x\right)}\right)dx, x, \sqrt{x}\right)}{\sqrt{-f}} - \sqrt{g}\,\mathrm{Subst}$$

$$= \frac{\mathrm{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\,x}\,dx, x, \sqrt{x}\right)}{2\sqrt{-f}\sqrt[4]{g}} - \frac{\mathrm{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt[4]{-f}-\sqrt[4]{g}\,x}\,dx, x, \sqrt{x}\right)}{2\sqrt{-f}\sqrt[4]{g}} - \frac{\mathrm{Subst}\left(\int -\right.}{}$$

$$= -\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\right.}{e\sqrt[4]{-f}+d\sqrt[4]{}}\right.}{2\sqrt{-f}\sqrt{g}}$$

$$= -\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\right.}{e\sqrt[4]{-f}+d\sqrt[4]{}}\right.}{2\sqrt{-f}\sqrt{g}}$$

$$= -\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\right.}{e\sqrt[4]{-f}+d\sqrt[4]{}}\right.}{2\sqrt{-f}\sqrt{g}}$$

Mathematica [C] time = 0.30, size = 422, normalized size = 0.78

$$\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)}{d\sqrt[4]{g}+e\sqrt[4]{-f}}\right) - \log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-i\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}+id\sqrt[4]{g}}\right) - \log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e(}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*Sqrt[x])^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) - g^(1/4)*Sqrt[x]))/(e*(-f)^(1
/4) + d*g^(1/4))] - Log[c*(d + e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) - I*g^(1/4)
*Sqrt[x]))/(e*(-f)^(1/4) + I*d*g^(1/4))] - Log[c*(d + e*Sqrt[x])^p]*Log[(e*
((-f)^(1/4) + I*g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4) - I*d*g^(1/4))] + Log[c*(d
+ e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) + g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4) - d*g^
(1/4))] + p*PolyLog[2, -((g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) - d*g^(1/4
)))] - p*PolyLog[2, (I*g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) + I*d*g^(1/4)
)] - p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(I*e*(-f)^(1/4) + d*g^(1/4))] +
 p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) + d*g^(1/4))])/(2*Sqr
t[-f]*Sqrt[g])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(e\sqrt{x}+d\right)^{p}c\right)}{gx^{2}+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^(1/2))^p)/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log((e*sqrt(x) + d)^p*c)/(g*x^2 + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(e\sqrt{x}+d\right)^{p}c\right)}{gx^{2}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^(1/2))^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((e*sqrt(x) + d)^p*c)/(g*x^2 + f), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\sqrt{x}+d\right)^{p}\right)}{g\,x^{2}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^(1/2)+d)^p)/(g*x^2+f),x)
[Out] int(ln(c*(e*x^(1/2)+d)^p)/(g*x^2+f),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(e\sqrt{x}+d\right)^{p}c\right)}{gx^{2}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^(1/2))^p)/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(log((e*sqrt(x) + d)^p*c)/(g*x^2 + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(d+e\,\sqrt{x}\right)^{p}\right)}{g\,x^{2}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^(1/2))^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e*x^(1/2))^p)/(f + g*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**(1/2))**p)/(g*x**2+f),x)
[Out] Timed out
```

3.267 $\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=561

$$-\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}+\sqrt[4]{g}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}\right.}{d\sqrt[4]{-}}\right.}{2\sqrt{-f}\sqrt{g}}$$

```
[Out] 1/2*ln(c*(d+e/x^(1/2))^p)*ln(e*(g^(1/4)-(-f)^(1/4)/x^(1/2))/(d*(-f)^(1/4)+e
*g^(1/4)))/(-f)^(1/2)/g^(1/2)+1/2*ln(c*(d+e/x^(1/2))^p)*ln(-e*(g^(1/4)+(-f)
^(1/4)/x^(1/2))/(d*(-f)^(1/4)-e*g^(1/4)))/(-f)^(1/2)/g^(1/2)-1/2*ln(c*(d+e/
x^(1/2))^p)*ln(e*(g^(1/4)-(-(-f)^(1/2))^(1/2)/x^(1/2))/(e*g^(1/4)+d*(-(-f)^
(1/2))^(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*ln(c*(d+e/x^(1/2))^p)*ln(-e*(g^(1/4)+
(-(-f)^(1/2))^(1/2)/x^(1/2))/(-e*g^(1/4)+d*(-(-f)^(1/2))^(1/2)))/(-f)^(1/2)
/g^(1/2)+1/2*p*polylog(2,(-f)^(1/4)*(d+e/x^(1/2))/(d*(-f)^(1/4)-e*g^(1/4)))
/(-f)^(1/2)/g^(1/2)+1/2*p*polylog(2,(-f)^(1/4)*(d+e/x^(1/2))/(d*(-f)^(1/4)+
e*g^(1/4)))/(-f)^(1/2)/g^(1/2)-1/2*p*polylog(2,(d+e/x^(1/2))*(-(-f)^(1/2))^
(1/2)/(-e*g^(1/4)+d*(-(-f)^(1/2))^(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*p*polylog(
2,(d+e/x^(1/2))*(-(-f)^(1/2))^(1/2)/(e*g^(1/4)+d*(-(-f)^(1/2))^(1/2)))/(-f)
^(1/2)/g^(1/2)
```

Rubi [A] time = 1.11, antiderivative size = 561, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2472, 2475, 263, 275, 205, 2416, 260, 2394, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-f}}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[4]{-f}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-f}}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[4]{-}}{d\sqrt[4]{}}\right.}{2\sqrt{-f}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e/Sqrt[x])^p]/(f + g*x^2),x]
```

```
[Out] -(Log[c*(d + e/Sqrt[x])^p]*Log[(e*(g^(1/4) - Sqrt[-Sqrt[-f]]/Sqrt[x]))/(d*S
qrt[-Sqrt[-f]] + e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) - (Log[c*(d + e/Sqrt[x])
^p]*Log[-((e*(g^(1/4) + Sqrt[-Sqrt[-f]]/Sqrt[x]))/(d*Sqrt[-Sqrt[-f]] - e*g^
(1/4)))])/(2*Sqrt[-f]*Sqrt[g]) + (Log[c*(d + e/Sqrt[x])^p]*Log[(e*(g^(1/4)
- (-f)^(1/4)/Sqrt[x]))/(d*(-f)^(1/4) + e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) +
(Log[c*(d + e/Sqrt[x])^p]*Log[-((e*(g^(1/4) + (-f)^(1/4)/Sqrt[x]))/(d*(-f)^
(1/4) - e*g^(1/4)))])/(2*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, (Sqrt[-Sqrt[-f]]
*(d + e/Sqrt[x]))/(d*Sqrt[-Sqrt[-f]] - e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) +
(p*PolyLog[2, ((-f)^(1/4)*(d + e/Sqrt[x]))/(d*(-f)^(1/4) - e*g^(1/4))])/(2*
Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, (Sqrt[-Sqrt[-f]]*(d + e/Sqrt[x]))/(d*Sqrt
[-Sqrt[-f]] + e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[2, ((-f)^(1/4)
*(d + e/Sqrt[x]))/(d*(-f)^(1/4) + e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2472

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Su
bst[Int[x^(k - 1)*(f + g*x^(k*s))^r*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x],
 x, x^(1/k)], x] /; IntegerQ[k*s]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q,
r, s}, x] && FractionQ[n]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx = 2\,\text{Subst}\left(\int \frac{x\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^4}\,dx, x, \sqrt{x}\right)$$

$$= -\left(2\,\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{\left(f+\frac{g}{x^4}\right)x^3}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\,\text{Subst}\left(\int\left(-\frac{fx\log\left(c(d+ex)^p\right)}{2\sqrt{-f}\sqrt{g}\left(\sqrt{-f}\sqrt{g}-fx^2\right)} - \frac{fx\log\left(c(d+ex)^p\right)}{2\sqrt{-f}\sqrt{g}\left(\sqrt{-f}\sqrt{g}+fx^2\right)}\right)dx, x\right.\right.$$

$$= -\frac{\sqrt{-f}\,\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{\sqrt{-f}\sqrt{g}-fx^2}\,dx, x, \frac{1}{\sqrt{x}}\right)}{\sqrt{g}} - \frac{\sqrt{-f}\,\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{\sqrt{-f}\sqrt{g}+fx^2}\,dx, x, \frac{1}{\sqrt{x}}\right)}{\sqrt{g}}$$

$$= -\frac{\sqrt{-f}\,\text{Subst}\left(\int\left(\frac{\sqrt{-\sqrt{-f}}\log(c(d+ex)^p)}{2f\left(\sqrt[4]{g}-\sqrt{-\sqrt{-f}}\,x\right)} - \frac{\sqrt{-\sqrt{-f}}\log(c(d+ex)^p)}{2f\left(\sqrt[4]{g}+\sqrt{-\sqrt{-f}}\,x\right)}\right)dx, x, \frac{1}{\sqrt{x}}\right)}{\sqrt{g}} - \frac{\sqrt{-f}\,\text{Subs}}{}$$

$$= -\frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt[4]{g}-\sqrt{-\sqrt{-f}}\,x}\,dx, x, \frac{1}{\sqrt{x}}\right)}{2\sqrt{-\sqrt{-f}}\sqrt{g}} + \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt[4]{g}+\sqrt{-\sqrt{-f}}\,x}\,dx, x, \frac{1}{\sqrt{x}}\right)}{2\sqrt{-\sqrt{-f}}\sqrt{g}} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\sqrt[4]{g}+\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

$$= -\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\sqrt[4]{g}+\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

$$= -\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\sqrt[4]{g}+\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

Mathematica [C] time = 0.58, size = 912, normalized size = 1.63

$$\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\sqrt[4]{g}\sqrt{x}-\sqrt[4]{-f}\right) - p\log\left(-\frac{\sqrt[4]{g}\left(\sqrt{x}\,d+e\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)\log\left(-\sqrt[4]{g}\sqrt{x}-\sqrt[4]{-f}\right) + p\log\left(\frac{f\sqrt[4]{g}\sqrt{x}}{(-f)^{5/4}}\right)\log\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/Sqrt[x])^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e/Sqrt[x])^p]*Log[-(-f)^(1/4) - g^(1/4)*Sqrt[x]] - p*Log[-((g^(
1/4)*(e + d*Sqrt[x]))/(d*(-f)^(1/4) - e*g^(1/4)))]*Log[-(-f)^(1/4) - g^(1/4
)*Sqrt[x]] - Log[c*(d + e/Sqrt[x])^p]*Log[(-I)*(-f)^(1/4) - g^(1/4)*Sqrt[x]
```

```
] + p*Log[(I*g^(1/4)*(e + d*Sqrt[x]))/(d*(-f)^(1/4) + I*e*g^(1/4))]*Log[(-I
)*(-f)^(1/4) - g^(1/4)*Sqrt[x]] - Log[c*(d + e/Sqrt[x])^p]*Log[I*(-f)^(1/4)
 - g^(1/4)*Sqrt[x]] + p*Log[(g^(1/4)*(e + d*Sqrt[x]))/(I*d*(-f)^(1/4) + e*g
^(1/4))]*Log[I*(-f)^(1/4) - g^(1/4)*Sqrt[x]] + Log[c*(d + e/Sqrt[x])^p]*Log
[(-f)^(1/4) - g^(1/4)*Sqrt[x]] - p*Log[(g^(1/4)*(e + d*Sqrt[x]))/(d*(-f)^(1
/4) + e*g^(1/4))]*Log[(-f)^(1/4) - g^(1/4)*Sqrt[x]] - p*Log[I*(-f)^(1/4) -
g^(1/4)*Sqrt[x]]*Log[((-I)*g^(1/4)*Sqrt[x])/(-f)^(1/4)] - p*Log[(-I)*(-f)^(
1/4) - g^(1/4)*Sqrt[x]]*Log[(I*g^(1/4)*Sqrt[x])/(-f)^(1/4)] + p*Log[(-f)^(1
/4) - g^(1/4)*Sqrt[x]]*Log[(g^(1/4)*Sqrt[x])/(-f)^(1/4)] + p*Log[-(-f)^(1/4
) - g^(1/4)*Sqrt[x]]*Log[(f*g^(1/4)*Sqrt[x])/(-f)^(5/4)] - p*PolyLog[2, (d*
((-f)^(1/4) - g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) + e*g^(1/4))] + p*PolyLog[2,
(d*((-f)^(1/4) - I*g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) + I*e*g^(1/4))] + p*Poly
Log[2, (d*((-f)^(1/4) + I*g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) - I*e*g^(1/4))] -
 p*PolyLog[2, (d*((-f)^(1/4) + g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) - e*g^(1/4))
] - p*PolyLog[2, 1 - (I*g^(1/4)*Sqrt[x])/(-f)^(1/4)] - p*PolyLog[2, 1 + (I*
g^(1/4)*Sqrt[x])/(-f)^(1/4)] + p*PolyLog[2, 1 + (g^(1/4)*Sqrt[x])/(-f)^(1/4
)] + p*PolyLog[2, 1 + (f*g^(1/4)*Sqrt[x])/(-f)^(5/4)])/(2*Sqrt[-f]*Sqrt[g])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^{p}\right)}{gx^2+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x^(1/2))^p)/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log(c*((d*x + e*sqrt(x))/x)^p)/(g*x^2 + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{p}\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x^(1/2))^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log(c*(d + e/sqrt(x))^p)/(g*x^2 + f), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{p}\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/x^(1/2))^p)/(g*x^2+f),x)
[Out] int(ln(c*(d+e/x^(1/2))^p)/(g*x^2+f),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{p}\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x^(1/2))^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] integrate(log(c*(d + e/sqrt(x))^p)/(g*x^2 + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/x^(1/2))^p)/(f + g*x^2),x)
```

```
[Out] int(log(c*(d + e/x^(1/2))^p)/(f + g*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/x**(1/2))**p)/(g*x**2+f),x)
```

```
[Out] Timed out
```

3.268 $\int (f+gx^2)^3 \log \left(c \left(d+ex^2\right)^p\right) \, dx$

Optimal. Leaf size=338

$$f^3x \log \left(c \left(d+ex^2\right)^p\right)+f^2gx^3 \log \left(c \left(d+ex^2\right)^p\right)+\frac{3}{5}fg^2x^5 \log \left(c \left(d+ex^2\right)^p\right)+\frac{1}{7}g^3x^7 \log \left(c \left(d+ex^2\right)^p\right)-\frac{2d^{3/2}f^2gp}{e}$$

```
[Out] -2*f^3*p*x+2*d*f^2*g*p*x/e-6/5*d^2*f*g^2*p*x/e^2+2/7*d^3*g^3*p*x/e^3-2/3*f^
2*g*p*x^3+2/5*d*f*g^2*p*x^3/e-2/21*d^2*g^3*p*x^3/e^2-6/25*f*g^2*p*x^5+2/35*
d*g^3*p*x^5/e-2/49*g^3*p*x^7-2*d^(3/2)*f^2*g*p*arctan(x*e^(1/2)/d^(1/2))/e^
(3/2)+6/5*d^(5/2)*f*g^2*p*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)-2/7*d^(7/2)*g^3
*p*arctan(x*e^(1/2)/d^(1/2))/e^(7/2)+f^3*x*ln(c*(e*x^2+d)^p)+f^2*g*x^3*ln(c
*(e*x^2+d)^p)+3/5*f*g^2*x^5*ln(c*(e*x^2+d)^p)+1/7*g^3*x^7*ln(c*(e*x^2+d)^p)
+2*f^3*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2471, 2448, 321, 205, 2455, 302}

$$f^2gx^3 \log \left(c \left(d+ex^2\right)^p\right)+f^3x \log \left(c \left(d+ex^2\right)^p\right)+\frac{3}{5}fg^2x^5 \log \left(c \left(d+ex^2\right)^p\right)+\frac{1}{7}g^3x^7 \log \left(c \left(d+ex^2\right)^p\right)-\frac{2d^{3/2}f^2gp}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)^3*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f^3*p*x + (2*d*f^2*g*p*x)/e - (6*d^2*f*g^2*p*x)/(5*e^2) + (2*d^3*g^3*p*x
)/(7*e^3) - (2*f^2*g*p*x^3)/3 + (2*d*f*g^2*p*x^3)/(5*e) - (2*d^2*g^3*p*x^3)
/(21*e^2) - (6*f*g^2*p*x^5)/25 + (2*d*g^3*p*x^5)/(35*e) - (2*g^3*p*x^7)/49
+ (2*Sqrt[d]*f^3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*f^2*g*
p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2) + (6*d^(5/2)*f*g^2*p*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(5*e^(5/2)) - (2*d^(7/2)*g^3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
7*e^(7/2)) + f^3*x*Log[c*(d + e*x^2)^p] + f^2*g*x^3*Log[c*(d + e*x^2)^p] +
(3*f*g^2*x^5*Log[c*(d + e*x^2)^p])/5 + (g^3*x^7*Log[c*(d + e*x^2)^p])/7
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
```

```
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int \left(f+gx^2\right)^3 \log\left(c\left(d+ex^2\right)^p\right) dx = \int \left(f^3 \log\left(c\left(d+ex^2\right)^p\right) + 3f^2gx^2 \log\left(c\left(d+ex^2\right)^p\right) + 3fg^2x^4 \log\left(c\left(d\right.\right.\right.$$
$$= f^3 \int \log\left(c\left(d+ex^2\right)^p\right) dx + \left(3f^2g\right) \int x^2 \log\left(c\left(d+ex^2\right)^p\right) dx + \left(3fg\right.$$
$$= f^3x \log\left(c\left(d+ex^2\right)^p\right) + f^2gx^3 \log\left(c\left(d+ex^2\right)^p\right) + \frac{3}{5}fg^2x^5 \log\left(c\left(d+e\right.\right.$$
$$= -2f^3px + f^3x \log\left(c\left(d+ex^2\right)^p\right) + f^2gx^3 \log\left(c\left(d+ex^2\right)^p\right) + \frac{3}{5}fg^2x^5 \mathrm{lo}$$
$$= -2f^3px + \frac{2df^2gpx}{e} - \frac{6d^2fg^2px}{5e^2} + \frac{2d^3g^3px}{7e^3} - \frac{2}{3}f^2gpx^3 + \frac{2dfg^2px^3}{5e} - $$
$$= -2f^3px + \frac{2df^2gpx}{e} - \frac{6d^2fg^2px}{5e^2} + \frac{2d^3g^3px}{7e^3} - \frac{2}{3}f^2gpx^3 + \frac{2dfg^2px^3}{5e} - $$

Mathematica [A] time = 0.28, size = 215, normalized size = 0.64

$$\frac{1}{35}x\left(35f^3+35f^2gx^2+21fg^2x^4+5g^3x^6\right)\log\left(c\left(d+ex^2\right)^p\right) - \frac{2px\left(-525d^3g^3+35d^2eg^2\left(63f+5gx^2\right)-105de\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)^3*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-2*p*x*(-525*d^3*g^3 + 35*d^2*e*g^2*(63*f + 5*g*x^2) - 105*d*e^2*g*(35*f^2
 + 7*f*g*x^2 + g^2*x^4) + e^3*(3675*f^3 + 1225*f^2*g*x^2 + 441*f*g^2*x^4 +
75*g^3*x^6)))/(3675*e^3) - (2*Sqrt[d]*(-35*e^3*f^3 + 35*d*e^2*f^2*g - 21*d^
2*e*f*g^2 + 5*d^3*g^3)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(35*e^(7/2)) + (x*(35
*f^3 + 35*f^2*g*x^2 + 21*f*g^2*x^4 + 5*g^3*x^6)*Log[c*(d + e*x^2)^p])/35
```

fricas [A] time = 0.92, size = 596, normalized size = 1.76

$$\left[-\frac{150\,e^3g^3px^7 + 42\left(21\,e^3fg^2 - 5\,de^2g^3\right)px^5 + 70\left(35\,e^3f^2g - 21\,de^2fg^2 + 5\,d^2eg^3\right)px^3 + 105\left(35\,e^3f^3 - 35\,de^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^3*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/3675*(150*e^3*g^3*p*x^7 + 42*(21*e^3*f*g^2 - 5*d*e^2*g^3)*p*x^5 + 70*(3
5*e^3*f^2*g - 21*d*e^2*f*g^2 + 5*d^2*e*g^3)*p*x^3 + 105*(35*e^3*f^3 - 35*d*
e^2*f^2*g + 21*d^2*e*f*g^2 - 5*d^3*g^3)*p*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqr
t(-d/e) - d)/(e*x^2 + d)) + 210*(35*e^3*f^3 - 35*d*e^2*f^2*g + 21*d^2*e*f*g
^2 - 5*d^3*g^3)*p*x - 105*(5*e^3*g^3*p*x^7 + 21*e^3*f*g^2*p*x^5 + 35*e^3*f^
2*g*p*x^3 + 35*e^3*f^3*p*x)*log(e*x^2 + d) - 105*(5*e^3*g^3*x^7 + 21*e^3*f*
g^2*x^5 + 35*e^3*f^2*g*x^3 + 35*e^3*f^3*x)*log(c))/e^3, -1/3675*(150*e^3*g^
3*p*x^7 + 42*(21*e^3*f*g^2 - 5*d*e^2*g^3)*p*x^5 + 70*(35*e^3*f^2*g - 21*d*e
^2*f*g^2 + 5*d^2*e*g^3)*p*x^3 - 210*(35*e^3*f^3 - 35*d*e^2*f^2*g + 21*d^2*e
*f*g^2 - 5*d^3*g^3)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 210*(35*e^3*f^3 -
 35*d*e^2*f^2*g + 21*d^2*e*f*g^2 - 5*d^3*g^3)*p*x - 105*(5*e^3*g^3*p*x^7 +
21*e^3*f*g^2*p*x^5 + 35*e^3*f^2*g*p*x^3 + 35*e^3*f^3*p*x)*log(e*x^2 + d) -
105*(5*e^3*g^3*x^7 + 21*e^3*f*g^2*x^5 + 35*e^3*f^2*g*x^3 + 35*e^3*f^3*x)*lo
g(c))/e^3]
```

giac [A] time = 0.21, size = 309, normalized size = 0.91

$$-\frac{2\left(5\,d^{4}g^{3}p-21\,d^{3}fg^{2}pe+35\,d^{2}f^{2}gpe^{2}-35\,df^{3}pe^{3}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{35\,\sqrt{d}}+\frac{1}{3675}\left(525\,g^{3}px^{7}e^{3}\log\left(x^{2}e+d\right)-150\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^3*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -2/35*(5*d^4*g^3*p - 21*d^3*f*g^2*p*e + 35*d^2*f^2*g*p*e^2 - 35*d*f^3*p*e^3
)*arctan(x*e^(1/2)/sqrt(d))*e^(-7/2)/sqrt(d) + 1/3675*(525*g^3*p*x^7*e^3*lo
g(x^2*e + d) - 150*g^3*p*x^7*e^3 + 525*g^3*x^7*e^3*log(c) + 210*d*g^3*p*x^5
*e^2 + 2205*f*g^2*p*x^5*e^3*log(x^2*e + d) - 882*f*g^2*p*x^5*e^3 - 350*d^2*
g^3*p*x^3*e + 2205*f*g^2*x^5*e^3*log(c) + 1470*d*f*g^2*p*x^3*e^2 + 3675*f^2
*g*p*x^3*e^3*log(x^2*e + d) + 1050*d^3*g^3*p*x - 2450*f^2*g*p*x^3*e^3 - 441
0*d^2*f*g^2*p*x*e + 3675*f^2*g*x^3*e^3*log(c) + 7350*d*f^2*g*p*x*e^2 + 3675
*f^3*p*x*e^3*log(x^2*e + d) - 7350*f^3*p*x*e^3 + 3675*f^3*x*e^3*log(c))*e^(
-3)
```

maple [C] time = 0.52, size = 995, normalized size = 2.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^3*ln(c*(e*x^2+d)^p),x)
```

```
[Out] 1/7*ln(c)*g^3*x^7+ln(c)*f^3*x-2*f^3*p*x-2/49*g^3*p*x^7+ln(c)*f^2*g*x^3+3/5*
ln(c)*f*g^2*x^5+(1/7*g^3*x^7+3/5*f*g^2*x^5+f^2*g*x^3+f^3*x)*ln((e*x^2+d)^p)
+1/e*(-d*e)^(1/2)*p*ln((-d*e)^(1/2)*x-d)*f^3-1/e*(-d*e)^(1/2)*p*ln(-(-d*e)^
(1/2)*x-d)*f^3-1/14*I*Pi*g^3*x^7*csgn(I*c*(e*x^2+d)^p)^3-1/2*I*Pi*f^3*csgn(
I*c*(e*x^2+d)^p)^3*x-2/3*f^2*g*p*x^3-6/25*f*g^2*p*x^5-3/5/e^3*(-d*e)^(1/2)*
p*ln(-(-d*e)^(1/2)*x-d)*f*g^2*d^2-1/e^2*(-d*e)^(1/2)*p*ln((-d*e)^(1/2)*x-d)
*f^2*g*d+3/5/e^3*(-d*e)^(1/2)*p*ln((-d*e)^(1/2)*x-d)*f*g^2*d^2+1/e^2*(-d*e)
^(1/2)*p*ln(-(-d*e)^(1/2)*x-d)*f^2*g*d-1/2*I*Pi*f^3*csgn(I*(e*x^2+d)^p)*csg
n(I*c*(e*x^2+d)^p)*csgn(I*c)*x+1/2*I*Pi*f^2*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(
I*c*(e*x^2+d)^p)^2+1/2*I*Pi*f^2*g*x^3*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/1
4*I*Pi*g^3*x^7*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+3/10*I*P
i*f*g^2*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+3/10*I*Pi*f*g^2*x^5
*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2/7*d^3*g^3*p*x/e^3-2/21*d^2*g^3*p*x^3/e
^2+2/35*d*g^3*p*x^5/e-3/10*I*Pi*f*g^2*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x
```

```
^2+d)^p)*csgn(I*c)-1/2*I*Pi*f^2*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d
)^p)*csgn(I*c)+1/7/e^4*(-d*e)^(1/2)*p*ln(-(-d*e)^(1/2)*x-d)*g^3*d^3-1/7/e^4
*(-d*e)^(1/2)*p*ln((-d*e)^(1/2)*x-d)*g^3*d^3+1/2*I*Pi*f^3*csgn(I*c*(e*x^2+d
)^p)^2*csgn(I*c)*x+1/14*I*Pi*g^3*x^7*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)
^p)^2+1/2*I*Pi*f^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*x+2*d*f^2*g*
p*x/e-6/5*d^2*f*g^2*p*x/e^2+2/5*d*f*g^2*p*x^3/e+1/14*I*Pi*g^3*x^7*csgn(I*c*
(e*x^2+d)^p)^2*csgn(I*c)-3/10*I*Pi*f*g^2*x^5*csgn(I*c*(e*x^2+d)^p)^3-1/2*I*
Pi*f^2*g*x^3*csgn(I*c*(e*x^2+d)^p)^3
```

maxima [A] time = 1.01, size = 227, normalized size = 0.67

$$\frac{2}{3675}\,ep\left(\frac{105\left(35\,de^3f^3-35\,d^2e^2f^2g+21\,d^3efg^2-5\,d^4g^3\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^4}-\frac{75\,e^3g^3x^7+21\left(21\,e^3fg^2-5\,de^2g^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^3*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] 2/3675*e*p*(105*(35*d*e^3*f^3 - 35*d^2*e^2*f^2*g + 21*d^3*e*f*g^2 - 5*d^4*g
^3)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^4) - (75*e^3*g^3*x^7 + 21*(21*e^3*f*
g^2 - 5*d*e^2*g^3)*x^5 + 35*(35*e^3*f^2*g - 21*d*e^2*f*g^2 + 5*d^2*e*g^3)*x
^3 + 105*(35*e^3*f^3 - 35*d*e^2*f^2*g + 21*d^2*e*f*g^2 - 5*d^3*g^3)*x)/e^4)
 + 1/35*(5*g^3*x^7 + 21*f*g^2*x^5 + 35*f^2*g*x^3 + 35*f^3*x)*log((e*x^2 + d
)^p*c)
```

mupad [B] time = 0.38, size = 298, normalized size = 0.88

$$x^3\left(\frac{d\left(\frac{6fg^2p}{5}-\frac{2dg^3p}{7e}\right)}{3e}-\frac{2f^2gp}{3}\right)-x\left(2f^3p+\frac{d\left(\frac{d\left(\frac{6fg^2p}{5}-\frac{2dg^3p}{7e}\right)}{e}-2f^2gp\right)}{e}\right)-x^5\left(\frac{6fg^2p}{25}-\frac{2dg^3p}{35e}\right)+\ln\left(c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^2)^3,x)
```

```
[Out] x^3*((d*((6*f*g^2*p)/5 - (2*d*g^3*p)/(7*e)))/(3*e) - (2*f^2*g*p)/3) - x*(2*
f^3*p + (d*((d*((6*f*g^2*p)/5 - (2*d*g^3*p)/(7*e)))/e - 2*f^2*g*p))/e) - x^
5*((6*f*g^2*p)/25 - (2*d*g^3*p)/(35*e)) + log(c*(d + e*x^2)^p)*(f^3*x + (g^
3*x^7)/7 + f^2*g*x^3 + (3*f*g^2*x^5)/5) - (2*g^3*p*x^7)/49 - (2*d^(1/2)*p*a
tan((d^(1/2)*e^(1/2)*p*x*(5*d^3*g^3 - 35*e^3*f^3 + 35*d*e^2*f^2*g - 21*d^2*
e*f*g^2))/(5*d^4*g^3*p - 35*d*e^3*f^3*p - 21*d^3*e*f*g^2*p + 35*d^2*e^2*f^2
*g*p))*(5*d^3*g^3 - 35*e^3*f^3 + 35*d*e^2*f^2*g - 21*d^2*e*f*g^2))/(35*e^(7
/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**3*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

3.269 $\int \left(f+gx^2\right)^2 \log \left(c \left(d+ex^2\right)^p\right) \, dx$

Optimal. Leaf size=221

$$f^2 x \log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3} f g x^3 \log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5} g^2 x^5 \log\left(c\left(d+ex^2\right)^p\right)-\frac{4 d^{3/2} f g p \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2 d^{5/2} g^2 p \tan^{-}}{5e^{5/2}}$$

[Out] -2*f^2*p*x+4/3*d*f*g*p*x/e-2/5*d^2*g^2*p*x/e^2-4/9*f*g*p*x^3+2/15*d*g^2*p*x^3/e-2/25*g^2*p*x^5-4/3*d^(3/2)*f*g*p*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)+2/5*d^(5/2)*g^2*p*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)+f^2*x*ln(c*(e*x^2+d)^p)+2/3*f*g*x^3*ln(c*(e*x^2+d)^p)+1/5*g^2*x^5*ln(c*(e*x^2+d)^p)+2*f^2*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)

Rubi [A] time = 0.17, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2471, 2448, 321, 205, 2455, 302}

$$f^2 x \log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3} f g x^3 \log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5} g^2 x^5 \log\left(c\left(d+ex^2\right)^p\right)-\frac{4 d^{3/2} f g p \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}-\frac{2 d^2 g^2 p x}{5e^2}+\frac{2d^{5}}{}$$

Antiderivative was successfully verified.

[In] Int[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]

[Out] -2*f^2*p*x + (4*d*f*g*p*x)/(3*e) - (2*d^2*g^2*p*x)/(5*e^2) - (4*f*g*p*x^3)/9 + (2*d*g^2*p*x^3)/(15*e) - (2*g^2*p*x^5)/25 + (2*Sqrt[d]*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (4*d^(3/2)*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(3/2)) + (2*d^(5/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*e^(5/2)) + f^2*x*Log[c*(d + e*x^2)^p] + (2*f*g*x^3*Log[c*(d + e*x^2)^p])/3 + (g^2*x^5*Log[c*(d + e*x^2)^p])/5

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int \left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(f^2\log\left(c\left(d+ex^2\right)^p\right)+2fgx^2\log\left(c\left(d+ex^2\right)^p\right)+g^2x^4\log\left(c\left(d+e\right.\right.\right.$$

$$= f^2\int \log\left(c\left(d+ex^2\right)^p\right)\,dx+(2fg)\int x^2\log\left(c\left(d+ex^2\right)^p\right)\,dx+g^2\int x$$

$$= f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log\left(c\left(d+e\right.\right.$$

$$= -2f^2px+f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log$$

$$= -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5+\frac{2\sqrt{d}}{}$$

$$= -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5+\frac{2\sqrt{d}}{}$$

Mathematica [A] time = 0.13, size = 151, normalized size = 0.68

$$\frac{\sqrt{e}\,x\left(15e^2\left(15f^2+10fgx^2+3g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right)-2p\left(45d^2g^2-15deg\left(10f+gx^2\right)+e^2\left(225f^2+50fg\right.\right.\right.}{225e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (30*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]] + Sqrt[e]*x*(-2*p*(45*d^2*g^2 - 15*d*e*g*(10*f + g*x^2) + e^2*(225*f^2
 + 50*f*g*x^2 + 9*g^2*x^4)) + 15*e^2*(15*f^2 + 10*f*g*x^2 + 3*g^2*x^4)*Log[
c*(d + e*x^2)^p]))/(225*e^(5/2))
```

fricas [A] time = 0.67, size = 404, normalized size = 1.83

$$\left[-\frac{18e^2g^2px^5+10\left(10e^2fg-3deg^2\right)px^3-15\left(15e^2f^2-10defg+3d^2g^2\right)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2+2ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)+30\left(15\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/225*(18*e^2*g^2*p*x^5 + 10*(10*e^2*f*g - 3*d*e*g^2)*p*x^3 - 15*(15*e^2*
f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) -
d)/(e*x^2 + d)) + 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*x - 15*(3*e^2*
g^2*p*x^5 + 10*e^2*f*g*p*x^3 + 15*e^2*f^2*p*x)*log(e*x^2 + d) - 15*(3*e^2*g
^2*x^5 + 10*e^2*f*g*x^3 + 15*e^2*f^2*x)*log(c))/e^2, -1/225*(18*e^2*g^2*p*x
^5 + 10*(10*e^2*f*g - 3*d*e*g^2)*p*x^3 - 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^
2*g^2)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 30*(15*e^2*f^2 - 10*d*e*f*g +
3*d^2*g^2)*p*x - 15*(3*e^2*g^2*p*x^5 + 10*e^2*f*g*p*x^3 + 15*e^2*f^2*p*x)*l
og(e*x^2 + d) - 15*(3*e^2*g^2*x^5 + 10*e^2*f*g*x^3 + 15*e^2*f^2*x)*log(c))/
e^2]
```

giac [A] time = 0.19, size = 201, normalized size = 0.91

$$\frac{2\left(3\,d^3g^2p-10\,d^2fgpe+15\,df^2pe^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{15\,\sqrt{d}}+\frac{1}{225}\left(45\,g^2px^5e^2\log\left(x^2e+d\right)-18\,g^2px^5e^2+45\,g^2x^5e^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 2/15*(3*d^3*g^2*p - 10*d^2*f*g*p*e + 15*d*f^2*p*e^2)*arctan(x*e^(1/2)/sqrt(
d))*e^(-5/2)/sqrt(d) + 1/225*(45*g^2*p*x^5*e^2*log(x^2*e + d) - 18*g^2*p*x^
5*e^2 + 45*g^2*x^5*e^2*log(c) + 30*d*g^2*p*x^3*e + 150*f*g*p*x^3*e^2*log(x^
2*e + d) - 100*f*g*p*x^3*e^2 + 150*f*g*x^3*e^2*log(c) - 90*d^2*g^2*p*x + 30
0*d*f*g*p*x*e + 225*f^2*p*x*e^2*log(x^2*e + d) - 450*f^2*p*x*e^2 + 225*f^2*
x*e^2*log(c))*e^(-2)
```

maple [C] time = 0.50, size = 686, normalized size = 3.10

$$\frac{g^2x^5\ln(c)}{5}+f^2x\ln(c)-2f^2px-\frac{2g^2px^5}{25}+\frac{2fgx^3\ln(c)}{3}+\frac{i\pi g^2x^5\mathrm{csgn}(ic)\,\mathrm{csgn}\left(ic\left(ex^2+d\right)^p\right)^2}{10}+\frac{i\pi g^2x^5\mathrm{csgn}\left(i\left(ex^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p),x)
```

```
[Out] 1/5*ln(c)*g^2*x^5+ln(c)*f^2*x-2*f^2*p*x-2/25*g^2*p*x^5+2/3*ln(c)*f*g*x^3-4/
9*f*g*p*x^3+(1/5*g^2*x^5+2/3*f*g*x^3+f^2*x)*ln((e*x^2+d)^p)-2/5*d^2*g^2*p*x
/e^2+2/15*d*g^2*p*x^3/e-1/3*I*Pi*f*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^
2+d)^p)*csgn(I*c)-1/5/e^3*(-d*e)^(1/2)*p*ln((-d*e)^(1/2)*x+d)*g^2*d^2+1/5/e
^3*(-d*e)^(1/2)*p*ln(-(-d*e)^(1/2)*x+d)*g^2*d^2+1/10*I*Pi*g^2*x^5*csgn(I*c*
(e*x^2+d)^p)^2*csgn(I*c)+1/10*I*Pi*g^2*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*
x^2+d)^p)^2-1/3*I*Pi*f*g*x^3*csgn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*f^2*csgn(I*c*
(e*x^2+d)^p)^2*csgn(I*c)*x+1/2*I*Pi*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2
+d)^p)^2*x-1/e*(-d*e)^(1/2)*p*ln((-d*e)^(1/2)*x+d)*f^2+1/e*(-d*e)^(1/2)*p*l
n(-(-d*e)^(1/2)*x+d)*f^2-1/10*I*Pi*g^2*x^5*csgn(I*c*(e*x^2+d)^p)^3-1/2*I*Pi
*f^2*csgn(I*c*(e*x^2+d)^p)^3*x+2/3/e^2*(-d*e)^(1/2)*p*ln((-d*e)^(1/2)*x+d)*
d*f*g-2/3/e^2*(-d*e)^(1/2)*p*ln(-(-d*e)^(1/2)*x+d)*d*f*g-1/10*I*Pi*g^2*x^5*
csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/3*I*Pi*f*g*x^3*csgn(I
*c*(e*x^2+d)^p)^2*csgn(I*c)+1/3*I*Pi*f*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(
e*x^2+d)^p)^2-1/2*I*Pi*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I
*c)*x+4/3*d*f*g*p*x/e
```

maxima [A] time = 1.01, size = 150, normalized size = 0.68

$$\frac{2}{225}\,ep\left(\frac{15\left(15\,de^2f^2-10\,d^2efg+3\,d^3g^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^3}-\frac{9\,e^2g^2x^5+5\left(10\,e^2fg-3\,deg^2\right)x^3+15\left(15\,e^2f^2-10\,de\right.}{e^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] 2/225*e*p*(15*(15*d*e^2*f^2 - 10*d^2*e*f*g + 3*d^3*g^2)*arctan(e*x/sqrt(d*e
))/(sqrt(d*e)*e^3) - (9*e^2*g^2*x^5 + 5*(10*e^2*f*g - 3*d*e*g^2)*x^3 + 15*(
15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*x)/e^3) + 1/15*(3*g^2*x^5 + 10*f*g*x^3
 + 15*f^2*x)*log((e*x^2 + d)^p*c)
```

mupad [B] time = 0.34, size = 193, normalized size = 0.87

$$\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(f^2\,x+\frac{2\,f\,g\,x^3}{3}+\frac{g^2\,x^5}{5}\right)-x\,\left(2\,f^2\,p-\frac{d\,\left(\frac{4\,f\,g\,p}{3}-\frac{2\,d\,g^2\,p}{5\,e}\right)}{e}\right)-x^3\,\left(\frac{4\,f\,g\,p}{9}-\frac{2\,d\,g^2\,p}{15\,e}\right)-\frac{2\,g^2\,p\,x^5}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*(f^2*x + (g^2*x^5)/5 + (2*f*g*x^3)/3) - x*(2*f^2*p - (
d*((4*f*g*p)/3 - (2*d*g^2*p)/(5*e)))/e) - x^3*((4*f*g*p)/9 - (2*d*g^2*p)/(1
5*e)) - (2*g^2*p*x^5)/25 + (2*d^(1/2)*p*atan((d^(1/2)*e^(1/2)*p*x*(3*d^2*g^
2 + 15*e^2*f^2 - 10*d*e*f*g))/(3*d^3*g^2*p + 15*d*e^2*f^2*p - 10*d^2*e*f*g*
p))*(3*d^2*g^2 + 15*e^2*f^2 - 10*d*e*f*g))/(15*e^(5/2))
```

sympy [A] time = 89.77, size = 415, normalized size = 1.88

$$\begin{cases} \frac{id^{\frac{5}{2}}g^2p\log\left(d+ex^2\right)}{5e^3\sqrt{\frac{1}{e}}}-\frac{2id^{\frac{5}{2}}g^2p\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{5e^3\sqrt{\frac{1}{e}}}-\frac{2id^{\frac{3}{2}}fgp\log\left(d+ex^2\right)}{3e^2\sqrt{\frac{1}{e}}}+\frac{4id^{\frac{3}{2}}fgp\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{3e^2\sqrt{\frac{1}{e}}}+\frac{i\sqrt{d}f^2p\log\left(d+ex^2\right)}{e\sqrt{\frac{1}{e}}}-\frac{2i\sqrt{d}f \\ \left(f^2x+\frac{2fgx^3}{3}+\frac{g^2x^5}{5}\right)\log\left(cd^p\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((I*d**(5/2)*g**2*p*log(d + e*x**2)/(5*e**3*sqrt(1/e)) - 2*I*d**(5
/2)*g**2*p*log(-I*sqrt(d)*sqrt(1/e) + x)/(5*e**3*sqrt(1/e)) - 2*I*d**(3/2)*
f*g*p*log(d + e*x**2)/(3*e**2*sqrt(1/e)) + 4*I*d**(3/2)*f*g*p*log(-I*sqrt(d
)*sqrt(1/e) + x)/(3*e**2*sqrt(1/e)) + I*sqrt(d)*f**2*p*log(d + e*x**2)/(e*s
qrt(1/e)) - 2*I*sqrt(d)*f**2*p*log(-I*sqrt(d)*sqrt(1/e) + x)/(e*sqrt(1/e))
- 2*d**2*g**2*p*x/(5*e**2) + 4*d*f*g*p*x/(3*e) + 2*d*g**2*p*x**3/(15*e) + f
**2*p*x*log(d + e*x**2) - 2*f**2*p*x + f**2*x*log(c) + 2*f*g*p*x**3*log(d +
 e*x**2)/3 - 4*f*g*p*x**3/9 + 2*f*g*x**3*log(c)/3 + g**2*p*x**5*log(d + e*x
**2)/5 - 2*g**2*p*x**5/25 + g**2*x**5*log(c)/5, Ne(e, 0)), ((f**2*x + 2*f*g
*x**3/3 + g**2*x**5/5)*log(c*d**p), True))
```

3.270 $\int \left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=117

$$fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{2dgpx}{3e}-2fpx-\frac{2}{9}gpx^3$$

```
[Out] -2*f*p*x+2/3*d*g*p*x/e-2/9*g*p*x^3-2/3*d^(3/2)*g*p*arctan(x*e^(1/2)/d^(1/2)
)/e^(3/2)+f*x*ln(c*(e*x^2+d)^p)+1/3*g*x^3*ln(c*(e*x^2+d)^p)+2*f*p*arctan(x*
e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2471, 2448, 321, 205, 2455, 302}

$$fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{2dgpx}{3e}-2fpx-\frac{2}{9}gpx^3$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^
(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int (f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)dx = \int \left(f\log\left(c\left(d+ex^2\right)^p\right)+gx^2\log\left(c\left(d+ex^2\right)^p\right)\right)dx$$
$$= f\int \log\left(c\left(d+ex^2\right)^p\right)dx + g\int x^2\log\left(c\left(d+ex^2\right)^p\right)dx$$
$$= fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)-(2efp)\int\frac{x^2}{d+ex^2}dx-$$
$$= -2fpx+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)+(2dfp)\int\frac{}{d+}$$
$$= -2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+fx\log\left(c\left(d+ex^2\right)^p\right)$$
$$= -2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}$$

Mathematica [A] time = 0.04, size = 117, normalized size = 1.00

$$fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{2dgpx}{3e}-2fpx-\frac{2}{9}g$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^
(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

fricas [A] time = 0.78, size = 220, normalized size = 1.88

$$\left[-\frac{2\,egpx^3+3\left(3\,ef-dg\right)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2-2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)+6\left(3\,ef-dg\right)px-3\left(egpx^3+3\,efpx\right)\log\left(ex^2+d\right)-3}{9\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/9*(2*e*g*p*x^3 + 3*(3*e*f - d*g)*p*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-
d/e) - d)/(e*x^2 + d)) + 6*(3*e*f - d*g)*p*x - 3*(e*g*p*x^3 + 3*e*f*p*x)*lo
g(e*x^2 + d) - 3*(e*g*x^3 + 3*e*f*x)*log(c))/e, -1/9*(2*e*g*p*x^3 - 6*(3*e*
f - d*g)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 6*(3*e*f - d*g)*p*x - 3*(e*g
*p*x^3 + 3*e*f*p*x)*log(e*x^2 + d) - 3*(e*g*x^3 + 3*e*f*x)*log(c))/e]
```

giac [A] time = 0.18, size = 109, normalized size = 0.93

$$-\frac{2\left(d^{2}gp-3\,dfpe\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{3\sqrt{d}}+\frac{1}{9}\left(3\,gpx^{3}e\log\left(x^{2}e+d\right)-2\,gpx^{3}e+3\,gx^{3}e\log(c)+9\,fpxe\log\left(x^{2}e+d\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -2/3*(d^2*g*p - 3*d*f*p*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/sqrt(d) + 1/9
*(3*g*p*x^3*e*log(x^2*e + d) - 2*g*p*x^3*e + 3*g*x^3*e*log(c) + 9*f*p*x*e*l
og(x^2*e + d) + 6*d*g*p*x - 18*f*p*x*e + 9*f*x*e*log(c))*e^(-1)
```

maple [C] time = 0.49, size = 416, normalized size = 3.56

$$-\frac{i\pi g\,x^{3}\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\mathrm{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)}{6}+\frac{i\pi g\,x^{3}\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)^{2}}{6}+\frac{i\pi g\,x^{3}\mathrm{csgn}\left(i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p),x)
```

```
[Out] (1/3*g*x^3+f*x)*ln((e*x^2+d)^p)-1/6*I*Pi*g*x^3*csgn(I*c*(e*x^2+d)^p)^3-1/2*
I*Pi*f*csgn(I*c*(e*x^2+d)^p)^3*x-1/6*I*Pi*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*
c*(e*x^2+d)^p)*csgn(I*c)+1/2*I*Pi*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*x+1/6
*I*Pi*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I*Pi*f*csgn(I*(
e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*x+1/2*I*Pi*f*csgn(I*(e*x^2+d)^p
)*csgn(I*c*(e*x^2+d)^p)^2*x+1/6*I*Pi*g*x^3*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c
)+1/3*ln(c)*g*x^3-2/9*g*p*x^3+ln(c)*f*x+1/3/e^2*(-d*e)^(1/2)*p*ln(-d-(-d*e)
^(1/2)*x)*d*g-1/e*(-d*e)^(1/2)*p*ln(-d-(-d*e)^(1/2)*x)*f-1/3/e^2*(-d*e)^(1/
2)*p*ln(-d+(-d*e)^(1/2)*x)*d*g+1/e*(-d*e)^(1/2)*p*ln(-d+(-d*e)^(1/2)*x)*f+2
/3*d*g*p*x/e-2*f*p*x
```

maxima [A] time = 1.04, size = 85, normalized size = 0.73

$$\frac{2}{9}\,ep\left(\frac{3\left(3\,def-d^{2}g\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^{2}}-\frac{egx^{3}+3\left(3\,ef-dg\right)x}{e^{2}}\right)+\frac{1}{3}\left(gx^{3}+3\,fx\right)\log\left(\left(ex^{2}+d\right)^{p}c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] 2/9*e*p*(3*(3*d*e*f - d^2*g)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^2) - (e*g*x
^3 + 3*(3*e*f - d*g)*x)/e^2) + 1/3*(g*x^3 + 3*f*x)*log((e*x^2 + d)^p*c)
```

mupad [B] time = 0.32, size = 97, normalized size = 0.83

$$\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)\left(\frac{g\,x^{3}}{3}+f\,x\right)-x\left(2\,f\,p-\frac{2\,d\,g\,p}{3\,e}\right)-\frac{2\,g\,p\,x^{3}}{9}-\frac{2\,\sqrt{d}\,p\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,p\,x\,\left(d\,g-3\,e\,f\right)}{d^{2}\,g\,p-3\,d\,e\,f\,p}\right)\left(d\,g-3\,e\,f\right)}{3\,e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

```
[Out] log(c*(d + e*x^2)^p)*(f*x + (g*x^3)/3) - x*(2*f*p - (2*d*g*p)/(3*e)) - (2*g
*p*x^3)/9 - (2*d^(1/2)*p*atan((d^(1/2)*e^(1/2)*p*x*(d*g - 3*e*f))/(d^2*g*p
 - 3*d*e*f*p))*(d*g - 3*e*f))/(3*e^(3/2))
```

sympy [A] time = 23.52, size = 228, normalized size = 1.95

$$\begin{cases} -\frac{id^{\frac{3}{2}}gp\log\left(d+ex^2\right)}{3e^2\sqrt{\frac{1}{e}}} + \frac{2id^{\frac{3}{2}}gp\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{3e^2\sqrt{\frac{1}{e}}} + \frac{i\sqrt{d}fp\log\left(d+ex^2\right)}{e\sqrt{\frac{1}{e}}} - \frac{2i\sqrt{d}fp\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{e\sqrt{\frac{1}{e}}} + \frac{2dgpx}{3e} + fpx\log\left(d+ex^2\right) \\ \left(fx+\frac{gx^3}{3}\right)\log\left(cd^p\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((-I*d**(3/2)*g*p*log(d + e*x**2)/(3*e**2*sqrt(1/e)) + 2*I*d**(3/2
)*g*p*log(-I*sqrt(d)*sqrt(1/e) + x)/(3*e**2*sqrt(1/e)) + I*sqrt(d)*f*p*log(
d + e*x**2)/(e*sqrt(1/e)) - 2*I*sqrt(d)*f*p*log(-I*sqrt(d)*sqrt(1/e) + x)/(
e*sqrt(1/e)) + 2*d*g*p*x/(3*e) + f*p*x*log(d + e*x**2) - 2*f*p*x + f*x*log(
c) + g*p*x**3*log(d + e*x**2)/3 - 2*g*p*x**3/9 + g*x**3*log(c)/3, Ne(e, 0))
, ((f*x + g*x**3/3)*log(c*d**p), True))
```

3.271 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=533

$$\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\,\sqrt{g}}+\frac{ip\text{Li}_2\left(\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}+1\right)}{2\sqrt{f}\,\sqrt{g}}+\frac{ip\text{Li}_2\left(1-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{2\sqrt{f}\,\sqrt{g}}-\frac{p\,t}{}$$

```
[Out] arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(1/2)/g^(1/2)+2*p*arctan(x*g^
(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan
(x*g^(1/2)/f^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-p*arcta
n(x*g^(1/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-I*p*pol
ylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2
,1+2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2
)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1-2*((-d)^
(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(
-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {205, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\,\sqrt{g}+i\sqrt{e}\,\sqrt{f}\right)}\right)}{2\sqrt{f}\,\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\,\sqrt{g}+i\sqrt{e}\,\sqrt{f}\right)}\right)}{2\sqrt{f}\,\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\right.}{\sqrt{f}\,\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] (2*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*S
qrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[
g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt
[f]]*Log[c*(d + e*x^2)^p])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (
2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sq
rt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1
 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d
]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\,\sqrt{g}} - (2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}\,\sqrt{g}\,\left(d+ex^2\right)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\,\sqrt{g}} - \frac{(2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\,\sqrt{g}} - \frac{(2ep)\int \left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{\left(\sqrt{e}\,p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{\sqrt{f}\,\sqrt{g}} - \frac{\left(\sqrt{e}\,p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\,\ldots}{\ldots}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\,\ldots}{\ldots}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\,\ldots}{\ldots}$$

<u>**Mathematica**</u> [A] time = 0.27, size = 564, normalized size = 1.06

$$-\frac{i\left(2i\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right) + p\text{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}-i\sqrt{-d}\,\sqrt{g}}\right) + p\text{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}+i\sqrt{-d}\,\sqrt{g}}\right) - p\text{Li}_2\left(\frac{\sqrt{e}\left(i\sqrt{g}\,x+\sqrt{f}\right)}{\sqrt{e}\,\sqrt{f}-i\sqrt{-d}\,\sqrt{g}}\right) - p\ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] ((-1/2*I)*(p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt
[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + p*Log[(Sqrt[g]*(Sqrt[-d] +
Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x
)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] +
 Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-
d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]
*x)/Sqrt[f]] + (2*I)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + p*P
olyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*S
qrt[g])] + p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f]
+ I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sq
rt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*S
qrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/(Sqrt[f]*Sqrt[g])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

maple [C] time = 0.07, size = 504, normalized size = 0.95

$$-\frac{i\pi\arctan\left(\frac{gx}{\sqrt{fg}}\right)\operatorname{csgn}(ic)\operatorname{csgn}\left(i\left(ex^2+d\right)^p\right)\operatorname{csgn}\left(ic\left(ex^2+d\right)^p\right)}{2\sqrt{fg}}+\frac{i\pi\arctan\left(\frac{gx}{\sqrt{fg}}\right)\operatorname{csgn}(ic)\operatorname{csgn}\left(ic\left(ex^2\right.\right.}{2\sqrt{fg}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f),x)
[Out] (-p*ln(e*x^2+d)+ln((e*x^2+d)^p))/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)+1/2*
p/g*sum(1/_alpha*(ln(-_alpha+x)*ln(e*x^2+d)-ln(-_alpha+x)*(ln((RootOf(_Z^2*
e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*
e*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-
x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))-dilog((RootOf(
_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_a
lpha*e*g+d*g-e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,i
ndex=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2))),_alpha
=RootOf(_Z^2*g+f))+1/2*I/(f*g)^(1/2)*Pi*arctan(1/(f*g)^(1/2)*g*x)*csgn(I*(e
*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I/(f*g)^(1/2)*Pi*arctan(1/(f*g)^(1/2
)*g*x)*csgn(I*c)*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)-1/2*I/(f*g)^(1/2
)*Pi*arctan(1/(f*g)^(1/2)*g*x)*csgn(I*c*(e*x^2+d)^p)^3+1/2*I/(f*g)^(1/2)*Pi
*arctan(1/(f*g)^(1/2)*g*x)*csgn(I*c)*csgn(I*c*(e*x^2+d)^p)^2+1/(f*g)^(1/2)*
arctan(1/(f*g)^(1/2)*g*x)*ln(c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^2),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Integral(log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

3.272 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=751

$$\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^{3/2}\sqrt{g}}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{ip\mathrm{Li}_2\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\right.}\right.}{4f^{3/2}\sqrt{g}}$$

```
[Out] p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/f/(-d*g+e*f)+1/2*arctan(x*g^(1/
2)/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(3/2)/g^(1/2)+p*arctan(x*g^(1/2)/f^(1/2))*l
n(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*p*arctan(x*g^(1/2)/f
^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/
(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*p*arctan(x*g^(1
/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/
2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*I*p*polylog
(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(3/2)/g^(1/2)+1/4*I*p*polylog(2,1+2
*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^
(1/2)-(-d)^(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)+1/4*I*p*polylog(2,1-2*((-d)^(1/2
)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^
(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*e*p*ln((-f)^(1/2)-x*g^(1/2))/(-d*g+e*f)
/(-f)^(1/2)/g^(1/2)+1/2*e*p*ln((-f)^(1/2)+x*g^(1/2))/(-d*g+e*f)/(-f)^(1/2)/
g^(1/2)-1/4*ln(c*(e*x^2+d)^p)/f/g^(1/2)/((-f)^(1/2)-x*g^(1/2))+1/4*ln(c*(e*
x^2+d)^p)/f/g^(1/2)/((-f)^(1/2)+x*g^(1/2))
```

Rubi [A] time = 1.02, antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {2471, 2463, 801, 635, 205, 260, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$\frac{ip\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4f^{3/2}\sqrt{g}}+\frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4f^{3/2}\sqrt{g}}-\frac{ip\,\mathrm{PolyLog}\left(2,\right.}{2f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

```
[Out] (Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(f*(e*f - d*g)) - (e*p*Log[
Sqrt[-f] - Sqrt[g]*x])/(2*Sqrt[-f]*Sqrt[g]*(e*f - d*g)) + (p*ArcTan[(Sqrt[g
]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g]) -
 (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]
*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*
f^(3/2)*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(S
qrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*
Sqrt[g]*x))])/(2*f^(3/2)*Sqrt[g]) + (e*p*Log[Sqrt[-f] + Sqrt[g]*x])/(2*Sqrt
[-f]*Sqrt[g]*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] - S
qrt[g]*x)) + Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] + Sqrt[g]*x)) + (A
rcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*f^(3/2)*Sqrt[g]) - ((I/
2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g])
 + ((I/4)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*S
qrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*Sqrt
[g]) + ((I/4)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(
(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*
Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int\left(-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}+gx\right)^2}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2f\left(-fg-g^2x^2\right)}\right)dx$$

$$= -\frac{g\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(\sqrt{-f}\sqrt{g}-gx\right)^2}dx}{4f}-\frac{g\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(\sqrt{-f}\sqrt{g}+gx\right)^2}dx}{4f}-\frac{g\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{-fg-g^2x^2}dx}{2f}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^{3/2}\sqrt{g}}+ -$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^{3/2}\sqrt{g}}- -$$

$$= -\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c\left(d\right.\right.}{4f\sqrt{g}\left(\sqrt{-}\right.}$$

$$= -\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c\left(d\right.\right.}{4f\sqrt{g}\left(\sqrt{-}\right.}$$

$$= \frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}}-\frac{p\tan}{}$$

$$= \frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}}-\frac{p\tan}{}$$

$$= \frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}}-\frac{p\tan}{}$$

<u>**Mathematica**</u> [A] time = 3.88, size = 1236, normalized size = 1.65

$$\frac{1}{2}\left(\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(\log\left(c\left(ex^2+d\right)^p\right)-p\log\left(ex^2+d\right)\right)}{f^{3/2}\sqrt{g}}+\frac{x\left(\log\left(c\left(ex^2+d\right)^p\right)-p\log\left(ex^2+d\right)\right)}{f\left(gx^2+f\right)}+\frac{1}{2}p\left(\frac{i\left(\frac{\log\left(x-\frac{i\sqrt{}}{\sqrt{}}\right.}{i\sqrt{g}x+\sqrt{}}\right.}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

```
[Out] ((x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(f*(f + g*x^2)) + (ArcTan
[(Sqrt[g]*x)/Sqrt[f]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(f^(3/2
```

```
)*Sqrt[g]) + (p*((I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]/(Sqrt[f] + I*Sqrt[g]*x
) + (Sqrt[e]*(-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[I*Sqrt[f] - Sqrt[g]*x]))/(S
qrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])))/(f*Sqrt[g]) + (I*(Log[(I*Sqrt[d])/Sqrt[
e] + x]/(Sqrt[f] + I*Sqrt[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] + Sqrt[e]*x] + L
og[I*Sqrt[f] - Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])))/(f*Sqrt[g
]) + ((-I)*(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Log[((-I)*Sqrt[d])/Sqrt[e] +
 x] + Sqrt[e]*(I*Sqrt[f] + Sqrt[g]*x)*(Log[I*Sqrt[d] - Sqrt[e]*x] - Log[I*S
qrt[f] + Sqrt[g]*x]))/(f*(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Sqrt[g]*(Sqrt[
f] - I*Sqrt[g]*x)) - (-(Log[(I*Sqrt[d])/Sqrt[e] + x]/(I*Sqrt[f] + Sqrt[g]*x
)) - (I*Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[I*Sqrt[f] + Sqrt[g]*x]))/
(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g]))/(f*Sqrt[g]) + 2*(x/(f^2 + f*g*x^2) + A
rcTan[(Sqrt[g]*x)/Sqrt[f]]/(f^(3/2)*Sqrt[g]))*(-Log[((-I)*Sqrt[d])/Sqrt[e]
+ x] - Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[d + e*x^2]) + (I*(Log[(I*Sqrt[d])
/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt
[d]*Sqrt[g])] + PolyLog[2, -((Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqr
t[f] - Sqrt[d]*Sqrt[g]))]))/(f^(3/2)*Sqrt[g]) - (I*(Log[(I*Sqrt[d])/Sqrt[e]
 + x]*Log[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt
[g])] + PolyLog[2, (Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqr
t[d]*Sqrt[g])]))/(f^(3/2)*Sqrt[g]) - (I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Lo
g[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])] +
PolyLog[2, -((Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*S
qrt[g]))]))/(f^(3/2)*Sqrt[g]) + (I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sq
rt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])] + PolyL
og[2, (Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])
]))/(f^(3/2)*Sqrt[g])))/2)/2
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{g^2x^4+2\,fgx^2+f^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

maple [F] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^2)^2,x)
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
[Out] Timed out
```

3.273 $\int \left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=945

$$\frac{8}{125}g^2p^2x^5+\frac{1}{5}g^2\log^2\left(c\left(ex^2+d\right)^p\right)x^5-\frac{4}{25}g^2p\log\left(c\left(ex^2+d\right)^p\right)x^5-\frac{64dg^2p^2x^3}{225e}+\frac{16}{27}fgp^2x^3+\frac{2}{3}fg\log^2\left(c\left(ex^2\right.\right.$$

```
[Out] 8*f^2*p^2*x+8/125*g^2*p^2*x^5+1/5*g^2*x^5*ln(c*(e*x^2+d)^p)^2+f^2*x*ln(c*(e
*x^2+d)^p)^2+8/3*d*f*g*p*x*ln(c*(e*x^2+d)^p)/e-8/3*d^(3/2)*f*g*p*arctan(x*e
^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(3/2)-16/3*d^(3/2)*f*g*p^2*arctan(x*e^(
1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(3/2)-8/3*I*d^(3/2)*f*g
*p^2*arctan(x*e^(1/2)/d^(1/2))^2/e^(3/2)-8/3*I*d^(3/2)*f*g*p^2*polylog(2,1-
2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(3/2)+184/75*d^2*g^2*p^2*x/e^2-64/225*d*
g^2*p^2*x^3/e-184/75*d^(5/2)*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)-8/9*
f*g*p*x^3*ln(c*(e*x^2+d)^p)-8*f^2*p^2*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(
1/2)-4*f^2*p*x*ln(c*(e*x^2+d)^p)-4/25*g^2*p*x^5*ln(c*(e*x^2+d)^p)+2/3*f*g*x
^3*ln(c*(e*x^2+d)^p)^2+16/27*f*g*p^2*x^3+4/5*d^(5/2)*g^2*p*arctan(x*e^(1/2)
/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(5/2)+8/5*d^(5/2)*g^2*p^2*arctan(x*e^(1/2)/d^
(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(5/2)+4*f^2*p*arctan(x*e^(1/2)
/d^(1/2))*ln(c*(e*x^2+d)^p)*d^(1/2)/e^(1/2)+8*f^2*p^2*arctan(x*e^(1/2)/d^(1
/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)+4*I*f^2*p^2*polylo
g(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)+4/5*I*d^(5/2)*g^2*p^
2*arctan(x*e^(1/2)/d^(1/2))^2/e^(5/2)+4/5*I*d^(5/2)*g^2*p^2*polylog(2,1-2*d
^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(5/2)+4*I*f^2*p^2*arctan(x*e^(1/2)/d^(1/2))
^2*d^(1/2)/e^(1/2)+64/9*d^(3/2)*f*g*p^2*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)-4
/5*d^2*g^2*p*x*ln(c*(e*x^2+d)^p)/e^2+4/15*d*g^2*p*x^3*ln(c*(e*x^2+d)^p)/e-6
4/9*d*f*g*p^2*x/e
```

Rubi [A] time = 1.25, antiderivative size = 945, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 15, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2471, 2450, 2476, 2448, 321, 205, 2470, 12, 4920, 4854, 2402, 2315, 2457, 2455, 302}

$$\frac{8}{125}g^2p^2x^5+\frac{1}{5}g^2\log^2\left(c\left(ex^2+d\right)^p\right)x^5-\frac{4}{25}g^2p\log\left(c\left(ex^2+d\right)^p\right)x^5-\frac{64dg^2p^2x^3}{225e}+\frac{16}{27}fgp^2x^3+\frac{2}{3}fg\log^2\left(c\left(ex^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f^2*p^2*x - (64*d*f*g*p^2*x)/(9*e) + (184*d^2*g^2*p^2*x)/(75*e^2) + (16*f
*g*p^2*x^3)/27 - (64*d*g^2*p^2*x^3)/(225*e) + (8*g^2*p^2*x^5)/125 - (8*Sqrt
[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (64*d^(3/2)*f*g*p^2*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/(9*e^(3/2)) - (184*d^(5/2)*g^2*p^2*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(75*e^(5/2)) + ((4*I)*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt
[d]]^2)/Sqrt[e] - (((8*I)/3)*d^(3/2)*f*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)
/e^(3/2) + (((4*I)/5)*d^(5/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(5/2
) + (8*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
 + I*Sqrt[e]*x)])/Sqrt[e] - (16*d^(3/2)*f*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(3*e^(3/2)) + (8*d^(5/2)*g^2*p^2
*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(5*e
^(5/2)) - 4*f^2*p*x*Log[c*(d + e*x^2)^p] + (8*d*f*g*p*x*Log[c*(d + e*x^2)^p
])/(3*e) - (4*d^2*g^2*p*x*Log[c*(d + e*x^2)^p])/(5*e^2) - (8*f*g*p*x^3*Log[
c*(d + e*x^2)^p])/9 + (4*d*g^2*p*x^3*Log[c*(d + e*x^2)^p])/(15*e) - (4*g^2*
p*x^5*Log[c*(d + e*x^2)^p])/25 + (4*Sqrt[d]*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d
]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - (8*d^(3/2)*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]]*Log[c*(d + e*x^2)^p])/(3*e^(3/2)) + (4*d^(5/2)*g^2*p*ArcTan[(Sqrt[e]*x
```

```
)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(5*e^(5/2)) + f^2*x*Log[c*(d + e*x^2)^p]^2
 + (2*f*g*x^3*Log[c*(d + e*x^2)^p]^2)/3 + (g^2*x^5*Log[c*(d + e*x^2)^p]^2)/
5 + ((4*I)*Sqrt[d]*f^2*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*
x)])/Sqrt[e] - (((8*I)/3)*d^(3/2)*f*g*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[
d] + I*Sqrt[e]*x)])/e^(3/2) + (((4*I)/5)*d^(5/2)*g^2*p^2*PolyLog[2, 1 - (2*
Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/e^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
```

```
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right) dx = \int \left(f^2\log^2\left(c\left(d+ex^2\right)^p\right)+2fgx^2\log^2\left(c\left(d+ex^2\right)^p\right)+g^2x^4\log^2\left(c\left(d+\right.\right.\right.$$

$$= f^2\int \log^2\left(c\left(d+ex^2\right)^p\right)dx+(2fg)\int x^2\log^2\left(c\left(d+ex^2\right)^p\right)dx+g^2\int x$$

$$= f^2x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log^2\left(c\left(d+\right.\right.$$

$$= f^2x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log^2\left(c\left(d+\right.\right.$$

$$= f^2x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log^2\left(c\left(d+\right.\right.$$

$$= -4f^2px\log\left(c\left(d+ex^2\right)^p\right)+\frac{8dfgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e}-\frac{4d^2g^2px\log\left(c\left(\right.\right.}{5e^2}$$

$$= 8f^2p^2x-\frac{16dfgp^2x}{3e}+\frac{8d^2g^2p^2x}{5e^2}-4f^2px\log\left(c\left(d+ex^2\right)^p\right)+\frac{8dfgpx\log}{}$$

$$= 8f^2p^2x-\frac{64dfgp^2x}{9e}+\frac{184d^2g^2p^2x}{75e^2}+\frac{16}{27}fgp^2x^3-\frac{64dg^2p^2x^3}{225e}+\frac{8}{125}g^2p^2$$

$$= 8f^2p^2x-\frac{64dfgp^2x}{9e}+\frac{184d^2g^2p^2x}{75e^2}+\frac{16}{27}fgp^2x^3-\frac{64dg^2p^2x^3}{225e}+\frac{8}{125}g^2p^2$$

$$= 8f^2p^2x-\frac{64dfgp^2x}{9e}+\frac{184d^2g^2p^2x}{75e^2}+\frac{16}{27}fgp^2x^3-\frac{64dg^2p^2x^3}{225e}+\frac{8}{125}g^2p^2$$

$$= 8f^2p^2x-\frac{64dfgp^2x}{9e}+\frac{184d^2g^2p^2x}{75e^2}+\frac{16}{27}fgp^2x^3-\frac{64dg^2p^2x^3}{225e}+\frac{8}{125}g^2p^2$$

Mathematica [A] time = 0.53, size = 435, normalized size = 0.46

$$\frac{\sqrt{e}x\left(-60p\left(45d^2g^2-15deg\left(10f+gx^2\right)+e^2\left(225f^2+50fgx^2+9g^2x^4\right)\right)\log\left(c\left(d+ex^2\right)^p\right)+225e^2\left(15f^2+10f\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((900*I)*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p^2*ArcTan[(Sqrt[e]*
x)/Sqrt[d]]^2 + 60*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2*(225*e^2*f^2 -
 200*d*e*f*g + 69*d^2*g^2)*p + 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*L
og[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + 15*(15*e^2*f^2 - 10*d*e*f*g + 3*d
^2*g^2)*Log[c*(d + e*x^2)^p]) + Sqrt[e]*x*(8*p^2*(1035*d^2*g^2 - 120*d*e*g*
```

```
(25*f + g*x^2) + e^2*(3375*f^2 + 250*f*g*x^2 + 27*g^2*x^4)) - 60*p*(45*d^2*
g^2 - 15*d*e*g*(10*f + g*x^2) + e^2*(225*f^2 + 50*f*g*x^2 + 9*g^2*x^4))*Log
[c*(d + e*x^2)^p] + 225*e^2*(15*f^2 + 10*f*g*x^2 + 3*g^2*x^4)*Log[c*(d + e*
x^2)^p]^2) + (900*I)*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p^2*Poly
Log[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)])/(3375*e^(5/2))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(g^2x^4 + 2\,fgx^2 + f^2\right)\log\left(\left(ex^2+d\right)^p c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(gx^2+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)^2, x)
```

maple [F] time = 0.75, size = 0, normalized size = 0.00

$$\int \left(g\,x^2+f\right)^2 \ln\left(c\left(e\,x^2+d\right)^p\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15}\left(3\,g^2p^2x^5 + 10\,fgp^2x^3 + 15\,f^2p^2x\right)\log\left(ex^2+d\right)^2 + \int \frac{15\,eg^2x^6\log(c)^2 + 15\left(2\,efg+dg^2\right)x^4\log(c)^2 + 15\,df$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*g^2*p^2*x^5 + 10*f*g*p^2*x^3 + 15*f^2*p^2*x)*log(e*x^2 + d)^2 + int
egrate(1/15*(15*e*g^2*x^6*log(c)^2 + 15*(2*e*f*g + d*g^2)*x^4*log(c)^2 + 15
*d*f^2*log(c)^2 + 15*(e*f^2 + 2*d*f*g)*x^2*log(c)^2 - 2*(3*(2*e*g^2*p^2 - 5
*e*g^2*p*log(c))*x^6 + 5*(4*e*f*g*p^2 - 3*(2*e*f*g*p + d*g^2*p)*log(c))*x^4
 - 15*d*f^2*p*log(c) + 15*(2*e*f^2*p^2 - (e*f^2*p + 2*d*f*g*p)*log(c))*x^2)
*log(e*x^2 + d))/(e*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(e\,x^2+d\right)^p\right)^2 \left(g\,x^2+f\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2*(f + g*x^2)^2,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^2*(f + g*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f + gx^2\right)^2 \log\left(c\left(d + ex^2\right)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)**2,x)
[Out] Integral((f + g*x**2)**2*log(c*(d + e*x**2)**p)**2, x)
```

3.274 $\int \left(f+gx^2\right)\log^2\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=548

$$-\frac{4d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{3e^{3/2}}+fx\log^2\left(c\left(d+ex^2\right)^p\right)-4fpx\log\left(c\left(d+ex^2\right)^p\right)+\frac{4\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right.}{\sqrt{\cdot}}$$

```
[Out] 8*f*p^2*x-32/9*d*g*p^2*x/e+8/27*g*p^2*x^3+32/9*d^(3/2)*g*p^2*arctan(x*e^(1/
2)/d^(1/2))/e^(3/2)+4*I*f*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*
d^(1/2)/e^(1/2)-4*f*p*x*ln(c*(e*x^2+d)^p)+4/3*d*g*p*x*ln(c*(e*x^2+d)^p)/e-4
/9*g*p*x^3*ln(c*(e*x^2+d)^p)-4/3*d^(3/2)*g*p*arctan(x*e^(1/2)/d^(1/2))*ln(c
*(e*x^2+d)^p)/e^(3/2)+f*x*ln(c*(e*x^2+d)^p)^2+1/3*g*x^3*ln(c*(e*x^2+d)^p)^2
-8/3*d^(3/2)*g*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1
/2)))/e^(3/2)+4*I*f*p^2*arctan(x*e^(1/2)/d^(1/2))^2*d^(1/2)/e^(1/2)-8*f*p^2
*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)-4/3*I*d^(3/2)*g*p^2*polylog(2,1-
2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(3/2)+4*f*p*arctan(x*e^(1/2)/d^(1/2))*ln
(c*(e*x^2+d)^p)*d^(1/2)/e^(1/2)+8*f*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1
/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)-4/3*I*d^(3/2)*g*p^2*arctan(x*e^(
1/2)/d^(1/2))^2/e^(3/2)
```

Rubi [A] time = 0.72, antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {2471, 2450, 2476, 2448, 321, 205, 2470, 12, 4920, 4854, 2402, 2315, 2457, 2455, 302}

$$-\frac{4id^{3/2}gp^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}}+\frac{4i\sqrt{d}fp^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}-\frac{4d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex\right.\right.}{3e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f*p^2*x - (32*d*g*p^2*x)/(9*e) + (8*g*p^2*x^3)/27 - (8*Sqrt[d]*f*p^2*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (32*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/(9*e^(3/2)) + ((4*I)*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sq
rt[e] - (((4*I)/3)*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(3/2) + (
8*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/Sqrt[e] - (8*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sq
rt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(3*e^(3/2)) - 4*f*p*x*Log[c*(d + e*x^2)^p]
 + (4*d*g*p*x*Log[c*(d + e*x^2)^p])/(3*e) - (4*g*p*x^3*Log[c*(d + e*x^2)^p]
)/9 + (4*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt
[e] - (4*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(3*e
^(3/2)) + f*x*Log[c*(d + e*x^2)^p]^2 + (g*x^3*Log[c*(d + e*x^2)^p]^2)/3 + (
(4*I)*Sqrt[d]*f*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sq
rt[e] - (((4*I)/3)*d^(3/2)*g*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/e^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int (f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)dx = \int \left(f\log^2\left(c\left(d+ex^2\right)^p\right)+gx^2\log^2\left(c\left(d+ex^2\right)^p\right)\right)dx$$

$$= f\int \log^2\left(c\left(d+ex^2\right)^p\right)dx + g\int x^2\log^2\left(c\left(d+ex^2\right)^p\right)dx$$

$$= fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^2\left(c\left(d+ex^2\right)^p\right)-(4efp)\int \frac{x^2\log\left(c\left(d+\right.\right.}{d+ex}$$

$$= fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^2\left(c\left(d+ex^2\right)^p\right)-(4efp)\int\left(\frac{\log\left(c\left(d+\right.\right.}{e}\right.$$

$$= fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^2\left(c\left(d+ex^2\right)^p\right)-(4fp)\int\log\left(c\left(d+ex\right.\right.$$

$$= -4fpx\log\left(c\left(d+ex^2\right)^p\right)+\frac{4dgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e}-\frac{4}{9}gpx^3\log\left(c\left(d+e\right.\right.$$

$$= 8fp^2x-\frac{8dgp^2x}{3e}-4fpx\log\left(c\left(d+ex^2\right)^p\right)+\frac{4dgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e}-\frac{4}{9}$$

$$= 8fp^2x-\frac{32dgp^2x}{9e}+\frac{8}{27}gp^2x^3-\frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{8d^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{}}\right.}{3e^{3/2}}$$

$$= 8fp^2x-\frac{32dgp^2x}{9e}+\frac{8}{27}gp^2x^3-\frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{32d^{3/2}gp^2\tan^{-1}\left(\right.}{9e^{3/2}}$$

$$= 8fp^2x-\frac{32dgp^2x}{9e}+\frac{8}{27}gp^2x^3-\frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{32d^{3/2}gp^2\tan^{-1}\left(\right.}{9e^{3/2}}$$

$$= 8fp^2x-\frac{32dgp^2x}{9e}+\frac{8}{27}gp^2x^3-\frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{32d^{3/2}gp^2\tan^{-1}\left(\right.}{9e^{3/2}}$$

Mathematica [A] time = 0.28, size = 281, normalized size = 0.51

$$\sqrt{e}x\left(9e\left(3f+gx^2\right)\log^2\left(c\left(d+ex^2\right)^p\right)-12p\left(-3dg+9ef+egx^2\right)\log\left(c\left(d+ex^2\right)^p\right)+8p^2\left(-12dg+27ef+egx\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((-36*I)*Sqrt[d]*(-3*e*f + d*g)*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 - 12*Sqrt
[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(2*(9*e*f - 4*d*g)*p + 6*(-3*e*f + d*g)*p
*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + (-9*e*f + 3*d*g)*Log[c*(d + e*x
^2)^p]) + Sqrt[e]*x*(8*p^2*(27*e*f - 12*d*g + e*g*x^2) - 12*p*(9*e*f - 3*d*
g + e*g*x^2)*Log[c*(d + e*x^2)^p] + 9*e*(3*f + g*x^2)*Log[c*(d + e*x^2)^p]^
2) - (36*I)*Sqrt[d]*(-3*e*f + d*g)*p^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/(
(-I)*Sqrt[d] + Sqrt[e]*x)])/(27*e^(3/2))
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(gx^2+f\right)\log\left(\left(ex^2+d\right)^p c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral((g*x^2 + f)*log((e*x^2 + d)^p*c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(gx^2+f\right)\log\left(\left(ex^2+d\right)^p c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
[Out] integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)^2, x)
```

maple [F] time = 1.52, size = 0, normalized size = 0.00

$$\int \left(g\,x^2+f\right)\ln\left(c\left(e\,x^2+d\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)^2,x)
[Out] int((g*x^2+f)*ln(c*(e*x^2+d)^p)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(gp^2x^3+3\,fp^2x\right)\log\left(ex^2+d\right)^2+\int \frac{3\,egx^4\log(c)^2+3\left(ef+dg\right)x^2\log(c)^2+3\,df\log(c)^2-2\left(\left(2\,egp^2-3\,eg\right.\right.}{3\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
[Out] 1/3*(g*p^2*x^3 + 3*f*p^2*x)*log(e*x^2 + d)^2 + integrate(1/3*(3*e*g*x^4*log
(c)^2 + 3*(e*f + d*g)*x^2*log(c)^2 + 3*d*f*log(c)^2 - 2*((2*e*g*p^2 - 3*e*g
*p*log(c))*x^4 - 3*d*f*p*log(c) + 3*(2*e*f*p^2 - (e*f*p + d*g*p)*log(c))*x^
2)*log(e*x^2 + d))/(e*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(e\,x^2+d\right)^p\right)^2\left(g\,x^2+f\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2*(f + g*x^2),x)
[Out] int(log(c*(d + e*x^2)^p)^2*(f + g*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)**2,x)
[Out] Integral((f + g*x**2)*log(c*(d + e*x**2)**p)**2, x)
```

3.275 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^2/(g*x^2+f),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^2),x]
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^2/(f + g*x^2), x]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx=\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$$

Mathematica [A] time = 2.84, size = 0, normalized size = 0.00

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2),x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)^2}{gx^2+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)^2/(g*x^2 + f), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)^2}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)^2/(g*x^2 + f), x)
```

maple [A] time = 18.03, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^{p}\right)^{2}}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f),x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^{p}c\right)^{2}}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)^2/(g*x^2 + f), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^{p}\right)^{2}}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2/(f + g*x^2),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^2/(f + g*x^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+ex^2\right)^{p}\right)^{2}}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**2/(g*x**2+f),x)
```

```
[Out] Integral(log(c*(d + e*x**2)**p)**2/(f + g*x**2), x)
```

3.276 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2,x]
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2, x]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx = \int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$$

Mathematica [A] time = 8.95, size = 0, normalized size = 0.00

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2,x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2, x]
```

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)^2}{g^2x^4+2\,fgx^2+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)^2/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)^2}{\left(gx^2+f\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)^2/(g*x^2 + f)^2, x)
```

maple [A] time = 41.74, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}{\left(g\,x^{2}+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x)
[Out] int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^{2}+d\right)^{p}c\right)^{2}}{\left(gx^{2}+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x, algorithm="maxima")
[Out] integrate(log((e*x^2 + d)^p*c)^2/(g*x^2 + f)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}{\left(g\,x^{2}+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2/(f + g*x^2)^2,x)
[Out] int(log(c*(d + e*x^2)^p)^2/(f + g*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**2/(g*x**2+f)**2,x)
[Out] Timed out
```

3.277 $\int \left(f+gx^2\right)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=683

$$-\frac{2dp(dg-3ef)\text{Int}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2},x\right)}{e}+\frac{32d^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{3e^{3/2}}+24fp^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{24\sqrt{\ }}{}$$

```
[Out] -48*f*p^3*x+208/9*d*g*p^3*x/e-16/27*g*p^3*x^3-208/9*d^(3/2)*g*p^3*arctan(x*
e^(1/2)/d^(1/2))/e^(3/2)-24*I*f*p^3*polylog(2,(-d^(1/2)+I*x*e^(1/2))/(d^(1/
2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)+24*f*p^2*x*ln(c*(e*x^2+d)^p)-32/3*d*g*p^2*
x*ln(c*(e*x^2+d)^p)/e+8/9*g*p^2*x^3*ln(c*(e*x^2+d)^p)+32/3*d^(3/2)*g*p^2*ar
ctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(3/2)-6*f*p*x*ln(c*(e*x^2+d)^p)
^2+2*d*g*p*x*ln(c*(e*x^2+d)^p)^2/e-2/3*g*p*x^3*ln(c*(e*x^2+d)^p)^2+f*x*ln(c
*(e*x^2+d)^p)^3+1/3*g*x^3*ln(c*(e*x^2+d)^p)^3+64/3*d^(3/2)*g*p^3*arctan(x*e
^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(3/2)-24*I*f*p^3*arct
an(x*e^(1/2)/d^(1/2))^2*d^(1/2)/e^(1/2)+48*f*p^3*arctan(x*e^(1/2)/d^(1/2))*
d^(1/2)/e^(1/2)+32/3*I*d^(3/2)*g*p^3*polylog(2,(-d^(1/2)+I*x*e^(1/2))/(d^(1
/2)+I*x*e^(1/2)))/e^(3/2)-24*f*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)
^p)*d^(1/2)/e^(1/2)-48*f*p^3*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2
)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)+32/3*I*d^(3/2)*g*p^3*arctan(x*e^(1/2)/d^(1/
2))^2/e^(3/2)-2*d*(d*g-3*e*f)*p*Unintegrable(ln(c*(e*x^2+d)^p)^2/(e*x^2+d),
x)/e
```

Rubi **[A]** time = 1.39, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(f+gx^2\right)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x^2)*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] -48*f*p^3*x + (208*d*g*p^3*x)/(9*e) - (16*g*p^3*x^3)/27 + (48*Sqrt[d]*f*p^3
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (208*d^(3/2)*g*p^3*ArcTan[(Sqrt[e]*
x)/Sqrt[d]])/(9*e^(3/2)) - ((24*I)*Sqrt[d]*f*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]
]^2)/Sqrt[e] + (((32*I)/3)*d^(3/2)*g*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(
3/2) - (48*Sqrt[d]*f*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[
d] + I*Sqrt[e]*x)])/Sqrt[e] + (64*d^(3/2)*g*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(3*e^(3/2)) + 24*f*p^2*x*Log[c*(
d + e*x^2)^p] - (32*d*g*p^2*x*Log[c*(d + e*x^2)^p])/(3*e) + (8*g*p^2*x^3*Lo
g[c*(d + e*x^2)^p])/9 - (24*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c
*(d + e*x^2)^p])/Sqrt[e] + (32*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Lo
g[c*(d + e*x^2)^p])/(3*e^(3/2)) - 6*f*p*x*Log[c*(d + e*x^2)^p]^2 + (2*d*g*p
*x*Log[c*(d + e*x^2)^p]^2)/e - (2*g*p*x^3*Log[c*(d + e*x^2)^p]^2)/3 + f*x*L
og[c*(d + e*x^2)^p]^3 + (g*x^3*Log[c*(d + e*x^2)^p]^3)/3 - ((24*I)*Sqrt[d]*
f*p^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + (((32*
I)/3)*d^(3/2)*g*p^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/e^
(3/2) + 6*d*f*p*Defer[Int][Log[c*(d + e*x^2)^p]^2/(d + e*x^2), x] - (2*d^2*
g*p*Defer[Int][Log[c*(d + e*x^2)^p]^2/(d + e*x^2), x])/e
```

Rubi steps

$$\int \left(f+gx^2\right)\log^3\left(c\left(d+ex^2\right)^p\right)dx = \int \left(f\log^3\left(c\left(d+ex^2\right)^p\right)+gx^2\log^3\left(c\left(d+ex^2\right)^p\right)\right)dx$$

$$= f\int \log^3\left(c\left(d+ex^2\right)^p\right)dx + g\int x^2\log^3\left(c\left(d+ex^2\right)^p\right)dx$$

$$= fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^3\left(c\left(d+ex^2\right)^p\right)-(6efp)\int \frac{x^2\log^2\left(c\right.}{d+}$$

$$= fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^3\left(c\left(d+ex^2\right)^p\right)-(6efp)\int\left(\frac{\log^2\left(c\left(\right.\right.}{}\right.$$

$$= fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^3\left(c\left(d+ex^2\right)^p\right)-(6fp)\int \log^2\left(c\left(d+\right.\right.$$

$$= -6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2dgpx\log^2\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{2}{3}gpx^3\log^2\left(c\right.$$

$$= -6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2dgpx\log^2\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{2}{3}gpx^3\log^2\left(c\right.$$

$$= -6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2dgpx\log^2\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{2}{3}gpx^3\log^2\left(c\right.$$

$$= 24fp^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{32dgp^2x\log\left(c\left(d+ex^2\right)^p\right)}{3e}+\frac{8}{9}gp^2x^3\log\left(c\right.$$

$$= -48fp^3x+\frac{64dgp^3x}{3e}+24fp^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{32dgp^2x\log\left(c\left(d+\right.\right.}{3e}$$

$$= -48fp^3x+\frac{208dgp^3x}{9e}-\frac{16}{27}gp^3x^3+\frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{64d^{3/2}gp^3}{3}$$

$$= -48fp^3x+\frac{208dgp^3x}{9e}-\frac{16}{27}gp^3x^3+\frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{208d^{3/2}gp}{9}$$

$$= -48fp^3x+\frac{208dgp^3x}{9e}-\frac{16}{27}gp^3x^3+\frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{208d^{3/2}gp}{9}$$

$$= -48fp^3x+\frac{208dgp^3x}{9e}-\frac{16}{27}gp^3x^3+\frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{208d^{3/2}gp}{9}$$

Mathematica [A] time = 4.57, size = 1460, normalized size = 2.14

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] (g*p^3*x*(-18*(d + e*x^2)*HypergeometricPFQ[{-1/2, 1, 1, 1, 1}, {2, 2, 2, 2
}, (d + e*x^2)/d] + 18*(d + e*x^2)*HypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2
, 2}, (d + e*x^2)/d]*Log[d + e*x^2] - 9*(d + e*x^2)*HypergeometricPFQ[{-1/2
, 1, 1}, {2, 2}, (d + e*x^2)/d]*Log[d + e*x^2]^2 + 2*d*Log[d + e*x^2]^3 - 2
*d*Sqrt[1 - (d + e*x^2)/d]*Log[d + e*x^2]^3 + 2*(d + e*x^2)*Sqrt[1 - (d + e
*x^2)/d]*Log[d + e*x^2]^3))/(6*e*Sqrt[1 - (d + e*x^2)/d]) + (2*d*g*p*x*(-(p
*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/e + (6*Sqrt[d]*f*p*ArcTan[(Sqrt
[e]*x)/Sqrt[d]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/Sqrt[e] - (
2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-(p*Log[d + e*x^2]) + Log[c*(d +
 e*x^2)^p])^2)/e^(3/2) + 3*f*p*x*Log[d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[
c*(d + e*x^2)^p])^2 + g*p*x^3*Log[d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[c*(
d + e*x^2)^p])^2 + f*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2*(-6*p
 - p*Log[d + e*x^2] + Log[c*(d + e*x^2)^p]) + (g*x^3*(-(p*Log[d + e*x^2]) +
 Log[c*(d + e*x^2)^p])^2*(-2*p - p*Log[d + e*x^2] + Log[c*(d + e*x^2)^p]))/
3 + 3*f*p^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])*(x*Log[d + e*x^2]^
2 - (4*((-I)*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 + Sqrt[e]*x*(-2 + Log[d
+ e*x^2]) - Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2 + 2*Log[(2*Sqrt[d])/(Sq
rt[d] + I*Sqrt[e]*x)] + Log[d + e*x^2]) - I*Sqrt[d]*PolyLog[2, (I*Sqrt[d] +
 Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/Sqrt[e]) + 3*g*p^2*(-(p*Log[d + e
*x^2]) + Log[c*(d + e*x^2)^p])*((x^3*Log[d + e*x^2]^2)/3 - (4*((9*I)*d^(3/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 + 3*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-8
 + 6*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + 3*Log[d + e*x^2]) + Sqrt[e]
*x*(24*d - 2*e*x^2 + (-9*d + 3*e*x^2)*Log[d + e*x^2]) + (9*I)*d^(3/2)*PolyL
og[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/(27*e^(3/2))) +
 (f*p^3*(-48*Sqrt[-d^2]*Sqrt[d + e*x^2]*Sqrt[1 - d/(d + e*x^2)]*ArcSin[Sqrt
[d]/Sqrt[d + e*x^2]] - 6*Sqrt[-d^2]*Sqrt[1 - d/(d + e*x^2)]*(8*Sqrt[d]*Hype
rgeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, d/(d + e*x^2)] + 4*Sqr
t[d]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, d/(d + e*x^2)]*Log[d +
e*x^2] + Sqrt[d + e*x^2]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]]*Log[d + e*x^2]^2)
+ Sqrt[-d]*e*x^2*(-48 + 24*Log[d + e*x^2] - 6*Log[d + e*x^2]^2 + Log[d + e*
x^2]^3) + 24*d*Sqrt[e*x^2]*ArcTanh[Sqrt[e*x^2]/Sqrt[-d]]*(Log[d + e*x^2] -
Log[(d + e*x^2)/d]) + 6*(-d)^(3/2)*Sqrt[1 - (d + e*x^2)/d]*(Log[(d + e*x^2)
/d]^2 - 4*Log[(d + e*x^2)/d]*Log[(1 + Sqrt[1 - (d + e*x^2)/d])/2] + 2*Log[(
1 + Sqrt[1 - (d + e*x^2)/d])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[1 - (d + e*x^2)
/d]/2])))/(Sqrt[-d]*e*x)
```

fricas [A] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(gx^2+f\right)\log\left(\left(ex^2+d\right)^p c\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
```

```
[Out] integral((g*x^2 + f)*log((e*x^2 + d)^p*c)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(gx^2+f\right)\log\left(\left(ex^2+d\right)^p c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)^3, x)
```

maple [A] time = 76.68, size = 0, normalized size = 0.00

$$\int \left(g\,x^2+f\right)\ln\left(c\left(e\,x^2+d\right)^p\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)^3,x)
```

```
[Out] int((g*x^2+f)*ln(c*(e*x^2+d)^p)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(gp^3x^3 + 3\,fp^3x\right)\log\left(ex^2+d\right)^3 + \int \frac{egx^4\log(c)^3 + \left(ef+dg\right)x^2\log(c)^3 + df\log(c)^3 - \left(\left(2\,egp^3 - 3\,egp^2\log(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(g*p^3*x^3 + 3*f*p^3*x)*log(e*x^2 + d)^3 + integrate((e*g*x^4*log(c)^3
+ (e*f + d*g)*x^2*log(c)^3 + d*f*log(c)^3 - ((2*e*g*p^3 - 3*e*g*p^2*log(c))
*x^4 - 3*d*f*p^2*log(c) + 3*(2*e*f*p^3 - (e*f*p^2 + d*g*p^2)*log(c))*x^2)*l
og(e*x^2 + d)^2 + 3*(e*g*p*x^4*log(c)^2 + d*f*p*log(c)^2 + (e*f*p + d*g*p)*
x^2*log(c)^2)*log(e*x^2 + d))/(e*x^2 + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(e\,x^2+d\right)^p\right)^3\,\left(g\,x^2+f\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^3*(f + g*x^2),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^3*(f + g*x^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)**3,x)
```

```
[Out] Integral((f + g*x**2)*log(c*(d + e*x**2)**p)**3, x)
```

3.278 $\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^3/(g*x^2+f),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^2),x]
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^3/(f + g*x^2), x]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx = \int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$$

Mathematica [A] time = 4.12, size = 0, normalized size = 0.00

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2),x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2), x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)^3}{gx^2+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)^3/(g*x^2 + f), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)^3}{gx^2+f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)^3/(g*x^2 + f), x)
```

maple [A] time = 4.94, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{3}}{g\,x^{2}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f),x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^{2}+d\right)^{p}c\right)^{3}}{gx^{2}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)^3/(g*x^2 + f), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{3}}{g\,x^{2}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^3/(f + g*x^2),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^3/(f + g*x^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+ex^{2}\right)^{p}\right)^{3}}{f+gx^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**3/(g*x**2+f),x)
```

```
[Out] Integral(log(c*(d + e*x**2)**p)**3/(f + g*x**2), x)
```

3.279 $\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2,x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2, x]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$$

Mathematica [A] time = 18.32, size = 0, normalized size = 0.00

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2,x]
```

```
[Out] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2, x]
```

fricas [A] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)^3}{g^2x^4+2\,fgx^2+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)^3/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)^3}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)^3/(g*x^2 + f)^2, x)
```

maple [A] time = 32.64, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^3}{\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)^3}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)^3/(g*x^2 + f)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^3}{\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^3/(f + g*x^2)^2,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^3/(f + g*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**3/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

3.280 $\int \frac{(f+gx^2)^2}{\log\left(c\left(d+ex^2\right)^p\right)} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(f+gx^2)^2}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

[Out] Unintegrable((g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx^2)^2}{\log\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(f + g*x^2)^2/Log[c*(d + e*x^2)^p],x]

[Out] Defer[Int][(f + g*x^2)^2/Log[c*(d + e*x^2)^p], x]

Rubi steps

$$\int \frac{(f+gx^2)^2}{\log\left(c\left(d+ex^2\right)^p\right)} dx = \int \frac{(f+gx^2)^2}{\log\left(c\left(d+ex^2\right)^p\right)} dx$$

Mathematica [A] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{(f+gx^2)^2}{\log\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p],x]

[Out] Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p], x]

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{g^2x^4+2fgx^2+f^2}{\log\left(\left(ex^2+d\right)^pc\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")

[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)/log((e*x^2 + d)^p*c), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx^2+f)^2}{\log\left(\left(ex^2+d\right)^pc\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)^2/log((e*x^2 + d)^p*c), x)
```

maple [A] time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{\left(g\,x^{2}+f\right)^{2}}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

```
[Out] int((g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx^{2}+f\right)^{2}}{\log\left(\left(ex^{2}+d\right)^{p}c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] integrate((g*x^2 + f)^2/log((e*x^2 + d)^p*c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(g\,x^{2}+f\right)^{2}}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x^2)^2/log(c*(d + e*x^2)^p),x)
```

```
[Out] int((f + g*x^2)^2/log(c*(d + e*x^2)^p), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx^{2}\right)^{2}}{\log\left(c\left(d+ex^{2}\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2/ln(c*(e*x**2+d)**p),x)
```

```
[Out] Integral((f + g*x**2)**2/log(c*(d + e*x**2)**p), x)
```

3.281 $\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x^2)/Log[c*(d + e*x^2)^p],x]
```

```
[Out] Defer[Int][(f + g*x^2)/Log[c*(d + e*x^2)^p], x]
```

Rubi steps

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)} dx = \int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)} dx$$

Mathematica [A] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p],x]
```

```
[Out] Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p], x]
```

fricas [A] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{gx^2+f}{\log\left(\left(ex^2+d\right)^p c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral((g*x^2 + f)/log((e*x^2 + d)^p*c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx^2+f}{\log\left(\left(ex^2+d\right)^p c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate((g*x^2 + f)/log((e*x^2 + d)^p*c), x)
```

maple [A] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{g\,x^{2}+f}{\ln \left(c \left(e\,x^{2}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)/ln(c*(e*x^2+d)^p),x)
[Out] int((g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx^{2}+f}{\log \left(\left(ex^{2}+d\right)^{p} c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate((g*x^2 + f)/log((e*x^2 + d)^p*c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{g\,x^{2}+f}{\ln \left(c \left(e\,x^{2}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x^2)/log(c*(d + e*x^2)^p),x)
[Out] int((f + g*x^2)/log(c*(d + e*x^2)^p), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx^{2}}{\log \left(c \left(d+ex^{2}\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)/ln(c*(e*x**2+d)**p),x)
[Out] Integral((f + g*x**2)/log(c*(d + e*x**2)**p), x)
```

3.282 $\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]),x]
[Out] Defer[Int][1/((f + g*x^2)*Log[c*(d + e*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]),x]
[Out] Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]), x]
```

fricas [A] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{(gx^2+f)\log\left(\left(ex^2+d\right)^p c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx^2+f)\log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)), x)
```

maple [A] time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(g\,x^{2}+f\right)\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p),x)
[Out] int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx^{2}+f\right)\log\left(\left(ex^{2}+d\right)^{p}c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)\,\left(g\,x^{2}+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^2)),x)
[Out] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(f+gx^{2}\right)\log\left(c\left(d+ex^{2}\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**2+f)/ln(c*(e*x**2+d)**p),x)
[Out] Integral(1/((f + g*x**2)*log(c*(d + e*x**2)**p)), x)
```

3.283 $\int \frac{1}{\left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(\frac{1}{\left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]),x]
[Out] Defer[Int][1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{\left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{\left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A] time = 2.63, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]),x]
[Out] Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]), x]
```

fricas [A] time = 0.66, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{\left(g^2x^4+2\,fgx^2+f^2\right)\log\left(\left(ex^2+d\right)^p c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral(1/((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx^2+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate(1/((g*x^2 + f)^2*log((e*x^2 + d)^p*c)), x)
```

maple [A] time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(g\,x^2+f\right)^2 \ln\left(c\left(e\,x^2+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
[Out] int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx^2+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate(1/((g*x^2 + f)^2*log((e*x^2 + d)^p*c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^2)^2),x)
[Out] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**2+f)**2/ln(c*(e*x**2+d)**p),x)
[Out] Timed out
```

3.284 $\int \frac{(f+gx^2)^2}{\log^2\left(c(d+ex^2)^p\right)} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

[Out] Unintegrable((g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2,x]

[Out] Defer[Int][(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2, x]

Rubi steps

$$\int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx = \int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Mathematica [A] time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2,x]

[Out] Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2, x]

fricas [A] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{g^2x^4+2\,fgx^2+f^2}{\log\left(\left(ex^2+d\right)^pc\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")

[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)/log((e*x^2 + d)^p*c)^2, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx^2+f)^2}{\log\left(\left(ex^2+d\right)^pc\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)^2/log((e*x^2 + d)^p*c)^2, x)
```

maple [A] time = 4.07, size = 0, normalized size = 0.00

$$\int \frac{\left(g\,x^{2}+f\right)^{2}}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{eg^{2}x^{6}+\left(2\,efg+dg^{2}\right)x^{4}+df^{2}+\left(ef^{2}+2\,dfg\right)x^{2}}{2\left(ep^{2}x\log\left(ex^{2}+d\right)+epx\log(c)\right)}+\int\frac{5\,eg^{2}x^{6}+3\left(2\,efg+dg^{2}\right)x^{4}-df^{2}+\left(ef^{2}+2\,dfg\right)x^{2}}{2\left(ep^{2}x^{2}\log\left(ex^{2}+d\right)+epx^{2}\log(c)\right)}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*g^2*x^6 + (2*e*f*g + d*g^2)*x^4 + d*f^2 + (e*f^2 + 2*d*f*g)*x^2)/(e
*p^2*x*log(e*x^2 + d) + e*p*x*log(c)) + integrate(1/2*(5*e*g^2*x^6 + 3*(2*e
*f*g + d*g^2)*x^4 - d*f^2 + (e*f^2 + 2*d*f*g)*x^2)/(e*p^2*x^2*log(e*x^2 + d
) + e*p*x^2*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(g\,x^{2}+f\right)^{2}}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x^2)^2/log(c*(d + e*x^2)^p)^2,x)
```

```
[Out] int((f + g*x^2)^2/log(c*(d + e*x^2)^p)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx^{2}\right)^{2}}{\log\left(c\left(d+ex^{2}\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**2)**2/log(c*(d + e*x**2)**p)**2, x)
```

3.285 $\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x^2)/Log[c*(d + e*x^2)^p]^2,x]
[Out] Defer[Int][(f + g*x^2)/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi steps

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx = \int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Mathematica [A] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p]^2,x]
[Out] Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p]^2, x]
```

fricas [A] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{gx^2+f}{\log\left(\left(ex^2+d\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral((g*x^2 + f)/log((e*x^2 + d)^p*c)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx^2+f}{\log\left(\left(ex^2+d\right)^p c\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)/log((e*x^2 + d)^p*c)^2, x)
```

maple [A] time = 4.25, size = 0, normalized size = 0.00

$$\int \frac{g\,x^{2}+f}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{egx^{4}+\left(ef+dg\right)x^{2}+df}{2\left(ep^{2}x\log\left(ex^{2}+d\right)+epx\log(c)\right)}+\int\frac{3\,egx^{4}+\left(ef+dg\right)x^{2}-df}{2\left(ep^{2}x^{2}\log\left(ex^{2}+d\right)+epx^{2}\log(c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*g*x^4 + (e*f + d*g)*x^2 + d*f)/(e*p^2*x*log(e*x^2 + d) + e*p*x*log(
c)) + integrate(1/2*(3*e*g*x^4 + (e*f + d*g)*x^2 - d*f)/(e*p^2*x^2*log(e*x^
2 + d) + e*p*x^2*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{g\,x^{2}+f}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x^2)/log(c*(d + e*x^2)^p)^2,x)
```

```
[Out] int((f + g*x^2)/log(c*(d + e*x^2)^p)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx^{2}}{\log\left(c\left(d+ex^{2}\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**2)/log(c*(d + e*x**2)**p)**2, x)
```

3.286 $\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2),x]
[Out] Defer[Int][1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A] time = 4.60, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2),x]
[Out] Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2), x]
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{(gx^2+f)\log\left(\left(ex^2+d\right)^pc\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx^2+f)\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)^2), x)
```

maple [A] time = 5.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(g\,x^{2}+f\right) \ln \left(c \left(e\,x^{2}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ex^{2}+d}{2\left(egpx^{3}\log(c)+efpx\log(c)+\left(egp^{2}x^{3}+efp^{2}x\right)\log\left(ex^{2}+d\right)\right)}-\int\frac{eg}{2\left(eg^{2}px^{6}\log(c)+2\,efgpx^{4}\log(c)+ef\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*x^2 + d)/(e*g*p*x^3*log(c) + e*f*p*x*log(c) + (e*g*p^2*x^3 + e*f*p^
2*x)*log(e*x^2 + d)) - integrate(1/2*(e*g*x^4 - (e*f - 3*d*g)*x^2 + d*f)/(e
*g^2*p*x^6*log(c) + 2*e*f*g*p*x^4*log(c) + e*f^2*p*x^2*log(c) + (e*g^2*p^2*
x^6 + 2*e*f*g*p^2*x^4 + e*f^2*p^2*x^2)*log(e*x^2 + d)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}^2\,\left(g\,x^2+f\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^2)),x)
```

```
[Out] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(f + g x^{2}\right) \log{\left(c \left(d + e x^{2}\right)^{p} \right)}^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**2+f)/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral(1/((f + g*x**2)*log(c*(d + e*x**2)**p)**2), x)
```

3.287 $\int \frac{1}{(f+gx^2)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{\left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

[Out] Unintegrable(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2),x]

[Out] Defer[Int][1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2), x]

Rubi steps

$$\int \frac{1}{\left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} \, dx = \int \frac{1}{\left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Mathematica [A] time = 7.73, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2),x]

[Out] Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2), x]

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(g^2x^4+2\,fgx^2+f^2\right)\log\left(\left(ex^2+d\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")

[Out] integral(1/((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx^2+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^2 + f)^2*log((e*x^2 + d)^p*c)^2), x)
```

maple [A] time = 4.73, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(g\,x^{2}+f\right)^{2} \ln \left(c \left(e\,x^{2}+d\right)^{p}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ex^{2}+d}{2\left(eg^{2}px^{5}\log(c)+2\,efgpx^{3}\log(c)+ef^{2}px\log(c)+\left(eg^{2}p^{2}x^{5}+2\,efgp^{2}x^{3}+ef^{2}p^{2}x\right)\log\left(ex^{2}+d\right)\right)}-\int \frac{}{2\left(eg\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*x^2 + d)/(e*g^2*p*x^5*log(c) + 2*e*f*g*p*x^3*log(c) + e*f^2*p*x*log
(c) + (e*g^2*p^2*x^5 + 2*e*f*g*p^2*x^3 + e*f^2*p^2*x)*log(e*x^2 + d)) - int
egrate(1/2*(3*e*g*x^4 - (e*f - 5*d*g)*x^2 + d*f)/(e*g^3*p*x^8*log(c) + 3*e*
f*g^2*p*x^6*log(c) + 3*e*f^2*g*p*x^4*log(c) + e*f^3*p*x^2*log(c) + (e*g^3*p
^2*x^8 + 3*e*f*g^2*p^2*x^6 + 3*e*f^2*g*p^2*x^4 + e*f^3*p^2*x^2)*log(e*x^2 +
 d)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}^2\,{\left(g\,x^2+f\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^2)^2),x)
```

```
[Out] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**2+f)**2/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Timed out
```

3.288 $\int \left(f+gx^3\right)^3 \log \left(c \left(d+ex^2\right)^p\right) dx$

Optimal. Leaf size=366

$$f^3x\log\left(c\left(d+ex^2\right)^p\right)+\frac{3}{4}f^2gx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{3}{7}fg^2x^7\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^3x^{10}\log\left(c\left(d+ex^2\right)^p\right)-\frac{6d^{7/2}f}{}$$

```
[Out] -2*f^3*p*x+6/7*d^3*f*g^2*p*x/e^3+3/4*d*f^2*g*p*x^2/e-1/10*d^4*g^3*p*x^2/e^4
-2/7*d^2*f*g^2*p*x^3/e^2-3/8*f^2*g*p*x^4+1/20*d^3*g^3*p*x^4/e^3+6/35*d*f*g^
2*p*x^5/e-1/30*d^2*g^3*p*x^6/e^2-6/49*f*g^2*p*x^7+1/40*d*g^3*p*x^8/e-1/50*g
^3*p*x^10-6/7*d^(7/2)*f*g^2*p*arctan(x*e^(1/2)/d^(1/2))/e^(7/2)-3/4*d^2*f^2
*g*p*ln(e*x^2+d)/e^2+1/10*d^5*g^3*p*ln(e*x^2+d)/e^5+f^3*x*ln(c*(e*x^2+d)^p)
+3/4*f^2*g*x^4*ln(c*(e*x^2+d)^p)+3/7*f*g^2*x^7*ln(c*(e*x^2+d)^p)+1/10*g^3*x
^10*ln(c*(e*x^2+d)^p)+2*f^3*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {2471, 2448, 321, 205, 2454, 2395, 43, 2455, 302}

$$\frac{3}{4}f^2gx^4\log\left(c\left(d+ex^2\right)^p\right)+f^3x\log\left(c\left(d+ex^2\right)^p\right)+\frac{3}{7}fg^2x^7\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^3x^{10}\log\left(c\left(d+ex^2\right)^p\right)-\frac{3d^2f^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^3)^3*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f^3*p*x + (6*d^3*f*g^2*p*x)/(7*e^3) + (3*d*f^2*g*p*x^2)/(4*e) - (d^4*g^3
*p*x^2)/(10*e^4) - (2*d^2*f*g^2*p*x^3)/(7*e^2) - (3*f^2*g*p*x^4)/8 + (d^3*g
^3*p*x^4)/(20*e^3) + (6*d*f*g^2*p*x^5)/(35*e) - (d^2*g^3*p*x^6)/(30*e^2) -
(6*f*g^2*p*x^7)/49 + (d*g^3*p*x^8)/(40*e) - (g^3*p*x^10)/50 + (2*Sqrt[d]*f^
3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (6*d^(7/2)*f*g^2*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(7*e^(7/2)) - (3*d^2*f^2*g*p*Log[d + e*x^2])/(4*e^2) + (d^5
*g^3*p*Log[d + e*x^2])/(10*e^5) + f^3*x*Log[c*(d + e*x^2)^p] + (3*f^2*g*x^4
*Log[c*(d + e*x^2)^p])/4 + (3*f*g^2*x^7*Log[c*(d + e*x^2)^p])/7 + (g^3*x^10
*Log[c*(d + e*x^2)^p])/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int \left(f+gx^3\right)^3 \log \left(c\left(d+ex^2\right)^p\right) dx = \int \left(f^3 \log \left(c\left(d+ex^2\right)^p\right) + 3f^2gx^3 \log \left(c\left(d+ex^2\right)^p\right) + 3fg^2x^6 \log \left(c\left(d+\right.\right.\right.$$

$$= f^3 \int \log \left(c\left(d+ex^2\right)^p\right) dx + \left(3f^2g\right) \int x^3 \log \left(c\left(d+ex^2\right)^p\right) dx + \left(3fg^2\right)$$

$$= f^3x \log \left(c\left(d+ex^2\right)^p\right) + \frac{3}{7}fg^2x^7 \log \left(c\left(d+ex^2\right)^p\right) + \frac{1}{2}\left(3f^2g\right) \text{Subst}\left(\int x\right.$$

$$= -2f^3px + f^3x \log \left(c\left(d+ex^2\right)^p\right) + \frac{3}{4}f^2gx^4 \log \left(c\left(d+ex^2\right)^p\right) + \frac{3}{7}fg^2x^7 \log$$

$$= -2f^3px + \frac{6d^3fg^2px}{7e^3} - \frac{2d^2fg^2px^3}{7e^2} + \frac{6dfg^2px^5}{35e} - \frac{6}{49}fg^2px^7 + \frac{2\sqrt{d}\,f^3p\tan}{\sqrt{e}}$$

$$= -2f^3px + \frac{6d^3fg^2px}{7e^3} + \frac{3df^2gpx^2}{4e} - \frac{d^4g^3px^2}{10e^4} - \frac{2d^2fg^2px^3}{7e^2} - \frac{3}{8}f^2gpx^4 + \frac{d}{}$$

Mathematica [A] time = 0.25, size = 258, normalized size = 0.70

$$\frac{210e^5x\left(140f^3+105f^2gx^3+60fg^2x^6+14g^3x^9\right)\log\left(c\left(d+ex^2\right)^p\right) - 8400\sqrt{d}\,e^{3/2}fp\left(3d^3g^2-7e^3f^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)^3*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-(e*p*x*(2940*d^4*g^3*x + 140*d^2*e^2*g^2*x^2*(60*f + 7*g*x^3) - 210*d^3*e
*g^2*(120*f + 7*g*x^3) - 105*d*e^3*g*x*(210*f^2 + 48*f*g*x^3 + 7*g^2*x^6) +
 3*e^4*(19600*f^3 + 3675*f^2*g*x^3 + 1200*f*g^2*x^6 + 196*g^3*x^9))) - 8400
*Sqrt[d]*e^(3/2)*f*(-7*e^3*f^2 + 3*d^3*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]] +
 1470*d^2*g*(-15*e^3*f^2 + 2*d^3*g^2)*p*Log[d + e*x^2] + 210*e^5*x*(140*f^3
 + 105*f^2*g*x^3 + 60*f*g^2*x^6 + 14*g^3*x^9)*Log[c*(d + e*x^2)^p])/(29400*
e^5)
```

fricas [A] time = 0.82, size = 708, normalized size = 1.93

$$\left[-\frac{588\,e^5g^3px^{10} - 735\,de^4g^3px^8 + 3600\,e^5fg^2px^7 + 980\,d^2e^3g^3px^6 - 5040\,de^4fg^2px^5 + 8400\,d^2e^3fg^2px^3 + 735\,(1}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/29400*(588*e^5*g^3*p*x^10 - 735*d*e^4*g^3*p*x^8 + 3600*e^5*f*g^2*p*x^7
+ 980*d^2*e^3*g^3*p*x^6 - 5040*d*e^4*f*g^2*p*x^5 + 8400*d^2*e^3*f*g^2*p*x^3
 + 735*(15*e^5*f^2*g - 2*d^3*e^2*g^3)*p*x^4 - 1470*(15*d*e^4*f^2*g - 2*d^4*
e*g^3)*p*x^2 + 4200*(7*e^5*f^3 - 3*d^3*e^2*f*g^2)*p*sqrt(-d/e)*log((e*x^2 -
 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) + 8400*(7*e^5*f^3 - 3*d^3*e^2*f*g^2)*p*
x - 210*(14*e^5*g^3*p*x^10 + 60*e^5*f*g^2*p*x^7 + 105*e^5*f^2*g*p*x^4 + 140
*e^5*f^3*p*x - 7*(15*d^2*e^3*f^2*g - 2*d^5*g^3)*p)*log(e*x^2 + d) - 210*(14
*e^5*g^3*x^10 + 60*e^5*f*g^2*x^7 + 105*e^5*f^2*g*x^4 + 140*e^5*f^3*x)*log(c
))/e^5, -1/29400*(588*e^5*g^3*p*x^10 - 735*d*e^4*g^3*p*x^8 + 3600*e^5*f*g^2
*p*x^7 + 980*d^2*e^3*g^3*p*x^6 - 5040*d*e^4*f*g^2*p*x^5 + 8400*d^2*e^3*f*g^
2*p*x^3 + 735*(15*e^5*f^2*g - 2*d^3*e^2*g^3)*p*x^4 - 1470*(15*d*e^4*f^2*g -
 2*d^4*e*g^3)*p*x^2 - 8400*(7*e^5*f^3 - 3*d^3*e^2*f*g^2)*p*sqrt(d/e)*arctan
```

```
(e*x*sqrt(d/e)/d) + 8400*(7*e^5*f^3 - 3*d^3*e^2*f*g^2)*p*x - 210*(14*e^5*g^
3*p*x^10 + 60*e^5*f*g^2*p*x^7 + 105*e^5*f^2*g*p*x^4 + 140*e^5*f^3*p*x - 7*(
15*d^2*e^3*f^2*g - 2*d^5*g^3)*p)*log(e*x^2 + d) - 210*(14*e^5*g^3*x^10 + 60
*e^5*f*g^2*x^7 + 105*e^5*f^2*g*x^4 + 140*e^5*f^3*x)*log(c))/e^5]
```

giac [A] time = 0.23, size = 354, normalized size = 0.97

$$\frac{1}{20}\left(2\,d^5g^3p-15\,d^2f^2gpe^3\right)e^{(-5)}\log\left(x^2e+d\right)-\frac{2\left(3\,d^4fg^2p-7\,df^3pe^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{7\sqrt{d}}+\frac{1}{29400}\left(2940\,g^3px\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/20*(2*d^5*g^3*p - 15*d^2*f^2*g*p*e^3)*e^(-5)*log(x^2*e + d) - 2/7*(3*d^4*
f*g^2*p - 7*d*f^3*p*e^3)*arctan(x*e^(1/2)/sqrt(d))*e^(-7/2)/sqrt(d) + 1/294
00*(2940*g^3*p*x^10*e^4*log(x^2*e + d) - 588*g^3*p*x^10*e^4 + 2940*g^3*x^10
*e^4*log(c) + 735*d*g^3*p*x^8*e^3 - 980*d^2*g^3*p*x^6*e^2 + 12600*f*g^2*p*x
^7*e^4*log(x^2*e + d) - 3600*f*g^2*p*x^7*e^4 + 1470*d^3*g^3*p*x^4*e + 12600
*f*g^2*x^7*e^4*log(c) + 5040*d*f*g^2*p*x^5*e^3 - 2940*d^4*g^3*p*x^2 - 8400*
d^2*f*g^2*p*x^3*e^2 + 22050*f^2*g*p*x^4*e^4*log(x^2*e + d) - 11025*f^2*g*p*
x^4*e^4 + 25200*d^3*f*g^2*p*x*e + 22050*f^2*g*x^4*e^4*log(c) + 22050*d*f^2*
g*p*x^2*e^3 + 29400*f^3*p*x*e^4*log(x^2*e + d) - 58800*f^3*p*x*e^4 + 29400*
f^3*x*e^4*log(c))*e^(-4)
```

maple [C] time = 0.71, size = 1311, normalized size = 3.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^3*ln(c*(e*x^2+d)^p),x)
```

```
[Out] f^3*x*ln(c)+1/7/e^5*p*ln(-3*d^4*e*f*g^2+7*d*e^4*f^3-(-9*d^7*e^3*f^2*g^4+42*
d^4*e^6*f^4*g^2-49*d*e^9*f^6)^(1/2)*x)*(-9*d^7*e^3*f^2*g^4+42*d^4*e^6*f^4*g
^2-49*d*e^9*f^6)^(1/2)-1/7/e^5*p*ln(-3*d^4*e*f*g^2+7*d*e^4*f^3+(-9*d^7*e^3*
f^2*g^4+42*d^4*e^6*f^4*g^2-49*d*e^9*f^6)^(1/2)*x)*(-9*d^7*e^3*f^2*g^4+42*d^
4*e^6*f^4*g^2-49*d*e^9*f^6)^(1/2)+3/7*ln(c)*f*g^2*x^7+3/4*ln(c)*f^2*g*x^4-1
/50*g^3*p*x^10-2*f^3*p*x+1/10*ln(c)*g^3*x^10+3/8*I*Pi*f^2*g*x^4*csgn(I*(e*x
^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+3/14*I*Pi*f*g^2*x^7*csgn(I*c*(e*x^2+d)^p)^
2*csgn(I*c)+3/8*I*Pi*f^2*g*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/20*I*Pi*
g^3*x^10*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+3/14*I*Pi*f*g^
2*x^7*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I*Pi*f^3*x*csgn(I*c*(
e*x^2+d)^p)^3-3/8*f^2*g*p*x^4-6/49*f*g^2*p*x^7-1/10*d^4*g^3*p*x^2/e^4+1/20*
d^3*g^3*p*x^4/e^3-1/30*d^2*g^3*p*x^6/e^2+1/40*d*g^3*p*x^8/e-1/2*I*Pi*f^3*x*
csgn(I*c)*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)+(1/10*g^3*x^10+3/7*f*g^
2*x^7+3/4*f^2*g*x^4+f^3*x)*ln((e*x^2+d)^p)-3/4/e^2*p*ln(-3*d^4*e*f*g^2+7*d*
e^4*f^3-(-9*d^7*e^3*f^2*g^4+42*d^4*e^6*f^4*g^2-49*d*e^9*f^6)^(1/2)*x)*d^2*f
^2*g-3/4/e^2*p*ln(-3*d^4*e*f*g^2+7*d*e^4*f^3+(-9*d^7*e^3*f^2*g^4+42*d^4*e^6
*f^4*g^2-49*d*e^9*f^6)^(1/2)*x)*d^2*f^2*g+1/20*I*Pi*g^3*x^10*csgn(I*(e*x^2+
d)^p)*csgn(I*c*(e*x^2+d)^p)^2-3/14*I*Pi*f*g^2*x^7*csgn(I*c*(e*x^2+d)^p)^3+1
/20*I*Pi*g^3*x^10*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-3/8*I*Pi*f^2*g*x^4*csgn
(I*c*(e*x^2+d)^p)^3+1/10/e^5*p*ln(-3*d^4*e*f*g^2+7*d*e^4*f^3-(-9*d^7*e^3*f^
2*g^4+42*d^4*e^6*f^4*g^2-49*d*e^9*f^6)^(1/2)*x)*d^5*g^3+1/10/e^5*p*ln(-3*d^
4*e*f*g^2+7*d*e^4*f^3+(-9*d^7*e^3*f^2*g^4+42*d^4*e^6*f^4*g^2-49*d*e^9*f^6)^
(1/2)*x)*d^5*g^3-3/14*I*Pi*f*g^2*x^7*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)
^p)*csgn(I*c)-3/8*I*Pi*f^2*g*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*
csgn(I*c)-1/20*I*Pi*g^3*x^10*csgn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*f^3*x*csgn(I*
c)*csgn(I*c*(e*x^2+d)^p)^2+1/2*I*Pi*f^3*x*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x
```

```
^2+d)^p)^2+6/7*d^3*f*g^2*p*x/e^3+3/4*d*f^2*g*p*x^2/e-2/7*d^2*f*g^2*p*x^3/e^
2+6/35*d*f*g^2*p*x^5/e
```

maxima [A] time = 1.00, size = 278, normalized size = 0.76

$$\frac{1}{29400}\,ep\left(\frac{8400\left(7\,de^{3}f^{3}-3\,d^{4}fg^{2}\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^{4}}-\frac{588\,e^{4}g^{3}x^{10}-735\,de^{3}g^{3}x^{8}+3600\,e^{4}fg^{2}x^{7}+980\,d^{2}e^{2}g^{3}x^{6}-}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] 1/29400*e*p*(8400*(7*d*e^3*f^3 - 3*d^4*f*g^2)*arctan(e*x/sqrt(d*e))/(sqrt(d
*e)*e^4) - (588*e^4*g^3*x^10 - 735*d*e^3*g^3*x^8 + 3600*e^4*f*g^2*x^7 + 980
*d^2*e^2*g^3*x^6 - 5040*d*e^3*f*g^2*x^5 + 8400*d^2*e^2*f*g^2*x^3 + 735*(15*
e^4*f^2*g - 2*d^3*e*g^3)*x^4 - 1470*(15*d*e^3*f^2*g - 2*d^4*g^3)*x^2 + 8400
*(7*e^4*f^3 - 3*d^3*e*f*g^2)*x)/e^5 - 1470*(15*d^2*e^3*f^2*g - 2*d^5*g^3)*l
og(e*x^2 + d)/e^6) + 1/140*(14*g^3*x^10 + 60*f*g^2*x^7 + 105*f^2*g*x^4 + 14
0*f^3*x)*log((e*x^2 + d)^p*c)
```

mupad [B] time = 3.33, size = 316, normalized size = 0.86

$$\frac{g^{3}\,x^{10}\,\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{10}-2\,f^{3}\,p\,x-\frac{g^{3}\,p\,x^{10}}{50}+f^{3}\,x\,\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)+\frac{3\,f^{2}\,g\,x^{4}\,\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{4}+\frac{3\,f\,g^{2}\,x^{7}\,\ln\left(c\left(e\right.\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^3)^3,x)
```

```
[Out] (g^3*x^10*log(c*(d + e*x^2)^p))/10 - 2*f^3*p*x - (g^3*p*x^10)/50 + f^3*x*lo
g(c*(d + e*x^2)^p) + (3*f^2*g*x^4*log(c*(d + e*x^2)^p))/4 + (3*f*g^2*x^7*lo
g(c*(d + e*x^2)^p))/7 - (3*f^2*g*p*x^4)/8 - (6*f*g^2*p*x^7)/49 + (d*g^3*p*x
^8)/(40*e) + (2*d^(1/2)*f^3*p*atan((e^(1/2)*x)/d^(1/2)))/e^(1/2) + (d^5*g^3
*p*log(d + e*x^2))/(10*e^5) - (d^2*g^3*p*x^6)/(30*e^2) + (d^3*g^3*p*x^4)/(2
0*e^3) - (d^4*g^3*p*x^2)/(10*e^4) - (6*d^(7/2)*f*g^2*p*atan((e^(1/2)*x)/d^(
1/2)))/(7*e^(7/2)) - (3*d^2*f^2*g*p*log(d + e*x^2))/(4*e^2) - (2*d^2*f*g^2*
p*x^3)/(7*e^2) + (3*d*f^2*g*p*x^2)/(4*e) + (6*d*f*g^2*p*x^5)/(35*e) + (6*d^
3*f*g^2*p*x)/(7*e^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**3*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

3.289 $\int \left(f+gx^3\right)^2 \log \left(c \left(d+ex^2\right)^p\right) \, dx$

Optimal. Leaf size=231

$$f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}fgx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{7/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}}+\frac{2d^3g^2px}{7e^3}-$$

```
[Out] -2*f^2*p*x+2/7*d^3*g^2*p*x/e^3+1/2*d*f*g*p*x^2/e-2/21*d^2*g^2*p*x^3/e^2-1/4
*f*g*p*x^4+2/35*d*g^2*p*x^5/e-2/49*g^2*p*x^7-2/7*d^(7/2)*g^2*p*arctan(x*e^(
1/2)/d^(1/2))/e^(7/2)-1/2*d^2*f*g*p*ln(e*x^2+d)/e^2+f^2*x*ln(c*(e*x^2+d)^p)
+1/2*f*g*x^4*ln(c*(e*x^2+d)^p)+1/7*g^2*x^7*ln(c*(e*x^2+d)^p)+2*f^2*p*arctan
(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {2471, 2448, 321, 205, 2454, 2395, 43, 2455, 302}

$$f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}fgx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2fgp\log\left(d+ex^2\right)}{2e^2}-\frac{2d^2g^2px^3}{21e^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^3)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f^2*p*x + (2*d^3*g^2*p*x)/(7*e^3) + (d*f*g*p*x^2)/(2*e) - (2*d^2*g^2*p*x
^3)/(21*e^2) - (f*g*p*x^4)/4 + (2*d*g^2*p*x^5)/(35*e) - (2*g^2*p*x^7)/49 +
(2*Sqrt[d]*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(7/2)*g^2*p*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(7*e^(7/2)) - (d^2*f*g*p*Log[d + e*x^2])/(2*e^2)
 + f^2*x*Log[c*(d + e*x^2)^p] + (f*g*x^4*Log[c*(d + e*x^2)^p])/2 + (g^2*x^7
*Log[c*(d + e*x^2)^p])/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\begin{aligned}\int \left(f+gx^3\right)^2 \log\left(c\left(d+ex^2\right)^p\right) dx &= \int \left(f^2\log\left(c\left(d+ex^2\right)^p\right)+2fgx^3\log\left(c\left(d+ex^2\right)^p\right)+g^2x^6\log\left(c\left(d+ex^2\right)\right.\right.\\ &= f^2\int \log\left(c\left(d+ex^2\right)^p\right) dx+(2fg)\int x^3\log\left(c\left(d+ex^2\right)^p\right) dx+g^2\int x^6 \text{lo}\\ &= f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)^p\right)+(fg)\,\text{Subst}\left(\int x\log\left(c\right.\right.\\ &= -2f^2px+f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}fgx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(\right.\\ &= -2f^2px+\frac{2d^3g^2px}{7e^3}-\frac{2d^2g^2px^3}{21e^2}+\frac{2dg^2px^5}{35e}-\frac{2}{49}g^2px^7+\frac{2\sqrt{d}\,f^2p\tan^{-1}\left(\right.}{\sqrt{e}}\\ &= -2f^2px+\frac{2d^3g^2px}{7e^3}+\frac{dfgpx^2}{2e}-\frac{2d^2g^2px^3}{21e^2}-\frac{1}{4}fgpx^4+\frac{2dg^2px^5}{35e}-\frac{2}{49}g^2px\end{aligned}$$

Mathematica [A] time = 0.21, size = 178, normalized size = 0.77

$$\frac{1}{14}x\left(14f^2+7fgx^3+2g^2x^6\right)\log\left(c\left(d+ex^2\right)^p\right)-\frac{2\sqrt{d}\,p\left(d^3g^2-7e^3f^2\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{7e^{7/2}}-\frac{d^2fgp\log\left(d+ex^2\right)}{2e^2}+\frac{px\,(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (p*x*(840*d^3*g^2 - 280*d^2*e*g^2*x^2 + 42*d*e^2*g*x*(35*f + 4*g*x^3) - 15*
e^3*(392*f^2 + 49*f*g*x^3 + 8*g^2*x^6)))/(2940*e^3) - (2*Sqrt[d]*(-7*e^3*f^
2 + d^3*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(7*e^(7/2)) - (d^2*f*g*p*Log[d
+ e*x^2])/(2*e^2) + (x*(14*f^2 + 7*f*g*x^3 + 2*g^2*x^6)*Log[c*(d + e*x^2)^p
])/14
```

fricas [A] time = 0.76, size = 454, normalized size = 1.97

$$\left[-\frac{120\,e^3g^2px^7-168\,de^2g^2px^5+735\,e^3fgpx^4+280\,d^2eg^2px^3-1470\,de^2fgpx^2+420\left(7\,e^3f^2-d^3g^2\right)p\sqrt{-\frac{d}{e}}\,\mathrm{lo}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/2940*(120*e^3*g^2*p*x^7 - 168*d*e^2*g^2*p*x^5 + 735*e^3*f*g*p*x^4 + 280
*d^2*e*g^2*p*x^3 - 1470*d*e^2*f*g*p*x^2 + 420*(7*e^3*f^2 - d^3*g^2)*p*sqrt(
-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) + 840*(7*e^3*f^2 - d^
3*g^2)*p*x - 210*(2*e^3*g^2*p*x^7 + 7*e^3*f*g*p*x^4 + 14*e^3*f^2*p*x - 7*d^
2*e*f*g*p)*log(e*x^2 + d) - 210*(2*e^3*g^2*x^7 + 7*e^3*f*g*x^4 + 14*e^3*f^2
*x)*log(c))/e^3, -1/2940*(120*e^3*g^2*p*x^7 - 168*d*e^2*g^2*p*x^5 + 735*e^3
*f*g*p*x^4 + 280*d^2*e*g^2*p*x^3 - 1470*d*e^2*f*g*p*x^2 - 840*(7*e^3*f^2 -
d^3*g^2)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 840*(7*e^3*f^2 - d^3*g^2)*p*
x - 210*(2*e^3*g^2*p*x^7 + 7*e^3*f*g*p*x^4 + 14*e^3*f^2*p*x - 7*d^2*e*f*g*p
)*log(e*x^2 + d) - 210*(2*e^3*g^2*x^7 + 7*e^3*f*g*x^4 + 14*e^3*f^2*x)*log(c
))/e^3]
```

giac [A] time = 0.25, size = 225, normalized size = 0.97

$$-\frac{1}{2}\,d^2fgpe^{(-2)}\log\left(x^2e+d\right)-\frac{2\left(d^4g^2p-7\,df^2pe^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{7\sqrt{d}}+\frac{1}{2940}\left(420\,g^2px^7e^3\log\left(x^2e+d\right)-120\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -1/2*d^2*f*g*p*e^(-2)*log(x^2*e + d) - 2/7*(d^4*g^2*p - 7*d*f^2*p*e^3)*arct
an(x*e^(1/2)/sqrt(d))*e^(-7/2)/sqrt(d) + 1/2940*(420*g^2*p*x^7*e^3*log(x^2*
e + d) - 120*g^2*p*x^7*e^3 + 420*g^2*x^7*e^3*log(c) + 168*d*g^2*p*x^5*e^2 -
 280*d^2*g^2*p*x^3*e + 1470*f*g*p*x^4*e^3*log(x^2*e + d) - 735*f*g*p*x^4*e^
3 + 1470*f*g*x^4*e^3*log(c) + 840*d^3*g^2*p*x + 1470*d*f*g*p*x^2*e^2 + 2940
*f^2*p*x*e^3*log(x^2*e + d) - 5880*f^2*p*x*e^3 + 2940*f^2*x*e^3*log(c))*e^(
-3)
```

maple [C] time = 0.54, size = 869, normalized size = 3.76

$$-\frac{i\pi fg\,x^4\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)}{4}+\frac{g^2x^7\ln(c)}{7}+f^2x\ln(c)-\frac{d^2fgp\ln\left(-d^4g^2+7d\,e^3f^2\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p),x)
```

```
[Out] -1/4*I*Pi*f*g*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/7*l
n(c)*g^2*x^7+f^2*x*ln(c)-1/2/e^2*p*ln(-d^4*g^2+7*d*e^3*f^2-(-d^7*e*g^4+14*d
^4*e^4*f^2*g^2-49*d*e^7*f^4)^(1/2)*x)*d^2*f*g-1/2/e^2*p*ln(-d^4*g^2+7*d*e^3
*f^2+(-d^7*e*g^4+14*d^4*e^4*f^2*g^2-49*d*e^7*f^4)^(1/2)*x)*d^2*f*g+1/14*I*P
i*g^2*x^7*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-2/49*g^2*p*x^7-2*f^2*
p*x+(1/7*g^2*x^7+1/2*f*g*x^4+f^2*x)*ln((e*x^2+d)^p)+1/7/e^4*p*ln(-d^4*g^2+7
*d*e^3*f^2-(-d^7*e*g^4+14*d^4*e^4*f^2*g^2-49*d*e^7*f^4)^(1/2)*x)*(-d^7*e*g^
4+14*d^4*e^4*f^2*g^2-49*d*e^7*f^4)^(1/2)-1/7/e^4*p*ln(-d^4*g^2+7*d*e^3*f^2+
(-d^7*e*g^4+14*d^4*e^4*f^2*g^2-49*d*e^7*f^4)^(1/2)*x)*(-d^7*e*g^4+14*d^4*e^
4*f^2*g^2-49*d*e^7*f^4)^(1/2)+1/2*ln(c)*f*g*x^4-1/4*f*g*p*x^4+2/7*d^3*g^2*p
*x/e^3-2/21*d^2*g^2*p*x^3/e^2+2/35*d*g^2*p*x^5/e+1/14*I*Pi*g^2*x^7*csgn(I*c
*(e*x^2+d)^p)^2*csgn(I*c)-1/4*I*Pi*f*g*x^4*csgn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi
*f^2*x*csgn(I*c)*csgn(I*c*(e*x^2+d)^p)^2+1/2*I*Pi*f^2*x*csgn(I*(e*x^2+d)^p)
*csgn(I*c*(e*x^2+d)^p)^2-1/14*I*Pi*g^2*x^7*csgn(I*c*(e*x^2+d)^p)^3-1/2*I*Pi
*f^2*x*csgn(I*c*(e*x^2+d)^p)^3-1/2*I*Pi*f^2*x*csgn(I*c)*csgn(I*(e*x^2+d)^p)
*csgn(I*c*(e*x^2+d)^p)+1/4*I*Pi*f*g*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1
/14*I*Pi*g^2*x^7*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/4*I*
Pi*f*g*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/2*d*f*g*p*x^2/e
```

maxima [A] time = 0.99, size = 178, normalized size = 0.77

$$-\frac{1}{2940}\left(\frac{1470\,d^2 fg\log\left(ex^2+d\right)}{e^3}-\frac{840\left(7\,de^3f^2-d^4g^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^4}+\frac{120\,e^3g^2x^7-168\,de^2g^2x^5+735\,e^3fgx^4}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -1/2940*(1470*d^2*f*g*log(e*x^2 + d)/e^3 - 840*(7*d*e^3*f^2 - d^4*g^2)*arct
an(e*x/sqrt(d*e))/(sqrt(d*e)*e^4) + (120*e^3*g^2*x^7 - 168*d*e^2*g^2*x^5 +
735*e^3*f*g*x^4 + 280*d^2*e*g^2*x^3 - 1470*d*e^2*f*g*x^2 + 840*(7*e^3*f^2 -
 d^3*g^2)*x)/e^4)*e*p + 1/14*(2*g^2*x^7 + 7*f*g*x^4 + 14*f^2*x)*log((e*x^2
+ d)^p*c)
```

mupad [B] time = 2.77, size = 317, normalized size = 1.37

$$\frac{g^2\,x^7\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{7}-2\,f^2\,p\,x-\frac{2\,g^2\,p\,x^7}{49}+f^2\,x\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)+\frac{f\,g\,x^4\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{2}-\frac{f\,g\,p\,x^4}{4}+\frac{2\,d\,g^2\,p\,x^5}{35\,e}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^3)^2,x)
```

```
[Out] (g^2*x^7*log(c*(d + e*x^2)^p))/7 - 2*f^2*p*x - (2*g^2*p*x^7)/49 + f^2*x*log
(c*(d + e*x^2)^p) + (f*g*x^4*log(c*(d + e*x^2)^p))/2 - (f*g*p*x^4)/4 + (2*d
*g^2*p*x^5)/(35*e) + (2*d^3*g^2*p*x)/(7*e^3) - (2*d^(1/2)*f^2*p*atan((7*d^(
1/2)*e^(7/2)*f^2*p*x)/(d^4*g^2*p - 7*d*e^3*f^2*p) - (d^(7/2)*e^(1/2)*g^2*p*
x)/(d^4*g^2*p - 7*d*e^3*f^2*p)))/e^(1/2) + (2*d^(7/2)*g^2*p*atan((7*d^(1/2)
*e^(7/2)*f^2*p*x)/(d^4*g^2*p - 7*d*e^3*f^2*p) - (d^(7/2)*e^(1/2)*g^2*p*x)/(
d^4*g^2*p - 7*d*e^3*f^2*p)))/(7*e^(7/2)) - (2*d^2*g^2*p*x^3)/(21*e^2) + (d*
f*g*p*x^2)/(2*e) - (d^2*f*g*p*log(d + e*x^2))/(2*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

3.290 $\int \left(f+gx^3\right)\log \left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=110

$$fx\log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log \left(c\left(d+ex^2\right)^p\right)-\frac{d^2gp\log \left(d+ex^2\right)}{4e^2}+\frac{2\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{dgpx^2}{4e}-2fpx-\frac{1}{8}gpx^4$$

```
[Out] -2*f*p*x+1/4*d*g*p*x^2/e-1/8*g*p*x^4-1/4*d^2*g*p*ln(e*x^2+d)/e^2+f*x*ln(c*(
e*x^2+d)^p)+1/4*g*x^4*ln(c*(e*x^2+d)^p)+2*f*p*arctan(x*e^(1/2)/d^(1/2))*d^(
1/2)/e^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2471, 2448, 321, 205, 2454, 2395, 43}

$$fx\log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log \left(c\left(d+ex^2\right)^p\right)-\frac{d^2gp\log \left(d+ex^2\right)}{4e^2}+\frac{2\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{dgpx^2}{4e}-2fpx-\frac{1}{8}gpx^4$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^3)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (d*g*p*x^2)/(4*e) - (g*p*x^4)/8 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/Sqrt[e] - (d^2*g*p*Log[d + e*x^2])/(4*e^2) + f*x*Log[c*(d + e
*x^2)^p] + (g*x^4*Log[c*(d + e*x^2)^p])/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
```

```
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\begin{aligned}\int \left(f+gx^3\right)\log\left(c\left(d+ex^2\right)^p\right)dx &= \int \left(f\log\left(c\left(d+ex^2\right)^p\right)+gx^3\log\left(c\left(d+ex^2\right)^p\right)\right)dx\\ &= f\int \log\left(c\left(d+ex^2\right)^p\right)dx+g\int x^3\log\left(c\left(d+ex^2\right)^p\right)dx\\ &= fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}g\,\mathrm{Subst}\left(\int x\log\left(c(d+ex)^p\right)dx,x,x^2\right)-(2efp\\ &= -2fpx+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log\left(c\left(d+ex^2\right)^p\right)+(2dfp)\int\frac{}{d+}\\ &= -2fpx+\frac{2\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log\left(c\left(d+\right.\right.\\ &= -2fpx+\frac{dgpx^2}{4e}-\frac{1}{8}gpx^4+\frac{2\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{d^2gp\log\left(d+ex^2\right)}{4e^2}+\end{aligned}$$

Mathematica [A] time = 0.05, size = 110, normalized size = 1.00

$$fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2gp\log\left(d+ex^2\right)}{4e^2}+\frac{2\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{dgpx^2}{4e}-2fpx-\frac{1}{8}gpx$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (d*g*p*x^2)/(4*e) - (g*p*x^4)/8 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/Sqrt[e] - (d^2*g*p*Log[d + e*x^2])/(4*e^2) + f*x*Log[c*(d + e
*x^2)^p] + (g*x^4*Log[c*(d + e*x^2)^p])/4
```

fricas [A] time = 0.61, size = 250, normalized size = 2.27

$$\left[-\frac{e^2gpx^4-2degpx^2-8e^2fp\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2+2ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)+16e^2fpx-2\left(e^2gpx^4+4e^2fpx-d^2gp\right)\log\left(ex^2+d\right.}{8e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/8*(e^2*g*p*x^4 - 2*d*e*g*p*x^2 - 8*e^2*f*p*sqrt(-d/e)*log((e*x^2 + 2*e*
x*sqrt(-d/e) - d)/(e*x^2 + d)) + 16*e^2*f*p*x - 2*(e^2*g*p*x^4 + 4*e^2*f*p*
x - d^2*g*p)*log(e*x^2 + d) - 2*(e^2*g*x^4 + 4*e^2*f*x)*log(c))/e^2, -1/8*(
e^2*g*p*x^4 - 2*d*e*g*p*x^2 - 16*e^2*f*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d)
+ 16*e^2*f*p*x - 2*(e^2*g*p*x^4 + 4*e^2*f*p*x - d^2*g*p)*log(e*x^2 + d) - 2
*(e^2*g*x^4 + 4*e^2*f*x)*log(c))/e^2]
```

giac [A] time = 0.19, size = 117, normalized size = 1.06

$$-\frac{1}{4}d^2gpe^{(-2)}\log\left(x^2e+d\right)+2\sqrt{d}fp\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}+\frac{1}{8}\left(2gpx^4e\log\left(x^2e+d\right)-gpx^4e+2gx^4e\log(c)+2dgp\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -1/4*d^2*g*p*e^(-2)*log(x^2*e + d) + 2*sqrt(d)*f*p*arctan(x*e^(1/2)/sqrt(d)
)*e^(-1/2) + 1/8*(2*g*p*x^4*e*log(x^2*e + d) - g*p*x^4*e + 2*g*x^4*e*log(c)
 + 2*d*g*p*x^2 + 8*f*p*x*e*log(x^2*e + d) - 16*f*p*x*e + 8*f*x*e*log(c))*e^
(-1)
```

maple [C] time = 0.55, size = 402, normalized size = 3.65

$$-\frac{i\pi g x^4\mathrm{csgn}(ic)\,\mathrm{csgn}\left(i\left(ex^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(ex^2+d\right)^p\right)}{8}+\frac{i\pi g x^4\mathrm{csgn}(ic)\,\mathrm{csgn}\left(ic\left(ex^2+d\right)^p\right)^2}{8}+\frac{i\pi g x^4\mathrm{csgn}\left(i\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)*ln(c*(e*x^2+d)^p),x)
```

```
[Out] (1/4*g*x^4+f*x)*ln((e*x^2+d)^p)-1/8*I*Pi*g*x^4*csgn(I*c*(e*x^2+d)^p)^3+1/2*
I*Pi*f*x*csgn(I*c)*csgn(I*c*(e*x^2+d)^p)^2+1/8*I*csgn(I*c)*csgn(I*c*(e*x^2+
d)^p)^2*x^4*g*Pi+1/2*I*Pi*f*x*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1
/8*I*Pi*g*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I*Pi*
f*x*csgn(I*c*(e*x^2+d)^p)^3+1/8*I*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*(e*x^2+d)^
p)*x^4*g*Pi-1/2*I*Pi*f*x*csgn(I*c)*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p
)+1/4*ln(c)*g*x^4-1/8*g*p*x^4+1/4*d*g*p*x^2/e+f*x*ln(c)+1/e*p*ln(d-(-d*e)^(
1/2)*x)*f*(-d*e)^(1/2)-1/4/e^2*p*ln(d-(-d*e)^(1/2)*x)*d^2*g-1/e*p*ln(d+(-d*
e)^(1/2)*x)*f*(-d*e)^(1/2)-1/4/e^2*p*ln(d+(-d*e)^(1/2)*x)*d^2*g-2*f*p*x
```

maxima [A] time = 1.00, size = 92, normalized size = 0.84

$$\frac{1}{8}\left(\frac{16\,df\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e}-\frac{2\,d^2g\log\left(ex^2+d\right)}{e^3}-\frac{egx^4-2\,dgx^2+16\,efx}{e^2}\right)ep+\frac{1}{4}\left(gx^4+4\,fx\right)\log\left(\left(ex^2+d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] 1/8*(16*d*f*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e) - 2*d^2*g*log(e*x^2 + d)/e^
3 - (e*g*x^4 - 2*d*g*x^2 + 16*e*f*x)/e^2)*e*p + 1/4*(g*x^4 + 4*f*x)*log((e*
x^2 + d)^p*c)
```

mupad [B] time = 0.92, size = 94, normalized size = 0.85

$$f\,x\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)-\frac{g\,p\,x^4}{8}-2\,f\,p\,x+\frac{g\,x^4\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{4}+\frac{d\,g\,p\,x^2}{4\,e}+\frac{2\,\sqrt{d}\,f\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{d^2\,g\,p\,\ln\left(e\,x^2+d\right)}{4\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^3),x)
```

```
[Out] f*x*log(c*(d + e*x^2)^p) - (g*p*x^4)/8 - 2*f*p*x + (g*x^4*log(c*(d + e*x^2)
^p))/4 + (d*g*p*x^2)/(4*e) + (2*d^(1/2)*f*p*atan((e^(1/2)*x)/d^(1/2)))/e^(1
/2) - (d^2*g*p*log(d + e*x^2))/(4*e^2)
```

sympy [A] time = 48.27, size = 175, normalized size = 1.59

$$\begin{cases} \frac{i\sqrt{d}fp\log\left(d+ex^2\right)}{e\sqrt{\frac{1}{e}}} - \frac{2i\sqrt{d}fp\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{e\sqrt{\frac{1}{e}}} - \frac{d^2gp\log\left(d+ex^2\right)}{4e^2} + \frac{dgpx^2}{4e} + fpx\log\left(d+ex^2\right) - 2fpx + fx\log(c) + \frac{gpx}{} \\ \left(fx + \frac{gx^4}{4}\right)\log\left(cd^p\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((I*sqrt(d)*f*p*log(d + e*x**2)/(e*sqrt(1/e)) - 2*I*sqrt(d)*f*p*lo
g(-I*sqrt(d)*sqrt(1/e) + x)/(e*sqrt(1/e)) - d**2*g*p*log(d + e*x**2)/(4*e**
2) + d*g*p*x**2/(4*e) + f*p*x*log(d + e*x**2) - 2*f*p*x + f*x*log(c) + g*p*
x**4*log(d + e*x**2)/4 - g*p*x**4/8 + g*x**4*log(c)/4, Ne(e, 0)), ((f*x + g
*x**4/4)*log(c*d**p), True))
```

3.291 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$

Optimal. Leaf size=1165

$$-\frac{p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)\log\left(-\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}-\frac{p\log\left(-\frac{\sqrt[3]{g}\left(\sqrt{e}x+\sqrt{-d}\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}+\frac{\log\left(c\left(ex^2+d\right)^p\right)\log\left(-\right.}{3f^{2/3}\sqrt[3]{g}}$$

```
[Out] 1/3*ln(-f^(1/3)-g^(1/3)*x)*ln(c*(e*x^2+d)^p)/f^(2/3)/g^(1/3)+1/3*(-1)^(2/3)
*ln(-f^(1/3)+(-1)^(1/3)*g^(1/3)*x)*ln(c*(e*x^2+d)^p)/f^(2/3)/g^(1/3)-1/3*(-
1)^(1/3)*ln(-f^(1/3)-(-1)^(2/3)*g^(1/3)*x)*ln(c*(e*x^2+d)^p)/f^(2/3)/g^(1/3
)-1/3*p*ln(-f^(1/3)-g^(1/3)*x)*ln(g^(1/3)*((-d)^(1/2)-x*e^(1/2))/(g^(1/3)*(
-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)-1/3*(-1)^(2/3)*p*ln(-f^(1/3)+(-
1)^(1/3)*g^(1/3)*x)*ln(-(-1)^(1/3)*g^(1/3)*((-d)^(1/2)-x*e^(1/2))/(-(-1)^(1
/3)*g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)+1/3*(-1)^(1/3)*p*l
n(-f^(1/3)-(-1)^(2/3)*g^(1/3)*x)*ln((-1)^(2/3)*g^(1/3)*((-d)^(1/2)-x*e^(1/2
))/((-1)^(2/3)*g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)-1/3*p*l
n(-f^(1/3)-g^(1/3)*x)*ln(-g^(1/3)*((-d)^(1/2)+x*e^(1/2))/(-g^(1/3)*(-d)^(1/
2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)-1/3*(-1)^(2/3)*p*ln(-f^(1/3)+(-1)^(1/3
)*g^(1/3)*x)*ln((-1)^(1/3)*g^(1/3)*((-d)^(1/2)+x*e^(1/2))/((-1)^(1/3)*g^(1/
3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)+1/3*(-1)^(1/3)*p*ln(-f^(1/3
)-(-1)^(2/3)*g^(1/3)*x)*ln(-(-1)^(2/3)*g^(1/3)*((-d)^(1/2)+x*e^(1/2))/(-(-1
)^(2/3)*g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)-1/3*p*polylog(
2,(f^(1/3)+g^(1/3)*x)*e^(1/2)/(-g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3
)/g^(1/3)-1/3*p*polylog(2,(f^(1/3)+g^(1/3)*x)*e^(1/2)/(g^(1/3)*(-d)^(1/2)+f
^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)-1/3*(-1)^(2/3)*p*polylog(2,(f^(1/3)-(-1)^(
1/3)*g^(1/3)*x)*e^(1/2)/(-(-1)^(1/3)*g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f
^(2/3)/g^(1/3)-1/3*(-1)^(2/3)*p*polylog(2,(f^(1/3)-(-1)^(1/3)*g^(1/3)*x)*e^
(1/2)/((-1)^(1/3)*g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)+1/3*
(-1)^(1/3)*p*polylog(2,(f^(1/3)+(-1)^(2/3)*g^(1/3)*x)*e^(1/2)/(-(-1)^(2/3)*
g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)+1/3*(-1)^(1/3)*p*polyl
og(2,(f^(1/3)+(-1)^(2/3)*g^(1/3)*x)*e^(1/2)/((-1)^(2/3)*g^(1/3)*(-d)^(1/2)+
f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)
```

Rubi [A] time = 1.60, antiderivative size = 1165, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2471, 2462, 260, 2416, 2394, 2393, 2391}

$$-\frac{p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)\log\left(-\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}-\frac{p\log\left(-\frac{\sqrt[3]{g}\left(\sqrt{e}x+\sqrt{-d}\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}+\frac{\log\left(c\left(ex^2+d\right)^p\right)\log\left(-\right.}{3f^{2/3}\sqrt[3]{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^3),x]
```

```
[Out] -(p*Log[(g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3
))]*Log[-f^(1/3) - g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) - (p*Log[-((g^(1/3)*(Sqr
t[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - Sqrt[-d]*g^(1/3)))]*Log[-f^(1/3) - g
^(1/3)*x])/(3*f^(2/3)*g^(1/3)) - ((-1)^(2/3)*p*Log[-(((-1)^(1/3)*g^(1/3)*(S
qrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3)))]*Log
[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) - ((-1)^(2/3)*p*Log[
((-1)^(1/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (-1)^(1/3)*S
qrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x])/(3*f^(2/3)*g^(1/3))
 + ((-1)^(1/3)*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f
^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x]
```

```
)/(3*f^(2/3)*g^(1/3)) + ((-1)^(1/3)*p*Log[-(((-1)^(2/3)*g^(1/3)*(Sqrt[-d] +
 Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqrt[-d]*g^(1/3)))]*Log[-f^(1/3)
 - (-1)^(2/3)*g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) + (Log[-f^(1/3) - g^(1/3)*x]*
Log[c*(d + e*x^2)^p])/(3*f^(2/3)*g^(1/3)) + ((-1)^(2/3)*Log[-f^(1/3) + (-1)
^(1/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p])/(3*f^(2/3)*g^(1/3)) - ((-1)^(1/3)*L
og[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p])/(3*f^(2/3)*g^(1/3
)) - (p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^(1/3)*x))/(Sqrt[e]*f^(1/3) - Sqrt[
-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) - (p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^(1
/3)*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) - ((-1)^
(2/3)*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/3)*x))/(Sqrt[e]*f^(1
/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) - ((-1)^(2/3)*p*Po
lyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)
^(1/3)*Sqrt[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) + ((-1)^(1/3)*p*PolyLog[2, (
Sqrt[e]*(f^(1/3) + (-1)^(2/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqr
t[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) + ((-1)^(1/3)*p*PolyLog[2, (Sqrt[e]*(f
^(1/3) + (-1)^(2/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1
/3))])/(3*f^(2/3)*g^(1/3))
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
```

```
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx = \int \left(-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{3f^{2/3}\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{3f^{2/3}\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{3f^{2/3}\left(-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x\right)}\right.$$

$$= -\frac{\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{-\sqrt[3]{f}-\sqrt[3]{g}x}\,dx}{3f^{2/3}}-\frac{\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x}\,dx}{3f^{2/3}}-\frac{\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x}\,dx}{3f^{2/3}}$$

$$= \frac{\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)\log\left(c\left(d+ex^2\right)^p\right)}{3f^{2/3}\sqrt[3]{g}}+\frac{(-1)^{2/3}\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)\log\left(c\left(d+ex\right.\right.}{3f^{2/3}\sqrt[3]{g}}$$

$$= \frac{\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)\log\left(c\left(d+ex^2\right)^p\right)}{3f^{2/3}\sqrt[3]{g}}+\frac{(-1)^{2/3}\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)\log\left(c\left(d+ex\right.\right.}{3f^{2/3}\sqrt[3]{g}}$$

$$= \frac{\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)\log\left(c\left(d+ex^2\right)^p\right)}{3f^{2/3}\sqrt[3]{g}}+\frac{(-1)^{2/3}\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)\log\left(c\left(d+ex\right.\right.}{3f^{2/3}\sqrt[3]{g}}$$

$$= -\frac{p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}\sqrt[3]{f}+\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}}-\frac{p\log\left(-\frac{\sqrt[3]{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}\right.}{3f^{2/3}\sqrt[3]{g}}$$

$$= -\frac{p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}\sqrt[3]{f}+\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}}-\frac{p\log\left(-\frac{\sqrt[3]{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}\right.}{3f^{2/3}\sqrt[3]{g}}$$

$$= -\frac{p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}\sqrt[3]{f}+\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}}-\frac{p\log\left(-\frac{\sqrt[3]{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}\right.}{3f^{2/3}\sqrt[3]{g}}$$

Mathematica [A] time = 0.83, size = 990, normalized size = 0.85

$$\frac{-p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)\log\left(-\sqrt[3]{g}x-\sqrt[3]{f}\right)-p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{e}x+\sqrt{-d}\right)}{\sqrt{-d}\sqrt[3]{g}-\sqrt{e}\sqrt[3]{f}}\right)\log\left(-\sqrt[3]{g}x-\sqrt[3]{f}\right)+\log\left(c\left(ex^2+d\right)^p\right)\log\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^3),x]
```

```
[Out] (-(p*Log[(g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/
3))]*Log[-f^(1/3) - g^(1/3)*x]) - p*Log[(g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(-
(Sqrt[e]*f^(1/3)) + Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - g^(1/3)*x] - (-1)^(2/
3)*p*Log[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(-(Sqrt[e]*f^(1/3)) +
(-1)^(1/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x] - (-1)^(
```

```
2/3)*p*Log[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (
-1)^(1/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x] + (-1)^(1
/3)*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (-
1)^(2/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x] + (-1)^(1/
3)*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(-(Sqrt[e]*f^(1/3)) +
(-1)^(2/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x] + Log[-f
^(1/3) - g^(1/3)*x]*Log[c*(d + e*x^2)^p] + (-1)^(2/3)*Log[-f^(1/3) + (-1)^(
1/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p] - (-1)^(1/3)*Log[-f^(1/3) - (-1)^(2/3)
*g^(1/3)*x]*Log[c*(d + e*x^2)^p] - p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^(1/3)
*x))/(Sqrt[e]*f^(1/3) - Sqrt[-d]*g^(1/3))] - p*PolyLog[2, (Sqrt[e]*(f^(1/3)
 + g^(1/3)*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3))] - (-1)^(2/3)*p*PolyLog
[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3
)*Sqrt[-d]*g^(1/3))] - (-1)^(2/3)*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/
3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(1/3)*Sqrt[-d]*g^(1/3))] + (-1)^(1/3
)*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + (-1)^(2/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3)
- (-1)^(2/3)*Sqrt[-d]*g^(1/3))] + (-1)^(1/3)*p*PolyLog[2, (Sqrt[e]*(f^(1/3)
 + (-1)^(2/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1/3))])
/(3*f^(2/3)*g^(1/3))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^3+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^3 + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^3+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^3 + f), x)
```

maple [C] time = 0.90, size = 1180, normalized size = 1.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^3+f),x)
```

```
[Out] 1/3*(-p*ln(e*x^2+d)+ln((e*x^2+d)^p))/g/(f/g)^(2/3)*ln(x+(f/g)^(1/3))-1/6*(-
p*ln(e*x^2+d)+ln((e*x^2+d)^p))/g/(f/g)^(2/3)*ln(x^2-(f/g)^(1/3)*x+(f/g)^(2/
3))+1/3*(-p*ln(e*x^2+d)+ln((e*x^2+d)^p))/g/(f/g)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(f/g)^(1/3)*x-1))+1/3*p/g*sum(1/_alpha^2*(ln(-_alpha+x)*ln(e*x^2+
d)-ln(-_alpha+x)*(ln((RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=1)-x+_
alpha)/RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=1))+ln((RootOf(_Z^2*e
+2*_Z*_alpha*e+_alpha^2*e+d,index=2)-x+_alpha)/RootOf(_Z^2*e+2*_Z*_alpha*e+
_alpha^2*e+d,index=2)))-dilog((RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,ind
ex=1)-x+_alpha)/RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=1))-dilog((R
ootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=2)-x+_alpha)/RootOf(_Z^2*e+2*
_Z*_alpha*e+_alpha^2*e+d,index=2))),_alpha=RootOf(_Z^3*g+f))-1/12*I*Pi*csgn
```

```
(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g/(f/g)^(2/3)*ln(x^2-(f/g)^(1/3)*x+(f/g)^(2/3
))+1/12*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/g/(f/g)^(2
/3)*ln(x^2-(f/g)^(1/3)*x+(f/g)^(2/3))-1/12*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*
c*(e*x^2+d)^p)^2/g/(f/g)^(2/3)*ln(x^2-(f/g)^(1/3)*x+(f/g)^(2/3))-1/6*I*Pi*c
sgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/g/(f/g)^(2/3)*ln(x+(f/g)
^(1/3))-1/6*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/g/(f/g
)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(f/g)^(1/3)*x-1))+1/12*I*Pi*csgn(I*c*
(e*x^2+d)^p)^3/g/(f/g)^(2/3)*ln(x^2-(f/g)^(1/3)*x+(f/g)^(2/3))-1/6*I*Pi*csg
n(I*c*(e*x^2+d)^p)^3/g/(f/g)^(2/3)*ln(x+(f/g)^(1/3))+1/6*I*Pi*csgn(I*(e*x^2
+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/g/(f/g)^(2/3)*ln(x+(f/g)^(1/3))+1/6*I*Pi*csg
n(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/g/(f/g)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(f/g)^(1/3)*x-1))+1/6*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g/(f
/g)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(f/g)^(1/3)*x-1))+1/6*I*Pi*csgn(I*c
*(e*x^2+d)^p)^2*csgn(I*c)/g/(f/g)^(2/3)*ln(x+(f/g)^(1/3))-1/6*I*Pi*csgn(I*c
*(e*x^2+d)^p)^3/g/(f/g)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(f/g)^(1/3)*x-1
))+1/3*ln(c)/g/(f/g)^(2/3)*ln(x+(f/g)^(1/3))-1/6*ln(c)/g/(f/g)^(2/3)*ln(x^2
-(f/g)^(1/3)*x+(f/g)^(2/3))+1/3*ln(c)/g/(f/g)^(2/3)*3^(1/2)*arctan(1/3*3^(1
/2)*(2/(f/g)^(1/3)*x-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^3+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f),x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^3 + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{g\,x^3+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^3),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**3+f),x)
```

```
[Out] Timed out
```

3.292 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} dx$

Optimal. Leaf size=1861

result too large to display

```
[Out] -2/9*e*p*ln(f^(1/3)+g^(1/3)*x)/f/(e*f^(2/3)+d*g^(2/3))/g^(1/3)+1/9*e*p*ln(e
*x^2+d)/f/(e*f^(2/3)+d*g^(2/3))/g^(1/3)+4/9*p*ln(2*f^(1/3)-g^(1/3)*x*(1-I*3
^(1/2)))*ln(-g^(1/3)*(3^(1/2)+I)*((-d)^(1/2)-x*e^(1/2))/(-g^(1/3)*(3^(1/2)+
I)*(-d)^(1/2)+2*I*f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)/(1-I*3^(1/2))+4/9*p*ln(
2*f^(1/3)-g^(1/3)*x*(1-I*3^(1/2)))*ln(g^(1/3)*(3^(1/2)+I)*((-d)^(1/2)+x*e^(
1/2))/(g^(1/3)*(3^(1/2)+I)*(-d)^(1/2)+2*I*f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)
/(1-I*3^(1/2))+4/9*p*ln(2*f^(1/3)-g^(1/3)*x*(1+I*3^(1/2)))*ln(-g^(1/3)*(1+I
*3^(1/2))*((-d)^(1/2)-x*e^(1/2))/(-g^(1/3)*(1+I*3^(1/2))*(-d)^(1/2)+2*f^(1/
3)*e^(1/2)))/f^(5/3)/g^(1/3)/(1+I*3^(1/2))+4/9*p*ln(2*f^(1/3)-g^(1/3)*x*(1+
I*3^(1/2)))*ln(g^(1/3)*(1+I*3^(1/2))*((-d)^(1/2)+x*e^(1/2))/(g^(1/3)*(1+I*3
^(1/2))*(-d)^(1/2)+2*f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)/(1+I*3^(1/2))+2/9*p*
arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/f^(4/3)/(e*f^(2/3)+d*g^(2/3))+4/9
*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/f^(4/3)/(2*e*f^(2/3)-d*g^(2/3)
*(1+I*3^(1/2)))+4/9*(-1)^(1/3)*e*p*ln(f^(1/3)+(-1)^(2/3)*g^(1/3)*x)/f/g^(1/
3)/(2*e*f^(2/3)-d*g^(2/3)*(1+I*3^(1/2)))-2/9*(-1)^(1/3)*e*p*ln(e*x^2+d)/f/g
^(1/3)/(2*e*f^(2/3)-d*g^(2/3)*(1+I*3^(1/2)))-ln(c*(e*x^2+d)^p)/(1+(-1)^(1/3
))^4/f^(4/3)/g^(1/3)/((-1)^(2/3)*f^(1/3)+g^(1/3)*x)+1/9*(-1)^(1/3)*ln(c*(e*
x^2+d)^p)/f^(4/3)/g^(1/3)/(f^(1/3)+(-1)^(2/3)*g^(1/3)*x)-2/9*p*ln(f^(1/3)+g
^(1/3)*x)*ln(g^(1/3)*((-d)^(1/2)-x*e^(1/2))/(g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(
1/2)))/f^(5/3)/g^(1/3)-2/9*p*ln(f^(1/3)+g^(1/3)*x)*ln(-g^(1/3)*((-d)^(1/2)+
x*e^(1/2))/(-g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)-4/9*ln(c*
(e*x^2+d)^p)*ln(2*f^(1/3)-g^(1/3)*x*(1-I*3^(1/2)))/f^(5/3)/g^(1/3)/(1-I*3^(
1/2))+4/9*p*polylog(2,(2*f^(1/3)-g^(1/3)*x*(1-I*3^(1/2)))*e^(1/2)/(g^(1/3)*
(1-I*3^(1/2))*(-d)^(1/2)+2*f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)/(1-I*3^(1/2))+
4/9*p*polylog(2,(2*f^(1/3)-g^(1/3)*x*(1-I*3^(1/2)))*e^(1/2)/(I*g^(1/3)*(3^(
1/2)+I)*(-d)^(1/2)+2*f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)/(1-I*3^(1/2))-4/9*ln
(c*(e*x^2+d)^p)*ln(2*f^(1/3)-g^(1/3)*x*(1+I*3^(1/2)))/f^(5/3)/g^(1/3)/(1+I*
3^(1/2))+4/9*p*polylog(2,(2*f^(1/3)-g^(1/3)*x*(1+I*3^(1/2)))*e^(1/2)/(-g^(1
/3)*(1+I*3^(1/2))*(-d)^(1/2)+2*f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)/(1+I*3^(1/
2))+4/9*p*polylog(2,(2*f^(1/3)-g^(1/3)*x*(1+I*3^(1/2)))*e^(1/2)/(g^(1/3)*(1
+I*3^(1/2))*(-d)^(1/2)+2*f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)/(1+I*3^(1/2))-1/
9*ln(c*(e*x^2+d)^p)/f^(4/3)/g^(1/3)/(f^(1/3)+g^(1/3)*x)+2/9*ln(f^(1/3)+g^(1
/3)*x)*ln(c*(e*x^2+d)^p)/f^(5/3)/g^(1/3)-2/9*p*polylog(2,(f^(1/3)+g^(1/3)*x
)*e^(1/2)/(-g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)-2/9*p*poly
log(2,(f^(1/3)+g^(1/3)*x)*e^(1/2)/(g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(
5/3)/g^(1/3)+2*(-1)^(1/3)*e*p*ln(f^(1/3)-(-1)^(1/3)*g^(1/3)*x)/(1+(-1)^(1/3
))^4/f/(e*f^(2/3)+(-1)^(2/3)*d*g^(2/3))/g^(1/3)-(-1)^(1/3)*e*p*ln(e*x^2+d)/
(1+(-1)^(1/3))^4/f/(e*f^(2/3)+(-1)^(2/3)*d*g^(2/3))/g^(1/3)+2*(-1)^(2/3)*p*
arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/(1+(-1)^(1/3))^4/f^(4/3)/(e*f^(2/
3)+(-1)^(2/3)*d*g^(2/3))
```

Rubi [A] time = 2.89, antiderivative size = 1863, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2471, 2463, 801, 635, 205, 260, 2462, 2416, 2394, 2393, 2391}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^3)^2,x]
```

```
[Out] (2*Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(9*f^(4/3)*(e*f^(2/3) + d
*g^(2/3))) + (2*(-1)^(2/3)*Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
```

```
(1 + (-1)^(1/3))^4*f^(4/3)*(e*f^(2/3) + (-1)^(2/3)*d*g^(2/3))) + (4*Sqrt[d]
*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(9*f^(4/3)*(2*e*f^(2/3) + I*(I - Sq
rt[3])*d*g^(2/3))) - (2*e*p*Log[f^(1/3) + g^(1/3)*x])/(9*f*(e*f^(2/3) + d*g
^(2/3))*g^(1/3)) - (2*p*Log[(g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/
3) + Sqrt[-d]*g^(1/3))]*Log[f^(1/3) + g^(1/3)*x])/(9*f^(5/3)*g^(1/3)) - (2*
p*Log[-((g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - Sqrt[-d]*g^(1/3
)))]*Log[f^(1/3) + g^(1/3)*x])/(9*f^(5/3)*g^(1/3)) + (2*(-1)^(1/3)*e*p*Log[
f^(1/3) - (-1)^(1/3)*g^(1/3)*x])/((1 + (-1)^(1/3))^4*f*(e*f^(2/3) + (-1)^(2
/3)*d*g^(2/3))*g^(1/3)) + ((2*I)*Sqrt[3]*p*Log[-(((-1)^(1/3)*g^(1/3)*(Sqrt[
-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3)))]*Log[-f^
(1/3) + (-1)^(1/3)*g^(1/3)*x])/((1 + (-1)^(1/3))^5*f^(5/3)*g^(1/3)) + ((2*I
)*Sqrt[3]*p*Log[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3
) + (-1)^(1/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x])/((1
 + (-1)^(1/3))^5*f^(5/3)*g^(1/3)) + (4*(-1)^(1/3)*e*p*Log[f^(1/3) + (-1)^(2
/3)*g^(1/3)*x])/(9*f*(2*e*f^(2/3) - (1 + I*Sqrt[3])*d*g^(2/3))*g^(1/3)) - (
2*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (-1)
^(2/3)*Sqrt[-d]*g^(1/3))]*Log[f^(1/3) + (-1)^(2/3)*g^(1/3)*x])/((1 + (-1)^(
1/3))^4*f^(5/3)*g^(1/3)) - (2*p*Log[-(((-1)^(2/3)*g^(1/3)*(Sqrt[-d] + Sqrt[
e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqrt[-d]*g^(1/3)))]*Log[f^(1/3) + (-1)
^(2/3)*g^(1/3)*x])/((1 + (-1)^(1/3))^4*f^(5/3)*g^(1/3)) + (e*p*Log[d + e*x^
2])/(9*f*(e*f^(2/3) + d*g^(2/3))*g^(1/3)) - ((-1)^(1/3)*e*p*Log[d + e*x^2])
/((1 + (-1)^(1/3))^4*f*(e*f^(2/3) + (-1)^(2/3)*d*g^(2/3))*g^(1/3)) - (2*(-1
)^(1/3)*e*p*Log[d + e*x^2])/(9*f*(2*e*f^(2/3) - (1 + I*Sqrt[3])*d*g^(2/3))*
g^(1/3)) - Log[c*(d + e*x^2)^p]/(9*f^(4/3)*g^(1/3)*(f^(1/3) + g^(1/3)*x)) -
 Log[c*(d + e*x^2)^p]/((1 + (-1)^(1/3))^4*f^(4/3)*g^(1/3)*((-1)^(2/3)*f^(1/
3) + g^(1/3)*x)) + ((-1)^(1/3)*Log[c*(d + e*x^2)^p])/(9*f^(4/3)*g^(1/3)*(f^
(1/3) + (-1)^(2/3)*g^(1/3)*x)) + (2*Log[f^(1/3) + g^(1/3)*x]*Log[c*(d + e*x
^2)^p])/(9*f^(5/3)*g^(1/3)) - ((2*I)*Sqrt[3]*Log[-f^(1/3) + (-1)^(1/3)*g^(1
/3)*x]*Log[c*(d + e*x^2)^p])/((1 + (-1)^(1/3))^5*f^(5/3)*g^(1/3)) + (2*Log[
f^(1/3) + (-1)^(2/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p])/((1 + (-1)^(1/3))^4*f
^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^(1/3)*x))/(Sqrt[e]*
f^(1/3) - Sqrt[-d]*g^(1/3))]])/(9*f^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e
]*(f^(1/3) + g^(1/3)*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3))]])/(9*f^(5/3)*
g^(1/3)) + ((2*I)*Sqrt[3]*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/
3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3))]])/((1 + (-1)^(1/3))^
5*f^(5/3)*g^(1/3)) + ((2*I)*Sqrt[3]*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(
1/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(1/3)*Sqrt[-d]*g^(1/3))]])/((1 + (-
1)^(1/3))^5*f^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + (-1)^(2/
3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqrt[-d]*g^(1/3))]])/((1 + (-1)
^(1/3))^4*f^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + (-1)^(2/3)
*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1/3))]])/((1 + (-1)^(
1/3))^4*f^(5/3)*g^(1/3))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx = \int\left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{9f^{4/3}\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)^2}+\frac{2\log\left(c\left(d+ex^2\right)^p\right)}{9f^{5/3}\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}+\frac{(-1)^{2/3}\log\left(c\left(d+ex^2\right)^p\right)}{\left(1+\sqrt[3]{-1}\right)^4 f^{4/3}\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right.}\right.$$

$$=\frac{2\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt[3]{f}+\sqrt[3]{g}x}dx}{9f^{5/3}}+\frac{2\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x}dx}{9f^{5/3}}-\frac{\left(2(-1)^{5/6}\sqrt{3}\right)\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x}dx}{\left(1+\sqrt[3]{-1}\right)^5 f^{5/3}}+\frac{\int\frac{\log}{(}}{}$$

$$=-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}+\frac{\sqrt[3]{-1}\log\left(c\left(d+ex^2\right)^p\right)}{9f^{4/3}\sqrt[3]{g}\left((-1)^{2/3}\sqrt[3]{f}+\sqrt[3]{g}x\right)}+\frac{\sqrt[3]{-1}\log\left(c\left(d+ex^2\right)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{\cdot}\right.}$$

$$=-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}+\frac{\sqrt[3]{-1}\log\left(c\left(d+ex^2\right)^p\right)}{9f^{4/3}\sqrt[3]{g}\left((-1)^{2/3}\sqrt[3]{f}+\sqrt[3]{g}x\right)}+\frac{\sqrt[3]{-1}\log\left(c\left(d+ex^2\right)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{\cdot}\right.}$$

$$=-\frac{2ep\log\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}{9f\left(ef^{2/3}+dg^{2/3}\right)\sqrt[3]{g}}-\frac{2(-1)^{2/3}ep\log\left(\sqrt[3]{f}-\sqrt[3]{-1}\sqrt[3]{g}x\right)}{9f\left(ef^{2/3}+(-1)^{2/3}dg^{2/3}\right)\sqrt[3]{g}}+\frac{2\sqrt[3]{-1}ep\log\left(\sqrt[3]{f}+(-1\right.}{9f\left(ef^{2/3}-\sqrt[3]{-1}dg^{2}\right.}$$

$$=-\frac{2ep\log\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}{9f\left(ef^{2/3}+dg^{2/3}\right)\sqrt[3]{g}}-\frac{2p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}\sqrt[3]{f}+\sqrt{-d}\sqrt[3]{g}}\right)\log\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}{9f^{5/3}\sqrt[3]{g}}-\frac{2p\log\left(-\frac{\sqrt[3]{g}\left(\sqrt{-}\right.}{\sqrt{e}\sqrt[3]{f}}\right.}{}$$

$$=\frac{2\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9f^{4/3}\left(ef^{2/3}+dg^{2/3}\right)}+\frac{2\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9f^{4/3}\left(ef^{2/3}-\sqrt[3]{-1}dg^{2/3}\right)}+\frac{2\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9f^{4/3}\left(ef^{2/3}+(-1)^{2/3}dg^{2/3}\right)}-\frac{2e}{9f}$$

$$=\frac{2\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9f^{4/3}\left(ef^{2/3}+dg^{2/3}\right)}+\frac{2\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9f^{4/3}\left(ef^{2/3}-\sqrt[3]{-1}dg^{2/3}\right)}+\frac{2\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9f^{4/3}\left(ef^{2/3}+(-1)^{2/3}dg^{2/3}\right)}-\frac{2e}{9f}$$

Mathematica [A] time = 7.14, size = 2168, normalized size = 1.16

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^3)^2,x]
```

```
[Out] (x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(3*f*(f + g*x^3)) + (2*Arc
Tan[(-f^(1/3) + 2*g^(1/3)*x)/(Sqrt[3]*f^(1/3))]*(-(p*Log[d + e*x^2]) + Log[
c*(d + e*x^2)^p]))/(3*Sqrt[3]*f^(5/3)*g^(1/3)) + (2*Log[f^(1/3) + g^(1/3)*x
]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(9*f^(5/3)*g^(1/3)) - ((-(p
*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])*Log[f^(2/3) - f^(1/3)*g^(1/3)*x +
g^(2/3)*x^2])/(9*f^(5/3)*g^(1/3)) + p*(-1/3*((-1 + (-1)^(1/3))*(-(Log[((-I)
*Sqrt[d])/Sqrt[e] + x]/((-1)^(2/3)*f^(1/3) + g^(1/3)*x)) + (Sqrt[e]*(Log[I*
Sqrt[d] - Sqrt[e]*x] - Log[-((-1)^(2/3)*f^(1/3)) - g^(1/3)*x]))/((-1)^(2/3)
*Sqrt[e]*f^(1/3) + I*Sqrt[d]*g^(1/3))))/((1 + (-1)^(1/3))^2*f^(4/3)*g^(1/3)
) - ((-1 + (-1)^(1/3))*(-(Log[(I*Sqrt[d])/Sqrt[e] + x]/((-1)^(2/3)*f^(1/3)
+ g^(1/3)*x)) + (Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[-((-1)^(2/3)*f^(
```

```
1/3)) - g^(1/3)*x]))/((-1)^(2/3)*Sqrt[e]*f^(1/3) - I*Sqrt[d]*g^(1/3))))/(3*
(1 + (-1)^(1/3))^2*f^(4/3)*g^(1/3)) + ((-1)^(1/3)*(-(Log[((-I)*Sqrt[d])/Sqr
t[e] + x]/(f^(1/3) + g^(1/3)*x)) + (Sqrt[e]*(Log[I*Sqrt[d] - Sqrt[e]*x] - L
og[f^(1/3) + g^(1/3)*x]))/(Sqrt[e]*f^(1/3) + I*Sqrt[d]*g^(1/3))))/(3*(1 + (
-1)^(1/3))^2*f^(4/3)*g^(1/3)) + ((-1)^(1/3)*(-(Log[(I*Sqrt[d])/Sqrt[e] + x]
/(f^(1/3) + g^(1/3)*x)) + (Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[f^(1/3
) + g^(1/3)*x]))/(Sqrt[e]*f^(1/3) - I*Sqrt[d]*g^(1/3))))/(3*(1 + (-1)^(1/3)
)^2*f^(4/3)*g^(1/3)) - (Log[((-I)*Sqrt[d])/Sqrt[e] + x]/((-1)^(1/3)*f^(1/3)
 - g^(1/3)*x) + (Sqrt[e]*(-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[f^(1/3) + (-1)^
(2/3)*g^(1/3)*x]))/((-1)^(1/3)*Sqrt[e]*f^(1/3) - I*Sqrt[d]*g^(1/3)))/(3*(1
+ (-1)^(1/3))^2*f^(4/3)*g^(1/3)) - (Log[(I*Sqrt[d])/Sqrt[e] + x]/((-1)^(1/3
)*f^(1/3) - g^(1/3)*x) + (Sqrt[e]*(-Log[I*Sqrt[d] + Sqrt[e]*x] + Log[f^(1/3
) + (-1)^(2/3)*g^(1/3)*x]))/((-1)^(1/3)*Sqrt[e]*f^(1/3) + I*Sqrt[d]*g^(1/3)
))/(3*(1 + (-1)^(1/3))^2*f^(4/3)*g^(1/3)) + ((-Log[((-I)*Sqrt[d])/Sqrt[e] +
 x] - Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[d + e*x^2])*((3*f^(2/3)*x)/(f + g*
x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*g^(1/3)*x)/f^(1/3))/Sqrt[3]])/g^(1/3) + (2
*Log[f^(1/3) + g^(1/3)*x])/g^(1/3) - Log[f^(2/3) - f^(1/3)*g^(1/3)*x + g^(2
/3)*x^2]/g^(1/3)))/(9*f^(5/3)) - (2*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[((-1)
^(2/3)*f^(1/3) + g^(1/3)*x)/((-1)^(2/3)*f^(1/3) - (I*Sqrt[d]*g^(1/3))/Sqrt[
e])] + PolyLog[2, -((g^(1/3)*(Sqrt[d] - I*Sqrt[e]*x))/((-1)^(1/6)*Sqrt[e]*f
^(1/3) - Sqrt[d]*g^(1/3)))]))/(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3)) - (2*(
-1 + (-1)^(1/3))*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[-((-((-1)^(1/3)*f^(1/3))
 + g^(1/3)*x)/((-1)^(1/3)*f^(1/3) + (I*Sqrt[d]*g^(1/3))/Sqrt[e]))] + PolyLo
g[2, -((g^(1/3)*(Sqrt[d] - I*Sqrt[e]*x))/((-1)^(5/6)*Sqrt[e]*f^(1/3) - Sqrt
[d]*g^(1/3)))]))/(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3)) + (2*(-1)^(1/3)*(Lo
g[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(f^(1/3) + g^(1/3)*x)/(f^(1/3) + (I*Sqrt[
d]*g^(1/3))/Sqrt[e])] + PolyLog[2, (I*g^(1/3)*(Sqrt[d] + I*Sqrt[e]*x))/(Sqr
t[e]*f^(1/3) + I*Sqrt[d]*g^(1/3))]))/(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3))
 - (2*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[((-1)^(2/3)*f^(1/3) + g^(1/3)*x)
/((-1)^(2/3)*f^(1/3) + (I*Sqrt[d]*g^(1/3))/Sqrt[e])] + PolyLog[2, (g^(1/3)*
(Sqrt[d] + I*Sqrt[e]*x))/((-1)^(1/6)*Sqrt[e]*f^(1/3) + Sqrt[d]*g^(1/3))]))/
(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3)) - (2*(-1 + (-1)^(1/3))*(Log[((-I)*Sq
rt[d])/Sqrt[e] + x]*Log[(-((-1)^(1/3)*f^(1/3)) + g^(1/3)*x)/(-((-1)^(1/3)*f
^(1/3)) + (I*Sqrt[d]*g^(1/3))/Sqrt[e])] + PolyLog[2, (g^(1/3)*(Sqrt[d] + I*
Sqrt[e]*x))/((-1)^(5/6)*Sqrt[e]*f^(1/3) + Sqrt[d]*g^(1/3))]))/(3*(1 + (-1)^
(1/3))^2*f^(5/3)*g^(1/3)) + (2*(-1)^(1/3)*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log
[(f^(1/3) + g^(1/3)*x)/(f^(1/3) - (I*Sqrt[d]*g^(1/3))/Sqrt[e])] + PolyLog[2
, -((g^(1/3)*(I*Sqrt[d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - I*Sqrt[d]*g^(1/3))
)]))/(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3)))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{g^2x^6+2\,fgx^3+f^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g^2*x^6 + 2*f*g*x^3 + f^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^3+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^3 + f)^2, x)
```

maple [F] time = 1.29, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{\left(g\,x^{3}+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^3+f)^2,x)
[Out] int(ln(c*(e*x^2+d)^p)/(g*x^3+f)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^{2}+d\right)^{p}c\right)}{\left(gx^{3}+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f)^2,x, algorithm="maxima")
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^3 + f)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{\left(g\,x^{3}+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^3)^2,x)
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^3)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**3+f)**2,x)
[Out] Timed out
```

3.293 $\int \left(f+gx^3\right)^3 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=1221

$$\frac{1}{10}g^3\log^2\left(c\left(ex^2+d\right)^p\right)x^{10}+\frac{24}{343}fg^2p^2x^7+\frac{3}{7}fg^2\log^2\left(c\left(ex^2+d\right)^p\right)x^7-\frac{12}{49}fg^2p\log\left(c\left(ex^2+d\right)^p\right)x^7-\frac{288dfg}{122}$$

```
[Out] 8*f^3*p^2*x+1/10*g^3*x^10*ln(c*(e*x^2+d)^p)^2+f^3*x*ln(c*(e*x^2+d)^p)^2+d^4
*g^3*p^2*x^2/e^4-d^4*g^3*p*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e^5+d^3*g^3*p*(e*x^2
+d)^2*ln(c*(e*x^2+d)^p)/e^5+4*I*f^3*p^2*arctan(x*e^(1/2)/d^(1/2))^2*d^(1/2)
/e^(1/2)+4*I*f^3*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e
^(1/2)+1408/245*d^(7/2)*f*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))/e^(7/2)-3/4*f^2
*g*p*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)/e^2-2/3*d^2*g^3*p*(e*x^2+d)^3*ln(c*(e*x^
2+d)^p)/e^5+1/4*d*g^3*p*(e*x^2+d)^4*ln(c*(e*x^2+d)^p)/e^5+1/5*d^5*g^3*p*ln(
e*x^2+d)*ln(c*(e*x^2+d)^p)/e^5-3/2*d*f^2*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)^2/e^
2+4*f^3*p*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)*d^(1/2)/e^(1/2)+8*f^3
*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/
e^(1/2)-12/7*I*d^(7/2)*f*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))^2/e^(7/2)-12/7*I
*d^(7/2)*f*g^2*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(7/2)+12/
7*d^3*f*g^2*p*x*ln(c*(e*x^2+d)^p)/e^3+3/8*f^2*g*p^2*(e*x^2+d)^2/e^2-1/2*d^3
*g^3*p^2*(e*x^2+d)^2/e^5+2/9*d^2*g^3*p^2*(e*x^2+d)^3/e^5-1/16*d*g^3*p^2*(e*
x^2+d)^4/e^5-1/10*d^5*g^3*p^2*ln(e*x^2+d)^2/e^5-12/49*f*g^2*p*x^7*ln(c*(e*x
^2+d)^p)-1/25*g^3*p*(e*x^2+d)^5*ln(c*(e*x^2+d)^p)/e^5+3/4*f^2*g*(e*x^2+d)^2
*ln(c*(e*x^2+d)^p)^2/e^2-8*f^3*p^2*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2
)-1408/245*d^3*f*g^2*p^2*x/e^3-3*d*f^2*g*p^2*x^2/e+568/735*d^2*f*g^2*p^2*x^
3/e^2-4*f^3*p*x*ln(c*(e*x^2+d)^p)+3/7*f*g^2*x^7*ln(c*(e*x^2+d)^p)^2+24/343*
f*g^2*p^2*x^7+1/125*g^3*p^2*(e*x^2+d)^5/e^5-4/7*d^2*f*g^2*p*x^3*ln(c*(e*x^2
+d)^p)/e^2+12/35*d*f*g^2*p*x^5*ln(c*(e*x^2+d)^p)/e+3*d*f^2*g*p*(e*x^2+d)*ln
(c*(e*x^2+d)^p)/e^2-12/7*d^(7/2)*f*g^2*p*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*
x^2+d)^p)/e^(7/2)-24/7*d^(7/2)*f*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(
1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(7/2)-288/1225*d*f*g^2*p^2*x^5/e
```

Rubi [A] time = 1.64, antiderivative size = 1139, normalized size of antiderivative = 0.93, number of steps used = 55, number of rules used = 29, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.208, Rules used = {2471, 2450, 2476, 2448, 321, 205, 2470, 12, 4920, 4854, 2402, 2315, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2457, 2455, 302, 2398, 2411, 43, 2334, 14, 2301}

$$\frac{1}{10}g^3\log^2\left(c\left(ex^2+d\right)^p\right)x^{10}+\frac{24}{343}fg^2p^2x^7+\frac{3}{7}fg^2\log^2\left(c\left(ex^2+d\right)^p\right)x^7-\frac{12}{49}fg^2p\log\left(c\left(ex^2+d\right)^p\right)x^7-\frac{288dfg}{122}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^3)^3*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f^3*p^2*x - (1408*d^3*f*g^2*p^2*x)/(245*e^3) - (3*d*f^2*g*p^2*x^2)/e + (d
^4*g^3*p^2*x^2)/e^4 + (568*d^2*f*g^2*p^2*x^3)/(735*e^2) - (288*d*f*g^2*p^2*
x^5)/(1225*e) + (24*f*g^2*p^2*x^7)/343 + (3*f^2*g*p^2*(d + e*x^2)^2)/(8*e^2
) - (d^3*g^3*p^2*(d + e*x^2)^2)/(2*e^5) + (2*d^2*g^3*p^2*(d + e*x^2)^3)/(9*
e^5) - (d*g^3*p^2*(d + e*x^2)^4)/(16*e^5) + (g^3*p^2*(d + e*x^2)^5)/(125*e^
5) - (8*Sqrt[d]*f^3*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (1408*d^(7/2
)*f*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(245*e^(7/2)) + ((4*I)*Sqrt[d]*f^3
*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e] - (((12*I)/7)*d^(7/2)*f*g^2*p^2
*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(7/2) + (8*Sqrt[d]*f^3*p^2*ArcTan[(Sqrt[e
]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] - (24*d^(7/
2)*f*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[
```

```
e]*x)])/(7*e^(7/2)) - (d^5*g^3*p^2*Log[d + e*x^2]^2)/(10*e^5) - 4*f^3*p*x*L
og[c*(d + e*x^2)^p] + (12*d^3*f*g^2*p*x*Log[c*(d + e*x^2)^p])/(7*e^3) - (4*
d^2*f*g^2*p*x^3*Log[c*(d + e*x^2)^p])/(7*e^2) + (12*d*f*g^2*p*x^5*Log[c*(d
+ e*x^2)^p])/(35*e) - (12*f*g^2*p*x^7*Log[c*(d + e*x^2)^p])/49 + (3*d*f^2*g
*p*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 - (3*f^2*g*p*(d + e*x^2)^2*Log[c*(
d + e*x^2)^p])/(4*e^2) + (4*Sqrt[d]*f^3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c
*(d + e*x^2)^p])/Sqrt[e] - (12*d^(7/2)*f*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*
Log[c*(d + e*x^2)^p])/(7*e^(7/2)) - (g^3*p*((300*d^4*(d + e*x^2))/e^5 - (30
0*d^3*(d + e*x^2)^2)/e^5 + (200*d^2*(d + e*x^2)^3)/e^5 - (75*d*(d + e*x^2)^
4)/e^5 + (12*(d + e*x^2)^5)/e^5 - (60*d^5*Log[d + e*x^2])/e^5)*Log[c*(d + e
*x^2)^p])/300 + f^3*x*Log[c*(d + e*x^2)^p]^2 + (3*f*g^2*x^7*Log[c*(d + e*x^
2)^p]^2)/7 + (g^3*x^10*Log[c*(d + e*x^2)^p]^2)/10 - (3*d*f^2*g*(d + e*x^2)*
Log[c*(d + e*x^2)^p]^2)/(2*e^2) + (3*f^2*g*(d + e*x^2)^2*Log[c*(d + e*x^2)^
p]^2)/(4*e^2) + ((4*I)*Sqrt[d]*f^3*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d]
+ I*Sqrt[e]*x)])/Sqrt[e] - (((12*I)/7)*d^(7/2)*f*g^2*p^2*PolyLog[2, 1 - (2*
Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/e^(7/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
```

```
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(f+gx^3\right)^3 \log^2\left(c\left(d+ex^2\right)^p\right) dx = \int \left(f^3 \log^2\left(c\left(d+ex^2\right)^p\right) + 3f^2gx^3 \log^2\left(c\left(d+ex^2\right)^p\right) + 3fg^2x^6 \log^2\left(c\left(\right.\right.\right.$$

$$= f^3 \int \log^2\left(c\left(d+ex^2\right)^p\right) dx + \left(3f^2g\right) \int x^3 \log^2\left(c\left(d+ex^2\right)^p\right) dx + \left(3fg^2\right.$$

$$= f^3 x \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{3}{7} fg^2x^7 \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{1}{2}\left(3f^2g\right) \text{Subst}\left(\int\right.$$

$$= f^3 x \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{3}{7} fg^2x^7 \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{1}{10} g^3x^{10} \log^2\left(c\left(d\right.\right.$$

$$= f^3 x \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{3}{7} fg^2x^7 \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{1}{10} g^3x^{10} \log^2\left(c\left(d\right.\right.$$

$$= -4f^3px \log\left(c\left(d+ex^2\right)^p\right) + \frac{12d^3fg^2px \log\left(c\left(d+ex^2\right)^p\right)}{7e^3} - \frac{4d^2fg^2px^3 \text{lo}}{}$$

$$= 8f^3p^2x - \frac{24d^3fg^2p^2x}{7e^3} - 4f^3px \log\left(c\left(d+ex^2\right)^p\right) + \frac{12d^3fg^2px \log\left(c\left(d+\right.\right.}{7e^3}$$

$$= 8f^3p^2x - \frac{1408d^3fg^2p^2x}{245e^3} - \frac{3df^2gp^2x^2}{e} + \frac{568d^2fg^2p^2x^3}{735e^2} - \frac{288dfg^2p^2x^5}{1225e} +$$

$$= 8f^3p^2x - \frac{1408d^3fg^2p^2x}{245e^3} - \frac{3df^2gp^2x^2}{e} + \frac{d^4g^3p^2x^2}{e^4} + \frac{568d^2fg^2p^2x^3}{735e^2} - \frac{28}{}$$

$$= 8f^3p^2x - \frac{1408d^3fg^2p^2x}{245e^3} - \frac{3df^2gp^2x^2}{e} + \frac{d^4g^3p^2x^2}{e^4} + \frac{568d^2fg^2p^2x^3}{735e^2} - \frac{28}{}$$

$$= 8f^3p^2x - \frac{1408d^3fg^2p^2x}{245e^3} - \frac{3df^2gp^2x^2}{e} + \frac{d^4g^3p^2x^2}{e^4} + \frac{568d^2fg^2p^2x^3}{735e^2} - \frac{28}{}$$

Mathematica [A] time = 0.99, size = 1020, normalized size = 0.84

$$\frac{1}{125}g^3p^2x^{10}+\frac{1}{10}g^3\log^2\left(c\left(ex^2+d\right)^p\right)x^{10}-\frac{1}{25}g^3p\log\left(c\left(ex^2+d\right)^p\right)x^{10}-\frac{9dg^3p^2x^8}{400e}+\frac{dg^3p\log\left(c\left(ex^2+d\right)^p\right)x^8}{20e}+\frac{}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)^3*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f^3*p^2*x - (1408*d^3*f*g^2*p^2*x)/(245*e^3) - (9*d*f^2*g*p^2*x^2)/(4*e)
+ (137*d^4*g^3*p^2*x^2)/(300*e^4) + (568*d^2*f*g^2*p^2*x^3)/(735*e^2) + (3*
f^2*g*p^2*x^4)/8 - (77*d^3*g^3*p^2*x^4)/(600*e^3) - (288*d*f*g^2*p^2*x^5)/(
```

```
1225*e) + (47*d^2*g^3*p^2*x^6)/(900*e^2) + (24*f*g^2*p^2*x^7)/343 - (9*d*g^
3*p^2*x^8)/(400*e) + (g^3*p^2*x^10)/125 - (((4*I)/7)*Sqrt[d]*f*(-7*e^3*f^2
+ 3*d^3*g^2)*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(7/2) + (3*d^2*f^2*g*p^2*
Log[d + e*x^2])/(4*e^2) - (77*d^5*g^3*p^2*Log[d + e*x^2])/(300*e^5) + (3*d^
2*f^2*g*p*Log[c*(d + e*x^2)^p])/(2*e^2) - (d^5*g^3*p*Log[c*(d + e*x^2)^p])/
(5*e^5) - 4*f^3*p*x*Log[c*(d + e*x^2)^p] + (12*d^3*f*g^2*p*x*Log[c*(d + e*x
^2)^p])/(7*e^3) + (3*d*f^2*g*p*x^2*Log[c*(d + e*x^2)^p])/(2*e) - (d^4*g^3*p
*x^2*Log[c*(d + e*x^2)^p])/(5*e^4) - (4*d^2*f*g^2*p*x^3*Log[c*(d + e*x^2)^p
])/(7*e^2) - (3*f^2*g*p*x^4*Log[c*(d + e*x^2)^p])/4 + (d^3*g^3*p*x^4*Log[c*
(d + e*x^2)^p])/(10*e^3) + (12*d*f*g^2*p*x^5*Log[c*(d + e*x^2)^p])/(35*e) -
 (d^2*g^3*p*x^6*Log[c*(d + e*x^2)^p])/(15*e^2) - (12*f*g^2*p*x^7*Log[c*(d +
 e*x^2)^p])/49 + (d*g^3*p*x^8*Log[c*(d + e*x^2)^p])/(20*e) - (g^3*p*x^10*Lo
g[c*(d + e*x^2)^p])/25 - (3*d^2*f^2*g*Log[c*(d + e*x^2)^p]^2)/(4*e^2) + (d^
5*g^3*Log[c*(d + e*x^2)^p]^2)/(10*e^5) + f^3*x*Log[c*(d + e*x^2)^p]^2 + (3*
f^2*g*x^4*Log[c*(d + e*x^2)^p]^2)/4 + (3*f*g^2*x^7*Log[c*(d + e*x^2)^p]^2)/
7 + (g^3*x^10*Log[c*(d + e*x^2)^p]^2)/10 - (4*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x
)/Sqrt[d]]*(490*e^3*f^2*p - 352*d^3*g^2*p - 70*(7*e^3*f^2 - 3*d^3*g^2)*p*Lo
g[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] - 35*(7*e^3*f^2 - 3*d^3*g^2)*Log[c*(
d + e*x^2)^p]))/(245*e^(7/2)) - (((4*I)/7)*Sqrt[d]*f*(-7*e^3*f^2 + 3*d^3*g^
2)*p^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)])/e^(7
/2)
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(g^3x^9+3\,fg^2x^6+3\,f^2gx^3+f^3\right)\log\left(\left(ex^2+d\right)^pc\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g^3*x^9 + 3*f*g^2*x^6 + 3*f^2*g*x^3 + f^3)*log((e*x^2 + d)^p*c)^2
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(gx^3+f\right)^3\log\left(\left(ex^2+d\right)^pc\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)^3*log((e*x^2 + d)^p*c)^2, x)
```

maple [F] time = 1.63, size = 0, normalized size = 0.00

$$\int \left(g\,x^3+f\right)^3\ln\left(c\left(e\,x^2+d\right)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^3*ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^3+f)^3*ln(c*(e*x^2+d)^p)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{140}\left(14\,g^3p^2x^{10}+60\,fg^2p^2x^7+105\,f^2gp^2x^4+140\,f^3p^2x\right)\log\left(ex^2+d\right)^2+\int\frac{35\,eg^3x^{11}\log(c)^2+35\,dg^3x^9\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/140*(14*g^3*p^2*x^10 + 60*f*g^2*p^2*x^7 + 105*f^2*g*p^2*x^4 + 140*f^3*p^2
*x)*log(e*x^2 + d)^2 + integrate(1/35*(35*e*g^3*x^11*log(c)^2 + 35*d*g^3*x^
9*log(c)^2 + 105*e*f*g^2*x^8*log(c)^2 + 105*d*f*g^2*x^6*log(c)^2 + 105*e*f^
2*g*x^5*log(c)^2 + 105*d*f^2*g*x^3*log(c)^2 + 35*e*f^3*x^2*log(c)^2 + 35*d*
f^3*log(c)^2 + (70*d*g^3*p*x^9*log(c) - 14*(e*g^3*p^2 - 5*e*g^3*p*log(c))*x
^11 + 210*d*f*g^2*p*x^6*log(c) - 30*(2*e*f*g^2*p^2 - 7*e*f*g^2*p*log(c))*x^
8 + 210*d*f^2*g*p*x^3*log(c) - 105*(e*f^2*g*p^2 - 2*e*f^2*g*p*log(c))*x^5 +
 70*d*f^3*p*log(c) - 70*(2*e*f^3*p^2 - e*f^3*p*log(c))*x^2)*log(e*x^2 + d))
/(e*x^2 + d), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(e\,x^2+d\right)^p\right)^2\left(g\,x^3+f\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^3,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**3*ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Timed out
```

3.294 $\int \left(f + gx^3\right)^2 \log^2\left(c\left(d + ex^2\right)^p\right) dx$

Optimal. Leaf size=835

$$\frac{8}{343}g^2p^2x^7+\frac{1}{7}g^2\log^2\left(c\left(ex^2+d\right)^p\right)x^7-\frac{4}{49}g^2p\log\left(c\left(ex^2+d\right)^p\right)x^7-\frac{96dg^2p^2x^5}{1225e}+\frac{4dg^2p\log\left(c\left(ex^2+d\right)^p\right)x^5}{35e}+$$

```
[Out] 8*f^2*p^2*x+8/343*g^2*p^2*x^7+1/7*g^2*x^7*ln(c*(e*x^2+d)^p)^2+f^2*x*ln(c*(e
*x^2+d)^p)^2-d*f*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)^2/e^2+4/7*d^3*g^2*p*x*ln(c*(
e*x^2+d)^p)/e^3-4/21*d^2*g^2*p*x^3*ln(c*(e*x^2+d)^p)/e^2+4/35*d*g^2*p*x^5*l
n(c*(e*x^2+d)^p)/e-1/2*f*g*p*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)/e^2-4/7*d^(7/2)*
g^2*p*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(7/2)-8/7*d^(7/2)*g^2*p
^2*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(7/2)-4/
7*I*d^(7/2)*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))^2/e^(7/2)-4/7*I*d^(7/2)*g^2*p
^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(7/2)-1408/735*d^3*g^2*p^
2*x/e^3+568/2205*d^2*g^2*p^2*x^3/e^2-96/1225*d*g^2*p^2*x^5/e+1/4*f*g*p^2*(e
*x^2+d)^2/e^2-8*f^2*p^2*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)+1408/735*
d^(7/2)*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))/e^(7/2)+1/2*f*g*(e*x^2+d)^2*ln(c*
(e*x^2+d)^p)^2/e^2-4*f^2*p*x*ln(c*(e*x^2+d)^p)-4/49*g^2*p*x^7*ln(c*(e*x^2+d
)^p)+4*f^2*p*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)*d^(1/2)/e^(1/2)+8*
f^2*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/
2)/e^(1/2)+4*I*f^2*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)
/e^(1/2)+4*I*f^2*p^2*arctan(x*e^(1/2)/d^(1/2))^2*d^(1/2)/e^(1/2)+2*d*f*g*p*
(e*x^2+d)*ln(c*(e*x^2+d)^p)/e^2-2*d*f*g*p^2*x^2/e
```

Rubi [A] time = 1.08, antiderivative size = 835, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 23, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.958, Rules used = {2471, 2450, 2476, 2448, 321, 205, 2470, 12, 4920, 4854, 2402, 2315, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2457, 2455, 302}

$$\frac{8}{343}g^2p^2x^7+\frac{1}{7}g^2\log^2\left(c\left(ex^2+d\right)^p\right)x^7-\frac{4}{49}g^2p\log\left(c\left(ex^2+d\right)^p\right)x^7-\frac{96dg^2p^2x^5}{1225e}+\frac{4dg^2p\log\left(c\left(ex^2+d\right)^p\right)x^5}{35e}+$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f^2*p^2*x - (1408*d^3*g^2*p^2*x)/(735*e^3) - (2*d*f*g*p^2*x^2)/e + (568*d
^2*g^2*p^2*x^3)/(2205*e^2) - (96*d*g^2*p^2*x^5)/(1225*e) + (8*g^2*p^2*x^7)/
343 + (f*g*p^2*(d + e*x^2)^2)/(4*e^2) - (8*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*
x)/Sqrt[d]])/Sqrt[e] + (1408*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
735*e^(7/2)) + ((4*I)*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e
] - (((4*I)/7)*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(7/2) + (8*
Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/Sqrt[e] - (8*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*
Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(7*e^(7/2)) - 4*f^2*p*x*Log[c*(d + e*x^2
)^p] + (4*d^3*g^2*p*x*Log[c*(d + e*x^2)^p])/(7*e^3) - (4*d^2*g^2*p*x^3*Log[
c*(d + e*x^2)^p])/(21*e^2) + (4*d*g^2*p*x^5*Log[c*(d + e*x^2)^p])/(35*e) -
(4*g^2*p*x^7*Log[c*(d + e*x^2)^p])/49 + (2*d*f*g*p*(d + e*x^2)*Log[c*(d + e
*x^2)^p])/e^2 - (f*g*p*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(2*e^2) + (4*Sqr
t[d]*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - (4*d
^(7/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(7*e^(7/2))
+ f^2*x*Log[c*(d + e*x^2)^p]^2 + (g^2*x^7*Log[c*(d + e*x^2)^p]^2)/7 - (d*f*
g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/e^2 + (f*g*(d + e*x^2)^2*Log[c*(d + e
*x^2)^p]^2)/(2*e^2) + ((4*I)*Sqrt[d]*f^2*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sq
rt[d] + I*Sqrt[e]*x)])/Sqrt[e] - (((4*I)/7)*d^(7/2)*g^2*p^2*PolyLog[2, 1 -
(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/e^(7/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
```

```
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right) dx = \int \left(f^2 \log^2\left(c\left(d+ex^2\right)^p\right) + 2fgx^3 \log^2\left(c\left(d+ex^2\right)^p\right) + g^2x^6 \log^2\left(c\left(d\right.\right.\right.$$

$$= f^2 \int \log^2\left(c\left(d+ex^2\right)^p\right) dx + (2fg) \int x^3 \log^2\left(c\left(d+ex^2\right)^p\right) dx + g^2$$

$$= f^2x \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{1}{7}g^2x^7 \log^2\left(c\left(d+ex^2\right)^p\right) + (fg)\,\text{Subst}\left(\int x\right.$$

$$= f^2x \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{1}{7}g^2x^7 \log^2\left(c\left(d+ex^2\right)^p\right) + (fg)\,\text{Subst}\left(\int\left(\right.\right.$$

$$= f^2x \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{1}{7}g^2x^7 \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{(fg)\,\text{Subst}\left(\int(d\right.}{}$$

$$= -4f^2px \log\left(c\left(d+ex^2\right)^p\right) + \frac{4d^3g^2px \log\left(c\left(d+ex^2\right)^p\right)}{7e^3} - \frac{4d^2g^2px^3 \log}{2}$$

$$= 8f^2p^2x - \frac{8d^3g^2p^2x}{7e^3} - 4f^2px \log\left(c\left(d+ex^2\right)^p\right) + \frac{4d^3g^2px \log\left(c\left(d+e\right.\right.}{7e^3}$$

$$= 8f^2p^2x - \frac{1408d^3g^2p^2x}{735e^3} - \frac{2dfgp^2x^2}{e} + \frac{568d^2g^2p^2x^3}{2205e^2} - \frac{96dg^2p^2x^5}{1225e} + \frac{8}{34}$$

$$= 8f^2p^2x - \frac{1408d^3g^2p^2x}{735e^3} - \frac{2dfgp^2x^2}{e} + \frac{568d^2g^2p^2x^3}{2205e^2} - \frac{96dg^2p^2x^5}{1225e} + \frac{8}{34}$$

$$= 8f^2p^2x - \frac{1408d^3g^2p^2x}{735e^3} - \frac{2dfgp^2x^2}{e} + \frac{568d^2g^2p^2x^3}{2205e^2} - \frac{96dg^2p^2x^5}{1225e} + \frac{8}{34}$$

$$= 8f^2p^2x - \frac{1408d^3g^2p^2x}{735e^3} - \frac{2dfgp^2x^2}{e} + \frac{568d^2g^2p^2x^3}{2205e^2} - \frac{96dg^2p^2x^5}{1225e} + \frac{8}{34}$$

Mathematica [A] time = 0.57, size = 475, normalized size = 0.57

$$\frac{-1680\sqrt{d}\,p \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-105\left(7e^3f^2-d^3g^2\right)\log\left(c\left(d+ex^2\right)^p\right) - 210p\left(7e^3f^2-d^3g^2\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right) + 2p\left(73\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((-176400*I)*Sqrt[d]*(-7*e^3*f^2 + d^3*g^2)*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
^2 - 1680*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(2*(735*e^3*f^2 - 176*d^3*g
^2)*p - 210*(7*e^3*f^2 - d^3*g^2)*p*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)
] - 105*(7*e^3*f^2 - d^3*g^2)*Log[c*(d + e*x^2)^p]) + Sqrt[e]*(p^2*x*(-5913
60*d^3*g^2 + 79520*d^2*e*g^2*x^2 - 378*d*e^2*g*x*(1225*f + 64*g*x^3) + 225*
```

```
e^3*(10976*f^2 + 343*f*g*x^3 + 32*g^2*x^6)) + 154350*d^2*e*f*g*p^2*Log[d +
e*x^2] - 210*p*(-840*d^3*g^2*x + 70*d^2*e*g*(-21*f + 4*g*x^3) - 42*d*e^2*g*
x^2*(35*f + 4*g*x^3) + 15*e^3*x*(392*f^2 + 49*f*g*x^3 + 8*g^2*x^6))*Log[c*(
d + e*x^2)^p] + 22050*(-7*d^2*e*f*g + e^3*x*(14*f^2 + 7*f*g*x^3 + 2*g^2*x^6
))*Log[c*(d + e*x^2)^p]^2) - (176400*I)*Sqrt[d]*(-7*e^3*f^2 + d^3*g^2)*p^2*
PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)])/(308700*e^(
7/2))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(g^2x^6 + 2\,fgx^3 + f^2\right)\log\left(\left(ex^2 + d\right)^p c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^6 + 2*f*g*x^3 + f^2)*log((e*x^2 + d)^p*c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(gx^3 + f\right)^2 \log\left(\left(ex^2 + d\right)^p c\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)^2*log((e*x^2 + d)^p*c)^2, x)
```

maple [F] time = 2.49, size = 0, normalized size = 0.00

$$\int \left(g\,x^3 + f\right)^2 \ln\left(c\left(e\,x^2 + d\right)^p\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{14}\left(2\,g^2p^2x^7 + 7\,fgp^2x^4 + 14\,f^2p^2x\right)\log\left(ex^2 + d\right)^2 + \int \frac{7\,eg^2x^8\log(c)^2 + 7\,dg^2x^6\log(c)^2 + 14\,efgx^5\log(c)^2 + 14}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/14*(2*g^2*p^2*x^7 + 7*f*g*p^2*x^4 + 14*f^2*p^2*x)*log(e*x^2 + d)^2 + inte
grate(1/7*(7*e*g^2*x^8*log(c)^2 + 7*d*g^2*x^6*log(c)^2 + 14*e*f*g*x^5*log(c
)^2 + 14*d*f*g*x^3*log(c)^2 + 7*e*f^2*x^2*log(c)^2 + 7*d*f^2*log(c)^2 + 2*(
7*d*g^2*p*x^6*log(c) - (2*e*g^2*p^2 - 7*e*g^2*p*log(c))*x^8 + 14*d*f*g*p*x^
3*log(c) - 7*(e*f*g*p^2 - 2*e*f*g*p*log(c))*x^5 + 7*d*f^2*p*log(c) - 7*(2*e
*f^2*p^2 - e*f^2*p*log(c))*x^2)*log(e*x^2 + d))/(e*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(e\,x^2 + d\right)^p\right)^2 \left(g\,x^3 + f\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^2,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f + gx^3\right)^2 \log\left(c\left(d + ex^2\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**2*ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**3)**2*log(c*(d + e*x**2)**p)**2, x)
```

3.295 $\int \left(f+gx^3\right)\log^2\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=395

$$\frac{g\left(d+ex^2\right)^2\log^2\left(c\left(d+ex^2\right)^p\right)}{4e^2}-\frac{dg\left(d+ex^2\right)\log^2\left(c\left(d+ex^2\right)^p\right)}{2e^2}-\frac{gp\left(d+ex^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4e^2}+\frac{dgp\left(d+ex^2\right)}{}$$

```
[Out] 8*f*p^2*x-d*g*p^2*x^2/e+1/8*g*p^2*(e*x^2+d)^2/e^2-4*f*p*x*ln(c*(e*x^2+d)^p)
+d*g*p*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e^2-1/4*g*p*(e*x^2+d)^2*ln(c*(e*x^2+d)^p
)/e^2+f*x*ln(c*(e*x^2+d)^p)^2-1/2*d*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)^2/e^2+1/4
*g*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)^2/e^2-8*f*p^2*arctan(x*e^(1/2)/d^(1/2))*d^
(1/2)/e^(1/2)+4*I*f*p^2*arctan(x*e^(1/2)/d^(1/2))^2*d^(1/2)/e^(1/2)+4*f*p*a
rctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)*d^(1/2)/e^(1/2)+8*f*p^2*arctan(x
*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)+4*I*f
*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 20, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {2471, 2450, 2476, 2448, 321, 205, 2470, 12, 4920, 4854, 2402, 2315, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{4i\sqrt{d}fp^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}+\frac{g\left(d+ex^2\right)^2\log^2\left(c\left(d+ex^2\right)^p\right)}{4e^2}-\frac{dg\left(d+ex^2\right)\log^2\left(c\left(d+ex^2\right)^p\right)}{2e^2}-\frac{gp\left(d+e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^3)*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f*p^2*x - (d*g*p^2*x^2)/e + (g*p^2*(d + e*x^2)^2)/(8*e^2) - (8*Sqrt[d]*f*
p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + ((4*I)*Sqrt[d]*f*p^2*ArcTan[(Sqr
t[e]*x)/Sqrt[d]]^2)/Sqrt[e] + (8*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*
Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] - 4*f*p*x*Log[c*(d + e*x^
2)^p] + (d*g*p*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 - (g*p*(d + e*x^2)^2*L
og[c*(d + e*x^2)^p])/(4*e^2) + (4*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*L
og[c*(d + e*x^2)^p])/Sqrt[e] + f*x*Log[c*(d + e*x^2)^p]^2 - (d*g*(d + e*x^2
)*Log[c*(d + e*x^2)^p]^2)/(2*e^2) + (g*(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^2
)/(4*e^2) + ((4*I)*Sqrt[d]*f*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/Sqrt[e]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
```

```
e, n, p}, x]
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(f+gx^3\right)\log^2\left(c\left(d+ex^2\right)^p\right)dx=\int \left(f\log^2\left(c\left(d+ex^2\right)^p\right)+gx^3\log^2\left(c\left(d+ex^2\right)^p\right)\right)dx$$

$$=f\int \log^2\left(c\left(d+ex^2\right)^p\right)dx+g\int x^3\log^2\left(c\left(d+ex^2\right)^p\right)dx$$

$$=fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}g\,\text{Subst}\left(\int x\log^2\left(c(d+ex)^p\right)dx,x,x^2\right)-(4e$$

$$=fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}g\,\text{Subst}\left(\int\left(-\frac{d\log^2\left(c(d+ex)^p\right)}{e}+\frac{(d+ex)\,l}{}\right.\right.$$

$$=fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{g\,\text{Subst}\left(\int(d+ex)\log^2\left(c(d+ex)^p\right)dx,x,x^2\right)}{2e}$$

$$=-4fpx\log\left(c\left(d+ex^2\right)^p\right)+\frac{4\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}}+fx$$

$$=8fp^2x-4fpx\log\left(c\left(d+ex^2\right)^p\right)+\frac{4\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}}$$

$$=8fp^2x-\frac{dgp^2x^2}{e}+\frac{gp^2\left(d+ex^2\right)^2}{8e^2}-\frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{4i\sqrt{d}fp^2t}{}$$

$$=8fp^2x-\frac{dgp^2x^2}{e}+\frac{gp^2\left(d+ex^2\right)^2}{8e^2}-\frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{4i\sqrt{d}fp^2t}{}$$

$$=8fp^2x-\frac{dgp^2x^2}{e}+\frac{gp^2\left(d+ex^2\right)^2}{8e^2}-\frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{4i\sqrt{d}fp^2t}{}$$

$$=8fp^2x-\frac{dgp^2x^2}{e}+\frac{gp^2\left(d+ex^2\right)^2}{8e^2}-\frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{4i\sqrt{d}fp^2t}{}$$

Mathematica [A] time = 0.17, size = 415, normalized size = 1.05

$$-egp\left(\frac{d^2\log^2\left(c\left(d+ex^2\right)^p\right)}{4e^3p}+\frac{d\left(px^2-\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e}\right)}{2e^2}+\frac{x^4\log\left(c\left(d+ex^2\right)^p\right)}{4e}+\frac{1}{8}p\left(-\frac{2d^2\log\left(d+ex^2\right)}{e^3}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f*p^2*x - (8*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + ((4*I)*
Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e] + (8*Sqrt[d]*f*p^2*Arc
Tan[(Sqrt[e]*x)/Sqrt[d]]*Log[((2*I)*Sqrt[d])/(I*Sqrt[d] - Sqrt[e]*x)])/Sqrt
[e] - 4*f*p*x*Log[c*(d + e*x^2)^p] + (4*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] + f*x*Log[c*(d + e*x^2)^p]^2 + (g*x^4*Lo
g[c*(d + e*x^2)^p]^2)/4 - e*g*p*((p*((2*d*x^2)/e^2 - x^4/e - (2*d^2*Log[d +
 e*x^2])/e^3))/8 + (x^4*Log[c*(d + e*x^2)^p])/(4*e) + (d^2*Log[c*(d + e*x^2
```

```
)^p]^2)/(4*e^3*p) + (d*(p*x^2 - ((d + e*x^2)*Log[c*(d + e*x^2)^p])/e))/(2*e
^2)) + ((4*I)*Sqrt[d]*f*p^2*PolyLog[2, -((I*Sqrt[d] + Sqrt[e]*x)/(I*Sqrt[d]
 - Sqrt[e]*x))])/Sqrt[e]
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(gx^3+f\right)\log\left(\left(ex^2+d\right)^p c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g*x^3 + f)*log((e*x^2 + d)^p*c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(gx^3+f\right)\log\left(\left(ex^2+d\right)^p c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)*log((e*x^2 + d)^p*c)^2, x)
```

maple [F] time = 1.83, size = 0, normalized size = 0.00

$$\int \left(g\,x^3+f\right)\ln\left(c\left(e\,x^2+d\right)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)*ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^3+f)*ln(c*(e*x^2+d)^p)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\left(gp^2x^4+4\,fp^2x\right)\log\left(ex^2+d\right)^2+\int \frac{egx^5\log(c)^2+dgx^3\log(c)^2+efx^2\log(c)^2+df\log(c)^2+\left(2\,dgpx^3\log(c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(g*p^2*x^4 + 4*f*p^2*x)*log(e*x^2 + d)^2 + integrate((e*g*x^5*log(c)^2
+ d*g*x^3*log(c)^2 + e*f*x^2*log(c)^2 + d*f*log(c)^2 + (2*d*g*p*x^3*log(c)
- (e*g*p^2 - 2*e*g*p*log(c))*x^5 + 2*d*f*p*log(c) - 2*(2*e*f*p^2 - e*f*p*lo
g(c))*x^2)*log(e*x^2 + d))/(e*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(e\,x^2+d\right)^p\right)^2\,\left(g\,x^3+f\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2*(f + g*x^3),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^2*(f + g*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f+gx^3\right)\log\left(c\left(d+ex^2\right)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)*ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**3)*log(c*(d + e*x**2)**p)**2, x)
```

3.296 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^2/(g*x^3+f),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^3),x]
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^2/(f + g*x^3), x]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} dx = \int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} dx$$

Mathematica [A] time = 13.83, size = 0, normalized size = 0.00

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3),x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3), x]
```

fricas [A] time = 1.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)^2}{gx^3+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f),x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)^2/(g*x^3 + f), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)^2}{gx^3+f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f),x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)^2/(g*x^3 + f), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}{g\,x^{3}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f),x)
[Out] int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^{2}+d\right)^{p}c\right)^{2}}{gx^{3}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f),x, algorithm="maxima")
[Out] integrate(log((e*x^2 + d)^p*c)^2/(g*x^3 + f), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}{g\,x^{3}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2/(f + g*x^3),x)
[Out] int(log(c*(d + e*x^2)^p)^2/(f + g*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**2/(g*x**3+f),x)
[Out] Timed out
```

3.297 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2,x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2, x]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} \, dx = \int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} \, dx$$

Mathematica [A] time = 22.94, size = 0, normalized size = 0.00

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2,x]
```

```
[Out] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2, x]
```

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)^2}{g^2x^6+2\,fgx^3+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)^2/(g^2*x^6 + 2*f*g*x^3 + f^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)^2}{\left(gx^3+f\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)^2/(g*x^3 + f)^2, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}{\left(g\,x^{3}+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}{\left(g\,x^{3}+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2/(f + g*x^3)^2,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^2/(f + g*x^3)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**2/(g*x**3+f)**2,x)
```

```
[Out] Timed out
```

3.298 $\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right) dx$

Optimal. Leaf size=1126

$$-\frac{48g^2p^3x^7}{2401}+\frac{1}{7}g^2\log^3\left(c\left(ex^2+d\right)^p\right)x^7-\frac{6}{49}g^2p\log^2\left(c\left(ex^2+d\right)^p\right)x^7+\frac{24}{343}g^2p^2\log\left(c\left(ex^2+d\right)^p\right)x^7+\frac{5232dg^2p^3}{42875e}$$

```
[Out] -48*f^2*p^3*x-48/2401*g^2*p^3*x^7+1/7*g^2*x^7*ln(c*(e*x^2+d)^p)^3+f^2*x*ln(
c*(e*x^2+d)^p)^3-d*f*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)^3/e^2+1408/245*I*d^(7/2)
*g^2*p^3*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(7/2)+1408/245*I*d^
(7/2)*g^2*p^3*arctan(x*e^(1/2)/d^(1/2))^2/e^(7/2)-24*I*f^2*p^3*arctan(x*e^(
1/2)/d^(1/2))^2*d^(1/2)/e^(1/2)-24*I*f^2*p^3*polylog(2,1-2*d^(1/2)/(d^(1/2)
+I*x*e^(1/2)))*d^(1/2)/e^(1/2)-1408/245*d^3*g^2*p^2*x*ln(c*(e*x^2+d)^p)/e^3
+568/735*d^2*g^2*p^2*x^3*ln(c*(e*x^2+d)^p)/e^2-288/1225*d*g^2*p^2*x^5*ln(c*
(e*x^2+d)^p)/e+3/4*f*g*p^2*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)/e^2+1408/245*d^(7/
2)*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(7/2)+6/7*d^3*g^2*
p*x*ln(c*(e*x^2+d)^p)^2/e^3-2/7*d^2*g^2*p*x^3*ln(c*(e*x^2+d)^p)^2/e^2+6/35*
d*g^2*p*x^5*ln(c*(e*x^2+d)^p)^2/e-3/4*f*g*p*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)^2
/e^2+2816/245*d^(7/2)*g^2*p^3*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/
2)+I*x*e^(1/2)))/e^(7/2)-24*f^2*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d
)^p)*d^(1/2)/e^(1/2)-48*f^2*p^3*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(
1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)+351136/25725*d^3*g^2*p^3*x/e^3-55456/771
75*d^2*g^2*p^3*x^3/e^2+5232/42875*d*g^2*p^3*x^5/e-3/8*f*g*p^3*(e*x^2+d)^2/e
^2-351136/25725*d^(7/2)*g^2*p^3*arctan(x*e^(1/2)/d^(1/2))/e^(7/2)+1/2*f*g*(
e*x^2+d)^2*ln(c*(e*x^2+d)^p)^3/e^2+48*f^2*p^3*arctan(x*e^(1/2)/d^(1/2))*d^(
1/2)/e^(1/2)-6/7*d^4*g^2*p*Unintegrable(ln(c*(e*x^2+d)^p)^2/(e*x^2+d),x)/e^
3+24*f^2*p^2*x*ln(c*(e*x^2+d)^p)+24/343*g^2*p^2*x^7*ln(c*(e*x^2+d)^p)-6*f^2
*p*x*ln(c*(e*x^2+d)^p)^2-6/49*g^2*p*x^7*ln(c*(e*x^2+d)^p)^2+6*d*f^2*p*Unint
egrable(ln(c*(e*x^2+d)^p)^2/(e*x^2+d),x)-6*d*f*g*p^2*(e*x^2+d)*ln(c*(e*x^2+
d)^p)/e^2+3*d*f*g*p*(e*x^2+d)*ln(c*(e*x^2+d)^p)^2/e^2+6*d*f*g*p^3*x^2/e
```

Rubi [A] time = 2.65, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] -48*f^2*p^3*x + (351136*d^3*g^2*p^3*x)/(25725*e^3) + (6*d*f*g*p^3*x^2)/e -
(55456*d^2*g^2*p^3*x^3)/(77175*e^2) + (5232*d*g^2*p^3*x^5)/(42875*e) - (48*
g^2*p^3*x^7)/2401 - (3*f*g*p^3*(d + e*x^2)^2)/(8*e^2) + (48*Sqrt[d]*f^2*p^3
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (351136*d^(7/2)*g^2*p^3*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(25725*e^(7/2)) - ((24*I)*Sqrt[d]*f^2*p^3*ArcTan[(Sqrt[e]
*x)/Sqrt[d]]^2)/Sqrt[e] + (((1408*I)/245)*d^(7/2)*g^2*p^3*ArcTan[(Sqrt[e]*x
)/Sqrt[d]]^2)/e^(7/2) - (48*Sqrt[d]*f^2*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log
[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + (2816*d^(7/2)*g^2*p^3*ArcT
an[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(245*e^(7
/2)) + 24*f^2*p^2*x*Log[c*(d + e*x^2)^p] - (1408*d^3*g^2*p^2*x*Log[c*(d + e
*x^2)^p])/(245*e^3) + (568*d^2*g^2*p^2*x^3*Log[c*(d + e*x^2)^p])/(735*e^2)
- (288*d*g^2*p^2*x^5*Log[c*(d + e*x^2)^p])/(1225*e) + (24*g^2*p^2*x^7*Log[c
*(d + e*x^2)^p])/343 - (6*d*f*g*p^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 +
 (3*f*g*p^2*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(4*e^2) - (24*Sqrt[d]*f^2*p
^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] + (1408*d^(7/2
)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(245*e^(7/2)) -
```

```
 6*f^2*p*x*Log[c*(d + e*x^2)^p]^2 + (6*d^3*g^2*p*x*Log[c*(d + e*x^2)^p]^2)/
(7*e^3) - (2*d^2*g^2*p*x^3*Log[c*(d + e*x^2)^p]^2)/(7*e^2) + (6*d*g^2*p*x^5
*Log[c*(d + e*x^2)^p]^2)/(35*e) - (6*g^2*p*x^7*Log[c*(d + e*x^2)^p]^2)/49 +
 (3*d*f*g*p*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/e^2 - (3*f*g*p*(d + e*x^2)^
2*Log[c*(d + e*x^2)^p]^2)/(4*e^2) + f^2*x*Log[c*(d + e*x^2)^p]^3 + (g^2*x^7
*Log[c*(d + e*x^2)^p]^3)/7 - (d*f*g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^3)/e^2
 + (f*g*(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^3)/(2*e^2) - ((24*I)*Sqrt[d]*f^2
*p^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + (((1408
*I)/245)*d^(7/2)*g^2*p^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)
])/e^(7/2) + 6*d*f^2*p*Defer[Int][Log[c*(d + e*x^2)^p]^2/(d + e*x^2), x] -
(6*d^4*g^2*p*Defer[Int][Log[c*(d + e*x^2)^p]^2/(d + e*x^2), x])/(7*e^3)
```

Rubi steps

$$\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right) dx = \int \left(f^2 \log^3\left(c\left(d+ex^2\right)^p\right) + 2fgx^3 \log^3\left(c\left(d+ex^2\right)^p\right) + g^2x^6 \log^3\left(c\left(d+\right.\right.\right.$$

$$= f^2 \int \log^3\left(c\left(d+ex^2\right)^p\right) dx + (2fg) \int x^3 \log^3\left(c\left(d+ex^2\right)^p\right) dx + g^2 \int x$$

$$= f^2 x \log^3\left(c\left(d+ex^2\right)^p\right) + \frac{1}{7} g^2 x^7 \log^3\left(c\left(d+ex^2\right)^p\right) + (fg)\,\text{Subst}\left(\int x \log\right.$$

$$= f^2 x \log^3\left(c\left(d+ex^2\right)^p\right) + \frac{1}{7} g^2 x^7 \log^3\left(c\left(d+ex^2\right)^p\right) + (fg)\,\text{Subst}\left(\int \left(-\frac{d}{}\right.\right.$$

$$= f^2 x \log^3\left(c\left(d+ex^2\right)^p\right) + \frac{1}{7} g^2 x^7 \log^3\left(c\left(d+ex^2\right)^p\right) + \frac{(fg)\,\text{Subst}\left(\int (d+e\right.}{}$$

$$= -6f^2px \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{6d^3g^2px \log^2\left(c\left(d+ex^2\right)^p\right)}{7e^3} - \frac{2d^2g^2px^3 \log}{7}$$

$$= -6f^2px \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{6d^3g^2px \log^2\left(c\left(d+ex^2\right)^p\right)}{7e^3} - \frac{2d^2g^2px^3 \log}{7}$$

$$= -6f^2px \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{6d^3g^2px \log^2\left(c\left(d+ex^2\right)^p\right)}{7e^3} - \frac{2d^2g^2px^3 \log}{7}$$

$$= \frac{6dfgp^3x^2}{e} - \frac{3fgp^3\left(d+ex^2\right)^2}{8e^2} + 24f^2p^2x \log\left(c\left(d+ex^2\right)^p\right) - \frac{1408d^3g^2p^2}{}$$

$$= -48f^2p^3x + \frac{2816d^3g^2p^3x}{245e^3} + \frac{6dfgp^3x^2}{e} - \frac{3fgp^3\left(d+ex^2\right)^2}{8e^2} + 24f^2p^2x \log$$

$$= -48f^2p^3x + \frac{351136d^3g^2p^3x}{25725e^3} + \frac{6dfgp^3x^2}{e} - \frac{55456d^2g^2p^3x^3}{77175e^2} + \frac{5232dg^2p^3}{42875e}$$

$$= -48f^2p^3x + \frac{351136d^3g^2p^3x}{25725e^3} + \frac{6dfgp^3x^2}{e} - \frac{55456d^2g^2p^3x^3}{77175e^2} + \frac{5232dg^2p^3}{42875e}$$

$$= -48f^2p^3x + \frac{351136d^3g^2p^3x}{25725e^3} + \frac{6dfgp^3x^2}{e} - \frac{55456d^2g^2p^3x^3}{77175e^2} + \frac{5232dg^2p^3}{42875e}$$

$$= -48f^2p^3x + \frac{351136d^3g^2p^3x}{25725e^3} + \frac{6dfgp^3x^2}{e} - \frac{55456d^2g^2p^3x^3}{77175e^2} + \frac{5232dg^2p^3}{42875e}$$

Mathematica [A] time = 9.32, size = 2727, normalized size = 2.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] (g^2*p^3*x*(168*d^2*(d + e*x^2)*HypergeometricPFQ[{-5/2, 1, 1, 1}, {2, 2, 2
}, (d + e*x^2)/d] - 280*d^2*(d + e*x^2)*HypergeometricPFQ[{-3/2, 1, 1, 1},
{2, 2, 2}, (d + e*x^2)/d] - 112*d^2*(d + e*x^2)*HypergeometricPFQ[{-5/2, 1,
 1, 1, 1}, {2, 2, 2, 2}, (d + e*x^2)/d] + 280*d^2*(d + e*x^2)*Hypergeometri
cPFQ[{-3/2, 1, 1, 1, 1}, {2, 2, 2, 2}, (d + e*x^2)/d] - 210*d^2*(d + e*x^2)
*HypergeometricPFQ[{-1/2, 1, 1, 1, 1}, {2, 2, 2, 2}, (d + e*x^2)/d] + 16*d^
3*Log[d + e*x^2] - 16*d^3*Sqrt[1 - (d + e*x^2)/d]*Log[d + e*x^2] + 48*d^2*(
d + e*x^2)*Sqrt[1 - (d + e*x^2)/d]*Log[d + e*x^2] - 48*d*(d + e*x^2)^2*Sqrt
[1 - (d + e*x^2)/d]*Log[d + e*x^2] + 16*(d + e*x^2)^3*Sqrt[1 - (d + e*x^2)/
d]*Log[d + e*x^2] + 112*d^2*(d + e*x^2)*HypergeometricPFQ[{-5/2, 1, 1, 1},
{2, 2, 2}, (d + e*x^2)/d]*Log[d + e*x^2] - 280*d^2*(d + e*x^2)*Hypergeometr
icPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^2)/d]*Log[d + e*x^2] + 210*d^2*(
d + e*x^2)*HypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^2)/d]*Log
[d + e*x^2] - 32*d^3*Log[d + e*x^2]^2 + 32*d^3*Sqrt[1 - (d + e*x^2)/d]*Log[
d + e*x^2]^2 - 68*d^2*(d + e*x^2)*Sqrt[1 - (d + e*x^2)/d]*Log[d + e*x^2]^2
 + 40*d*(d + e*x^2)^2*Sqrt[1 - (d + e*x^2)/d]*Log[d + e*x^2]^2 - 4*(d + e*x^
2)^3*Sqrt[1 - (d + e*x^2)/d]*Log[d + e*x^2]^2 - 105*d^2*(d + e*x^2)*Hyperge
ometricPFQ[{-1/2, 1, 1}, {2, 2}, (d + e*x^2)/d]*Log[d + e*x^2]^2 + 10*d^3*L
og[d + e*x^2]^3 - 10*d^3*Sqrt[1 - (d + e*x^2)/d]*Log[d + e*x^2]^3 + 30*d^2*
(d + e*x^2)*Sqrt[1 - (d + e*x^2)/d]*Log[d + e*x^2]^3 - 30*d*(d + e*x^2)^2*S
qrt[1 - (d + e*x^2)/d]*Log[d + e*x^2]^3 + 10*(d + e*x^2)^3*Sqrt[1 - (d + e*
x^2)/d]*Log[d + e*x^2]^3 + 140*d^2*(d + e*x^2)*HypergeometricPFQ[{-3/2, 1,
1}, {2, 2}, (d + e*x^2)/d]*Log[d + e*x^2]*(2 + Log[d + e*x^2]) - 56*d^2*(d
 + e*x^2)*HypergeometricPFQ[{-5/2, 1, 1}, {2, 2}, (d + e*x^2)/d]*(1 + 3*Log[
d + e*x^2] + Log[d + e*x^2]^2)))/(70*e^3*Sqrt[1 - (d + e*x^2)/d]) + (f*g*p^
3*(d + e*x^2)*(-8*d*(-6 + 6*Log[d + e*x^2] - 3*Log[d + e*x^2]^2 + Log[d + e
*x^2]^3) + (d + e*x^2)*(-3 + 6*Log[d + e*x^2] - 6*Log[d + e*x^2]^2 + 4*Log[
d + e*x^2]^3)))/(8*e^2) + 6*f*g*p^2*((x^4*Log[d + e*x^2]^2)/4 - e*((3*d*x^2
)/(4*e^2) - x^4/(8*e) - (3*d^2*Log[d + e*x^2])/(4*e^3) - (d*x^2*Log[d + e*x
^2])/(2*e^2) + (x^4*Log[d + e*x^2])/(4*e) + (d^2*Log[d + e*x^2]^2)/(4*e^3))
)*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]) + (3*d*f*g*p*x^2*(-(p*Log[d
 + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(2*e) - (2*d^2*g^2*p*x^3*(-(p*Log[d +
e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(7*e^2) + (6*d*g^2*p*x^5*(-(p*Log[d + e*
x^2]) + Log[c*(d + e*x^2)^p])^2)/(35*e) - (3*d^2*f*g*p*Log[d + e*x^2]*(-(p*
Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(2*e^2) + (3*p*x*(14*f^2 + 7*f*g
*x^3 + 2*g^2*x^6)*Log[d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p
])^2)/14 + (f*g*x^4*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2*(-3*p +
2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])))/4 + (g^2*x^7*(-(p*Log[d +
e*x^2]) + Log[c*(d + e*x^2)^p])^2*(-6*p + 7*(-(p*Log[d + e*x^2]) + Log[c*(d
 + e*x^2)^p])))/49 + (x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2*(-42
*e^3*f^2*p + 6*d^3*g^2*p + 7*e^3*f^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^
2)^p])))/(7*e^3) - (6*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-7*d*e^3*f^2*p*(-(p*Log[
d + e*x^2]) + Log[c*(d + e*x^2)^p])^2 + d^4*g^2*p*(-(p*Log[d + e*x^2]) + Lo
g[c*(d + e*x^2)^p])^2))/(7*Sqrt[d]*e^(7/2)) + 3*f^2*p^2*(-(p*Log[d + e*x^2]
) + Log[c*(d + e*x^2)^p])*(x*Log[d + e*x^2]^2 - (4*((-I)*Sqrt[d]*ArcTan[(Sq
rt[e]*x)/Sqrt[d]]^2 + Sqrt[e]*x*(-2 + Log[d + e*x^2]) - Sqrt[d]*ArcTan[(Sqr
t[e]*x)/Sqrt[d]]*(-2 + 2*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + Log[d +
 e*x^2]) - I*Sqrt[d]*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqr
t[e]*x)]))/Sqrt[e]) + 3*g^2*p^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]
)*((x^7*Log[d + e*x^2]^2)/7 - (4*((11025*I)*d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt
[d]]^2 + 105*d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-352 + 210*Log[(2*Sqrt[d]
)/(Sqrt[d] + I*Sqrt[e]*x)] + 105*Log[d + e*x^2]) + Sqrt[e]*x*(36960*d^3 - 4
970*d^2*e*x^2 + 1512*d*e^2*x^4 - 450*e^3*x^6 - 105*(105*d^3 - 35*d^2*e*x^2
+ 21*d*e^2*x^4 - 15*e^3*x^6)*Log[d + e*x^2]) + (11025*I)*d^(7/2)*PolyLog[2,
 (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/(77175*e^(7/2))) + (
f^2*p^3*(-48*Sqrt[-d^2]*Sqrt[d + e*x^2]*Sqrt[1 - d/(d + e*x^2)]*ArcSin[Sqrt
```

```
[d]/Sqrt[d + e*x^2]] - 6*Sqrt[-d^2]*Sqrt[1 - d/(d + e*x^2)]*(8*Sqrt[d]*Hype
rgeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, d/(d + e*x^2)] + 4*Sqr
t[d]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, d/(d + e*x^2)]*Log[d +
e*x^2] + Sqrt[d + e*x^2]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]]*Log[d + e*x^2]^2)
+ Sqrt[-d]*e*x^2*(-48 + 24*Log[d + e*x^2] - 6*Log[d + e*x^2]^2 + Log[d + e*
x^2]^3) + 24*d*Sqrt[e*x^2]*ArcTanh[Sqrt[e*x^2]/Sqrt[-d]]*(Log[d + e*x^2] -
Log[(d + e*x^2)/d]) + 6*(-d)^(3/2)*Sqrt[1 - (d + e*x^2)/d]*(Log[(d + e*x^2)
/d]^2 - 4*Log[(d + e*x^2)/d]*Log[(1 + Sqrt[1 - (d + e*x^2)/d])/2] + 2*Log[(
1 + Sqrt[1 - (d + e*x^2)/d])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[1 - (d + e*x^2)
/d]/2])))/(Sqrt[-d]*e*x)
```

fricas [A] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(g^2x^6 + 2\,fgx^3 + f^2\right)\log\left(\left(ex^2+d\right)^p c\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^6 + 2*f*g*x^3 + f^2)*log((e*x^2 + d)^p*c)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(gx^3+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)^2*log((e*x^2 + d)^p*c)^3, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(g\,x^3+f\right)^2 \ln\left(c\left(e\,x^2+d\right)^p\right)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p)^3,x)
```

```
[Out] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p)^3,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*sqrt(e)>0)', see `assume?`
for more details)Is 4*d^2-4*sqrt(e) positive or negative?
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(e\,x^2+d\right)^p\right)^3 \left(g\,x^3+f\right)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^3*(f + g*x^3)^2,x)
[Out] int(log(c*(d + e*x^2)^p)^3*(f + g*x^3)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f + gx^3\right)^2 \log\left(c\left(d + ex^2\right)^p\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**2*ln(c*(e*x**2+d)**p)**3,x)
[Out] Integral((f + g*x**3)**2*log(c*(d + e*x**2)**p)**3, x)
```

3.299 $\int \left(f+gx^3\right)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=518

$$6dfp\text{Int}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2},x\right)+\frac{3gp^2\left(d+ex^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{8e^2}-\frac{3dgp^2\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e^2}+\frac{g\left(d+e\right.}{}$$

```
[Out] -48*f*p^3*x+3*d*g*p^3*x^2/e-3/16*g*p^3*(e*x^2+d)^2/e^2+24*f*p^2*x*ln(c*(e*x
^2+d)^p)-3*d*g*p^2*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e^2+3/8*g*p^2*(e*x^2+d)^2*ln
(c*(e*x^2+d)^p)/e^2-6*f*p*x*ln(c*(e*x^2+d)^p)^2+3/2*d*g*p*(e*x^2+d)*ln(c*(e
*x^2+d)^p)^2/e^2-3/8*g*p*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)^2/e^2+f*x*ln(c*(e*x^
2+d)^p)^3-1/2*d*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)^3/e^2+1/4*g*(e*x^2+d)^2*ln(c*
(e*x^2+d)^p)^3/e^2+48*f*p^3*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)-24*I*
f*p^3*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)-24*f*p^2
*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)*d^(1/2)/e^(1/2)-48*f*p^3*arcta
n(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)-24
*I*f*p^3*arctan(x*e^(1/2)/d^(1/2))^2*d^(1/2)/e^(1/2)+6*d*f*p*Unintegrable(l
n(c*(e*x^2+d)^p)^2/(e*x^2+d),x)
```

Rubi [A] time = 0.77, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(f+gx^3\right)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x^3)*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] -48*f*p^3*x + (3*d*g*p^3*x^2)/e - (3*g*p^3*(d + e*x^2)^2)/(16*e^2) + (48*Sq
rt[d]*f*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - ((24*I)*Sqrt[d]*f*p^3*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e] - (48*Sqrt[d]*f*p^3*ArcTan[(Sqrt[e]*x)
/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + 24*f*p^2*x*Lo
g[c*(d + e*x^2)^p] - (3*d*g*p^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 + (3*
g*p^2*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(8*e^2) - (24*Sqrt[d]*f*p^2*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - 6*f*p*x*Log[c*(d + e
*x^2)^p]^2 + (3*d*g*p*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/(2*e^2) - (3*g*p*
(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^2)/(8*e^2) + f*x*Log[c*(d + e*x^2)^p]^3
- (d*g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^3)/(2*e^2) + (g*(d + e*x^2)^2*Log[c
*(d + e*x^2)^p]^3)/(4*e^2) - ((24*I)*Sqrt[d]*f*p^3*PolyLog[2, 1 - (2*Sqrt[d
])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + 6*d*f*p*Defer[Int][Log[c*(d + e*x^2)
^p]^2/(d + e*x^2), x]
```

Rubi steps

$$\int \left(f+gx^3\right)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(f\log^3\left(c\left(d+ex^2\right)^p\right)+gx^3\log^3\left(c\left(d+ex^2\right)^p\right)\right)dx$$

$$= f\int \log^3\left(c\left(d+ex^2\right)^p\right)dx + g\int x^3\log^3\left(c\left(d+ex^2\right)^p\right)dx$$

$$= fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}g\,\text{Subst}\left(\int x\log^3\left(c(d+ex)^p\right)dx,x,x^2\right)-(6\ldots$$

$$= fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}g\,\text{Subst}\left(\int\left(-\frac{d\log^3\left(c(d+ex)^p\right)}{e}+\frac{(d+ex)\,\mathrm{l}\ldots}{}\right.\right.$$

$$= fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{g\,\text{Subst}\left(\int (d+ex)\log^3\left(c(d+ex)^p\right)dx,x,x^2\right)}{2e}\ldots$$

$$= -6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{g\,\text{Subst}\left(\int x\log^3\ldots\right.}{2\ldots}$$

$$= -6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+fx\log^3\left(c\left(d+ex^2\right)^p\right)-\frac{dg\left(d+ex^2\right)\log^3\ldots}{2e^2}$$

$$= -6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{3dgp\left(d+ex^2\right)\log^2\left(c\left(d+ex^2\right)^p\right)}{2e^2}-\frac{3gp\,(\ldots}{}$$

$$= \frac{3dgp^3x^2}{e}-\frac{3gp^3\left(d+ex^2\right)^2}{16e^2}+24fp^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{3dgp^2\left(d+ex\ldots\right.}{}$$

$$= -48fp^3x+\frac{3dgp^3x^2}{e}-\frac{3gp^3\left(d+ex^2\right)^2}{16e^2}+24fp^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{3\ldots}{}$$

$$= -48fp^3x+\frac{3dgp^3x^2}{e}-\frac{3gp^3\left(d+ex^2\right)^2}{16e^2}+\frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{24i\sqrt{\ldots}}{}$$

$$= -48fp^3x+\frac{3dgp^3x^2}{e}-\frac{3gp^3\left(d+ex^2\right)^2}{16e^2}+\frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{24i\sqrt{\ldots}}{}$$

$$= -48fp^3x+\frac{3dgp^3x^2}{e}-\frac{3gp^3\left(d+ex^2\right)^2}{16e^2}+\frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{24i\sqrt{\ldots}}{}$$

$$= -48fp^3x+\frac{3dgp^3x^2}{e}-\frac{3gp^3\left(d+ex^2\right)^2}{16e^2}+\frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{24i\sqrt{\ldots}}{}$$

Mathematica [A] time = 4.54, size = 1146, normalized size = 2.21

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x^3)*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] (g*p^3*(d + e*x^2)*(45*d - 3*e*x^2 + (-42*d + 6*e*x^2)*Log[d + e*x^2] + 6*(
3*d - e*x^2)*Log[d + e*x^2]^2 - 4*(d - e*x^2)*Log[d + e*x^2]^3))/(16*e^2) -
```

```
(3*g*p^2*(e*x^2*(-6*d + e*x^2) + (6*d^2 + 4*d*e*x^2 - 2*e^2*x^4)*Log[d + e
*x^2] - 2*(d^2 - e^2*x^4)*Log[d + e*x^2]^2)*(p*Log[d + e*x^2] - Log[c*(d +
e*x^2)^p]))/(8*e^2) + (3*d*g*p*x^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)
^p])^2)/(4*e) + (6*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-(p*Log[d + e*x
^2]) + Log[c*(d + e*x^2)^p])^2)/Sqrt[e] - (3*d^2*g*p*Log[d + e*x^2]*(-(p*Lo
g[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(4*e^2) + (3*p*x*(4*f + g*x^3)*Log
[d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/4 - (g*x^4*(3*p
 + 2*p*Log[d + e*x^2] - 2*Log[c*(d + e*x^2)^p])*(-(p*Log[d + e*x^2]) + Log[
c*(d + e*x^2)^p])^2)/8 + f*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2
*(-6*p - p*Log[d + e*x^2] + Log[c*(d + e*x^2)^p]) - (3*f*p^2*(p*Log[d + e*x
^2] - Log[c*(d + e*x^2)^p])*((4*I)*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 +
4*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2 + 2*Log[(2*Sqrt[d])/(Sqrt[d] + I*
Sqrt[e]*x)] + Log[d + e*x^2]) + Sqrt[e]*x*(8 - 4*Log[d + e*x^2] + Log[d + e
*x^2]^2) + (4*I)*Sqrt[d]*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] +
 Sqrt[e]*x)]))/Sqrt[e] + (f*p^3*(-48*Sqrt[-d^2]*Sqrt[(e*x^2)/(d + e*x^2)]*S
qrt[d + e*x^2]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]] + Sqrt[-d]*e*x^2*(-48 + 24*L
og[d + e*x^2] - 6*Log[d + e*x^2]^2 + Log[d + e*x^2]^3) - 6*Sqrt[-d^2]*Sqrt[
(e*x^2)/(d + e*x^2)]*(8*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/
2, 3/2, 3/2}, d/(d + e*x^2)] + Log[d + e*x^2]*(4*Sqrt[d]*HypergeometricPFQ[
{1/2, 1/2, 1/2}, {3/2, 3/2}, d/(d + e*x^2)] + Sqrt[d + e*x^2]*ArcSin[Sqrt[d
]/Sqrt[d + e*x^2]]*Log[d + e*x^2])) + 24*d*Sqrt[e*x^2]*ArcTanh[Sqrt[e*x^2]/
Sqrt[-d]]*(Log[d + e*x^2] - Log[1 + (e*x^2)/d]) + 6*(-d)^(3/2)*Sqrt[-((e*x^
2)/d)]*(Log[1 + (e*x^2)/d]^2 - 4*Log[1 + (e*x^2)/d]*Log[(1 + Sqrt[-((e*x^2)
/d)])/2] + 2*Log[(1 + Sqrt[-((e*x^2)/d)])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[-(
(e*x^2)/d)]/2])))/(Sqrt[-d]*e*x)
```

fricas [A] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(gx^3+f\right)\log\left(\left(ex^2+d\right)^p c\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
```

```
[Out] integral((g*x^3 + f)*log((e*x^2 + d)^p*c)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(gx^3+f\right)\log\left(\left(ex^2+d\right)^p c\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)*log((e*x^2 + d)^p*c)^3, x)
```

maple [A] time = 175.46, size = 0, normalized size = 0.00

$$\int \left(g\,x^3+f\right)\ln\left(c\left(e\,x^2+d\right)^p\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)*ln(c*(e*x^2+d)^p)^3,x)
```

```
[Out] int((g*x^3+f)*ln(c*(e*x^2+d)^p)^3,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*sqrt(e)>0)', see `assume?`
for more details)Is 4*d^2-4*sqrt(e) positive or negative?
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(e\,x^2+d\right)^p\right)^3\,\left(g\,x^3+f\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^3*(f + g*x^3),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^3*(f + g*x^3), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f+gx^3\right)\log\left(c\left(d+ex^2\right)^p\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)*ln(c*(e*x**2+d)**p)**3,x)
```

```
[Out] Integral((f + g*x**3)*log(c*(d + e*x**2)**p)**3, x)
```

3.300 $\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^3/(g*x^3+f),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^3),x]
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^3/(f + g*x^3), x]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx=\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$$

Mathematica [A] time = 17.40, size = 0, normalized size = 0.00

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3),x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3), x]
```

fricas [A] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)^3}{gx^3+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f),x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)^3/(g*x^3 + f), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)^3}{gx^3+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f),x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)^3/(g*x^3 + f), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c \left(e\,x^{2}+d\right)^{p}\right)^{3}}{g\,x^{3}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f),x)
[Out] int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(\left(ex^{2}+d\right)^{p} c\right)^{3}}{gx^{3}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f),x, algorithm="maxima")
[Out] integrate(log((e*x^2 + d)^p*c)^3/(g*x^3 + f), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln \left(c \left(e\,x^{2}+d\right)^{p}\right)^{3}}{g\,x^{3}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^3/(f + g*x^3),x)
[Out] int(log(c*(d + e*x^2)^p)^3/(f + g*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**3/(g*x**3+f),x)
[Out] Timed out
```

3.301 $\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(\frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2,x]
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2, x]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx = \int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$$

Mathematica [A] time = 39.13, size = 0, normalized size = 0.00

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2,x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2, x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)^3}{g^2x^6+2\,fgx^3+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)^3/(g^2*x^6 + 2*f*g*x^3 + f^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)^3}{\left(gx^3+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)^3/(g*x^3 + f)^2, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c \left(e\,x^{2}+d\right)^{p}\right)^{3}}{\left(g\,x^{3}+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x)
[Out] int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln \left(c \left(e\,x^{2}+d\right)^{p}\right)^{3}}{\left(g\,x^{3}+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^3/(f + g*x^3)^2,x)
[Out] int(log(c*(d + e*x^2)^p)^3/(f + g*x^3)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**3/(g*x**3+f)**2,x)
[Out] Timed out
```

3.302 $\int \frac{(f+gx^3)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(\frac{(f+gx^3)^2}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

[Out] Unintegrable((g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx^3)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[(f + g*x^3)^2/Log[c*(d + e*x^2)^p],x]

[Out] Defer[Int][(f + g*x^3)^2/Log[c*(d + e*x^2)^p], x]

Rubi steps

$$\int \frac{(f+gx^3)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(f+gx^3)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(f+gx^3)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p],x]

[Out] Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p], x]

fricas [A] time = 0.66, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{g^2x^6+2\,fgx^3+f^2}{\log\left(\left(ex^2+d\right)^pc\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")

[Out] integral((g^2*x^6 + 2*f*g*x^3 + f^2)/log((e*x^2 + d)^p*c), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx^3+f)^2}{\log\left(\left(ex^2+d\right)^pc\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate((g*x^3 + f)^2/log((e*x^2 + d)^p*c), x)
```

maple [A] time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{\left(g\,x^{3}+f\right)^{2}}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
[Out] int((g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx^{3}+f\right)^{2}}{\log\left(\left(ex^{2}+d\right)^{p}c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate((g*x^3 + f)^2/log((e*x^2 + d)^p*c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(g\,x^{3}+f\right)^{2}}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x^3)^2/log(c*(d + e*x^2)^p),x)
[Out] int((f + g*x^3)^2/log(c*(d + e*x^2)^p), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx^{3}\right)^{2}}{\log\left(c\left(d+ex^{2}\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**2/ln(c*(e*x**2+d)**p),x)
[Out] Integral((f + g*x**3)**2/log(c*(d + e*x**2)**p), x)
```

3.303 $\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

[Out] Unintegrable((g*x^3+f)/ln(c*(e*x^2+d)^p),x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[(f + g*x^3)/Log[c*(d + e*x^2)^p],x]

[Out] Defer[Int][(f + g*x^3)/Log[c*(d + e*x^2)^p], x]

Rubi steps

$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p],x]

[Out] Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p], x]

fricas [A] time = 1.28, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{gx^3+f}{\log\left(\left(ex^2+d\right)^p c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")

[Out] integral((g*x^3 + f)/log((e*x^2 + d)^p*c), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx^3+f}{\log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate((g*x^3 + f)/log((e*x^2 + d)^p*c), x)
```

maple [A] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{g\,x^3+f}{\ln\left(c\left(e\,x^2+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)/ln(c*(e*x^2+d)^p),x)
[Out] int((g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx^3+f}{\log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate((g*x^3 + f)/log((e*x^2 + d)^p*c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{g\,x^3+f}{\ln\left(c\left(e\,x^2+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x^3)/log(c*(d + e*x^2)^p),x)
[Out] int((f + g*x^3)/log(c*(d + e*x^2)^p), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)/ln(c*(e*x**2+d)**p),x)
[Out] Integral((f + g*x**3)/log(c*(d + e*x**2)**p), x)
```

3.304 $\int \frac{1}{(f+gx^3)\log\left(c(d+ex^2)^p\right)} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

[Out] Unintegrable(1/(g*x^3+f)/ln(c*(e*x^2+d)^p),x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]),x]

[Out] Defer[Int][1/((f + g*x^3)*Log[c*(d + e*x^2)^p]), x]

Rubi steps

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)} dx = \int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)} dx$$

Mathematica [A] time = 2.91, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]),x]

[Out] Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]), x]

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{(gx^3+f)\log\left(\left(ex^2+d\right)^p c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")

[Out] integral(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx^3+f)\log\left(\left(ex^2+d\right)^p c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)), x)
```

maple [A] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(g\,x^{3}+f\right)\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p),x)
[Out] int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx^{3}+f\right)\log\left(\left(ex^{2}+d\right)^{p}c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)\,\left(g\,x^{3}+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^3)),x)
[Out] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**3+f)/ln(c*(e*x**2+d)**p),x)
[Out] Timed out
```

3.305 $\int \frac{1}{(f+gx^3)^2 \log\left(c\left(d+ex^2\right)^p\right)} \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{\left(f+gx^3\right)^2 \log\left(c\left(d+ex^2\right)^p\right)}, x\right)$$

```
[Out] Unintegrable(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(f+gx^3\right)^2 \log\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]),x]
[Out] Defer[Int][1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{\left(f+gx^3\right)^2 \log\left(c\left(d+ex^2\right)^p\right)} \, dx = \int \frac{1}{\left(f+gx^3\right)^2 \log\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Mathematica [A] time = 8.77, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(f+gx^3\right)^2 \log\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]),x]
[Out] Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]), x]
```

fricas [A] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(g^2x^6+2\,fgx^3+f^2\right)\log\left(\left(ex^2+d\right)^p c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral(1/((g^2*x^6 + 2*f*g*x^3 + f^2)*log((e*x^2 + d)^p*c)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx^3+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate(1/((g*x^3 + f)^2*log((e*x^2 + d)^p*c)), x)
```

maple [A] time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(g\,x^{3}+f\right)^{2} \ln \left(c \left(e\,x^{2}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
[Out] int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx^{3}+f\right)^{2} \log \left(\left(ex^{2}+d\right)^{p} c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate(1/((g*x^3 + f)^2*log((e*x^2 + d)^p*c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\ln \left(c \left(e\,x^{2}+d\right)^{p}\right) \left(g\,x^{3}+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^3)^2),x)
[Out] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^3)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**3+f)**2/ln(c*(e*x**2+d)**p),x)
[Out] Timed out
```

3.306 $\int \frac{(f+gx^3)^2}{\log^2\left(c(d+ex^2)^p\right)} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

[Out] Unintegrable((g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2,x]

[Out] Defer[Int][(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2, x]

Rubi steps

$$\int \frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx = \int \frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Mathematica [A] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2,x]

[Out] Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2, x]

fricas [A] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{g^2x^6+2\,fgx^3+f^2}{\log\left(\left(ex^2+d\right)^pc\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")

[Out] integral((g^2*x^6 + 2*f*g*x^3 + f^2)/log((e*x^2 + d)^p*c)^2, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx^3+f)^2}{\log\left(\left(ex^2+d\right)^pc\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)^2/log((e*x^2 + d)^p*c)^2, x)
```

maple [A] time = 4.56, size = 0, normalized size = 0.00

$$\int \frac{\left(g\,x^{3}+f\right)^{2}}{\ln \left(c \left(e\,x^{2}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{eg^{2}x^{8}+dg^{2}x^{6}+2\,efgx^{5}+2\,dfgx^{3}+ef^{2}x^{2}+df^{2}}{2\left(ep^{2}x\log\left(ex^{2}+d\right)+epx\log(c)\right)}+\int \frac{7\,eg^{2}x^{8}+5\,dg^{2}x^{6}+8\,efgx^{5}+4\,dfgx^{3}+ef^{2}x^{2}-df^{2}}{2\left(ep^{2}x^{2}\log\left(ex^{2}+d\right)+epx^{2}\log(c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*g^2*x^8 + d*g^2*x^6 + 2*e*f*g*x^5 + 2*d*f*g*x^3 + e*f^2*x^2 + d*f^2
)/(e*p^2*x*log(e*x^2 + d) + e*p*x*log(c)) + integrate(1/2*(7*e*g^2*x^8 + 5*
d*g^2*x^6 + 8*e*f*g*x^5 + 4*d*f*g*x^3 + e*f^2*x^2 - d*f^2)/(e*p^2*x^2*log(e
*x^2 + d) + e*p*x^2*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(g\,x^{3}+f\right)^{2}}{\ln \left(c \left(e\,x^{2}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x^3)^2/log(c*(d + e*x^2)^p)^2,x)
```

```
[Out] int((f + g*x^3)^2/log(c*(d + e*x^2)^p)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx^{3}\right)^{2}}{\log \left(c\left(d+ex^{2}\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**2/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**3)**2/log(c*(d + e*x**2)**p)**2, x)
```

3.307 $\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x^3)/Log[c*(d + e*x^2)^p]^2,x]
[Out] Defer[Int][(f + g*x^3)/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi steps

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx = \int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Mathematica [A] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p]^2,x]
[Out] Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p]^2, x]
```

fricas [A] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{gx^3+f}{\log\left(\left(ex^2+d\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral((g*x^3 + f)/log((e*x^2 + d)^p*c)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx^3+f}{\log\left(\left(ex^2+d\right)^p c\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)/log((e*x^2 + d)^p*c)^2, x)
```

maple [A] time = 4.75, size = 0, normalized size = 0.00

$$\int \frac{g\,x^{3}+f}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{egx^{5}+dgx^{3}+efx^{2}+df}{2\left(ep^{2}x\log\left(ex^{2}+d\right)+epx\log(c)\right)}+\int \frac{4\,egx^{5}+2\,dgx^{3}+efx^{2}-df}{2\left(ep^{2}x^{2}\log\left(ex^{2}+d\right)+epx^{2}\log(c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*g*x^5 + d*g*x^3 + e*f*x^2 + d*f)/(e*p^2*x*log(e*x^2 + d) + e*p*x*lo
g(c)) + integrate(1/2*(4*e*g*x^5 + 2*d*g*x^3 + e*f*x^2 - d*f)/(e*p^2*x^2*lo
g(e*x^2 + d) + e*p*x^2*log(c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{g\,x^{3}+f}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x^3)/log(c*(d + e*x^2)^p)^2,x)
```

```
[Out] int((f + g*x^3)/log(c*(d + e*x^2)^p)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx^{3}}{\log\left(c\left(d+ex^{2}\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**3)/log(c*(d + e*x**2)**p)**2, x)
```

3.308 $\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2),x]
```

```
[Out] Defer[Int][1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A] time = 8.15, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2),x]
```

```
[Out] Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2), x]
```

fricas [A] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{(gx^3+f)\log\left(\left(ex^2+d\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx^3+f)\log\left(\left(ex^2+d\right)^p c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)^2), x)
```

maple [A] time = 4.98, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(g\,x^{3}+f\right)\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ex^{2}+d}{2\left(egpx^{4}\log(c)+efpx\log(c)+\left(egp^{2}x^{4}+efp^{2}x\right)\log\left(ex^{2}+d\right)\right)}-\int\frac{2\,e}{2\left(eg^{2}px^{8}\log(c)+2\,efgpx^{5}\log(c)+ef\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*x^2 + d)/(e*g*p*x^4*log(c) + e*f*p*x*log(c) + (e*g*p^2*x^4 + e*f*p^
2*x)*log(e*x^2 + d)) - integrate(1/2*(2*e*g*x^5 + 4*d*g*x^3 - e*f*x^2 + d*f
)/(e*g^2*p*x^8*log(c) + 2*e*f*g*p*x^5*log(c) + e*f^2*p*x^2*log(c) + (e*g^2*
p^2*x^8 + 2*e*f*g*p^2*x^5 + e*f^2*p^2*x^2)*log(e*x^2 + d)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}\,\left(g\,x^{3}+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^3)),x)
```

```
[Out] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**3+f)/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Timed out
```

3.309 $\int \frac{1}{(f+gx^3)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{\left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2),x]
[Out] Defer[Int][1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{\left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} dx = \int \frac{1}{\left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Mathematica [A] time = 14.48, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2),x]
[Out] Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2), x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(g^2x^6+2\,fgx^3+f^2\right)\log\left(\left(ex^2+d\right)^p c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral(1/((g^2*x^6 + 2*f*g*x^3 + f^2)*log((e*x^2 + d)^p*c)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx^3+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^3 + f)^2*log((e*x^2 + d)^p*c)^2), x)
```

maple [A] time = 5.96, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(g\,x^{3}+f\right)^{2} \ln \left(c \left(e\,x^{2}+d\right)^{p}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ex^{2}+d}{2\left(eg^{2}px^{7}\log(c)+2\,efgpx^{4}\log(c)+ef^{2}px\log(c)+\left(eg^{2}p^{2}x^{7}+2\,efgp^{2}x^{4}+ef^{2}p^{2}x\right)\log\left(ex^{2}+d\right)\right)}-\int\frac{}{2\left(eg\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*x^2 + d)/(e*g^2*p*x^7*log(c) + 2*e*f*g*p*x^4*log(c) + e*f^2*p*x*log
(c) + (e*g^2*p^2*x^7 + 2*e*f*g*p^2*x^4 + e*f^2*p^2*x)*log(e*x^2 + d)) - int
egrate(1/2*(5*e*g*x^5 + 7*d*g*x^3 - e*f*x^2 + d*f)/(e*g^3*p*x^11*log(c) + 3
*e*f*g^2*p*x^8*log(c) + 3*e*f^2*g*p*x^5*log(c) + e*f^3*p*x^2*log(c) + (e*g^
3*p^2*x^11 + 3*e*f*g^2*p^2*x^8 + 3*e*f^2*g*p^2*x^5 + e*f^3*p^2*x^2)*log(e*x
^2 + d)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}^2\,{\left(g\,x^3+f\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^2),x)
```

```
[Out] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**3+f)**2/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Timed out
```

3.310 $\int x^5 \left(f+gx^2\right) \log \left(c \left(d+ex^2\right)^p\right) \, dx$

Optimal. Leaf size=142

$$\frac{1}{6}fx^6 \log \left(c \left(d+ex^2\right)^p\right)+\frac{1}{8}gx^8 \log \left(c \left(d+ex^2\right)^p\right)+\frac{d^3p(4ef-3dg)\log \left(d+ex^2\right)}{24e^4}-\frac{d^2px^2(4ef-3dg)}{24e^3}+\frac{dpx^4(4ef-}{48e^2}$$

```
[Out] -1/24*d^2*(-3*d*g+4*e*f)*p*x^2/e^3+1/48*d*(-3*d*g+4*e*f)*p*x^4/e^2-1/72*(-3
*d*g+4*e*f)*p*x^6/e-1/32*g*p*x^8+1/24*d^3*(-3*d*g+4*e*f)*p*ln(e*x^2+d)/e^4+
1/6*f*x^6*ln(c*(e*x^2+d)^p)+1/8*g*x^8*ln(c*(e*x^2+d)^p)
```

Rubi [A] time = 0.23, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2475, 43, 2414, 12, 77}

$$\frac{1}{6}fx^6 \log \left(c \left(d+ex^2\right)^p\right)+\frac{1}{8}gx^8 \log \left(c \left(d+ex^2\right)^p\right)-\frac{d^2px^2(4ef-3dg)}{24e^3}+\frac{d^3p(4ef-3dg)\log \left(d+ex^2\right)}{24e^4}+\frac{dpx^4(4ef-}{48e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -(d^2*(4*e*f - 3*d*g)*p*x^2)/(24*e^3) + (d*(4*e*f - 3*d*g)*p*x^4)/(48*e^2)
- ((4*e*f - 3*d*g)*p*x^6)/(72*e) - (g*p*x^8)/32 + (d^3*(4*e*f - 3*d*g)*p*Lo
g[d + e*x^2])/(24*e^4) + (f*x^6*Log[c*(d + e*x^2)^p])/6 + (g*x^8*Log[c*(d +
 e*x^2)^p])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
```

```
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int x^5\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)dx=\frac{1}{2}\text{Subst}\left(\int x^2(f+gx)\log\left(c(d+ex)^p\right)dx,x,x^2\right)$$
$$=\frac{1}{6}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}gx^8\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{2}(ep)\,\text{Subst}\left(\int \right.$$
$$=\frac{1}{6}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}gx^8\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{24}(ep)\,\text{Subst}\left(\int \right.$$
$$=\frac{1}{6}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}gx^8\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{24}(ep)\,\text{Subst}\left(\int \right.$$
$$=-\frac{d^2(4ef-3dg)px^2}{24e^3}+\frac{d(4ef-3dg)px^4}{48e^2}-\frac{(4ef-3dg)px^6}{72e}-\frac{1}{32}gpx^8+$$

Mathematica [A] time = 0.04, size = 170, normalized size = 1.20

$$\frac{1}{6}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}gx^8\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^4gp\log\left(d+ex^2\right)}{8e^4}+\frac{d^3fp\log\left(d+ex^2\right)}{6e^3}+\frac{d^3gpx^2}{8e^3}-\frac{d^2fpx^2}{6e^2}-$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -1/6*(d^2*f*p*x^2)/e^2 + (d^3*g*p*x^2)/(8*e^3) + (d*f*p*x^4)/(12*e) - (d^2*
g*p*x^4)/(16*e^2) - (f*p*x^6)/18 + (d*g*p*x^6)/(24*e) - (g*p*x^8)/32 + (d^3
*f*p*Log[d + e*x^2])/(6*e^3) - (d^4*g*p*Log[d + e*x^2])/(8*e^4) + (f*x^6*Lo
g[c*(d + e*x^2)^p])/6 + (g*x^8*Log[c*(d + e*x^2)^p])/8
```

fricas [A] time = 0.51, size = 152, normalized size = 1.07

$$-\frac{9\,e^4gpx^8+4\left(4\,e^4f-3\,de^3g\right)px^6-6\left(4\,de^3f-3\,d^2e^2g\right)px^4+12\left(4\,d^2e^2f-3\,d^3eg\right)px^2-12\left(3\,e^4gpx^8+4\,e^4f\right.}{288\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/288*(9*e^4*g*p*x^8 + 4*(4*e^4*f - 3*d*e^3*g)*p*x^6 - 6*(4*d*e^3*f - 3*d^
2*e^2*g)*p*x^4 + 12*(4*d^2*e^2*f - 3*d^3*e*g)*p*x^2 - 12*(3*e^4*g*p*x^8 + 4
*e^4*f*p*x^6 + (4*d^3*e*f - 3*d^4*g)*p)*log(e*x^2 + d) - 12*(3*e^4*g*x^8 +
4*e^4*f*x^6)*log(c))/e^4
```

giac [B] time = 0.19, size = 308, normalized size = 2.17

$$\frac{1}{288}\left(36\,gx^8e\log(c)+48\,fx^6e\log(c)+8\left(6\left(x^2e+d\right)^3e^{(-2)}\log\left(x^2e+d\right)-18\left(x^2e+d\right)^2de^{(-2)}\log\left(x^2e+d\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/288*(36*g*x^8*e*log(c) + 48*f*x^6*e*log(c) + 8*(6*(x^2*e + d)^3*e^(-2)*lo
g(x^2*e + d) - 18*(x^2*e + d)^2*d*e^(-2)*log(x^2*e + d) + 18*(x^2*e + d)*d^
```

```
2*e^(-2)*log(x^2*e + d) - 2*(x^2*e + d)^3*e^(-2) + 9*(x^2*e + d)^2*d*e^(-2)
 - 18*(x^2*e + d)*d^2*e^(-2))*f*p + 3*(12*(x^2*e + d)^4*e^(-3)*log(x^2*e +
d) - 48*(x^2*e + d)^3*d*e^(-3)*log(x^2*e + d) + 72*(x^2*e + d)^2*d^2*e^(-3)
*log(x^2*e + d) - 48*(x^2*e + d)*d^3*e^(-3)*log(x^2*e + d) - 3*(x^2*e + d)^
4*e^(-3) + 16*(x^2*e + d)^3*d*e^(-3) - 36*(x^2*e + d)^2*d^2*e^(-3) + 48*(x^
2*e + d)*d^3*e^(-3))*g*p)*e^(-1)
```

maple [C] time = 0.51, size = 413, normalized size = 2.91

$$-\frac{i\pi g\,x^8\text{csgn}\,(ic)\,\text{csgn}\left(i\left(e\,x^2+d\right)^p\right)\text{csgn}\left(ic\left(e\,x^2+d\right)^p\right)}{16}+\frac{i\pi g\,x^8\text{csgn}\,(ic)\,\text{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2}{16}+\frac{i\pi g\,x^8\text{csgn}\left(i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(g*x^2+f)*ln(c*(e*x^2+d)^p),x)
```

```
[Out] (1/8*g*x^8+1/6*f*x^6)*ln((e*x^2+d)^p)-1/16*I*Pi*g*x^8*csgn(I*c*(e*x^2+d)^p)
^3+1/16*I*Pi*g*x^8*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/16*I*Pi*g*
x^8*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/12*I*Pi*f*x^6*csg
n(I*c*(e*x^2+d)^p)^3+1/12*I*Pi*f*x^6*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/12
*I*Pi*f*x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/16*I*Pi*g*x^8*csg
n(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/12*I*Pi*f*x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c
*(e*x^2+d)^p)*csgn(I*c)+1/8*ln(c)*g*x^8-1/32*g*p*x^8+1/6*ln(c)*f*x^6+1/24/e
*d*g*p*x^6-1/18*f*p*x^6-1/16/e^2*d^2*g*p*x^4+1/12/e*d*f*p*x^4+1/8/e^3*d^3*g
*p*x^2-1/6/e^2*d^2*f*p*x^2-1/8/e^4*ln(e*x^2+d)*d^4*g*p+1/6/e^3*ln(e*x^2+d)*
d^3*f*p
```

maxima [A] time = 0.46, size = 132, normalized size = 0.93

$$-\frac{1}{288}\,ep\left(\frac{9\,e^3gx^8+4\,\left(4\,e^3f-3\,de^2g\right)x^6-6\,\left(4\,de^2f-3\,d^2eg\right)x^4+12\,\left(4\,d^2ef-3\,d^3g\right)x^2}{e^4}-\frac{12\,\left(4\,d^3ef-3\,d^4g\right)\log}{e^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -1/288*e*p*((9*e^3*g*x^8 + 4*(4*e^3*f - 3*d*e^2*g)*x^6 - 6*(4*d*e^2*f - 3*d
^2*e*g)*x^4 + 12*(4*d^2*e*f - 3*d^3*g)*x^2)/e^4 - 12*(4*d^3*e*f - 3*d^4*g)*
log(e*x^2 + d)/e^5) + 1/24*(3*g*x^8 + 4*f*x^6)*log((e*x^2 + d)^p*c)
```

mupad [B] time = 0.32, size = 127, normalized size = 0.89

$$\ln\left(c\left(e\,x^2+d\right)^p\right)\left(\frac{g\,x^8}{8}+\frac{f\,x^6}{6}\right)-x^6\left(\frac{f\,p}{18}-\frac{d\,g\,p}{24\,e}\right)-\frac{g\,p\,x^8}{32}-\frac{\ln\left(e\,x^2+d\right)\left(3\,d^4\,g\,p-4\,d^3\,e\,f\,p\right)}{24\,e^4}+\frac{d\,x^4\left(\frac{f\,p}{3}-\frac{d\,g\,p}{4\,e}\right)}{4\,e}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f*x^6)/6 + (g*x^8)/8) - x^6*((f*p)/18 - (d*g*p)/(24*
e)) - (g*p*x^8)/32 - (log(d + e*x^2)*(3*d^4*g*p - 4*d^3*e*f*p))/(24*e^4) +
(d*x^4*((f*p)/3 - (d*g*p)/(4*e)))/(4*e) - (d^2*x^2*((f*p)/3 - (d*g*p)/(4*e)
))/(2*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

3.311 $\int x^3 \left(f+gx^2\right) \log \left(c \left(d+ex^2\right)^p\right) \, dx$

Optimal. Leaf size=119

$$\frac{1}{4} f x^4 \log \left(c \left(d+e x^2\right)^p\right)+\frac{1}{6} g x^6 \log \left(c \left(d+e x^2\right)^p\right)-\frac{d^2 p (3 e f-2 d g) \log \left(d+e x^2\right)}{12 e^3}+\frac{d p x^2 (3 e f-2 d g)}{12 e^2}-\frac{p x^4 (3 e f-}{24 e}$$

```
[Out] 1/12*d*(-2*d*g+3*e*f)*p*x^2/e^2-1/24*(-2*d*g+3*e*f)*p*x^4/e-1/18*g*p*x^6-1/
12*d^2*(-2*d*g+3*e*f)*p*ln(e*x^2+d)/e^3+1/4*f*x^4*ln(c*(e*x^2+d)^p)+1/6*g*x
^6*ln(c*(e*x^2+d)^p)
```

Rubi [A] time = 0.18, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2475, 43, 2414, 12, 77}

$$\frac{1}{4} f x^4 \log \left(c \left(d+e x^2\right)^p\right)+\frac{1}{6} g x^6 \log \left(c \left(d+e x^2\right)^p\right)-\frac{d^2 p (3 e f-2 d g) \log \left(d+e x^2\right)}{12 e^3}+\frac{d p x^2 (3 e f-2 d g)}{12 e^2}-\frac{p x^4 (3 e f-}{24 e}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (d*(3*e*f - 2*d*g)*p*x^2)/(12*e^2) - ((3*e*f - 2*d*g)*p*x^4)/(24*e) - (g*p*
x^6)/18 - (d^2*(3*e*f - 2*d*g)*p*Log[d + e*x^2])/(12*e^3) + (f*x^4*Log[c*(d
 + e*x^2)^p])/4 + (g*x^6*Log[c*(d + e*x^2)^p])/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
```

```
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)dx &= \frac{1}{2}\text{Subst}\left(\int x(f+gx)\log\left(c(d+ex)^p\right)dx,x,x^2\right)\\ &= \frac{1}{4}fx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{6}gx^6\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{2}(ep)\text{Subst}\left(\int \frac{x^2(3}{6(}\right.\\ &= \frac{1}{4}fx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{6}gx^6\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{12}(ep)\text{Subst}\left(\int \frac{x^2(}{}\right.\\ &= \frac{1}{4}fx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{6}gx^6\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{12}(ep)\text{Subst}\left(\int\left(\frac{d(}{}\right.\right.\\ &= \frac{d(3ef-2dg)px^2}{12e^2}-\frac{(3ef-2dg)px^4}{24e}-\frac{1}{18}gpx^6-\frac{d^2(3ef-2dg)p\log\left(d+\right.}{12e^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 140, normalized size = 1.18

$$\frac{1}{4}fx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{6}gx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{d^3gp\log\left(d+ex^2\right)}{6e^3}-\frac{d^2fp\log\left(d+ex^2\right)}{4e^2}-\frac{d^2gpx^2}{6e^2}+\frac{dfpx^2}{4e}+\frac{dgp}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (d*f*p*x^2)/(4*e) - (d^2*g*p*x^2)/(6*e^2) - (f*p*x^4)/8 + (d*g*p*x^4)/(12*e
) - (g*p*x^6)/18 - (d^2*f*p*Log[d + e*x^2])/(4*e^2) + (d^3*g*p*Log[d + e*x^
2])/(6*e^3) + (f*x^4*Log[c*(d + e*x^2)^p])/4 + (g*x^6*Log[c*(d + e*x^2)^p])
/6
```

fricas [A] time = 0.69, size = 128, normalized size = 1.08

$$-\frac{4e^3gpx^6+3\left(3e^3f-2de^2g\right)px^4-6\left(3de^2f-2d^2eg\right)px^2-6\left(2e^3gpx^6+3e^3fpx^4-\left(3d^2ef-2d^3g\right)p\right)\log\left(ex^2\right.}{72e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/72*(4*e^3*g*p*x^6 + 3*(3*e^3*f - 2*d*e^2*g)*p*x^4 - 6*(3*d*e^2*f - 2*d^2
*e*g)*p*x^2 - 6*(2*e^3*g*p*x^6 + 3*e^3*f*p*x^4 - (3*d^2*e*f - 2*d^3*g)*p)*l
og(e*x^2 + d) - 6*(2*e^3*g*x^6 + 3*e^3*f*x^4)*log(c))/e^3
```

giac [B] time = 0.23, size = 235, normalized size = 1.97

$$\frac{1}{72}\left(12gx^6e\log(c)+9\left(2\left(x^2e+d\right)^2\log\left(x^2e+d\right)-4\left(x^2e+d\right)d\log\left(x^2e+d\right)-\left(x^2e+d\right)^2+4\left(x^2e+d\right)d\right)fpe^{(-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/72*(12*g*x^6*e*log(c) + 9*(2*(x^2*e + d)^2*log(x^2*e + d) - 4*(x^2*e + d)
*d*log(x^2*e + d) - (x^2*e + d)^2 + 4*(x^2*e + d)*d)*f*p*e^(-1) + 18*((x^2*
e + d)^2 - 2*(x^2*e + d)*d)*f*e^(-1)*log(c) + 2*(6*(x^2*e + d)^3*e^(-2)*log
```

```
(x^2*e + d) - 18*(x^2*e + d)^2*d*e^(-2)*log(x^2*e + d) + 18*(x^2*e + d)*d^2
*e^(-2)*log(x^2*e + d) - 2*(x^2*e + d)^3*e^(-2) + 9*(x^2*e + d)^2*d*e^(-2)
- 18*(x^2*e + d)*d^2*e^(-2))*g*p)*e^(-1)
```

maple [C] time = 0.50, size = 387, normalized size = 3.25

$$-\frac{i\pi g\,x^6\mathrm{csgn}\,(ic)\,\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)}{12}+\frac{i\pi g\,x^6\mathrm{csgn}\,(ic)\,\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2}{12}+\frac{i\pi g\,x^6\mathrm{csgn}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(g*x^2+f)*ln(c*(e*x^2+d)^p),x)
```

```
[Out] (1/6*g*x^6+1/4*f*x^4)*ln((e*x^2+d)^p)+1/8*I*Pi*f*x^4*csgn(I*(e*x^2+d)^p)*cs
gn(I*c*(e*x^2+d)^p)^2+1/12*I*Pi*g*x^6*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/1
2*I*Pi*g*x^6*csgn(I*c*(e*x^2+d)^p)^3+1/8*I*Pi*f*x^4*csgn(I*c*(e*x^2+d)^p)^2
*csgn(I*c)-1/8*I*Pi*f*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*
c)-1/8*I*Pi*f*x^4*csgn(I*c*(e*x^2+d)^p)^3+1/12*I*Pi*g*x^6*csgn(I*(e*x^2+d)^
p)*csgn(I*c*(e*x^2+d)^p)^2-1/12*I*Pi*g*x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*
x^2+d)^p)*csgn(I*c)+1/6*ln(c)*g*x^6-1/18*g*p*x^6+1/4*ln(c)*f*x^4+1/12/e*d*g
*p*x^4-1/8*f*p*x^4-1/6/e^2*d^2*g*p*x^2+1/4/e*d*f*p*x^2+1/6/e^3*ln(e*x^2+d)*
d^3*g*p-1/4/e^2*ln(e*x^2+d)*d^2*f*p
```

maxima [A] time = 0.46, size = 108, normalized size = 0.91

$$-\frac{1}{72}\,ep\left(\frac{4\,e^2gx^6+3\left(3\,e^2f-2\,deg\right)x^4-6\left(3\,def-2\,d^2g\right)x^2}{e^3}+\frac{6\left(3\,d^2ef-2\,d^3g\right)\log\left(ex^2+d\right)}{e^4}\right)+\frac{1}{12}\left(2\,gx^6+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -1/72*e*p*((4*e^2*g*x^6 + 3*(3*e^2*f - 2*d*e*g)*x^4 - 6*(3*d*e*f - 2*d^2*g)
*x^2)/e^3 + 6*(3*d^2*e*f - 2*d^3*g)*log(e*x^2 + d)/e^4) + 1/12*(2*g*x^6 + 3
*f*x^4)*log((e*x^2 + d)^p*c)
```

mupad [B] time = 0.31, size = 103, normalized size = 0.87

$$\ln\left(c\left(e\,x^2+d\right)^p\right)\left(\frac{g\,x^6}{6}+\frac{f\,x^4}{4}\right)-x^4\left(\frac{f\,p}{8}-\frac{d\,g\,p}{12\,e}\right)-\frac{g\,p\,x^6}{18}+\frac{\ln\left(e\,x^2+d\right)\left(2\,d^3\,g\,p-3\,d^2\,e\,f\,p\right)}{12\,e^3}+\frac{d\,x^2\left(\frac{f\,p}{2}-\frac{d\,g\,p}{3\,e}\right.}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f*x^4)/4 + (g*x^6)/6) - x^4*((f*p)/8 - (d*g*p)/(12*e
)) - (g*p*x^6)/18 + (log(d + e*x^2)*(2*d^3*g*p - 3*d^2*e*f*p))/(12*e^3) + (
d*x^2*((f*p)/2 - (d*g*p)/(3*e)))/(2*e)
```

sympy [A] time = 162.65, size = 170, normalized size = 1.43

$$\begin{cases}\frac{d^3gp\log\left(d+ex^2\right)}{6e^3}-\frac{d^2fp\log\left(d+ex^2\right)}{4e^2}-\frac{d^2gpx^2}{6e^2}+\frac{dfpx^2}{4e}+\frac{dgpx^4}{12e}+\frac{fpx^4\log\left(d+ex^2\right)}{4}-\frac{fpx^4}{8}+\frac{fx^4\log\left(c\right)}{4}+\frac{gpx^6\log\left(d+ex^2\right)}{6}-\frac{gp}{18} \\ \left(\frac{fx^4}{4}+\frac{gx^6}{6}\right)\log\left(cd^p\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((d**3*g*p*log(d + e*x**2)/(6*e**3) - d**2*f*p*log(d + e*x**2)/(4*
e**2) - d**2*g*p*x**2/(6*e**2) + d*f*p*x**2/(4*e) + d*g*p*x**4/(12*e) + f*p
*x**4*log(d + e*x**2)/4 - f*p*x**4/8 + f*x**4*log(c)/4 + g*p*x**6*log(d + e
*x**2)/6 - g*p*x**6/18 + g*x**6*log(c)/6, Ne(e, 0)), ((f*x**4/4 + g*x**6/6)
*log(c*d**p), True))
```

3.312 $\int x\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=94

$$\frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4g}-\frac{p(ef-dg)^2\log\left(d+ex^2\right)}{4e^2g}-\frac{px^2(ef-dg)}{4e}-\frac{p\left(f+gx^2\right)^2}{8g}$$

```
[Out] -1/4*(-d*g+e*f)*p*x^2/e-1/8*p*(g*x^2+f)^2/g-1/4*(-d*g+e*f)^2*p*ln(e*x^2+d)/
e^2/g+1/4*(g*x^2+f)^2*ln(c*(e*x^2+d)^p)/g
```

Rubi [A] time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2475, 2395, 43}

$$\frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4g}-\frac{p(ef-dg)^2\log\left(d+ex^2\right)}{4e^2g}-\frac{px^2(ef-dg)}{4e}-\frac{p\left(f+gx^2\right)^2}{8g}$$

Antiderivative was successfully verified.

```
[In] Int[x*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -((e*f - d*g)*p*x^2)/(4*e) - (p*(f + g*x^2)^2)/(8*g) - ((e*f - d*g)^2*p*Log
[d + e*x^2])/(4*e^2*g) + ((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/(4*g)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int x\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx = \frac{1}{2}\text{Subst}\left(\int (f+gx)\log\left(c(d+ex)^p\right)\,dx,x,x^2\right)$$
$$= \frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4g} - \frac{(ep)\,\text{Subst}\left(\int \frac{(f+gx)^2}{d+ex}\,dx,x,x^2\right)}{4g}$$
$$= \frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4g} - \frac{(ep)\,\text{Subst}\left(\int \left(\frac{g(ef-dg)}{e^2}+\frac{(ef-dg)^2}{e^2(d+ex)}+\frac{g(f+gx)}{e}\right.\right.}{4g}$$
$$= -\frac{(ef-dg)px^2}{4e} - \frac{p\left(f+gx^2\right)^2}{8g} - \frac{(ef-dg)^2p\log\left(d+ex^2\right)}{4e^2g} + \frac{\left(f+gx^2\right)^2\text{lo}}{}$$

Mathematica [A] time = 0.05, size = 98, normalized size = 1.04

$$\frac{1}{2}f\left(\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e}-px^2\right)+\frac{1}{4}gx^4\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2gp\log\left(d+ex^2\right)}{4e^2}+\frac{dgpx^2}{4e}-\frac{1}{8}gpx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (d*g*p*x^2)/(4*e) - (g*p*x^4)/8 - (d^2*g*p*Log[d + e*x^2])/(4*e^2) + (g*x^4
*Log[c*(d + e*x^2)^p])/4 + (f*(-(p*x^2) + ((d + e*x^2)*Log[c*(d + e*x^2)^p]
)/e))/2
```

fricas [A] time = 0.76, size = 99, normalized size = 1.05

$$-\frac{e^2gpx^4+2\left(2e^2f-deg\right)px^2-2\left(e^2gpx^4+2e^2fpx^2+\left(2def-d^2g\right)p\right)\log\left(ex^2+d\right)-2\left(e^2gx^4+2e^2fx^2\right)\log(c}{8e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/8*(e^2*g*p*x^4 + 2*(2*e^2*f - d*e*g)*p*x^2 - 2*(e^2*g*p*x^4 + 2*e^2*f*p*
x^2 + (2*d*e*f - d^2*g)*p)*log(e*x^2 + d) - 2*(e^2*g*x^4 + 2*e^2*f*x^2)*log
(c))/e^2
```

giac [A] time = 0.17, size = 148, normalized size = 1.57

$$\frac{1}{8}\left(\left(2\left(x^2e+d\right)^2\log\left(x^2e+d\right)-4\left(x^2e+d\right)d\log\left(x^2e+d\right)-\left(x^2e+d\right)^2+4\left(x^2e+d\right)d\right)gpe^{(-1)}+2\left(\left(x^2e+d\right)^2-2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/8*((2*(x^2*e + d)^2*log(x^2*e + d) - 4*(x^2*e + d)*d*log(x^2*e + d) - (x^
2*e + d)^2 + 4*(x^2*e + d)*d)*g*p*e^(-1) + 2*((x^2*e + d)^2 - 2*(x^2*e + d)
*d)*g*e^(-1)*log(c) - 4*(x^2*e - (x^2*e + d)*log(x^2*e + d) + d)*f*p + 4*(x
^2*e + d)*f*log(c))*e^(-1)
```

maple [C] time = 0.48, size = 361, normalized size = 3.84

$$-\frac{i\pi g\,x^4\text{csgn}(ic)\,\text{csgn}\left(i\left(e\,x^2+d\right)^p\right)\text{csgn}\left(ic\left(e\,x^2+d\right)^p\right)}{8}+\frac{i\pi g\,x^4\text{csgn}(ic)\,\text{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2}{8}+\frac{i\pi g\,x^4\text{csgn}\left(i\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g*x^2+f)*ln(c*(e*x^2+d)^p),x)
```

```
[Out] (1/4*g*x^4+1/2*f*x^2)*ln((e*x^2+d)^p)+1/8*I*Pi*g*x^4*csgn(I*c*(e*x^2+d)^p)^
2*csgn(I*c)-1/4*I*Pi*f*x^2*csgn(I*c*(e*x^2+d)^p)^3-1/8*I*Pi*g*x^4*csgn(I*c*
(e*x^2+d)^p)^3+1/4*I*Pi*f*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1
/8*I*Pi*g*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/8*I*Pi*
g*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/4*I*Pi*f*x^2*csgn(I*c*(
e*x^2+d)^p)^2*csgn(I*c)-1/4*I*Pi*f*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+
d)^p)*csgn(I*c)+1/4*ln(c)*g*x^4-1/8*g*p*x^4+1/2*ln(c)*f*x^2+1/4*d/e*g*p*x^2
-1/2*f*p*x^2-1/4*d^2*g*p*ln(e*x^2+d)/e^2+1/2/e*ln(e*x^2+d)*d*f*p
```

maxima [A] time = 0.47, size = 99, normalized size = 1.05

$$-\frac{ep\left(\frac{eg^2x^4+2\left(2\,efg-dg^2\right)x^2}{e^2}+\frac{2\left(e^2f^2-2\,defg+d^2g^2\right)\log\left(ex^2+d\right)}{e^3}\right)}{8\,g}+\frac{\left(gx^2+f\right)^2\log\left(\left(ex^2+d\right)^pc\right)}{4\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -1/8*e*p*((e*g^2*x^4 + 2*(2*e*f*g - d*g^2)*x^2)/e^2 + 2*(e^2*f^2 - 2*d*e*f*
g + d^2*g^2)*log(e*x^2 + d)/e^3)/g + 1/4*(g*x^2 + f)^2*log((e*x^2 + d)^p*c)
/g
```

mupad [B] time = 0.31, size = 78, normalized size = 0.83

$$\ln\left(c\left(e\,x^2+d\right)^p\right)\left(\frac{g\,x^4}{4}+\frac{f\,x^2}{2}\right)-x^2\left(\frac{f\,p}{2}-\frac{d\,g\,p}{4\,e}\right)-\frac{g\,p\,x^4}{8}-\frac{\ln\left(e\,x^2+d\right)\left(d^2\,g\,p-2\,d\,e\,f\,p\right)}{4\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f*x^2)/2 + (g*x^4)/4) - x^2*((f*p)/2 - (d*g*p)/(4*e)
) - (g*p*x^4)/8 - (log(d + e*x^2)*(d^2*g*p - 2*d*e*f*p))/(4*e^2)
```

sympy [A] time = 44.85, size = 139, normalized size = 1.48

$$\begin{cases} -\frac{d^2gp\log\left(d+ex^2\right)}{4e^2}+\frac{dfp\log\left(d+ex^2\right)}{2e}+\frac{dgpx^2}{4e}+\frac{fpx^2\log\left(d+ex^2\right)}{2}-\frac{fpx^2}{2}+\frac{fx^2\log\left(c\right)}{2}+\frac{gpx^4\log\left(d+ex^2\right)}{4}-\frac{gpx^4}{8}+\frac{gx^4\log\left(c\right)}{4} & \\ \left(\frac{fx^2}{2}+\frac{gx^4}{4}\right)\log\left(cd^p\right) & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((-d**2*g*p*log(d + e*x**2)/(4*e**2) + d*f*p*log(d + e*x**2)/(2*e)
 + d*g*p*x**2/(4*e) + f*p*x**2*log(d + e*x**2)/2 - f*p*x**2/2 + f*x**2*log(
c)/2 + g*p*x**4*log(d + e*x**2)/4 - g*p*x**4/8 + g*x**4*log(c)/4, Ne(e, 0))
, ((f*x**2/2 + g*x**4/4)*log(c*d**p), True))
```

3.313 $\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x}\,dx$

Optimal. Leaf size=82

$$\frac{1}{2}f\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{g\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2e}+\frac{1}{2}fp\text{Li}_2\left(\frac{ex^2}{d}+1\right)-\frac{1}{2}gpx^2$$

```
[Out] -1/2*g*p*x^2+1/2*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e+1/2*f*ln(-e*x^2/d)*ln(c*(e
*x^2+d)^p)+1/2*f*p*polylog(2,1+e*x^2/d)
```

Rubi [A] time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2475, 43, 2416, 2389, 2295, 2394, 2315}

$$\frac{1}{2}fp\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)+\frac{1}{2}f\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{g\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2e}-\frac{1}{2}gpx^2$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x,x]
```

```
[Out] -(g*p*x^2)/2 + (g*(d + e*x^2)*Log[c*(d + e*x^2)^p])/(2*e) + (f*Log[-((e*x^2
)/d)]*Log[c*(d + e*x^2)^p])/2 + (f*p*PolyLog[2, 1 + (e*x^2)/d])/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)\log\left(c(d+ex)^p\right)}{x}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(g\log\left(c(d+ex)^p\right)+\frac{f\log\left(c(d+ex)^p\right)}{x}\right)dx,x,x^2\right)\\
&= \frac{1}{2}f\,\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x}\,dx,x,x^2\right)+\frac{1}{2}g\,\text{Subst}\left(\int \log\left(c(d+ex)^p\right)\right.\\
&= \frac{1}{2}f\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{g\,\text{Subst}\left(\int \log\left(cx^p\right)\,dx,x,d+ex^2\right)}{2e}-\\
&= -\frac{1}{2}gpx^2+\frac{g\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2e}+\frac{1}{2}f\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right.\right.
\end{aligned}$$

Mathematica [A] time = 0.02, size = 80, normalized size = 0.98

$$\frac{1}{2}f\left(\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+p\,\text{Li}_2\left(\frac{ex^2+d}{d}\right)\right)+\frac{1}{2}g\left(\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e}-px^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x,x]
```

```
[Out] (g*(-(p*x^2) + ((d + e*x^2)*Log[c*(d + e*x^2)^p])/e))/2 + (f*(Log[-((e*x^2)
/d)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, (d + e*x^2)/d]))/2
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(gx^2+f\right)\log\left(\left(ex^2+d\right)^p c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x,x, algorithm="fricas")
```

```
[Out] integral((g*x^2 + f)*log((e*x^2 + d)^p*c)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx^2+f\right)\log\left(\left(ex^2+d\right)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)/x, x)
```

maple [C] time = 0.32, size = 419, normalized size = 5.11

$$-\frac{i\pi g\,x^2\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)}{4}+\frac{i\pi g\,x^2\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2}{4}+\frac{i\pi g\,x^2\mathrm{csgn}\left(i\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x,x)
```

```
[Out] 1/2*ln((e*x^2+d)^p)*g*x^2+ln((e*x^2+d)^p)*f*ln(x)-1/2*g*p*x^2+1/2*p/e*g*d*l
n(e*x^2+d)-p*f*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p*f*ln(x)*ln((e*x
+(-d*e)^(1/2))/(-d*e)^(1/2))-p*f*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p*
f*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(
I*c*(e*x^2+d)^p)*csgn(I*c)*f*ln(x)+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*
c)*f*ln(x)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f*ln(x)+1/4*I*Pi*csgn(I*c*(e*x^
2+d)^p)^2*csgn(I*c)*g*x^2-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p
)*csgn(I*c)*g*x^2+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*g*x^
2+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f*ln(x)-1/4*I*Pi*csg
n(I*c*(e*x^2+d)^p)^3*g*x^2+1/2*ln(c)*g*x^2+ln(c)*f*ln(x)
```

maxima [A] time = 1.26, size = 91, normalized size = 1.11

$$\frac{1}{2}\left(\log\left(ex^2+d\right)\log\left(-\frac{ex^2+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{ex^2+d}{d}\right)\right)fp+f\log(c)\log(x)-\frac{\left(egp-eg\log(c)\right)x^2-\left(egpx^2+dgp\right)\mathrm{lo}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(log(e*x^2 + d)*log(-(e*x^2 + d)/d + 1) + dilog((e*x^2 + d)/d))*f*p + f
*log(c)*log(x) - 1/2*((e*g*p - e*g*log(c))*x^2 - (e*g*p*x^2 + d*g*p)*log(e*
x^2 + d))/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)\left(g\,x^2+f\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x,x)
```

```
[Out] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x,x)
```

```
[Out] Integral((f + g*x**2)*log(c*(d + e*x**2)**p)/x, x)
```

3.314 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx$

Optimal. Leaf size=93

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)-\frac{efp\log\left(d+ex^2\right)}{2d}+\frac{efp\log(x)}{d}+\frac{1}{2}gp\text{Li}_2\left(\frac{ex^2}{d}+1\right)$$

```
[Out] e*f*p*ln(x)/d-1/2*e*f*p*ln(e*x^2+d)/d-1/2*f*ln(c*(e*x^2+d)^p)/x^2+1/2*g*ln(
-e*x^2/d)*ln(c*(e*x^2+d)^p)+1/2*g*p*polylog(2,1+e*x^2/d)
```

Rubi [A] time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2475, 43, 2416, 2395, 36, 29, 31, 2394, 2315}

$$\frac{1}{2}gp\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)-\frac{efp\log\left(d+ex^2\right)}{2d}+\frac{efp\log}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^3,x]
```

```
[Out] (e*f*p*Log[x])/d - (e*f*p*Log[d + e*x^2])/(2*d) - (f*Log[c*(d + e*x^2)^p])/
(2*x^2) + (g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/2 + (g*p*PolyLog[2, 1
+ (e*x^2)/d])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)\log\left(c(d+ex)^p\right)}{x^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(\frac{f\log\left(c(d+ex)^p\right)}{x^2}+\frac{g\log\left(c(d+ex)^p\right)}{x}\right)dx,x,x^2\right)\\
&= \frac{1}{2}f\,\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x^2}\,dx,x,x^2\right)+\frac{1}{2}g\,\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x}\,dx,\right.\\
&= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}(efp)\,\text{Subst}\left(\right.\\
&= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}gp\text{Li}_2\left(1+\frac{ex^2}{d}\right.\\
&= \frac{efp\log(x)}{d}-\frac{efp\log\left(d+ex^2\right)}{2d}-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log
\end{aligned}$$

Mathematica [A] time = 0.03, size = 92, normalized size = 0.99

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2x^2}+\frac{1}{2}g\left(\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+p\text{Li}_2\left(\frac{ex^2+d}{d}\right)\right)-\frac{efp\log\left(d+ex^2\right)}{2d}+\frac{efp\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^3,x]
```

```
[Out] (e*f*p*Log[x])/d - (e*f*p*Log[d + e*x^2])/(2*d) - (f*Log[c*(d + e*x^2)^p])/
(2*x^2) + (g*(Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, (d + e*
x^2)/d]))/2
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(gx^2+f\right)\log\left(\left(ex^2+d\right)^pc\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^3,x, algorithm="fricas")
```

```
[Out] integral((g*x^2 + f)*log((e*x^2 + d)^p*c)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(gx^2+f\right)\log\left(\left(ex^2+d\right)^pc\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^3,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)/x^3, x)
```

maple [C] time = 0.26, size = 421, normalized size = 4.53

$$-\frac{i\pi g\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(ex^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(ex^2+d\right)^p\right)\ln(x)}{2}+\frac{i\pi g\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(ex^2+d\right)^p\right)^2\ln(x)}{2}+\frac{i\pi g\,\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^3,x)
```

```
[Out] ln((e*x^2+d)^p)*g*ln(x)-1/2*ln((e*x^2+d)^p)*f/x^2-1/2*e*f*p*ln(e*x^2+d)/d+e
*f*p*ln(x)/d-p*g*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p*g*ln(x)*ln((e
*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p*g*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-
p*g*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3
*f/x^2-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f/x^2+1/4*I*Pi*
csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f/x^2+1/2*I*Pi*csgn(I*c
*(e*x^2+d)^p)^2*csgn(I*c)*g*ln(x)-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c
)*f/x^2-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*g*ln(x
)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*g*ln(x)+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csg
n(I*c*(e*x^2+d)^p)^2*g*ln(x)+ln(c)*g*ln(x)-1/2*ln(c)*f/x^2
```

maxima [A] time = 1.27, size = 93, normalized size = 1.00

$$\frac{1}{2}\left(\log\left(ex^2+d\right)\log\left(-\frac{ex^2+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{ex^2+d}{d}\right)\right)gp+\frac{\left(efp+dg\log(c)\right)\log(x)}{d}-\frac{df\log(c)+\left(efpx^2+dfp\right.}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(log(e*x^2 + d)*log(-(e*x^2 + d)/d + 1) + dilog((e*x^2 + d)/d))*g*p + (
e*f*p + d*g*log(c))*log(x)/d - 1/2*(d*f*log(c) + (e*f*p*x^2 + d*f*p)*log(e*
x^2 + d))/(d*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(g\,x^2+f\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^3,x)
```

```
[Out] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**3,x)
```

```
[Out] Integral((f + g*x**2)*log(c*(d + e*x**2)**p)/x**3, x)
```

3.315 $\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^5}\,dx$

Optimal. Leaf size=93

$$-\frac{(f+gx^2)^2\log\left(c\left(d+ex^2\right)^p\right)}{4fx^4}+\frac{p(ef-dg)^2\log\left(d+ex^2\right)}{4d^2f}-\frac{ep\log(x)(ef-2dg)}{2d^2}-\frac{efp}{4dx^2}$$

```
[Out] -1/4*e*f*p/d/x^2-1/2*e*(-2*d*g+e*f)*p*ln(x)/d^2+1/4*(-d*g+e*f)^2*p*ln(e*x^2
+d)/d^2/f-1/4*(g*x^2+f)^2*ln(c*(e*x^2+d)^p)/f/x^4
```

Rubi [A] time = 0.14, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2475, 37, 2414, 12, 88}

$$-\frac{(f+gx^2)^2\log\left(c\left(d+ex^2\right)^p\right)}{4fx^4}+\frac{p(ef-dg)^2\log\left(d+ex^2\right)}{4d^2f}-\frac{ep\log(x)(ef-2dg)}{2d^2}-\frac{efp}{4dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^5,x]
```

```
[Out] -(e*f*p)/(4*d*x^2) - (e*(e*f - 2*d*g)*p*Log[x])/(2*d^2) + ((e*f - d*g)^2*p*
Log[d + e*x^2])/(4*d^2*f) - ((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/(4*f*x^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
```

```
|| IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)\log(c(d+ex)^p)}{x^3}\,dx,x,x^2\right)$$
$$= -\frac{(f+gx^2)^2\log\left(c\left(d+ex^2\right)^p\right)}{4fx^4} - \frac{1}{2}(ep)\,\text{Subst}\left(\int -\frac{(f+gx)^2}{2fx^2(d+ex)}\,dx,x,x^2\right)$$
$$= -\frac{(f+gx^2)^2\log\left(c\left(d+ex^2\right)^p\right)}{4fx^4} + \frac{(ep)\,\text{Subst}\left(\int \frac{(f+gx)^2}{x^2(d+ex)}\,dx,x,x^2\right)}{4f}$$
$$= -\frac{(f+gx^2)^2\log\left(c\left(d+ex^2\right)^p\right)}{4fx^4} + \frac{(ep)\,\text{Subst}\left(\int \left(\frac{f^2}{dx^2} + \frac{f(-ef+2dg)}{d^2x} + \frac{(-ef+dg)^2}{d^2(d+ex)}\right.\right.}{4f}$$
$$= -\frac{efp}{4dx^2} - \frac{e(ef-2dg)p\log(x)}{2d^2} + \frac{(ef-dg)^2p\log\left(d+ex^2\right)}{4d^2f} - \frac{(f+gx^2)^2\log}{4f}$$

Mathematica [A] time = 0.04, size = 105, normalized size = 1.13

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{4x^4} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2x^2} + \frac{1}{4}efp\left(\frac{e\log\left(d+ex^2\right)}{d^2} - \frac{2e\log(x)}{d^2} - \frac{1}{dx^2}\right) - \frac{egp\log\left(d+ex^2\right)}{2d} + \frac{egp\,\mathrm{lc}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^5,x]
```

```
[Out] (e*g*p*Log[x])/d - (e*g*p*Log[d + e*x^2])/(2*d) + (e*f*p*(-(1/(d*x^2)) - (2
*e*Log[x])/d^2 + (e*Log[d + e*x^2])/d^2))/4 - (f*Log[c*(d + e*x^2)^p])/(4*x
^4) - (g*Log[c*(d + e*x^2)^p])/(2*x^2)
```

fricas [A] time = 0.68, size = 97, normalized size = 1.04

$$-\frac{2\left(e^2f-2\,deg\right)px^4\log(x) + defpx^2 + \left(2\,d^2gpx^2 - \left(e^2f-2\,deg\right)px^4 + d^2fp\right)\log\left(ex^2+d\right) + \left(2\,d^2gx^2+d^2f\right)\log}{4\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(e^2*f - 2*d*e*g)*p*x^4*log(x) + d*e*f*p*x^2 + (2*d^2*g*p*x^2 - (e^
2*f - 2*d*e*g)*p*x^4 + d^2*f*p)*log(e*x^2 + d) + (2*d^2*g*x^2 + d^2*f)*log(
c))/(d^2*x^4)
```

giac [B] time = 0.20, size = 322, normalized size = 3.46

$$-\frac{\left(2\left(x^2e+d\right)^2dgpe^2\log\left(x^2e+d\right) - 2\left(x^2e+d\right)d^2gpe^2\log\left(x^2e+d\right) - 2\left(x^2e+d\right)^2dgpe^2\log\left(x^2e\right) + 4\left(x^2e+d\right)d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(2*(x^2*e + d)^2*d*g*p*e^2*log(x^2*e + d) - 2*(x^2*e + d)*d^2*g*p*e^2*
log(x^2*e + d) - 2*(x^2*e + d)^2*d*g*p*e^2*log(x^2*e) + 4*(x^2*e + d)*d^2*g
*p*e^2*log(x^2*e) - 2*d^3*g*p*e^2*log(x^2*e) - (x^2*e + d)^2*f*p*e^3*log(x^
2*e + d) + 2*(x^2*e + d)*d*f*p*e^3*log(x^2*e + d) + (x^2*e + d)^2*f*p*e^3*l
og(x^2*e) - 2*(x^2*e + d)*d*f*p*e^3*log(x^2*e) + d^2*f*p*e^3*log(x^2*e) + 2
*(x^2*e + d)*d^2*g*e^2*log(c) - 2*d^3*g*e^2*log(c) + (x^2*e + d)*d*f*p*e^3
- d^2*f*p*e^3 + d^2*f*e^3*log(c))*e^(-1)/((x^2*e + d)^2*d^2 - 2*(x^2*e + d)
*d^3 + d^4)
```

maple [C] time = 0.40, size = 392, normalized size = 4.22

$$-\frac{\left(2g\,x^2+f\right)\ln\left(\left(e\,x^2+d\right)^p\right)}{4x^4}-\frac{-8degp\,x^4\ln(x)+4degp\,x^4\ln\left(e\,x^2+d\right)+4e^2fp\,x^4\ln(x)-2e^2fp\,x^4\ln\left(e\,x^2+d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^5,x)
```

```
[Out] -1/4*(2*g*x^2+f)/x^4*ln((e*x^2+d)^p)-1/8*(2*I*Pi*d^2*g*x^2*csgn(I*(e*x^2+d)
^p)*csgn(I*c*(e*x^2+d)^p)^2-2*I*Pi*d^2*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(
e*x^2+d)^p)*csgn(I*c)-2*I*Pi*d^2*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+2*I*Pi*d^2*g
*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-8*ln(x)*d*e*g*p*x^4+4*ln(x)*e^2*f*p*
x^4+4*ln(e*x^2+d)*d*e*g*p*x^4-2*ln(e*x^2+d)*e^2*f*p*x^4+I*Pi*d^2*f*csgn(I*(
e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-I*Pi*d^2*f*csgn(I*(e*x^2+d)^p)*csgn(I*c
*(e*x^2+d)^p)*csgn(I*c)-I*Pi*d^2*f*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*d^2*f*csgn(
I*c*(e*x^2+d)^p)^2*csgn(I*c)+4*ln(c)*d^2*g*x^2+2*d*e*f*p*x^2+2*ln(c)*d^2*f)
/d^2/x^4
```

maxima [A] time = 0.47, size = 77, normalized size = 0.83

$$\frac{1}{4}\,ep\left(\frac{\left(ef-2\,dg\right)\log\left(ex^2+d\right)}{d^2}-\frac{\left(ef-2\,dg\right)\log\left(x^2\right)}{d^2}-\frac{f}{dx^2}\right)-\frac{\left(2\,gx^2+f\right)\log\left(\left(ex^2+d\right)^pc\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*e*p*((e*f - 2*d*g)*log(e*x^2 + d)/d^2 - (e*f - 2*d*g)*log(x^2)/d^2 - f/
(d*x^2)) - 1/4*(2*g*x^2 + f)*log((e*x^2 + d)^p*c)/x^4
```

mupad [B] time = 0.37, size = 85, normalized size = 0.91

$$\frac{\ln\left(e\,x^2+d\right)\,\left(e^2\,f\,p-2\,d\,e\,g\,p\right)}{4\,d^2}-\frac{\ln\left(c\,\left(e\,x^2+d\right)^p\right)\,\left(\frac{g\,x^2}{2}+\frac{f}{4}\right)}{x^4}-\frac{\ln(x)\,\left(e^2\,f\,p-2\,d\,e\,g\,p\right)}{2\,d^2}-\frac{e\,f\,p}{4\,d\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^5,x)
```

```
[Out] (log(d + e*x^2)*(e^2*f*p - 2*d*e*g*p))/(4*d^2) - (log(c*(d + e*x^2)^p)*(f/4
 + (g*x^2)/2))/x^4 - (log(x)*(e^2*f*p - 2*d*e*g*p))/(2*d^2) - (e*f*p)/(4*d*
x^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**5,x)
```

```
[Out] Timed out
```

3.316 $\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^7}\,dx$

Optimal. Leaf size=125

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{6x^6}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4x^4}-\frac{e^2p(2ef-3dg)\log\left(d+ex^2\right)}{12d^3}+\frac{e^2p\log(x)(2ef-3dg)}{6d^3}+\frac{ep(2ef-3dg)}{12d^2x^2}$$

```
[Out] -1/12*e*f*p/d/x^4+1/12*e*(-3*d*g+2*e*f)*p/d^2/x^2+1/6*e^2*(-3*d*g+2*e*f)*p*
ln(x)/d^3-1/12*e^2*(-3*d*g+2*e*f)*p*ln(e*x^2+d)/d^3-1/6*f*ln(c*(e*x^2+d)^p)
/x^6-1/4*g*ln(c*(e*x^2+d)^p)/x^4
```

Rubi [A] time = 0.16, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2475, 43, 2414, 12, 77}

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{6x^6}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4x^4}-\frac{e^2p(2ef-3dg)\log\left(d+ex^2\right)}{12d^3}+\frac{e^2p\log(x)(2ef-3dg)}{6d^3}+\frac{ep(2ef-3dg)}{12d^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^7,x]
```

```
[Out] -(e*f*p)/(12*d*x^4) + (e*(2*e*f - 3*d*g)*p)/(12*d^2*x^2) + (e^2*(2*e*f - 3*
d*g)*p*Log[x])/(6*d^3) - (e^2*(2*e*f - 3*d*g)*p*Log[d + e*x^2])/(12*d^3) -
(f*Log[c*(d + e*x^2)^p])/(6*x^6) - (g*Log[c*(d + e*x^2)^p])/(4*x^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
```

```
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)\log\left(c(d+ex)^p\right)}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4x^4} - \frac{1}{2}(ep)\text{Subst}\left(\int \frac{-2f-3g}{6x^3(d+e}\right.$$

$$= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4x^4} - \frac{1}{12}(ep)\text{Subst}\left(\int \frac{-2f-3}{x^3(d+e}\right.$$

$$= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4x^4} - \frac{1}{12}(ep)\text{Subst}\left(\int \left(-\frac{2f}{dx^3}+\right.\right.$$

$$= -\frac{efp}{12dx^4} + \frac{e(2ef-3dg)p}{12d^2x^2} + \frac{e^2(2ef-3dg)p\log(x)}{6d^3} - \frac{e^2(2ef-3dg)p\log(}{12d^3}$$

Mathematica [A] time = 0.07, size = 130, normalized size = 1.04

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4x^4} + \frac{1}{4}egp\left(\frac{e\log\left(d+ex^2\right)}{d^2} - \frac{2e\log(x)}{d^2} - \frac{1}{dx^2}\right) + \frac{1}{6}efp\left(-\frac{e^2\log\left(d+ex\right.}{d^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^7,x]
```

```
[Out] (e*g*p*(-(1/(d*x^2)) - (2*e*Log[x])/d^2 + (e*Log[d + e*x^2])/d^2))/4 + (e*f
*p*(-1/2*1/(d*x^4) + e/(d^2*x^2) + (2*e^2*Log[x])/d^3 - (e^2*Log[d + e*x^2]
)/d^3))/6 - (f*Log[c*(d + e*x^2)^p])/(6*x^6) - (g*Log[c*(d + e*x^2)^p])/(4*
x^4)
```

fricas [A] time = 0.69, size = 129, normalized size = 1.03

$$\frac{2\left(2e^3f-3de^2g\right)px^6\log(x) - d^2efpx^2 + \left(2de^2f-3d^2eg\right)px^4 - \left(\left(2e^3f-3de^2g\right)px^6 + 3d^3gpx^2 + 2d^3fp\right)\log}{12d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(2*(2*e^3*f - 3*d*e^2*g)*p*x^6*log(x) - d^2*e*f*p*x^2 + (2*d*e^2*f - 3
*d^2*e*g)*p*x^4 - ((2*e^3*f - 3*d*e^2*g)*p*x^6 + 3*d^3*g*p*x^2 + 2*d^3*f*p)
*log(e*x^2 + d) - (3*d^3*g*x^2 + 2*d^3*f)*log(c))/(d^3*x^6)
```

giac [B] time = 0.24, size = 515, normalized size = 4.12

$$\frac{\left(3\left(x^2e+d\right)^3dgpe^3\log\left(x^2e+d\right) - 9\left(x^2e+d\right)^2d^2gpe^3\log\left(x^2e+d\right) + 6\left(x^2e+d\right)d^3gpe^3\log\left(x^2e+d\right) - 3\left(x^2e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*(3*(x^2*e + d)^3*d*g*p*e^3*log(x^2*e + d) - 9*(x^2*e + d)^2*d^2*g*p*e^
3*log(x^2*e + d) + 6*(x^2*e + d)*d^3*g*p*e^3*log(x^2*e + d) - 3*(x^2*e + d)
^3*d*g*p*e^3*log(x^2*e) + 9*(x^2*e + d)^2*d^2*g*p*e^3*log(x^2*e) - 9*(x^2*e
 + d)*d^3*g*p*e^3*log(x^2*e) + 3*d^4*g*p*e^3*log(x^2*e) - 3*(x^2*e + d)^2*d
^2*g*p*e^3 + 6*(x^2*e + d)*d^3*g*p*e^3 - 3*d^4*g*p*e^3 - 2*(x^2*e + d)^3*f*
p*e^4*log(x^2*e + d) + 6*(x^2*e + d)^2*d*f*p*e^4*log(x^2*e + d) - 6*(x^2*e
+ d)*d^2*f*p*e^4*log(x^2*e + d) + 2*(x^2*e + d)^3*f*p*e^4*log(x^2*e) - 6*(x
^2*e + d)^2*d*f*p*e^4*log(x^2*e) + 6*(x^2*e + d)*d^2*f*p*e^4*log(x^2*e) - 2
*d^3*f*p*e^4*log(x^2*e) - 3*(x^2*e + d)*d^3*g*e^3*log(c) + 3*d^4*g*e^3*log(
c) + 2*(x^2*e + d)^2*d*f*p*e^4 - 5*(x^2*e + d)*d^2*f*p*e^4 + 3*d^3*f*p*e^4
- 2*d^3*f*e^4*log(c))*e^(-1)/((x^2*e + d)^3*d^3 - 3*(x^2*e + d)^2*d^4 + 3*(
x^2*e + d)*d^5 - d^6)
```

maple [C] time = 0.41, size = 428, normalized size = 3.42

$$-\frac{\left(3g\,x^{2}+2f\right)\ln\left(\left(e\,x^{2}+d\right)^{p}\right)}{12x^{6}}-\frac{12d\,e^{2}gp\,x^{6}\ln(x)-6d\,e^{2}gp\,x^{6}\ln\left(-e\,x^{2}-d\right)-8e^{3}fp\,x^{6}\ln(x)+4e^{3}fp\,x^{6}\ln\left(-e\,x^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^7,x)
```

```
[Out] -1/12*(3*g*x^2+2*f)/x^6*ln((e*x^2+d)^p)-1/24*(12*ln(x)*d*e^2*g*p*x^6-8*ln(x
)*e^3*f*p*x^6-6*ln(-e*x^2-d)*d*e^2*g*p*x^6+4*ln(-e*x^2-d)*e^3*f*p*x^6-2*I*P
i*d^3*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+3*I*Pi*d^3*g*x^
2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+2*I*Pi*d^3*f*csgn(I*c*(e*x^2+
d)^p)^2*csgn(I*c)-3*I*Pi*d^3*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p
)*csgn(I*c)+3*I*Pi*d^3*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-2*I*Pi*d^3*f
*csgn(I*c*(e*x^2+d)^p)^3-3*I*Pi*d^3*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+2*I*Pi*d^
3*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+6*d^2*e*g*p*x^4-4*d*e^2*f*p
*x^4+6*ln(c)*d^3*g*x^2+2*d^2*e*f*p*x^2+4*ln(c)*d^3*f)/d^3/x^6
```

maxima [A] time = 0.45, size = 104, normalized size = 0.83

$$-\frac{1}{12}\,ep\left(\frac{\left(2\,e^{2}f-3\,deg\right)\log\left(ex^{2}+d\right)}{d^{3}}-\frac{\left(2\,e^{2}f-3\,deg\right)\log\left(x^{2}\right)}{d^{3}}-\frac{\left(2\,ef-3\,dg\right)x^{2}-df}{d^{2}x^{4}}\right)-\frac{\left(3\,gx^{2}+2\,f\right)\log\left(\left(ex^{2}\right.\right.}{12\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*e*p*((2*e^2*f - 3*d*e*g)*log(e*x^2 + d)/d^3 - (2*e^2*f - 3*d*e*g)*log
(x^2)/d^3 - ((2*e*f - 3*d*g)*x^2 - d*f)/(d^2*x^4)) - 1/12*(3*g*x^2 + 2*f)*l
og((e*x^2 + d)^p*c)/x^6
```

mupad [B] time = 0.37, size = 113, normalized size = 0.90

$$\frac{\ln(x)\,\left(2\,e^{3}\,f\,p-3\,d\,e^{2}\,g\,p\right)}{6\,d^{3}}-\frac{\ln\left(c\,\left(e\,x^{2}+d\right)^{p}\right)\,\left(\frac{g\,x^{2}}{4}+\frac{f}{6}\right)}{x^{6}}-\frac{\ln\left(e\,x^{2}+d\right)\,\left(2\,e^{3}\,f\,p-3\,d\,e^{2}\,g\,p\right)}{12\,d^{3}}-\frac{\frac{e\,f\,p}{2\,d}+\frac{e\,p\,x^{2}\,\left(3\,d\,g-2\,e\,f\right.}{2\,d^{2}}}{6\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^7,x)
```

```
[Out] (log(x)*(2*e^3*f*p - 3*d*e^2*g*p))/(6*d^3) - (log(c*(d + e*x^2)^p)*(f/6 + (
g*x^2)/4))/x^6 - (log(d + e*x^2)*(2*e^3*f*p - 3*d*e^2*g*p))/(12*d^3) - ((e*
f*p)/(2*d) + (e*p*x^2*(3*d*g - 2*e*f))/(2*d^2))/(6*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**7,x)
```

```
[Out] Timed out
```

3.317 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9}\,dx$

Optimal. Leaf size=148

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{8x^8}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{6x^6}+\frac{e^3p(3ef-4dg)\log\left(d+ex^2\right)}{24d^4}-\frac{e^3p\log(x)(3ef-4dg)}{12d^4}-\frac{e^2p(3ef-4dg}{24d^3x^2}$$

```
[Out] -1/24*e*f*p/d/x^6+1/48*e*(-4*d*g+3*e*f)*p/d^2/x^4-1/24*e^2*(-4*d*g+3*e*f)*p
/d^3/x^2-1/12*e^3*(-4*d*g+3*e*f)*p*ln(x)/d^4+1/24*e^3*(-4*d*g+3*e*f)*p*ln(e
*x^2+d)/d^4-1/8*f*ln(c*(e*x^2+d)^p)/x^8-1/6*g*ln(c*(e*x^2+d)^p)/x^6
```

Rubi [A] time = 0.20, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2475, 43, 2414, 12, 77}

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{8x^8}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{6x^6}-\frac{e^2p(3ef-4dg)}{24d^3x^2}+\frac{e^3p(3ef-4dg)\log\left(d+ex^2\right)}{24d^4}-\frac{e^3p\log(x)(3ef-4dg}{12d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^9,x]
```

```
[Out] -(e*f*p)/(24*d*x^6) + (e*(3*e*f - 4*d*g)*p)/(48*d^2*x^4) - (e^2*(3*e*f - 4*
d*g)*p)/(24*d^3*x^2) - (e^3*(3*e*f - 4*d*g)*p*Log[x])/(12*d^4) + (e^3*(3*e*
f - 4*d*g)*p*Log[d + e*x^2])/(24*d^4) - (f*Log[c*(d + e*x^2)^p])/(8*x^8) -
(g*Log[c*(d + e*x^2)^p])/(6*x^6)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{x^9}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(f+gx)\log\left(c(d+ex)^p\right)}{x^5}\,dx, x, x^2\right)$$
$$= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{8x^8} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{1}{2}(ep)\,\text{Subst}\left(\int \frac{-3f-4}{12x^4(d+}\right.$$
$$= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{8x^8} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{1}{24}(ep)\,\text{Subst}\left(\int \frac{-3f-4}{x^4(d+e}\right.$$
$$= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{8x^8} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{1}{24}(ep)\,\text{Subst}\left(\int \left(-\frac{3f}{dx^4}+\right.\right.$$
$$= -\frac{efp}{24dx^6} + \frac{e(3ef-4dg)p}{48d^2x^4} - \frac{e^2(3ef-4dg)p}{24d^3x^2} - \frac{e^3(3ef-4dg)p\log(x)}{12d^4} + \frac{e^3}{}$$

Mathematica [A] time = 0.12, size = 158, normalized size = 1.07

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{8x^8}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{6x^6}+\frac{1}{6}egp\left(-\frac{e^2\log\left(d+ex^2\right)}{d^3}+\frac{2e^2\log(x)}{d^3}+\frac{e}{d^2x^2}-\frac{1}{2dx^4}\right)+\frac{1}{8}efp\left(\frac{e^3}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^9,x]
```

```
[Out] (e*g*p*(-1/2*1/(d*x^4) + e/(d^2*x^2) + (2*e^2*Log[x])/d^3 - (e^2*Log[d + e*
x^2])/d^3))/6 + (e*f*p*(-1/3*1/(d*x^6) + e/(2*d^2*x^4) - e^2/(d^3*x^2) - (2
*e^3*Log[x])/d^4 + (e^3*Log[d + e*x^2])/d^4))/8 - (f*Log[c*(d + e*x^2)^p])/
(8*x^8) - (g*Log[c*(d + e*x^2)^p])/(6*x^6)
```

fricas [A] time = 0.63, size = 155, normalized size = 1.05

$$-\frac{4\left(3e^4f-4de^3g\right)px^8\log(x)+2d^3efpx^2+2\left(3de^3f-4d^2e^2g\right)px^6-\left(3d^2e^2f-4d^3eg\right)px^4-2\left(\left(3e^4f-4de\right.\right.}{48d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^9,x, algorithm="fricas")
```

```
[Out] -1/48*(4*(3*e^4*f - 4*d*e^3*g)*p*x^8*log(x) + 2*d^3*e*f*p*x^2 + 2*(3*d*e^3*
f - 4*d^2*e^2*g)*p*x^6 - (3*d^2*e^2*f - 4*d^3*e*g)*p*x^4 - 2*((3*e^4*f - 4*
d*e^3*g)*p*x^8 - 4*d^4*g*p*x^2 - 3*d^4*f*p)*log(e*x^2 + d) + 2*(4*d^4*g*x^2
 + 3*d^4*f)*log(c))/(d^4*x^8)
```

giac [B] time = 0.20, size = 674, normalized size = 4.55

$$-\frac{\left(8\left(x^2e+d\right)^4dgpe^4\log\left(x^2e+d\right)-32\left(x^2e+d\right)^3d^2gpe^4\log\left(x^2e+d\right)+48\left(x^2e+d\right)^2d^3gpe^4\log\left(x^2e+d\right)-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^9,x, algorithm="giac")
```

```
[Out] -1/48*(8*(x^2*e + d)^4*d*g*p*e^4*log(x^2*e + d) - 32*(x^2*e + d)^3*d^2*g*p*
e^4*log(x^2*e + d) + 48*(x^2*e + d)^2*d^3*g*p*e^4*log(x^2*e + d) - 24*(x^2*
e + d)*d^4*g*p*e^4*log(x^2*e + d) - 8*(x^2*e + d)^4*d*g*p*e^4*log(x^2*e) +
32*(x^2*e + d)^3*d^2*g*p*e^4*log(x^2*e) - 48*(x^2*e + d)^2*d^3*g*p*e^4*log(
x^2*e) + 32*(x^2*e + d)*d^4*g*p*e^4*log(x^2*e) - 8*d^5*g*p*e^4*log(x^2*e) -
 8*(x^2*e + d)^3*d^2*g*p*e^4 + 28*(x^2*e + d)^2*d^3*g*p*e^4 - 32*(x^2*e + d
)*d^4*g*p*e^4 + 12*d^5*g*p*e^4 - 6*(x^2*e + d)^4*f*p*e^5*log(x^2*e + d) + 2
4*(x^2*e + d)^3*d*f*p*e^5*log(x^2*e + d) - 36*(x^2*e + d)^2*d^2*f*p*e^5*log
(x^2*e + d) + 24*(x^2*e + d)*d^3*f*p*e^5*log(x^2*e + d) + 6*(x^2*e + d)^4*f
*p*e^5*log(x^2*e) - 24*(x^2*e + d)^3*d*f*p*e^5*log(x^2*e) + 36*(x^2*e + d)^
2*d^2*f*p*e^5*log(x^2*e) - 24*(x^2*e + d)*d^3*f*p*e^5*log(x^2*e) + 6*d^4*f*
p*e^5*log(x^2*e) + 8*(x^2*e + d)*d^4*g*e^4*log(c) - 8*d^5*g*e^4*log(c) + 6*
(x^2*e + d)^3*d*f*p*e^5 - 21*(x^2*e + d)^2*d^2*f*p*e^5 + 26*(x^2*e + d)*d^3
*f*p*e^5 - 11*d^4*f*p*e^5 + 6*d^4*f*e^5*log(c))*e^(-1)/((x^2*e + d)^4*d^4 -
 4*(x^2*e + d)^3*d^5 + 6*(x^2*e + d)^2*d^6 - 4*(x^2*e + d)*d^7 + d^8)
```

maple [C] time = 0.51, size = 448, normalized size = 3.03

$$-\frac{\left(4g\,x^2+3f\right)\ln\left(\left(e\,x^2+d\right)^p\right)}{24x^8}-\frac{-16d\,e^3gp\,x^8\ln(x)+8d\,e^3gp\,x^8\ln\left(e\,x^2+d\right)+12e^4fp\,x^8\ln(x)-6e^4fp\,x^8\ln\left(e\,x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^9,x)
```

```
[Out] -1/24*(4*g*x^2+3*f)/x^8*ln((e*x^2+d)^p)-1/48*(-16*ln(x)*d*e^3*g*p*x^8+12*ln
(x)*e^4*f*p*x^8+8*ln(e*x^2+d)*d*e^3*g*p*x^8-6*ln(e*x^2+d)*e^4*f*p*x^8-3*I*P
i*d^4*f*csgn(I*c*(e*x^2+d)^p)^3+3*I*Pi*d^4*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I
*c)-4*I*Pi*d^4*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+3*I*Pi*d^4*f*csgn(I*(e*x^2+d)^
p)*csgn(I*c*(e*x^2+d)^p)^2-8*d^2*e^2*g*p*x^6+6*d*e^3*f*p*x^6+4*I*Pi*d^4*g*x
^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+4*I*Pi*d^4*g*x^2*csgn(I*c*(e
*x^2+d)^p)^2*csgn(I*c)-3*I*Pi*d^4*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^
p)*csgn(I*c)-4*I*Pi*d^4*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csg
n(I*c)+4*d^3*e*g*p*x^4-3*d^2*e^2*f*p*x^4+8*ln(c)*d^4*g*x^2+2*d^3*e*f*p*x^2+
6*ln(c)*d^4*f)/d^4/x^8
```

maxima [A] time = 0.45, size = 132, normalized size = 0.89

$$\frac{1}{48}ep\left(\frac{2\left(3e^3f-4de^2g\right)\log\left(ex^2+d\right)}{d^4}-\frac{2\left(3e^3f-4de^2g\right)\log\left(x^2\right)}{d^4}-\frac{2\left(3e^2f-4deg\right)x^4+2d^2f-\left(3def-4d^2\right.}{d^3x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^9,x, algorithm="maxima")
```

```
[Out] 1/48*e*p*(2*(3*e^3*f - 4*d*e^2*g)*log(e*x^2 + d)/d^4 - 2*(3*e^3*f - 4*d*e^2
*g)*log(x^2)/d^4 - (2*(3*e^2*f - 4*d*e*g)*x^4 + 2*d^2*f - (3*d*e*f - 4*d^2*
g)*x^2)/(d^3*x^6)) - 1/24*(4*g*x^2 + 3*f)*log((e*x^2 + d)^p*c)/x^8
```

mupad [B] time = 0.40, size = 134, normalized size = 0.91

$$\frac{\ln\left(e\,x^2+d\right)\,\left(3\,e^4\,f\,p-4\,d\,e^3\,g\,p\right)}{24\,d^4}-\frac{\ln\left(c\,\left(e\,x^2+d\right)^p\right)\,\left(\frac{g\,x^2}{6}+\frac{f}{8}\right)}{x^8}-\frac{\frac{e\,f\,p}{2\,d}+\frac{e\,p\,x^2\,\left(4\,d\,g-3\,e\,f\right)}{4\,d^2}-\frac{e^2\,p\,x^4\,\left(4\,d\,g-3\,e\,f\right)}{2\,d^3}}{12\,x^6}-\frac{\ln(x)\,\left(3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^9,x)
```

```
[Out] (log(d + e*x^2)*(3*e^4*f*p - 4*d*e^3*g*p))/(24*d^4) - (log(c*(d + e*x^2)^p)
*(f/8 + (g*x^2)/6))/x^8 - ((e*f*p)/(2*d) + (e*p*x^2*(4*d*g - 3*e*f))/(4*d^2
) - (e^2*p*x^4*(4*d*g - 3*e*f))/(2*d^3))/(12*x^6) - (log(x)*(3*e^4*f*p - 4*
d*e^3*g*p))/(12*d^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**9,x)
```

```
[Out] Timed out
```

3.318 $\int x^2 \left(f+gx^2\right) \log \left(c \left(d+ex^2\right)^p\right) \, dx$

Optimal. Leaf size=154

$$\frac{1}{3} f x^3 \log \left(c \left(d+e x^2\right)^p\right)+\frac{1}{5} g x^5 \log \left(c \left(d+e x^2\right)^p\right)-\frac{2 d^{3/2} f p \tan ^{-1}\left(\frac{\sqrt{e} x}{\sqrt{d}}\right)}{3 e^{3/2}}+\frac{2 d^{5/2} g p \tan ^{-1}\left(\frac{\sqrt{e} x}{\sqrt{d}}\right)}{5 e^{5/2}}-\frac{2 d^2 g p x}{5 e^2}+\frac{2 d f p x}{3 e}+$$

```
[Out] 2/3*d*f*p*x/e-2/5*d^2*g*p*x/e^2-2/9*f*p*x^3+2/15*d*g*p*x^3/e-2/25*g*p*x^5-2
/3*d^(3/2)*f*p*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)+2/5*d^(5/2)*g*p*arctan(x*e
^(1/2)/d^(1/2))/e^(5/2)+1/3*f*x^3*ln(c*(e*x^2+d)^p)+1/5*g*x^5*ln(c*(e*x^2+d
)^p)
```

Rubi [A] time = 0.13, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2476, 2455, 302, 205}

$$\frac{1}{3} f x^3 \log \left(c \left(d+e x^2\right)^p\right)+\frac{1}{5} g x^5 \log \left(c \left(d+e x^2\right)^p\right)-\frac{2 d^{3/2} f p \tan ^{-1}\left(\frac{\sqrt{e} x}{\sqrt{d}}\right)}{3 e^{3/2}}-\frac{2 d^2 g p x}{5 e^2}+\frac{2 d^{5/2} g p \tan ^{-1}\left(\frac{\sqrt{e} x}{\sqrt{d}}\right)}{5 e^{5/2}}+\frac{2 d f p x}{3 e}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (2*d*f*p*x)/(3*e) - (2*d^2*g*p*x)/(5*e^2) - (2*f*p*x^3)/9 + (2*d*g*p*x^3)/(
15*e) - (2*g*p*x^5)/25 - (2*d^(3/2)*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(
3/2)) + (2*d^(5/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*e^(5/2)) + (f*x^3*Lo
g[c*(d + e*x^2)^p])/3 + (g*x^5*Log[c*(d + e*x^2)^p])/5
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int x^2 \left(f+gx^2\right) \log \left(c \left(d+ex^2\right)^p\right) dx = \int \left(fx^2 \log \left(c \left(d+ex^2\right)^p\right) + gx^4 \log \left(c \left(d+ex^2\right)^p\right)\right) dx$$
$$= f \int x^2 \log \left(c \left(d+ex^2\right)^p\right) dx + g \int x^4 \log \left(c \left(d+ex^2\right)^p\right) dx$$
$$= \frac{1}{3} fx^3 \log \left(c \left(d+ex^2\right)^p\right) + \frac{1}{5} gx^5 \log \left(c \left(d+ex^2\right)^p\right) - \frac{1}{3}(2efp) \int \frac{x^4}{d+ex}$$
$$= \frac{1}{3} fx^3 \log \left(c \left(d+ex^2\right)^p\right) + \frac{1}{5} gx^5 \log \left(c \left(d+ex^2\right)^p\right) - \frac{1}{3}(2efp) \int \left(-\frac{d}{e^2}\right.$$
$$= \frac{2dfpx}{3e} - \frac{2d^2gpx}{5e^2} - \frac{2}{9} fpx^3 + \frac{2dgpx^3}{15e} - \frac{2}{25} gpx^5 + \frac{1}{3} fx^3 \log \left(c \left(d+ex^2\right.\right.$$
$$= \frac{2dfpx}{3e} - \frac{2d^2gpx}{5e^2} - \frac{2}{9} fpx^3 + \frac{2dgpx^3}{15e} - \frac{2}{25} gpx^5 - \frac{2d^{3/2} fp \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}$$

Mathematica [A] time = 0.06, size = 118, normalized size = 0.77

$$\frac{\sqrt{e}x \left(15e^2x^2 \left(5f+3gx^2\right) \log \left(c \left(d+ex^2\right)^p\right) - 2p \left(45d^2g - 15de \left(5f+gx^2\right) + e^2x^2 \left(25f+9gx^2\right)\right)\right) + 30d^{3/2}p(3}{225e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (30*d^(3/2)*(-5*e*f + 3*d*g)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[e]*x*(-2*
p*(45*d^2*g - 15*d*e*(5*f + g*x^2) + e^2*x^2*(25*f + 9*g*x^2)) + 15*e^2*x^2
*(5*f + 3*g*x^2)*Log[c*(d + e*x^2)^p]))/(225*e^(5/2))
```

fricas [A] time = 0.78, size = 300, normalized size = 1.95

$$\left[-\frac{18\,e^2gpx^5 + 10\left(5\,e^2f - 3\,deg\right)px^3 + 15\left(5\,def - 3\,d^2g\right)p\sqrt{-\frac{d}{e}}\,\log\left(\frac{ex^2+2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right) - 30\left(5\,def - 3\,d^2g\right)px - 1}{225\,e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/225*(18*e^2*g*p*x^5 + 10*(5*e^2*f - 3*d*e*g)*p*x^3 + 15*(5*d*e*f - 3*d^
2*g)*p*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) - 30*(5*d
*e*f - 3*d^2*g)*p*x - 15*(3*e^2*g*p*x^5 + 5*e^2*f*p*x^3)*log(e*x^2 + d) - 1
5*(3*e^2*g*x^5 + 5*e^2*f*x^3)*log(c))/e^2, -1/225*(18*e^2*g*p*x^5 + 10*(5*e
^2*f - 3*d*e*g)*p*x^3 + 30*(5*d*e*f - 3*d^2*g)*p*sqrt(d/e)*arctan(e*x*sqrt(
d/e)/d) - 30*(5*d*e*f - 3*d^2*g)*p*x - 15*(3*e^2*g*p*x^5 + 5*e^2*f*p*x^3)*l
og(e*x^2 + d) - 15*(3*e^2*g*x^5 + 5*e^2*f*x^3)*log(c))/e^2]
```

giac [A] time = 0.18, size = 138, normalized size = 0.90

$$\frac{2\left(3\,d^3gp - 5\,d^2fpe\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{15\,\sqrt{d}} + \frac{1}{225}\left(45\,gpx^5e^2\log\left(x^2e+d\right) - 18\,gpx^5e^2 + 45\,gx^5e^2\log(c) + 30\,dgpx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 2/15*(3*d^3*g*p - 5*d^2*f*p*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d) +
 1/225*(45*g*p*x^5*e^2*log(x^2*e + d) - 18*g*p*x^5*e^2 + 45*g*x^5*e^2*log(c
) + 30*d*g*p*x^3*e + 75*f*p*x^3*e^2*log(x^2*e + d) - 50*f*p*x^3*e^2 + 75*f*
x^3*e^2*log(c) - 90*d^2*g*p*x + 150*d*f*p*x*e)*e^(-2)
```

maple [C] time = 0.55, size = 453, normalized size = 2.94

$$-\frac{i\pi g\,x^5\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)}{10}+\frac{i\pi g\,x^5\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2}{10}+\frac{i\pi g\,x^5\mathrm{csgn}\left(i\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g*x^2+f)*ln(c*(e*x^2+d)^p),x)
```

```
[Out] (1/5*g*x^5+1/3*f*x^3)*ln((e*x^2+d)^p)-1/6*I*Pi*f*x^3*csgn(I*(e*x^2+d)^p)*cs
gn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/6*I*Pi*f*x^3*csgn(I*c*(e*x^2+d)^p)^2*csgn(I
*c)+1/6*I*Pi*f*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/10*I*Pi*g*
x^5*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/6*I*Pi*f*x^3*csgn(I*c*(e*x^2+d)^p)^
3+1/10*I*Pi*g*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/10*I*Pi*g*x
^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/10*I*Pi*g*x^5*csgn
(I*c*(e*x^2+d)^p)^3+1/5*ln(c)*g*x^5-2/25*g*p*x^5+1/3*ln(c)*f*x^3+2/15*d*g*p
*x^3/e-2/9*f*p*x^3+1/5/e^3*(-d*e)^(1/2)*p*d^2*ln(d-(-d*e)^(1/2)*x)*g-1/3/e^
2*(-d*e)^(1/2)*p*d*ln(d-(-d*e)^(1/2)*x)*f-1/5/e^3*(-d*e)^(1/2)*p*d^2*ln(d+(
-d*e)^(1/2)*x)*g+1/3/e^2*(-d*e)^(1/2)*p*d*ln(d+(-d*e)^(1/2)*x)*f-2/5*d^2*g*
p*x/e^2+2/3*d*f*p*x/e
```

maxima [A] time = 0.99, size = 112, normalized size = 0.73

$$-\frac{2}{225}\,ep\left(\frac{15\left(5\,d^2ef-3\,d^3g\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^3}+\frac{9\,e^2gx^5+5\left(5\,e^2f-3\,deg\right)x^3-15\left(5\,def-3\,d^2g\right)x}{e^3}\right)+\frac{1}{15}\left(3\,gx^5+5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -2/225*e*p*(15*(5*d^2*e*f - 3*d^3*g)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^3)
+ (9*e^2*g*x^5 + 5*(5*e^2*f - 3*d*e*g)*x^3 - 15*(5*d*e*f - 3*d^2*g)*x)/e^3)
 + 1/15*(3*g*x^5 + 5*f*x^3)*log((e*x^2 + d)^p*c)
```

mupad [B] time = 0.32, size = 126, normalized size = 0.82

$$\ln\left(c\left(e\,x^2+d\right)^p\right)\left(\frac{g\,x^5}{5}+\frac{f\,x^3}{3}\right)-x^3\left(\frac{2\,f\,p}{9}-\frac{2\,d\,g\,p}{15\,e}\right)-\frac{2\,g\,p\,x^5}{25}+\frac{d\,x\left(\frac{2\,f\,p}{3}-\frac{2\,d\,g\,p}{5\,e}\right)}{e}+\frac{2\,d^{3/2}\,p\,\mathrm{atan}\left(\frac{d^{3/2}\,\sqrt{e}\,p\,x\left(3\,d\,g-5\right.}{3\,d^3\,g\,p-5\,d^2\,e\,f\,p}\right.}{15\,e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f*x^3)/3 + (g*x^5)/5) - x^3*((2*f*p)/9 - (2*d*g*p)/(
15*e)) - (2*g*p*x^5)/25 + (d*x*((2*f*p)/3 - (2*d*g*p)/(5*e)))/e + (2*d^(3/2
)*p*atan((d^(3/2)*e^(1/2)*p*x*(3*d*g - 5*e*f))/(3*d^3*g*p - 5*d^2*e*f*p))*(
3*d*g - 5*e*f))/(15*e^(5/2))
```

sympy [A] time = 90.17, size = 277, normalized size = 1.80

$$\begin{cases}\frac{id^{\frac{5}{2}}gp\log\left(d+ex^2\right)}{5e^3\sqrt{\frac{1}{e}}}-\frac{2id^{\frac{5}{2}}gp\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{5e^3\sqrt{\frac{1}{e}}}-\frac{id^{\frac{3}{2}}fp\log\left(d+ex^2\right)}{3e^2\sqrt{\frac{1}{e}}}+\frac{2id^{\frac{3}{2}}fp\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{3e^2\sqrt{\frac{1}{e}}}-\frac{2d^2gpx}{5e^2}+\frac{2dfpx}{3e}+\frac{2dgpx^3}{15e}+\frac{fpx^3\log}{} \\ \left(\frac{fx^3}{3}+\frac{gx^5}{5}\right)\log\left(cd^p\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((I*d**(5/2)*g*p*log(d + e*x**2)/(5*e**3*sqrt(1/e)) - 2*I*d**(5/2)
*g*p*log(-I*sqrt(d)*sqrt(1/e) + x)/(5*e**3*sqrt(1/e)) - I*d**(3/2)*f*p*log(
d + e*x**2)/(3*e**2*sqrt(1/e)) + 2*I*d**(3/2)*f*p*log(-I*sqrt(d)*sqrt(1/e)
+ x)/(3*e**2*sqrt(1/e)) - 2*d**2*g*p*x/(5*e**2) + 2*d*f*p*x/(3*e) + 2*d*g*p
*x**3/(15*e) + f*p*x**3*log(d + e*x**2)/3 - 2*f*p*x**3/9 + f*x**3*log(c)/3
+ g*p*x**5*log(d + e*x**2)/5 - 2*g*p*x**5/25 + g*x**5*log(c)/5, Ne(e, 0)),
((f*x**3/3 + g*x**5/5)*log(c*d**p), True))
```

3.319 $\int \left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=117

$$fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{2dgpx}{3e}-2fpx-\frac{2}{9}gpx^3$$

```
[Out] -2*f*p*x+2/3*d*g*p*x/e-2/9*g*p*x^3-2/3*d^(3/2)*g*p*arctan(x*e^(1/2)/d^(1/2)
)/e^(3/2)+f*x*ln(c*(e*x^2+d)^p)+1/3*g*x^3*ln(c*(e*x^2+d)^p)+2*f*p*arctan(x*
e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2471, 2448, 321, 205, 2455, 302}

$$fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{2dgpx}{3e}-2fpx-\frac{2}{9}gpx^3$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^
(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int \left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)dx = \int \left(f\log\left(c\left(d+ex^2\right)^p\right)+gx^2\log\left(c\left(d+ex^2\right)^p\right)\right)dx$$
$$= f\int \log\left(c\left(d+ex^2\right)^p\right)dx+g\int x^2\log\left(c\left(d+ex^2\right)^p\right)dx$$
$$= fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)-(2efp)\int \frac{x^2}{d+ex^2}dx-$$
$$= -2fpx+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)+(2dfp)\int \frac{}{d+}$$
$$= -2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+fx\log\left(c\left(d+ex^2\right)^p\right)$$
$$= -2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}$$

Mathematica [A] time = 0.04, size = 117, normalized size = 1.00

$$fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{2dgpx}{3e}-2fpx-\frac{2}{9}g$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^
(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

fricas [A] time = 0.85, size = 220, normalized size = 1.88

$$\left[-\frac{2\,egpx^3+3\left(3\,ef-dg\right)p\sqrt{-\frac{d}{e}}\,\log\left(\frac{ex^2-2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)+6\left(3\,ef-dg\right)px-3\left(egpx^3+3\,efpx\right)\log\left(ex^2+d\right)-3\,(}{9\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/9*(2*e*g*p*x^3 + 3*(3*e*f - d*g)*p*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-
d/e) - d)/(e*x^2 + d)) + 6*(3*e*f - d*g)*p*x - 3*(e*g*p*x^3 + 3*e*f*p*x)*lo
g(e*x^2 + d) - 3*(e*g*x^3 + 3*e*f*x)*log(c))/e, -1/9*(2*e*g*p*x^3 - 6*(3*e*
f - d*g)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 6*(3*e*f - d*g)*p*x - 3*(e*g
*p*x^3 + 3*e*f*p*x)*log(e*x^2 + d) - 3*(e*g*x^3 + 3*e*f*x)*log(c))/e]
```

giac [A] time = 0.17, size = 109, normalized size = 0.93

$$-\frac{2\left(d^{2}gp-3\,dfpe\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{3\,\sqrt{d}}+\frac{1}{9}\left(3\,gpx^{3}e\log\left(x^{2}e+d\right)-2\,gpx^{3}e+3\,gx^{3}e\log(c)+9\,fpxe\log\left(x^{2}e+d\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -2/3*(d^2*g*p - 3*d*f*p*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/sqrt(d) + 1/9
*(3*g*p*x^3*e*log(x^2*e + d) - 2*g*p*x^3*e + 3*g*x^3*e*log(c) + 9*f*p*x*e*l
og(x^2*e + d) + 6*d*g*p*x - 18*f*p*x*e + 9*f*x*e*log(c))*e^(-1)
```

maple [C] time = 0.09, size = 416, normalized size = 3.56

$$-\frac{i\pi g\,x^{3}\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\mathrm{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)}{6}+\frac{i\pi g\,x^{3}\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)^{2}}{6}+\frac{i\pi g\,x^{3}\mathrm{csgn}\left(i\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p),x)
```

```
[Out] (1/3*g*x^3+f*x)*ln((e*x^2+d)^p)+1/6*I*Pi*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c
*(e*x^2+d)^p)^2+1/6*I*Pi*g*x^3*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/2*I*Pi*f
*csgn(I*c*(e*x^2+d)^p)^3*x-1/2*I*Pi*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d
)^p)*csgn(I*c)*x-1/6*I*Pi*g*x^3*csgn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*f*csgn(I*(
e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*x-1/6*I*Pi*g*x^3*csgn(I*(e*x^2+d)^p)*cs
gn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/2*I*Pi*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*
x+1/3*g*x^3*ln(c)-2/9*g*p*x^3+f*x*ln(c)+1/3/e^2*(-d*e)^(1/2)*p*ln(-d-(-d*e)
^(1/2)*x)*d*g-1/e*(-d*e)^(1/2)*p*ln(-d-(-d*e)^(1/2)*x)*f-1/3/e^2*(-d*e)^(1/
2)*p*ln(-d+(-d*e)^(1/2)*x)*d*g+1/e*(-d*e)^(1/2)*p*ln(-d+(-d*e)^(1/2)*x)*f+2
/3*d/e*g*p*x-2*f*p*x
```

maxima [A] time = 0.99, size = 85, normalized size = 0.73

$$\frac{2}{9}\,ep\left(\frac{3\left(3\,def-d^{2}g\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^{2}}-\frac{egx^{3}+3\left(3\,ef-dg\right)x}{e^{2}}\right)+\frac{1}{3}\left(gx^{3}+3\,fx\right)\log\left(\left(ex^{2}+d\right)^{p}c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] 2/9*e*p*(3*(3*d*e*f - d^2*g)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^2) - (e*g*x
^3 + 3*(3*e*f - d*g)*x)/e^2) + 1/3*(g*x^3 + 3*f*x)*log((e*x^2 + d)^p*c)
```

mupad [B] time = 0.00, size = 97, normalized size = 0.83

$$\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)\left(\frac{g\,x^{3}}{3}+f\,x\right)-x\left(2\,f\,p-\frac{2\,d\,g\,p}{3\,e}\right)-\frac{2\,g\,p\,x^{3}}{9}-\frac{2\,\sqrt{d}\,p\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,p\,x\left(d\,g-3\,e\,f\right)}{d^{2}\,g\,p-3\,d\,e\,f\,p}\right)\left(d\,g-3\,e\,f\right)}{3\,e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

```
[Out] log(c*(d + e*x^2)^p)*(f*x + (g*x^3)/3) - x*(2*f*p - (2*d*g*p)/(3*e)) - (2*g
*p*x^3)/9 - (2*d^(1/2)*p*atan((d^(1/2)*e^(1/2)*p*x*(d*g - 3*e*f))/(d^2*g*p
- 3*d*e*f*p))*(d*g - 3*e*f))/(3*e^(3/2))
```

sympy [A] time = 24.13, size = 228, normalized size = 1.95

$$\begin{cases} -\frac{id^{\frac{3}{2}}gp\log\left(d+ex^2\right)}{3e^2\sqrt{\frac{1}{e}}}+\frac{2id^{\frac{3}{2}}gp\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{3e^2\sqrt{\frac{1}{e}}}+\frac{i\sqrt{d}fp\log\left(d+ex^2\right)}{e\sqrt{\frac{1}{e}}}-\frac{2i\sqrt{d}fp\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{e\sqrt{\frac{1}{e}}}+\frac{2dgpx}{3e}+fpx\log\left(d+ex^2\right) \\ \left(fx+\frac{gx^3}{3}\right)\log\left(cd^p\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((-I*d**(3/2)*g*p*log(d + e*x**2)/(3*e**2*sqrt(1/e)) + 2*I*d**(3/2
)*g*p*log(-I*sqrt(d)*sqrt(1/e) + x)/(3*e**2*sqrt(1/e)) + I*sqrt(d)*f*p*log(
d + e*x**2)/(e*sqrt(1/e)) - 2*I*sqrt(d)*f*p*log(-I*sqrt(d)*sqrt(1/e) + x)/(
e*sqrt(1/e)) + 2*d*g*p*x/(3*e) + f*p*x*log(d + e*x**2) - 2*f*p*x + f*x*log(
c) + g*p*x**3*log(d + e*x**2)/3 - 2*g*p*x**3/9 + g*x**3*log(c)/3, Ne(e, 0))
, ((f*x + g*x**3/3)*log(c*d**p), True))
```

3.320 $\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=72

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x}+gx\log\left(c\left(d+ex^2\right)^p\right)+\frac{2p(dg+ef)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}-2gpx$$

```
[Out] -2*g*p*x-f*ln(c*(e*x^2+d)^p)/x+g*x*ln(c*(e*x^2+d)^p)+2*(d*g+e*f)*p*arctan(x
*e^(1/2)/d^(1/2))/d^(1/2)/e^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2476, 2448, 321, 205, 2455}

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x}+gx\log\left(c\left(d+ex^2\right)^p\right)+\frac{2\sqrt{e}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{d}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-2gpx$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^2,x]
```

```
[Out] -2*g*p*x + (2*Sqrt[e]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[d]
*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (f*Log[c*(d + e*x^2)^p])/x + g*
x*Log[c*(d + e*x^2)^p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
```

```
& IntegerQ[s]
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\,dx &= \int \left(g\log\left(c\left(d+ex^2\right)^p\right)+\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\right)dx\\
&= f\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\,dx+g\int \log\left(c\left(d+ex^2\right)^p\right)dx\\
&= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x}+gx\log\left(c\left(d+ex^2\right)^p\right)+(2efp)\int \frac{1}{d+ex^2}\,dx-(2\\
&= -2gpx+\frac{2\sqrt{e}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x}+gx\log\left(c\left(d+ex^2\right.\right.\\
&= -2gpx+\frac{2\sqrt{e}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{d}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{f\log\left(c\left(d+ex^2\right)\right.}{x}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.86

$$\left(gx-\frac{f}{x}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{2p(dg+ef)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}}-2gpx$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^2,x]
```

```
[Out] -2*g*p*x + (2*(e*f + d*g)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e])
+ (-(f/x) + g*x)*Log[c*(d + e*x^2)^p]
```

fricas [A] time = 0.71, size = 199, normalized size = 2.76

$$\left[-\frac{2\,degpx^2+\sqrt{-de}\left(ef+dg\right)px\log\left(\frac{ex^2-2\,\sqrt{-de}\,x-d}{ex^2+d}\right)-\left(degpx^2-defp\right)\log\left(ex^2+d\right)-\left(degx^2-def\right)\log(c)}{dex},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^2,x, algorithm="fricas")
```

```
[Out] [-(2*d*e*g*p*x^2 + sqrt(-d*e)*(e*f + d*g)*p*x*log((e*x^2 - 2*sqrt(-d*e)*x -
 d)/(e*x^2 + d)) - (d*e*g*p*x^2 - d*e*f*p)*log(e*x^2 + d) - (d*e*g*x^2 - d*
e*f)*log(c))/(d*e*x), -(2*d*e*g*p*x^2 - 2*sqrt(d*e)*(e*f + d*g)*p*x*arctan(
sqrt(d*e)*x/d) - (d*e*g*p*x^2 - d*e*f*p)*log(e*x^2 + d) - (d*e*g*x^2 - d*e*
f)*log(c))/(d*e*x)]
```

giac [A] time = 0.19, size = 78, normalized size = 1.08

$$\frac{2\left(dgp+fpe\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{\sqrt{d}}+\frac{gpx^2\log\left(x^2e+d\right)-2\,gpx^2+gx^2\log(c)-fp\log\left(x^2e+d\right)-f\log(c)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^2,x, algorithm="giac")

[Out] 2*(d*g*p + f*p*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/sqrt(d) + (g*p*x^2*log(x^2*e + d) - 2*g*p*x^2 + g*x^2*log(c) - f*p*log(x^2*e + d) - f*log(c))/x

maple [C] time = 0.77, size = 403, normalized size = 5.60

$$-\frac{\left(-g\,x^{2}+f\right)\ln\left(\left(e\,x^{2}+d\right)^{p}\right)}{x}+\frac{-i\pi g\,x^{2}\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\mathrm{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)+i\pi g\,x^{2}\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^2,x)

[Out] -(-g*x^2+f)/x*ln((e*x^2+d)^p)+1/2*(I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*g*x^2-I*Pi*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-I*Pi*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-I*Pi*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+I*Pi*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+I*Pi*f*csgn(I*c*(e*x^2+d)^p)^3-I*Pi*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2*g*x^2*ln(c)-4*g*p*x^2-2*ln(c)*f+2*sum(_R*ln((2*d^2*g^2*p^2+4*d*e*f*g*p^2+2*e^2*f^2*p^2+3*_R^2*d*e)*x+(-d^2*g*p-d*e*f*p)*_R),_R=RootOf(d^2*g^2*p^2+2*d*e*f*g*p^2+e^2*f^2*p^2+_Z^2*d*e))*x)/x

maxima [A] time = 0.99, size = 61, normalized size = 0.85

$$-2\,ep\left(\frac{gx}{e}-\frac{\left(ef+dg\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e}\right)+\left(gx-\frac{f}{x}\right)\log\left(\left(ex^{2}+d\right)^{p}c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^2,x, algorithm="maxima")

[Out] -2*e*p*(g*x/e - (e*f + d*g)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e)) + (g*x - f/x)*log((e*x^2 + d)^p*c)

mupad [B] time = 0.33, size = 83, normalized size = 1.15

$$\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)\left(2\,g\,x-\frac{g\,x^{2}+f}{x}\right)-2\,g\,p\,x+\frac{2\,p\,\mathrm{atan}\left(\frac{2\,\sqrt{e}\,p\,x\,\left(d\,g+e\,f\right)}{\sqrt{d}\,\left(2\,d\,g\,p+2\,e\,f\,p\right)}\right)\,\left(d\,g+e\,f\right)}{\sqrt{d}\,\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^2,x)

[Out] log(c*(d + e*x^2)^p)*(2*g*x - (f + g*x^2)/x) - 2*g*p*x + (2*p*atan((2*e^(1/2)*p*x*(d*g + e*f))/(d^(1/2)*(2*d*g*p + 2*e*f*p)))*(d*g + e*f))/(d^(1/2)*e^(1/2))

sympy [A] time = 46.00, size = 262, normalized size = 3.64

$$\begin{cases}\left(-\frac{f}{x}+gx\right)\log\left(0^{p}c\right)\\ -\frac{fp\log\left(e\right)}{x}-\frac{2fp\log\left(x\right)}{x}-\frac{2fp}{x}-\frac{f\log\left(c\right)}{x}+gpx\log\left(e\right)+2gpx\log\left(x\right)-2gpx+gx\log\left(c\right)\\ \left(-\frac{f}{x}+gx\right)\log\left(cd^{p}\right)\\ \frac{i\sqrt{d}gp\log\left(d+ex^{2}\right)}{e\sqrt{\frac{1}{e}}}-\frac{2i\sqrt{d}gp\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{e\sqrt{\frac{1}{e}}}-\frac{fp\log\left(d+ex^{2}\right)}{x}-\frac{f\log\left(c\right)}{x}+gpx\log\left(d+ex^{2}\right)-2gpx+gx\log\left(c\right)+\frac{ifp\log\left(\right.}{\sqrt{d}\,\sqrt{}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**2,x)
```

```
[Out] Piecewise(((-f/x + g*x)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), (-f*p*log(e)/x -
 2*f*p*log(x)/x - 2*f*p/x - f*log(c)/x + g*p*x*log(e) + 2*g*p*x*log(x) - 2*
g*p*x + g*x*log(c), Eq(d, 0)), ((-f/x + g*x)*log(c*d**p), Eq(e, 0)), (I*sqr
t(d)*g*p*log(d + e*x**2)/(e*sqrt(1/e)) - 2*I*sqrt(d)*g*p*log(-I*sqrt(d)*sqr
t(1/e) + x)/(e*sqrt(1/e)) - f*p*log(d + e*x**2)/x - f*log(c)/x + g*p*x*log(
d + e*x**2) - 2*g*p*x + g*x*log(c) + I*f*p*log(d + e*x**2)/(sqrt(d)*sqrt(1/
e)) - 2*I*f*p*log(-I*sqrt(d)*sqrt(1/e) + x)/(sqrt(d)*sqrt(1/e)), True))
```

3.321 $\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=108

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{x}-\frac{2e^{3/2}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{2efp}{3dx}+\frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}$$

```
[Out] -2/3*e*f*p/d/x-2/3*e^(3/2)*f*p*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)-1/3*f*ln(c
*(e*x^2+d)^p)/x^3-g*ln(c*(e*x^2+d)^p)/x+2*g*p*arctan(x*e^(1/2)/d^(1/2))*e^(
1/2)/d^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2476, 2455, 325, 205}

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{x}-\frac{2e^{3/2}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{2efp}{3dx}+\frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^4,x]
```

```
[Out] (-2*e*f*p)/(3*d*x) - (2*e^(3/2)*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)
) + (2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (f*Log[c*(d + e*x
^2)^p])/(3*x^3) - (g*Log[c*(d + e*x^2)^p])/x
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^4}\,dx = \int \left(\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x^4} + \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\right) dx$$

$$= f\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^4}\,dx + g\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\,dx$$

$$= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{x} + \frac{1}{3}(2efp)\int \frac{1}{x^2\left(d+ex^2\right)}\,d$$

$$= -\frac{2efp}{3dx} + \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} - \frac{g\log\left(c\left(d+ex^2\right)\right.}{x}$$

$$= -\frac{2efp}{3dx} - \frac{2e^{3/2}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} + \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{f\log\left(c\left(d+ex^2\right)\right.}{3x^3}$$

Mathematica [C] time = 0.04, size = 96, normalized size = 0.89

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{x} - \frac{2efp\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{ex^2}{d}\right)}{3dx} + \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^4,x]
```

```
[Out] (2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (2*e*f*p*Hypergeometr
ic2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f*Log[c*(d + e*x^2)^p])/(3*x^
3) - (g*Log[c*(d + e*x^2)^p])/x
```

fricas [A] time = 0.80, size = 191, normalized size = 1.77

$$\left[-\frac{(ef-3dg)px^3\sqrt{-\frac{e}{d}}\log\left(\frac{ex^2+2dx\sqrt{-\frac{e}{d}}-d}{ex^2+d}\right) + 2efpx^2 + \left(3dgpx^2+dfp\right)\log\left(ex^2+d\right) + \left(3dgx^2+df\right)\log(c)}{3dx^3},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/3*((e*f - 3*d*g)*p*x^3*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(e
*x^2 + d)) + 2*e*f*p*x^2 + (3*d*g*p*x^2 + d*f*p)*log(e*x^2 + d) + (3*d*g*x^
2 + d*f)*log(c))/(d*x^3), -1/3*(2*(e*f - 3*d*g)*p*x^3*sqrt(e/d)*arctan(x*sq
rt(e/d)) + 2*e*f*p*x^2 + (3*d*g*p*x^2 + d*f*p)*log(e*x^2 + d) + (3*d*g*x^2
+ d*f)*log(c))/(d*x^3)]
```

giac [A] time = 0.18, size = 92, normalized size = 0.85

$$\frac{2\left(3dgpe-fpe^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{3d^{\frac{3}{2}}} - \frac{3dgpx^2\log\left(x^2e+d\right) + 2fpx^2e + 3dgx^2\log(c) + dfp\log\left(x^2e+d\right) + d}{3dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(3*d*g*p*e - f*p*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(3/2) - 1/3*
(3*d*g*p*x^2*log(x^2*e + d) + 2*f*p*x^2*e + 3*d*g*x^2*log(c) + d*f*p*log(x^
2*e + d) + d*f*log(c))/(d*x^3)
```

maple [C] time = 0.66, size = 430, normalized size = 3.98

$$-\frac{\left(3g\,x^2+f\right)\ln\left(\left(e\,x^2+d\right)^p\right)}{3x^3}+\frac{3i\pi dg\,x^2\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)-3i\pi dg\,x^2\mathrm{csgn}\left(ic\right)\mathrm{csg}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^4,x)
```

```
[Out] -1/3*(3*g*x^2+f)/x^3*ln((e*x^2+d)^p)+1/6*(-3*I*Pi*d*g*x^2*csgn(I*(e*x^2+d)^
p)*csgn(I*c*(e*x^2+d)^p)^2+3*I*Pi*d*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x
^2+d)^p)*csgn(I*c)+3*I*Pi*d*g*x^2*csgn(I*c*(e*x^2+d)^p)^3-3*I*Pi*d*g*x^2*cs
gn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-I*Pi*d*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^
2+d)^p)^2+I*Pi*d*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+I*Pi
*d*f*csgn(I*c*(e*x^2+d)^p)^3-I*Pi*d*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-6*l
n(c)*d*g*x^2+2*sum(_R*ln((18*d^2*e*g^2*p^2-12*d*e^2*f*g*p^2+2*e^3*f^2*p^2+3
*_R^2*d^3)*x+(-3*d^3*g*p+d^2*e*f*p)*_R),_R=RootOf(9*d^2*e*g^2*p^2-6*d*e^2*f
*g*p^2+e^3*f^2*p^2+_Z^2*d^3))*d*x^3-4*e*f*p*x^2-2*ln(c)*d*f)/d/x^3
```

maxima [A] time = 0.99, size = 65, normalized size = 0.60

$$-\frac{2}{3}\,ep\left(\frac{\left(ef-3\,dg\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,d}+\frac{f}{dx}\right)-\frac{\left(3\,gx^2+f\right)\log\left(\left(ex^2+d\right)^pc\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -2/3*e*p*((e*f - 3*d*g)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d) + f/(d*x)) - 1/
3*(3*g*x^2 + f)*log((e*x^2 + d)^p*c)/x^3
```

mupad [B] time = 0.37, size = 65, normalized size = 0.60

$$\frac{2\,\sqrt{e}\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,d\,g-e\,f\right)}{3\,d^{3/2}}-\frac{2\,e\,f\,p}{3\,d\,x}-\frac{\ln\left(c\,\left(e\,x^2+d\right)^p\right)\,\left(g\,x^2+\frac{f}{3}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^4,x)
```

```
[Out] (2*e^(1/2)*p*atan((e^(1/2)*x)/d^(1/2))*(3*d*g - e*f))/(3*d^(3/2)) - (2*e*f*
p)/(3*d*x) - (log(c*(d + e*x^2)^p)*(f/3 + g*x^2))/x^3
```

sympy [A] time = 105.07, size = 1454, normalized size = 13.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**4,x)
```

```
[Out] Piecewise(((-f/(3*x**3) - g/x)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((-f/(3*x
**3) - g/x)*log(c*d**p), Eq(e, 0)), (-f*p*log(e)/(3*x**3) - 2*f*p*log(x)/(3
*x**3) - 2*f*p/(9*x**3) - f*log(c)/(3*x**3) - g*p*log(e)/x - 2*g*p*log(x)/x
```

```
 - 2*g*p/x - g*log(c)/x, Eq(d, 0)), (-I*d**(5/2)*f*p*sqrt(1/e)*log(d + e*x*
*2)/(3*I*d**(5/2)*x**3*sqrt(1/e) + 3*I*d**(3/2)*e*x**5*sqrt(1/e)) - I*d**(5
/2)*f*sqrt(1/e)*log(c)/(3*I*d**(5/2)*x**3*sqrt(1/e) + 3*I*d**(3/2)*e*x**5*s
qrt(1/e)) - 3*I*d**(5/2)*g*p*x**2*sqrt(1/e)*log(d + e*x**2)/(3*I*d**(5/2)*x
**3*sqrt(1/e) + 3*I*d**(3/2)*e*x**5*sqrt(1/e)) - 3*I*d**(5/2)*g*x**2*sqrt(1
/e)*log(c)/(3*I*d**(5/2)*x**3*sqrt(1/e) + 3*I*d**(3/2)*e*x**5*sqrt(1/e)) -
I*d**(3/2)*f*p*x**2*sqrt(1/e)*log(d + e*x**2)/(3*I*d**(5/2)*x**3*sqrt(1/e)/
e + 3*I*d**(3/2)*x**5*sqrt(1/e)) - 2*I*d**(3/2)*f*p*x**2*sqrt(1/e)/(3*I*d**
(5/2)*x**3*sqrt(1/e)/e + 3*I*d**(3/2)*x**5*sqrt(1/e)) - I*d**(3/2)*f*x**2*s
qrt(1/e)*log(c)/(3*I*d**(5/2)*x**3*sqrt(1/e)/e + 3*I*d**(3/2)*x**5*sqrt(1/e
)) - 3*I*d**(3/2)*g*p*x**4*sqrt(1/e)*log(d + e*x**2)/(3*I*d**(5/2)*x**3*sqr
t(1/e)/e + 3*I*d**(3/2)*x**5*sqrt(1/e)) - 3*I*d**(3/2)*g*x**4*sqrt(1/e)*log
(c)/(3*I*d**(5/2)*x**3*sqrt(1/e)/e + 3*I*d**(3/2)*x**5*sqrt(1/e)) - 2*I*sqr
t(d)*e*f*p*x**4*sqrt(1/e)/(3*I*d**(5/2)*x**3*sqrt(1/e)/e + 3*I*d**(3/2)*x**
5*sqrt(1/e)) - 3*d**2*g*p*x**3*log(d + e*x**2)/(3*I*d**(5/2)*x**3*sqrt(1/e)
 + 3*I*d**(3/2)*e*x**5*sqrt(1/e)) + 6*d**2*g*p*x**3*log(-I*sqrt(d)*sqrt(1/e
) + x)/(3*I*d**(5/2)*x**3*sqrt(1/e) + 3*I*d**(3/2)*e*x**5*sqrt(1/e)) - 3*d*
*2*g*x**3*log(c)/(3*I*d**(5/2)*x**3*sqrt(1/e) + 3*I*d**(3/2)*e*x**5*sqrt(1/
e)) + d*f*p*x**3*log(d + e*x**2)/(3*I*d**(5/2)*x**3*sqrt(1/e)/e + 3*I*d**(3
/2)*x**5*sqrt(1/e)) - 2*d*f*p*x**3*log(-I*sqrt(d)*sqrt(1/e) + x)/(3*I*d**(5
/2)*x**3*sqrt(1/e)/e + 3*I*d**(3/2)*x**5*sqrt(1/e)) + d*f*x**3*log(c)/(3*I*
d**(5/2)*x**3*sqrt(1/e)/e + 3*I*d**(3/2)*x**5*sqrt(1/e)) - 3*d*g*p*x**5*log
(d + e*x**2)/(3*I*d**(5/2)*x**3*sqrt(1/e)/e + 3*I*d**(3/2)*x**5*sqrt(1/e))
+ 6*d*g*p*x**5*log(-I*sqrt(d)*sqrt(1/e) + x)/(3*I*d**(5/2)*x**3*sqrt(1/e)/e
 + 3*I*d**(3/2)*x**5*sqrt(1/e)) - 3*d*g*x**5*log(c)/(3*I*d**(5/2)*x**3*sqrt
(1/e)/e + 3*I*d**(3/2)*x**5*sqrt(1/e)) + e*f*p*x**5*log(d + e*x**2)/(3*I*d*
*(5/2)*x**3*sqrt(1/e)/e + 3*I*d**(3/2)*x**5*sqrt(1/e)) - 2*e*f*p*x**5*log(-
I*sqrt(d)*sqrt(1/e) + x)/(3*I*d**(5/2)*x**3*sqrt(1/e)/e + 3*I*d**(3/2)*x**5
*sqrt(1/e)) + e*f*x**5*log(c)/(3*I*d**(5/2)*x**3*sqrt(1/e)/e + 3*I*d**(3/2)
*x**5*sqrt(1/e)), True))
```

3.322 $\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^6}\,dx$

Optimal. Leaf size=140

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{5x^5}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{3x^3}+\frac{2e^{5/2}fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{2e^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}}+\frac{2e^2fp}{5d^2x}-\frac{2efp}{15dx^3}-\frac{2egp}{3dx}$$

```
[Out] -2/15*e*f*p/d/x^3+2/5*e^2*f*p/d^2/x-2/3*e*g*p/d/x+2/5*e^(5/2)*f*p*arctan(x*
e^(1/2)/d^(1/2))/d^(5/2)-2/3*e^(3/2)*g*p*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)-
1/5*f*ln(c*(e*x^2+d)^p)/x^5-1/3*g*ln(c*(e*x^2+d)^p)/x^3
```

Rubi [A] time = 0.12, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2476, 2455, 325, 205}

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{5x^5}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{3x^3}+\frac{2e^2fp}{5d^2x}+\frac{2e^{5/2}fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{2e^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{2efp}{15dx^3}-\frac{2egp}{3dx}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^6,x]
```

```
[Out] (-2*e*f*p)/(15*d*x^3) + (2*e^2*f*p)/(5*d^2*x) - (2*e*g*p)/(3*d*x) + (2*e^(5
/2)*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*d^(5/2)) - (2*e^(3/2)*g*p*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(3*d^(3/2)) - (f*Log[c*(d + e*x^2)^p])/(5*x^5) - (g*Log
[c*(d + e*x^2)^p])/(3*x^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}{x^6}\,dx = \int \left(\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x^6} + \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{x^4}\right)dx$$

$$= f\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^6}\,dx + g\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^4}\,dx$$

$$= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{5x^5} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} + \frac{1}{5}(2efp)\int \frac{1}{x^4\left(d+ex^2\right)}\,d$$

$$= -\frac{2efp}{15dx^3} - \frac{2egp}{3dx} - \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{5x^5} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} - \frac{\left(2e^2fp\right)}{}$$

$$= -\frac{2efp}{15dx^3} + \frac{2e^2fp}{5d^2x} - \frac{2egp}{3dx} - \frac{2e^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} - \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{5x^5} -$$

$$= -\frac{2efp}{15dx^3} + \frac{2e^2fp}{5d^2x} - \frac{2egp}{3dx} + \frac{2e^{5/2}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}} - \frac{2e^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} -$$

Mathematica [C] time = 0.01, size = 101, normalized size = 0.72

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{5x^5} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} - \frac{2efp\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{ex^2}{d}\right)}{15dx^3} - \frac{2egp\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{ex^2}{d}\right)}{3dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^6,x]
```

```
[Out] (-2*e*f*p*Hypergeometric2F1[-3/2, 1, -1/2, -((e*x^2)/d)])/(15*d*x^3) - (2*e
*g*p*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f*Log[c*(d +
 e*x^2)^p])/(5*x^5) - (g*Log[c*(d + e*x^2)^p])/(3*x^3)
```

fricas [A] time = 0.86, size = 259, normalized size = 1.85

$$\left[-\frac{\left(3e^2f-5deg\right)px^5\sqrt{-\frac{e}{d}}\log\left(\frac{ex^2-2dx\sqrt{-\frac{e}{d}}-d}{ex^2+d}\right) + 2defpx^2 - 2\left(3e^2f-5deg\right)px^4 + \left(5d^2gpx^2+3d^2fp\right)\log\left(e\right.}{15d^2x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^6,x, algorithm="fricas")
```

```
[Out] [-1/15*((3*e^2*f - 5*d*e*g)*p*x^5*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-e/d)
- d)/(e*x^2 + d)) + 2*d*e*f*p*x^2 - 2*(3*e^2*f - 5*d*e*g)*p*x^4 + (5*d^2*g*
p*x^2 + 3*d^2*f*p)*log(e*x^2 + d) + (5*d^2*g*x^2 + 3*d^2*f)*log(c))/(d^2*x^
5), 1/15*(2*(3*e^2*f - 5*d*e*g)*p*x^5*sqrt(e/d)*arctan(x*sqrt(e/d)) - 2*d*e
*f*p*x^2 + 2*(3*e^2*f - 5*d*e*g)*p*x^4 - (5*d^2*g*p*x^2 + 3*d^2*f*p)*log(e*
x^2 + d) - (5*d^2*g*x^2 + 3*d^2*f)*log(c))/(d^2*x^5)]
```

giac [A] time = 0.21, size = 122, normalized size = 0.87

$$-\frac{2\left(5dgpe^2-3fpe^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{15d^{\frac{5}{2}}} - \frac{10dgpx^4e - 6fpx^4e^2 + 5d^2gpx^2\log\left(x^2e+d\right) + 2dfpx^2e + 5d^2gx^2}{15d^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^6,x, algorithm="giac")
```

```
[Out] -2/15*(5*d*g*p*e^2 - 3*f*p*e^3)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(5/2)
- 1/15*(10*d*g*p*x^4*e - 6*f*p*x^4*e^2 + 5*d^2*g*p*x^2*log(x^2*e + d) + 2*d
*f*p*x^2*e + 5*d^2*g*x^2*log(c) + 3*d^2*f*p*log(x^2*e + d) + 3*d^2*f*log(c)
)/(d^2*x^5)
```

maple [C] time = 0.62, size = 483, normalized size = 3.45

$$-\frac{\left(5g\,x^{2}+3f\right)\ln\left(\left(e\,x^{2}+d\right)^{p}\right)}{15x^{5}}+\frac{5i\pi\,d^{2}g\,x^{2}\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\mathrm{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)-5i\pi\,d^{2}g\,x^{2}\mathrm{csgn}\left(ic\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^6,x)
```

```
[Out] -1/15*(5*g*x^2+3*f)/x^5*ln((e*x^2+d)^p)+1/30*(-5*I*Pi*d^2*g*x^2*csgn(I*(e*x
^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+5*I*Pi*d^2*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(
I*c*(e*x^2+d)^p)*csgn(I*c)+5*I*Pi*d^2*g*x^2*csgn(I*c*(e*x^2+d)^p)^3-5*I*Pi*
d^2*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-3*I*Pi*f*csgn(I*(e*x^2+d)^p)*cs
gn(I*c*(e*x^2+d)^p)^2*d^2+3*I*Pi*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p
)*csgn(I*c)*d^2+3*I*Pi*f*csgn(I*c*(e*x^2+d)^p)^3*d^2-3*I*Pi*f*csgn(I*c*(e*x
^2+d)^p)^2*csgn(I*c)*d^2+2*sum(_R*ln((50*d^2*e^3*g^2*p^2-60*d*e^4*f*g*p^2+1
8*e^5*f^2*p^2+3*_R^2*d^5)*x+(5*d^4*e*g*p-3*d^3*e^2*f*p)*_R),_R=RootOf(25*d^
2*e^3*g^2*p^2-30*d*e^4*f*g*p^2+9*e^5*f^2*p^2+_Z^2*d^5))*d^2*x^5-20*d*e*g*p*
x^4+12*e^2*f*p*x^4-10*d^2*g*x^2*ln(c)-4*d*e*f*p*x^2-6*d^2*f*ln(c))/d^2/x^5
```

maxima [A] time = 1.01, size = 88, normalized size = 0.63

$$\frac{2}{15}\,ep\left(\frac{\left(3\,e^{2}f-5\,deg\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,d^{2}}+\frac{\left(3\,ef-5\,dg\right)x^{2}-df}{d^{2}x^{3}}\right)-\frac{\left(5\,gx^{2}+3\,f\right)\log\left(\left(ex^{2}+d\right)^{p}c\right)}{15\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^6,x, algorithm="maxima")
```

```
[Out] 2/15*e*p*((3*e^2*f - 5*d*e*g)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2) + ((3*e
*f - 5*d*g)*x^2 - d*f)/(d^2*x^3)) - 1/15*(5*g*x^2 + 3*f)*log((e*x^2 + d)^p*
c)/x^5
```

mupad [B] time = 0.38, size = 88, normalized size = 0.63

$$-\frac{\frac{2\,e\,f\,p}{d}+\frac{2\,e\,p\,x^{2}\,\left(5\,d\,g-3\,e\,f\right)}{d^{2}}}{15\,x^{3}}-\frac{\ln\left(c\,\left(e\,x^{2}+d\right)^{p}\right)\,\left(\frac{g\,x^{2}}{3}+\frac{f}{5}\right)}{x^{5}}-\frac{2\,e^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(5\,d\,g-3\,e\,f\right)}{15\,d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^6,x)
```

```
[Out] - ((2*e*f*p)/d + (2*e*p*x^2*(5*d*g - 3*e*f))/d^2)/(15*x^3) - (log(c*(d + e*
x^2)^p)*(f/5 + (g*x^2)/3))/x^5 - (2*e^(3/2)*p*atan((e^(1/2)*x)/d^(1/2))*(5*
d*g - 3*e*f))/(15*d^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**6,x)
```

```
[Out] Timed out
```

3.323 $\int x^5 \left(f+gx^2\right)^2 \log \left(c \left(d+ex^2\right)^p\right) \, dx$

Optimal. Leaf size=251

$$\frac{1}{6}f^2x^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}fgx^8\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^2x^{10}\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2px^2(ef-dg)^2}{2e^4}-\frac{p\left(d+ex^2\right)}{}$$

```
[Out] -1/2*d^2*(-d*g+e*f)^2*p*x^2/e^4+1/4*d*(-2*d*g+e*f)*(-d*g+e*f)*p*(e*x^2+d)^2
/e^5-1/18*(6*d^2*g^2-6*d*e*f*g+e^2*f^2)*p*(e*x^2+d)^3/e^5-1/16*g*(-2*d*g+e*
f)*p*(e*x^2+d)^4/e^5-1/50*g^2*p*(e*x^2+d)^5/e^5+1/60*d^3*(6*d^2*g^2-15*d*e*
f*g+10*e^2*f^2)*p*ln(e*x^2+d)/e^5+1/6*f^2*x^6*ln(c*(e*x^2+d)^p)+1/4*f*g*x^8
*ln(c*(e*x^2+d)^p)+1/10*g^2*x^10*ln(c*(e*x^2+d)^p)
```

Rubi [A] time = 0.47, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2475, 43, 2414, 12, 893}

$$\frac{1}{6}f^2x^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}fgx^8\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^2x^{10}\log\left(c\left(d+ex^2\right)^p\right)-\frac{p\left(d+ex^2\right)^3\left(6d^2g^2-6defg\right.}{18e^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -(d^2*(e*f - d*g)^2*p*x^2)/(2*e^4) + (d*(e*f - 2*d*g)*(e*f - d*g)*p*(d + e*
x^2)^2)/(4*e^5) - ((e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)*p*(d + e*x^2)^3)/(18*e
^5) - (g*(e*f - 2*d*g)*p*(d + e*x^2)^4)/(16*e^5) - (g^2*p*(d + e*x^2)^5)/(5
0*e^5) + (d^3*(10*e^2*f^2 - 15*d*e*f*g + 6*d^2*g^2)*p*Log[d + e*x^2])/(60*e
^5) + (f^2*x^6*Log[c*(d + e*x^2)^p])/6 + (f*g*x^8*Log[c*(d + e*x^2)^p])/4 +
 (g^2*x^10*Log[c*(d + e*x^2)^p])/10
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
```

```
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int x^5 \left(f+gx^2\right)^2 \log \left(c \left(d+ex^2\right)^p\right) dx = \frac{1}{2} \text{Subst}\left(\int x^2 (f+gx)^2 \log \left(c(d+ex)^p\right) dx, x, x^2\right)$$

$$= \frac{1}{6} f^2 x^6 \log \left(c \left(d+ex^2\right)^p\right) + \frac{1}{4} fgx^8 \log \left(c \left(d+ex^2\right)^p\right) + \frac{1}{10} g^2 x^{10} \log \left(c \left(d\right.\right.$$

$$= \frac{1}{6} f^2 x^6 \log \left(c \left(d+ex^2\right)^p\right) + \frac{1}{4} fgx^8 \log \left(c \left(d+ex^2\right)^p\right) + \frac{1}{10} g^2 x^{10} \log \left(c \left(d\right.\right.$$

$$= \frac{1}{6} f^2 x^6 \log \left(c \left(d+ex^2\right)^p\right) + \frac{1}{4} fgx^8 \log \left(c \left(d+ex^2\right)^p\right) + \frac{1}{10} g^2 x^{10} \log \left(c \left(d\right.\right.$$

$$= -\frac{d^2 (ef-dg)^2 px^2}{2e^4} + \frac{d(ef-2dg)(ef-dg)p \left(d+ex^2\right)^2}{4e^5} - \frac{\left(e^2 f^2 - 6defg\right.}{}$$

Mathematica [A] time = 0.19, size = 205, normalized size = 0.82

$$\frac{60e^5x^6 \left(10f^2+15fgx^2+6g^2x^4\right) \log \left(c \left(d+ex^2\right)^p\right) + 60d^3p \left(6d^2g^2-15defg+10e^2f^2\right) \log \left(d+ex^2\right) - epx^2 \left(360\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-(e*p*x^2*(360*d^4*g^2 - 180*d^3*e*g*(5*f + g*x^2) - 30*d*e^3*x^2*(10*f^2
+ 10*f*g*x^2 + 3*g^2*x^4) + 30*d^2*e^2*(20*f^2 + 15*f*g*x^2 + 4*g^2*x^4) +
e^4*x^4*(200*f^2 + 225*f*g*x^2 + 72*g^2*x^4))) + 60*d^3*(10*e^2*f^2 - 15*d*
e*f*g + 6*d^2*g^2)*p*Log[d + e*x^2] + 60*e^5*x^6*(10*f^2 + 15*f*g*x^2 + 6*g
^2*x^4)*Log[c*(d + e*x^2)^p])/(3600*e^5)
```

fricas [A] time = 0.57, size = 262, normalized size = 1.04

$$-\frac{72\,e^5g^2px^{10} + 45\left(5\,e^5fg - 2\,de^4g^2\right)px^8 + 20\left(10\,e^5f^2 - 15\,de^4fg + 6\,d^2e^3g^2\right)px^6 - 30\left(10\,de^4f^2 - 15\,d^2e^3fg + 6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/3600*(72*e^5*g^2*p*x^10 + 45*(5*e^5*f*g - 2*d*e^4*g^2)*p*x^8 + 20*(10*e^
5*f^2 - 15*d*e^4*f*g + 6*d^2*e^3*g^2)*p*x^6 - 30*(10*d*e^4*f^2 - 15*d^2*e^3
*f*g + 6*d^3*e^2*g^2)*p*x^4 + 60*(10*d^2*e^3*f^2 - 15*d^3*e^2*f*g + 6*d^4*e
*g^2)*p*x^2 - 60*(6*e^5*g^2*p*x^10 + 15*e^5*f*g*p*x^8 + 10*e^5*f^2*p*x^6 +
(10*d^3*e^2*f^2 - 15*d^4*e*f*g + 6*d^5*g^2)*p)*log(e*x^2 + d) - 60*(6*e^5*g
^2*x^10 + 15*e^5*f*g*x^8 + 10*e^5*f^2*x^6)*log(c))/e^5
```

giac [B] time = 0.22, size = 534, normalized size = 2.13

$$\frac{1}{3600}\left(360\,g^2x^{10}e\log(c)+900\,fgx^8e\log(c)+600\,f^2x^6e\log(c)+100\left(6\left(x^2e+d\right)^3e^{(-2)}\log\left(x^2e+d\right)-18\left(x^2e\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/3600*(360*g^2*x^10*e*log(c) + 900*f*g*x^8*e*log(c) + 600*f^2*x^6*e*log(c)
 + 100*(6*(x^2*e + d)^3*e^(-2)*log(x^2*e + d) - 18*(x^2*e + d)^2*d*e^(-2)*l
og(x^2*e + d) + 18*(x^2*e + d)*d^2*e^(-2)*log(x^2*e + d) - 2*(x^2*e + d)^3*
e^(-2) + 9*(x^2*e + d)^2*d*e^(-2) - 18*(x^2*e + d)*d^2*e^(-2))*f^2*p + 75*(
12*(x^2*e + d)^4*e^(-3)*log(x^2*e + d) - 48*(x^2*e + d)^3*d*e^(-3)*log(x^2*
e + d) + 72*(x^2*e + d)^2*d^2*e^(-3)*log(x^2*e + d) - 48*(x^2*e + d)*d^3*e^
(-3)*log(x^2*e + d) - 3*(x^2*e + d)^4*e^(-3) + 16*(x^2*e + d)^3*d*e^(-3) -
36*(x^2*e + d)^2*d^2*e^(-3) + 48*(x^2*e + d)*d^3*e^(-3))*f*g*p + 6*(60*(x^2
*e + d)^5*e^(-4)*log(x^2*e + d) - 300*(x^2*e + d)^4*d*e^(-4)*log(x^2*e + d)
 + 600*(x^2*e + d)^3*d^2*e^(-4)*log(x^2*e + d) - 600*(x^2*e + d)^2*d^3*e^(-
4)*log(x^2*e + d) + 300*(x^2*e + d)*d^4*e^(-4)*log(x^2*e + d) - 12*(x^2*e +
 d)^5*e^(-4) + 75*(x^2*e + d)^4*d*e^(-4) - 200*(x^2*e + d)^3*d^2*e^(-4) + 3
00*(x^2*e + d)^2*d^3*e^(-4) - 300*(x^2*e + d)*d^4*e^(-4))*g^2*p)*e^(-1)
```

maple [C] time = 0.48, size = 687, normalized size = 2.74

$$\frac{f^2x^6\ln(c)}{6}+\frac{g^2x^{10}\ln(c)}{10}-\frac{g^2p\,x^{10}}{50}-\frac{f^2p\,x^6}{18}+\frac{fg\,x^8\ln(c)}{4}+\left(\frac{1}{10}g^2x^{10}+\frac{1}{4}fg\,x^8+\frac{1}{6}f^2x^6\right)\ln\left(\left(e\,x^2+d\right)^p\right)+\frac{i\pi\,f^2x^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x)
```

```
[Out] 1/6*ln(c)*f^2*x^6+1/10*ln(c)*g^2*x^10-1/50*g^2*p*x^10-1/18*f^2*p*x^6+1/4*ln
(c)*f*g*x^8+(1/10*g^2*x^10+1/4*f*g*x^8+1/6*f^2*x^6)*ln((e*x^2+d)^p)-1/8*I*P
i*f*g*x^8*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/12/e*d*f*g*
p*x^6-1/8/e^2*d^2*f*g*p*x^4+1/4/e^3*d^3*f*g*p*x^2-1/4/e^4*ln(e*x^2+d)*d^4*f
*g*p+1/20*I*Pi*g^2*x^10*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/20*I*
Pi*g^2*x^10*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/8*I*Pi*f*g*x^8*csgn(I*c*(e*
x^2+d)^p)^3+1/12*I*Pi*f^2*x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1
/12*I*Pi*f^2*x^6*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/16*f*g*p*x^8+1/40/e*d*
g^2*p*x^8-1/30/e^2*d^2*g^2*p*x^6+1/20/e^3*d^3*g^2*p*x^4+1/12/e*d*f^2*p*x^4-
1/10/e^4*d^4*g^2*p*x^2-1/6/e^2*d^2*f^2*p*x^2+1/10/e^5*ln(e*x^2+d)*d^5*g^2*p
+1/6/e^3*ln(e*x^2+d)*d^3*f^2*p-1/20*I*Pi*g^2*x^10*csgn(I*c*(e*x^2+d)^p)^3-1
/12*I*Pi*f^2*x^6*csgn(I*c*(e*x^2+d)^p)^3-1/12*I*Pi*f^2*x^6*csgn(I*(e*x^2+d)
^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/20*I*Pi*g^2*x^10*csgn(I*(e*x^2+d)^p)*
csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/8*I*Pi*f*g*x^8*csgn(I*(e*x^2+d)^p)*csgn(I
*c*(e*x^2+d)^p)^2+1/8*I*Pi*f*g*x^8*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)
```

maxima [A] time = 0.47, size = 223, normalized size = 0.89

$$-\frac{1}{3600}\,ep\left(\frac{72\,e^4g^2x^{10}+45\left(5\,e^4fg-2\,de^3g^2\right)x^8+20\left(10\,e^4f^2-15\,de^3fg+6\,d^2e^2g^2\right)x^6-30\left(10\,de^3f^2-15\,d^2e\right.}{e^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -1/3600*e*p*((72*e^4*g^2*x^10 + 45*(5*e^4*f*g - 2*d*e^3*g^2)*x^8 + 20*(10*e
^4*f^2 - 15*d*e^3*f*g + 6*d^2*e^2*g^2)*x^6 - 30*(10*d*e^3*f^2 - 15*d^2*e^2*
f*g + 6*d^3*e*g^2)*x^4 + 60*(10*d^2*e^2*f^2 - 15*d^3*e*f*g + 6*d^4*g^2)*x^2
```

```
)/e^5 - 60*(10*d^3*e^2*f^2 - 15*d^4*e*f*g + 6*d^5*g^2)*log(e*x^2 + d)/e^6)
+ 1/60*(6*g^2*x^10 + 15*f*g*x^8 + 10*f^2*x^6)*log((e*x^2 + d)^p*c)
```

mupad [B] time = 0.36, size = 224, normalized size = 0.89

$$\ln\left(c\left(e\,x^2+d\right)^p\right)\left(\frac{f^2\,x^6}{6}+\frac{f\,g\,x^8}{4}+\frac{g^2\,x^{10}}{10}\right)-x^6\left(\frac{f^2\,p}{18}-\frac{d\left(\frac{f\,g\,p}{2}-\frac{d\,g^2\,p}{5\,e}\right)}{6\,e}\right)-x^8\left(\frac{f\,g\,p}{16}-\frac{d\,g^2\,p}{40\,e}\right)-\frac{g^2\,p\,x^{10}}{50}+\frac{\ln\left(e\,x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f^2*x^6)/6 + (g^2*x^10)/10 + (f*g*x^8)/4) - x^6*((f^
2*p)/18 - (d*((f*g*p)/2 - (d*g^2*p)/(5*e)))/(6*e)) - x^8*((f*g*p)/16 - (d*g
^2*p)/(40*e)) - (g^2*p*x^10)/50 + (log(d + e*x^2)*(6*d^5*g^2*p + 10*d^3*e^2
*f^2*p - 15*d^4*e*f*g*p))/(60*e^5) + (d*x^4*((f^2*p)/3 - (d*((f*g*p)/2 - (d
*g^2*p)/(5*e)))/e))/(4*e) - (d^2*x^2*((f^2*p)/3 - (d*((f*g*p)/2 - (d*g^2*p)
/(5*e)))/e))/(2*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

3.324 $\int x^3 \left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right) dx$

Optimal. Leaf size=210

$$\frac{1}{4}f^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}fgx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}g^2x^8\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2p\left(3d^2g^2-8defg+6e^2f^2\right)\mathrm{lo}}{24e^4}$$

```
[Out] 1/2*d*(-d*g+e*f)^2*p*x^2/e^3-1/8*(-3*d*g+e*f)*(-d*g+e*f)*p*(e*x^2+d)^2/e^4-
1/18*g*(-3*d*g+2*e*f)*p*(e*x^2+d)^3/e^4-1/32*g^2*p*(e*x^2+d)^4/e^4-1/24*d^2
*(3*d^2*g^2-8*d*e*f*g+6*e^2*f^2)*p*ln(e*x^2+d)/e^4+1/4*f^2*x^4*ln(c*(e*x^2+
d)^p)+1/3*f*g*x^6*ln(c*(e*x^2+d)^p)+1/8*g^2*x^8*ln(c*(e*x^2+d)^p)
```

Rubi [A] time = 0.36, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2475, 43, 2414, 12, 893}

$$\frac{1}{4}f^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}fgx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}g^2x^8\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2p\left(3d^2g^2-8defg+6e^2f^2\right)\mathrm{lo}}{24e^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (d*(e*f - d*g)^2*p*x^2)/(2*e^3) - ((e*f - 3*d*g)*(e*f - d*g)*p*(d + e*x^2)^
2)/(8*e^4) - (g*(2*e*f - 3*d*g)*p*(d + e*x^2)^3)/(18*e^4) - (g^2*p*(d + e*x
^2)^4)/(32*e^4) - (d^2*(6*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2)*p*Log[d + e*x^2]
)/(24*e^4) + (f^2*x^4*Log[c*(d + e*x^2)^p])/4 + (f*g*x^6*Log[c*(d + e*x^2)^
p])/3 + (g^2*x^8*Log[c*(d + e*x^2)^p])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int x^3 \left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right) dx = \frac{1}{2}\,\text{Subst}\left(\int x(f+gx)^2 \log\left(c(d+ex)^p\right) dx, x, x^2\right)$$

$$= \frac{1}{4}f^2x^4 \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{3}fgx^6 \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{8}g^2x^8 \log\left(c\left(d+\right.\right.$$

$$= \frac{1}{4}f^2x^4 \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{3}fgx^6 \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{8}g^2x^8 \log\left(c\left(d+\right.\right.$$

$$= \frac{1}{4}f^2x^4 \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{3}fgx^6 \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{8}g^2x^8 \log\left(c\left(d+\right.\right.$$

$$= \frac{d(ef-dg)^2px^2}{2e^3} - \frac{(ef-3dg)(ef-dg)p\left(d+ex^2\right)^2}{8e^4} - \frac{g(2ef-3dg)p\left(d+\right.}{18e^4}$$

Mathematica [A] time = 0.14, size = 173, normalized size = 0.82

$$\frac{12e^4x^4\left(6f^2+8fgx^2+3g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right)-12d^2p\left(3d^2g^2-8defg+6e^2f^2\right)\log\left(d+ex^2\right)+epx^2\left(36d^3g^2-\right.}{288e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (e*p*x^2*(36*d^3*g^2 - 6*d^2*e*g*(16*f + 3*g*x^2) + 12*d*e^2*(6*f^2 + 4*f*g
*x^2 + g^2*x^4) - e^3*x^2*(36*f^2 + 32*f*g*x^2 + 9*g^2*x^4)) - 12*d^2*(6*e^
2*f^2 - 8*d*e*f*g + 3*d^2*g^2)*p*Log[d + e*x^2] + 12*e^4*x^4*(6*f^2 + 8*f*g
*x^2 + 3*g^2*x^4)*Log[c*(d + e*x^2)^p])/(288*e^4)
```

fricas [A] time = 0.72, size = 224, normalized size = 1.07

$$-\frac{9\,e^4g^2px^8+4\left(8\,e^4fg-3\,de^3g^2\right)px^6+6\left(6\,e^4f^2-8\,de^3fg+3\,d^2e^2g^2\right)px^4-12\left(6\,de^3f^2-8\,d^2e^2fg+3\,d^3eg^2\right)px^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/288*(9*e^4*g^2*p*x^8 + 4*(8*e^4*f*g - 3*d*e^3*g^2)*p*x^6 + 6*(6*e^4*f^2
- 8*d*e^3*f*g + 3*d^2*e^2*g^2)*p*x^4 - 12*(6*d*e^3*f^2 - 8*d^2*e^2*f*g + 3*
d^3*e*g^2)*p*x^2 - 12*(3*e^4*g^2*p*x^8 + 8*e^4*f*g*p*x^6 + 6*e^4*f^2*p*x^4
- (6*d^2*e^2*f^2 - 8*d^3*e*f*g + 3*d^4*g^2)*p)*log(e*x^2 + d) - 12*(3*e^4*g
^2*x^8 + 8*e^4*f*g*x^6 + 6*e^4*f^2*x^4)*log(c))/e^4
```

giac [B] time = 0.19, size = 418, normalized size = 1.99

$$\frac{1}{288}\left(36\,g^2x^8e\log(c)+96\,fgx^6e\log(c)+36\left(2\left(x^2e+d\right)^2\log\left(x^2e+d\right)-4\left(x^2e+d\right)d\log\left(x^2e+d\right)-\left(x^2e+d\right)^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/288*(36*g^2*x^8*e*log(c) + 96*f*g*x^6*e*log(c) + 36*(2*(x^2*e + d)^2*log(
x^2*e + d) - 4*(x^2*e + d)*d*log(x^2*e + d) - (x^2*e + d)^2 + 4*(x^2*e + d)
*d)*f^2*p*e^(-1) + 72*((x^2*e + d)^2 - 2*(x^2*e + d)*d)*f^2*e^(-1)*log(c) +
 16*(6*(x^2*e + d)^3*e^(-2)*log(x^2*e + d) - 18*(x^2*e + d)^2*d*e^(-2)*log(
x^2*e + d) + 18*(x^2*e + d)*d^2*e^(-2)*log(x^2*e + d) - 2*(x^2*e + d)^3*e^(
-2) + 9*(x^2*e + d)^2*d*e^(-2) - 18*(x^2*e + d)*d^2*e^(-2))*f*g*p + 3*(12*(
x^2*e + d)^4*e^(-3)*log(x^2*e + d) - 48*(x^2*e + d)^3*d*e^(-3)*log(x^2*e +
d) + 72*(x^2*e + d)^2*d^2*e^(-3)*log(x^2*e + d) - 48*(x^2*e + d)*d^3*e^(-3)
*log(x^2*e + d) - 3*(x^2*e + d)^4*e^(-3) + 16*(x^2*e + d)^3*d*e^(-3) - 36*(
x^2*e + d)^2*d^2*e^(-3) + 48*(x^2*e + d)*d^3*e^(-3))*g^2*p)*e^(-1)
```

maple [C] time = 0.47, size = 643, normalized size = 3.06

$$\frac{fg\,x^6\ln(c)}{3}-\frac{f^2p\,x^4}{8}-\frac{g^2p\,x^8}{32}+\frac{g^2x^8\ln(c)}{8}+\frac{f^2x^4\ln(c)}{4}+\frac{d^3fgp\ln\left(e\,x^2+d\right)}{3e^3}+\left(\frac{1}{8}g^2x^8+\frac{1}{3}fg\,x^6+\frac{1}{4}f^2x^4\right)\ln\left(\left(e\,x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x)
```

```
[Out] 1/3*ln(c)*f*g*x^6-1/8*f^2*p*x^4-1/32*g^2*p*x^8+1/8*ln(c)*g^2*x^8+1/4*ln(c)*
f^2*x^4+1/3/e^3*ln(e*x^2+d)*d^3*f*g*p+1/8*I*Pi*f^2*x^4*csgn(I*c*(e*x^2+d)^p
)^2*csgn(I*c)+1/16*I*Pi*g^2*x^8*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2
+1/16*I*Pi*g^2*x^8*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/6*I*Pi*f*g*x^6*csgn(
I*c*(e*x^2+d)^p)^3+1/8*I*Pi*f^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^
p)^2+(1/8*g^2*x^8+1/3*f*g*x^6+1/4*f^2*x^4)*ln((e*x^2+d)^p)-1/6*I*Pi*f*g*x^6
*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/9*f*g*p*x^6-1/16*I*P
i*g^2*x^8*csgn(I*c*(e*x^2+d)^p)^3-1/8*I*Pi*f^2*x^4*csgn(I*c*(e*x^2+d)^p)^3+
1/24/e*d*g^2*p*x^6-1/16/e^2*d^2*g^2*p*x^4+1/8/e^3*d^3*g^2*p*x^2+1/4/e*d*f^2
*p*x^2-1/8/e^4*ln(e*x^2+d)*d^4*g^2*p-1/4/e^2*ln(e*x^2+d)*d^2*f^2*p-1/16*I*P
i*g^2*x^8*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/6*I*Pi*f*g*
x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/6*I*Pi*f*g*x^6*csgn(I*c*(
e*x^2+d)^p)^2*csgn(I*c)-1/8*I*Pi*f^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^
2+d)^p)*csgn(I*c)+1/6/e*d*f*g*p*x^4-1/3/e^2*d^2*f*g*p*x^2
```

maxima [A] time = 0.47, size = 185, normalized size = 0.88

$$-\frac{1}{288}\,ep\left(\frac{9\,e^3g^2x^8+4\left(8\,e^3fg-3\,de^2g^2\right)x^6+6\left(6\,e^3f^2-8\,de^2fg+3\,d^2eg^2\right)x^4-12\left(6\,de^2f^2-8\,d^2efg+3\,d^3g^2\right.}{e^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -1/288*e*p*((9*e^3*g^2*x^8 + 4*(8*e^3*f*g - 3*d*e^2*g^2)*x^6 + 6*(6*e^3*f^2
 - 8*d*e^2*f*g + 3*d^2*e*g^2)*x^4 - 12*(6*d*e^2*f^2 - 8*d^2*e*f*g + 3*d^3*g
^2)*x^2)/e^4 + 12*(6*d^2*e^2*f^2 - 8*d^3*e*f*g + 3*d^4*g^2)*log(e*x^2 + d)/
e^5) + 1/24*(3*g^2*x^8 + 8*f*g*x^6 + 6*f^2*x^4)*log((e*x^2 + d)^p*c)
```

mupad [B] time = 0.34, size = 184, normalized size = 0.88

$$\ln\left(c\left(e\,x^2+d\right)^p\right)\left(\frac{f^2\,x^4}{4}+\frac{f\,g\,x^6}{3}+\frac{g^2\,x^8}{8}\right)-x^4\left(\frac{f^2\,p}{8}-\frac{d\left(\frac{2\,f\,g\,p}{3}-\frac{d\,g^2\,p}{4\,e}\right)}{4\,e}\right)-x^6\left(\frac{f\,g\,p}{9}-\frac{d\,g^2\,p}{24\,e}\right)-\frac{g^2\,p\,x^8}{32}-\frac{\ln\left(e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f^2*x^4)/4 + (g^2*x^8)/8 + (f*g*x^6)/3) - x^4*((f^2*
p)/8 - (d*((2*f*g*p)/3 - (d*g^2*p)/(4*e)))/(4*e)) - x^6*((f*g*p)/9 - (d*g^2
*p)/(24*e)) - (g^2*p*x^8)/32 - (log(d + e*x^2)*(3*d^4*g^2*p + 6*d^2*e^2*f^2
*p - 8*d^3*e*f*g*p))/(24*e^4) + (d*x^2*((f^2*p)/2 - (d*((2*f*g*p)/3 - (d*g^
2*p)/(4*e)))/e))/(2*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

3.325 $\int x\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=124

$$\frac{\left(f+gx^2\right)^3\log\left(c\left(d+ex^2\right)^p\right)}{6g}-\frac{p(ef-dg)^3\log\left(d+ex^2\right)}{6e^3g}-\frac{px^2(ef-dg)^2}{6e^2}-\frac{p\left(f+gx^2\right)^2(ef-dg)}{12eg}-\frac{p\left(f+gx^2\right)^3}{18g}$$

```
[Out] -1/6*(-d*g+e*f)^2*p*x^2/e^2-1/12*(-d*g+e*f)*p*(g*x^2+f)^2/e/g-1/18*p*(g*x^2
+f)^3/g-1/6*(-d*g+e*f)^3*p*ln(e*x^2+d)/e^3/g+1/6*(g*x^2+f)^3*ln(c*(e*x^2+d)
^p)/g
```

Rubi [A] time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2475, 2395, 43}

$$\frac{\left(f+gx^2\right)^3\log\left(c\left(d+ex^2\right)^p\right)}{6g}-\frac{px^2(ef-dg)^2}{6e^2}-\frac{p(ef-dg)^3\log\left(d+ex^2\right)}{6e^3g}-\frac{p\left(f+gx^2\right)^2(ef-dg)}{12eg}-\frac{p\left(f+gx^2\right)^3}{18g}$$

Antiderivative was successfully verified.

```
[In] Int[x*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -((e*f - d*g)^2*p*x^2)/(6*e^2) - ((e*f - d*g)*p*(f + g*x^2)^2)/(12*e*g) - (
p*(f + g*x^2)^3)/(18*g) - ((e*f - d*g)^3*p*Log[d + e*x^2])/(6*e^3*g) + ((f
+ g*x^2)^3*Log[c*(d + e*x^2)^p])/(6*g)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int x\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)dx = \frac{1}{2}\text{Subst}\left(\int (f+gx)^2\log\left(c(d+ex)^p\right)dx,x,x^2\right)$$
$$= \frac{\left(f+gx^2\right)^3\log\left(c\left(d+ex^2\right)^p\right)}{6g} - \frac{(ep)\,\text{Subst}\left(\int\frac{(f+gx)^3}{d+ex}dx,x,x^2\right)}{6g}$$
$$= \frac{\left(f+gx^2\right)^3\log\left(c\left(d+ex^2\right)^p\right)}{6g} - \frac{(ep)\,\text{Subst}\left(\int\left(\frac{g(ef-dg)^2}{e^3}+\frac{(ef-dg)^3}{e^3(d+ex)}+\frac{g(ef-
\end{aligned}$$
$$= -\frac{(ef-dg)^2px^2}{6e^2} - \frac{(ef-dg)p\left(f+gx^2\right)^2}{12eg} - \frac{p\left(f+gx^2\right)^3}{18g} - \frac{(ef-dg)^3p\log}{6e^3g}$$

Mathematica [A] time = 0.11, size = 135, normalized size = 1.09

$$\frac{e\left(6e\left(3df^2+ex^2\left(3f^2+3fgx^2+g^2x^4\right)\right)\log\left(c\left(d+ex^2\right)^p\right)-px^2\left(6d^2g^2-3deg\left(6f+gx^2\right)+e^2\left(18f^2+9fgx^2+\right.\right.\right.}{36e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (6*d^2*g*(-3*e*f + d*g)*p*Log[d + e*x^2] + e*(-(p*x^2*(6*d^2*g^2 - 3*d*e*g*
(6*f + g*x^2) + e^2*(18*f^2 + 9*f*g*x^2 + 2*g^2*x^4))) + 6*e*(3*d*f^2 + e*x
^2*(3*f^2 + 3*f*g*x^2 + g^2*x^4))*Log[c*(d + e*x^2)^p]))/(36*e^3)
```

fricas [A] time = 0.67, size = 180, normalized size = 1.45

$$-\frac{2e^3g^2px^6+3\left(3e^3fg-de^2g^2\right)px^4+6\left(3e^3f^2-3de^2fg+d^2eg^2\right)px^2-6\left(e^3g^2px^6+3e^3fgpx^4+3e^3f^2px^2+\left(3\right.\right.}{36e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/36*(2*e^3*g^2*p*x^6 + 3*(3*e^3*f*g - d*e^2*g^2)*p*x^4 + 6*(3*e^3*f^2 - 3
*d*e^2*f*g + d^2*e*g^2)*p*x^2 - 6*(e^3*g^2*p*x^6 + 3*e^3*f*g*p*x^4 + 3*e^3*
f^2*p*x^2 + (3*d*e^2*f^2 - 3*d^2*e*f*g + d^3*g^2)*p)*log(e*x^2 + d) - 6*(e^
3*g^2*x^6 + 3*e^3*f*g*x^4 + 3*e^3*f^2*x^2)*log(c))/e^3
```

giac [B] time = 0.22, size = 288, normalized size = 2.32

$$\frac{1}{36}\left(6g^2x^6e\log(c)+9\left(2\left(x^2e+d\right)^2\log\left(x^2e+d\right)-4\left(x^2e+d\right)d\log\left(x^2e+d\right)-\left(x^2e+d\right)^2+4\left(x^2e+d\right)d\right)fgpe^{(-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/36*(6*g^2*x^6*e*log(c) + 9*(2*(x^2*e + d)^2*log(x^2*e + d) - 4*(x^2*e + d
)*d*log(x^2*e + d) - (x^2*e + d)^2 + 4*(x^2*e + d)*d)*f*g*p*e^(-1) + 18*((x
^2*e + d)^2 - 2*(x^2*e + d)*d)*f*g*e^(-1)*log(c) - 18*(x^2*e - (x^2*e + d)*
log(x^2*e + d) + d)*f^2*p + (6*(x^2*e + d)^3*e^(-2)*log(x^2*e + d) - 18*(x^
2*e + d)^2*d*e^(-2)*log(x^2*e + d) + 18*(x^2*e + d)*d^2*e^(-2)*log(x^2*e +
d) - 2*(x^2*e + d)^3*e^(-2) + 9*(x^2*e + d)^2*d*e^(-2) - 18*(x^2*e + d)*d^2
*e^(-2))*g^2*p + 18*(x^2*e + d)*f^2*log(c))*e^(-1)
```

maple [C] time = 0.46, size = 599, normalized size = 4.83

$$\frac{fgx^4\ln(c)}{2}-\frac{g^2px^6}{18}-\frac{f^2px^2}{2}+\frac{g^2x^6\ln(c)}{6}+\frac{f^2x^2\ln(c)}{2}+\left(\frac{1}{6}g^2x^6+\frac{1}{2}fgx^4+\frac{1}{2}f^2x^2\right)\ln\left(\left(ex^2+d\right)^p\right)-\frac{d^2fgp\ln\left(ex^2\right.}{2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x)
```

```
[Out] 1/2*ln(c)*f*g*x^4-1/18*g^2*p*x^6-1/2*f^2*p*x^2+1/6*ln(c)*g^2*x^6+1/2*ln(c)*
f^2*x^2+(1/6*g^2*x^6+1/2*f*g*x^4+1/2*f^2*x^2)*ln((e*x^2+d)^p)-1/2*d^2*f*g*p
*ln(e*x^2+d)/e^2-1/4*f*g*p*x^4+1/4*I*Pi*f^2*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*
c*(e*x^2+d)^p)^2+1/4*I*Pi*f^2*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/4*I*P
i*f*g*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/12*I*Pi*g^2
*x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/12*I*Pi*g^2*x^6*csgn(I*c
*(e*x^2+d)^p)^2*csgn(I*c)-1/4*I*Pi*f*g*x^4*csgn(I*c*(e*x^2+d)^p)^3-1/12*I*P
i*g^2*x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/4*I*Pi*f*g*
x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/4*I*Pi*f*g*x^4*csgn(I*c*(
e*x^2+d)^p)^2*csgn(I*c)-1/4*I*Pi*f^2*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^
2+d)^p)*csgn(I*c)+1/12/e*d*g^2*p*x^4-1/6/e^2*d^2*g^2*p*x^2+1/6/e^3*ln(e*x^2
+d)*d^3*g^2*p+1/2/e*ln(e*x^2+d)*d*f^2*p-1/12*I*Pi*g^2*x^6*csgn(I*c*(e*x^2+d
)^p)^3-1/4*I*Pi*f^2*x^2*csgn(I*c*(e*x^2+d)^p)^3+1/2*d/e*f*g*p*x^2
```

maxima [A] time = 0.48, size = 152, normalized size = 1.23

$$\frac{\left(gx^2+f\right)^3\log\left(\left(ex^2+d\right)^pc\right)}{6g}-\frac{ep\left(\frac{2e^2g^3x^6+3\left(3e^2fg^2-deg^3\right)x^4+6\left(3e^2f^2g-3defg^2+d^2g^3\right)x^2}{e^3}+\frac{6\left(e^3f^3-3de^2f^2g+3d^2efg^2-d^3g^3\right)\log\ldots}{e^4}\ldots\right.}{36g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] 1/6*(g*x^2 + f)^3*log((e*x^2 + d)^p*c)/g - 1/36*e*p*((2*e^2*g^3*x^6 + 3*(3*
e^2*f*g^2 - d*e*g^3)*x^4 + 6*(3*e^2*f^2*g - 3*d*e*f*g^2 + d^2*g^3)*x^2)/e^3
 + 6*(e^3*f^3 - 3*d*e^2*f^2*g + 3*d^2*e*f*g^2 - d^3*g^3)*log(e*x^2 + d)/e^4
)/g
```

mupad [B] time = 0.33, size = 142, normalized size = 1.15

$$\ln\left(c\left(e\,x^2+d\right)^p\right)\left(\frac{f^2\,x^2}{2}+\frac{f\,g\,x^4}{2}+\frac{g^2\,x^6}{6}\right)-x^2\left(\frac{f^2\,p}{2}-\frac{d\left(f\,g\,p-\frac{d\,g^2\,p}{3\,e}\right)}{2\,e}\right)-x^4\left(\frac{f\,g\,p}{4}-\frac{d\,g^2\,p}{12\,e}\right)+\frac{\ln\left(e\,x^2+d\right)\left(\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f^2*x^2)/2 + (g^2*x^6)/6 + (f*g*x^4)/2) - x^2*((f^2*
p)/2 - (d*(f*g*p - (d*g^2*p)/(3*e)))/(2*e)) - x^4*((f*g*p)/4 - (d*g^2*p)/(1
2*e)) + (log(d + e*x^2)*(d^3*g^2*p + 3*d*e^2*f^2*p - 3*d^2*e*f*g*p))/(6*e^3
) - (g^2*p*x^6)/18
```

sympy [A] time = 174.00, size = 260, normalized size = 2.10

$$\begin{cases}\frac{d^3g^2p\log\left(d+ex^2\right)}{6e^3}-\frac{d^2fgp\log\left(d+ex^2\right)}{2e^2}-\frac{d^2g^2px^2}{6e^2}+\frac{df^2p\log\left(d+ex^2\right)}{2e}+\frac{dfgpx^2}{2e}+\frac{dg^2px^4}{12e}+\frac{f^2px^2\log\left(d+ex^2\right)}{2}-\frac{f^2px^2}{2}+\frac{f^2x^2\log\left(\ldots\right.}{2}\ldots \\ \left(\frac{f^2x^2}{2}+\frac{fgx^4}{2}+\frac{g^2x^6}{6}\right)\log\left(cd^p\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((d**3*g**2*p*log(d + e*x**2)/(6*e**3) - d**2*f*g*p*log(d + e*x**2
)/(2*e**2) - d**2*g**2*p*x**2/(6*e**2) + d*f**2*p*log(d + e*x**2)/(2*e) + d
```

```
*f*g*p*x**2/(2*e) + d*g**2*p*x**4/(12*e) + f**2*p*x**2*log(d + e*x**2)/2 -
f**2*p*x**2/2 + f**2*x**2*log(c)/2 + f*g*p*x**4*log(d + e*x**2)/2 - f*g*p*x
**4/4 + f*g*x**4*log(c)/2 + g**2*p*x**6*log(d + e*x**2)/6 - g**2*p*x**6/18
+ g**2*x**6*log(c)/6, Ne(e, 0)), ((f**2*x**2/2 + f*g*x**4/2 + g**2*x**6/6)*
log(c*d**p), True))
```

3.326 $\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x} dx$

Optimal. Leaf size=153

$$\frac{1}{2}f^2\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{fg\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e}+\frac{1}{4}g^2x^4\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2g^2p\log\left(d+\right.}{4e^2}$$

```
[Out] -f*g*p*x^2+1/4*d*g^2*p*x^2/e-1/8*g^2*p*x^4-1/4*d^2*g^2*p*ln(e*x^2+d)/e^2+1/
4*g^2*x^4*ln(c*(e*x^2+d)^p)+f*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e+1/2*f^2*ln(-e
*x^2/d)*ln(c*(e*x^2+d)^p)+1/2*f^2*p*polylog(2,1+e*x^2/d)
```

Rubi [A] time = 0.20, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2475, 43, 2416, 2389, 2295, 2394, 2315, 2395}

$$\frac{1}{2}f^2p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)+\frac{1}{2}f^2\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{fg\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e}+\frac{1}{4}g^2x^4\log\left(c\right.$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x,x]
```

```
[Out] -(f*g*p*x^2) + (d*g^2*p*x^2)/(4*e) - (g^2*p*x^4)/8 - (d^2*g^2*p*Log[d + e*x
^2])/(4*e^2) + (g^2*x^4*Log[c*(d + e*x^2)^p])/4 + (f*g*(d + e*x^2)*Log[c*(d
 + e*x^2)^p])/e + (f^2*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/2 + (f^2*p*P
olyLog[2, 1 + (e*x^2)/d])/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(f+gx)^2\log\left(c(d+ex)^p\right)}{x}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(2fg\log\left(c(d+ex)^p\right)+\frac{f^2\log\left(c(d+ex)^p\right)}{x}+g^2x\log\left(c(d+ex\right.\right.\right.$$
$$= \frac{1}{2}f^2\,\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x}\,dx,x,x^2\right)+(fg)\,\text{Subst}\left(\int \log\left(c(d+ex)^p\right)\right.$$
$$= \frac{1}{4}g^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}f^2\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{(fg)\,\text{Subst}}{}$$
$$= -fgpx^2+\frac{1}{4}g^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{fg\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e}+\frac{1}{2}f$$
$$= -fgpx^2+\frac{dg^2px^2}{4e}-\frac{1}{8}g^2px^4-\frac{d^2g^2p\log\left(d+ex^2\right)}{4e^2}+\frac{1}{4}g^2x^4\log\left(c\left(d+ex^2\right.\right.$$

Mathematica [A] time = 0.09, size = 121, normalized size = 0.79

$$\frac{2e\log\left(c\left(d+ex^2\right)^p\right)\left(2ef^2\log\left(-\frac{ex^2}{d}\right)+g\left(4df+4efx^2+egx^4\right)\right)-2d^2g^2p\log\left(d+ex^2\right)+4e^2f^2p\text{Li}_2\left(\frac{ex^2}{d}+1\right)-}{8e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x,x]
```

```
[Out] (-(e*g*p*x^2*(8*e*f - 2*d*g + e*g*x^2)) - 2*d^2*g^2*p*Log[d + e*x^2] + 2*e*
(g*(4*d*f + 4*e*f*x^2 + e*g*x^4) + 2*e*f^2*Log[-((e*x^2)/d)])*Log[c*(d + e*
x^2)^p] + 4*e^2*f^2*p*PolyLog[2, 1 + (e*x^2)/d])/(8*e^2)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(g^2x^4 + 2\,fgx^2 + f^2\right)\log\left(\left(ex^2 + d\right)^p c\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx^2 + f\right)^2 \log\left(\left(ex^2 + d\right)^p c\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x, x)
```

maple [C] time = 0.30, size = 652, normalized size = 4.26

$$-\frac{i\pi g^2x^4\text{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^3}{8}-\frac{i\pi f^2\text{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^3\ln(x)}{2}+\frac{g^2x^4\ln(c)}{4}+\frac{dfgp\ln\left(e\,x^2+d\right)}{e}+\frac{g^2x^4\ln\left(\left(e\right.\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x,x)
```

```
[Out] -1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*x^2*f*g+1/4*l
n(c)*x^4*g^2+p/e*g*d*ln(e*x^2+d)*f-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e
*x^2+d)^p)*csgn(I*c)*f^2*ln(x)-1/8*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2
+d)^p)*csgn(I*c)*x^4*g^2+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)
^2*x^2*f*g+1/4*ln((e*x^2+d)^p)*x^4*g^2-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f^2
*ln(x)-1/8*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*x^4*g^2+ln((e*x^2+d)^p)*f^2*ln(x)-1
/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*x^2*f*g+1/8*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*cs
gn(I*c)*x^4*g^2+1/8*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*x^4*g^
2+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f^2*ln(x)+1/2*I*Pi*c
sgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f^2*ln(x)-1/8*g^2*p*x^4-p*f^2*dilog((-e*x+
(-d*e)^(1/2))/(-d*e)^(1/2))-f*g*p*x^2-p*f^2*dilog((e*x+(-d*e)^(1/2))/(-d*e)
^(1/2))+ln((e*x^2+d)^p)*x^2*f*g+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*
x^2*f*g+ln(c)*f^2*ln(x)+ln(c)*x^2*f*g-p*f^2*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-
d*e)^(1/2))-p*f^2*ln(x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-1/4*d^2*g^2*p*l
n(e*x^2+d)/e^2+1/4*d*g^2*p*x^2/e
```

maxima [A] time = 1.30, size = 161, normalized size = 1.05

$$\frac{1}{2}\left(\log\left(ex^2+d\right)\log\left(-\frac{ex^2+d}{d}+1\right)+\text{Li}_2\left(\frac{ex^2+d}{d}\right)\right)f^2p+f^2\log(c)\log(x)-\frac{\left(e^2g^2p-2\,e^2g^2\log(c)\right)x^4+2\left(4\,e^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(log(e*x^2 + d)*log(-(e*x^2 + d)/d + 1) + dilog((e*x^2 + d)/d))*f^2*p +
 f^2*log(c)*log(x) - 1/8*((e^2*g^2*p - 2*e^2*g^2*log(c))*x^4 + 2*(4*e^2*f*g
```

```
*p - d*e*g^2*p - 4*e^2*f*g*log(c))*x^2 - 2*(e^2*g^2*p*x^4 + 4*e^2*f*g*p*x^2
 + 4*d*e*f*g*p - d^2*g^2*p)*log(e*x^2 + d))/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)\left(g\,x^2+f\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x,x)
```

```
[Out] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x,x)
```

```
[Out] Integral((f + g*x**2)**2*log(c*(d + e*x**2)**p)/x, x)
```

3.327 $\int \frac{\left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^3} dx$

Optimal. Leaf size=135

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{2x^2}+fg\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{g^2\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2e}-\frac{ef^2p\log\left(d+ex^2\right)}{2d}$$

```
[Out] -1/2*g^2*p*x^2+e*f^2*p*ln(x)/d-1/2*e*f^2*p*ln(e*x^2+d)/d-1/2*f^2*ln(c*(e*x^
2+d)^p)/x^2+1/2*g^2*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e+f*g*ln(-e*x^2/d)*ln(c*(e*
x^2+d)^p)+f*g*p*polylog(2,1+e*x^2/d)
```

Rubi [A] time = 0.19, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2475, 43, 2416, 2389, 2295, 2395, 36, 29, 31, 2394, 2315}

$$fgp\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{2x^2}+fg\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{g^2\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^3,x]
```

```
[Out] -(g^2*p*x^2)/2 + (e*f^2*p*Log[x])/d - (e*f^2*p*Log[d + e*x^2])/(2*d) - (f^2
*Log[c*(d + e*x^2)^p])/(2*x^2) + (g^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/(2*
e) + f*g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + f*g*p*PolyLog[2, 1 + (e*x
^2)/d]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\left(f+gx^2\right)^2 \log \left(c\left(d+ex^2\right)^p\right)}{x^3} dx = \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)^2 \log (c(d+ex)^p)}{x^2} dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(g^2 \log (c(d+ex)^p) + \frac{f^2 \log (c(d+ex)^p)}{x^2} + \frac{2fg \log (c(d+$$
$$= \frac{1}{2}f^2\,\text{Subst}\left(\int \frac{\log (c(d+ex)^p)}{x^2} dx, x, x^2\right) + (fg)\,\text{Subst}\left(\int \frac{\log (c(d+ex}{x}\right.$$
$$= -\frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{2x^2} + fg \log \left(-\frac{ex^2}{d}\right) \log \left(c\left(d+ex^2\right)^p\right) + \frac{g^2\,\text{Subst}(}{}$$
$$= -\frac{1}{2}g^2px^2 - \frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{2x^2} + \frac{g^2\left(d+ex^2\right) \log \left(c\left(d+ex^2\right)^p\right)}{2e} + fg$$
$$= -\frac{1}{2}g^2px^2 + \frac{ef^2p\log(x)}{d} - \frac{ef^2p \log \left(d+ex^2\right)}{2d} - \frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{2x^2} +$$

Mathematica [A] time = 0.09, size = 126, normalized size = 0.93

$$\frac{1}{2}\left(-\frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{x^2} + 2fg\left(\log \left(-\frac{ex^2}{d}\right) \log \left(c\left(d+ex^2\right)^p\right) + p\text{Li}_2\left(\frac{ex^2}{d}+1\right)\right) + \frac{g^2\left(d+ex^2\right) \log \left(c\left(d+e\right.\right.}{e}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^3,x]
```

```
[Out] (-(g^2*p*x^2) + (e*f^2*p*(2*Log[x] - Log[d + e*x^2]))/d - (f^2*Log[c*(d + e
*x^2)^p])/x^2 + (g^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e + 2*f*g*(Log[-((e*
x^2)/d)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, 1 + (e*x^2)/d]))/2
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(g^2x^4 + 2\,fgx^2 + f^2\right) \log \left(\left(ex^2+d\right)^p c\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^3,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx^2+f\right)^2 \log \left(\left(ex^2+d\right)^p c\right)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^3,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x^3, x)
```

maple [C] time = 0.31, size = 642, normalized size = 4.76

$$\frac{d\,g^2p\ln\left(e\,x^2+d\right)}{2e}-2fgp\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)-2fgp\ln(x)\ln\left(\frac{ex+\sqrt{-de}}{\sqrt{-de}}\right)-\frac{f^2\ln(c)}{2x^2}+\frac{g^2x^2\ln\left(\left(e\,x^2+d\right)^p\right)}{2}-\frac{f}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^3,x)
```

```
[Out] 1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f^2/x^2+1/2*p/
e*d*ln(e*x^2+d)*g^2-2*p*f*g*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-2*p*
f*g*ln(x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+I*Pi*csgn(I*c*(e*x^2+d)^p)^2*
csgn(I*c)*f*g*ln(x)-I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c
)*f*g*ln(x)-1/2*ln(c)*f^2/x^2+1/2*ln((e*x^2+d)^p)*x^2*g^2-1/2*ln((e*x^2+d)^
p)*f^2/x^2-1/2*g^2*p*x^2+1/2*ln(c)*x^2*g^2-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2
*csgn(I*c)*f^2/x^2+2*ln((e*x^2+d)^p)*f*g*ln(x)-2*p*f*g*dilog((-e*x+(-d*e)^(
1/2))/(-d*e)^(1/2))-2*p*f*g*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+1/4*I*Pi
*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*x^2*g^2+1/4*I*Pi*csgn(I*c*(e*x
^2+d)^p)^2*csgn(I*c)*x^2*g^2-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d
)^p)^2*f^2/x^2-I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f*g*ln(x)+2*ln(c)*f*g*ln(x)+I*P
i*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f*g*ln(x)-1/4*I*Pi*csgn(I*(e*
x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*x^2*g^2+1/4*I*Pi*csgn(I*c*(e*x^2+
d)^p)^3*f^2/x^2-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*x^2*g^2+e*f^2*p*ln(x)/d-1/
2*e*f^2*p*ln(e*x^2+d)/d
```

maxima [A] time = 0.75, size = 155, normalized size = 1.15

$$\left(\log\left(ex^2+d\right)\log\left(-\frac{ex^2+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{ex^2+d}{d}\right)\right)fgp+\frac{\left(ef^2p+2\,dfg\log(c)\right)\log(x)}{d}-\frac{\left(deg^2p-deg^2\log(c)\right)x^4+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^3,x, algorithm="maxima")
```

```
[Out] (log(e*x^2 + d)*log(-(e*x^2 + d)/d + 1) + dilog((e*x^2 + d)/d))*f*g*p + (e*
f^2*p + 2*d*f*g*log(c))*log(x)/d - 1/2*((d*e*g^2*p - d*e*g^2*log(c))*x^4 +
d*e*f^2*log(c) - (d*e*g^2*p*x^4 - d*e*f^2*p - (e^2*f^2*p - d^2*g^2*p)*x^2)*
log(e*x^2 + d))/(d*e*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(g\,x^2+f\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^3,x)
```

```
[Out] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**3,x)
```

```
[Out] Integral((f + g*x**2)**2*log(c*(d + e*x**2)**p)/x**3, x)
```

3.328 $\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^5} \, dx$

Optimal. Leaf size=172

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{4x^4}-\frac{fg \log\left(c\left(d+ex^2\right)^p\right)}{x^2}+\frac{1}{2}g^2 \log\left(-\frac{ex^2}{d}\right) \log\left(c\left(d+ex^2\right)^p\right)+\frac{e^2f^2p \log\left(d+ex^2\right)}{4d^2}-\frac{e^2f^2p}{2}$$

```
[Out] -1/4*e*f^2*p/d/x^2-1/2*e^2*f^2*p*ln(x)/d^2+2*e*f*g*p*ln(x)/d+1/4*e^2*f^2*p*
ln(e*x^2+d)/d^2-e*f*g*p*ln(e*x^2+d)/d-1/4*f^2*ln(c*(e*x^2+d)^p)/x^4-f*g*ln(
c*(e*x^2+d)^p)/x^2+1/2*g^2*ln(-e*x^2/d)*ln(c*(e*x^2+d)^p)+1/2*g^2*p*polylog
(2,1+e*x^2/d)
```

Rubi [A] time = 0.22, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2475, 43, 2416, 2395, 44, 36, 29, 31, 2394, 2315}

$$\frac{1}{2}g^2p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{4x^4}-\frac{fg \log\left(c\left(d+ex^2\right)^p\right)}{x^2}+\frac{1}{2}g^2 \log\left(-\frac{ex^2}{d}\right) \log\left(c\left(d+ex^2\right)^p\right)+$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^5,x]
```

```
[Out] -(e*f^2*p)/(4*d*x^2) - (e^2*f^2*p*Log[x])/(2*d^2) + (2*e*f*g*p*Log[x])/d +
(e^2*f^2*p*Log[d + e*x^2])/(4*d^2) - (e*f*g*p*Log[d + e*x^2])/d - (f^2*Log[
c*(d + e*x^2)^p])/(4*x^4) - (f*g*Log[c*(d + e*x^2)^p])/x^2 + (g^2*Log[-((e*
x^2)/d)]*Log[c*(d + e*x^2)^p])/2 + (g^2*p*PolyLog[2, 1 + (e*x^2)/d])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\left(f+gx^2\right)^2 \log \left(c \left(d+ex^2\right)^p\right)}{x^5} dx = \frac{1}{2} \text{Subst}\left(\int \frac{(f+gx)^2 \log \left(c(d+ex)^p\right)}{x^3} dx, x, x^2\right)$$

$$= \frac{1}{2} \text{Subst}\left(\int \left(\frac{f^2 \log \left(c(d+ex)^p\right)}{x^3} + \frac{2fg \log \left(c(d+ex)^p\right)}{x^2} + \frac{g^2 \log \left(c(d+ex\right.}{x}\right.\right.$$

$$= \frac{1}{2} f^2 \text{Subst}\left(\int \frac{\log \left(c(d+ex)^p\right)}{x^3} dx, x, x^2\right) + (fg) \text{Subst}\left(\int \frac{\log \left(c(d+ex)^p\right)}{x^2}\right.$$

$$= -\frac{f^2 \log \left(c \left(d+ex^2\right)^p\right)}{4x^4} - \frac{fg \log \left(c \left(d+ex^2\right)^p\right)}{x^2} + \frac{1}{2} g^2 \log \left(-\frac{ex^2}{d}\right) \log \left(c \left(d\right.\right.$$

$$= -\frac{f^2 \log \left(c \left(d+ex^2\right)^p\right)}{4x^4} - \frac{fg \log \left(c \left(d+ex^2\right)^p\right)}{x^2} + \frac{1}{2} g^2 \log \left(-\frac{ex^2}{d}\right) \log \left(c \left(d\right.\right.$$

$$= -\frac{ef^2p}{4dx^2} - \frac{e^2f^2p \log(x)}{2d^2} + \frac{2efgp \log(x)}{d} + \frac{e^2f^2p \log \left(d+ex^2\right)}{4d^2} - \frac{efgp \log \left(\right.}{d}$$

Mathematica [A] time = 0.12, size = 148, normalized size = 0.86

$$\frac{1}{4}\left(-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{x^4}-\frac{4fg\log\left(c\left(d+ex^2\right)^p\right)}{x^2}+2g^2\left(\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+p\text{Li}_2\left(\frac{ex^2}{d}+1\right)\right)-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^5,x]
```

```
[Out] ((4*e*f*g*p*(2*Log[x] - Log[d + e*x^2]))/d - (e*f^2*p*(d + 2*e*x^2*Log[x] -
 e*x^2*Log[d + e*x^2]))/(d^2*x^2) - (f^2*Log[c*(d + e*x^2)^p])/x^4 - (4*f*g
*Log[c*(d + e*x^2)^p])/x^2 + 2*g^2*(Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p]
+ p*PolyLog[2, 1 + (e*x^2)/d]))/4
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(g^2x^4+2\,fgx^2+f^2\right)\log\left(\left(ex^2+d\right)^pc\right)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^5,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(gx^2+f\right)^2\log\left(\left(ex^2+d\right)^pc\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^5,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x^5, x)
```

maple [C] time = 0.26, size = 663, normalized size = 3.85

$$-g^2p\,\text{dilog}\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)-g^2p\,\text{dilog}\left(\frac{ex+\sqrt{-de}}{\sqrt{-de}}\right)-\frac{fg\ln\left(\left(e\,x^2+d\right)^p\right)}{x^2}+g^2\ln(c)\ln(x)-\frac{f^2\ln(c)}{4x^4}+g^2\ln(x)\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^5,x)
```

```
[Out] -ln((e*x^2+d)^p)*f*g/x^2-p*g^2*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p*g^
2*dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+ln(c)*g^2*ln(x)-1/4*ln(c)*f^2/x^4+
1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f*g/x^2+ln((e*
x^2+d)^p)*g^2*ln(x)-1/4*ln((e*x^2+d)^p)*f^2/x^4-1/8*I*Pi*csgn(I*c*(e*x^2+d)
^p)^2*csgn(I*c)*f^2/x^4+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^
2*g^2*ln(x)-1/8*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f^2/x^4+1/
2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f*g/x^2+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csg
n(I*c)*g^2*ln(x)-p*g^2*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p*g^2*ln(
x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-ln(c)*f*g/x^2-1/2*I*Pi*csgn(I*(e*x^2
+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*g^2*ln(x)-1/2*I*Pi*csgn(I*(e*x^2+d)^
p)*csgn(I*c*(e*x^2+d)^p)^2*f*g/x^2-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*
c)*f*g/x^2+1/8*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f^2
/x^4+1/8*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f^2/x^4-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p
```

```
)^3*g^2*ln(x)-1/2*e^2*f^2*p*ln(x)/d^2+1/4*e^2*f^2*p*ln(e*x^2+d)/d^2+2*e*f*g
*p*ln(x)/d-e*f*g*p*ln(e*x^2+d)/d-1/4*e*f^2*p/d/x^2
```

maxima [A] time = 1.30, size = 166, normalized size = 0.97

$$\frac{1}{2}\left(\log\left(ex^2+d\right)\log\left(-\frac{ex^2+d}{d}+1\right)+\text{Li}_2\left(\frac{ex^2+d}{d}\right)\right)g^2p-\frac{\left(e^2f^2p-4\,defgp-2\,d^2g^2\log(c)\right)\log(x)}{2\,d^2}-\frac{d^2f^2\log(c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*(log(e*x^2 + d)*log(-(e*x^2 + d)/d + 1) + dilog((e*x^2 + d)/d))*g^2*p -
 1/2*(e^2*f^2*p - 4*d*e*f*g*p - 2*d^2*g^2*log(c))*log(x)/d^2 - 1/4*(d^2*f^2
*log(c) + (d*e*f^2*p + 4*d^2*f*g*log(c))*x^2 + (4*d^2*f*g*p*x^2 + d^2*f^2*p
 - (e^2*f^2*p - 4*d*e*f*g*p)*x^4)*log(e*x^2 + d))/(d^2*x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)\left(g\,x^2+f\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^5,x)
```

```
[Out] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**5,x)
```

```
[Out] Integral((f + g*x**2)**2*log(c*(d + e*x**2)**p)/x**5, x)
```

3.329 $\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^7} dx$

Optimal. Leaf size=130

$$-\frac{(f+gx^2)^3 \log\left(c\left(d+ex^2\right)^p\right)}{6fx^6}-\frac{p(ef-dg)^3 \log\left(d+ex^2\right)}{6d^3f}+\frac{efp(ef-3dg)}{6d^2x^2}+\frac{ep\log(x)\left(3d^2g^2-3defg+e^2f^2\right)}{3d^3}$$

```
[Out] -1/12*e*f^2*p/d/x^4+1/6*e*f*(-3*d*g+e*f)*p/d^2/x^2+1/3*e*(3*d^2*g^2-3*d*e*f
*g+e^2*f^2)*p*ln(x)/d^3-1/6*(-d*g+e*f)^3*p*ln(e*x^2+d)/d^3/f-1/6*(g*x^2+f)^
3*ln(c*(e*x^2+d)^p)/f/x^6
```

Rubi [A] time = 0.21, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2475, 37, 2414, 12, 88}

$$-\frac{(f+gx^2)^3 \log\left(c\left(d+ex^2\right)^p\right)}{6fx^6}+\frac{ep\log(x)\left(3d^2g^2-3defg+e^2f^2\right)}{3d^3}+\frac{efp(ef-3dg)}{6d^2x^2}-\frac{p(ef-dg)^3 \log\left(d+ex^2\right)}{6d^3f}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^7,x]
```

```
[Out] -(e*f^2*p)/(12*d*x^4) + (e*f*(e*f - 3*d*g)*p)/(6*d^2*x^2) + (e*(e^2*f^2 - 3
*d*e*f*g + 3*d^2*g^2)*p*Log[x])/(3*d^3) - ((e*f - d*g)^3*p*Log[d + e*x^2])/
(6*d^3*f) - ((f + g*x^2)^3*Log[c*(d + e*x^2)^p])/(6*f*x^6)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
```

```
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^7}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(f+gx)^2\log(c(d+ex)^p)}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{(f+gx^2)^3\log\left(c\left(d+ex^2\right)^p\right)}{6fx^6} - \frac{1}{2}(ep)\operatorname{Subst}\left(\int -\frac{(f+gx)^3}{3fx^3(d+ex)}\,dx, x, x^2\right)$$
$$= -\frac{(f+gx^2)^3\log\left(c\left(d+ex^2\right)^p\right)}{6fx^6} + \frac{(ep)\operatorname{Subst}\left(\int \frac{(f+gx)^3}{x^3(d+ex)}\,dx, x, x^2\right)}{6f}$$
$$= -\frac{(f+gx^2)^3\log\left(c\left(d+ex^2\right)^p\right)}{6fx^6} + \frac{(ep)\operatorname{Subst}\left(\int \left(\frac{f^3}{dx^3} + \frac{f^2(-ef+3dg)}{d^2x^2} + \frac{f(e^2f^2-}{}\right.\right.}{6f}$$
$$= -\frac{ef^2p}{12dx^4} + \frac{ef(ef-3dg)p}{6d^2x^2} + \frac{e\left(e^2f^2-3defg+3d^2g^2\right)p\log(x)}{3d^3} - \frac{(ef-dg)^3}{}$$

Mathematica [A] time = 0.13, size = 141, normalized size = 1.08

$$-\frac{2d^3\left(f^2+3fgx^2+3g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right) - 4epx^6\log(x)\left(3d^2g^2-3defg+e^2f^2\right) + 2epx^6\left(3d^2g^2-3defg+e\right.}{12d^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^7,x]
```

```
[Out] -1/12*(d*e*f*p*x^2*(-2*e*f*x^2 + d*(f + 6*g*x^2)) - 4*e*(e^2*f^2 - 3*d*e*f*
g + 3*d^2*g^2)*p*x^6*Log[x] + 2*e*(e^2*f^2 - 3*d*e*f*g + 3*d^2*g^2)*p*x^6*L
og[d + e*x^2] + 2*d^3*(f^2 + 3*f*g*x^2 + 3*g^2*x^4)*Log[c*(d + e*x^2)^p])/(
d^3*x^6)
```

fricas [A] time = 0.82, size = 183, normalized size = 1.41

$$\frac{4\left(e^3f^2-3\,de^2fg+3\,d^2eg^2\right)px^6\log(x) - d^2ef^2px^2 + 2\left(de^2f^2-3\,d^2efg\right)px^4 - 2\left(3\,d^3g^2px^4+3\,d^3fgpx^2+\left(e^3f^2\right.\right.}{12\,d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(4*(e^3*f^2 - 3*d*e^2*f*g + 3*d^2*e*g^2)*p*x^6*log(x) - d^2*e*f^2*p*x^
2 + 2*(d*e^2*f^2 - 3*d^2*e*f*g)*p*x^4 - 2*(3*d^3*g^2*p*x^4 + 3*d^3*f*g*p*x^
2 + (e^3*f^2 - 3*d*e^2*f*g + 3*d^2*e*g^2)*p*x^6 + d^3*f^2*p)*log(e*x^2 + d)
 - 2*(3*d^3*g^2*x^4 + 3*d^3*f*g*x^2 + d^3*f^2)*log(c))/(d^3*x^6)
```

giac [B] time = 0.23, size = 791, normalized size = 6.08

$$-\frac{\left(6\left(x^2e+d\right)^3d^2g^2pe^2\log\left(x^2e+d\right) - 12\left(x^2e+d\right)^2d^3g^2pe^2\log\left(x^2e+d\right) + 6\left(x^2e+d\right)d^4g^2pe^2\log\left(x^2e+d\right) - 6\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(6*(x^2*e + d)^3*d^2*g^2*p*e^2*log(x^2*e + d) - 12*(x^2*e + d)^2*d^3*
g^2*p*e^2*log(x^2*e + d) + 6*(x^2*e + d)*d^4*g^2*p*e^2*log(x^2*e + d) - 6*(
x^2*e + d)^3*d^2*g^2*p*e^2*log(x^2*e) + 18*(x^2*e + d)^2*d^3*g^2*p*e^2*log(
x^2*e) - 18*(x^2*e + d)*d^4*g^2*p*e^2*log(x^2*e) + 6*d^5*g^2*p*e^2*log(x^2*
e) - 6*(x^2*e + d)^3*d*f*g*p*e^3*log(x^2*e + d) + 18*(x^2*e + d)^2*d^2*f*g*
p*e^3*log(x^2*e + d) - 12*(x^2*e + d)*d^3*f*g*p*e^3*log(x^2*e + d) + 6*(x^2
*e + d)^3*d*f*g*p*e^3*log(x^2*e) - 18*(x^2*e + d)^2*d^2*f*g*p*e^3*log(x^2*e
) + 18*(x^2*e + d)*d^3*f*g*p*e^3*log(x^2*e) - 6*d^4*f*g*p*e^3*log(x^2*e) +
6*(x^2*e + d)^2*d^3*g^2*e^2*log(c) - 12*(x^2*e + d)*d^4*g^2*e^2*log(c) + 6*
d^5*g^2*e^2*log(c) + 6*(x^2*e + d)^2*d^2*f*g*p*e^3 - 12*(x^2*e + d)*d^3*f*g
*p*e^3 + 6*d^4*f*g*p*e^3 + 2*(x^2*e + d)^3*f^2*p*e^4*log(x^2*e + d) - 6*(x^
2*e + d)^2*d*f^2*p*e^4*log(x^2*e + d) + 6*(x^2*e + d)*d^2*f^2*p*e^4*log(x^2
*e + d) - 2*(x^2*e + d)^3*f^2*p*e^4*log(x^2*e) + 6*(x^2*e + d)^2*d*f^2*p*e^
4*log(x^2*e) - 6*(x^2*e + d)*d^2*f^2*p*e^4*log(x^2*e) + 2*d^3*f^2*p*e^4*log
(x^2*e) + 6*(x^2*e + d)*d^3*f*g*e^3*log(c) - 6*d^4*f*g*e^3*log(c) - 2*(x^2*
e + d)^2*d*f^2*p*e^4 + 5*(x^2*e + d)*d^2*f^2*p*e^4 - 3*d^3*f^2*p*e^4 + 2*d^
3*f^2*e^4*log(c))*e^(-1)/((x^2*e + d)^3*d^3 - 3*(x^2*e + d)^2*d^4 + 3*(x^2*
e + d)*d^5 - d^6)
```

maple [C] time = 0.46, size = 656, normalized size = 5.05

$$-\frac{\left(3g^2x^4+3fg\,x^2+f^2\right)\ln\left(\left(e\,x^2+d\right)^p\right)}{6x^6}+\frac{12d^2e\,g^2p\,x^6\ln(x)-6d^2e\,g^2p\,x^6\ln\left(e\,x^2+d\right)-12d\,e^2fgp\,x^6\ln(x)+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^7,x)
```

```
[Out] -1/6*(3*g^2*x^4+3*f*g*x^2+f^2)/x^6*ln((e*x^2+d)^p)+1/12*(-3*I*Pi*d^3*g^2*x^
4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-I*Pi*d^3*f^2*csgn(I*c*(e*x^2+d)^p)^2*cs
gn(I*c)+3*I*Pi*d^3*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I
*c)+3*I*Pi*d^3*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+12*ln(x)*d^2*e*g^2*p*x^6-12*
ln(x)*d*e^2*f*g*p*x^6+4*ln(x)*e^3*f^2*p*x^6-6*ln(e*x^2+d)*d^2*e*g^2*p*x^6+6
*ln(e*x^2+d)*d*e^2*f*g*p*x^6-2*ln(e*x^2+d)*e^3*f^2*p*x^6+3*I*Pi*d^3*g^2*x^4
*csgn(I*c*(e*x^2+d)^p)^3-3*I*Pi*d^3*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e
*x^2+d)^p)^2-3*I*Pi*d^3*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2
-3*I*Pi*d^3*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-6*ln(c)*d^3*g^2*x^4+3
*I*Pi*d^3*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+I*Pi*
d^3*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+I*Pi*d^3*f^2*cs
gn(I*c*(e*x^2+d)^p)^3-I*Pi*d^3*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p
)^2-6*d^2*e*f*g*p*x^4+2*d*e^2*f^2*p*x^4-6*ln(c)*d^3*f*g*x^2-d^2*e*f^2*p*x^2
-2*ln(c)*d^3*f^2)/d^3/x^6
```

maxima [A] time = 0.49, size = 137, normalized size = 1.05

$$-\frac{1}{12}\,ep\left(\frac{2\left(e^2f^2-3\,defg+3\,d^2g^2\right)\log\left(ex^2+d\right)}{d^3}-\frac{2\left(e^2f^2-3\,defg+3\,d^2g^2\right)\log\left(x^2\right)}{d^3}+\frac{df^2-2\left(ef^2-3\,dfg\cdots\right.}{d^2x^4}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*e*p*(2*(e^2*f^2 - 3*d*e*f*g + 3*d^2*g^2)*log(e*x^2 + d)/d^3 - 2*(e^2*
f^2 - 3*d*e*f*g + 3*d^2*g^2)*log(x^2)/d^3 + (d*f^2 - 2*(e*f^2 - 3*d*f*g)*x^
2)/(d^2*x^4)) - 1/6*(3*g^2*x^4 + 3*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)/x^6
```

mupad [B] time = 0.41, size = 151, normalized size = 1.16

$$\frac{\ln(x)\,\left(3\,p\,d^2\,e\,g^2-3\,p\,d\,e^2\,f\,g+p\,e^3\,f^2\right)}{3\,d^3}-\frac{\ln\left(c\,\left(e\,x^2+d\right)^p\right)\,\left(\frac{f^2}{6}+\frac{f\,g\,x^2}{2}+\frac{g^2\,x^4}{2}\right)}{x^6}-\frac{\ln\left(e\,x^2+d\right)\,\left(3\,p\,d^2\,e\,g^2-3\,p\cdots\right.}{6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^7,x)
```

```
[Out] (log(x)*(e^3*f^2*p + 3*d^2*e*g^2*p - 3*d*e^2*f*g*p))/(3*d^3) - (log(c*(d +
e*x^2)^p)*(f^2/6 + (g^2*x^4)/2 + (f*g*x^2)/2))/x^6 - (log(d + e*x^2)*(e^3*f
^2*p + 3*d^2*e*g^2*p - 3*d*e^2*f*g*p))/(6*d^3) - ((e*f^2*p)/(4*d) + (e*f*p*
x^2*(3*d*g - e*f))/(2*d^2))/(3*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**7,x)
```

```
[Out] Timed out
```

3.330 $\int \frac{\left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^9} \, dx$

Optimal. Leaf size=216

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{8x^8}-\frac{fg \log\left(c\left(d+ex^2\right)^p\right)}{3x^6}-\frac{g^2 \log\left(c\left(d+ex^2\right)^p\right)}{4x^4}+\frac{efp(3ef-8dg)}{48d^2x^4}+\frac{e^2p\left(6d^2g^2-8defg+3\right.}{24d}$$

```
[Out] -1/24*e*f^2*p/d/x^6+1/48*e*f*(-8*d*g+3*e*f)*p/d^2/x^4-1/24*e*(6*d^2*g^2-8*d
*e*f*g+3*e^2*f^2)*p/d^3/x^2-1/12*e^2*(6*d^2*g^2-8*d*e*f*g+3*e^2*f^2)*p*ln(x
)/d^4+1/24*e^2*(6*d^2*g^2-8*d*e*f*g+3*e^2*f^2)*p*ln(e*x^2+d)/d^4-1/8*f^2*ln
(c*(e*x^2+d)^p)/x^8-1/3*f*g*ln(c*(e*x^2+d)^p)/x^6-1/4*g^2*ln(c*(e*x^2+d)^p)
/x^4
```

Rubi [A] time = 0.29, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2475, 43, 2414, 12, 893}

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{8x^8}-\frac{fg \log\left(c\left(d+ex^2\right)^p\right)}{3x^6}-\frac{g^2 \log\left(c\left(d+ex^2\right)^p\right)}{4x^4}-\frac{ep\left(6d^2g^2-8defg+3e^2f^2\right)}{24d^3x^2}+\frac{e^2p\left(6d^2g^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^9,x]
```

```
[Out] -(e*f^2*p)/(24*d*x^6) + (e*f*(3*e*f - 8*d*g)*p)/(48*d^2*x^4) - (e*(3*e^2*f^
2 - 8*d*e*f*g + 6*d^2*g^2)*p)/(24*d^3*x^2) - (e^2*(3*e^2*f^2 - 8*d*e*f*g +
6*d^2*g^2)*p*Log[x])/(12*d^4) + (e^2*(3*e^2*f^2 - 8*d*e*f*g + 6*d^2*g^2)*p*
Log[d + e*x^2])/(24*d^4) - (f^2*Log[c*(d + e*x^2)^p])/(8*x^8) - (f*g*Log[c*
(d + e*x^2)^p])/(3*x^6) - (g^2*Log[c*(d + e*x^2)^p])/(4*x^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
```

```
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^9}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(f+gx)^2\log(c(d+ex)^p)}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{8x^8} - \frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{3x^6} - \frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4} - \frac{1}{2}$$

$$= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{8x^8} - \frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{3x^6} - \frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4} - \frac{1}{24}$$

$$= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{8x^8} - \frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{3x^6} - \frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4} - \frac{1}{24}$$

$$= -\frac{ef^2p}{24dx^6} + \frac{ef(3ef-8dg)p}{48d^2x^4} - \frac{e\left(3e^2f^2-8defg+6d^2g^2\right)p}{24d^3x^2} - \frac{e^2\left(3e^2f^2-8d\right.}{}$$

Mathematica [**A**] time = 0.18, size = 184, normalized size = 0.85

$$-\frac{2d^4\left(3f^2+8fgx^2+6g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right)+4e^2px^8\log(x)\left(6d^2g^2-8defg+3e^2f^2\right)-2e^2px^8\left(6d^2g^2-8def\right.}{48d^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^9,x]
```

```
[Out] -1/48*(d*e*p*x^2*(6*e^2*f^2*x^4 - d*e*f*x^2*(3*f + 16*g*x^2) + 2*d^2*(f^2 +
 4*f*g*x^2 + 6*g^2*x^4)) + 4*e^2*(3*e^2*f^2 - 8*d*e*f*g + 6*d^2*g^2)*p*x^8*
Log[x] - 2*e^2*(3*e^2*f^2 - 8*d*e*f*g + 6*d^2*g^2)*p*x^8*Log[d + e*x^2] + 2
*d^4*(3*f^2 + 8*f*g*x^2 + 6*g^2*x^4)*Log[c*(d + e*x^2)^p])/(d^4*x^8)
```

fricas [**A**] time = 0.76, size = 230, normalized size = 1.06

$$-\frac{4\left(3e^4f^2-8de^3fg+6d^2e^2g^2\right)px^8\log(x)+2d^3ef^2px^2+2\left(3de^3f^2-8d^2e^2fg+6d^3eg^2\right)px^6-\left(3d^2e^2f^2-8d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^9,x, algorithm="fricas")
```

```
[Out] -1/48*(4*(3*e^4*f^2 - 8*d*e^3*f*g + 6*d^2*e^2*g^2)*p*x^8*log(x) + 2*d^3*e*f
^2*p*x^2 + 2*(3*d*e^3*f^2 - 8*d^2*e^2*f*g + 6*d^3*e*g^2)*p*x^6 - (3*d^2*e^2
*f^2 - 8*d^3*e*f*g)*p*x^4 + 2*(6*d^4*g^2*p*x^4 - (3*e^4*f^2 - 8*d*e^3*f*g +
 6*d^2*e^2*g^2)*p*x^8 + 8*d^4*f*g*p*x^2 + 3*d^4*f^2*p)*log(e*x^2 + d) + 2*(
6*d^4*g^2*x^4 + 8*d^4*f*g*x^2 + 3*d^4*f^2)*log(c))/(d^4*x^8)
```

giac [B] time = 0.24, size = 1089, normalized size = 5.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^9,x, algorithm="giac")
```

```
[Out] 1/48*(12*(x^2*e + d)^4*d^2*g^2*p*e^3*log(x^2*e + d) - 48*(x^2*e + d)^3*d^3*
g^2*p*e^3*log(x^2*e + d) + 60*(x^2*e + d)^2*d^4*g^2*p*e^3*log(x^2*e + d) -
24*(x^2*e + d)*d^5*g^2*p*e^3*log(x^2*e + d) - 12*(x^2*e + d)^4*d^2*g^2*p*e^
3*log(x^2*e) + 48*(x^2*e + d)^3*d^3*g^2*p*e^3*log(x^2*e) - 72*(x^2*e + d)^2
*d^4*g^2*p*e^3*log(x^2*e) + 48*(x^2*e + d)*d^5*g^2*p*e^3*log(x^2*e) - 12*d^
6*g^2*p*e^3*log(x^2*e) - 12*(x^2*e + d)^3*d^3*g^2*p*e^3 + 36*(x^2*e + d)^2*
d^4*g^2*p*e^3 - 36*(x^2*e + d)*d^5*g^2*p*e^3 + 12*d^6*g^2*p*e^3 - 16*(x^2*e
 + d)^4*d*f*g*p*e^4*log(x^2*e + d) + 64*(x^2*e + d)^3*d^2*f*g*p*e^4*log(x^2
*e + d) - 96*(x^2*e + d)^2*d^3*f*g*p*e^4*log(x^2*e + d) + 48*(x^2*e + d)*d^
4*f*g*p*e^4*log(x^2*e + d) + 16*(x^2*e + d)^4*d*f*g*p*e^4*log(x^2*e) - 64*(
x^2*e + d)^3*d^2*f*g*p*e^4*log(x^2*e) + 96*(x^2*e + d)^2*d^3*f*g*p*e^4*log(
x^2*e) - 64*(x^2*e + d)*d^4*f*g*p*e^4*log(x^2*e) + 16*d^5*f*g*p*e^4*log(x^2
*e) - 12*(x^2*e + d)^2*d^4*g^2*e^3*log(c) + 24*(x^2*e + d)*d^5*g^2*e^3*log(
c) - 12*d^6*g^2*e^3*log(c) + 16*(x^2*e + d)^3*d^2*f*g*p*e^4 - 56*(x^2*e + d
)^2*d^3*f*g*p*e^4 + 64*(x^2*e + d)*d^4*f*g*p*e^4 - 24*d^5*f*g*p*e^4 + 6*(x^
2*e + d)^4*f^2*p*e^5*log(x^2*e + d) - 24*(x^2*e + d)^3*d*f^2*p*e^5*log(x^2*
e + d) + 36*(x^2*e + d)^2*d^2*f^2*p*e^5*log(x^2*e + d) - 24*(x^2*e + d)*d^3
*f^2*p*e^5*log(x^2*e + d) - 6*(x^2*e + d)^4*f^2*p*e^5*log(x^2*e) + 24*(x^2*
e + d)^3*d*f^2*p*e^5*log(x^2*e) - 36*(x^2*e + d)^2*d^2*f^2*p*e^5*log(x^2*e)
 + 24*(x^2*e + d)*d^3*f^2*p*e^5*log(x^2*e) - 6*d^4*f^2*p*e^5*log(x^2*e) - 1
6*(x^2*e + d)*d^4*f*g*e^4*log(c) + 16*d^5*f*g*e^4*log(c) - 6*(x^2*e + d)^3*
d*f^2*p*e^5 + 21*(x^2*e + d)^2*d^2*f^2*p*e^5 - 26*(x^2*e + d)*d^3*f^2*p*e^5
 + 11*d^4*f^2*p*e^5 - 6*d^4*f^2*e^5*log(c))*e^(-1)/((x^2*e + d)^4*d^4 - 4*(
x^2*e + d)^3*d^5 + 6*(x^2*e + d)^2*d^6 - 4*(x^2*e + d)*d^7 + d^8)
```

maple [C] time = 0.51, size = 713, normalized size = 3.30

$$-\frac{\left(6g^2x^4+8fg\,x^2+3f^2\right)\ln\left(\left(e\,x^2+d\right)^p\right)}{24x^8}-\frac{6d^4f^2\ln(c)-8i\pi\,d^4fg\,x^2\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^3-3i\pi\,d^4f^2\mathrm{csgn}\left(ic\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^9,x)
```

```
[Out] -1/24*(6*g^2*x^4+8*f*g*x^2+3*f^2)/x^8*ln((e*x^2+d)^p)-1/48*(6*ln(c)*d^4*f^2
+3*I*Pi*d^4*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+3*I*Pi*d^4*f^2*
csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+12*ln(c)*d^4*g^2*x^4-8*I*Pi*d^4*f*g*x^2*c
sgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+12*d^3*e*g^2*p*x^6+6*d*e
^3*f^2*p*x^6-3*d^2*e^2*f^2*p*x^4+2*d^3*e*f^2*p*x^2-6*I*Pi*d^4*g^2*x^4*csgn(
I*c*(e*x^2+d)^p)^3-3*I*Pi*d^4*f^2*csgn(I*c*(e*x^2+d)^p)^3+6*I*Pi*d^4*g^2*x^
4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-6*ln(-e*x^2-d)*e^4*f^2*p*x^8+12*ln(x)*e
^4*f^2*p*x^8+16*ln(c)*d^4*f*g*x^2-16*d^2*e^2*f*g*p*x^6+8*d^3*e*f*g*p*x^4-8*
I*Pi*d^4*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^3-3*I*Pi*d^4*f^2*csgn(I*(e*x^2+d)^p)
*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+6*I*Pi*d^4*g^2*x^4*csgn(I*(e*x^2+d)^p)*csg
n(I*c*(e*x^2+d)^p)^2-6*I*Pi*d^4*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2
+d)^p)*csgn(I*c)+8*I*Pi*d^4*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^
p)^2+8*I*Pi*d^4*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-12*ln(-e*x^2-d)*d
^2*e^2*g^2*p*x^8+24*ln(x)*d^2*e^2*g^2*p*x^8+16*ln(-e*x^2-d)*d*e^3*f*g*p*x^8
-32*ln(x)*d*e^3*f*g*p*x^8)/d^4/x^8
```

maxima [A] time = 0.49, size = 183, normalized size = 0.85

$$\frac{1}{48}ep\left(\frac{2\left(3e^3f^2-8de^2fg+6d^2eg^2\right)\log\left(ex^2+d\right)}{d^4}-\frac{2\left(3e^3f^2-8de^2fg+6d^2eg^2\right)\log\left(x^2\right)}{d^4}-\frac{2\left(3e^2f^2-8de\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^9,x, algorithm="maxima")
```

```
[Out] 1/48*e*p*(2*(3*e^3*f^2 - 8*d*e^2*f*g + 6*d^2*e*g^2)*log(e*x^2 + d)/d^4 - 2*
(3*e^3*f^2 - 8*d*e^2*f*g + 6*d^2*e*g^2)*log(x^2)/d^4 - (2*(3*e^2*f^2 - 8*d*
e*f*g + 6*d^2*g^2)*x^4 + 2*d^2*f^2 - (3*d*e*f^2 - 8*d^2*f*g)*x^2)/(d^3*x^6)
) - 1/24*(6*g^2*x^4 + 8*f*g*x^2 + 3*f^2)*log((e*x^2 + d)^p*c)/x^8
```

mupad [B] time = 0.43, size = 190, normalized size = 0.88

$$\frac{\ln\left(e\,x^2+d\right)\,\left(6\,p\,d^2\,e^2\,g^2-8\,p\,d\,e^3\,f\,g+3\,p\,e^4\,f^2\right)}{24\,d^4}-\frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{f^2}{8}+\frac{f\,g\,x^2}{3}+\frac{g^2\,x^4}{4}\right)}{x^8}-\frac{\frac{e\,f^2\,p}{2\,d}+\frac{e\,p\,x^4\,\left(6\,d^2\,g^2-8\,d\,e\right.}{2\,d^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^9,x)
```

```
[Out] (log(d + e*x^2)*(3*e^4*f^2*p + 6*d^2*e^2*g^2*p - 8*d*e^3*f*g*p))/(24*d^4) -
 (log(c*(d + e*x^2)^p)*(f^2/8 + (g^2*x^4)/4 + (f*g*x^2)/3))/x^8 - ((e*f^2*p
)/(2*d) + (e*p*x^4*(6*d^2*g^2 + 3*e^2*f^2 - 8*d*e*f*g))/(2*d^3) + (e*f*p*x^
2*(8*d*g - 3*e*f))/(4*d^2))/(12*x^6) - (log(x)*(3*e^4*f^2*p + 6*d^2*e^2*g^2
*p - 8*d*e^3*f*g*p))/(12*d^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**9,x)
```

```
[Out] Timed out
```

3.331 $\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^{11}} dx$

Optimal. Leaf size=253

$$-\frac{f^2 \log \left(c \left(d+e x^2\right)^p\right)}{10 x^{10}}-\frac{f g \log \left(c \left(d+e x^2\right)^p\right)}{4 x^8}-\frac{g^2 \log \left(c \left(d+e x^2\right)^p\right)}{6 x^6}+\frac{e f p (2 e f-5 d g)}{60 d^2 x^6}-\frac{e^3 p \left(10 d^2 g^2-15 d e f g+\right.}{60 d}$$

```
[Out] -1/40*e*f^2*p/d/x^8+1/60*e*f*(-5*d*g+2*e*f)*p/d^2/x^6-1/120*e*(10*d^2*g^2-1
5*d*e*f*g+6*e^2*f^2)*p/d^3/x^4+1/60*e^2*(10*d^2*g^2-15*d*e*f*g+6*e^2*f^2)*p
/d^4/x^2+1/30*e^3*(10*d^2*g^2-15*d*e*f*g+6*e^2*f^2)*p*ln(x)/d^5-1/60*e^3*(1
0*d^2*g^2-15*d*e*f*g+6*e^2*f^2)*p*ln(e*x^2+d)/d^5-1/10*f^2*ln(c*(e*x^2+d)^p
)/x^10-1/4*f*g*ln(c*(e*x^2+d)^p)/x^8-1/6*g^2*ln(c*(e*x^2+d)^p)/x^6
```

Rubi [A] time = 0.33, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2475, 43, 2414, 12, 893}

$$-\frac{f^2 \log \left(c \left(d+e x^2\right)^p\right)}{10 x^{10}}-\frac{f g \log \left(c \left(d+e x^2\right)^p\right)}{4 x^8}-\frac{g^2 \log \left(c \left(d+e x^2\right)^p\right)}{6 x^6}+\frac{e^2 p \left(10 d^2 g^2-15 d e f g+6 e^2 f^2\right)}{60 d^4 x^2}-\frac{e p \left(10 d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^11,x]
```

```
[Out] -(e*f^2*p)/(40*d*x^8) + (e*f*(2*e*f - 5*d*g)*p)/(60*d^2*x^6) - (e*(6*e^2*f^
2 - 15*d*e*f*g + 10*d^2*g^2)*p)/(120*d^3*x^4) + (e^2*(6*e^2*f^2 - 15*d*e*f*
g + 10*d^2*g^2)*p)/(60*d^4*x^2) + (e^3*(6*e^2*f^2 - 15*d*e*f*g + 10*d^2*g^2
)*p*Log[x])/(30*d^5) - (e^3*(6*e^2*f^2 - 15*d*e*f*g + 10*d^2*g^2)*p*Log[d +
 e*x^2])/(60*d^5) - (f^2*Log[c*(d + e*x^2)^p])/(10*x^10) - (f*g*Log[c*(d +
e*x^2)^p])/(4*x^8) - (g^2*Log[c*(d + e*x^2)^p])/(6*x^6)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2414

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
```

```
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^{11}}\,dx = \frac{1}{2}\mathrm{Subst}\left(\int \frac{(f+gx)^2\log(c(d+ex)^p)}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{10x^{10}} - \frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{4x^8} - \frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{1}{2}$$

$$= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{10x^{10}} - \frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{4x^8} - \frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{1}{60}$$

$$= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{10x^{10}} - \frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{4x^8} - \frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{6x^6} - \frac{1}{60}$$

$$= -\frac{ef^2p}{40dx^8} + \frac{ef(2ef-5dg)p}{60d^2x^6} - \frac{e\left(6e^2f^2-15defg+10d^2g^2\right)p}{120d^3x^4} + \frac{e^2\left(6e^2f^2-\right.}{}$$

Mathematica [A] time = 0.24, size = 215, normalized size = 0.85

$$-\frac{2d^5\left(6f^2+15fgx^2+10g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right)-4e^3px^{10}\log(x)\left(10d^2g^2-15defg+6e^2f^2\right)+2e^3px^{10}\left(10d^2g^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^11,x]
```

```
[Out] -1/120*(d*e*p*x^2*(-12*e^3*f^2*x^6 + 6*d*e^2*f*x^4*(f + 5*g*x^2) + d^3*(3*f
^2 + 10*f*g*x^2 + 10*g^2*x^4) - d^2*e*x^2*(4*f^2 + 15*f*g*x^2 + 20*g^2*x^4)
) - 4*e^3*(6*e^2*f^2 - 15*d*e*f*g + 10*d^2*g^2)*p*x^10*Log[x] + 2*e^3*(6*e^
2*f^2 - 15*d*e*f*g + 10*d^2*g^2)*p*x^10*Log[d + e*x^2] + 2*d^5*(6*f^2 + 15*
f*g*x^2 + 10*g^2*x^4)*Log[c*(d + e*x^2)^p])/(d^5*x^10)
```

fricas [A] time = 0.67, size = 268, normalized size = 1.06

$$\frac{4\left(6e^5f^2-15de^4fg+10d^2e^3g^2\right)px^{10}\log(x)-3d^4ef^2px^2+2\left(6de^4f^2-15d^2e^3fg+10d^3e^2g^2\right)px^8-\left(6d^2e^3f^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^11,x, algorithm="fricas")
```

```
[Out] 1/120*(4*(6*e^5*f^2 - 15*d*e^4*f*g + 10*d^2*e^3*g^2)*p*x^10*log(x) - 3*d^4*
e*f^2*p*x^2 + 2*(6*d*e^4*f^2 - 15*d^2*e^3*f*g + 10*d^3*e^2*g^2)*p*x^8 - (6*
d^2*e^3*f^2 - 15*d^3*e^2*f*g + 10*d^4*e*g^2)*p*x^6 + 2*(2*d^3*e^2*f^2 - 5*d
^4*e*f*g)*p*x^4 - 2*(10*d^5*g^2*p*x^4 + (6*e^5*f^2 - 15*d*e^4*f*g + 10*d^2*
```

```
e^3*g^2)*p*x^10 + 15*d^5*f*g*p*x^2 + 6*d^5*f^2*p)*log(e*x^2 + d) - 2*(10*d^
5*g^2*x^4 + 15*d^5*f*g*x^2 + 6*d^5*f^2)*log(c))/(d^5*x^10)
```

giac [B] time = 0.30, size = 1340, normalized size = 5.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^11,x, algorithm="giac")
```

```
[Out] -1/120*(20*(x^2*e + d)^5*d^2*g^2*p*e^4*log(x^2*e + d) - 100*(x^2*e + d)^4*d
^3*g^2*p*e^4*log(x^2*e + d) + 200*(x^2*e + d)^3*d^4*g^2*p*e^4*log(x^2*e + d
) - 180*(x^2*e + d)^2*d^5*g^2*p*e^4*log(x^2*e + d) + 60*(x^2*e + d)*d^6*g^2
*p*e^4*log(x^2*e + d) - 20*(x^2*e + d)^5*d^2*g^2*p*e^4*log(x^2*e) + 100*(x^
2*e + d)^4*d^3*g^2*p*e^4*log(x^2*e) - 200*(x^2*e + d)^3*d^4*g^2*p*e^4*log(x
^2*e) + 200*(x^2*e + d)^2*d^5*g^2*p*e^4*log(x^2*e) - 100*(x^2*e + d)*d^6*g^
2*p*e^4*log(x^2*e) + 20*d^7*g^2*p*e^4*log(x^2*e) - 20*(x^2*e + d)^4*d^3*g^2
*p*e^4 + 90*(x^2*e + d)^3*d^4*g^2*p*e^4 - 150*(x^2*e + d)^2*d^5*g^2*p*e^4 +
 110*(x^2*e + d)*d^6*g^2*p*e^4 - 30*d^7*g^2*p*e^4 - 30*(x^2*e + d)^5*d*f*g*
p*e^5*log(x^2*e + d) + 150*(x^2*e + d)^4*d^2*f*g*p*e^5*log(x^2*e + d) - 300
*(x^2*e + d)^3*d^3*f*g*p*e^5*log(x^2*e + d) + 300*(x^2*e + d)^2*d^4*f*g*p*e
^5*log(x^2*e + d) - 120*(x^2*e + d)*d^5*f*g*p*e^5*log(x^2*e + d) + 30*(x^2*
e + d)^5*d*f*g*p*e^5*log(x^2*e) - 150*(x^2*e + d)^4*d^2*f*g*p*e^5*log(x^2*e
) + 300*(x^2*e + d)^3*d^3*f*g*p*e^5*log(x^2*e) - 300*(x^2*e + d)^2*d^4*f*g*
p*e^5*log(x^2*e) + 150*(x^2*e + d)*d^5*f*g*p*e^5*log(x^2*e) - 30*d^6*f*g*p*
e^5*log(x^2*e) + 20*(x^2*e + d)^2*d^5*g^2*e^4*log(c) - 40*(x^2*e + d)*d^6*g
^2*e^4*log(c) + 20*d^7*g^2*e^4*log(c) + 30*(x^2*e + d)^4*d^2*f*g*p*e^5 - 13
5*(x^2*e + d)^3*d^3*f*g*p*e^5 + 235*(x^2*e + d)^2*d^4*f*g*p*e^5 - 185*(x^2*
e + d)*d^5*f*g*p*e^5 + 55*d^6*f*g*p*e^5 + 12*(x^2*e + d)^5*f^2*p*e^6*log(x^
2*e + d) - 60*(x^2*e + d)^4*d*f^2*p*e^6*log(x^2*e + d) + 120*(x^2*e + d)^3*
d^2*f^2*p*e^6*log(x^2*e + d) - 120*(x^2*e + d)^2*d^3*f^2*p*e^6*log(x^2*e +
d) + 60*(x^2*e + d)*d^4*f^2*p*e^6*log(x^2*e + d) - 12*(x^2*e + d)^5*f^2*p*e
^6*log(x^2*e) + 60*(x^2*e + d)^4*d*f^2*p*e^6*log(x^2*e) - 120*(x^2*e + d)^3
*d^2*f^2*p*e^6*log(x^2*e) + 120*(x^2*e + d)^2*d^3*f^2*p*e^6*log(x^2*e) - 60
*(x^2*e + d)*d^4*f^2*p*e^6*log(x^2*e) + 12*d^5*f^2*p*e^6*log(x^2*e) + 30*(x
^2*e + d)*d^5*f*g*e^5*log(c) - 30*d^6*f*g*e^5*log(c) - 12*(x^2*e + d)^4*d*f
^2*p*e^6 + 54*(x^2*e + d)^3*d^2*f^2*p*e^6 - 94*(x^2*e + d)^2*d^3*f^2*p*e^6
+ 77*(x^2*e + d)*d^4*f^2*p*e^6 - 25*d^5*f^2*p*e^6 + 12*d^5*f^2*e^6*log(c))*
e^(-1)/((x^2*e + d)^5*d^5 - 5*(x^2*e + d)^4*d^6 + 10*(x^2*e + d)^3*d^7 - 10
*(x^2*e + d)^2*d^8 + 5*(x^2*e + d)*d^9 - d^10)
```

maple [C] time = 0.53, size = 748, normalized size = 2.96

$$-\frac{\left(10g^2x^4+15fg\,x^2+6f^2\right)\ln\left(\left(e\,x^2+d\right)^p\right)}{60x^{10}}+\frac{-12d^5f^2\ln(c)-30d^2e^3fgp\,x^8+15d^3e^2fgp\,x^6-10d^4efgp\,x^4+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^11,x)
```

```
[Out] -1/60*(10*g^2*x^4+15*f*g*x^2+6*f^2)/x^10*ln((e*x^2+d)^p)+1/120*(-12*ln(c)*d
^5*f^2-30*d^2*e^3*f*g*p*x^8+15*d^3*e^2*f*g*p*x^6-10*d^4*e*f*g*p*x^4+20*d^3*
e^2*g^2*p*x^8+12*d*e^4*f^2*p*x^8-10*d^4*e*g^2*p*x^6-6*d^2*e^3*f^2*p*x^6+4*d
^3*e^2*f^2*p*x^4-3*d^4*e*f^2*p*x^2+24*ln(x)*e^5*f^2*p*x^10-12*ln(e*x^2+d)*e
^5*f^2*p*x^10-30*ln(c)*d^5*f*g*x^2-20*ln(c)*d^5*g^2*x^4+15*I*Pi*d^5*f*g*x^2
*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+10*I*Pi*d^5*g^2*x^4*cs
gn(I*c*(e*x^2+d)^p)^3-6*I*Pi*d^5*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)
^p)^2-15*I*Pi*d^5*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-6*I*Pi*d^5*f^2*
csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+6*I*Pi*d^5*f^2*csgn(I*(e*x^2+d)^p)*csgn(I
```

```
*c*(e*x^2+d)^p)*csgn(I*c)+6*I*Pi*d^5*f^2*csgn(I*c*(e*x^2+d)^p)^3+40*ln(x)*d
^2*e^3*g^2*p*x^10-60*ln(x)*d*e^4*f*g*p*x^10+30*ln(e*x^2+d)*d*e^4*f*g*p*x^10
-15*I*Pi*d^5*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+10*I*Pi*d^
5*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-20*ln(e*x^2+d
)*d^2*e^3*g^2*p*x^10-10*I*Pi*d^5*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+
15*I*Pi*d^5*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^3-10*I*Pi*d^5*g^2*x^4*csgn(I*(e*x
^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2)/d^5/x^10
```

maxima [A] time = 0.50, size = 223, normalized size = 0.88

$$-\frac{1}{120}\,ep\left(\frac{2\left(6\,e^4f^2-15\,de^3fg+10\,d^2e^2g^2\right)\log\left(ex^2+d\right)}{d^5}-\frac{2\left(6\,e^4f^2-15\,de^3fg+10\,d^2e^2g^2\right)\log\left(x^2\right)}{d^5}-\frac{2\left(6\,e^3f^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^11,x, algorithm="maxima")
```

```
[Out] -1/120*e*p*(2*(6*e^4*f^2 - 15*d*e^3*f*g + 10*d^2*e^2*g^2)*log(e*x^2 + d)/d^
5 - 2*(6*e^4*f^2 - 15*d*e^3*f*g + 10*d^2*e^2*g^2)*log(x^2)/d^5 - (2*(6*e^3*
f^2 - 15*d*e^2*f*g + 10*d^2*e*g^2)*x^6 - 3*d^3*f^2 - (6*d*e^2*f^2 - 15*d^2*
e*f*g + 10*d^3*g^2)*x^4 + 2*(2*d^2*e*f^2 - 5*d^3*f*g)*x^2)/(d^4*x^8)) - 1/6
0*(10*g^2*x^4 + 15*f*g*x^2 + 6*f^2)*log((e*x^2 + d)^p*c)/x^10
```

mupad [B] time = 0.46, size = 225, normalized size = 0.89

$$\frac{\ln(x)\,\left(10\,p\,d^2\,e^3\,g^2-15\,p\,d\,e^4\,f\,g+6\,p\,e^5\,f^2\right)}{30\,d^5}-\frac{\ln\left(c\,\left(e\,x^2+d\right)^p\right)\,\left(\frac{f^2}{10}+\frac{f\,g\,x^2}{4}+\frac{g^2\,x^4}{6}\right)}{x^{10}}-\frac{\ln\left(e\,x^2+d\right)\,\left(10\,p\,d^2\,e^3\,g^2\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^11,x)
```

```
[Out] (log(x)*(6*e^5*f^2*p + 10*d^2*e^3*g^2*p - 15*d*e^4*f*g*p))/(30*d^5) - (log(
c*(d + e*x^2)^p)*(f^2/10 + (g^2*x^4)/6 + (f*g*x^2)/4))/x^10 - (log(d + e*x^
2)*(6*e^5*f^2*p + 10*d^2*e^3*g^2*p - 15*d*e^4*f*g*p))/(60*d^5) - ((3*e*f^2*
p)/(4*d) - (e^2*p*x^6*(10*d^2*g^2 + 6*e^2*f^2 - 15*d*e*f*g))/(2*d^4) + (e*p
*x^4*(10*d^2*g^2 + 6*e^2*f^2 - 15*d*e*f*g))/(4*d^3) + (e*f*p*x^2*(5*d*g - 2
*e*f))/(2*d^2))/(30*x^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**11,x)
```

```
[Out] Timed out
```

3.332 $\int x^2 \left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=278

$$\frac{1}{3}f^2x^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{5}fgx^5\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{4d^{5/2}fg}{}$$

```
[Out] 2/3*d*f^2*p*x/e-4/5*d^2*f*g*p*x/e^2+2/7*d^3*g^2*p*x/e^3-2/9*f^2*p*x^3+4/15*
d*f*g*p*x^3/e-2/21*d^2*g^2*p*x^3/e^2-4/25*f*g*p*x^5+2/35*d*g^2*p*x^5/e-2/49
*g^2*p*x^7-2/3*d^(3/2)*f^2*p*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)+4/5*d^(5/2)*
f*g*p*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)-2/7*d^(7/2)*g^2*p*arctan(x*e^(1/2)/
d^(1/2))/e^(7/2)+1/3*f^2*x^3*ln(c*(e*x^2+d)^p)+2/5*f*g*x^5*ln(c*(e*x^2+d)^p
)+1/7*g^2*x^7*ln(c*(e*x^2+d)^p)
```

Rubi [A] time = 0.24, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2476, 2455, 302, 205}

$$\frac{1}{3}f^2x^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{5}fgx^5\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}-\frac{4d^2fgp}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (2*d*f^2*p*x)/(3*e) - (4*d^2*f*g*p*x)/(5*e^2) + (2*d^3*g^2*p*x)/(7*e^3) - (
2*f^2*p*x^3)/9 + (4*d*f*g*p*x^3)/(15*e) - (2*d^2*g^2*p*x^3)/(21*e^2) - (4*f
*g*p*x^5)/25 + (2*d*g^2*p*x^5)/(35*e) - (2*g^2*p*x^7)/49 - (2*d^(3/2)*f^2*p
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(3/2)) + (4*d^(5/2)*f*g*p*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/(5*e^(5/2)) - (2*d^(7/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(7*e^(7/2)) + (f^2*x^3*Log[c*(d + e*x^2)^p])/3 + (2*f*g*x^5*Log[c*(d + e*x^
2)^p])/5 + (g^2*x^7*Log[c*(d + e*x^2)^p])/7
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int x^2 \left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(f^2x^2 \log\left(c\left(d+ex^2\right)^p\right) + 2fgx^4 \log\left(c\left(d+ex^2\right)^p\right) + g^2x^6 \log\left(c\left(d+\right.\right.\right.$$

$$= f^2 \int x^2 \log\left(c\left(d+ex^2\right)^p\right)\,dx + (2fg)\int x^4 \log\left(c\left(d+ex^2\right)^p\right)\,dx + g^2 \int$$

$$= \frac{1}{3}f^2x^3 \log\left(c\left(d+ex^2\right)^p\right) + \frac{2}{5}fgx^5 \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{7}g^2x^7 \log\left(c\left(d+\right.\right.\right.$$

$$= \frac{1}{3}f^2x^3 \log\left(c\left(d+ex^2\right)^p\right) + \frac{2}{5}fgx^5 \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{7}g^2x^7 \log\left(c\left(d+\right.\right.\right.$$

$$= \frac{2df^2px}{3e} - \frac{4d^2fgpx}{5e^2} + \frac{2d^3g^2px}{7e^3} - \frac{2}{9}f^2px^3 + \frac{4dfgpx^3}{15e} - \frac{2d^2g^2px^3}{21e^2} - \frac{4}{25}$$

$$= \frac{2df^2px}{3e} - \frac{4d^2fgpx}{5e^2} + \frac{2d^3g^2px}{7e^3} - \frac{2}{9}f^2px^3 + \frac{4dfgpx^3}{15e} - \frac{2d^2g^2px^3}{21e^2} - \frac{4}{25}$$

Mathematica [A] time = 0.18, size = 188, normalized size = 0.68

$$\frac{\sqrt{e}\,x\left(105e^3x^2\left(35f^2+42fgx^2+15g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right)+2p\left(1575d^3g^2-105d^2eg\left(42f+5gx^2\right)+105de^2\left(35\right.\right.\right.}{110}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-210*d^(3/2)*(35*e^2*f^2 - 42*d*e*f*g + 15*d^2*g^2)*p*ArcTan[(Sqrt[e]*x)/S
qrt[d]] + Sqrt[e]*x*(2*p*(1575*d^3*g^2 - 105*d^2*e*g*(42*f + 5*g*x^2) + 105
*d*e^2*(35*f^2 + 14*f*g*x^2 + 3*g^2*x^4) - e^3*x^2*(1225*f^2 + 882*f*g*x^2
+ 225*g^2*x^4)) + 105*e^3*x^2*(35*f^2 + 42*f*g*x^2 + 15*g^2*x^4)*Log[c*(d +
 e*x^2)^p]))/(11025*e^(7/2))
```

fricas [A] time = 0.64, size = 492, normalized size = 1.77

$$\left[-\frac{450\,e^3g^2px^7+126\left(14\,e^3fg-5\,de^2g^2\right)px^5+70\left(35\,e^3f^2-42\,de^2fg+15\,d^2eg^2\right)px^3-105\left(35\,de^2f^2-42\,d^2efg\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/11025*(450*e^3*g^2*p*x^7 + 126*(14*e^3*f*g - 5*d*e^2*g^2)*p*x^5 + 70*(3
5*e^3*f^2 - 42*d*e^2*f*g + 15*d^2*e*g^2)*p*x^3 - 105*(35*d*e^2*f^2 - 42*d^2
*e*f*g + 15*d^3*g^2)*p*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2
 + d)) - 210*(35*d*e^2*f^2 - 42*d^2*e*f*g + 15*d^3*g^2)*p*x - 105*(15*e^3*g
^2*p*x^7 + 42*e^3*f*g*p*x^5 + 35*e^3*f^2*p*x^3)*log(e*x^2 + d) - 105*(15*e^
3*g^2*x^7 + 42*e^3*f*g*x^5 + 35*e^3*f^2*x^3)*log(c))/e^3, -1/11025*(450*e^3
*g^2*p*x^7 + 126*(14*e^3*f*g - 5*d*e^2*g^2)*p*x^5 + 70*(35*e^3*f^2 - 42*d*e
^2*f*g + 15*d^2*e*g^2)*p*x^3 + 210*(35*d*e^2*f^2 - 42*d^2*e*f*g + 15*d^3*g^
2)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) - 210*(35*d*e^2*f^2 - 42*d^2*e*f*g +
 15*d^3*g^2)*p*x - 105*(15*e^3*g^2*p*x^7 + 42*e^3*f*g*p*x^5 + 35*e^3*f^2*p*
x^3)*log(e*x^2 + d) - 105*(15*e^3*g^2*x^7 + 42*e^3*f*g*x^5 + 35*e^3*f^2*x^3
)*log(c))/e^3]
```

giac [A] time = 0.25, size = 246, normalized size = 0.88

$$-\frac{2\left(15\,d^{4}g^{2}p-42\,d^{3}fgpe+35\,d^{2}f^{2}pe^{2}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{105\,\sqrt{d}}+\frac{1}{11025}\left(1575\,g^{2}px^{7}e^{3}\log\left(x^{2}e+d\right)-450\,g^{2}px^{7}e^{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -2/105*(15*d^4*g^2*p - 42*d^3*f*g*p*e + 35*d^2*f^2*p*e^2)*arctan(x*e^(1/2)/
sqrt(d))*e^(-7/2)/sqrt(d) + 1/11025*(1575*g^2*p*x^7*e^3*log(x^2*e + d) - 45
0*g^2*p*x^7*e^3 + 1575*g^2*x^7*e^3*log(c) + 630*d*g^2*p*x^5*e^2 + 4410*f*g*
p*x^5*e^3*log(x^2*e + d) - 1764*f*g*p*x^5*e^3 - 1050*d^2*g^2*p*x^3*e + 4410
*f*g*x^5*e^3*log(c) + 2940*d*f*g*p*x^3*e^2 + 3675*f^2*p*x^3*e^3*log(x^2*e +
 d) + 3150*d^3*g^2*p*x - 2450*f^2*p*x^3*e^3 - 8820*d^2*f*g*p*x*e + 3675*f^2
*x^3*e^3*log(c) + 7350*d*f^2*p*x*e^2)*e^(-3)
```

maple [C] time = 0.54, size = 761, normalized size = 2.74

$$\frac{g^{2}x^{7}\ln(c)}{7}+\frac{f^{2}x^{3}\ln(c)}{3}+\frac{2fg\,x^{5}\ln(c)}{5}-\frac{2g^{2}p\,x^{7}}{49}-\frac{2f^{2}p\,x^{3}}{9}-\frac{\sqrt{-de}\;d\,f^{2}p\ln\left(-d+\sqrt{-de}\;x\right)}{3e^{2}}+\frac{\sqrt{-de}\;d^{3}g^{2}p\ln\left(-d-\right.}{7e^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x)
```

```
[Out] 1/7*g^2*x^7*ln(c)+1/3*ln(c)*f^2*x^3+2/5*ln(c)*f*g*x^5-2/49*g^2*p*x^7-2/9*f^
2*p*x^3-1/3/e^2*(-d*e)^(1/2)*p*d*ln(-d+(-d*e)^(1/2)*x)*f^2+1/7/e^4*(-d*e)^(
1/2)*p*d^3*ln(-d-(-d*e)^(1/2)*x)*g^2-1/7/e^4*(-d*e)^(1/2)*p*d^3*ln(-d+(-d*e
)^(1/2)*x)*g^2+1/3/e^2*(-d*e)^(1/2)*p*d*ln(-d-(-d*e)^(1/2)*x)*f^2-1/5*I*Pi*
f*g*x^5*csgn(I*c*(e*x^2+d)^p)^3+1/14*I*Pi*g^2*x^7*csgn(I*(e*x^2+d)^p)*csgn(
I*c*(e*x^2+d)^p)^2+1/14*I*Pi*g^2*x^7*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/6*
I*Pi*f^2*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/6*I*Pi*f^2*x^3*c
sgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-4/25*f*g*p*x^5+2/7*d^3/e^3*g^2*p*x+(1/7*g^
2*x^7+2/5*f*g*x^5+1/3*f^2*x^3)*ln((e*x^2+d)^p)-2/21*d^2/e^2*g^2*p*x^3+2/35*
d/e*g^2*p*x^5+2/5/e^3*(-d*e)^(1/2)*p*d^2*ln(-d+(-d*e)^(1/2)*x)*f*g-2/5/e^3*
(-d*e)^(1/2)*p*d^2*ln(-d-(-d*e)^(1/2)*x)*f*g-1/6*I*Pi*f^2*x^3*csgn(I*(e*x^2
+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/5*I*Pi*f*g*x^5*csgn(I*(e*x^2+d)^p)
*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+2/3*d*f^2*p*x/e-1/14*I*Pi*g^2*x^7*csgn(I*c
*(e*x^2+d)^p)^3-1/6*I*Pi*f^2*x^3*csgn(I*c*(e*x^2+d)^p)^3-1/14*I*Pi*g^2*x^7*
csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/5*I*Pi*f*g*x^5*csgn(I
*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/5*I*Pi*f*g*x^5*csgn(I*c*(e*x^2+d)^p
)^2*csgn(I*c)-4/5*d^2*f*g*p*x/e^2+4/15*d*f*g*p*x^3/e
```

maxima [A] time = 1.02, size = 189, normalized size = 0.68

$$-\frac{2}{11025}\,ep\left(\frac{105\left(35\,d^{2}e^{2}f^{2}-42\,d^{3}efg+15\,d^{4}g^{2}\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^{4}}+\frac{225\,e^{3}g^{2}x^{7}+63\left(14\,e^{3}fg-5\,de^{2}g^{2}\right)x^{5}+35\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -2/11025*e*p*(105*(35*d^2*e^2*f^2 - 42*d^3*e*f*g + 15*d^4*g^2)*arctan(e*x/s
qrt(d*e))/(sqrt(d*e)*e^4) + (225*e^3*g^2*x^7 + 63*(14*e^3*f*g - 5*d*e^2*g^2
)*x^5 + 35*(35*e^3*f^2 - 42*d*e^2*f*g + 15*d^2*e*g^2)*x^3 - 105*(35*d*e^2*f
^2 - 42*d^2*e*f*g + 15*d^3*g^2)*x)/e^4) + 1/105*(15*g^2*x^7 + 42*f*g*x^5 +
35*f^2*x^3)*log((e*x^2 + d)^p*c)
```

mupad [B] time = 0.36, size = 235, normalized size = 0.85

$$\ln\left(c\left(e\,x^2+d\right)^p\right)\left(\frac{f^2\,x^3}{3}+\frac{2\,f\,g\,x^5}{5}+\frac{g^2\,x^7}{7}\right)-x^3\left(\frac{2\,f^2\,p}{9}-\frac{d\left(\frac{4\,f\,g\,p}{5}-\frac{2\,d\,g^2\,p}{7\,e}\right)}{3\,e}\right)-x^5\left(\frac{4\,f\,g\,p}{25}-\frac{2\,d\,g^2\,p}{35\,e}\right)-\frac{2\,g^2\,p\,x^7}{49}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f^2*x^3)/3 + (g^2*x^7)/7 + (2*f*g*x^5)/5) - x^3*((2*
f^2*p)/9 - (d*((4*f*g*p)/5 - (2*d*g^2*p)/(7*e)))/(3*e)) - x^5*((4*f*g*p)/25
 - (2*d*g^2*p)/(35*e)) - (2*g^2*p*x^7)/49 + (d*x*((2*f^2*p)/3 - (d*((4*f*g*
p)/5 - (2*d*g^2*p)/(7*e)))/e))/e - (2*d^(3/2)*p*atan((d^(3/2)*e^(1/2)*p*x*(
15*d^2*g^2 + 35*e^2*f^2 - 42*d*e*f*g))/(15*d^4*g^2*p + 35*d^2*e^2*f^2*p - 4
2*d^3*e*f*g*p))*(15*d^2*g^2 + 35*e^2*f^2 - 42*d*e*f*g))/(105*e^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

3.333 $\int \left(f+gx^2\right)^2 \log \left(c \left(d+ex^2\right)^p\right) \, dx$

Optimal. Leaf size=221

$$f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log\left(c\left(d+ex^2\right)^p\right)-\frac{4d^{3/2}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}g^2p\,\mathrm{t}}{5e}$$

```
[Out] -2*f^2*p*x+4/3*d*f*g*p*x/e-2/5*d^2*g^2*p*x/e^2-4/9*f*g*p*x^3+2/15*d*g^2*p*x
^3/e-2/25*g^2*p*x^5-4/3*d^(3/2)*f*g*p*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)+2/5
*d^(5/2)*g^2*p*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)+f^2*x*ln(c*(e*x^2+d)^p)+2/
3*f*g*x^3*ln(c*(e*x^2+d)^p)+1/5*g^2*x^5*ln(c*(e*x^2+d)^p)+2*f^2*p*arctan(x*
e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2471, 2448, 321, 205, 2455, 302}

$$f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log\left(c\left(d+ex^2\right)^p\right)-\frac{4d^{3/2}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}-\frac{2d^2g^2px}{5e^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f^2*p*x + (4*d*f*g*p*x)/(3*e) - (2*d^2*g^2*p*x)/(5*e^2) - (4*f*g*p*x^3)/
9 + (2*d*g^2*p*x^3)/(15*e) - (2*g^2*p*x^5)/25 + (2*Sqrt[d]*f^2*p*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/Sqrt[e] - (4*d^(3/2)*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(3*e^(3/2)) + (2*d^(5/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*e^(5/2)) + f
^2*x*Log[c*(d + e*x^2)^p] + (2*f*g*x^3*Log[c*(d + e*x^2)^p])/3 + (g^2*x^5*L
og[c*(d + e*x^2)^p])/5
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int \left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(f^2\log\left(c\left(d+ex^2\right)^p\right)+2fgx^2\log\left(c\left(d+ex^2\right)^p\right)+g^2x^4\log\left(c\left(d+ex^2\right)\right.\right.$$

$$= f^2\int \log\left(c\left(d+ex^2\right)^p\right)\,dx+(2fg)\int x^2\log\left(c\left(d+ex^2\right)^p\right)\,dx+g^2\int x^4\,\mathrm{lc}$$

$$= f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log\left(c\left(d+ex^2\right)\right.$$

$$= -2f^2px+f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log\left(\right.$$

$$= -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5+\frac{2\sqrt{d}\,f^2p}{}$$

$$= -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5+\frac{2\sqrt{d}\,f^2p}{}$$

Mathematica [A] time = 0.14, size = 151, normalized size = 0.68

$$\frac{\sqrt{e}\,x\left(15e^2\left(15f^2+10fgx^2+3g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right)-2p\left(45d^2g^2-15deg\left(10f+gx^2\right)+e^2\left(225f^2+50fgx^2\right.\right.\right.}{225e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (30*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]] + Sqrt[e]*x*(-2*p*(45*d^2*g^2 - 15*d*e*g*(10*f + g*x^2) + e^2*(225*f^2
 + 50*f*g*x^2 + 9*g^2*x^4)) + 15*e^2*(15*f^2 + 10*f*g*x^2 + 3*g^2*x^4)*Log[
c*(d + e*x^2)^p]))/(225*e^(5/2))
```

fricas [A] time = 1.13, size = 404, normalized size = 1.83

$$\left[-\frac{18e^2g^2px^5+10\left(10e^2fg-3deg^2\right)px^3-15\left(15e^2f^2-10defg+3d^2g^2\right)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2+2ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)+30\left(15e^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/225*(18*e^2*g^2*p*x^5 + 10*(10*e^2*f*g - 3*d*e*g^2)*p*x^3 - 15*(15*e^2*
f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) -
d)/(e*x^2 + d)) + 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*x - 15*(3*e^2*
g^2*p*x^5 + 10*e^2*f*g*p*x^3 + 15*e^2*f^2*p*x)*log(e*x^2 + d) - 15*(3*e^2*g
^2*x^5 + 10*e^2*f*g*x^3 + 15*e^2*f^2*x)*log(c))/e^2, -1/225*(18*e^2*g^2*p*x
^5 + 10*(10*e^2*f*g - 3*d*e*g^2)*p*x^3 - 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^
2*g^2)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 30*(15*e^2*f^2 - 10*d*e*f*g +
3*d^2*g^2)*p*x - 15*(3*e^2*g^2*p*x^5 + 10*e^2*f*g*p*x^3 + 15*e^2*f^2*p*x)*l
og(e*x^2 + d) - 15*(3*e^2*g^2*x^5 + 10*e^2*f*g*x^3 + 15*e^2*f^2*x)*log(c))/
e^2]
```

giac [A] time = 0.19, size = 201, normalized size = 0.91

$$\frac{2\left(3\,d^3g^2p-10\,d^2fgpe+15\,df^2pe^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{15\,\sqrt{d}}+\frac{1}{225}\left(45\,g^2px^5e^2\log\left(x^2e+d\right)-18\,g^2px^5e^2+45\,g^2x^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 2/15*(3*d^3*g^2*p - 10*d^2*f*g*p*e + 15*d*f^2*p*e^2)*arctan(x*e^(1/2)/sqrt(
d))*e^(-5/2)/sqrt(d) + 1/225*(45*g^2*p*x^5*e^2*log(x^2*e + d) - 18*g^2*p*x^
5*e^2 + 45*g^2*x^5*e^2*log(c) + 30*d*g^2*p*x^3*e + 150*f*g*p*x^3*e^2*log(x^
2*e + d) - 100*f*g*p*x^3*e^2 + 150*f*g*x^3*e^2*log(c) - 90*d^2*g^2*p*x + 30
0*d*f*g*p*x*e + 225*f^2*p*x*e^2*log(x^2*e + d) - 450*f^2*p*x*e^2 + 225*f^2*
x*e^2*log(c))*e^(-2)
```

maple [C] time = 0.09, size = 686, normalized size = 3.10

$$-\frac{i\pi\,g^2x^5\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^3}{10}-\frac{\sqrt{-de}\,f^2p\ln\left(d+\sqrt{-de}\,x\right)}{e}+\frac{\sqrt{-de}\,f^2p\ln\left(d-\sqrt{-de}\,x\right)}{e}-2f^2px-\frac{2g^2p\,x^5}{25}+\frac{g^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p),x)
```

```
[Out] -1/3*I*Pi*f*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I
*Pi*f^2*csgn(I*c*(e*x^2+d)^p)^3*x-1/e*(-d*e)^(1/2)*p*ln(d+(-d*e)^(1/2)*x)*f
^2+1/e*(-d*e)^(1/2)*p*ln(d-(-d*e)^(1/2)*x)*f^2-1/10*I*Pi*g^2*x^5*csgn(I*c*(
e*x^2+d)^p)^3-2*f^2*p*x-2/25*g^2*p*x^5+1/5*ln(c)*g^2*x^5+ln(c)*f^2*x+1/5/e^
3*(-d*e)^(1/2)*p*ln(d-(-d*e)^(1/2)*x)*g^2*d^2+(1/5*g^2*x^5+2/3*f*g*x^3+f^2*
x)*ln((e*x^2+d)^p)-1/3*I*Pi*f*g*x^3*csgn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*f^2*cs
gn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*x+1/2*I*Pi*f^2*csgn(I*c*(e*x^2+d)
^p)^2*csgn(I*c)*x-1/5/e^3*(-d*e)^(1/2)*p*ln(d+(-d*e)^(1/2)*x)*g^2*d^2+1/10*
I*Pi*g^2*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/10*I*Pi*g^2*x^5*
csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2/3*f*g*x^3*ln(c)-4/9*f*g*p*x^3-2/5*d^2/e
^2*g^2*p*x+2/15*d/e*g^2*p*x^3+4/3*d/e*f*g*p*x+2/3/e^2*(-d*e)^(1/2)*p*ln(d+(
-d*e)^(1/2)*x)*d*f*g-2/3/e^2*(-d*e)^(1/2)*p*ln(d-(-d*e)^(1/2)*x)*d*f*g-1/10
*I*Pi*g^2*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/3*I*Pi*
f*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/3*I*Pi*f*g*x^3*csgn(I
*c*(e*x^2+d)^p)^2*csgn(I*c)-1/2*I*Pi*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^
2+d)^p)*csgn(I*c)*x
```

maxima [A] time = 1.02, size = 150, normalized size = 0.68

$$\frac{2}{225}\,ep\left(\frac{15\left(15\,de^2f^2-10\,d^2efg+3\,d^3g^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^3}-\frac{9\,e^2g^2x^5+5\left(10\,e^2fg-3\,deg^2\right)x^3+15\left(15\,e^2f^2-10\right.}{e^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] 2/225*e*p*(15*(15*d*e^2*f^2 - 10*d^2*e*f*g + 3*d^3*g^2)*arctan(e*x/sqrt(d*e
))/(sqrt(d*e)*e^3) - (9*e^2*g^2*x^5 + 5*(10*e^2*f*g - 3*d*e*g^2)*x^3 + 15*(
15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*x)/e^3) + 1/15*(3*g^2*x^5 + 10*f*g*x^3
 + 15*f^2*x)*log((e*x^2 + d)^p*c)
```

mupad [B] time = 0.00, size = 193, normalized size = 0.87

$$\ln\left(c\left(e\,x^2+d\right)^p\right)\left(f^2\,x+\frac{2\,f\,g\,x^3}{3}+\frac{g^2\,x^5}{5}\right)-x\left(2\,f^2\,p-\frac{d\left(\frac{4\,f\,g\,p}{3}-\frac{2\,d\,g^2\,p}{5\,e}\right)}{e}\right)-x^3\left(\frac{4\,f\,g\,p}{9}-\frac{2\,d\,g^2\,p}{15\,e}\right)-\frac{2\,g^2\,p\,x^5}{25}+\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*(f^2*x + (g^2*x^5)/5 + (2*f*g*x^3)/3) - x*(2*f^2*p - (
d*((4*f*g*p)/3 - (2*d*g^2*p)/(5*e)))/e) - x^3*((4*f*g*p)/9 - (2*d*g^2*p)/(1
5*e)) - (2*g^2*p*x^5)/25 + (2*d^(1/2)*p*atan((d^(1/2)*e^(1/2)*p*x*(3*d^2*g^
2 + 15*e^2*f^2 - 10*d*e*f*g))/(3*d^3*g^2*p + 15*d*e^2*f^2*p - 10*d^2*e*f*g*
p))*(3*d^2*g^2 + 15*e^2*f^2 - 10*d*e*f*g))/(15*e^(5/2))
```

sympy [A] time = 95.79, size = 415, normalized size = 1.88

$$\begin{cases} \frac{id^{\frac{5}{2}}g^2p\log\left(d+ex^2\right)}{5e^3\sqrt{\frac{1}{e}}}-\frac{2id^{\frac{5}{2}}g^2p\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{5e^3\sqrt{\frac{1}{e}}}-\frac{2id^{\frac{3}{2}}fgp\log\left(d+ex^2\right)}{3e^2\sqrt{\frac{1}{e}}}+\frac{4id^{\frac{3}{2}}fgp\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{3e^2\sqrt{\frac{1}{e}}}+\frac{i\sqrt{d}f^2p\log\left(d+ex^2\right)}{e\sqrt{\frac{1}{e}}}-\frac{2i\sqrt{d}f^2p\,\mathrm{lo}}{} \\ \left(f^2x+\frac{2fgx^3}{3}+\frac{g^2x^5}{5}\right)\log\left(cd^p\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((I*d**(5/2)*g**2*p*log(d + e*x**2)/(5*e**3*sqrt(1/e)) - 2*I*d**(5
/2)*g**2*p*log(-I*sqrt(d)*sqrt(1/e) + x)/(5*e**3*sqrt(1/e)) - 2*I*d**(3/2)*
f*g*p*log(d + e*x**2)/(3*e**2*sqrt(1/e)) + 4*I*d**(3/2)*f*g*p*log(-I*sqrt(d
)*sqrt(1/e) + x)/(3*e**2*sqrt(1/e)) + I*sqrt(d)*f**2*p*log(d + e*x**2)/(e*s
qrt(1/e)) - 2*I*sqrt(d)*f**2*p*log(-I*sqrt(d)*sqrt(1/e) + x)/(e*sqrt(1/e))
- 2*d**2*g**2*p*x/(5*e**2) + 4*d*f*g*p*x/(3*e) + 2*d*g**2*p*x**3/(15*e) + f
**2*p*x*log(d + e*x**2) - 2*f**2*p*x + f**2*x*log(c) + 2*f*g*p*x**3*log(d +
 e*x**2)/3 - 4*f*g*p*x**3/9 + 2*f*g*x**3*log(c)/3 + g**2*p*x**5*log(d + e*x
**2)/5 - 2*g**2*p*x**5/25 + g**2*x**5*log(c)/5, Ne(e, 0)), ((f**2*x + 2*f*g
*x**3/3 + g**2*x**5/5)*log(c*d**p), True))
```

3.334 $\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^2} dx$

Optimal. Leaf size=178

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{x}+2fgx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}g^2x^3\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{e}f^2p\tan$$

```
[Out] -4*f*g*p*x+2/3*d*g^2*p*x/e-2/9*g^2*p*x^3-2/3*d^(3/2)*g^2*p*arctan(x*e^(1/2)
/d^(1/2))/e^(3/2)-f^2*ln(c*(e*x^2+d)^p)/x+2*f*g*x*ln(c*(e*x^2+d)^p)+1/3*g^2
*x^3*ln(c*(e*x^2+d)^p)+4*f*g*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)+2*
f^2*p*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2476, 2448, 321, 205, 2455, 302}

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{x}+2fgx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}g^2x^3\log\left(c\left(d+ex^2\right)^p\right)-\frac{2d^{3/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{e}f^2p\tan$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^2,x]
```

```
[Out] -4*f*g*p*x + (2*d*g^2*p*x)/(3*e) - (2*g^2*p*x^3)/9 + (2*Sqrt[e]*f^2*p*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (4*Sqrt[d]*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/Sqrt[e] - (2*d^(3/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(3/2)) -
(f^2*Log[c*(d + e*x^2)^p])/x + 2*f*g*x*Log[c*(d + e*x^2)^p] + (g^2*x^3*Log[
c*(d + e*x^2)^p])/3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
```

```
+ 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx^2)^2\log\left(c\left(d+ex^2\right)^p\right)}{x^2}dx = \int \left(2fg\log\left(c\left(d+ex^2\right)^p\right)+\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{x^2}+g^2x^2\log\left(c\left(d+ex^2\right)^p\right)\right.$$

$$= f^2\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2}dx+(2fg)\int \log\left(c\left(d+ex^2\right)^p\right)dx+g^2\int x^2\log$$

$$= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{x}+2fgx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}g^2x^3\log\left(c\left(d+ex^2\right)^p\right.$$

$$= -4fgpx+\frac{2\sqrt{e}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{x}+2fgx\log\left(c\left(d+\right.\right.$$

$$= -4fgpx+\frac{2dg^2px}{3e}-\frac{2}{9}g^2px^3+\frac{2\sqrt{e}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{4\sqrt{d}fgp\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{d}}\right.}{\sqrt{e}}$$

$$= -4fgpx+\frac{2dg^2px}{3e}-\frac{2}{9}g^2px^3+\frac{2\sqrt{e}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{4\sqrt{d}fgp\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{d}}\right.}{\sqrt{e}}$$

Mathematica [A] time = 0.15, size = 112, normalized size = 0.63

$$\frac{1}{9}\left(\left(-\frac{9f^2}{x}+18fgx+3g^2x^3\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{6p\left(-d^2g^2+6defg+3e^2f^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}}-\frac{2gpx\left(-3dg+18ef\right.}{e}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^2,x]
```

```
[Out] ((-2*g*p*x*(18*e*f - 3*d*g + e*g*x^2))/e + (6*(3*e^2*f^2 + 6*d*e*f*g - d^2*
g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(3/2)) + ((-9*f^2)/x + 18*f*
g*x + 3*g^2*x^3)*Log[c*(d + e*x^2)^p])/9
```

fricas [A] time = 0.93, size = 366, normalized size = 2.06

$$\left[-\frac{2de^2g^2px^4-3\left(3e^2f^2+6defg-d^2g^2\right)\sqrt{-de}\,px\log\left(\frac{ex^2+2\sqrt{-de}x-d}{ex^2+d}\right)+6\left(6de^2fg-d^2eg^2\right)px^2-3\left(de^2g^2px^4+\right.}{9de^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/9*(2*d*e^2*g^2*p*x^4 - 3*(3*e^2*f^2 + 6*d*e*f*g - d^2*g^2)*sqrt(-d*e)*p
*x*log((e*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(6*d*e^2*f*g - d^2*e*g
^2)*p*x^2 - 3*(d*e^2*g^2*p*x^4 + 6*d*e^2*f*g*p*x^2 - 3*d*e^2*f^2*p)*log(e*x
^2 + d) - 3*(d*e^2*g^2*x^4 + 6*d*e^2*f*g*x^2 - 3*d*e^2*f^2)*log(c))/(d*e^2*
x), -1/9*(2*d*e^2*g^2*p*x^4 - 6*(3*e^2*f^2 + 6*d*e*f*g - d^2*g^2)*sqrt(d*e)
*p*x*arctan(sqrt(d*e)*x/d) + 6*(6*d*e^2*f*g - d^2*e*g^2)*p*x^2 - 3*(d*e^2*g
^2*p*x^4 + 6*d*e^2*f*g*p*x^2 - 3*d*e^2*f^2*p)*log(e*x^2 + d) - 3*(d*e^2*g^2
*x^4 + 6*d*e^2*f*g*x^2 - 3*d*e^2*f^2)*log(c))/(d*e^2*x)]
```

giac [A] time = 0.18, size = 168, normalized size = 0.94

$$-\frac{2\left(d^2g^2p-6\,dfgpe-3\,f^2pe^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{3\,\sqrt{d}}+\frac{\left(3\,g^2px^4e\log\left(x^2e+d\right)-2\,g^2px^4e+3\,g^2x^4e\log(c)+18\,f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^2,x, algorithm="giac")
```

```
[Out] -2/3*(d^2*g^2*p - 6*d*f*g*p*e - 3*f^2*p*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-
3/2)/sqrt(d) + 1/9*(3*g^2*p*x^4*e*log(x^2*e + d) - 2*g^2*p*x^4*e + 3*g^2*x^
4*e*log(c) + 18*f*g*p*x^2*e*log(x^2*e + d) + 6*d*g^2*p*x^2 - 36*f*g*p*x^2*e
 + 18*f*g*x^2*e*log(c) - 9*f^2*p*e*log(x^2*e + d) - 9*f^2*e*log(c))*e^(-1)/
x
```

maple [C] time = 0.59, size = 742, normalized size = 4.17

$$-\frac{\left(-g^2x^4-6fg\,x^2+3f^2\right)\ln\left(\left(e\,x^2+d\right)^p\right)}{3x}-\frac{3i\pi d\,e^2g^2x^4\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)-3i\pi}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^2,x)
```

```
[Out] -1/3*(-g^2*x^4-6*f*g*x^2+3*f^2)/x*ln((e*x^2+d)^p)-1/18*(-18*I*Pi*f*g*csgn(I
*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*x^2*e^2*d-18*I*Pi*f*g*csgn(I*c*(e*x^2
+d)^p)^2*csgn(I*c)*x^2*e^2*d+18*I*Pi*f*g*csgn(I*c*(e*x^2+d)^p)^3*x^2*e^2*d+
9*I*Pi*d*e^2*f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-3*I*Pi*g^2*x^4*csgn(I*c*
(e*x^2+d)^p)^2*csgn(I*c)*e^2*d+18*I*Pi*f*g*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*
x^2+d)^p)*csgn(I*c)*x^2*e^2*d-9*I*Pi*d*e^2*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c
*(e*x^2+d)^p)*csgn(I*c)+9*I*Pi*d*e^2*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^
2+d)^p)^2-6*ln(c)*g^2*x^4*e^2*d+3*I*Pi*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c
*(e*x^2+d)^p)*csgn(I*c)*e^2*d+3*I*Pi*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^3*e^2*d-
3*I*Pi*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*e^2*d-9*I*Pi*d*e
^2*f^2*csgn(I*c*(e*x^2+d)^p)^3+4*d*e^2*g^2*p*x^4-36*ln(c)*f*g*x^2*e^2*d-6*(
-d*e)^(1/2)*d^2*p*ln(-d-(-d*e)^(1/2)*x)*g^2*x+36*(-d*e)^(1/2)*p*ln(-d-(-d*e
)^(1/2)*x)*f*g*e*d*x+18*(-d*e)^(1/2)*p*ln(-d-(-d*e)^(1/2)*x)*f^2*e^2*x+6*(-
d*e)^(1/2)*d^2*p*ln(d-(-d*e)^(1/2)*x)*g^2*x-36*(-d*e)^(1/2)*p*ln(d-(-d*e)^(
1/2)*x)*f*g*e*d*x-18*(-d*e)^(1/2)*p*ln(d-(-d*e)^(1/2)*x)*f^2*e^2*x-12*d^2*e
*g^2*p*x^2+72*d*f*g*p*x^2*e^2+18*ln(c)*d*e^2*f^2)/e^2/d/x
```

maxima [A] time = 1.04, size = 112, normalized size = 0.63

$$\frac{2}{9}\,ep\left(\frac{3\left(3\,e^2f^2+6\,defg-d^2g^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^2}-\frac{eg^2x^3+3\left(6\,efg-dg^2\right)x}{e^2}\right)+\frac{1}{3}\left(g^2x^3+6\,fgx-\frac{3\,f^2}{x}\right)\log\left(\left(ex^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^2,x, algorithm="maxima")
```

```
[Out] 2/9*e*p*(3*(3*e^2*f^2 + 6*d*e*f*g - d^2*g^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*
e)*e^2) - (e*g^2*x^3 + 3*(6*e*f*g - d*g^2)*x)/e^2) + 1/3*(g^2*x^3 + 6*f*g*x
 - 3*f^2/x)*log((e*x^2 + d)^p*c)
```

mupad [B] time = 0.37, size = 180, normalized size = 1.01

$$\frac{2\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,p\,x\,\left(-d^2\,g^2+6\,d\,e\,f\,g+3\,e^2\,f^2\right)}{\sqrt{d}\,\left(-p\,d^2\,g^2+6\,p\,d\,e\,f\,g+3\,p\,e^2\,f^2\right)}\right)\,\left(-d^2\,g^2+6\,d\,e\,f\,g+3\,e^2\,f^2\right)}{3\,\sqrt{d}\,e^{3/2}}-x\,\left(4\,f\,g\,p-\frac{2\,d\,g^2\,p}{3\,e}\right)-\frac{2\,g^2\,p\,x^3}{9}-\ln\left(c\,\left(e\,x^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^2,x)
```

```
[Out] (2*p*atan((e^(1/2)*p*x*(3*e^2*f^2 - d^2*g^2 + 6*d*e*f*g))/(d^(1/2)*(3*e^2*f
^2*p - d^2*g^2*p + 6*d*e*f*g*p)))*(3*e^2*f^2 - d^2*g^2 + 6*d*e*f*g))/(3*d^(
1/2)*e^(3/2)) - x*(4*f*g*p - (2*d*g^2*p)/(3*e)) - (2*g^2*p*x^3)/9 - log(c*(
d + e*x^2)^p)*((f^2 + g^2*x^4 + 2*f*g*x^2)/x - ((4*g^2*x^4)/3 + 4*f*g*x^2)/
x)
```

sympy [A] time = 172.74, size = 510, normalized size = 2.87

$$\begin{cases} \left(-\frac{f^2}{x}+2fgx+\frac{g^2x^3}{3}\right)\log\left(0^pc\right) \\ -\frac{f^2p\log(e)}{x}-\frac{2f^2p\log(x)}{x}-\frac{2f^2p}{x}-\frac{f^2\log(c)}{x}+2fgpx\log(e)+4fgpx\log(x)-4fgpx+2fgx\log(c)+\frac{g^2px^3\log(e)}{3}+ \\ \left(-\frac{f^2}{x}+2fgx+\frac{g^2x^3}{3}\right)\log\left(cd^p\right) \\ -\frac{id^{\frac{3}{2}}g^2p\log\left(d+ex^2\right)}{3e^2\sqrt{\frac{1}{e}}}+\frac{2id^{\frac{3}{2}}g^2p\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{3e^2\sqrt{\frac{1}{e}}}+\frac{2i\sqrt{d}fgp\log\left(d+ex^2\right)}{e\sqrt{\frac{1}{e}}}-\frac{4i\sqrt{d}fgp\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+x\right)}{e\sqrt{\frac{1}{e}}}+\frac{2dg^2px}{3e}-\frac{f^2p\log\left(d+ex^2\right)}{x}- \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**2,x)
```

```
[Out] Piecewise(((-f**2/x + 2*f*g*x + g**2*x**3/3)*log(0**p*c), Eq(d, 0) & Eq(e,
0)), (-f**2*p*log(e)/x - 2*f**2*p*log(x)/x - 2*f**2*p/x - f**2*log(c)/x + 2
*f*g*p*x*log(e) + 4*f*g*p*x*log(x) - 4*f*g*p*x + 2*f*g*x*log(c) + g**2*p*x*
*3*log(e)/3 + 2*g**2*p*x**3*log(x)/3 - 2*g**2*p*x**3/9 + g**2*x**3*log(c)/3
, Eq(d, 0)), ((-f**2/x + 2*f*g*x + g**2*x**3/3)*log(c*d**p), Eq(e, 0)), (-I
*d**(3/2)*g**2*p*log(d + e*x**2)/(3*e**2*sqrt(1/e)) + 2*I*d**(3/2)*g**2*p*l
og(-I*sqrt(d)*sqrt(1/e) + x)/(3*e**2*sqrt(1/e)) + 2*I*sqrt(d)*f*g*p*log(d +
 e*x**2)/(e*sqrt(1/e)) - 4*I*sqrt(d)*f*g*p*log(-I*sqrt(d)*sqrt(1/e) + x)/(e
*sqrt(1/e)) + 2*d*g**2*p*x/(3*e) - f**2*p*log(d + e*x**2)/x - f**2*log(c)/x
 + 2*f*g*p*x*log(d + e*x**2) - 4*f*g*p*x + 2*f*g*x*log(c) + g**2*p*x**3*log
(d + e*x**2)/3 - 2*g**2*p*x**3/9 + g**2*x**3*log(c)/3 + I*f**2*p*log(d + e*
x**2)/(sqrt(d)*sqrt(1/e)) - 2*I*f**2*p*log(-I*sqrt(d)*sqrt(1/e) + x)/(sqrt(
d)*sqrt(1/e)), True))
```

3.335 $\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^4} \, dx$

Optimal. Leaf size=169

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{x}+g^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{2e^{3/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{2ef^2p}{3dx}+\frac{4\sqrt{e}fg}{}$$

```
[Out] -2/3*e*f^2*p/d/x-2*g^2*p*x-2/3*e^(3/2)*f^2*p*arctan(x*e^(1/2)/d^(1/2))/d^(3
/2)-1/3*f^2*ln(c*(e*x^2+d)^p)/x^3-2*f*g*ln(c*(e*x^2+d)^p)/x+g^2*x*ln(c*(e*x
^2+d)^p)+2*g^2*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)+4*f*g*p*arctan(x
*e^(1/2)/d^(1/2))*e^(1/2)/d^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2476, 2448, 321, 205, 2455, 325}

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{x}+g^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{2e^{3/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{2ef^2p}{3dx}+\frac{4\sqrt{e}fg}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^4,x]
```

```
[Out] (-2*e*f^2*p)/(3*d*x) - 2*g^2*p*x - (2*e^(3/2)*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(3*d^(3/2)) + (4*Sqrt[e]*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] +
 (2*Sqrt[d]*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (f^2*Log[c*(d + e*
x^2)^p])/(3*x^3) - (2*f*g*Log[c*(d + e*x^2)^p])/x + g^2*x*Log[c*(d + e*x^2)
^p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx^2)^2\log\left(c\left(d+ex^2\right)^p\right)}{x^4}dx = \int \left(g^2\log\left(c\left(d+ex^2\right)^p\right) + \frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{x^4} + \frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\right)$$

$$= f^2\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^4}dx + (2fg)\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2}dx + g^2\int \log\left(c\right.$$

$$= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} - \frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{x} + g^2x\log\left(c\left(d+ex^2\right)^p\right) +$$

$$= -\frac{2ef^2p}{3dx} - 2g^2px + \frac{4\sqrt{e}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} - \frac{2fg\log}{}$$

$$= -\frac{2ef^2p}{3dx} - 2g^2px - \frac{2e^{3/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} + \frac{4\sqrt{e}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{d}g^2}{}$$

Mathematica [C] time = 0.14, size = 113, normalized size = 0.67

$$-\frac{\left(f^2+6fgx^2-3g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} - \frac{2ef^2p\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{ex^2}{d}\right)}{3dx} + \frac{2gp(dg+2ef)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}} - 2g^2px$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^4,x]
```

```
[Out] -2*g^2*p*x + (2*g*(2*e*f + d*g)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqr
t[e]) - (2*e*f^2*p*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) -
 ((f^2 + 6*f*g*x^2 - 3*g^2*x^4)*Log[c*(d + e*x^2)^p])/(3*x^3)
```

fricas [A] time = 0.98, size = 350, normalized size = 2.07

$$\left[-\frac{6d^2eg^2px^4 + 2de^2f^2px^2 - \left(e^2f^2 - 6defg - 3d^2g^2\right)\sqrt{-de}\,px^3\log\left(\frac{ex^2-2\sqrt{-de}\,x-d}{ex^2+d}\right) - \left(3d^2eg^2px^4 - 6d^2efgpx^2 - \right.}{3d^2ex^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/3*(6*d^2*e*g^2*p*x^4 + 2*d*e^2*f^2*p*x^2 - (e^2*f^2 - 6*d*e*f*g - 3*d^2
*g^2)*sqrt(-d*e)*p*x^3*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - (3*d
```

```
^2*e*g^2*p*x^4 - 6*d^2*e*f*g*p*x^2 - d^2*e*f^2*p)*log(e*x^2 + d) - (3*d^2*e
*g^2*x^4 - 6*d^2*e*f*g*x^2 - d^2*e*f^2)*log(c))/(d^2*e*x^3), -1/3*(6*d^2*e*
g^2*p*x^4 + 2*d*e^2*f^2*p*x^2 + 2*(e^2*f^2 - 6*d*e*f*g - 3*d^2*g^2)*sqrt(d*
e)*p*x^3*arctan(sqrt(d*e)*x/d) - (3*d^2*e*g^2*p*x^4 - 6*d^2*e*f*g*p*x^2 - d
^2*e*f^2*p)*log(e*x^2 + d) - (3*d^2*e*g^2*x^4 - 6*d^2*e*f*g*x^2 - d^2*e*f^2
)*log(c))/(d^2*e*x^3)]
```

giac [A] time = 0.18, size = 154, normalized size = 0.91

$$\frac{2\left(3\,d^2g^2p+6\,dfgpe-f^2pe^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{3\,d^{\frac{3}{2}}}+\frac{3\,dg^2px^4\log\left(x^2e+d\right)-6\,dg^2px^4+3\,dg^2x^4\log(c)-6\,dfg\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(3*d^2*g^2*p + 6*d*f*g*p*e - f^2*p*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1
/2)/d^(3/2) + 1/3*(3*d*g^2*p*x^4*log(x^2*e + d) - 6*d*g^2*p*x^4 + 3*d*g^2*x
^4*log(c) - 6*d*f*g*p*x^2*log(x^2*e + d) - 2*f^2*p*x^2*e - 6*d*f*g*x^2*log(
c) - d*f^2*p*log(x^2*e + d) - d*f^2*log(c))/(d*x^3)
```

maple [C] time = 0.62, size = 740, normalized size = 4.38

$$-\frac{\left(-3g^2x^4+6fg\,x^2+f^2\right)\ln\left(\left(e\,x^2+d\right)^p\right)}{3x^3}+\frac{-3i\pi\,d^2e\,g^2x^4\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)+\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^4,x)
```

```
[Out] -1/3*(-3*g^2*x^4+6*f*g*x^2+f^2)/x^3*ln((e*x^2+d)^p)+1/6*(-3*I*Pi*g^2*x^4*cs
gn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*e*d^2-3*I*Pi*g^2*x^4*csgn
(I*c*(e*x^2+d)^p)^3*e*d^2+6*I*Pi*d^2*e*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+I*Pi
*d^2*e*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+6*I*Pi*d^2*e
*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+I*Pi*d^2*e*f^2
*csgn(I*c*(e*x^2+d)^p)^3-6*I*Pi*d^2*e*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*
(e*x^2+d)^p)^2-6*I*Pi*d^2*e*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+6*ln(
c)*g^2*x^4*e*d^2-I*Pi*d^2*e*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2
+3*I*Pi*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*e*d^2-I*Pi*d^2*e*f^2*csgn
(I*c*(e*x^2+d)^p)^2*csgn(I*c)+3*I*Pi*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(
e*x^2+d)^p)^2*e*d^2+6*(-d*e)^(1/2)*p*ln(d-(-d*e)^(1/2)*x)*g^2*d^2*x^3+12*(-
d*e)^(1/2)*p*ln(d-(-d*e)^(1/2)*x)*f*g*e*d*x^3-2*(-d*e)^(1/2)*e^2*p*ln(d-(-d
*e)^(1/2)*x)*f^2*x^3-6*(-d*e)^(1/2)*p*ln(-d-(-d*e)^(1/2)*x)*g^2*d^2*x^3-12*
(-d*e)^(1/2)*p*ln(-d-(-d*e)^(1/2)*x)*f*g*e*d*x^3+2*(-d*e)^(1/2)*e^2*p*ln(-d
-(-d*e)^(1/2)*x)*f^2*x^3-12*d^2*e*g^2*p*x^4-12*ln(c)*d^2*e*f*g*x^2-4*d*e^2*
f^2*p*x^2-2*ln(c)*d^2*e*f^2)/e/d^2/x^3
```

maxima [A] time = 1.01, size = 105, normalized size = 0.62

$$-\frac{2}{3}\left(\frac{3\,g^2x}{e}+\frac{f^2}{dx}+\frac{\left(e^2f^2-6\,defg-3\,d^2g^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,de}\right)ep+\frac{1}{3}\left(3\,g^2x-\frac{6\,fgx^2+f^2}{x^3}\right)\log\left(\left(ex^2+d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -2/3*(3*g^2*x/e + f^2/(d*x) + (e^2*f^2 - 6*d*e*f*g - 3*d^2*g^2)*arctan(e*x/
sqrt(d*e))/(sqrt(d*e)*d*e))*e*p + 1/3*(3*g^2*x - (6*f*g*x^2 + f^2)/x^3)*log
((e*x^2 + d)^p*c)
```

mupad [B] time = 0.39, size = 108, normalized size = 0.64

$$\ln\left(c\left(e\,x^2+d\right)^p\right)\left(\frac{8\,g^2\,x}{3}-\frac{\frac{f^2}{3}+2\,f\,g\,x^2+\frac{5\,g^2\,x^4}{3}}{x^3}\right)-2\,g^2\,p\,x+\frac{2\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(3\,d^2\,g^2+6\,d\,e\,f\,g-e^2\,f^2\right)}{3\,d^{3/2}\,\sqrt{e}}-\frac{2\,e\,f^2\,p}{3\,d\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^4,x)
```

```
[Out] log(c*(d + e*x^2)^p)*((8*g^2*x)/3 - (f^2/3 + (5*g^2*x^4)/3 + 2*f*g*x^2)/x^3
) - 2*g^2*p*x + (2*p*atan((e^(1/2)*x)/d^(1/2))*(3*d^2*g^2 - e^2*f^2 + 6*d*e
*f*g))/(3*d^(3/2)*e^(1/2)) - (2*e*f^2*p)/(3*d*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**4,x)
```

```
[Out] Timed out
```

3.336 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^6} dx$

Optimal. Leaf size=200

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{5x^5}-\frac{2fg \log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{g^2 \log\left(c\left(d+ex^2\right)^p\right)}{x}+\frac{2e^{5/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{4e^{3/2}fgp\tan^{-1}\left(\right.}{3d^{3/2}}$$

```
[Out] -2/15*e*f^2*p/d/x^3+2/5*e^2*f^2*p/d^2/x-4/3*e*f*g*p/d/x+2/5*e^(5/2)*f^2*p*a
rctan(x*e^(1/2)/d^(1/2))/d^(5/2)-4/3*e^(3/2)*f*g*p*arctan(x*e^(1/2)/d^(1/2)
)/d^(3/2)-1/5*f^2*ln(c*(e*x^2+d)^p)/x^5-2/3*f*g*ln(c*(e*x^2+d)^p)/x^3-g^2*l
n(c*(e*x^2+d)^p)/x+2*g^2*p*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2476, 2455, 325, 205}

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{5x^5}-\frac{2fg \log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{g^2 \log\left(c\left(d+ex^2\right)^p\right)}{x}+\frac{2e^2f^2p}{5d^2x}+\frac{2e^{5/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{4e^{3/2}fg}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^6,x]
```

```
[Out] (-2*e*f^2*p)/(15*d*x^3) + (2*e^2*f^2*p)/(5*d^2*x) - (4*e*f*g*p)/(3*d*x) + (
2*e^(5/2)*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*d^(5/2)) - (4*e^(3/2)*f*g*p
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)) + (2*Sqrt[e]*g^2*p*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/Sqrt[d] - (f^2*Log[c*(d + e*x^2)^p])/(5*x^5) - (2*f*g*Log[c*
(d + e*x^2)^p])/(3*x^3) - (g^2*Log[c*(d + e*x^2)^p])/x
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^6}\,dx = \int \left(\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{x^6} + \frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{x^4} + \frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\right.$$

$$= f^2\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^6}\,dx + (2fg)\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^4}\,dx + g^2\int \frac{\log\left(c\right.}{}$$

$$= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{5x^5} - \frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} - \frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{x} +$$

$$= -\frac{2ef^2p}{15dx^3} - \frac{4efgp}{3dx} + \frac{2\sqrt{e}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{5x^5} - \frac{2fg\log}{}$$

$$= -\frac{2ef^2p}{15dx^3} + \frac{2e^2f^2p}{5d^2x} - \frac{4efgp}{3dx} - \frac{4e^{3/2}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} + \frac{2\sqrt{e}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}$$

$$= -\frac{2ef^2p}{15dx^3} + \frac{2e^2f^2p}{5d^2x} - \frac{4efgp}{3dx} + \frac{2e^{5/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}} - \frac{4e^{3/2}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}$$

Mathematica [C] time = 0.07, size = 156, normalized size = 0.78

$$-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{5x^5} - \frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} - \frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{x} - \frac{2ef^2p\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{ex^2}{d}\right)}{15dx^3} - \frac{4efgp\,{}_2F_1\left(-\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^6,x]
```

```
[Out] (2*Sqrt[e]*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (2*e*f^2*p*Hypergeo
metric2F1[-3/2, 1, -1/2, -((e*x^2)/d)])/(15*d*x^3) - (4*e*f*g*p*Hypergeomet
ric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f^2*Log[c*(d + e*x^2)^p])/(5
*x^5) - (2*f*g*Log[c*(d + e*x^2)^p])/(3*x^3) - (g^2*Log[c*(d + e*x^2)^p])/x
```

fricas [A] time = 0.56, size = 351, normalized size = 1.76

$$\left[\frac{\left(3e^2f^2 - 10defg + 15d^2g^2\right)px^5\sqrt{-\frac{e}{d}}\log\left(\frac{ex^2+2dx\sqrt{-\frac{e}{d}}-d}{ex^2+d}\right) - 2def^2px^2 + 2\left(3e^2f^2 - 10defg\right)px^4 - \left(15d^2g^2px^4\right.}{15d^2x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^6,x, algorithm="fricas")
```

```
[Out] [1/15*((3*e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2)*p*x^5*sqrt(-e/d)*log((e*x^2 +
2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) - 2*d*e*f^2*p*x^2 + 2*(3*e^2*f^2 - 10*d*
e*f*g)*p*x^4 - (15*d^2*g^2*p*x^4 + 10*d^2*f*g*p*x^2 + 3*d^2*f^2*p)*log(e*x^
2 + d) - (15*d^2*g^2*x^4 + 10*d^2*f*g*x^2 + 3*d^2*f^2)*log(c))/(d^2*x^5), 1
/15*(2*(3*e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2)*p*x^5*sqrt(e/d)*arctan(x*sqrt(
e/d)) - 2*d*e*f^2*p*x^2 + 2*(3*e^2*f^2 - 10*d*e*f*g)*p*x^4 - (15*d^2*g^2*p*
x^4 + 10*d^2*f*g*p*x^2 + 3*d^2*f^2*p)*log(e*x^2 + d) - (15*d^2*g^2*x^4 + 10
*d^2*f*g*x^2 + 3*d^2*f^2)*log(c))/(d^2*x^5)]
```

giac [A] time = 0.19, size = 181, normalized size = 0.90

$$\frac{2\left(15\,d^2g^2pe-10\,dfgpe^2+3\,f^2pe^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{15\,d^{\frac{5}{2}}}-\frac{15\,d^2g^2px^4\log\left(x^2e+d\right)+20\,dfgpx^4e+15\,d^2g^2x^4\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^6,x, algorithm="giac")
```

```
[Out] 2/15*(15*d^2*g^2*p*e - 10*d*f*g*p*e^2 + 3*f^2*p*e^3)*arctan(x*e^(1/2)/sqrt(
d))*e^(-1/2)/d^(5/2) - 1/15*(15*d^2*g^2*p*x^4*log(x^2*e + d) + 20*d*f*g*p*x
^4*e + 15*d^2*g^2*x^4*log(c) - 6*f^2*p*x^4*e^2 + 10*d^2*f*g*p*x^2*log(x^2*e
 + d) + 2*d*f^2*p*x^2*e + 10*d^2*f*g*x^2*log(c) + 3*d^2*f^2*p*log(x^2*e + d
) + 3*d^2*f^2*log(c))/(d^2*x^5)
```

maple [C] time = 0.80, size = 753, normalized size = 3.76

$$-\frac{\left(15g^2x^4+10fg\,x^2+3f^2\right)\ln\left(\left(e\,x^2+d\right)^p\right)}{15x^5}+\frac{-3i\pi\,d^2f^2\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2-15i\pi\,d^2g^2x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^6,x)
```

```
[Out] -1/15*(15*g^2*x^4+10*f*g*x^2+3*f^2)/x^5*ln((e*x^2+d)^p)+1/30*(-3*I*Pi*f^2*c
sgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*d^2-15*I*Pi*d^2*g^2*x^4*csgn(I*c
*(e*x^2+d)^p)^2*csgn(I*c)-15*I*Pi*d^2*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*
(e*x^2+d)^p)^2+10*I*Pi*d^2*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p
)*csgn(I*c)+10*I*Pi*d^2*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+15*I*Pi*d^2*g^2*x^4
*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-10*I*Pi*d^2*f*g*x^2*cs
gn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-10*I*Pi*d^2*f*g*x^2*csgn(I*c*(e*x
^2+d)^p)^2*csgn(I*c)-30*ln(c)*d^2*g^2*x^4+15*I*Pi*d^2*g^2*x^4*csgn(I*c*(e*x
^2+d)^p)^3+3*I*Pi*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*d
^2-3*I*Pi*f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*d^2+3*I*Pi*f^2*csgn(I*c*(e*
x^2+d)^p)^3*d^2-40*d*e*f*g*p*x^4+12*e^2*f^2*p*x^4+2*sum(_R*ln((450*d^4*e*g^
4*p^2-600*d^3*e^2*f*g^3*p^2+380*d^2*e^3*f^2*g^2*p^2-120*d*e^4*f^3*g*p^2+18*
e^5*f^4*p^2+3*_R^2*d^5)*x+(-15*d^5*g^2*p+10*d^4*e*f*g*p-3*d^3*e^2*f^2*p)*_R
),_R=RootOf(225*d^4*e*g^4*p^2-300*d^3*e^2*f*g^3*p^2+190*d^2*e^3*f^2*g^2*p^2
-60*d*e^4*f^3*g*p^2+9*e^5*f^4*p^2+_Z^2*d^5))*d^2*x^5-20*ln(c)*d^2*f*g*x^2-4
*d*e*f^2*p*x^2-6*ln(c)*f^2*d^2)/d^2/x^5
```

maxima [A] time = 1.03, size = 116, normalized size = 0.58

$$\frac{2}{15}\,ep\left(\frac{\left(3\,e^2f^2-10\,defg+15\,d^2g^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,d^2}-\frac{df^2-\left(3\,ef^2-10\,dfg\right)x^2}{d^2x^3}\right)-\frac{\left(15\,g^2x^4+10\,fgx^2+3\,f^2\right)\log}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^6,x, algorithm="maxima")
```

```
[Out] 2/15*e*p*((3*e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2)*arctan(e*x/sqrt(d*e))/(sqrt
(d*e)*d^2) - (d*f^2 - (3*e*f^2 - 10*d*f*g)*x^2)/(d^2*x^3)) - 1/15*(15*g^2*x
^4 + 10*f*g*x^2 + 3*f^2)*log((e*x^2 + d)^p*c)/x^5
```

mupad [B] time = 0.39, size = 115, normalized size = 0.58

$$\frac{2\,\sqrt{e}\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(15\,d^2\,g^2-10\,d\,e\,f\,g+3\,e^2\,f^2\right)}{15\,d^{5/2}}-\frac{\ln\left(c\left(e\,x^2+d\right)^p\right)\left(\frac{f^2}{5}+\frac{2\,f\,g\,x^2}{3}+g^2\,x^4\right)}{x^5}-\frac{\frac{2\,e\,f^2\,p}{d}+\frac{2\,e\,f\,p\,x^2\,(1}{d}}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^6,x)
```

```
[Out] (2*e^(1/2)*p*atan((e^(1/2)*x)/d^(1/2))*(15*d^2*g^2 + 3*e^2*f^2 - 10*d*e*f*g
))/(15*d^(5/2)) - (log(c*(d + e*x^2)^p)*(f^2/5 + g^2*x^4 + (2*f*g*x^2)/3))/
x^5 - ((2*e*f^2*p)/d + (2*e*f*p*x^2*(10*d*g - 3*e*f))/d^2)/(15*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**6,x)
```

```
[Out] Timed out
```

3.337 $\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^8} dx$

Optimal. Leaf size=252

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{7x^7}-\frac{2fg \log\left(c\left(d+ex^2\right)^p\right)}{5x^5}-\frac{g^2 \log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{2e^{7/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7d^{7/2}}+\frac{4e^{5/2}fgp\tan^{-1}\left(\right.}{5d^{5/2}}$$

```
[Out] -2/35*e*f^2*p/d/x^5+2/21*e^2*f^2*p/d^2/x^3-4/15*e*f*g*p/d/x^3-2/7*e^3*f^2*p
/d^3/x+4/5*e^2*f*g*p/d^2/x-2/3*e*g^2*p/d/x-2/7*e^(7/2)*f^2*p*arctan(x*e^(1/
2)/d^(1/2))/d^(7/2)+4/5*e^(5/2)*f*g*p*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)-2/3
*e^(3/2)*g^2*p*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)-1/7*f^2*ln(c*(e*x^2+d)^p)/
x^7-2/5*f*g*ln(c*(e*x^2+d)^p)/x^5-1/3*g^2*ln(c*(e*x^2+d)^p)/x^3
```

Rubi [A] time = 0.21, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2476, 2455, 325, 205}

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{7x^7}-\frac{2fg \log\left(c\left(d+ex^2\right)^p\right)}{5x^5}-\frac{g^2 \log\left(c\left(d+ex^2\right)^p\right)}{3x^3}+\frac{2e^2f^2p}{21d^2x^3}-\frac{2e^3f^2p}{7d^3x}-\frac{2e^{7/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^8,x]
```

```
[Out] (-2*e*f^2*p)/(35*d*x^5) + (2*e^2*f^2*p)/(21*d^2*x^3) - (4*e*f*g*p)/(15*d*x^
3) - (2*e^3*f^2*p)/(7*d^3*x) + (4*e^2*f*g*p)/(5*d^2*x) - (2*e*g^2*p)/(3*d*x
) - (2*e^(7/2)*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(7*d^(7/2)) + (4*e^(5/2)*
f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*d^(5/2)) - (2*e^(3/2)*g^2*p*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(3*d^(3/2)) - (f^2*Log[c*(d + e*x^2)^p])/(7*x^7) - (2*f
*g*Log[c*(d + e*x^2)^p])/(5*x^5) - (g^2*Log[c*(d + e*x^2)^p])/(3*x^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
```

& IntegerQ[s]

Rubi steps

$$\int \frac{\left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^8}\,dx = \int \left(\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^8} + \frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{x^6} + \frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{x^4}\right.$$

$$= f^2 \int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^8}\,dx + (2fg)\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^6}\,dx + g^2 \int \frac{\log\left(c\right.}{}$$

$$= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{7x^7} - \frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{5x^5} - \frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} +$$

$$= -\frac{2ef^2p}{35dx^5} - \frac{4efgp}{15dx^3} - \frac{2eg^2p}{3dx} - \frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{7x^7} - \frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{5x^5}$$

$$= -\frac{2ef^2p}{35dx^5} + \frac{2e^2f^2p}{21d^2x^3} - \frac{4efgp}{15dx^3} + \frac{4e^2fgp}{5d^2x} - \frac{2eg^2p}{3dx} - \frac{2e^{3/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} - f$$

$$= -\frac{2ef^2p}{35dx^5} + \frac{2e^2f^2p}{21d^2x^3} - \frac{4efgp}{15dx^3} - \frac{2e^3f^2p}{7d^3x} + \frac{4e^2fgp}{5d^2x} - \frac{2eg^2p}{3dx} + \frac{4e^{5/2}fgp\tan^{-1}}{5d^{5/2}}$$

$$= -\frac{2ef^2p}{35dx^5} + \frac{2e^2f^2p}{21d^2x^3} - \frac{4efgp}{15dx^3} - \frac{2e^3f^2p}{7d^3x} + \frac{4e^2fgp}{5d^2x} - \frac{2eg^2p}{3dx} - \frac{2e^{7/2}f^2p\tan^{-1}}{7d^{7/2}}$$

Mathematica [C] time = 0.03, size = 161, normalized size = 0.64

$$-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{7x^7} - \frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{5x^5} - \frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{3x^3} - \frac{2ef^2p\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\frac{ex^2}{d}\right)}{35dx^5} - \frac{4efgp\,{}_2F_1\left(-\right.}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^8,x]
```

```
[Out] (-2*e*f^2*p*Hypergeometric2F1[-5/2, 1, -3/2, -((e*x^2)/d)])/(35*d*x^5) - (4
*e*f*g*p*Hypergeometric2F1[-3/2, 1, -1/2, -((e*x^2)/d)])/(15*d*x^3) - (2*e*
g^2*p*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f^2*Log[c*(
d + e*x^2)^p])/(7*x^7) - (2*f*g*Log[c*(d + e*x^2)^p])/(5*x^5) - (g^2*Log[c*
(d + e*x^2)^p])/(3*x^3)
```

fricas [A] time = 0.86, size = 429, normalized size = 1.70

$$\left[\frac{\left(15\,e^3f^2 - 42\,de^2fg + 35\,d^2eg^2\right)px^7\sqrt{-\frac{e}{d}}\,\log\left(\frac{ex^2-2\,dx\sqrt{-\frac{e}{d}}-d}{ex^2+d}\right) - 6\,d^2ef^2px^2 - 2\left(15\,e^3f^2 - 42\,de^2fg + 35\,d^2eg^2\right)p}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^8,x, algorithm="fricas")
```

```
[Out] [1/105*((15*e^3*f^2 - 42*d*e^2*f*g + 35*d^2*e*g^2)*p*x^7*sqrt(-e/d)*log((e*
x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) - 6*d^2*e*f^2*p*x^2 - 2*(15*e^3*f^
2 - 42*d*e^2*f*g + 35*d^2*e*g^2)*p*x^6 + 2*(5*d*e^2*f^2 - 14*d^2*e*f*g)*p*x
```

```
^4 - (35*d^3*g^2*p*x^4 + 42*d^3*f*g*p*x^2 + 15*d^3*f^2*p)*log(e*x^2 + d) -
(35*d^3*g^2*x^4 + 42*d^3*f*g*x^2 + 15*d^3*f^2)*log(c))/(d^3*x^7), -1/105*(2
*(15*e^3*f^2 - 42*d*e^2*f*g + 35*d^2*e*g^2)*p*x^7*sqrt(e/d)*arctan(x*sqrt(e
/d)) + 6*d^2*e*f^2*p*x^2 + 2*(15*e^3*f^2 - 42*d*e^2*f*g + 35*d^2*e*g^2)*p*x
^6 - 2*(5*d*e^2*f^2 - 14*d^2*e*f*g)*p*x^4 + (35*d^3*g^2*p*x^4 + 42*d^3*f*g*
p*x^2 + 15*d^3*f^2*p)*log(e*x^2 + d) + (35*d^3*g^2*x^4 + 42*d^3*f*g*x^2 + 1
5*d^3*f^2)*log(c))/(d^3*x^7)]
```

giac [A] time = 0.19, size = 222, normalized size = 0.88

$$-\frac{2\left(35\,d^2g^2pe^2-42\,dfgpe^3+15\,f^2pe^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{105\,d^{\frac{7}{2}}}-\frac{70\,d^2g^2px^6e-84\,dfgpx^6e^2+35\,d^3g^2px^4\log\left(x^2e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^8,x, algorithm="giac")
```

```
[Out] -2/105*(35*d^2*g^2*p*e^2 - 42*d*f*g*p*e^3 + 15*f^2*p*e^4)*arctan(x*e^(1/2)/
sqrt(d))*e^(-1/2)/d^(7/2) - 1/105*(70*d^2*g^2*p*x^6*e - 84*d*f*g*p*x^6*e^2
+ 35*d^3*g^2*p*x^4*log(x^2*e + d) + 30*f^2*p*x^6*e^3 + 28*d^2*f*g*p*x^4*e +
 35*d^3*g^2*x^4*log(c) - 10*d*f^2*p*x^4*e^2 + 42*d^3*f*g*p*x^2*log(x^2*e +
d) + 6*d^2*f^2*p*x^2*e + 42*d^3*f*g*x^2*log(c) + 15*d^3*f^2*p*log(x^2*e + d
) + 15*d^3*f^2*log(c))/(d^3*x^7)
```

maple [C] time = 0.53, size = 784, normalized size = 3.11

$$-\frac{\left(35g^2x^4+42fg\,x^2+15f^2\right)\ln\left(\left(e\,x^2+d\right)^p\right)}{105x^7}-\frac{30d^4f^2\ln(c)-42i\pi\,d^4fg\,x^2\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^3-15i\pi\,d^4f^2\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^8,x)
```

```
[Out] -1/105*(35*g^2*x^4+42*f*g*x^2+15*f^2)/x^7*ln((e*x^2+d)^p)-1/210*(30*d^4*f^2
*ln(c)+70*d^4*g^2*x^4*ln(c)-42*I*Pi*d^4*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^3-15*
I*Pi*d^4*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-35*I*Pi*d^
4*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^3+15*I*Pi*d^4*f^2*csgn(I*(e*x^2+d)^p)*csgn(
I*c*(e*x^2+d)^p)^2+140*d^3*e*g^2*p*x^6+60*d*e^3*f^2*p*x^6-20*d^2*e^2*f^2*p*
x^4+12*d^3*e*f^2*p*x^2+42*I*Pi*d^4*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c
)-35*I*Pi*d^4*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+4
2*I*Pi*d^4*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+15*I*Pi*d^4*
f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-70*(-d*e)^(1/2)*p*e*ln(-e*x+(-d*e)^(1
/2))*g^2*d^2*x^7+70*(-d*e)^(1/2)*p*e*ln(-e*x-(-d*e)^(1/2))*g^2*d^2*x^7+84*d
^4*f*g*x^2*ln(c)-42*I*Pi*d^4*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)
^p)*csgn(I*c)+35*I*Pi*d^4*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)
^2+35*I*Pi*d^4*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-168*d^2*e^2*f*g*p*
x^6+56*d^3*e*f*g*p*x^4+84*(-d*e)^(1/2)*p*e^2*ln(-e*x+(-d*e)^(1/2))*f*g*d*x^
7-84*(-d*e)^(1/2)*p*e^2*ln(-e*x-(-d*e)^(1/2))*f*g*d*x^7-15*I*Pi*d^4*f^2*csg
n(I*c*(e*x^2+d)^p)^3-30*(-d*e)^(1/2)*p*e^3*ln(-e*x+(-d*e)^(1/2))*f^2*x^7+30
*(-d*e)^(1/2)*p*e^3*ln(-e*x-(-d*e)^(1/2))*f^2*x^7)/d^4/x^7
```

maxima [A] time = 1.03, size = 151, normalized size = 0.60

$$-\frac{2}{105}\,ep\left(\frac{\left(15\,e^3f^2-42\,de^2fg+35\,d^2eg^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,d^3}+\frac{\left(15\,e^2f^2-42\,defg+35\,d^2g^2\right)x^4+3\,d^2f^2-\left(5\,def^2\right.}{d^3x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^8,x, algorithm="maxima")
```

```
[Out] -2/105*e*p*((15*e^3*f^2 - 42*d*e^2*f*g + 35*d^2*e*g^2)*arctan(e*x/sqrt(d*e)
)/(sqrt(d*e)*d^3) + ((15*e^2*f^2 - 42*d*e*f*g + 35*d^2*g^2)*x^4 + 3*d^2*f^2
 - (5*d*e*f^2 - 14*d^2*f*g)*x^2)/(d^3*x^5)) - 1/105*(35*g^2*x^4 + 42*f*g*x^
2 + 15*f^2)*log((e*x^2 + d)^p*c)/x^7
```

mupad [B] time = 0.43, size = 149, normalized size = 0.59

$$-\frac{\frac{6\,e\,f^2\,p}{d}+\frac{2\,e\,p\,x^4\,\left(35\,d^2\,g^2-42\,d\,e\,f\,g+15\,e^2\,f^2\right)}{d^3}+\frac{2\,e\,f\,p\,x^2\,\left(14\,d\,g-5\,e\,f\right)}{d^2}}{105\,x^5}-\frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{f^2}{7}+\frac{2\,f\,g\,x^2}{5}+\frac{g^2\,x^4}{3}\right)}{x^7}-\frac{2\,e^{3/2}\,p\,\mathrm{atan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^8,x)
```

```
[Out] - ((6*e*f^2*p)/d + (2*e*p*x^4*(35*d^2*g^2 + 15*e^2*f^2 - 42*d*e*f*g))/d^3 +
 (2*e*f*p*x^2*(14*d*g - 5*e*f))/d^2)/(105*x^5) - (log(c*(d + e*x^2)^p)*(f^2
/7 + (g^2*x^4)/3 + (2*f*g*x^2)/5))/x^7 - (2*e^(3/2)*p*atan((e^(1/2)*x)/d^(1
/2))*(35*d^2*g^2 + 15*e^2*f^2 - 42*d*e*f*g))/(105*d^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**8,x)
```

```
[Out] Timed out
```

3.338 $\int \frac{x^5 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$

Optimal. Leaf size=188

$$\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right) \log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^3} - \frac{f\left(d+ex^2\right) \log\left(c\left(d+ex^2\right)^p\right)}{2eg^2} + \frac{x^4 \log\left(c\left(d+ex^2\right)^p\right)}{4g} - \frac{d^2 p \log\left(d+ex^2\right)}{4e^2 g} +$$

```
[Out] 1/2*f*p*x^2/g^2+1/4*d*p*x^2/e/g-1/8*p*x^4/g-1/4*d^2*p*ln(e*x^2+d)/e^2/g+1/4
*x^4*ln(c*(e*x^2+d)^p)/g-1/2*f*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e/g^2+1/2*f^2*ln
(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/g^3+1/2*f^2*p*polylog(2,-g*(e*x^
2+d)/(-d*g+e*f))/g^3
```

Rubi [A] time = 0.28, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2475, 43, 2416, 2389, 2295, 2395, 2394, 2393, 2391}

$$\frac{f^2 p \text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2g^3} + \frac{f^2 \log\left(c\left(d+ex^2\right)^p\right) \log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^3} - \frac{f\left(d+ex^2\right) \log\left(c\left(d+ex^2\right)^p\right)}{2eg^2} + \frac{x^4 \log\left(c\left(d\right.\right.}{4g}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (f*p*x^2)/(2*g^2) + (d*p*x^2)/(4*e*g) - (p*x^4)/(8*g) - (d^2*p*Log[d + e*x^
2])/(4*e^2*g) + (x^4*Log[c*(d + e*x^2)^p])/(4*g) - (f*(d + e*x^2)*Log[c*(d
+ e*x^2)^p])/(2*e*g^2) + (f^2*Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f
 - d*g)])/(2*g^3) + (f^2*p*PolyLog[2, -((g*(d + e*x^2))/(e*f - d*g))])/(2*g
^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{x^5 \log \left(c \left(d+e x^2\right)^p\right)}{f+g x^2} \, dx = \frac{1}{2} \text{Subst}\left(\int \frac{x^2 \log \left(c (d+e x)^p\right)}{f+g x} \, dx,x,x^2\right)$$

$$= \frac{1}{2} \text{Subst}\left(\int \left(-\frac{f \log \left(c (d+e x)^p\right)}{g^2}+\frac{x \log \left(c (d+e x)^p\right)}{g}+\frac{f^2 \log \left(c (d+e x)^p\right)}{g^2 (f+g x)}\right) \, dx,x,x$$

$$= -\frac{f \text{Subst}\left(\int \log \left(c (d+e x)^p\right) \, dx,x,x^2\right)}{2 g^2}+\frac{f^2 \text{Subst}\left(\int \frac{\log \left(c (d+e x)^p\right)}{f+g x} \, dx,x,x^2\right)}{2 g^2}+\text{Subs}$$

$$= \frac{x^4 \log \left(c \left(d+e x^2\right)^p\right)}{4 g}+\frac{f^2 \log \left(c \left(d+e x^2\right)^p\right) \log \left(\frac{e \left(f+g x^2\right)}{e f-d g}\right)}{2 g^3}-\frac{f \text{Subst}\left(\int \log \left(c x^p\right) \right.}{2 e g^2}$$

$$= \frac{f p x^2}{2 g^2}+\frac{x^4 \log \left(c \left(d+e x^2\right)^p\right)}{4 g}-\frac{f \left(d+e x^2\right) \log \left(c \left(d+e x^2\right)^p\right)}{2 e g^2}+\frac{f^2 \log \left(c \left(d+e x^2\right)\right.}{2 g^3}$$

$$= \frac{f p x^2}{2 g^2}+\frac{d p x^2}{4 e g}-\frac{p x^4}{8 g}-\frac{d^2 p \log \left(d+e x^2\right)}{4 e^2 g}+\frac{x^4 \log \left(c \left(d+e x^2\right)^p\right)}{4 g}-\frac{f \left(d+e x^2\right) \log \left(\right.}{2 e g^2}$$

Mathematica [A] time = 0.13, size = 143, normalized size = 0.76

$$\frac{e\log\left(c\left(d+ex^2\right)^p\right)\left(4ef^2\log\left(\frac{e(f+gx^2)}{ef-dg}\right)+2g\left(-2df-2efx^2+egx^4\right)\right)-2d^2g^2p\log\left(d+ex^2\right)+4e^2f^2p\mathrm{Li}_2\left(\frac{g(e}{d}\right.}{8e^2g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (e*g*p*x^2*(4*e*f + 2*d*g - e*g*x^2) - 2*d^2*g^2*p*Log[d + e*x^2] + e*Log[c
*(d + e*x^2)^p]*(2*g*(-2*d*f - 2*e*f*x^2 + e*g*x^4) + 4*e*f^2*Log[(e*(f + g
*x^2))/(e*f - d*g)]) + 4*e^2*f^2*p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g
)])/(8*e^2*g^3)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^5\log\left(\left(ex^2+d\right)^pc\right)}{gx^2+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(x^5*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5\log\left(\left(ex^2+d\right)^pc\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(x^5*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

maple [C] time = 1.12, size = 902, normalized size = 4.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*ln(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

```
[Out] 1/4*ln((e*x^2+d)^p)/g*x^4-1/2*ln((e*x^2+d)^p)/g^2*f*x^2+1/2*ln((e*x^2+d)^p)
*f^2/g^3*ln(g*x^2+f)-1/2*p*f^2/g^3*sum(ln(-_alpha+x)*ln(g*x^2+f)-ln(-_alpha
+x)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(
_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*
e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,inde
x=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/Ro
otOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z
*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+
e*f,index=2)),_alpha=RootOf(_Z^2*e+d))-1/8*p*x^4/g+1/4*d*p*x^2/e/g+1/2*f*p*
x^2/g^2-1/4*d^2*p*ln(e*x^2+d)/e^2/g-1/2*p/e/g^2*d*ln(e*x^2+d)*f+1/4*I*Pi*cs
gn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/g^2*f*x^2-1/4*I*Pi*csgn(I
*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/g^2*f*x^2-1/8*I*Pi*csgn(I*(e*x^2+d)^p
)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/g*x^4-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*cs
gn(I*c)/g^2*f*x^2-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f^2/g^3*ln(g*x^2+f)+1/8*
I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/g*x^4+1/4*I*Pi*csgn(I*(e*x
```

```
^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f^2/g^3*ln(g*x^2+f)-1/4*I*Pi*csgn(I*(e*x^2
+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f^2/g^3*ln(g*x^2+f)+1/8*I*Pi*csgn(I*
c*(e*x^2+d)^p)^2*csgn(I*c)/g*x^4-1/8*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g*x^4+1/4
*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f^2/g^3*ln(g*x^2+f)+1/4*I*Pi*csgn(I
*c*(e*x^2+d)^p)^3/g^2*f*x^2+1/4*ln(c)/g*x^4-1/2*ln(c)/g^2*f*x^2+1/2*ln(c)*f
^2/g^3*ln(g*x^2+f)
```

maxima [A] time = 1.31, size = 182, normalized size = 0.97

$$\frac{\left(\log\left(ex^2+d\right)\log\left(\frac{egx^2+dg}{ef-dg}+1\right)+\mathrm{Li}_2\left(-\frac{egx^2+dg}{ef-dg}\right)\right)f^2p}{2\,g^3}+\frac{f^2\log\left(gx^2+f\right)\log(c)}{2\,g^3}-\frac{\left(e^2gp-2\,e^2g\log(c)\right)x^4-2\left(2\,e^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*(log(e*x^2 + d)*log((e*g*x^2 + d*g)/(e*f - d*g) + 1) + dilog(-(e*g*x^2
+ d*g)/(e*f - d*g)))*f^2*p/g^3 + 1/2*f^2*log(g*x^2 + f)*log(c)/g^3 - 1/8*((
e^2*g*p - 2*e^2*g*log(c))*x^4 - 2*(2*e^2*f*p + d*e*g*p - 2*e^2*f*log(c))*x^
2 - 2*(e^2*g*p*x^4 - 2*e^2*f*p*x^2 - 2*d*e*f*p - d^2*g*p)*log(e*x^2 + d))/(
e^2*g^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
```

```
[Out] int((x^5*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Integral(x**5*log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

3.339 $\int \frac{x^3 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$

Optimal. Leaf size=112

$$-\frac{f\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2g^2}+\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg}-\frac{fp\text{Li}_2\left(-\frac{g\left(ex^2+d\right)}{ef-dg}\right)}{2g^2}-\frac{px^2}{2g}$$

```
[Out] -1/2*p*x^2/g+1/2*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e/g-1/2*f*ln(c*(e*x^2+d)^p)*ln
(e*(g*x^2+f)/(-d*g+e*f))/g^2-1/2*f*p*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/g^2
```

Rubi [A] time = 0.19, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2475, 43, 2416, 2389, 2295, 2394, 2393, 2391}

$$-\frac{fp\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2g^2}-\frac{f\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2g^2}+\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg}-\frac{px^2}{2g}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] -(p*x^2)/(2*g) + ((d + e*x^2)*Log[c*(d + e*x^2)^p])/(2*e*g) - (f*Log[c*(d +
 e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*g^2) - (f*p*PolyLog[2, -((g
*(d + e*x^2))/(e*f - d*g))])/(2*g^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{x^3 \log \left(c \left(d+e x^2\right)^p\right)}{f+g x^2} \, dx = \frac{1}{2} \text{Subst}\left(\int \frac{x \log \left(c (d+e x)^p\right)}{f+g x} \, dx,x,x^2\right)$$

$$= \frac{1}{2} \text{Subst}\left(\int \left(\frac{\log \left(c (d+e x)^p\right)}{g}-\frac{f \log \left(c (d+e x)^p\right)}{g (f+g x)}\right) \, dx,x,x^2\right)$$

$$= \frac{\text{Subst}\left(\int \log \left(c (d+e x)^p\right) \, dx,x,x^2\right)}{2 g}-\frac{f \text{Subst}\left(\int \frac{\log \left(c (d+e x)^p\right)}{f+g x} \, dx,x,x^2\right)}{2 g}$$

$$= -\frac{f \log \left(c \left(d+e x^2\right)^p\right) \log \left(\frac{e \left(f+g x^2\right)}{e f-d g}\right)}{2 g^2}+\frac{\text{Subst}\left(\int \log \left(c x^p\right) \, dx,x,d+e x^2\right)}{2 e g}+\frac{(e f p) \text{Su}}{}$$

$$= -\frac{p x^2}{2 g}+\frac{\left(d+e x^2\right) \log \left(c \left(d+e x^2\right)^p\right)}{2 e g}-\frac{f \log \left(c \left(d+e x^2\right)^p\right) \log \left(\frac{e \left(f+g x^2\right)}{e f-d g}\right)}{2 g^2}+\frac{(f p) \text{Su}}{}$$

$$= -\frac{p x^2}{2 g}+\frac{\left(d+e x^2\right) \log \left(c \left(d+e x^2\right)^p\right)}{2 e g}-\frac{f \log \left(c \left(d+e x^2\right)^p\right) \log \left(\frac{e \left(f+g x^2\right)}{e f-d g}\right)}{2 g^2}-\frac{f p \text{Li}_2\left(\right.}{}$$

Mathematica [A] time = 0.04, size = 91, normalized size = 0.81

$$-\frac{-\log \left(c \left(d+e x^2\right)^p\right) \left(-e f \log \left(\frac{e \left(f+g x^2\right)}{e f-d g}\right)+d g+e g x^2\right)+e f p \text{Li}_2\left(\frac{g \left(e x^2+d\right)}{d g-e f}\right)+e g p x^2}{2 e g^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] -1/2*(e*g*p*x^2 - Log[c*(d + e*x^2)^p]*(d*g + e*g*x^2 - e*f*Log[(e*(f + g*x
^2))/(e*f - d*g)]) + e*f*p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)])/(e*g
^2)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{3}\log\left(\left(ex^{2}+d\right)^{p}c\right)}{gx^{2}+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(x^3*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\log\left(\left(ex^{2}+d\right)^{p}c\right)}{gx^{2}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(x^3*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

maple [C] time = 0.77, size = 672, normalized size = 6.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

```
[Out] 1/2*ln((e*x^2+d)^p)/g*x^2-1/2*ln((e*x^2+d)^p)*f/g^2*ln(g*x^2+f)-1/2*p*x^2/g
+1/2*p/e/g*d*ln(e*x^2+d)+1/2*p*f/g^2*sum(ln(-_alpha+x)*ln(g*x^2+f)-ln(-_alp
ha+x)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootO
f(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alph
a*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,in
dex=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/
RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*
_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*
g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(
I*c*(e*x^2+d)^p)^2/g*x^2-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)
^2*f/g^2*ln(g*x^2+f)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csg
n(I*c)/g*x^2+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f
/g^2*ln(g*x^2+f)-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g*x^2+1/4*I*Pi*csgn(I*c*(
e*x^2+d)^p)^3*f/g^2*ln(g*x^2+f)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/
g*x^2-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f/g^2*ln(g*x^2+f)+1/2*ln(c
)/g*x^2-1/2*ln(c)*f/g^2*ln(g*x^2+f)
```

maxima [A] time = 1.32, size = 123, normalized size = 1.10

$$-\frac{\left(\log\left(ex^{2}+d\right)\log\left(\frac{egx^{2}+dg}{ef-dg}+1\right)+\mathrm{Li}_2\left(-\frac{egx^{2}+dg}{ef-dg}\right)\right)fp}{2\,g^{2}}-\frac{f\log\left(gx^{2}+f\right)\log(c)}{2\,g^{2}}-\frac{\left(ep-e\log(c)\right)x^{2}-\left(epx^{2}+dp\right.}{2\,eg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] -1/2*(log(e*x^2 + d)*log((e*g*x^2 + d*g)/(e*f - d*g) + 1) + dilog(-(e*g*x^2
 + d*g)/(e*f - d*g)))*f*p/g^2 - 1/2*f*log(g*x^2 + f)*log(c)/g^2 - 1/2*((e*p
 - e*log(c))*x^2 - (e*p*x^2 + d*p)*log(e*x^2 + d))/(e*g)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \ln\left(c\left(e\,x^2+d\right)^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
[Out] int((x^3*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
[Out] Integral(x**3*log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

3.340 $\int \frac{x\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=70

$$\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g}+\frac{p\text{Li}_2\left(-\frac{g(ex^2+d)}{ef-dg}\right)}{2g}$$

```
[Out] 1/2*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/g+1/2*p*polylog(2,-g*(e*x^
2+d)/(-d*g+e*f))/g
```

Rubi [A] time = 0.10, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2475, 2394, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2g}+\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*g) + (p*PolyLog[
2, -((g*(d + e*x^2))/(e*f - d*g))])/(2*g)
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{x \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{f+gx}\,dx, x, x^2\right)$$
$$= \frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g} - \frac{(ep)\operatorname{Subst}\left(\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx, x, x^2\right)}{2g}$$
$$= \frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g} - \frac{p\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex^2\right)}{2g}$$
$$= \frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g} + \frac{p\operatorname{Li}_2\left(-\frac{g(d+ex^2)}{ef-dg}\right)}{2g}$$

Mathematica [A] time = 0.01, size = 64, normalized size = 0.91

$$\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)+p\operatorname{Li}_2\left(\frac{g(ex^2+d)}{dg-ef}\right)}{2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)] + p*PolyLog[2, (g*(d
 + e*x^2))/(-(e*f) + d*g)])/(2*g)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{x\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(x*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(x*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

maple [C] time = 0.69, size = 472, normalized size = 6.74

$$-\frac{i\pi \operatorname{csgn}(ic)\operatorname{csgn}\left(i\left(e\,x^2+d\right)^p\right)\operatorname{csgn}\left(ic\left(e\,x^2+d\right)^p\right)\ln\left(g\,x^2+f\right)}{4g}+\frac{i\pi \operatorname{csgn}(ic)\operatorname{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2\ln\left(g\,x^2+\right.}{4g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

```
[Out] 1/2/g*ln(g*x^2+f)*ln((e*x^2+d)^p)-1/2/g*p*sum(ln(-_alpha+x)*ln(g*x^2+f)-ln(
-_alpha+x)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/
RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*
_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e
*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_al
pha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e
*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e
*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))+1/4*I/g*ln(g*x^2+f)*Pi*csgn(I
*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/4*I/g*ln(g*x^2+f)*Pi*csgn(I*(e*x^2+
d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/4*I/g*ln(g*x^2+f)*Pi*csgn(I*c*(e*x^
2+d)^p)^3+1/4*I/g*ln(g*x^2+f)*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/2/g*ln
(g*x^2+f)*ln(c)
```

maxima [B] time = 0.49, size = 138, normalized size = 1.97

$$\frac{ep\left(\frac{\log\left(ex^2+d\right)\log\left(gx^2+f\right)}{e}-\frac{\log\left(gx^2+f\right)\log\left(-\frac{egx^2+ef}{ef-dg}+1\right)+\mathrm{Li}_2\left(\frac{egx^2+ef}{ef-dg}\right)}{e}\right)}{2g}-\frac{p\log\left(ex^2+d\right)\log\left(gx^2+f\right)}{2g}+\frac{\log\left(gx^2+f\right)\log}{2g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*e*p*(log(e*x^2 + d)*log(g*x^2 + f)/e - (log(g*x^2 + f)*log(-(e*g*x^2 +
e*f)/(e*f - d*g) + 1) + dilog((e*g*x^2 + e*f)/(e*f - d*g)))/e)/g - 1/2*p*lo
g(e*x^2 + d)*log(g*x^2 + f)/g + 1/2*log(g*x^2 + f)*log((e*x^2 + d)^p*c)/g
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
```

```
[Out] int((x*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Integral(x*log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

3.341 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x\left(f+gx^2\right)}\,dx$

Optimal. Leaf size=119

$$-\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f}+\frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f}-\frac{p\text{Li}_2\left(-\frac{g\left(ex^2+d\right)}{ef-dg}\right)}{2f}+\frac{p\text{Li}_2\left(\frac{ex^2}{d}+1\right)}{2f}$$

```
[Out] 1/2*ln(-e*x^2/d)*ln(c*(e*x^2+d)^p)/f-1/2*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(
-d*g+e*f))/f-1/2*p*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/f+1/2*p*polylog(2,1+e
*x^2/d)/f
```

Rubi [A] time = 0.21, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2475, 36, 29, 31, 2416, 2394, 2315, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2f}+\frac{p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{2f}-\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f}+\frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)),x]
```

```
[Out] (Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/(2*f) - (Log[c*(d + e*x^2)^p]*Log[
(e*(f + g*x^2))/(e*f - d*g)])/(2*f) - (p*PolyLog[2, -((g*(d + e*x^2))/(e*f
- d*g))])/(2*f) + (p*PolyLog[2, 1 + (e*x^2)/d])/(2*f)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x\left(f+gx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x(f+gx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{\log\left(c(d+ex)^p\right)}{fx} - \frac{g\log\left(c(d+ex)^p\right)}{f(f+gx)}\right)dx, x, x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^2\right)}{2f} - \frac{g\operatorname{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^2\right)}{2f}\\
&= \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f} - \frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f} - \frac{(ep)\operatorname{Subst}\left(\int \frac{\mathrm{lo}}{}\right.}{2}\\
&= \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f} - \frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f} + \frac{p\operatorname{Li}_2\left(1+\frac{ex^2}{d}\right)}{2f} +\\
&= \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f} - \frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f} - \frac{p\operatorname{Li}_2\left(-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2f}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 92, normalized size = 0.77

$$\frac{\log\left(c\left(d+ex^2\right)^p\right)\left(\log\left(-\frac{ex^2}{d}\right)-\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)\right)-p\operatorname{Li}_2\left(\frac{g\left(ex^2+d\right)}{dg-ef}\right)+p\operatorname{Li}_2\left(\frac{ex^2}{d}+1\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)),x]
```

```
[Out] (Log[c*(d + e*x^2)^p]*(Log[-((e*x^2)/d)] - Log[(e*(f + g*x^2))/(e*f - d*g)]
) - p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^2)
/d])/(2*f)
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^3+fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^3 + f*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)*x), x)
```

maple [C] time = 0.64, size = 732, normalized size = 6.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x/(g*x^2+f),x)
```

```
[Out] -1/2*ln((e*x^2+d)^p)/f*ln(g*x^2+f)+ln((e*x^2+d)^p)/f*ln(x)-p/f*ln(x)*ln((-e
*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p/f*ln(x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)
)-p/f*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p/f*dilog((e*x+(-d*e)^(1/2))/
(-d*e)^(1/2))+1/2*p/f*sum(ln(-_alpha+x)*ln(g*x^2+f)-ln(-_alpha+x)*(ln((Root
Of(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z
*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,i
ndex=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)))-dilog(
(RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g
+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d
*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2))
,_alpha=RootOf(_Z^2*e+d))-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p
)*csgn(I*c)/f*ln(x)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f*ln(g*x^2+f)-1/4*I*Pi
*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f*ln(g*x^2+f)-1/2*I*Pi*csgn(I*c*(e*x^2+d
)^p)^3/f*ln(x)+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f*ln(x)-1/4*I*Pi*
csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f*ln(g*x^2+f)+1/4*I*Pi*csgn(I*(
e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f*ln(g*x^2+f)+1/2*I*Pi*csgn(I*(
e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f*ln(x)-1/2*ln(c)/f*ln(g*x^2+f)+1/f*ln(
c)*ln(x)
```

maxima [A] time = 1.26, size = 140, normalized size = 1.18

$$-\frac{1}{2}ep\left(\frac{2\log\left(\frac{ex^2}{d}+1\right)\log(x)+\text{Li}_2\left(-\frac{ex^2}{d}\right)}{ef}-\frac{\log\left(gx^2+f\right)\log\left(-\frac{egx^2+ef}{ef-dg}+1\right)+\text{Li}_2\left(\frac{egx^2+ef}{ef-dg}\right)}{ef}\right)-\frac{1}{2}\left(\frac{\log\left(gx^2+f\right)}{f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] -1/2*e*p*((2*log(e*x^2/d + 1)*log(x) + dilog(-e*x^2/d))/(e*f) - (log(g*x^2
+ f)*log(-(e*g*x^2 + e*f)/(e*f - d*g) + 1) + dilog((e*g*x^2 + e*f)/(e*f - d
*g)))/(e*f)) - 1/2*(log(g*x^2 + f)/f - log(x^2)/f)*log((e*x^2 + d)^p*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(e x^2+d\right)^p\right)}{x\left(g x^2+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(x*(f + g*x^2)),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)/(x*(f + g*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x/(g*x**2+f),x)
```

```
[Out] Timed out
```

3.342 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^3\left(f+gx^2\right)}\,dx$

Optimal. Leaf size=176

$$-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2}+\frac{g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f^2}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2fx^2}+\frac{gp\text{Li}_2\left(-\frac{g\left(ex^2+d\right)}{ef-dg}\right)}{2f^2}-\frac{gp\text{L}}{}$$

```
[Out] e*p*ln(x)/d/f-1/2*e*p*ln(e*x^2+d)/d/f-1/2*ln(c*(e*x^2+d)^p)/f/x^2-1/2*g*ln(
-e*x^2/d)*ln(c*(e*x^2+d)^p)/f^2+1/2*g*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*
g+e*f))/f^2+1/2*g*p*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/f^2-1/2*g*p*polylog(
2,1+e*x^2/d)/f^2
```

Rubi [A] time = 0.27, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2475, 44, 2416, 2395, 36, 29, 31, 2394, 2315, 2393, 2391}

$$\frac{gp\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2f^2}-\frac{gp\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{2f^2}-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2}+\frac{g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f}{ef}\right.}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)),x]
```

```
[Out] (e*p*Log[x])/(d*f) - (e*p*Log[d + e*x^2])/(2*d*f) - Log[c*(d + e*x^2)^p]/(2
*f*x^2) - (g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/(2*f^2) + (g*Log[c*(d
+ e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*f^2) + (g*p*PolyLog[2, -((
g*(d + e*x^2))/(e*f - d*g))])/(2*f^2) - (g*p*PolyLog[2, 1 + (e*x^2)/d])/(2*
f^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^3\left(f+gx^2\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x^2(f+gx)}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{\log\left(c(d+ex)^p\right)}{fx^2} - \frac{g\log\left(c(d+ex)^p\right)}{f^2x} + \frac{g^2\log\left(c(d+ex)^p\right)}{f^2(f+gx)}\right)dx,x,x^2\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^2}\,dx,x,x^2\right)}{2f} - \frac{g\operatorname{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^2\right)}{2f^2} + \frac{g^2\operatorname{Subst}\left(\int \frac{\log(c(}{f+}\right.}{2f^{}}$$
$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2fx^2} - \frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2} + \frac{g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+g}{ef-}\right.}{2f^2}$$
$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2fx^2} - \frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2} + \frac{g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+g}{ef-}\right.}{2f^2}$$
$$= \frac{ep\log(x)}{df} - \frac{ep\log\left(d+ex^2\right)}{2df} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{2fx^2} - \frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2} +$$

Mathematica [A] time = 0.07, size = 147, normalized size = 0.84

$$\frac{g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right) - \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x^2} - g\left(\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right) + p\text{Li}_2\left(\frac{ex^2}{d}+1\right)\right) + gp\text{Li}_2\left(\frac{g}{}\right.}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)),x]
```

```
[Out] ((e*f*p*(2*Log[x] - Log[d + e*x^2]))/d - (f*Log[c*(d + e*x^2)^p])/x^2 + g*L
og[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)] + g*p*PolyLog[2, (g*(d
 + e*x^2))/(-(e*f) + d*g)] - g*(Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + p*
PolyLog[2, 1 + (e*x^2)/d]))/(2*f^2)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^5+fx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^5 + f*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)*x^3), x)
```

maple [C] time = 0.68, size = 942, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x)
```

```
[Out] -1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*g/f^2*ln(g*x^
2+f)+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*g/f^2*ln(
x)-1/2*ln((e*x^2+d)^p)/f/x^2-ln((e*x^2+d)^p)*g/f^2*ln(x)-1/2*p*g/f^2*sum(ln
(-_alpha+x)*ln(g*x^2+f)-ln(-_alpha+x)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-
d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)
)+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^
2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha
*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,ind
ex=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/Ro
otOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))+p*
g/f^2*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+p*g/f^2*dilog((e*x+(-d*e)^(1/
2))/(-d*e)^(1/2))-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*g/f^2*ln(g*x^2+f)-1/4*I*
Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f/x^2+1/2*ln((e*x^2+d)^p)*g/f^2*ln(g*x
^2+f)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f/x^2+1/2*I*Pi*c
sgn(I*c*(e*x^2+d)^p)^3*g/f^2*ln(x)-1/2*ln(c)/f/x^2-ln(c)*g/f^2*ln(x)+1/2*ln
(c)*g/f^2*ln(g*x^2+f)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*
g/f^2*ln(g*x^2+f)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*g/f^2*ln(x)+1/
4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f/x^2+e*p*ln(x)/d/f-1/2*e*p*ln(e*x^2+d)/d/f+
p*g/f^2*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+p*g/f^2*ln(x)*ln((e*x+(-
d*e)^(1/2))/(-d*e)^(1/2))+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*g/f^2*
ln(g*x^2+f)-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*g/f^2*ln(x
)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f/x^2
```

maxima [A] time = 1.26, size = 178, normalized size = 1.01

$$\frac{1}{2}\,ep\left(\frac{\left(2\,\log\left(\frac{ex^2}{d}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ex^2}{d}\right)\right)g}{ef^2}-\frac{\left(\log\left(gx^2+f\right)\log\left(-\frac{egx^2+ef}{ef-dg}+1\right)+\mathrm{Li}_2\left(\frac{egx^2+ef}{ef-dg}\right)\right)g}{ef^2}-\frac{\log\left(ex^2\right.}{df}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*e*p*((2*log(e*x^2/d + 1)*log(x) + dilog(-e*x^2/d))*g/(e*f^2) - (log(g*x
^2 + f)*log(-(e*g*x^2 + e*f)/(e*f - d*g) + 1) + dilog((e*g*x^2 + e*f)/(e*f
- d*g)))*g/(e*f^2) - log(e*x^2 + d)/(d*f) + 2*log(x)/(d*f)) + 1/2*(g*log(g*
x^2 + f)/f^2 - g*log(x^2)/f^2 - 1/(f*x^2))*log((e*x^2 + d)^p*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{x^3\,\left(g\,x^2+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(x^3*(f + g*x^2)),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)/(x^3*(f + g*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**3/(g*x**2+f),x)
```

```
[Out] Timed out
```

3.343 $\int \frac{x^4 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$

Optimal. Leaf size=667

$$\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{g^{5/2}}-\frac{fx\log\left(c\left(d+ex^2\right)^p\right)}{g^2}+\frac{x^3\log\left(c\left(d+ex^2\right)^p\right)}{3g}-\frac{2d^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g}+\frac{if^{3/2}pI}{}$$

```
[Out] 2*f*p*x/g^2+2/3*d*p*x/e/g-2/9*p*x^3/g-2/3*d^(3/2)*p*arctan(x*e^(1/2)/d^(1/2
))/e^(3/2)/g-f*x*ln(c*(e*x^2+d)^p)/g^2+1/3*x^3*ln(c*(e*x^2+d)^p)/g+f^(3/2)*
arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)/g^(5/2)+2*f^(3/2)*p*arctan(x*g^
(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/g^(5/2)-f^(3/2)*p*arctan
(x*g^(1/2)/f^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/g^(5/2)-f^(3/2)*p*arcta
n(x*g^(1/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/g^(5/2)-I*f^(3/2)*p*pol
ylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/g^(5/2)+1/2*I*f^(3/2)*p*polylog(2
,1+2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2
)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/g^(5/2)+1/2*I*f^(3/2)*p*polylog(2,1-2*((-d)^
(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(
-d)^(1/2)*g^(1/2)))/g^(5/2)-2*f*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/g^2/e^(
1/2)
```

Rubi [A] time = 0.72, antiderivative size = 667, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {2476, 2448, 321, 205, 2455, 302, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$\frac{if^{3/2}p\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2g^{5/2}}+\frac{if^{3/2}p\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2g^{5/2}}-\frac{if^{3/2}pP}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (2*f*p*x)/g^2 + (2*d*p*x)/(3*e*g) - (2*p*x^3)/(9*g) - (2*Sqrt[d]*f*p*ArcTan
[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g^2) - (2*d^(3/2)*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(3*e^(3/2)*g) + (2*f^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/2) - (f^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[
f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - S
qrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) - (f^(3/2)*p*ArcTan[(Sq
rt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e
]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) - (f*x*Log
[c*(d + e*x^2)^p])/g^2 + (x^3*Log[c*(d + e*x^2)^p])/(3*g) + (f^(3/2)*ArcTan
[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/g^(5/2) - (I*f^(3/2)*p*PolyLog[
2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/2) + ((I/2)*f^(3/2)*p*Pol
yLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f]
- Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) + ((I/2)*f^(3/2)*p*P
olyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f
] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
```

```
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{x^4 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx = \int \left(-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{g^2}+\frac{x^2\log\left(c\left(d+ex^2\right)^p\right)}{g}+\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{g^2\left(f+gx^2\right)}\right)dx$$

$$= -\frac{f\int \log\left(c\left(d+ex^2\right)^p\right)dx}{g^2}+\frac{f^2\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}dx}{g^2}+\frac{\int x^2\log\left(c\left(d+ex^2\right)^p\right)dx}{g}$$

$$= -\frac{fx\log\left(c\left(d+ex^2\right)^p\right)}{g^2}+\frac{x^3\log\left(c\left(d+ex^2\right)^p\right)}{3g}+\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right.}{g^{5/2}}$$

$$= \frac{2fpx}{g^2}-\frac{fx\log\left(c\left(d+ex^2\right)^p\right)}{g^2}+\frac{x^3\log\left(c\left(d+ex^2\right)^p\right)}{3g}+\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d\right.\right.}{g^{5/2}}$$

$$= \frac{2fpx}{g^2}+\frac{2dpx}{3eg}-\frac{2px^3}{9g}-\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}-\frac{fx\log\left(c\left(d+ex^2\right)^p\right)}{g^2}+\frac{x^3\log\left(c\left(d\right.\right.}{3g}$$

$$= \frac{2fpx}{g^2}+\frac{2dpx}{3eg}-\frac{2px^3}{9g}-\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}-\frac{2d^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g}-\frac{fx\log\left(c\left(d+\right.\right.}{g^2}$$

$$= \frac{2fpx}{g^2}+\frac{2dpx}{3eg}-\frac{2px^3}{9g}-\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}-\frac{2d^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g}+\frac{2f^{3/2}p\tan^{-1}}{}$$

$$= \frac{2fpx}{g^2}+\frac{2dpx}{3eg}-\frac{2px^3}{9g}-\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}-\frac{2d^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g}+\frac{2f^{3/2}p\tan^{-1}}{}$$

$$= \frac{2fpx}{g^2}+\frac{2dpx}{3eg}-\frac{2px^3}{9g}-\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}-\frac{2d^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g}+\frac{2f^{3/2}p\tan^{-1}}{}$$

<u>**Mathematica**</u> **[A]** time = 0.59, size = 691, normalized size = 1.04

$$\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{g^{5/2}}-\frac{fx\log\left(c\left(d+ex^2\right)^p\right)}{g^2}+\frac{x^3\log\left(c\left(d+ex^2\right)^p\right)}{3g}+\frac{2dp\left(\sqrt{e}x-\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{3e^{3/2}g}-$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (-2*p*x^3)/(9*g) + (2*d*p*(Sqrt[e]*x - Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
)/(3*e^(3/2)*g) + (2*f*p*(x - (Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
))/g^2 - (f*x*Log[c*(d + e*x^2)^p])/g^2 + (x^3*Log[c*(d + e*x^2)^p])/(3*g)
+ (f^(3/2)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/g^(5/2) - ((I/
2)*f^(3/2)*p*(Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqr
t[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + Log[(Sqrt[g]*(Sqrt[-d] + S
qrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)
```

```
/Sqrt[f]] - Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sq
rt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - Log[(Sqrt[g]*(Sqrt[-d] +
Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/S
qrt[f]] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I
*Sqrt[-d]*Sqrt[g])] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]
*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])] - PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*
x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - PolyLog[2, (Sqrt[e]*(Sqrt[f]
+ I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/g^(5/2)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4\log\left(\left(ex^2+d\right)^pc\right)}{gx^2+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(x^4*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\log\left(\left(ex^2+d\right)^pc\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(x^4*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

maple [C] time = 0.50, size = 1011, normalized size = 1.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

```
[Out] -ln(c)/g^2*x*f+1/3*(-p*ln(e*x^2+d)+ln((e*x^2+d)^p))/g*x^3+1/3*p/g*x^3*ln(e*
x^2+d)+ln(c)*f^2/g^2/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)-(-p*ln(e*x^2+d)+
ln((e*x^2+d)^p))/g^2*x*f-2/9*p*x^3/g+1/6*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*
(e*x^2+d)^p)^2/g*x^3+1/6*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g*x^3+1/2*I
*Pi*csgn(I*c*(e*x^2+d)^p)^3/g^2*x*f-2*p*f/g^2*d/(d*e)^(1/2)*arctan(1/(d*e)^
(1/2)*e*x)-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f^2
/g^2/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csg
n(I*c*(e*x^2+d)^p)*csgn(I*c)/g^2*x*f+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*
(e*x^2+d)^p)^2*f^2/g^2/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)-2/3*p/g*d^2/e/
(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn
(I*c)*f^2/g^2/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)+1/3*ln(c)/g*x^3-1/2*I*P
i*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g^2*x*f-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^
3*f^2/g^2/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)+2*f*p*x/g^2-p*f/g^2*x*ln(e*
x^2+d)+2/3*d*p*x/e/g+p*Sum(1/2*(ln(-_alpha+x)*ln(e*x^2+d)-2*e*(1/2*ln(-_alp
ha+x)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootO
f(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alph
a*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,in
dex=2)))/e+1/2*(dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_
alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+dilog((RootOf(_Z^2
*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha
*e*g+d*g-e*f,index=2)))/e))*f^2/g^3/_alpha,_alpha=RootOf(_Z^2*g+f))+(-p*ln(
```

```
e*x^2+d)+ln((e*x^2+d)^p))*f^2/g^2/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)-1/6
*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g*x^3-1/6*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(
e*x^2+d)^p)*csgn(I*c)/g*x^3-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)
^p)^2/g^2*x*f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \log \left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(x^4*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \ln \left(c \left(e\,x^2+d\right)^p\right)}{g\,x^2+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
[Out] int((x^4*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \log \left(c \left(d+ex^2\right)^p\right)}{f+gx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
[Out] Integral(x**4*log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

3.344 $\int \frac{x^2 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$

Optimal. Leaf size=585

$$-\frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{g^{3/2}}+\frac{x\log\left(c\left(d+ex^2\right)^p\right)}{g}-\frac{i\sqrt{f}p\text{Li}_2\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}+1\right)}{2g^{3/2}}-\frac{i\sqrt{f}p\text{Li}}{}$$

```
[Out] -2*p*x/g+x*ln(c*(e*x^2+d)^p)/g+2*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/g/e^(1
/2)-arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)*f^(1/2)/g^(3/2)-2*p*arctan(
x*g^(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))*f^(1/2)/g^(3/2)+p*ar
ctan(x*g^(1/2)/f^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/
2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))*f^(1/2)/g^(3/2)+p*a
rctan(x*g^(1/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/
2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))*f^(1/2)/g^(3/2)+I*p
*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))*f^(1/2)/g^(3/2)-1/2*I*p*polyl
og(2,1+2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^
(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))*f^(1/2)/g^(3/2)-1/2*I*p*polylog(2,1-2*((
-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/
2)+(-d)^(1/2)*g^(1/2)))*f^(1/2)/g^(3/2)
```

Rubi [A] time = 0.59, antiderivative size = 585, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2476, 2448, 321, 205, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$-\frac{i\sqrt{f}p\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2g^{3/2}}-\frac{i\sqrt{f}p\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2g^{3/2}}+\frac{i\sqrt{f}p}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (-2*p*x)/g + (2*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g) - (2*Sqr
t[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)]
)/g^(3/2) + (Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/g^(3/2) + (Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqr
t[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g]
)*(Sqrt[f] - I*Sqrt[g]*x))])/g^(3/2) + (x*Log[c*(d + e*x^2)^p])/g - (Sqrt[f
]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/g^(3/2) + (I*Sqrt[f]*p*
PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(3/2) - ((I/2)*Sqrt[
f]*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*
Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(3/2) - ((I/2)*Sqr
t[f]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e
]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
```

```
/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{x^2 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx = \int \left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{g\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \log\left(c\left(d+ex^2\right)^p\right)dx}{g} - \frac{f\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}dx}{g}$$

$$= \frac{x\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{g^{3/2}} - \frac{(2ep)\int \frac{x^2}{d+ex^2}dx}{g} + \cdots$$

$$= -\frac{2px}{g} + \frac{x\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{g^{3/2}} + \frac{\left(2e\sqrt{f}p\right)\int \cdots}{\cdots}$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g} + \frac{x\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\cdots\right.\right.}{g^{3/2}}$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g} + \frac{x\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\cdots\right.\right.}{g^{3/2}}$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g} - \frac{2\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{3/2}} + \frac{\sqrt{f}p\tan^{-1}\left(\cdots\right.}{\cdots}$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g} - \frac{2\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{3/2}} + \frac{\sqrt{f}p\tan^{-1}\left(\cdots\right.}{\cdots}$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g} - \frac{2\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{3/2}} + \frac{\sqrt{f}p\tan^{-1}\left(\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.29, size = 680, normalized size = 1.16

$$\frac{-2\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right) + 2\sqrt{g}x\log\left(c\left(d+ex^2\right)^p\right) + i\sqrt{f}p\text{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right) + i\sqrt{f}p\text{Li}_2\left(\frac{\sqrt{e}\cdots}{\sqrt{e}\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (-4*Sqrt[g]*p*x + (4*Sqrt[d]*Sqrt[g]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
 + I*Sqrt[f]*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sq
rt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + I*Sqrt[f]*p*Log[(Sqrt[g]*
```

```
(Sqrt[-d] + Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 -
(I*Sqrt[g]*x)/Sqrt[f]] - I*Sqrt[f]*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(
(-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] -
I*Sqrt[f]*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[
-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] + 2*Sqrt[g]*x*Log[c*(d + e*x^2
)^p] - 2*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + I*Sqrt[
f]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt
[-d]*Sqrt[g])] + I*Sqrt[f]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(
Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])] - I*Sqrt[f]*p*PolyLog[2, (Sqrt[e]*(S
qrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - I*Sqrt[f]*
p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d
]*Sqrt[g])])/(2*g^(3/2))
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(x^2*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

maple [C] time = 0.50, size = 746, normalized size = 1.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

```
[Out] (-p*ln(e*x^2+d)+ln((e*x^2+d)^p))/g*x-(-p*ln(e*x^2+d)+ln((e*x^2+d)^p))*f/g/(
f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)+p/g*x*ln(e*x^2+d)-2*p*x/g+2*p/g*d/(d*e
)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)+p*Sum(-1/2*(ln(-_alpha+x)*ln(e*x^2+d)-2*e
*(1/2*ln(-_alpha+x)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x
+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*
e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*
e*g+d*g-e*f,index=2)))/e+1/2*(dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*
f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+dilo
g((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e
*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e))*f/g^2/_alpha,_alpha=RootOf(_Z^2*g
+f))-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f/g/(f*g)^(1/2)*arctan(1/(f
*g)^(1/2)*g*x)-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f/g/(f*
g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f/g/(f*
g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*
c)/g*x-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/g*x+1/2
*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/g*x-1/2*I*Pi*csgn(I*c*(e*
x^2+d)^p)^3/g*x+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c
```

```
)*f/g/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)+ln(c)/g*x-ln(c)*f/g/(f*g)^(1/2)
*arctan(1/(f*g)^(1/2)*g*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] integrate(x^2*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
```

```
[Out] int((x^2*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Integral(x**2*log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

3.345 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=533

$$\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}+\frac{ip\text{Li}_2\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}+1\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}}-\frac{p\,\text{t}}{}$$

```
[Out] arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(1/2)/g^(1/2)+2*p*arctan(x*g^
(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan
(x*g^(1/2)/f^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-p*arcta
n(x*g^(1/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-I*p*pol
ylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2
,1+2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2
)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1-2*((-d)^
(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(
-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {205, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\right.}{\sqrt{f}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] (2*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*S
qrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[
g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt
[f]]*Log[c*(d + e*x^2)^p])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (
2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sq
rt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1
 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d
]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\,\sqrt{g}} - (2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}\,\sqrt{g}\left(d+ex^2\right)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\,\sqrt{g}} - \frac{(2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\,\sqrt{g}} - \frac{(2ep)\int \left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{\left(\sqrt{e}\,p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{\sqrt{f}\,\sqrt{g}} - \frac{\left(\sqrt{e}\,p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{f}\,\sqrt{g}} - \cdots$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{f}\,\sqrt{g}} - \cdots$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{f}\,\sqrt{g}} - \cdots$$

Mathematica [**A**] time = 0.15, size = 564, normalized size = 1.06

$$-\frac{i\left(2i\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right) + p\,\mathrm{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}-i\sqrt{-d}\,\sqrt{g}}\right) + p\,\mathrm{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}+i\sqrt{-d}\,\sqrt{g}}\right) - p\,\mathrm{Li}_2\left(\frac{\sqrt{e}\left(i\sqrt{g}\,x+\sqrt{f}\right)}{\sqrt{e}\,\sqrt{f}-i\sqrt{-d}\,\sqrt{g}}\right) - p\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] ((-1/2*I)*(p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt
[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + p*Log[(Sqrt[g]*(Sqrt[-d] +
Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x
)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] +
 Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-
d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]
*x)/Sqrt[f]] + (2*I)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + p*P
olyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*S
qrt[g])] + p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f]
+ I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sq
rt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*S
qrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/(Sqrt[f]*Sqrt[g])
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

maple [C] time = 0.09, size = 504, normalized size = 0.95

$$-\frac{i\pi \arctan\left(\frac{gx}{\sqrt{fg}}\right)\operatorname{csgn}\left(ic\right)\operatorname{csgn}\left(i\left(e\,x^2+d\right)^p\right)\operatorname{csgn}\left(ic\left(e\,x^2+d\right)^p\right)}{2\sqrt{fg}}+\frac{i\pi \arctan\left(\frac{gx}{\sqrt{fg}}\right)\operatorname{csgn}\left(ic\right)\operatorname{csgn}\left(ic\left(e\,x^2\right.\right.}{2\sqrt{fg}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

```
[Out] (-p*ln(e*x^2+d)+ln((e*x^2+d)^p))/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)+1/2*
p/g*sum(1/_alpha*(ln(-_alpha+x)*ln(e*x^2+d)-ln(-_alpha+x)*(ln((RootOf(_Z^2*
e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*
e*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-
x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))-dilog((RootOf(
_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_a
lpha*e*g+d*g-e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,i
ndex=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2))),_alpha
=RootOf(_Z^2*g+f))+1/2*I/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)*Pi*csgn(I*(e
*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g
*x)*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I/(f*g)^(1/2
)*arctan(1/(f*g)^(1/2)*g*x)*Pi*csgn(I*c*(e*x^2+d)^p)^3+1/2*I/(f*g)^(1/2)*ar
ctan(1/(f*g)^(1/2)*g*x)*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/(f*g)^(1/2)*
arctan(1/(f*g)^(1/2)*g*x)*ln(c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^2),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Integral(log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

3.346 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2\left(f+gx^2\right)}\,dx$

Optimal. Leaf size=581

$$-\frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{3/2}}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{fx}-\frac{i\sqrt{g}p\text{Li}_2\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}+1\right)}{2f^{3/2}}-\frac{i\sqrt{g}p\text{Li}_2\left(\right.}{}$$

```
[Out] -ln(c*(e*x^2+d)^p)/f/x+2*p*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/f/d^(1/2)-arct
an(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)*g^(1/2)/f^(3/2)-2*p*arctan(x*g^(1/2
)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))*g^(1/2)/f^(3/2)+p*arctan(x*g
^(1/2)/f^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g
^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))*g^(1/2)/f^(3/2)+p*arctan(x*
g^(1/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g
^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))*g^(1/2)/f^(3/2)+I*p*polylog
(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))*g^(1/2)/f^(3/2)-1/2*I*p*polylog(2,1+2
*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^
(1/2)-(-d)^(1/2)*g^(1/2)))*g^(1/2)/f^(3/2)-1/2*I*p*polylog(2,1-2*((-d)^(1/2
)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^
(1/2)*g^(1/2)))*g^(1/2)/f^(3/2)
```

Rubi [A] time = 0.60, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2476, 2455, 205, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$-\frac{i\sqrt{g}p\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2f^{3/2}}-\frac{i\sqrt{g}p\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2f^{3/2}}+\frac{i\sqrt{g}p\text{I}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)),x]
```

```
[Out] (2*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f) - (2*Sqrt[g]*p*ArcTan
[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(3/2) + (
Sqrt[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - S
qrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))
])/f^(3/2) + (Sqrt[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/f^(3/2) - Log[c*(d + e*x^2)^p]/(f*x) - (Sqrt[g]*ArcTan[(Sqr
t[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/f^(3/2) + (I*Sqrt[g]*p*PolyLog[2, 1
- (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(3/2) - ((I/2)*Sqrt[g]*p*PolyLog[
2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqr
t[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(3/2) - ((I/2)*Sqrt[g]*p*PolyLo
g[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + S
qrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2\left(f+gx^2\right)}dx = \int \left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{fx^2} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{f\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2}dx}{f} - \frac{g\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}dx}{f}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{fx} - \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{3/2}} + \frac{(2ep)\int \frac{1}{d+ex^2}dx}{f} + \frac{(2eg \ldots}{\ldots}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{fx} - \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{3/2}} + \frac{(2 \ldots}{\ldots}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{fx} - \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{3/2}} + \frac{(2 \ldots}{\ldots}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{fx} - \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{3/2}} - \frac{(\sqrt{\ldots}}{\ldots}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{2\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}} + \frac{\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\ldots\right.}{\ldots}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{2\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}} + \frac{\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\ldots\right.}{\ldots}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{2\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}} + \frac{\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.29, size = 673, normalized size = 1.16

$$\frac{-2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right) - \frac{2\sqrt{f}\log\left(c\left(d+ex^2\right)^p\right)}{x} + i\sqrt{g}p\text{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right) + i\sqrt{g}p\text{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\ldots\right.}{\sqrt{e}\sqrt{f}+i\sqrt{-d}\ldots}\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)),x]
```

```
[Out] ((4*Sqrt[e]*Sqrt[f]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + I*Sqrt[g]*p*Lo
g[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*
Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + I*Sqrt[g]*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[
e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqr
t[f]] - I*Sqrt[g]*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt
[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - I*Sqrt[g]*p*Log[(
```

```
Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log
[1 + (I*Sqrt[g]*x)/Sqrt[f]] - (2*Sqrt[f]*Log[c*(d + e*x^2)^p])/x - 2*Sqrt[g
]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + I*Sqrt[g]*p*PolyLog[2,
 (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])]
+ I*Sqrt[g]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f]
 + I*Sqrt[-d]*Sqrt[g])] - I*Sqrt[g]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt
[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - I*Sqrt[g]*p*PolyLog[2, (S
qrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])])/(2
*f^(3/2))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^4+fx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^4 + f*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)*x^2), x)
```

maple [C] time = 0.40, size = 755, normalized size = 1.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x)
```

```
[Out] -(-p*ln(e*x^2+d)+ln((e*x^2+d)^p))/f/x-(-p*ln(e*x^2+d)+ln((e*x^2+d)^p))/f*g/
(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)-p/f/x*ln(e*x^2+d)+2*p/f*e/(d*e)^(1/2)
*arctan(1/(d*e)^(1/2)*e*x)+p*Sum(-1/2*(ln(-_alpha+x)*ln(e*x^2+d)-2*e*(1/2*l
n(-_alpha+x)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha
)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_
Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g
-e*f,index=2)))/e+1/2*(dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index
=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+dilog((Root
Of(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z
*_alpha*e*g+d*g-e*f,index=2)))/e))/f/_alpha,_alpha=RootOf(_Z^2*g+f))-1/2*I*
Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f/x-1/2*I*Pi*csgn(I*(e*x^2+d
)^p)*csgn(I*c*(e*x^2+d)^p)^2/f*g/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)+1/2*
I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f/x+1/2*I*Pi*csgn(
I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f*g/(f*g)^(1/2)*arctan(1/(f*
g)^(1/2)*g*x)+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f/x+1/2*I*Pi*csgn(I*c*(e*x^2
+d)^p)^3/f*g/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)-1/2*I*Pi*csgn(I*c*(e*x^2
+d)^p)^2*csgn(I*c)/f/x-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f*g/(f*g)
^(1/2)*arctan(1/(f*g)^(1/2)*g*x)-ln(c)/f/x-ln(c)/f*g/(f*g)^(1/2)*arctan(1/(
f*g)^(1/2)*g*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{x^2\left(g\,x^2+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(x^2*(f + g*x^2)),x)
[Out] int(log(c*(d + e*x^2)^p)/(x^2*(f + g*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2\left(f+gx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**2/(g*x**2+f),x)
[Out] Integral(log(c*(d + e*x**2)**p)/(x**2*(f + g*x**2)), x)
```

3.347 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^4\left(f+gx^2\right)}\,dx$

Optimal. Leaf size=651

$$\frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{5/2}}+\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{f^2x}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{3fx^3}-\frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}f}+\frac{ig^{3/2}p\mathrm{Li}_2\left(\frac{\ldots}{(i\sqrt{e}\ldots}\right.}{\ldots}$$

```
[Out] -2/3*e*p/d/f/x-2/3*e^(3/2)*p*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/f-1/3*ln(c*(
e*x^2+d)^p)/f/x^3+g*ln(c*(e*x^2+d)^p)/f^2/x+g^(3/2)*arctan(x*g^(1/2)/f^(1/2
))*ln(c*(e*x^2+d)^p)/f^(5/2)+2*g^(3/2)*p*arctan(x*g^(1/2)/f^(1/2))*ln(2*f^(
1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(5/2)-g^(3/2)*p*arctan(x*g^(1/2)/f^(1/2))*ln(
-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*
f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(5/2)-g^(3/2)*p*arctan(x*g^(1/2)/f^(1/2))*ln
(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*
f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(5/2)-I*g^(3/2)*p*polylog(2,1-2*f^(1/2)/(f^(
1/2)-I*x*g^(1/2)))/f^(5/2)+1/2*I*g^(3/2)*p*polylog(2,1+2*((-d)^(1/2)-x*e^(1
/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^
(1/2)))/f^(5/2)+1/2*I*g^(3/2)*p*polylog(2,1-2*((-d)^(1/2)+x*e^(1/2))*f^(1/2
)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(
5/2)-2*g*p*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/f^2/d^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 651, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2476, 2455, 325, 205, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$\frac{ig^{3/2}p\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2f^{5/2}}+\frac{ig^{3/2}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2f^{5/2}}-\frac{ig^{3/2}p\,\mathrm{Poly}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^4*(f + g*x^2)),x]
```

```
[Out] (-2*e*p)/(3*d*f*x) - (2*e^(3/2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)*f
) - (2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f^2) + (2*g^(3/2)*
p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(
5/2) - (g^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt
[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqr
t[g]*x))])/f^(5/2) - (g^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*
Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sq
rt[f] - I*Sqrt[g]*x))])/f^(5/2) - Log[c*(d + e*x^2)^p]/(3*f*x^3) + (g*Log[c
*(d + e*x^2)^p])/(f^2*x) + (g^(3/2)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d +
e*x^2)^p])/f^(5/2) - (I*g^(3/2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*S
qrt[g]*x)])/f^(5/2) + ((I/2)*g^(3/2)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(S
qrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*
Sqrt[g]*x))])/f^(5/2) + ((I/2)*g^(3/2)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/f^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
```

```
c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^4\left(f+gx^2\right)}\,dx = \int \left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{fx^4} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{f^2x^2} + \frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{f^2\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^4}\,dx}{f} - \frac{g\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\,dx}{f^2} + \frac{g^2\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx}{f^2}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{3fx^3} + \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{f^2x} + \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{5/2}} + \frac{(2e \cdots}{\cdots}$$

$$= -\frac{2ep}{3dfx} - \frac{2\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{3fx^3} + \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{f^2x} + \frac{g^{3/2}\tan^{-1} \cdots}{\cdots}$$

$$= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{3fx^3} + \frac{g\log\left(c\left(d \cdots\right.\right.}{f^2 \cdots}$$

$$= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{3fx^3} + \frac{g\log\left(c\left(d \cdots\right.\right.}{f^2 \cdots}$$

$$= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2} + \frac{2g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g} \cdots}\right.}{f^{5/2}}$$

$$= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2} + \frac{2g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g} \cdots}\right.}{f^{5/2}}$$

$$= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2} + \frac{2g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g} \cdots}\right.}{f^{5/2}}$$

Mathematica [C] time = 0.26, size = 754, normalized size = 1.16

$$\frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{5/2}}+\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{f^2x}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{3fx^3}-\frac{2eg^{3/2}p\left(i\left(\frac{\text{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right)}{\sqrt{e}}+\frac{\log\left(1-\ldots\right.}{\ldots}\right.\right.}{4\sqrt{\ldots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^4*(f + g*x^2)),x]
```

```
[Out] (-2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f^2) - (2*e*p*Hyperge
ometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*f*x) - Log[c*(d + e*x^2)^p]/(3
*f*x^3) + (g*Log[c*(d + e*x^2)^p])/(f^2*x) + (g^(3/2)*ArcTan[(Sqrt[g]*x)/Sq
rt[f]]*Log[c*(d + e*x^2)^p])/f^(5/2) - (2*e*g^(3/2)*p*(((I/4)*((Log[(Sqrt[g
]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (
I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[e] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x
))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])]/Sqrt[e]))/Sqrt[e] + ((I/4)*((Log
[-((Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g]))
]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[e] + PolyLog[2, (Sqrt[e]*(Sqrt[f] -
I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]/Sqrt[e]))/Sqrt[e] - (
(I/4)*((Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*
Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[e] + PolyLog[2, (Sqrt[e]*(Sq
rt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])]/Sqrt[e]))/Sqr
t[e] - ((I/4)*((Log[-((Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] -
 Sqrt[-d]*Sqrt[g]))]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[e] + PolyLog[2, (
Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]/Sq
rt[e]))/Sqrt[e]))/f^(5/2)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{gx^6+fx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^6 + f*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)*x^4), x)
```

maple [C] time = 0.59, size = 1005, normalized size = 1.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x)
```

```
[Out] (-p*ln(e*x^2+d)+ln((e*x^2+d)^p))*g^2/f^2/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g
*x)-1/3*(-p*ln(e*x^2+d)+ln((e*x^2+d)^p))/f/x^3+(-p*ln(e*x^2+d)+ln((e*x^2+d)
^p))*g/f^2/x+p*g/f^2/x*ln(e*x^2+d)-2*p*g/f^2*e/(d*e)^(1/2)*arctan(1/(d*e)^(
1/2)*e*x)+p*Sum(1/2*(ln(-_alpha+x)*ln(e*x^2+d)-2*e*(1/2*ln(-_alpha+x)*(ln((
RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+
2*_Z*_alpha*e*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e
*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e+
1/2*(dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/Root
Of(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+dilog((RootOf(_Z^2*e*g+2*_Z*_
alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*
f,index=2)))/e))*g/f^2/_alpha,_alpha=RootOf(_Z^2*g+f))-1/3*p/f/x^3*ln(e*x^2
+d)-2/3*e*p/d/f/x-2/3*p/f*e^2/d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)-1/6*I
*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f/x^3+1/6*I*Pi*csgn(I*c*(e*
x^2+d)^p)^3/f/x^3-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I
*c)*g/f^2/x+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*g^2/f^2/(f*g)^(1/2)*
arctan(1/(f*g)^(1/2)*g*x)+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*g/f^2/
x+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*g^2/f^2/(f*g)^(1/2)*
arctan(1/(f*g)^(1/2)*g*x)+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p
)^2*g/f^2/x-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*g/f^2/x-1/2*I*Pi*csgn(I*c*(e*x
^2+d)^p)^3*g^2/f^2/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)+1/6*I*Pi*csgn(I*(e
*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f/x^3-1/6*I*Pi*csgn(I*c*(e*x^2+d
)^p)^2*csgn(I*c)/f/x^3-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*c
sgn(I*c)*g^2/f^2/(f*g)^(1/2)*arctan(1/(f*g)^(1/2)*g*x)+ln(c)*g^2/f^2/(f*g)^
(1/2)*arctan(1/(f*g)^(1/2)*g*x)-1/3*ln(c)/f/x^3+ln(c)*g/f^2/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{x^4\,\left(g\,x^2+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(x^4*(f + g*x^2)),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)/(x^4*(f + g*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**4/(g*x**2+f),x)
```

```
[Out] Timed out
```

3.348 $\int \frac{x^5 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$

Optimal. Leaf size=199

$$-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{2g^3\left(f+gx^2\right)}-\frac{f \log\left(c\left(d+ex^2\right)^p\right) \log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{g^3}+\frac{\left(d+ex^2\right) \log\left(c\left(d+ex^2\right)^p\right)}{2eg^2}+\frac{ef^2 p \log\left(d+ex^2\right)}{2g^3(ef-dg)}-$$

```
[Out] -1/2*p*x^2/g^2+1/2*e*f^2*p*ln(e*x^2+d)/g^3/(-d*g+e*f)+1/2*(e*x^2+d)*ln(c*(e
*x^2+d)^p)/e/g^2-1/2*f^2*ln(c*(e*x^2+d)^p)/g^3/(g*x^2+f)-1/2*e*f^2*p*ln(g*x
^2+f)/g^3/(-d*g+e*f)-f*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/g^3-f*p
*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/g^3
```

Rubi [A] time = 0.28, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2475, 43, 2416, 2389, 2295, 2395, 36, 31, 2394, 2393, 2391}

$$-\frac{fp\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{g^3}-\frac{f^2 \log\left(c\left(d+ex^2\right)^p\right)}{2g^3\left(f+gx^2\right)}-\frac{f \log\left(c\left(d+ex^2\right)^p\right) \log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{g^3}+\frac{\left(d+ex^2\right) \log\left(c\left(d+e\right.\right.}{2eg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] -(p*x^2)/(2*g^2) + (e*f^2*p*Log[d + e*x^2])/(2*g^3*(e*f - d*g)) + ((d + e*x
^2)*Log[c*(d + e*x^2)^p])/(2*e*g^2) - (f^2*Log[c*(d + e*x^2)^p])/(2*g^3*(f
+ g*x^2)) - (e*f^2*p*Log[f + g*x^2])/(2*g^3*(e*f - d*g)) - (f*Log[c*(d + e*
x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/g^3 - (f*p*PolyLog[2, -((g*(d + e
*x^2))/(e*f - d*g))])/g^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{x^5 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2\log(c(d+ex)^p)}{(f+gx)^2}dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{g^2} + \frac{f^2\log(c(d+ex)^p)}{g^2(f+gx)^2} - \frac{2f\log(c(d+ex)^p)}{g^2(f+gx)}\right) dx, x,$$

$$= \frac{\text{Subst}\left(\int \log(c(d+ex)^p)\, dx, x, x^2\right)}{2g^2} - \frac{f\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx} dx, x, x^2\right)}{g^2} + \frac{f^2\,\text{Sub}}{}$$

$$= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{2g^3\left(f+gx^2\right)} - \frac{f\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{g^3} + \frac{\text{Subst}\left(\int \log(cx^p\right.}{2eg}$$

$$= -\frac{px^2}{2g^2} + \frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg^2} - \frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{2g^3\left(f+gx^2\right)} - \frac{f\log\left(c\left(d+ex^2\right)\right.}{g^3}$$

$$= -\frac{px^2}{2g^2} + \frac{ef^2p\log\left(d+ex^2\right)}{2g^3(ef-dg)} + \frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg^2} - \frac{f^2\log\left(c\left(d+ex^2\right)^p\right.}{2g^3\left(f+gx^2\right)}$$

Mathematica [A] time = 0.22, size = 166, normalized size = 0.83

$$-\frac{\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{g\left(f+gx^2\right)} + \frac{2f\left(\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right) + p\text{Li}_2\left(\frac{g\left(ex^2+d\right)}{dg-ef}\right)\right)}{g} - \frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e} + \frac{ef^2p\left(\log\left(d+ex^2\right)-\log\left(f+gx^2\right)\right)}{g(dg-ef)} + p}{2g^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] -1/2*(p*x^2 - ((d + e*x^2)*Log[c*(d + e*x^2)^p])/e + (f^2*Log[c*(d + e*x^2)
^p])/(g*(f + g*x^2)) + (e*f^2*p*(Log[d + e*x^2] - Log[f + g*x^2]))/(g*(-(e*
f) + d*g)) + (2*f*(Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)] +
p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)]))/g)/g^2
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^5\log\left(\left(ex^2+d\right)^p c\right)}{g^2x^4+2\,fgx^2+f^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(x^5*log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

maple [C] time = 0.76, size = 985, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

```
[Out] 1/2*ln((e*x^2+d)^p)/g^2*x^2-1/2*ln((e*x^2+d)^p)*f^2/g^3/(g*x^2+f)-ln((e*x^2
+d)^p)*f/g^3*ln(g*x^2+f)-1/2*p*x^2/g^2+1/2*p*e/g^3*f^2/(d*g-e*f)*ln(g*x^2+f
)+1/2*p/e/g/(d*g-e*f)*ln(e*x^2+d)*d^2-1/2*p/g^2/(d*g-e*f)*ln(e*x^2+d)*d*f-1
/2*p*e/g^3/(d*g-e*f)*ln(e*x^2+d)*f^2+p*f/g^3*sum(ln(-_alpha+x)*ln(g*x^2+f)-
ln(-_alpha+x)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alph
a)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*
_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*
g+e*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+
_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^
2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alph
a*e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))+1/2*I*Pi*csgn(I*c*(e*x^2+d
)^p)^3*f/g^3*ln(g*x^2+f)+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)
*csgn(I*c)*f/g^3*ln(g*x^2+f)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d
)^p)^2/g^2*x^2+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)
*f^2/g^3/(g*x^2+f)-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g^2*x^2-1/4*I*Pi*csgn(I
*c*(e*x^2+d)^p)^2*csgn(I*c)*f^2/g^3/(g*x^2+f)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*
csgn(I*c*(e*x^2+d)^p)^2*f^2/g^3/(g*x^2+f)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn
(I*c*(e*x^2+d)^p)*csgn(I*c)/g^2*x^2+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f^2/g^
3/(g*x^2+f)-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f/g^3*ln(g
*x^2+f)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f/g^3*ln(g*x^2+f)+1/4*I*
Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g^2*x^2+1/2*ln(c)/g^2*x^2-1/2*ln(c)*f^
2/g^3/(g*x^2+f)-ln(c)*f/g^3*ln(g*x^2+f)
```

maxima [A] time = 0.80, size = 337, normalized size = 1.69

$$-\frac{\left(ef^{2}p+2\left(ef^{2}-dfg\right)\log(c)\right)\log\left(gx^{2}+f\right)}{2\left(efg^{3}-dg^{4}\right)}-\frac{\left(e^{2}fg^{2}p-deg^{3}p-\left(e^{2}fg^{2}-deg^{3}\right)\log(c)\right)x^{4}+\left(e^{2}f^{2}gp-defg^{2}p-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*f^2*p + 2*(e*f^2 - d*f*g)*log(c))*log(g*x^2 + f)/(e*f*g^3 - d*g^4)
- 1/2*((e^2*f*g^2*p - d*e*g^3*p - (e^2*f*g^2 - d*e*g^3)*log(c))*x^4 + (e^2*
f^2*g*p - d*e*f*g^2*p - (e^2*f^2*g - d*e*f*g^2)*log(c))*x^2 - (2*d*e*f^2*g*
p - d^2*f*g^2*p + (e^2*f*g^2*p - d*e*g^3*p)*x^4 + (2*e^2*f^2*g*p - d^2*g^3*
p)*x^2)*log(e*x^2 + d) + (e^2*f^3 - d*e*f^2*g)*log(c))/(e^2*f^2*g^3 - d*e*f
*g^4 + (e^2*f*g^4 - d*e*g^5)*x^2) - (log(e*x^2 + d)*log((e*g*x^2 + d*g)/(e*
f - d*g) + 1) + dilog(-(e*g*x^2 + d*g)/(e*f - d*g)))*f*p/g^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{{\left(g\,x^2+f\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
```

```
[Out] int((x^5*log(c*(d + e*x^2)^p))/(f + g*x^2)^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

3.349 $\int \frac{x^3 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$

Optimal. Leaf size=155

$$\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2g^2\left(f+gx^2\right)}+\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2g^2}+\frac{p\text{Li}_2\left(-\frac{g\left(ex^2+d\right)}{ef-dg}\right)}{2g^2}-\frac{efp\log\left(d+ex^2\right)}{2g^2(ef-dg)}+\frac{efp\log\left(f+gx^2\right)}{2g^2(ef-dg)}$$

```
[Out] -1/2*e*f*p*ln(e*x^2+d)/g^2/(-d*g+e*f)+1/2*f*ln(c*(e*x^2+d)^p)/g^2/(g*x^2+f)
+1/2*e*f*p*ln(g*x^2+f)/g^2/(-d*g+e*f)+1/2*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/
(-d*g+e*f))/g^2+1/2*p*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/g^2
```

Rubi [A] time = 0.22, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2475, 43, 2416, 2395, 36, 31, 2394, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2g^2}+\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2g^2\left(f+gx^2\right)}+\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2g^2}-\frac{efp\log\left(d+ex^2\right)}{2g^2(ef-dg)}+\frac{efp\log\left(f-\right.}{2g^2(ef-}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] -(e*f*p*Log[d + e*x^2])/(2*g^2*(e*f - d*g)) + (f*Log[c*(d + e*x^2)^p])/(2*g
^2*(f + g*x^2)) + (e*f*p*Log[f + g*x^2])/(2*g^2*(e*f - d*g)) + (Log[c*(d +
e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*g^2) + (p*PolyLog[2, -((g*(d
 + e*x^2))/(e*f - d*g))])/(2*g^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{x^3 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{(f+gx)^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{f\log(c(d+ex)^p)}{g(f+gx)^2}+\frac{\log(c(d+ex)^p)}{g(f+gx)}\right)dx, x, x^2\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^2\right)}{2g} - \frac{f\operatorname{Subst}\left(\int \frac{\log(c(d+ex)^p)}{(f+gx)^2}\,dx, x, x^2\right)}{2g}$$

$$= \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2g^2\left(f+gx^2\right)} + \frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2g^2} - \frac{(ep)\operatorname{Subst}\left(\int \frac{\log\left(\frac{e(f+gx}{ef-dg}\right.}{d+ex}\right.}{2g^2}$$

$$= \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2g^2\left(f+gx^2\right)} + \frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2g^2} - \frac{p\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\right.}{2g^2}$$

$$= -\frac{efp\log\left(d+ex^2\right)}{2g^2(ef-dg)} + \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2g^2\left(f+gx^2\right)} + \frac{efp\log\left(f+gx^2\right)}{2g^2(ef-dg)} + \frac{\log\left(c\left(d+ex^2\right.\right.}{}$$

Mathematica [A] time = 0.09, size = 131, normalized size = 0.85

$$\frac{\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}+\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)+p\text{Li}_2\left(\frac{g\left(ex^2+d\right)}{dg-ef}\right)+\frac{efp\log\left(d+ex^2\right)}{dg-ef}+\frac{efp\log\left(f+gx^2\right)}{ef-dg}}{2g^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] ((e*f*p*Log[d + e*x^2])/(-(e*f) + d*g) + (f*Log[c*(d + e*x^2)^p])/(f + g*x^
2) + (e*f*p*Log[f + g*x^2])/(e*f - d*g) + Log[c*(d + e*x^2)^p]*Log[(e*(f +
g*x^2))/(e*f - d*g)] + p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)])/(2*g^2
)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3\log\left(\left(ex^2+d\right)^p c\right)}{g^2x^4+2\,fgx^2+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

maple [C] time = 0.76, size = 732, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

```
[Out] 1/2*ln((e*x^2+d)^p)*f/g^2/(g*x^2+f)+1/2*ln((e*x^2+d)^p)/g^2*ln(g*x^2+f)-1/2
*p/g^2*sum(ln(-_alpha+x)*ln(g*x^2+f)-ln(-_alpha+x)*(ln((RootOf(_Z^2*e*g+2*_
Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g
+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alph
a)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)))-dilog((RootOf(_Z^2*e*
g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*
g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)
-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)),_alpha=RootOf(
_Z^2*e+d))-1/2*p*e*f/g^2/(d*g-e*f)*ln(g*x^2+f)+1/2*p*e*f/g^2/(d*g-e*f)*ln(e
*x^2+d)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f/g^2/(g*x^2+f
)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/g^2*ln(g*x^2+f)-1/4*
I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f/g^2/(g*x^2+f)-1/
4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/g^2*ln(g*x^2+f)-
1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f/g^2/(g*x^2+f)-1/4*I*Pi*csgn(I*c*(e*x^2+d
```

```
)^p)^3/g^2*ln(g*x^2+f)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f/g^2/(g*
x^2+f)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g^2*ln(g*x^2+f)+1/2*ln(c)
*f/g^2/(g*x^2+f)+1/2*ln(c)/g^2*ln(g*x^2+f)
```

maxima [A] time = 1.29, size = 181, normalized size = 1.17

$$\frac{\left(efp+\left(ef-dg\right)\log(c)\right)\log\left(gx^2+f\right)}{2\left(efg^2-dg^3\right)}-\frac{\left(efgpx^2+dfgp\right)\log\left(ex^2+d\right)-\left(ef^2-dfg\right)\log(c)}{2\left(ef^2g^2-dfg^3+\left(efg^3-dg^4\right)x^2\right)}+\frac{\left(\log\left(ex^2+d\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(e*f*p + (e*f - d*g)*log(c))*log(g*x^2 + f)/(e*f*g^2 - d*g^3) - 1/2*((e
*f*g*p*x^2 + d*f*g*p)*log(e*x^2 + d) - (e*f^2 - d*f*g)*log(c))/(e*f^2*g^2 -
 d*f*g^3 + (e*f*g^3 - d*g^4)*x^2) + 1/2*(log(e*x^2 + d)*log((e*g*x^2 + d*g)
/(e*f - d*g) + 1) + dilog(-(e*g*x^2 + d*g)/(e*f - d*g)))*p/g^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{{\left(g\,x^2+f\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
```

```
[Out] int((x^3*log(c*(d + e*x^2)^p))/(f + g*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

3.350 $\int \frac{x \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$

Optimal. Leaf size=83

$$-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2g\left(f+gx^2\right)}+\frac{ep\log\left(d+ex^2\right)}{2g(ef-dg)}-\frac{ep\log\left(f+gx^2\right)}{2g(ef-dg)}$$

```
[Out] 1/2*e*p*ln(e*x^2+d)/g/(-d*g+e*f)-1/2*ln(c*(e*x^2+d)^p)/g/(g*x^2+f)-1/2*e*p*
ln(g*x^2+f)/g/(-d*g+e*f)
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2475, 2395, 36, 31}

$$-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2g\left(f+gx^2\right)}+\frac{ep\log\left(d+ex^2\right)}{2g(ef-dg)}-\frac{ep\log\left(f+gx^2\right)}{2g(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] (e*p*Log[d + e*x^2])/(2*g*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(2*g*(f + g*x
^2)) - (e*p*Log[f + g*x^2])/(2*g*(e*f - d*g))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{x\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}dx = \frac{1}{2}\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{(f+gx)^2}dx, x, x^2\right)$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2g\left(f+gx^2\right)} + \frac{(ep)\,\text{Subst}\left(\int \frac{1}{(d+ex)(f+gx)}dx, x, x^2\right)}{2g}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2g\left(f+gx^2\right)} - \frac{(ep)\,\text{Subst}\left(\int \frac{1}{f+gx}dx, x, x^2\right)}{2(ef-dg)} + \frac{\left(e^2p\right)\text{Subst}\left(\int \frac{1}{d+ex}dx, x, x\right.}{2g(ef-dg)}$$

$$= \frac{ep\log\left(d+ex^2\right)}{2g(ef-dg)} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{2g\left(f+gx^2\right)} - \frac{ep\log\left(f+gx^2\right)}{2g(ef-dg)}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.76

$$\frac{\frac{ep\left(\log\left(d+ex^2\right)-\log\left(f+gx^2\right)\right)}{ef-dg} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}}{2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] (-(Log[c*(d + e*x^2)^p]/(f + g*x^2)) + (e*p*(Log[d + e*x^2] - Log[f + g*x^2
]))/(e*f - d*g))/(2*g)
```

fricas [A] time = 0.70, size = 91, normalized size = 1.10

$$\frac{\left(egpx^2+dgp\right)\log\left(ex^2+d\right)-\left(egpx^2+efp\right)\log\left(gx^2+f\right)-\left(ef-dg\right)\log(c)}{2\left(ef^2g-dfg^2+\left(efg^2-dg^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((e*g*p*x^2 + d*g*p)*log(e*x^2 + d) - (e*g*p*x^2 + e*f*p)*log(g*x^2 + f
) - (e*f - d*g)*log(c))/(e*f^2*g - d*f*g^2 + (e*f*g^2 - d*g^3)*x^2)
```

giac [B] time = 0.21, size = 182, normalized size = 2.19

$$-\frac{\left(x^2e+d\right)gpe\log\left(x^2e+d\right)-\left(x^2e+d\right)gpe\log\left(\left(x^2e+d\right)g-dg+fe\right)+dgpe\log\left(\left(x^2e+d\right)g-dg+fe\right)-fp}{2\left(\left(x^2e+d\right)dg^3-d^2g^3-\left(x^2e+d\right)fg^2e+2dfg^2e-f^2g\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] -1/2*((x^2*e + d)*g*p*e*log(x^2*e + d) - (x^2*e + d)*g*p*e*log((x^2*e + d)*
g - d*g + f*e) + d*g*p*e*log((x^2*e + d)*g - d*g + f*e) - f*p*e^2*log((x^2*
e + d)*g - d*g + f*e) + d*g*e*log(c) - f*e^2*log(c))/((x^2*e + d)*d*g^3 - d
^2*g^3 - (x^2*e + d)*f*g^2*e + 2*d*f*g^2*e - f^2*g*e^2)
```

maple [C] time = 0.54, size = 371, normalized size = 4.47

$$-\frac{\ln\left(\left(e\,x^2+d\right)^p\right)}{2\left(g\,x^2+f\right)g} - \frac{2egp\,x^2\ln\left(-e\,x^2-d\right)-2egp\,x^2\ln\left(g\,x^2+f\right)-i\pi dg\,\text{csgn}(ic)\,\text{csgn}\left(i\left(e\,x^2+d\right)^p\right)\text{csgn}\left(ic\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

```
[Out] -1/2/g/(g*x^2+f)*ln((e*x^2+d)^p)-1/4*(I*Pi*d*g*csgn(I*(e*x^2+d)^p)*csgn(I*c
*(e*x^2+d)^p)^2-I*Pi*d*g*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c
)-I*Pi*d*g*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*d*g*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*
c)-I*Pi*e*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+I*Pi*e*f*csgn(I*(e*
x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+I*Pi*e*f*csgn(I*c*(e*x^2+d)^p)^3-
I*Pi*e*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-2*ln(g*x^2+f)*e*g*p*x^2+2*ln(-e*
x^2-d)*e*g*p*x^2-2*ln(g*x^2+f)*e*f*p+2*ln(-e*x^2-d)*e*f*p+2*ln(c)*d*g-2*ln(
c)*e*f)/g/(g*x^2+f)/(d*g-e*f)
```

maxima [A] time = 0.46, size = 74, normalized size = 0.89

$$\frac{ep\left(\frac{\log\left(ex^2+d\right)}{ef-dg}-\frac{\log\left(gx^2+f\right)}{ef-dg}\right)}{2g}-\frac{\log\left(\left(ex^2+d\right)^pc\right)}{2\left(gx^2+f\right)g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] 1/2*e*p*(log(e*x^2 + d)/(e*f - d*g) - log(g*x^2 + f)/(e*f - d*g))/g - 1/2*l
og((e*x^2 + d)^p*c)/((g*x^2 + f)*g)
```

mupad [B] time = 1.46, size = 80, normalized size = 0.96

$$-\frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{2g\,\left(g\,x^2+f\right)}-\frac{e\,p\,\mathrm{atan}\left(\frac{x^2\,\left(d\,g\,1\mathrm{i}-e\,f\,1\mathrm{i}\right)}{2\,d\,f+d\,g\,x^2+e\,f\,x^2}\right)\,1\mathrm{i}}{d\,g^2-e\,f\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
```

```
[Out] - log(c*(d + e*x^2)^p)/(2*g*(f + g*x^2)) - (e*p*atan((x^2*(d*g*1i - e*f*1i)
)/(2*d*f + d*g*x^2 + e*f*x^2))*1i)/(d*g^2 - e*f*g)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

3.351 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x\left(f+gx^2\right)^2} dx$

Optimal. Leaf size=201

$$-\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f^2}+\frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f\left(f+gx^2\right)}-\frac{p\text{Li}_2\left(-\frac{g\left(ex^2+d\right)}{ef-dg}\right)}{2f^2}+\frac{p\text{Li}_2}{}$$

```
[Out] -1/2*e*p*ln(e*x^2+d)/f/(-d*g+e*f)+1/2*ln(c*(e*x^2+d)^p)/f/(g*x^2+f)+1/2*ln(
-e*x^2/d)*ln(c*(e*x^2+d)^p)/f^2+1/2*e*p*ln(g*x^2+f)/f/(-d*g+e*f)-1/2*ln(c*(
e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/f^2-1/2*p*polylog(2,-g*(e*x^2+d)/(-d
*g+e*f))/f^2+1/2*p*polylog(2,1+e*x^2/d)/f^2
```

Rubi [A] time = 0.28, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2475, 44, 2416, 2394, 2315, 2395, 36, 31, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2f^2}+\frac{p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{2f^2}-\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f^2}+\frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+e\right.\right.}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)^2),x]
```

```
[Out] -(e*p*Log[d + e*x^2])/(2*f*(e*f - d*g)) + Log[c*(d + e*x^2)^p]/(2*f*(f + g*
x^2)) + (Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/(2*f^2) + (e*p*Log[f + g*x
^2])/(2*f*(e*f - d*g)) - (Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d
*g)])/(2*f^2) - (p*PolyLog[2, -((g*(d + e*x^2))/(e*f - d*g))])/(2*f^2) + (p
*PolyLog[2, 1 + (e*x^2)/d])/(2*f^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x\left(f+gx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x(f+gx)^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{\log\left(c(d+ex)^p\right)}{f^2x} - \frac{g\log\left(c(d+ex)^p\right)}{f(f+gx)^2} - \frac{g\log\left(c(d+ex)^p\right)}{f^2(f+gx)}\right)dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^2\right)}{2f^2} - \frac{g\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^2\right)}{2f^2} - \frac{g\,\text{Subst}\left(\int \frac{\log(\cdots}{(\cdots}\right.}{}$$

$$= \frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f\left(f+gx^2\right)} + \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2} - \frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right.}{ef-dg}\right.}{2f^2}$$

$$= \frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f\left(f+gx^2\right)} + \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2} - \frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right.}{ef-dg}\right.}{2f^2}$$

$$= -\frac{ep\log\left(d+ex^2\right)}{2f(ef-dg)} + \frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f\left(f+gx^2\right)} + \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2} + \frac{ep\log\left(f\right.}{2f(ef-\right.}$$

Mathematica [A] time = 0.11, size = 170, normalized size = 0.85

$$\frac{\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} - \log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right) + \log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right) - p\,\text{Li}_2\left(\frac{g\left(ex^2+d\right)}{dg-ef}\right) + \frac{efp\log\left(d+ex\right.}{dg-ef}}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)^2),x]
```

```
[Out] ((e*f*p*Log[d + e*x^2])/(-(e*f) + d*g) + (f*Log[c*(d + e*x^2)^p])/(f + g*x^
2) + Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + (e*f*p*Log[f + g*x^2])/(e*f -
 d*g) - Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)] - p*PolyLog[2
, (g*(d + e*x^2))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^2)/d])/(2*f^2)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{g^2x^5+2\,fgx^3+f^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g^2*x^5 + 2*f*g*x^3 + f^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)^2*x), x)
```

maple [C] time = 0.72, size = 984, normalized size = 4.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x)
```

```
[Out] 1/2*ln(c)/f/(g*x^2+f)-1/2*ln((e*x^2+d)^p)/f^2*ln(g*x^2+f)+ln((e*x^2+d)^p)/f
^2*ln(x)+1/2*ln((e*x^2+d)^p)/f/(g*x^2+f)-1/2*ln(c)/f^2*ln(g*x^2+f)+ln(c)/f^
2*ln(x)-p/f^2*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p/f^2*dilog((e*x+(-d*
e)^(1/2))/(-d*e)^(1/2))+1/2*p/f^2*sum(ln(-_alpha+x)*ln(g*x^2+f)-ln(-_alpha+
x)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_
Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e
*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index
=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/Roo
tOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*
_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e
*f,index=2)),_alpha=RootOf(_Z^2*e+d))+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn
(I*c)/f/(g*x^2+f)+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f^2*
ln(x)-p/f^2*ln(x)*ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))-p/f^2*ln(x)*ln((e*x+
(-d*e)^(1/2))/(-d*e)^(1/2))-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f^2*
ln(g*x^2+f)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f/
(g*x^2+f)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f/(g*x^2+f)+
1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f^2*ln(x)-1/4*I*Pi*csgn(I*(e*x^2
+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f^2*ln(g*x^2+f)-1/2*I*Pi*csgn(I*(e*x^2+d)^p)
*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f^2*ln(x)-1/2*p*e/f/(d*g-e*f)*ln(g*x^2+f)+
1/2*p*e/f/(d*g-e*f)*ln(e*x^2+d)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f^2*ln(g*x
^2+f)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f^2*ln(x)-1/4*I*Pi*csgn(I*c*(e*x^2+d
)^p)^3/f/(g*x^2+f)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(
I*c)/f^2*ln(g*x^2+f)
```

maxima [A] time = 1.27, size = 197, normalized size = 0.98

$$-\frac{1}{2}\,ep\left(\frac{\log\left(ex^2+d\right)}{ef^2-dfg}-\frac{\log\left(gx^2+f\right)}{ef^2-dfg}+\frac{2\,\log\left(\frac{ex^2}{d}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ex^2}{d}\right)}{ef^2}-\frac{\log\left(gx^2+f\right)\log\left(-\frac{egx^2+ef}{ef-dg}+1\right)+}{ef^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] -1/2*e*p*(log(e*x^2 + d)/(e*f^2 - d*f*g) - log(g*x^2 + f)/(e*f^2 - d*f*g) +
 (2*log(e*x^2/d + 1)*log(x) + dilog(-e*x^2/d))/(e*f^2) - (log(g*x^2 + f)*lo
g(-(e*g*x^2 + e*f)/(e*f - d*g) + 1) + dilog((e*g*x^2 + e*f)/(e*f - d*g)))/(
e*f^2)) + 1/2*(1/(f*g*x^2 + f^2) - log(g*x^2 + f)/f^2 + log(x^2)/f^2)*log((
e*x^2 + d)^p*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{x\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(x*(f + g*x^2)^2),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)/(x*(f + g*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x/(g*x**2+f)**2,x)
[Out] Timed out
```

3.352 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^3\left(f+gx^2\right)^2} dx$

Optimal. Leaf size=251

$$-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^3}+\frac{g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{f^3}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2f^2\left(f+gx^2\right)}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f^2x^2}+-$$

```
[Out] e*p*ln(x)/d/f^2-1/2*e*p*ln(e*x^2+d)/d/f^2+1/2*e*g*p*ln(e*x^2+d)/f^2/(-d*g+e
*f)-1/2*ln(c*(e*x^2+d)^p)/f^2/x^2-1/2*g*ln(c*(e*x^2+d)^p)/f^2/(g*x^2+f)-g*l
n(-e*x^2/d)*ln(c*(e*x^2+d)^p)/f^3-1/2*e*g*p*ln(g*x^2+f)/f^2/(-d*g+e*f)+g*ln
(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/f^3+g*p*polylog(2,-g*(e*x^2+d)/(
-d*g+e*f))/f^3-g*p*polylog(2,1+e*x^2/d)/f^3
```

Rubi [A] time = 0.34, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2475, 44, 2416, 2395, 36, 29, 31, 2394, 2315, 2393, 2391}

$$\frac{gp\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{f^3}-\frac{gp\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{f^3}-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^3}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2f^2\left(f+gx^2\right)}+\frac{g\log(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)^2),x]
```

```
[Out] (e*p*Log[x])/(d*f^2) - (e*p*Log[d + e*x^2])/(2*d*f^2) + (e*g*p*Log[d + e*x^
2])/(2*f^2*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(2*f^2*x^2) - (g*Log[c*(d +
e*x^2)^p])/(2*f^2*(f + g*x^2)) - (g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])
/f^3 - (e*g*p*Log[f + g*x^2])/(2*f^2*(e*f - d*g)) + (g*Log[c*(d + e*x^2)^p]
*Log[(e*(f + g*x^2))/(e*f - d*g)])/f^3 + (g*p*PolyLog[2, -((g*(d + e*x^2))/
(e*f - d*g))])/f^3 - (g*p*PolyLog[2, 1 + (e*x^2)/d])/f^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^3\left(f+gx^2\right)^2}dx = \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x^2(f+gx)^2}dx,x,x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\log\left(c(d+ex)^p\right)}{f^2x^2} - \frac{2g\log\left(c(d+ex)^p\right)}{f^3x} + \frac{g^2\log\left(c(d+ex)^p\right)}{f^2(f+gx)^2} + \frac{2g^2\log\left(c\right.}{f^3(f\right.}\right.\right.$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^2}dx,x,x^2\right)}{2f^2} - \frac{g\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}dx,x,x^2\right)}{f^3} + \frac{g^2\,\text{Subst}\left(\int \frac{\log(c(\right.}{f+}\right.}{f^3}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f^2x^2} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2f^2\left(f+gx^2\right)} - \frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^3} + \frac{g\log\left(c\right.}{}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f^2x^2} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2f^2\left(f+gx^2\right)} - \frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^3} + \frac{g\log\left(c\right.}{}$$

$$= \frac{ep\log(x)}{df^2} - \frac{ep\log\left(d+ex^2\right)}{2df^2} + \frac{egp\log\left(d+ex^2\right)}{2f^2(ef-dg)} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f^2x^2} - \frac{g\log\left(c\left(d+e\right.\right.}{2f^2\left(f+g\right.}$$

Mathematica [A] time = 0.17, size = 208, normalized size = 0.83

$$\frac{2g\left(\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)+p\text{Li}_2\left(\frac{g\left(ex^2+d\right)}{dg-ef}\right)\right) - \frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} - \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x^2} - 2g\left(\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(\right.\right.\right.}{2f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)^2),x]
```

```
[Out] ((e*f*p*(2*Log[x] - Log[d + e*x^2]))/d - (f*Log[c*(d + e*x^2)^p])/x^2 - (f*
g*Log[c*(d + e*x^2)^p])/(f + g*x^2) + (e*f*g*p*(Log[d + e*x^2] - Log[f + g*
x^2]))/(e*f - d*g) + 2*g*(Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d
*g)] + p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)]) - 2*g*(Log[-((e*x^2)/d
)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, 1 + (e*x^2)/d]))/(2*f^3)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{g^2x^7+2\,fgx^5+f^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g^2*x^7 + 2*f*g*x^5 + f^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)^2*x^3), x)
```

maple [C] time = 0.68, size = 1216, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x)
```

```
[Out] -1/2*ln((e*x^2+d)^p)/f^2/x^2-I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)
^2*g/f^3*ln(x)-1/2*ln(c)/f^2/x^2+I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d
)^p)*csgn(I*c)*g/f^3*ln(x)-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^
p)*csgn(I*c)*g/f^3*ln(g*x^2+f)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2
+d)^p)*csgn(I*c)*g/f^2/(g*x^2+f)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f^2/x^2-1
/2*ln(c)*g/f^2/(g*x^2+f)+ln(c)*g/f^3*ln(g*x^2+f)-2*ln(c)*g/f^3*ln(x)+2*p*g/
f^3*dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+2*p*g/f^3*dilog((e*x+(-d*e)^(1/
2))/(-d*e)^(1/2))-p*g/f^3*sum(ln(-_alpha+x)*ln(g*x^2+f)-ln(-_alpha+x)*(ln((
RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+
2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e
*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)))-di
log((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2
*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e
*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index
=2)),_alpha=RootOf(_Z^2*e+d))-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*g/
f^2/(g*x^2+f)-1/2*ln((e*x^2+d)^p)*g/f^2/(g*x^2+f)+ln((e*x^2+d)^p)*g/f^3*ln(
g*x^2+f)-2*ln((e*x^2+d)^p)*g/f^3*ln(x)+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*
c*(e*x^2+d)^p)^2*g/f^3*ln(g*x^2+f)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e
*x^2+d)^p)*csgn(I*c)/f^2/x^2+1/2*p*e/f^2*g/(d*g-e*f)*ln(g*x^2+f)-p*e/f^2/(d
*g-e*f)*ln(e*x^2+d)*g+1/2*p*e^2/f/d/(d*g-e*f)*ln(e*x^2+d)+I*Pi*csgn(I*c*(e*
x^2+d)^p)^3*g/f^3*ln(x)-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f^2/x^2-
1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f^2/x^2-1/2*I*Pi*csgn(
I*c*(e*x^2+d)^p)^3*g/f^3*ln(g*x^2+f)+2*p*g/f^3*ln(x)*ln((-e*x+(-d*e)^(1/2))
/(-d*e)^(1/2))+2*p*g/f^3*ln(x)*ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+e*p*ln(x
)/d/f^2+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*g/f^2/(g*x^2+f)+1/2*I*Pi*csgn(I*c*
(e*x^2+d)^p)^2*csgn(I*c)*g/f^3*ln(g*x^2+f)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csg
n(I*c*(e*x^2+d)^p)^2*g/f^2/(g*x^2+f)-I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)
*g/f^3*ln(x)
```

maxima [A] time = 0.74, size = 295, normalized size = 1.18

$$-\frac{1}{2}\left(f\left(\frac{e\log\left(ex^2+d\right)}{def^3-d^2f^2g}-\frac{g\log\left(gx^2+f\right)}{ef^4-df^3g}-\frac{\log\left(x^2\right)}{df^3}\right)-2g\left(\frac{\log\left(ex^2+d\right)}{ef^3-df^2g}-\frac{\log\left(gx^2+f\right)}{ef^3-df^2g}\right)-\frac{2\left(2\log\left(\frac{ex^2}{d}+1\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(f*(e*log(e*x^2 + d)/(d*e*f^3 - d^2*f^2*g) - g*log(g*x^2 + f)/(e*f^4 -
 d*f^3*g) - log(x^2)/(d*f^3)) - 2*g*(log(e*x^2 + d)/(e*f^3 - d*f^2*g) - log
(g*x^2 + f)/(e*f^3 - d*f^2*g)) - 2*(2*log(e*x^2/d + 1)*log(x) + dilog(-e*x^
2/d))*g/(e*f^3) + 2*(log(g*x^2 + f)*log(-(e*g*x^2 + e*f)/(e*f - d*g) + 1) +
 dilog((e*g*x^2 + e*f)/(e*f - d*g)))*g/(e*f^3))*e*p - 1/2*((2*g*x^2 + f)/(f
^2*g*x^4 + f^3*x^2) - 2*g*log(g*x^2 + f)/f^3 + 2*g*log(x^2)/f^3)*log((e*x^2
 + d)^p*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{x^3\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(x^3*(f + g*x^2)^2),x)
[Out] int(log(c*(d + e*x^2)^p)/(x^3*(f + g*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**3/(g*x**2+f)**2,x)
[Out] Timed out
```

3.353 $\int \frac{x^4 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$

Optimal. Leaf size=802

$$-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)(-f)^{3/2}}{2g^{5/2}(ef-dg)}+\frac{ep\log\left(\sqrt{g}x+\sqrt{-f}\right)(-f)^{3/2}}{2g^{5/2}(ef-dg)}-\frac{2px}{g^2}+\frac{\sqrt{d}\sqrt{e}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g^2(ef-dg)}+\frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{d}}\right.}{\sqrt{e}g^2}$$

```
[Out] -2*p*x/g^2+x*ln(c*(e*x^2+d)^p)/g^2-1/2*e*(-f)^(3/2)*p*ln((-f)^(1/2)-x*g^(1/
2))/g^(5/2)/(-d*g+e*f)+1/2*e*(-f)^(3/2)*p*ln((-f)^(1/2)+x*g^(1/2))/g^(5/2)/
(-d*g+e*f)+2*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/g^2/e^(1/2)+f*p*arctan(x*e
^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/g^2/(-d*g+e*f)-3/2*arctan(x*g^(1/2)/f^(1/2)
)*ln(c*(e*x^2+d)^p)*f^(1/2)/g^(5/2)-3*p*arctan(x*g^(1/2)/f^(1/2))*ln(2*f^(1
/2)/(f^(1/2)-I*x*g^(1/2)))*f^(1/2)/g^(5/2)+3/2*p*arctan(x*g^(1/2)/f^(1/2))*
ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/
2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))*f^(1/2)/g^(5/2)+3/2*p*arctan(x*g^(1/2)/f^(1
/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e
^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))*f^(1/2)/g^(5/2)+3/2*I*p*polylog(2,1-2*f
^(1/2)/(f^(1/2)-I*x*g^(1/2)))*f^(1/2)/g^(5/2)-3/4*I*p*polylog(2,1+2*((-d)^(
1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-
d)^(1/2)*g^(1/2)))*f^(1/2)/g^(5/2)-3/4*I*p*polylog(2,1-2*((-d)^(1/2)+x*e^(1
/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^
(1/2)))*f^(1/2)/g^(5/2)-1/4*f*ln(c*(e*x^2+d)^p)/g^(5/2)/((-f)^(1/2)-x*g^(1/
2))+1/4*f*ln(c*(e*x^2+d)^p)/g^(5/2)/((-f)^(1/2)+x*g^(1/2))
```

Rubi [A] time = 1.69, antiderivative size = 802, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {2476, 2448, 321, 205, 2471, 2463, 801, 635, 260, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)(-f)^{3/2}}{2g^{5/2}(ef-dg)}+\frac{ep\log\left(\sqrt{g}x+\sqrt{-f}\right)(-f)^{3/2}}{2g^{5/2}(ef-dg)}-\frac{2px}{g^2}+\frac{\sqrt{d}\sqrt{e}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g^2(ef-dg)}+\frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{d}}\right.}{\sqrt{e}g^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] (-2*p*x)/g^2 + (2*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g^2) + (S
qrt[d]*Sqrt[e]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(g^2*(e*f - d*g)) - (e*(-f)
^(3/2)*p*Log[Sqrt[-f] - Sqrt[g]*x])/(2*g^(5/2)*(e*f - d*g)) - (3*Sqrt[f]*p*
ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/
2) + (3*Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt
[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqr
t[g]*x))])/(2*g^(5/2)) + (3*Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sq
rt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g
])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*g^(5/2)) + (e*(-f)^(3/2)*p*Log[Sqrt[-f] +
Sqrt[g]*x])/(2*g^(5/2)*(e*f - d*g)) + (x*Log[c*(d + e*x^2)^p])/g^2 - (f*Log
[c*(d + e*x^2)^p])/(4*g^(5/2)*(Sqrt[-f] - Sqrt[g]*x)) + (f*Log[c*(d + e*x^2
)^p])/(4*g^(5/2)*(Sqrt[-f] + Sqrt[g]*x)) - (3*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sq
rt[f]]*Log[c*(d + e*x^2)^p])/(2*g^(5/2)) + (((3*I)/2)*Sqrt[f]*p*PolyLog[2,
1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/2) - (((3*I)/4)*Sqrt[f]*p*Po
lyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f]
 - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) - (((3*I)/4)*Sqrt[f
]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*S
qrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{x^4 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{g^2} + \frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{g^2\left(f+gx^2\right)^2} - \frac{2f\log\left(c\left(d+ex^2\right)^p\right)}{g^2\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \log\left(c\left(d+ex^2\right)^p\right)dx}{g^2} - \frac{(2f)\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}dx}{g^2} + \frac{f^2\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}dx}{g^2}$$

$$= \frac{x\log\left(c\left(d+ex^2\right)^p\right)}{g^2} - \frac{2\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{g^{5/2}} + \frac{f^2\int\left(-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2}\right.}{}$$

$$= -\frac{2px}{g^2} + \frac{x\log\left(c\left(d+ex^2\right)^p\right)}{g^2} - \frac{2\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{g^{5/2}} - \frac{f\int\frac{\log\left(c\left(d+ex\right.\right.}{\left(\sqrt{-f}\sqrt{g}-\right.}}{4g}$$

$$= -\frac{2px}{g^2} + \frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2} + \frac{x\log\left(c\left(d+ex^2\right)^p\right)}{g^2} - \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}\,x\right)} + \frac{f\log\left(\right.}{4g^{5/2}\left(\right.}$$

$$= -\frac{2px}{g^2} + \frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2} + \frac{x\log\left(c\left(d+ex^2\right)^p\right)}{g^2} - \frac{f\log\left(c\left(d+ex^2\right)^p\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}\,x\right)} + \frac{f\log\left(\right.}{4g^{5/2}\left(\right.}$$

$$= -\frac{2px}{g^2} + \frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2} - \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2g^{5/2}(ef-dg)} - \frac{4\sqrt{f}\,p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\mathrm{lo}}{g^{5/2}}$$

$$= -\frac{2px}{g^2} + \frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2} - \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2g^{5/2}(ef-dg)} - \frac{4\sqrt{f}\,p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\mathrm{lo}}{g^{5/2}}$$

$$= -\frac{2px}{g^2} + \frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2} + \frac{\sqrt{d}\sqrt{e}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{g^2(ef-dg)} - \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}\,x\right.}{2g^{5/2}(ef-dg)}$$

$$= -\frac{2px}{g^2} + \frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2} + \frac{\sqrt{d}\sqrt{e}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{g^2(ef-dg)} - \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}\,x\right.}{2g^{5/2}(ef-dg)}$$

$$= -\frac{2px}{g^2} + \frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2} + \frac{\sqrt{d}\sqrt{e}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{g^2(ef-dg)} - \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}\,x\right.}{2g^{5/2}(ef-dg)}$$

Mathematica [A] time = 4.54, size = 1349, normalized size = 1.68

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] ((6*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(p*Log[d + e*x^2] - Log[c*(d + e*x^
2)^p]))/g^(5/2) + (4*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/g^2 +
(2*f*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(g^2*(f + g*x^2)) + p*
((4*(((-I)*Sqrt[d])/Sqrt[e] + x)*(-1 + Log[((-I)*Sqrt[d])/Sqrt[e] + x]))/g^
2 + (4*((I*Sqrt[d])/Sqrt[e] + x)*(-1 + Log[(I*Sqrt[d])/Sqrt[e] + x]))/g^2 +
 (I*f*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]/(Sqrt[f] + I*Sqrt[g]*x) + (Sqrt[e]*(
-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[I*Sqrt[f] - Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f]
 - Sqrt[d]*Sqrt[g])))/g^(5/2) + (I*f*(Log[(I*Sqrt[d])/Sqrt[e] + x]/(Sqrt[f]
 + I*Sqrt[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] + Sqrt[e]*x] + Log[I*Sqrt[f] - S
qrt[g]*x]))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])))/g^(5/2) + (f*((-I)*(Sqrt[
e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Log[((-I)*Sqrt[d])/Sqrt[e] + x] + Sqrt[e]*(I*
Sqrt[f] + Sqrt[g]*x)*(Log[I*Sqrt[d] - Sqrt[e]*x] - Log[I*Sqrt[f] + Sqrt[g]*
x])))/((Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*g^(5/2)*(Sqrt[f] - I*Sqrt[g]*x))
 - (f*(-(Log[(I*Sqrt[d])/Sqrt[e] + x]/(I*Sqrt[f] + Sqrt[g]*x)) - (I*Sqrt[e]
*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[I*Sqrt[f] + Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f
] - Sqrt[d]*Sqrt[g])))/g^(5/2) + 4*((x*(2 + f/(f + g*x^2)))/(2*g^2) - (3*Sq
rt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(2*g^(5/2)))*(-Log[((-I)*Sqrt[d])/Sqrt[e
] + x] - Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[d + e*x^2]) - ((3*I)*Sqrt[f]*(L
og[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*
Sqrt[f] - Sqrt[d]*Sqrt[g])] + PolyLog[2, -((Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x)
)/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g]))]))/g^(5/2) + ((3*I)*Sqrt[f]*(Log[(I*
Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f
] + Sqrt[d]*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[
e]*Sqrt[f] + Sqrt[d]*Sqrt[g])]))/g^(5/2) + ((3*I)*Sqrt[f]*(Log[((-I)*Sqrt[d
])/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sq
rt[d]*Sqrt[g])] + PolyLog[2, -((Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*S
qrt[f] - Sqrt[d]*Sqrt[g]))]))/g^(5/2) - ((3*I)*Sqrt[f]*(Log[((-I)*Sqrt[d])/
Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[
d]*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f
] + Sqrt[d]*Sqrt[g])]))/g^(5/2)))/4
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4 \log\left(\left(ex^2+d\right)^p c\right)}{g^2x^4+2\,fgx^2+f^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4*log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

maple [F] time = 1.19, size = 0, normalized size = 0.00

$$\int \frac{x^4 \ln\left(c\left(e\,x^2+d\right)^p\right)}{\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

```
[Out] int(x^4*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \log \left(\left(ex^2+d\right)^p c \right)}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^4*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \ln \left(c \left(e\,x^2+d\right)^p \right)}{\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
```

```
[Out] int((x^4*log(c*(d + e*x^2)^p))/(f + g*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

3.354 $\int \frac{x^2 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$

Optimal. Leaf size=746

$$\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}\,x\right)}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}\,x\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2\sqrt{f}\,g^{3/2}}+\frac{ip\text{Li}_2\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right.}{4\sqrt{f}\,g^{3/2}}$$

```
[Out] -p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/g/(-d*g+e*f)-1/2*e*p*ln((-f)^(
1/2)-x*g^(1/2))*(-f)^(1/2)/g^(3/2)/(-d*g+e*f)+1/2*e*p*ln((-f)^(1/2)+x*g^(1/
2))*(-f)^(1/2)/g^(3/2)/(-d*g+e*f)+1/2*arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2
+d)^p)/g^(3/2)/f^(1/2)+p*arctan(x*g^(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*
x*g^(1/2)))/g^(3/2)/f^(1/2)-1/2*p*arctan(x*g^(1/2)/f^(1/2))*ln(-2*((-d)^(1/
2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)
^(1/2)*g^(1/2)))/g^(3/2)/f^(1/2)-1/2*p*arctan(x*g^(1/2)/f^(1/2))*ln(2*((-d)
^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+
(-d)^(1/2)*g^(1/2)))/g^(3/2)/f^(1/2)-1/2*I*p*polylog(2,1-2*f^(1/2)/(f^(1/2)
-I*x*g^(1/2)))/g^(3/2)/f^(1/2)+1/4*I*p*polylog(2,1+2*((-d)^(1/2)-x*e^(1/2))
*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2
)))/g^(3/2)/f^(1/2)+1/4*I*p*polylog(2,1-2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^
(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/g^(3/2)
/f^(1/2)+1/4*ln(c*(e*x^2+d)^p)/g^(3/2)/((-f)^(1/2)-x*g^(1/2))-1/4*ln(c*(e*x
^2+d)^p)/g^(3/2)/((-f)^(1/2)+x*g^(1/2))
```

Rubi [A] time = 1.51, antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {2476, 2471, 2463, 801, 635, 205, 260, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4\sqrt{f}\,g^{3/2}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4\sqrt{f}\,g^{3/2}}-\frac{ip\text{PolyLog}\left(2,\right.}{2\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] -((Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(g*(e*f - d*g))) - (e*Sqr
t[-f]*p*Log[Sqrt[-f] - Sqrt[g]*x])/(2*g^(3/2)*(e*f - d*g)) + (p*ArcTan[(Sqr
t[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*g^(3/2)
) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt
[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/
(2*Sqrt[f]*g^(3/2)) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]
*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
 I*Sqrt[g]*x))])/(2*Sqrt[f]*g^(3/2)) + (e*Sqrt[-f]*p*Log[Sqrt[-f] + Sqrt[g]
*x])/(2*g^(3/2)*(e*f - d*g)) + Log[c*(d + e*x^2)^p]/(4*g^(3/2)*(Sqrt[-f] -
Sqrt[g]*x)) - Log[c*(d + e*x^2)^p]/(4*g^(3/2)*(Sqrt[-f] + Sqrt[g]*x)) + (Ar
cTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*Sqrt[f]*g^(3/2)) - ((I/2
)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*g^(3/2))
+ ((I/4)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sq
rt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*g^(3/
2)) + ((I/4)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((
I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*g
^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{x^2 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \left(-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{g\left(f+gx^2\right)^2}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{g\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx}{g} - \frac{f\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx}{g}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\,g^{3/2}} - \frac{f\int\left(-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\,\sqrt{g}-gx\right)^2}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\,\sqrt{g}+gx\right)^2}-\frac{g\log\left(c\left(d+e\right.\right.}{2f\left(-fg-g\right.}\right.}{g}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\,g^{3/2}} + \frac{1}{4}\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(\sqrt{-f}\,\sqrt{g}-gx\right)^2}\,dx + \frac{1}{4}\int \frac{\log\left(c\left(d+ex^2\right.\right.}{\left(\sqrt{-f}\,\sqrt{g}+gx\right.}$$

$$= \frac{\log\left(c\left(d+ex^2\right)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}\,x\right)} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}\,x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2\sqrt{f}\,g^{3/2}} + (ep$$

$$= \frac{\log\left(c\left(d+ex^2\right)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}\,x\right)} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}\,x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2\sqrt{f}\,g^{3/2}} + \frac{(\sqrt{}}{}$$

$$= -\frac{e\sqrt{-f}\,p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2g^{3/2}(ef-dg)} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,g^{3/2}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log}{}$$

$$= -\frac{e\sqrt{-f}\,p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2g^{3/2}(ef-dg)} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,g^{3/2}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log}{}$$

$$= -\frac{\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{g(ef-dg)} - \frac{e\sqrt{-f}\,p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2g^{3/2}(ef-dg)} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right.}{\sqrt{f}\,g^{3/2}}$$

$$= -\frac{\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{g(ef-dg)} - \frac{e\sqrt{-f}\,p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2g^{3/2}(ef-dg)} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right.}{\sqrt{f}\,g^{3/2}}$$

$$= -\frac{\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{g(ef-dg)} - \frac{e\sqrt{-f}\,p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2g^{3/2}(ef-dg)} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right.}{\sqrt{f}\,g^{3/2}}$$

Mathematica [A] time = 3.52, size = 1231, normalized size = 1.65

$$\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(\log\left(c\left(ex^2+d\right)^p\right)-p\log\left(ex^2+d\right)\right)}{2\sqrt{f}g^{3/2}}+\frac{px\log\left(ex^2+d\right)-x\log\left(c\left(ex^2+d\right)^p\right)}{2g^2x^2+2fg}+\frac{1}{4}p\left(-\frac{i\left(\frac{\log\left(x-\frac{i\sqrt{d}}{\sqrt{e}}\right)}{i\sqrt{g}x+\sqrt{f}}\cdots\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/
(2*Sqrt[f]*g^(3/2)) + (p*x*Log[d + e*x^2] - x*Log[c*(d + e*x^2)^p])/(2*f*g
+ 2*g^2*x^2) + (p*(((-I)*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]/(Sqrt[f] + I*Sqrt
[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[I*Sqrt[f] - Sqrt[g]*x]
))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])))/g^(3/2) - (I*(Log[(I*Sqrt[d])/Sqrt
[e] + x]/(Sqrt[f] + I*Sqrt[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] + Sqrt[e]*x] +
Log[I*Sqrt[f] - Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])))/g^(3/2)
+ (-((Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Log[((-I)*Sqrt[d])/Sqrt[e] + x]) +
 Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x)*(Log[I*Sqrt[d] - Sqrt[e]*x] - Log[I*Sqrt[f
] + Sqrt[g]*x]))/((Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*g^(3/2)*(I*Sqrt[f] +
Sqrt[g]*x)) + (-(Log[(I*Sqrt[d])/Sqrt[e] + x]/(I*Sqrt[f] + Sqrt[g]*x)) - (I
*Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[I*Sqrt[f] + Sqrt[g]*x]))/(Sqrt[e
]*Sqrt[f] - Sqrt[d]*Sqrt[g]))/g^(3/2) + 4*(-1/2*x/(g*(f + g*x^2)) + ArcTan[
(Sqrt[g]*x)/Sqrt[f]]/(2*Sqrt[f]*g^(3/2)))*(-Log[((-I)*Sqrt[d])/Sqrt[e] + x]
 - Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[d + e*x^2]) + (I*(Log[(I*Sqrt[d])/Sqr
t[e] + x]*Log[(Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*
Sqrt[g])] + PolyLog[2, -((Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f]
 - Sqrt[d]*Sqrt[g]))]))/(Sqrt[f]*g^(3/2)) - (I*(Log[(I*Sqrt[d])/Sqrt[e] + x
]*Log[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])
] + PolyLog[2, (Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]
*Sqrt[g])]))/(Sqrt[f]*g^(3/2)) - (I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(S
qrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])] + Poly
Log[2, -((Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[
g]))]))/(Sqrt[f]*g^(3/2)) + (I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e
]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])] + PolyLog[2
, (Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])]))/
(Sqrt[f]*g^(3/2))))/4
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2\log\left(\left(ex^2+d\right)^pc\right)}{g^2x^4+2fgx^2+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\log\left(\left(ex^2+d\right)^pc\right)}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

maple **[F]** time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{x^2 \ln \left(c \left(e\,x^2+d\right)^p\right)}{\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

```
[Out] int(x^2*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log \left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\ln \left(c\,{\left(e\,x^2+d\right)}^p\right)}{{\left(g\,x^2+f\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
```

```
[Out] int((x^2*log(c*(d + e*x^2)^p))/(f + g*x^2)^2, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

3.355 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=751

$$\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^{3/2}\sqrt{g}}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{ip\text{Li}_2\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\right.}\right.}{4f^{3/2}\sqrt{g}}$$

```
[Out] p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/f/(-d*g+e*f)+1/2*arctan(x*g^(1/
2)/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(3/2)/g^(1/2)+p*arctan(x*g^(1/2)/f^(1/2))*l
n(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*p*arctan(x*g^(1/2)/f
^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/
(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*p*arctan(x*g^(1
/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/
2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*I*p*polylog
(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(3/2)/g^(1/2)+1/4*I*p*polylog(2,1+2
*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^
(1/2)-(-d)^(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)+1/4*I*p*polylog(2,1-2*((-d)^(1/2
)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^
(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*e*p*ln((-f)^(1/2)-x*g^(1/2))/(-d*g+e*f)
/(-f)^(1/2)/g^(1/2)+1/2*e*p*ln((-f)^(1/2)+x*g^(1/2))/(-d*g+e*f)/(-f)^(1/2)/
g^(1/2)-1/4*ln(c*(e*x^2+d)^p)/f/g^(1/2)/((-f)^(1/2)-x*g^(1/2))+1/4*ln(c*(e*
x^2+d)^p)/f/g^(1/2)/((-f)^(1/2)+x*g^(1/2))
```

Rubi [A] time = 0.82, antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {2471, 2463, 801, 635, 205, 260, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4f^{3/2}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4f^{3/2}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,\right.}{2f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

```
[Out] (Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(f*(e*f - d*g)) - (e*p*Log[
Sqrt[-f] - Sqrt[g]*x])/(2*Sqrt[-f]*Sqrt[g]*(e*f - d*g)) + (p*ArcTan[(Sqrt[g
]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g]) -
 (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]
*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*
f^(3/2)*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(S
qrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*
Sqrt[g]*x))])/(2*f^(3/2)*Sqrt[g]) + (e*p*Log[Sqrt[-f] + Sqrt[g]*x])/(2*Sqrt
[-f]*Sqrt[g]*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] - S
qrt[g]*x)) + Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] + Sqrt[g]*x)) + (A
rcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*f^(3/2)*Sqrt[g]) - ((I/
2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g])
 + ((I/4)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*S
qrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*Sqrt
[g]) + ((I/4)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(
(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*
Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \left(-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}+gx\right)^2}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2f\left(-fg-g^2x^2\right)}\right)dx$$

$$= -\frac{g\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(\sqrt{-f}\sqrt{g}-gx\right)^2}dx}{4f}-\frac{g\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(\sqrt{-f}\sqrt{g}+gx\right)^2}dx}{4f}-\frac{g\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{-fg-g^2x^2}dx}{2f}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^{3/2}\sqrt{g}}+\cdots$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^{3/2}\sqrt{g}}-\cdots$$

$$= -\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c\left(d\cdots\right.\right.}{4f\sqrt{g}\left(\sqrt{-\cdots}\right.}$$

$$= -\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c\left(d\cdots\right.\right.}{4f\sqrt{g}\left(\sqrt{-\cdots}\right.}$$

$$= \frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}}-\frac{p\tan\cdots}{\cdots}$$

$$= \frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}}-\frac{p\tan\cdots}{\cdots}$$

$$= \frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}}-\frac{p\tan\cdots}{\cdots}$$

Mathematica [A] time = 3.19, size = 1236, normalized size = 1.65

$$\frac{1}{2}\left(\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(\log\left(c\left(ex^2+d\right)^p\right)-p\log\left(ex^2+d\right)\right)}{f^{3/2}\sqrt{g}}+\frac{x\left(\log\left(c\left(ex^2+d\right)^p\right)-p\log\left(ex^2+d\right)\right)}{f\left(gx^2+f\right)}+\frac{1}{2}p\left(\frac{i\left(\frac{\log\left(x-\frac{i\cdots}{\cdots}\right)}{i\sqrt{g}x+\cdots}\right.}{\cdots}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

```
[Out] ((x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(f*(f + g*x^2)) + (ArcTan
[(Sqrt[g]*x)/Sqrt[f]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(f^(3/2
```

```
)*Sqrt[g]) + (p*((I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]/(Sqrt[f] + I*Sqrt[g]*x
) + (Sqrt[e]*(-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[I*Sqrt[f] - Sqrt[g]*x]))/(S
qrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])))/(f*Sqrt[g]) + (I*(Log[(I*Sqrt[d])/Sqrt[
e] + x]/(Sqrt[f] + I*Sqrt[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] + Sqrt[e]*x] + L
og[I*Sqrt[f] - Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])))/(f*Sqrt[g
]) + ((-I)*(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Log[((-I)*Sqrt[d])/Sqrt[e] +
 x] + Sqrt[e]*(I*Sqrt[f] + Sqrt[g]*x)*(Log[I*Sqrt[d] - Sqrt[e]*x] - Log[I*S
qrt[f] + Sqrt[g]*x]))/(f*(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Sqrt[g]*(Sqrt[
f] - I*Sqrt[g]*x)) - (-(Log[(I*Sqrt[d])/Sqrt[e] + x]/(I*Sqrt[f] + Sqrt[g]*x
)) - (I*Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[I*Sqrt[f] + Sqrt[g]*x]))/
(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g]))/(f*Sqrt[g]) + 2*(x/(f^2 + f*g*x^2) + A
rcTan[(Sqrt[g]*x)/Sqrt[f]]/(f^(3/2)*Sqrt[g]))*(-Log[((-I)*Sqrt[d])/Sqrt[e]
+ x] - Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[d + e*x^2]) + (I*(Log[(I*Sqrt[d])
/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt
[d]*Sqrt[g])] + PolyLog[2, -((Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqr
t[f] - Sqrt[d]*Sqrt[g]))]))/(f^(3/2)*Sqrt[g]) - (I*(Log[(I*Sqrt[d])/Sqrt[e]
 + x]*Log[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt
[g])] + PolyLog[2, (Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqr
t[d]*Sqrt[g])]))/(f^(3/2)*Sqrt[g]) - (I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Lo
g[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])] +
PolyLog[2, -((Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*S
qrt[g]))]))/(f^(3/2)*Sqrt[g]) + (I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sq
rt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])] + PolyL
og[2, (Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])
]))/(f^(3/2)*Sqrt[g])))/2)/2
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{g^2x^4+2\,fgx^2+f^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x)
```

```
[Out] int(1/(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^2)^2,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

3.356 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=803

$$-\frac{\sqrt{d}\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f^2(ef-dg)}+\frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2}-\frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)}-\frac{3\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}}+\cdots$$

```
[Out] -ln(c*(e*x^2+d)^p)/f^2/x+2*p*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/f^2/d^(1/2)-
g*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/f^2/(-d*g+e*f)-3/2*arctan(x*g
^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)*g^(1/2)/f^(5/2)-1/2*e*p*ln((-f)^(1/2)-x*g
^(1/2))*g^(1/2)/(-f)^(3/2)/(-d*g+e*f)-3*p*arctan(x*g^(1/2)/f^(1/2))*ln(2*f^
(1/2)/(f^(1/2)-I*x*g^(1/2)))*g^(1/2)/f^(5/2)+1/2*e*p*ln((-f)^(1/2)+x*g^(1/2
))*g^(1/2)/(-f)^(3/2)/(-d*g+e*f)+3/2*p*arctan(x*g^(1/2)/f^(1/2))*ln(-2*((-d
)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)
-(-d)^(1/2)*g^(1/2)))*g^(1/2)/f^(5/2)+3/2*p*arctan(x*g^(1/2)/f^(1/2))*ln(2*
((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(
1/2)+(-d)^(1/2)*g^(1/2)))*g^(1/2)/f^(5/2)+3/2*I*p*polylog(2,1-2*f^(1/2)/(f^
(1/2)-I*x*g^(1/2)))*g^(1/2)/f^(5/2)-3/4*I*p*polylog(2,1+2*((-d)^(1/2)-x*e^(
1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g
^(1/2)))*g^(1/2)/f^(5/2)-3/4*I*p*polylog(2,1-2*((-d)^(1/2)+x*e^(1/2))*f^(1/
2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))*g^
(1/2)/f^(5/2)+1/4*ln(c*(e*x^2+d)^p)*g^(1/2)/f^2/((-f)^(1/2)-x*g^(1/2))-1/4*
ln(c*(e*x^2+d)^p)*g^(1/2)/f^2/((-f)^(1/2)+x*g^(1/2))
```

Rubi [A] time = 1.54, antiderivative size = 803, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2476, 2455, 205, 2471, 2463, 801, 635, 260, 2470, 12, 4928, 4856, 2402, 2315, 2447}

$$-\frac{\sqrt{d}\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f^2(ef-dg)}+\frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2}-\frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)}-\frac{3\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)^2),x]
```

```
[Out] (2*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f^2) - (Sqrt[d]*Sqrt[e]*
g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(f^2*(e*f - d*g)) - (e*Sqrt[g]*p*Log[Sqrt[
-f] - Sqrt[g]*x])/(2*(-f)^(3/2)*(e*f - d*g)) - (3*Sqrt[g]*p*ArcTan[(Sqrt[g]
*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(5/2) + (3*Sqrt[g]
*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*
x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*f
^(5/2)) + (3*Sqrt[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(
Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I
*Sqrt[g]*x))])/(2*f^(5/2)) + (e*Sqrt[g]*p*Log[Sqrt[-f] + Sqrt[g]*x])/(2*(-f
)^(3/2)*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(f^2*x) + (Sqrt[g]*Log[c*(d + e
*x^2)^p])/(4*f^2*(Sqrt[-f] - Sqrt[g]*x)) - (Sqrt[g]*Log[c*(d + e*x^2)^p])/(
4*f^2*(Sqrt[-f] + Sqrt[g]*x)) - (3*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[
c*(d + e*x^2)^p])/(2*f^(5/2)) + (((3*I)/2)*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt
[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(5/2) - (((3*I)/4)*Sqrt[g]*p*PolyLog[2, 1
+ (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]
*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(5/2) - (((3*I)/4)*Sqrt[g]*p*PolyLog
[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sq
rt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[((f + g*x)^(r + 1)*(a + b*Log[c*(d + e*x^n
)^p]))/(g*(r + 1)), x] - Dist[(b*e*n*p)/(g*(r + 1)), Int[(x^(n - 1)*(f + g*
x)^(r + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2\left(f+gx^2\right)^2}\,dx = \int \left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{f^2x^2} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{f\left(f+gx^2\right)^2} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{f^2\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2}dx}{f^2} - \frac{g\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}dx}{f^2} - \frac{g\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}dx}{f}$$

$$= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{f^2x} - \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{5/2}} - \frac{g\int\left(-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2} - \cdots\right.}{\cdots}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f^2x} - \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{5/2}} + \frac{g^2\int\cdots}{\cdots}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f^2x} + \frac{\sqrt{g}\log\left(c\left(d+ex^2\right)^p\right)}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\sqrt{g}\log\left(c\left(d+ex^2\cdots\right.\right.}{4f^2\left(\sqrt{-f}+\sqrt{g}\cdots\right.}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f^2x} + \frac{\sqrt{g}\log\left(c\left(d+ex^2\right)^p\right)}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\sqrt{g}\log\left(c\left(d+ex^2\cdots\right.\right.}{4f^2\left(\sqrt{-f}+\sqrt{g}\cdots\right.}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{2\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} + \cdots$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{2\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} + \cdots$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\sqrt{d}\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f^2(ef-dg)} - \frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{3\sqrt{g}p\tan^{-}\cdots}{\cdots}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\sqrt{d}\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f^2(ef-dg)} - \frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{3\sqrt{g}p\tan^{-}\cdots}{\cdots}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\sqrt{d}\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f^2(ef-dg)} - \frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{3\sqrt{g}p\tan^{-}\cdots}{\cdots}$$

Mathematica [A] time = 4.82, size = 1438, normalized size = 1.79

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)^2),x]
```

```
[Out] ((4*p*Log[d + e*x^2] - 4*Log[c*(d + e*x^2)^p])/(f^2*x) + (2*g*x*(p*Log[d +
e*x^2] - Log[c*(d + e*x^2)^p]))/(f^2*(f + g*x^2)) + (6*Sqrt[g]*ArcTan[(Sqrt
[g]*x)/Sqrt[f]]*(p*Log[d + e*x^2] - Log[c*(d + e*x^2)^p]))/f^(5/2) + p*(((4
*I)*(Sqrt[e]*x*Log[x] + I*Sqrt[d]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] - Sqrt[e]
*x*Log[I*Sqrt[d] - Sqrt[e]*x]))/(Sqrt[d]*f^2*x) - (4*(I*Sqrt[e]*x*Log[x] +
Sqrt[d]*Log[(I*Sqrt[d])/Sqrt[e] + x] - I*Sqrt[e]*x*Log[I*Sqrt[d] + Sqrt[e]*
x]))/(Sqrt[d]*f^2*x) - (I*Sqrt[g]*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]/(Sqrt[f]
 + I*Sqrt[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[I*Sqrt[f] - S
qrt[g]*x]))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])))/f^2 - (I*Sqrt[g]*(Log[(I*
Sqrt[d])/Sqrt[e] + x]/(Sqrt[f] + I*Sqrt[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] +
Sqrt[e]*x] + Log[I*Sqrt[f] - Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g
])))/f^2 + (Sqrt[g]*(-((Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Log[((-I)*Sqrt[d
])/Sqrt[e] + x]) + Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x)*(Log[I*Sqrt[d] - Sqrt[e]
*x] - Log[I*Sqrt[f] + Sqrt[g]*x])))/(f^2*(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g]
)*(I*Sqrt[f] + Sqrt[g]*x)) + (Sqrt[g]*(-(Log[(I*Sqrt[d])/Sqrt[e] + x]/(I*Sq
rt[f] + Sqrt[g]*x)) - (I*Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[I*Sqrt[f
] + Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])))/f^2 + 4*(-1/2*(2 + (
g*x^2)/(f + g*x^2))/(f^2*x) - (3*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(2*f^
(5/2)))*(-Log[((-I)*Sqrt[d])/Sqrt[e] + x] - Log[(I*Sqrt[d])/Sqrt[e] + x] +
Log[d + e*x^2]) - ((3*I)*Sqrt[g]*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]
*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])] + PolyLog[2,
 -((Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g]))])
)/f^(5/2) + ((3*I)*Sqrt[g]*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt
[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])] + PolyLog[2, (Sqrt
[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])]))/f^(5/2)
 + ((3*I)*Sqrt[g]*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] +
I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])] + PolyLog[2, -((Sqrt[g]*
(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g]))]))/f^(5/2) -
((3*I)*Sqrt[g]*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] - I*S
qrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(Sqrt
[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])]))/f^(5/2)))/4
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^2+d\right)^p c\right)}{g^2x^6+2\,fgx^4+f^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g^2*x^6 + 2*f*g*x^4 + f^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)^2*x^2), x)
```

maple [F] time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{\left(g\,x^2+f\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,\left(e\,x^2+d\right)^p\right)}{x^2\,\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(x^2*(f + g*x^2)^2),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)/(x^2*(f + g*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**2/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

3.357 $\int \frac{\log\left(c\left(a+bx^2\right)^n\right)}{a+bx^2}\,dx$

Optimal. Leaf size=163

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}}+\frac{in\text{Li}_2\left(1-\frac{2\sqrt{a}}{i\sqrt{b}\,x+\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{in\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}}+\frac{2n\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] I*n*arctan(x*b^(1/2)/a^(1/2))^2/a^(1/2)/b^(1/2)+arctan(x*b^(1/2)/a^(1/2))*l
n(c*(b*x^2+a)^n)/a^(1/2)/b^(1/2)+2*n*arctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)
/(a^(1/2)+I*x*b^(1/2)))/a^(1/2)/b^(1/2)+I*n*polylog(2,1-2*a^(1/2)/(a^(1/2)+
I*x*b^(1/2)))/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {205, 2470, 12, 4920, 4854, 2402, 2315}

$$\frac{in\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{a}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}}+\frac{in\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}}+\frac{2n\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^n]/(a + b*x^2),x]
```

```
[Out] (I*n*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/(Sqrt[a]*Sqrt[b]) + (2*n*ArcTan[(Sqrt[b
]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b]) +
 (ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^n])/(Sqrt[a]*Sqrt[b]) + (I*
n*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+bx^2\right)^n\right)}{a+bx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} - (2bn)\int \frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\left(a+bx^2\right)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} - \frac{\left(2\sqrt{b}n\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a+bx^2}\,dx}{\sqrt{a}}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} + \frac{(2n)\int \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{i-\frac{\sqrt{b}x}{\sqrt{a}}}\,dx}{a}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{2n\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} -$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{2n\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} +$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{2n\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} +$$

Mathematica [A] time = 0.05, size = 128, normalized size = 0.79

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(\log\left(c\left(a+bx^2\right)^n\right)+2n\log\left(\frac{2i}{-\frac{\sqrt{b}x}{\sqrt{a}}+i}\right)+in\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)+in\text{Li}_2\left(\frac{\sqrt{b}x+i\sqrt{a}}{\sqrt{b}x-i\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^n]/(a + b*x^2),x]
```

```
[Out] (ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(I*n*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*n*Log[(2*
I)/(I - (Sqrt[b]*x)/Sqrt[a])] + Log[c*(a + b*x^2)^n]) + I*n*PolyLog[2, (I*S
qrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(bx^2+a\right)^n c\right)}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^n)/(b*x^2+a),x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^n*c)/(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^n c\right)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^n)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^n*c)/(b*x^2 + a), x)
```

maple [F] time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^n\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^n)/(b*x^2+a),x)
[Out] int(ln(c*(b*x^2+a)^n)/(b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(bx^2+a\right)^n c\right)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^n)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(log((b*x^2 + a)^n*c)/(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^n\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^n)/(a + b*x^2),x)
[Out] int(log(c*(a + b*x^2)^n)/(a + b*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^2\right)^n\right)}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**n)/(b*x**2+a),x)
```

```
[Out] Integral(log(c*(a + b*x**2)**n)/(a + b*x**2), x)
```

3.358 $\int \frac{\log(1-x^2)}{2-x^2}\,dx$

Optimal. Leaf size=239

$$-\frac{\text{Li}_2\left(1-\frac{2\sqrt{2}}{x+\sqrt{2}}\right)}{\sqrt{2}}+\frac{\text{Li}_2\left(\frac{4(1-x)}{(2-\sqrt{2})(x+\sqrt{2})}+1\right)}{2\sqrt{2}}+\frac{\text{Li}_2\left(1-\frac{4(x+1)}{(2+\sqrt{2})(x+\sqrt{2})}\right)}{2\sqrt{2}}+\frac{\log\left(1-x^2\right)\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+\sqrt{2}\log\left(\frac{2\sqrt{2}}{x+\sqrt{2}}\right.$$

```
[Out] 1/2*arctanh(1/2*x*2^(1/2))*ln(-x^2+1)*2^(1/2)-1/2*arctanh(1/2*x*2^(1/2))*ln
(-4*(1-x)/(2-2^(1/2))/(x+2^(1/2)))*2^(1/2)-1/2*arctanh(1/2*x*2^(1/2))*ln(4*
(1+x)/(2+2^(1/2))/(x+2^(1/2)))*2^(1/2)+1/4*polylog(2,1+4*(1-x)/(2-2^(1/2))/
(x+2^(1/2)))*2^(1/2)-1/2*polylog(2,1-2*2^(1/2)/(x+2^(1/2)))*2^(1/2)+1/4*pol
ylog(2,1-4*(1+x)/(2+2^(1/2))/(x+2^(1/2)))*2^(1/2)+arctanh(1/2*x*2^(1/2))*ln
(2*2^(1/2)/(x+2^(1/2)))*2^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {206, 2470, 12, 5992, 5920, 2402, 2315, 2447}

$$-\frac{\text{PolyLog}\left(2,1-\frac{2\sqrt{2}}{x+\sqrt{2}}\right)}{\sqrt{2}}+\frac{\text{PolyLog}\left(2,\frac{4(1-x)}{(2-\sqrt{2})(x+\sqrt{2})}+1\right)}{2\sqrt{2}}+\frac{\text{PolyLog}\left(2,1-\frac{4(x+1)}{(2+\sqrt{2})(x+\sqrt{2})}\right)}{2\sqrt{2}}+\frac{\log\left(1-x^2\right)\tanh}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 - x^2]/(2 - x^2),x]
```

```
[Out] Sqrt[2]*ArcTanh[x/Sqrt[2]]*Log[(2*Sqrt[2])/(Sqrt[2] + x)] - (ArcTanh[x/Sqrt
[2]]*Log[(-4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))])/Sqrt[2] - (ArcTanh[x/
Sqrt[2]]*Log[(4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))])/Sqrt[2] + (ArcTanh
[x/Sqrt[2]]*Log[1 - x^2])/Sqrt[2] - PolyLog[2, 1 - (2*Sqrt[2])/(Sqrt[2] + x
)]/Sqrt[2] + PolyLog[2, 1 + (4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))]/(2*S
qrt[2]) + PolyLog[2, 1 - (4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))]/(2*Sqrt
[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rubi steps

$$\int \frac{\log\left(1-x^2\right)}{2-x^2}\,dx = \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(1-x^2\right)}{\sqrt{2}} + 2\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}\left(1-x^2\right)}\,dx$$

$$= \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(1-x^2\right)}{\sqrt{2}} + \sqrt{2}\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}\,dx$$

$$= \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(1-x^2\right)}{\sqrt{2}} + \sqrt{2}\int \left(-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2(-1+x)} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2(1+x)}\right)dx$$

$$= \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(1-x^2\right)}{\sqrt{2}} - \frac{\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{-1+x}\,dx}{\sqrt{2}} - \frac{\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{1+x}\,dx}{\sqrt{2}}$$

$$= \sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\ldots\right.}{\sqrt{2}}$$

$$= \sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\ldots\right.}{\sqrt{2}}$$

$$= \sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\ldots\right.}{\sqrt{2}}$$

Mathematica [A] time = 0.13, size = 248, normalized size = 1.04

$$\frac{\text{Li}_2\left(\frac{x-1}{-1-\sqrt{2}}\right)+\log\left(1-\frac{x-1}{-1-\sqrt{2}}\right)\log(x-1)}{2\sqrt{2}}-\frac{\text{Li}_2\left(\frac{x-1}{-1+\sqrt{2}}\right)+\log\left(1-\frac{x-1}{\sqrt{2}-1}\right)\log(x-1)}{2\sqrt{2}}+\frac{\text{Li}_2\left(\frac{x+1}{1-\sqrt{2}}\right)+\log(x+1}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 - x^2]/(2 - x^2),x]
```

```
[Out] -1/2*((Log[Sqrt[2] - x] - Log[Sqrt[2] + x])*(-Log[-1 + x] - Log[1 + x] + Lo
g[1 - x^2]))/Sqrt[2] + (Log[1 - (-1 + x)/(-1 - Sqrt[2])]*Log[-1 + x] + Poly
Log[2, (-1 + x)/(-1 - Sqrt[2])])/(2*Sqrt[2]) - (Log[1 - (-1 + x)/(-1 + Sqrt
[2])]*Log[-1 + x] + PolyLog[2, (-1 + x)/(-1 + Sqrt[2])])/(2*Sqrt[2]) + (Log
[1 + x]*Log[1 - (1 + x)/(1 - Sqrt[2])] + PolyLog[2, (1 + x)/(1 - Sqrt[2])])
/(2*Sqrt[2]) - (Log[1 + x]*Log[1 - (1 + x)/(1 + Sqrt[2])] + PolyLog[2, (1 +
 x)/(1 + Sqrt[2])])/(2*Sqrt[2])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\log\left(-x^2+1\right)}{x^2-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-x^2+1)/(-x^2+2),x, algorithm="fricas")
```

```
[Out] integral(-log(-x^2 + 1)/(x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\log\left(-x^2+1\right)}{x^2-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-x^2+1)/(-x^2+2),x, algorithm="giac")
```

```
[Out] integrate(-log(-x^2 + 1)/(x^2 - 2), x)
```

maple [A] time = 0.34, size = 214, normalized size = 0.90

$$-\frac{\sqrt{2}\,\ln\left(\frac{x-1}{-\sqrt{2}-1}\right)\ln\left(x+\sqrt{2}\right)}{4}+\frac{\sqrt{2}\,\ln\left(\frac{x-1}{\sqrt{2}-1}\right)\ln\left(x-\sqrt{2}\right)}{4}+\frac{\sqrt{2}\,\ln\left(\frac{x+1}{1+\sqrt{2}}\right)\ln\left(x-\sqrt{2}\right)}{4}-\frac{\sqrt{2}\,\ln\left(\frac{x+1}{-\sqrt{2}+1}\right)\ln(}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-x^2+1)/(-x^2+2),x)
```

```
[Out] -1/4*2^(1/2)*ln(-x^2+1)*ln(x-2^(1/2))+1/4*2^(1/2)*ln(x-2^(1/2))*ln((x+1)/(1
+2^(1/2)))+1/4*2^(1/2)*ln(x-2^(1/2))*ln((x-1)/(2^(1/2)-1))+1/4*2^(1/2)*dilo
g((x+1)/(1+2^(1/2)))+1/4*2^(1/2)*dilog((x-1)/(2^(1/2)-1))+1/4*2^(1/2)*ln(-x
^2+1)*ln(x+2^(1/2))-1/4*2^(1/2)*ln(x+2^(1/2))*ln((x+1)/(-2^(1/2)+1))-1/4*2^
(1/2)*ln(x+2^(1/2))*ln((x-1)/(-2^(1/2)-1))-1/4*2^(1/2)*dilog((x-1)/(-2^(1/2
)-1))-1/4*2^(1/2)*dilog((x+1)/(-2^(1/2)+1))
```

maxima [A] time = 1.06, size = 208, normalized size = 0.87

$$\frac{1}{4}\sqrt{2}\left(\left(\log\left(2x+2\sqrt{2}\right)-\log\left(2x-2\sqrt{2}\right)\right)\log\left(-x^2+1\right)-\log\left(x+\sqrt{2}\right)\log\left(-\frac{x+\sqrt{2}}{\sqrt{2}+1}+1\right)+\log\left(x-\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-x^2+1)/(-x^2+2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*((log(2*x + 2*sqrt(2)) - log(2*x - 2*sqrt(2)))*log(-x^2 + 1) -
log(x + sqrt(2))*log(-(x + sqrt(2))/(sqrt(2) + 1) + 1) + log(x - sqrt(2))*l
og((x - sqrt(2))/(sqrt(2) + 1) + 1) - log(x + sqrt(2))*log(-(x + sqrt(2))/(
sqrt(2) - 1) + 1) + log(x - sqrt(2))*log((x - sqrt(2))/(sqrt(2) - 1) + 1) -
 dilog((x + sqrt(2))/(sqrt(2) + 1)) + dilog(-(x - sqrt(2))/(sqrt(2) + 1)) -
 dilog((x + sqrt(2))/(sqrt(2) - 1)) + dilog(-(x - sqrt(2))/(sqrt(2) - 1)))
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\ln\left(1-x^2\right)}{x^2-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-log(1 - x^2)/(x^2 - 2),x)
```

```
[Out] -int(log(1 - x^2)/(x^2 - 2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\log\left(1-x^2\right)}{x^2-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-x**2+1)/(-x**2+2),x)
```

```
[Out] -Integral(log(1 - x**2)/(x**2 - 2), x)
```

3.359 $\int \frac{\log(d+ex^2)}{1-x^2}\,dx$

Optimal. Leaf size=217

$$\frac{1}{2}\text{Li}_2\left(1-\frac{2\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{-d}-\sqrt{e}\right)(x+1)}\right)+\frac{1}{2}\text{Li}_2\left(1-\frac{2\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(\sqrt{-d}+\sqrt{e}\right)(x+1)}\right)+\tanh^{-1}(x)\log\left(d+ex^2\right)-\tanh^{-1}(x)\log\left(\frac{\ldots}{(x\ldots}\right.$$

```
[Out] 2*arctanh(x)*ln(2/(1+x))+arctanh(x)*ln(e*x^2+d)-arctanh(x)*ln(2*((-d)^(1/2)
-x*e^(1/2))/(1+x)/((-d)^(1/2)-e^(1/2)))-arctanh(x)*ln(2*((-d)^(1/2)+x*e^(1/
2))/(1+x)/((-d)^(1/2)+e^(1/2)))-polylog(2,1-2/(1+x))+1/2*polylog(2,1-2*((-d
)^(1/2)-x*e^(1/2))/(1+x)/((-d)^(1/2)-e^(1/2)))+1/2*polylog(2,1-2*((-d)^(1/2
)+x*e^(1/2))/(1+x)/((-d)^(1/2)+e^(1/2)))
```

Rubi [A] time = 0.25, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {206, 2470, 5992, 5920, 2402, 2315, 2447}

$$\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2\left(\sqrt{-d}-\sqrt{e}\,x\right)}{(x+1)\left(\sqrt{-d}-\sqrt{e}\right)}\right)+\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2\left(\sqrt{-d}+\sqrt{e}\,x\right)}{(x+1)\left(\sqrt{-d}+\sqrt{e}\right)}\right)-\text{PolyLog}\left(2,1-\frac{2}{x+1}\right)+\text{ta}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Log[d + e*x^2]/(1 - x^2),x]
```

```
[Out] 2*ArcTanh[x]*Log[2/(1 + x)] - ArcTanh[x]*Log[(2*(Sqrt[-d] - Sqrt[e]*x))/((S
qrt[-d] - Sqrt[e])*(1 + x))] - ArcTanh[x]*Log[(2*(Sqrt[-d] + Sqrt[e]*x))/((
Sqrt[-d] + Sqrt[e])*(1 + x))] + ArcTanh[x]*Log[d + e*x^2] - PolyLog[2, 1 -
2/(1 + x)] + PolyLog[2, 1 - (2*(Sqrt[-d] - Sqrt[e]*x))/((Sqrt[-d] - Sqrt[e]
)*(1 + x))]/2 + PolyLog[2, 1 - (2*(Sqrt[-d] + Sqrt[e]*x))/((Sqrt[-d] + Sqrt
[e])*(1 + x))]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
```

```
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(d+ex^2\right)}{1-x^2}\,dx &= \tanh^{-1}(x)\log\left(d+ex^2\right)-(2e)\int \frac{x\tanh^{-1}(x)}{d+ex^2}\,dx\\ &= \tanh^{-1}(x)\log\left(d+ex^2\right)-(2e)\int\left(-\frac{\tanh^{-1}(x)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{\tanh^{-1}(x)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx\\ &= \tanh^{-1}(x)\log\left(d+ex^2\right)+\sqrt{e}\int\frac{\tanh^{-1}(x)}{\sqrt{-d}-\sqrt{e}\,x}\,dx-\sqrt{e}\int\frac{\tanh^{-1}(x)}{\sqrt{-d}+\sqrt{e}\,x}\,dx\\ &= 2\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)-\tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{-d}-\sqrt{e}\right)(1+x)}\right)-\tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}\right.}{\left(\sqrt{-d}+\right.}\right.\\ &= 2\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)-\tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{-d}-\sqrt{e}\right)(1+x)}\right)-\tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}\right.}{\left(\sqrt{-d}+\right.}\right.\\ &= 2\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)-\tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{-d}-\sqrt{e}\right)(1+x)}\right)-\tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}\right.}{\left(\sqrt{-d}+\right.}\right.\end{aligned}$$

Mathematica [C] time = 0.13, size = 468, normalized size = 2.16

$$\frac{1}{2}\left(-\mathrm{Li}_2\left(\frac{\sqrt{d}-i\sqrt{e}\,x}{\sqrt{d}-i\sqrt{e}}\right)+\mathrm{Li}_2\left(\frac{\sqrt{d}-i\sqrt{e}\,x}{\sqrt{d}+i\sqrt{e}}\right)+\mathrm{Li}_2\left(\frac{i\sqrt{e}\,x+\sqrt{d}}{\sqrt{d}-i\sqrt{e}}\right)-\mathrm{Li}_2\left(\frac{i\sqrt{e}\,x+\sqrt{d}}{\sqrt{d}+i\sqrt{e}}\right)-\log(1-x)\log\left(d+ex^2\right)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d + e*x^2]/(1 - x^2),x]
```

```
[Out] (Log[1 - x]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] - Log[(Sqrt[e]*(-1 + x))/(I*Sqr
t[d] - Sqrt[e])]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] - Log[1 + x]*Log[((-I)*Sqr
t[d])/Sqrt[e] + x] + Log[((-I)*Sqrt[e]*(1 + x))/(Sqrt[d] - I*Sqrt[e])]*Log[
((-I)*Sqrt[d])/Sqrt[e] + x] + Log[1 - x]*Log[(I*Sqrt[d])/Sqrt[e] + x] - Log
[(Sqrt[e]*(-1 + x))/((-I)*Sqrt[d] - Sqrt[e])]*Log[(I*Sqrt[d])/Sqrt[e] + x]
- Log[1 + x]*Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[(I*Sqrt[e]*(1 + x))/(Sqrt[d
] + I*Sqrt[e])]*Log[(I*Sqrt[d])/Sqrt[e] + x] - Log[1 - x]*Log[d + e*x^2] +
Log[1 + x]*Log[d + e*x^2] - PolyLog[2, (Sqrt[d] - I*Sqrt[e]*x)/(Sqrt[d] - I
```

```
*Sqrt[e])] + PolyLog[2, (Sqrt[d] - I*Sqrt[e]*x)/(Sqrt[d] + I*Sqrt[e])] + Po
lyLog[2, (Sqrt[d] + I*Sqrt[e]*x)/(Sqrt[d] - I*Sqrt[e])] - PolyLog[2, (Sqrt[
d] + I*Sqrt[e]*x)/(Sqrt[d] + I*Sqrt[e])])/2
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\log\left(ex^2+d\right)}{x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*x^2+d)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-log(e*x^2 + d)/(x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\log\left(ex^2+d\right)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*x^2+d)/(-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-log(e*x^2 + d)/(x^2 - 1), x)
```

maple [A] time = 0.09, size = 282, normalized size = 1.30

$$-\frac{\ln\left(\frac{-(x+1)e+e+\sqrt{-de}}{e+\sqrt{-de}}\right)\ln(x+1)}{2}-\frac{\ln\left(\frac{(x+1)e-e+\sqrt{-de}}{-e+\sqrt{-de}}\right)\ln(x+1)}{2}+\frac{\ln\left(\frac{-(x-1)e-e+\sqrt{-de}}{-e+\sqrt{-de}}\right)\ln(x-1)}{2}+\frac{\ln\left(\frac{(x-1)e+e+\sqrt{-de}}{e+\sqrt{-de}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*x^2+d)/(-x^2+1),x)
```

```
[Out] -1/2*ln(x-1)*ln(e*x^2+d)+1/2*ln(x-1)*ln((-(x-1)*e+(-d*e)^(1/2)-e)/(-e+(-d*e
)^(1/2)))+1/2*ln(x-1)*ln(((x-1)*e+(-d*e)^(1/2)+e)/(e+(-d*e)^(1/2)))+1/2*dil
og((-(x-1)*e+(-d*e)^(1/2)-e)/(-e+(-d*e)^(1/2)))+1/2*dilog(((x-1)*e+(-d*e)^(
1/2)+e)/(e+(-d*e)^(1/2)))+1/2*ln(x+1)*ln(e*x^2+d)-1/2*ln(x+1)*ln((-e*(x+1)+
(-d*e)^(1/2)+e)/(e+(-d*e)^(1/2)))-1/2*ln(x+1)*ln((e*(x+1)+(-d*e)^(1/2)-e)/(
-e+(-d*e)^(1/2)))-1/2*dilog((-e*(x+1)+(-d*e)^(1/2)+e)/(e+(-d*e)^(1/2)))-1/2
*dilog((e*(x+1)+(-d*e)^(1/2)-e)/(-e+(-d*e)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\log\left(ex^2+d\right)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*x^2+d)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate(log(e*x^2 + d)/(x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\ln\left(e\,x^2+d\right)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-log(d + e*x^2)/(x^2 - 1),x)
```

```
[Out] -int(log(d + e*x^2)/(x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\log\left(d+ex^{2}\right)}{x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*x**2+d)/(-x**2+1),x)
```

```
[Out] -Integral(log(d + e*x**2)/(x**2 - 1), x)
```

3.360 $\int \frac{\left(f+gx^{3n}\right)\log\left(c(d+ex^n)^p\right)}{x}\,dx$

Optimal. Leaf size=144

$$\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{gx^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3n}+\frac{d^3gp\log\left(d+ex^n\right)}{3e^3n}-\frac{d^2gpx^n}{3e^2n}+\frac{fp\text{Li}_2\left(\frac{ex^n}{d}+1\right)}{n}+\frac{dgpx^2}{6en}$$

```
[Out] -1/3*d^2*g*p*x^n/e^2/n+1/6*d*g*p*x^(2*n)/e/n-1/9*g*p*x^(3*n)/n+1/3*d^3*g*p*
ln(d+e*x^n)/e^3/n+1/3*g*x^(3*n)*ln(c*(d+e*x^n)^p)/n+f*ln(-e*x^n/d)*ln(c*(d+
e*x^n)^p)/n+f*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A] time = 0.17, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2475, 14, 2416, 2394, 2315, 2395, 43}

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{gx^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3n}-\frac{d^2gpx^n}{3e^2n}+\frac{d^3gp\log\left(d+ex^n\right)}{3e^3n}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^(3*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -(d^2*g*p*x^n)/(3*e^2*n) + (d*g*p*x^(2*n))/(6*e*n) - (g*p*x^(3*n))/(9*n) +
(d^3*g*p*Log[d + e*x^n])/(3*e^3*n) + (g*x^(3*n)*Log[c*(d + e*x^n)^p])/(3*n)
 + (f*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*x^
n)/d])/n
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\left(f+gx^{3n}\right)\log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(f+gx^3\right)\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{f\log(c(d+ex)^p)}{x}+gx^2\log\left(c(d+ex)^p\right)\right)dx,x,x^n\right)}{n}$$

$$= \frac{f\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}+\frac{g\,\text{Subst}\left(\int x^2\log\left(c(d+ex)^p\right)dx,x,x^n\right)}{n}$$

$$= \frac{gx^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{(efp)\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{gx^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}$$

$$= -\frac{d^2gpx^n}{3e^2n}+\frac{dgpx^{2n}}{6en}-\frac{gpx^{3n}}{9n}+\frac{d^3gp\log\left(d+ex^n\right)}{3e^3n}+\frac{gx^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3n}$$

Mathematica [A] time = 0.17, size = 118, normalized size = 0.82

$$\frac{18f\left(\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)+p\text{Li}_2\left(\frac{ex^n}{d}+1\right)\right)+6gx^{3n}\log\left(c\left(d+ex^n\right)^p\right)-\frac{gp\left(ex^n\left(6d^2-3dex^n+2e^2x^{2n}\right)-6d^3\log(d+ex^n)\right)}{e^3}}{18n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^(3*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-((g*p*(e*x^n*(6*d^2 - 3*d*e*x^n + 2*e^2*x^(2*n)) - 6*d^3*Log[d + e*x^n]))
/e^3) + 6*g*x^(3*n)*Log[c*(d + e*x^n)^p] + 18*f*(Log[-((e*x^n)/d)]*Log[c*(d
 + e*x^n)^p] + p*PolyLog[2, 1 + (e*x^n)/d]))/(18*n)
```

fricas [A] time = 0.81, size = 148, normalized size = 1.03

$$-\frac{18\,e^3fnp\log(x)\log\left(\frac{ex^n+d}{d}\right)-18\,e^3fn\log(c)\log(x)-3\,de^2gpx^{2\,n}+6\,d^2egpx^n+18\,e^3fp\text{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+2\left(e^3\right.}{18\,e^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(3*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/18*(18*e^3*f*n*p*log(x)*log((e*x^n + d)/d) - 18*e^3*f*n*log(c)*log(x) -
3*d*e^2*g*p*x^(2*n) + 6*d^2*e*g*p*x^n + 18*e^3*f*p*dilog(-(e*x^n + d)/d + 1
) + 2*(e^3*g*p - 3*e^3*g*log(c))*x^(3*n) - 6*(3*e^3*f*n*p*log(x) + e^3*g*p*
x^(3*n) + d^3*g*p)*log(e*x^n + d))/(e^3*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx^{3n}+f\right)\log\left((ex^n+d)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(3*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^(3*n) + f)*log((e*x^n + d)^p*c)/x, x)
```

maple [C] time = 3.01, size = 428, normalized size = 2.97

$$-\frac{i\pi f\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(e\,x^n+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^n+d\right)^p\right)\ln(x)}{2}+\frac{i\pi f\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(e\,x^n+d\right)^p\right)^2\ln(x)}{2}+\frac{i\pi f\,\mathrm{cs}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^(3*n))*ln(c*(e*x^n+d)^p)/x,x)
```

```
[Out] 1/3*(g*(x^n)^3+3*f*ln(x)*n)/n*ln((e*x^n+d)^p)+1/6*I*Pi*csgn(I*(e*x^n+d)^p)*
csgn(I*c*(e*x^n+d)^p)^2*g*(x^n)^3/n-1/6*I*Pi*csgn(I*c*(e*x^n+d)^p)^3*g*(x^n
)^3/n+1/2*I*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2*ln(x)*f-1/2*I*Pi
*csgn(I*c*(e*x^n+d)^p)^3*ln(x)*f-1/2*I*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x
^n+d)^p)*csgn(I*c)*ln(x)*f+1/2*I*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)*ln(x)
*f+1/6*I*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)*g*(x^n)^3/n-1/6*I*Pi*csgn(I*(
e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)*g*(x^n)^3/n+f*ln(c)*ln(x)+1/3*l
n(c)*g*(x^n)^3/n-1/9*p/n*g*(x^n)^3+1/6*p/e/n*g*(x^n)^2*d-1/3*d^2*g*p*x^n/e^
2/n+1/3*d^3*g*p*ln(e*x^n+d)/e^3/n-p/n*f*dilog((e*x^n+d)/d)-p*f*ln(x)*ln((e*
x^n+d)/d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{9\,e^3fn^2p\log(x)^2-3\,de^2gpx^{2n}+6\,d^2egpx^n+2\left(e^3gp-3\,e^3g\log(c)\right)x^{3n}-6\left(3\,e^3fn\log(x)+e^3gx^{3n}\right)\log\left((e\right.}{18\,e^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(3*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/18*(9*e^3*f*n^2*p*log(x)^2 - 3*d*e^2*g*p*x^(2*n) + 6*d^2*e*g*p*x^n + 2*(
e^3*g*p - 3*e^3*g*log(c))*x^(3*n) - 6*(3*e^3*f*n*log(x) + e^3*g*x^(3*n))*lo
g((e*x^n + d)^p) - 6*(d^3*g*n*p + 3*e^3*f*n*log(c))*log(x))/(e^3*n) + integ
rate(1/3*(3*d*e^3*f*n*p*log(x) - d^4*g*p)/(e^4*x*x^n + d*e^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,\left(f+g\,x^{3\,n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g*x^(3*n)))/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)*(f + g*x^(3*n)))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx^{3n}\right)\log\left(c\left(d+ex^{n}\right)^{p}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**(3*n))*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral((f + g*x**(3*n))*log(c*(d + e*x**n)**p)/x, x)
```

3.361 $\int \frac{(f+gx^{2n})\log(c(d+ex^n)^p)}{x}\,dx$

Optimal. Leaf size=124

$$\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{gx^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}-\frac{d^2gp\log\left(d+ex^n\right)}{2e^2n}+\frac{fp\text{Li}_2\left(\frac{ex^n}{d}+1\right)}{n}+\frac{dgpx^n}{2en}-\frac{gpx^{2n}}{4n}$$

```
[Out] 1/2*d*g*p*x^n/e/n-1/4*g*p*x^(2*n)/n-1/2*d^2*g*p*ln(d+e*x^n)/e^2/n+1/2*g*x^(
2*n)*ln(c*(d+e*x^n)^p)/n+f*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+f*p*polylog(2,1
+e*x^n/d)/n
```

Rubi [A] time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2475, 14, 2416, 2394, 2315, 2395, 43}

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{gx^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}-\frac{d^2gp\log\left(d+ex^n\right)}{2e^2n}+\frac{dgpx^n}{2en}-$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (d*g*p*x^n)/(2*e*n) - (g*p*x^(2*n))/(4*n) - (d^2*g*p*Log[d + e*x^n])/(2*e^2
*n) + (g*x^(2*n)*Log[c*(d + e*x^n)^p])/(2*n) + (f*Log[-((e*x^n)/d)]*Log[c*(
d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\left(f+gx^{2n}\right)\log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(f+gx^2\right)\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{f\log(c(d+ex)^p)}{x}+gx\log\left(c(d+ex)^p\right)\right)dx,x,x^n\right)}{n}$$
$$= \frac{f\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}+\frac{g\,\text{Subst}\left(\int x\log\left(c(d+ex)^p\right)dx,x,x^n\right)}{n}$$
$$= \frac{gx^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{(efp)\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{gx^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}$$
$$= \frac{dgpx^n}{2en}-\frac{gpx^{2n}}{4n}-\frac{d^2gp\log\left(d+ex^n\right)}{2e^2n}+\frac{gx^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{f\log\left(-\frac{e}{}\right.}{}$$

Mathematica [A] time = 0.12, size = 100, normalized size = 0.81

$$\frac{2e^2\log\left(c\left(d+ex^n\right)^p\right)\left(2f\log\left(-\frac{ex^n}{d}\right)+gx^{2n}\right)-2d^2gp\log\left(d+ex^n\right)+4e^2fp\text{Li}_2\left(\frac{ex^n}{d}+1\right)-egpx^n\left(ex^n-2d\right)}{4e^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-(e*g*p*x^n*(-2*d + e*x^n)) - 2*d^2*g*p*Log[d + e*x^n] + 2*e^2*(g*x^(2*n)
+ 2*f*Log[-((e*x^n)/d)])*Log[c*(d + e*x^n)^p] + 4*e^2*f*p*PolyLog[2, 1 + (e
*x^n)/d])/(4*e^2*n)
```

fricas [A] time = 0.63, size = 133, normalized size = 1.07

$$-\frac{4\,e^2fnp\log(x)\log\left(\frac{ex^n+d}{d}\right)-4\,e^2fn\log(c)\log(x)-2\,degpx^n+4\,e^2fp\text{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+\left(e^2gp-2\,e^2g\log(c)\right)x^2}{4\,e^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/4*(4*e^2*f*n*p*log(x)*log((e*x^n + d)/d) - 4*e^2*f*n*log(c)*log(x) - 2*d
*e*g*p*x^n + 4*e^2*f*p*dilog(-(e*x^n + d)/d + 1) + (e^2*g*p - 2*e^2*g*log(c
))*x^(2*n) - 2*(2*e^2*f*n*p*log(x) + e^2*g*p*x^(2*n) - d^2*g*p)*log(e*x^n +
 d))/(e^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx^{2n}+f\right)\log\left((ex^n+d)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^(2*n) + f)*log((e*x^n + d)^p*c)/x, x)
```

maple [C] time = 2.91, size = 410, normalized size = 3.31

$$-\frac{i\pi f\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(e\,x^n+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^n+d\right)^p\right)\ln(x)}{2}+\frac{i\pi f\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(e\,x^n+d\right)^p\right)^2\ln(x)}{2}+\frac{i\pi f\,\mathrm{cs}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^(2*n))*ln(c*(e*x^n+d)^p)/x,x)
```

```
[Out] 1/2*(2*f*n*ln(x)+g*(x^n)^2)/n*ln((e*x^n+d)^p)-1/4*I*Pi*csgn(I*(e*x^n+d)^p)*
csgn(I*c*(e*x^n+d)^p)*csgn(I*c)*g*(x^n)^2/n-1/2*I*Pi*f*csgn(I*c)*csgn(I*(e*
x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*ln(x)+1/4*I*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn
(I*c)*g*(x^n)^2/n-1/2*I*Pi*f*csgn(I*c*(e*x^n+d)^p)^3*ln(x)+1/2*I*Pi*f*csgn(
I*c)*csgn(I*c*(e*x^n+d)^p)^2*ln(x)+1/4*I*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e
*x^n+d)^p)^2*g*(x^n)^2/n-1/4*I*Pi*csgn(I*c*(e*x^n+d)^p)^3*g*(x^n)^2/n+1/2*I
*Pi*f*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2*ln(x)+f*ln(c)*ln(x)+1/2*l
n(c)*g*(x^n)^2/n-1/4*p/n*g*(x^n)^2+1/2*d*g*p*x^n/e/n-1/2*d^2*g*p*ln(e*x^n+d
)/e^2/n-p/n*f*dilog((e*x^n+d)/d)-f*p*ln(x)*ln((e*x^n+d)/d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,e^2fn^2p\log(x)^2-2\,degpx^n+\left(e^2gp-2\,e^2g\log(c)\right)x^{2n}-2\left(2\,e^2fn\log(x)+e^2gx^{2n}\right)\log\left((ex^n+d)^p\right)+2\left(d^2\right.}{4\,e^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(2*e^2*f*n^2*p*log(x)^2 - 2*d*e*g*p*x^n + (e^2*g*p - 2*e^2*g*log(c))*x
^(2*n) - 2*(2*e^2*f*n*log(x) + e^2*g*x^(2*n))*log((e*x^n + d)^p) + 2*(d^2*g
*n*p - 2*e^2*f*n*log(c))*log(x))/(e^2*n) + integrate(1/2*(2*d*e^2*f*n*p*log
(x) + d^3*g*p)/(e^3*x*x^n + d*e^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,\left(f+g\,x^{2\,n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g*x^(2*n)))/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)*(f + g*x^(2*n)))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+g x^{2 n}\right) \log \left(c\left(d+e x^{n}\right)^{p}\right)}{x} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**(2*n))*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral((f + g*x**(2*n))*log(c*(d + e*x**n)**p)/x, x)
```

3.362 $\int \frac{(f+gx^n)\log(c(d+ex^n)^p)}{x}\,dx$

Optimal. Leaf size=83

$$\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{g\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{en}+\frac{fp\text{Li}_2\left(\frac{ex^n}{d}+1\right)}{n}-\frac{gpx^n}{n}$$

```
[Out] -g*p*x^n/n+g*(d+e*x^n)*ln(c*(d+e*x^n)^p)/e/n+f*ln(-e*x^n/d)*ln(c*(d+e*x^n)^
p)/n+f*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A] time = 0.11, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2475, 43, 2416, 2389, 2295, 2394, 2315}

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{g\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{en}-\frac{gpx^n}{n}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -((g*p*x^n)/n) + (g*(d + e*x^n)*Log[c*(d + e*x^n)^p])/(e*n) + (f*Log[-((e*x
^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^n)\log(c(d+ex^n)^p)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{(f+gx)\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(g\log(c(d+ex)^p) + \frac{f\log(c(d+ex)^p)}{x}\right)dx, x, x^n\right)}{n}$$
$$= \frac{f\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} + \frac{g\,\text{Subst}\left(\int \log(c(d+ex)^p)\,dx, x, x^n\right)}{n}$$
$$= \frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n} + \frac{g\,\text{Subst}\left(\int \log(cx^p)\,dx, x, d+ex^n\right)}{en} - \frac{(efp}{}$$
$$= -\frac{gpx^n}{n} + \frac{g(d+ex^n)\log\left(c(d+ex^n)^p\right)}{en} + \frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n} + -$$

Mathematica [A] time = 0.06, size = 68, normalized size = 0.82

$$\frac{\log\left(c(d+ex^n)^p\right)\left(ef\log\left(-\frac{ex^n}{d}\right)+dg+egx^n\right)+efp\text{Li}_2\left(\frac{ex^n}{d}+1\right)-egpx^n}{en}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-(e*g*p*x^n) + (d*g + e*g*x^n + e*f*Log[-((e*x^n)/d)])*Log[c*(d + e*x^n)^p
] + e*f*p*PolyLog[2, 1 + (e*x^n)/d])/(e*n)
```

fricas [A] time = 0.49, size = 100, normalized size = 1.20

$$-\frac{efnp\log(x)\log\left(\frac{ex^n+d}{d}\right)-efn\log(c)\log(x)+efp\text{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+\left(egp-eg\log(c)\right)x^n-\left(efnp\log(x)+egpx\right.}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -(e*f*n*p*log(x)*log((e*x^n + d)/d) - e*f*n*log(c)*log(x) + e*f*p*dilog(-(e
*x^n + d)/d + 1) + (e*g*p - e*g*log(c))*x^n - (e*f*n*p*log(x) + e*g*p*x^n +
 d*g*p)*log(e*x^n + d))/(e*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx^n+f)\log\left((ex^n+d)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^n + f)*log((e*x^n + d)^p*c)/x, x)
```

maple [C] time = 3.50, size = 376, normalized size = 4.53

$$-\frac{i\pi f \operatorname{csgn}(ic)\operatorname{csgn}\left(i\left(e x^{n}+d\right)^{p}\right)\operatorname{csgn}\left(ic\left(e x^{n}+d\right)^{p}\right)\ln(x)}{2}+\frac{i\pi f \operatorname{csgn}(ic)\operatorname{csgn}\left(ic\left(e x^{n}+d\right)^{p}\right)^{2}\ln(x)}{2}+\frac{i\pi f \operatorname{cs}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^n)*ln(c*(e*x^n+d)^p)/x,x)
```

```
[Out] (f*n*ln(x)+g*x^n)/n*ln((e*x^n+d)^p)+1/2*I*Pi*f*csgn(I*(e*x^n+d)^p)*csgn(I*c
*(e*x^n+d)^p)^2*ln(x)+1/2*I*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2*
g*x^n/n-1/2*I*Pi*f*csgn(I*c)*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*ln(x
)-1/2*I*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)*g*x^n/n-1/2*
I*Pi*f*csgn(I*c*(e*x^n+d)^p)^3*ln(x)-1/2*I*Pi*csgn(I*c*(e*x^n+d)^p)^3*g*x^n
/n+1/2*I*Pi*f*csgn(I*c)*csgn(I*c*(e*x^n+d)^p)^2*ln(x)+1/2*I*Pi*csgn(I*c*(e*
x^n+d)^p)^2*csgn(I*c)*g*x^n/n+f*ln(c)*ln(x)+ln(c)*g*x^n/n-g*p*x^n/n+p/e/n*g
*d*ln(e*x^n+d)-p/n*f*dilog((e*x^n+d)/d)-f*p*ln(x)*ln((e*x^n+d)/d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{efn^{2}p\log(x)^{2}+2\left(egp-eg\log(c)\right)x^{n}-2\left(efn\log(x)+egx^{n}\right)\log\left(\left(ex^{n}+d\right)^{p}\right)-2\left(dgnp+efn\log(c)\right)\log(x}{2\,en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(e*f*n^2*p*log(x)^2 + 2*(e*g*p - e*g*log(c))*x^n - 2*(e*f*n*log(x) + e
*g*x^n)*log((e*x^n + d)^p) - 2*(d*g*n*p + e*f*n*log(c))*log(x))/(e*n) + int
egrate((d*e*f*n*p*log(x) - d^2*g*p)/(e^2*x*x^n + d*e*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(d+e\,x^{n}\right)^{p}\right)\,\left(f+g\,x^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g*x^n))/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)*(f + g*x^n))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx^{n}\right)\log\left(c\left(d+ex^{n}\right)^{p}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**n)*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral((f + g*x**n)*log(c*(d + e*x**n)**p)/x, x)
```

3.363 $\int \frac{(f+gx^{-n})\log(c(d+ex^n)^p)}{x}\,dx$

Optimal. Leaf size=97

$$\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{gx^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{fp\text{Li}_2\left(\frac{ex^n}{d}+1\right)}{n}-\frac{egp\log\left(d+ex^n\right)}{dn}+\frac{egp\log(x)}{d}$$

```
[Out] e*g*p*ln(x)/d-e*g*p*ln(d+e*x^n)/d/n-g*ln(c*(d+e*x^n)^p)/n/(x^n)+f*ln(-e*x^n
/d)*ln(c*(d+e*x^n)^p)/n+f*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A] time = 0.14, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2475, 14, 2416, 2395, 36, 29, 31, 2394, 2315}

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{gx^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{egp\log\left(d+ex^n\right)}{dn}+\frac{egp\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g/x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (e*g*p*Log[x])/d - (e*g*p*Log[d + e*x^n])/(d*n) - (g*Log[c*(d + e*x^n)^p])/
(n*x^n) + (f*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1
+ (e*x^n)/d])/n
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(f+gx^{-n}\right)\log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(f+\frac{g}{x}\right)\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{g\log(c(d+ex)^p)}{x^2}+\frac{f\log(c(d+ex)^p)}{x}\right)dx, x, x^n\right)}{n}\\
&= \frac{f\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}+\frac{g\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^2}\,dx, x, x^n\right)}{n}\\
&= -\frac{gx^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{(efp)\,\text{Subs}}{}\\
&= -\frac{gx^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{fp\text{Li}_2\left(1+\right.}{n}\\
&= \frac{egp\log(x)}{d}-\frac{egp\log\left(d+ex^n\right)}{dn}-\frac{gx^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)}{}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 87, normalized size = 0.90

$$
\frac{f\left(\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)+p\text{Li}_2\left(\frac{ex^n}{d}+1\right)\right)-gx^{-n}\log\left(c\left(d+ex^n\right)^p\right)+\frac{egp\left(n\log(x)-\log\left(d+ex^n\right)\right)}{d}}{n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g/x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] ((e*g*p*(n*Log[x] - Log[d + e*x^n]))/d - (g*Log[c*(d + e*x^n)^p])/x^n + f*(
Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, 1 + (e*x^n)/d]))/n
```

fricas [A] time = 0.46, size = 114, normalized size = 1.18

$$-\frac{dfnpx^n \log(x)\log\left(\frac{ex^n+d}{d}\right) + dfpx^n\text{Li}_2\left(-\frac{ex^n+d}{d}+1\right) + dg\log(c) - \left(egnp + dfn\log(c)\right)x^n\log(x) + \left(dgp - \left(dfnp\right.\right.}{dnx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -(d*f*n*p*x^n*log(x)*log((e*x^n + d)/d) + d*f*p*x^n*dilog(-(e*x^n + d)/d +
1) + d*g*log(c) - (e*g*n*p + d*f*n*log(c))*x^n*log(x) + (d*g*p - (d*f*n*p*l
og(x) - e*g*p)*x^n)*log(e*x^n + d))/(d*n*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + \frac{g}{x^n}\right)\log\left((ex^n + d)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((f + g/x^n)*log((e*x^n + d)^p*c)/x, x)
```

maple [C] time = 3.41, size = 423, normalized size = 4.36

$$-\frac{i\pi f\,\mathrm{csgn}\,(ic)\,\mathrm{csgn}\left(i\,(e\,x^n+d)^p\right)\mathrm{csgn}\left(ic\,(e\,x^n+d)^p\right)\ln\left(x^n\right)}{2n}+\frac{i\pi f\,\mathrm{csgn}\,(ic)\,\mathrm{csgn}\left(ic\,(e\,x^n+d)^p\right)^2\ln\left(x^n\right)}{2n}+\frac{i\pi f\,\mathrm{cs}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/(x^n))*ln(c*(e*x^n+d)^p)/x,x)
```

```
[Out] (f*ln(x)*n*x^n-g)/n/(x^n)*ln((e*x^n+d)^p)+1/2*I/n*Pi*csgn(I*(e*x^n+d)^p)*cs
gn(I*c*(e*x^n+d)^p)^2*ln(x^n)*f-1/2*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*
x^n+d)^p)^2*g/(x^n)-1/2*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*cs
gn(I*c)*ln(x^n)*f+1/2*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn
(I*c)*g/(x^n)-1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3*ln(x^n)*f+1/2*I/n*Pi*csgn(
I*c*(e*x^n+d)^p)^3*g/(x^n)+1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)*ln(
x^n)*f-1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)*g/(x^n)+1/n*ln(c)*ln(x^
n)*f-1/n*ln(c)*g/(x^n)-p/n*f*dilog((e*x^n+d)/d)-f*p*ln(x)*ln((e*x^n+d)/d)+p
*e/n*g/d*ln(x^n)-e*g*p*ln(e*x^n+d)/d/n
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(fn^2p\log(x)^2 - 2\,fn\log(c)\log(x)\right)x^n - 2\left(fnx^n\log(x) - g\right)\log\left((ex^n+d)^p\right) + 2\,g\log(c)}{2\,nx^n}+\int \frac{dfnp\log(x) + egp}{exx^n + dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/2*((f*n^2*p*log(x)^2 - 2*f*n*log(c)*log(x))*x^n - 2*(f*n*x^n*log(x) - g)
*log((e*x^n + d)^p) + 2*g*log(c))/(n*x^n) + integrate((d*f*n*p*log(x) + e*g
*p)/(e*x*x^n + d*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,(d+e\,x^n)^p\right)\,\left(f+\frac{g}{x^n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g/x^n))/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)*(f + g/x^n))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n}\left(fx^{n}+g\right)\log\left(c\left(d+ex^{n}\right)^{p}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x**n))*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral(x**(-n)*(f*x**n + g)*log(c*(d + e*x**n)**p)/x, x)
```

3.364 $\int \frac{\left(f+gx^{-2n}\right)\log\left(c(d+ex^n)^p\right)}{x}\,dx$

Optimal. Leaf size=126

$$\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{gx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{e^2gp\log\left(d+ex^n\right)}{2d^2n}-\frac{e^2gp\log(x)}{2d^2}+\frac{fp\text{Li}_2\left(\frac{ex^n}{d}+1\right)}{n}-\frac{egpx}{2d}$$

```
[Out] -1/2*e*g*p/d/n/(x^n)-1/2*e^2*g*p*ln(x)/d^2+1/2*e^2*g*p*ln(d+e*x^n)/d^2/n-1/
2*g*ln(c*(d+e*x^n)^p)/n/(x^(2*n))+f*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+f*p*po
lylog(2,1+e*x^n/d)/n
```

Rubi [A] time = 0.17, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2475, 14, 2416, 2395, 44, 2394, 2315}

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{gx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{e^2gp\log\left(d+ex^n\right)}{2d^2n}-\frac{e^2gp\log(}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g/x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -(e*g*p)/(2*d*n*x^n) - (e^2*g*p*Log[x])/(2*d^2) + (e^2*g*p*Log[d + e*x^n])/
(2*d^2*n) - (g*Log[c*(d + e*x^n)^p])/(2*n*x^(2*n)) + (f*Log[-((e*x^n)/d)]*L
og[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\left(f+gx^{-2n}\right)\log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(f+\frac{g}{x^2}\right)\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{g\log(c(d+ex)^p)}{x^3}+\frac{f\log(c(d+ex)^p)}{x}\right)dx,x,x^n\right)}{n}$$
$$= \frac{f\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}+\frac{g\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^3}\,dx,x,x^n\right)}{n}$$
$$= -\frac{gx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{(efp)\,\text{Sub}}{}$$
$$= -\frac{gx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{fp\text{Li}_2\left(1\right.}{n}$$
$$= -\frac{egpx^{-n}}{2dn}-\frac{e^2gp\log(x)}{2d^2}+\frac{e^2gp\log\left(d+ex^n\right)}{2d^2n}-\frac{gx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}$$

Mathematica [A] time = 0.20, size = 104, normalized size = 0.83

$$-\frac{-2f\left(\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)+p\text{Li}_2\left(\frac{ex^n}{d}+1\right)\right)+gx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)+\frac{egpx^{-n}\left(-ex^n\log(d+ex^n)+d+enx^n\log\right.}{d^2}}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g/x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -1/2*((e*g*p*(d + e*n*x^n*Log[x] - e*x^n*Log[d + e*x^n]))/(d^2*x^n) + (g*Lo
g[c*(d + e*x^n)^p])/x^(2*n) - 2*f*(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] +
 p*PolyLog[2, 1 + (e*x^n)/d]))/n
```

fricas [A] time = 0.45, size = 150, normalized size = 1.19

$$-\frac{2\,d^2fnpx^{2\,n}\log(x)\log\left(\frac{ex^n+d}{d}\right)+2\,d^2fpx^{2\,n}\text{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+degpx^n+d^2g\log(c)+\left(e^2gnp-2\,d^2fn\log(c)\right)}{2\,d^2nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*f*n*p*x^(2*n)*log(x)*log((e*x^n + d)/d) + 2*d^2*f*p*x^(2*n)*dil
og(-(e*x^n + d)/d + 1) + d*e*g*p*x^n + d^2*g*log(c) + (e^2*g*n*p - 2*d^2*f*
n*log(c))*x^(2*n)*log(x) + (d^2*g*p - (2*d^2*f*n*p*log(x) + e^2*g*p)*x^(2*n
))*log(e*x^n + d))/(d^2*n*x^(2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + \frac{g}{x^{2n}}\right)\log\left((ex^n + d)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((f + g/x^(2*n))*log((e*x^n + d)^p*c)/x, x)
```

maple [C] time = 3.44, size = 448, normalized size = 3.56

$$-\frac{i\pi f \operatorname{csgn}(ic)\operatorname{csgn}\left(i(e x^n + d)^p\right)\operatorname{csgn}\left(ic(e x^n + d)^p\right)\ln(x^n)}{2n} + \frac{i\pi f \operatorname{csgn}(ic)\operatorname{csgn}\left(ic(e x^n + d)^p\right)^2\ln(x^n)}{2n} + \frac{i\pi f \operatorname{cs}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/(x^(2*n)))*ln(c*(e*x^n+d)^p)/x,x)
```

```
[Out] 1/2*(2*f*ln(x)*n*(x^n)^2-g)/n/(x^n)^2*ln((e*x^n+d)^p)+1/4*I/n*Pi*csgn(I*c*(
e*x^n+d)^p)^3*g/(x^n)^2-1/2*I*Pi*f/n*csgn(I*c)*csgn(I*(e*x^n+d)^p)*csgn(I*c
*(e*x^n+d)^p)*ln(x^n)-1/4*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^
2*g/(x^n)^2-1/2*I*Pi*f/n*csgn(I*c*(e*x^n+d)^p)^3*ln(x^n)+1/4*I/n*Pi*csgn(I*
(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)*g/(x^n)^2-1/4*I/n*Pi*csgn(I*c*
(e*x^n+d)^p)^2*csgn(I*c)*g/(x^n)^2+1/2*I*Pi*f/n*csgn(I*c)*csgn(I*c*(e*x^n+d
)^p)^2*ln(x^n)+1/2*I*Pi*f/n*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2*ln(
x^n)+f/n*ln(c)*ln(x^n)-1/2/n*ln(c)*g/(x^n)^2-1/2*e*g*p/d/n/(x^n)-1/2*p*e^2/
n*g/d^2*ln(x^n)+1/2*e^2*g*p*ln(e*x^n+d)/d^2/n-p/n*f*dilog((e*x^n+d)/d)-f*p*
ln(x)*ln((e*x^n+d)/d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{egpx^n + dg\log(c) + \left(dfn^2p\log(x)^2 - 2\,dfn\log(c)\log(x)\right)x^{2n} - \left(2\,dfnx^{2n}\log(x) - dg\right)\log\left((ex^n + d)^p\right)}{2\,dnx^{2n}} + \int \frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(e*g*p*x^n + d*g*log(c) + (d*f*n^2*p*log(x)^2 - 2*d*f*n*log(c)*log(x))
*x^(2*n) - (2*d*f*n*x^(2*n)*log(x) - d*g)*log((e*x^n + d)^p))/(d*n*x^(2*n))
 + integrate(1/2*(2*d^2*f*n*p*log(x) - e^2*g*p)/(d*e*x*x^n + d^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,(d + e\,x^n)^p\right)\,\left(f + \frac{g}{x^{2n}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g/x^(2*n)))/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)*(f + g/x^(2*n)))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-2n} \left(fx^{2n} + g\right) \log \left(c \left(d + ex^{n}\right)^{p}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x**(2*n)))*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral(x**(-2*n)*(f*x**(2*n) + g)*log(c*(d + e*x**n)**p)/x, x)
```

3.365 $\int \frac{\left(f+gx^{3n}\right)^2 \log\left(c(d+ex^n)^p\right)}{x}\,dx$

Optimal. Leaf size=327

$$\frac{f^2 \log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{2fgx^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3n}+\frac{g^2x^{6n}\log\left(c\left(d+ex^n\right)^p\right)}{6n}-\frac{d^6g^2p\log\left(d+ex^n\right)}{6e^6n}+\frac{d^5g^2p}{6e^5n}$$

```
[Out] -2/3*d^2*f*g*p*x^n/e^2/n+1/6*d^5*g^2*p*x^n/e^5/n+1/3*d*f*g*p*x^(2*n)/e/n-1/
12*d^4*g^2*p*x^(2*n)/e^4/n-2/9*f*g*p*x^(3*n)/n+1/18*d^3*g^2*p*x^(3*n)/e^3/n
-1/24*d^2*g^2*p*x^(4*n)/e^2/n+1/30*d*g^2*p*x^(5*n)/e/n-1/36*g^2*p*x^(6*n)/n
+2/3*d^3*f*g*p*ln(d+e*x^n)/e^3/n-1/6*d^6*g^2*p*ln(d+e*x^n)/e^6/n+2/3*f*g*x^
(3*n)*ln(c*(d+e*x^n)^p)/n+1/6*g^2*x^(6*n)*ln(c*(d+e*x^n)^p)/n+f^2*ln(-e*x^n
/d)*ln(c*(d+e*x^n)^p)/n+f^2*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A] time = 0.33, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2475, 266, 43, 2416, 2394, 2315, 2395}

$$\frac{f^2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{2fgx^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3n}+\frac{g^2x^{6n}\log\left(c\left(d+ex^n\right)^p\right)}{6n}-$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^(3*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-2*d^2*f*g*p*x^n)/(3*e^2*n) + (d^5*g^2*p*x^n)/(6*e^5*n) + (d*f*g*p*x^(2*n)
)/(3*e*n) - (d^4*g^2*p*x^(2*n))/(12*e^4*n) - (2*f*g*p*x^(3*n))/(9*n) + (d^3
*g^2*p*x^(3*n))/(18*e^3*n) - (d^2*g^2*p*x^(4*n))/(24*e^2*n) + (d*g^2*p*x^(5
*n))/(30*e*n) - (g^2*p*x^(6*n))/(36*n) + (2*d^3*f*g*p*Log[d + e*x^n])/(3*e^
3*n) - (d^6*g^2*p*Log[d + e*x^n])/(6*e^6*n) + (2*f*g*x^(3*n)*Log[c*(d + e*x
^n)^p])/(3*n) + (g^2*x^(6*n)*Log[c*(d + e*x^n)^p])/(6*n) + (f^2*Log[-((e*x^
n)/d)]*Log[c*(d + e*x^n)^p])/n + (f^2*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\left(f+gx^{3n}\right)^2 \log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\mathrm{Subst}\left(\int \frac{\left(f+gx^3\right)^2 \log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\mathrm{Subst}\left(\int \left(\frac{f^2\log(c(d+ex)^p)}{x} + 2fgx^2\log\left(c(d+ex)^p\right) + g^2x^5\log\left(c(d+ex)^p\right)\right.}{n}$$

$$= \frac{f^2\,\mathrm{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} + \frac{(2fg)\,\mathrm{Subst}\left(\int x^2\log\left(c(d+ex)^p\right)\right.}{n}$$

$$= \frac{2fgx^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3n} + \frac{g^2x^{6n}\log\left(c\left(d+ex^n\right)^p\right)}{6n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\mathrm{lo}}{n}$$

$$= \frac{2fgx^{3n}\log\left(c\left(d+ex^n\right)^p\right)}{3n} + \frac{g^2x^{6n}\log\left(c\left(d+ex^n\right)^p\right)}{6n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\mathrm{lo}}{n}$$

$$= -\frac{2d^2fgpx^n}{3e^2n} + \frac{d^5g^2px^n}{6e^5n} + \frac{dfgpx^{2n}}{3en} - \frac{d^4g^2px^{2n}}{12e^4n} - \frac{2fgpx^{3n}}{9n} + \frac{d^3g^2px^{3n}}{18e^3n}$$

Mathematica [A] time = 0.31, size = 209, normalized size = 0.64

$$\frac{60e^6\log\left(c\left(d+ex^n\right)^p\right)\left(6f^2\log\left(-\frac{ex^n}{d}\right)+gx^{3n}\left(4f+gx^{3n}\right)\right)-60d^3gp\left(d^3g-4e^3f\right)\log\left(d+ex^n\right)-egpx^n\left(-60\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^(3*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-(e*g*p*x^n*(-60*d^5*g + 30*d^4*e*g*x^n - 20*d^3*e^2*g*x^(2*n) + 10*e^5*x^
(2*n)*(8*f + g*x^(3*n)) - 12*d*e^4*x^n*(10*f + g*x^(3*n)) + 15*d^2*e^3*(16*
f + g*x^(3*n)))) - 60*d^3*g*(-4*e^3*f + d^3*g)*p*Log[d + e*x^n] + 60*e^6*(g
```

```
*x^(3*n)*(4*f + g*x^(3*n)) + 6*f^2*Log[-((e*x^n)/d)])*Log[c*(d + e*x^n)^p]
+ 360*e^6*f^2*p*PolyLog[2, 1 + (e*x^n)/d])/(360*e^6*n)
```

fricas [A] time = 0.47, size = 291, normalized size = 0.89

$$-\frac{360\,e^{6}f^{2}np\log(x)\log\left(\frac{ex^{n}+d}{d}\right)-360\,e^{6}f^{2}n\log(c)\log(x)-12\,de^{5}g^{2}px^{5\,n}+15\,d^{2}e^{4}g^{2}px^{4\,n}+360\,e^{6}f^{2}p\mathrm{Li}_2\left(-\frac{ex^{n}+d}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(3*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/360*(360*e^6*f^2*n*p*log(x)*log((e*x^n + d)/d) - 360*e^6*f^2*n*log(c)*lo
g(x) - 12*d*e^5*g^2*p*x^(5*n) + 15*d^2*e^4*g^2*p*x^(4*n) + 360*e^6*f^2*p*di
log(-(e*x^n + d)/d + 1) - 30*(4*d*e^5*f*g - d^4*e^2*g^2)*p*x^(2*n) + 60*(4*
d^2*e^4*f*g - d^5*e*g^2)*p*x^n + 10*(e^6*g^2*p - 6*e^6*g^2*log(c))*x^(6*n)
- 20*(12*e^6*f*g*log(c) - (4*e^6*f*g - d^3*e^3*g^2)*p)*x^(3*n) - 60*(6*e^6*
f^2*n*p*log(x) + e^6*g^2*p*x^(6*n) + 4*e^6*f*g*p*x^(3*n) + (4*d^3*e^3*f*g -
 d^6*g^2)*p)*log(e*x^n + d))/(e^6*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx^{3\,n}+f\right)^{2}\log\left(\left(ex^{n}+d\right)^{p}c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(3*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^(3*n) + f)^2*log((e*x^n + d)^p*c)/x, x)
```

maple [C] time = 4.04, size = 795, normalized size = 2.43

$$\frac{\left(6f^{2}n\ln(x)+4fg\,x^{3n}+g^{2}x^{6n}\right)\ln\left(\left(e\,x^{n}+d\right)^{p}\right)}{6n}-\frac{g^{2}p\,x^{6n}}{36n}+\frac{g^{2}x^{6n}\ln(c)}{6n}-\frac{f^{2}p\,\mathrm{dilog}\left(\frac{e\,x^{n}+d}{d}\right)}{n}+\frac{f^{2}\ln(c)\ln\left(x^{n}\right)}{n}+\frac{d^{5}g^{2}}{6e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^(3*n))^2*ln(c*(e*x^n+d)^p)/x,x)
```

```
[Out] 1/6*(g^2*(x^n)^6+4*f*g*(x^n)^3+6*f^2*ln(x)*n)/n*ln((e*x^n+d)^p)+1/n*ln(c)*f
^2*ln(x^n)+1/6*d^5*g^2*p*x^n/e^5/n-p/n*f^2*dilog((e*x^n+d)/d)-p*f^2*ln(x)*l
n((e*x^n+d)/d)+1/6/n*ln(c)*(x^n)^6*g^2-1/36*p/n*g^2*(x^n)^6-1/3*I/n*Pi*csgn
(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)*(x^n)^3*f*g+1/2*I/n*Pi*csgn
(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2*f^2*ln(x^n)+1/12*I/n*Pi*csgn(I*c*(e
*x^n+d)^p)^2*csgn(I*c)*(x^n)^6*g^2-2/3*d^2*f*g*p*x^n/e^2/n+2/3*d^3*f*g*p*ln
(e*x^n+d)/e^3/n-1/3*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3*(x^n)^3*f*g+1/12*I/n*Pi*
csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2*(x^n)^6*g^2-1/6*d^6*g^2*p*ln(e*
x^n+d)/e^6/n-2/9*p/n*f*g*(x^n)^3-1/2*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e
*x^n+d)^p)*csgn(I*c)*f^2*ln(x^n)-1/12*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(
e*x^n+d)^p)*csgn(I*c)*(x^n)^6*g^2+1/3*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(
e*x^n+d)^p)^2*(x^n)^3*f*g-1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3*f^2*ln(x^n)+1/
3*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)*(x^n)^3*f*g+1/2*I/n*Pi*csgn(I*c*
(e*x^n+d)^p)^2*csgn(I*c)*f^2*ln(x^n)+2/3/n*ln(c)*(x^n)^3*f*g+1/30*p/e/n*g^2
*(x^n)^5*d-1/24*p/e^2/n*g^2*d^2*(x^n)^4+1/18*p/e^3/n*g^2*d^3*(x^n)^3-1/12*p
/e^4/n*g^2*(x^n)^2*d^4-1/12*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3*(x^n)^6*g^2+1/3*
p/e/n*f*g*(x^n)^2*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{180\,e^{6}f^{2}n^{2}p\log(x)^{2}-12\,de^{5}g^{2}px^{5\,n}+15\,d^{2}e^{4}g^{2}px^{4\,n}+10\left(e^{6}g^{2}p-6\,e^{6}g^{2}\log(c)\right)x^{6\,n}+20\left(4\,e^{6}fgp-d^{3}e^{3}g^{2}p-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(3*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/360*(180*e^6*f^2*n^2*p*log(x)^2 - 12*d*e^5*g^2*p*x^(5*n) + 15*d^2*e^4*g^
2*p*x^(4*n) + 10*(e^6*g^2*p - 6*e^6*g^2*log(c))*x^(6*n) + 20*(4*e^6*f*g*p -
 d^3*e^3*g^2*p - 12*e^6*f*g*log(c))*x^(3*n) - 30*(4*d*e^5*f*g*p - d^4*e^2*g
^2*p)*x^(2*n) + 60*(4*d^2*e^4*f*g*p - d^5*e*g^2*p)*x^n - 60*(6*e^6*f^2*n*lo
g(x) + e^6*g^2*x^(6*n) + 4*e^6*f*g*x^(3*n))*log((e*x^n + d)^p) - 60*(4*d^3*
e^3*f*g*n*p - d^6*g^2*n*p + 6*e^6*f^2*n*log(c))*log(x))/(e^6*n) + integrate
(1/6*(6*d*e^6*f^2*n*p*log(x) - 4*d^4*e^3*f*g*p + d^7*g^2*p)/(e^7*x*x^n + d*
e^6*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,(d+e\,x^n)^p\right)\,\left(f+g\,x^{3\,n}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g*x^(3*n))^2)/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)*(f + g*x^(3*n))^2)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**(3*n))**2*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Timed out
```

3.366 $\int \frac{\left(f+gx^{2n}\right)^2 \log\left(c(d+ex^n)^p\right)}{x}\,dx$

Optimal. Leaf size=254

$$\frac{f^2 \log \left(-\frac{ex^n}{d}\right) \log \left(c\left(d+ex^n\right)^p\right)}{n}+\frac{fgx^{2n} \log \left(c\left(d+ex^n\right)^p\right)}{n}+\frac{g^2x^{4n} \log \left(c\left(d+ex^n\right)^p\right)}{4n}-\frac{d^4g^2p \log \left(d+ex^n\right)}{4e^4n}+\frac{d^3g^2px}{4e^3n}$$

```
[Out] d*f*g*p*x^n/e/n+1/4*d^3*g^2*p*x^n/e^3/n-1/2*f*g*p*x^(2*n)/n-1/8*d^2*g^2*p*x
^(2*n)/e^2/n+1/12*d*g^2*p*x^(3*n)/e/n-1/16*g^2*p*x^(4*n)/n-d^2*f*g*p*ln(d+e
*x^n)/e^2/n-1/4*d^4*g^2*p*ln(d+e*x^n)/e^4/n+f*g*x^(2*n)*ln(c*(d+e*x^n)^p)/n
+1/4*g^2*x^(4*n)*ln(c*(d+e*x^n)^p)/n+f^2*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+f
^2*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A] time = 0.27, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2475, 266, 43, 2416, 2394, 2315, 2395}

$$\frac{f^2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f^2 \log \left(-\frac{ex^n}{d}\right) \log \left(c\left(d+ex^n\right)^p\right)}{n}+\frac{fgx^{2n} \log \left(c\left(d+ex^n\right)^p\right)}{n}+\frac{g^2x^{4n} \log \left(c\left(d+ex^n\right)^p\right)}{4n}-\frac{d}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (d*f*g*p*x^n)/(e*n) + (d^3*g^2*p*x^n)/(4*e^3*n) - (f*g*p*x^(2*n))/(2*n) - (
d^2*g^2*p*x^(2*n))/(8*e^2*n) + (d*g^2*p*x^(3*n))/(12*e*n) - (g^2*p*x^(4*n))
/(16*n) - (d^2*f*g*p*Log[d + e*x^n])/(e^2*n) - (d^4*g^2*p*Log[d + e*x^n])/(
4*e^4*n) + (f*g*x^(2*n)*Log[c*(d + e*x^n)^p])/n + (g^2*x^(4*n)*Log[c*(d + e
*x^n)^p])/(4*n) + (f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f^2*p*P
olyLog[2, 1 + (e*x^n)/d])/n
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\left(f+gx^{2n}\right)^2\log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(f+gx^2\right)^2\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{f^2\log(c(d+ex)^p)}{x}+2fgx\log\left(c(d+ex)^p\right)+g^2x^3\log\left(c(d+ex)^p\right)\right.}{n}$$

$$= \frac{f^2\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}+\frac{(2fg)\,\text{Subst}\left(\int x\log\left(c(d+ex)^p\right)\,d\right.}{n}$$

$$= \frac{fgx^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{g^2x^{4n}\log\left(c\left(d+ex^n\right)^p\right)}{4n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log}{n}$$

$$= \frac{fgx^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{g^2x^{4n}\log\left(c\left(d+ex^n\right)^p\right)}{4n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log}{n}$$

$$= \frac{dfgpx^n}{en}+\frac{d^3g^2px^n}{4e^3n}-\frac{fgpx^{2n}}{2n}-\frac{d^2g^2px^{2n}}{8e^2n}+\frac{dg^2px^{3n}}{12en}-\frac{g^2px^{4n}}{16n}-\frac{d^2fg}{}$$

Mathematica [A] time = 0.27, size = 171, normalized size = 0.67

$$\frac{12e^4\log\left(c\left(d+ex^n\right)^p\right)\left(4f^2\log\left(-\frac{ex^n}{d}\right)+gx^{2n}\left(4f+gx^{2n}\right)\right)-12d^2gp\left(d^2g+4e^2f\right)\log\left(d+ex^n\right)-egpx^n\left(-12d\right.}{48e^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-(e*g*p*x^n*(-12*d^3*g + 6*d^2*e*g*x^n + 3*e^3*x^n*(8*f + g*x^(2*n)) - 4*d
*e^2*(12*f + g*x^(2*n)))) - 12*d^2*g*(4*e^2*f + d^2*g)*p*Log[d + e*x^n] + 1
2*e^4*(g*x^(2*n)*(4*f + g*x^(2*n)) + 4*f^2*Log[-((e*x^n)/d)])*Log[c*(d + e*
x^n)^p] + 48*e^4*f^2*p*PolyLog[2, 1 + (e*x^n)/d])/(48*e^4*n)
```

fricas [A] time = 0.47, size = 242, normalized size = 0.95

$$-\frac{48\,e^4f^2np\log(x)\log\left(\frac{ex^n+d}{d}\right)-48\,e^4f^2n\log(c)\log(x)-4\,de^3g^2px^{3\,n}+48\,e^4f^2p\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)-12\left(4\,de^3fg+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/48*(48*e^4*f^2*n*p*log(x)*log((e*x^n + d)/d) - 48*e^4*f^2*n*log(c)*log(x
) - 4*d*e^3*g^2*p*x^(3*n) + 48*e^4*f^2*p*dilog(-(e*x^n + d)/d + 1) - 12*(4*
d*e^3*f*g + d^3*e*g^2)*p*x^n + 3*(e^4*g^2*p - 4*e^4*g^2*log(c))*x^(4*n) - 6
*(8*e^4*f*g*log(c) - (4*e^4*f*g + d^2*e^2*g^2)*p)*x^(2*n) - 12*(4*e^4*f^2*n
*p*log(x) + e^4*g^2*p*x^(4*n) + 4*e^4*f*g*p*x^(2*n) - (4*d^2*e^2*f*g + d^4*
g^2)*p)*log(e*x^n + d))/(e^4*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx^{2\,n}+f\right)^2\log\left((ex^n+d)^pc\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^(2*n) + f)^2*log((e*x^n + d)^p*c)/x, x)
```

maple [C] time = 3.79, size = 734, normalized size = 2.89

$$\frac{\left(4f^2n\ln(x)+4fg\,x^{2n}+g^2x^{4n}\right)\ln\left((e\,x^n+d)^p\right)}{4n}+\frac{g^2x^{4n}\ln(c)}{4n}+\frac{fg\,x^{2n}\ln(c)}{n}-\frac{g^2p\,x^{4n}}{16n}-\frac{f^2p\,\mathrm{dilog}\left(\frac{e\,x^n+d}{d}\right)}{n}+\frac{f^2\ln(c)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^(2*n))^2*ln(c*(e*x^n+d)^p)/x,x)
```

```
[Out] 1/4*(g^2*(x^n)^4+4*f^2*n*ln(x)+4*f*g*(x^n)^2)/n*ln((e*x^n+d)^p)-1/16*p/n*g^
2*(x^n)^4+1/4/n*ln(c)*(x^n)^4*g^2+f^2/n*ln(c)*ln(x^n)+1/n*ln(c)*(x^n)^2*f*g
-1/2*p/n*f*g*(x^n)^2+1/8*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2
*(x^n)^4*g^2-p/n*f^2*dilog((e*x^n+d)/d)-f^2*p*ln(x)*ln((e*x^n+d)/d)+1/2*I*P
i*f^2/n*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2*ln(x^n)-1/4*d^4*g^2*p*l
n(e*x^n+d)/e^4/n-1/2*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(
I*c)*(x^n)^2*f*g-1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3*(x^n)^2*f*g-1/2*I*Pi*f^
2/n*csgn(I*c)*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*ln(x^n)-1/2*I*Pi*f^
2/n*csgn(I*c*(e*x^n+d)^p)^3*ln(x^n)+1/2*I*Pi*f^2/n*csgn(I*c)*csgn(I*c*(e*x^
n+d)^p)^2*ln(x^n)+1/2*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2*(x
^n)^2*f*g-1/8*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)*(x
^n)^4*g^2+1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)*(x^n)^2*f*g+1/4*d^3*
g^2*p*x^n/e^3/n+d*f*g*p*x^n/e/n+1/12*p/e/n*g^2*(x^n)^3*d-1/8*p/e^2/n*g^2*(x
^n)^2*d^2-1/8*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3*(x^n)^4*g^2+1/8*I/n*Pi*csgn(I*
c*(e*x^n+d)^p)^2*csgn(I*c)*(x^n)^4*g^2-d^2*f*g*p*ln(e*x^n+d)/e^2/n
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{24\,e^4f^2n^2p\log(x)^2-4\,de^3g^2px^{3\,n}+3\left(e^4g^2p-4\,e^4g^2\log(c)\right)x^{4\,n}+6\left(4\,e^4fgp+d^2e^2g^2p-8\,e^4fg\log(c)\right)x^{2\,n}-1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/48*(24*e^4*f^2*n^2*p*log(x)^2 - 4*d*e^3*g^2*p*x^(3*n) + 3*(e^4*g^2*p - 4
*e^4*g^2*log(c))*x^(4*n) + 6*(4*e^4*f*g*p + d^2*e^2*g^2*p - 8*e^4*f*g*log(c
))*x^(2*n) - 12*(4*d*e^3*f*g*p + d^3*e*g^2*p)*x^n - 12*(4*e^4*f^2*n*log(x)
+ e^4*g^2*x^(4*n) + 4*e^4*f*g*x^(2*n))*log((e*x^n + d)^p) + 12*(4*d^2*e^2*f
*g*n*p + d^4*g^2*n*p - 4*e^4*f^2*n*log(c))*log(x))/(e^4*n) + integrate(1/4*
(4*d*e^4*f^2*n*p*log(x) + 4*d^3*e^2*f*g*p + d^5*g^2*p)/(e^5*x*x^n + d*e^4*x
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln \left(c\,{\left(d+e\,x^n\right)}^p\right)\,{\left(f+g\,x^{2\,n}\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g*x^(2*n))^2)/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)*(f + g*x^(2*n))^2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + g x^{2 n}\right)^{2} \log{\left(c \left(d + e x^{n}\right)^{p} \right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**(2*n))**2*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral((f + g*x**(2*n))**2*log(c*(d + e*x**n)**p)/x, x)
```

3.367 $\int \frac{\left(f+gx^n\right)^2 \log\left(c(d+ex^n)^p\right)}{x}\,dx$

Optimal. Leaf size=176

$$\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{2fg\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{en}+\frac{g^2x^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}-\frac{d^2g^2p\log\left(d+ex^n\right)}{2e^2n}+$$

```
[Out] -2*f*g*p*x^n/n+1/2*d*g^2*p*x^n/e/n-1/4*g^2*p*x^(2*n)/n-1/2*d^2*g^2*p*ln(d+e
*x^n)/e^2/n+1/2*g^2*x^(2*n)*ln(c*(d+e*x^n)^p)/n+2*f*g*(d+e*x^n)*ln(c*(d+e*x
^n)^p)/e/n+f^2*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+f^2*p*polylog(2,1+e*x^n/d)/
n
```

Rubi [A] time = 0.20, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2475, 43, 2416, 2389, 2295, 2394, 2315, 2395}

$$\frac{f^2p\,\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}+\frac{2fg\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{en}+\frac{g^2x^{2n}\log\left(c\left(d+ex\right.\right.}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-2*f*g*p*x^n)/n + (d*g^2*p*x^n)/(2*e*n) - (g^2*p*x^(2*n))/(4*n) - (d^2*g^2
*p*Log[d + e*x^n])/(2*e^2*n) + (g^2*x^(2*n)*Log[c*(d + e*x^n)^p])/(2*n) + (
2*f*g*(d + e*x^n)*Log[c*(d + e*x^n)^p])/(e*n) + (f^2*Log[-((e*x^n)/d)]*Log[
c*(d + e*x^n)^p])/n + (f^2*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\left(f+gx^n\right)^2 \log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{(f+gx)^2\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(2fg\log\left(c(d+ex)^p\right)+\frac{f^2\log(c(d+ex)^p)}{x}+g^2x\log\left(c(d+ex)^p\right)\right) d$$

$$= \frac{f^2\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} + \frac{(2fg)\,\text{Subst}\left(\int \log\left(c(d+ex)^p\right) dx,\right.}{n}$$

$$= \frac{g^2x^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{(2fg)\,\text{Subs}}{}$$

$$= -\frac{2fgpx^n}{n} + \frac{g^2x^{2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n} + \frac{2fg\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{en} +$$

$$= -\frac{2fgpx^n}{n} + \frac{dg^2px^n}{2en} - \frac{g^2px^{2n}}{4n} - \frac{d^2g^2p\log\left(d+ex^n\right)}{2e^2n} + \frac{g^2x^{2n}\log\left(c\left(d+\right.\right.}{2n}$$

Mathematica [A] time = 0.19, size = 124, normalized size = 0.70

$$\frac{2e\log\left(c\left(d+ex^n\right)^p\right)\left(2ef^2\log\left(-\frac{ex^n}{d}\right)+4dfg+egx^n\left(4f+gx^n\right)\right)-2d^2g^2p\log\left(d+ex^n\right)+4e^2f^2p\text{Li}_2\left(\frac{ex^n}{d}+1\right.}{4e^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-(e*g*p*x^n*(8*e*f - 2*d*g + e*g*x^n)) - 2*d^2*g^2*p*Log[d + e*x^n] + 2*e*
(4*d*f*g + e*g*x^n*(4*f + g*x^n) + 2*e*f^2*Log[-((e*x^n)/d)])*Log[c*(d + e*
x^n)^p] + 4*e^2*f^2*p*PolyLog[2, 1 + (e*x^n)/d])/(4*e^2*n)
```

fricas [A] time = 0.46, size = 192, normalized size = 1.09

$$-\frac{4\,e^2f^2np\log(x)\log\left(\frac{ex^n+d}{d}\right)-4\,e^2f^2n\log(c)\log(x)+4\,e^2f^2p\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+\left(e^2g^2p-2\,e^2g^2\log(c)\right)x^{2\,n}-2\left(\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/4*(4*e^2*f^2*n*p*log(x)*log((e*x^n + d)/d) - 4*e^2*f^2*n*log(c)*log(x) +
 4*e^2*f^2*p*dilog(-(e*x^n + d)/d + 1) + (e^2*g^2*p - 2*e^2*g^2*log(c))*x^(
2*n) - 2*(4*e^2*f*g*log(c) - (4*e^2*f*g - d*e*g^2)*p)*x^n - 2*(2*e^2*f^2*n*
p*log(x) + e^2*g^2*p*x^(2*n) + 4*e^2*f*g*p*x^n + (4*d*e*f*g - d^2*g^2)*p)*l
og(e*x^n + d))/(e^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(gx^n+f\right)^2\log\left(\left(ex^n+d\right)^pc\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^n + f)^2*log((e*x^n + d)^p*c)/x, x)
```

maple [C] time = 3.80, size = 665, normalized size = 3.78

$$-\frac{i\pi\,f^2\mathrm{csgn}\,(ic)\,\mathrm{csgn}\left(i\,(e\,x^n+d)^p\right)\mathrm{csgn}\left(ic\,(e\,x^n+d)^p\right)\ln\left(x^n\right)}{2n}+\frac{i\pi\,f^2\mathrm{csgn}\,(ic)\,\mathrm{csgn}\left(ic\,(e\,x^n+d)^p\right)^2\ln\left(x^n\right)}{2n}+\frac{i\pi\,f}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^n)^2*ln(c*(e*x^n+d)^p)/x,x)
```

```
[Out] 1/2*(2*f^2*n*ln(x)+g^2*(x^n)^2+4*f*g*x^n)/n*ln((e*x^n+d)^p)-I/n*Pi*csgn(I*(
e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)*x^n*f*g+1/2*I*Pi*f^2/n*csgn(I*c
)*csgn(I*c*(e*x^n+d)^p)^2*ln(x^n)+I/n*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)*
x^n*f*g+1/2*I*Pi*f^2/n*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2*ln(x^n)-
1/4*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)*(x^n)^2*g^2-
I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3*x^n*f*g-1/2*I*Pi*f^2/n*csgn(I*c)*csgn(I*(e*x
^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*ln(x^n)-1/2*I*Pi*f^2/n*csgn(I*c*(e*x^n+d)^p)
^3*ln(x^n)-1/4*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3*(x^n)^2*g^2+1/4*I/n*Pi*csgn(I
*c*(e*x^n+d)^p)^2*csgn(I*c)*(x^n)^2*g^2+I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c
*(e*x^n+d)^p)^2*x^n*f*g+1/4*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p
)^2*(x^n)^2*g^2+1/2/n*ln(c)*(x^n)^2*g^2+2/n*ln(c)*x^n*f*g+f^2/n*ln(c)*ln(x^
n)-1/4*p/n*g^2*(x^n)^2+1/2*d*g^2*p*x^n/e/n-1/2*d^2*g^2*p*ln(e*x^n+d)/e^2/n-
p/n*f^2*dilog((e*x^n+d)/d)-f^2*p*ln(x)*ln((e*x^n+d)/d)-2*f*g*p*x^n/n+2*p/e/
n*f*g*d*ln(e*x^n+d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,e^2f^2n^2p\log(x)^2+\left(e^2g^2p-2\,e^2g^2\log(c)\right)x^{2\,n}+2\left(4\,e^2fgp-deg^2p-4\,e^2fg\log(c)\right)x^n-2\left(2\,e^2f^2n\log(x)+e^2\right.}{4\,e^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(2*e^2*f^2*n^2*p*log(x)^2 + (e^2*g^2*p - 2*e^2*g^2*log(c))*x^(2*n) + 2
*(4*e^2*f*g*p - d*e*g^2*p - 4*e^2*f*g*log(c))*x^n - 2*(2*e^2*f^2*n*log(x) +
```

```
e^2*g^2*x^(2*n) + 4*e^2*f*g*x^n)*log((e*x^n + d)^p) - 2*(4*d*e*f*g*n*p - d
^2*g^2*n*p + 2*e^2*f^2*n*log(c))*log(x))/(e^2*n) + integrate(1/2*(2*d*e^2*f
^2*n*p*log(x) - 4*d^2*e*f*g*p + d^3*g^2*p)/(e^3*x*x^n + d*e^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,{\left(f+g\,x^n\right)}^2}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g*x^n)^2)/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)*(f + g*x^n)^2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + g x^{n}\right)^{2} \log{\left(c \left(d + e x^{n}\right)^{p} \right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**n)**2*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral((f + g*x**n)**2*log(c*(d + e*x**n)**p)/x, x)
```

3.368 $\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x} dx$

Optimal. Leaf size=193

$$\frac{f^2 \log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{2fgx^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{g^2x^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{e^2g^2p\log\left(d+ex^n\right)}{2d^2n}-\frac{e^2g^2}{}$$

```
[Out] -1/2*e*g^2*p/d/n/(x^n)+2*e*f*g*p*ln(x)/d-1/2*e^2*g^2*p*ln(x)/d^2-2*e*f*g*p*
ln(d+e*x^n)/d/n+1/2*e^2*g^2*p*ln(d+e*x^n)/d^2/n-1/2*g^2*ln(c*(d+e*x^n)^p)/n
/(x^(2*n))-2*f*g*ln(c*(d+e*x^n)^p)/n/(x^n)+f^2*ln(-e*x^n/d)*ln(c*(d+e*x^n)^
p)/n+f^2*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A] time = 0.24, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2475, 263, 43, 2416, 2395, 44, 36, 29, 31, 2394, 2315}

$$\frac{f^2 p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{2fgx^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{g^2x^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+$$

Antiderivative was successfully verified.

```
[In] Int[((f + g/x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -(e*g^2*p)/(2*d*n*x^n) + (2*e*f*g*p*Log[x])/d - (e^2*g^2*p*Log[x])/(2*d^2)
- (2*e*f*g*p*Log[d + e*x^n])/(d*n) + (e^2*g^2*p*Log[d + e*x^n])/(2*d^2*n) -
 (g^2*Log[c*(d + e*x^n)^p])/(2*n*x^(2*n)) - (2*f*g*Log[c*(d + e*x^n)^p])/(n
*x^n) + (f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f^2*p*PolyLog[2,
1 + (e*x^n)/d])/n
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(f+\frac{g}{x}\right)^2 \log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{g^2\log(c(d+ex)^p)}{x^3} + \frac{2fg\log(c(d+ex)^p)}{x^2} + \frac{f^2\log(c(d+ex)^p)}{x}\right)dx, x, x^n\right)}{n}$$
$$= \frac{f^2\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} + \frac{(2fg)\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^2}\,dx, x, x^n\right)}{n}$$
$$= -\frac{g^2x^{-2n}\log(c(d+ex^n)^p)}{2n} - \frac{2fgx^{-n}\log(c(d+ex^n)^p)}{n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\text{lo}}{n}$$
$$= -\frac{g^2x^{-2n}\log(c(d+ex^n)^p)}{2n} - \frac{2fgx^{-n}\log(c(d+ex^n)^p)}{n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\text{lo}}{n}$$
$$= -\frac{eg^2px^{-n}}{2dn} + \frac{2efgp\log(x)}{d} - \frac{e^2g^2p\log(x)}{2d^2} - \frac{2efgp\log(d+ex^n)}{dn} + \frac{e^2g^2p\,\text{l}}{}$$

Mathematica [A] time = 0.39, size = 150, normalized size = 0.78

$$-\frac{-2f^2\left(\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p) + p\text{Li}_2\left(\frac{ex^n}{d}+1\right)\right) + 4fgx^{-n}\log(c(d+ex^n)^p) + g^2x^{-2n}\log(c(d+ex^n)^p) +}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g/x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -1/2*((-4*e*f*g*p*(n*Log[x] - Log[d + e*x^n]))/d + (e*g^2*p*(d/x^n + e*n*Lo
g[x] - e*Log[d + e*x^n]))/d^2 + (g^2*Log[c*(d + e*x^n)^p])/x^(2*n) + (4*f*g
*Log[c*(d + e*x^n)^p])/x^n - 2*f^2*(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]
+ p*PolyLog[2, 1 + (e*x^n)/d]))/n
```

fricas [A] time = 0.48, size = 210, normalized size = 1.09

$$-\frac{2\,d^2f^2npx^{2\,n}\log(x)\log\left(\frac{ex^n+d}{d}\right) + 2\,d^2f^2px^{2\,n}\text{Li}_2\left(-\frac{ex^n+d}{d}+1\right) + d^2g^2\log(c) - \left(2\,d^2f^2n\log(c) + \left(4\,defg - e^2g^2\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*f^2*n*p*x^(2*n)*log(x)*log((e*x^n + d)/d) + 2*d^2*f^2*p*x^(2*n)
*dilog(-(e*x^n + d)/d + 1) + d^2*g^2*log(c) - (2*d^2*f^2*n*log(c) + (4*d*e*
f*g - e^2*g^2)*n*p)*x^(2*n)*log(x) + (d*e*g^2*p + 4*d^2*f*g*log(c))*x^n + (
4*d^2*f*g*p*x^n + d^2*g^2*p - (2*d^2*f^2*n*p*log(x) - (4*d*e*f*g - e^2*g^2)
*p)*x^(2*n))*log(e*x^n + d))/(d^2*n*x^(2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+\frac{g}{x^n}\right)^2 \log\left((ex^n+d)^p c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

[Out] integrate((f + g/x^n)^2*log((e*x^n + d)^p*c)/x, x)

maple [C] time = 3.43, size = 693, normalized size = 3.59

$$-\frac{i\pi f^2\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(e\,x^n+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^n+d\right)^p\right)\ln\left(x^n\right)}{2n}+\frac{i\pi f^2\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(e\,x^n+d\right)^p\right)^2\ln\left(x^n\right)}{2n}+$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f+g/(x^n))^2*ln(c*(e*x^n+d)^p)/x,x)

[Out] 1/2*(2*f^2*ln(x)*n*(x^n)^2-4*f*g*x^n-g^2)/n/(x^n)^2*ln((e*x^n+d)^p)-I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2*f*g/(x^n)+1/2*I*Pi*f^2/n*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2*ln(x^n)+I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3*f*g/(x^n)-1/4*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)*g^2/(x^n)^2+1/4*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)*g^2/(x^n)^2+I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)*f*g/(x^n)+1/4*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3*g^2/(x^n)^2-1/2*I*Pi*f^2/n*csgn(I*c*(e*x^n+d)^p)^3*ln(x^n)+1/2*I*Pi*f^2/n*csgn(I*c)*csgn(I*c*(e*x^n+d)^p)^2*ln(x^n)-I/n*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)*f*g/(x^n)-1/2*I*Pi*f^2/n*csgn(I*c)*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*ln(x^n)-1/4*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2*g^2/(x^n)^2+f^2/n*ln(c)*ln(x^n)-2/n*ln(c)*f*g/(x^n)-1/2/n*ln(c)*g^2/(x^n)^2+2*p*e/n*f*g/d*ln(x^n)-2*e*f*g*p*ln(e*x^n+d)/d/n-1/2*e*g^2*p/d/n/(x^n)-1/2*p*e^2/n*g^2/d^2*ln(x^n)+1/2*e^2*g^2*p*ln(e*x^n+d)/d^2/n-p/n*f^2*dilog((e*x^n+d)/d)-f^2*p*ln(x)*ln((e*x^n+d)/d)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{dg^2\log(c)+\left(df^2n^2p\log(x)^2-2\,df^2n\log(c)\log(x)\right)x^{2\,n}+\left(eg^2p+4\,dfg\log(c)\right)x^n-\left(2\,df^2nx^{2\,n}\log(x)-4\right.}{2\,dnx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")

[Out] -1/2*(d*g^2*log(c) + (d*f^2*n^2*p*log(x)^2 - 2*d*f^2*n*log(c)*log(x))*x^(2*n) + (e*g^2*p + 4*d*f*g*log(c))*x^n - (2*d*f^2*n*x^(2*n)*log(x) - 4*d*f*g*x^n - d*g^2)*log((e*x^n + d)^p))/(d*n*x^(2*n)) + integrate(1/2*(2*d^2*f^2*n*p*log(x) + 4*d*e*f*g*p - e^2*g^2*p)/(d*e*x*x^n + d^2*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,{\left(f+\frac{g}{x^n}\right)}^2}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((log(c*(d + e*x^n)^p)*(f + g/x^n)^2)/x,x)

[Out] int((log(c*(d + e*x^n)^p)*(f + g/x^n)^2)/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-2n}\left(fx^n+g\right)^2\log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f+g/(x**n))**2*ln(c*(d+e*x**n)**p)/x,x)

[Out] Integral(x**(-2*n)*(f*x**n + g)**2*log(c*(d + e*x**n)**p)/x, x)

3.369 $\int \frac{(f+gx^{-2n})^2 \log(c(d+ex^n)^p)}{x} dx$

Optimal. Leaf size=257

$$\frac{f^2 \log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{fgx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{g^2x^{-4n}\log\left(c\left(d+ex^n\right)^p\right)}{4n}+\frac{e^4g^2p\log\left(d+ex^n\right)}{4d^4n}-\frac{e^4g^2p}{4}$$

```
[Out] -1/12*e*g^2*p/d/n/(x^(3*n))+1/8*e^2*g^2*p/d^2/n/(x^(2*n))-e*f*g*p/d/n/(x^n)
-1/4*e^3*g^2*p/d^3/n/(x^n)-e^2*f*g*p*ln(x)/d^2-1/4*e^4*g^2*p*ln(x)/d^4+e^2*
f*g*p*ln(d+e*x^n)/d^2/n+1/4*e^4*g^2*p*ln(d+e*x^n)/d^4/n-1/4*g^2*ln(c*(d+e*x
^n)^p)/n/(x^(4*n))-f*g*ln(c*(d+e*x^n)^p)/n/(x^(2*n))+f^2*ln(-e*x^n/d)*ln(c*
(d+e*x^n)^p)/n+f^2*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A] time = 0.32, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2475, 263, 266, 43, 2416, 2395, 44, 2394, 2315}

$$\frac{f^2 p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{fgx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{g^2x^{-4n}\log\left(c\left(d+ex^n\right)^p\right)}{4n}+$$

Antiderivative was successfully verified.

```
[In] Int[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -(e*g^2*p)/(12*d*n*x^(3*n)) + (e^2*g^2*p)/(8*d^2*n*x^(2*n)) - (e*f*g*p)/(d*
n*x^n) - (e^3*g^2*p)/(4*d^3*n*x^n) - (e^2*f*g*p*Log[x])/d^2 - (e^4*g^2*p*Lo
g[x])/(4*d^4) + (e^2*f*g*p*Log[d + e*x^n])/(d^2*n) + (e^4*g^2*p*Log[d + e*x
^n])/(4*d^4*n) - (g^2*Log[c*(d + e*x^n)^p])/(4*n*x^(4*n)) - (f*g*Log[c*(d +
 e*x^n)^p])/(n*x^(2*n)) + (f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n +
(f^2*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\left(f+gx^{-2n}\right)^2 \log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(f+\frac{g}{x^2}\right)^2 \log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{g^2\log(c(d+ex)^p)}{x^5} + \frac{2fg\log(c(d+ex)^p)}{x^3} + \frac{f^2\log(c(d+ex)^p)}{x}\right) dx, x, x^n\right)}{n}$$

$$= \frac{f^2\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} + \frac{(2fg)\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^3}\,dx, x, x\right.}{n}$$

$$= -\frac{g^2x^{-4n}\log\left(c\left(d+ex^n\right)^p\right)}{4n} - \frac{fgx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right.}{}$$

$$= -\frac{g^2x^{-4n}\log\left(c\left(d+ex^n\right)^p\right)}{4n} - \frac{fgx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right.}{}$$

$$= -\frac{eg^2px^{-3n}}{12dn} + \frac{e^2g^2px^{-2n}}{8d^2n} - \frac{efgpx^{-n}}{dn} - \frac{e^3g^2px^{-n}}{4d^3n} - \frac{e^2fgp\log(x)}{d^2} - \frac{e^4g^2}{}$$

Mathematica [A] time = 0.58, size = 188, normalized size = 0.73

$$-\frac{-24f^2\left(\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)+p\text{Li}_2\left(\frac{ex^n}{d}+1\right)\right)+24fgx^{-2n}\log\left(c\left(d+ex^n\right)^p\right)+6g^2x^{-4n}\log\left(c\left(d+ex^n\right)^p\right)\cdots}{24n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -1/24*((24*e*f*g*p*(d/x^n + e*n*Log[x] - e*Log[d + e*x^n]))/d^2 + (e*g^2*p*
((d*(2*d^2 - 3*d*e*x^n + 6*e^2*x^(2*n)))/x^(3*n) + 6*e^3*n*Log[x] - 6*e^3*L
og[d + e*x^n]))/d^4 + (6*g^2*Log[c*(d + e*x^n)^p])/x^(4*n) + (24*f*g*Log[c*
(d + e*x^n)^p])/x^(2*n) - 24*f^2*(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] +
p*PolyLog[2, 1 + (e*x^n)/d]))/n
```

fricas [A] time = 0.47, size = 265, normalized size = 1.03

$$-\frac{24\,d^4f^2npx^{4\,n}\log(x)\log\left(\frac{ex^n+d}{d}\right)+24\,d^4f^2px^{4\,n}\text{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+2\,d^3eg^2px^n+6\,d^4g^2\log(c)+6\left(4\,d^3efg+de\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/24*(24*d^4*f^2*n*p*x^(4*n)*log(x)*log((e*x^n + d)/d) + 24*d^4*f^2*p*x^(4
*n)*dilog(-(e*x^n + d)/d + 1) + 2*d^3*e*g^2*p*x^n + 6*d^4*g^2*log(c) + 6*(4
*d^3*e*f*g + d*e^3*g^2)*p*x^(3*n) - 6*(4*d^4*f^2*n*log(c) - (4*d^2*e^2*f*g
+ e^4*g^2)*n*p)*x^(4*n)*log(x) - 3*(d^2*e^2*g^2*p - 8*d^4*f*g*log(c))*x^(2*
n) + 6*(4*d^4*f*g*p*x^(2*n) + d^4*g^2*p - (4*d^4*f^2*n*p*log(x) + (4*d^2*e^
2*f*g + e^4*g^2)*p)*x^(4*n))*log(e*x^n + d))/(d^4*n*x^(4*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+\frac{g}{x^{2\,n}}\right)^2\log\left(\left(ex^n+d\right)^pc\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((f + g/x^(2*n))^2*log((e*x^n + d)^p*c)/x, x)
```

maple [C] time = 3.64, size = 755, normalized size = 2.94

$$-\frac{e\,g^2p\,x^{-3n}}{12dn}-\frac{f^2p\,\text{dilog}\left(\frac{e\,x^n+d}{d}\right)}{n}+\frac{f^2\ln(c)\ln\left(x^n\right)}{n}+\frac{\left(4f^2n\,x^{4n}\ln(x)-4fg\,x^{2n}-g^2\right)x^{-4n}\ln\left(\left(e\,x^n+d\right)^p\right)}{4n}-\frac{g^2x^{-4n}\ln\cdots}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/(x^(2*n)))^2*ln(c*(e*x^n+d)^p)/x,x)
```

```
[Out] 1/8*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)*g^2/(x^n)^4+
f^2/n*ln(c)*ln(x^n)-1/12*p*e/n*g^2/d/(x^n)^3+1/8*p*e^2/n*g^2/d^2/(x^n)^2-1/
4*p*e^4/n*g^2/d^4*ln(x^n)+1/8*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3*g^2/(x^n)^4+1/
4*(4*f^2*ln(x)*n*(x^n)^4-4*f*g*(x^n)^2-g^2)/n/(x^n)^4*ln((e*x^n+d)^p)-1/4/n
*ln(c)*g^2/(x^n)^4-1/4*e^3*g^2*p/d^3/n/(x^n)-p/n*f^2*dilog((e*x^n+d)/d)-f^2
*p*ln(x)*ln((e*x^n+d)/d)-e*f*g*p/d/n/(x^n)+1/2*I*Pi*f^2/n*csgn(I*(e*x^n+d)^
p)*csgn(I*c*(e*x^n+d)^p)^2*ln(x^n)-1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(
I*c)*f*g/(x^n)^2-p*e^2/n*f*g/d^2*ln(x^n)-1/8*I/n*Pi*csgn(I*(e*x^n+d)^p)*csg
n(I*c*(e*x^n+d)^p)^2*g^2/(x^n)^4+1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3*f*g/(x^
```

```
n)^2-1/8*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)*g^2/(x^n)^4+1/4*e^4*g^2*p
*ln(e*x^n+d)/d^4/n-1/2*I*Pi*f^2/n*csgn(I*c)*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e
*x^n+d)^p)*ln(x^n)-1/2*I*Pi*f^2/n*csgn(I*c*(e*x^n+d)^p)^3*ln(x^n)+1/2*I*Pi*
f^2/n*csgn(I*c)*csgn(I*c*(e*x^n+d)^p)^2*ln(x^n)-1/2*I/n*Pi*csgn(I*(e*x^n+d)
^p)*csgn(I*c*(e*x^n+d)^p)^2*f*g/(x^n)^2+1/2*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn
(I*c*(e*x^n+d)^p)*csgn(I*c)*f*g/(x^n)^2-1/n*ln(c)*f*g/(x^n)^2+e^2*f*g*p*ln(
e*x^n+d)/d^2/n
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,d^2eg^2px^n+6\,d^3g^2\log(c)+12\left(d^3f^2n^2p\log(x)^2-2\,d^3f^2n\log(c)\log(x)\right)x^{4\,n}+6\left(4\,d^2efgp+e^3g^2p\right)x^{3\,n}-3}{24\,d^3nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/24*(2*d^2*e*g^2*p*x^n + 6*d^3*g^2*log(c) + 12*(d^3*f^2*n^2*p*log(x)^2 -
2*d^3*f^2*n*log(c)*log(x))*x^(4*n) + 6*(4*d^2*e*f*g*p + e^3*g^2*p)*x^(3*n)
- 3*(d*e^2*g^2*p - 8*d^3*f*g*log(c))*x^(2*n) - 6*(4*d^3*f^2*n*x^(4*n)*log(x
) - 4*d^3*f*g*x^(2*n) - d^3*g^2)*log((e*x^n + d)^p))/(d^3*n*x^(4*n)) + inte
grate(1/4*(4*d^4*f^2*n*p*log(x) - 4*d^2*e^2*f*g*p - e^4*g^2*p)/(d^3*e*x*x^n
 + d^4*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,{\left(f+\frac{g}{x^{2\,n}}\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g/x^(2*n))^2)/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)*(f + g/x^(2*n))^2)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x**(2*n)))**2*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Timed out
```

3.370 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx$

Optimal. Leaf size=266

$$-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x^n\right)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2fn}-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}\,x^n\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2fn}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn}-\frac{p\mathrm{Li}_2}{}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/f/n-1/2*ln(c*(d+e*x^n)^p)*ln(e*((-f)^(1/2)-x
^n*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/f/n-1/2*ln(c*(d+e*x^n)^p)*ln(e*((-f)^
(1/2)+x^n*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/f/n+p*polylog(2,1+e*x^n/d)/f/n
-1/2*p*polylog(2,-(d+e*x^n)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f/n-1/2*p*pol
ylog(2,(d+e*x^n)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f/n
```

Rubi [A] time = 0.42, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2475, 266, 36, 29, 31, 2416, 2394, 2315, 260, 2393, 2391}

$$-\frac{p\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}\left(d+ex^n\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2fn}-\frac{p\mathrm{PolyLog}\left(2,\frac{\sqrt{g}\left(d+ex^n\right)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2fn}+\frac{p\mathrm{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{fn}-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x^n\right)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2fn}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))),x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f*n) - (Log[c*(d + e*x^n)^p]*Log[
(e*(Sqrt[-f] - Sqrt[g]*x^n))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f*n) - (Log[c*(d
 + e*x^n)^p]*Log[(e*(Sqrt[-f] + Sqrt[g]*x^n))/(e*Sqrt[-f] - d*Sqrt[g])])/(2
*f*n) - (p*PolyLog[2, -((Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] - d*Sqrt[g]))])/(
2*f*n) - (p*PolyLog[2, (Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*
f*n) + (p*PolyLog[2, 1 + (e*x^n)/d])/(f*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log \left(c \left(d+e x^n\right)^p\right)}{x \left(f+g x^{2 n}\right)} \, dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x\left(f+gx^2\right)} \, dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{fx} - \frac{gx\log(c(d+ex)^p)}{f\left(f+gx^2\right)}\right) dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x} \, dx, x, x^n\right)}{fn} - \frac{g \,\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{f+gx^2} \, dx, x, x^n\right)}{fn}$$

$$= \frac{\log \left(-\frac{ex^n}{d}\right) \log \left(c \left(d+e x^n\right)^p\right)}{fn} - \frac{g\,\text{Subst}\left(\int \left(-\frac{\log(c(d+ex)^p)}{2\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\log(c(d+ex)^p)}{2\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}\right) dx, x, x\right.}{fn}$$

$$= \frac{\log \left(-\frac{ex^n}{d}\right) \log \left(c \left(d+e x^n\right)^p\right)}{fn} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn} + \frac{\sqrt{g}\ \text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{-f}-\sqrt{g}x} \, dx, x, x^n\right)}{2fn} -$$

$$= \frac{\log \left(-\frac{ex^n}{d}\right) \log \left(c \left(d+e x^n\right)^p\right)}{fn} - \frac{\log \left(c \left(d+e x^n\right)^p\right) \log \left(\frac{e\left(\sqrt{-f}-\sqrt{g}x^n\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fn} - \frac{\log \left(c \left(d+e x^n\right)\right.}{}$$

$$= \frac{\log \left(-\frac{ex^n}{d}\right) \log \left(c \left(d+e x^n\right)^p\right)}{fn} - \frac{\log \left(c \left(d+e x^n\right)^p\right) \log \left(\frac{e\left(\sqrt{-f}-\sqrt{g}x^n\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fn} - \frac{\log \left(c \left(d+e x^n\right)\right.}{}$$

$$= \frac{\log \left(-\frac{ex^n}{d}\right) \log \left(c \left(d+e x^n\right)^p\right)}{fn} - \frac{\log \left(c \left(d+e x^n\right)^p\right) \log \left(\frac{e\left(\sqrt{-f}-\sqrt{g}x^n\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fn} - \frac{\log \left(c \left(d+e x^n\right)\right.}{}$$

Mathematica [F] time = 5.13, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c \left(d+e x^n\right)^p\right)}{x \left(f+g x^{2 n}\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))), x]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log \left(\left(e x^n+d\right)^p c\right)}{g x x^{2 n}+f x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x, algorithm="fricas")
[Out] integral(log((e*x^n + d)^p*c)/(g*x*x^(2*n) + f*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(\left(e x^n+d\right)^p c\right)}{\left(g x^{2 n}+f\right) x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((g*x^(2*n) + f)*x), x)
```

maple [C] time = 0.59, size = 695, normalized size = 2.61

$$-\frac{i\pi \operatorname{csgn}(ic)\operatorname{csgn}\left(i(e\,x^n+d)^p\right)\operatorname{csgn}\left(ic(e\,x^n+d)^p\right)\ln(x^n)}{2fn}+\frac{i\pi \operatorname{csgn}(ic)\operatorname{csgn}\left(i(e\,x^n+d)^p\right)\operatorname{csgn}\left(ic(e\,x^n+\right.}{4fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)/x/(f+g*x^(2*n)),x)
```

```
[Out] -1/2/n*ln((e*x^n+d)^p)/f*ln(f+g*(x^n)^2)+1/n*ln((e*x^n+d)^p)/f*ln(x^n)-1/f*
p/n*dilog((e*x^n+d)/d)-1/n*p/f*ln(x^n)*ln((e*x^n+d)/d)+1/2/n*p/f*ln(e*x^n+d
)*ln(f+g*(x^n)^2)-1/2/n*p/f*ln(e*x^n+d)*ln(((-f*g)^(1/2)*e-(e*x^n+d)*g+d*g)
/(d*g+(-f*g)^(1/2)*e))-1/2/n*p/f*ln(e*x^n+d)*ln(((-f*g)^(1/2)*e+(e*x^n+d)*g
-d*g)/(-d*g+(-f*g)^(1/2)*e))-1/2/n*p/f*dilog(((-f*g)^(1/2)*e-(e*x^n+d)*g+d*
g)/(d*g+(-f*g)^(1/2)*e))-1/2/n*p/f*dilog(((-f*g)^(1/2)*e+(e*x^n+d)*g-d*g)/(
-d*g+(-f*g)^(1/2)*e))+1/4*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3/f*ln(f+g*(x^n)^2)-
1/4*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)/f*ln(f+g*(x^n)^2)+1/2*I/n*Pi*c
sgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2/f*ln(x^n)+1/2*I/n*Pi*csgn(I*c*(e
*x^n+d)^p)^2*csgn(I*c)/f*ln(x^n)-1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3/f*ln(x^
n)+1/4*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)/f*ln(f+g*
(x^n)^2)-1/4*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2/f*ln(f+g*(x
^n)^2)-1/2*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)/f*ln(
x^n)-1/2/n*ln(c)/f*ln(f+g*(x^n)^2)+1/n*ln(c)/f*ln(x^n)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left((ex^n+d)^p c\right)}{\left(gx^{2n}+f\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((g*x^(2*n) + f)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{x\,\left(f+g\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^(2*n))),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**(2*n)),x)
```

```
[Out] Timed out
```

3.371 $\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^n\right)}\,dx$

Optimal. Leaf size=121

$$-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right)}{fn}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn}-\frac{p\text{Li}_2\left(-\frac{g\left(ex^n+d\right)}{ef-dg}\right)}{fn}+\frac{p\text{Li}_2\left(\frac{ex^n}{d}+1\right)}{fn}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/f/n-ln(c*(d+e*x^n)^p)*ln(e*(f+g*x^n)/(-d*g+e
*f))/f/n-p*polylog(2,-g*(d+e*x^n)/(-d*g+e*f))/f/n+p*polylog(2,1+e*x^n/d)/f/
n
```

Rubi [A] time = 0.20, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2475, 36, 29, 31, 2416, 2394, 2315, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,-\frac{g\left(d+ex^n\right)}{ef-dg}\right)}{fn}+\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{fn}-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right)}{fn}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)),x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f*n) - (Log[c*(d + e*x^n)^p]*Log[
(e*(f + g*x^n))/(e*f - d*g)])/(f*n) - (p*PolyLog[2, -((g*(d + e*x^n))/(e*f
- d*g))])/(f*n) + (p*PolyLog[2, 1 + (e*x^n)/d])/(f*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^n\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x(f+gx)}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{fx} - \frac{g\log(c(d+ex)^p)}{f(f+gx)}\right)dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{fn} - \frac{g\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^n\right)}{fn}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right)}{fn} - \frac{(ep)\,\text{Subst}\left(\int \frac{\log}{d}\right.}{fn}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right)}{fn} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn} +$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{fn} - \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right)}{fn} - \frac{p\text{Li}_2\left(-\frac{g(d+ex^n)}{ef-dg}\right)}{fn}$$

Mathematica [A] time = 0.08, size = 92, normalized size = 0.76

$$\frac{\log\left(c\left(d+ex^n\right)^p\right)\left(\log\left(-\frac{ex^n}{d}\right)-\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right)\right)-p\text{Li}_2\left(\frac{g\left(ex^n+d\right)}{dg-ef}\right)+p\text{Li}_2\left(\frac{ex^n}{d}+1\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)),x]
```

```
[Out] (Log[c*(d + e*x^n)^p]*(Log[-((e*x^n)/d)] - Log[(e*(f + g*x^n))/(e*f - d*g)]
) - p*PolyLog[2, (g*(d + e*x^n))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^n)
/d])/(f*n)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left((ex^n+d)^p c\right)}{gxx^n+fx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/(g*x*x^n + f*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left((ex^n+d)^p c\right)}{\left(gx^n+f\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((g*x^n + f)*x), x)
```

maple [C] time = 0.59, size = 532, normalized size = 4.40

$$-\frac{i\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}\left(i(ex^n+d)^p\right)\mathrm{csgn}\left(ic(ex^n+d)^p\right)\ln(x^n)}{2fn}+\frac{i\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}\left(i(ex^n+d)^p\right)\mathrm{csgn}\left(ic(ex^n+d)^p\right.}{2fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)/x/(f+g*x^n),x)
```

```
[Out] -1/n*ln((e*x^n+d)^p)/f*ln(f+g*x^n)+1/f/n*ln(x^n)*ln((e*x^n+d)^p)-1/f*p/n*di
log((e*x^n+d)/d)-1/f/n*p*ln(x^n)*ln((e*x^n+d)/d)+1/n*p/f*dilog(((f+g*x^n)*e
+d*g-e*f)/(d*g-e*f))+1/n*p/f*ln(f+g*x^n)*ln(((f+g*x^n)*e+d*g-e*f)/(d*g-e*f)
)+1/2*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)/f*ln(f+g*x
^n)+1/2*I*Pi/f/n*csgn(I*c)*csgn(I*c*(e*x^n+d)^p)^2*ln(x^n)-1/2*I*Pi/f/n*csg
n(I*c*(e*x^n+d)^p)^3*ln(x^n)-1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)/f
*ln(f+g*x^n)+1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3/f*ln(f+g*x^n)-1/2*I/n*Pi*cs
gn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2/f*ln(f+g*x^n)-1/2*I*Pi/f/n*csgn(I
*c)*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*ln(x^n)+1/2*I*Pi/f/n*csgn(I*(
e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2*ln(x^n)-1/n*ln(c)/f*ln(f+g*x^n)+1/f/n*l
n(c)*ln(x^n)
```

maxima [A] time = 0.89, size = 154, normalized size = 1.27

$$-enp\left(\frac{\log(x^n)\log\left(\frac{ex^n}{d}+1\right)+\mathrm{Li}_2\left(-\frac{ex^n}{d}\right)}{efn^2}-\frac{\log\left(gx^n+f\right)\log\left(-\frac{egx^n+ef}{ef-dg}+1\right)+\mathrm{Li}_2\left(\frac{egx^n+ef}{ef-dg}\right)}{efn^2}\right)-\left(\frac{\log\left(gx^n+f\right)}{fn}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n),x, algorithm="maxima")
```

```
[Out] -e*n*p*((log(x^n)*log(e*x^n/d + 1) + dilog(-e*x^n/d))/(e*f*n^2) - (log(g*x^
n + f)*log(-(e*g*x^n + e*f)/(e*f - d*g) + 1) + dilog((e*g*x^n + e*f)/(e*f -
```

```
 d*g)))/(e*f*n^2)) - (log(g*x^n + f)/(f*n) - log(x^n)/(f*n))*log((e*x^n + d
)^p*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{x\,\left(f+g\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^n)),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**n),x)
```

```
[Out] Timed out
```

3.372 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx$

Optimal. Leaf size=70

$$\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(fx^n+g\right)}{df-eg}\right)}{fn}+\frac{p\text{Li}_2\left(\frac{f(ex^n+d)}{df-eg}\right)}{fn}$$

```
[Out] ln(c*(d+e*x^n)^p)*ln(-e*(g+f*x^n)/(d*f-e*g))/f/n+p*polylog(2,f*(d+e*x^n)/(d
*f-e*g))/f/n
```

Rubi [A] time = 0.16, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2475, 2412, 2394, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,\frac{f(d+ex^n)}{df-eg}\right)}{fn}+\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(fx^n+g\right)}{df-eg}\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)),x]
```

```
[Out] (Log[c*(d + e*x^n)^p]*Log[-((e*(g + f*x^n))/(d*f - e*g))])/(f*n) + (p*PolyL
og[2, (f*(d + e*x^n))/(d*f - e*g)])/(f*n)
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2412

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
```

```
|| IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-n}\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\left(f+\frac{g}{x}\right)x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{g+fx}\,dx, x, x^n\right)}{n}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{fn} - \frac{(ep)\,\text{Subst}\left(\int \frac{\log\left(\frac{e(g+fx)}{-df+eg}\right)}{d+ex}\,dx, x, x^n\right)}{fn}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{fn} - \frac{p\,\text{Subst}\left(\int \frac{\log\left(1+\frac{fx}{-df+eg}\right)}{x}\,dx, x, d+ex^n\right)}{fn}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{fn} + \frac{p\text{Li}_2\left(\frac{f(d+ex^n)}{df-eg}\right)}{fn}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 0.91

$$\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e(fx^n+g)}{eg-df}\right)+p\text{Li}_2\left(\frac{f(ex^n+d)}{df-eg}\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)),x]
```

```
[Out] (Log[c*(d + e*x^n)^p]*Log[(e*(g + f*x^n))/(-(d*f) + e*g)] + p*PolyLog[2, (f
*(d + e*x^n))/(d*f - e*g)])/(f*n)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^n\log\left(\left(ex^n+d\right)^p c\right)}{fxx^n+gx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x, algorithm="fricas")
```

```
[Out] integral(x^n*log((e*x^n + d)^p*c)/(f*x*x^n + g*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^n+d\right)^p c\right)}{\left(f+\frac{g}{x^n}\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((f + g/x^n)*x), x)
```

maple [C] time = 0.60, size = 298, normalized size = 4.26

$$-\frac{i\pi\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(e\,x^{n}+d\right)^{p}\right)\mathrm{csgn}\left(ic\left(e\,x^{n}+d\right)^{p}\right)\ln\left(f\,x^{n}+g\right)}{2fn}+\frac{i\pi\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(e\,x^{n}+d\right)^{p}\right)^{2}\ln\left(f\,x^{n}+g\right.}{2fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)/x/(f+g/(x^n)),x)
```

```
[Out] 1/n*ln(g+f*x^n)/f*ln((e*x^n+d)^p)-1/n/f*p*dilog(((g+f*x^n)*e+d*f-e*g)/(d*f-
e*g))-1/n/f*p*ln(g+f*x^n)*ln(((g+f*x^n)*e+d*f-e*g)/(d*f-e*g))+1/2*I/n*ln(g+
f*x^n)/f*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2-1/2*I/n*ln(g+f*x^n)
/f*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)-1/2*I/n*ln(g+f*x^
n)/f*Pi*csgn(I*c*(e*x^n+d)^p)^3+1/2*I/n*ln(g+f*x^n)/f*Pi*csgn(I*c*(e*x^n+d)
^p)^2*csgn(I*c)+1/n*ln(g+f*x^n)/f*ln(c)
```

maxima [A] time = 0.86, size = 112, normalized size = 1.60

$$\left(\frac{\log\left(f+\frac{g}{x^{n}}\right)}{fn}-\frac{\log\left(\frac{1}{x^{n}}\right)}{fn}\right)\log\left(\left(ex^{n}+d\right)^{p}c\right)-\frac{\left(\log\left(fx^{n}+g\right)\log\left(\frac{efx^{n}+eg}{df-eg}+1\right)+\mathrm{Li}_{2}\left(-\frac{efx^{n}+eg}{df-eg}\right)\right)p}{fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x, algorithm="maxima")
```

```
[Out] (log(f + g/x^n)/(f*n) - log(1/(x^n))/(f*n))*log((e*x^n + d)^p*c) - (log(f*x
^n + g)*log((e*f*x^n + e*g)/(d*f - e*g) + 1) + dilog(-(e*f*x^n + e*g)/(d*f
- e*g)))*p/(f*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^{n}\right)}^{p}\right)}{x\,\left(f+\frac{g}{x^{n}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^n)),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.373 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx$

Optimal. Leaf size=221

$$\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}\,x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn}+\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(\sqrt{-f}\,x^n+\sqrt{g}\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn}+\frac{p\mathrm{Li}_2\left(\frac{\sqrt{-f}\,(ex^n+d)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn}+\frac{p\mathrm{Li}_2\left(\frac{\sqrt{-f}\,(e\cdots}{\sqrt{-f}\,d\cdots}\right)}{2fn}$$

```
[Out] 1/2*ln(c*(d+e*x^n)^p)*ln(-e*(x^n*(-f)^(1/2)+g^(1/2))/(d*(-f)^(1/2)-e*g^(1/2
)))/f/n+1/2*ln(c*(d+e*x^n)^p)*ln(e*(-x^n*(-f)^(1/2)+g^(1/2))/(d*(-f)^(1/2)+
e*g^(1/2)))/f/n+1/2*p*polylog(2,(d+e*x^n)*(-f)^(1/2)/(d*(-f)^(1/2)-e*g^(1/2
)))/f/n+1/2*p*polylog(2,(d+e*x^n)*(-f)^(1/2)/(d*(-f)^(1/2)+e*g^(1/2)))/f/n
```

Rubi [A] time = 0.41, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2475, 263, 260, 2416, 2394, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-f}\,(d+ex^n)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-f}\,(d+ex^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn}+\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}\,x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn}+\frac{\log\left(c\left(d+ex^n\right)^{p}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))),x]
```

```
[Out] (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[g] - Sqrt[-f]*x^n))/(d*Sqrt[-f] + e*Sqrt
[g])])/(2*f*n) + (Log[c*(d + e*x^n)^p]*Log[-((e*(Sqrt[g] + Sqrt[-f]*x^n))/(
d*Sqrt[-f] - e*Sqrt[g]))])/(2*f*n) + (p*PolyLog[2, (Sqrt[-f]*(d + e*x^n))/(
d*Sqrt[-f] - e*Sqrt[g])])/(2*f*n) + (p*PolyLog[2, (Sqrt[-f]*(d + e*x^n))/(d
*Sqrt[-f] + e*Sqrt[g])])/(2*f*n)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\left(f+\frac{g}{x^2}\right)x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-f}\log(c(d+ex)^p)}{2f\left(\sqrt{g}-\sqrt{-f}\,x\right)}+\frac{\sqrt{-f}\log(c(d+ex)^p)}{2f\left(\sqrt{g}+\sqrt{-f}\,x\right)}\right)dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{g}-\sqrt{-f}\,x}\,dx, x, x^n\right)}{2\sqrt{-f}\,n} - \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{g}+\sqrt{-f}\,x}\,dx, x, x^n\right)}{2\sqrt{-f}\,n}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}\,x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn} + \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(\sqrt{g}+\sqrt{-f}\,x^n\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn} - \frac{(ep)\,\text{S}}{}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}\,x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn} + \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(\sqrt{g}+\sqrt{-f}\,x^n\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn} - \frac{p\,\text{Sub}}{}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}\,x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn} + \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(\sqrt{g}+\sqrt{-f}\,x^n\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn} + \frac{p\text{Li}_2\left(\right.}{}$$

Mathematica [F] time = 1.45, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))), x]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{2n}\log\left((ex^n+d)^p c\right)}{fxx^{2n}+gx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x, algorithm="fricas")
```

```
[Out] integral(x^(2*n)*log((e*x^n + d)^p*c)/(f*x*x^(2*n) + g*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left((ex^n+d)^p c\right)}{\left(f+\frac{g}{x^{2n}}\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((f + g/x^(2*n))*x), x)
```

maple [C] time = 0.67, size = 461, normalized size = 2.09

$$-\frac{i\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}\left(i(ex^n+d)^p\right)\mathrm{csgn}\left(ic(ex^n+d)^p\right)\ln\left(fx^{2n}+g\right)}{4fn}+\frac{i\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}\left(ic(ex^n+d)^p\right)^2\ln\left(fx^2\right.}{4fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)/x/(f+g/(x^(2*n))),x)
```

```
[Out] 1/2/n/f*ln(f*(x^n)^2+g)*ln((e*x^n+d)^p)-1/2/n/f*p*ln(e*x^n+d)*ln(f*(x^n)^2+
g)+1/2/n/f*p*ln(e*x^n+d)*ln(((-f*g)^(1/2)*e-(e*x^n+d)*f+d*f)/((-f*g)^(1/2)*
e+d*f))+1/2/n/f*p*ln(e*x^n+d)*ln(((-f*g)^(1/2)*e+(e*x^n+d)*f-d*f)/((-f*g)^(
1/2)*e-d*f))+1/2/n/f*p*dilog(((-f*g)^(1/2)*e-(e*x^n+d)*f+d*f)/((-f*g)^(1/2)
*e+d*f))+1/2/n/f*p*dilog(((-f*g)^(1/2)*e+(e*x^n+d)*f-d*f)/((-f*g)^(1/2)*e-d
*f))+1/4*I/n/f*ln(f*(x^n)^2+g)*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)
^2-1/4*I/n/f*ln(f*(x^n)^2+g)*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*c
sgn(I*c)-1/4*I/n/f*ln(f*(x^n)^2+g)*Pi*csgn(I*c*(e*x^n+d)^p)^3+1/4*I/n/f*ln(
f*(x^n)^2+g)*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)+1/2/n/f*ln(f*(x^n)^2+g)*l
n(c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left((ex^n+d)^p c\right)}{\left(f+\frac{g}{x^{2n}}\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((f + g/x^(2*n))*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{x\,\left(f+\frac{g}{x^{2n}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^(2*n))),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**(2*n))),x)
```

```
[Out] Timed out
```

3.374 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx$

Optimal. Leaf size=419

$$-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x^n\right)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2f^2n}-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}\,x^n\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{f^2n}+\cdots$$

```
[Out] -1/2*e^2*p*ln(d+e*x^n)/f/(d^2*g+e^2*f)/n+1/2*ln(c*(d+e*x^n)^p)/f/n/(f+g*x^(
2*n))+ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/f^2/n+1/4*e^2*p*ln(f+g*x^(2*n))/f/(d^2
*g+e^2*f)/n-1/2*ln(c*(d+e*x^n)^p)*ln(e*((-f)^(1/2)-x^n*g^(1/2))/(e*(-f)^(1/
2)+d*g^(1/2)))/f^2/n-1/2*ln(c*(d+e*x^n)^p)*ln(e*((-f)^(1/2)+x^n*g^(1/2))/(e
*(-f)^(1/2)-d*g^(1/2)))/f^2/n+p*polylog(2,1+e*x^n/d)/f^2/n-1/2*p*polylog(2,
-(d+e*x^n)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f^2/n-1/2*p*polylog(2,(d+e*x^n
)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f^2/n-1/2*d*e*p*arctan(x^n*g^(1/2)/f^(1
/2))*g^(1/2)/f^(3/2)/(d^2*g+e^2*f)/n
```

Rubi [A] time = 0.57, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.518, Rules used = {2475, 266, 44, 2416, 2394, 2315, 2413, 706, 31, 635, 205, 260, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,-\frac{\sqrt{g}\left(d+ex^n\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2n}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{g}\left(d+ex^n\right)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2f^2n}+\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{f^2n}-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{-f}\cdots\right.}{d\sqrt{g}\cdots}\right.}{2f^2n}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))^2),x]
```

```
[Out] -(d*e*Sqrt[g]*p*ArcTan[(Sqrt[g]*x^n)/Sqrt[f]])/(2*f^(3/2)*(e^2*f + d^2*g)*n
) - (e^2*p*Log[d + e*x^n])/(2*f*(e^2*f + d^2*g)*n) + Log[c*(d + e*x^n)^p]/(
2*f*n*(f + g*x^(2*n))) + (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f^2*n) -
 (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[-f] - Sqrt[g]*x^n))/(e*Sqrt[-f] + d*Sqr
t[g])])/(2*f^2*n) - (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[-f] + Sqrt[g]*x^n))/
(e*Sqrt[-f] - d*Sqrt[g])])/(2*f^2*n) + (e^2*p*Log[f + g*x^(2*n)])/(4*f*(e^2
*f + d^2*g)*n) - (p*PolyLog[2, -((Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] - d*Sqrt
[g]))])/(2*f^2*n) - (p*PolyLog[2, (Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] + d*Sqr
t[g])])/(2*f^2*n) + (p*PolyLog[2, 1 + (e*x^n)/d])/(f^2*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[((f + g*x^r)^(q + 1)*(a
+ b*Log[c*(d + e*x)^n])^p)/(g*r*(q + 1)), x] - Dist[(b*e*n*p)/(g*r*(q + 1))
, Int[((f + g*x^r)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x
], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && Ne
Q[q, -1] && IGtQ[p, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x\left(f+gx^2\right)^2}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{f^2x} - \frac{gx\log(c(d+ex)^p)}{f\left(f+gx^2\right)^2} - \frac{gx\log(c(d+ex)^p)}{f^2\left(f+gx^2\right)}\right) dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{f^2n} - \frac{g\,\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{f+gx^2}\,dx, x, x^n\right)}{f^2n} - \frac{g\,\text{Subst}\left(\int \frac{x\,\mathrm{l}}{}\right.}{}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)}{2fn\left(f+gx^{2n}\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{f^2n} - \frac{g\,\text{Subst}\left(\int\left(-\frac{\log(c(d+ex)^p)}{2\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \right.\right.}{f^2n}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)}{2fn\left(f+gx^{2n}\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{f^2n} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{f^2n} + \frac{\sqrt{g}\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{e^2p\log\left(d+ex^n\right)}{2f\left(e^2f+d^2g\right)n} + \frac{\log\left(c\left(d+ex^n\right)^p\right)}{2fn\left(f+gx^{2n}\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{f^2n} - \frac{\log\left(c\left(d\right.\right.}{}$$

$$= -\frac{de\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{g}x^n}{\sqrt{f}}\right)}{2f^{3/2}\left(e^2f+d^2g\right)n} - \frac{e^2p\log\left(d+ex^n\right)}{2f\left(e^2f+d^2g\right)n} + \frac{\log\left(c\left(d+ex^n\right)^p\right)}{2fn\left(f+gx^{2n}\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\right.}{f^2n}$$

$$= -\frac{de\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{g}x^n}{\sqrt{f}}\right)}{2f^{3/2}\left(e^2f+d^2g\right)n} - \frac{e^2p\log\left(d+ex^n\right)}{2f\left(e^2f+d^2g\right)n} + \frac{\log\left(c\left(d+ex^n\right)^p\right)}{2fn\left(f+gx^{2n}\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\right.}{f^2n}$$

Mathematica [F] time = 8.11, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))^2),x]
```

```
[Out] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))^2), x]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left((ex^n + d)^p c\right)}{g^2 x x^{4n} + 2 f g x x^{2n} + f^2 x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/(g^2*x*x^(4*n) + 2*f*g*x*x^(2*n) + f^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left((ex^n + d)^p c\right)}{\left(gx^{2n} + f\right)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((g*x^(2*n) + f)^2*x), x)
```

maple [C] time = 0.60, size = 1036, normalized size = 2.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)/x/(g*x^(2*n)+f)^2,x)
```

```
[Out] -1/2/n*p*e/f*g/(d^2*g+e^2*f)*d/(f*g)^(1/2)*arctan(x^n*g/(f*g)^(1/2))+1/n*ln
((e*x^n+d)^p)/f^2*ln(x^n)+1/2/n*ln((e*x^n+d)^p)/f/(f+g*(x^n)^2)+1/4*I/n*Pi*
csgn(I*c*(e*x^n+d)^p)^3/f^2*ln(f+g*(x^n)^2)-1/2/n*ln((e*x^n+d)^p)/f^2*ln(f+
g*(x^n)^2)-1/2*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)/f
^2*ln(x^n)-1/4*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)/f
/(f+g*(x^n)^2)+1/4*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*
c)/f^2*ln(f+g*(x^n)^2)+1/n*ln(c)/f^2*ln(x^n)-1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^
p)^3/f^2*ln(x^n)+1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)/f^2*ln(x^n)-1
/n*p/f^2*dilog((e*x^n+d)/d)-1/2/n*p/f^2*dilog((d*g+(-f*g)^(1/2)*e-(e*x^n+d)
*g)/(d*g+(-f*g)^(1/2)*e))-1/2/n*p/f^2*dilog((-d*g+(-f*g)^(1/2)*e+(e*x^n+d)*
g)/(-d*g+(-f*g)^(1/2)*e))-1/4*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3/f/(f+g*(x^n)^2
)+1/2/n*ln(c)/f/(f+g*(x^n)^2)-1/2/n*ln(c)/f^2*ln(f+g*(x^n)^2)-1/4*I/n*Pi*cs
gn(I*c*(e*x^n+d)^p)^2*csgn(I*c)/f^2*ln(f+g*(x^n)^2)+1/4/n*p*e^2/f/(d^2*g+e^
2*f)*ln(f+g*(x^n)^2)-1/n*p/f^2*ln(x^n)*ln((e*x^n+d)/d)+1/2/n*p/f^2*ln(e*x^n
+d)*ln(f+g*(x^n)^2)-1/2/n*p/f^2*ln(e*x^n+d)*ln((d*g+(-f*g)^(1/2)*e-(e*x^n+d
)*g)/(d*g+(-f*g)^(1/2)*e))-1/2/n*p/f^2*ln(e*x^n+d)*ln((-d*g+(-f*g)^(1/2)*e+
(e*x^n+d)*g)/(-d*g+(-f*g)^(1/2)*e))+1/4*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn
(I*c)/f/(f+g*(x^n)^2)-1/2*e^2*p*ln(e*x^n+d)/f/(d^2*g+e^2*f)/n-1/4*I/n*Pi*cs
gn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2/f^2*ln(f+g*(x^n)^2)+1/2*I/n*Pi*cs
gn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2/f^2*ln(x^n)+1/4*I/n*Pi*csgn(I*(e*
x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2/f/(f+g*(x^n)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left((ex^n + d)^p c\right)}{\left(gx^{2n} + f\right)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x, algorithm="maxima")
[Out] integrate(log((e*x^n + d)^p*c)/((g*x^(2*n) + f)^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)}{x\,{\left(f+g\,x^{2\,n}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^(2*n))^2),x)
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^(2*n))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**(2*n))**2,x)
[Out] Timed out
```

3.375 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)^2}\,dx$

Optimal. Leaf size=204

$$-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right)}{f^2n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{f^2n}+\frac{\log\left(c\left(d+ex^n\right)^p\right)}{fn\left(f+gx^n\right)}-\frac{p\text{Li}_2\left(-\frac{g(ex^n+d)}{ef-dg}\right)}{f^2n}+\frac{p\text{Li}_2\left(\frac{ex^n}{d}\right.}{f^2n}$$

```
[Out] -e*p*ln(d+e*x^n)/f/(-d*g+e*f)/n+ln(c*(d+e*x^n)^p)/f/n/(f+g*x^n)+ln(-e*x^n/d
)*ln(c*(d+e*x^n)^p)/f^2/n+e*p*ln(f+g*x^n)/f/(-d*g+e*f)/n-ln(c*(d+e*x^n)^p)*
ln(e*(f+g*x^n)/(-d*g+e*f))/f^2/n-p*polylog(2,-g*(d+e*x^n)/(-d*g+e*f))/f^2/n
+p*polylog(2,1+e*x^n/d)/f^2/n
```

Rubi [A] time = 0.27, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2475, 44, 2416, 2394, 2315, 2395, 36, 31, 2393, 2391}

$$-\frac{p\text{PolyLog}\left(2,-\frac{g(d+ex^n)}{ef-dg}\right)}{f^2n}+\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{f^2n}-\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right)}{f^2n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right.}{f^2n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)^2),x]
```

```
[Out] -((e*p*Log[d + e*x^n])/(f*(e*f - d*g)*n)) + Log[c*(d + e*x^n)^p]/(f*n*(f +
g*x^n)) + (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f^2*n) + (e*p*Log[f + g
*x^n])/(f*(e*f - d*g)*n) - (Log[c*(d + e*x^n)^p]*Log[(e*(f + g*x^n))/(e*f -
 d*g)])/(f^2*n) - (p*PolyLog[2, -((g*(d + e*x^n))/(e*f - d*g))])/(f^2*n) +
(p*PolyLog[2, 1 + (e*x^n)/d])/(f^2*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^n\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x(f+gx)^2}\,dx,x,x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{f^2x} - \frac{g\log(c(d+ex)^p)}{f(f+gx)^2} - \frac{g\log(c(d+ex)^p)}{f^2(f+gx)}\right)dx,x,x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{f^2n} - \frac{g\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx,x,x^n\right)}{f^2n} - \frac{g\,\text{Subst}\left(\int \frac{\log(c(d+}{(f+g}\right.}{fn}$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)}{fn\left(f+gx^n\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{f^2n} - \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right)}{f^2n} -$$

$$= \frac{\log\left(c\left(d+ex^n\right)^p\right)}{fn\left(f+gx^n\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{f^2n} - \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right)}{f^2n} +$$

$$= -\frac{ep\log\left(d+ex^n\right)}{f(ef-dg)n} + \frac{\log\left(c\left(d+ex^n\right)^p\right)}{fn\left(f+gx^n\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{f^2n} + \frac{ep\log\left(f+g\right.}{f(ef-dg)n}$$

Mathematica [A] time = 0.16, size = 171, normalized size = 0.84

$$\frac{\frac{f\log\left(c\left(d+ex^n\right)^p\right)}{f+gx^n} - \log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(f+gx^n\right)}{ef-dg}\right) + \log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right) - p\text{Li}_2\left(\frac{g\left(ex^n+d\right)}{dg-ef}\right) - \frac{efp\log\left(d+ex^n\right)}{ef-dg} +}{f^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)^2),x]
```

```
[Out] (-((e*f*p*Log[d + e*x^n])/(e*f - d*g)) + (f*Log[c*(d + e*x^n)^p])/(f + g*x^
n) + Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + (e*f*p*Log[f + g*x^n])/(e*f -
 d*g) - Log[c*(d + e*x^n)^p]*Log[(e*(f + g*x^n))/(e*f - d*g)] - p*PolyLog[2
, (g*(d + e*x^n))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^n)/d])/(f^2*n)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^n+d\right)^pc\right)}{g^2xx^{2n}+2fgxx^n+f^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/(g^2*x*x^(2*n) + 2*f*g*x*x^n + f^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^n+d\right)^pc\right)}{\left(gx^n+f\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((g*x^n + f)^2*x), x)
```

maple [C] time = 0.71, size = 805, normalized size = 3.95

$$\frac{ep\ln\left(e\,x^{n}+d\right)}{\left(dg-ef\right)fn}-\frac{ep\ln\left(g\,x^{n}+f\right)}{\left(dg-ef\right)fn}-\frac{i\pi\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(e\,x^{n}+d\right)^{p}\right)\mathrm{csgn}\left(ic\left(e\,x^{n}+d\right)^{p}\right)}{2\left(g\,x^{n}+f\right)fn}+\frac{i\pi\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ic\left(e\right.\right.}{2\left(g\,x^{n}+f\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)/x/(g*x^n+f)^2,x)
```

```
[Out] 1/n*ln((e*x^n+d)^p)/f/(g*x^n+f)-1/n*ln((e*x^n+d)^p)/f^2*ln(g*x^n+f)+1/f^2/n
*ln(x^n)*ln((e*x^n+d)^p)-1/n*p*e/f/(d*g-e*f)*ln(g*x^n+f)+1/n*p*e/f/(d*g-e*f
)*ln(e*x^n+d)-1/n*p/f^2*dilog((e*x^n+d)/d)-1/f^2/n*p*ln(x^n)*ln((e*x^n+d)/d
)+1/n*p/f^2*dilog((d*g-e*f+(g*x^n+f)*e)/(d*g-e*f))+1/n*p/f^2*ln(g*x^n+f)*ln
((d*g-e*f+(g*x^n+f)*e)/(d*g-e*f))-1/2*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(
e*x^n+d)^p)*csgn(I*c)/f/(g*x^n+f)+1/2*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(
e*x^n+d)^p)*csgn(I*c)/f^2*ln(g*x^n+f)+1/2*I*Pi/f^2/n*csgn(I*c)*csgn(I*c*(e*
x^n+d)^p)^2*ln(x^n)-1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3/f/(g*x^n+f)+1/2*I*Pi
/f^2/n*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2*ln(x^n)+1/2*I/n*Pi*csgn(
I*c*(e*x^n+d)^p)^3/f^2*ln(g*x^n+f)+1/2*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*
(e*x^n+d)^p)^2/f/(g*x^n+f)-1/2*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d
)^p)^2/f^2*ln(g*x^n+f)-1/2*I*Pi/f^2/n*csgn(I*c)*csgn(I*(e*x^n+d)^p)*csgn(I*
c*(e*x^n+d)^p)*ln(x^n)+1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)/f/(g*x^
n+f)-1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)/f^2*ln(g*x^n+f)-1/2*I*Pi/
f^2/n*csgn(I*c*(e*x^n+d)^p)^3*ln(x^n)+1/n*ln(c)/f/(g*x^n+f)-1/n*ln(c)/f^2*l
n(g*x^n+f)+1/f^2/n*ln(c)*ln(x^n)
```

maxima [A] time = 0.90, size = 233, normalized size = 1.14

$$-enp\left(\frac{\log\left(\frac{ex^{n}+d}{e}\right)}{ef^{2}n^{2}-dfgn^{2}}-\frac{\log\left(\frac{gx^{n}+f}{g}\right)}{ef^{2}n^{2}-dfgn^{2}}+\frac{\log\left(x^{n}\right)\log\left(\frac{ex^{n}}{d}+1\right)+\mathrm{Li}_{2}\left(-\frac{ex^{n}}{d}\right)}{ef^{2}n^{2}}-\frac{\log\left(gx^{n}+f\right)\log\left(-\frac{egx^{n}+ef}{ef-dg}+\right.}{ef^{2}n^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x, algorithm="maxima")
```

```
[Out] -e*n*p*(log((e*x^n + d)/e)/(e*f^2*n^2 - d*f*g*n^2) - log((g*x^n + f)/g)/(e*
f^2*n^2 - d*f*g*n^2) + (log(x^n)*log(e*x^n/d + 1) + dilog(-e*x^n/d))/(e*f^2
*n^2) - (log(g*x^n + f)*log(-(e*g*x^n + e*f)/(e*f - d*g) + 1) + dilog((e*g*
x^n + e*f)/(e*f - d*g)))/(e*f^2*n^2)) + (1/(f*g*n*x^n + f^2*n) - log(g*x^n
+ f)/(f^2*n) + log(x^n)/(f^2*n))*log((e*x^n + d)^p*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(d+e\,x^{n}\right)}^{p}\right)}{x\,{\left(f+g\,x^{n}\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^n)^2),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^n)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**n)**2,x)
```

```
[Out] Timed out
```

3.376 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)^2}\,dx$

Optimal. Leaf size=156

$$\frac{g\log\left(c\left(d+ex^n\right)^p\right)}{f^2n\left(fx^n+g\right)}+\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(fx^n+g\right)}{df-eg}\right)}{f^2n}+\frac{p\text{Li}_2\left(\frac{f(ex^n+d)}{df-eg}\right)}{f^2n}+\frac{egp\log\left(d+ex^n\right)}{f^2n(df-eg)}-\frac{egp\log\left(fx^n+g\right)}{f^2n(df-eg)}$$

```
[Out] e*g*p*ln(d+e*x^n)/f^2/(d*f-e*g)/n+g*ln(c*(d+e*x^n)^p)/f^2/n/(g+f*x^n)-e*g*p
*ln(g+f*x^n)/f^2/(d*f-e*g)/n+ln(c*(d+e*x^n)^p)*ln(-e*(g+f*x^n)/(d*f-e*g))/f
^2/n+p*polylog(2,f*(d+e*x^n)/(d*f-e*g))/f^2/n
```

Rubi [A] time = 0.28, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2475, 263, 43, 2416, 2395, 36, 31, 2394, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,\frac{f(d+ex^n)}{df-eg}\right)}{f^2n}+\frac{g\log\left(c\left(d+ex^n\right)^p\right)}{f^2n\left(fx^n+g\right)}+\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(fx^n+g\right)}{df-eg}\right)}{f^2n}+\frac{egp\log\left(d+ex^n\right)}{f^2n(df-eg)}-\frac{egp\log(f}{f^2n(df}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)^2),x]
```

```
[Out] (e*g*p*Log[d + e*x^n])/(f^2*(d*f - e*g)*n) + (g*Log[c*(d + e*x^n)^p])/(f^2*
n*(g + f*x^n)) - (e*g*p*Log[g + f*x^n])/(f^2*(d*f - e*g)*n) + (Log[c*(d + e
*x^n)^p]*Log[-((e*(g + f*x^n))/(d*f - e*g))])/(f^2*n) + (p*PolyLog[2, (f*(d
 + e*x^n))/(d*f - e*g)])/(f^2*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\left(f+\frac{g}{x}\right)^2 x}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{g\log(c(d+ex)^p)}{f(g+fx)^2} + \frac{\log(c(d+ex)^p)}{f(g+fx)}\right)dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{g+fx}\,dx, x, x^n\right)}{fn} - \frac{g\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{(g+fx)^2}\,dx, x, x^n\right)}{fn}$$
$$= \frac{g\log\left(c(d+ex^n)^p\right)}{f^2n\left(g+fx^n\right)} + \frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e\left(g+fx^n\right)}{df-eg}\right)}{f^2n} - \frac{(ep)\,\text{Subst}\left(\int \frac{\log\left(\frac{e(g+fx)}{-df+eg}\right)}{d+ex}\right.}{f^2n}$$
$$= \frac{g\log\left(c(d+ex^n)^p\right)}{f^2n\left(g+fx^n\right)} + \frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e\left(g+fx^n\right)}{df-eg}\right)}{f^2n} - \frac{p\,\text{Subst}\left(\int \frac{\log\left(1+\frac{fx}{-df+eg}\right)}{x}\right.}{f^2n}$$
$$= \frac{egp\log(d+ex^n)}{f^2(df-eg)n} + \frac{g\log\left(c(d+ex^n)^p\right)}{f^2n\left(g+fx^n\right)} - \frac{egp\log\left(g+fx^n\right)}{f^2(df-eg)n} + \frac{\log\left(c(d+ex^n)^p\right)\log}{f^2n}$$

Mathematica [B] time = 1.51, size = 433, normalized size = 2.78

$$\frac{g\log\left(f-fx^{-n}\right)\log\left(c(d+ex^n)^p\right) - fx^n\log\left(c(d+ex^n)^p\right) + fx^n\log\left(f-fx^{-n}\right)\log\left(c(d+ex^n)^p\right) - p\log(dx}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)^2),x]
```

```
[Out] (g*p*Log[f - f/x^n] + f*p*x^n*Log[f - f/x^n] - g*n*p*Log[x]*Log[f - f/x^n]
- f*n*p*x^n*Log[x]*Log[f - f/x^n] - p*Log[e + d/x^n]*(-(f*x^n) + (g + f*x^n
)*Log[f - f/x^n]) - f*x^n*Log[c*(d + e*x^n)^p] + g*Log[f - f/x^n]*Log[c*(d
+ e*x^n)^p] + f*x^n*Log[f - f/x^n]*Log[c*(d + e*x^n)^p] + g*n*p*Log[x]*Log[
1 + (f*x^n)/g] + f*n*p*x^n*Log[x]*Log[1 + (f*x^n)/g] + p*(g + f*x^n)*PolyLo
g[2, -((f*x^n)/g)])/(f^2*n*(g + f*x^n)) - (p*(-((d*f*Log[e + d/x^n])/(d*f -
 e*g)) + (f*x^n*Log[e + d/x^n])/(g + f*x^n) + Log[-(d/(e*x^n))]*Log[e + d/x
^n] + (d*f*Log[f + g/x^n])/(d*f - e*g) - Log[e + d/x^n]*Log[(d*(f + g/x^n))
/(d*f - e*g)] - PolyLog[2, -((g*(e + d/x^n))/(d*f - e*g))] + PolyLog[2, 1 +
 d/(e*x^n)]))/(f^2*n)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left((ex^n+d)^p c\right)}{f^2x + \frac{2fgxx^n}{x^{2n}} + \frac{g^2x}{x^{2n}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/(f^2*x + 2*f*g*x*x^n/x^(2*n) + g^2*x/x^(2*n))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left((ex^n+d)^p c\right)}{\left(f+\frac{g}{x^n}\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((f + g/x^n)^2*x), x)
```

maple [C] time = 0.62, size = 589, normalized size = 3.78

$$-\frac{egp\ln\left(f\,x^n+g\right)}{\left(df-eg\right)f^2n}+\frac{egp\ln\left(df-eg+\left(f\,x^n+g\right)e\right)}{\left(df-eg\right)f^2n}-\frac{i\pi g\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(e\,x^n+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^n+d\right)^p\right)}{2\left(f\,x^n+g\right)f^2n}+\frac{i\pi g\,\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)/x/(f+g/(x^n))^2,x)
```

```
[Out] 1/n*ln((e*x^n+d)^p)*g/f^2/(f*x^n+g)+1/n*ln((e*x^n+d)^p)/f^2*ln(f*x^n+g)-1/n
*p/f^2*dilog((d*f-e*g+(f*x^n+g)*e)/(d*f-e*g))-1/n*p/f^2*ln(f*x^n+g)*ln((d*f
-e*g+(f*x^n+g)*e)/(d*f-e*g))-e*g*p*ln(f*x^n+g)/f^2/(d*f-e*g)/n+1/n*p*e/f^2*
g/(d*f-e*g)*ln(d*f-e*g+(f*x^n+g)*e)+1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn
(I*c)/f^2*ln(f*x^n+g)+1/2*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^
2/f^2*ln(f*x^n+g)+1/2*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2*g/
f^2/(f*x^n+g)+1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)*g/f^2/(f*x^n+g)-
1/2*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3*g/f^2/(f*x^n+g)-1/2*I/n*Pi*csgn(I*(e*x^n
+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)/f^2*ln(f*x^n+g)-1/2*I/n*Pi*csgn(I*(e
*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)*g/f^2/(f*x^n+g)-1/2*I/n*Pi*csgn(
I*c*(e*x^n+d)^p)^3/f^2*ln(f*x^n+g)+1/n*ln(c)*g/f^2/(f*x^n+g)+1/n*ln(c)/f^2*
ln(f*x^n+g)
```

maxima [A] time = 0.88, size = 209, normalized size = 1.34

$$enp\left(\frac{d\log\left(\frac{ex^n+d}{e}\right)}{def^2n^2-e^2fgn^2}-\frac{g\log\left(\frac{fx^n+g}{f}\right)}{df^3n^2-ef^2gn^2}-\frac{\log\left(fx^n+g\right)\log\left(\frac{efx^n+eg}{df-eg}+1\right)+\mathrm{Li}_2\left(-\frac{efx^n+eg}{df-eg}\right)}{ef^2n^2}\right)-\left(\frac{1}{f^2n+\frac{fgn}{x^n}}-\frac{\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x, algorithm="maxima")
```

```
[Out] e*n*p*(d*log((e*x^n + d)/e)/(d*e*f^2*n^2 - e^2*f*g*n^2) - g*log((f*x^n + g)
/f)/(d*f^3*n^2 - e*f^2*g*n^2) - (log(f*x^n + g)*log((e*f*x^n + e*g)/(d*f -
e*g) + 1) + dilog(-(e*f*x^n + e*g)/(d*f - e*g)))/(e*f^2*n^2)) - (1/(f^2*n +
 f*g*n/x^n) - log(f + g/x^n)/(f^2*n) + log(1/(x^n))/(f^2*n))*log((e*x^n + d
)^p*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,\left(d+e\,x^n\right)^p\right)}{x\,\left(f+\frac{g}{x^n}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^n)^2),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^n)^2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**n))**2,x)
[Out] Timed out
```

3.377 $\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx$

Optimal. Leaf size=377

$$\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}\,x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n}+\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(-\frac{e\left(\sqrt{-f}\,x^n+\sqrt{g}\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f^2n}+\frac{g\log\left(c\left(d+ex^n\right)^p\right)}{2f^2n\left(fx^{2n}+g\right)}-\frac{de\sqrt{g}\,p\tan^{-1}}{2f^{3/2}n\left(d^2f\right.}$$

```
[Out] -1/2*e^2*g*p*ln(d+e*x^n)/f^2/(d^2*f+e^2*g)/n+1/2*g*ln(c*(d+e*x^n)^p)/f^2/n/
(g+f*x^(2*n))+1/4*e^2*g*p*ln(g+f*x^(2*n))/f^2/(d^2*f+e^2*g)/n+1/2*ln(c*(d+e
*x^n)^p)*ln(-e*(x^n*(-f)^(1/2)+g^(1/2))/(d*(-f)^(1/2)-e*g^(1/2)))/f^2/n+1/2
*ln(c*(d+e*x^n)^p)*ln(e*(-x^n*(-f)^(1/2)+g^(1/2))/(d*(-f)^(1/2)+e*g^(1/2)))
/f^2/n+1/2*p*polylog(2,(d+e*x^n)*(-f)^(1/2)/(d*(-f)^(1/2)-e*g^(1/2)))/f^2/n
+1/2*p*polylog(2,(d+e*x^n)*(-f)^(1/2)/(d*(-f)^(1/2)+e*g^(1/2)))/f^2/n-1/2*d
*e*p*arctan(x^n*f^(1/2)/g^(1/2))*g^(1/2)/f^(3/2)/(d^2*f+e^2*g)/n
```

Rubi [A] time = 0.61, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.518, Rules used = {2475, 263, 266, 43, 2416, 2413, 706, 31, 635, 205, 260, 2394, 2393, 2391}

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-f}\,(d+ex^n)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f^2n}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-f}\,(d+ex^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n}+\frac{g\log\left(c\left(d+ex^n\right)^p\right)}{2f^2n\left(fx^{2n}+g\right)}+\frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}\,x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))^2),x]
```

```
[Out] -(d*e*Sqrt[g]*p*ArcTan[(Sqrt[f]*x^n)/Sqrt[g]])/(2*f^(3/2)*(d^2*f + e^2*g)*n
) - (e^2*g*p*Log[d + e*x^n])/(2*f^2*(d^2*f + e^2*g)*n) + (g*Log[c*(d + e*x^
n)^p])/(2*f^2*n*(g + f*x^(2*n))) + (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[g] -
Sqrt[-f]*x^n))/(d*Sqrt[-f] + e*Sqrt[g])])/(2*f^2*n) + (Log[c*(d + e*x^n)^p]
*Log[-((e*(Sqrt[g] + Sqrt[-f]*x^n))/(d*Sqrt[-f] - e*Sqrt[g]))])/(2*f^2*n) +
 (e^2*g*p*Log[g + f*x^(2*n)])/(4*f^2*(d^2*f + e^2*g)*n) + (p*PolyLog[2, (Sq
rt[-f]*(d + e*x^n))/(d*Sqrt[-f] - e*Sqrt[g])])/(2*f^2*n) + (p*PolyLog[2, (S
qrt[-f]*(d + e*x^n))/(d*Sqrt[-f] + e*Sqrt[g])])/(2*f^2*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[((f + g*x^r)^(q + 1)*(a
+ b*Log[c*(d + e*x)^n])^p)/(g*r*(q + 1)), x] - Dist[(b*e*n*p)/(g*r*(q + 1))
, Int[((f + g*x^r)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x
], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && Ne
Q[q, -1] && IGtQ[p, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\left(f+\frac{g}{x^2}\right)^2 x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{gx\log(c(d+ex)^p)}{f\left(g+fx^2\right)^2}+\frac{x\log(c(d+ex)^p)}{f\left(g+fx^2\right)}\right)dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{g+fx^2}\,dx, x, x^n\right)}{fn} - \frac{g\,\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{\left(g+fx^2\right)^2}\,dx, x, x^n\right)}{fn}$$

$$= \frac{g\log\left(c\left(d+ex^n\right)^p\right)}{2f^2n\left(g+fx^{2n}\right)} + \frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-f}\log(c(d+ex)^p)}{2f\left(\sqrt{g}-\sqrt{-f}\,x\right)}+\frac{\sqrt{-f}\log(c(d+ex)^p)}{2f\left(\sqrt{g}+\sqrt{-f}\,x\right)}\right)dx, x, x^n\right)}{fn} - \frac{(egp}{}$$

$$= \frac{g\log\left(c\left(d+ex^n\right)^p\right)}{2f^2n\left(g+fx^{2n}\right)} - \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{g}-\sqrt{-f}\,x}\,dx, x, x^n\right)}{2(-f)^{3/2}n} + \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{g}+\sqrt{-f}\,x}\,dx, x, x^n\right)}{2(-f)^{3/2}n}$$

$$= -\frac{e^2gp\log\left(d+ex^n\right)}{2f^2\left(d^2f+e^2g\right)n} + \frac{g\log\left(c\left(d+ex^n\right)^p\right)}{2f^2n\left(g+fx^{2n}\right)} + \frac{\log\left(c\left(d+ex^n\right)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}\,x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n} + \frac{\text{lo}}{}$$

$$= -\frac{de\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{f}\,x^n}{\sqrt{g}}\right)}{2f^{3/2}\left(d^2f+e^2g\right)n} - \frac{e^2gp\log\left(d+ex^n\right)}{2f^2\left(d^2f+e^2g\right)n} + \frac{g\log\left(c\left(d+ex^n\right)^p\right)}{2f^2n\left(g+fx^{2n}\right)} + \frac{\log\left(c\left(d+ex^n\right)^p\right)}{2}$$

$$= -\frac{de\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{f}\,x^n}{\sqrt{g}}\right)}{2f^{3/2}\left(d^2f+e^2g\right)n} - \frac{e^2gp\log\left(d+ex^n\right)}{2f^2\left(d^2f+e^2g\right)n} + \frac{g\log\left(c\left(d+ex^n\right)^p\right)}{2f^2n\left(g+fx^{2n}\right)} + \frac{\log\left(c\left(d+ex^n\right)^p\right)}{2}$$

Mathematica [F] time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))^2),x]
```

```
[Out] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))^2), x]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log\left((ex^n+d)^p c\right)}{f^2x+\frac{2fgxx^{2n}}{x^{4n}}+\frac{g^2x}{x^{4n}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/(f^2*x + 2*f*g*x*x^(2*n)/x^(4*n) + g^2*x/x^(4
*n)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left((ex^n+d)^p c\right)}{\left(f+\frac{g}{x^{2n}}\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((f + g/x^(2*n))^2*x), x)
```

maple [C] time = 0.71, size = 810, normalized size = 2.15

$$-\frac{degp\arctan\left(\frac{f x^n}{\sqrt{fg}}\right)}{2\left(d^2f+e^2g\right)\sqrt{fg}\,fn}-\frac{e^2gp\ln\left(e x^n+d\right)}{2\left(d^2f+e^2g\right)f^2n}+\frac{e^2gp\ln\left(f x^{2n}+g\right)}{4\left(d^2f+e^2g\right)f^2n}-\frac{i\pi g\,\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(i\left(e x^n+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\right.\right.}{4\left(f x^{2n}+g\right)f^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)/x/(f+g/(x^(2*n)))^2,x)
```

```
[Out] 1/2/n*ln((e*x^n+d)^p)*g/f^2/(f*(x^n)^2+g)+1/2/n*ln((e*x^n+d)^p)/f^2*ln(f*(x
^n)^2+g)-1/2/n*p/f^2*ln(e*x^n+d)*ln(f*(x^n)^2+g)+1/2/n*p/f^2*ln(e*x^n+d)*ln
((d*f+(-f*g)^(1/2)*e-(e*x^n+d)*f)/(d*f+(-f*g)^(1/2)*e))+1/2/n*p/f^2*ln(e*x^
n+d)*ln((-d*f+(-f*g)^(1/2)*e+(e*x^n+d)*f)/(-d*f+(-f*g)^(1/2)*e))+1/2/n*p/f^
2*dilog((d*f+(-f*g)^(1/2)*e-(e*x^n+d)*f)/(d*f+(-f*g)^(1/2)*e))+1/2/n*p/f^2*
dilog((-d*f+(-f*g)^(1/2)*e+(e*x^n+d)*f)/(-d*f+(-f*g)^(1/2)*e))+1/4/n*p*e^2*
g/f^2/(d^2*f+e^2*g)*ln(f*(x^n)^2+g)-1/2/n*p*e*g/f/(d^2*f+e^2*g)*d/(f*g)^(1/
2)*arctan(x^n*f/(f*g)^(1/2))-1/2*e^2*g*p*ln(e*x^n+d)/f^2/(d^2*f+e^2*g)/n+1/
4*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)^2*g/f^2/(f*(x^n)^2+g)+1/
4*I/n*Pi*csgn(I*c*(e*x^n+d)^p)^2*csgn(I*c)*g/f^2/(f*(x^n)^2+g)-1/4*I/n*Pi*c
sgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)/f^2*ln(f*(x^n)^2+g)-1/4*
I/n*Pi*csgn(I*c*(e*x^n+d)^p)^3*g/f^2/(f*(x^n)^2+g)-1/4*I/n*Pi*csgn(I*(e*x^n
+d)^p)*csgn(I*c*(e*x^n+d)^p)*csgn(I*c)*g/f^2/(f*(x^n)^2+g)+1/4*I/n*Pi*csgn(
I*c*(e*x^n+d)^p)^2*csgn(I*c)/f^2*ln(f*(x^n)^2+g)-1/4*I/n*Pi*csgn(I*c*(e*x^n
+d)^p)^3/f^2*ln(f*(x^n)^2+g)+1/4*I/n*Pi*csgn(I*(e*x^n+d)^p)*csgn(I*c*(e*x^n
+d)^p)^2/f^2*ln(f*(x^n)^2+g)+1/2/n*ln(c)*g/f^2/(f*(x^n)^2+g)+1/2/n*ln(c)/f^
2*ln(f*(x^n)^2+g)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left((ex^n+d)^p c\right)}{\left(f+\frac{g}{x^{2n}}\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((f + g/x^(2*n))^2*x), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)}{x\,{\left(f+\frac{g}{x^{2\,n}}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^(2*n))^2),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^(2*n))^2), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**(2*n)))**2,x)
```

```
[Out] Timed out
```

3.378 $\int \frac{\log(c(d+ex^n))}{x(ce-(1-cd)x^{-n})}\,dx$

Optimal. Leaf size=25

$$-\frac{\text{Li}_2\left(1-c\left(ex^n+d\right)\right)}{cen}$$

```
[Out] -polylog(2,1-c*(d+e*x^n))/c/e/n
```

Rubi [A] time = 0.16, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2475, 2412, 2393, 2391}

$$-\frac{\text{PolyLog}\left(2,1-c\left(d+ex^n\right)\right)}{cen}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)]/(x*(c*e - (1 - c*d)/x^n)),x]
```

```
[Out] -(PolyLog[2, 1 - c*(d + e*x^n)]/(c*e*n))
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2412

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log (c (d+e x^n))}{x \left(c e-(1-c d) x^{-n}\right)} \, dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{\left(ce+\frac{-1+cd}{x}\right)x} \, dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{-1+cd+cex} \, dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x} \, dx, x, -1+cd+cex^n\right)}{cen}$$
$$= -\frac{\text{Li}_2(1-c(d+ex^n))}{cen}$$

Mathematica [A] time = 0.07, size = 26, normalized size = 1.04

$$-\frac{\text{Li}_2\left(-cex^n-cd+1\right)}{cen}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)]/(x*(c*e - (1 - c*d)/x^n)),x]
[Out] -(PolyLog[2, 1 - c*d - c*e*x^n]/(c*e*n))
```

fricas [A] time = 0.45, size = 25, normalized size = 1.00

$$-\frac{\text{Li}_2\left(-cex^n-cd+1\right)}{cen}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n))/x/(c*e+(c*d-1)/(x^n)),x, algorithm="fricas")
[Out] -dilog(-c*e*x^n - c*d + 1)/(c*e*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left((ex^n+d)c\right)}{\left(ce+\frac{cd-1}{x^n}\right)x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n))/x/(c*e+(c*d-1)/(x^n)),x, algorithm="giac")
[Out] integrate(log((e*x^n + d)*c)/((c*e + (c*d - 1)/x^n)*x), x)
```

maple [A] time = 0.09, size = 23, normalized size = 0.92

$$-\frac{\text{dilog}\left(ce\,x^n+cd\right)}{cen}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d))/x/(c*e+(c*d-1)/(x^n)),x)
[Out] -1/n*dilog(c*e*x^n+c*d)/c/e
```

maxima [B] time = 0.88, size = 106, normalized size = 4.24

$$\left(\frac{\log\left(ce+\frac{cd-1}{x^n}\right)}{cen}-\frac{\log\left(\frac{1}{x^n}\right)}{cen}\right)\log\left((ex^n+d)c\right)-\frac{\log\left(cex^n+cd\right)\log\left(cex^n+cd-1\right)+\text{Li}_2\left(-cex^n-cd+1\right)}{cen}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n))/x/(c*e+(c*d-1)/(x^n)),x, algorithm="maxima")
```

```
[Out] (log(c*e + (c*d - 1)/x^n)/(c*e*n) - log(1/(x^n))/(c*e*n))*log((e*x^n + d)*c
) - (log(c*e*x^n + c*d)*log(c*e*x^n + c*d - 1) + dilog(-c*e*x^n - c*d + 1))
/(c*e*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\,\left(d+e\,x^n\right)\right)}{x\,\left(c\,e+\frac{c\,d-1}{x^n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n))/(x*(c*e + (c*d - 1)/x^n)),x)
```

```
[Out] int(log(c*(d + e*x^n))/(x*(c*e + (c*d - 1)/x^n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n))/x/(c*e+(c*d-1)/(x**n)),x)
```

```
[Out] Exception raised: TypeError
```

3.379 $\int \frac{x^{-1+n}\log(c(d+ex^n))}{-1+cd+cex^n}\,dx$

Optimal. Leaf size=25

$$-\frac{\text{Li}_2\left(1-c\left(ex^n+d\right)\right)}{cen}$$

```
[Out] -polylog(2,1-c*(d+e*x^n))/c/e/n
```

Rubi [A] time = 0.10, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2475, 2393, 2391}

$$-\frac{\text{PolyLog}\left(2,1-c\left(d+ex^n\right)\right)}{cen}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*Log[c*(d + e*x^n)])/(-1 + c*d + c*e*x^n),x]
```

```
[Out] -(PolyLog[2, 1 - c*(d + e*x^n)]/(c*e*n))
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+n}\log\left(c\left(d+ex^n\right)\right)}{-1+cd+cex^n}\,dx &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{-1+cd+cex}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,-1+cd+cex^n\right)}{cen}\\
&= -\frac{\text{Li}_2\left(1-c\left(d+ex^n\right)\right)}{cen}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.04

$$-\frac{\text{Li}_2\left(-cex^n-cd+1\right)}{cen}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*Log[c*(d + e*x^n)])/(-1 + c*d + c*e*x^n),x]
[Out] -(PolyLog[2, 1 - c*d - c*e*x^n]/(c*e*n))
```

fricas [A] time = 0.46, size = 25, normalized size = 1.00

$$-\frac{\text{Li}_2\left(-cex^n - cd + 1\right)}{cen}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*log(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x, algorithm="fricas")
[Out] -dilog(-c*e*x^n - c*d + 1)/(c*e*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{n-1}\log\left((ex^n + d)c\right)}{cex^n + cd - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*log(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x, algorithm="giac")
[Out] integrate(x^(n - 1)*log((e*x^n + d)*c)/(c*e*x^n + c*d - 1), x)
```

maple [A] time = 0.08, size = 23, normalized size = 0.92

$$-\frac{\text{dilog}\left(ce\,x^n + cd\right)}{cen}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*ln(c*(e*x^n+d))/(-1+c*d+c*e*x^n),x)
[Out] -1/n*dilog(c*e*x^n+c*d)/c/e
```

maxima [B] time = 0.52, size = 109, normalized size = 4.36

$$\frac{\log\left(cex^n + cd - 1\right)\log\left((ex^n + d)c\right)}{cen} - \frac{\log\left(cex^n + cd - 1\right)\log\left(ex^n + d\right)}{cen} + \frac{\log\left(-cex^n - cd + 1\right)\log\left(ex^n + d\right) + \text{Li}}{cen}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*log(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x, algorithm="maxima")
[Out] log(c*e*x^n + c*d - 1)*log((e*x^n + d)*c)/(c*e*n) - log(c*e*x^n + c*d - 1)*
log(e*x^n + d)/(c*e*n) + (log(-c*e*x^n - c*d + 1)*log(e*x^n + d) + dilog(c*
e*x^n + c*d))/(c*e*n)
```

mupad [B] time = 0.65, size = 21, normalized size = 0.84

$$-\frac{\text{Li}_2\left(c\,\left(d + e\,x^n\right)\right)}{c\,e\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*log(c*(d + e*x^n)))/(c*d + c*e*x^n - 1),x)
[Out] -dilog(c*(d + e*x^n))/(c*e*n)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*ln(c*(d+e*x**n))/(-1+c*d+c*e*x**n),x)
[Out] Exception raised: TypeError
```

3.380 $\int \frac{\log(c(d+ex^{-n}))}{x(ce-(1-cd)x^n)}\,dx$

Optimal. Leaf size=26

$$\frac{\text{Li}_2\left(1-c\left(ex^{-n}+d\right)\right)}{cen}$$

[Out] polylog(2,1-c*(d+e/(x^n)))/c/e/n

Rubi [A] time = 0.16, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2475, 2412, 2393, 2391}

$$\frac{\text{PolyLog}\left(2,1-c\left(d+ex^{-n}\right)\right)}{cen}$$

Antiderivative was successfully verified.

[In] Int[Log[c*(d + e/x^n)]/(x*(c*e - (1 - c*d)*x^n)),x]

[Out] PolyLog[2, 1 - c*(d + e/x^n)]/(c*e*n)

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2412

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log(c(d+ex^{-n}))}{x(ce-(1-cd)x^n)}\,dx = -\frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{\left(ce+\frac{-1+cd}{x}\right)x}\,dx, x, x^{-n}\right)}{n}$$
$$= -\frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{-1+cd+cex}\,dx, x, x^{-n}\right)}{n}$$
$$= -\frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, -1+cd+cex^{-n}\right)}{cen}$$
$$= \frac{\text{Li}_2(1-cd-cex^{-n})}{cen}$$

Mathematica [A] time = 0.07, size = 34, normalized size = 1.31

$$\frac{\text{Li}_2(-x^{-n}(cdx^n-x^n+ce))}{cen}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/x^n)]/(x*(c*e - (1 - c*d)*x^n)),x]
[Out] PolyLog[2, -((c*e - x^n + c*d*x^n)/x^n)]/(c*e*n)
```

fricas [A] time = 0.44, size = 30, normalized size = 1.15

$$\frac{\text{Li}_2\left(-\frac{cdx^n+ce}{x^n}+1\right)}{cen}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x, algorithm="fricas")
[Out] dilog(-(c*d*x^n + c*e)/x^n + 1)/(c*e*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(d+\frac{e}{x^n}\right)\right)}{(ce+(cd-1)x^n)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x, algorithm="giac")
[Out] integrate(log(c*(d + e/x^n))/((c*e + (c*d - 1)*x^n)*x), x)
```

maple [A] time = 0.08, size = 24, normalized size = 0.92

$$\frac{\text{dilog}(ce\,x^{-n}+cd)}{cen}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x)
[Out] 1/n*dilog(c*d+c*e/(x^n))/c/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$n\int \frac{\log(x)}{cdxx^n+cex}\,dx+\frac{\log(dx^n+e)\log(x)+\log(c)\log(x)-\log(x)\log(x^n)}{ce}-\frac{\log(c)\log\left(\frac{ce+(cd-1)x^n}{cd-1}\right)}{cen}-\frac{\log(dx^n}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x, algorithm="maxima")
```

```
[Out] n*integrate(log(x)/(c*d*x*x^n + c*e*x), x) + (log(d*x^n + e)*log(x) + log(c
)*log(x) - log(x)*log(x^n))/(c*e) - log(c)*log((c*e + (c*d - 1)*x^n)/(c*d -
 1))/(c*e*n) - (log(d*x^n + e)*log((c*d*e + (c*d^2 - d)*x^n - e)/e + 1) + d
ilog(-(c*d*e + (c*d^2 - d)*x^n - e)/e))/(c*e*n) + (log(x^n)*log((c*d - 1)*x
^n/(c*e) + 1) + dilog(-(c*d - 1)*x^n/(c*e)))/(c*e*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\,\left(d+\frac{e}{x^n}\right)\right)}{x\,\left(c\,e+x^n\,\left(c\,d-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/x^n))/(x*(c*e + x^n*(c*d - 1))),x)
```

```
[Out] int(log(c*(d + e/x^n))/(x*(c*e + x^n*(c*d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/(x**n)))/x/(c*e-(-c*d+1)*x**n),x)
```

```
[Out] Timed out
```

3.381 $\int \frac{\left(f+gx^{2n}\right)^2 \log^q\left(c(d+ex^n)^p\right)}{x}\,dx$

Optimal. Leaf size=608

$$f^2\text{Int}\left(\frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x},x\right)-\frac{d^3g^2\left(d+ex^n\right)\left(c\left(d+ex^n\right)^p\right)^{-1/p}\log^q\left(c\left(d+ex^n\right)^p\right)\left(-\frac{\log\left(c(d+ex^n)^p\right)}{p}\right)^{-q}\Gamma\left(q+1,-\right.}{e^4n}$$

```
[Out] 4^(-1-q)*g^2*(d+e*x^n)^4*GAMMA(1+q,-4*ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p
)^q/e^4/n/((c*(d+e*x^n)^p)^(4/p))/((-ln(c*(d+e*x^n)^p)/p)^q)-d*g^2*(d+e*x^n
)^3*GAMMA(1+q,-3*ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/(3^q)/e^4/n/((c*(
d+e*x^n)^p)^(3/p))/((-ln(c*(d+e*x^n)^p)/p)^q)+f*g*(d+e*x^n)^2*GAMMA(1+q,-2*
ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/(2^q)/e^2/n/((c*(d+e*x^n)^p)^(2/p)
)/((-ln(c*(d+e*x^n)^p)/p)^q)+3*2^(-1-q)*d^2*g^2*(d+e*x^n)^2*GAMMA(1+q,-2*ln
(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/e^4/n/((c*(d+e*x^n)^p)^(2/p))/((-ln(
c*(d+e*x^n)^p)/p)^q)-2*d*f*g*(d+e*x^n)*GAMMA(1+q,-ln(c*(d+e*x^n)^p)/p)*ln(c
*(d+e*x^n)^p)^q/e^2/n/((c*(d+e*x^n)^p)^(1/p))/((-ln(c*(d+e*x^n)^p)/p)^q)-d^
3*g^2*(d+e*x^n)*GAMMA(1+q,-ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/e^4/n/(
(c*(d+e*x^n)^p)^(1/p))/((-ln(c*(d+e*x^n)^p)/p)^q)+f^2*Unintegrable(ln(c*(d+
e*x^n)^p)^q/x,x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(f+gx^{2n}\right)^2\log^q\left(c\left(d+ex^n\right)^p\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

```
[Out] Defer[Int][((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi steps

$$\int \frac{\left(f+gx^{2n}\right)^2\log^q\left(c\left(d+ex^n\right)^p\right)}{x}\,dx=\int \frac{\left(f+gx^{2n}\right)^2\log^q\left(c\left(d+ex^n\right)^p\right)}{x}\,dx$$

Mathematica [A] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx^{2n}\right)^2\log^q\left(c\left(d+ex^n\right)^p\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

```
[Out] Integrate[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(g^2x^{4\,n}+2\,fgx^{2\,n}+f^2\right)\log\left(\left(ex^n+d\right)^pc\right)^q}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^(4*n) + 2*f*g*x^(2*n) + f^2)*log((e*x^n + d)^p*c)^q/x, x)
```

giac [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx^{2n} + f\right)^2 \log\left(\left(ex^n + d\right)^p c\right)^q}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^(2*n) + f)^2*log((e*x^n + d)^p*c)^q/x, x)
```

maple [**A**] time = 50.02, size = 0, normalized size = 0.00

$$\int \frac{\left(g\,x^{2n} + f\right)^2 \ln\left(c\left(e\,x^n + d\right)^p\right)^q}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^(2*n)+f)^2*ln(c*(e*x^n+d)^p)^q/x,x)
```

```
[Out] int((g*x^(2*n)+f)^2*ln(c*(e*x^n+d)^p)^q/x,x)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(d + e\,x^n\right)}^p\right)^q \left(f + g\,x^{2n}\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)^q*(f + g*x^(2*n))^2)/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)^q*(f + g*x^(2*n))^2)/x, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**(2*n))**2*ln(c*(d+e*x**n)**p)**q/x,x)
```

```
[Out] Timed out
```

3.382 $\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x} dx$

Optimal. Leaf size=307

$$f^2\text{Int}\left(\frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x},x\right)+\frac{g^2 2^{-q-1}\left(d+ex^n\right)^2\left(c\left(d+ex^n\right)^p\right)^{-2/p}\log^q\left(c\left(d+ex^n\right)^p\right)\left(-\frac{\log\left(c(d+ex^n)^p\right)}{p}\right)^{-q}\Gamma\left(q+\right.}{e^2 n}$$

```
[Out] 2^(-1-q)*g^2*(d+e*x^n)^2*GAMMA(1+q,-2*ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p
)^q/e^2/n/((c*(d+e*x^n)^p)^(2/p))/((-ln(c*(d+e*x^n)^p)/p)^q)+2*f*g*(d+e*x^n
)*GAMMA(1+q,-ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/e/n/((c*(d+e*x^n)^p)^
(1/p))/((-ln(c*(d+e*x^n)^p)/p)^q)-d*g^2*(d+e*x^n)*GAMMA(1+q,-ln(c*(d+e*x^n)
^p)/p)*ln(c*(d+e*x^n)^p)^q/e^2/n/((c*(d+e*x^n)^p)^(1/p))/((-ln(c*(d+e*x^n)^
p)/p)^q)+f^2*Unintegrable(ln(c*(d+e*x^n)^p)^q/x,x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(f+gx^n\right)^2\log^q\left(c\left(d+ex^n\right)^p\right)}{x}dx$$

Verification is Not applicable to the result.

```
[In] Int[((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
[Out] Defer[Int][((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi steps

$$\int \frac{\left(f+gx^n\right)^2\log^q\left(c\left(d+ex^n\right)^p\right)}{x}dx=\int \frac{\left(f+gx^n\right)^2\log^q\left(c\left(d+ex^n\right)^p\right)}{x}dx$$

Mathematica [A] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx^n\right)^2\log^q\left(c\left(d+ex^n\right)^p\right)}{x}dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
[Out] Integrate[((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(g^2x^{2n}+2fgx^n+f^2\right)\log\left(\left(ex^n+d\right)^pc\right)^q}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
[Out] integral((g^2*x^(2*n) + 2*f*g*x^n + f^2)*log((e*x^n + d)^p*c)^q/x, x)
```

giac [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx^n+f\right)^2 \log\left(\left((ex^n+d)^p c\right)^q\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
[Out] integrate((g*x^n + f)^2*log((e*x^n + d)^p*c)^q/x, x)
```

maple [**A**] time = 47.95, size = 0, normalized size = 0.00

$$\int \frac{\left(g\,x^n+f\right)^2 \ln\left(c\,(e\,x^n+d)^p\right)^q}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^n+f)^2*ln(c*(e*x^n+d)^p)^q/x,x)
[Out] int((g*x^n+f)^2*ln(c*(e*x^n+d)^p)^q/x,x)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,(d+e\,x^n)^p\right)^q\,\left(f+g\,x^n\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)^q*(f + g*x^n)^2)/x,x)
[Out] int((log(c*(d + e*x^n)^p)^q*(f + g*x^n)^2)/x, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**n)**2*ln(c*(d+e*x**n)**p)**q/x,x)
[Out] Timed out
```

3.383 $\int \frac{(f+gx^{-n})^2 \log^q(c(d+ex^n)^p)}{x} dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{(f+gx^{-n})^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x},x\right)$$

```
[Out] Unintegrable((f+g/(x^n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx^{-n})^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

```
[Out] Defer[Int][((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi steps

$$\int \frac{(f+gx^{-n})^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x} dx = \int \frac{(f+gx^{-n})^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x} dx$$

Mathematica [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{(f+gx^{-n})^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

```
[Out] Integrate[((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(f^2x^{2n}+2\,fgx^n+g^2\right)\log\left(\left(ex^n+d\right)^pc\right)^q}{xx^{2n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
```

```
[Out] integral((f^2*x^(2*n) + 2*f*g*x^n + g^2)*log((e*x^n + d)^p*c)^q/(x*x^(2*n))
, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+\frac{g}{x^n}\right)^2 \log\left(\left(ex^n+d\right)^pc\right)^q}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
```

```
[Out] integrate((f + g/x^n)^2*log((e*x^n + d)^p*c)^q/x, x)
```

maple [A] time = 39.88, size = 0, normalized size = 0.00

$$\int \frac{\left(g\,x^{-n}+f\right)^{2}\ln\left(c\left(e\,x^{n}+d\right)^{p}\right)^{q}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/(x^n))^2*ln(c*(e*x^n+d)^p)^q/x,x)
```

```
[Out] int((f+g/(x^n))^2*ln(c*(e*x^n+d)^p)^q/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(c\left(d+e\,x^{n}\right)^{p}\right)^{q}\left(f+\frac{g}{x^{n}}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)^q*(f + g/x^n)^2)/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)^q*(f + g/x^n)^2)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x**n))**2*ln(c*(d+e*x**n)**p)**q/x,x)
```

```
[Out] Timed out
```

3.384 $\int \frac{\left(f+gx^{-2n}\right)^2 \log^q\left(c(d+ex^n)^p\right)}{x}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\left(f+gx^{-2n}\right)^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x},x\right)$$

```
[Out] Unintegrable((f+g/(x^(2*n)))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(f+gx^{-2n}\right)^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
[Out] Defer[Int][((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi steps

$$\int \frac{\left(f+gx^{-2n}\right)^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x}\,dx=\int \frac{\left(f+gx^{-2n}\right)^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x}\,dx$$

Mathematica [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx^{-2n}\right)^2 \log^q\left(c\left(d+ex^n\right)^p\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
[Out] Integrate[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(f^2x^{4n}+2\,fgx^{2n}+g^2\right)\log\left(\left(ex^n+d\right)^pc\right)^q}{xx^{4n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
[Out] integral((f^2*x^(4*n) + 2*f*g*x^(2*n) + g^2)*log((e*x^n + d)^p*c)^q/(x*x^(4
*n)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+\frac{g}{x^{2n}}\right)^2 \log\left(\left(ex^n+d\right)^pc\right)^q}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
```

```
[Out] integrate((f + g/x^(2*n))^2*log((e*x^n + d)^p*c)^q/x, x)
```

maple [A] time = 40.45, size = 0, normalized size = 0.00

$$\int \frac{\left(g\,x^{-2n}+f\right)^{2} \ln\left(c\left(e\,x^{n}+d\right)^{p}\right)^{q}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/(x^(2*n)))^2*ln(c*(e*x^n+d)^p)^q/x,x)
```

```
[Out] int((f+g/(x^(2*n)))^2*ln(c*(e*x^n+d)^p)^q/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(c\left(d+e\,x^{n}\right)^{p}\right)^{q}\,\left(f+\frac{g}{x^{2\,n}}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)^q*(f + g/x^(2*n))^2)/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)^q*(f + g/x^(2*n))^2)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x**(2*n)))**2*ln(c*(d+e*x**n)**p)**q/x,x)
```

```
[Out] Timed out
```

3.385 $\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)} dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{2n}\right)},x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{2n}\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))),x]
[Out] Defer[Int][Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))), x]
```

Rubi steps

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{2n}\right)} dx = \int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{2n}\right)} dx$$

Mathematica [A] time = 2.44, size = 0, normalized size = 0.00

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{2n}\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^n+d\right)^p c\right)^q}{gxx^{2n}+fx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x, algorithm="fricas")
[Out] integral(log((e*x^n + d)^p*c)^q/(g*x*x^(2*n) + f*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^n+d\right)^p c\right)^q}{\left(gx^{2n}+f\right)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^q/((g*x^(2*n) + f)*x), x)
```

maple [A] time = 11.84, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\,(e\,x^{n}+d)^{p}\right)^{q}}{\left(g\,x^{2n}+f\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)^q/x/(g*x^(2*n)+f),x)
```

```
[Out] int(ln(c*(e*x^n+d)^p)^q/x/(g*x^(2*n)+f),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^q}{x\,\left(f+g\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^q/(x*(f + g*x^(2*n))),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^q/(x*(f + g*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g*x**(2*n)),x)
```

```
[Out] Timed out
```

3.386 $\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^n\right)},x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)),x]
[Out] Defer[Int][Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)), x]
```

Rubi steps

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^n\right)}\,dx=\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^n\right)}\,dx$$

Mathematica [A] time = 1.96, size = 0, normalized size = 0.00

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\left(ex^n+d\right)^p c\right)^q}{gxx^n+fx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x, algorithm="fricas")
[Out] integral(log((e*x^n + d)^p*c)^q/(g*x*x^n + f*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^n+d\right)^p c\right)^q}{\left(gx^n+f\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^q/((g*x^n + f)*x), x)
```

maple [A] time = 19.94, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\,(e\,x^{n}+d)^{p}\right)^{q}}{\left(g\,x^{n}+f\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)^q/x/(g*x^n+f),x)
```

```
[Out] int(ln(c*(e*x^n+d)^p)^q/x/(g*x^n+f),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(c\,(d+e\,x^{n})^{p}\right)^{q}}{x\,\left(f+g\,x^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^q/(x*(f + g*x^n)),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^q/(x*(f + g*x^n)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\,(d+ex^{n})^{p}\right)^{q}}{x\left(f+gx^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g*x**n),x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)**q/(x*(f + g*x**n)), x)
```

3.387 $\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-n})}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-n}\right)},x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-n}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)),x]
[Out] Defer[Int][Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)), x]
```

Rubi steps

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-n}\right)}\,dx=\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-n}\right)}\,dx$$

Mathematica [A] time = 1.96, size = 0, normalized size = 0.00

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-n}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)), x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^n\log\left(\left(ex^n+d\right)^pc\right)^q}{fxx^n+gx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x, algorithm="fricas")
[Out] integral(x^n*log((e*x^n + d)^p*c)^q/(f*x*x^n + g*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^n+d\right)^pc\right)^q}{\left(f+\frac{g}{x^n}\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^q/((f + g/x^n)*x), x)
```

maple [A] time = 26.70, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c \left(e \,x^{n}+d \right)^{p}\right)^{q}}{\left(g \,x^{-n}+f \right) x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)^q/x/(f+g/(x^n)),x)
```

```
[Out] int(ln(c*(e*x^n+d)^p)^q/x/(f+g/(x^n)),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln \left(c\,{\left(d+e\,x^n\right)}^p\right)^q}{x\,\left(f+\frac{g}{x^n}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^q/(x*(f + g/x^n)),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^q/(x*(f + g/x^n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g/(x**n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.388 $\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-2n})} dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)},x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))),x]
[Out] Defer[Int][Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))), x]
```

Rubi steps

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx=\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx$$

Mathematica [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\log^q\left(c\left(d+ex^n\right)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))), x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{2n}\log\left(\left(ex^n+d\right)^p c\right)^q}{fxx^{2n}+gx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x, algorithm="fricas")
[Out] integral(x^(2*n)*log((e*x^n + d)^p*c)^q/(f*x*x^(2*n) + g*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(ex^n+d\right)^p c\right)^q}{\left(f+\frac{g}{x^{2n}}\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^q/((f + g/x^(2*n))*x), x)
```

maple [A] time = 17.19, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^{n}+d\right)^{p}\right)^{q}}{\left(g\,x^{-2n}+f\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^n+d)^p)^q/x/(f+g/(x^(2*n))),x)
```

```
[Out] int(ln(c*(e*x^n+d)^p)^q/x/(f+g/(x^(2*n))),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(c\left(d+e\,x^{n}\right)^{p}\right)^{q}}{x\left(f+\frac{g}{x^{2\,n}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^q/(x*(f + g/x^(2*n))),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^q/(x*(f + g/x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g/(x**(2*n))),x)
```

```
[Out] Timed out
```

3.389 $\int \frac{\log(x)\log(d+ex^m)}{x}\,dx$

Optimal. Leaf size=69

$$\frac{\text{Li}_3\left(-\frac{ex^m}{d}\right)}{m^2}-\frac{\log(x)\text{Li}_2\left(-\frac{ex^m}{d}\right)}{m}+\frac{1}{2}\log^2(x)\log\left(d+ex^m\right)-\frac{1}{2}\log^2(x)\log\left(\frac{ex^m}{d}+1\right)$$

```
[Out] 1/2*ln(x)^2*ln(d+e*x^m)-1/2*ln(x)^2*ln(1+e*x^m/d)-ln(x)*polylog(2,-e*x^m/d)
/m+polylog(3,-e*x^m/d)/m^2
```

Rubi [A] time = 0.12, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2375, 2337, 2374, 6589}

$$\frac{\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}-\frac{\log(x)\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}+\frac{1}{2}\log^2(x)\log\left(d+ex^m\right)-\frac{1}{2}\log^2(x)\log\left(\frac{ex^m}{d}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Log[x]*Log[d + e*x^m])/x,x]
```

```
[Out] (Log[x]^2*Log[d + e*x^m])/2 - (Log[x]^2*Log[1 + (e*x^m)/d])/2 - (Log[x]*Pol
yLog[2, -((e*x^m)/d)])/m + PolyLog[3, -((e*x^m)/d)]/m^2
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log(x)\log(d+ex^m)}{x}\,dx = \frac{1}{2}\log^2(x)\log(d+ex^m) - \frac{1}{2}(em)\int \frac{x^{-1+m}\log^2(x)}{d+ex^m}\,dx$$
$$= \frac{1}{2}\log^2(x)\log(d+ex^m) - \frac{1}{2}\log^2(x)\log\left(1+\frac{ex^m}{d}\right) + \int \frac{\log(x)\log\left(1+\frac{ex^m}{d}\right)}{x}\,dx$$
$$= \frac{1}{2}\log^2(x)\log(d+ex^m) - \frac{1}{2}\log^2(x)\log\left(1+\frac{ex^m}{d}\right) - \frac{\log(x)\text{Li}_2\left(-\frac{ex^m}{d}\right)}{m} + \frac{\int \frac{\text{Li}_2\left(-\frac{ex^m}{d}\right)}{x}}{m}$$
$$= \frac{1}{2}\log^2(x)\log(d+ex^m) - \frac{1}{2}\log^2(x)\log\left(1+\frac{ex^m}{d}\right) - \frac{\log(x)\text{Li}_2\left(-\frac{ex^m}{d}\right)}{m} + \frac{\text{Li}_3\left(-\frac{ex^m}{d}\right)}{m^2}$$

Mathematica [A] time = 0.06, size = 75, normalized size = 1.09

$$\frac{\text{Li}_3\left(-\frac{dx^{-m}}{e}\right)}{m^2} + \frac{\log(x)\text{Li}_2\left(-\frac{dx^{-m}}{e}\right)}{m} - \frac{1}{6}\log^2(x)\left(3\log\left(\frac{dx^{-m}}{e}+1\right) - 3\log(d+ex^m) + m\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[x]*Log[d + e*x^m])/x,x]
```

```
[Out] -1/6*(Log[x]^2*(m*Log[x] + 3*Log[1 + d/(e*x^m)] - 3*Log[d + e*x^m])) + (Log
[x]*PolyLog[2, -(d/(e*x^m))])/m + PolyLog[3, -(d/(e*x^m))]/m^2
```

fricas [C] time = 0.43, size = 76, normalized size = 1.10

$$\frac{m^2\log(ex^m+d)\log(x)^2 - m^2\log(x)^2\log\left(\frac{ex^m+d}{d}\right) - 2\,m\text{Li}_2\left(-\frac{ex^m+d}{d}+1\right)\log(x) + 2\,\text{polylog}\left(3,-\frac{ex^m}{d}\right)}{2\,m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log(d+e*x^m)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(m^2*log(e*x^m + d)*log(x)^2 - m^2*log(x)^2*log((e*x^m + d)/d) - 2*m*di
log(-(e*x^m + d)/d + 1)*log(x) + 2*polylog(3, -e*x^m/d))/m^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(ex^m+d)\log(x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log(d+e*x^m)/x,x, algorithm="giac")
```

```
[Out] integrate(log(e*x^m + d)*log(x)/x, x)
```

maple [A] time = 1.78, size = 66, normalized size = 0.96

$$-\frac{\ln(x)^2\ln\left(\frac{e\,x^m}{d}+1\right)}{2} + \frac{\ln(x)^2\ln(e\,x^m+d)}{2} - \frac{\text{polylog}\left(2,-\frac{e\,x^m}{d}\right)\ln(x)}{m} + \frac{\text{polylog}\left(3,-\frac{e\,x^m}{d}\right)}{m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*ln(e*x^m+d)/x,x)
```

```
[Out] 1/2*ln(x)^2*ln(e*x^m+d)-1/2*ln(x)^2*ln(1+e*x^m/d)-ln(x)*polylog(2,-e*x^m/d)
/m+polylog(3,-e*x^m/d)/m^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\,m\log(x)^3 + dm\int\frac{\log(x)^2}{2\,(exx^m + dx)}\,dx + \frac{1}{2}\,\log\left(ex^m + d\right)\log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log(d+e*x^m)/x,x, algorithm="maxima")
```

```
[Out] -1/6*m*log(x)^3 + d*m*integrate(1/2*log(x)^2/(e*x*x^m + d*x), x) + 1/2*log(
e*x^m + d)*log(x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\ln\left(d+e\,x^m\right)\,\ln(x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d + e*x^m)*log(x))/x,x)
```

```
[Out] int((log(d + e*x^m)*log(x))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*ln(d+e*x**m)/x,x)
```

```
[Out] Timed out
```

3.390 $\int \frac{\log\left(\frac{a+x}{x}\right)}{x} dx$

Optimal. Leaf size=8

$$\text{Li}_2\left(-\frac{a}{x}\right)$$

[Out] polylog(2,-a/x)

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.50, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2447}

$$\text{PolyLog}\left(2, 1 - \frac{a+x}{x}\right)$$

Antiderivative was successfully verified.

[In] Int[Log[(a + x)/x]/x,x]

[Out] PolyLog[2, 1 - (a + x)/x]

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x} dx = \text{Li}_2\left(1 - \frac{a+x}{x}\right)$$

Mathematica [B] time = 0.00, size = 34, normalized size = 4.25

$$-\text{Li}_2\left(-\frac{-a-x}{x}\right) - \log\left(-\frac{a}{x}\right)\log\left(\frac{a+x}{x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Log[(a + x)/x]/x,x]

[Out] -(Log[-(a/x)]*Log[(a + x)/x]) - PolyLog[2, -((-a - x)/x)]

fricas [A] time = 0.42, size = 11, normalized size = 1.38

$$\text{Li}_2\left(-\frac{a+x}{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((a+x)/x)/x,x, algorithm="fricas")

[Out] dilog(-(a + x)/x + 1)

giac [B] time = 0.27, size = 68, normalized size = 8.50

$$-\frac{a^3\left(\frac{1}{\frac{a+x}{x}-1} - \log\left(\frac{|a+x|}{|x|}\right) + \log\left(\left|\frac{a+x}{x} - 1\right|\right)\right) + \frac{a^3\log\left(\frac{a+x}{x}\right)}{\left(\frac{a+x}{x}-1\right)^2}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+x)/x)/x,x, algorithm="giac")
```

```
[Out] -1/2*(a^3*(1/((a + x)/x - 1) - log(abs(a + x)/abs(x)) + log(abs((a + x)/x -
 1))) + a^3*log((a + x)/x)/((a + x)/x - 1)^2)/a^2
```

maple [A] time = 0.07, size = 9, normalized size = 1.12

$$\mathrm{dilog}\left(\frac{a}{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((a+x)/x)/x,x)
```

```
[Out] dilog(1+a/x)
```

maxima [B] time = 0.46, size = 59, normalized size = 7.38

$$-\left(\log\left(a+x\right)-\log(x)\right)\log(x)+\log\left(a+x\right)\log(x)-\frac{1}{2}\log(x)^2+\log(x)\log\left(\frac{a+x}{x}\right)-\log(x)\log\left(\frac{x}{a}+1\right)-\mathrm{Li}_2\left(-\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+x)/x)/x,x, algorithm="maxima")
```

```
[Out] -(log(a + x) - log(x))*log(x) + log(a + x)*log(x) - 1/2*log(x)^2 + log(x)*l
og((a + x)/x) - log(x)*log(x/a + 1) - dilog(-x/a)
```

mupad [B] time = 0.31, size = 8, normalized size = 1.00

$$\mathrm{polylog}\left(2,-\frac{a}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((a + x)/x)/x,x)
```

```
[Out] polylog(2, -a/x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{a}{x}+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((a+x)/x)/x,x)
```

```
[Out] Integral(log(a/x + 1)/x, x)
```

3.391 $\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx$

Optimal. Leaf size=12

$$\frac{1}{2}\text{Li}_2\left(-\frac{a}{x^2}\right)$$

```
[Out] 1/2*polylog(2,-a/x^2)
```

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2461, 2391}

$$\frac{1}{2}\text{PolyLog}\left(2,-\frac{a}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + x^2)/x^2]/x,x]
[Out] PolyLog[2, -(a/x^2)]/2
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] && !BinomialMatchQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx &= \int \frac{\log\left(1+\frac{a}{x^2}\right)}{x}\,dx\\ &= \frac{1}{2}\text{Li}_2\left(-\frac{a}{x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{2}\text{Li}_2\left(-\frac{a}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + x^2)/x^2]/x,x]
[Out] PolyLog[2, -(a/x^2)]/2
```

fricas [A] time = 0.42, size = 15, normalized size = 1.25

$$\frac{1}{2}\,\text{Li}_2\left(-\frac{x^2+a}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((x^2+a)/x^2)/x,x, algorithm="fricas")
[Out] 1/2*dilog(-(x^2 + a)/x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{x^2+a}{x^2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((x^2+a)/x^2)/x,x, algorithm="giac")
[Out] integrate(log((x^2 + a)/x^2)/x, x)
```

maple [B] time = 0.13, size = 76, normalized size = 6.33

$$-\ln\left(\frac{1}{x}\right)\ln\left(\frac{a}{x^2}+1\right)+\ln\left(\frac{1}{x}\right)\ln\left(-\frac{\sqrt{-a}}{x}+1\right)+\ln\left(\frac{1}{x}\right)\ln\left(\frac{\sqrt{-a}}{x}+1\right)+\mathrm{dilog}\left(-\frac{\sqrt{-a}}{x}+1\right)+\mathrm{dilog}\left(\frac{\sqrt{-a}}{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((x^2+a)/x^2)/x,x)
[Out] -ln(1/x)*ln(a/x^2+1)+ln(1/x)*ln(1+1/x*(-a)^(1/2))+ln(1/x)*ln(1-1/x*(-a)^(1/
2))+dilog(1+1/x*(-a)^(1/2))+dilog(1-1/x*(-a)^(1/2))
```

maxima [B] time = 0.47, size = 69, normalized size = 5.75

$$-\left(\log\left(x^2+a\right)-2\log(x)\right)\log(x)+\log\left(x^2+a\right)\log(x)-\log(x)^2-\log(x)\log\left(\frac{x^2}{a}+1\right)+\log(x)\log\left(\frac{x^2+a}{x^2}\right)-\frac{1}{2}\,\mathrm{I}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((x^2+a)/x^2)/x,x, algorithm="maxima")
[Out] -(log(x^2 + a) - 2*log(x))*log(x) + log(x^2 + a)*log(x) - log(x)^2 - log(x)
*log(x^2/a + 1) + log(x)*log((x^2 + a)/x^2) - 1/2*dilog(-x^2/a)
```

mupad [B] time = 0.29, size = 10, normalized size = 0.83

$$\frac{\mathrm{polylog}\left(2,-\frac{a}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((a + x^2)/x^2)/x,x)
[Out] polylog(2, -a/x^2)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{a}{x^2}+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((x**2+a)/x**2)/x,x)
[Out] Integral(log(a/x**2 + 1)/x, x)
```

3.392 $\int \frac{\log(x^{-n}(a+x^n))}{x}\,dx$

Optimal. Leaf size=14

$$\frac{\operatorname{Li}_2(-ax^{-n})}{n}$$

```
[Out] polylog(2,-a/(x^n))/n
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2461, 2391}

$$\frac{\operatorname{PolyLog}(2,-ax^{-n})}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + x^n)/x^n]/x,x]
```

```
[Out] PolyLog[2, -(a/x^n)]/n
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] && !BinomialMatchQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x^{-n}(a+x^n))}{x}\,dx &= \int \frac{\log(1+ax^{-n})}{x}\,dx\\ &= \frac{\operatorname{Li}_2(-ax^{-n})}{n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{\operatorname{Li}_2(-ax^{-n})}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + x^n)/x^n]/x,x]
```

```
[Out] PolyLog[2, -(a/x^n)]/n
```

fricas [B] time = 0.43, size = 61, normalized size = 4.36

$$\frac{n^2\log(x)^2 - 2\,n\log(x)\log\left(\frac{a+x^n}{a}\right) + 2\,n\log(x)\log\left(\frac{a+x^n}{x^n}\right) - 2\operatorname{Li}_2\left(-\frac{a+x^n}{a}+1\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+x^n)/(x^n))/x,x, algorithm="fricas")
```

```
[Out] 1/2*(n^2*log(x)^2 - 2*n*log(x)*log((a + x^n)/a) + 2*n*log(x)*log((a + x^n)/
x^n) - 2*dilog(-(a + x^n)/a + 1))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{a+x^n}{x^n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+x^n)/(x^n))/x,x, algorithm="giac")
```

```
[Out] integrate(log((a + x^n)/x^n)/x, x)
```

maple [A] time = 0.08, size = 15, normalized size = 1.07

$$\frac{\operatorname{dilog}\left(a\,x^{-n}+1\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((a+x^n)/(x^n))/x,x)
```

```
[Out] 1/n*dilog(1+a/(x^n))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$an\int \frac{\log(x)}{ax+xx^n}\,dx + \log\left(a+x^n\right)\log(x) - \log(x)\log\left(x^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+x^n)/(x^n))/x,x, algorithm="maxima")
```

```
[Out] a*n*integrate(log(x)/(a*x + x*x^n), x) + log(a + x^n)*log(x) - log(x)*log(x
^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\ln\left(\frac{a+x^n}{x^n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((a + x^n)/x^n)/x,x)
```

```
[Out] int(log((a + x^n)/x^n)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(ax^{-n}+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((a+x**n)/(x**n))/x,x)
```

```
[Out] Integral(log(a*x**(-n) + 1)/x, x)
```

3.393 $\int \frac{\log\left(\frac{a+bx}{x}\right)}{x} dx$

Optimal. Leaf size=35

$$-\mathrm{Li}_2\left(\frac{a}{bx}+1\right)-\log\left(\frac{a}{x}+b\right)\log\left(-\frac{a}{bx}\right)$$

```
[Out] -ln(a/x+b)*ln(-a/b/x)-polylog(2,1+a/b/x)
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2461, 2454, 2394, 2315}

$$-\text{PolyLog}\left(2,\frac{a}{bx}+1\right)-\log\left(\frac{a}{x}+b\right)\log\left(-\frac{a}{bx}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + b*x)/x]/x,x]
```

```
[Out] -(Log[b + a/x]*Log[-(a/(b*x))]) - PolyLog[2, 1 + a/(b*x)]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx = \int \frac{\log\left(b+\frac{a}{x}\right)}{x}\,dx$$
$$= -\text{Subst}\left(\int \frac{\log(b+ax)}{x}\,dx, x, \frac{1}{x}\right)$$
$$= -\log\left(b+\frac{a}{x}\right)\log\left(-\frac{a}{bx}\right) + a\,\text{Subst}\left(\int \frac{\log\left(-\frac{ax}{b}\right)}{b+ax}\,dx, x, \frac{1}{x}\right)$$
$$= -\log\left(b+\frac{a}{x}\right)\log\left(-\frac{a}{bx}\right) - \text{Li}_2\left(1+\frac{a}{bx}\right)$$

Mathematica [A] time = 0.00, size = 36, normalized size = 1.03

$$-\text{Li}_2\left(\frac{\frac{a}{x}+b}{b}\right) - \log\left(\frac{a}{x}+b\right)\log\left(-\frac{a}{bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + b*x)/x]/x,x]
[Out] -(Log[b + a/x]*Log[-(a/(b*x))]) - PolyLog[2, (b + a/x)/b]
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\frac{bx+a}{x}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x+a)/x)/x,x, algorithm="fricas")
[Out] integral(log((b*x + a)/x)/x, x)
```

giac [B] time = 0.66, size = 204, normalized size = 5.83

$$\frac{a^3\left(\frac{\log\left(\frac{|bx+a|}{|x|}\right)}{b^2} - \frac{\log\left(\left|-b+\frac{bx+a}{x}\right|\right)}{b^2} + \frac{1}{\left(b-\frac{bx+a}{x}\right)b}\right) - \frac{a^3\log\left(-\left(a-\frac{b}{\frac{\left(a-\frac{b}{\frac{b}{a}-\frac{bx+a}{ax}}\right)\left(\frac{b}{a}-\frac{bx+a}{ax}\right)}{a}+\frac{b}{a}}\right)\left(\frac{\left(a-\frac{b}{\frac{b}{a}-\frac{bx+a}{ax}}\right)\left(\frac{b}{a}-\frac{bx+a}{ax}\right)}{a}+\frac{b}{a}\right)\right)}{\left(b-\frac{bx+a}{x}\right)^2}}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x+a)/x)/x,x, algorithm="giac")
[Out] 1/2*(a^3*(log(abs(b*x + a)/abs(x))/b^2 - log(abs(-b + (b*x + a)/x))/b^2 + 1
/((b - (b*x + a)/x)*b)) - a^3*log(-(a - b/((a - b/(b/a - (b*x + a)/(a*x)))*
(b/a - (b*x + a)/(a*x))/a + b/a))*((a - b/(b/a - (b*x + a)/(a*x)))*(b/a - (
b*x + a)/(a*x))/a + b/a))/(b - (b*x + a)/x)^2)/a^2
```

maple [A] time = 0.08, size = 34, normalized size = 0.97

$$-\ln\left(-\frac{a}{bx}\right)\ln\left(b+\frac{a}{x}\right) - \text{dilog}\left(-\frac{a}{bx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((b*x+a)/x)/x,x)
```

```
[Out] -dilog(-a/b/x)-ln(b+a/x)*ln(-a/b/x)
```

maxima [A] time = 0.48, size = 67, normalized size = 1.91

$$-\left(\log\left(bx+a\right)-\log(x)\right)\log(x)+\log\left(bx+a\right)\log(x)-\log\left(\frac{bx}{a}+1\right)\log(x)-\frac{1}{2}\log(x)^2+\log(x)\log\left(\frac{bx+a}{x}\right)-\mathrm{Li}_2\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x+a)/x)/x,x, algorithm="maxima")
```

```
[Out] -(log(b*x + a) - log(x))*log(x) + log(b*x + a)*log(x) - log(b*x/a + 1)*log(
x) - 1/2*log(x)^2 + log(x)*log((b*x + a)/x) - dilog(-b*x/a)
```

mupad [B] time = 0.41, size = 37, normalized size = 1.06

$$-\mathrm{polylog}\left(2,\frac{a}{b\,x}+1\right)-\ln\left(\frac{a+b\,x}{x}\right)\,\ln\left(-\frac{a}{b\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((a + b*x)/x)/x,x)
```

```
[Out] - polylog(2, a/(b*x) + 1) - log((a + b*x)/x)*log(-a/(b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(\frac{a}{x}+b\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((b*x+a)/x)/x,x)
```

```
[Out] Integral(log(a/x + b)/x, x)
```

3.394 $\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x} dx$

Optimal. Leaf size=39

$$-\frac{1}{2}\text{Li}_2\left(\frac{a}{bx^2}+1\right)-\frac{1}{2}\log\left(\frac{a}{x^2}+b\right)\log\left(-\frac{a}{bx^2}\right)$$

```
[Out] -1/2*ln(b+a/x^2)*ln(-a/b/x^2)-1/2*polylog(2,1+a/b/x^2)
```

Rubi [A] time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2461, 2454, 2394, 2315}

$$-\frac{1}{2}\text{PolyLog}\left(2,\frac{a}{bx^2}+1\right)-\frac{1}{2}\log\left(\frac{a}{x^2}+b\right)\log\left(-\frac{a}{bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + b*x^2)/x^2]/x,x]
```

```
[Out] -(Log[b + a/x^2]*Log[-(a/(b*x^2))])/2 - PolyLog[2, 1 + a/(b*x^2)]/2
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx &= \int \frac{\log\left(b+\frac{a}{x^2}\right)}{x}\,dx \\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{\log(b+ax)}{x}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= -\frac{1}{2}\log\left(b+\frac{a}{x^2}\right)\log\left(-\frac{a}{bx^2}\right)+\frac{1}{2}a\operatorname{Subst}\left(\int \frac{\log\left(-\frac{ax}{b}\right)}{b+ax}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{1}{2}\log\left(b+\frac{a}{x^2}\right)\log\left(-\frac{a}{bx^2}\right)-\frac{1}{2}\operatorname{Li}_2\left(1+\frac{a}{bx^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 40, normalized size = 1.03

$$
-\frac{1}{2}\operatorname{Li}_2\left(\frac{\frac{a}{x^2}+b}{b}\right)-\frac{1}{2}\log\left(\frac{a}{x^2}+b\right)\log\left(-\frac{a}{bx^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + b*x^2)/x^2]/x,x]
[Out] -1/2*(Log[b + a/x^2]*Log[-(a/(b*x^2))]) - PolyLog[2, (b + a/x^2)/b]/2
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$
\operatorname{integral}\left(\frac{\log\left(\frac{bx^2+a}{x^2}\right)}{x}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x^2+a)/x^2)/x,x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)/x^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\log\left(\frac{bx^2+a}{x^2}\right)}{x}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x^2+a)/x^2)/x,x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)/x^2)/x, x)
```

maple [B] time = 0.09, size = 108, normalized size = 2.77

$$
\ln\left(\frac{1}{x}\right)\ln\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{1}{x}\right)\ln\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)-\ln\left(\frac{1}{x}\right)\ln\left(b+\frac{a}{x^2}\right)+\operatorname{dilog}\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\operatorname{dilog}\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((b*x^2+a)/x^2)/x,x)
[Out] -ln(1/x)*ln(b+a/x^2)+ln(1/x)*ln((-a/x+(-a*b)^(1/2))/(-a*b)^(1/2))+ln(1/x)*l
n((a/x+(-a*b)^(1/2))/(-a*b)^(1/2))+dilog((-a/x+(-a*b)^(1/2))/(-a*b)^(1/2))+
dilog((a/x+(-a*b)^(1/2))/(-a*b)^(1/2))
```

maxima [B] time = 0.47, size = 77, normalized size = 1.97

$$-\left(\log\left(bx^2+a\right)-2\,\log(x)\right)\log(x)+\log\left(bx^2+a\right)\log(x)-\log\left(\frac{bx^2}{a}+1\right)\log(x)-\log(x)^2+\log(x)\log\left(\frac{bx^2+a}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x^2+a)/x^2)/x,x, algorithm="maxima")
```

```
[Out] -(log(b*x^2 + a) - 2*log(x))*log(x) + log(b*x^2 + a)*log(x) - log(b*x^2/a +
 1)*log(x) - log(x)^2 + log(x)*log((b*x^2 + a)/x^2) - 1/2*dilog(-b*x^2/a)
```

mupad [B] time = 0.44, size = 33, normalized size = 0.85

$$-\frac{\mathrm{Li}_2\left(-\frac{a}{b\,x^2}\right)}{2}-\frac{\ln\left(b+\frac{a}{x^2}\right)\,\ln\left(-\frac{a}{b\,x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((a + b*x^2)/x^2)/x,x)
```

```
[Out] - dilog(-a/(b*x^2))/2 - (log(b + a/x^2)*log(-a/(b*x^2)))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{a}{x^2}+b\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((b*x**2+a)/x**2)/x,x)
```

```
[Out] Integral(log(a/x**2 + b)/x, x)
```

3.395 $\int \frac{\log(x^{-n}(a+bx^n))}{x}\,dx$

Optimal. Leaf size=47

$$-\frac{\text{Li}_2\left(\frac{ax^{-n}}{b}+1\right)}{n}-\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log\left(ax^{-n}+b\right)}{n}$$

[Out] -ln(-a/b/(x^n))*ln(b+a/(x^n))/n-polylog(2,1+a/b/(x^n))/n

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2461, 2454, 2394, 2315}

$$-\frac{\text{PolyLog}\left(2,\frac{ax^{-n}}{b}+1\right)}{n}-\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log\left(ax^{-n}+b\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[Log[(a + b*x^n)/x^n]/x,x]

[Out] -((Log[-(a/(b*x^n))]*Log[b + a/x^n])/n) - PolyLog[2, 1 + a/(b*x^n)]/n

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] && !BinomialMatchQ[v, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(x^{-n}\left(a+bx^n\right)\right)}{x}\,dx &= \int \frac{\log\left(b+ax^{-n}\right)}{x}\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{\log(b+ax)}{x}\,dx, x, x^{-n}\right)}{n} \\
&= -\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log\left(b+ax^{-n}\right)}{n} + \frac{a\,\text{Subst}\left(\int \frac{\log\left(-\frac{ax}{b}\right)}{b+ax}\,dx, x, x^{-n}\right)}{n} \\
&= -\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log\left(b+ax^{-n}\right)}{n} - \frac{\text{Li}_2\left(1+\frac{ax^{-n}}{b}\right)}{n}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.94

$$
-\frac{\text{Li}_2\left(\frac{ax^{-n}+b}{b}\right)+\log\left(-\frac{ax^{-n}}{b}\right)\log\left(ax^{-n}+b\right)}{n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + b*x^n)/x^n]/x,x]
[Out] -((Log[-(a/(b*x^n))]*Log[b + a/x^n] + PolyLog[2, (b + a/x^n)/b])/n)
```

fricas [A] time = 0.45, size = 67, normalized size = 1.43

$$
\frac{n^2\log(x)^2 - 2\,n\log(x)\log\left(\frac{bx^n+a}{a}\right) + 2\,n\log(x)\log\left(\frac{bx^n+a}{x^n}\right) - 2\,\text{Li}_2\left(-\frac{bx^n+a}{a}+1\right)}{2\,n}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+b*x^n)/(x^n))/x,x, algorithm="fricas")
[Out] 1/2*(n^2*log(x)^2 - 2*n*log(x)*log((b*x^n + a)/a) + 2*n*log(x)*log((b*x^n +
 a)/x^n) - 2*dilog(-(b*x^n + a)/a + 1))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\log\left(\frac{bx^n+a}{x^n}\right)}{x}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+b*x^n)/(x^n))/x,x, algorithm="giac")
[Out] integrate(log((b*x^n + a)/x^n)/x, x)
```

maple [A] time = 0.08, size = 46, normalized size = 0.98

$$
-\frac{\ln\left(-\frac{a\,x^{-n}}{b}\right)\ln\left(a\,x^{-n}+b\right)}{n} - \frac{\text{dilog}\left(-\frac{a\,x^{-n}}{b}\right)}{n}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((b*x^n+a)/(x^n))/x,x)
[Out] -ln(-a/b/(x^n))*ln(b+a/(x^n))/n-1/n*dilog(-a/b/(x^n))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$an\int \frac{\log(x)}{bxx^n + ax}\,dx + \log(bx^n + a)\log(x) - \log(x)\log(x^n)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+b*x^n)/(x^n))/x,x, algorithm="maxima")
```

```
[Out] a*n*integrate(log(x)/(b*x*x^n + a*x), x) + log(b*x^n + a)*log(x) - log(x)*l
og(x^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(\frac{a+b\,x^n}{x^n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((a + b*x^n)/x^n)/x,x)
```

```
[Out] int(log((a + b*x^n)/x^n)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(ax^{-n} + b)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((a+b*x**n)/(x**n))/x,x)
```

```
[Out] Integral(log(a*x**(-n) + b)/x, x)
```

3.396 $\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx$

Optimal. Leaf size=105

$$-\frac{\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{d}+\frac{\log\left(\frac{a}{x}+b\right)\log(c+dx)}{d}-\frac{\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{d}+\frac{\text{Li}_2\left(\frac{dx}{c}+1\right)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}$$

```
[Out] ln(a/x+b)*ln(d*x+c)/d+ln(-d*x/c)*ln(d*x+c)/d-ln(-d*(b*x+a)/(-a*d+b*c))*ln(d
*x+c)/d-polylog(2,b*(d*x+c)/(-a*d+b*c))/d+polylog(2,1+d*x/c)/d
```

Rubi [A] time = 0.17, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2465, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$-\frac{\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{d}+\frac{\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{d}+\frac{\log\left(\frac{a}{x}+b\right)\log(c+dx)}{d}-\frac{\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{d}+\frac{\log\left(-\frac{dx}{c}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + b*x)/x]/(c + d*x),x]
```

```
[Out] (Log[b + a/x]*Log[c + d*x])/d + (Log[-((d*x)/c)]*Log[c + d*x])/d - (Log[-((
d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/d - PolyLog[2, (b*(c + d*x))/(b*c
- a*d)]/d + PolyLog[2, 1 + (d*x)/c]/d
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*(u_)^(r_.), x_Symbol] :> In
t[ExpandToSum[u, x]^r*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a,
 b, c, p, q, r}, x] && LinearQ[u, x] && BinomialQ[v, x] &&  !(LinearMatchQ[
u, x] && BinomialMatchQ[v, x])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx &= \int \frac{\log\left(b+\frac{a}{x}\right)}{c+dx}\,dx\\
&= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{a\int \frac{\log(c+dx)}{\left(b+\frac{a}{x}\right)x^2}\,dx}{d}\\
&= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{a\int \left(\frac{\log(c+dx)}{ax}-\frac{b\log(c+dx)}{a(a+bx)}\right)dx}{d}\\
&= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{\int \frac{\log(c+dx)}{x}\,dx}{d} - \frac{b\int \frac{\log(c+dx)}{a+bx}\,dx}{d}\\
&= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{d} - \int \frac{\log\left(-\frac{dx}{c}\right)}{c+dx}\\
&= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{d} + \frac{\mathrm{Li}_2\left(1+\frac{dx}{c}\right)}{d}\\
&= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{d} - \frac{\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 80, normalized size = 0.76

$$
\frac{-\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(-\log\left(\frac{d(a+bx)}{ad-bc}\right)+\log\left(\frac{a}{x}+b\right)+\log\left(-\frac{dx}{c}\right)\right)+\mathrm{Li}_2\left(\frac{dx}{c}+1\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + b*x)/x]/(c + d*x),x]
```

```
[Out] ((Log[b + a/x] + Log[-((d*x)/c)] - Log[(d*(a + b*x))/(-(b*c) + a*d)])*Log[c
 + d*x] - PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + PolyLog[2, 1 + (d*x)/c])/
d
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\log\left(\frac{bx+a}{x}\right)}{dx+c}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x+a)/x)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)/x)/(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{bx+a}{x}\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x+a)/x)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)/x)/(d*x + c), x)
```

maple [A] time = 0.09, size = 114, normalized size = 1.09

$$\frac{\ln\left(\frac{ad-bc+\left(b+\frac{a}{x}\right)c}{ad-bc}\right)\ln\left(b+\frac{a}{x}\right)}{d}-\frac{\ln\left(-\frac{a}{bx}\right)\ln\left(b+\frac{a}{x}\right)}{d}+\frac{\operatorname{dilog}\left(\frac{ad-bc+\left(b+\frac{a}{x}\right)c}{ad-bc}\right)}{d}-\frac{\operatorname{dilog}\left(-\frac{a}{bx}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((b*x+a)/x)/(d*x+c),x)
```

```
[Out] -1/d*ln(b+a/x)*ln(-a/b/x)-1/d*dilog(-a/b/x)+1/d*dilog(((b+a/x)*c+a*d-b*c)/(
a*d-b*c))+1/d*ln(b+a/x)*ln(((b+a/x)*c+a*d-b*c)/(a*d-b*c))
```

maxima [A] time = 0.49, size = 124, normalized size = 1.18

$$-\frac{\left(\log\left(bx+a\right)-\log(x)\right)\log\left(dx+c\right)}{d}+\frac{\log\left(dx+c\right)\log\left(\frac{bx+a}{x}\right)}{d}-\frac{\log\left(\frac{dx}{c}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{dx}{c}\right)}{d}+\frac{\log\left(bx+a\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x+a)/x)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -(log(b*x + a) - log(x))*log(d*x + c)/d + log(d*x + c)*log((b*x + a)/x)/d -
 (log(d*x/c + 1)*log(x) + dilog(-d*x/c))/d + (log(b*x + a)*log((b*d*x + a*d
)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(\frac{a+b\,x}{x}\right)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((a + b*x)/x)/(c + d*x),x)
```

```
[Out] int(log((a + b*x)/x)/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{a}{x}+b\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((b*x+a)/x)/(d*x+c),x)
```

```
[Out] Integral(log(a/x + b)/(c + d*x), x)
```

3.397 $\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx$

Optimal. Leaf size=227

$$-\frac{\text{Li}_2\left(\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{d}-\frac{\text{Li}_2\left(\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}\right)}{d}+\frac{\log\left(\frac{a}{x^2}+b\right)\log(c+dx)}{d}-\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}d+\sqrt{b}c}\right)}{d}-\frac{\log(c+dx)\log\left(-\right.}{d}$$

```
[Out] ln(b+a/x^2)*ln(d*x+c)/d+2*ln(-d*x/c)*ln(d*x+c)/d-ln(d*x+c)*ln(d*((-a)^(1/2)
-x*b^(1/2))/(d*(-a)^(1/2)+c*b^(1/2)))/d-ln(d*x+c)*ln(-d*((-a)^(1/2)+x*b^(1/
2))/(-d*(-a)^(1/2)+c*b^(1/2)))/d+2*polylog(2,1+d*x/c)/d-polylog(2,(d*x+c)*b
^(1/2)/(-d*(-a)^(1/2)+c*b^(1/2)))/d-polylog(2,(d*x+c)*b^(1/2)/(d*(-a)^(1/2)
+c*b^(1/2)))/d
```

Rubi [A] time = 0.38, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2465, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$-\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{d}-\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{-a}d+\sqrt{b}c}\right)}{d}+\frac{2\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{d}+\frac{\log\left(\frac{a}{x^2}+b\right)\log(c+dx)}{d}-\frac{\log(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + b*x^2)/x^2]/(c + d*x),x]
```

```
[Out] (Log[b + a/x^2]*Log[c + d*x])/d + (2*Log[-((d*x)/c)]*Log[c + d*x])/d - (Log
[(d*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*c + Sqrt[-a]*d)]*Log[c + d*x])/d - (Lo
g[-((d*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*c - Sqrt[-a]*d))]*Log[c + d*x])/d -
 PolyLog[2, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c - Sqrt[-a]*d)]/d - PolyLog[2, (S
qrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[-a]*d)]/d + (2*PolyLog[2, 1 + (d*x)/c])
/d
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*(u_)^(r_.), x_Symbol] :> In
t[ExpandToSum[u, x]^r*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a,
 b, c, p, q, r}, x] && LinearQ[u, x] && BinomialQ[v, x] &&  !(LinearMatchQ[
u, x] && BinomialMatchQ[v, x])
```

Rubi steps

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx = \int \frac{\log\left(b+\frac{a}{x^2}\right)}{c+dx}\,dx$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{(2a)\int \frac{\log(c+dx)}{\left(b+\frac{a}{x^2}\right)x^3}\,dx}{d}$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{(2a)\int \left(\frac{\log(c+dx)}{ax} - \frac{bx\log(c+dx)}{a\left(a+bx^2\right)}\right)dx}{d}$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\int \frac{\log(c+dx)}{x}\,dx}{d} - \frac{(2b)\int \frac{x\log(c+dx)}{a+bx^2}\,dx}{d}$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - 2\int \frac{\log\left(-\frac{dx}{c}\right)}{c+dx}\,dx - \frac{(2b)\int \left(-\frac{\log}{2\sqrt{b}\left(\sqrt{\right.}}\right.}{}$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} + \frac{2\mathrm{Li}_2\left(1+\frac{dx}{c}\right)}{d} + \frac{\sqrt{b}\int \frac{\log(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{d}$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,c+\sqrt{-a}\,d}\right)\log(c+dx)}{d} - \frac{\log}{}$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,c+\sqrt{-a}\,d}\right)\log(c+dx)}{d} - \frac{\log}{}$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,c+\sqrt{-a}\,d}\right)\log(c+dx)}{d} - \frac{\log}{}$$

Mathematica [A] time = 0.11, size = 228, normalized size = 1.00

$$-\frac{\text{Li}_2\left(\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{d}-\frac{\text{Li}_2\left(\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}\right)}{d}+\frac{\log\left(\frac{a}{x^2}+b\right)\log(c+dx)}{d}-\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}d+\sqrt{b}c}\right)}{d}-\frac{\log(c+dx)\log\left(-\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + b*x^2)/x^2]/(c + d*x),x]
```

```
[Out] (Log[b + a/x^2]*Log[c + d*x])/d + (2*Log[-((d*x)/c)]*Log[c + d*x])/d - (Log
[(d*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*c + Sqrt[-a]*d)]*Log[c + d*x])/d - (Lo
g[-((d*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*c - Sqrt[-a]*d))]*Log[c + d*x])/d +
 (2*PolyLog[2, (c + d*x)/c])/d - PolyLog[2, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c
- Sqrt[-a]*d)]/d - PolyLog[2, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[-a]*d)]
/d
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\frac{bx^2+a}{x^2}\right)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x^2+a)/x^2)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)/x^2)/(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(\frac{bx^2+a}{x^2}\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x^2+a)/x^2)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)/x^2)/(d*x + c), x)
```

maple [A] time = 0.14, size = 335, normalized size = 1.48

$$\frac{\ln\left(\frac{1}{x}\right)\ln\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{d}+\frac{\ln\left(\frac{1}{x}\right)\ln\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{d}-\frac{\ln\left(\frac{1}{x}\right)\ln\left(b+\frac{a}{x^2}\right)}{d}-\frac{\ln\left(\frac{ad-\left(d+\frac{c}{x}\right)a+\sqrt{-ab}\,c}{ad+\sqrt{-ab}\,c}\right)\ln\left(d+\frac{c}{x}\right)}{d}-\frac{\ln\left(\frac{-ad+\left(d+\frac{c}{x}\right)a+\sqrt{-}}{-ad+\sqrt{-ab}\,c}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((b*x^2+a)/x^2)/(d*x+c),x)
```

```
[Out] 1/d*ln(d+c/x)*ln(b+a/x^2)-1/d*ln(d+c/x)*ln((c*(-a*b)^(1/2)-a*(d+c/x)+a*d)/(
c*(-a*b)^(1/2)+a*d))-1/d*ln(d+c/x)*ln((c*(-a*b)^(1/2)+a*(d+c/x)-a*d)/(c*(-a
*b)^(1/2)-a*d))-1/d*dilog((c*(-a*b)^(1/2)-a*(d+c/x)+a*d)/(c*(-a*b)^(1/2)+a*
d))-1/d*dilog((c*(-a*b)^(1/2)+a*(d+c/x)-a*d)/(c*(-a*b)^(1/2)-a*d))-1/d*ln(1
/x)*ln(b+a/x^2)+1/d*ln(1/x)*ln((-a/x+(-a*b)^(1/2))/(-a*b)^(1/2))+1/d*ln(1/x
)*ln((a/x+(-a*b)^(1/2))/(-a*b)^(1/2))+1/d*dilog((-a/x+(-a*b)^(1/2))/(-a*b)^
(1/2))+1/d*dilog((a/x+(-a*b)^(1/2))/(-a*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(\frac{bx^2+a}{x^2}\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x^2+a)/x^2)/(d*x+c),x, algorithm="maxima")
[Out] integrate(log((b*x^2 + a)/x^2)/(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(\frac{b\,x^2+a}{x^2}\right)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((a + b*x^2)/x^2)/(c + d*x),x)
[Out] int(log((a + b*x^2)/x^2)/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{a}{x^2}+b\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((b*x**2+a)/x**2)/(d*x+c),x)
[Out] Integral(log(a/x**2 + b)/(c + d*x), x)
```

3.398 $\int \frac{\log(x^{-n}(a+bx^n))}{c+dx}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{\log\left(ax^{-n}+b\right)}{c+dx},x\right)$$

[Out] Unintegrable(ln(b+a/(x^n))/(d*x+c),x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(x^{-n}\left(a+bx^n\right)\right)}{c+dx}\,dx$$

Verification is Not applicable to the result.

[In] Int[Log[(a + b*x^n)/x^n]/(c + d*x),x]

[Out] Defer[Int][Log[b + a/x^n]/(c + d*x), x]

Rubi steps

$$\int \frac{\log\left(x^{-n}\left(a+bx^n\right)\right)}{c+dx}\,dx=\int \frac{\log\left(b+ax^{-n}\right)}{c+dx}\,dx$$

Mathematica [A] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{\log\left(x^{-n}\left(a+bx^n\right)\right)}{c+dx}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Log[(a + b*x^n)/x^n]/(c + d*x),x]

[Out] Integrate[Log[(a + b*x^n)/x^n]/(c + d*x), x]

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\frac{bx^n+a}{x^n}\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((a+b*x^n)/(x^n))/(d*x+c),x, algorithm="fricas")

[Out] integral(log((b*x^n + a)/x^n)/(d*x + c), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{bx^n+a}{x^n}\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((a+b*x^n)/(x^n))/(d*x+c),x, algorithm="giac")

[Out] integrate(log((b*x^n + a)/x^n)/(d*x + c), x)

maple [A] time = 1.68, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\left(b\,x^{n}+a\right)x^{-n}\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((b*x^n+a)/(x^n))/(d*x+c),x)
```

```
[Out] int(ln((b*x^n+a)/(x^n))/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{bx^{n}+a}{x^{n}}\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+b*x^n)/(x^n))/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^n + a)/x^n)/(d*x + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\ln\left(\frac{a+b\,x^{n}}{x^{n}}\right)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((a + b*x^n)/x^n)/(c + d*x),x)
```

```
[Out] int(log((a + b*x^n)/x^n)/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(ax^{-n}+b\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((a+b*x**n)/(x**n))/(d*x+c),x)
```

```
[Out] Integral(log(a*x**(-n) + b)/(c + d*x), x)
```

3.399 $\int (fx)^q \left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)\,dx$

Optimal. Leaf size=92

$$\frac{(fx)^{q+1}\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{f(q+1)}-\frac{bemnx^{m+1}(fx)^q\,{}_2F_1\left(1,\frac{m+q+1}{m};\frac{2m+q+1}{m};-\frac{ex^m}{d}\right)}{d(q+1)(m+q+1)}$$

```
[Out] -b*e*m*n*x^(1+m)*(f*x)^q*hypergeom([1, (1+m+q)/m],[(1+2*m+q)/m],-e*x^m/d)/d
/(1+q)/(1+m+q)+(f*x)^(1+q)*(a+b*ln(c*(d+e*x^m)^n))/f/(1+q)
```

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2455, 20, 364}

$$\frac{(fx)^{q+1}\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{f(q+1)}-\frac{bemnx^{m+1}(fx)^q\,{}_2F_1\left(1,\frac{m+q+1}{m};\frac{2m+q+1}{m};-\frac{ex^m}{d}\right)}{d(q+1)(m+q+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^q*(a + b*Log[c*(d + e*x^m)^n]),x]
```

```
[Out] -((b*e*m*n*x^(1 + m)*(f*x)^q*Hypergeometric2F1[1, (1 + m + q)/m, (1 + 2*m +
 q)/m, -((e*x^m)/d)])/(d*(1 + q)*(1 + m + q))) + ((f*x)^(1 + q)*(a + b*Log[
c*(d + e*x^m)^n]))/(f*(1 + q))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (fx)^q\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)\,dx &= \frac{(fx)^{1+q}\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{f(1+q)}-\frac{(bemn)\int\frac{x^{-1+m}(fx)^{1+q}}{d+ex^m}\,dx}{f(1+q)}\\
&= \frac{(fx)^{1+q}\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{f(1+q)}-\frac{\left(bemnx^{-q}(fx)^q\right)\int\frac{x^{m+q}}{d+ex^m}\,dx}{1+q}\\
&= -\frac{bemnx^{1+m}(fx)^q\,{}_2F_1\left(1,\frac{1+m+q}{m};\frac{1+2m+q}{m};-\frac{ex^m}{d}\right)}{d(1+q)(1+m+q)}+\frac{(fx)^{1+q}\left(a+b\log\left(c\left(d\cdot\right.\right.\right.}{f(1+q)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 82, normalized size = 0.89

$$\frac{x(fx)^q\left(d(m+q+1)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)-bemnx^m\,{}_2F_1\left(1,\frac{m+q+1}{m};\frac{2m+q+1}{m};-\frac{ex^m}{d}\right)\right)}{d(q+1)(m+q+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^q*(a + b*Log[c*(d + e*x^m)^n]),x]
```

```
[Out] (x*(f*x)^q*(-(b*e*m*n*x^m*Hypergeometric2F1[1, (1 + m + q)/m, (1 + 2*m + q)
/m, -((e*x^m)/d)]) + d*(1 + m + q)*(a + b*Log[c*(d + e*x^m)^n])))/(d*(1 + q
)*(1 + m + q))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fx\right)^q b\log\left(\left(ex^m+d\right)^n c\right)+\left(fx\right)^q a,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^q*(a+b*log(c*(d+e*x^m)^n)),x, algorithm="fricas")
```

```
[Out] integral((f*x)^q*b*log((e*x^m + d)^n*c) + (f*x)^q*a, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(ex^m+d\right)^n c\right)+a\right)\left(fx\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^q*(a+b*log(c*(d+e*x^m)^n)),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^m + d)^n*c) + a)*(f*x)^q, x)
```

maple [F] time = 2.22, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(e\,x^m+d\right)^n\right)+a\right)\left(fx\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^q*(a+b*ln(c*(e*x^m+d)^n)),x)
```

```
[Out] int((f*x)^q*(a+b*ln(c*(e*x^m+d)^n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(d^2f^qm^2n\int\frac{x^q}{\left(m\left(q+1\right)-q^2-2q-1\right)e^2x^{2m}+2\left(m\left(q+1\right)-q^2-2q-1\right)dex^m+\left(m\left(q+1\right)-q^2-2q-1\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^q*(a+b*log(c*(d+e*x^m)^n)),x, algorithm="maxima")
```

```
[Out] (d^2*f^q*m^2*n*integrate(x^q/((m*(q + 1) - q^2 - 2*q - 1)*e^2*x^(2*m) + 2*(
m*(q + 1) - q^2 - 2*q - 1)*d*e*x^m + (m*(q + 1) - q^2 - 2*q - 1)*d^2), x) -
 (((m*(q + 1) - q^2 - 2*q - 1)*e*f^q*x*x^m + (m*(q + 1) - q^2 - 2*q - 1)*d*
f^q*x)*x^q*log((e*x^m + d)^n) + (((m*(q + 1) - q^2 - 2*q - 1)*e*f^q*log(c)
- (m^2*n - m*n*(q + 1))*e*f^q)*x*x^m - (d*f^q*m^2*n - (m*(q + 1) - q^2 - 2*
q - 1)*d*f^q*log(c))*x)*x^q)/((q^3 - (q^2 + 2*q + 1)*m + 3*q^2 + 3*q + 1)*e
*x^m + (q^3 - (q^2 + 2*q + 1)*m + 3*q^2 + 3*q + 1)*d))*b + (f*x)^(q + 1)*a/
(f*(q + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (f\,x)^q\,\left(a+b\,\ln\left(c\,(d+e\,x^m)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^q*(a + b*log(c*(d + e*x^m)^n)),x)
[Out] int((f*x)^q*(a + b*log(c*(d + e*x^m)^n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^q\left(a+b\log\left(c\,(d+ex^m)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**q*(a+b*ln(c*(d+e*x**m)**n)),x)
[Out] Integral((f*x)**q*(a + b*log(c*(d + e*x**m)**n)), x)
```

3.400 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) dx$

Optimal. Leaf size=166

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-\frac{bd^8n\log\left(d+e\sqrt{x}\right)}{4e^8}+\frac{bd^7n\sqrt{x}}{4e^7}-\frac{bd^6nx}{8e^6}+\frac{bd^5nx^{3/2}}{12e^5}-\frac{bd^4nx^2}{16e^4}+\frac{bd^3nx^{5/2}}{20e^3}-\frac{bd^2nx^3}{24e^2}$$

```
[Out] -1/8*b*d^6*n*x/e^6+1/12*b*d^5*n*x^(3/2)/e^5-1/16*b*d^4*n*x^2/e^4+1/20*b*d^3
*n*x^(5/2)/e^3-1/24*b*d^2*n*x^3/e^2+1/28*b*d*n*x^(7/2)/e-1/32*b*n*x^4-1/4*b
*d^8*n*ln(d+e*x^(1/2))/e^8+1/4*x^4*(a+b*ln(c*(d+e*x^(1/2))^n))+1/4*b*d^7*n*
x^(1/2)/e^7
```

Rubi [A] time = 0.14, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+\frac{bd^5nx^{3/2}}{12e^5}-\frac{bd^4nx^2}{16e^4}+\frac{bd^3nx^{5/2}}{20e^3}-\frac{bd^2nx^3}{24e^2}+\frac{bd^7n\sqrt{x}}{4e^7}-\frac{bd^6nx}{8e^6}-\frac{bd^8n\log\left(d+e\sqrt{x}\right)}{4e^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (b*d^7*n*Sqrt[x])/(4*e^7) - (b*d^6*n*x)/(8*e^6) + (b*d^5*n*x^(3/2))/(12*e^5
) - (b*d^4*n*x^2)/(16*e^4) + (b*d^3*n*x^(5/2))/(20*e^3) - (b*d^2*n*x^3)/(24
*e^2) + (b*d*n*x^(7/2))/(28*e) - (b*n*x^4)/32 - (b*d^8*n*Log[d + e*Sqrt[x]]
)/(4*e^8) + (x^4*(a + b*Log[c*(d + e*Sqrt[x])^n]))/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx = 2\,\text{Subst}\left(\int x^7\left(a+b\log\left(c(d+ex)^n\right)\right)dx,x,\sqrt{x}\right)$$
$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-\frac{1}{4}(ben)\,\text{Subst}\left(\int\frac{x^8}{d+ex}dx,x,\sqrt{x}\right)$$
$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-\frac{1}{4}(ben)\,\text{Subst}\left(\int\left(-\frac{d^7}{e^8}+\frac{d^6x}{e^7}-\frac{d^5x^2}{e^6}+\right.\right.$$
$$= \frac{bd^7n\sqrt{x}}{4e^7}-\frac{bd^6nx}{8e^6}+\frac{bd^5nx^{3/2}}{12e^5}-\frac{bd^4nx^2}{16e^4}+\frac{bd^3nx^{5/2}}{20e^3}-\frac{bd^2nx^3}{24e^2}+\frac{bdnx^{7/2}}{28e}-$$

Mathematica [A] time = 0.14, size = 159, normalized size = 0.96

$$\frac{ax^4}{4}+\frac{1}{4}bx^4\log\left(c\left(d+e\sqrt{x}\right)^n\right)-\frac{1}{4}ben\left(\frac{d^8\log\left(d+e\sqrt{x}\right)}{e^9}-\frac{d^7\sqrt{x}}{e^8}+\frac{d^6x}{2e^7}-\frac{d^5x^{3/2}}{3e^6}+\frac{d^4x^2}{4e^5}-\frac{d^3x^{5/2}}{5e^4}+\frac{d^2x^3}{6e^3}-\frac{dx^{7/2}}{7e^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (a*x^4)/4 - (b*e*n*(-((d^7*Sqrt[x])/e^8) + (d^6*x)/(2*e^7) - (d^5*x^(3/2))/
(3*e^6) + (d^4*x^2)/(4*e^5) - (d^3*x^(5/2))/(5*e^4) + (d^2*x^3)/(6*e^3) - (
d*x^(7/2))/(7*e^2) + x^4/(8*e) + (d^8*Log[d + e*Sqrt[x]])/e^9))/4 + (b*x^4*
Log[c*(d + e*Sqrt[x])^n])/4
```

fricas [A] time = 0.44, size = 148, normalized size = 0.89

$$\frac{840\,be^8x^4\log(c)-140\,bd^2e^6nx^3-210\,bd^4e^4nx^2-420\,bd^6e^2nx-105\left(be^8n-8\,ae^8\right)x^4+840\left(be^8nx^4-bd^8n\right)\log}{3360\,e^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/3360*(840*b*e^8*x^4*log(c) - 140*b*d^2*e^6*n*x^3 - 210*b*d^4*e^4*n*x^2 -
420*b*d^6*e^2*n*x - 105*(b*e^8*n - 8*a*e^8)*x^4 + 840*(b*e^8*n*x^4 - b*d^8*
n)*log(e*sqrt(x) + d) + 8*(15*b*d*e^7*n*x^3 + 21*b*d^3*e^5*n*x^2 + 35*b*d^5
*e^3*n*x + 105*b*d^7*e*n)*sqrt(x))/e^8
```

giac [B] time = 0.20, size = 357, normalized size = 2.15

$$\frac{1}{3360}\left(840\,bx^4e\log(c)+840\,ax^4e+\left(840\left(\sqrt{x}e+d\right)^8e^{(-7)}\log\left(\sqrt{x}e+d\right)-6720\left(\sqrt{x}e+d\right)^7de^{(-7)}\log\left(\sqrt{x}e+d\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/3360*(840*b*x^4*e*log(c) + 840*a*x^4*e + (840*(sqrt(x)*e + d)^8*e^(-7)*lo
g(sqrt(x)*e + d) - 6720*(sqrt(x)*e + d)^7*d*e^(-7)*log(sqrt(x)*e + d) + 235
20*(sqrt(x)*e + d)^6*d^2*e^(-7)*log(sqrt(x)*e + d) - 47040*(sqrt(x)*e + d)^
5*d^3*e^(-7)*log(sqrt(x)*e + d) + 58800*(sqrt(x)*e + d)^4*d^4*e^(-7)*log(sq
rt(x)*e + d) - 47040*(sqrt(x)*e + d)^3*d^5*e^(-7)*log(sqrt(x)*e + d) + 2352
0*(sqrt(x)*e + d)^2*d^6*e^(-7)*log(sqrt(x)*e + d) - 6720*(sqrt(x)*e + d)*d^
7*e^(-7)*log(sqrt(x)*e + d) - 105*(sqrt(x)*e + d)^8*e^(-7) + 960*(sqrt(x)*e
 + d)^7*d*e^(-7) - 3920*(sqrt(x)*e + d)^6*d^2*e^(-7) + 9408*(sqrt(x)*e + d)
^5*d^3*e^(-7) - 14700*(sqrt(x)*e + d)^4*d^4*e^(-7) + 15680*(sqrt(x)*e + d)^
3*d^5*e^(-7) - 11760*(sqrt(x)*e + d)^2*d^6*e^(-7) + 6720*(sqrt(x)*e + d)*d^
7*e^(-7))*b*n)*e^(-1)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\left(e\sqrt{x}+d\right)^{n}\right)+a\right)x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(e*x^(1/2)+d)^n)),x)
```

```
[Out] int(x^3*(a+b*ln(c*(e*x^(1/2)+d)^n)),x)
```

maxima [A] time = 0.50, size = 128, normalized size = 0.77

$$\frac{1}{4}bx^{4}\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+\frac{1}{4}ax^{4}-\frac{1}{3360}ben\left(\frac{840\,d^{8}\log\left(e\sqrt{x}+d\right)}{e^{9}}+\frac{105\,e^{7}x^{4}-120\,de^{6}x^{\frac{7}{2}}+140\,d^{2}e^{5}x^{3}-168\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log((e*sqrt(x) + d)^n*c) + 1/4*a*x^4 - 1/3360*b*e*n*(840*d^8*log(
e*sqrt(x) + d)/e^9 + (105*e^7*x^4 - 120*d*e^6*x^(7/2) + 140*d^2*e^5*x^3 - 1
68*d^3*e^4*x^(5/2) + 210*d^4*e^3*x^2 - 280*d^5*e^2*x^(3/2) + 420*d^6*e*x -
840*d^7*sqrt(x))/e^8)
```

mupad [B] time = 0.51, size = 137, normalized size = 0.83

$$\frac{a\,x^{4}}{4}-\frac{b\,n\,x^{4}}{32}+\frac{b\,x^{4}\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^{n}\right)}{4}+\frac{b\,d\,n\,x^{7/2}}{28\,e}-\frac{b\,d^{6}\,n\,x}{8\,e^{6}}-\frac{b\,d^{8}\,n\,\ln\left(d+e\,\sqrt{x}\right)}{4\,e^{8}}-\frac{b\,d^{2}\,n\,x^{3}}{24\,e^{2}}-\frac{b\,d^{4}\,n\,x^{2}}{16\,e^{4}}+\frac{b\,d^{3}\,n\,x^{5}}{20\,e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/2))^n)),x)
```

```
[Out] (a*x^4)/4 - (b*n*x^4)/32 + (b*x^4*log(c*(d + e*x^(1/2))^n))/4 + (b*d*n*x^(7
/2))/(28*e) - (b*d^6*n*x)/(8*e^6) - (b*d^8*n*log(d + e*x^(1/2)))/(4*e^8) -
(b*d^2*n*x^3)/(24*e^2) - (b*d^4*n*x^2)/(16*e^4) + (b*d^3*n*x^(5/2))/(20*e^3
) + (b*d^5*n*x^(3/2))/(12*e^5) + (b*d^7*n*x^(1/2))/(4*e^7)
```

sympy [A] time = 24.20, size = 155, normalized size = 0.93

$$\frac{ax^{4}}{4}+b\left(-\frac{en\left(\frac{2d^{8}\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for}\ e=0\\ \frac{\log\left(d+e\sqrt{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^{8}}-\frac{2d^{7}\sqrt{x}}{e^{8}}+\frac{d^{6}x}{e^{7}}-\frac{2d^{5}x^{\frac{3}{2}}}{3e^{6}}+\frac{d^{4}x^{2}}{2e^{5}}-\frac{2d^{3}x^{\frac{5}{2}}}{5e^{4}}+\frac{d^{2}x^{3}}{3e^{3}}-\frac{2dx^{\frac{7}{2}}}{7e^{2}}+\frac{x^{4}}{4e}\right)}{8}+\frac{x^{4}\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/2))**n)),x)
```

```
[Out] a*x**4/4 + b*(-e*n*(2*d**8*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqrt
(x))/e, True))/e**8 - 2*d**7*sqrt(x)/e**8 + d**6*x/e**7 - 2*d**5*x**(3/2)/(
3*e**6) + d**4*x**2/(2*e**5) - 2*d**3*x**(5/2)/(5*e**4) + d**2*x**3/(3*e**3
) - 2*d*x**(7/2)/(7*e**2) + x**4/(4*e))/8 + x**4*log(c*(d + e*sqrt(x))**n)/
4)
```

3.401 $\int x^2 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right) \, dx$

Optimal. Leaf size=134

$$\frac{1}{3} x^3 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right)-\frac{b d^6 n \log \left(d+e \sqrt{x}\right)}{3 e^6}+\frac{b d^5 n \sqrt{x}}{3 e^5}-\frac{b d^4 n x}{6 e^4}+\frac{b d^3 n x^{3/2}}{9 e^3}-\frac{b d^2 n x^2}{12 e^2}+\frac{b d n x^{5/2}}{15 e}-\frac{1}{18} b n x^3$$

```
[Out] -1/6*b*d^4*n*x/e^4+1/9*b*d^3*n*x^(3/2)/e^3-1/12*b*d^2*n*x^2/e^2+1/15*b*d*n*
x^(5/2)/e-1/18*b*n*x^3-1/3*b*d^6*n*ln(d+e*x^(1/2))/e^6+1/3*x^3*(a+b*ln(c*(d
+e*x^(1/2))^n))+1/3*b*d^5*n*x^(1/2)/e^5
```

Rubi [A] time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$\frac{1}{3} x^3 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right)+\frac{b d^3 n x^{3/2}}{9 e^3}-\frac{b d^2 n x^2}{12 e^2}+\frac{b d^5 n \sqrt{x}}{3 e^5}-\frac{b d^4 n x}{6 e^4}-\frac{b d^6 n \log \left(d+e \sqrt{x}\right)}{3 e^6}+\frac{b d n x^{5/2}}{15 e}-\frac{1}{18} b n x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (b*d^5*n*Sqrt[x])/(3*e^5) - (b*d^4*n*x)/(6*e^4) + (b*d^3*n*x^(3/2))/(9*e^3)
 - (b*d^2*n*x^2)/(12*e^2) + (b*d*n*x^(5/2))/(15*e) - (b*n*x^3)/18 - (b*d^6*
n*Log[d + e*Sqrt[x]])/(3*e^6) + (x^3*(a + b*Log[c*(d + e*Sqrt[x])^n]))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx = 2\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)dx, x, \sqrt{x}\right)$$
$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) - \frac{1}{3}(ben)\,\text{Subst}\left(\int \frac{x^6}{d+ex}dx, x, \sqrt{x}\right)$$
$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) - \frac{1}{3}(ben)\,\text{Subst}\left(\int \left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3x^2}{e^4}+\right.\right.$$
$$= \frac{bd^5n\sqrt{x}}{3e^5} - \frac{bd^4nx}{6e^4} + \frac{bd^3nx^{3/2}}{9e^3} - \frac{bd^2nx^2}{12e^2} + \frac{bdnx^{5/2}}{15e} - \frac{1}{18}bnx^3 - \frac{bd^6n\log(}{3e}$$

Mathematica [A] time = 0.09, size = 131, normalized size = 0.98

$$\frac{ax^3}{3}+\frac{1}{3}bx^3\log\left(c\left(d+e\sqrt{x}\right)^n\right)-\frac{1}{3}ben\left(\frac{d^6\log\left(d+e\sqrt{x}\right)}{e^7}-\frac{d^5\sqrt{x}}{e^6}+\frac{d^4x}{2e^5}-\frac{d^3x^{3/2}}{3e^4}+\frac{d^2x^2}{4e^3}-\frac{dx^{5/2}}{5e^2}+\frac{x^3}{6e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (a*x^3)/3 - (b*e*n*(-((d^5*Sqrt[x])/e^6) + (d^4*x)/(2*e^5) - (d^3*x^(3/2))/
(3*e^4) + (d^2*x^2)/(4*e^3) - (d*x^(5/2))/(5*e^2) + x^3/(6*e) + (d^6*Log[d
+ e*Sqrt[x]])/e^7))/3 + (b*x^3*Log[c*(d + e*Sqrt[x])^n])/3
```

fricas [A] time = 0.43, size = 122, normalized size = 0.91

$$\frac{60\,be^6x^3\log(c) - 15\,bd^2e^4nx^2 - 30\,bd^4e^2nx - 10\left(be^6n - 6\,ae^6\right)x^3 + 60\left(be^6nx^3 - bd^6n\right)\log\left(e\sqrt{x}+d\right) + 4\left(3\,bde^5\right.}{180\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/180*(60*b*e^6*x^3*log(c) - 15*b*d^2*e^4*n*x^2 - 30*b*d^4*e^2*n*x - 10*(b*
e^6*n - 6*a*e^6)*x^3 + 60*(b*e^6*n*x^3 - b*d^6*n)*log(e*sqrt(x) + d) + 4*(3
*b*d*e^5*n*x^2 + 5*b*d^3*e^3*n*x + 15*b*d^5*e*n)*sqrt(x))/e^6
```

giac [B] time = 0.21, size = 271, normalized size = 2.02

$$\frac{1}{180}\left(60\,bx^3e\log(c) + 60\,ax^3e + \left(60\left(\sqrt{x}e+d\right)^6e^{(-5)}\log\left(\sqrt{x}e+d\right) - 360\left(\sqrt{x}e+d\right)^5de^{(-5)}\log\left(\sqrt{x}e+d\right) + 900\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/180*(60*b*x^3*e*log(c) + 60*a*x^3*e + (60*(sqrt(x)*e + d)^6*e^(-5)*log(sq
rt(x)*e + d) - 360*(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt(x)*e + d) + 900*(sqr
t(x)*e + d)^4*d^2*e^(-5)*log(sqrt(x)*e + d) - 1200*(sqrt(x)*e + d)^3*d^3*e^
(-5)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x)*e +
d) - 360*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x)*e + d) - 10*(sqrt(x)*e + d)
^6*e^(-5) + 72*(sqrt(x)*e + d)^5*d*e^(-5) - 225*(sqrt(x)*e + d)^4*d^2*e^(-5
) + 400*(sqrt(x)*e + d)^3*d^3*e^(-5) - 450*(sqrt(x)*e + d)^2*d^4*e^(-5) + 3
60*(sqrt(x)*e + d)*d^5*e^(-5))*b*n)*e^(-1)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\left(e\sqrt{x}+d\right)^n\right)+a\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*(e*x^(1/2)+d)^n)+a),x)
```

```
[Out] int(x^2*(b*ln(c*(e*x^(1/2)+d)^n)+a),x)
```

maxima [A] time = 0.49, size = 106, normalized size = 0.79

$$\frac{1}{3}bx^3\log\left(\left(e\sqrt{x}+d\right)^n c\right)+\frac{1}{3}ax^3-\frac{1}{180}ben\left(\frac{60\,d^6\log\left(e\sqrt{x}+d\right)}{e^7}+\frac{10\,e^5x^3-12\,de^4x^{\frac{5}{2}}+15\,d^2e^3x^2-20\,d^3e^2x^{\frac{3}{2}}-\cdots}{e^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3*log((e*sqrt(x) + d)^n*c) + 1/3*a*x^3 - 1/180*b*e*n*(60*d^6*log(e*
sqrt(x) + d)/e^7 + (10*e^5*x^3 - 12*d*e^4*x^(5/2) + 15*d^2*e^3*x^2 - 20*d^3
*e^2*x^(3/2) + 30*d^4*e*x - 60*d^5*sqrt(x))/e^6)
```

mupad [B] time = 0.42, size = 111, normalized size = 0.83

$$\frac{a\,x^3}{3}-\frac{b\,n\,x^3}{18}+\frac{b\,x^3\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)}{3}+\frac{b\,d\,n\,x^{5/2}}{15\,e}-\frac{b\,d^4\,n\,x}{6\,e^4}-\frac{b\,d^6\,n\,\ln\left(d+e\,\sqrt{x}\right)}{3\,e^6}-\frac{b\,d^2\,n\,x^2}{12\,e^2}+\frac{b\,d^3\,n\,x^{3/2}}{9\,e^3}+\frac{b\,d^5\,n\cdots}{3\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/2))^n)),x)
```

```
[Out] (a*x^3)/3 - (b*n*x^3)/18 + (b*x^3*log(c*(d + e*x^(1/2))^n))/3 + (b*d*n*x^(5
/2))/(15*e) - (b*d^4*n*x)/(6*e^4) - (b*d^6*n*log(d + e*x^(1/2)))/(3*e^6) -
(b*d^2*n*x^2)/(12*e^2) + (b*d^3*n*x^(3/2))/(9*e^3) + (b*d^5*n*x^(1/2))/(3*e
^5)
```

sympy [A] time = 9.66, size = 128, normalized size = 0.96

$$\frac{ax^3}{3}+b\left(-\frac{en\left(\frac{2d^6\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^6}-\frac{2d^5\sqrt{x}}{e^6}+\frac{d^4x}{e^5}-\frac{2d^3x^{\frac{3}{2}}}{3e^4}+\frac{d^2x^2}{2e^3}-\frac{2dx^{\frac{5}{2}}}{5e^2}+\frac{x^3}{3e}\right)}{6}+\frac{x^3\log\left(c\left(d+e\sqrt{x}\right)^n\cdots\right)}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**n)),x)
```

```
[Out] a*x**3/3 + b*(-e*n*(2*d**6*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqrt
(x))/e, True))/e**6 - 2*d**5*sqrt(x)/e**6 + d**4*x/e**5 - 2*d**3*x**(3/2)/(
3*e**4) + d**2*x**2/(2*e**3) - 2*d*x**(5/2)/(5*e**2) + x**3/(3*e))/6 + x**3
*log(c*(d + e*sqrt(x))**n)/3)
```

3.402 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx$

Optimal. Leaf size=102

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-\frac{bd^4n\log\left(d+e\sqrt{x}\right)}{2e^4}+\frac{bd^3n\sqrt{x}}{2e^3}-\frac{bd^2nx}{4e^2}+\frac{bdnx^{3/2}}{6e}-\frac{1}{8}bnx^2$$

```
[Out] -1/4*b*d^2*n*x/e^2+1/6*b*d*n*x^(3/2)/e-1/8*b*n*x^2-1/2*b*d^4*n*ln(d+e*x^(1/
2))/e^4+1/2*x^2*(a+b*ln(c*(d+e*x^(1/2))^n))+1/2*b*d^3*n*x^(1/2)/e^3
```

Rubi [A] time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2454, 2395, 43}

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+\frac{bd^3n\sqrt{x}}{2e^3}-\frac{bd^2nx}{4e^2}-\frac{bd^4n\log\left(d+e\sqrt{x}\right)}{2e^4}+\frac{bdnx^{3/2}}{6e}-\frac{1}{8}bnx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (b*d^3*n*Sqrt[x])/(2*e^3) - (b*d^2*n*x)/(4*e^2) + (b*d*n*x^(3/2))/(6*e) - (
b*n*x^2)/8 - (b*d^4*n*Log[d + e*Sqrt[x]])/(2*e^4) + (x^2*(a + b*Log[c*(d +
e*Sqrt[x])^n]))/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx = 2\,\text{Subst}\left(\int x^3\left(a+b\log\left(c(d+ex)^n\right)\right)dx, x, \sqrt{x}\right)$$
$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) - \frac{1}{2}(ben)\,\text{Subst}\left(\int \frac{x^4}{d+ex}dx, x, \sqrt{x}\right)$$
$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) - \frac{1}{2}(ben)\,\text{Subst}\left(\int\left(-\frac{d^3}{e^4}+\frac{d^2x}{e^3}-\frac{dx^2}{e^2}+\right.\right.$$
$$= \frac{bd^3n\sqrt{x}}{2e^3} - \frac{bd^2nx}{4e^2} + \frac{bdnx^{3/2}}{6e} - \frac{1}{8}bnx^2 - \frac{bd^4n\log\left(d+e\sqrt{x}\right)}{2e^4} + \frac{1}{2}x^2\left(a+\right.$$

Mathematica [A] time = 0.03, size = 107, normalized size = 1.05

$$\frac{ax^2}{2} + \frac{1}{2}bx^2\log\left(c\left(d+e\sqrt{x}\right)^n\right) - \frac{bd^4n\log\left(d+e\sqrt{x}\right)}{2e^4} + \frac{bd^3n\sqrt{x}}{2e^3} - \frac{bd^2nx}{4e^2} + \frac{bdnx^{3/2}}{6e} - \frac{1}{8}bnx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (b*d^3*n*Sqrt[x])/(2*e^3) - (b*d^2*n*x)/(4*e^2) + (b*d*n*x^(3/2))/(6*e) + (
a*x^2)/2 - (b*n*x^2)/8 - (b*d^4*n*Log[d + e*Sqrt[x]])/(2*e^4) + (b*x^2*Log[
c*(d + e*Sqrt[x])^n])/2
```

fricas [A] time = 0.44, size = 95, normalized size = 0.93

$$\frac{12\,be^4x^2\log(c) - 6\,bd^2e^2nx - 3\left(be^4n - 4\,ae^4\right)x^2 + 12\left(be^4nx^2 - bd^4n\right)\log\left(e\sqrt{x}+d\right) + 4\left(bde^3nx + 3\,bd^3en\right)\sqrt{}}{24\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/24*(12*b*e^4*x^2*log(c) - 6*b*d^2*e^2*n*x - 3*(b*e^4*n - 4*a*e^4)*x^2 + 1
2*(b*e^4*n*x^2 - b*d^4*n)*log(e*sqrt(x) + d) + 4*(b*d*e^3*n*x + 3*b*d^3*e*n
)*sqrt(x))/e^4
```

giac [B] time = 0.20, size = 185, normalized size = 1.81

$$\frac{1}{24}\left(12\,bx^2e\log(c) + 12\,ax^2e + \left(12\left(\sqrt{x}e+d\right)^4e^{(-3)}\log\left(\sqrt{x}e+d\right) - 48\left(\sqrt{x}e+d\right)^3de^{(-3)}\log\left(\sqrt{x}e+d\right) + 72\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/24*(12*b*x^2*e*log(c) + 12*a*x^2*e + (12*(sqrt(x)*e + d)^4*e^(-3)*log(sqr
t(x)*e + d) - 48*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*e + d) + 72*(sqrt(x
)*e + d)^2*d^2*e^(-3)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)*d^3*e^(-3)*lo
g(sqrt(x)*e + d) - 3*(sqrt(x)*e + d)^4*e^(-3) + 16*(sqrt(x)*e + d)^3*d*e^(-
3) - 36*(sqrt(x)*e + d)^2*d^2*e^(-3) + 48*(sqrt(x)*e + d)*d^3*e^(-3))*b*n)*
e^(-1)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(e\sqrt{x}+d\right)^n\right)+a\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(e*x^(1/2)+d)^n)+a),x)
```

[Out] int(x*(b*ln(c*(e*x^(1/2)+d)^n)+a),x)

maxima [A] time = 0.50, size = 84, normalized size = 0.82

$$-\frac{1}{24}\,ben\left(\frac{12\,d^4\log\left(e\sqrt{x}+d\right)}{e^5}+\frac{3\,e^3x^2-4\,de^2x^{\frac{3}{2}}+6\,d^2ex-12\,d^3\sqrt{x}}{e^4}\right)+\frac{1}{2}\,bx^2\log\left(\left(e\sqrt{x}+d\right)^nc\right)+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="maxima")

[Out] -1/24*b*e*n*(12*d^4*log(e*sqrt(x) + d)/e^5 + (3*e^3*x^2 - 4*d*e^2*x^(3/2) + 6*d^2*e*x - 12*d^3*sqrt(x))/e^4) + 1/2*b*x^2*log((e*sqrt(x) + d)^n*c) + 1/2*a*x^2

mupad [B] time = 0.41, size = 85, normalized size = 0.83

$$\frac{a\,x^2}{2}-\frac{b\,n\,x^2}{8}+\frac{b\,x^2\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{2}-\frac{b\,d^2\,n\,x}{4\,e^2}+\frac{b\,d\,n\,x^{3/2}}{6\,e}-\frac{b\,d^4\,n\,\ln\left(d+e\,\sqrt{x}\right)}{2\,e^4}+\frac{b\,d^3\,n\,\sqrt{x}}{2\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + b*log(c*(d + e*x^(1/2))^n)),x)

[Out] (a*x^2)/2 - (b*n*x^2)/8 + (b*x^2*log(c*(d + e*x^(1/2))^n))/2 - (b*d^2*n*x)/(4*e^2) + (b*d*n*x^(3/2))/(6*e) - (b*d^4*n*log(d + e*x^(1/2)))/(2*e^4) + (b*d^3*n*x^(1/2))/(2*e^3)

sympy [A] time = 4.68, size = 100, normalized size = 0.98

$$\frac{ax^2}{2}+b\left(-\frac{en\left(\frac{2d^4\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^4}-\frac{2d^3\sqrt{x}}{e^4}+\frac{d^2x}{e^3}-\frac{2dx^{\frac{3}{2}}}{3e^2}+\frac{x^2}{2e}\right)}{4}+\frac{x^2\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))**n)),x)

[Out] a*x**2/2 + b*(-e*n*(2*d**4*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqrt(x))/e, True))/e**4 - 2*d**3*sqrt(x)/e**4 + d**2*x/e**3 - 2*d*x**(3/2)/(3*e**2) + x**2/(2*e))/4 + x**2*log(c*(d + e*sqrt(x))**n)/2)

3.403 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx$

Optimal. Leaf size=60

$$ax+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)-\frac{bd^2n\log\left(d+e\sqrt{x}\right)}{e^2}+\frac{bdn\sqrt{x}}{e}-\frac{bnx}{2}$$

```
[Out] a*x-1/2*b*n*x-b*d^2*n*ln(d+e*x^(1/2))/e^2+b*x*ln(c*(d+e*x^(1/2))^n)+b*d*n*x
^(1/2)/e
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2448, 266, 43}

$$ax+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)-\frac{bd^2n\log\left(d+e\sqrt{x}\right)}{e^2}+\frac{bdn\sqrt{x}}{e}-\frac{bnx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e*Sqrt[x])^n],x]
```

```
[Out] (b*d*n*Sqrt[x])/e + a*x - (b*n*x)/2 - (b*d^2*n*Log[d + e*Sqrt[x]])/e^2 + b*
x*Log[c*(d + e*Sqrt[x])^n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx &= ax+b\int \log\left(c\left(d+e\sqrt{x}\right)^n\right)dx\\
&= ax+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)-\frac{1}{2}(ben)\int \frac{\sqrt{x}}{d+e\sqrt{x}}\,dx\\
&= ax+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)-(ben)\,\text{Subst}\left(\int \frac{x^2}{d+ex}\,dx,x,\sqrt{x}\right)\\
&= ax+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)-(ben)\,\text{Subst}\left(\int \left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx,\right.\\
&= \frac{bdn\sqrt{x}}{e}+ax-\frac{bnx}{2}-\frac{bd^2n\log\left(d+e\sqrt{x}\right)}{e^2}+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 1.00

$$ax + bx\log\left(c\left(d + e\sqrt{x}\right)^n\right) - \frac{bd^2n\log\left(d + e\sqrt{x}\right)}{e^2} + \frac{bdn\sqrt{x}}{e} - \frac{bnx}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e*Sqrt[x])^n],x]
```

```
[Out] (b*d*n*Sqrt[x])/e + a*x - (b*n*x)/2 - (b*d^2*n*Log[d + e*Sqrt[x]])/e^2 + b*
x*Log[c*(d + e*Sqrt[x])^n]
```

fricas [A] time = 0.42, size = 65, normalized size = 1.08

$$\frac{2\,be^2x\log(c) + 2\,bden\sqrt{x} - \left(be^2n - 2\,ae^2\right)x + 2\left(be^2nx - bd^2n\right)\log\left(e\sqrt{x} + d\right)}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(1/2))^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*e^2*x*log(c) + 2*b*d*e*n*sqrt(x) - (b*e^2*n - 2*a*e^2)*x + 2*(b*e^
2*n*x - b*d^2*n)*log(e*sqrt(x) + d))/e^2
```

giac [B] time = 0.17, size = 107, normalized size = 1.78

$$\frac{1}{2}\left(\left(2\left(\sqrt{x}e + d\right)^2\log\left(\sqrt{x}e + d\right) - 4\left(\sqrt{x}e + d\right)d\log\left(\sqrt{x}e + d\right) - \left(\sqrt{x}e + d\right)^2 + 4\left(\sqrt{x}e + d\right)d\right)ne^{(-1)} + 2\left(\left(\sqrt{x}e -\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(1/2))^n),x, algorithm="giac")
```

```
[Out] 1/2*((2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) - 4*(sqrt(x)*e + d)*d*log(sqrt
(x)*e + d) - (sqrt(x)*e + d)^2 + 4*(sqrt(x)*e + d)*d)*n*e^(-1) + 2*((sqrt(x
)*e + d)^2 - 2*(sqrt(x)*e + d)*d)*e^(-1)*log(c))*b*e^(-1) + a*x
```

maple [A] time = 0.08, size = 53, normalized size = 0.88

$$-\frac{b\,d^2n\ln\left(e\sqrt{x} + d\right)}{e^2} - \frac{bnx}{2} + bx\ln\left(c\left(e\sqrt{x} + d\right)^n\right) + \frac{bdn\sqrt{x}}{e} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*ln(c*(e*x^(1/2)+d)^n)+a,x)
```

```
[Out] a*x-1/2*b*n*x-b*d^2*n*ln(e*x^(1/2)+d)/e^2+b*x*ln(c*(e*x^(1/2)+d)^n)+b*d*n*x
^(1/2)/e
```

maxima [A] time = 0.47, size = 57, normalized size = 0.95

$$-\frac{1}{2}\left(en\left(\frac{2\,d^2\log\left(e\sqrt{x} + d\right)}{e^3} + \frac{ex - 2\,d\sqrt{x}}{e^2}\right) - 2\,x\log\left(\left(e\sqrt{x} + d\right)^nc\right)\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(1/2))^n),x, algorithm="maxima")
```

```
[Out] -1/2*(e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 + (e*x - 2*d*sqrt(x))/e^2) - 2*x*lo
g((e*sqrt(x) + d)^n*c))*b + a*x
```

mupad [B] time = 0.43, size = 52, normalized size = 0.87

$$a\,x + b\,x\,\ln\left(c\left(d + e\,\sqrt{x}\right)^n\right) - \frac{b\,n\,\left(e^2\,x + 2\,d^2\,\ln\left(d + e\,\sqrt{x}\right) - 2\,d\,e\,\sqrt{x}\right)}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*log(c*(d + e*x^(1/2))^n),x)
```

```
[Out] a*x + b*x*log(c*(d + e*x^(1/2))^n) - (b*n*(e^2*x + 2*d^2*log(d + e*x^(1/2))
 - 2*d*e*x^(1/2)))/(2*e^2)
```

sympy [A] time = 1.86, size = 66, normalized size = 1.10

$$ax + b\left(-\frac{en\left(\frac{2d^2\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^2}-\frac{2d\sqrt{x}}{e^2}+\frac{x}{e}\right)}{2}+x\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e*x**(1/2))**n),x)
```

```
[Out] a*x + b*(-e*n*(2*d**2*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqrt(x))/
e, True))/e**2 - 2*d*sqrt(x)/e**2 + x/e)/2 + x*log(c*(d + e*sqrt(x))**n))
```

3.404 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=51

$$2\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+2bn\,\mathrm{Li}_2\left(\frac{\sqrt{x}\,e}{d}+1\right)$$

```
[Out] 2*ln(-e*x^(1/2)/d)*(a+b*ln(c*(d+e*x^(1/2))^n))+2*b*n*polylog(2,1+e*x^(1/2)/
d)
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2454, 2394, 2315}

$$2bn\,\mathrm{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)+2\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x,x]
```

```
[Out] 2*(a + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)] + 2*b*n*PolyLog[2,
 1 + (e*Sqrt[x])/d]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}\,dx &= 2\,\mathrm{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx,x,\sqrt{x}\right)\\
&= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)-(2ben)\,\mathrm{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,\sqrt{x}\right)\\
&= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)+2bn\,\mathrm{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 53, normalized size = 1.04

$$a\log(x)+2b\log\left(-\frac{e\sqrt{x}}{d}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)+2bn\text{Li}_2\left(\frac{d+e\sqrt{x}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x,x]
```

```
[Out] 2*b*Log[c*(d + e*Sqrt[x])^n]*Log[-((e*Sqrt[x])/d)] + a*Log[x] + 2*b*n*PolyL
og[2, (d + e*Sqrt[x])/d]
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x,x, algorithm="fricas")
```

```
[Out] integral((b*log((e*sqrt(x) + d)^n*c) + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)/x, x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int\frac{b\ln\left(c\left(e\sqrt{x}+d\right)^n\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/2)+d)^n)+a)/x,x)
```

```
[Out] int((b*ln(c*(e*x^(1/2)+d)^n)+a)/x,x)
```

maxima [B] time = 1.14, size = 107, normalized size = 2.10

$$-2\left(\log\left(\frac{e\sqrt{x}}{d}+1\right)\log\left(\sqrt{x}\right)+\text{Li}_2\left(-\frac{e\sqrt{x}}{d}\right)\right)bn+\frac{bdn\log\left(e\sqrt{x}+d\right)\log(x)+\left(bd\log(c)+ad\right)\log(x)-\frac{benx\log}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x,x, algorithm="maxima")
```

```
[Out] -2*(log(e*sqrt(x)/d + 1)*log(sqrt(x)) + dilog(-e*sqrt(x)/d))*b*n + (b*d*n*l
og(e*sqrt(x) + d)*log(x) + (b*d*log(c) + a*d)*log(x) - (b*e*n*x*log(x) - 2*
b*e*n*x)/sqrt(x))/d + 2*(b*e*n*sqrt(x)*log(sqrt(x)) - b*e*n*sqrt(x))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \log \left(c \left(d + e\sqrt{x}\right)^{n}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))/x, x)
```

3.405 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=70

$$-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}+\frac{be^2n\log\left(d+e\sqrt{x}\right)}{d^2}-\frac{be^2n\log(x)}{2d^2}-\frac{ben}{d\sqrt{x}}$$

```
[Out] -1/2*b*e^2*n*ln(x)/d^2+b*e^2*n*ln(d+e*x^(1/2))/d^2+(-a-b*ln(c*(d+e*x^(1/2))
^n))/x-b*e*n/d/x^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}+\frac{be^2n\log\left(d+e\sqrt{x}\right)}{d^2}-\frac{be^2n\log(x)}{2d^2}-\frac{ben}{d\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^2,x]
```

```
[Out] -((b*e*n)/(d*Sqrt[x])) + (b*e^2*n*Log[d + e*Sqrt[x]])/d^2 - (a + b*Log[c*(d
 + e*Sqrt[x])^n])/x - (b*e^2*n*Log[x])/(2*d^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx = 2\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x} + (ben)\,\text{Subst}\left(\int \frac{1}{x^2(d+ex)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x} + (ben)\,\text{Subst}\left(\int \left(\frac{1}{dx^2}-\frac{e}{d^2x}+\frac{e^2}{d^2(d+ex)}\right)dx, x, \sqrt{x}\right)$$
$$= -\frac{ben}{d\sqrt{x}} + \frac{be^2n\log\left(d+e\sqrt{x}\right)}{d^2} - \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x} - \frac{be^2n\log(x)}{2d^2}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.96

$$-\frac{a}{x} - \frac{b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x} + ben\left(\frac{e\log\left(d+e\sqrt{x}\right)}{d^2} - \frac{e\log(x)}{2d^2} - \frac{1}{d\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^2,x]
```

```
[Out] -(a/x) - (b*Log[c*(d + e*Sqrt[x])^n])/x + b*e*n*(-(1/(d*Sqrt[x])) + (e*Log[
d + e*Sqrt[x]])/d^2 - (e*Log[x])/(2*d^2))
```

fricas [A] time = 0.44, size = 65, normalized size = 0.93

$$-\frac{be^2nx\log\left(\sqrt{x}\right) + bden\sqrt{x} + bd^2\log(c) + ad^2 - \left(be^2nx - bd^2n\right)\log\left(e\sqrt{x}+d\right)}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^2,x, algorithm="fricas")
```

```
[Out] -(b*e^2*n*x*log(sqrt(x)) + b*d*e*n*sqrt(x) + b*d^2*log(c) + a*d^2 - (b*e^2*
n*x - b*d^2*n)*log(e*sqrt(x) + d))/(d^2*x)
```

giac [B] time = 0.18, size = 187, normalized size = 2.67

$$\frac{\left(\left(\sqrt{x}e+d\right)^2bne^3\log\left(\sqrt{x}e+d\right) - 2\left(\sqrt{x}e+d\right)bdne^3\log\left(\sqrt{x}e+d\right) - \left(\sqrt{x}e+d\right)^2bne^3\log\left(\sqrt{x}e\right) + 2\left(\sqrt{x}e+d\right)b\right.}{\left(\sqrt{x}e+d\right)^2d^2 - 2\left(\sqrt{x}e+d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^2,x, algorithm="giac")
```

```
[Out] ((sqrt(x)*e + d)^2*b*n*e^3*log(sqrt(x)*e + d) - 2*(sqrt(x)*e + d)*b*d*n*e^3
*log(sqrt(x)*e + d) - (sqrt(x)*e + d)^2*b*n*e^3*log(sqrt(x)*e) + 2*(sqrt(x)
*e + d)*b*d*n*e^3*log(sqrt(x)*e) - b*d^2*n*e^3*log(sqrt(x)*e) - (sqrt(x)*e
+ d)*b*d*n*e^3 + b*d^2*n*e^3 - b*d^2*e^3*log(c) - a*d^2*e^3)*e^(-1)/((sqrt(
x)*e + d)^2*d^2 - 2*(sqrt(x)*e + d)*d^3 + d^4)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{b\ln\left(c\left(e\sqrt{x}+d\right)^n\right)+a}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/2)+d)^n)+a)/x^2,x)
```

```
[Out] int((b*ln(c*(e*x^(1/2)+d)^n)+a)/x^2,x)
```

maxima [A] time = 0.48, size = 61, normalized size = 0.87

$$\frac{1}{2}\,ben\left(\frac{2\,e\log\left(e\sqrt{x}+d\right)}{d^2}-\frac{e\log(x)}{d^2}-\frac{2}{d\sqrt{x}}\right)-\frac{b\log\left(\left(e\sqrt{x}+d\right)^n c\right)}{x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*e*n*(2*e*log(e*sqrt(x) + d)/d^2 - e*log(x)/d^2 - 2/(d*sqrt(x))) - b*l
og((e*sqrt(x) + d)^n*c)/x - a/x
```

mupad [B] time = 0.80, size = 58, normalized size = 0.83

$$\frac{2\,b\,e^2\,n\,\mathrm{atanh}\left(\frac{2\,e\,\sqrt{x}}{d}+1\right)}{d^2}-\frac{b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)}{x}-\frac{b\,e\,n}{d\,\sqrt{x}}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))/x^2,x)
```

```
[Out] (2*b*e^2*n*atanh((2*e*x^(1/2))/d + 1))/d^2 - (b*log(c*(d + e*x^(1/2))^n))/x
 - (b*e*n)/(d*x^(1/2)) - a/x
```

sympy [A] time = 63.06, size = 554, normalized size = 7.91

$$\begin{cases} -\frac{2ad^3\sqrt{x}}{2d^3x^{\frac{3}{2}}+2d^2ex^2}-\frac{2ad^2ex}{2d^3x^{\frac{3}{2}}+2d^2ex^2}-\frac{2bd^3n\sqrt{x}\log\left(d+e\sqrt{x}\right)}{2d^3x^{\frac{3}{2}}+2d^2ex^2}-\frac{2bd^3\sqrt{x}\log(c)}{2d^3x^{\frac{3}{2}}+2d^2ex^2}-\frac{2bd^2enx\log\left(d+e\sqrt{x}\right)}{2d^3x^{\frac{3}{2}}+2d^2ex^2}-\frac{2bd^2enx}{2d^3x^{\frac{3}{2}}+2d^2ex^2}-\frac{2bd^2ex\log(}{2d^3x^{\frac{3}{2}}+2d^2e} \\ -\frac{a}{x}-\frac{bn\log(e)}{x}-\frac{bn\log(x)}{2x}-\frac{bn}{2x}-\frac{b\log(c)}{x} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x**2,x)
```

```
[Out] Piecewise((-2*a*d**3*sqrt(x)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) - 2*a*d**2*e
*x/(2*d**3*x**(3/2) + 2*d**2*e*x**2) - 2*b*d**3*n*sqrt(x)*log(d + e*sqrt(x)
)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) - 2*b*d**3*sqrt(x)*log(c)/(2*d**3*x**(3
/2) + 2*d**2*e*x**2) - 2*b*d**2*e*n*x*log(d + e*sqrt(x))/(2*d**3*x**(3/2) +
 2*d**2*e*x**2) - 2*b*d**2*e*n*x/(2*d**3*x**(3/2) + 2*d**2*e*x**2) - 2*b*d*
*2*e*x*log(c)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) - b*d*e**2*n*x**(3/2)*log(x
)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) + 2*b*d*e**2*n*x**(3/2)*log(d + e*sqrt(
x))/(2*d**3*x**(3/2) + 2*d**2*e*x**2) - 2*b*d*e**2*n*x**(3/2)/(2*d**3*x**(3
/2) + 2*d**2*e*x**2) + 2*b*d*e**2*x**(3/2)*log(c)/(2*d**3*x**(3/2) + 2*d**2
*e*x**2) - b*e**3*n*x**2*log(x)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) + 2*b*e**
3*n*x**2*log(d + e*sqrt(x))/(2*d**3*x**(3/2) + 2*d**2*e*x**2) + 2*b*e**3*x*
*2*log(c)/(2*d**3*x**(3/2) + 2*d**2*e*x**2), Ne(d, 0)), (-a/x - b*n*log(e)/
x - b*n*log(x)/(2*x) - b*n/(2*x) - b*log(c)/x, True))
```

3.406 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=109

$$-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2}+\frac{be^4n\log\left(d+e\sqrt{x}\right)}{2d^4}-\frac{be^4n\log(x)}{4d^4}-\frac{be^3n}{2d^3\sqrt{x}}+\frac{be^2n}{4d^2x}-\frac{ben}{6dx^{3/2}}$$

```
[Out] -1/6*b*e*n/d/x^(3/2)+1/4*b*e^2*n/d^2/x-1/4*b*e^4*n*ln(x)/d^4+1/2*b*e^4*n*ln
(d+e*x^(1/2))/d^4+1/2*(-a-b*ln(c*(d+e*x^(1/2))^n))/x^2-1/2*b*e^3*n/d^3/x^(1
/2)
```

Rubi [A] time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2}-\frac{be^3n}{2d^3\sqrt{x}}+\frac{be^2n}{4d^2x}+\frac{be^4n\log\left(d+e\sqrt{x}\right)}{2d^4}-\frac{be^4n\log(x)}{4d^4}-\frac{ben}{6dx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^3,x]
```

```
[Out] -(b*e*n)/(6*d*x^(3/2)) + (b*e^2*n)/(4*d^2*x) - (b*e^3*n)/(2*d^3*Sqrt[x]) +
(b*e^4*n*Log[d + e*Sqrt[x]])/(2*d^4) - (a + b*Log[c*(d + e*Sqrt[x])^n])/(2*
x^2) - (b*e^4*n*Log[x])/(4*d^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx = 2\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^5}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\,\text{Subst}\left(\int \frac{1}{x^4(d+ex)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\,\text{Subst}\left(\int \left(\frac{1}{dx^4} - \frac{e}{d^2x^3} + \frac{e^2}{d^3x^2} - \frac{e^3}{d^4x} +$$
$$= -\frac{ben}{6dx^{3/2}} + \frac{be^2n}{4d^2x} - \frac{be^3n}{2d^3\sqrt{x}} + \frac{be^4n\log\left(d+e\sqrt{x}\right)}{2d^4} - \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2}$$

Mathematica [A] time = 0.04, size = 104, normalized size = 0.95

$$-\frac{a}{2x^2} - \frac{b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2} + \frac{1}{2}ben\left(\frac{e^3\log\left(d+e\sqrt{x}\right)}{d^4} - \frac{e^3\log(x)}{2d^4} - \frac{e^2}{d^3\sqrt{x}} + \frac{e}{2d^2x} - \frac{1}{3dx^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*Log[c*(d + e*Sqrt[x])^n])/(2*x^2) + (b*e*n*(-1/3*1/(d*x^(3/
2)) + e/(2*d^2*x) - e^2/(d^3*Sqrt[x]) + (e^3*Log[d + e*Sqrt[x]])/d^4 - (e^3
*Log[x])/(2*d^4)))/2
```

fricas [A] time = 0.44, size = 97, normalized size = 0.89

$$-\frac{6\,be^4nx^2\log\left(\sqrt{x}\right) - 3\,bd^2e^2nx + 6\,bd^4\log(c) + 6\,ad^4 - 6\left(be^4nx^2 - bd^4n\right)\log\left(e\sqrt{x}+d\right) + 2\left(3\,bde^3nx + bd^3\right.}{12\,d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b*e^4*n*x^2*log(sqrt(x)) - 3*b*d^2*e^2*n*x + 6*b*d^4*log(c) + 6*a*
d^4 - 6*(b*e^4*n*x^2 - b*d^4*n)*log(e*sqrt(x) + d) + 2*(3*b*d*e^3*n*x + b*d
^3*e*n)*sqrt(x))/(d^4*x^2)
```

giac [B] time = 0.19, size = 366, normalized size = 3.36

$$\frac{\left(6\left(\sqrt{x}e+d\right)^4bne^5\log\left(\sqrt{x}e+d\right) - 24\left(\sqrt{x}e+d\right)^3bdne^5\log\left(\sqrt{x}e+d\right) + 36\left(\sqrt{x}e+d\right)^2bd^2ne^5\log\left(\sqrt{x}e+d\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/12*(6*(sqrt(x)*e + d)^4*b*n*e^5*log(sqrt(x)*e + d) - 24*(sqrt(x)*e + d)^3
*b*d*n*e^5*log(sqrt(x)*e + d) + 36*(sqrt(x)*e + d)^2*b*d^2*n*e^5*log(sqrt(x
)*e + d) - 24*(sqrt(x)*e + d)*b*d^3*n*e^5*log(sqrt(x)*e + d) - 6*(sqrt(x)*e
 + d)^4*b*n*e^5*log(sqrt(x)*e) + 24*(sqrt(x)*e + d)^3*b*d*n*e^5*log(sqrt(x)
*e) - 36*(sqrt(x)*e + d)^2*b*d^2*n*e^5*log(sqrt(x)*e) + 24*(sqrt(x)*e + d)*
b*d^3*n*e^5*log(sqrt(x)*e) - 6*b*d^4*n*e^5*log(sqrt(x)*e) - 6*(sqrt(x)*e +
d)^3*b*d*n*e^5 + 21*(sqrt(x)*e + d)^2*b*d^2*n*e^5 - 26*(sqrt(x)*e + d)*b*d^
```

```
3*n*e^5 + 11*b*d^4*n*e^5 - 6*b*d^4*e^5*log(c) - 6*a*d^4*e^5)*e^(-1)/((sqrt(
x)*e + d)^4*d^4 - 4*(sqrt(x)*e + d)^3*d^5 + 6*(sqrt(x)*e + d)^2*d^6 - 4*(sq
rt(x)*e + d)*d^7 + d^8)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{b \ln \left(c \left(e\sqrt{x}+d\right)^{n}\right)+a}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/2)+d)^n)+a)/x^3,x)
```

```
[Out] int((b*ln(c*(e*x^(1/2)+d)^n)+a)/x^3,x)
```

maxima [A] time = 0.49, size = 84, normalized size = 0.77

$$\frac{1}{12}\,ben\left(\frac{6\,e^{3}\log\left(e\sqrt{x}+d\right)}{d^{4}}-\frac{3\,e^{3}\log(x)}{d^{4}}-\frac{6\,e^{2}x-3\,de\sqrt{x}+2\,d^{2}}{d^{3}x^{\frac{3}{2}}}\right)-\frac{b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)}{2\,x^{2}}-\frac{a}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*b*e*n*(6*e^3*log(e*sqrt(x) + d)/d^4 - 3*e^3*log(x)/d^4 - (6*e^2*x - 3*
d*e*sqrt(x) + 2*d^2)/(d^3*x^(3/2))) - 1/2*b*log((e*sqrt(x) + d)^n*c)/x^2 -
1/2*a/x^2
```

mupad [B] time = 0.63, size = 83, normalized size = 0.76

$$\frac{b\,e^{4}\,n\,\mathrm{atanh}\left(\frac{2\,e\,\sqrt{x}}{d}+1\right)}{d^{4}}-\frac{\frac{b\,e\,n}{3\,d}+\frac{b\,e^{3}\,n\,x}{d^{3}}-\frac{b\,e^{2}\,n\,\sqrt{x}}{2\,d^{2}}}{2\,x^{3/2}}-\frac{b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^{n}\right)}{2\,x^{2}}-\frac{a}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))/x^3,x)
```

```
[Out] (b*e^4*n*atanh((2*e*x^(1/2))/d + 1))/d^4 - ((b*e*n)/(3*d) + (b*e^3*n*x)/d^3
 - (b*e^2*n*x^(1/2))/(2*d^2))/(2*x^(3/2)) - (b*log(c*(d + e*x^(1/2))^n))/(2
*x^2) - a/(2*x^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x**3,x)
```

```
[Out] Timed out
```

3.407 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=141

$$-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3}+\frac{be^6n\log\left(d+e\sqrt{x}\right)}{3d^6}-\frac{be^6n\log(x)}{6d^6}-\frac{be^5n}{3d^5\sqrt{x}}+\frac{be^4n}{6d^4x}-\frac{be^3n}{9d^3x^{3/2}}+\frac{be^2n}{12d^2x^2}-\frac{ben}{15dx^{5/2}}$$

```
[Out] -1/15*b*e*n/d/x^(5/2)+1/12*b*e^2*n/d^2/x^2-1/9*b*e^3*n/d^3/x^(3/2)+1/6*b*e^
4*n/d^4/x-1/6*b*e^6*n*ln(x)/d^6+1/3*b*e^6*n*ln(d+e*x^(1/2))/d^6+1/3*(-a-b*l
n(c*(d+e*x^(1/2))^n))/x^3-1/3*b*e^5*n/d^5/x^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3}-\frac{be^3n}{9d^3x^{3/2}}+\frac{be^2n}{12d^2x^2}-\frac{be^5n}{3d^5\sqrt{x}}+\frac{be^4n}{6d^4x}+\frac{be^6n\log\left(d+e\sqrt{x}\right)}{3d^6}-\frac{be^6n\log(x)}{6d^6}-\frac{ben}{15dx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^4,x]
```

```
[Out] -(b*e*n)/(15*d*x^(5/2)) + (b*e^2*n)/(12*d^2*x^2) - (b*e^3*n)/(9*d^3*x^(3/2)
) + (b*e^4*n)/(6*d^4*x) - (b*e^5*n)/(3*d^5*Sqrt[x]) + (b*e^6*n*Log[d + e*Sq
rt[x]])/(3*d^6) - (a + b*Log[c*(d + e*Sqrt[x])^n])/(3*x^3) - (b*e^6*n*Log[x
])/(6*d^6)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx = 2\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^7}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3} + \frac{1}{3}(ben)\,\text{Subst}\left(\int \frac{1}{x^6(d+ex)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3} + \frac{1}{3}(ben)\,\text{Subst}\left(\int \left(\frac{1}{dx^6} - \frac{e}{d^2x^5} + \frac{e^2}{d^3x^4} - \frac{e^3}{d^4x^3} + \right.\right.$$
$$= -\frac{ben}{15dx^{5/2}} + \frac{be^2n}{12d^2x^2} - \frac{be^3n}{9d^3x^{3/2}} + \frac{be^4n}{6d^4x} - \frac{be^5n}{3d^5\sqrt{x}} + \frac{be^6n\log\left(d+e\sqrt{x}\right)}{3d^6} - \frac{a+b}{}$$

Mathematica [A] time = 0.14, size = 132, normalized size = 0.94

$$-\frac{a}{3x^3} - \frac{b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3} + \frac{1}{3}ben\left(\frac{e^5\log\left(d+e\sqrt{x}\right)}{d^6} - \frac{e^5\log(x)}{2d^6} - \frac{e^4}{d^5\sqrt{x}} + \frac{e^3}{2d^4x} - \frac{e^2}{3d^3x^{3/2}} + \frac{e}{4d^2x^2} - \frac{1}{5dx^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*Log[c*(d + e*Sqrt[x])^n])/(3*x^3) + (b*e*n*(-1/5*1/(d*x^(5/
2)) + e/(4*d^2*x^2) - e^2/(3*d^3*x^(3/2)) + e^3/(2*d^4*x) - e^4/(d^5*Sqrt[x
]) + (e^5*Log[d + e*Sqrt[x]])/d^6 - (e^5*Log[x])/(2*d^6)))/3
```

fricas [A] time = 0.45, size = 124, normalized size = 0.88

$$-\frac{60\,be^6nx^3\log\left(\sqrt{x}\right) - 30\,bd^2e^4nx^2 - 15\,bd^4e^2nx + 60\,bd^6\log(c) + 60\,ad^6 - 60\left(be^6nx^3 - bd^6n\right)\log\left(e\sqrt{x}+d\right) + }{180\,d^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^4,x, algorithm="fricas")
```

```
[Out] -1/180*(60*b*e^6*n*x^3*log(sqrt(x)) - 30*b*d^2*e^4*n*x^2 - 15*b*d^4*e^2*n*x
 + 60*b*d^6*log(c) + 60*a*d^6 - 60*(b*e^6*n*x^3 - b*d^6*n)*log(e*sqrt(x) +
d) + 4*(15*b*d*e^5*n*x^2 + 5*b*d^3*e^3*n*x + 3*b*d^5*e*n)*sqrt(x))/(d^6*x^3
)
```

giac [B] time = 0.21, size = 542, normalized size = 3.84

$$\frac{\left(60\left(\sqrt{x}e+d\right)^6bne^7\log\left(\sqrt{x}e+d\right) - 360\left(\sqrt{x}e+d\right)^5bdne^7\log\left(\sqrt{x}e+d\right) + 900\left(\sqrt{x}e+d\right)^4bd^2ne^7\log\left(\sqrt{x}e+d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^4,x, algorithm="giac")
```

```
[Out] 1/180*(60*(sqrt(x)*e + d)^6*b*n*e^7*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d
)^5*b*d*n*e^7*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^4*b*d^2*n*e^7*log(sq
rt(x)*e + d) - 1200*(sqrt(x)*e + d)^3*b*d^3*n*e^7*log(sqrt(x)*e + d) + 900*
(sqrt(x)*e + d)^2*b*d^4*n*e^7*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)*b*d^
5*n*e^7*log(sqrt(x)*e + d) - 60*(sqrt(x)*e + d)^6*b*n*e^7*log(sqrt(x)*e) +
360*(sqrt(x)*e + d)^5*b*d*n*e^7*log(sqrt(x)*e) - 900*(sqrt(x)*e + d)^4*b*d^
```

```
2*n*e^7*log(sqrt(x)*e) + 1200*(sqrt(x)*e + d)^3*b*d^3*n*e^7*log(sqrt(x)*e)
- 900*(sqrt(x)*e + d)^2*b*d^4*n*e^7*log(sqrt(x)*e) + 360*(sqrt(x)*e + d)*b*
d^5*n*e^7*log(sqrt(x)*e) - 60*b*d^6*n*e^7*log(sqrt(x)*e) - 60*(sqrt(x)*e +
d)^5*b*d*n*e^7 + 330*(sqrt(x)*e + d)^4*b*d^2*n*e^7 - 740*(sqrt(x)*e + d)^3*
b*d^3*n*e^7 + 855*(sqrt(x)*e + d)^2*b*d^4*n*e^7 - 522*(sqrt(x)*e + d)*b*d^5
*n*e^7 + 137*b*d^6*n*e^7 - 60*b*d^6*e^7*log(c) - 60*a*d^6*e^7)*e^(-1)/((sqr
t(x)*e + d)^6*d^6 - 6*(sqrt(x)*e + d)^5*d^7 + 15*(sqrt(x)*e + d)^4*d^8 - 20
*(sqrt(x)*e + d)^3*d^9 + 15*(sqrt(x)*e + d)^2*d^10 - 6*(sqrt(x)*e + d)*d^11
 + d^12)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{b \ln\left(c \left(e\sqrt{x}+d\right)^{n}\right)+a}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/2)+d)^n)+a)/x^4,x)
```

```
[Out] int((b*ln(c*(e*x^(1/2)+d)^n)+a)/x^4,x)
```

maxima [A] time = 0.49, size = 106, normalized size = 0.75

$$\frac{1}{180}ben\left(\frac{60\,e^5\log\left(e\sqrt{x}+d\right)}{d^6}-\frac{30\,e^5\log(x)}{d^6}-\frac{60\,e^4x^2-30\,de^3x^{\frac{3}{2}}+20\,d^2e^2x-15\,d^3e\sqrt{x}+12\,d^4}{d^5x^{\frac{5}{2}}}\right)-\frac{b\log\left(\left(e\sqrt{}\right.\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^4,x, algorithm="maxima")
```

```
[Out] 1/180*b*e*n*(60*e^5*log(e*sqrt(x) + d)/d^6 - 30*e^5*log(x)/d^6 - (60*e^4*x^
2 - 30*d*e^3*x^(3/2) + 20*d^2*e^2*x - 15*d^3*e*sqrt(x) + 12*d^4)/(d^5*x^(5/
2))) - 1/3*b*log((e*sqrt(x) + d)^n*c)/x^3 - 1/3*a/x^3
```

mupad [B] time = 0.76, size = 110, normalized size = 0.78

$$\frac{2\,b\,e^6\,n\,\mathrm{atanh}\left(\frac{2\,e\,\sqrt{x}}{d}+1\right)}{3\,d^6}-\frac{\frac{b\,e\,n}{5\,d}+\frac{b\,e^3\,n\,x}{3\,d^3}-\frac{b\,e^2\,n\,\sqrt{x}}{4\,d^2}+\frac{b\,e^5\,n\,x^2}{d^5}-\frac{b\,e^4\,n\,x^{3/2}}{2\,d^4}}{3\,x^{5/2}}-\frac{b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{3\,x^3}-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))/x^4,x)
```

```
[Out] (2*b*e^6*n*atanh((2*e*x^(1/2))/d + 1))/(3*d^6) - ((b*e*n)/(5*d) + (b*e^3*n*
x)/(3*d^3) - (b*e^2*n*x^(1/2))/(4*d^2) + (b*e^5*n*x^2)/d^5 - (b*e^4*n*x^(3/
2))/(2*d^4))/(3*x^(5/2)) - (b*log(c*(d + e*x^(1/2))^n))/(3*x^3) - a/(3*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x**4,x)
```

```
[Out] Timed out
```

3.408 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=480

$$-\frac{2bd^6n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^6}+\frac{4bd^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^6}-\frac{5bd^4n\left(d+e\sqrt{\right.}}{}$$

```
[Out] 1/3*b^2*d^6*n^2*ln(d+e*x^(1/2))^2/e^6-2/3*b*d^6*n*ln(d+e*x^(1/2))*(a+b*ln(c
*(d+e*x^(1/2))^n))/e^6+1/3*x^3*(a+b*ln(c*(d+e*x^(1/2))^n))^2-4*b^2*d^5*n^2*
x^(1/2)/e^5+4*b*d^5*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))/e^6+5/2*b^2
*d^4*n^2*(d+e*x^(1/2))^2/e^6-5*b*d^4*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(
1/2))^2/e^6-40/27*b^2*d^3*n^2*(d+e*x^(1/2))^3/e^6+40/9*b*d^3*n*(a+b*ln(c*(d
+e*x^(1/2))^n))*(d+e*x^(1/2))^3/e^6+5/8*b^2*d^2*n^2*(d+e*x^(1/2))^4/e^6-5/2
*b*d^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^4/e^6-4/25*b^2*d*n^2*(d+
e*x^(1/2))^5/e^6+4/5*b*d*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^5/e^6+
1/54*b^2*n^2*(d+e*x^(1/2))^6/e^6-1/9*b*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x
^(1/2))^6/e^6
```

Rubi [A] time = 0.48, antiderivative size = 355, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$\frac{1}{90}bn\left(\frac{360d^5\left(d+e\sqrt{x}\right)}{e^6}-\frac{450d^4\left(d+e\sqrt{x}\right)^2}{e^6}+\frac{400d^3\left(d+e\sqrt{x}\right)^3}{e^6}-\frac{225d^2\left(d+e\sqrt{x}\right)^4}{e^6}-\frac{60d^6\log\left(d+e\sqrt{x}\right)}{e^6}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (5*b^2*d^4*n^2*(d + e*Sqrt[x])^2)/(2*e^6) - (40*b^2*d^3*n^2*(d + e*Sqrt[x])
^3)/(27*e^6) + (5*b^2*d^2*n^2*(d + e*Sqrt[x])^4)/(8*e^6) - (4*b^2*d*n^2*(d
+ e*Sqrt[x])^5)/(25*e^6) + (b^2*n^2*(d + e*Sqrt[x])^6)/(54*e^6) - (4*b^2*d^
5*n^2*Sqrt[x])/e^5 + (b^2*d^6*n^2*Log[d + e*Sqrt[x]]^2)/(3*e^6) + (b*n*((36
0*d^5*(d + e*Sqrt[x]))/e^6 - (450*d^4*(d + e*Sqrt[x])^2)/e^6 + (400*d^3*(d
+ e*Sqrt[x])^3)/e^6 - (225*d^2*(d + e*Sqrt[x])^4)/e^6 + (72*d*(d + e*Sqrt[x
])^5)/e^6 - (10*(d + e*Sqrt[x])^6)/e^6 - (60*d^6*Log[d + e*Sqrt[x]])/e^6)*(
a + b*Log[c*(d + e*Sqrt[x])^n]))/90 + (x^3*(a + b*Log[c*(d + e*Sqrt[x])^n])
^2)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 dx = 2\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^2 dx, x, \sqrt{x}\right)$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 - \frac{1}{3}(2ben)\,\text{Subst}\left(\int \frac{x^6\left(a+b\log\left(c(d+\right.\right.}{d+ex}\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 - \frac{1}{3}(2bn)\,\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6\left(a+b\log\right.}{x}\right.$$

$$= \frac{1}{90}bn\left(\frac{360d^5\left(d+e\sqrt{x}\right)}{e^6} - \frac{450d^4\left(d+e\sqrt{x}\right)^2}{e^6} + \frac{400d^3\left(d+e\sqrt{x}\right)^3}{e^6} - \frac{225}{}\right.$$

$$= \frac{1}{90}bn\left(\frac{360d^5\left(d+e\sqrt{x}\right)}{e^6} - \frac{450d^4\left(d+e\sqrt{x}\right)^2}{e^6} + \frac{400d^3\left(d+e\sqrt{x}\right)^3}{e^6} - \frac{225}{}\right.$$

$$= \frac{1}{90}bn\left(\frac{360d^5\left(d+e\sqrt{x}\right)}{e^6} - \frac{450d^4\left(d+e\sqrt{x}\right)^2}{e^6} + \frac{400d^3\left(d+e\sqrt{x}\right)^3}{e^6} - \frac{225}{}\right.$$

$$= \frac{5b^2d^4n^2\left(d+e\sqrt{x}\right)^2}{2e^6} - \frac{40b^2d^3n^2\left(d+e\sqrt{x}\right)^3}{27e^6} + \frac{5b^2d^2n^2\left(d+e\sqrt{x}\right)^4}{8e^6} - \frac{4b}{}$$

$$= \frac{5b^2d^4n^2\left(d+e\sqrt{x}\right)^2}{2e^6} - \frac{40b^2d^3n^2\left(d+e\sqrt{x}\right)^3}{27e^6} + \frac{5b^2d^2n^2\left(d+e\sqrt{x}\right)^4}{8e^6} - \frac{4b}{}$$

Mathematica [A] time = 0.33, size = 295, normalized size = 0.61

$$\frac{e\sqrt{x}\left(1800a^2e^5x^{5/2} + 60abn\left(60d^5 - 30d^4e\sqrt{x} + 20d^3e^2x - 15d^2e^3x^{3/2} + 12de^4x^2 - 10e^5x^{5/2}\right) + b^2n^2\left(-8820d^5 + 2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (e*Sqrt[x]*(1800*a^2*e^5*x^(5/2) + 60*a*b*n*(60*d^5 - 30*d^4*e*Sqrt[x] + 20
*d^3*e^2*x - 15*d^2*e^3*x^(3/2) + 12*d*e^4*x^2 - 10*e^5*x^(5/2)) + b^2*n^2*
(-8820*d^5 + 2610*d^4*e*Sqrt[x] - 1140*d^3*e^2*x + 555*d^2*e^3*x^(3/2) - 26
4*d*e^4*x^2 + 100*e^5*x^(5/2))) - 60*b*(60*a*(d^6 - e^6*x^3) + b*n*(-147*d^
6 - 60*d^5*e*Sqrt[x] + 30*d^4*e^2*x - 20*d^3*e^3*x^(3/2) + 15*d^2*e^4*x^2 -
 12*d*e^5*x^(5/2) + 10*e^6*x^3))*Log[c*(d + e*Sqrt[x])^n] - 1800*b^2*(d^6 -
 e^6*x^3)*Log[c*(d + e*Sqrt[x])^n]^2)/(5400*e^6)
```

fricas [A] time = 0.48, size = 487, normalized size = 1.01

$$\frac{1800\,b^2e^6x^3\log(c)^2 + 100\left(b^2e^6n^2 - 6\,abe^6n + 18\,a^2e^6\right)x^3 + 15\left(37\,b^2d^2e^4n^2 - 60\,abd^2e^4n\right)x^2 + 1800\left(b^2e^6n^2x^3 - b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/5400*(1800*b^2*e^6*x^3*log(c)^2 + 100*(b^2*e^6*n^2 - 6*a*b*e^6*n + 18*a^2
*e^6)*x^3 + 15*(37*b^2*d^2*e^4*n^2 - 60*a*b*d^2*e^4*n)*x^2 + 1800*(b^2*e^6*
n^2*x^3 - b^2*d^6*n^2)*log(e*sqrt(x) + d)^2 + 90*(29*b^2*d^4*e^2*n^2 - 20*a
```

```
*b*d^4*e^2*n)*x - 60*(15*b^2*d^2*e^4*n^2*x^2 + 30*b^2*d^4*e^2*n^2*x - 147*b
^2*d^6*n^2 + 60*a*b*d^6*n + 10*(b^2*e^6*n^2 - 6*a*b*e^6*n)*x^3 - 60*(b^2*e^
6*n*x^3 - b^2*d^6*n)*log(c) - 4*(3*b^2*d*e^5*n^2*x^2 + 5*b^2*d^3*e^3*n^2*x
+ 15*b^2*d^5*e*n^2)*sqrt(x))*log(e*sqrt(x) + d) - 300*(3*b^2*d^2*e^4*n*x^2
+ 6*b^2*d^4*e^2*n*x + 2*(b^2*e^6*n - 6*a*b*e^6)*x^3)*log(c) - 12*(735*b^2*d
^5*e*n^2 - 300*a*b*d^5*e*n + 2*(11*b^2*d*e^5*n^2 - 30*a*b*d*e^5*n)*x^2 + 5*
(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x - 20*(3*b^2*d*e^5*n*x^2 + 5*b^2*d
^3*e^3*n*x + 15*b^2*d^5*e*n)*log(c))*sqrt(x))/e^6
```

giac [B] time = 0.25, size = 956, normalized size = 1.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] 1/5400*(1800*b^2*x^3*e*log(c)^2 + 3600*a*b*x^3*e*log(c) + 1800*a^2*x^3*e +
(1800*(sqrt(x)*e + d)^6*e^(-5)*log(sqrt(x)*e + d)^2 - 10800*(sqrt(x)*e + d)
^5*d*e^(-5)*log(sqrt(x)*e + d)^2 + 27000*(sqrt(x)*e + d)^4*d^2*e^(-5)*log(s
qrt(x)*e + d)^2 - 36000*(sqrt(x)*e + d)^3*d^3*e^(-5)*log(sqrt(x)*e + d)^2 +
 27000*(sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x)*e + d)^2 - 10800*(sqrt(x)*e
 + d)*d^5*e^(-5)*log(sqrt(x)*e + d)^2 - 600*(sqrt(x)*e + d)^6*e^(-5)*log(sq
rt(x)*e + d) + 4320*(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt(x)*e + d) - 13500*(
sqrt(x)*e + d)^4*d^2*e^(-5)*log(sqrt(x)*e + d) + 24000*(sqrt(x)*e + d)^3*d^
3*e^(-5)*log(sqrt(x)*e + d) - 27000*(sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x
)*e + d) + 21600*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x)*e + d) + 100*(sqrt(
x)*e + d)^6*e^(-5) - 864*(sqrt(x)*e + d)^5*d*e^(-5) + 3375*(sqrt(x)*e + d)^
4*d^2*e^(-5) - 8000*(sqrt(x)*e + d)^3*d^3*e^(-5) + 13500*(sqrt(x)*e + d)^2*
d^4*e^(-5) - 21600*(sqrt(x)*e + d)*d^5*e^(-5))*b^2*n^2 + 60*(60*(sqrt(x)*e
+ d)^6*e^(-5)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt(
x)*e + d) + 900*(sqrt(x)*e + d)^4*d^2*e^(-5)*log(sqrt(x)*e + d) - 1200*(sqr
t(x)*e + d)^3*d^3*e^(-5)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^2*d^4*e^(
-5)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x)*e + d)
- 10*(sqrt(x)*e + d)^6*e^(-5) + 72*(sqrt(x)*e + d)^5*d*e^(-5) - 225*(sqrt(x
)*e + d)^4*d^2*e^(-5) + 400*(sqrt(x)*e + d)^3*d^3*e^(-5) - 450*(sqrt(x)*e +
 d)^2*d^4*e^(-5) + 360*(sqrt(x)*e + d)*d^5*e^(-5))*b^2*n*log(c) + 60*(60*(s
qrt(x)*e + d)^6*e^(-5)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)^5*d*e^(-5)*
log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^4*d^2*e^(-5)*log(sqrt(x)*e + d) -
1200*(sqrt(x)*e + d)^3*d^3*e^(-5)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^
2*d^4*e^(-5)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x
)*e + d) - 10*(sqrt(x)*e + d)^6*e^(-5) + 72*(sqrt(x)*e + d)^5*d*e^(-5) - 22
5*(sqrt(x)*e + d)^4*d^2*e^(-5) + 400*(sqrt(x)*e + d)^3*d^3*e^(-5) - 450*(sq
rt(x)*e + d)^2*d^4*e^(-5) + 360*(sqrt(x)*e + d)*d^5*e^(-5))*a*b*n)*e^(-1)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(b \ln \left(c \left(e\sqrt{x}+d\right)^{n}\right)+a\right)^{2} x^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*(e*x^(1/2)+d)^n)+a)^2,x)
```

```
[Out] int(x^2*(b*ln(c*(e*x^(1/2)+d)^n)+a)^2,x)
```

maxima [A] time = 0.52, size = 324, normalized size = 0.68

$$\frac{1}{3}\,b^2x^3\log\left(\left(e\sqrt{x}+d\right)^nc\right)^2+\frac{2}{3}\,abx^3\log\left(\left(e\sqrt{x}+d\right)^nc\right)+\frac{1}{3}\,a^2x^3-\frac{1}{90}\,aben\left(\frac{60\,d^6\log\left(e\sqrt{x}+d\right)}{e^7}+\frac{10\,e^5x^3-12\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*log((e*sqrt(x) + d)^n*c)^2 + 2/3*a*b*x^3*log((e*sqrt(x) + d)^n*
c) + 1/3*a^2*x^3 - 1/90*a*b*e*n*(60*d^6*log(e*sqrt(x) + d)/e^7 + (10*e^5*x^
3 - 12*d*e^4*x^(5/2) + 15*d^2*e^3*x^2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*x - 6
0*d^5*sqrt(x))/e^6) - 1/5400*(60*e*n*(60*d^6*log(e*sqrt(x) + d)/e^7 + (10*e
^5*x^3 - 12*d*e^4*x^(5/2) + 15*d^2*e^3*x^2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*
x - 60*d^5*sqrt(x))/e^6)*log((e*sqrt(x) + d)^n*c) - (100*e^6*x^3 - 264*d*e^
5*x^(5/2) + 555*d^2*e^4*x^2 + 1800*d^6*log(e*sqrt(x) + d)^2 - 1140*d^3*e^3*
x^(3/2) + 2610*d^4*e^2*x + 8820*d^6*log(e*sqrt(x) + d) - 8820*d^5*e*sqrt(x)
)*n^2/e^6)*b^2
```

mupad [B] time = 1.78, size = 434, normalized size = 0.90

$$\frac{a^2\,x^3}{3}+\frac{b^2\,x^3\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^2}{3}+\frac{b^2\,n^2\,x^3}{54}+\frac{2\,a\,b\,x^3\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{3}-\frac{b^2\,d^6\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^2}{3\,e^6}-\frac{a\,b\,n\,x^3}{9}-\frac{b^2\,n\,x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/2))^n))^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^3*log(c*(d + e*x^(1/2))^n)^2)/3 + (b^2*n^2*x^3)/54 + (
2*a*b*x^3*log(c*(d + e*x^(1/2))^n))/3 - (b^2*d^6*log(c*(d + e*x^(1/2))^n)^2
)/(3*e^6) - (a*b*n*x^3)/9 - (b^2*n*x^3*log(c*(d + e*x^(1/2))^n))/9 + (49*b^
2*d^6*n^2*log(d + e*x^(1/2)))/(30*e^6) + (37*b^2*d^2*n^2*x^2)/(360*e^2) - (
19*b^2*d^3*n^2*x^(3/2))/(90*e^3) - (49*b^2*d^5*n^2*x^(1/2))/(30*e^5) - (11*
b^2*d*n^2*x^(5/2))/(225*e) + (29*b^2*d^4*n^2*x)/(60*e^4) - (b^2*d^2*n*x^2*l
og(c*(d + e*x^(1/2))^n))/(6*e^2) + (2*b^2*d^3*n*x^(3/2)*log(c*(d + e*x^(1/2
))^n))/(9*e^3) + (2*b^2*d^5*n*x^(1/2)*log(c*(d + e*x^(1/2))^n))/(3*e^5) + (
2*a*b*d*n*x^(5/2))/(15*e) - (a*b*d^4*n*x)/(3*e^4) - (2*a*b*d^6*n*log(d + e*
x^(1/2)))/(3*e^6) + (2*b^2*d*n*x^(5/2)*log(c*(d + e*x^(1/2))^n))/(15*e) - (
b^2*d^4*n*x*log(c*(d + e*x^(1/2))^n))/(3*e^4) - (a*b*d^2*n*x^2)/(6*e^2) + (
2*a*b*d^3*n*x^(3/2))/(9*e^3) + (2*a*b*d^5*n*x^(1/2))/(3*e^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**n))**2,x)
```

```
[Out] Integral(x**2*(a + b*log(c*(d + e*sqrt(x))**n))**2, x)
```

3.409 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=342

$$-\frac{bd^4n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}+\frac{4bd^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}-\frac{3bd^2n\left(d+e\right.}{}$$

```
[Out] 1/2*b^2*d^4*n^2*ln(d+e*x^(1/2))^2/e^4-b*d^4*n*ln(d+e*x^(1/2))*(a+b*ln(c*(d+
e*x^(1/2))^n))/e^4+1/2*x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^2-4*b^2*d^3*n^2*x^(1
/2)/e^3+4*b*d^3*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))/e^4+3/2*b^2*d^2
*n^2*(d+e*x^(1/2))^2/e^4-3*b*d^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2)
)^2/e^4-4/9*b^2*d*n^2*(d+e*x^(1/2))^3/e^4+4/3*b*d*n*(a+b*ln(c*(d+e*x^(1/2))
^n))*(d+e*x^(1/2))^3/e^4+1/16*b^2*n^2*(d+e*x^(1/2))^4/e^4-1/4*b*n*(a+b*ln(c
*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^4/e^4
```

Rubi [A] time = 0.36, antiderivative size = 263, normalized size of antiderivative = 0.77, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$\frac{1}{12}bn\left(\frac{48d^3\left(d+e\sqrt{x}\right)}{e^4}-\frac{36d^2\left(d+e\sqrt{x}\right)^2}{e^4}-\frac{12d^4\log\left(d+e\sqrt{x}\right)}{e^4}+\frac{16d\left(d+e\sqrt{x}\right)^3}{e^4}-\frac{3\left(d+e\sqrt{x}\right)^4}{e^4}\right)\left(a+b\,\mathrm{l}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (3*b^2*d^2*n^2*(d + e*Sqrt[x])^2)/(2*e^4) - (4*b^2*d*n^2*(d + e*Sqrt[x])^3)
/(9*e^4) + (b^2*n^2*(d + e*Sqrt[x])^4)/(16*e^4) - (4*b^2*d^3*n^2*Sqrt[x])/e
^3 + (b^2*d^4*n^2*Log[d + e*Sqrt[x]]^2)/(2*e^4) + (b*n*((48*d^3*(d + e*Sqrt
[x]))/e^4 - (36*d^2*(d + e*Sqrt[x])^2)/e^4 + (16*d*(d + e*Sqrt[x])^3)/e^4 -
 (3*(d + e*Sqrt[x])^4)/e^4 - (12*d^4*Log[d + e*Sqrt[x]])/e^4)*(a + b*Log[c*
(d + e*Sqrt[x])^n]))/12 + (x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx = 2\,\text{Subst}\left(\int x^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, \sqrt{x}\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 - (ben)\,\text{Subst}\left(\int \frac{x^4\left(a+b\log(c(d+\right.}{d+ex}\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 - (bn)\,\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^4\left(a+b\log\right.}{x}\right.$$

$$= \frac{1}{12}bn\left(\frac{48d^3\left(d+e\sqrt{x}\right)}{e^4} - \frac{36d^2\left(d+e\sqrt{x}\right)^2}{e^4} + \frac{16d\left(d+e\sqrt{x}\right)^3}{e^4} - \frac{3\left(d+\right.}{e}\right.$$

$$= \frac{1}{12}bn\left(\frac{48d^3\left(d+e\sqrt{x}\right)}{e^4} - \frac{36d^2\left(d+e\sqrt{x}\right)^2}{e^4} + \frac{16d\left(d+e\sqrt{x}\right)^3}{e^4} - \frac{3\left(d+\right.}{e}\right.$$

$$= \frac{1}{12}bn\left(\frac{48d^3\left(d+e\sqrt{x}\right)}{e^4} - \frac{36d^2\left(d+e\sqrt{x}\right)^2}{e^4} + \frac{16d\left(d+e\sqrt{x}\right)^3}{e^4} - \frac{3\left(d+\right.}{e}\right.$$

$$= \frac{3b^2d^2n^2\left(d+e\sqrt{x}\right)^2}{2e^4} - \frac{4b^2dn^2\left(d+e\sqrt{x}\right)^3}{9e^4} + \frac{b^2n^2\left(d+e\sqrt{x}\right)^4}{16e^4} - \frac{4b^2d^3}{e}$$

$$= \frac{3b^2d^2n^2\left(d+e\sqrt{x}\right)^2}{2e^4} - \frac{4b^2dn^2\left(d+e\sqrt{x}\right)^3}{9e^4} + \frac{b^2n^2\left(d+e\sqrt{x}\right)^4}{16e^4} - \frac{4b^2d^3}{e}$$

Mathematica [A] time = 0.21, size = 223, normalized size = 0.65

$$\frac{e\sqrt{x}\left(72a^2e^3x^{3/2} + 12abn\left(12d^3 - 6d^2e\sqrt{x} + 4de^2x - 3e^3x^{3/2}\right) + b^2n^2\left(-300d^3 + 78d^2e\sqrt{x} - 28de^2x + 9e^3x^{3/2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (e*Sqrt[x]*(72*a^2*e^3*x^(3/2) + 12*a*b*n*(12*d^3 - 6*d^2*e*Sqrt[x] + 4*d*e
^2*x - 3*e^3*x^(3/2)) + b^2*n^2*(-300*d^3 + 78*d^2*e*Sqrt[x] - 28*d*e^2*x +
 9*e^3*x^(3/2))) - 12*b*(12*a*(d^4 - e^4*x^2) + b*n*(-25*d^4 - 12*d^3*e*Sqr
t[x] + 6*d^2*e^2*x - 4*d*e^3*x^(3/2) + 3*e^4*x^2))*Log[c*(d + e*Sqrt[x])^n]
 - 72*b^2*(d^4 - e^4*x^2)*Log[c*(d + e*Sqrt[x])^n]^2)/(144*e^4)
```

fricas [A] time = 0.45, size = 357, normalized size = 1.04

$$\frac{72\,b^2e^4x^2\log(c)^2 + 9\left(b^2e^4n^2 - 4\,abe^4n + 8\,a^2e^4\right)x^2 + 72\left(b^2e^4n^2x^2 - b^2d^4n^2\right)\log\left(e\sqrt{x}+d\right)^2 + 6\left(13\,b^2d^2e^2n^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/144*(72*b^2*e^4*x^2*log(c)^2 + 9*(b^2*e^4*n^2 - 4*a*b*e^4*n + 8*a^2*e^4)*
x^2 + 72*(b^2*e^4*n^2*x^2 - b^2*d^4*n^2)*log(e*sqrt(x) + d)^2 + 6*(13*b^2*d
^2*e^2*n^2 - 12*a*b*d^2*e^2*n)*x - 12*(6*b^2*d^2*e^2*n^2*x - 25*b^2*d^4*n^2
 + 12*a*b*d^4*n + 3*(b^2*e^4*n^2 - 4*a*b*e^4*n)*x^2 - 12*(b^2*e^4*n*x^2 - b
^2*d^4*n)*log(c) - 4*(b^2*d*e^3*n^2*x + 3*b^2*d^3*e*n^2)*sqrt(x))*log(e*sqr
```

```
t(x) + d) - 36*(2*b^2*d^2*e^2*n*x + (b^2*e^4*n - 4*a*b*e^4)*x^2)*log(c) - 4
*(75*b^2*d^3*e*n^2 - 36*a*b*d^3*e*n + (7*b^2*d*e^3*n^2 - 12*a*b*d*e^3*n)*x
- 12*(b^2*d*e^3*n*x + 3*b^2*d^3*e*n)*log(c))*sqrt(x))/e^4
```

giac [B] time = 0.25, size = 642, normalized size = 1.88

$$\frac{1}{144}\left(72\,b^2x^2e\log(c)^2 + 144\,abx^2e\log(c) + \left(72\left(\sqrt{x}\,e + d\right)^4 e^{(-3)}\log\left(\sqrt{x}\,e + d\right)^2 - 288\left(\sqrt{x}\,e + d\right)^3 de^{(-3)}\log\left(\sqrt{x}\,e\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] 1/144*(72*b^2*x^2*e*log(c)^2 + 144*a*b*x^2*e*log(c) + (72*(sqrt(x)*e + d)^4
*e^(-3)*log(sqrt(x)*e + d)^2 - 288*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*e
 + d)^2 + 432*(sqrt(x)*e + d)^2*d^2*e^(-3)*log(sqrt(x)*e + d)^2 - 288*(sqrt
(x)*e + d)*d^3*e^(-3)*log(sqrt(x)*e + d)^2 - 36*(sqrt(x)*e + d)^4*e^(-3)*lo
g(sqrt(x)*e + d) + 192*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*e + d) - 432*
(sqrt(x)*e + d)^2*d^2*e^(-3)*log(sqrt(x)*e + d) + 576*(sqrt(x)*e + d)*d^3*e
^(-3)*log(sqrt(x)*e + d) + 9*(sqrt(x)*e + d)^4*e^(-3) - 64*(sqrt(x)*e + d)^
3*d*e^(-3) + 216*(sqrt(x)*e + d)^2*d^2*e^(-3) - 576*(sqrt(x)*e + d)*d^3*e^(
-3))*b^2*n^2 + 72*a^2*x^2*e + 12*(12*(sqrt(x)*e + d)^4*e^(-3)*log(sqrt(x)*e
 + d) - 48*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*e + d) + 72*(sqrt(x)*e +
d)^2*d^2*e^(-3)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)*d^3*e^(-3)*log(sqrt
(x)*e + d) - 3*(sqrt(x)*e + d)^4*e^(-3) + 16*(sqrt(x)*e + d)^3*d*e^(-3) - 3
6*(sqrt(x)*e + d)^2*d^2*e^(-3) + 48*(sqrt(x)*e + d)*d^3*e^(-3))*b^2*n*log(c
) + 12*(12*(sqrt(x)*e + d)^4*e^(-3)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)
^3*d*e^(-3)*log(sqrt(x)*e + d) + 72*(sqrt(x)*e + d)^2*d^2*e^(-3)*log(sqrt(x
)*e + d) - 48*(sqrt(x)*e + d)*d^3*e^(-3)*log(sqrt(x)*e + d) - 3*(sqrt(x)*e
+ d)^4*e^(-3) + 16*(sqrt(x)*e + d)^3*d*e^(-3) - 36*(sqrt(x)*e + d)^2*d^2*e^
(-3) + 48*(sqrt(x)*e + d)*d^3*e^(-3))*a*b*n)*e^(-1)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\left(e\sqrt{x}+d\right)^{n}\right)+a\right)^{2} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(e*x^(1/2)+d)^n)+a)^2,x)
```

```
[Out] int(x*(b*ln(c*(e*x^(1/2)+d)^n)+a)^2,x)
```

maxima [A] time = 0.55, size = 257, normalized size = 0.75

$$\frac{1}{2}\,b^2x^2\log\left(\left(e\sqrt{x}+d\right)^n c\right)^2 - \frac{1}{12}\,aben\left(\frac{12\,d^4\log\left(e\sqrt{x}+d\right)}{e^5} + \frac{3\,e^3x^2 - 4\,de^2x^{\frac{3}{2}} + 6\,d^2ex - 12\,d^3\sqrt{x}}{e^4}\right) + abx^2\log\left(\left(e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log((e*sqrt(x) + d)^n*c)^2 - 1/12*a*b*e*n*(12*d^4*log(e*sqrt(x)
 + d)/e^5 + (3*e^3*x^2 - 4*d*e^2*x^(3/2) + 6*d^2*e*x - 12*d^3*sqrt(x))/e^4)
 + a*b*x^2*log((e*sqrt(x) + d)^n*c) + 1/2*a^2*x^2 - 1/144*(12*e*n*(12*d^4*l
og(e*sqrt(x) + d)/e^5 + (3*e^3*x^2 - 4*d*e^2*x^(3/2) + 6*d^2*e*x - 12*d^3*s
qrt(x))/e^4)*log((e*sqrt(x) + d)^n*c) - (9*e^4*x^2 + 72*d^4*log(e*sqrt(x) +
 d)^2 - 28*d*e^3*x^(3/2) + 78*d^2*e^2*x + 300*d^4*log(e*sqrt(x) + d) - 300*
d^3*e*sqrt(x))*n^2/e^4)*b^2
```

mupad [B] time = 0.56, size = 420, normalized size = 1.23

$$x\left(\frac{d\left(\frac{d\left(2a^2-abn+\frac{b^2n^2}{4}\right)}{e}-\frac{d\left(6a^2-b^2n^2\right)}{3e}\right)}{2e}+\frac{b^2d^2n^2}{4e^2}\right)-x^{3/2}\left(\frac{d\left(2a^2-abn+\frac{b^2n^2}{4}\right)}{3e}-\frac{d\left(6a^2-b^2n^2\right)}{9e}\right)+\ln\left(c\left(d+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(1/2))^n))^2,x)
```

```
[Out] x*((d*((d*(2*a^2 + (b^2*n^2)/4 - a*b*n))/e - (d*(6*a^2 - b^2*n^2))/(3*e)))/
(2*e) + (b^2*d^2*n^2)/(4*e^2)) - x^(3/2)*((d*(2*a^2 + (b^2*n^2)/4 - a*b*n))
/(3*e) - (d*(6*a^2 - b^2*n^2))/(9*e)) + log(c*(d + e*x^(1/2))^n)^2*((b^2*x^
2)/2 - (b^2*d^4)/(2*e^4)) + x^2*(a^2/2 + (b^2*n^2)/16 - (a*b*n)/4) - log(c*
(d + e*x^(1/2))^n)*(x^(3/2)*((b*d*(4*a - b*n))/(3*e) - (4*a*b*d)/(3*e)) - (
b*x^2*(4*a - b*n))/4 + (d^2*x^(1/2)*((b*d*(4*a - b*n))/e - (4*a*b*d)/e))/e^
2 - (d*x*((b*d*(4*a - b*n))/e - (4*a*b*d)/e))/(2*e)) - x^(1/2)*((d*((d*((d*
(2*a^2 + (b^2*n^2)/4 - a*b*n))/e - (d*(6*a^2 - b^2*n^2))/(3*e)))/e + (b^2*d
^2*n^2)/(2*e^2)))/e + (b^2*d^3*n^2)/e^3) + (log(d + e*x^(1/2))*(25*b^2*d^4*
n^2 - 12*a*b*d^4*n))/(12*e^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))**n))**2,x)
```

```
[Out] Integral(x*(a + b*log(c*(d + e*sqrt(x))**n))**2, x)
```

3.410 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=195

$$-\frac{bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^2}+\frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2}-\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(\right.\right.}{e^2}$$

```
[Out] 4*a*b*d*n*x^(1/2)/e-4*b^2*d*n^2*x^(1/2)/e+4*b^2*d*n*ln(c*(d+e*x^(1/2))^n)*(
d+e*x^(1/2))/e^2-2*d*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))/e^2+1/2*b^
2*n^2*(d+e*x^(1/2))^2/e^2-b*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^2/e
^2+(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^2/e^2
```

Rubi [A] time = 0.18, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2451, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$-\frac{bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^2}+\frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2}-\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(\right.\right.}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (b^2*n^2*(d + e*Sqrt[x])^2)/(2*e^2) + (4*a*b*d*n*Sqrt[x])/e - (4*b^2*d*n^2*
Sqrt[x])/e + (4*b^2*d*n*(d + e*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n])/e^2 - (b*
n*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/e^2 - (2*d*(d + e*Sqr
t[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/e^2 + ((d + e*Sqrt[x])^2*(a + b*L
og[c*(d + e*Sqrt[x])^n])^2)/e^2
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx = 2\,\text{Subst}\left(\int x\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx,x,\sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int\left(-\frac{d\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e}+\frac{(d+ex)\left(a+b\log\left(c(d+ex)^\right.\right.}{e}\right.\right.$$
$$= \frac{2\,\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx,x,\sqrt{x}\right)}{e} - \frac{(2d)\,\text{Subst}\left(\int\left(\right.\right.}{}$$
$$= \frac{2\,\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)^2\,dx,x,d+e\sqrt{x}\right)}{e^2} - \frac{(2d)\,\text{Subst}\left(\int\left(a+b\log\right.\right.}{}$$
$$= -\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2} + \frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(\right.\right.\right.}{e^2}$$
$$= \frac{b^2n^2\left(d+e\sqrt{x}\right)^2}{2e^2} + \frac{4abdn\sqrt{x}}{e} - \frac{bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right.\right.}{e^2}$$
$$= \frac{b^2n^2\left(d+e\sqrt{x}\right)^2}{2e^2} + \frac{4abdn\sqrt{x}}{e} - \frac{4b^2dn^2\sqrt{x}}{e} + \frac{4b^2dn\left(d+e\sqrt{x}\right)\log\left(c\left(d\right.\right.}{e^2}$$

Mathematica [A] time = 0.08, size = 150, normalized size = 0.77

$$\frac{-2a^2\left(d^2-e^2x\right)+2b\left(d+e\sqrt{x}\right)\left(-2ad+2ae\sqrt{x}+3bdn-ben\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)-2abn\left(d-e\sqrt{x}\right)^2-2}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (-2*a*b*n*(d - e*Sqrt[x])^2 + b^2*e*n^2*(-6*d + e*Sqrt[x])*Sqrt[x] - 2*a^2*
(d^2 - e^2*x) + 2*b*(d + e*Sqrt[x])*(-2*a*d + 3*b*d*n + 2*a*e*Sqrt[x] - b*e
```

```
*n*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n] - 2*b^2*(d^2 - e^2*x)*Log[c*(d + e*Sqr
t[x])^n]^2)/(2*e^2)
```

fricas [A] time = 0.44, size = 225, normalized size = 1.15

$$\frac{2\,b^2e^2x\log(c)^2 + 2\left(b^2e^2n^2x - b^2d^2n^2\right)\log\left(e\sqrt{x} + d\right)^2 - 2\left(b^2e^2n - 2\,abe^2\right)x\log(c) + \left(b^2e^2n^2 - 2\,abe^2n + 2\,a^2e^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*e^2*x*log(c)^2 + 2*(b^2*e^2*n^2*x - b^2*d^2*n^2)*log(e*sqrt(x) +
 d)^2 - 2*(b^2*e^2*n - 2*a*b*e^2)*x*log(c) + (b^2*e^2*n^2 - 2*a*b*e^2*n + 2
*a^2*e^2)*x + 2*(2*b^2*d*e*n^2*sqrt(x) + 3*b^2*d^2*n^2 - 2*a*b*d^2*n - (b^2
*e^2*n^2 - 2*a*b*e^2*n)*x + 2*(b^2*e^2*n*x - b^2*d^2*n)*log(c))*log(e*sqrt(
x) + d) - 2*(3*b^2*d*e*n^2 - 2*b^2*d*e*n*log(c) - 2*a*b*d*e*n)*sqrt(x))/e^2
```

giac [B] time = 0.19, size = 361, normalized size = 1.85

$$\frac{1}{2}\left(\left(2\left(\sqrt{x}e + d\right)^2\log\left(\sqrt{x}e + d\right)^2 - 4\left(\sqrt{x}e + d\right)d\log\left(\sqrt{x}e + d\right)^2 - 2\left(\sqrt{x}e + d\right)^2\log\left(\sqrt{x}e + d\right) + 8\left(\sqrt{x}e + d\right)d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)^2 - 4*(sqrt(x)*e + d)*d*log(sq
rt(x)*e + d)^2 - 2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) + 8*(sqrt(x)*e + d)
*d*log(sqrt(x)*e + d) + (sqrt(x)*e + d)^2 - 8*(sqrt(x)*e + d)*d)*b^2*n^2*e^
(-1) + 2*(2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) - 4*(sqrt(x)*e + d)*d*log(
sqrt(x)*e + d) - (sqrt(x)*e + d)^2 + 4*(sqrt(x)*e + d)*d)*b^2*n*e^(-1)*log(
c) + 2*((sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)*d)*b^2*e^(-1)*log(c)^2 + 2*(2
*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) - 4*(sqrt(x)*e + d)*d*log(sqrt(x)*e +
 d) - (sqrt(x)*e + d)^2 + 4*(sqrt(x)*e + d)*d)*a*b*n*e^(-1) + 4*((sqrt(x)*e
 + d)^2 - 2*(sqrt(x)*e + d)*d)*a*b*e^(-1)*log(c) + 2*((sqrt(x)*e + d)^2 - 2
*(sqrt(x)*e + d)*d)*a^2*e^(-1))*e^(-1)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\left(e\sqrt{x} + d\right)^n\right) + a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/2)+d)^n)+a)^2,x)
```

```
[Out] int((b*ln(c*(e*x^(1/2)+d)^n)+a)^2,x)
```

maxima [A] time = 0.54, size = 179, normalized size = 0.92

$$-\left(en\left(\frac{2\,d^2\log\left(e\sqrt{x} + d\right)}{e^3} + \frac{ex - 2\,d\sqrt{x}}{e^2}\right) - 2\,x\log\left(\left(e\sqrt{x} + d\right)^n c\right)\right)ab - \frac{1}{2}\left(2\,en\left(\frac{2\,d^2\log\left(e\sqrt{x} + d\right)}{e^3} + \frac{ex - 2\,d\sqrt{x}}{e^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] -(e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 + (e*x - 2*d*sqrt(x))/e^2) - 2*x*log((e
*sqrt(x) + d)^n*c))*a*b - 1/2*(2*e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 + (e*x -
 2*d*sqrt(x))/e^2)*log((e*sqrt(x) + d)^n*c) - 2*x*log((e*sqrt(x) + d)^n*c)^
```

```
2 - (2*d^2*log(e*sqrt(x) + d)^2 + e^2*x + 6*d^2*log(e*sqrt(x) + d) - 6*d*e*
sqrt(x))*n^2/e^2)*b^2 + a^2*x
```

mupad [B] time = 0.47, size = 186, normalized size = 0.95

$$x\left(a^2-a\,b\,n+\frac{b^2\,n^2}{2}\right)-\sqrt{x}\,\left(\frac{d\,\left(2\,a^2-2\,a\,b\,n+b^2\,n^2\right)}{e}-\frac{2\,d\,\left(a^2-b^2\,n^2\right)}{e}\right)+\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)^2\,\left(b^2\,x-\frac{b^2\,d^2}{e^2}\right)\cdot$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^2,x)
```

```
[Out] x*(a^2 + (b^2*n^2)/2 - a*b*n) - x^(1/2)*((d*(2*a^2 + b^2*n^2 - 2*a*b*n))/e
- (2*d*(a^2 - b^2*n^2))/e) + log(c*(d + e*x^(1/2))^n)^2*(b^2*x - (b^2*d^2)/
e^2) - log(c*(d + e*x^(1/2))^n)*(x^(1/2)*((2*b*d*(2*a - b*n))/e - (4*a*b*d)
/e) - b*x*(2*a - b*n)) + (log(d + e*x^(1/2))*(3*b^2*d^2*n^2 - 2*a*b*d^2*n))
/e^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**2, x)
```

3.411 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx$

Optimal. Leaf size=93

$$4bn\text{Li}_2\left(\frac{\sqrt{x}\,e}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+2\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-4b^2n^2\text{Li}_3\left(\frac{\sqrt{x}\,e}{d}+1\right)$$

```
[Out] 2*ln(-e*x^(1/2)/d)*(a+b*ln(c*(d+e*x^(1/2))^n))^2+4*b*n*(a+b*ln(c*(d+e*x^(1/
2))^n))*polylog(2,1+e*x^(1/2)/d)-4*b^2*n^2*polylog(3,1+e*x^(1/2)/d)
```

Rubi [A] time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2454, 2396, 2433, 2374, 6589}

$$4bn\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-4b^2n^2\text{PolyLog}\left(3,\frac{e\sqrt{x}}{d}+1\right)+2\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x,x]
```

```
[Out] 2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[-((e*Sqrt[x])/d)] + 4*b*n*(a + b*L
og[c*(d + e*Sqrt[x])^n])*PolyLog[2, 1 + (e*Sqrt[x])/d] - 4*b^2*n^2*PolyLog[
3, 1 + (e*Sqrt[x])/d]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx = 2\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx, x, \sqrt{x}\right)$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right) - (4ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{\cdots}\right.$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right) - (4bn)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(\cdots\right.\right.}{\cdots}\right.$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right) + 4bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right.\right.\right.$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right) + 4bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right.\right.\right.$$

Mathematica [B] time = 0.14, size = 195, normalized size = 2.10

$$2bn\left(\log(x)\left(\log\left(d+e\sqrt{x}\right)-\log\left(\frac{e\sqrt{x}}{d}+1\right)\right)-2\text{Li}_2\left(-\frac{e\sqrt{x}}{d}\right)\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)-bn\log\left(d+e\sqrt{x}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])*((Log[d + e*Sqrt[
x]] - Log[1 + (e*Sqrt[x])/d])*Log[x] - 2*PolyLog[2, -((e*Sqrt[x])/d)]) + 2*
b^2*n^2*(Log[d + e*Sqrt[x]]^2*Log[-((e*Sqrt[x])/d)] + 2*Log[d + e*Sqrt[x]]*
PolyLog[2, 1 + (e*Sqrt[x])/d] - 2*PolyLog[3, 1 + (e*Sqrt[x])/d])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\log\left(\left(e\sqrt{x}+d\right)^n c\right)^2+2\,ab\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*sqrt(x) + d)^n*c)^2 + 2*a*b*log((e*sqrt(x) + d)^n*c) +
 a^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^2/x, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(e\sqrt{x}+d\right)^{n}\right)+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/2)+d)^n)+a)^2/x,x)
```

```
[Out] int((b*ln(c*(e*x^(1/2)+d)^n)+a)^2/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^{2}n^{2}\log\left(e\sqrt{x}+d\right)^{2}\log(x)+\int -\frac{\left(b^{2}enx\log(x)-2\left(b^{2}e\log(c)+abe\right)x-2\left(b^{2}d\log(c)+abd\right)\sqrt{x}\right)n\log\left(e\sqrt{x}+d\right.}{ex^{2}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*n^2*log(e*sqrt(x) + d)^2*log(x) + integrate(-((b^2*e*n*x*log(x) - 2*(b^
2*e*log(c) + a*b*e)*x - 2*(b^2*d*log(c) + a*b*d)*sqrt(x))*n*log(e*sqrt(x) +
 d) - (b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x - (b^2*d*log(c)^2 + 2*a*b
*d*log(c) + a^2*d)*sqrt(x))/(e*x^2 + d*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^{n}\right)\right)}^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**2/x, x)
```

3.412 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=155

$$-\frac{2be^2n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2}-\frac{2ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2\sqrt{x}}-\frac{\left(a+b\log\left(c\right.\right.}{}$$

```
[Out] b^2*e^2*n^2*ln(x)/d^2-(a+b*ln(c*(d+e*x^(1/2))^n))^2/x-2*b*e^2*n*(a+b*ln(c*(
d+e*x^(1/2))^n))*ln(1-d/(d+e*x^(1/2)))/d^2+2*b^2*e^2*n^2*polylog(2,d/(d+e*x
^(1/2)))/d^2-2*b*e*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))/d^2/x^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 176, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31}

$$-\frac{2b^2e^2n^2\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)}{d^2}+\frac{e^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2}-\frac{2be^2n\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^2,x]
```

```
[Out] (-2*b*e*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(d^2*Sqrt[x]) +
 (e^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/d^2 - (a + b*Log[c*(d + e*Sqrt[x]
)^n])^2/x - (2*b*e^2*n*(a + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d
)])/d^2 + (b^2*e^2*n^2*Log[x])/d^2 - (2*b^2*e^2*n^2*PolyLog[2, 1 + (e*Sqrt[
x])/d])/d^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
```

```
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}dx = 2\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^3}dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x} + (2ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(d+ex)} d\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x} + (2bn)\,\text{Subst}\left(\int \frac{a+b\log\left(cx^n\right)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}dx, x, d\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x} + \frac{(2bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}dx, x, d+e\sqrt{x}\right.}{d}$$

$$= -\frac{2ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2\sqrt{x}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right.\right.}{x}$$

$$= -\frac{2ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2\sqrt{x}} + \frac{e^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right.\right.\right.}{d^2}$$

$$= -\frac{2ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2\sqrt{x}} + \frac{e^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right.\right.\right.}{d^2}$$

Mathematica [A] time = 0.19, size = 188, normalized size = 1.21

$$2\left(ben\left(\frac{e\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2bd^2n} - \frac{e\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2} - \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{d\sqrt{x}} - \right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^2,x]
```

```
[Out] 2*(-1/2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x + b*e*n*(-((a + b*Log[c*(d + e
*Sqrt[x])^n])/(d*Sqrt[x])) + (e*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*b*d^
2*n) - (e*(a + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)])/d^2 + (b*
e*n*(-(Log[d + e*Sqrt[x]]/d) + Log[x]/(2*d)))/d - (b*e*n*PolyLog[2, (d + e*
Sqrt[x])/d])/d^2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\log\left(\left(e\sqrt{x}+d\right)^n c\right)^2 + 2\,ab\log\left(\left(e\sqrt{x}+d\right)^n c\right) + a^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*sqrt(x) + d)^n*c)^2 + 2*a*b*log((e*sqrt(x) + d)^n*c) +
 a^2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(\left(e\sqrt{x} + d\right)^{n} c\right) + a\right)^{2}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^2,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^2/x^2, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c \left(e\sqrt{x} + d\right)^{n}\right) + a\right)^{2}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/2)+d)^n)+a)^2/x^2,x)
[Out] int((b*ln(c*(e*x^(1/2)+d)^n)+a)^2/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(\log\left(\frac{e\sqrt{x}}{d} + 1\right)\log\left(\sqrt{x}\right) + \mathrm{Li}_2\left(-\frac{e\sqrt{x}}{d}\right)\right)b^2e^2n^2}{d^2} + \frac{2\left(abe^2n - \left(e^2n^2 - e^2n\log(c)\right)b^2\right)\log\left(e\sqrt{x} + d\right)}{d^2} - \frac{2\left(b^2e^2n\log(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^2,x, algorithm="maxima")
[Out] 2*(log(e*sqrt(x)/d + 1)*log(sqrt(x)) + dilog(-e*sqrt(x)/d))*b^2*e^2*n^2/d^2
 + 2*(a*b*e^2*n - (e^2*n^2 - e^2*n*log(c))*b^2)*log(e*sqrt(x) + d)/d^2 - 2*
(b^2*e^2*n*log(c) + a*b*e^2*n)*log(sqrt(x))/d^2 + integrate((b^2*e^4*n^2*x
+ b^2*d^2*e^2*n^2)/x, x)/d^4 + 1/3*(2*b^2*e^5*n^2*x^(3/2) - 6*b^2*d^2*e^3*n
^2*sqrt(x)*log(sqrt(x)) - 3*b^2*d*e^4*n^2*x + 12*b^2*d^2*e^3*n^2*sqrt(x))/d
^5 - 1/3*(3*b^2*d^3*e^2*n^2*x^(3/2)*log(e*sqrt(x) + d)^2 + 2*b^2*e^5*n^2*x^
3 - 3*b^2*d^2*e^3*n^2*x^2*log(x) + 3*b^2*d^5*n^2*sqrt(x)*log(e*sqrt(x) + d)
^2 + 12*b^2*d^2*e^3*n^2*x^2 - 3*(2*b^2*d^3*e^2*n*x^(3/2)*log(e*sqrt(x) + d)
 - 2*b^2*d^4*e*n*x - (b^2*d^3*e^2*n*x*log(x) + 2*b^2*d^5*log(c) + 2*a*b*d^5
)*sqrt(x))*n*log(e*sqrt(x) + d) + 6*(b^2*d^4*e*n*log(c) + a*b*d^4*e*n)*x)/(
d^5*x^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\ln\left(c\,\left(d + e\,\sqrt{x}\right)^{n}\right)\right)^{2}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(c\left(d + e\sqrt{x}\right)^{n}\right)\right)^{2}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**2/x**2, x)
```

3.413 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=293

$$-\frac{be^4n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4}-\frac{be^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4\sqrt{x}}+\frac{be^2n\left(a+b\log\left(\right.\right.}{2d^{}}$$

```
[Out] -1/6*b^2*e^2*n^2/d^2/x+11/12*b^2*e^4*n^2*ln(x)/d^4-5/6*b^2*e^4*n^2*ln(d+e*x
^(1/2))/d^4-1/3*b*e*n*(a+b*ln(c*(d+e*x^(1/2))^n))/d/x^(3/2)+1/2*b*e^2*n*(a+
b*ln(c*(d+e*x^(1/2))^n))/d^2/x-1/2*(a+b*ln(c*(d+e*x^(1/2))^n))^2/x^2-b*e^4*
n*(a+b*ln(c*(d+e*x^(1/2))^n))*ln(1-d/(d+e*x^(1/2)))/d^4+b^2*e^4*n^2*polylog
(2,d/(d+e*x^(1/2)))/d^4+5/6*b^2*e^3*n^2/d^3/x^(1/2)-b*e^3*n*(a+b*ln(c*(d+e*
x^(1/2))^n))*(d+e*x^(1/2))/d^4/x^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 318, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$-\frac{b^2e^4n^2\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)}{d^4}+\frac{e^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2d^4}-\frac{be^4n\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4}-\frac{be^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^3,x]
```

```
[Out] -(b^2*e^2*n^2)/(6*d^2*x) + (5*b^2*e^3*n^2)/(6*d^3*Sqrt[x]) - (5*b^2*e^4*n^2
*Log[d + e*Sqrt[x]])/(6*d^4) - (b*e*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(3*
d*x^(3/2)) + (b*e^2*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*d^2*x) - (b*e^3*
n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(d^4*Sqrt[x]) + (e^4*(a
 + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*d^4) - (a + b*Log[c*(d + e*Sqrt[x])^n]
)^2/(2*x^2) - (b*e^4*n*(a + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d
)])/d^4 + (11*b^2*e^4*n^2*Log[x])/(12*d^4) - (b^2*e^4*n^2*PolyLog[2, 1 + (e
*Sqrt[x])/d])/d^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
```

```
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx = 2\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^5}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2} + (ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4(d+ex)}\,dx, x\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2} + (bn)\,\text{Subst}\left(\int \frac{a+b\log\left(cx^n\right)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt{}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2} + \frac{(bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt{x}\right)}{d} -$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3dx^{3/2}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2} - \frac{(ben)\,\text{Sub}}{}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3dx^{3/2}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} - \frac{\left(a+\right.}{}$$

$$= -\frac{b^2e^2n^2}{6d^2x} + \frac{b^2e^3n^2}{3d^3\sqrt{x}} - \frac{b^2e^4n^2\log\left(d+e\sqrt{x}\right)}{3d^4} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right.}{3dx^{3/2}}$$

$$= -\frac{b^2e^2n^2}{6d^2x} + \frac{5b^2e^3n^2}{6d^3\sqrt{x}} - \frac{5b^2e^4n^2\log\left(d+e\sqrt{x}\right)}{6d^4} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right.\right.}{3dx^{3/2}}$$

$$= -\frac{b^2e^2n^2}{6d^2x} + \frac{5b^2e^3n^2}{6d^3\sqrt{x}} - \frac{5b^2e^4n^2\log\left(d+e\sqrt{x}\right)}{6d^4} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right.\right.}{3dx^{3/2}}$$

Mathematica [A] time = 0.36, size = 353, normalized size = 1.20

$$-\frac{\frac{e\sqrt{x}\left(4bd^3n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-6bd^2en\sqrt{x}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-6e^3x^{3/2}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2+12be^3nx^{3/2}\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^3,x]
```

```
[Out] -1/12*(6*(a + b*Log[c*(d + e*Sqrt[x])^n])^2 + (e*Sqrt[x]*(4*b*d^3*n*(a + b*
Log[c*(d + e*Sqrt[x])^n]) - 6*b*d^2*e*n*Sqrt[x]*(a + b*Log[c*(d + e*Sqrt[x]
)^n]) + 12*b*d*e^2*n*x*(a + b*Log[c*(d + e*Sqrt[x])^n]) - 6*e^3*x^(3/2)*(a
+ b*Log[c*(d + e*Sqrt[x])^n])^2 + 12*b*e^3*n*x^(3/2)*(a + b*Log[c*(d + e*Sq
rt[x])^n])*Log[-((e*Sqrt[x])/d)] + 6*b^2*e^3*n^2*x^(3/2)*(2*Log[d + e*Sqrt[
x]] - Log[x]) - 3*b^2*e^2*n^2*x*(2*d - 2*e*Sqrt[x]*Log[d + e*Sqrt[x]] + e*S
```

```
qrt[x]*Log[x]) + 2*b^2*e*n^2*Sqrt[x]*(d*(d - 2*e*Sqrt[x]) + 2*e^2*x*Log[d +
 e*Sqrt[x]] - e^2*x*Log[x]) + 12*b^2*e^3*n^2*x^(3/2)*PolyLog[2, 1 + (e*Sqrt
[x])/d]))/d^4)/x^2
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \log\left(\left(e\sqrt{x}+d\right)^n c\right)^2 + 2\,ab\log\left(\left(e\sqrt{x}+d\right)^n c\right) + a^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*sqrt(x) + d)^n*c)^2 + 2*a*b*log((e*sqrt(x) + d)^n*c) +
 a^2)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^2/x^3, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(e\sqrt{x}+d\right)^n\right)+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/2)+d)^n)+a)^2/x^3,x)
```

```
[Out] int((b*ln(c*(e*x^(1/2)+d)^n)+a)^2/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2n^2\log\left(e\sqrt{x}+d\right)^2}{2\,x^2}+\int \frac{\left(b^2enx+4\left(b^2e\log(c)+abe\right)x+4\left(b^2d\log(c)+abd\right)\sqrt{x}\right)n\log\left(e\sqrt{x}+d\right)+2\left(b^2e\right.}{2\left(ex^4+dx^{\frac{7}{2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*n^2*log(e*sqrt(x) + d)^2/x^2 + integrate(1/2*((b^2*e*n*x + 4*(b^2*
e*log(c) + a*b*e)*x + 4*(b^2*d*log(c) + a*b*d)*sqrt(x))*n*log(e*sqrt(x) + d
) + 2*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x + 2*(b^2*d*log(c)^2 + 2*a
*b*d*log(c) + a^2*d)*sqrt(x))/(e*x^4 + d*x^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\left(d+e\,\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^3,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(c\left(d + e\sqrt{x}\right)^{n}\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**2/x**3, x)
```

3.414 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=408

$$-\frac{2be^6n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^6}-\frac{2be^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^6\sqrt{x}}+\frac{be^4n\left(a+b\right.}{}$$

```
[Out] -1/30*b^2*e^2*n^2/d^2/x^2+1/10*b^2*e^3*n^2/d^3/x^(3/2)-47/180*b^2*e^4*n^2/d
^4/x+137/180*b^2*e^6*n^2*ln(x)/d^6-77/90*b^2*e^6*n^2*ln(d+e*x^(1/2))/d^6-2/
15*b*e*n*(a+b*ln(c*(d+e*x^(1/2))^n))/d/x^(5/2)+1/6*b*e^2*n*(a+b*ln(c*(d+e*x
^(1/2))^n))/d^2/x^2-2/9*b*e^3*n*(a+b*ln(c*(d+e*x^(1/2))^n))/d^3/x^(3/2)+1/3
*b*e^4*n*(a+b*ln(c*(d+e*x^(1/2))^n))/d^4/x-1/3*(a+b*ln(c*(d+e*x^(1/2))^n))^
2/x^3-2/3*b*e^6*n*(a+b*ln(c*(d+e*x^(1/2))^n))*ln(1-d/(d+e*x^(1/2)))/d^6+2/3
*b^2*e^6*n^2*polylog(2,d/(d+e*x^(1/2)))/d^6+77/90*b^2*e^5*n^2/d^5/x^(1/2)-2
/3*b*e^5*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))/d^6/x^(1/2)
```

Rubi [A] time = 1.03, antiderivative size = 432, normalized size of antiderivative = 1.06, number of steps used = 26, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$-\frac{2b^2e^6n^2\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)}{3d^6}-\frac{2be^3n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9d^3x^{3/2}}+\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{6d^2x^2}+\frac{e^6\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^4,x]
```

```
[Out] -(b^2*e^2*n^2)/(30*d^2*x^2) + (b^2*e^3*n^2)/(10*d^3*x^(3/2)) - (47*b^2*e^4*
n^2)/(180*d^4*x) + (77*b^2*e^5*n^2)/(90*d^5*Sqrt[x]) - (77*b^2*e^6*n^2*Log[
d + e*Sqrt[x]])/(90*d^6) - (2*b*e*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(15*d
*x^(5/2)) + (b*e^2*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(6*d^2*x^2) - (2*b*e
^3*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(9*d^3*x^(3/2)) + (b*e^4*n*(a + b*Lo
g[c*(d + e*Sqrt[x])^n]))/(3*d^4*x) - (2*b*e^5*n*(d + e*Sqrt[x])*(a + b*Log[
c*(d + e*Sqrt[x])^n]))/(3*d^6*Sqrt[x]) + (e^6*(a + b*Log[c*(d + e*Sqrt[x])^
n])^2)/(3*d^6) - (a + b*Log[c*(d + e*Sqrt[x])^n])^2/(3*x^3) - (2*b*e^6*n*(a
 + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)])/(3*d^6) + (137*b^2*e^
6*n^2*Log[x])/(180*d^6) - (2*b^2*e^6*n^2*PolyLog[2, 1 + (e*Sqrt[x])/d])/(3*
d^6)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx = 2\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^7}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(2ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^6(d+ex)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(2bn)\,\text{Subst}\left(\int \frac{a+b\log\left(cx^n\right)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} + \frac{(2bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+e\sqrt{x}\right.}{3d}$$

$$= -\frac{2ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{15dx^{5/2}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} - \frac{(2ben}{}$$

$$= -\frac{2ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{15dx^{5/2}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{6d^2x^2} - $$

$$= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{2b^2e^3n^2}{45d^3x^{3/2}} - \frac{b^2e^4n^2}{15d^4x} + \frac{2b^2e^5n^2}{15d^5\sqrt{x}} - \frac{2b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{15d^6} - \frac{2b}{}$$

$$= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{b^2e^3n^2}{10d^3x^{3/2}} - \frac{3b^2e^4n^2}{20d^4x} + \frac{3b^2e^5n^2}{10d^5\sqrt{x}} - \frac{3b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{10d^6} - \frac{2}{}$$

$$= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{b^2e^3n^2}{10d^3x^{3/2}} - \frac{47b^2e^4n^2}{180d^4x} + \frac{47b^2e^5n^2}{90d^5\sqrt{x}} - \frac{47b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{90d^6}$$

$$= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{b^2e^3n^2}{10d^3x^{3/2}} - \frac{47b^2e^4n^2}{180d^4x} + \frac{77b^2e^5n^2}{90d^5\sqrt{x}} - \frac{77b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{90d^6}$$

$$= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{b^2e^3n^2}{10d^3x^{3/2}} - \frac{47b^2e^4n^2}{180d^4x} + \frac{77b^2e^5n^2}{90d^5\sqrt{x}} - \frac{77b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{90d^6}$$

Mathematica [A] time = 0.29, size = 538, normalized size = 1.32

$$-\frac{60a^2d^6-60a^2e^6x^3+120abd^6\log\left(c\left(d+e\sqrt{x}\right)^n\right)-120abe^6x^3\log\left(c\left(d+e\sqrt{x}\right)^n\right)+24abd^5en\sqrt{x}-30abd^4e^2nx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^4,x]
```

```
[Out] -1/180*(60*a^2*d^6 + 24*a*b*d^5*e*n*Sqrt[x] - 30*a*b*d^4*e^2*n*x + 6*b^2*d^
4*e^2*n^2*x + 40*a*b*d^3*e^3*n*x^(3/2) - 18*b^2*d^3*e^3*n^2*x^(3/2) - 60*a*
b*d^2*e^4*n*x^2 + 47*b^2*d^2*e^4*n^2*x^2 + 120*a*b*d*e^5*n*x^(5/2) - 154*b^
2*d*e^5*n^2*x^(5/2) - 60*a^2*e^6*x^3 + 274*b^2*e^6*n^2*x^3*Log[d + e*Sqrt[x
]] + 120*a*b*d^6*Log[c*(d + e*Sqrt[x])^n] + 24*b^2*d^5*e*n*Sqrt[x]*Log[c*(d
 + e*Sqrt[x])^n] - 30*b^2*d^4*e^2*n*x*Log[c*(d + e*Sqrt[x])^n] + 40*b^2*d^3
*e^3*n*x^(3/2)*Log[c*(d + e*Sqrt[x])^n] - 60*b^2*d^2*e^4*n*x^2*Log[c*(d + e
*Sqrt[x])^n] + 120*b^2*d*e^5*n*x^(5/2)*Log[c*(d + e*Sqrt[x])^n] - 120*a*b*e
^6*x^3*Log[c*(d + e*Sqrt[x])^n] + 60*b^2*d^6*Log[c*(d + e*Sqrt[x])^n]^2 - 6
0*b^2*e^6*x^3*Log[c*(d + e*Sqrt[x])^n]^2 + 120*a*b*e^6*n*x^3*Log[-((e*Sqrt[
x])/d)] + 120*b^2*e^6*n*x^3*Log[c*(d + e*Sqrt[x])^n]*Log[-((e*Sqrt[x])/d)]
- 137*b^2*e^6*n^2*x^3*Log[x] + 120*b^2*e^6*n^2*x^3*PolyLog[2, 1 + (e*Sqrt[x
])/d])/(d^6*x^3)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^2\log\left(\left(e\sqrt{x}+d\right)^nc\right)^2+2\,ab\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*sqrt(x) + d)^n*c)^2 + 2*a*b*log((e*sqrt(x) + d)^n*c) +
 a^2)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^2/x^4, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\left(e\sqrt{x}+d\right)^n\right)+a\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/2)+d)^n)+a)^2/x^4,x)
```

```
[Out] int((b*ln(c*(e*x^(1/2)+d)^n)+a)^2/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2n^2\log\left(e\sqrt{x}+d\right)^2}{3\,x^3}+\int\frac{\left(b^2enx+6\left(b^2e\log(c)+abe\right)x+6\left(b^2d\log(c)+abd\right)\sqrt{x}\right)n\log\left(e\sqrt{x}+d\right)+3\left(b^2e\right.}{3\left(ex^5+dx^{\frac{9}{2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*n^2*log(e*sqrt(x) + d)^2/x^3 + integrate(1/3*((b^2*e*n*x + 6*(b^2*
e*log(c) + a*b*e)*x + 6*(b^2*d*log(c) + a*b*d)*sqrt(x))*n*log(e*sqrt(x) + d
) + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x + 3*(b^2*d*log(c)^2 + 2*a
*b*d*log(c) + a^2*d)*sqrt(x))/(e*x^5 + d*x^(9/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^4,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x**4,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**2/x**4, x)
```

3.415 $\int x^2 \left(a + b\log\left(c\left(d + e\sqrt{x}\right)^n\right)\right)^3 dx$

Optimal. Leaf size=907

$$-\frac{b^3n^3\left(d+e\sqrt{x}\right)^6}{108e^6}+\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\left(d+e\sqrt{x}\right)^6}{3e^6}-\frac{bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\left(d+e\sqrt{x}\right)^6}{6e^6}+\frac{b^2n^2(}{}$$

```
[Out] 1/3*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))^6/e^6+12*b^3*d^5*n^3*x^(1/2
)/e^5-15/4*b^3*d^4*n^3*(d+e*x^(1/2))^2/e^6+40/27*b^3*d^3*n^3*(d+e*x^(1/2))^
3/e^6-15/32*b^3*d^2*n^3*(d+e*x^(1/2))^4/e^6+12/125*b^3*d*n^3*(d+e*x^(1/2))^
5/e^6+1/18*b^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^6/e^6-1/6*b*n*
(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^6/e^6-2*d^5*(a+b*ln(c*(d+e*x^(1
/2))^n))^3*(d+e*x^(1/2))/e^6+5*d^4*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/
2))^2/e^6-20/3*d^3*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))^3/e^6+5*d^2*
(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))^4/e^6-2*d*(a+b*ln(c*(d+e*x^(1/2
))^n))^3*(d+e*x^(1/2))^5/e^6-1/108*b^3*n^3*(d+e*x^(1/2))^6/e^6-12*a*b^2*d^5
*n^2*x^(1/2)/e^5-12*b^3*d^5*n^2*ln(c*(d+e*x^(1/2))^n)*(d+e*x^(1/2))/e^6+6*b
*d^5*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))/e^6+15/2*b^2*d^4*n^2*(a+
b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^2/e^6-15/2*b*d^4*n*(a+b*ln(c*(d+e*x^
(1/2))^n))^2*(d+e*x^(1/2))^2/e^6-40/9*b^2*d^3*n^2*(a+b*ln(c*(d+e*x^(1/2))^n
))*(d+e*x^(1/2))^3/e^6+20/3*b*d^3*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1
/2))^3/e^6+15/8*b^2*d^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^4/e^6
-15/4*b*d^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^4/e^6-12/25*b^2*d
*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^5/e^6+6/5*b*d*n*(a+b*ln(c*(d
+e*x^(1/2))^n))^2*(d+e*x^(1/2))^5/e^6
```

Rubi [A] time = 1.01, antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$-\frac{b^3n^3\left(d+e\sqrt{x}\right)^6}{108e^6}+\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\left(d+e\sqrt{x}\right)^6}{3e^6}-\frac{bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\left(d+e\sqrt{x}\right)^6}{6e^6}+\frac{b^2n^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (-15*b^3*d^4*n^3*(d + e*Sqrt[x])^2)/(4*e^6) + (40*b^3*d^3*n^3*(d + e*Sqrt[x
])^3)/(27*e^6) - (15*b^3*d^2*n^3*(d + e*Sqrt[x])^4)/(32*e^6) + (12*b^3*d*n^
3*(d + e*Sqrt[x])^5)/(125*e^6) - (b^3*n^3*(d + e*Sqrt[x])^6)/(108*e^6) - (1
2*a*b^2*d^5*n^2*Sqrt[x])/e^5 + (12*b^3*d^5*n^3*Sqrt[x])/e^5 - (12*b^3*d^5*n
^2*(d + e*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n])/e^6 + (15*b^2*d^4*n^2*(d + e*S
qrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*e^6) - (40*b^2*d^3*n^2*(d +
e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(9*e^6) + (15*b^2*d^2*n^2*(d
 + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(8*e^6) - (12*b^2*d*n^2*(
d + e*Sqrt[x])^5*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(25*e^6) + (b^2*n^2*(d +
 e*Sqrt[x])^6*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(18*e^6) + (6*b*d^5*n*(d +
e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/e^6 - (15*b*d^4*n*(d + e*Sqr
t[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*e^6) + (20*b*d^3*n*(d + e*Sq
rt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(3*e^6) - (15*b*d^2*n*(d + e*S
qrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(4*e^6) + (6*b*d*n*(d + e*Sqr
t[x])^5*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(5*e^6) - (b*n*(d + e*Sqrt[x])^
6*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(6*e^6) - (2*d^5*(d + e*Sqrt[x])*(a +
 b*Log[c*(d + e*Sqrt[x])^n])^3)/e^6 + (5*d^4*(d + e*Sqrt[x])^2*(a + b*Log[c
*(d + e*Sqrt[x])^n])^3)/e^6 - (20*d^3*(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e
*Sqrt[x])^n])^3)/(3*e^6) + (5*d^2*(d + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqr
```

```
t[x])^n])^3)/e^6 - (2*d*(d + e*Sqrt[x])^5*(a + b*Log[c*(d + e*Sqrt[x])^n])^
3)/e^6 + ((d + e*Sqrt[x])^6*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/(3*e^6)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3 dx = 2\,\text{Subst}\left(\int x^5 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^5} + \frac{5d^4(d+ex)\left(a+b\log(c(d+\right.}{e^5}\right.\right.$$

$$= \frac{2\,\text{Subst}\left(\int (d+ex)^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3 dx, x, \sqrt{x}\right)}{e^5} - \frac{(10d)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{2\,\text{Subst}\left(\int x^5\left(a+b\log\left(cx^n\right)\right)^3 dx, x, d+e\sqrt{x}\right)}{e^6} - \frac{(10d)\,\text{Subst}\left(\int x^4\left(a+\right.\right.}{}$$

$$= -\frac{2d^5\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6} + \frac{5d^4\left(d+e\sqrt{x}\right)^2\left(a+b\log\right.}{e^6}$$

$$= \frac{6bd^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^6} - \frac{15bd^4n\left(d+e\sqrt{x}\right)^2\left(a+\right.}{2e}$$

$$= -\frac{15b^3d^4n^3\left(d+e\sqrt{x}\right)^2}{4e^6} + \frac{40b^3d^3n^3\left(d+e\sqrt{x}\right)^3}{27e^6} - \frac{15b^3d^2n^3\left(d+e\sqrt{x}\right)^4}{32e^6}$$

$$= -\frac{15b^3d^4n^3\left(d+e\sqrt{x}\right)^2}{4e^6} + \frac{40b^3d^3n^3\left(d+e\sqrt{x}\right)^3}{27e^6} - \frac{15b^3d^2n^3\left(d+e\sqrt{x}\right)^4}{32e^6}$$

Mathematica [A] time = 0.48, size = 577, normalized size = 0.64

$$\frac{-36000a^3\left(d^6-e^6x^3\right)-60b\left(1800a^2\left(d^6-e^6x^3\right)-60abn\left(147d^6+60d^5e\sqrt{x}-30d^4e^2x+20d^3e^3x^{3/2}-15d^2e^4x^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (b^3*e*n^3*Sqrt[x]*(809340*d^5 - 140070*d^4*e*Sqrt[x] + 41180*d^3*e^2*x - 1
3785*d^2*e^3*x^(3/2) + 4368*d*e^4*x^2 - 1000*e^5*x^(5/2)) + 1800*a^2*b*n*(1
47*d^6 + 60*d^5*e*Sqrt[x] - 30*d^4*e^2*x + 20*d^3*e^3*x^(3/2) - 15*d^2*e^4*
x^2 + 12*d*e^5*x^(5/2) - 10*e^6*x^3) - 36000*a^3*(d^6 - e^6*x^3) + 60*a*b^2
*n^2*(8111*d^6 - 8820*d^5*e*Sqrt[x] + 2610*d^4*e^2*x - 1140*d^3*e^3*x^(3/2)
 + 555*d^2*e^4*x^2 - 264*d*e^5*x^(5/2) + 100*e^6*x^3) - 60*b*(b^2*n^2*(1348
9*d^6 + 8820*d^5*e*Sqrt[x] - 2610*d^4*e^2*x + 1140*d^3*e^3*x^(3/2) - 555*d^
2*e^4*x^2 + 264*d*e^5*x^(5/2) - 100*e^6*x^3) - 60*a*b*n*(147*d^6 + 60*d^5*e
*Sqrt[x] - 30*d^4*e^2*x + 20*d^3*e^3*x^(3/2) - 15*d^2*e^4*x^2 + 12*d*e^5*x^
(5/2) - 10*e^6*x^3) + 1800*a^2*(d^6 - e^6*x^3))*Log[c*(d + e*Sqrt[x])^n] -
1800*b^2*(60*a*(d^6 - e^6*x^3) + b*n*(-147*d^6 - 60*d^5*e*Sqrt[x] + 30*d^4*
e^2*x - 20*d^3*e^3*x^(3/2) + 15*d^2*e^4*x^2 - 12*d*e^5*x^(5/2) + 10*e^6*x^3
))*Log[c*(d + e*Sqrt[x])^n]^2 - 36000*b^3*(d^6 - e^6*x^3)*Log[c*(d + e*Sqrt
[x])^n]^3)/(108000*e^6)
```

fricas [A] time = 0.53, size = 1197, normalized size = 1.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/108000*(36000*b^3*e^6*x^3*log(c)^3 - 1000*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2
+ 18*a^2*b*e^6*n - 36*a^3*e^6)*x^3 + 36000*(b^3*e^6*n^3*x^3 - b^3*d^6*n^3)*
```

```
log(e*sqrt(x) + d)^3 - 15*(919*b^3*d^2*e^4*n^3 - 2220*a*b^2*d^2*e^4*n^2 + 1
800*a^2*b*d^2*e^4*n)*x^2 - 1800*(15*b^3*d^2*e^4*n^3*x^2 + 30*b^3*d^4*e^2*n^
3*x - 147*b^3*d^6*n^3 + 60*a*b^2*d^6*n^2 + 10*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^
2)*x^3 - 60*(b^3*e^6*n^2*x^3 - b^3*d^6*n^2)*log(c) - 4*(3*b^3*d*e^5*n^3*x^2
 + 5*b^3*d^3*e^3*n^3*x + 15*b^3*d^5*e*n^3)*sqrt(x))*log(e*sqrt(x) + d)^2 -
9000*(3*b^3*d^2*e^4*n*x^2 + 6*b^3*d^4*e^2*n*x + 2*(b^3*e^6*n - 6*a*b^2*e^6)
*x^3)*log(c)^2 - 30*(4669*b^3*d^4*e^2*n^3 - 5220*a*b^2*d^4*e^2*n^2 + 1800*a
^2*b*d^4*e^2*n)*x - 60*(13489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a^2*b
*d^6*n - 100*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2 + 18*a^2*b*e^6*n)*x^3 - 15*(37*
b^3*d^2*e^4*n^3 - 60*a*b^2*d^2*e^4*n^2)*x^2 - 1800*(b^3*e^6*n*x^3 - b^3*d^6
*n)*log(c)^2 - 90*(29*b^3*d^4*e^2*n^3 - 20*a*b^2*d^4*e^2*n^2)*x + 60*(15*b^
3*d^2*e^4*n^2*x^2 + 30*b^3*d^4*e^2*n^2*x - 147*b^3*d^6*n^2 + 60*a*b^2*d^6*n
 + 10*(b^3*e^6*n^2 - 6*a*b^2*e^6*n)*x^3)*log(c) + 12*(735*b^3*d^5*e*n^3 - 3
00*a*b^2*d^5*e*n^2 + 2*(11*b^3*d*e^5*n^3 - 30*a*b^2*d*e^5*n^2)*x^2 + 5*(19*
b^3*d^3*e^3*n^3 - 20*a*b^2*d^3*e^3*n^2)*x - 20*(3*b^3*d*e^5*n^2*x^2 + 5*b^3
*d^3*e^3*n^2*x + 15*b^3*d^5*e*n^2)*log(c))*sqrt(x))*log(e*sqrt(x) + d) + 30
0*(20*(b^3*e^6*n^2 - 6*a*b^2*e^6*n + 18*a^2*b*e^6)*x^3 + 3*(37*b^3*d^2*e^4*
n^2 - 60*a*b^2*d^2*e^4*n)*x^2 + 18*(29*b^3*d^4*e^2*n^2 - 20*a*b^2*d^4*e^2*n
)*x)*log(c) + 4*(202335*b^3*d^5*e*n^3 - 132300*a*b^2*d^5*e*n^2 + 27000*a^2*
b*d^5*e*n + 12*(91*b^3*d*e^5*n^3 - 330*a*b^2*d*e^5*n^2 + 450*a^2*b*d*e^5*n)
*x^2 + 1800*(3*b^3*d*e^5*n*x^2 + 5*b^3*d^3*e^3*n*x + 15*b^3*d^5*e*n)*log(c)
^2 + 5*(2059*b^3*d^3*e^3*n^3 - 3420*a*b^2*d^3*e^3*n^2 + 1800*a^2*b*d^3*e^3*
n)*x - 180*(735*b^3*d^5*e*n^2 - 300*a*b^2*d^5*e*n + 2*(11*b^3*d*e^5*n^2 - 3
0*a*b^2*d*e^5*n)*x^2 + 5*(19*b^3*d^3*e^3*n^2 - 20*a*b^2*d^3*e^3*n)*x)*log(c
))*sqrt(x))/e^6
```

giac [B] time = 0.36, size = 2223, normalized size = 2.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="giac")
```

```
[Out] 1/108000*(36000*b^3*x^3*e*log(c)^3 + 108000*a*b^2*x^3*e*log(c)^2 + 108000*a
^2*b*x^3*e*log(c) + (36000*(sqrt(x)*e + d)^6*e^(-5)*log(sqrt(x)*e + d)^3 -
216000*(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt(x)*e + d)^3 + 540000*(sqrt(x)*e
+ d)^4*d^2*e^(-5)*log(sqrt(x)*e + d)^3 - 720000*(sqrt(x)*e + d)^3*d^3*e^(-5
)*log(sqrt(x)*e + d)^3 + 540000*(sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x)*e
+ d)^3 - 216000*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x)*e + d)^3 - 18000*(sq
rt(x)*e + d)^6*e^(-5)*log(sqrt(x)*e + d)^2 + 129600*(sqrt(x)*e + d)^5*d*e^(
-5)*log(sqrt(x)*e + d)^2 - 405000*(sqrt(x)*e + d)^4*d^2*e^(-5)*log(sqrt(x)*
e + d)^2 + 720000*(sqrt(x)*e + d)^3*d^3*e^(-5)*log(sqrt(x)*e + d)^2 - 81000
0*(sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x)*e + d)^2 + 648000*(sqrt(x)*e + d
)*d^5*e^(-5)*log(sqrt(x)*e + d)^2 + 6000*(sqrt(x)*e + d)^6*e^(-5)*log(sqrt(
x)*e + d) - 51840*(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt(x)*e + d) + 202500*(s
qrt(x)*e + d)^4*d^2*e^(-5)*log(sqrt(x)*e + d) - 480000*(sqrt(x)*e + d)^3*d^
3*e^(-5)*log(sqrt(x)*e + d) + 810000*(sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(
x)*e + d) - 1296000*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x)*e + d) - 1000*(s
qrt(x)*e + d)^6*e^(-5) + 10368*(sqrt(x)*e + d)^5*d*e^(-5) - 50625*(sqrt(x)*
e + d)^4*d^2*e^(-5) + 160000*(sqrt(x)*e + d)^3*d^3*e^(-5) - 405000*(sqrt(x)
*e + d)^2*d^4*e^(-5) + 1296000*(sqrt(x)*e + d)*d^5*e^(-5))*b^3*n^3 + 36000*
a^3*x^3*e + 60*(1800*(sqrt(x)*e + d)^6*e^(-5)*log(sqrt(x)*e + d)^2 - 10800*
(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt(x)*e + d)^2 + 27000*(sqrt(x)*e + d)^4*d
^2*e^(-5)*log(sqrt(x)*e + d)^2 - 36000*(sqrt(x)*e + d)^3*d^3*e^(-5)*log(sqr
t(x)*e + d)^2 + 27000*(sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x)*e + d)^2 - 1
0800*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x)*e + d)^2 - 600*(sqrt(x)*e + d)^
6*e^(-5)*log(sqrt(x)*e + d) + 4320*(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt(x)*e
 + d) - 13500*(sqrt(x)*e + d)^4*d^2*e^(-5)*log(sqrt(x)*e + d) + 24000*(sqrt
(x)*e + d)^3*d^3*e^(-5)*log(sqrt(x)*e + d) - 27000*(sqrt(x)*e + d)^2*d^4*e^
(-5)*log(sqrt(x)*e + d) + 21600*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x)*e +
```

```
d) + 100*(sqrt(x)*e + d)^6*e^(-5) - 864*(sqrt(x)*e + d)^5*d*e^(-5) + 3375*(
sqrt(x)*e + d)^4*d^2*e^(-5) - 8000*(sqrt(x)*e + d)^3*d^3*e^(-5) + 13500*(sq
rt(x)*e + d)^2*d^4*e^(-5) - 21600*(sqrt(x)*e + d)*d^5*e^(-5))*b^3*n^2*log(c
) + 1800*(60*(sqrt(x)*e + d)^6*e^(-5)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e +
 d)^5*d*e^(-5)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^4*d^2*e^(-5)*log(sq
rt(x)*e + d) - 1200*(sqrt(x)*e + d)^3*d^3*e^(-5)*log(sqrt(x)*e + d) + 900*(
sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)*d^5*e^
(-5)*log(sqrt(x)*e + d) - 10*(sqrt(x)*e + d)^6*e^(-5) + 72*(sqrt(x)*e + d)^
5*d*e^(-5) - 225*(sqrt(x)*e + d)^4*d^2*e^(-5) + 400*(sqrt(x)*e + d)^3*d^3*e
^(-5) - 450*(sqrt(x)*e + d)^2*d^4*e^(-5) + 360*(sqrt(x)*e + d)*d^5*e^(-5))*
b^3*n*log(c)^2 + 60*(1800*(sqrt(x)*e + d)^6*e^(-5)*log(sqrt(x)*e + d)^2 - 1
0800*(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt(x)*e + d)^2 + 27000*(sqrt(x)*e + d
)^4*d^2*e^(-5)*log(sqrt(x)*e + d)^2 - 36000*(sqrt(x)*e + d)^3*d^3*e^(-5)*lo
g(sqrt(x)*e + d)^2 + 27000*(sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x)*e + d)^
2 - 10800*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x)*e + d)^2 - 600*(sqrt(x)*e
+ d)^6*e^(-5)*log(sqrt(x)*e + d) + 4320*(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt
(x)*e + d) - 13500*(sqrt(x)*e + d)^4*d^2*e^(-5)*log(sqrt(x)*e + d) + 24000*
(sqrt(x)*e + d)^3*d^3*e^(-5)*log(sqrt(x)*e + d) - 27000*(sqrt(x)*e + d)^2*d
^4*e^(-5)*log(sqrt(x)*e + d) + 21600*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x)
*e + d) + 100*(sqrt(x)*e + d)^6*e^(-5) - 864*(sqrt(x)*e + d)^5*d*e^(-5) + 3
375*(sqrt(x)*e + d)^4*d^2*e^(-5) - 8000*(sqrt(x)*e + d)^3*d^3*e^(-5) + 1350
0*(sqrt(x)*e + d)^2*d^4*e^(-5) - 21600*(sqrt(x)*e + d)*d^5*e^(-5))*a*b^2*n^
2 + 3600*(60*(sqrt(x)*e + d)^6*e^(-5)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e +
 d)^5*d*e^(-5)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^4*d^2*e^(-5)*log(sq
rt(x)*e + d) - 1200*(sqrt(x)*e + d)^3*d^3*e^(-5)*log(sqrt(x)*e + d) + 900*(
sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)*d^5*e^
(-5)*log(sqrt(x)*e + d) - 10*(sqrt(x)*e + d)^6*e^(-5) + 72*(sqrt(x)*e + d)^
5*d*e^(-5) - 225*(sqrt(x)*e + d)^4*d^2*e^(-5) + 400*(sqrt(x)*e + d)^3*d^3*e
^(-5) - 450*(sqrt(x)*e + d)^2*d^4*e^(-5) + 360*(sqrt(x)*e + d)*d^5*e^(-5))*
a*b^2*n*log(c) + 1800*(60*(sqrt(x)*e + d)^6*e^(-5)*log(sqrt(x)*e + d) - 360
*(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^4*d^2*
e^(-5)*log(sqrt(x)*e + d) - 1200*(sqrt(x)*e + d)^3*d^3*e^(-5)*log(sqrt(x)*e
 + d) + 900*(sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x)*e + d) - 360*(sqrt(x)*
e + d)*d^5*e^(-5)*log(sqrt(x)*e + d) - 10*(sqrt(x)*e + d)^6*e^(-5) + 72*(sq
rt(x)*e + d)^5*d*e^(-5) - 225*(sqrt(x)*e + d)^4*d^2*e^(-5) + 400*(sqrt(x)*e
 + d)^3*d^3*e^(-5) - 450*(sqrt(x)*e + d)^2*d^4*e^(-5) + 360*(sqrt(x)*e + d)
*d^5*e^(-5))*a^2*b*n)*e^(-1)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\left(e\sqrt{x}+d\right)^{n}\right)+a\right)^{3} x^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*(e*x^(1/2)+d)^n)+a)^3,x)
```

```
[Out] int(x^2*(b*ln(c*(e*x^(1/2)+d)^n)+a)^3,x)
```

maxima [A] time = 0.56, size = 666, normalized size = 0.73

$$\frac{1}{3}\, b^{3} x^{3} \log\left(\left(e\sqrt{x}+d\right)^{n} c\right)^{3} + a b^{2} x^{3} \log\left(\left(e\sqrt{x}+d\right)^{n} c\right)^{2} + a^{2} b x^{3} \log\left(\left(e\sqrt{x}+d\right)^{n} c\right) + \frac{1}{3}\, a^{3} x^{3} - \frac{1}{60}\, a^{2} b e n \left(\frac{60\, d^{6} \log\left(e\right.}{e^{7}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3*log((e*sqrt(x) + d)^n*c)^3 + a*b^2*x^3*log((e*sqrt(x) + d)^n*c)
^2 + a^2*b*x^3*log((e*sqrt(x) + d)^n*c) + 1/3*a^3*x^3 - 1/60*a^2*b*e*n*(60*
```

```
d^6*log(e*sqrt(x) + d)/e^7 + (10*e^5*x^3 - 12*d*e^4*x^(5/2) + 15*d^2*e^3*x^
2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*x - 60*d^5*sqrt(x))/e^6) - 1/1800*(60*e*n
*(60*d^6*log(e*sqrt(x) + d)/e^7 + (10*e^5*x^3 - 12*d*e^4*x^(5/2) + 15*d^2*e
^3*x^2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*x - 60*d^5*sqrt(x))/e^6)*log((e*sqrt
(x) + d)^n*c) - (100*e^6*x^3 - 264*d*e^5*x^(5/2) + 555*d^2*e^4*x^2 + 1800*d
^6*log(e*sqrt(x) + d)^2 - 1140*d^3*e^3*x^(3/2) + 2610*d^4*e^2*x + 8820*d^6*
log(e*sqrt(x) + d) - 8820*d^5*e*sqrt(x))*n^2/e^6)*a*b^2 - 1/108000*(1800*e*
n*(60*d^6*log(e*sqrt(x) + d)/e^7 + (10*e^5*x^3 - 12*d*e^4*x^(5/2) + 15*d^2*
e^3*x^2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*x - 60*d^5*sqrt(x))/e^6)*log((e*sqr
t(x) + d)^n*c)^2 + e*n*((1000*e^6*x^3 + 36000*d^6*log(e*sqrt(x) + d)^3 - 43
68*d*e^5*x^(5/2) + 13785*d^2*e^4*x^2 + 264600*d^6*log(e*sqrt(x) + d)^2 - 41
180*d^3*e^3*x^(3/2) + 140070*d^4*e^2*x + 809340*d^6*log(e*sqrt(x) + d) - 80
9340*d^5*e*sqrt(x))*n^2/e^7 - 60*(100*e^6*x^3 - 264*d*e^5*x^(5/2) + 555*d^2
*e^4*x^2 + 1800*d^6*log(e*sqrt(x) + d)^2 - 1140*d^3*e^3*x^(3/2) + 2610*d^4*
e^2*x + 8820*d^6*log(e*sqrt(x) + d) - 8820*d^5*e*sqrt(x))*n*log((e*sqrt(x)
+ d)^n*c)/e^7))*b^3
```

mupad [B] time = 8.18, size = 976, normalized size = 1.08

$$\frac{a^3\,x^3}{3}+\frac{b^3\,x^3\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^3}{3}-\frac{b^3\,n^3\,x^3}{108}+a\,b^2\,x^3\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^2-\frac{b^3\,n\,x^3\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^2}{6}+\frac{b^3\,n^2\,x^3\,\mathrm{ln}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/2))^n))^3,x)
```

```
[Out] (a^3*x^3)/3 + (b^3*x^3*log(c*(d + e*x^(1/2))^n)^3)/3 - (b^3*n^3*x^3)/108 +
a*b^2*x^3*log(c*(d + e*x^(1/2))^n)^2 - (b^3*n*x^3*log(c*(d + e*x^(1/2))^n)^
2)/6 + (b^3*n^2*x^3*log(c*(d + e*x^(1/2))^n))/18 + (a*b^2*n^2*x^3)/18 - (b^
3*d^6*log(c*(d + e*x^(1/2))^n)^3)/(3*e^6) + a^2*b*x^3*log(c*(d + e*x^(1/2))
^n) - (a^2*b*n*x^3)/6 - (a*b^2*n*x^3*log(c*(d + e*x^(1/2))^n))/3 - (13489*b
^3*d^6*n^3*log(d + e*x^(1/2)))/(1800*e^6) - (919*b^3*d^2*n^3*x^2)/(7200*e^2
) + (2059*b^3*d^3*n^3*x^(3/2))/(5400*e^3) + (13489*b^3*d^5*n^3*x^(1/2))/(18
00*e^5) - (a*b^2*d^6*log(c*(d + e*x^(1/2))^n)^2)/e^6 + (49*b^3*d^6*n*log(c*
(d + e*x^(1/2))^n)^2)/(20*e^6) + (91*b^3*d*n^3*x^(5/2))/(2250*e) - (4669*b^
3*d^4*n^3*x)/(3600*e^4) - (a^2*b*d^6*n*log(d + e*x^(1/2)))/e^6 + (b^3*d*n*x
^(5/2)*log(c*(d + e*x^(1/2))^n)^2)/(5*e) - (11*b^3*d*n^2*x^(5/2)*log(c*(d +
 e*x^(1/2))^n))/(75*e) - (b^3*d^4*n*x*log(c*(d + e*x^(1/2))^n)^2)/(2*e^4) +
 (29*b^3*d^4*n^2*x*log(c*(d + e*x^(1/2))^n))/(20*e^4) - (a^2*b*d^2*n*x^2)/(
4*e^2) - (11*a*b^2*d*n^2*x^(5/2))/(75*e) + (29*a*b^2*d^4*n^2*x)/(20*e^4) +
(a^2*b*d^3*n*x^(3/2))/(3*e^3) + (a^2*b*d^5*n*x^(1/2))/e^5 + (49*a*b^2*d^6*n
^2*log(d + e*x^(1/2)))/(10*e^6) - (b^3*d^2*n*x^2*log(c*(d + e*x^(1/2))^n)^2
)/(4*e^2) + (37*b^3*d^2*n^2*x^2*log(c*(d + e*x^(1/2))^n))/(120*e^2) + (b^3*
d^3*n*x^(3/2)*log(c*(d + e*x^(1/2))^n)^2)/(3*e^3) - (19*b^3*d^3*n^2*x^(3/2)
*log(c*(d + e*x^(1/2))^n))/(30*e^3) + (b^3*d^5*n*x^(1/2)*log(c*(d + e*x^(1/
2))^n)^2)/e^5 - (49*b^3*d^5*n^2*x^(1/2)*log(c*(d + e*x^(1/2))^n))/(10*e^5)
+ (37*a*b^2*d^2*n^2*x^2)/(120*e^2) - (19*a*b^2*d^3*n^2*x^(3/2))/(30*e^3) -
(49*a*b^2*d^5*n^2*x^(1/2))/(10*e^5) + (a^2*b*d*n*x^(5/2))/(5*e) - (a^2*b*d^
4*n*x)/(2*e^4) + (2*a*b^2*d*n*x^(5/2)*log(c*(d + e*x^(1/2))^n))/(5*e) - (a*
b^2*d^4*n*x*log(c*(d + e*x^(1/2))^n))/e^4 - (a*b^2*d^2*n*x^2*log(c*(d + e*x
^(1/2))^n))/(2*e^2) + (2*a*b^2*d^3*n*x^(3/2)*log(c*(d + e*x^(1/2))^n))/(3*e
^3) + (2*a*b^2*d^5*n*x^(1/2)*log(c*(d + e*x^(1/2))^n))/e^5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**n))**3,x)
```

```
[Out] Integral(x**2*(a + b*log(c*(d + e*sqrt(x))**n))**3, x)
```

3.416 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=595

$$\frac{9b^2d^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^4}+\frac{3b^2n^2\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{16e^4}-\frac{4b^2dn^2\left(d+e\right.}{}$$

```
[Out] -12*a*b^2*d^3*n^2*x^(1/2)/e^3+12*b^3*d^3*n^3*x^(1/2)/e^3-12*b^3*d^3*n^2*ln(
c*(d+e*x^(1/2))^n)*(d+e*x^(1/2))/e^4+6*b*d^3*n*(a+b*ln(c*(d+e*x^(1/2))^n))^
2*(d+e*x^(1/2))/e^4-2*d^3*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))/e^4-9
/4*b^3*d^2*n^3*(d+e*x^(1/2))^2/e^4+9/2*b^2*d^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^
n))*(d+e*x^(1/2))^2/e^4-9/2*b*d^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1
/2))^2/e^4+3*d^2*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))^2/e^4+4/9*b^3*
d*n^3*(d+e*x^(1/2))^3/e^4-4/3*b^2*d*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^
(1/2))^3/e^4+2*b*d*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^3/e^4-2*d*
(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))^3/e^4-3/64*b^3*n^3*(d+e*x^(1/2)
)^4/e^4+3/16*b^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^4/e^4-3/8*b*
n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^4/e^4+1/2*(a+b*ln(c*(d+e*x^(1
/2))^n))^3*(d+e*x^(1/2))^4/e^4
```

Rubi [A] time = 0.62, antiderivative size = 595, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{9b^2d^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^4}+\frac{3b^2n^2\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{16e^4}-\frac{4b^2dn^2\left(d+e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (-9*b^3*d^2*n^3*(d + e*Sqrt[x])^2)/(4*e^4) + (4*b^3*d*n^3*(d + e*Sqrt[x])^3
)/(9*e^4) - (3*b^3*n^3*(d + e*Sqrt[x])^4)/(64*e^4) - (12*a*b^2*d^3*n^2*Sqrt
[x])/e^3 + (12*b^3*d^3*n^3*Sqrt[x])/e^3 - (12*b^3*d^3*n^2*(d + e*Sqrt[x])*L
og[c*(d + e*Sqrt[x])^n])/e^4 + (9*b^2*d^2*n^2*(d + e*Sqrt[x])^2*(a + b*Log[
c*(d + e*Sqrt[x])^n]))/(2*e^4) - (4*b^2*d*n^2*(d + e*Sqrt[x])^3*(a + b*Log[
c*(d + e*Sqrt[x])^n]))/(3*e^4) + (3*b^2*n^2*(d + e*Sqrt[x])^4*(a + b*Log[c*
(d + e*Sqrt[x])^n]))/(16*e^4) + (6*b*d^3*n*(d + e*Sqrt[x])*(a + b*Log[c*(d
+ e*Sqrt[x])^n])^2)/e^4 - (9*b*d^2*n*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*
Sqrt[x])^n])^2)/(2*e^4) + (2*b*d*n*(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sq
rt[x])^n])^2)/e^4 - (3*b*n*(d + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n
])^2)/(8*e^4) - (2*d^3*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/
e^4 + (3*d^2*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^4 - (2
*d*(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^4 + ((d + e*Sqrt
[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/(2*e^4)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx = 2\,\mathrm{Subst}\left(\int x^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \sqrt{x}\right)$$
$$= 2\,\mathrm{Subst}\left(\int\left(-\frac{d^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^3}+\frac{3d^2(d+ex)\left(a+b\log\left(c(d\right.\right.}{e^3}\right.\right.$$
$$= \frac{2\,\mathrm{Subst}\left(\int(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \sqrt{x}\right)}{e^3}-(6d)\,\mathrm{Subst}\left(\right.$$
$$= \frac{2\,\mathrm{Subst}\left(\int x^3\left(a+b\log\left(cx^n\right)\right)^3\,dx, x, d+e\sqrt{x}\right)}{e^4}-(6d)\,\mathrm{Subst}\left(\int x^2\left(a\right.\right.$$
$$= -\frac{2d^3\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4}+\frac{3d^2\left(d+e\sqrt{x}\right)^2\left(a+b\,l\right.}{e^4}$$
$$= \frac{6bd^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^4}-\frac{9bd^2n\left(d+e\sqrt{x}\right)^2\left(a+\right.}{2}$$
$$= -\frac{9b^3d^2n^3\left(d+e\sqrt{x}\right)^2}{4e^4}+\frac{4b^3dn^3\left(d+e\sqrt{x}\right)^3}{9e^4}-\frac{3b^3n^3\left(d+e\sqrt{x}\right)^4}{64e^4}-12a$$
$$= -\frac{9b^3d^2n^3\left(d+e\sqrt{x}\right)^2}{4e^4}+\frac{4b^3dn^3\left(d+e\sqrt{x}\right)^3}{9e^4}-\frac{3b^3n^3\left(d+e\sqrt{x}\right)^4}{64e^4}-12a$$

Mathematica [A] time = 0.31, size = 433, normalized size = 0.73

$$\frac{-288a^3\left(d^4-e^4x^2\right)-12b\left(72a^2\left(d^4-e^4x^2\right)-12abn\left(25d^4+12d^3e\sqrt{x}-6d^2e^2x+4de^3x^{3/2}-3e^4x^2\right)+b^2n^2\left(41\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (b^3*e*n^3*Sqrt[x]*(4980*d^3 - 690*d^2*e*Sqrt[x] + 148*d*e^2*x - 27*e^3*x^(
3/2)) + 72*a^2*b*n*(25*d^4 + 12*d^3*e*Sqrt[x] - 6*d^2*e^2*x + 4*d*e^3*x^(3/
2) - 3*e^4*x^2) - 288*a^3*(d^4 - e^4*x^2) + 12*a*b^2*n^2*(161*d^4 - 300*d^3
*e*Sqrt[x] + 78*d^2*e^2*x - 28*d*e^3*x^(3/2) + 9*e^4*x^2) - 12*b*(b^2*n^2*(
415*d^4 + 300*d^3*e*Sqrt[x] - 78*d^2*e^2*x + 28*d*e^3*x^(3/2) - 9*e^4*x^2)
- 12*a*b*n*(25*d^4 + 12*d^3*e*Sqrt[x] - 6*d^2*e^2*x + 4*d*e^3*x^(3/2) - 3*e
^4*x^2) + 72*a^2*(d^4 - e^4*x^2))*Log[c*(d + e*Sqrt[x])^n] - 72*b^2*(12*a*(
d^4 - e^4*x^2) + b*n*(-25*d^4 - 12*d^3*e*Sqrt[x] + 6*d^2*e^2*x - 4*d*e^3*x^
(3/2) + 3*e^4*x^2))*Log[c*(d + e*Sqrt[x])^n]^2 - 288*b^3*(d^4 - e^4*x^2)*Lo
g[c*(d + e*Sqrt[x])^n]^3)/(576*e^4)
```

fricas [A] time = 0.54, size = 861, normalized size = 1.45

$$\frac{288\,b^3e^4x^2\log(c)^3+288\left(b^3e^4n^3x^2-b^3d^4n^3\right)\log\left(e\sqrt{x}+d\right)^3-9\left(3\,b^3e^4n^3-12\,ab^2e^4n^2+24\,a^2be^4n-32\,a^3e^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/576*(288*b^3*e^4*x^2*log(c)^3 + 288*(b^3*e^4*n^3*x^2 - b^3*d^4*n^3)*log(e
*sqrt(x) + d)^3 - 9*(3*b^3*e^4*n^3 - 12*a*b^2*e^4*n^2 + 24*a^2*b*e^4*n - 32
*a^3*e^4)*x^2 - 72*(6*b^3*d^2*e^2*n^3*x - 25*b^3*d^4*n^3 + 12*a*b^2*d^4*n^2
```

```
 + 3*(b^3*e^4*n^3 - 4*a*b^2*e^4*n^2)*x^2 - 12*(b^3*e^4*n^2*x^2 - b^3*d^4*n^
2)*log(c) - 4*(b^3*d*e^3*n^3*x + 3*b^3*d^3*e*n^3)*sqrt(x))*log(e*sqrt(x) +
d)^2 - 216*(2*b^3*d^2*e^2*n*x + (b^3*e^4*n - 4*a*b^2*e^4)*x^2)*log(c)^2 - 6
*(115*b^3*d^2*e^2*n^3 - 156*a*b^2*d^2*e^2*n^2 + 72*a^2*b*d^2*e^2*n)*x - 12*
(415*b^3*d^4*n^3 - 300*a*b^2*d^4*n^2 + 72*a^2*b*d^4*n - 9*(b^3*e^4*n^3 - 4*
a*b^2*e^4*n^2 + 8*a^2*b*e^4*n)*x^2 - 72*(b^3*e^4*n*x^2 - b^3*d^4*n)*log(c)^
2 - 6*(13*b^3*d^2*e^2*n^3 - 12*a*b^2*d^2*e^2*n^2)*x + 12*(6*b^3*d^2*e^2*n^2
*x - 25*b^3*d^4*n^2 + 12*a*b^2*d^4*n + 3*(b^3*e^4*n^2 - 4*a*b^2*e^4*n)*x^2)
*log(c) + 4*(75*b^3*d^3*e*n^3 - 36*a*b^2*d^3*e*n^2 + (7*b^3*d*e^3*n^3 - 12*
a*b^2*d*e^3*n^2)*x - 12*(b^3*d*e^3*n^2*x + 3*b^3*d^3*e*n^2)*log(c))*sqrt(x)
)*log(e*sqrt(x) + d) + 36*(3*(b^3*e^4*n^2 - 4*a*b^2*e^4*n + 8*a^2*b*e^4)*x^
2 + 2*(13*b^3*d^2*e^2*n^2 - 12*a*b^2*d^2*e^2*n)*x)*log(c) + 4*(1245*b^3*d^3
*e*n^3 - 900*a*b^2*d^3*e*n^2 + 216*a^2*b*d^3*e*n + 72*(b^3*d*e^3*n*x + 3*b^
3*d^3*e*n)*log(c)^2 + (37*b^3*d*e^3*n^3 - 84*a*b^2*d*e^3*n^2 + 72*a^2*b*d*e
^3*n)*x - 12*(75*b^3*d^3*e*n^2 - 36*a*b^2*d^3*e*n + (7*b^3*d*e^3*n^2 - 12*a
*b^2*d*e^3*n)*x)*log(c))*sqrt(x))/e^4
```

giac [B] time = 0.30, size = 1483, normalized size = 2.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="giac")
```

```
[Out] 1/576*(288*b^3*x^2*e*log(c)^3 + 864*a*b^2*x^2*e*log(c)^2 + (288*(sqrt(x)*e
+ d)^4*e^(-3)*log(sqrt(x)*e + d)^3 - 1152*(sqrt(x)*e + d)^3*d*e^(-3)*log(sq
rt(x)*e + d)^3 + 1728*(sqrt(x)*e + d)^2*d^2*e^(-3)*log(sqrt(x)*e + d)^3 - 1
152*(sqrt(x)*e + d)*d^3*e^(-3)*log(sqrt(x)*e + d)^3 - 216*(sqrt(x)*e + d)^4
*e^(-3)*log(sqrt(x)*e + d)^2 + 1152*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*
e + d)^2 - 2592*(sqrt(x)*e + d)^2*d^2*e^(-3)*log(sqrt(x)*e + d)^2 + 3456*(s
qrt(x)*e + d)*d^3*e^(-3)*log(sqrt(x)*e + d)^2 + 108*(sqrt(x)*e + d)^4*e^(-3
)*log(sqrt(x)*e + d) - 768*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*e + d) +
2592*(sqrt(x)*e + d)^2*d^2*e^(-3)*log(sqrt(x)*e + d) - 6912*(sqrt(x)*e + d)
*d^3*e^(-3)*log(sqrt(x)*e + d) - 27*(sqrt(x)*e + d)^4*e^(-3) + 256*(sqrt(x)
*e + d)^3*d*e^(-3) - 1296*(sqrt(x)*e + d)^2*d^2*e^(-3) + 6912*(sqrt(x)*e +
d)*d^3*e^(-3))*b^3*n^3 + 12*(72*(sqrt(x)*e + d)^4*e^(-3)*log(sqrt(x)*e + d)
^2 - 288*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*e + d)^2 + 432*(sqrt(x)*e +
 d)^2*d^2*e^(-3)*log(sqrt(x)*e + d)^2 - 288*(sqrt(x)*e + d)*d^3*e^(-3)*log(
sqrt(x)*e + d)^2 - 36*(sqrt(x)*e + d)^4*e^(-3)*log(sqrt(x)*e + d) + 192*(sq
rt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*e + d) - 432*(sqrt(x)*e + d)^2*d^2*e^(-
3)*log(sqrt(x)*e + d) + 576*(sqrt(x)*e + d)*d^3*e^(-3)*log(sqrt(x)*e + d) +
 9*(sqrt(x)*e + d)^4*e^(-3) - 64*(sqrt(x)*e + d)^3*d*e^(-3) + 216*(sqrt(x)*
e + d)^2*d^2*e^(-3) - 576*(sqrt(x)*e + d)*d^3*e^(-3))*b^3*n^2*log(c) + 864*
a^2*b*x^2*e*log(c) + 72*(12*(sqrt(x)*e + d)^4*e^(-3)*log(sqrt(x)*e + d) - 4
8*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*e + d) + 72*(sqrt(x)*e + d)^2*d^2*
e^(-3)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)*d^3*e^(-3)*log(sqrt(x)*e + d
) - 3*(sqrt(x)*e + d)^4*e^(-3) + 16*(sqrt(x)*e + d)^3*d*e^(-3) - 36*(sqrt(x
)*e + d)^2*d^2*e^(-3) + 48*(sqrt(x)*e + d)*d^3*e^(-3))*b^3*n*log(c)^2 + 12*
(72*(sqrt(x)*e + d)^4*e^(-3)*log(sqrt(x)*e + d)^2 - 288*(sqrt(x)*e + d)^3*d
*e^(-3)*log(sqrt(x)*e + d)^2 + 432*(sqrt(x)*e + d)^2*d^2*e^(-3)*log(sqrt(x)
*e + d)^2 - 288*(sqrt(x)*e + d)*d^3*e^(-3)*log(sqrt(x)*e + d)^2 - 36*(sqrt(
x)*e + d)^4*e^(-3)*log(sqrt(x)*e + d) + 192*(sqrt(x)*e + d)^3*d*e^(-3)*log(
sqrt(x)*e + d) - 432*(sqrt(x)*e + d)^2*d^2*e^(-3)*log(sqrt(x)*e + d) + 576*
(sqrt(x)*e + d)*d^3*e^(-3)*log(sqrt(x)*e + d) + 9*(sqrt(x)*e + d)^4*e^(-3)
- 64*(sqrt(x)*e + d)^3*d*e^(-3) + 216*(sqrt(x)*e + d)^2*d^2*e^(-3) - 576*(s
qrt(x)*e + d)*d^3*e^(-3))*a*b^2*n^2 + 288*a^3*x^2*e + 144*(12*(sqrt(x)*e +
d)^4*e^(-3)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*
e + d) + 72*(sqrt(x)*e + d)^2*d^2*e^(-3)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e
 + d)*d^3*e^(-3)*log(sqrt(x)*e + d) - 3*(sqrt(x)*e + d)^4*e^(-3) + 16*(sqrt
(x)*e + d)^3*d*e^(-3) - 36*(sqrt(x)*e + d)^2*d^2*e^(-3) + 48*(sqrt(x)*e + d
```

```
)*d^3*e^(-3))*a*b^2*n*log(c) + 72*(12*(sqrt(x)*e + d)^4*e^(-3)*log(sqrt(x)*
e + d) - 48*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*e + d) + 72*(sqrt(x)*e +
 d)^2*d^2*e^(-3)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)*d^3*e^(-3)*log(sqr
t(x)*e + d) - 3*(sqrt(x)*e + d)^4*e^(-3) + 16*(sqrt(x)*e + d)^3*d*e^(-3) -
36*(sqrt(x)*e + d)^2*d^2*e^(-3) + 48*(sqrt(x)*e + d)*d^3*e^(-3))*a^2*b*n)*e
^(-1)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(b \ln \left(c \left(e\sqrt{x}+d\right)^{n}\right)+a\right)^{3} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(e*x^(1/2)+d)^n)+a)^3,x)
```

```
[Out] int(x*(b*ln(c*(e*x^(1/2)+d)^n)+a)^3,x)
```

maxima [A] time = 0.57, size = 536, normalized size = 0.90

$$\frac{1}{2}\,b^{3}x^{2}\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)^{3}+\frac{3}{2}\,ab^{2}x^{2}\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)^{2}-\frac{1}{8}\,a^{2}ben\left(\frac{12\,d^{4}\log\left(e\sqrt{x}+d\right)}{e^{5}}+\frac{3\,e^{3}x^{2}-4\,de^{2}x^{\frac{3}{2}}+6}{e^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*log((e*sqrt(x) + d)^n*c)^3 + 3/2*a*b^2*x^2*log((e*sqrt(x) + d)^
n*c)^2 - 1/8*a^2*b*e*n*(12*d^4*log(e*sqrt(x) + d)/e^5 + (3*e^3*x^2 - 4*d*e^
2*x^(3/2) + 6*d^2*e*x - 12*d^3*sqrt(x))/e^4) + 3/2*a^2*b*x^2*log((e*sqrt(x)
 + d)^n*c) + 1/2*a^3*x^2 - 1/48*(12*e*n*(12*d^4*log(e*sqrt(x) + d)/e^5 + (3
*e^3*x^2 - 4*d*e^2*x^(3/2) + 6*d^2*e*x - 12*d^3*sqrt(x))/e^4)*log((e*sqrt(x
) + d)^n*c) - (9*e^4*x^2 + 72*d^4*log(e*sqrt(x) + d)^2 - 28*d*e^3*x^(3/2) +
 78*d^2*e^2*x + 300*d^4*log(e*sqrt(x) + d) - 300*d^3*e*sqrt(x))*n^2/e^4)*a*
b^2 - 1/576*(72*e*n*(12*d^4*log(e*sqrt(x) + d)/e^5 + (3*e^3*x^2 - 4*d*e^2*x
^(3/2) + 6*d^2*e*x - 12*d^3*sqrt(x))/e^4)*log((e*sqrt(x) + d)^n*c)^2 + e*n*
((288*d^4*log(e*sqrt(x) + d)^3 + 27*e^4*x^2 + 1800*d^4*log(e*sqrt(x) + d)^2
 - 148*d*e^3*x^(3/2) + 690*d^2*e^2*x + 4980*d^4*log(e*sqrt(x) + d) - 4980*d
^3*e*sqrt(x))*n^2/e^5 - 12*(9*e^4*x^2 + 72*d^4*log(e*sqrt(x) + d)^2 - 28*d*
e^3*x^(3/2) + 78*d^2*e^2*x + 300*d^4*log(e*sqrt(x) + d) - 300*d^3*e*sqrt(x)
)*n*log((e*sqrt(x) + d)^n*c)/e^5))*b^3
```

mupad [B] time = 0.77, size = 840, normalized size = 1.41

$$\ln\left(c\left(d+e\,\sqrt{x}\right)^{n}\right)^{3}\,\left(\frac{b^{3}\,x^{2}}{2}-\frac{b^{3}\,d^{4}}{2\,e^{4}}\right)-x^{3/2}\left(\frac{d\,\left(2\,a^{3}-\frac{3\,a^{2}\,b\,n}{2}+\frac{3\,a\,b^{2}\,n^{2}}{4}-\frac{3\,b^{3}\,n^{3}}{16}\right)}{3\,e}-\frac{d\,\left(24\,a^{3}-12\,a\,b^{2}\,n^{2}+7\,b^{3}\,n^{3}\right.}{36\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(1/2))^n))^3,x)
```

```
[Out] log(c*(d + e*x^(1/2))^n)^3*((b^3*x^2)/2 - (b^3*d^4)/(2*e^4)) - x^(3/2)*((d*
(2*a^3 - (3*b^3*n^3)/16 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/2))/(3*e) - (d*(24*
a^3 + 7*b^3*n^3 - 12*a*b^2*n^2))/(36*e)) - log(c*(d + e*x^(1/2))^n)^2*((x^(
3/2)*((b^2*d*(4*a - b*n))/e - (4*a*b^2*d)/e))/2 - (3*b^2*x^2*(4*a - b*n))/8
 + (d*(12*a*b^2*d^3 - 25*b^3*d^3*n))/(8*e^4) + (d^2*x^(1/2)*((6*b^2*d*(4*a
- b*n))/e - (24*a*b^2*d)/e))/(4*e^2) - (d*x*((6*b^2*d*(4*a - b*n))/e - (24*
a*b^2*d)/e))/(8*e)) + x*((d*((d*(2*a^3 - (3*b^3*n^3)/16 + (3*a*b^2*n^2)/4 -
 (3*a^2*b*n)/2))/e - (d*(24*a^3 + 7*b^3*n^3 - 12*a*b^2*n^2))/(12*e)))/(2*e)
 + (b^2*d^2*n^2*(12*a - 13*b*n))/(16*e^2)) - x^(1/2)*((d*((d*((d*(2*a^3 - (
3*b^3*n^3)/16 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/2))/e - (d*(24*a^3 + 7*b^3*n^
3 - 12*a*b^2*n^2))/(12*e)))/e + (b^2*d^2*n^2*(12*a - 13*b*n))/(8*e^2)))/e +
 (b^2*d^3*n^2*(12*a - 25*b*n))/(4*e^3)) + x^2*(a^3/2 - (3*b^3*n^3)/64 + (3*
a*b^2*n^2)/16 - (3*a^2*b*n)/8) + (log(c*(d + e*x^(1/2))^n)*((x^(3/2)*(16*b*
d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*a*b*n)))/(12*e^2)
 - (x*((d*(16*b*d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*a
*b*n)))/e - 24*b^3*d^2*e^2*n^2))/(8*e^2) + (x^(1/2)*((d*((d*(16*b*d*e^3*(6*
a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*a*b*n)))/e - 24*b^3*d^2*e^
2*n^2))/e - 48*b^3*d^3*e*n^2))/(4*e^2) + (3*b*e^2*x^2*(8*a^2 + b^2*n^2 - 4*
a*b*n))/4))/(4*e^2) - (log(d + e*x^(1/2))*(415*b^3*d^4*n^3 - 300*a*b^2*d^4*
n^2 + 72*a^2*b*d^4*n))/(48*e^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))**n))**3,x)
```

```
[Out] Integral(x*(a + b*log(c*(d + e*sqrt(x))**n))**3, x)
```

3.417 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3 dx$

Optimal. Leaf size=284

$$\frac{3b^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^2}-\frac{12ab^2dn^2\sqrt{x}}{e}-\frac{3bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^2}+\cdots$$

```
[Out] -12*a*b^2*d*n^2*x^(1/2)/e+12*b^3*d*n^3*x^(1/2)/e-12*b^3*d*n^2*ln(c*(d+e*x^(
1/2))^n)*(d+e*x^(1/2))/e^2+6*b*d*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/
2))/e^2-2*d*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))/e^2-3/4*b^3*n^3*(d+
e*x^(1/2))^2/e^2+3/2*b^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^2/e^
2-3/2*b*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^2/e^2+(a+b*ln(c*(d+e*
x^(1/2))^n))^3*(d+e*x^(1/2))^2/e^2
```

Rubi [A] time = 0.25, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2451, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{3b^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^2}-\frac{12ab^2dn^2\sqrt{x}}{e}-\frac{3bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^2}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (-3*b^3*n^3*(d + e*Sqrt[x])^2)/(4*e^2) - (12*a*b^2*d*n^2*Sqrt[x])/e + (12*b
^3*d*n^3*Sqrt[x])/e - (12*b^3*d*n^2*(d + e*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n
])/e^2 + (3*b^2*n^2*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*
e^2) + (6*b*d*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/e^2 - (
3*b*n*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*e^2) - (2*d*
(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^2 + ((d + e*Sqrt[x])^
2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^2
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx = 2\,\text{Subst}\left(\int x\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int\left(-\frac{d\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e}+\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e}\right.\right.$$

$$= \frac{2\,\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \sqrt{x}\right)}{e} - \frac{(2d)\,\text{Subst}\left(\int\left(a\right.\right.}{}$$

$$= \frac{2\,\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)^3\,dx, x, d+e\sqrt{x}\right)}{e^2} - \frac{(2d)\,\text{Subst}\left(\int\left(a+b\log\left(\right.\right.\right.}{e^2}$$

$$= -\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^2} + \frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+\right.\right.\right.}{e^2}$$

$$= \frac{6bdn\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2} - \frac{3bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(\right.\right.}{2e^2}$$

$$= -\frac{3b^3n^3\left(d+e\sqrt{x}\right)^2}{4e^2} - \frac{12ab^2dn^2\sqrt{x}}{e} + \frac{3b^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\right.\right.\right.}{2e^2}$$

$$= -\frac{3b^3n^3\left(d+e\sqrt{x}\right)^2}{4e^2} - \frac{12ab^2dn^2\sqrt{x}}{e} + \frac{12b^3dn^3\sqrt{x}}{e} - \frac{12b^3dn^2\left(d+e\sqrt{x}\right)\log}{e^2}$$

Mathematica [A] time = 0.21, size = 241, normalized size = 0.85

$$\frac{4\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3 - 8d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3 + 24bdn\left(\left(d+e\sqrt{x}\right)\left(a+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (-8*d*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^3 + 4*(d + e*Sqrt[x]
)^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3 + 24*b*d*n*((d + e*Sqrt[x])*(a + b*L
og[c*(d + e*Sqrt[x])^n])^2 - 2*b*n*(e*(a - b*n)*Sqrt[x] + b*(d + e*Sqrt[x])
*Log[c*(d + e*Sqrt[x])^n])) - 3*b*n*(2*(d + e*Sqrt[x])^2*(a + b*Log[c*(d +
e*Sqrt[x])^n])^2 + b*n*(b*e*n*(2*d*Sqrt[x] + e*x) - 2*(d + e*Sqrt[x])^2*(a
+ b*Log[c*(d + e*Sqrt[x])^n]))))/(4*e^2)
```

fricas [B] time = 0.46, size = 527, normalized size = 1.86

$$\frac{4\,b^3e^2x\log(c)^3 + 4\left(b^3e^2n^3x - b^3d^2n^3\right)\log\left(e\sqrt{x}+d\right)^3 - 6\left(b^3e^2n - 2\,ab^2e^2\right)x\log(c)^2 + 6\left(2\,b^3den^3\sqrt{x} + 3\,b^3d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^3*e^2*x*log(c)^3 + 4*(b^3*e^2*n^3*x - b^3*d^2*n^3)*log(e*sqrt(x) +
 d)^3 - 6*(b^3*e^2*n - 2*a*b^2*e^2)*x*log(c)^2 + 6*(2*b^3*d*e*n^3*sqrt(x) +
 3*b^3*d^2*n^3 - 2*a*b^2*d^2*n^2 - (b^3*e^2*n^3 - 2*a*b^2*e^2*n^2)*x + 2*(b
^3*e^2*n^2*x - b^3*d^2*n^2)*log(c))*log(e*sqrt(x) + d)^2 + 6*(b^3*e^2*n^2 -
 2*a*b^2*e^2*n + 2*a^2*b*e^2)*x*log(c) - (3*b^3*e^2*n^3 - 6*a*b^2*e^2*n^2 +
 6*a^2*b*e^2*n - 4*a^3*e^2)*x - 6*(7*b^3*d^2*n^3 - 6*a*b^2*d^2*n^2 + 2*a^2*
b*d^2*n - 2*(b^3*e^2*n*x - b^3*d^2*n)*log(c)^2 - (b^3*e^2*n^3 - 2*a*b^2*e^2
*n^2 + 2*a^2*b*e^2*n)*x - 2*(3*b^3*d^2*n^2 - 2*a*b^2*d^2*n - (b^3*e^2*n^2 -
 2*a*b^2*e^2*n)*x)*log(c) + 2*(3*b^3*d*e*n^3 - 2*b^3*d*e*n^2*log(c) - 2*a*b
^2*d*e*n^2)*sqrt(x))*log(e*sqrt(x) + d) + 6*(7*b^3*d*e*n^3 + 2*b^3*d*e*n*lo
g(c)^2 - 6*a*b^2*d*e*n^2 + 2*a^2*b*d*e*n - 2*(3*b^3*d*e*n^2 - 2*a*b^2*d*e*n
)*log(c))*sqrt(x))/e^2
```

giac [B] time = 0.26, size = 763, normalized size = 2.69

$$\frac{1}{4}\left(\left(4\left(\sqrt{x}e+d\right)^2\log\left(\sqrt{x}e+d\right)^3 - 8\left(\sqrt{x}e+d\right)d\log\left(\sqrt{x}e+d\right)^3 - 6\left(\sqrt{x}e+d\right)^2\log\left(\sqrt{x}e+d\right)^2 + 24\left(\sqrt{x}e+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="giac")
```

```
[Out] 1/4*((4*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)^3 - 8*(sqrt(x)*e + d)*d*log(sq
rt(x)*e + d)^3 - 6*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)^2 + 24*(sqrt(x)*e +
 d)*d*log(sqrt(x)*e + d)^2 + 6*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) - 48*(s
qrt(x)*e + d)*d*log(sqrt(x)*e + d) - 3*(sqrt(x)*e + d)^2 + 48*(sqrt(x)*e +
d)*d)*b^3*n^3*e^(-1) + 6*(2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)^2 - 4*(sqr
t(x)*e + d)*d*log(sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)
 + 8*(sqrt(x)*e + d)*d*log(sqrt(x)*e + d) + (sqrt(x)*e + d)^2 - 8*(sqrt(x)*
e + d)*d)*b^3*n^2*e^(-1)*log(c) + 6*(2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)
 - 4*(sqrt(x)*e + d)*d*log(sqrt(x)*e + d) - (sqrt(x)*e + d)^2 + 4*(sqrt(x)*
e + d)*d)*b^3*n*e^(-1)*log(c)^2 + 4*((sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)*
d)*b^3*e^(-1)*log(c)^3 + 6*(2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)^2 - 4*(s
qrt(x)*e + d)*d*log(sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)^2*log(sqrt(x)*e +
d) + 8*(sqrt(x)*e + d)*d*log(sqrt(x)*e + d) + (sqrt(x)*e + d)^2 - 8*(sqrt(x
)*e + d)*d)*a*b^2*n^2*e^(-1) + 12*(2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) -
 4*(sqrt(x)*e + d)*d*log(sqrt(x)*e + d) - (sqrt(x)*e + d)^2 + 4*(sqrt(x)*e
+ d)*d)*a*b^2*n*e^(-1)*log(c) + 12*((sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)*d
)*a*b^2*e^(-1)*log(c)^2 + 6*(2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) - 4*(sq
rt(x)*e + d)*d*log(sqrt(x)*e + d) - (sqrt(x)*e + d)^2 + 4*(sqrt(x)*e + d)*d
)*a^2*b*n*e^(-1) + 12*((sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)*d)*a^2*b*e^(-1
)*log(c) + 4*((sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)*d)*a^3*e^(-1))*e^(-1)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(b \ln\left(c\left(e\sqrt{x}+d\right)^{n}\right)+a\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/2)+d)^n)+a)^3,x)
```

```
[Out] int((b*ln(c*(e*x^(1/2)+d)^n)+a)^3,x)
```

maxima [A] time = 0.90, size = 381, normalized size = 1.34

$$-\frac{3}{2}\left(en\left(\frac{2\,d^{2}\log\left(e\sqrt{x}+d\right)}{e^{3}}+\frac{ex-2\,d\sqrt{x}}{e^{2}}\right)-2\,x\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)\right)a^{2}b-\frac{3}{2}\left(2\,en\left(\frac{2\,d^{2}\log\left(e\sqrt{x}+d\right)}{e^{3}}+\frac{ex-2\,d\sqrt{x}}{e^{2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="maxima")
```

```
[Out] -3/2*(e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 + (e*x - 2*d*sqrt(x))/e^2) - 2*x*lo
g((e*sqrt(x) + d)^n*c))*a^2*b - 3/2*(2*e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 +
(e*x - 2*d*sqrt(x))/e^2)*log((e*sqrt(x) + d)^n*c) - 2*x*log((e*sqrt(x) + d)
^n*c)^2 - (2*d^2*log(e*sqrt(x) + d)^2 + e^2*x + 6*d^2*log(e*sqrt(x) + d) -
6*d*e*sqrt(x))*n^2/e^2)*a*b^2 - 1/4*(6*e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 +
(e*x - 2*d*sqrt(x))/e^2)*log((e*sqrt(x) + d)^n*c)^2 - 4*x*log((e*sqrt(x) +
d)^n*c)^3 + e*n*((4*d^2*log(e*sqrt(x) + d)^3 + 18*d^2*log(e*sqrt(x) + d)^2
+ 3*e^2*x + 42*d^2*log(e*sqrt(x) + d) - 42*d*e*sqrt(x))*n^2/e^3 - 6*(2*d^2*
log(e*sqrt(x) + d)^2 + e^2*x + 6*d^2*log(e*sqrt(x) + d) - 6*d*e*sqrt(x))*n*
log((e*sqrt(x) + d)^n*c)/e^3))*b^3 + a^3*x
```

mupad [B] time = 0.60, size = 350, normalized size = 1.23

$$x\left(a^{3}-\frac{3\,a^{2}\,b\,n}{2}+\frac{3\,a\,b^{2}\,n^{2}}{2}-\frac{3\,b^{3}\,n^{3}}{4}\right)-\sqrt{x}\left(\frac{d\left(2\,a^{3}-3\,a^{2}\,b\,n+3\,a\,b^{2}\,n^{2}-\frac{3\,b^{3}\,n^{3}}{2}\right)}{e}-\frac{d\left(2\,a^{3}-6\,a\,b^{2}\,n^{2}+9\,b^{3}\,n^{3}\right.}{e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^3,x)
```

```
[Out] x*(a^3 - (3*b^3*n^3)/4 + (3*a*b^2*n^2)/2 - (3*a^2*b*n)/2) - x^(1/2)*((d*(2*
a^3 - (3*b^3*n^3)/2 + 3*a*b^2*n^2 - 3*a^2*b*n))/e - (d*(2*a^3 + 9*b^3*n^3 -
 6*a*b^2*n^2))/e) + log(c*(d + e*x^(1/2))^n)^3*(b^3*x - (b^3*d^2)/e^2) - lo
g(c*(d + e*x^(1/2))^n)*(x^(1/2)*((3*b*d*(2*a^2 + b^2*n^2 - 2*a*b*n))/e - (6
*b*d*(a^2 - b^2*n^2))/e) - (3*b*x*(2*a^2 + b^2*n^2 - 2*a*b*n))/2) - log(c*(
d + e*x^(1/2))^n)^2*(x^(1/2)*((3*b^2*d*(2*a - b*n))/e - (6*a*b^2*d)/e) + (3
*d*(2*a*b^2*d - 3*b^3*d*n))/(2*e^2) - (3*b^2*x*(2*a - b*n))/2) - (log(d + e
*x^(1/2))*(21*b^3*d^2*n^3 - 18*a*b^2*d^2*n^2 + 6*a^2*b*d^2*n))/(2*e^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**3, x)
```

3.418 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx$

Optimal. Leaf size=135

$$-12b^2n^2\text{Li}_3\left(\frac{\sqrt{x}e}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+6bn\text{Li}_2\left(\frac{\sqrt{x}e}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2+2\log\left(-\frac{e}{}\right.$$

```
[Out] 2*ln(-e*x^(1/2)/d)*(a+b*ln(c*(d+e*x^(1/2))^n))^3+6*b*n*(a+b*ln(c*(d+e*x^(1/
2))^n))^2*polylog(2,1+e*x^(1/2)/d)-12*b^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*p
olylog(3,1+e*x^(1/2)/d)+12*b^3*n^3*polylog(4,1+e*x^(1/2)/d)
```

Rubi [A] time = 0.20, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2454, 2396, 2433, 2374, 2383, 6589}

$$-12b^2n^2\text{PolyLog}\left(3,\frac{e\sqrt{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+6bn\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x,x]
```

```
[Out] 2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3*Log[-((e*Sqrt[x])/d)] + 6*b*n*(a + b*L
og[c*(d + e*Sqrt[x])^n])^2*PolyLog[2, 1 + (e*Sqrt[x])/d] - 12*b^2*n^2*(a +
b*Log[c*(d + e*Sqrt[x])^n])*PolyLog[3, 1 + (e*Sqrt[x])/d] + 12*b^3*n^3*Poly
Log[4, 1 + (e*Sqrt[x])/d]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx = 2\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x}\,dx, x, \sqrt{x}\right)$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right) - (6ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\left(a\right.}{}\right.$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right) - (6bn)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(cx\right.\right.}{}\right.$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right.$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right.$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right.$$

Mathematica [B] time = 0.17, size = 333, normalized size = 2.47

$$6b^2n^2\left(-2\text{Li}_3\left(\frac{\sqrt{x}e}{d}+1\right)+2\text{Li}_2\left(\frac{\sqrt{x}e}{d}+1\right)\log\left(d+e\sqrt{x}\right)+\log\left(-\frac{e\sqrt{x}}{d}\right)\log^2\left(d+e\sqrt{x}\right)\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2*((Log[d + e*Sqr
t[x]] - Log[1 + (e*Sqrt[x])/d])*Log[x] - 2*PolyLog[2, -((e*Sqrt[x])/d)]) +
6*b^2*n^2*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])*(Log[d
+ e*Sqrt[x]]^2*Log[-((e*Sqrt[x])/d)] + 2*Log[d + e*Sqrt[x]]*PolyLog[2, 1 +
(e*Sqrt[x])/d] - 2*PolyLog[3, 1 + (e*Sqrt[x])/d]) + 2*b^3*n^3*(Log[d + e*Sq
rt[x]]^3*Log[-((e*Sqrt[x])/d)] + 3*Log[d + e*Sqrt[x]]^2*PolyLog[2, 1 + (e*S
qrt[x])/d] - 6*Log[d + e*Sqrt[x]]*PolyLog[3, 1 + (e*Sqrt[x])/d] + 6*PolyLog
[4, 1 + (e*Sqrt[x])/d])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^{3}\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)^{3}+3\,ab^{2}\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)^{2}+3\,a^{2}b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+a^{3}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*sqrt(x) + d)^n*c)^3 + 3*a*b^2*log((e*sqrt(x) + d)^n*c)
^2 + 3*a^2*b*log((e*sqrt(x) + d)^n*c) + a^3)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+a\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^3/x, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(e\sqrt{x}+d\right)^{n}\right)+a\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/2)+d)^n)+a)^3/x,x)
```

```
[Out] int((b*ln(c*(e*x^(1/2)+d)^n)+a)^3/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^{3}n^{3}\log\left(e\sqrt{x}+d\right)^{3}\log(x)+\int -\frac{3\left(b^{3}enx\log(x)-2\left(b^{3}e\log(c)+ab^{2}e\right)x-2\left(b^{3}d\log(c)+ab^{2}d\right)\sqrt{x}\right)n^{2}\log\left(e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*n^3*log(e*sqrt(x) + d)^3*log(x) + integrate(-1/2*(3*(b^3*e*n*x*log(x) -
 2*(b^3*e*log(c) + a*b^2*e)*x - 2*(b^3*d*log(c) + a*b^2*d)*sqrt(x))*n^2*log
(e*sqrt(x) + d)^2 - 6*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b
^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*sqrt(x))*n*log(e*sqrt(x) + d) -
 2*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x - 2*(
b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*sqrt(x))/(e
*x^2 + d*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^{n}\right)\right)}^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^3/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \log \left(c \left(d + e\sqrt{x}\right)^{n}\right)\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**3/x, x)
```

3.419 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=263

$$\frac{6b^2e^2n^2\text{Li}_2\left(\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2}+\frac{6b^2e^2n^2\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2}-\frac{3be^2n\log\left(1\right.}{}$$

```
[Out] 6*b^2*e^2*n^2*ln(-e*x^(1/2)/d)*(a+b*ln(c*(d+e*x^(1/2))^n))/d^2-(a+b*ln(c*(d
+e*x^(1/2))^n))^3/x-3*b*e^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*ln(1-d/(d+e*x^(
1/2)))/d^2+6*b^2*e^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*polylog(2,d/(d+e*x^(1/
2)))/d^2+6*b^3*e^2*n^3*polylog(2,1+e*x^(1/2)/d)/d^2+6*b^3*e^2*n^3*polylog(3
,d/(d+e*x^(1/2)))/d^2-3*b*e*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))/d
^2/x^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 283, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391}

$$-\frac{6b^2e^2n^2\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2}+\frac{6b^3e^2n^3\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)}{d^2}+\frac{6b^3e^2n^3\text{PolyLog}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^2,x]
```

```
[Out] (-3*b*e*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(d^2*Sqrt[x])
 + (e^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/d^2 - (a + b*Log[c*(d + e*Sqrt[
x])^n])^3/x + (6*b^2*e^2*n^2*(a + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt
[x])/d)])/d^2 - (3*b*e^2*n*(a + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[-((e*Sqrt
[x])/d)])/d^2 + (6*b^3*e^2*n^3*PolyLog[2, 1 + (e*Sqrt[x])/d])/d^2 - (6*b^2*
e^2*n^2*(a + b*Log[c*(d + e*Sqrt[x])^n])*PolyLog[2, 1 + (e*Sqrt[x])/d])/d^2
 + (6*b^3*e^2*n^3*PolyLog[3, 1 + (e*Sqrt[x])/d])/d^2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
```

```
, p}, x] && GtQ[p, 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx = 2\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x} + (3ben)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)}{x^2(d+ex)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x} + (3bn)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(cx^n\right)\right)^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x,\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x} + \frac{(3bn)\,\text{Subst}\left(\int \frac{\left(a+b\log(cx^n)\right)^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\right.}{d}$$

$$= -\frac{3ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2\sqrt{x}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right.\right.}{x}$$

$$= -\frac{3ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2\sqrt{x}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right.\right.}{x}$$

$$= -\frac{3ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2\sqrt{x}} + \frac{e^2\left(a+b\log\left(c\left(d+e\sqrt{\right.\right.\right.}{d^2}$$

$$= -\frac{3ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2\sqrt{x}} + \frac{e^2\left(a+b\log\left(c\left(d+e\sqrt{\right.\right.\right.}{d^2}$$

Mathematica [B] time = 0.75, size = 536, normalized size = 2.04

$$\frac{3b^2n^2\left(-2e^2x\text{Li}_2\left(\frac{\sqrt{x}e}{d}+1\right)-2e^2x\left(\log\left(d+e\sqrt{x}\right)-1\right)\log\left(-\frac{e\sqrt{x}}{d}\right)+\left(d+e\sqrt{x}\right)\log\left(d+e\sqrt{x}\right)\left(\left(e\sqrt{x}-d\right)\log\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^2,x]
```

```
[Out] (-3*b*d*e*n*Sqrt[x]*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n
])^2 - 3*b*d^2*n*Log[d + e*Sqrt[x]]*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(
d + e*Sqrt[x])^n])^2 + 3*b*e^2*n*x*Log[d + e*Sqrt[x]]*(a - b*n*Log[d + e*Sq
rt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2 - d^2*(a - b*n*Log[d + e*Sqrt[x]] +
b*Log[c*(d + e*Sqrt[x])^n])^3 - (3*b*e^2*n*x*(a - b*n*Log[d + e*Sqrt[x]] +
b*Log[c*(d + e*Sqrt[x])^n])^2*Log[x])/2 + 3*b^2*n^2*(a - b*n*Log[d + e*Sqrt
[x]] + b*Log[c*(d + e*Sqrt[x])^n])*((d + e*Sqrt[x])*Log[d + e*Sqrt[x]]*(-2*
e*Sqrt[x] + (-d + e*Sqrt[x])*Log[d + e*Sqrt[x]]) - 2*e^2*x*(-1 + Log[d + e*
Sqrt[x]])*Log[-((e*Sqrt[x])/d)] - 2*e^2*x*PolyLog[2, 1 + (e*Sqrt[x])/d]) +
b^3*n^3*((d + e*Sqrt[x])*Log[d + e*Sqrt[x]]^2*(-3*e*Sqrt[x] + (-d + e*Sqrt[
x])*Log[d + e*Sqrt[x]]) - 3*e^2*x*(-2 + Log[d + e*Sqrt[x]])*Log[d + e*Sqrt[
```

```
x]]*Log[-((e*Sqrt[x])/d)] - 6*e^2*x*(-1 + Log[d + e*Sqrt[x]])*PolyLog[2, 1
+ (e*Sqrt[x])/d] + 6*e^2*x*PolyLog[3, 1 + (e*Sqrt[x])/d]))/(d^2*x)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^3\log\left(\left(e\sqrt{x}+d\right)^n c\right)^3+3\,ab^2\log\left(\left(e\sqrt{x}+d\right)^n c\right)^2+3\,a^2b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*sqrt(x) + d)^n*c)^3 + 3*a*b^2*log((e*sqrt(x) + d)^n*c)
^2 + 3*a^2*b*log((e*sqrt(x) + d)^n*c) + a^3)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^3/x^2, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\left(e\sqrt{x}+d\right)^n\right)+a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/2)+d)^n)+a)^3/x^2,x)
```

```
[Out] int((b*ln(c*(e*x^(1/2)+d)^n)+a)^3/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,b^3d^2n^3\sqrt{x}\,\log\left(e\sqrt{x}+d\right)^3-3\left(2\,b^3e^2nx^{\frac{3}{2}}\log\left(e\sqrt{x}+d\right)-2\,b^3denx-\left(b^3e^2nx\log(x)+2\,b^3d^2\log(c)+2\,ab^2d^2\right.\right.}{2\,d^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^3*d^2*n^3*sqrt(x)*log(e*sqrt(x) + d)^3 - 3*(2*b^3*e^2*n*x^(3/2)*l
og(e*sqrt(x) + d) - 2*b^3*d*e*n*x - (b^3*e^2*n*x*log(x) + 2*b^3*d^2*log(c)
+ 2*a*b^2*d^2)*sqrt(x))*n^2*log(e*sqrt(x) + d)^2)/(d^2*x^(3/2)) - integrate
(1/2*(3*(2*b^3*e^3*n^2*x^(5/2)*log(e*sqrt(x) + d) - 2*b^3*d*e^2*n^2*x^2 - 2
*(b^3*d^2*e*log(c)^2 + 2*a*b^2*d^2*e*log(c) + a^2*b*d^2*e)*x^(3/2) - 2*(b^3
*d^3*log(c)^2 + 2*a*b^2*d^3*log(c) + a^2*b*d^3)*x - (b^3*e^3*n^2*x^2*log(x)
 + 2*(b^3*d^2*e*n*log(c) + a*b^2*d^2*e*n)*x)*sqrt(x))*n*log(e*sqrt(x) + d)
- 2*(b^3*d^2*e*log(c)^3 + 3*a*b^2*d^2*e*log(c)^2 + 3*a^2*b*d^2*e*log(c) + a
^3*d^2*e)*x^(3/2) - 2*(b^3*d^3*log(c)^3 + 3*a*b^2*d^3*log(c)^2 + 3*a^2*b*d^
3*log(c) + a^3*d^3)*x)/(d^2*e*x^(7/2) + d^3*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\left(d+e\,\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^3/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^3/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3/x**2,x)
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**3/x**2, x)
```

3.420 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=573

$$\frac{3b^2e^4n^2\operatorname{Li}_2\left(\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4}+\frac{5b^2e^4n^2\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^4}+\frac{3b^2e^4n^2\,\mathrm{lo}}{}$$

```
[Out] -3/2*b^3*e^4*n^3*ln(x)/d^4+1/2*b^3*e^4*n^3*ln(d+e*x^(1/2))/d^4-1/2*b^2*e^2*
n^2*(a+b*ln(c*(d+e*x^(1/2))^n))/d^2/x+3*b^2*e^4*n^2*ln(-e*x^(1/2)/d)*(a+b*l
n(c*(d+e*x^(1/2))^n))/d^4-1/2*b*e*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2/d/x^(3/2)
+3/4*b*e^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2/d^2/x-1/2*(a+b*ln(c*(d+e*x^(1/2)
)^n))^3/x^2+5/2*b^2*e^4*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*ln(1-d/(d+e*x^(1/2)
))/d^4-3/2*b*e^4*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*ln(1-d/(d+e*x^(1/2)))/d^4-
5/2*b^3*e^4*n^3*polylog(2,d/(d+e*x^(1/2)))/d^4+3*b^2*e^4*n^2*(a+b*ln(c*(d+e
*x^(1/2))^n))*polylog(2,d/(d+e*x^(1/2)))/d^4+3*b^3*e^4*n^3*polylog(2,1+e*x^
(1/2)/d)/d^4+3*b^3*e^4*n^3*polylog(3,d/(d+e*x^(1/2)))/d^4-1/2*b^3*e^3*n^3/d
^3/x^(1/2)+5/2*b^2*e^3*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))/d^4/x^
(1/2)-3/2*b*e^3*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))/d^4/x^(1/2)
```

Rubi [A] time = 1.50, antiderivative size = 550, normalized size of antiderivative = 0.96, number of steps used = 35, number of rules used = 17, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.708, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31, 44}

$$-\frac{3b^2e^4n^2\operatorname{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4}+\frac{11b^3e^4n^3\operatorname{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)}{2d^4}+\frac{3b^3e^4n^3\operatorname{PolyLog}(3}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^3,x]
```

```
[Out] -(b^3*e^3*n^3)/(2*d^3*Sqrt[x]) + (b^3*e^4*n^3*Log[d + e*Sqrt[x]])/(2*d^4) -
 (b^2*e^2*n^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*d^2*x) + (5*b^2*e^3*n^2*
(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*d^4*Sqrt[x]) - (5*b*e^
4*n*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(4*d^4) - (b*e*n*(a + b*Log[c*(d +
e*Sqrt[x])^n])^2)/(2*d*x^(3/2)) + (3*b*e^2*n*(a + b*Log[c*(d + e*Sqrt[x])^n
])^2)/(4*d^2*x) - (3*b*e^3*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n
])^2)/(2*d^4*Sqrt[x]) + (e^4*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/(2*d^4) -
(a + b*Log[c*(d + e*Sqrt[x])^n])^3/(2*x^2) + (11*b^2*e^4*n^2*(a + b*Log[c*(
d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)])/(2*d^4) - (3*b*e^4*n*(a + b*Log[c
*(d + e*Sqrt[x])^n])^2*Log[-((e*Sqrt[x])/d)])/(2*d^4) - (3*b^3*e^4*n^3*Log[
x])/(2*d^4) + (11*b^3*e^4*n^3*PolyLog[2, 1 + (e*Sqrt[x])/d])/(2*d^4) - (3*b
^2*e^4*n^2*(a + b*Log[c*(d + e*Sqrt[x])^n])*PolyLog[2, 1 + (e*Sqrt[x])/d])/
d^4 + (3*b^3*e^4*n^3*PolyLog[3, 1 + (e*Sqrt[x])/d])/d^4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
```

```
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx = 2\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x^5}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} + \frac{1}{2}(3ben)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right.}{x^4(d+ex)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} + \frac{1}{2}(3bn)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(cx^n\right)\right)^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx,\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} + \frac{(3bn)\,\text{Subst}\left(\int \frac{\left(a+b\log(cx^n)\right)^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\right.}{2d}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2dx^{3/2}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} - \frac{(3ben}{}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2dx^{3/2}} + \frac{3be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{4d^2x} -$$

$$= -\frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2dx^{3/2}} +$$

$$= -\frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} + \frac{5b^2e^3n^2\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d\right.\right.\right.}{2d^4\sqrt{x}}$$

$$= -\frac{b^3e^3n^3}{2d^3\sqrt{x}} + \frac{b^3e^4n^3\log\left(d+e\sqrt{x}\right)}{2d^4} - \frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} +$$

$$= -\frac{b^3e^3n^3}{2d^3\sqrt{x}} + \frac{b^3e^4n^3\log\left(d+e\sqrt{x}\right)}{2d^4} - \frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} +$$

Mathematica [A] time = 1.17, size = 841, normalized size = 1.47

$$-\frac{2\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3 d^4 + 6bn\log\left(d+e\sqrt{x}\right)\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^3,x]
```

```
[Out] -1/4*(2*b*d^3*e*n*Sqrt[x]*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt
[x])^n])^2 - 3*b*d^2*e^2*n*x*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*S
qrt[x])^n])^2 + 6*b*d*e^3*n*x^(3/2)*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(
d + e*Sqrt[x])^n])^2 + 6*b*d^4*n*Log[d + e*Sqrt[x]]*(a - b*n*Log[d + e*Sqrt
[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2 - 6*b*e^4*n*x^2*Log[d + e*Sqrt[x]]*(a
- b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2 + 2*d^4*(a - b*n*L
```

```
og[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^3 + 3*b*e^4*n*x^2*(a - b*n*
Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[x] - 2*b^2*n^2*(a -
b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])*(-3*(d^4 - e^4*x^2)*Lo
g[d + e*Sqrt[x]]^2 + e^2*x*(-d^2 + 5*d*e*Sqrt[x] + 11*e^2*x*Log[-((e*Sqrt[x
])/d)]) - Log[d + e*Sqrt[x]]*(2*d^3*e*Sqrt[x] - 3*d^2*e^2*x + 6*d*e^3*x^(3/
2) + 11*e^4*x^2 + 6*e^4*x^2*Log[-((e*Sqrt[x])/d)]) - 6*e^4*x^2*PolyLog[2, 1
 + (e*Sqrt[x])/d]) + b^3*n^3*(d^2*e^2*x*(2 - 3*Log[d + e*Sqrt[x]])*Log[d +
e*Sqrt[x]] + 2*d^3*e*Sqrt[x]*Log[d + e*Sqrt[x]]^2 + 2*d^4*Log[d + e*Sqrt[x]
]^3 + 2*d*e^3*x^(3/2)*(1 - 5*Log[d + e*Sqrt[x]] + 3*Log[d + e*Sqrt[x]]^2) +
 12*e^4*x^2*(-Log[d + e*Sqrt[x]] + Log[-((e*Sqrt[x])/d)]) + 11*e^4*x^2*(Log
[d + e*Sqrt[x]]*(Log[d + e*Sqrt[x]] - 2*Log[-((e*Sqrt[x])/d)]) - 2*PolyLog[
2, 1 + (e*Sqrt[x])/d]) - 2*e^4*x^2*(Log[d + e*Sqrt[x]]^2*(Log[d + e*Sqrt[x]
] - 3*Log[-((e*Sqrt[x])/d)]) - 6*Log[d + e*Sqrt[x]]*PolyLog[2, 1 + (e*Sqrt[
x])/d] + 6*PolyLog[3, 1 + (e*Sqrt[x])/d])))/(d^4*x^2)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\log\left(\left(e\sqrt{x}+d\right)^n c\right)^3+3\,ab^2\log\left(\left(e\sqrt{x}+d\right)^n c\right)^2+3\,a^2b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*sqrt(x) + d)^n*c)^3 + 3*a*b^2*log((e*sqrt(x) + d)^n*c)
^2 + 3*a^2*b*log((e*sqrt(x) + d)^n*c) + a^3)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^3/x^3, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\left(e\sqrt{x}+d\right)^n\right)+a\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/2)+d)^n)+a)^3/x^3,x)
```

```
[Out] int((b*ln(c*(e*x^(1/2)+d)^n)+a)^3/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3n^3\log\left(e\sqrt{x}+d\right)^3}{2\,x^2}+\int\frac{3\left(b^3enx+4\left(b^3e\log(c)+ab^2e\right)x+4\left(b^3d\log(c)+ab^2d\right)\sqrt{x}\right)n^2\log\left(e\sqrt{x}+d\right)^2+12\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^3*n^3*log(e*sqrt(x) + d)^3/x^2 + integrate(1/4*(3*(b^3*e*n*x + 4*(b^
3*e*log(c) + a*b^2*e)*x + 4*(b^3*d*log(c) + a*b^2*d)*sqrt(x))*n^2*log(e*sqr
t(x) + d)^2 + 12*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*
log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*sqrt(x))*n*log(e*sqrt(x) + d) + 4*(b
^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x + 4*(b^3*d
*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*sqrt(x))/(e*x^4
+ d*x^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^3/x^3,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^3/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \log{\left(c \left(d + e \sqrt{x}\right)^{n} \right)}\right)^{3}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3/x**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**3/x**3, x)
```

3.421 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right) dx$

Optimal. Leaf size=171

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{be^8n\log\left(d+\frac{e}{\sqrt{x}}\right)}{4d^8}-\frac{be^8n\log(x)}{8d^8}+\frac{be^7n\sqrt{x}}{4d^7}-\frac{be^6nx}{8d^6}+\frac{be^5nx^{3/2}}{12d^5}-\frac{be^4nx^2}{16d^4}+\frac{be^3nx^{5/2}}{20d^3}-$$

```
[Out] -1/8*b*e^6*n*x/d^6+1/12*b*e^5*n*x^(3/2)/d^5-1/16*b*e^4*n*x^2/d^4+1/20*b*e^3
*n*x^(5/2)/d^3-1/24*b*e^2*n*x^3/d^2+1/28*b*e*n*x^(7/2)/d-1/8*b*e^8*n*ln(x)/
d^8-1/4*b*e^8*n*ln(d+e/x^(1/2))/d^8+1/4*x^4*(a+b*ln(c*(d+e/x^(1/2))^n))+1/4
*b*e^7*n*x^(1/2)/d^7
```

Rubi [A] time = 0.13, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)+\frac{be^5nx^{3/2}}{12d^5}-\frac{be^4nx^2}{16d^4}+\frac{be^3nx^{5/2}}{20d^3}-\frac{be^2nx^3}{24d^2}+\frac{be^7n\sqrt{x}}{4d^7}-\frac{be^6nx}{8d^6}-\frac{be^8n\log\left(d+\frac{e}{\sqrt{x}}\right)}{4d^8}-\frac{be^8}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (b*e^7*n*Sqrt[x])/(4*d^7) - (b*e^6*n*x)/(8*d^6) + (b*e^5*n*x^(3/2))/(12*d^5
) - (b*e^4*n*x^2)/(16*d^4) + (b*e^3*n*x^(5/2))/(20*d^3) - (b*e^2*n*x^3)/(24
*d^2) + (b*e*n*x^(7/2))/(28*d) - (b*e^8*n*Log[d + e/Sqrt[x]])/(4*d^8) + (x^
4*(a + b*Log[c*(d + e/Sqrt[x])^n]))/4 - (b*e^8*n*Log[x])/(8*d^8)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx = -\left(2\,\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^9}dx, x, \frac{1}{\sqrt{x}}\right)\right)$$
$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right) - \frac{1}{4}(ben)\,\text{Subst}\left(\int\frac{1}{x^8(d+ex)}dx, x, \frac{1}{\sqrt{x}}\right.$$
$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right) - \frac{1}{4}(ben)\,\text{Subst}\left(\int\left(\frac{1}{dx^8}-\frac{e}{d^2x^7}+\frac{e^2}{d^3x^6}\right.\right.$$
$$= \frac{be^7n\sqrt{x}}{4d^7} - \frac{be^6nx}{8d^6} + \frac{be^5nx^{3/2}}{12d^5} - \frac{be^4nx^2}{16d^4} + \frac{be^3nx^{5/2}}{20d^3} - \frac{be^2nx^3}{24d^2} + \frac{benx^{7/2}}{28d}$$

Mathematica [A] time = 0.14, size = 158, normalized size = 0.92

$$\frac{ax^4}{4}+\frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{1}{4}ben\left(\frac{e^7\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^8}+\frac{e^7\log(x)}{2d^8}-\frac{e^6\sqrt{x}}{d^7}+\frac{e^5x}{2d^6}-\frac{e^4x^{3/2}}{3d^5}+\frac{e^3x^2}{4d^4}-\frac{e^2x^{5/2}}{5d^3}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (a*x^4)/4 + (b*x^4*Log[c*(d + e/Sqrt[x])^n])/4 - (b*e*n*(-((e^6*Sqrt[x])/d^
7) + (e^5*x)/(2*d^6) - (e^4*x^(3/2))/(3*d^5) + (e^3*x^2)/(4*d^4) - (e^2*x^(
5/2))/(5*d^3) + (e*x^3)/(6*d^2) - x^(7/2)/(7*d) + (e^7*Log[d + e/Sqrt[x]])/
d^8 + (e^7*Log[x])/(2*d^8)))/4
```

fricas [A] time = 0.45, size = 179, normalized size = 1.05

$$\frac{420\,bd^8x^4\log(c) - 70\,bd^6e^2nx^3 + 420\,ad^8x^4 - 105\,bd^4e^4nx^2 - 210\,bd^2e^6nx - 420\,bd^8n\log\left(\sqrt{x}\right) + 420\left(bd^8 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/1680*(420*b*d^8*x^4*log(c) - 70*b*d^6*e^2*n*x^3 + 420*a*d^8*x^4 - 105*b*d
^4*e^4*n*x^2 - 210*b*d^2*e^6*n*x - 420*b*d^8*n*log(sqrt(x)) + 420*(b*d^8 -
b*e^8)*n*log(d*sqrt(x) + e) + 420*(b*d^8*n*x^4 - b*d^8*n)*log((d*x + e*sqrt
(x))/x) + 4*(15*b*d^7*e*n*x^3 + 21*b*d^5*e^3*n*x^2 + 35*b*d^3*e^5*n*x + 105
*b*d*e^7*n)*sqrt(x))/d^8
```

giac [A] time = 0.38, size = 272, normalized size = 1.59

$$\frac{1}{4}bx^4\log(c)+\frac{1}{4}ax^4-\frac{1}{1680}\left(\left(\frac{420\log\left(\frac{|d\sqrt{x}+e|}{\sqrt{|x|}}\right)}{d^8}-\frac{420\log\left(\left|-d+\frac{d\sqrt{x}+e}{\sqrt{x}}\right|\right)}{d^8}+\frac{1089\,d^7-\frac{4683\left(d\sqrt{x}+e\right)d^6}{\sqrt{x}}+\frac{9639\left(d\sqrt{\right.}{x}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*log(c) + 1/4*a*x^4 - 1/1680*((420*log(abs(d*sqrt(x) + e)/sqrt(abs
(x)))/d^8 - 420*log(abs(-d + (d*sqrt(x) + e)/sqrt(x)))/d^8 + (1089*d^7 - 46
```

```
83*(d*sqrt(x) + e)*d^6/sqrt(x) + 9639*(d*sqrt(x) + e)^2*d^5/x - 11165*(d*sq
rt(x) + e)^3*d^4/x^(3/2) + 7490*(d*sqrt(x) + e)^4*d^3/x^2 - 2730*(d*sqrt(x)
 + e)^5*d^2/x^(5/2) + 420*(d*sqrt(x) + e)^6*d/x^3)/((d - (d*sqrt(x) + e)/sq
rt(x))^7*d^8))*e^9 - 420*e^9*log((d*e^(-1) - (d*sqrt(x) + e)*e^(-1)/sqrt(x)
)*(d/(d*e^(-1) - (d*sqrt(x) + e)*e^(-1)/sqrt(x)) - e))/(d - (d*sqrt(x) + e)
/sqrt(x))^8)*b*n*e^(-1)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \left(b \ln \left(c \left(d + \frac{e}{\sqrt{x}}\right)^n\right) + a\right) x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

maxima [A] time = 0.73, size = 118, normalized size = 0.69

$$\frac{1}{4}\,bx^4\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{4}\,ax^4-\frac{1}{1680}\,ben\left(\frac{420\,e^7\log\left(d\sqrt{x}+e\right)}{d^8}-\frac{60\,d^6x^{\frac{7}{2}}-70\,d^5ex^3+84\,d^4e^2x^{\frac{5}{2}}-105\,d^3e^3x^2}{d^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log(c*(d + e/sqrt(x))^n) + 1/4*a*x^4 - 1/1680*b*e*n*(420*e^7*log(
d*sqrt(x) + e)/d^8 - (60*d^6*x^(7/2) - 70*d^5*e*x^3 + 84*d^4*e^2*x^(5/2) -
105*d^3*e^3*x^2 + 140*d^2*e^4*x^(3/2) - 210*d*e^5*x + 420*e^6*sqrt(x))/d^7)
```

mupad [B] time = 0.98, size = 140, normalized size = 0.82

$$\frac{\frac{b\,d\,e^7\,n\,\sqrt{x}}{4}-\frac{b\,d^2\,e^6\,n\,x}{8}+\frac{b\,d^7\,e\,n\,x^{7/2}}{28}-\frac{b\,d^4\,e^4\,n\,x^2}{16}-\frac{b\,d^6\,e^2\,n\,x^3}{24}+\frac{b\,d^3\,e^5\,n\,x^{3/2}}{12}+\frac{b\,d^5\,e^3\,n\,x^{5/2}}{20}+\frac{b\,e^8\,n\,\mathrm{atan}\left(\frac{d\,1\mathrm{i}+\frac{e\,2\mathrm{i}}{\sqrt{x}}}{d}\right)1\mathrm{i}}{2}}{d^8}+\frac{a\,x^4}{4}+\frac{b\,x^4\,1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e/x^(1/2))^n)),x)
```

```
[Out] ((b*e^8*n*atan((d*1i + (e*2i)/x^(1/2))/d)*1i)/2 - (b*d^2*e^6*n*x)/8 + (b*d*
e^7*n*x^(1/2))/4 + (b*d^7*e*n*x^(7/2))/28 - (b*d^4*e^4*n*x^2)/16 - (b*d^6*e
^2*n*x^3)/24 + (b*d^3*e^5*n*x^(3/2))/12 + (b*d^5*e^3*n*x^(5/2))/20)/d^8 + (
a*x^4)/4 + (b*x^4*log(c*(d + e/x^(1/2))^n))/4
```

sympy [A] time = 134.39, size = 162, normalized size = 0.95

$$\frac{ax^4}{4}+b\left(\frac{en\left(\frac{2x^{\frac{7}{2}}}{7d}-\frac{ex^3}{3d^2}+\frac{2e^2x^{\frac{5}{2}}}{5d^3}-\frac{e^3x^2}{2d^4}+\frac{2e^4x^{\frac{3}{2}}}{3d^5}-\frac{e^5x}{d^6}+\frac{2e^6\sqrt{x}}{d^7}-\frac{2e^8\left(\begin{cases}\frac{1}{d\sqrt{x}} & \text{for } e=0\\ \frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{e} & \text{otherwise}\end{cases}\right)}{d^8}+\frac{2e^7\log\left(\frac{1}{\sqrt{x}}\right)}{d^8}\right)}{8}+\frac{x^4\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(1/2))**n)),x)
```

```
[Out] a*x**4/4 + b*(e*n*(2*x**(7/2)/(7*d) - e*x**3/(3*d**2) + 2*e**2*x**(5/2)/(5*
d**3) - e**3*x**2/(2*d**4) + 2*e**4*x**(3/2)/(3*d**5) - e**5*x/d**6 + 2*e**
6*sqrt(x)/d**7 - 2*e**8*Piecewise((1/(d*sqrt(x)), Eq(e, 0)), (log(d + e/sqr
t(x))/e, True))/d**8 + 2*e**7*log(1/sqrt(x))/d**8)/8 + x**4*log(c*(d + e/sq
rt(x))**n)/4)
```

3.422 $\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx$

Optimal. Leaf size=139

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{be^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3d^6}-\frac{be^6n\log(x)}{6d^6}+\frac{be^5n\sqrt{x}}{3d^5}-\frac{be^4nx}{6d^4}+\frac{be^3nx^{3/2}}{9d^3}-\frac{be^2nx^2}{12d^2}+\frac{benx^{5/2}}{15d}$$

```
[Out] -1/6*b*e^4*n*x/d^4+1/9*b*e^3*n*x^(3/2)/d^3-1/12*b*e^2*n*x^2/d^2+1/15*b*e*n*
x^(5/2)/d-1/6*b*e^6*n*ln(x)/d^6-1/3*b*e^6*n*ln(d+e/x^(1/2))/d^6+1/3*x^3*(a+
b*ln(c*(d+e/x^(1/2))^n))+1/3*b*e^5*n*x^(1/2)/d^5
```

Rubi [A] time = 0.10, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)+\frac{be^3nx^{3/2}}{9d^3}-\frac{be^2nx^2}{12d^2}+\frac{be^5n\sqrt{x}}{3d^5}-\frac{be^4nx}{6d^4}-\frac{be^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3d^6}-\frac{be^6n\log(x)}{6d^6}+\frac{benx^{5/2}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (b*e^5*n*Sqrt[x])/(3*d^5) - (b*e^4*n*x)/(6*d^4) + (b*e^3*n*x^(3/2))/(9*d^3)
 - (b*e^2*n*x^2)/(12*d^2) + (b*e*n*x^(5/2))/(15*d) - (b*e^6*n*Log[d + e/Sqr
t[x]])/(3*d^6) + (x^3*(a + b*Log[c*(d + e/Sqrt[x])^n]))/3 - (b*e^6*n*Log[x]
)/(6*d^6)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx = -\left(2\,\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^7}dx,x,\frac{1}{\sqrt{x}}\right)\right)$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{3}(ben)\,\text{Subst}\left(\int\frac{1}{x^6(d+ex)}dx,x,\frac{1}{\sqrt{x}}\right.$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{3}(ben)\,\text{Subst}\left(\int\left(\frac{1}{dx^6}-\frac{e}{d^2x^5}+\frac{e^2}{d^3x^4}\right.\right.$$

$$=\frac{be^5n\sqrt{x}}{3d^5}-\frac{be^4nx}{6d^4}+\frac{be^3nx^{3/2}}{9d^3}-\frac{be^2nx^2}{12d^2}+\frac{benx^{5/2}}{15d}-\frac{be^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3d^6}$$

Mathematica [A] time = 0.09, size = 130, normalized size = 0.94

$$\frac{ax^3}{3}+\frac{1}{3}bx^3\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{1}{3}ben\left(\frac{e^5\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^6}+\frac{e^5\log(x)}{2d^6}-\frac{e^4\sqrt{x}}{d^5}+\frac{e^3x}{2d^4}-\frac{e^2x^{3/2}}{3d^3}+\frac{ex^2}{4d^2}-\frac{x^{5/2}}{5d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (a*x^3)/3 + (b*x^3*Log[c*(d + e/Sqrt[x])^n])/3 - (b*e*n*(-((e^4*Sqrt[x])/d^
5) + (e^3*x)/(2*d^4) - (e^2*x^(3/2))/(3*d^3) + (e*x^2)/(4*d^2) - x^(5/2)/(5
*d) + (e^5*Log[d + e/Sqrt[x]])/d^6 + (e^5*Log[x])/(2*d^6)))/3
```

fricas [A] time = 0.45, size = 153, normalized size = 1.10

$$\frac{60\,bd^6x^3\log(c)-15\,bd^4e^2nx^2+60\,ad^6x^3-30\,bd^2e^4nx-60\,bd^6n\log\left(\sqrt{x}\right)+60\left(bd^6-be^6\right)n\log\left(d\sqrt{x}+e\right)+}{180\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/180*(60*b*d^6*x^3*log(c) - 15*b*d^4*e^2*n*x^2 + 60*a*d^6*x^3 - 30*b*d^2*e
^4*n*x - 60*b*d^6*n*log(sqrt(x)) + 60*(b*d^6 - b*e^6)*n*log(d*sqrt(x) + e)
+ 60*(b*d^6*n*x^3 - b*d^6*n)*log((d*x + e*sqrt(x))/x) + 4*(3*b*d^5*e*n*x^2
+ 5*b*d^3*e^3*n*x + 15*b*d*e^5*n)*sqrt(x))/d^6
```

giac [B] time = 0.39, size = 236, normalized size = 1.70

$$\frac{1}{3}bx^3\log(c)+\frac{1}{3}ax^3-\frac{1}{180}\left(\left(\frac{60\log\left(\frac{|d\sqrt{x}+e|}{\sqrt{|x|}}\right)}{d^6}-\frac{60\log\left(\left|-d+\frac{d\sqrt{x}+e}{\sqrt{x}}\right|\right)}{d^6}+\frac{137\,d^5-\frac{385\left(d\sqrt{x}+e\right)d^4}{\sqrt{x}}+\frac{470\left(d\sqrt{x}+e\right)^2d^3}{x}}{\left(d-\frac{d\sqrt{x}+e}{\sqrt{x}}\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*log(c) + 1/3*a*x^3 - 1/180*((60*log(abs(d*sqrt(x) + e)/sqrt(abs(x
)))/d^6 - 60*log(abs(-d + (d*sqrt(x) + e)/sqrt(x)))/d^6 + (137*d^5 - 385*(d
*sqrt(x) + e)*d^4/sqrt(x) + 470*(d*sqrt(x) + e)^2*d^3/x - 270*(d*sqrt(x) +
e)^3*d^2/x^(3/2) + 60*(d*sqrt(x) + e)^4*d/x^2)/((d - (d*sqrt(x) + e)/sqrt(x
```

```
))^5*d^6))*e^7 - 60*e^7*log((d*e^(-1) - (d*sqrt(x) + e)*e^(-1)/sqrt(x))*(d/
(d*e^(-1) - (d*sqrt(x) + e)*e^(-1)/sqrt(x)) - e))/(d - (d*sqrt(x) + e)/sqrt
(x))^6)*b*n*e^(-1)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*(d+e/x^(1/2))^n)+a),x)
```

```
[Out] int(x^2*(b*ln(c*(d+e/x^(1/2))^n)+a),x)
```

maxima [A] time = 0.71, size = 96, normalized size = 0.69

$$\frac{1}{3}bx^3\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+\frac{1}{3}ax^3-\frac{1}{180}ben\left(\frac{60\,e^5\log\left(d\sqrt{x}+e\right)}{d^6}-\frac{12\,d^4x^{\frac{5}{2}}-15\,d^3ex^2+20\,d^2e^2x^{\frac{3}{2}}-30\,de^3x+60\,e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3*log(c*(d + e/sqrt(x))^n) + 1/3*a*x^3 - 1/180*b*e*n*(60*e^5*log(d*
sqrt(x) + e)/d^6 - (12*d^4*x^(5/2) - 15*d^3*e*x^2 + 20*d^2*e^2*x^(3/2) - 30
*d*e^3*x + 60*e^4*sqrt(x))/d^5)
```

mupad [B] time = 0.69, size = 106, normalized size = 0.76

$$\frac{a\,x^3}{3}+\frac{b\left(60\,d^6\,x^3\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)-120\,e^6\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,\sqrt{x}}+1\right)-15\,d^4\,e^2\,n\,x^2+20\,d^3\,e^3\,n\,x^{3/2}-30\,d^2\,e^4\,n\,x+60\right.}{180\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e/x^(1/2))^n)),x)
```

```
[Out] (a*x^3)/3 + (b*(60*d^6*x^3*log(c*(d + e/x^(1/2))^n) - 120*e^6*n*atanh((2*e)
/(d*x^(1/2)) + 1) - 15*d^4*e^2*n*x^2 + 20*d^3*e^3*n*x^(3/2) - 30*d^2*e^4*n*
x + 60*d*e^5*n*x^(1/2) + 12*d^5*e*n*x^(5/2)))/(180*d^6)
```

sympy [A] time = 49.02, size = 134, normalized size = 0.96

$$\frac{ax^3}{3}+b\left(\frac{en\left(\frac{2x^{\frac{5}{2}}}{5d}-\frac{ex^2}{2d^2}+\frac{2e^2x^{\frac{3}{2}}}{3d^3}-\frac{e^3x}{d^4}+\frac{2e^4\sqrt{x}}{d^5}-\frac{2e^6\left(\begin{cases}\frac{1}{d\sqrt{x}} & \text{for}\ e=0\\ \frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{e} & \text{otherwise}\end{cases}\right)}{d^6}+\frac{2e^5\log\left(\frac{1}{\sqrt{x}}\right)}{d^6}\right)}{6}+\frac{x^3\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(1/2))**n)),x)
```

```
[Out] a*x**3/3 + b*(e*n*(2*x**(5/2)/(5*d) - e*x**2/(2*d**2) + 2*e**2*x**(3/2)/(3*
d**3) - e**3*x/d**4 + 2*e**4*sqrt(x)/d**5 - 2*e**6*Piecewise((1/(d*sqrt(x))
, Eq(e, 0)), (log(d + e/sqrt(x))/e, True))/d**6 + 2*e**5*log(1/sqrt(x))/d**
6)/6 + x**3*log(c*(d + e/sqrt(x))**n)/3)
```

3.423 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx$

Optimal. Leaf size=107

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{be^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}-\frac{be^4n\log(x)}{4d^4}+\frac{be^3n\sqrt{x}}{2d^3}-\frac{be^2nx}{4d^2}+\frac{benx^{3/2}}{6d}$$

```
[Out] -1/4*b*e^2*n*x/d^2+1/6*b*e*n*x^(3/2)/d-1/4*b*e^4*n*ln(x)/d^4-1/2*b*e^4*n*ln
(d+e/x^(1/2))/d^4+1/2*x^2*(a+b*ln(c*(d+e/x^(1/2))^n))+1/2*b*e^3*n*x^(1/2)/d
^3
```

Rubi [A] time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2454, 2395, 44}

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)+\frac{be^3n\sqrt{x}}{2d^3}-\frac{be^2nx}{4d^2}-\frac{be^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}-\frac{be^4n\log(x)}{4d^4}+\frac{benx^{3/2}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (b*e^3*n*Sqrt[x])/(2*d^3) - (b*e^2*n*x)/(4*d^2) + (b*e*n*x^(3/2))/(6*d) - (
b*e^4*n*Log[d + e/Sqrt[x]])/(2*d^4) + (x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])
)/2 - (b*e^4*n*Log[x])/(4*d^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx=-\left(2\,\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^5}dx,x,\frac{1}{\sqrt{x}}\right)\right)$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{2}(ben)\,\text{Subst}\left(\int\frac{1}{x^4(d+ex)}dx,x,\frac{1}{\sqrt{x}}$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{2}(ben)\,\text{Subst}\left(\int\left(\frac{1}{dx^4}-\frac{e}{d^2x^3}+\frac{e^2}{d^3x^2}\right.\right.$$
$$=\frac{be^3n\sqrt{x}}{2d^3}-\frac{be^2nx}{4d^2}+\frac{benx^{3/2}}{6d}-\frac{be^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d\right.\right.\right.$$

Mathematica [A] time = 0.03, size = 102, normalized size = 0.95

$$\frac{ax^2}{2}+\frac{1}{2}bx^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{1}{2}ben\left(\frac{e^3\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^4}+\frac{e^3\log(x)}{2d^4}-\frac{e^2\sqrt{x}}{d^3}+\frac{ex}{2d^2}-\frac{x^{3/2}}{3d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*Log[c*(d + e/Sqrt[x])^n])/2 - (b*e*n*(-((e^2*Sqrt[x])/d^
3) + (e*x)/(2*d^2) - x^(3/2)/(3*d) + (e^3*Log[d + e/Sqrt[x]])/d^4 + (e^3*Lo
g[x])/(2*d^4)))/2
```

fricas [A] time = 0.46, size = 126, normalized size = 1.18

$$\frac{6\,bd^4x^2\log(c)-3\,bd^2e^2nx+6\,ad^4x^2-6\,bd^4n\log\left(\sqrt{x}\right)+6\left(bd^4-be^4\right)n\log\left(d\sqrt{x}+e\right)+6\left(bd^4nx^2-bd^4n\right)\text{l}}{12\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*b*d^4*x^2*log(c) - 3*b*d^2*e^2*n*x + 6*a*d^4*x^2 - 6*b*d^4*n*log(sq
rt(x)) + 6*(b*d^4 - b*e^4)*n*log(d*sqrt(x) + e) + 6*(b*d^4*n*x^2 - b*d^4*n)
*log((d*x + e*sqrt(x))/x) + 2*(b*d^3*e*n*x + 3*b*d*e^3*n)*sqrt(x))/d^4
```

giac [A] time = 0.34, size = 81, normalized size = 0.76

$$\frac{1}{2}\,bx^2\log(c)+\frac{1}{12}\left(6\,x^2\log\left(d+\frac{e}{\sqrt{x}}\right)+\left(\frac{2\,d^2x^{\frac{3}{2}}-3\,dxe+6\,\sqrt{x}e^2}{d^3}-\frac{6\,e^3\log\left(\left|d\sqrt{x}+e\right|\right)}{d^4}\right)e\right)bn+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*log(c) + 1/12*(6*x^2*log(d + e/sqrt(x)) + ((2*d^2*x^(3/2) - 3*d*x
*e + 6*sqrt(x)*e^2)/d^3 - 6*e^3*log(abs(d*sqrt(x) + e))/d^4)*e)*b*n + 1/2*a
*x^2
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(d+e/x^(1/2))^n)+a),x)
[Out] int(x*(b*ln(c*(d+e/x^(1/2))^n)+a),x)
```

maxima [A] time = 0.71, size = 74, normalized size = 0.69

$$-\frac{1}{12}\,ben\left(\frac{6\,e^{3}\log\left(d\sqrt{x}+e\right)}{d^{4}}-\frac{2\,d^{2}x^{\frac{3}{2}}-3\,dex+6\,e^{2}\sqrt{x}}{d^{3}}\right)+\frac{1}{2}\,bx^{2}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+\frac{1}{2}\,ax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="maxima")
[Out] -1/12*b*e*n*(6*e^3*log(d*sqrt(x) + e)/d^4 - (2*d^2*x^(3/2) - 3*d*e*x + 6*e^
2*sqrt(x))/d^3) + 1/2*b*x^2*log(c*(d + e/sqrt(x))^n) + 1/2*a*x^2
```

mupad [B] time = 0.83, size = 86, normalized size = 0.80

$$\frac{x^{3/2}\left(\frac{b\,e\,n}{3\,d}-\frac{b\,e^{2}\,n}{2\,d^{2}\,\sqrt{x}}+\frac{b\,e^{3}\,n}{d^{3}\,x}\right)}{2}+\frac{a\,x^{2}}{2}+\frac{b\,x^{2}\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{2}-\frac{b\,e^{4}\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,\sqrt{x}}+1\right)}{d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/2))^n)),x)
[Out] (x^(3/2)*((b*e*n)/(3*d) - (b*e^2*n)/(2*d^2*x^(1/2)) + (b*e^3*n)/(d^3*x)))/2
 + (a*x^2)/2 + (b*x^2*log(c*(d + e/x^(1/2))^n))/2 - (b*e^4*n*atanh((2*e)/(d
*x^(1/2)) + 1))/d^4
```

sympy [A] time = 17.26, size = 88, normalized size = 0.82

$$\frac{ax^{2}}{2}+b\left(\frac{en\left(\frac{2x^{\frac{3}{2}}}{3d}-\frac{ex}{d^{2}}-\frac{2e^{3}\left(\begin{cases}\frac{\sqrt{x}}{e} & \text{for}\: d=0\\\frac{\log\left(d\sqrt{x}+e\right)}{d} & \text{otherwise}\end{cases}\right)}{d^{3}}+\frac{2e^{2}\sqrt{x}}{d^{3}}\right)}{4}+\frac{x^{2}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))**n)),x)
[Out] a*x**2/2 + b*(e*n*(2*x**(3/2)/(3*d) - e*x/d**2 - 2*e**3*Piecewise((sqrt(x)/
e, Eq(d, 0)), (log(d*sqrt(x) + e)/d, True))/d**3 + 2*e**2*sqrt(x)/d**3)/4 +
 x**2*log(c*(d + e/sqrt(x))**n)/2)
```

3.424 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$

Optimal. Leaf size=53

$$ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{be^2n\log\left(d\sqrt{x}+e\right)}{d^2}+\frac{ben\sqrt{x}}{d}$$

```
[Out] a*x+b*x*ln(c*(d+e/x^(1/2))^n)-b*e^2*n*ln(e+d*x^(1/2))/d^2+b*e*n*x^(1/2)/d
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2448, 263, 190, 43}

$$ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{be^2n\log\left(d\sqrt{x}+e\right)}{d^2}+\frac{ben\sqrt{x}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e/Sqrt[x])^n],x]
```

```
[Out] (b*e*n*Sqrt[x])/d + a*x + b*x*Log[c*(d + e/Sqrt[x])^n] - (b*e^2*n*Log[e + d
*Sqrt[x]])/d^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx = ax+b\int \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)dx$$
$$= ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{2}(ben)\int \frac{1}{\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}}dx$$
$$= ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{2}(ben)\int \frac{1}{e+d\sqrt{x}}dx$$
$$= ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+(ben)\,\text{Subst}\left(\int \frac{x}{e+dx}dx,x,\sqrt{x}\right)$$
$$= ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+(ben)\,\text{Subst}\left(\int \left(\frac{1}{d}-\frac{e}{d(e+dx)}\right)dx,x,\sqrt{x}\right)$$
$$= \frac{ben\sqrt{x}}{d}+ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{be^2n\log\left(e+d\sqrt{x}\right)}{d^2}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 1.17

$$ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-ben\left(\frac{e\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^2}+\frac{e\log(x)}{2d^2}-\frac{\sqrt{x}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e/Sqrt[x])^n],x]
```

```
[Out] a*x + b*x*Log[c*(d + e/Sqrt[x])^n] - b*e*n*(-(Sqrt[x]/d) + (e*Log[d + e/Sqr
t[x]])/d^2 + (e*Log[x])/(2*d^2))
```

fricas [A] time = 0.46, size = 90, normalized size = 1.70

$$\frac{bd^2x\log(c)-bd^2n\log\left(\sqrt{x}\right)+bden\sqrt{x}+ad^2x+\left(bd^2-be^2\right)n\log\left(d\sqrt{x}+e\right)+\left(bd^2nx-bd^2n\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(1/2))^n),x, algorithm="fricas")
```

```
[Out] (b*d^2*x*log(c) - b*d^2*n*log(sqrt(x)) + b*d*e*n*sqrt(x) + a*d^2*x + (b*d^2
 - b*e^2)*n*log(d*sqrt(x) + e) + (b*d^2*n*x - b*d^2*n)*log((d*x + e*sqrt(x)
)/x))/d^2
```

giac [A] time = 0.25, size = 56, normalized size = 1.06

$$-\left(\left(\left(\frac{e\log\left(\left|d\sqrt{x}+e\right|\right)}{d^2}-\frac{\sqrt{x}}{d}\right)e-x\log\left(d+\frac{e}{\sqrt{x}}\right)\right)n-x\log(c)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(1/2))^n),x, algorithm="giac")
```

```
[Out] -(((e*log(abs(d*sqrt(x) + e))/d^2 - sqrt(x)/d)*e - x*log(d + e/sqrt(x)))*n
- x*log(c))*b + a*x
```

maple [A] time = 0.08, size = 94, normalized size = 1.77

$$-\frac{b\,e^2n\ln\left(d\sqrt{x}+e\right)}{2d^2}+\frac{b\,e^2n\ln\left(d\sqrt{x}-e\right)}{2d^2}-\frac{b\,e^2n\ln\left(d^2x-e^2\right)}{2d^2}+bx\ln\left(c\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)^n\right)+\frac{ben\sqrt{x}}{d}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*ln(c*(d+e/x^(1/2))^n)+a,x)
```

```
[Out] a*x+x*b*ln(c*((e+d*x^(1/2))/x^(1/2))^n)+b*e*n*x^(1/2)/d-1/2*b*e^2*n*ln(e+d*
x^(1/2))/d^2+1/2*b*e^2*n/d^2*ln(d*x^(1/2)-e)-1/2*b*e^2*n*ln(d^2*x-e^2)/d^2
```

maxima [A] time = 0.66, size = 48, normalized size = 0.91

$$-\left(en\left(\frac{e\log\left(d\sqrt{x}+e\right)}{d^2}-\frac{\sqrt{x}}{d}\right)-x\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(1/2))^n),x, algorithm="maxima")
```

```
[Out] -(e*n*(e*log(d*sqrt(x) + e)/d^2 - sqrt(x)/d) - x*log(c*(d + e/sqrt(x))^n))*
b + a*x
```

mupad [B] time = 0.38, size = 44, normalized size = 0.83

$$a\,x+b\,x\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{b\,e\,n\,\left(e\,\ln\left(e+d\,\sqrt{x}\right)-d\,\sqrt{x}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*log(c*(d + e/x^(1/2))^n),x)
```

```
[Out] a*x + b*x*log(c*(d + e/x^(1/2))^n) - (b*e*n*(e*log(e + d*x^(1/2)) - d*x^(1/
2)))/d^2
```

sympy [A] time = 8.42, size = 76, normalized size = 1.43

$$ax+b\left(\frac{en\left(\frac{2\sqrt{x}}{d}-\frac{2e^2\left(\begin{cases}\frac{1}{d\sqrt{x}} & \text{for } e=0\\ \frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{e} & \text{otherwise}\end{cases}\right)}{d^2}+\frac{2e\log\left(\frac{1}{\sqrt{x}}\right)}{d^2}\right)}{2}+x\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e/x**(1/2))**n),x)
```

```
[Out] a*x + b*(e*n*(2*sqrt(x)/d - 2*e**2*Piecewise((1/(d*sqrt(x)), Eq(e, 0)), (lo
g(d + e/sqrt(x))/e, True))/d**2 + 2*e*log(1/sqrt(x))/d**2)/2 + x*log(c*(d +
 e/sqrt(x))**n))
```

3.425 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=51

$$-2\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-2bn\text{Li}_2\left(\frac{e}{d\sqrt{x}}+1\right)$$

```
[Out] -2*(a+b*ln(c*(d+e/x^(1/2))^n))*ln(-e/d/x^(1/2))-2*b*n*polylog(2,1+e/d/x^(1/
2))
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2394, 2315}

$$-2bn\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)-2\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x,x]
```

```
[Out] -2*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))] - 2*b*n*PolyLog[2
, 1 + e/(d*Sqrt[x])]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx,x,\frac{1}{\sqrt{x}}\right)\right)\\ &= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right)+(2ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,\right.\\ &= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right)-2bn\text{Li}_2\left(1+\frac{e}{d\sqrt{x}}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 53, normalized size = 1.04

$$a\log(x)-2b\log\left(-\frac{e}{d\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-2bn\text{Li}_2\left(\frac{d+\frac{e}{\sqrt{x}}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x,x]
```

```
[Out] -2*b*Log[c*(d + e/Sqrt[x])^n]*Log[-(e/(d*Sqrt[x]))] + a*Log[x] - 2*b*n*Poly
Log[2, (d + e/Sqrt[x])/d]
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*((d*x + e*sqrt(x))/x)^n) + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)/x, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int\frac{b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/2))^n)+a)/x,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/2))^n)+a)/x,x)
```

maxima [B] time = 2.69, size = 123, normalized size = 2.41

$$-2\left(\log\left(\frac{d\sqrt{x}}{e}+1\right)\log\left(\sqrt{x}\right)+\text{Li}_2\left(-\frac{d\sqrt{x}}{e}\right)\right)bn+\frac{4ben\log\left(d\sqrt{x}+e\right)\log(x)+ben\log(x)^2+4bdn\sqrt{x}\log(x)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x,x, algorithm="maxima")
```

```
[Out] -2*(log(d*sqrt(x)/e + 1)*log(sqrt(x)) + dilog(-d*sqrt(x)/e))*b*n + 1/4*(4*b
*e*n*log(d*sqrt(x) + e)*log(x) + b*e*n*log(x)^2 + 4*b*d*n*sqrt(x)*log(x) -
```

```
4*b*e*log(x)*log(x^(1/2*n)) - 8*b*d*n*sqrt(x) + 4*(b*e*log(c) + a*e)*log(x)
 - 4*(b*d*n*x*log(x) - 2*b*d*n*x)/sqrt(x))/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b\ \ln\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))/x,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/2))^n))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))/x, x)
```

3.426 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=65

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}+\frac{bd^2n\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^2}-\frac{bdn}{e\sqrt{x}}+\frac{bn}{2x}$$

```
[Out] 1/2*b*n/x+b*d^2*n*ln(d+e/x^(1/2))/e^2+(-a-b*ln(c*(d+e/x^(1/2))^n))/x-b*d*n/
e/x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}+\frac{bd^2n\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^2}-\frac{bdn}{e\sqrt{x}}+\frac{bn}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^2,x]
```

```
[Out] (b*n)/(2*x) - (b*d*n)/(e*Sqrt[x]) + (b*d^2*n*Log[d + e/Sqrt[x]])/e^2 - (a +
 b*Log[c*(d + e/Sqrt[x])^n])/x
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx = -\left(2\,\text{Subst}\left(\int x\left(a+b\log\left(c(d+ex)^n\right)\right)dx, x, \frac{1}{\sqrt{x}}\right)\right)$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x} + (ben)\,\text{Subst}\left(\int \frac{x^2}{d+ex}dx, x, \frac{1}{\sqrt{x}}\right)$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x} + (ben)\,\text{Subst}\left(\int \left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx, x, \frac{1}{\sqrt{x}}\right)$$
$$= \frac{bn}{2x} - \frac{bdn}{e\sqrt{x}} + \frac{bd^2n\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^2} - \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.05

$$-\frac{a}{x} - \frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x} + \frac{bd^2n\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^2} - \frac{bdn}{e\sqrt{x}} + \frac{bn}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^2,x]
```

```
[Out] -(a/x) + (b*n)/(2*x) - (b*d*n)/(e*Sqrt[x]) + (b*d^2*n*Log[d + e/Sqrt[x]])/e
^2 - (b*Log[c*(d + e/Sqrt[x])^n])/x
```

fricas [A] time = 0.42, size = 70, normalized size = 1.08

$$-\frac{2\,bden\sqrt{x} - be^2n + 2\,be^2\log(c) + 2\,ae^2 - 2\left(bd^2nx - be^2n\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)}{2\,e^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*e*n*sqrt(x) - b*e^2*n + 2*b*e^2*log(c) + 2*a*e^2 - 2*(b*d^2*n*x
 - b*e^2*n)*log((d*x + e*sqrt(x))/x))/(e^2*x)
```

giac [B] time = 0.22, size = 162, normalized size = 2.49

$$\frac{1}{2}\left(\frac{4\left(d\sqrt{x}+e\right)bdn\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{\sqrt{x}} - \frac{2\left(d\sqrt{x}+e\right)^2bn\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{x} - \frac{4\left(d\sqrt{x}+e\right)bdn}{\sqrt{x}} + \frac{4\left(d\sqrt{x}+e\right)bd\log(c)}{\sqrt{x}} + \frac{(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(d*sqrt(x) + e)*b*d*n*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) - 2*(d*sq
rt(x) + e)^2*b*n*log((d*sqrt(x) + e)/sqrt(x))/x - 4*(d*sqrt(x) + e)*b*d*n/s
qrt(x) + 4*(d*sqrt(x) + e)*b*d*log(c)/sqrt(x) + (d*sqrt(x) + e)^2*b*n/x - 2
*(d*sqrt(x) + e)^2*b*log(c)/x + 4*(d*sqrt(x) + e)*a*d/sqrt(x) - 2*(d*sqrt(x
) + e)^2*a/x)*e^(-2)
```

maple [A] time = 0.09, size = 63, normalized size = 0.97

$$\frac{b\,d^{2}n\ln\left(d+\frac{e}{\sqrt{x}}\right)}{e^{2}}-\frac{bdn}{e\sqrt{x}}+\frac{bn}{2x}-\frac{b\ln\left(c\,\mathrm{e}^{n\ln\left(d+\frac{e}{\sqrt{x}}\right)}\right)}{x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/2))^n)+a)/x^2,x)
```

```
[Out] -a/x-b/x*ln(c*exp(n*ln(d+e/x^(1/2))))+1/2*b*n/x+b*d^2*n*ln(d+e/x^(1/2))/e^2
-b*d*n/e/x^(1/2)
```

maxima [A] time = 0.81, size = 75, normalized size = 1.15

$$\frac{1}{2}\,ben\left(\frac{2\,d^{2}\log\left(d\sqrt{x}+e\right)}{e^{3}}-\frac{d^{2}\log(x)}{e^{3}}-\frac{2\,d\sqrt{x}-e}{e^{2}x}\right)-\frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d*sqrt(x) - e
)/(e^2*x)) - b*log(c*(d + e/sqrt(x))^n)/x - a/x
```

mupad [B] time = 0.43, size = 60, normalized size = 0.92

$$\frac{b\,n}{2\,x}-\frac{a}{x}-\frac{b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{x}-\frac{b\,d\,n}{e\,\sqrt{x}}+\frac{b\,d^{2}\,n\,\ln\left(d+\frac{e}{\sqrt{x}}\right)}{e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))/x^2,x)
```

```
[Out] (b*n)/(2*x) - a/x - (b*log(c*(d + e/x^(1/2))^n))/x - (b*d*n)/(e*x^(1/2)) +
(b*d^2*n*log(d + e/x^(1/2)))/e^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x**2,x)
```

```
[Out] Timed out
```

3.427 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=104

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}+\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4}-\frac{bd^3n}{2e^3\sqrt{x}}+\frac{bd^2n}{4e^2x}-\frac{bdn}{6ex^{3/2}}+\frac{bn}{8x^2}$$

```
[Out] 1/8*b*n/x^2-1/6*b*d*n/e/x^(3/2)+1/4*b*d^2*n/e^2/x+1/2*b*d^4*n*ln(d+e/x^(1/2
))/e^4+1/2*(-a-b*ln(c*(d+e/x^(1/2))^n))/x^2-1/2*b*d^3*n/e^3/x^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}-\frac{bd^3n}{2e^3\sqrt{x}}+\frac{bd^2n}{4e^2x}+\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4}-\frac{bdn}{6ex^{3/2}}+\frac{bn}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^3,x]
```

```
[Out] (b*n)/(8*x^2) - (b*d*n)/(6*e*x^(3/2)) + (b*d^2*n)/(4*e^2*x) - (b*d^3*n)/(2*
e^3*Sqrt[x]) + (b*d^4*n*Log[d + e/Sqrt[x]])/(2*e^4) - (a + b*Log[c*(d + e/S
qrt[x])^n])/(2*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx = -\left(2\,\text{Subst}\left(\int x^3\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx,x,\frac{1}{\sqrt{x}}\right)\right)$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\,\text{Subst}\left(\int\frac{x^4}{d+ex}\,dx,x,\frac{1}{\sqrt{x}}\right)$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\,\text{Subst}\left(\int\left(-\frac{d^3}{e^4}+\frac{d^2x}{e^3}-\frac{dx^2}{e^2}+\frac{x^3}{e}+\frac{}{e^4(}\right.\right.$$
$$= \frac{bn}{8x^2}-\frac{bdn}{6ex^{3/2}}+\frac{bd^2n}{4e^2x}-\frac{bd^3n}{2e^3\sqrt{x}}+\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right.}{2x^2}$$

Mathematica [A] time = 0.07, size = 109, normalized size = 1.05

$$-\frac{a}{2x^2}-\frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}+\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4}-\frac{bd^3n}{2e^3\sqrt{x}}+\frac{bd^2n}{4e^2x}-\frac{bdn}{6ex^{3/2}}+\frac{bn}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^3,x]
```

```
[Out] -1/2*a/x^2 + (b*n)/(8*x^2) - (b*d*n)/(6*e*x^(3/2)) + (b*d^2*n)/(4*e^2*x) -
(b*d^3*n)/(2*e^3*Sqrt[x]) + (b*d^4*n*Log[d + e/Sqrt[x]])/(2*e^4) - (b*Log[c
*(d + e/Sqrt[x])^n])/(2*x^2)
```

fricas [A] time = 0.42, size = 96, normalized size = 0.92

$$\frac{6\,bd^2e^2nx+3\,be^4n-12\,be^4\log(c)-12\,ae^4+12\left(bd^4nx^2-be^4n\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)-4\left(3\,bd^3enx+bde^3n\right)\sqrt{x}}{24\,e^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(6*b*d^2*e^2*n*x + 3*b*e^4*n - 12*b*e^4*log(c) - 12*a*e^4 + 12*(b*d^4*
n*x^2 - b*e^4*n)*log((d*x + e*sqrt(x))/x) - 4*(3*b*d^3*e*n*x + b*d*e^3*n)*s
qrt(x))/(e^4*x^2)
```

giac [B] time = 0.27, size = 349, normalized size = 3.36

$$\frac{1}{24}\left(\frac{48\left(d\sqrt{x}+e\right)bd^3n\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{\sqrt{x}}-\frac{72\left(d\sqrt{x}+e\right)^2bd^2n\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{x}-\frac{48\left(d\sqrt{x}+e\right)bd^3n}{\sqrt{x}}+\frac{48\left(d\sqrt{x}+e\right)b}{\sqrt{x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/24*(48*(d*sqrt(x) + e)*b*d^3*n*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) - 72*
(d*sqrt(x) + e)^2*b*d^2*n*log((d*sqrt(x) + e)/sqrt(x))/x - 48*(d*sqrt(x) +
e)*b*d^3*n/sqrt(x) + 48*(d*sqrt(x) + e)*b*d^3*log(c)/sqrt(x) + 48*(d*sqrt(x
) + e)^3*b*d*n*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 36*(d*sqrt(x) + e)^2*
b*d^2*n/x - 72*(d*sqrt(x) + e)^2*b*d^2*log(c)/x - 12*(d*sqrt(x) + e)^4*b*n*
```

```
log((d*sqrt(x) + e)/sqrt(x))/x^2 - 16*(d*sqrt(x) + e)^3*b*d*n/x^(3/2) + 48*
(d*sqrt(x) + e)*a*d^3/sqrt(x) + 48*(d*sqrt(x) + e)^3*b*d*log(c)/x^(3/2) + 3
*(d*sqrt(x) + e)^4*b*n/x^2 - 72*(d*sqrt(x) + e)^2*a*d^2/x - 12*(d*sqrt(x) +
 e)^4*b*log(c)/x^2 + 48*(d*sqrt(x) + e)^3*a*d/x^(3/2) - 12*(d*sqrt(x) + e)^
4*a/x^2)*e^(-4)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{b \ln\left(c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right) + a}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/2))^n)+a)/x^3,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/2))^n)+a)/x^3,x)
```

maxima [A] time = 0.62, size = 95, normalized size = 0.91

$$\frac{1}{24}\, ben\left(\frac{12\, d^{4} \log\left(d\sqrt{x} + e\right)}{e^{5}} - \frac{6\, d^{4} \log(x)}{e^{5}} - \frac{12\, d^{3} x^{\frac{3}{2}} - 6\, d^{2} e x + 4\, d e^{2} \sqrt{x} - 3\, e^{3}}{e^{4} x^{2}}\right) - \frac{b \log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^{n}\right)}{2\, x^{2}} - \frac{a}{2\, x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^3,x, algorithm="maxima")
```

```
[Out] 1/24*b*e*n*(12*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^(3
/2) - 6*d^2*e*x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2)) - 1/2*b*log(c*(d + e/
sqrt(x))^n)/x^2 - 1/2*a/x^2
```

mupad [B] time = 0.42, size = 87, normalized size = 0.84

$$\frac{b\, n}{8\, x^{2}} - \frac{a}{2\, x^{2}} - \frac{b\, \ln\left(c\left(d + \frac{e}{\sqrt{x}}\right)^{n}\right)}{2\, x^{2}} - \frac{b\, d\, n}{6\, e\, x^{3/2}} + \frac{b\, d^{4}\, n\, \ln\left(d + \frac{e}{\sqrt{x}}\right)}{2\, e^{4}} + \frac{b\, d^{2}\, n}{4\, e^{2}\, x} - \frac{b\, d^{3}\, n}{2\, e^{3}\, \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))/x^3,x)
```

```
[Out] (b*n)/(8*x^2) - a/(2*x^2) - (b*log(c*(d + e/x^(1/2))^n))/(2*x^2) - (b*d*n)/
(6*e*x^(3/2)) + (b*d^4*n*log(d + e/x^(1/2)))/(2*e^4) + (b*d^2*n)/(4*e^2*x)
- (b*d^3*n)/(2*e^3*x^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x**3,x)
```

```
[Out] Timed out
```

3.428 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=136

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3e^6}-\frac{bd^5n}{3e^5\sqrt{x}}+\frac{bd^4n}{6e^4x}-\frac{bd^3n}{9e^3x^{3/2}}+\frac{bd^2n}{12e^2x^2}-\frac{bdn}{15ex^{5/2}}+\frac{bn}{18x^3}$$

```
[Out] 1/18*b*n/x^3-1/15*b*d*n/e/x^(5/2)+1/12*b*d^2*n/e^2/x^2-1/9*b*d^3*n/e^3/x^(3
/2)+1/6*b*d^4*n/e^4/x+1/3*b*d^6*n*ln(d+e/x^(1/2))/e^6+1/3*(-a-b*ln(c*(d+e/x
^(1/2))^n))/x^3-1/3*b*d^5*n/e^5/x^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}-\frac{bd^3n}{9e^3x^{3/2}}+\frac{bd^2n}{12e^2x^2}-\frac{bd^5n}{3e^5\sqrt{x}}+\frac{bd^4n}{6e^4x}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3e^6}-\frac{bdn}{15ex^{5/2}}+\frac{bn}{18x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^4,x]
```

```
[Out] (b*n)/(18*x^3) - (b*d*n)/(15*e*x^(5/2)) + (b*d^2*n)/(12*e^2*x^2) - (b*d^3*n
)/(9*e^3*x^(3/2)) + (b*d^4*n)/(6*e^4*x) - (b*d^5*n)/(3*e^5*Sqrt[x]) + (b*d^
6*n*Log[d + e/Sqrt[x]])/(3*e^6) - (a + b*Log[c*(d + e/Sqrt[x])^n])/(3*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx = -\left(2\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}+\frac{1}{3}(ben)\,\text{Subst}\left(\int\frac{x^6}{d+ex}dx, x, \frac{1}{\sqrt{x}}\right)$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}+\frac{1}{3}(ben)\,\text{Subst}\left(\int\left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3x^2}{e^4}+\frac{d^2x^3}{e^3}-\frac{dx^4}{e^2}\right.\right.$$

$$= \frac{bn}{18x^3}-\frac{bdn}{15ex^{5/2}}+\frac{bd^2n}{12e^2x^2}-\frac{bd^3n}{9e^3x^{3/2}}+\frac{bd^4n}{6e^4x}-\frac{bd^5n}{3e^5\sqrt{x}}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3e^6}-\frac{a}{}$$

Mathematica [A] time = 0.09, size = 133, normalized size = 0.98

$$-\frac{a}{3x^3}-\frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}+\frac{1}{3}ben\left(\frac{d^6\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^7}-\frac{d^5}{e^6\sqrt{x}}+\frac{d^4}{2e^5x}-\frac{d^3}{3e^4x^{3/2}}+\frac{d^2}{4e^3x^2}-\frac{d}{5e^2x^{5/2}}+\frac{1}{6ex^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^4,x]
```

```
[Out] -1/3*a/x^3 + (b*e*n*(1/(6*e*x^3) - d/(5*e^2*x^(5/2)) + d^2/(4*e^3*x^2) - d^
3/(3*e^4*x^(3/2)) + d^4/(2*e^5*x) - d^5/(e^6*Sqrt[x]) + (d^6*Log[d + e/Sqrt
[x]])/e^7))/3 - (b*Log[c*(d + e/Sqrt[x])^n])/(3*x^3)
```

fricas [A] time = 0.42, size = 123, normalized size = 0.90

$$\frac{30\,bd^4e^2nx^2+15\,bd^2e^4nx+10\,be^6n-60\,be^6\log(c)-60\,ae^6+60\left(bd^6nx^3-be^6n\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)-4\left(15\,bd^5enx^2+\right.}{180\,e^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^4,x, algorithm="fricas")
```

```
[Out] 1/180*(30*b*d^4*e^2*n*x^2 + 15*b*d^2*e^4*n*x + 10*b*e^6*n - 60*b*e^6*log(c)
 - 60*a*e^6 + 60*(b*d^6*n*x^3 - b*e^6*n)*log((d*x + e*sqrt(x))/x) - 4*(15*b
*d^5*e*n*x^2 + 5*b*d^3*e^3*n*x + 3*b*d*e^5*n)*sqrt(x))/(e^6*x^3)
```

giac [B] time = 0.27, size = 535, normalized size = 3.93

$$\frac{1}{180}\left(\frac{360\left(d\sqrt{x}+e\right)bd^5n\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{\sqrt{x}}-\frac{900\left(d\sqrt{x}+e\right)^2bd^4n\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{x}-\frac{360\left(d\sqrt{x}+e\right)bd^5n}{\sqrt{x}}+\frac{360\left(d\sqrt{x}+\right.}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^4,x, algorithm="giac")
```

```
[Out] 1/180*(360*(d*sqrt(x) + e)*b*d^5*n*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) - 9
00*(d*sqrt(x) + e)^2*b*d^4*n*log((d*sqrt(x) + e)/sqrt(x))/x - 360*(d*sqrt(x
) + e)*b*d^5*n/sqrt(x) + 360*(d*sqrt(x) + e)*b*d^5*log(c)/sqrt(x) + 1200*(d
*sqrt(x) + e)^3*b*d^3*n*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 450*(d*sqrt(
```

```
x) + e)^2*b*d^4*n/x - 900*(d*sqrt(x) + e)^2*b*d^4*log(c)/x - 900*(d*sqrt(x)
 + e)^4*b*d^2*n*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 400*(d*sqrt(x) + e)^3*b*
d^3*n/x^(3/2) + 360*(d*sqrt(x) + e)*a*d^5/sqrt(x) + 1200*(d*sqrt(x) + e)^3*
b*d^3*log(c)/x^(3/2) + 360*(d*sqrt(x) + e)^5*b*d*n*log((d*sqrt(x) + e)/sqrt
(x))/x^(5/2) + 225*(d*sqrt(x) + e)^4*b*d^2*n/x^2 - 900*(d*sqrt(x) + e)^2*a*
d^4/x - 900*(d*sqrt(x) + e)^4*b*d^2*log(c)/x^2 - 60*(d*sqrt(x) + e)^6*b*n*l
og((d*sqrt(x) + e)/sqrt(x))/x^3 - 72*(d*sqrt(x) + e)^5*b*d*n/x^(5/2) + 1200
*(d*sqrt(x) + e)^3*a*d^3/x^(3/2) + 360*(d*sqrt(x) + e)^5*b*d*log(c)/x^(5/2)
 + 10*(d*sqrt(x) + e)^6*b*n/x^3 - 900*(d*sqrt(x) + e)^4*a*d^2/x^2 - 60*(d*s
qrt(x) + e)^6*b*log(c)/x^3 + 360*(d*sqrt(x) + e)^5*a*d/x^(5/2) - 60*(d*sqrt
(x) + e)^6*a/x^3)*e^(-6)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{b \ln\left(c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right) + a}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/2))^n)+a)/x^4,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/2))^n)+a)/x^4,x)
```

maxima [A] time = 0.81, size = 117, normalized size = 0.86

$$\frac{1}{180}\, ben\left(\frac{60\, d^{6} \log\left(d\sqrt{x} + e\right)}{e^{7}} - \frac{30\, d^{6} \log(x)}{e^{7}} - \frac{60\, d^{5} x^{\frac{5}{2}} - 30\, d^{4} e x^{2} + 20\, d^{3} e^{2} x^{\frac{3}{2}} - 15\, d^{2} e^{3} x + 12\, d e^{4} \sqrt{x} - 10\, e^{5}}{e^{6} x^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^4,x, algorithm="maxima")
```

```
[Out] 1/180*b*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 - 30*d^6*log(x)/e^7 - (60*d^5*x^
(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 15*d^2*e^3*x + 12*d*e^4*sqrt(x)
 - 10*e^5)/(e^6*x^3)) - 1/3*b*log(c*(d + e/sqrt(x))^n)/x^3 - 1/3*a/x^3
```

mupad [B] time = 0.44, size = 113, normalized size = 0.83

$$\frac{b\, n}{18\, x^{3}} - \frac{a}{3\, x^{3}} - \frac{b\, \ln\left(c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right)}{3\, x^{3}} - \frac{b\, d\, n}{15\, e\, x^{5/2}} + \frac{b\, d^{6}\, n\, \ln\left(d + \frac{e}{\sqrt{x}}\right)}{3\, e^{6}} + \frac{b\, d^{2}\, n}{12\, e^{2}\, x^{2}} + \frac{b\, d^{4}\, n}{6\, e^{4}\, x} - \frac{b\, d^{3}\, n}{9\, e^{3}\, x^{3/2}} - \frac{b\, d^{5}\, n}{3\, e^{5}\, \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))/x^4,x)
```

```
[Out] (b*n)/(18*x^3) - a/(3*x^3) - (b*log(c*(d + e/x^(1/2))^n))/(3*x^3) - (b*d*n)
/(15*e*x^(5/2)) + (b*d^6*n*log(d + e/x^(1/2)))/(3*e^6) + (b*d^2*n)/(12*e^2*
x^2) + (b*d^4*n)/(6*e^4*x) - (b*d^3*n)/(9*e^3*x^(3/2)) - (b*d^5*n)/(3*e^5*x
^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x**4,x)
```

```
[Out] Timed out
```

3.429 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=404

$$\frac{2be^6n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^6}+\frac{2be^5n\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^6}-\frac{be^4nx\left(a+b\log\ldots\right.}{3\ldots}$$

```
[Out] 47/180*b^2*e^4*n^2*x/d^4-1/10*b^2*e^3*n^2*x^(3/2)/d^3+1/30*b^2*e^2*n^2*x^2/
d^2+137/180*b^2*e^6*n^2*ln(x)/d^6+77/90*b^2*e^6*n^2*ln(d+e/x^(1/2))/d^6-1/3
*b*e^4*n*x*(a+b*ln(c*(d+e/x^(1/2))^n))/d^4+2/9*b*e^3*n*x^(3/2)*(a+b*ln(c*(d
+e/x^(1/2))^n))/d^3-1/6*b*e^2*n*x^2*(a+b*ln(c*(d+e/x^(1/2))^n))/d^2+2/15*b*
e*n*x^(5/2)*(a+b*ln(c*(d+e/x^(1/2))^n))/d+2/3*b*e^6*n*ln(1-d/(d+e/x^(1/2)))
*(a+b*ln(c*(d+e/x^(1/2))^n))/d^6+1/3*x^3*(a+b*ln(c*(d+e/x^(1/2))^n))^2-2/3*
b^2*e^6*n^2*polylog(2,d/(d+e/x^(1/2)))/d^6-77/90*b^2*e^5*n^2*x^(1/2)/d^5+2/
3*b*e^5*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))*x^(1/2)/d^6
```

Rubi [A] time = 1.01, antiderivative size = 428, normalized size of antiderivative = 1.06, number of steps used = 26, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$\frac{2b^2e^6n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)}{3d^6}+\frac{2be^3nx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9d^3}-\frac{be^2nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{6d^2}-\frac{e^6\left(a+b\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] (-77*b^2*e^5*n^2*Sqrt[x])/(90*d^5) + (47*b^2*e^4*n^2*x)/(180*d^4) - (b^2*e^
3*n^2*x^(3/2))/(10*d^3) + (b^2*e^2*n^2*x^2)/(30*d^2) + (77*b^2*e^6*n^2*Log[
d + e/Sqrt[x]])/(90*d^6) + (2*b*e^5*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*
(d + e/Sqrt[x])^n]))/(3*d^6) - (b*e^4*n*x*(a + b*Log[c*(d + e/Sqrt[x])^n]))
/(3*d^4) + (2*b*e^3*n*x^(3/2)*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(9*d^3) - (
b*e^2*n*x^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(6*d^2) + (2*b*e*n*x^(5/2)*(a
 + b*Log[c*(d + e/Sqrt[x])^n]))/(15*d) - (e^6*(a + b*Log[c*(d + e/Sqrt[x])^
n])^2)/(3*d^6) + (x^3*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/3 + (2*b*e^6*n*(a
 + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))])/(3*d^6) + (137*b^2*e^
6*n^2*Log[x])/(180*d^6) + (2*b^2*e^6*n^2*PolyLog[2, 1 + e/(d*Sqrt[x])])/(3*
d^6)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx = -\left(2\,\text{Subst}\left(\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^7}dx,x,\frac{1}{\sqrt{x}}\right)\right)$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - \frac{1}{3}(2ben)\,\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)\right.}{x^6(d+ex)}\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - \frac{1}{3}(2bn)\,\text{Subst}\left(\int\frac{a+b\log\left(cx^n\right)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}dx,x,\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - \frac{(2bn)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}dx,x,d+\frac{e}{\sqrt{x}}\right.}{3d}$$

$$= \frac{2benx^{5/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{15d} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 + $$

$$= -\frac{be^2nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{6d^2} + \frac{2benx^{5/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{15d}$$

$$= -\frac{2b^2e^5n^2\sqrt{x}}{15d^5} + \frac{b^2e^4n^2x}{15d^4} - \frac{2b^2e^3n^2x^{3/2}}{45d^3} + \frac{b^2e^2n^2x^2}{30d^2} + \frac{2b^2e^6n^2\log\left(d+\frac{e}{\sqrt{x}}\right.}{15d^6}$$

$$= -\frac{3b^2e^5n^2\sqrt{x}}{10d^5} + \frac{3b^2e^4n^2x}{20d^4} - \frac{b^2e^3n^2x^{3/2}}{10d^3} + \frac{b^2e^2n^2x^2}{30d^2} + \frac{3b^2e^6n^2\log\left(d+\frac{e}{\sqrt{x}}\right.}{10d^6}$$

$$= -\frac{47b^2e^5n^2\sqrt{x}}{90d^5} + \frac{47b^2e^4n^2x}{180d^4} - \frac{b^2e^3n^2x^{3/2}}{10d^3} + \frac{b^2e^2n^2x^2}{30d^2} + \frac{47b^2e^6n^2\log\left(d\right.}{90d^6}$$

$$= -\frac{77b^2e^5n^2\sqrt{x}}{90d^5} + \frac{47b^2e^4n^2x}{180d^4} - \frac{b^2e^3n^2x^{3/2}}{10d^3} + \frac{b^2e^2n^2x^2}{30d^2} + \frac{77b^2e^6n^2\log\left(d\right.}{90d^6}$$

$$= -\frac{77b^2e^5n^2\sqrt{x}}{90d^5} + \frac{47b^2e^4n^2x}{180d^4} - \frac{b^2e^3n^2x^{3/2}}{10d^3} + \frac{b^2e^2n^2x^2}{30d^2} + \frac{77b^2e^6n^2\log\left(d\right.}{90d^6}$$

Mathematica [A] time = 0.27, size = 540, normalized size = 1.34

$$60a^2d^6x^3+120abd^6x^3\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+24abd^5enx^{5/2}-30abd^4e^2nx^2+40abd^3e^3nx^{3/2}-60abd^2e^4nx-120$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] (120*a*b*d*e^5*n*Sqrt[x] - 154*b^2*d*e^5*n^2*Sqrt[x] - 60*a*b*d^2*e^4*n*x +
 47*b^2*d^2*e^4*n^2*x + 40*a*b*d^3*e^3*n*x^(3/2) - 18*b^2*d^3*e^3*n^2*x^(3/
2) - 30*a*b*d^4*e^2*n*x^2 + 6*b^2*d^4*e^2*n^2*x^2 + 24*a*b*d^5*e*n*x^(5/2)
+ 60*a^2*d^6*x^3 + 214*b^2*e^6*n^2*Log[d + e/Sqrt[x]] + 120*b^2*d*e^5*n*Sqr
t[x]*Log[c*(d + e/Sqrt[x])^n] - 60*b^2*d^2*e^4*n*x*Log[c*(d + e/Sqrt[x])^n]
 + 40*b^2*d^3*e^3*n*x^(3/2)*Log[c*(d + e/Sqrt[x])^n] - 30*b^2*d^4*e^2*n*x^2
*Log[c*(d + e/Sqrt[x])^n] + 24*b^2*d^5*e*n*x^(5/2)*Log[c*(d + e/Sqrt[x])^n]
 + 120*a*b*d^6*x^3*Log[c*(d + e/Sqrt[x])^n] + 60*b^2*d^6*x^3*Log[c*(d + e/S
qrt[x])^n]^2 - 120*a*b*e^6*n*Log[e + d*Sqrt[x]] + 60*b^2*e^6*n^2*Log[e + d*
Sqrt[x]] - 120*b^2*e^6*n*Log[c*(d + e/Sqrt[x])^n]*Log[e + d*Sqrt[x]] + 60*b
^2*e^6*n^2*Log[e + d*Sqrt[x]]^2 - 120*b^2*e^6*n^2*Log[e + d*Sqrt[x]]*Log[-(
(d*Sqrt[x])/e)] + 107*b^2*e^6*n^2*Log[x] - 120*b^2*e^6*n^2*PolyLog[2, 1 + (
d*Sqrt[x])/e])/(180*d^6)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^2\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)^2+2\,abx^2\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)+a^2x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*log(c*((d*x + e*sqrt(x))/x)^n)^2 + 2*a*b*x^2*log(c*((d*x +
 e*sqrt(x))/x)^n) + a^2*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2*x^2, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*(d+e/x^(1/2))^n)+a)^2,x)
```

```
[Out] int(x^2*(b*ln(c*(d+e/x^(1/2))^n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}b^2n^2x^3\log\left(d\sqrt{x}+e\right)^2-\int-\frac{3\left(b^2d\log(c)^2+2\,abd\log(c)+a^2d\right)x^3+3\left(b^2e\log(c)^2+2\,abe\log(c)+a^2e\right)x^{\frac{5}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*n^2*x^3*log(d*sqrt(x) + e)^2 - integrate(-1/3*(3*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^3 + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x
^(5/2) - (b^2*d*n*x^3 - 6*(b^2*d*log(c) + a*b*d)*x^3 - 6*(b^2*e*log(c) + a*
b*e)*x^(5/2) + 6*(b^2*d*x^3 + b^2*e*x^(5/2))*log(x^(1/2*n)))*n*log(d*sqrt(x
) + e) + 3*(b^2*d*x^3 + b^2*e*x^(5/2))*log(x^(1/2*n))^2 - 6*((b^2*d*log(c)
+ a*b*d)*x^3 + (b^2*e*log(c) + a*b*e)*x^(5/2))*log(x^(1/2*n)))/(d*x + e*sqr
t(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b \ln\left(c \left(d + \frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e/x^(1/2))^n))^2,x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e/x^(1/2))^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(1/2))**n))**2,x)
```

```
[Out] Timed out
```

3.430 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=288

$$\frac{be^4n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4}+\frac{be^3n\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4}-\frac{be^2nx\left(a+b\lo\right.}{2}$$

[Out] 1/6*b^2*e^2*n^2*x/d^2+11/12*b^2*e^4*n^2*ln(x)/d^4+5/6*b^2*e^4*n^2*ln(d+e/x^(1/2))/d^4-1/2*b*e^2*n*x*(a+b*ln(c*(d+e/x^(1/2))^n))/d^2+1/3*b*e*n*x^(3/2)*(a+b*ln(c*(d+e/x^(1/2))^n))/d+b*e^4*n*ln(1-d/(d+e/x^(1/2)))*(a+b*ln(c*(d+e/x^(1/2))^n))/d^4+1/2*x^2*(a+b*ln(c*(d+e/x^(1/2))^n))^2-b^2*e^4*n^2*polylog(2,d/(d+e/x^(1/2)))/d^4-5/6*b^2*e^3*n^2*x^(1/2)/d^3+b*e^3*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))*x^(1/2)/d^4

Rubi [A] time = 0.64, antiderivative size = 311, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$\frac{b^2e^4n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)}{d^4}-\frac{e^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d^4}+\frac{be^4n\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4}+\frac{be^3\cdots}{}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]

[Out] (-5*b^2*e^3*n^2*Sqrt[x])/(6*d^3) + (b^2*e^2*n^2*x)/(6*d^2) + (5*b^2*e^4*n^2*Log[d + e/Sqrt[x]])/(6*d^4) + (b*e^3*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/d^4 - (b*e^2*n*x*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2*d^2) + (b*e*n*x^(3/2)*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(3*d) - (e^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*d^4) + (x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/2 + (b*e^4*n*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))])/d^4 + (11*b^2*e^4*n^2*Log[x])/(12*d^4) + (b^2*e^4*n^2*PolyLog[2, 1 + e/(d*Sqrt[x])])/d^4

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
```

```
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx = -\left(2\,\text{Subst}\left(\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^5}dx,x,\frac{1}{\sqrt{x}}\right)\right)$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-(ben)\,\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right.}{x^4(d+ex)}\right.$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-(bn)\,\text{Subst}\left(\int\frac{a+b\log\left(cx^n\right)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}dx,x,\right.$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-\frac{(bn)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}dx,x,d+\right.}{d}$$

$$=\frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+$$

$$=-\frac{be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2}+\frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d}$$

$$=-\frac{b^2e^3n^2\sqrt{x}}{3d^3}+\frac{b^2e^2n^2x}{6d^2}+\frac{b^2e^4n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{3d^4}+\frac{be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\,(a}{}$$

$$=-\frac{5b^2e^3n^2\sqrt{x}}{6d^3}+\frac{b^2e^2n^2x}{6d^2}+\frac{5b^2e^4n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{6d^4}+\frac{be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}}{}$$

$$=-\frac{5b^2e^3n^2\sqrt{x}}{6d^3}+\frac{b^2e^2n^2x}{6d^2}+\frac{5b^2e^4n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{6d^4}+\frac{be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}}{}$$

Mathematica [A] time = 0.23, size = 321, normalized size = 1.11

$$\frac{1}{6}\left(\frac{ben\left(2ad^3x^{3/2}-3ad^2ex-6ae^3\log\left(d\sqrt{x}+e\right)+6ade^2\sqrt{x}+2bd^3x^{3/2}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-3bd^2ex\log\left(c\left(d\right.\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]

[Out] (3*x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b*e*n*(6*a*d*e^2*Sqrt[x] - 5*b
*d*e^2*n*Sqrt[x] - 3*a*d^2*e*x + b*d^2*e*n*x + 2*a*d^3*x^(3/2) + 8*b*e^3*n*
Log[d + e/Sqrt[x]] + 6*b*d*e^2*Sqrt[x]*Log[c*(d + e/Sqrt[x])^n] - 3*b*d^2*e
*x*Log[c*(d + e/Sqrt[x])^n] + 2*b*d^3*x^(3/2)*Log[c*(d + e/Sqrt[x])^n] - 6*
```

```
a*e^3*Log[e + d*Sqrt[x]] + 3*b*e^3*n*Log[e + d*Sqrt[x]] - 6*b*e^3*Log[c*(d
+ e/Sqrt[x])^n]*Log[e + d*Sqrt[x]] + 3*b*e^3*n*Log[e + d*Sqrt[x]]^2 - 6*b*e
^3*n*Log[e + d*Sqrt[x]]*Log[-((d*Sqrt[x])/e)] + 4*b*e^3*n*Log[x] - 6*b*e^3*
n*PolyLog[2, 1 + (d*Sqrt[x])/e]))/d^4)/6
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)^2+2\,abx\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)+a^2x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x*log(c*((d*x + e*sqrt(x))/x)^n)^2 + 2*a*b*x*log(c*((d*x + e*s
qrt(x))/x)^n) + a^2*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2*x, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(d+e/x^(1/2))^n)+a)^2,x)
```

```
[Out] int(x*(b*ln(c*(d+e/x^(1/2))^n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}b^2n^2x^2\log\left(d\sqrt{x}+e\right)^2-\int-\frac{2\left(b^2d\log(c)^2+2\,abd\log(c)+a^2d\right)x^2-\left(b^2dnx^2-4\left(b^2d\log(c)+abd\right)x^2-4\left(b^2e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*n^2*x^2*log(d*sqrt(x) + e)^2 - integrate(-1/2*(2*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^2 - (b^2*d*n*x^2 - 4*(b^2*d*log(c) + a*b*d)*x^2 -
 4*(b^2*e*log(c) + a*b*e)*x^(3/2) + 4*(b^2*d*x^2 + b^2*e*x^(3/2))*log(x^(1/
2*n)))*n*log(d*sqrt(x) + e) + 2*(b^2*d*x^2 + b^2*e*x^(3/2))*log(x^(1/2*n))^
2 + 2*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(3/2) - 4*((b^2*d*log(c)
+ a*b*d)*x^2 + (b^2*e*log(c) + a*b*e)*x^(3/2))*log(x^(1/2*n)))/(d*x + e*sqr
t(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/2))^n))^2,x)
```

```
[Out] int(x*(a + b*log(c*(d + e/x^(1/2))^n))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))**n))**2,x)
```

```
[Out] Integral(x*(a + b*log(c*(d + e/sqrt(x))**n))**2, x)
```

3.431 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=152

$$\frac{2be^2n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2}+\frac{2ben\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2}+x\left(a+b\log\left(c\left(\right.\right.\right.$$

[Out] b^2*e^2*n^2*ln(x)/d^2+2*b*e^2*n*ln(1-d/(d+e/x^(1/2)))*(a+b*ln(c*(d+e/x^(1/2))^n))/d^2+x*(a+b*ln(c*(d+e/x^(1/2))^n))^2-2*b^2*e^2*n^2*polylog(2,d/(d+e/x^(1/2)))/d^2+2*b*e*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))*x^(1/2)/d^2

Rubi [A] time = 0.35, antiderivative size = 174, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {2451, 2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31}

$$\frac{2b^2e^2n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)}{d^2}-\frac{e^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2}+\frac{2be^2n\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2}+\frac{2be}{\ }$$

Antiderivative was successfully verified.

[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]

[Out] (2*b*e*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/d^2 - (e^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/d^2 + x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (2*b*e^2*n*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))])/d^2 + (b^2*e^2*n^2*Log[x])/d^2 + (2*b^2*e^2*n^2*PolyLog[2, 1 + e/(d*Sqrt[x])])/d^2

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e, Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
```

```
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx = 2\,\text{Subst}\left(\int x\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^n\right)\right)^2 dx, x, \sqrt{x}\right)$$
$$= -\left(2\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^3} dx, x, \frac{1}{\sqrt{x}}\right)\right)$$
$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - (2ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(d+ex)} dx, x$$
$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - (2bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2} dx, x, d+\frac{e}{\sqrt{}}\right.$$
$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - \frac{(2bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2} dx, x, d+\frac{e}{\sqrt{x}}\right)}{d} +$$
$$= \frac{2ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)$$
$$= \frac{2ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} - \frac{e^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2}$$
$$= \frac{2ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} - \frac{e^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2}$$

Mathematica [A] time = 0.13, size = 170, normalized size = 1.12

$$\frac{ben\left(-2e\log\left(d\sqrt{x}+e\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)+2ad\sqrt{x}+2bd\sqrt{x}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+ben\left(\log\left(d\sqrt{x}+e\right)\right(1\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b*e*n*(2*a*d*Sqrt[x] + 2*b*d*Sqrt[x
]*Log[c*(d + e/Sqrt[x])^n] - 2*e*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[e + d
*Sqrt[x]] + b*e*n*(2*Log[d + e/Sqrt[x]] + Log[x]) + b*e*n*(Log[e + d*Sqrt[x
]]*(Log[e + d*Sqrt[x]] - 2*Log[-((d*Sqrt[x])/e)]) - 2*PolyLog[2, 1 + (d*Sqr
t[x])/e])))/d^2
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)^2 + 2\,ab\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right) + a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*log(c*((d*x + e*sqrt(x))/x)^n)^2 + 2*a*b*log(c*((d*x + e*sqrt(
x))/x)^n) + a^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/2))^n)+a)^2,x)
[Out] int((b*ln(c*(d+e/x^(1/2))^n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-2\left(en\left(\frac{e\log\left(d\sqrt{x}+e\right)}{d^2}-\frac{\sqrt{x}}{d}\right)-x\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)ab+\left(n^2x\log\left(d\sqrt{x}+e\right)^2-\int -\frac{dx\log(c)^2+e\sqrt{x}\,\log(c}{\quad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="maxima")
[Out] -2*(e*n*(e*log(d*sqrt(x) + e)/d^2 - sqrt(x)/d) - x*log(c*(d + e/sqrt(x))^n)
)*a*b + (n^2*x*log(d*sqrt(x) + e)^2 - integrate(-(d*x*log(c)^2 + e*sqrt(x)*
log(c)^2 - (d*n*x - 2*d*x*log(c) - 2*e*sqrt(x)*log(c) + 2*(d*x + e*sqrt(x))
*log(x^(1/2*n)))*n*log(d*sqrt(x) + e) + (d*x + e*sqrt(x))*log(x^(1/2*n))^2
- 2*(d*x*log(c) + e*sqrt(x)*log(c))*log(x^(1/2*n)))/(d*x + e*sqrt(x)), x))*
b^2 + a^2*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^2,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^n))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2,x)
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**2, x)
```

3.432 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx$

Optimal. Leaf size=93

$$-4bn\text{Li}_2\left(\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-2\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+4b^2n^2\text{Li}_3\left(\frac{e}{d\sqrt{x}}+1\right.$$

```
[Out] -2*(a+b*ln(c*(d+e/x^(1/2))^n))^2*ln(-e/d/x^(1/2))-4*b*n*(a+b*ln(c*(d+e/x^(1
/2))^n))*polylog(2,1+e/d/x^(1/2))+4*b^2*n^2*polylog(3,1+e/d/x^(1/2))
```

Rubi [A] time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2454, 2396, 2433, 2374, 6589}

$$-4bn\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)+4b^2n^2\text{PolyLog}\left(3,\frac{e}{d\sqrt{x}}+1\right)-2\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x,x]
```

```
[Out] -2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2*Log[-(e/(d*Sqrt[x]))] - 4*b*n*(a + b*
Log[c*(d + e/Sqrt[x])^n])*PolyLog[2, 1 + e/(d*Sqrt[x])] + 4*b^2*n^2*PolyLog
[3, 1 + e/(d*Sqrt[x])]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx = -\left(2\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right)+(4ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{}\right.$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right)+(4bn)\,\text{Subst}\left(\int \frac{\left(a+b\log\right.}{}\right.$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right)-4bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right)-4bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.$$

Mathematica [B] time = 0.36, size = 386, normalized size = 4.15

$$2bn\left(2\text{Li}_2\left(-\frac{e}{d\sqrt{x}}\right)+\log(x)\left(\log\left(d+\frac{e}{\sqrt{x}}\right)-\log\left(\frac{e}{d\sqrt{x}}+1\right)\right)\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-bn\log\left(d+\frac{e}{\sqrt{x}}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])*((Log[d + e/Sqrt[
x]] - Log[1 + e/(d*Sqrt[x])])*Log[x] + 2*PolyLog[2, -(e/(d*Sqrt[x]))]) + (b
^2*n^2*(24*Log[e/d + Sqrt[x]]^2*Log[-((d*Sqrt[x])/e)] + 12*Log[d + e/Sqrt[x
]]^2*Log[x] - 12*Log[e/d + Sqrt[x]]^2*Log[x] - 24*Log[d + e/Sqrt[x]]*Log[1
+ (d*Sqrt[x])/e]*Log[x] + 24*Log[e/d + Sqrt[x]]*Log[1 + (d*Sqrt[x])/e]*Log[
x] + 6*Log[d + e/Sqrt[x]]*Log[x]^2 - 6*Log[1 + (d*Sqrt[x])/e]*Log[x]^2 + Lo
g[x]^3 + 48*Log[e/d + Sqrt[x]]*PolyLog[2, 1 + (d*Sqrt[x])/e] - 48*(Log[d +
e/Sqrt[x]] - Log[e/d + Sqrt[x]])*PolyLog[2, -((d*Sqrt[x])/e)] - 48*PolyLog[
3, 1 + (d*Sqrt[x])/e] - 48*PolyLog[3, -((d*Sqrt[x])/e)]))/12
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)^2+2\,ab\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)+a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*((d*x + e*sqrt(x))/x)^n)^2 + 2*a*b*log(c*((d*x + e*sqrt
(x))/x)^n) + a^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2/x, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/2))^n)+a)^2/x,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/2))^n)+a)^2/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^{2}n^{2}\log\left(d\sqrt{x}+e\right)^{2}\log(x)-\int \frac{\left(b^{2}dnx\log(x)-2\left(b^{2}d\log(c)+abd\right)x+2\left(b^{2}dx+b^{2}e\sqrt{x}\right)\log\left(x^{\frac{1}{2}n}\right)-2\left(b^{2}e\log\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*n^2*log(d*sqrt(x) + e)^2*log(x) - integrate(((b^2*d*n*x*log(x) - 2*(b^2
*d*log(c) + a*b*d)*x + 2*(b^2*d*x + b^2*e*sqrt(x))*log(x^(1/2*n)) - 2*(b^2*
e*log(c) + a*b*e)*sqrt(x))*n*log(d*sqrt(x) + e) - (b^2*d*x + b^2*e*sqrt(x))
*log(x^(1/2*n))^2 - (b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x + 2*((b^2*d
*log(c) + a*b*d)*x + (b^2*e*log(c) + a*b*e)*sqrt(x))*log(x^(1/2*n)) - (b^2*
e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*sqrt(x))/(d*x^2 + e*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^2/x,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/2))^n))^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**2/x, x)
```

3.433 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=195

$$\frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^2}-\frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2}+\frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\cdot\right.\right.\right.}{e^2}$$

```
[Out] -4*b^2*d*n*ln(c*(d+e/x^(1/2))^n)*(d+e/x^(1/2))/e^2+2*d*(a+b*ln(c*(d+e/x^(1/
2))^n))^2*(d+e/x^(1/2))/e^2-1/2*b^2*n^2*(d+e/x^(1/2))^2/e^2+b*n*(a+b*ln(c*(
d+e/x^(1/2))^n))*(d+e/x^(1/2))^2/e^2-(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(
1/2))^2/e^2-4*a*b*d*n/e/x^(1/2)+4*b^2*d*n^2/e/x^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^2}-\frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2}+\frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\cdot\right.\right.\right.}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^2,x]
```

```
[Out] -(b^2*n^2*(d + e/Sqrt[x])^2)/(2*e^2) - (4*a*b*d*n)/(e*Sqrt[x]) + (4*b^2*d*n
^2)/(e*Sqrt[x]) - (4*b^2*d*n*(d + e/Sqrt[x])*Log[c*(d + e/Sqrt[x])^n])/e^2
+ (b*n*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/e^2 + (2*d*(d +
e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/e^2 - ((d + e/Sqrt[x])^2*(a
+ b*Log[c*(d + e/Sqrt[x])^n])^2)/e^2
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx = -\left(2\,\text{Subst}\left(\int x\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\,\text{Subst}\left(\int\left(-\frac{d\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{e}+\frac{(d+ex)\left(a+b\log\left(c(d+e\right.\right.}{e}\right.\right.\right.$$

$$= -\frac{2\,\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, \frac{1}{\sqrt{x}}\right)}{e}+\frac{(2d)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2\,\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)^2\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^2}+\frac{(2d)\,\text{Subst}\left(\int\left(a+b\log\right.\right.}{e}$$

$$= \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2}-\frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{}}\right.\right.\right.}{e^2}$$

$$= -\frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^2}-\frac{4abdn}{e\sqrt{x}}+\frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^2}+\frac{2}{}$$

$$= -\frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^2}-\frac{4abdn}{e\sqrt{x}}+\frac{4b^2dn^2}{e\sqrt{x}}-\frac{4b^2dn\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right.}{e^2}$$

Mathematica [C] time = 0.35, size = 298, normalized size = 1.53

$$-\frac{bn\left(-4d^2x\log\left(d\sqrt{x}+e\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-4d^2x\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-2e^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)+4ade\sqrt{x}+4bd\sqrt{x}\left(d\sqrt{x}+e\right)\log\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^2,x]
```

```
[Out] -1/2*(2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b*n*(4*a*d*e*Sqrt[x] - 4*b*d*
e*n*Sqrt[x] + b*n*(e*(e - 2*d*Sqrt[x]) + 2*d^2*x*Log[d + e/Sqrt[x]]) + 4*b*
d*(e + d*Sqrt[x])*Sqrt[x]*Log[c*(d + e/Sqrt[x])^n] - 2*e^2*(a + b*Log[c*(d
+ e/Sqrt[x])^n]) - 4*d^2*x*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[e + d*Sqrt[
x]] - 4*d^2*x*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))] - 4*b*
d^2*n*x*PolyLog[2, 1 + e/(d*Sqrt[x])] + 2*b*d^2*n*x*(Log[e + d*Sqrt[x]]*(Lo
g[e + d*Sqrt[x]] - 2*Log[-((d*Sqrt[x])/e)]) - 2*PolyLog[2, 1 + (d*Sqrt[x])/
e])))/e^2)/x
```

fricas [A] time = 0.43, size = 235, normalized size = 1.21

$$-\frac{b^2e^2n^2+2\,b^2e^2\log(c)^2-2\,abe^2n+2\,a^2e^2-2\left(b^2d^2n^2x-b^2e^2n^2\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)^2-2\left(b^2e^2n-2\,abe^2\right)\log(c)+2\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*e^2*n^2 + 2*b^2*e^2*log(c)^2 - 2*a*b*e^2*n + 2*a^2*e^2 - 2*(b^2*d
^2*n^2*x - b^2*e^2*n^2)*log((d*x + e*sqrt(x))/x)^2 - 2*(b^2*e^2*n - 2*a*b*e
^2)*log(c) + 2*(2*b^2*d*e*n^2*sqrt(x) - b^2*e^2*n^2 + 2*a*b*e^2*n + (3*b^2*
d^2*n^2 - 2*a*b*d^2*n)*x - 2*(b^2*d^2*n*x - b^2*e^2*n)*log(c))*log((d*x + e
*sqrt(x))/x) - 2*(3*b^2*d*e*n^2 - 2*b^2*d*e*n*log(c) - 2*a*b*d*e*n)*sqrt(x)
)/(e^2*x)
```

giac [B] time = 0.41, size = 503, normalized size = 2.58

$$\frac{1}{2}\left(\frac{4\left(d\sqrt{x}+e\right)b^2dn^2\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)^2}{\sqrt{x}}-\frac{2\left(d\sqrt{x}+e\right)^2b^2n^2\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)^2}{x}-\frac{8\left(d\sqrt{x}+e\right)b^2dn^2\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{\sqrt{x}}+\frac{8\left(d\sqrt{x}\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(d*sqrt(x) + e)*b^2*d*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/sqrt(x) - 2
*(d*sqrt(x) + e)^2*b^2*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x - 8*(d*sqrt(x)
+ e)*b^2*d*n^2*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 8*(d*sqrt(x) + e)*b^2
*d*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 2*(d*sqrt(x) + e)^2*b^2*
n^2*log((d*sqrt(x) + e)/sqrt(x))/x - 4*(d*sqrt(x) + e)^2*b^2*n*log(c)*log((
d*sqrt(x) + e)/sqrt(x))/x + 8*(d*sqrt(x) + e)*b^2*d*n^2/sqrt(x) - 8*(d*sqrt
(x) + e)*b^2*d*n*log(c)/sqrt(x) + 4*(d*sqrt(x) + e)*b^2*d*log(c)^2/sqrt(x)
+ 8*(d*sqrt(x) + e)*a*b*d*n*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) - (d*sqrt(
x) + e)^2*b^2*n^2/x + 2*(d*sqrt(x) + e)^2*b^2*n*log(c)/x - 2*(d*sqrt(x) + e
)^2*b^2*log(c)^2/x - 4*(d*sqrt(x) + e)^2*a*b*n*log((d*sqrt(x) + e)/sqrt(x))
/x - 8*(d*sqrt(x) + e)*a*b*d*n/sqrt(x) + 8*(d*sqrt(x) + e)*a*b*d*log(c)/sqr
t(x) + 2*(d*sqrt(x) + e)^2*a*b*n/x - 4*(d*sqrt(x) + e)^2*a*b*log(c)/x + 4*(
d*sqrt(x) + e)*a^2*d/sqrt(x) - 2*(d*sqrt(x) + e)^2*a^2/x)*e^(-2)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{2}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/2))^n)+a)^2/x^2,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/2))^n)+a)^2/x^2,x)
```

maxima [A] time = 0.79, size = 248, normalized size = 1.27

$$aben\left(\frac{2\,d^{2}\log\left(d\sqrt{x}+e\right)}{e^{3}}-\frac{d^{2}\log(x)}{e^{3}}-\frac{2\,d\sqrt{x}-e}{e^{2}x}\right)+\frac{1}{4}\left(4\,en\left(\frac{2\,d^{2}\log\left(d\sqrt{x}+e\right)}{e^{3}}-\frac{d^{2}\log(x)}{e^{3}}-\frac{2\,d\sqrt{x}-e}{e^{2}x}\right)\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] a*b*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d*sqrt(x) - e)/
(e^2*x)) + 1/4*(4*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d
*sqrt(x) - e)/(e^2*x))*log(c*(d + e/sqrt(x))^n) - (4*d^2*x*log(d*sqrt(x) +
e)^2 + d^2*x*log(x)^2 - 6*d^2*x*log(x) - 12*d*e*sqrt(x) + 2*e^2 - 4*(d^2*x*
log(x) - 3*d^2*x)*log(d*sqrt(x) + e))*n^2/(e^2*x))*b^2 - b^2*log(c*(d + e/s
qrt(x))^n)^2/x - 2*a*b*log(c*(d + e/sqrt(x))^n)/x - a^2/x
```

mupad [B] time = 0.47, size = 193, normalized size = 0.99

$$\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\left(\frac{\frac{2\,b\,d\,(2\,a-b\,n)}{e}-\frac{4\,a\,b\,d}{e}}{\sqrt{x}}-\frac{b\,(2\,a-b\,n)}{x}\right)-\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)^{2}\left(\frac{b^{2}}{x}-\frac{b^{2}\,d^{2}}{e^{2}}\right)+\frac{\frac{d\left(2\,a^{2}-2\,a\,b\,n+b^{2}\,n^{2}\right)}{e}-}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^2/x^2,x)
```

```
[Out] log(c*(d + e/x^(1/2))^n)*(((2*b*d*(2*a - b*n))/e - (4*a*b*d)/e)/x^(1/2) - (
b*(2*a - b*n))/x) - log(c*(d + e/x^(1/2))^n)^2*(b^2/x - (b^2*d^2)/e^2) + ((
d*(2*a^2 + b^2*n^2 - 2*a*b*n))/e - (2*d*(a^2 - b^2*n^2))/e)/x^(1/2) - (a^2
+ (b^2*n^2)/2 - a*b*n)/x - (log(d + e/x^(1/2))*(3*b^2*d^2*n^2 - 2*a*b*d^2*n
))/e^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**2/x**2, x)
```

3.434 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=341

$$\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4}-\frac{4bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4}+\frac{3bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+\right.}{e^4}$$

```
[Out] -1/2*b^2*d^4*n^2*ln(d+e/x^(1/2))^2/e^4+b*d^4*n*ln(d+e/x^(1/2))*(a+b*ln(c*(d
+e/x^(1/2))^n))/e^4-1/2*(a+b*ln(c*(d+e/x^(1/2))^n))^2/x^2-4*b*d^3*n*(a+b*ln
(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))/e^4-3/2*b^2*d^2*n^2*(d+e/x^(1/2))^2/e^4+
3*b*d^2*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^2/e^4+4/9*b^2*d*n^2*(d+
e/x^(1/2))^3/e^4-4/3*b*d*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^3/e^4-
1/16*b^2*n^2*(d+e/x^(1/2))^4/e^4+1/4*b*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x
^(1/2))^4/e^4+4*b^2*d^3*n^2/e^3/x^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 263, normalized size of antiderivative = 0.77, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$-\frac{1}{12}bn\left(\frac{48d^3\left(d+\frac{e}{\sqrt{x}}\right)}{e^4}-\frac{36d^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^4}-\frac{12d^4\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^4}+\frac{16d\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^4}-\frac{3\left(d+\frac{e}{\sqrt{x}}\right)^4}{e^4}\right)\left(a+b\log\left(c\left(d\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^3,x]
```

```
[Out] (-3*b^2*d^2*n^2*(d + e/Sqrt[x])^2)/(2*e^4) + (4*b^2*d*n^2*(d + e/Sqrt[x])^3
)/(9*e^4) - (b^2*n^2*(d + e/Sqrt[x])^4)/(16*e^4) + (4*b^2*d^3*n^2)/(e^3*Sqr
t[x]) - (b^2*d^4*n^2*Log[d + e/Sqrt[x]]^2)/(2*e^4) - (b*n*((48*d^3*(d + e/S
qrt[x]))/e^4 - (36*d^2*(d + e/Sqrt[x])^2)/e^4 + (16*d*(d + e/Sqrt[x])^3)/e^
4 - (3*(d + e/Sqrt[x])^4)/e^4 - (12*d^4*Log[d + e/Sqrt[x]])/e^4)*(a + b*Log
[c*(d + e/Sqrt[x])^n]))/12 - (a + b*Log[c*(d + e/Sqrt[x])^n])^2/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx = -\left(2\,\text{Subst}\left(\int x^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x^2} + (ben)\,\text{Subst}\left(\int \frac{x^4\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}\,dx\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x^2} + (bn)\,\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx\right.$$

$$= -\frac{1}{12}bn\left(\frac{48d^3\left(d+\frac{e}{\sqrt{x}}\right)}{e^4} - \frac{36d^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^4} + \frac{16d\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^4} - \frac{3\left(d+\frac{e}{\sqrt{x}}\right)^4}{e^4} - \right.$$

$$= -\frac{1}{12}bn\left(\frac{48d^3\left(d+\frac{e}{\sqrt{x}}\right)}{e^4} - \frac{36d^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^4} + \frac{16d\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^4} - \frac{3\left(d+\frac{e}{\sqrt{x}}\right)^4}{e^4} - \right.$$

$$= -\frac{1}{12}bn\left(\frac{48d^3\left(d+\frac{e}{\sqrt{x}}\right)}{e^4} - \frac{36d^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^4} + \frac{16d\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^4} - \frac{3\left(d+\frac{e}{\sqrt{x}}\right)^4}{e^4} - \right.$$

$$= -\frac{3b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^4} + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4} - \frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{16e^4} + \frac{4b^2d^3n^2}{e^3\sqrt{x}} - \frac{1}{12}$$

$$= -\frac{3b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^4} + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4} - \frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{16e^4} + \frac{4b^2d^3n^2}{e^3\sqrt{x}} - \frac{b^2}{}$$

Mathematica [C] time = 0.39, size = 473, normalized size = 1.39

$$-\frac{bn\left(-144ad^4x^2\log\left(d\sqrt{x}+e\right) - 144ad^4x^2\log\left(-\frac{e}{d\sqrt{x}}\right) + 144ad^3ex^{3/2} - 72ad^2e^2x + 48ade^3\sqrt{x} - 36ae^4 + 144bd^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^3,x]
```

```
[Out] -1/144*(72*e^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + b*n*(-36*a*e^4 + 9*b*e^
4*n + 48*a*d*e^3*Sqrt[x] - 28*b*d*e^3*n*Sqrt[x] - 72*a*d^2*e^2*x + 78*b*d^2
*e^2*n*x + 144*a*d^3*e*x^(3/2) - 300*b*d^3*e*n*x^(3/2) + 156*b*d^4*n*x^2*Lo
g[d + e/Sqrt[x]] - 36*b*e^4*Log[c*(d + e/Sqrt[x])^n] + 48*b*d*e^3*Sqrt[x]*L
og[c*(d + e/Sqrt[x])^n] - 72*b*d^2*e^2*x*Log[c*(d + e/Sqrt[x])^n] + 144*b*d
^3*e*x^(3/2)*Log[c*(d + e/Sqrt[x])^n] + 144*b*d^4*x^2*Log[c*(d + e/Sqrt[x])
^n] - 144*a*d^4*x^2*Log[e + d*Sqrt[x]] - 144*b*d^4*x^2*Log[c*(d + e/Sqrt[x]
)^n]*Log[e + d*Sqrt[x]] + 72*b*d^4*n*x^2*Log[e + d*Sqrt[x]]^2 - 144*a*d^4*x
^2*Log[-(e/(d*Sqrt[x]))] - 144*b*d^4*x^2*Log[c*(d + e/Sqrt[x])^n]*Log[-(e/(
d*Sqrt[x]))] - 144*b*d^4*n*x^2*Log[e + d*Sqrt[x]]*Log[-((d*Sqrt[x])/e)] - 1
```

```
44*b*d^4*n*x^2*PolyLog[2, 1 + e/(d*Sqrt[x])] - 144*b*d^4*n*x^2*PolyLog[2, 1
 + (d*Sqrt[x])/e]))/(e^4*x^2)
```

fricas [A] time = 0.44, size = 361, normalized size = 1.06

$$-\frac{9\,b^2e^4n^2+72\,b^2e^4\log(c)^2-36\,abe^4n+72\,a^2e^4-72\left(b^2d^4n^2x^2-b^2e^4n^2\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)^2+6\left(13\,b^2d^2e^2n^2-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/144*(9*b^2*e^4*n^2 + 72*b^2*e^4*log(c)^2 - 36*a*b*e^4*n + 72*a^2*e^4 - 7
2*(b^2*d^4*n^2*x^2 - b^2*e^4*n^2)*log((d*x + e*sqrt(x))/x)^2 + 6*(13*b^2*d^
2*e^2*n^2 - 12*a*b*d^2*e^2*n)*x - 36*(2*b^2*d^2*e^2*n*x + b^2*e^4*n - 4*a*b
*e^4)*log(c) - 12*(6*b^2*d^2*e^2*n^2*x + 3*b^2*e^4*n^2 - 12*a*b*e^4*n - (25
*b^2*d^4*n^2 - 12*a*b*d^4*n)*x^2 + 12*(b^2*d^4*n*x^2 - b^2*e^4*n)*log(c) -
4*(3*b^2*d^3*e*n^2*x + b^2*d*e^3*n^2)*sqrt(x))*log((d*x + e*sqrt(x))/x) - 4
*(7*b^2*d*e^3*n^2 - 12*a*b*d*e^3*n + 3*(25*b^2*d^3*e*n^2 - 12*a*b*d^3*e*n)*
x - 12*(3*b^2*d^3*e*n*x + b^2*d*e^3*n)*log(c))*sqrt(x))/(e^4*x^2)
```

giac [B] time = 0.44, size = 1071, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] 1/144*(288*(d*sqrt(x) + e)*b^2*d^3*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/sqrt(
x) - 432*(d*sqrt(x) + e)^2*b^2*d^2*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x - 5
76*(d*sqrt(x) + e)*b^2*d^3*n^2*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 576*(
d*sqrt(x) + e)*b^2*d^3*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 288*
(d*sqrt(x) + e)^3*b^2*d*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x^(3/2) + 432*(d
*sqrt(x) + e)^2*b^2*d^2*n^2*log((d*sqrt(x) + e)/sqrt(x))/x - 864*(d*sqrt(x)
 + e)^2*b^2*d^2*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x - 72*(d*sqrt(x) + e
)^4*b^2*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x^2 + 576*(d*sqrt(x) + e)*b^2*d^
3*n^2/sqrt(x) - 576*(d*sqrt(x) + e)*b^2*d^3*n*log(c)/sqrt(x) + 288*(d*sqrt(
x) + e)*b^2*d^3*log(c)^2/sqrt(x) - 192*(d*sqrt(x) + e)^3*b^2*d*n^2*log((d*s
qrt(x) + e)/sqrt(x))/x^(3/2) + 576*(d*sqrt(x) + e)*a*b*d^3*n*log((d*sqrt(x)
 + e)/sqrt(x))/sqrt(x) + 576*(d*sqrt(x) + e)^3*b^2*d*n*log(c)*log((d*sqrt(x
) + e)/sqrt(x))/x^(3/2) - 216*(d*sqrt(x) + e)^2*b^2*d^2*n^2/x + 432*(d*sqrt
(x) + e)^2*b^2*d^2*n*log(c)/x - 432*(d*sqrt(x) + e)^2*b^2*d^2*log(c)^2/x +
36*(d*sqrt(x) + e)^4*b^2*n^2*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 864*(d*sqrt
(x) + e)^2*a*b*d^2*n*log((d*sqrt(x) + e)/sqrt(x))/x - 144*(d*sqrt(x) + e)^4
*b^2*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^2 + 64*(d*sqrt(x) + e)^3*b^2*d
*n^2/x^(3/2) - 576*(d*sqrt(x) + e)*a*b*d^3*n/sqrt(x) - 192*(d*sqrt(x) + e)^
3*b^2*d*n*log(c)/x^(3/2) + 576*(d*sqrt(x) + e)*a*b*d^3*log(c)/sqrt(x) + 288
*(d*sqrt(x) + e)^3*b^2*d*log(c)^2/x^(3/2) + 576*(d*sqrt(x) + e)^3*a*b*d*n*l
og((d*sqrt(x) + e)/sqrt(x))/x^(3/2) - 9*(d*sqrt(x) + e)^4*b^2*n^2/x^2 + 432
*(d*sqrt(x) + e)^2*a*b*d^2*n/x + 36*(d*sqrt(x) + e)^4*b^2*n*log(c)/x^2 - 86
4*(d*sqrt(x) + e)^2*a*b*d^2*log(c)/x - 72*(d*sqrt(x) + e)^4*b^2*log(c)^2/x^
2 - 144*(d*sqrt(x) + e)^4*a*b*n*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 192*(d*s
qrt(x) + e)^3*a*b*d*n/x^(3/2) + 288*(d*sqrt(x) + e)*a^2*d^3/sqrt(x) + 576*(
d*sqrt(x) + e)^3*a*b*d*log(c)/x^(3/2) + 36*(d*sqrt(x) + e)^4*a*b*n/x^2 - 43
2*(d*sqrt(x) + e)^2*a^2*d^2/x - 144*(d*sqrt(x) + e)^4*a*b*log(c)/x^2 + 288*
(d*sqrt(x) + e)^3*a^2*d/x^(3/2) - 72*(d*sqrt(x) + e)^4*a^2/x^2)*e^(-4)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/2))^n)+a)^2/x^3,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/2))^n)+a)^2/x^3,x)
```

maxima [A] time = 0.88, size = 321, normalized size = 0.94

$$\frac{1}{12}\,aben\left(\frac{12\,d^4\log\left(d\sqrt{x}+e\right)}{e^5}-\frac{6\,d^4\log(x)}{e^5}-\frac{12\,d^3x^{\frac{3}{2}}-6\,d^2ex+4\,de^2\sqrt{x}-3\,e^3}{e^4x^2}\right)+\frac{1}{144}\left(12\,en\left(\frac{12\,d^4\log\left(d\sqrt{x}+\right.}{e^5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*a*b*e*n*(12*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^
(3/2) - 6*d^2*e*x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2)) + 1/144*(12*e*n*(12
*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^(3/2) - 6*d^2*e*
x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2))*log(c*(d + e/sqrt(x))^n) - (72*d^4*
x^2*log(d*sqrt(x) + e)^2 + 18*d^4*x^2*log(x)^2 - 150*d^4*x^2*log(x) - 300*d
^3*e*x^(3/2) + 78*d^2*e^2*x - 28*d*e^3*sqrt(x) + 9*e^4 - 12*(6*d^4*x^2*log(
x) - 25*d^4*x^2)*log(d*sqrt(x) + e))*n^2/(e^4*x^2))*b^2 - 1/2*b^2*log(c*(d
+ e/sqrt(x))^n)^2/x^2 - a*b*log(c*(d + e/sqrt(x))^n)/x^2 - 1/2*a^2/x^2
```

mupad [B] time = 0.56, size = 424, normalized size = 1.24

$$\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\left(\frac{\frac{b\,d\,(4\,a-b\,n)}{3\,e}-\frac{4\,a\,b\,d}{3\,e}}{x^{3/2}}-\frac{b\,(4\,a-b\,n)}{4\,x^2}-\frac{d\,\left(\frac{b\,d\,(4\,a-b\,n)}{e}-\frac{4\,a\,b\,d}{e}\right)}{2\,e\,x}+\frac{d^2\,\left(\frac{b\,d\,(4\,a-b\,n)}{e}-\frac{4\,a\,b\,d}{e}\right)}{e^2\,\sqrt{x}}\right)+\frac{d\left(2\,a^2-a\,b\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^2/x^3,x)
```

```
[Out] log(c*(d + e/x^(1/2))^n)*(((b*d*(4*a - b*n))/(3*e) - (4*a*b*d)/(3*e))/x^(3/
2) - (b*(4*a - b*n))/(4*x^2) - (d*((b*d*(4*a - b*n))/e - (4*a*b*d)/e))/(2*e
*x) + (d^2*((b*d*(4*a - b*n))/e - (4*a*b*d)/e))/(e^2*x^(1/2))) + ((d*(2*a^2
 + (b^2*n^2)/4 - a*b*n))/(3*e) - (d*(6*a^2 - b^2*n^2))/(9*e))/x^(3/2) - log
(c*(d + e/x^(1/2))^n)^2*(b^2/(2*x^2) - (b^2*d^4)/(2*e^4)) - (a^2/2 + (b^2*n
^2)/16 - (a*b*n)/4)/x^2 - ((d*((d*(2*a^2 + (b^2*n^2)/4 - a*b*n))/e - (d*(6*
a^2 - b^2*n^2))/(3*e)))/(2*e) + (b^2*d^2*n^2)/(4*e^2))/x + ((d*((d*((d*(2*a
^2 + (b^2*n^2)/4 - a*b*n))/e - (d*(6*a^2 - b^2*n^2))/(3*e)))/e + (b^2*d^2*n
^2)/(2*e^2)))/e + (b^2*d^3*n^2)/e^3)/x^(1/2) - (log(d + e/x^(1/2))*(25*b^2*
d^4*n^2 - 12*a*b*d^4*n))/(12*e^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x**3,x)
```

```
[Out] Timed out
```

3.435 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=480

$$\frac{2bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^6}-\frac{4bd^5n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6}+\frac{5bd^4n\left(d+\frac{e}{\sqrt{x}}\right)^2}{}$$

```
[Out] -1/3*b^2*d^6*n^2*ln(d+e/x^(1/2))^2/e^6+2/3*b*d^6*n*ln(d+e/x^(1/2))*(a+b*ln(
c*(d+e/x^(1/2))^n))/e^6-1/3*(a+b*ln(c*(d+e/x^(1/2))^n))^2/x^3-4*b*d^5*n*(a+
b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))/e^6-5/2*b^2*d^4*n^2*(d+e/x^(1/2))^2/
e^6+5*b*d^4*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^2/e^6+40/27*b^2*d^3
*n^2*(d+e/x^(1/2))^3/e^6-40/9*b*d^3*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1
/2))^3/e^6-5/8*b^2*d^2*n^2*(d+e/x^(1/2))^4/e^6+5/2*b*d^2*n*(a+b*ln(c*(d+e/x
^(1/2))^n))*(d+e/x^(1/2))^4/e^6+4/25*b^2*d*n^2*(d+e/x^(1/2))^5/e^6-4/5*b*d*
n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^5/e^6-1/54*b^2*n^2*(d+e/x^(1/2)
)^6/e^6+1/9*b*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^6/e^6+4*b^2*d^5*n
^2/e^5/x^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 355, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$-\frac{1}{90}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt{x}}\right)}{e^6}-\frac{450d^4\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^6}+\frac{400d^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^6}-\frac{225d^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{e^6}-\frac{60d^6\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^6}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^4,x]
```

```
[Out] (-5*b^2*d^4*n^2*(d + e/Sqrt[x])^2)/(2*e^6) + (40*b^2*d^3*n^2*(d + e/Sqrt[x]
)^3)/(27*e^6) - (5*b^2*d^2*n^2*(d + e/Sqrt[x])^4)/(8*e^6) + (4*b^2*d*n^2*(d
 + e/Sqrt[x])^5)/(25*e^6) - (b^2*n^2*(d + e/Sqrt[x])^6)/(54*e^6) + (4*b^2*d
^5*n^2)/(e^5*Sqrt[x]) - (b^2*d^6*n^2*Log[d + e/Sqrt[x]]^2)/(3*e^6) - (b*n*(
(360*d^5*(d + e/Sqrt[x]))/e^6 - (450*d^4*(d + e/Sqrt[x])^2)/e^6 + (400*d^3*
(d + e/Sqrt[x])^3)/e^6 - (225*d^2*(d + e/Sqrt[x])^4)/e^6 + (72*d*(d + e/Sqr
t[x])^5)/e^6 - (10*(d + e/Sqrt[x])^6)/e^6 - (60*d^6*Log[d + e/Sqrt[x]])/e^6
)*(a + b*Log[c*(d + e/Sqrt[x])^n]))/90 - (a + b*Log[c*(d + e/Sqrt[x])^n])^2
/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx = -\left(2\,\mathrm{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx,x,\frac{1}{\sqrt{x}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(2ben)\,\mathrm{Subst}\left(\int \frac{x^6\left(a+b\log\left(c(d+ex)\right.\right.}{d+ex}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(2bn)\,\mathrm{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6\left(a+b\log\left(cx\right.\right.}{x}\right.$$

$$= -\frac{1}{90}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt{x}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^6} - \frac{225d^2}{}\right.$$

$$= -\frac{1}{90}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt{x}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^6} - \frac{225d^2}{}\right.$$

$$= -\frac{1}{90}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt{x}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^6} - \frac{225d^2}{}\right.$$

$$= -\frac{5b^2d^4n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^6} + \frac{40b^2d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6} - \frac{5b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{8e^6} + \frac{4b^2d}{}$$

$$= -\frac{5b^2d^4n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^6} + \frac{40b^2d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6} - \frac{5b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{8e^6} + \frac{4b^2d}{}$$

Mathematica [C] time = 0.35, size = 692, normalized size = 1.44

$$\frac{-1800a^2e^6 - 3600abe^6\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right) + 3600abd^6nx^3\log\left(d\sqrt{x}+e\right) + 3600abd^6nx^3\log\left(-\frac{e}{d\sqrt{x}}\right) - 3600ab}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^4,x]
```

```
[Out] (-1800*a^2*e^6 + 600*a*b*e^6*n - 100*b^2*e^6*n^2 - 720*a*b*d*e^5*n*Sqrt[x]
+ 264*b^2*d*e^5*n^2*Sqrt[x] + 900*a*b*d^2*e^4*n*x - 555*b^2*d^2*e^4*n^2*x -
 1200*a*b*d^3*e^3*n*x^(3/2) + 1140*b^2*d^3*e^3*n^2*x^(3/2) + 1800*a*b*d^4*e
^2*n*x^2 - 2610*b^2*d^4*e^2*n^2*x^2 - 3600*a*b*d^5*e*n*x^(5/2) + 8820*b^2*d
^5*e*n^2*x^(5/2) - 5220*b^2*d^6*n^2*x^3*Log[d + e/Sqrt[x]] - 3600*a*b*e^6*L
og[c*(d + e/Sqrt[x])^n] + 600*b^2*e^6*n*Log[c*(d + e/Sqrt[x])^n] - 720*b^2*
d*e^5*n*Sqrt[x]*Log[c*(d + e/Sqrt[x])^n] + 900*b^2*d^2*e^4*n*x*Log[c*(d + e
/Sqrt[x])^n] - 1200*b^2*d^3*e^3*n*x^(3/2)*Log[c*(d + e/Sqrt[x])^n] + 1800*b
^2*d^4*e^2*n*x^2*Log[c*(d + e/Sqrt[x])^n] - 3600*b^2*d^5*e*n*x^(5/2)*Log[c*
(d + e/Sqrt[x])^n] - 3600*b^2*d^6*n*x^3*Log[c*(d + e/Sqrt[x])^n] - 1800*b^2
*e^6*Log[c*(d + e/Sqrt[x])^n]^2 + 3600*a*b*d^6*n*x^3*Log[e + d*Sqrt[x]] + 3
```

```
600*b^2*d^6*n*x^3*Log[c*(d + e/Sqrt[x])^n]*Log[e + d*Sqrt[x]] - 1800*b^2*d^
6*n^2*x^3*Log[e + d*Sqrt[x]]^2 + 3600*a*b*d^6*n*x^3*Log[-(e/(d*Sqrt[x]))] +
 3600*b^2*d^6*n*x^3*Log[c*(d + e/Sqrt[x])^n]*Log[-(e/(d*Sqrt[x]))] + 3600*b
^2*d^6*n^2*x^3*Log[e + d*Sqrt[x]]*Log[-((d*Sqrt[x])/e)] + 3600*b^2*d^6*n^2*
x^3*PolyLog[2, 1 + e/(d*Sqrt[x])] + 3600*b^2*d^6*n^2*x^3*PolyLog[2, 1 + (d*
Sqrt[x])/e])/(5400*e^6*x^3)
```

fricas [A] time = 0.50, size = 490, normalized size = 1.02

$$-\frac{100\,b^2e^6n^2+1800\,b^2e^6\log(c)^2-600\,abe^6n+1800\,a^2e^6+90\left(29\,b^2d^4e^2n^2-20\,abd^4e^2n\right)x^2-1800\left(b^2d^6n^2x^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/5400*(100*b^2*e^6*n^2 + 1800*b^2*e^6*log(c)^2 - 600*a*b*e^6*n + 1800*a^2
*e^6 + 90*(29*b^2*d^4*e^2*n^2 - 20*a*b*d^4*e^2*n)*x^2 - 1800*(b^2*d^6*n^2*x
^3 - b^2*e^6*n^2)*log((d*x + e*sqrt(x))/x)^2 + 15*(37*b^2*d^2*e^4*n^2 - 60*
a*b*d^2*e^4*n)*x - 300*(6*b^2*d^4*e^2*n*x^2 + 3*b^2*d^2*e^4*n*x + 2*b^2*e^6
*n - 12*a*b*e^6)*log(c) - 60*(30*b^2*d^4*e^2*n^2*x^2 + 15*b^2*d^2*e^4*n^2*x
 + 10*b^2*e^6*n^2 - 60*a*b*e^6*n - 3*(49*b^2*d^6*n^2 - 20*a*b*d^6*n)*x^3 +
60*(b^2*d^6*n*x^3 - b^2*e^6*n)*log(c) - 4*(15*b^2*d^5*e*n^2*x^2 + 5*b^2*d^3
*e^3*n^2*x + 3*b^2*d*e^5*n^2)*sqrt(x))*log((d*x + e*sqrt(x))/x) - 12*(22*b^
2*d*e^5*n^2 - 60*a*b*d*e^5*n + 15*(49*b^2*d^5*e*n^2 - 20*a*b*d^5*e*n)*x^2 +
 5*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x - 20*(15*b^2*d^5*e*n*x^2 + 5*b
^2*d^3*e^3*n*x + 3*b^2*d*e^5*n)*log(c))*sqrt(x))/(e^6*x^3)
```

giac [B] time = 0.45, size = 1639, normalized size = 3.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5400*(10800*(d*sqrt(x) + e)*b^2*d^5*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/sq
rt(x) - 27000*(d*sqrt(x) + e)^2*b^2*d^4*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/
x - 21600*(d*sqrt(x) + e)*b^2*d^5*n^2*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x)
+ 21600*(d*sqrt(x) + e)*b^2*d^5*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/sqrt(
x) + 36000*(d*sqrt(x) + e)^3*b^2*d^3*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x^(
3/2) + 27000*(d*sqrt(x) + e)^2*b^2*d^4*n^2*log((d*sqrt(x) + e)/sqrt(x))/x -
 54000*(d*sqrt(x) + e)^2*b^2*d^4*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x -
27000*(d*sqrt(x) + e)^4*b^2*d^2*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x^2 + 21
600*(d*sqrt(x) + e)*b^2*d^5*n^2/sqrt(x) - 21600*(d*sqrt(x) + e)*b^2*d^5*n*l
og(c)/sqrt(x) + 10800*(d*sqrt(x) + e)*b^2*d^5*log(c)^2/sqrt(x) - 24000*(d*s
qrt(x) + e)^3*b^2*d^3*n^2*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 21600*(d*s
qrt(x) + e)*a*b*d^5*n*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 72000*(d*sqrt(
x) + e)^3*b^2*d^3*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 10800*(d*
sqrt(x) + e)^5*b^2*d*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x^(5/2) - 13500*(d*
sqrt(x) + e)^2*b^2*d^4*n^2/x + 27000*(d*sqrt(x) + e)^2*b^2*d^4*n*log(c)/x -
 27000*(d*sqrt(x) + e)^2*b^2*d^4*log(c)^2/x + 13500*(d*sqrt(x) + e)^4*b^2*d
^2*n^2*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 54000*(d*sqrt(x) + e)^2*a*b*d^4*n
*log((d*sqrt(x) + e)/sqrt(x))/x - 54000*(d*sqrt(x) + e)^4*b^2*d^2*n*log(c)*
log((d*sqrt(x) + e)/sqrt(x))/x^2 - 1800*(d*sqrt(x) + e)^6*b^2*n^2*log((d*sq
rt(x) + e)/sqrt(x))^2/x^3 + 8000*(d*sqrt(x) + e)^3*b^2*d^3*n^2/x^(3/2) - 21
600*(d*sqrt(x) + e)*a*b*d^5*n/sqrt(x) - 24000*(d*sqrt(x) + e)^3*b^2*d^3*n*l
og(c)/x^(3/2) + 21600*(d*sqrt(x) + e)*a*b*d^5*log(c)/sqrt(x) + 36000*(d*sqr
t(x) + e)^3*b^2*d^3*log(c)^2/x^(3/2) - 4320*(d*sqrt(x) + e)^5*b^2*d*n^2*log
((d*sqrt(x) + e)/sqrt(x))/x^(5/2) + 72000*(d*sqrt(x) + e)^3*a*b*d^3*n*log((
d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 21600*(d*sqrt(x) + e)^5*b^2*d*n*log(c)*lo
```

```
g((d*sqrt(x) + e)/sqrt(x))/x^(5/2) - 3375*(d*sqrt(x) + e)^4*b^2*d^2*n^2/x^2
 + 27000*(d*sqrt(x) + e)^2*a*b*d^4*n/x + 13500*(d*sqrt(x) + e)^4*b^2*d^2*n*
log(c)/x^2 - 54000*(d*sqrt(x) + e)^2*a*b*d^4*log(c)/x - 27000*(d*sqrt(x) +
e)^4*b^2*d^2*log(c)^2/x^2 + 600*(d*sqrt(x) + e)^6*b^2*n^2*log((d*sqrt(x) +
e)/sqrt(x))/x^3 - 54000*(d*sqrt(x) + e)^4*a*b*d^2*n*log((d*sqrt(x) + e)/sqr
t(x))/x^2 - 3600*(d*sqrt(x) + e)^6*b^2*n*log(c)*log((d*sqrt(x) + e)/sqrt(x)
)/x^3 + 864*(d*sqrt(x) + e)^5*b^2*d*n^2/x^(5/2) - 24000*(d*sqrt(x) + e)^3*a
*b*d^3*n/x^(3/2) + 10800*(d*sqrt(x) + e)*a^2*d^5/sqrt(x) - 4320*(d*sqrt(x)
+ e)^5*b^2*d*n*log(c)/x^(5/2) + 72000*(d*sqrt(x) + e)^3*a*b*d^3*log(c)/x^(3
/2) + 10800*(d*sqrt(x) + e)^5*b^2*d*log(c)^2/x^(5/2) + 21600*(d*sqrt(x) + e
)^5*a*b*d*n*log((d*sqrt(x) + e)/sqrt(x))/x^(5/2) - 100*(d*sqrt(x) + e)^6*b^
2*n^2/x^3 + 13500*(d*sqrt(x) + e)^4*a*b*d^2*n/x^2 - 27000*(d*sqrt(x) + e)^2
*a^2*d^4/x + 600*(d*sqrt(x) + e)^6*b^2*n*log(c)/x^3 - 54000*(d*sqrt(x) + e)
^4*a*b*d^2*log(c)/x^2 - 1800*(d*sqrt(x) + e)^6*b^2*log(c)^2/x^3 - 3600*(d*s
qrt(x) + e)^6*a*b*n*log((d*sqrt(x) + e)/sqrt(x))/x^3 - 4320*(d*sqrt(x) + e)
^5*a*b*d*n/x^(5/2) + 36000*(d*sqrt(x) + e)^3*a^2*d^3/x^(3/2) + 21600*(d*sqr
t(x) + e)^5*a*b*d*log(c)/x^(5/2) + 600*(d*sqrt(x) + e)^6*a*b*n/x^3 - 27000*
(d*sqrt(x) + e)^4*a^2*d^2/x^2 - 3600*(d*sqrt(x) + e)^6*a*b*log(c)/x^3 + 108
00*(d*sqrt(x) + e)^5*a^2*d/x^(5/2) - 1800*(d*sqrt(x) + e)^6*a^2/x^3)*e^(-6)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{2}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/2))^n)+a)^2/x^4,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/2))^n)+a)^2/x^4,x)
```

maxima [A] time = 0.67, size = 387, normalized size = 0.81

$$\frac{1}{90}\, aben\left(\frac{60\, d^{6} \log \left(d \sqrt{x}+e\right)}{e^{7}}-\frac{30\, d^{6} \log(x)}{e^{7}}-\frac{60\, d^{5} x^{\frac{5}{2}}-30\, d^{4} e x^{2}+20\, d^{3} e^{2} x^{\frac{3}{2}}-15\, d^{2} e^{3} x+12\, d e^{4} \sqrt{x}-10\, e^{5}}{e^{6} x^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/90*a*b*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 - 30*d^6*log(x)/e^7 - (60*d^5*x
^(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 15*d^2*e^3*x + 12*d*e^4*sqrt(x
) - 10*e^5)/(e^6*x^3)) + 1/5400*(60*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 - 30
*d^6*log(x)/e^7 - (60*d^5*x^(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 15*
d^2*e^3*x + 12*d*e^4*sqrt(x) - 10*e^5)/(e^6*x^3))*log(c*(d + e/sqrt(x))^n)
- (1800*d^6*x^3*log(d*sqrt(x) + e)^2 + 450*d^6*x^3*log(x)^2 - 4410*d^6*x^3*
log(x) - 8820*d^5*e*x^(5/2) + 2610*d^4*e^2*x^2 - 1140*d^3*e^3*x^(3/2) + 555
*d^2*e^4*x - 264*d*e^5*sqrt(x) + 100*e^6 - 180*(10*d^6*x^3*log(x) - 49*d^6*
x^3)*log(d*sqrt(x) + e))*n^2/(e^6*x^3))*b^2 - 1/3*b^2*log(c*(d + e/sqrt(x))
^n)^2/x^3 - 2/3*a*b*log(c*(d + e/sqrt(x))^n)/x^3 - 1/3*a^2/x^3
```

mupad [B] time = 1.77, size = 440, normalized size = 0.92

$$\frac{b^{2}\, d^{6} \ln \left(c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)^{2}}{3\, e^{6}}-\frac{b^{2} \ln \left(c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)^{2}}{3\, x^{3}}-\frac{b^{2}\, n^{2}}{54\, x^{3}}-\frac{2\, a\, b\, \ln \left(c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{3\, x^{3}}-\frac{a^{2}}{3\, x^{3}}+\frac{a\, b\, n}{9\, x^{3}}+\frac{b^{2}\, n\, \ln \left(c \left(d+\frac{e}{\sqrt{x}}\right.\right.}{9\, x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^2/x^4,x)
```

```
[Out] (b^2*d^6*log(c*(d + e/x^(1/2))^n)^2)/(3*e^6) - (b^2*log(c*(d + e/x^(1/2))^n
)^2)/(3*x^3) - (b^2*n^2)/(54*x^3) - (2*a*b*log(c*(d + e/x^(1/2))^n))/(3*x^3
) - a^2/(3*x^3) + (a*b*n)/(9*x^3) + (b^2*n*log(c*(d + e/x^(1/2))^n))/(9*x^3
) - (49*b^2*d^6*n^2*log(d + e/x^(1/2)))/(30*e^6) - (37*b^2*d^2*n^2)/(360*e^
2*x^2) - (29*b^2*d^4*n^2)/(60*e^4*x) + (19*b^2*d^3*n^2)/(90*e^3*x^(3/2)) +
(49*b^2*d^5*n^2)/(30*e^5*x^(1/2)) + (11*b^2*d*n^2)/(225*e*x^(5/2)) + (b^2*d
^2*n*log(c*(d + e/x^(1/2))^n))/(6*e^2*x^2) + (b^2*d^4*n*log(c*(d + e/x^(1/2
))^n))/(3*e^4*x) - (2*b^2*d^3*n*log(c*(d + e/x^(1/2))^n))/(9*e^3*x^(3/2)) -
 (2*b^2*d^5*n*log(c*(d + e/x^(1/2))^n))/(3*e^5*x^(1/2)) - (2*a*b*d*n)/(15*e
*x^(5/2)) + (2*a*b*d^6*n*log(d + e/x^(1/2)))/(3*e^6) - (2*b^2*d*n*log(c*(d
+ e/x^(1/2))^n))/(15*e*x^(5/2)) + (a*b*d^2*n)/(6*e^2*x^2) + (a*b*d^4*n)/(3*
e^4*x) - (2*a*b*d^3*n)/(9*e^3*x^(3/2)) - (2*a*b*d^5*n)/(3*e^5*x^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x**4,x)
```

```
[Out] Timed out
```

3.436 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx$

Optimal. Leaf size=569

$$-\frac{3b^2e^4n^2\text{Li}_2\left(\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4}-\frac{5b^2e^4n^2\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^4}-\frac{3b^2e^4n^2\text{l}}{}$$

```
[Out] -3/2*b^3*e^4*n^3*ln(x)/d^4-1/2*b^3*e^4*n^3*ln(d+e/x^(1/2))/d^4+1/2*b^2*e^2*
n^2*x*(a+b*ln(c*(d+e/x^(1/2))^n))/d^2-5/2*b^2*e^4*n^2*ln(1-d/(d+e/x^(1/2)))
*(a+b*ln(c*(d+e/x^(1/2))^n))/d^4-3/4*b*e^2*n*x*(a+b*ln(c*(d+e/x^(1/2))^n))^
2/d^2+1/2*b*e*n*x^(3/2)*(a+b*ln(c*(d+e/x^(1/2))^n))^2/d+3/2*b*e^4*n*ln(1-d/
(d+e/x^(1/2)))*(a+b*ln(c*(d+e/x^(1/2))^n))^2/d^4+1/2*x^2*(a+b*ln(c*(d+e/x^(
1/2))^n))^3-3*b^2*e^4*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*ln(-e/d/x^(1/2))/d^4+
5/2*b^3*e^4*n^3*polylog(2,d/(d+e/x^(1/2)))/d^4-3*b^2*e^4*n^2*(a+b*ln(c*(d+e
/x^(1/2))^n))*polylog(2,d/(d+e/x^(1/2)))/d^4-3*b^3*e^4*n^3*polylog(2,1+e/d/
x^(1/2))/d^4-3*b^3*e^4*n^3*polylog(3,d/(d+e/x^(1/2)))/d^4+1/2*b^3*e^3*n^3*x
^(1/2)/d^3-5/2*b^2*e^3*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))*x^(1/2
)/d^4+3/2*b*e^3*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))*x^(1/2)/d^4
```

Rubi [A] time = 1.49, antiderivative size = 546, normalized size of antiderivative = 0.96, number of steps used = 35, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31, 44}

$$\frac{3b^2e^4n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4}-\frac{11b^3e^4n^3\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)}{2d^4}-\frac{3b^3e^4n^3\text{PolyLog}\left(3\right.}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
```

```
[Out] (b^3*e^3*n^3*Sqrt[x])/(2*d^3) - (b^3*e^4*n^3*Log[d + e/Sqrt[x]])/(2*d^4) -
(5*b^2*e^3*n^2*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2
*d^4) + (b^2*e^2*n^2*x*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2*d^2) + (5*b*e^4
*n*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(4*d^4) + (3*b*e^3*n*(d + e/Sqrt[x])
*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*d^4) - (3*b*e^2*n*x*(a + b*
Log[c*(d + e/Sqrt[x])^n])^2)/(4*d^2) + (b*e*n*x^(3/2)*(a + b*Log[c*(d + e/S
qrt[x])^n])^2)/(2*d) - (e^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/(2*d^4) + (
x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/2 - (11*b^2*e^4*n^2*(a + b*Log[c*(d
 + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))])/(2*d^4) + (3*b*e^4*n*(a + b*Log[c*
(d + e/Sqrt[x])^n])^2*Log[-(e/(d*Sqrt[x]))])/(2*d^4) - (3*b^3*e^4*n^3*Log[x
])/(2*d^4) - (11*b^3*e^4*n^3*PolyLog[2, 1 + e/(d*Sqrt[x])])/(2*d^4) + (3*b^
2*e^4*n^2*(a + b*Log[c*(d + e/Sqrt[x])^n])*PolyLog[2, 1 + e/(d*Sqrt[x])])/d
^4 - (3*b^3*e^4*n^3*PolyLog[3, 1 + e/(d*Sqrt[x])])/d^4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
```

```
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx = -\left(2\,\text{Subst}\left(\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x^5}dx,x,\frac{1}{\sqrt{x}}\right)\right)$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-\frac{1}{2}(3ben)\,\text{Subst}\left(\int\frac{\left(a+b\log\left(c(d+ex)\right.\right.}{x^4(d+ex)}\right.$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-\frac{1}{2}(3bn)\,\text{Subst}\left(\int\frac{\left(a+b\log\left(cx^n\right)\right)^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}dx,\right.$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-\frac{(3bn)\,\text{Subst}\left(\int\frac{\left(a+b\log\left(cx^n\right)\right)^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}dx,x,d+\right.}{2d}$$

$$=\frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3+-$$

$$=-\frac{3be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4d^2}+\frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d}$$

$$=\frac{b^2e^2n^2x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2}+\frac{3be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\right.\right.\right.}{2d^4}$$

$$=-\frac{5b^2e^3n^2\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^4}+\frac{b^2e^2n^2x\left(a+b\log\left(c\right.\right.}{2d^2}$$

$$=\frac{b^3e^3n^3\sqrt{x}}{2d^3}-\frac{b^3e^4n^3\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}-\frac{5b^2e^3n^2\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d\right.\right.\right.}{2d^4}$$

$$=\frac{b^3e^3n^3\sqrt{x}}{2d^3}-\frac{b^3e^4n^3\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}-\frac{5b^2e^3n^2\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d\right.\right.\right.}{2d^4}$$

Mathematica [A] time = 1.04, size = 777, normalized size = 1.37

$$\frac{-2b^2n^2\left(3\left(e^4-d^4x^2\right)\log^2\left(d+\frac{e}{\sqrt{x}}\right)+e^2\left(d^2(-x)+11e^2\log\left(-\frac{e}{d\sqrt{x}}\right)+5de\sqrt{x}\right)-e\log\left(d+\frac{e}{\sqrt{x}}\right)\left(2d^3x^{3/2}-3d^2e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
```

```
[Out] (6*b*d*e^3*n*Sqrt[x]*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^
n])^2 - 3*b*d^2*e^2*n*x*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x
])^n])^2 + 2*b*d^3*e*n*x^(3/2)*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e
/Sqrt[x])^n])^2 + 6*b*d^4*n*x^2*Log[d + e/Sqrt[x]]*(a - b*n*Log[d + e/Sqrt[
x]] + b*Log[c*(d + e/Sqrt[x])^n])^2 + 2*d^4*x^2*(a - b*n*Log[d + e/Sqrt[x]]
```

```
 + b*Log[c*(d + e/Sqrt[x])^n])^3 - 6*b*e^4*n*(a - b*n*Log[d + e/Sqrt[x]] +
b*Log[c*(d + e/Sqrt[x])^n])^2*Log[e + d*Sqrt[x]] - 2*b^2*n^2*(a - b*n*Log[d
 + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])*(3*(e^4 - d^4*x^2)*Log[d + e/Sq
rt[x]]^2 + e^2*(5*d*e*Sqrt[x] - d^2*x + 11*e^2*Log[-(e/(d*Sqrt[x]))]) - e*L
og[d + e/Sqrt[x]]*(11*e^3 + 6*d*e^2*Sqrt[x] - 3*d^2*e*x + 2*d^3*x^(3/2) + 6
*e^3*Log[-(e/(d*Sqrt[x]))]) - 6*e^4*PolyLog[2, 1 + e/(d*Sqrt[x])]) + b^3*n^
3*(d^2*e^2*x*(2 - 3*Log[d + e/Sqrt[x]])*Log[d + e/Sqrt[x]] + 2*d^3*e*x^(3/2
)*Log[d + e/Sqrt[x]]^2 + 2*d^4*x^2*Log[d + e/Sqrt[x]]^3 + 2*d*e^3*Sqrt[x]*(
1 - 5*Log[d + e/Sqrt[x]] + 3*Log[d + e/Sqrt[x]]^2) + 12*e^4*(-Log[d + e/Sqr
t[x]] + Log[-(e/(d*Sqrt[x]))]) + 11*e^4*(Log[d + e/Sqrt[x]]*(Log[d + e/Sqrt
[x]] - 2*Log[-(e/(d*Sqrt[x]))]) - 2*PolyLog[2, 1 + e/(d*Sqrt[x])]) - 2*e^4*
(Log[d + e/Sqrt[x]]^2*(Log[d + e/Sqrt[x]] - 3*Log[-(e/(d*Sqrt[x]))]) - 6*Lo
g[d + e/Sqrt[x]]*PolyLog[2, 1 + e/(d*Sqrt[x])] + 6*PolyLog[3, 1 + e/(d*Sqrt
[x])])))/(4*d^4)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^3 x \log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)^3 + 3\,ab^2 x \log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)^2 + 3\,a^2bx \log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right) + a^3x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*log(c*((d*x + e*sqrt(x))/x)^n)^3 + 3*a*b^2*x*log(c*((d*x + e
*sqrt(x))/x)^n)^2 + 3*a^2*b*x*log(c*((d*x + e*sqrt(x))/x)^n) + a^3*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right) + a\right)^3 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^3*x, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(b \ln\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right) + a\right)^3 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(d+e/x^(1/2))^n)+a)^3,x)
```

```
[Out] int(x*(b*ln(c*(d+e/x^(1/2))^n)+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,b^3n^3x^2\log\left(d\sqrt{x}+e\right)^3 - \int \frac{3\left(b^3dnx^2 - 4\left(b^3d\log(c) + ab^2d\right)x^2 - 4\left(b^3e\log(c) + ab^2e\right)x^{\frac{3}{2}} + 4\left(b^3dx^2 + b^3ex^{\frac{3}{2}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*n^3*x^2*log(d*sqrt(x) + e)^3 - integrate(1/4*(3*(b^3*d*n*x^2 - 4*(b
^3*d*log(c) + a*b^2*d)*x^2 - 4*(b^3*e*log(c) + a*b^2*e)*x^(3/2) + 4*(b^3*d*
```

```
x^2 + b^3*e*x^(3/2))*log(x^(1/2*n)))*n^2*log(d*sqrt(x) + e)^2 + 4*(b^3*d*x^
2 + b^3*e*x^(3/2))*log(x^(1/2*n))^3 - 4*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^
2 + 3*a^2*b*d*log(c) + a^3*d)*x^2 - 12*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c)
+ a^2*b*d)*x^2 + (b^3*d*x^2 + b^3*e*x^(3/2))*log(x^(1/2*n))^2 + (b^3*e*log(
c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(3/2) - 2*((b^3*d*log(c) + a*b^2*d)*x^
2 + (b^3*e*log(c) + a*b^2*e)*x^(3/2))*log(x^(1/2*n)))*n*log(d*sqrt(x) + e)
- 12*((b^3*d*log(c) + a*b^2*d)*x^2 + (b^3*e*log(c) + a*b^2*e)*x^(3/2))*log(
x^(1/2*n))^2 - 4*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) +
a^3*e)*x^(3/2) + 12*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^2 + (b
^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(3/2))*log(x^(1/2*n)))/(d*x +
 e*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/2))^n))^3,x)
[Out] int(x*(a + b*log(c*(d + e/x^(1/2))^n))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))**n))**3,x)
[Out] Integral(x*(a + b*log(c*(d + e/sqrt(x))**n))**3, x)
```

3.437 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx$

Optimal. Leaf size=260

$$-\frac{6b^2e^2n^2\text{Li}_2\left(\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2}-\frac{6b^2e^2n^2\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2}+\frac{3be^2n\log\left(1\right.}{}$$

```
[Out] 3*b*e^2*n*ln(1-d/(d+e/x^(1/2)))*(a+b*ln(c*(d+e/x^(1/2))^n))^2/d^2+x*(a+b*ln
(c*(d+e/x^(1/2))^n))^3-6*b^2*e^2*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*ln(-e/d/x^
(1/2))/d^2-6*b^2*e^2*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*polylog(2,d/(d+e/x^(1/
2)))/d^2-6*b^3*e^2*n^3*polylog(2,1+e/d/x^(1/2))/d^2-6*b^3*e^2*n^3*polylog(3
,d/(d+e/x^(1/2)))/d^2+3*b*e*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))*x
^(1/2)/d^2
```

Rubi [A] time = 0.62, antiderivative size = 281, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.650, Rules used = {2451, 2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391}

$$\frac{6b^2e^2n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2}-\frac{6b^3e^2n^3\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)}{d^2}-\frac{6b^3e^2n^3\text{PolyLog}\left(3,\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
```

```
[Out] (3*b*e*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/d^2 -
(e^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/d^2 + x*(a + b*Log[c*(d + e/Sqrt[x
])^n])^3 - (6*b^2*e^2*n^2*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[
x]))])/d^2 + (3*b*e^2*n*(a + b*Log[c*(d + e/Sqrt[x])^n])^2*Log[-(e/(d*Sqrt[
x]))])/d^2 - (6*b^3*e^2*n^3*PolyLog[2, 1 + e/(d*Sqrt[x])])/d^2 + (6*b^2*e^2
*n^2*(a + b*Log[c*(d + e/Sqrt[x])^n])*PolyLog[2, 1 + e/(d*Sqrt[x])])/d^2 -
(6*b^3*e^2*n^3*PolyLog[3, 1 + e/(d*Sqrt[x])])/d^2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
```

```
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx = 2\,\text{Subst}\left(\int x\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^n\right)\right)^3 dx, x, \sqrt{x}\right)$$

$$= -\left(2\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x^3} dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - (3ben)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^2(d+ex)}\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - (3bn)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(cx^n\right)\right)^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2} dx, x, \right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - \frac{(3bn)\,\text{Subst}\left(\int \frac{\left(a+b\log(cx^n)\right)^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2} dx, x, d+\frac{e}{\sqrt{x}}\right.}{d}$$

$$= \frac{3ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.$$

$$= \frac{3ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.$$

$$= \frac{3ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} - \frac{e^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.}{d^2}$$

$$= \frac{3ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} - \frac{e^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.}{d^2}$$

Mathematica [A] time = 0.73, size = 476, normalized size = 1.83

$$\frac{3b^2n^2\left(\left(d^2x-e^2\right)\log^2\left(d+\frac{e}{\sqrt{x}}\right)+2e^2\text{Li}_2\left(\frac{e}{d\sqrt{x}}+1\right)-2e^2\log\left(-\frac{e}{d\sqrt{x}}\right)+2e\log\left(d+\frac{e}{\sqrt{x}}\right)\left(e\log\left(-\frac{e}{d\sqrt{x}}\right)+d\sqrt{x}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
```

```
[Out] (3*b*d*e*n*Sqrt[x]*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n]
)^2 + 3*b*d^2*n*x*Log[d + e/Sqrt[x]]*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*
```

```
(d + e/Sqrt[x])^n])^2 + d^2*x*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/
Sqrt[x])^n])^3 - 3*b*e^2*n*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqr
t[x])^n])^2*Log[e + d*Sqrt[x]] + 3*b^2*n^2*(a - b*n*Log[d + e/Sqrt[x]] + b*
Log[c*(d + e/Sqrt[x])^n])*((-e^2 + d^2*x)*Log[d + e/Sqrt[x]]^2 - 2*e^2*Log[
-(e/(d*Sqrt[x]))] + 2*e*Log[d + e/Sqrt[x]]*(e + d*Sqrt[x] + e*Log[-(e/(d*Sq
rt[x]))]) + 2*e^2*PolyLog[2, 1 + e/(d*Sqrt[x])]) + b^3*n^3*(Log[d + e/Sqrt[
x]]*((-e^2 + d^2*x)*Log[d + e/Sqrt[x]]^2 - 6*e^2*Log[-(e/(d*Sqrt[x]))] + 3*
e*Log[d + e/Sqrt[x]]*(e + d*Sqrt[x] + e*Log[-(e/(d*Sqrt[x]))])) + 6*e^2*(-1
 + Log[d + e/Sqrt[x]])*PolyLog[2, 1 + e/(d*Sqrt[x])] - 6*e^2*PolyLog[3, 1 +
 e/(d*Sqrt[x])]))/d^2
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)^3+3\,ab^2\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)^2+3\,a^2b\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)+a^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*log(c*((d*x + e*sqrt(x))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*sqr
t(x))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*sqrt(x))/x)^n) + a^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^3, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/2))^n)+a)^3,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/2))^n)+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^3n^3x\log\left(d\sqrt{x}+e\right)^3-3\left(en\left(\frac{e\log\left(d\sqrt{x}+e\right)}{d^2}-\frac{\sqrt{x}}{d}\right)-x\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)a^2b+a^3x-\int\frac{3\left(b^3dnx-2\left(b^3d\log(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="maxima")
```

```
[Out] b^3*n^3*x*log(d*sqrt(x) + e)^3 - 3*(e*n*(e*log(d*sqrt(x) + e)/d^2 - sqrt(x)
/d) - x*log(c*(d + e/sqrt(x))^n))*a^2*b + a^3*x - integrate(1/2*(3*(b^3*d*n
*x - 2*(b^3*d*log(c) + a*b^2*d)*x + 2*(b^3*d*x + b^3*e*sqrt(x))*log(x^(1/2*
n)) - 2*(b^3*e*log(c) + a*b^2*e)*sqrt(x))*n^2*log(d*sqrt(x) + e)^2 + 2*(b^3
*d*x + b^3*e*sqrt(x))*log(x^(1/2*n))^3 - 6*((b^3*d*x + b^3*e*sqrt(x))*log(x
^(1/2*n))^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c))*x - 2*((b^3*d*log(c) + a*
```

```
b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*sqrt(x))*log(x^(1/2*n)) + (b^3*e*log(c)
^2 + 2*a*b^2*e*log(c))*sqrt(x))*n*log(d*sqrt(x) + e) - 6*((b^3*d*log(c) + a
*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*sqrt(x))*log(x^(1/2*n))^2 - 2*(b^3*d*l
og(c)^3 + 3*a*b^2*d*log(c)^2)*x + 6*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c))*x
+ (b^3*e*log(c)^2 + 2*a*b^2*e*log(c))*sqrt(x))*log(x^(1/2*n)) - 2*(b^3*e*lo
g(c)^3 + 3*a*b^2*e*log(c)^2)*sqrt(x))/(d*x + e*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b\,\ln\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^3,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/2))^n))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**3, x)
```

3.438 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx$

Optimal. Leaf size=135

$$12b^2n^2\mathrm{Li}_3\left(\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-6bn\mathrm{Li}_2\left(\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-2\log\left(-\frac{e}{d\sqrt{x}}\right)$$

```
[Out] -2*(a+b*ln(c*(d+e/x^(1/2))^n))^3*ln(-e/d/x^(1/2))-6*b*n*(a+b*ln(c*(d+e/x^(1
/2))^n))^2*polylog(2,1+e/d/x^(1/2))+12*b^2*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*
polylog(3,1+e/d/x^(1/2))-12*b^3*n^3*polylog(4,1+e/d/x^(1/2))
```

Rubi [A] time = 0.20, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2454, 2396, 2433, 2374, 2383, 6589}

$$12b^2n^2\mathrm{PolyLog}\left(3,\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-6bn\mathrm{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x,x]
```

```
[Out] -2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3*Log[-(e/(d*Sqrt[x]))] - 6*b*n*(a + b*
Log[c*(d + e/Sqrt[x])^n])^2*PolyLog[2, 1 + e/(d*Sqrt[x])] + 12*b^2*n^2*(a +
 b*Log[c*(d + e/Sqrt[x])^n])*PolyLog[3, 1 + e/(d*Sqrt[x])] - 12*b^3*n^3*Pol
yLog[4, 1 + e/(d*Sqrt[x])]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx = -\left(2\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right)+(6ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{\quad}\right.$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right)+(6bn)\,\text{Subst}\left(\int \frac{\left(a+b\log\right.}{\quad}\right.$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right)-6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right)-6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right)-6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.$$

Mathematica [B] time = 0.31, size = 532, normalized size = 3.94

$$6b^2n^2\left(-2\text{Li}_3\left(\frac{\sqrt{x}\,d}{e}+1\right)-2\text{Li}_3\left(-\frac{d\sqrt{x}}{e}\right)+2\text{Li}_2\left(\frac{\sqrt{x}\,d}{e}+1\right)\log\left(\frac{e}{d}+\sqrt{x}\right)-2\text{Li}_2\left(-\frac{d\sqrt{x}}{e}\right)\left(\log\left(d+\frac{e}{\sqrt{x}}\right)-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])^2*((Log[d + e/Sqr
t[x]] - Log[1 + e/(d*Sqrt[x])])*Log[x] + 2*PolyLog[2, -(e/(d*Sqrt[x]))]) +
6*b^2*n^2*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])*(Log[e/
d + Sqrt[x]]^2*Log[-((d*Sqrt[x])/e)] + (Log[d + e/Sqrt[x]]^2*Log[x])/2 - (L
og[e/d + Sqrt[x]]^2*Log[x])/2 - Log[d + e/Sqrt[x]]*Log[1 + (d*Sqrt[x])/e]*L
og[x] + Log[e/d + Sqrt[x]]*Log[1 + (d*Sqrt[x])/e]*Log[x] + (Log[d + e/Sqrt[
x]]*Log[x]^2)/4 - (Log[1 + (d*Sqrt[x])/e]*Log[x]^2)/4 + Log[x]^3/24 + 2*Log
[e/d + Sqrt[x]]*PolyLog[2, 1 + (d*Sqrt[x])/e] - 2*(Log[d + e/Sqrt[x]] - Log
```

```
[e/d + Sqrt[x]])*PolyLog[2, -((d*Sqrt[x])/e)] - 2*PolyLog[3, 1 + (d*Sqrt[x]
)/e] - 2*PolyLog[3, -((d*Sqrt[x])/e)]) - 2*b^3*n^3*(Log[d + e/Sqrt[x]]^3*Lo
g[-(e/(d*Sqrt[x]))] + 3*Log[d + e/Sqrt[x]]^2*PolyLog[2, 1 + e/(d*Sqrt[x])]
- 6*Log[d + e/Sqrt[x]]*PolyLog[3, 1 + e/(d*Sqrt[x])] + 6*PolyLog[4, 1 + e/(
d*Sqrt[x])])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)^3+3\,ab^2\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)^2+3\,a^2b\log\left(c\left(\frac{dx+e\sqrt{x}}{x}\right)^n\right)+a^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*((d*x + e*sqrt(x))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*sq
rt(x))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*sqrt(x))/x)^n) + a^3)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^3/x, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/2))^n)+a)^3/x,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/2))^n)+a)^3/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^3n^3\log\left(d\sqrt{x}+e\right)^3\log(x)-\int\frac{3\left(b^3dnx\log(x)-2\left(b^3d\log(c)+ab^2d\right)x+2\left(b^3dx+b^3e\sqrt{x}\right)\log\left(x^{\frac{1}{2}n}\right)-2\left(b^3e\log\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*n^3*log(d*sqrt(x) + e)^3*log(x) - integrate(1/2*(3*(b^3*d*n*x*log(x) -
2*(b^3*d*log(c) + a*b^2*d)*x + 2*(b^3*d*x + b^3*e*sqrt(x))*log(x^(1/2*n)) -
 2*(b^3*e*log(c) + a*b^2*e)*sqrt(x))*n^2*log(d*sqrt(x) + e)^2 + 2*(b^3*d*x
+ b^3*e*sqrt(x))*log(x^(1/2*n))^3 - 6*((b^3*d*x + b^3*e*sqrt(x))*log(x^(1/2
*n))^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x - 2*((b^3*d*log(c)
 + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*sqrt(x))*log(x^(1/2*n)) + (b^3*e*l
```

```
og(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*sqrt(x))*n*log(d*sqrt(x) + e) - 6*((b
^3*d*log(c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*sqrt(x))*log(x^(1/2*n))
^2 - 2*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x +
 6*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x + (b^3*e*log(c)^2 + 2*a
*b^2*e*log(c) + a^2*b*e)*sqrt(x))*log(x^(1/2*n)) - 2*(b^3*e*log(c)^3 + 3*a*
b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*sqrt(x))/(d*x^2 + e*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\ln\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^3/x,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/2))^n))^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**3/x, x)
```

3.439 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=285

$$-\frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^2}+\frac{12ab^2dn^2}{e\sqrt{x}}+\frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^2}-\frac{6bdn\left(d+\right.}{}$$

```
[Out] 12*b^3*d*n^2*ln(c*(d+e/x^(1/2))^n)*(d+e/x^(1/2))/e^2-6*b*d*n*(a+b*ln(c*(d+e
/x^(1/2))^n))^2*(d+e/x^(1/2))/e^2+2*d*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^
(1/2))/e^2+3/4*b^3*n^3*(d+e/x^(1/2))^2/e^2-3/2*b^2*n^2*(a+b*ln(c*(d+e/x^(1/
2))^n))*(d+e/x^(1/2))^2/e^2+3/2*b*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1
/2))^2/e^2-(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))^2/e^2+12*a*b^2*d*n^2
/e/x^(1/2)-12*b^3*d*n^3/e/x^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$-\frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^2}+\frac{12ab^2dn^2}{e\sqrt{x}}+\frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^2}-\frac{6bdn\left(d+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^2,x]
```

```
[Out] (3*b^3*n^3*(d + e/Sqrt[x])^2)/(4*e^2) + (12*a*b^2*d*n^2)/(e*Sqrt[x]) - (12*
b^3*d*n^3)/(e*Sqrt[x]) + (12*b^3*d*n^2*(d + e/Sqrt[x])*Log[c*(d + e/Sqrt[x]
)^n])/e^2 - (3*b^2*n^2*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/
(2*e^2) - (6*b*d*n*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/e^2
+ (3*b*n*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*e^2) + (2
*d*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^2 - ((d + e/Sqrt[x
])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^2
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx = -\left(2\,\text{Subst}\left(\int x\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\,\text{Subst}\left(\int\left(-\frac{d\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e} + \frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right.\right.}{e}\right.\right.\right.$$

$$= -\frac{2\,\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \frac{1}{\sqrt{x}}\right)}{e} + \frac{(2d)\,\text{Subst}\left(\int\left(a\right.\right.}{}$$

$$= -\frac{2\,\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)^3\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^2} + \frac{(2d)\,\text{Subst}\left(\int\left(a+b\log\left(c\right.\right.\right.}{e^2}$$

$$= \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2} - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right.\right.}{e^2}$$

$$= -\frac{6bdn\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2} + \frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(\right.\right.\right.}{2e^2}$$

$$= \frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^2} + \frac{12ab^2dn^2}{e\sqrt{x}} - \frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^2} -$$

$$= \frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^2} + \frac{12ab^2dn^2}{e\sqrt{x}} - \frac{12b^3dn^3}{e\sqrt{x}} + \frac{12b^3dn^2\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.}{e^2}$$

Mathematica [A] time = 0.67, size = 558, normalized size = 1.96

$$\frac{-4a^3e^2 - 6b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\left(e\left(2a^2e - 2abn\left(e-2d\sqrt{x}\right) + b^2n^2\left(e-6d\sqrt{x}\right)\right) + 2bd^2nx(3bn-2a)\log\left(d\sqrt{x}+e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^2,x]
```

```
[Out] (-4*a^3*e^2 + 6*a^2*b*e^2*n - 6*a*b^2*e^2*n^2 + 3*b^3*e^2*n^3 - 12*a^2*b*d*
e*n*Sqrt[x] + 36*a*b^2*d*e*n^2*Sqrt[x] - 42*b^3*d*e*n^3*Sqrt[x] - 8*b^3*d^2
*n^3*x*Log[d + e/Sqrt[x]]^3 - 4*b^3*e^2*Log[c*(d + e/Sqrt[x])^n]^3 + 12*a^2
*b*d^2*n*x*Log[e + d*Sqrt[x]] - 36*a*b^2*d^2*n^2*x*Log[e + d*Sqrt[x]] + 42*
b^3*d^2*n^3*x*Log[e + d*Sqrt[x]] + 6*b^2*d^2*n^2*x*Log[d + e/Sqrt[x]]*(-2*a
 + 3*b*n - 2*b*Log[c*(d + e/Sqrt[x])^n])*(2*Log[e + d*Sqrt[x]] - Log[x]) -
6*a^2*b*d^2*n*x*Log[x] + 18*a*b^2*d^2*n^2*x*Log[x] - 21*b^3*d^2*n^3*x*Log[x
] + 6*b^2*d^2*n^2*x*Log[d + e/Sqrt[x]]^2*(2*a - 3*b*n + 2*b*Log[c*(d + e/Sq
rt[x])^n] + 2*b*n*Log[e + d*Sqrt[x]] - b*n*Log[x]) + 6*b^2*Log[c*(d + e/Sqr
t[x])^n]^2*(e*(-2*a*e + b*n*(e - 2*d*Sqrt[x])) + 2*b*d^2*n*x*Log[e + d*Sqrt
[x]] - b*d^2*n*x*Log[x]) - 6*b*Log[c*(d + e/Sqrt[x])^n]*(e*(2*a^2*e + b^2*n
^2*(e - 6*d*Sqrt[x]) - 2*a*b*n*(e - 2*d*Sqrt[x])) + 2*b*d^2*n*(-2*a + 3*b*n
)*x*Log[e + d*Sqrt[x]] + b*d^2*n*(2*a - 3*b*n)*x*Log[x]))/(4*e^2*x)
```

fricas [B] time = 0.47, size = 541, normalized size = 1.90

$$\frac{3\,b^3e^2n^3 - 4\,b^3e^2\log(c)^3 - 6\,ab^2e^2n^2 + 6\,a^2be^2n - 4\,a^3e^2 + 4\left(b^3d^2n^3x - b^3e^2n^3\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)^3 + 6\left(b^3e^2n - 2\,ab^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*b^3*e^2*n^3 - 4*b^3*e^2*log(c)^3 - 6*a*b^2*e^2*n^2 + 6*a^2*b*e^2*n -
 4*a^3*e^2 + 4*(b^3*d^2*n^3*x - b^3*e^2*n^3)*log((d*x + e*sqrt(x))/x)^3 + 6
*(b^3*e^2*n - 2*a*b^2*e^2)*log(c)^2 - 6*(2*b^3*d*e*n^3*sqrt(x) - b^3*e^2*n^
3 + 2*a*b^2*e^2*n^2 + (3*b^3*d^2*n^3 - 2*a*b^2*d^2*n^2)*x - 2*(b^3*d^2*n^2*
x - b^3*e^2*n^2)*log(c))*log((d*x + e*sqrt(x))/x)^2 - 6*(b^3*e^2*n^2 - 2*a*
b^2*e^2*n + 2*a^2*b*e^2)*log(c) - 6*(b^3*e^2*n^3 - 2*a*b^2*e^2*n^2 + 2*a^2*
b*e^2*n - 2*(b^3*d^2*n*x - b^3*e^2*n)*log(c)^2 - (7*b^3*d^2*n^3 - 6*a*b^2*d
^2*n^2 + 2*a^2*b*d^2*n)*x - 2*(b^3*e^2*n^2 - 2*a*b^2*e^2*n - (3*b^3*d^2*n^2
 - 2*a*b^2*d^2*n)*x)*log(c) - 2*(3*b^3*d*e*n^3 - 2*b^3*d*e*n^2*log(c) - 2*a
*b^2*d*e*n^2)*sqrt(x))*log((d*x + e*sqrt(x))/x) - 6*(7*b^3*d*e*n^3 + 2*b^3*
d*e*n*log(c)^2 - 6*a*b^2*d*e*n^2 + 2*a^2*b*d*e*n - 2*(3*b^3*d*e*n^2 - 2*a*b
^2*d*e*n)*log(c))*sqrt(x))/(e^2*x)
```

giac [B] time = 0.65, size = 1127, normalized size = 3.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(8*(d*sqrt(x) + e)*b^3*d*n^3*log((d*sqrt(x) + e)/sqrt(x))^3/sqrt(x) - 4
*(d*sqrt(x) + e)^2*b^3*n^3*log((d*sqrt(x) + e)/sqrt(x))^3/x - 24*(d*sqrt(x)
 + e)*b^3*d*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/sqrt(x) + 24*(d*sqrt(x) + e)
*b^3*d*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/sqrt(x) + 6*(d*sqrt(x) + e
)^2*b^3*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/x - 12*(d*sqrt(x) + e)^2*b^3*n^2
*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/x + 48*(d*sqrt(x) + e)*b^3*d*n^3*log
((d*sqrt(x) + e)/sqrt(x))/sqrt(x) - 48*(d*sqrt(x) + e)*b^3*d*n^2*log(c)*log
((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 24*(d*sqrt(x) + e)*b^3*d*n*log(c)^2*log
((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 24*(d*sqrt(x) + e)*a*b^2*d*n^2*log((d*s
qrt(x) + e)/sqrt(x))^2/sqrt(x) - 6*(d*sqrt(x) + e)^2*b^3*n^3*log((d*sqrt(x)
 + e)/sqrt(x))/x + 12*(d*sqrt(x) + e)^2*b^3*n^2*log(c)*log((d*sqrt(x) + e)/
sqrt(x))/x - 12*(d*sqrt(x) + e)^2*b^3*n*log(c)^2*log((d*sqrt(x) + e)/sqrt(x
))/x - 12*(d*sqrt(x) + e)^2*a*b^2*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x - 48
*(d*sqrt(x) + e)*b^3*d*n^3/sqrt(x) + 48*(d*sqrt(x) + e)*b^3*d*n^2*log(c)/sq
rt(x) - 24*(d*sqrt(x) + e)*b^3*d*n*log(c)^2/sqrt(x) + 8*(d*sqrt(x) + e)*b^3
*d*log(c)^3/sqrt(x) - 48*(d*sqrt(x) + e)*a*b^2*d*n^2*log((d*sqrt(x) + e)/sq
rt(x))/sqrt(x) + 48*(d*sqrt(x) + e)*a*b^2*d*n*log(c)*log((d*sqrt(x) + e)/sq
rt(x))/sqrt(x) + 3*(d*sqrt(x) + e)^2*b^3*n^3/x - 6*(d*sqrt(x) + e)^2*b^3*n^
2*log(c)/x + 6*(d*sqrt(x) + e)^2*b^3*n*log(c)^2/x - 4*(d*sqrt(x) + e)^2*b^3
*log(c)^3/x + 12*(d*sqrt(x) + e)^2*a*b^2*n^2*log((d*sqrt(x) + e)/sqrt(x))/x
 - 24*(d*sqrt(x) + e)^2*a*b^2*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x + 48*
(d*sqrt(x) + e)*a*b^2*d*n^2/sqrt(x) - 48*(d*sqrt(x) + e)*a*b^2*d*n*log(c)/s
qrt(x) + 24*(d*sqrt(x) + e)*a*b^2*d*log(c)^2/sqrt(x) + 24*(d*sqrt(x) + e)*a
^2*b*d*n*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) - 6*(d*sqrt(x) + e)^2*a*b^2*n
^2/x + 12*(d*sqrt(x) + e)^2*a*b^2*n*log(c)/x - 12*(d*sqrt(x) + e)^2*a*b^2*l
og(c)^2/x - 12*(d*sqrt(x) + e)^2*a^2*b*n*log((d*sqrt(x) + e)/sqrt(x))/x - 2
4*(d*sqrt(x) + e)*a^2*b*d*n/sqrt(x) + 24*(d*sqrt(x) + e)*a^2*b*d*log(c)/sqr
t(x) + 6*(d*sqrt(x) + e)^2*a^2*b*n/x - 12*(d*sqrt(x) + e)^2*a^2*b*log(c)/x
+ 8*(d*sqrt(x) + e)*a^3*d/sqrt(x) - 4*(d*sqrt(x) + e)^2*a^3/x)*e^(-2)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{3}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/2))^n)+a)^3/x^2,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/2))^n)+a)^3/x^2,x)
```

maxima [B] time = 0.82, size = 568, normalized size = 1.99

$$\frac{3}{2}\,a^2ben\left(\frac{2\,d^2\log\left(d\sqrt{x}+e\right)}{e^3}-\frac{d^2\log(x)}{e^3}-\frac{2\,d\sqrt{x}-e}{e^2x}\right)-\frac{b^3\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^3}{x}+\frac{3}{4}\left(4\,en\left(\frac{2\,d^2\log\left(d\sqrt{x}+e\right)}{e^3}-\frac{d^2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] 3/2*a^2*b*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d*sqrt(x)
 - e)/(e^2*x)) - b^3*log(c*(d + e/sqrt(x))^n)^3/x + 3/4*(4*e*n*(2*d^2*log(d
*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d*sqrt(x) - e)/(e^2*x))*log(c*(d +
e/sqrt(x))^n) - (4*d^2*x*log(d*sqrt(x) + e)^2 + d^2*x*log(x)^2 - 6*d^2*x*lo
g(x) - 12*d*e*sqrt(x) + 2*e^2 - 4*(d^2*x*log(x) - 3*d^2*x)*log(d*sqrt(x) +
e))*n^2/(e^2*x))*a*b^2 + 1/8*(12*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*lo
g(x)/e^3 - (2*d*sqrt(x) - e)/(e^2*x))*log(c*(d + e/sqrt(x))^n)^2 + e*n*((8*
d^2*x*log(d*sqrt(x) + e)^3 - d^2*x*log(x)^3 + 9*d^2*x*log(x)^2 - 42*d^2*x*l
og(x) - 12*(d^2*x*log(x) - 3*d^2*x)*log(d*sqrt(x) + e)^2 - 84*d*e*sqrt(x) +
 6*e^2 + 6*(d^2*x*log(x)^2 - 6*d^2*x*log(x) + 14*d^2*x)*log(d*sqrt(x) + e))
*n^2/(e^3*x) - 6*(4*d^2*x*log(d*sqrt(x) + e)^2 + d^2*x*log(x)^2 - 6*d^2*x*l
og(x) - 12*d*e*sqrt(x) + 2*e^2 - 4*(d^2*x*log(x) - 3*d^2*x)*log(d*sqrt(x) +
 e))*n*log(c*(d + e/sqrt(x))^n)/(e^3*x)))*b^3 - 3*a*b^2*log(c*(d + e/sqrt(x
))^n)^2/x - 3*a^2*b*log(c*(d + e/sqrt(x))^n)/x - a^3/x
```

mupad [B] time = 0.62, size = 357, normalized size = 1.25

$$\frac{\frac{d\left(2\,a^3-3\,a^2\,b\,n+3\,a\,b^2\,n^2-\frac{3\,b^3\,n^3}{2}\right)}{e}-\frac{d\left(2\,a^3-6\,a\,b^2\,n^2+9\,b^3\,n^3\right)}{e}}{\sqrt{x}}-\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^3\left(\frac{b^3}{x}-\frac{b^3\,d^2}{e^2}\right)+\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\left(\frac{3\,b\,d\left(2\,a^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^3/x^2,x)
```

```
[Out] ((d*(2*a^3 - (3*b^3*n^3)/2 + 3*a*b^2*n^2 - 3*a^2*b*n))/e - (d*(2*a^3 + 9*b^
3*n^3 - 6*a*b^2*n^2))/e)/x^(1/2) - log(c*(d + e/x^(1/2))^n)^3*(b^3/x - (b^3
*d^2)/e^2) + log(c*(d + e/x^(1/2))^n)*(((3*b*d*(2*a^2 + b^2*n^2 - 2*a*b*n))
/e - (6*b*d*(a^2 - b^2*n^2))/e)/x^(1/2) - (3*b*(2*a^2 + b^2*n^2 - 2*a*b*n))
/(2*x)) + log(c*(d + e/x^(1/2))^n)^2*(((3*b^2*d*(2*a - b*n))/e - (6*a*b^2*d
)/e)/x^(1/2) - (3*b^2*(2*a - b*n))/(2*x) + (3*d*(2*a*b^2*d - 3*b^3*d*n))/(2
*e^2)) - (a^3 - (3*b^3*n^3)/4 + (3*a*b^2*n^2)/2 - (3*a^2*b*n)/2)/x + (log(d
 + e/x^(1/2))*(21*b^3*d^2*n^3 - 18*a*b^2*d^2*n^2 + 6*a^2*b*d^2*n))/(2*e^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**3/x**2, x)
```

3.440 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=595

$$-\frac{9b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^4}-\frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{16e^4}+\frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)}{}$$

```
[Out] 12*b^3*d^3*n^2*ln(c*(d+e/x^(1/2))^n)*(d+e/x^(1/2))/e^4-6*b*d^3*n*(a+b*ln(c*
(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))/e^4+2*d^3*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(
d+e/x^(1/2))/e^4+9/4*b^3*d^2*n^3*(d+e/x^(1/2))^2/e^4-9/2*b^2*d^2*n^2*(a+b*l
n(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^2/e^4+9/2*b*d^2*n*(a+b*ln(c*(d+e/x^(1/2
))^n))^2*(d+e/x^(1/2))^2/e^4-3*d^2*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/
2))^2/e^4-4/9*b^3*d*n^3*(d+e/x^(1/2))^3/e^4+4/3*b^2*d*n^2*(a+b*ln(c*(d+e/x^
(1/2))^n))*(d+e/x^(1/2))^3/e^4-2*b*d*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x
^(1/2))^3/e^4+2*d*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))^3/e^4+3/64*b^
3*n^3*(d+e/x^(1/2))^4/e^4-3/16*b^2*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(
1/2))^4/e^4+3/8*b*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))^4/e^4-1/2*(
a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))^4/e^4+12*a*b^2*d^3*n^2/e^3/x^(1/
2)-12*b^3*d^3*n^3/e^3/x^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 595, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$-\frac{9b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^4}-\frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{16e^4}+\frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^3,x]
```

```
[Out] (9*b^3*d^2*n^3*(d + e/Sqrt[x])^2)/(4*e^4) - (4*b^3*d*n^3*(d + e/Sqrt[x])^3)
/(9*e^4) + (3*b^3*n^3*(d + e/Sqrt[x])^4)/(64*e^4) + (12*a*b^2*d^3*n^2)/(e^3
*Sqrt[x]) - (12*b^3*d^3*n^3)/(e^3*Sqrt[x]) + (12*b^3*d^3*n^2*(d + e/Sqrt[x]
)*Log[c*(d + e/Sqrt[x])^n])/e^4 - (9*b^2*d^2*n^2*(d + e/Sqrt[x])^2*(a + b*L
og[c*(d + e/Sqrt[x])^n]))/(2*e^4) + (4*b^2*d*n^2*(d + e/Sqrt[x])^3*(a + b*L
og[c*(d + e/Sqrt[x])^n]))/(3*e^4) - (3*b^2*n^2*(d + e/Sqrt[x])^4*(a + b*Log
[c*(d + e/Sqrt[x])^n]))/(16*e^4) - (6*b*d^3*n*(d + e/Sqrt[x])*(a + b*Log[c*
(d + e/Sqrt[x])^n])^2)/e^4 + (9*b*d^2*n*(d + e/Sqrt[x])^2*(a + b*Log[c*(d +
 e/Sqrt[x])^n])^2)/(2*e^4) - (2*b*d*n*(d + e/Sqrt[x])^3*(a + b*Log[c*(d + e
/Sqrt[x])^n])^2)/e^4 + (3*b*n*(d + e/Sqrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x]
)^n])^2)/(8*e^4) + (2*d^3*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^
3)/e^4 - (3*d^2*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^4 +
 (2*d*(d + e/Sqrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^4 - ((d + e/S
qrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/(2*e^4)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
```

```
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}dx = -\left(2\,\text{Subst}\left(\int x^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3 dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\,\text{Subst}\left(\int\left(-\frac{d^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^3} + \frac{3d^2(d+ex)\left(a+b\log(c(d\right.}{e^3}\right.\right.\right.$$

$$= -\frac{2\,\text{Subst}\left(\int(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3 dx, x, \frac{1}{\sqrt{x}}\right)}{e^3} + \frac{(6d)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2\,\text{Subst}\left(\int x^3\left(a+b\log\left(cx^n\right)\right)^3 dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^4} + \frac{(6d)\,\text{Subst}\left(\int x^2\left(a+\right.\right.}{}$$

$$= \frac{2d^3\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4} - \frac{3d^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d\right.\right.\right.}{e^4}$$

$$= -\frac{6bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^4} + \frac{9bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\right.}{2e^4}$$

$$= \frac{9b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^4} - \frac{4b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4} + \frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{64e^4} + \frac{12ab^2d^3n^2}{e^3\sqrt{x}}$$

$$= \frac{9b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^4} - \frac{4b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4} + \frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{64e^4} + \frac{12ab^2d^3n^2}{e^3\sqrt{x}}$$

Mathematica [A] time = 1.10, size = 766, normalized size = 1.29

$$\frac{-288a^3e^4 - 12b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\left(72a^2e^4 + 12bd^4nx^2(25bn-12a)\log\left(d\sqrt{x}+e\right) + 6bd^4nx^2\log(x)(12a-25\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^3,x]
```

```
[Out] (-288*a^3*e^4 + 216*a^2*b*e^4*n - 108*a*b^2*e^4*n^2 + 27*b^3*e^4*n^3 - 288*
a^2*b*d*e^3*n*Sqrt[x] + 336*a*b^2*d*e^3*n^2*Sqrt[x] - 148*b^3*d*e^3*n^3*Sqr
t[x] + 432*a^2*b*d^2*e^2*n*x - 936*a*b^2*d^2*e^2*n^2*x + 690*b^3*d^2*e^2*n^
3*x - 864*a^2*b*d^3*e*n*x^(3/2) + 3600*a*b^2*d^3*e*n^2*x^(3/2) - 4980*b^3*d
^3*e*n^3*x^(3/2) - 576*b^3*d^4*n^3*x^2*Log[d + e/Sqrt[x]]^3 - 288*b^3*e^4*L
og[c*(d + e/Sqrt[x])^n]^3 + 864*a^2*b*d^4*n*x^2*Log[e + d*Sqrt[x]] - 3600*a
*b^2*d^4*n^2*x^2*Log[e + d*Sqrt[x]] + 4980*b^3*d^4*n^3*x^2*Log[e + d*Sqrt[x
]] + 72*b^2*d^4*n^2*x^2*Log[d + e/Sqrt[x]]*(-12*a + 25*b*n - 12*b*Log[c*(d
+ e/Sqrt[x])^n])*(2*Log[e + d*Sqrt[x]] - Log[x]) - 432*a^2*b*d^4*n*x^2*Log[
x] + 1800*a*b^2*d^4*n^2*x^2*Log[x] - 2490*b^3*d^4*n^3*x^2*Log[x] + 72*b^2*d
^4*n^2*x^2*Log[d + e/Sqrt[x]]^2*(12*a - 25*b*n + 12*b*Log[c*(d + e/Sqrt[x])
^n] + 12*b*n*Log[e + d*Sqrt[x]] - 6*b*n*Log[x]) + 72*b^2*Log[c*(d + e/Sqrt[
x])^n]^2*(e*(-12*a*e^3 + 3*b*e^3*n - 4*b*d*e^2*n*Sqrt[x] + 6*b*d^2*e*n*x -
12*b*d^3*n*x^(3/2)) + 12*b*d^4*n*x^2*Log[e + d*Sqrt[x]] - 6*b*d^4*n*x^2*Log
[x]) - 12*b*Log[c*(d + e/Sqrt[x])^n]*(72*a^2*e^4 + b^2*e*n^2*(9*e^3 - 28*d*
e^2*Sqrt[x] + 78*d^2*e*x - 300*d^3*x^(3/2)) - 12*a*b*e*n*(3*e^3 - 4*d*e^2*S
qrt[x] + 6*d^2*e*x - 12*d^3*x^(3/2)) + 12*b*d^4*n*(-12*a + 25*b*n)*x^2*Log[
e + d*Sqrt[x]] + 6*b*d^4*n*(12*a - 25*b*n)*x^2*Log[x]))/(576*e^4*x^2)
```

fricas [A] time = 0.46, size = 869, normalized size = 1.46

$$27\,b^3e^4n^3 - 288\,b^3e^4\log(c)^3 - 108\,ab^2e^4n^2 + 216\,a^2be^4n - 288\,a^3e^4 + 288\left(b^3d^4n^3x^2 - b^3e^4n^3\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)^3 + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/576*(27*b^3*e^4*n^3 - 288*b^3*e^4*log(c)^3 - 108*a*b^2*e^4*n^2 + 216*a^2*
b*e^4*n - 288*a^3*e^4 + 288*(b^3*d^4*n^3*x^2 - b^3*e^4*n^3)*log((d*x + e*sq
rt(x))/x)^3 + 216*(2*b^3*d^2*e^2*n*x + b^3*e^4*n - 4*a*b^2*e^4)*log(c)^2 +
72*(6*b^3*d^2*e^2*n^3*x + 3*b^3*e^4*n^3 - 12*a*b^2*e^4*n^2 - (25*b^3*d^4*n^
3 - 12*a*b^2*d^4*n^2)*x^2 + 12*(b^3*d^4*n^2*x^2 - b^3*e^4*n^2)*log(c) - 4*(
3*b^3*d^3*e*n^3*x + b^3*d*e^3*n^3)*sqrt(x))*log((d*x + e*sqrt(x))/x)^2 + 6*
(115*b^3*d^2*e^2*n^3 - 156*a*b^2*d^2*e^2*n^2 + 72*a^2*b*d^2*e^2*n)*x - 36*(
3*b^3*e^4*n^2 - 12*a*b^2*e^4*n + 24*a^2*b*e^4 + 2*(13*b^3*d^2*e^2*n^2 - 12*
a*b^2*d^2*e^2*n)*x)*log(c) - 12*(9*b^3*e^4*n^3 - 36*a*b^2*e^4*n^2 + 72*a^2*
b*e^4*n - (415*b^3*d^4*n^3 - 300*a*b^2*d^4*n^2 + 72*a^2*b*d^4*n)*x^2 - 72*(
b^3*d^4*n*x^2 - b^3*e^4*n)*log(c)^2 + 6*(13*b^3*d^2*e^2*n^3 - 12*a*b^2*d^2*
e^2*n^2)*x - 12*(6*b^3*d^2*e^2*n^2*x + 3*b^3*e^4*n^2 - 12*a*b^2*e^4*n - (25
*b^3*d^4*n^2 - 12*a*b^2*d^4*n)*x^2)*log(c) - 4*(7*b^3*d*e^3*n^3 - 12*a*b^2*
d*e^3*n^2 + 3*(25*b^3*d^3*e*n^3 - 12*a*b^2*d^3*e*n^2)*x - 12*(3*b^3*d^3*e*n
^2*x + b^3*d*e^3*n^2)*log(c))*sqrt(x))*log((d*x + e*sqrt(x))/x) - 4*(37*b^3
*d*e^3*n^3 - 84*a*b^2*d*e^3*n^2 + 72*a^2*b*d*e^3*n + 72*(3*b^3*d^3*e*n*x +
b^3*d*e^3*n)*log(c)^2 + 3*(415*b^3*d^3*e*n^3 - 300*a*b^2*d^3*e*n^2 + 72*a^2
*b*d^3*e*n)*x - 12*(7*b^3*d*e^3*n^2 - 12*a*b^2*d*e^3*n + 3*(25*b^3*d^3*e*n^
2 - 12*a*b^2*d^3*e*n)*x)*log(c))*sqrt(x))/(e^4*x^2)
```

giac [B] time = 0.72, size = 2389, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] 1/576*(1152*(d*sqrt(x) + e)*b^3*d^3*n^3*log((d*sqrt(x) + e)/sqrt(x))^3/sqrt
(x) - 1728*(d*sqrt(x) + e)^2*b^3*d^2*n^3*log((d*sqrt(x) + e)/sqrt(x))^3/x -
 3456*(d*sqrt(x) + e)*b^3*d^3*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/sqrt(x) +
3456*(d*sqrt(x) + e)*b^3*d^3*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/sqrt
(x) + 1152*(d*sqrt(x) + e)^3*b^3*d*n^3*log((d*sqrt(x) + e)/sqrt(x))^3/x^(3/
2) + 2592*(d*sqrt(x) + e)^2*b^3*d^2*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/x -
5184*(d*sqrt(x) + e)^2*b^3*d^2*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/x
- 288*(d*sqrt(x) + e)^4*b^3*n^3*log((d*sqrt(x) + e)/sqrt(x))^3/x^2 + 6912*(
d*sqrt(x) + e)*b^3*d^3*n^3*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) - 6912*(d*s
qrt(x) + e)*b^3*d^3*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 3456*
(d*sqrt(x) + e)*b^3*d^3*n*log(c)^2*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) - 1
152*(d*sqrt(x) + e)^3*b^3*d*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/x^(3/2) + 34
56*(d*sqrt(x) + e)*a*b^2*d^3*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/sqrt(x) + 3
456*(d*sqrt(x) + e)^3*b^3*d*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/x^(3/
2) - 2592*(d*sqrt(x) + e)^2*b^3*d^2*n^3*log((d*sqrt(x) + e)/sqrt(x))/x + 51
84*(d*sqrt(x) + e)^2*b^3*d^2*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x - 51
84*(d*sqrt(x) + e)^2*b^3*d^2*n*log(c)^2*log((d*sqrt(x) + e)/sqrt(x))/x + 21
6*(d*sqrt(x) + e)^4*b^3*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/x^2 - 5184*(d*sq
rt(x) + e)^2*a*b^2*d^2*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x - 864*(d*sqrt(x
) + e)^4*b^3*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/x^2 - 6912*(d*sqrt(x
) + e)*b^3*d^3*n^3/sqrt(x) + 6912*(d*sqrt(x) + e)*b^3*d^3*n^2*log(c)/sqrt(x
) - 3456*(d*sqrt(x) + e)*b^3*d^3*n*log(c)^2/sqrt(x) + 1152*(d*sqrt(x) + e)*
b^3*d^3*log(c)^3/sqrt(x) + 768*(d*sqrt(x) + e)^3*b^3*d*n^3*log((d*sqrt(x) +
```

```
e)/sqrt(x))/x^(3/2) - 6912*(d*sqrt(x) + e)*a*b^2*d^3*n^2*log((d*sqrt(x) +
e)/sqrt(x))/sqrt(x) - 2304*(d*sqrt(x) + e)^3*b^3*d*n^2*log(c)*log((d*sqrt(x
) + e)/sqrt(x))/x^(3/2) + 6912*(d*sqrt(x) + e)*a*b^2*d^3*n*log(c)*log((d*sq
rt(x) + e)/sqrt(x))/sqrt(x) + 3456*(d*sqrt(x) + e)^3*b^3*d*n*log(c)^2*log((
d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 3456*(d*sqrt(x) + e)^3*a*b^2*d*n^2*log((d
*sqrt(x) + e)/sqrt(x))^2/x^(3/2) + 1296*(d*sqrt(x) + e)^2*b^3*d^2*n^3/x - 2
592*(d*sqrt(x) + e)^2*b^3*d^2*n^2*log(c)/x + 2592*(d*sqrt(x) + e)^2*b^3*d^2
*n*log(c)^2/x - 1728*(d*sqrt(x) + e)^2*b^3*d^2*log(c)^3/x - 108*(d*sqrt(x)
+ e)^4*b^3*n^3*log((d*sqrt(x) + e)/sqrt(x))/x^2 + 5184*(d*sqrt(x) + e)^2*a*
b^2*d^2*n^2*log((d*sqrt(x) + e)/sqrt(x))/x + 432*(d*sqrt(x) + e)^4*b^3*n^2*
log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 10368*(d*sqrt(x) + e)^2*a*b^2*d^2
*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x - 864*(d*sqrt(x) + e)^4*b^3*n*log(
c)^2*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 864*(d*sqrt(x) + e)^4*a*b^2*n^2*log
((d*sqrt(x) + e)/sqrt(x))^2/x^2 - 256*(d*sqrt(x) + e)^3*b^3*d*n^3/x^(3/2) +
 6912*(d*sqrt(x) + e)*a*b^2*d^3*n^2/sqrt(x) + 768*(d*sqrt(x) + e)^3*b^3*d*n
^2*log(c)/x^(3/2) - 6912*(d*sqrt(x) + e)*a*b^2*d^3*n*log(c)/sqrt(x) - 1152*
(d*sqrt(x) + e)^3*b^3*d*n*log(c)^2/x^(3/2) + 3456*(d*sqrt(x) + e)*a*b^2*d^3
*log(c)^2/sqrt(x) + 1152*(d*sqrt(x) + e)^3*b^3*d*log(c)^3/x^(3/2) - 2304*(d
*sqrt(x) + e)^3*a*b^2*d*n^2*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 3456*(d*
sqrt(x) + e)*a^2*b*d^3*n*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 6912*(d*sqr
t(x) + e)^3*a*b^2*d*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 27*(d*s
qrt(x) + e)^4*b^3*n^3/x^2 - 2592*(d*sqrt(x) + e)^2*a*b^2*d^2*n^2/x - 108*(d
*sqrt(x) + e)^4*b^3*n^2*log(c)/x^2 + 5184*(d*sqrt(x) + e)^2*a*b^2*d^2*n*log
(c)/x + 216*(d*sqrt(x) + e)^4*b^3*n*log(c)^2/x^2 - 5184*(d*sqrt(x) + e)^2*a
*b^2*d^2*log(c)^2/x - 288*(d*sqrt(x) + e)^4*b^3*log(c)^3/x^2 + 432*(d*sqrt(
x) + e)^4*a*b^2*n^2*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 5184*(d*sqrt(x) + e)
^2*a^2*b*d^2*n*log((d*sqrt(x) + e)/sqrt(x))/x - 1728*(d*sqrt(x) + e)^4*a*b^
2*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^2 + 768*(d*sqrt(x) + e)^3*a*b^2*d
*n^2/x^(3/2) - 3456*(d*sqrt(x) + e)*a^2*b*d^3*n/sqrt(x) - 2304*(d*sqrt(x) +
 e)^3*a*b^2*d*n*log(c)/x^(3/2) + 3456*(d*sqrt(x) + e)*a^2*b*d^3*log(c)/sqrt
(x) + 3456*(d*sqrt(x) + e)^3*a*b^2*d*log(c)^2/x^(3/2) + 3456*(d*sqrt(x) + e
)^3*a^2*b*d*n*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) - 108*(d*sqrt(x) + e)^4*
a*b^2*n^2/x^2 + 2592*(d*sqrt(x) + e)^2*a^2*b*d^2*n/x + 432*(d*sqrt(x) + e)^
4*a*b^2*n*log(c)/x^2 - 5184*(d*sqrt(x) + e)^2*a^2*b*d^2*log(c)/x - 864*(d*s
qrt(x) + e)^4*a*b^2*log(c)^2/x^2 - 864*(d*sqrt(x) + e)^4*a^2*b*n*log((d*sqr
t(x) + e)/sqrt(x))/x^2 - 1152*(d*sqrt(x) + e)^3*a^2*b*d*n/x^(3/2) + 1152*(d
*sqrt(x) + e)*a^3*d^3/sqrt(x) + 3456*(d*sqrt(x) + e)^3*a^2*b*d*log(c)/x^(3/
2) + 216*(d*sqrt(x) + e)^4*a^2*b*n/x^2 - 1728*(d*sqrt(x) + e)^2*a^3*d^2/x -
 864*(d*sqrt(x) + e)^4*a^2*b*log(c)/x^2 + 1152*(d*sqrt(x) + e)^3*a^3*d/x^(3
/2) - 288*(d*sqrt(x) + e)^4*a^3/x^2)*e^(-4)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{3}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/2))^n)+a)^3/x^3,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/2))^n)+a)^3/x^3,x)
```

maxima [A] time = 0.89, size = 732, normalized size = 1.23

$$\frac{1}{8}\,a^{2}ben\left(\frac{12\,d^{4}\log\left(d\sqrt{x}+e\right)}{e^{5}}-\frac{6\,d^{4}\log(x)}{e^{5}}-\frac{12\,d^{3}x^{\frac{3}{2}}-6\,d^{2}ex+4\,de^{2}\sqrt{x}-3\,e^{3}}{e^{4}x^{2}}\right)+\frac{1}{48}\left(12\,en\left(\frac{12\,d^{4}\log\left(d\sqrt{x}\right.}{e^{5}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*b*e*n*(12*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x
^(3/2) - 6*d^2*e*x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2)) + 1/48*(12*e*n*(12
*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^(3/2) - 6*d^2*e*
x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2))*log(c*(d + e/sqrt(x))^n) - (72*d^4*
x^2*log(d*sqrt(x) + e)^2 + 18*d^4*x^2*log(x)^2 - 150*d^4*x^2*log(x) - 300*d
^3*e*x^(3/2) + 78*d^2*e^2*x - 28*d*e^3*sqrt(x) + 9*e^4 - 12*(6*d^4*x^2*log(
x) - 25*d^4*x^2)*log(d*sqrt(x) + e))*n^2/(e^4*x^2))*a*b^2 + 1/576*(72*e*n*(
12*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^(3/2) - 6*d^2*
e*x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2))*log(c*(d + e/sqrt(x))^n)^2 + e*n*
((288*d^4*x^2*log(d*sqrt(x) + e)^3 - 36*d^4*x^2*log(x)^3 + 450*d^4*x^2*log(
x)^2 - 2490*d^4*x^2*log(x) - 4980*d^3*e*x^(3/2) + 690*d^2*e^2*x - 148*d*e^3
*sqrt(x) + 27*e^4 - 72*(6*d^4*x^2*log(x) - 25*d^4*x^2)*log(d*sqrt(x) + e)^2
 + 12*(18*d^4*x^2*log(x)^2 - 150*d^4*x^2*log(x) + 415*d^4*x^2)*log(d*sqrt(x
) + e))*n^2/(e^5*x^2) - 12*(72*d^4*x^2*log(d*sqrt(x) + e)^2 + 18*d^4*x^2*lo
g(x)^2 - 150*d^4*x^2*log(x) - 300*d^3*e*x^(3/2) + 78*d^2*e^2*x - 28*d*e^3*s
qrt(x) + 9*e^4 - 12*(6*d^4*x^2*log(x) - 25*d^4*x^2)*log(d*sqrt(x) + e))*n*l
og(c*(d + e/sqrt(x))^n)/(e^5*x^2)))*b^3 - 1/2*b^3*log(c*(d + e/sqrt(x))^n)^
3/x^2 - 3/2*a*b^2*log(c*(d + e/sqrt(x))^n)^2/x^2 - 3/2*a^2*b*log(c*(d + e/s
qrt(x))^n)/x^2 - 1/2*a^3/x^2
```

mupad [B] time = 0.80, size = 846, normalized size = 1.42

$$\frac{\frac{d\left(2a^3-\frac{3a^2bn}{2}+\frac{3ab^2n^2}{4}-\frac{3b^3n^3}{16}\right)}{3e}-\frac{d\left(24a^3-12ab^2n^2+7b^3n^3\right)}{36e}}{x^{3/2}}-\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^3\left(\frac{b^3}{2x^2}-\frac{b^3d^4}{2e^4}\right)+\frac{d\left(\frac{d\left(\frac{d\left(2a^3-\frac{3a^2bn}{2}+\frac{3ab^2n^2}{4}-\frac{3b^3}{16}\right.}{e}\right.}{e}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^3/x^3,x)
```

```
[Out] ((d*(2*a^3 - (3*b^3*n^3)/16 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/2))/(3*e) - (d*
(24*a^3 + 7*b^3*n^3 - 12*a*b^2*n^2))/(36*e))/x^(3/2) - log(c*(d + e/x^(1/2)
)^n)^3*(b^3/(2*x^2) - (b^3*d^4)/(2*e^4)) + ((d*((d*((d*(2*a^3 - (3*b^3*n^3)
/16 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/2))/e - (d*(24*a^3 + 7*b^3*n^3 - 12*a*b
^2*n^2))/(12*e)))/e + (b^2*d^2*n^2*(12*a - 13*b*n))/(8*e^2)))/e + (b^2*d^3*
n^2*(12*a - 25*b*n))/(4*e^3))/x^(1/2) + log(c*(d + e/x^(1/2))^n)^2*(((b^2*d
*(4*a - b*n))/e - (4*a*b^2*d)/e)/(2*x^(3/2)) - (3*b^2*(4*a - b*n))/(8*x^2)
+ (d*(12*a*b^2*d^3 - 25*b^3*d^3*n))/(8*e^4) - (d*((6*b^2*d*(4*a - b*n))/e -
 (24*a*b^2*d)/e))/(8*e*x) + (d^2*((6*b^2*d*(4*a - b*n))/e - (24*a*b^2*d)/e)
)/(4*e^2*x^(1/2))) - ((d*((d*(2*a^3 - (3*b^3*n^3)/16 + (3*a*b^2*n^2)/4 - (3
*a^2*b*n)/2))/e - (d*(24*a^3 + 7*b^3*n^3 - 12*a*b^2*n^2))/(12*e)))/(2*e) +
(b^2*d^2*n^2*(12*a - 13*b*n))/(16*e^2))/x - (a^3/2 - (3*b^3*n^3)/64 + (3*a*
b^2*n^2)/16 - (3*a^2*b*n)/8)/x^2 - (log(c*(d + e/x^(1/2))^n)*((16*b*d*e^3*(
6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*a*b*n))/(12*e^2*x^(3/2))
 + ((d*((d*(16*b*d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*
a*b*n)))/e - 24*b^3*d^2*e^2*n^2))/e - 48*b^3*d^3*e*n^2)/(4*e^2*x^(1/2)) - (
(d*(16*b*d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*a*b*n)))
/e - 24*b^3*d^2*e^2*n^2)/(8*e^2*x) + (3*b*e^2*(8*a^2 + b^2*n^2 - 4*a*b*n))/
(4*x^2)))/(4*e^2) + (log(d + e/x^(1/2))*(415*b^3*d^4*n^3 - 300*a*b^2*d^4*n^
2 + 72*a^2*b*d^4*n))/(48*e^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x**3,x)
```

```
[Out] Timed out
```

3.441 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx$

Optimal. Leaf size=907

$$\frac{b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^6}{108e^6}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\left(d+\frac{e}{\sqrt{x}}\right)^6}{3e^6}+\frac{bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\left(d+\frac{e}{\sqrt{x}}\right)^6}{6e^6}-\frac{b^2n^2\left(a+b\log\right.}{}$$

```
[Out] -1/3*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))^6/e^6+12*b^3*d^5*n^2*ln(c*
(d+e/x^(1/2))^n)*(d+e/x^(1/2))/e^6-6*b*d^5*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*
(d+e/x^(1/2))/e^6-15/2*b^2*d^4*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2)
)^2/e^6+15/2*b*d^4*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))^2/e^6+40/9
*b^2*d^3*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^3/e^6-20/3*b*d^3*n*(
a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))^3/e^6-15/8*b^2*d^2*n^2*(a+b*ln(c
*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^4/e^6+15/4*b*d^2*n*(a+b*ln(c*(d+e/x^(1/2))
^n))^2*(d+e/x^(1/2))^4/e^6+12/25*b^2*d*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e
/x^(1/2))^5/e^6-6/5*b*d*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))^5/e^6
+12*a*b^2*d^5*n^2/e^5/x^(1/2)+2*d^5*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1
/2))/e^6-5*d^4*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))^2/e^6+20/3*d^3*(
a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))^3/e^6-5*d^2*(a+b*ln(c*(d+e/x^(1/
2))^n))^3*(d+e/x^(1/2))^4/e^6+2*d*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2
))^5/e^6+1/108*b^3*n^3*(d+e/x^(1/2))^6/e^6+15/4*b^3*d^4*n^3*(d+e/x^(1/2))^2
/e^6-40/27*b^3*d^3*n^3*(d+e/x^(1/2))^3/e^6+15/32*b^3*d^2*n^3*(d+e/x^(1/2))^
4/e^6-12/125*b^3*d*n^3*(d+e/x^(1/2))^5/e^6-1/18*b^2*n^2*(a+b*ln(c*(d+e/x^(1
/2))^n))*(d+e/x^(1/2))^6/e^6+1/6*b*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(
1/2))^6/e^6-12*b^3*d^5*n^3/e^5/x^(1/2)
```

Rubi [A] time = 1.01, antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^6}{108e^6}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\left(d+\frac{e}{\sqrt{x}}\right)^6}{3e^6}+\frac{bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\left(d+\frac{e}{\sqrt{x}}\right)^6}{6e^6}-\frac{b^2n^2\left(a+b\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^4,x]
```

```
[Out] (15*b^3*d^4*n^3*(d + e/Sqrt[x])^2)/(4*e^6) - (40*b^3*d^3*n^3*(d + e/Sqrt[x]
)^3)/(27*e^6) + (15*b^3*d^2*n^3*(d + e/Sqrt[x])^4)/(32*e^6) - (12*b^3*d*n^3
*(d + e/Sqrt[x])^5)/(125*e^6) + (b^3*n^3*(d + e/Sqrt[x])^6)/(108*e^6) + (12
*a*b^2*d^5*n^2)/(e^5*Sqrt[x]) - (12*b^3*d^5*n^3)/(e^5*Sqrt[x]) + (12*b^3*d^
5*n^2*(d + e/Sqrt[x])*Log[c*(d + e/Sqrt[x])^n])/e^6 - (15*b^2*d^4*n^2*(d +
e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2*e^6) + (40*b^2*d^3*n^2*(d
 + e/Sqrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(9*e^6) - (15*b^2*d^2*n^2
*(d + e/Sqrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(8*e^6) + (12*b^2*d*n^
2*(d + e/Sqrt[x])^5*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(25*e^6) - (b^2*n^2*(
d + e/Sqrt[x])^6*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(18*e^6) - (6*b*d^5*n*(d
 + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/e^6 + (15*b*d^4*n*(d + e/
Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*e^6) - (20*b*d^3*n*(d + e
/Sqrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(3*e^6) + (15*b*d^2*n*(d +
e/Sqrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(4*e^6) - (6*b*d*n*(d + e/
Sqrt[x])^5*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(5*e^6) + (b*n*(d + e/Sqrt[x
])^6*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(6*e^6) + (2*d^5*(d + e/Sqrt[x])*(
a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^6 - (5*d^4*(d + e/Sqrt[x])^2*(a + b*Lo
```

```
g[c*(d + e/Sqrt[x])^n])^3)/e^6 + (20*d^3*(d + e/Sqrt[x])^3*(a + b*Log[c*(d
+ e/Sqrt[x])^n])^3)/(3*e^6) - (5*d^2*(d + e/Sqrt[x])^4*(a + b*Log[c*(d + e/
Sqrt[x])^n])^3)/e^6 + (2*d*(d + e/Sqrt[x])^5*(a + b*Log[c*(d + e/Sqrt[x])^n
])^3)/e^6 - ((d + e/Sqrt[x])^6*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/(3*e^6)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx = -\left(2\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\,\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^5} + \frac{5d^4(d+ex)\left(a+b\log\left(c(d+\right.\right.}{e^5}\right.\right.\right.$$

$$= -\frac{2\,\text{Subst}\left(\int (d+ex)^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^5} + \frac{(10d)\,\text{Subst}\left(\int (\right.}{}$$

$$= -\frac{2\,\text{Subst}\left(\int x^5\left(a+b\log\left(cx^n\right)\right)^3\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} + \frac{(10d)\,\text{Subst}\left(\int x^4\left(a+b\,l\right.\right.}{e}$$

$$= \frac{2d^5\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6} - \frac{5d^4\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\right.\right.\right.}{e^6}$$

$$= -\frac{6bd^5n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^6} + \frac{15bd^4n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\right.}{2e^6}$$

$$= \frac{15b^3d^4n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^6} - \frac{40b^3d^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6} + \frac{15b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{32e^6} - \frac{12b^3dn}{}$$

$$= \frac{15b^3d^4n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^6} - \frac{40b^3d^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6} + \frac{15b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{32e^6} - \frac{12b^3dn}{}$$

Mathematica [A] time = 1.79, size = 950, normalized size = 1.05

$$\frac{-72000b^3n^3x^3\log^3\left(d+\frac{e}{\sqrt{x}}\right)d^6 + 809340b^3n^3x^3\log\left(\sqrt{x}\,d+e\right)d^6 - 529200ab^2n^2x^3\log\left(\sqrt{x}\,d+e\right)d^6 + 108000}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^4,x]

[Out] (-36000*a^3*e^6 + 18000*a^2*b*e^6*n - 6000*a*b^2*e^6*n^2 + 1000*b^3*e^6*n^3
 - 21600*a^2*b*d*e^5*n*Sqrt[x] + 15840*a*b^2*d*e^5*n^2*Sqrt[x] - 4368*b^3*d
*e^5*n^3*Sqrt[x] + 27000*a^2*b*d^2*e^4*n*x - 33300*a*b^2*d^2*e^4*n^2*x + 13
785*b^3*d^2*e^4*n^3*x - 36000*a^2*b*d^3*e^3*n*x^(3/2) + 68400*a*b^2*d^3*e^3
*n^2*x^(3/2) - 41180*b^3*d^3*e^3*n^3*x^(3/2) + 54000*a^2*b*d^4*e^2*n*x^2 -
156600*a*b^2*d^4*e^2*n^2*x^2 + 140070*b^3*d^4*e^2*n^3*x^2 - 108000*a^2*b*d^
5*e*n*x^(5/2) + 529200*a*b^2*d^5*e*n^2*x^(5/2) - 809340*b^3*d^5*e*n^3*x^(5/
2) - 72000*b^3*d^6*n^3*x^3*Log[d + e/Sqrt[x]]^3 - 36000*b^3*e^6*Log[c*(d +
e/Sqrt[x])^n]^3 + 108000*a^2*b*d^6*n*x^3*Log[e + d*Sqrt[x]] - 529200*a*b^2*
d^6*n^2*x^3*Log[e + d*Sqrt[x]] + 809340*b^3*d^6*n^3*x^3*Log[e + d*Sqrt[x]]
+ 5400*b^2*d^6*n^2*x^3*Log[d + e/Sqrt[x]]*(-20*a + 49*b*n - 20*b*Log[c*(d +
 e/Sqrt[x])^n])*(2*Log[e + d*Sqrt[x]] - Log[x]) - 54000*a^2*b*d^6*n*x^3*Log
[x] + 264600*a*b^2*d^6*n^2*x^3*Log[x] - 404670*b^3*d^6*n^3*x^3*Log[x] + 540
0*b^2*d^6*n^2*x^3*Log[d + e/Sqrt[x]]^2*(20*a - 49*b*n + 20*b*Log[c*(d + e/S
qrt[x])^n] + 20*b*n*Log[e + d*Sqrt[x]] - 10*b*n*Log[x]) + 1800*b^2*Log[c*(d
 + e/Sqrt[x])^n]^2*(e*(-60*a*e^5 + 10*b*e^5*n - 12*b*d*e^4*n*Sqrt[x] + 15*b
*d^2*e^3*n*x - 20*b*d^3*e^2*n*x^(3/2) + 30*b*d^4*e*n*x^2 - 60*b*d^5*n*x^(5/
2)) + 60*b*d^6*n*x^3*Log[e + d*Sqrt[x]] - 30*b*d^6*n*x^3*Log[x]) - 60*b*Log
[c*(d + e/Sqrt[x])^n]*(1800*a^2*e^6 + b^2*e*n^2*(100*e^5 - 264*d*e^4*Sqrt[x
```

```
] + 555*d^2*e^3*x - 1140*d^3*e^2*x^(3/2) + 2610*d^4*e*x^2 - 8820*d^5*x^(5/2
)) - 60*a*b*e*n*(10*e^5 - 12*d*e^4*Sqrt[x] + 15*d^2*e^3*x - 20*d^3*e^2*x^(3
/2) + 30*d^4*e*x^2 - 60*d^5*x^(5/2)) + 180*b*d^6*n*(-20*a + 49*b*n)*x^3*Log
[e + d*Sqrt[x]] + 90*b*d^6*n*(20*a - 49*b*n)*x^3*Log[x]))/(108000*e^6*x^3)
```

fricas [A] time = 0.48, size = 1203, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/108000*(1000*b^3*e^6*n^3 - 36000*b^3*e^6*log(c)^3 - 6000*a*b^2*e^6*n^2 +
18000*a^2*b*e^6*n - 36000*a^3*e^6 + 36000*(b^3*d^6*n^3*x^3 - b^3*e^6*n^3)*l
og((d*x + e*sqrt(x))/x)^3 + 30*(4669*b^3*d^4*e^2*n^3 - 5220*a*b^2*d^4*e^2*n
^2 + 1800*a^2*b*d^4*e^2*n)*x^2 + 9000*(6*b^3*d^4*e^2*n*x^2 + 3*b^3*d^2*e^4*
n*x + 2*b^3*e^6*n - 12*a*b^2*e^6)*log(c)^2 + 1800*(30*b^3*d^4*e^2*n^3*x^2 +
 15*b^3*d^2*e^4*n^3*x + 10*b^3*e^6*n^3 - 60*a*b^2*e^6*n^2 - 3*(49*b^3*d^6*n
^3 - 20*a*b^2*d^6*n^2)*x^3 + 60*(b^3*d^6*n^2*x^3 - b^3*e^6*n^2)*log(c) - 4*
(15*b^3*d^5*e*n^3*x^2 + 5*b^3*d^3*e^3*n^3*x + 3*b^3*d*e^5*n^3)*sqrt(x))*log
((d*x + e*sqrt(x))/x)^2 + 15*(919*b^3*d^2*e^4*n^3 - 2220*a*b^2*d^2*e^4*n^2
+ 1800*a^2*b*d^2*e^4*n)*x - 300*(20*b^3*e^6*n^2 - 120*a*b^2*e^6*n + 360*a^2
*b*e^6 + 18*(29*b^3*d^4*e^2*n^2 - 20*a*b^2*d^4*e^2*n)*x^2 + 3*(37*b^3*d^2*e
^4*n^2 - 60*a*b^2*d^2*e^4*n)*x)*log(c) - 60*(100*b^3*e^6*n^3 - 600*a*b^2*e^
6*n^2 + 1800*a^2*b*e^6*n - (13489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a
^2*b*d^6*n)*x^3 + 90*(29*b^3*d^4*e^2*n^3 - 20*a*b^2*d^4*e^2*n^2)*x^2 - 1800
*(b^3*d^6*n*x^3 - b^3*e^6*n)*log(c)^2 + 15*(37*b^3*d^2*e^4*n^3 - 60*a*b^2*d
^2*e^4*n^2)*x - 60*(30*b^3*d^4*e^2*n^2*x^2 + 15*b^3*d^2*e^4*n^2*x + 10*b^3*
e^6*n^2 - 60*a*b^2*e^6*n - 3*(49*b^3*d^6*n^2 - 20*a*b^2*d^6*n)*x^3)*log(c)
- 12*(22*b^3*d*e^5*n^3 - 60*a*b^2*d*e^5*n^2 + 15*(49*b^3*d^5*e*n^3 - 20*a*b
^2*d^5*e*n^2)*x^2 + 5*(19*b^3*d^3*e^3*n^3 - 20*a*b^2*d^3*e^3*n^2)*x - 20*(1
5*b^3*d^5*e*n^2*x^2 + 5*b^3*d^3*e^3*n^2*x + 3*b^3*d*e^5*n^2)*log(c))*sqrt(x
))*log((d*x + e*sqrt(x))/x) - 4*(1092*b^3*d*e^5*n^3 - 3960*a*b^2*d*e^5*n^2
+ 5400*a^2*b*d*e^5*n + 15*(13489*b^3*d^5*e*n^3 - 8820*a*b^2*d^5*e*n^2 + 180
0*a^2*b*d^5*e*n)*x^2 + 1800*(15*b^3*d^5*e*n*x^2 + 5*b^3*d^3*e^3*n*x + 3*b^3
*d*e^5*n)*log(c)^2 + 5*(2059*b^3*d^3*e^3*n^3 - 3420*a*b^2*d^3*e^3*n^2 + 180
0*a^2*b*d^3*e^3*n)*x - 180*(22*b^3*d*e^5*n^2 - 60*a*b^2*d*e^5*n + 15*(49*b^
3*d^5*e*n^2 - 20*a*b^2*d^5*e*n)*x^2 + 5*(19*b^3*d^3*e^3*n^2 - 20*a*b^2*d^3*
e^3*n)*x)*log(c))*sqrt(x))/(e^6*x^3)
```

giac [B] time = 0.86, size = 3651, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^4,x, algorithm="giac")
```

```
[Out] 1/108000*(216000*(d*sqrt(x) + e)*b^3*d^5*n^3*log((d*sqrt(x) + e)/sqrt(x))^3
/sqrt(x) - 540000*(d*sqrt(x) + e)^2*b^3*d^4*n^3*log((d*sqrt(x) + e)/sqrt(x)
)^3/x - 648000*(d*sqrt(x) + e)*b^3*d^5*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/s
qrt(x) + 648000*(d*sqrt(x) + e)*b^3*d^5*n^2*log(c)*log((d*sqrt(x) + e)/sqrt
(x))^2/sqrt(x) + 720000*(d*sqrt(x) + e)^3*b^3*d^3*n^3*log((d*sqrt(x) + e)/s
qrt(x))^3/x^(3/2) + 810000*(d*sqrt(x) + e)^2*b^3*d^4*n^3*log((d*sqrt(x) + e
)/sqrt(x))^2/x - 1620000*(d*sqrt(x) + e)^2*b^3*d^4*n^2*log(c)*log((d*sqrt(x
) + e)/sqrt(x))^2/x - 540000*(d*sqrt(x) + e)^4*b^3*d^2*n^3*log((d*sqrt(x) +
 e)/sqrt(x))^3/x^2 + 1296000*(d*sqrt(x) + e)*b^3*d^5*n^3*log((d*sqrt(x) + e
)/sqrt(x))/sqrt(x) - 1296000*(d*sqrt(x) + e)*b^3*d^5*n^2*log(c)*log((d*sqrt
(x) + e)/sqrt(x))/sqrt(x) + 648000*(d*sqrt(x) + e)*b^3*d^5*n*log(c)^2*log((
d*sqrt(x) + e)/sqrt(x))/sqrt(x) - 720000*(d*sqrt(x) + e)^3*b^3*d^3*n^3*log(
(d*sqrt(x) + e)/sqrt(x))^2/x^(3/2) + 648000*(d*sqrt(x) + e)*a*b^2*d^5*n^2*l
```

og((d*sqrt(x) + e)/sqrt(x))^2/sqrt(x) + 2160000*(d*sqrt(x) + e)^3*b^3*d^3*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/x^(3/2) + 216000*(d*sqrt(x) + e)^5*b^3*d*n^3*log((d*sqrt(x) + e)/sqrt(x))^3/x^(5/2) - 810000*(d*sqrt(x) + e)^2*b^3*d^4*n^3*log((d*sqrt(x) + e)/sqrt(x))/x + 1620000*(d*sqrt(x) + e)^2*b^3*d^4*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x - 1620000*(d*sqrt(x) + e)^2*b^3*d^4*n*log(c)^2*log((d*sqrt(x) + e)/sqrt(x))/x + 405000*(d*sqrt(x) + e)^4*b^3*d^2*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/x^2 - 1620000*(d*sqrt(x) + e)^2*a*b^2*d^4*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x - 1620000*(d*sqrt(x) + e)^4*b^3*d^2*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/x^2 - 36000*(d*sqrt(x) + e)^6*b^3*n^3*log((d*sqrt(x) + e)/sqrt(x))^3/x^3 - 1296000*(d*sqrt(x) + e)*b^3*d^5*n^3/sqrt(x) + 1296000*(d*sqrt(x) + e)*b^3*d^5*n^2*log(c)/sqrt(x) - 648000*(d*sqrt(x) + e)*b^3*d^5*n*log(c)^2/sqrt(x) + 216000*(d*sqrt(x) + e)*b^3*d^5*log(c)^3/sqrt(x) + 480000*(d*sqrt(x) + e)^3*b^3*d^3*n^3*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) - 1296000*(d*sqrt(x) + e)*a*b^2*d^5*n^2*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) - 1440000*(d*sqrt(x) + e)^3*b^3*d^3*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 1296000*(d*sqrt(x) + e)*a*b^2*d^5*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 2160000*(d*sqrt(x) + e)^3*b^3*d^3*n*log(c)^2*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) - 129600*(d*sqrt(x) + e)^5*b^3*d*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/x^(5/2) + 2160000*(d*sqrt(x) + e)^3*a*b^2*d^3*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x^(3/2) + 648000*(d*sqrt(x) + e)^5*b^3*d*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/x^(5/2) + 405000*(d*sqrt(x) + e)^2*b^3*d^4*n^3/x - 810000*(d*sqrt(x) + e)^2*b^3*d^4*n^2*log(c)/x + 810000*(d*sqrt(x) + e)^2*b^3*d^4*n*log(c)^2/x - 540000*(d*sqrt(x) + e)^2*b^3*d^4*log(c)^3/x - 202500*(d*sqrt(x) + e)^4*b^3*d^2*n^3*log((d*sqrt(x) + e)/sqrt(x))/x^2 + 1620000*(d*sqrt(x) + e)^2*a*b^2*d^4*n^2*log((d*sqrt(x) + e)/sqrt(x))/x + 810000*(d*sqrt(x) + e)^4*b^3*d^2*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 3240000*(d*sqrt(x) + e)^2*a*b^2*d^4*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x - 1620000*(d*sqrt(x) + e)^4*b^3*d^2*n*log(c)^2*log((d*sqrt(x) + e)/sqrt(x))/x^2 + 18000*(d*sqrt(x) + e)^6*b^3*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/x^3 - 1620000*(d*sqrt(x) + e)^4*a*b^2*d^2*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x^2 - 108000*(d*sqrt(x) + e)^6*b^3*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/x^3 - 160000*(d*sqrt(x) + e)^3*b^3*d^3*n^3/x^(3/2) + 1296000*(d*sqrt(x) + e)*a*b^2*d^5*n^2/sqrt(x) + 480000*(d*sqrt(x) + e)^3*b^3*d^3*n^2*log(c)/x^(3/2) - 1296000*(d*sqrt(x) + e)*a*b^2*d^5*n*log(c)/sqrt(x) - 720000*(d*sqrt(x) + e)^3*b^3*d^3*n*log(c)^2/x^(3/2) + 648000*(d*sqrt(x) + e)*a*b^2*d^5*log(c)^2/sqrt(x) + 720000*(d*sqrt(x) + e)^3*b^3*d^3*log(c)^3/x^(3/2) + 51840*(d*sqrt(x) + e)^5*b^3*d*n^3*log((d*sqrt(x) + e)/sqrt(x))/x^(5/2) - 1440000*(d*sqrt(x) + e)^3*a*b^2*d^3*n^2*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 648000*(d*sqrt(x) + e)*a^2*b*d^5*n*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) - 259200*(d*sqrt(x) + e)^5*b^3*d*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^(5/2) + 4320000*(d*sqrt(x) + e)^3*a*b^2*d^3*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 648000*(d*sqrt(x) + e)^5*b^3*d*n*log(c)^2*log((d*sqrt(x) + e)/sqrt(x))/x^(5/2) + 648000*(d*sqrt(x) + e)^5*a*b^2*d*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x^(5/2) + 50625*(d*sqrt(x) + e)^4*b^3*d^2*n^3/x^2 - 810000*(d*sqrt(x) + e)^2*a*b^2*d^4*n^2/x - 202500*(d*sqrt(x) + e)^4*b^3*d^2*n^2*log(c)/x^2 + 1620000*(d*sqrt(x) + e)^2*a*b^2*d^4*n*log(c)/x + 405000*(d*sqrt(x) + e)^4*b^3*d^2*n*log(c)^2/x^2 - 1620000*(d*sqrt(x) + e)^2*a*b^2*d^4*log(c)^2/x - 540000*(d*sqrt(x) + e)^4*b^3*d^2*log(c)^3/x^2 - 6000*(d*sqrt(x) + e)^6*b^3*n^3*log((d*sqrt(x) + e)/sqrt(x))/x^3 + 810000*(d*sqrt(x) + e)^4*a*b^2*d^2*n^2*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 1620000*(d*sqrt(x) + e)^2*a^2*b*d^4*n*log((d*sqrt(x) + e)/sqrt(x))/x + 36000*(d*sqrt(x) + e)^6*b^3*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^3 - 3240000*(d*sqrt(x) + e)^4*a*b^2*d^2*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 108000*(d*sqrt(x) + e)^6*b^3*n*log(c)^2*log((d*sqrt(x) + e)/sqrt(x))/x^3 - 108000*(d*sqrt(x) + e)^6*a*b^2*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x^3 - 10368*(d*sqrt(x) + e)^5*b^3*d*n^3/x^(5/2) + 480000*(d*sqrt(x) + e)^3*a*b^2*d^3*n^2/x^(3/2) - 648000*(d*sqrt(x) + e)*a^2*b*d^5*n/sqrt(x) + 51840*(d*sqrt(x) + e)^5*b^3*d*n^2*log(c)/x^(5/2) - 1440000*(d*sqrt(x) + e)^3*a*b^2*d^3*n*log(c)/x^(3/2) + 648000*(d*sqrt(x) + e)*a^2*b*d^5*log(c)/sqrt(x) - 129600*(d*sqrt(x)

```
+ e)^5*b^3*d*n*log(c)^2/x^(5/2) + 2160000*(d*sqrt(x) + e)^3*a*b^2*d^3*log(c
)^2/x^(3/2) + 216000*(d*sqrt(x) + e)^5*b^3*d*log(c)^3/x^(5/2) - 259200*(d*s
qrt(x) + e)^5*a*b^2*d*n^2*log((d*sqrt(x) + e)/sqrt(x))/x^(5/2) + 2160000*(d
*sqrt(x) + e)^3*a^2*b*d^3*n*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 1296000*
(d*sqrt(x) + e)^5*a*b^2*d*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^(5/2) + 1
000*(d*sqrt(x) + e)^6*b^3*n^3/x^3 - 202500*(d*sqrt(x) + e)^4*a*b^2*d^2*n^2/
x^2 + 810000*(d*sqrt(x) + e)^2*a^2*b*d^4*n/x - 6000*(d*sqrt(x) + e)^6*b^3*n
^2*log(c)/x^3 + 810000*(d*sqrt(x) + e)^4*a*b^2*d^2*n*log(c)/x^2 - 1620000*(
d*sqrt(x) + e)^2*a^2*b*d^4*log(c)/x + 18000*(d*sqrt(x) + e)^6*b^3*n*log(c)^
2/x^3 - 1620000*(d*sqrt(x) + e)^4*a*b^2*d^2*log(c)^2/x^2 - 36000*(d*sqrt(x)
 + e)^6*b^3*log(c)^3/x^3 + 36000*(d*sqrt(x) + e)^6*a*b^2*n^2*log((d*sqrt(x)
 + e)/sqrt(x))/x^3 - 1620000*(d*sqrt(x) + e)^4*a^2*b*d^2*n*log((d*sqrt(x) +
 e)/sqrt(x))/x^2 - 216000*(d*sqrt(x) + e)^6*a*b^2*n*log(c)*log((d*sqrt(x) +
 e)/sqrt(x))/x^3 + 51840*(d*sqrt(x) + e)^5*a*b^2*d*n^2/x^(5/2) - 720000*(d*
sqrt(x) + e)^3*a^2*b*d^3*n/x^(3/2) + 216000*(d*sqrt(x) + e)*a^3*d^5/sqrt(x)
 - 259200*(d*sqrt(x) + e)^5*a*b^2*d*n*log(c)/x^(5/2) + 2160000*(d*sqrt(x) +
 e)^3*a^2*b*d^3*log(c)/x^(3/2) + 648000*(d*sqrt(x) + e)^5*a*b^2*d*log(c)^2/
x^(5/2) + 648000*(d*sqrt(x) + e)^5*a^2*b*d*n*log((d*sqrt(x) + e)/sqrt(x))/x
^(5/2) - 6000*(d*sqrt(x) + e)^6*a*b^2*n^2/x^3 + 405000*(d*sqrt(x) + e)^4*a^
2*b*d^2*n/x^2 - 540000*(d*sqrt(x) + e)^2*a^3*d^4/x + 36000*(d*sqrt(x) + e)^
6*a*b^2*n*log(c)/x^3 - 1620000*(d*sqrt(x) + e)^4*a^2*b*d^2*log(c)/x^2 - 108
000*(d*sqrt(x) + e)^6*a*b^2*log(c)^2/x^3 - 108000*(d*sqrt(x) + e)^6*a^2*b*n
*log((d*sqrt(x) + e)/sqrt(x))/x^3 - 129600*(d*sqrt(x) + e)^5*a^2*b*d*n/x^(5
/2) + 720000*(d*sqrt(x) + e)^3*a^3*d^3/x^(3/2) + 648000*(d*sqrt(x) + e)^5*a
^2*b*d*log(c)/x^(5/2) + 18000*(d*sqrt(x) + e)^6*a^2*b*n/x^3 - 540000*(d*sqr
t(x) + e)^4*a^3*d^2/x^2 - 108000*(d*sqrt(x) + e)^6*a^2*b*log(c)/x^3 + 21600
0*(d*sqrt(x) + e)^5*a^3*d/x^(5/2) - 36000*(d*sqrt(x) + e)^6*a^3/x^3)*e^(-6)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{3}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/2))^n)+a)^3/x^4,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/2))^n)+a)^3/x^4,x)
```

maxima [A] time = 1.07, size = 864, normalized size = 0.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/60*a^2*b*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 - 30*d^6*log(x)/e^7 - (60*d^5
*x^(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 15*d^2*e^3*x + 12*d*e^4*sqrt
(x) - 10*e^5)/(e^6*x^3)) + 1/1800*(60*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 -
30*d^6*log(x)/e^7 - (60*d^5*x^(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 1
5*d^2*e^3*x + 12*d*e^4*sqrt(x) - 10*e^5)/(e^6*x^3))*log(c*(d + e/sqrt(x))^n
) - (1800*d^6*x^3*log(d*sqrt(x) + e)^2 + 450*d^6*x^3*log(x)^2 - 4410*d^6*x^
3*log(x) - 8820*d^5*e*x^(5/2) + 2610*d^4*e^2*x^2 - 1140*d^3*e^3*x^(3/2) + 5
55*d^2*e^4*x - 264*d*e^5*sqrt(x) + 100*e^6 - 180*(10*d^6*x^3*log(x) - 49*d^
6*x^3)*log(d*sqrt(x) + e))*n^2/(e^6*x^3))*a*b^2 + 1/108000*(1800*e*n*(60*d^
6*log(d*sqrt(x) + e)/e^7 - 30*d^6*log(x)/e^7 - (60*d^5*x^(5/2) - 30*d^4*e*x
^2 + 20*d^3*e^2*x^(3/2) - 15*d^2*e^3*x + 12*d*e^4*sqrt(x) - 10*e^5)/(e^6*x^
3))*log(c*(d + e/sqrt(x))^n)^2 + e*n*((36000*d^6*x^3*log(d*sqrt(x) + e)^3 -
 4500*d^6*x^3*log(x)^3 + 66150*d^6*x^3*log(x)^2 - 404670*d^6*x^3*log(x) - 8
```

```
09340*d^5*e*x^(5/2) + 140070*d^4*e^2*x^2 - 41180*d^3*e^3*x^(3/2) + 13785*d^
2*e^4*x - 4368*d*e^5*sqrt(x) + 1000*e^6 - 5400*(10*d^6*x^3*log(x) - 49*d^6*
x^3)*log(d*sqrt(x) + e)^2 + 60*(450*d^6*x^3*log(x)^2 - 4410*d^6*x^3*log(x)
+ 13489*d^6*x^3)*log(d*sqrt(x) + e))*n^2/(e^7*x^3) - 60*(1800*d^6*x^3*log(d
*sqrt(x) + e)^2 + 450*d^6*x^3*log(x)^2 - 4410*d^6*x^3*log(x) - 8820*d^5*e*x
^(5/2) + 2610*d^4*e^2*x^2 - 1140*d^3*e^3*x^(3/2) + 555*d^2*e^4*x - 264*d*e^
5*sqrt(x) + 100*e^6 - 180*(10*d^6*x^3*log(x) - 49*d^6*x^3)*log(d*sqrt(x) +
e))*n*log(c*(d + e/sqrt(x))^n)/(e^7*x^3)))*b^3 - 1/3*b^3*log(c*(d + e/sqrt(
x))^n)^3/x^3 - a*b^2*log(c*(d + e/sqrt(x))^n)^2/x^3 - a^2*b*log(c*(d + e/sq
rt(x))^n)/x^3 - 1/3*a^3/x^3
```

mupad [B] time = 8.18, size = 989, normalized size = 1.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^3/x^4,x)
```

```
[Out] (b^3*n^3)/(108*x^3) - (b^3*log(c*(d + e/x^(1/2))^n)^3)/(3*x^3) - a^3/(3*x^3
) - (a*b^2*log(c*(d + e/x^(1/2))^n)^2)/x^3 + (b^3*n*log(c*(d + e/x^(1/2))^n
)^2)/(6*x^3) - (b^3*n^2*log(c*(d + e/x^(1/2))^n))/(18*x^3) - (a*b^2*n^2)/(1
8*x^3) + (b^3*d^6*log(c*(d + e/x^(1/2))^n)^3)/(3*e^6) - (a^2*b*log(c*(d + e
/x^(1/2))^n))/x^3 + (a^2*b*n)/(6*x^3) + (a*b^2*n*log(c*(d + e/x^(1/2))^n))/
(3*x^3) + (13489*b^3*d^6*n^3*log(d + e/x^(1/2)))/(1800*e^6) + (919*b^3*d^2*
n^3)/(7200*e^2*x^2) + (4669*b^3*d^4*n^3)/(3600*e^4*x) - (2059*b^3*d^3*n^3)/
(5400*e^3*x^(3/2)) - (13489*b^3*d^5*n^3)/(1800*e^5*x^(1/2)) + (a*b^2*d^6*lo
g(c*(d + e/x^(1/2))^n)^2)/e^6 - (49*b^3*d^6*n*log(c*(d + e/x^(1/2))^n)^2)/(
20*e^6) - (91*b^3*d*n^3)/(2250*e*x^(5/2)) + (a^2*b*d^6*n*log(d + e/x^(1/2))
)/e^6 - (b^3*d*n*log(c*(d + e/x^(1/2))^n)^2)/(5*e*x^(5/2)) + (11*b^3*d*n^2*
log(c*(d + e/x^(1/2))^n))/(75*e*x^(5/2)) + (a^2*b*d^2*n)/(4*e^2*x^2) + (a^2
*b*d^4*n)/(2*e^4*x) + (11*a*b^2*d*n^2)/(75*e*x^(5/2)) - (a^2*b*d^3*n)/(3*e^
3*x^(3/2)) - (a^2*b*d^5*n)/(e^5*x^(1/2)) - (49*a*b^2*d^6*n^2*log(d + e/x^(1
/2)))/(10*e^6) + (b^3*d^2*n*log(c*(d + e/x^(1/2))^n)^2)/(4*e^2*x^2) - (37*b
^3*d^2*n^2*log(c*(d + e/x^(1/2))^n))/(120*e^2*x^2) + (b^3*d^4*n*log(c*(d +
e/x^(1/2))^n)^2)/(2*e^4*x) - (29*b^3*d^4*n^2*log(c*(d + e/x^(1/2))^n))/(20*
e^4*x) - (b^3*d^3*n*log(c*(d + e/x^(1/2))^n)^2)/(3*e^3*x^(3/2)) + (19*b^3*d
^3*n^2*log(c*(d + e/x^(1/2))^n))/(30*e^3*x^(3/2)) - (b^3*d^5*n*log(c*(d + e
/x^(1/2))^n)^2)/(e^5*x^(1/2)) + (49*b^3*d^5*n^2*log(c*(d + e/x^(1/2))^n))/(
10*e^5*x^(1/2)) - (37*a*b^2*d^2*n^2)/(120*e^2*x^2) - (29*a*b^2*d^4*n^2)/(20
*e^4*x) + (19*a*b^2*d^3*n^2)/(30*e^3*x^(3/2)) + (49*a*b^2*d^5*n^2)/(10*e^5*
x^(1/2)) - (a^2*b*d*n)/(5*e*x^(5/2)) - (2*a*b^2*d*n*log(c*(d + e/x^(1/2))^n
))/(5*e*x^(5/2)) + (a*b^2*d^2*n*log(c*(d + e/x^(1/2))^n))/(2*e^2*x^2) + (a*
b^2*d^4*n*log(c*(d + e/x^(1/2))^n))/(e^4*x) - (2*a*b^2*d^3*n*log(c*(d + e/x
^(1/2))^n))/(3*e^3*x^(3/2)) - (2*a*b^2*d^5*n*log(c*(d + e/x^(1/2))^n))/(e^5
*x^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x**4,x)
```

```
[Out] Timed out
```

3.442 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$

Optimal. Leaf size=234

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{bd^{12}n\log\left(d+e\sqrt[3]{x}\right)}{4e^{12}}+\frac{bd^{11}n\sqrt[3]{x}}{4e^{11}}-\frac{bd^{10}nx^{2/3}}{8e^{10}}+\frac{bd^9nx}{12e^9}-\frac{bd^8nx^{4/3}}{16e^8}+\frac{bd^7nx^{5/3}}{20e^7}-\frac{bd}{2}$$

```
[Out] 1/4*b*d^11*n*x^(1/3)/e^11-1/8*b*d^10*n*x^(2/3)/e^10+1/12*b*d^9*n*x/e^9-1/16
*b*d^8*n*x^(4/3)/e^8+1/20*b*d^7*n*x^(5/3)/e^7-1/24*b*d^6*n*x^2/e^6+1/28*b*d
^5*n*x^(7/3)/e^5-1/32*b*d^4*n*x^(8/3)/e^4+1/36*b*d^3*n*x^3/e^3-1/40*b*d^2*n
*x^(10/3)/e^2+1/44*b*d*n*x^(11/3)/e-1/48*b*n*x^4-1/4*b*d^12*n*ln(d+e*x^(1/3
))/e^12+1/4*x^4*(a+b*ln(c*(d+e*x^(1/3))^n))
```

Rubi [A] time = 0.19, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{bd^{10}nx^{2/3}}{8e^{10}}-\frac{bd^8nx^{4/3}}{16e^8}+\frac{bd^7nx^{5/3}}{20e^7}-\frac{bd^6nx^2}{24e^6}+\frac{bd^5nx^{7/3}}{28e^5}-\frac{bd^4nx^{8/3}}{32e^4}+\frac{bd^3nx^3}{36e^3}-\frac{bd^2n}{40}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] (b*d^11*n*x^(1/3))/(4*e^11) - (b*d^10*n*x^(2/3))/(8*e^10) + (b*d^9*n*x)/(12
*e^9) - (b*d^8*n*x^(4/3))/(16*e^8) + (b*d^7*n*x^(5/3))/(20*e^7) - (b*d^6*n*
x^2)/(24*e^6) + (b*d^5*n*x^(7/3))/(28*e^5) - (b*d^4*n*x^(8/3))/(32*e^4) + (
b*d^3*n*x^3)/(36*e^3) - (b*d^2*n*x^(10/3))/(40*e^2) + (b*d*n*x^(11/3))/(44*
e) - (b*n*x^4)/48 - (b*d^12*n*Log[d + e*x^(1/3)])/(4*e^12) + (x^4*(a + b*Lo
g[c*(d + e*x^(1/3))^n]))/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)dx=3\,\text{Subst}\left(\int x^{11}\left(a+b\log\left(c(d+ex)^n\right)\right)dx,x,\sqrt[3]{x}\right)$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{4}(ben)\,\text{Subst}\left(\int\frac{x^{12}}{d+ex}dx,x,\sqrt[3]{x}\right)$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{4}(ben)\,\text{Subst}\left(\int\left(-\frac{d^{11}}{e^{12}}+\frac{d^{10}x}{e^{11}}-\frac{d^9x^2}{e^{10}}\right.\right.$$
$$=\frac{bd^{11}n\sqrt[3]{x}}{4e^{11}}-\frac{bd^{10}nx^{2/3}}{8e^{10}}+\frac{bd^9nx}{12e^9}-\frac{bd^8nx^{4/3}}{16e^8}+\frac{bd^7nx^{5/3}}{20e^7}-\frac{bd^6nx^2}{24e^6}+\frac{bd^5nx}{28e^5}$$

Mathematica [A] time = 0.24, size = 219, normalized size = 0.94

$$\frac{ax^4}{4}+\frac{1}{4}bx^4\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-\frac{1}{4}ben\left(\frac{d^{12}\log\left(d+e\sqrt[3]{x}\right)}{e^{13}}-\frac{d^{11}\sqrt[3]{x}}{e^{12}}+\frac{d^{10}x^{2/3}}{2e^{11}}-\frac{d^9x}{3e^{10}}+\frac{d^8x^{4/3}}{4e^9}-\frac{d^7x^{5/3}}{5e^8}+\frac{d^6x^2}{6e^7}-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] (a*x^4)/4 - (b*e*n*(-((d^11*x^(1/3))/e^12) + (d^10*x^(2/3))/(2*e^11) - (d^9
*x)/(3*e^10) + (d^8*x^(4/3))/(4*e^9) - (d^7*x^(5/3))/(5*e^8) + (d^6*x^2)/(6
*e^7) - (d^5*x^(7/3))/(7*e^6) + (d^4*x^(8/3))/(8*e^5) - (d^3*x^3)/(9*e^4) +
 (d^2*x^(10/3))/(10*e^3) - (d*x^(11/3))/(11*e^2) + x^4/(12*e) + (d^12*Log[d
 + e*x^(1/3)])/e^13))/4 + (b*x^4*Log[c*(d + e*x^(1/3))^n])/4
```

fricas [A] time = 0.47, size = 201, normalized size = 0.86

$$\frac{27720\,be^{12}x^4\log(c)+3080\,bd^3e^9nx^3-4620\,bd^6e^6nx^2+9240\,bd^9e^3nx-2310\left(be^{12}n-12\,ae^{12}\right)x^4+27720\left(be^{12}n\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/110880*(27720*b*e^12*x^4*log(c) + 3080*b*d^3*e^9*n*x^3 - 4620*b*d^6*e^6*n
*x^2 + 9240*b*d^9*e^3*n*x - 2310*(b*e^12*n - 12*a*e^12)*x^4 + 27720*(b*e^12
*n*x^4 - b*d^12*n)*log(e*x^(1/3) + d) + 63*(40*b*d*e^11*n*x^3 - 55*b*d^4*e^
8*n*x^2 + 88*b*d^7*e^5*n*x - 220*b*d^10*e^2*n)*x^(2/3) - 198*(14*b*d^2*e^10
*n*x^3 - 20*b*d^5*e^7*n*x^2 + 35*b*d^8*e^4*n*x - 140*b*d^11*e*n)*x^(1/3))/e
^12
```

giac [B] time = 0.20, size = 529, normalized size = 2.26

$$\frac{1}{110880}\left(27720\,bx^4e\log(c)+27720\,ax^4e+\left(27720\left(x^{\frac{1}{3}}e+d\right)^{12}e^{(-11)}\log\left(x^{\frac{1}{3}}e+d\right)-332640\left(x^{\frac{1}{3}}e+d\right)^{11}de^{(-11)}\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/110880*(27720*b*x^4*e*log(c) + 27720*a*x^4*e + (27720*(x^(1/3)*e + d)^12*
e^(-11)*log(x^(1/3)*e + d) - 332640*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3
)*e + d) + 1829520*(x^(1/3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d) - 6098
400*(x^(1/3)*e + d)^9*d^3*e^(-11)*log(x^(1/3)*e + d) + 13721400*(x^(1/3)*e
+ d)^8*d^4*e^(-11)*log(x^(1/3)*e + d) - 21954240*(x^(1/3)*e + d)^7*d^5*e^(-
11)*log(x^(1/3)*e + d) + 25613280*(x^(1/3)*e + d)^6*d^6*e^(-11)*log(x^(1/3)
*e + d) - 21954240*(x^(1/3)*e + d)^5*d^7*e^(-11)*log(x^(1/3)*e + d) + 13721
```

```
400*(x^(1/3)*e + d)^4*d^8*e^(-11)*log(x^(1/3)*e + d) - 6098400*(x^(1/3)*e +
 d)^3*d^9*e^(-11)*log(x^(1/3)*e + d) + 1829520*(x^(1/3)*e + d)^2*d^10*e^(-1
1)*log(x^(1/3)*e + d) - 332640*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e +
 d) - 2310*(x^(1/3)*e + d)^12*e^(-11) + 30240*(x^(1/3)*e + d)^11*d*e^(-11)
- 182952*(x^(1/3)*e + d)^10*d^2*e^(-11) + 677600*(x^(1/3)*e + d)^9*d^3*e^(-
11) - 1715175*(x^(1/3)*e + d)^8*d^4*e^(-11) + 3136320*(x^(1/3)*e + d)^7*d^5
*e^(-11) - 4268880*(x^(1/3)*e + d)^6*d^6*e^(-11) + 4390848*(x^(1/3)*e + d)^
5*d^7*e^(-11) - 3430350*(x^(1/3)*e + d)^4*d^8*e^(-11) + 2032800*(x^(1/3)*e
+ d)^3*d^9*e^(-11) - 914760*(x^(1/3)*e + d)^2*d^10*e^(-11) + 332640*(x^(1/3
)*e + d)*d^11*e^(-11))*b*n)*e^(-1)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \left(b \ln \left(c \left(e x^{\frac{1}{3}}+d\right)^{n}\right)+a\right) x^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

maxima [A] time = 0.87, size = 172, normalized size = 0.74

$$\frac{1}{4}\, bx^{4} \log\left(\left(ex^{\frac{1}{3}}+d\right)^{n} c\right)+\frac{1}{4}\, ax^{4}-\frac{1}{110880}\, ben\left(\frac{27720\, d^{12} \log\left(ex^{\frac{1}{3}}+d\right)}{e^{13}}+\frac{2310\, e^{11} x^{4}-2520\, de^{10} x^{\frac{11}{3}}+2772\, d^{2} e\cdots}{\cdots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log((e*x^(1/3) + d)^n*c) + 1/4*a*x^4 - 1/110880*b*e*n*(27720*d^12
*log(e*x^(1/3) + d)/e^13 + (2310*e^11*x^4 - 2520*d*e^10*x^(11/3) + 2772*d^2
*e^9*x^(10/3) - 3080*d^3*e^8*x^3 + 3465*d^4*e^7*x^(8/3) - 3960*d^5*e^6*x^(7
/3) + 4620*d^6*e^5*x^2 - 5544*d^7*e^4*x^(5/3) + 6930*d^8*e^3*x^(4/3) - 9240
*d^9*e^2*x + 13860*d^10*e*x^(2/3) - 27720*d^11*x^(1/3))/e^12)
```

mupad [B] time = 0.65, size = 189, normalized size = 0.81

$$\frac{a\, x^4}{4}-\frac{b\, n\, x^4}{48}+\frac{b\, x^4\, \ln\left(c\, {\left(d+e\, x^{1/3}\right)}^n\right)}{4}+\frac{b\, d\, n\, x^{11/3}}{44\, e}+\frac{b\, d^9\, n\, x}{12\, e^9}-\frac{b\, d^{12}\, n\, \ln\left(d+e\, x^{1/3}\right)}{4\, e^{12}}+\frac{b\, d^3\, n\, x^3}{36\, e^3}-\frac{b\, d^6\, n\, x^2}{24\, e^6}-\frac{b\, d^2\, n\cdots}{40\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/3))^n)),x)
```

```
[Out] (a*x^4)/4 - (b*n*x^4)/48 + (b*x^4*log(c*(d + e*x^(1/3))^n))/4 + (b*d*n*x^(1
1/3))/(44*e) + (b*d^9*n*x)/(12*e^9) - (b*d^12*n*log(d + e*x^(1/3)))/(4*e^12
) + (b*d^3*n*x^3)/(36*e^3) - (b*d^6*n*x^2)/(24*e^6) - (b*d^2*n*x^(10/3))/(4
0*e^2) - (b*d^4*n*x^(8/3))/(32*e^4) + (b*d^5*n*x^(7/3))/(28*e^5) + (b*d^7*n
*x^(5/3))/(20*e^7) - (b*d^8*n*x^(4/3))/(16*e^8) - (b*d^10*n*x^(2/3))/(8*e^1
0) + (b*d^11*n*x^(1/3))/(4*e^11)
```

sympy [A] time = 81.57, size = 216, normalized size = 0.92

$$\frac{ax^4}{4}+b\left(-\frac{en\left(\frac{3d^{12}\left(\begin{cases}\frac{\sqrt[3]{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt[3]{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^{12}}-\frac{3d^{11}\sqrt[3]{x}}{e^{12}}+\frac{3d^{10}x^{\frac{2}{3}}}{2e^{11}}-\frac{d^9x}{e^{10}}+\frac{3d^8x^{\frac{4}{3}}}{4e^9}-\frac{3d^7x^{\frac{5}{3}}}{5e^8}+\frac{d^6x^2}{2e^7}-\frac{3d^5x^{\frac{7}{3}}}{7e^6}+\frac{3d^4x^{\frac{8}{3}}}{8e^5}-\frac{d^3x}{3e^{}}\right.}{12}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))**n)),x)
```

```
[Out] a*x**4/4 + b*(-e*n*(3*d**12*Piecewise((x**(1/3)/d, Eq(e, 0)), (log(d + e*x*
*(1/3))/e, True))/e**12 - 3*d**11*x**(1/3)/e**12 + 3*d**10*x**(2/3)/(2*e**1
1) - d**9*x/e**10 + 3*d**8*x**(4/3)/(4*e**9) - 3*d**7*x**(5/3)/(5*e**8) + d
**6*x**2/(2*e**7) - 3*d**5*x**(7/3)/(7*e**6) + 3*d**4*x**(8/3)/(8*e**5) - d
**3*x**3/(3*e**4) + 3*d**2*x**(10/3)/(10*e**3) - 3*d*x**(11/3)/(11*e**2) +
x**4/(4*e))/12 + x**4*log(c*(d + e*x**(1/3))**n)/4)
```

3.443 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$

Optimal. Leaf size=185

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)+\frac{bd^9n\log\left(d+e\sqrt[3]{x}\right)}{3e^9}-\frac{bd^8n\sqrt[3]{x}}{3e^8}+\frac{bd^7nx^{2/3}}{6e^7}-\frac{bd^6nx}{9e^6}+\frac{bd^5nx^{4/3}}{12e^5}-\frac{bd^4nx^{5/3}}{15e^4}+\frac{bd^3n}{18e}$$

```
[Out] -1/3*b*d^8*n*x^(1/3)/e^8+1/6*b*d^7*n*x^(2/3)/e^7-1/9*b*d^6*n*x/e^6+1/12*b*d
^5*n*x^(4/3)/e^5-1/15*b*d^4*n*x^(5/3)/e^4+1/18*b*d^3*n*x^2/e^3-1/21*b*d^2*n
*x^(7/3)/e^2+1/24*b*d*n*x^(8/3)/e-1/27*b*n*x^3+1/3*b*d^9*n*ln(d+e*x^(1/3))/
e^9+1/3*x^3*(a+b*ln(c*(d+e*x^(1/3))^n))
```

Rubi [A] time = 0.13, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)+\frac{bd^7nx^{2/3}}{6e^7}+\frac{bd^5nx^{4/3}}{12e^5}-\frac{bd^4nx^{5/3}}{15e^4}+\frac{bd^3nx^2}{18e^3}-\frac{bd^2nx^{7/3}}{21e^2}-\frac{bd^8n\sqrt[3]{x}}{3e^8}-\frac{bd^6nx}{9e^6}+\frac{bd^9n\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] -(b*d^8*n*x^(1/3))/(3*e^8) + (b*d^7*n*x^(2/3))/(6*e^7) - (b*d^6*n*x)/(9*e^6
) + (b*d^5*n*x^(4/3))/(12*e^5) - (b*d^4*n*x^(5/3))/(15*e^4) + (b*d^3*n*x^2)
/(18*e^3) - (b*d^2*n*x^(7/3))/(21*e^2) + (b*d*n*x^(8/3))/(24*e) - (b*n*x^3)
/27 + (b*d^9*n*Log[d + e*x^(1/3)])/(3*e^9) + (x^3*(a + b*Log[c*(d + e*x^(1/
3))^n]))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right) dx = 3\,\text{Subst}\left(\int x^8 \left(a+b\log\left(c(d+ex)^n\right)\right) dx, x, \sqrt[3]{x}\right)$$
$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right) - \frac{1}{3}(ben)\,\text{Subst}\left(\int \frac{x^9}{d+ex} dx, x, \sqrt[3]{x}\right)$$
$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right) - \frac{1}{3}(ben)\,\text{Subst}\left(\int \left(\frac{d^8}{e^9} - \frac{d^7x}{e^8} + \frac{d^6x^2}{e^7} - \frac{d}{-}\right.\right.$$
$$= -\frac{bd^8n\sqrt[3]{x}}{3e^8} + \frac{bd^7nx^{2/3}}{6e^7} - \frac{bd^6nx}{9e^6} + \frac{bd^5nx^{4/3}}{12e^5} - \frac{bd^4nx^{5/3}}{15e^4} + \frac{bd^3nx^2}{18e^3} - \frac{bd^2nx}{21e^2}$$

Mathematica [A] time = 0.14, size = 176, normalized size = 0.95

$$\frac{ax^3}{3}+\frac{1}{3}bx^3\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-\frac{1}{3}ben\left(-\frac{d^9\log\left(d+e\sqrt[3]{x}\right)}{e^{10}}+\frac{d^8\sqrt[3]{x}}{e^9}-\frac{d^7x^{2/3}}{2e^8}+\frac{d^6x}{3e^7}-\frac{d^5x^{4/3}}{4e^6}+\frac{d^4x^{5/3}}{5e^5}-\frac{d^3x^2}{6e^4}+\frac{d}{-}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] (a*x^3)/3 - (b*e*n*((d^8*x^(1/3))/e^9 - (d^7*x^(2/3))/(2*e^8) + (d^6*x)/(3*
e^7) - (d^5*x^(4/3))/(4*e^6) + (d^4*x^(5/3))/(5*e^5) - (d^3*x^2)/(6*e^4) +
(d^2*x^(7/3))/(7*e^3) - (d*x^(8/3))/(8*e^2) + x^3/(9*e) - (d^9*Log[d + e*x^
(1/3)])/e^10))/3 + (b*x^3*Log[c*(d + e*x^(1/3))^n])/3
```

fricas [A] time = 0.44, size = 161, normalized size = 0.87

$$\frac{2520\,be^9x^3\log(c) + 420\,bd^3e^6nx^2 - 840\,bd^6e^3nx - 280\left(be^9n - 9\,ae^9\right)x^3 + 2520\left(be^9nx^3 + bd^9n\right)\log\left(ex^{\frac{1}{3}} + d\right) +}{7560\,e^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/7560*(2520*b*e^9*x^3*log(c) + 420*b*d^3*e^6*n*x^2 - 840*b*d^6*e^3*n*x - 2
80*(b*e^9*n - 9*a*e^9)*x^3 + 2520*(b*e^9*n*x^3 + b*d^9*n)*log(e*x^(1/3) + d
) + 63*(5*b*d*e^8*n*x^2 - 8*b*d^4*e^5*n*x + 20*b*d^7*e^2*n)*x^(2/3) - 90*(4
*b*d^2*e^7*n*x^2 - 7*b*d^5*e^4*n*x + 28*b*d^8*e*n)*x^(1/3))/e^9
```

giac [B] time = 0.19, size = 400, normalized size = 2.16

$$\frac{1}{7560}\left(2520\,bx^3e\log(c) + 2520\,ax^3e + \left(2520\left(x^{\frac{1}{3}}e + d\right)^9 e^{(-8)}\log\left(x^{\frac{1}{3}}e + d\right) - 22680\left(x^{\frac{1}{3}}e + d\right)^8 de^{(-8)}\log\left(x^{\frac{1}{3}}e + d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/7560*(2520*b*x^3*e*log(c) + 2520*a*x^3*e + (2520*(x^(1/3)*e + d)^9*e^(-8)
*log(x^(1/3)*e + d) - 22680*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d) +
 90720*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d) - 211680*(x^(1/3)*e
+ d)^6*d^3*e^(-8)*log(x^(1/3)*e + d) + 317520*(x^(1/3)*e + d)^5*d^4*e^(-8)*
log(x^(1/3)*e + d) - 317520*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d)
 + 211680*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d) - 90720*(x^(1/3)*
e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d) + 22680*(x^(1/3)*e + d)*d^8*e^(-8)*l
og(x^(1/3)*e + d) - 280*(x^(1/3)*e + d)^9*e^(-8) + 2835*(x^(1/3)*e + d)^8*d
*e^(-8) - 12960*(x^(1/3)*e + d)^7*d^2*e^(-8) + 35280*(x^(1/3)*e + d)^6*d^3*
e^(-8) - 63504*(x^(1/3)*e + d)^5*d^4*e^(-8) + 79380*(x^(1/3)*e + d)^4*d^5*e
```

```
^(-8) - 70560*(x^(1/3)*e + d)^3*d^6*e^(-8) + 45360*(x^(1/3)*e + d)^2*d^7*e^
(-8) - 22680*(x^(1/3)*e + d)*d^8*e^(-8))*b*n)*e^(-1)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\left(e\,x^{\frac{1}{3}}+d\right)^{n}\right)+a\right)x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*(e*x^(1/3)+d)^n)+a),x)
```

```
[Out] int(x^2*(b*ln(c*(e*x^(1/3)+d)^n)+a),x)
```

maxima [A] time = 0.72, size = 140, normalized size = 0.76

$$\frac{1}{3}\,bx^{3}\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+\frac{1}{3}\,ax^{3}+\frac{1}{7560}\,ben\left(\frac{2520\,d^{9}\log\left(ex^{\frac{1}{3}}+d\right)}{e^{10}}-\frac{280\,e^{8}x^{3}-315\,de^{7}x^{\frac{8}{3}}+360\,d^{2}e^{6}x^{\frac{7}{3}}-420$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3*log((e*x^(1/3) + d)^n*c) + 1/3*a*x^3 + 1/7560*b*e*n*(2520*d^9*log
(e*x^(1/3) + d)/e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^2*e^6*x^(7/
3) - 420*d^3*e^5*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3) + 840*d^6*
e^2*x - 1260*d^7*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9)
```

mupad [B] time = 0.51, size = 150, normalized size = 0.81

$$\frac{a\,x^3}{3}-\frac{b\,n\,x^3}{27}+\frac{b\,x^3\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{3}+\frac{b\,d\,n\,x^{8/3}}{24\,e}-\frac{b\,d^6\,n\,x}{9\,e^6}+\frac{b\,d^9\,n\,\ln\left(d+e\,x^{1/3}\right)}{3\,e^9}+\frac{b\,d^3\,n\,x^2}{18\,e^3}-\frac{b\,d^2\,n\,x^{7/3}}{21\,e^2}-\frac{b\,d^4\,n}{15\,e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/3))^n)),x)
```

```
[Out] (a*x^3)/3 - (b*n*x^3)/27 + (b*x^3*log(c*(d + e*x^(1/3))^n))/3 + (b*d*n*x^(8
/3))/(24*e) - (b*d^6*n*x)/(9*e^6) + (b*d^9*n*log(d + e*x^(1/3)))/(3*e^9) +
(b*d^3*n*x^2)/(18*e^3) - (b*d^2*n*x^(7/3))/(21*e^2) - (b*d^4*n*x^(5/3))/(15
*e^4) + (b*d^5*n*x^(4/3))/(12*e^5) + (b*d^7*n*x^(2/3))/(6*e^7) - (b*d^8*n*x
^(1/3))/(3*e^8)
```

sympy [A] time = 19.48, size = 173, normalized size = 0.94

$$\frac{ax^3}{3}+b\left(-\frac{en\left(-\frac{3d^9\left(\begin{cases}\frac{\sqrt[3]{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt[3]{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^9}+\frac{3d^8\sqrt[3]{x}}{e^9}-\frac{3d^7x^{\frac{2}{3}}}{2e^8}+\frac{d^6x}{e^7}-\frac{3d^5x^{\frac{4}{3}}}{4e^6}+\frac{3d^4x^{\frac{5}{3}}}{5e^5}-\frac{d^3x^2}{2e^4}+\frac{3d^2x^{\frac{7}{3}}}{7e^3}-\frac{3dx^{\frac{8}{3}}}{8e^2}+\frac{x^3}{3e}\right)}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**n)),x)
```

```
[Out] a*x**3/3 + b*(-e*n*(-3*d**9*Piecewise((x**(1/3)/d, Eq(e, 0)), (log(d + e*x*
*(1/3))/e, True))/e**9 + 3*d**8*x**(1/3)/e**9 - 3*d**7*x**(2/3)/(2*e**8) +
d**6*x/e**7 - 3*d**5*x**(4/3)/(4*e**6) + 3*d**4*x**(5/3)/(5*e**5) - d**3*x*
*2/(2*e**4) + 3*d**2*x**(7/3)/(7*e**3) - 3*d*x**(8/3)/(8*e**2) + x**3/(3*e)
)/9 + x**3*log(c*(d + e*x**(1/3))**n)/3)
```

3.444 $\int x \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right) dx$

Optimal. Leaf size=136

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)}{2e^6}+\frac{bd^5n\sqrt[3]{x}}{2e^5}-\frac{bd^4nx^{2/3}}{4e^4}+\frac{bd^3nx}{6e^3}-\frac{bd^2nx^{4/3}}{8e^2}+\frac{bdnx^{5/3}}{10e}-\frac{1}{12}bn$$

```
[Out] 1/2*b*d^5*n*x^(1/3)/e^5-1/4*b*d^4*n*x^(2/3)/e^4+1/6*b*d^3*n*x/e^3-1/8*b*d^2
*n*x^(4/3)/e^2+1/10*b*d*n*x^(5/3)/e-1/12*b*n*x^2-1/2*b*d^6*n*ln(d+e*x^(1/3)
)/e^6+1/2*x^2*(a+b*ln(c*(d+e*x^(1/3))^n))
```

Rubi [A] time = 0.09, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2454, 2395, 43}

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{bd^4nx^{2/3}}{4e^4}-\frac{bd^2nx^{4/3}}{8e^2}+\frac{bd^5n\sqrt[3]{x}}{2e^5}+\frac{bd^3nx}{6e^3}-\frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)}{2e^6}+\frac{bdnx^{5/3}}{10e}-\frac{1}{12}bn$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] (b*d^5*n*x^(1/3))/(2*e^5) - (b*d^4*n*x^(2/3))/(4*e^4) + (b*d^3*n*x)/(6*e^3)
 - (b*d^2*n*x^(4/3))/(8*e^2) + (b*d*n*x^(5/3))/(10*e) - (b*n*x^2)/12 - (b*d
^6*n*Log[d + e*x^(1/3)])/(2*e^6) + (x^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)dx=3\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)dx,x,\sqrt[3]{x}\right)$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{2}(ben)\,\text{Subst}\left(\int\frac{x^6}{d+ex}dx,x,\sqrt[3]{x}\right)$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{2}(ben)\,\text{Subst}\left(\int\left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3x^2}{e^4}+\right.\right.$$
$$=\frac{bd^5n\sqrt[3]{x}}{2e^5}-\frac{bd^4nx^{2/3}}{4e^4}+\frac{bd^3nx}{6e^3}-\frac{bd^2nx^{4/3}}{8e^2}+\frac{bdnx^{5/3}}{10e}-\frac{1}{12}bnx^2-\frac{bd^6n\log(}{2}$$

Mathematica [A] time = 0.10, size = 133, normalized size = 0.98

$$\frac{ax^2}{2}+\frac{1}{2}bx^2\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-\frac{1}{2}ben\left(\frac{d^6\log\left(d+e\sqrt[3]{x}\right)}{e^7}-\frac{d^5\sqrt[3]{x}}{e^6}+\frac{d^4x^{2/3}}{2e^5}-\frac{d^3x}{3e^4}+\frac{d^2x^{4/3}}{4e^3}-\frac{dx^{5/3}}{5e^2}+\frac{x^2}{6e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] (a*x^2)/2 - (b*e*n*(-((d^5*x^(1/3))/e^6) + (d^4*x^(2/3))/(2*e^5) - (d^3*x)/
(3*e^4) + (d^2*x^(4/3))/(4*e^3) - (d*x^(5/3))/(5*e^2) + x^2/(6*e) + (d^6*Lo
g[d + e*x^(1/3)])/e^7))/2 + (b*x^2*Log[c*(d + e*x^(1/3))^n])/2
```

fricas [A] time = 0.46, size = 122, normalized size = 0.90

$$\frac{60\,be^6x^2\log(c)+20\,bd^3e^3nx-10\left(be^6n-6\,ae^6\right)x^2+60\left(be^6nx^2-bd^6n\right)\log\left(ex^{\frac{1}{3}}+d\right)+6\left(2\,bde^5nx-5\,bd^4e^2n\right)}{120\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/120*(60*b*e^6*x^2*log(c) + 20*b*d^3*e^3*n*x - 10*(b*e^6*n - 6*a*e^6)*x^2
+ 60*(b*e^6*n*x^2 - b*d^6*n)*log(e*x^(1/3) + d) + 6*(2*b*d*e^5*n*x - 5*b*d^
4*e^2*n)*x^(2/3) - 15*(b*d^2*e^4*n*x - 4*b*d^5*e*n)*x^(1/3))/e^6
```

giac [B] time = 0.19, size = 271, normalized size = 1.99

$$\frac{1}{120}\left(60\,bx^2e\log(c)+60\,ax^2e+\left(60\left(x^{\frac{1}{3}}e+d\right)^6e^{(-5)}\log\left(x^{\frac{1}{3}}e+d\right)-360\left(x^{\frac{1}{3}}e+d\right)^5de^{(-5)}\log\left(x^{\frac{1}{3}}e+d\right)+900\left(x^{\frac{1}{3}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/120*(60*b*x^2*e*log(c) + 60*a*x^2*e + (60*(x^(1/3)*e + d)^6*e^(-5)*log(x^
(1/3)*e + d) - 360*(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(
1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d) - 1200*(x^(1/3)*e + d)^3*d^3*e^
(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e +
d) - 360*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e + d) - 10*(x^(1/3)*e + d)
^6*e^(-5) + 72*(x^(1/3)*e + d)^5*d*e^(-5) - 225*(x^(1/3)*e + d)^4*d^2*e^(-5
) + 400*(x^(1/3)*e + d)^3*d^3*e^(-5) - 450*(x^(1/3)*e + d)^2*d^4*e^(-5) + 3
60*(x^(1/3)*e + d)*d^5*e^(-5))*b*n)*e^(-1)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(e\,x^{\frac{1}{3}}+d\right)^{n}\right)+a\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(e*x^(1/3)+d)^n)+a),x)
```

```
[Out] int(x*(b*ln(c*(e*x^(1/3)+d)^n)+a),x)
```

maxima [A] time = 0.68, size = 106, normalized size = 0.78

$$-\frac{1}{120}\,ben\left(\frac{60\,d^6\log\left(ex^{\frac{1}{3}}+d\right)}{e^7}+\frac{10\,e^5x^2-12\,de^4x^{\frac{5}{3}}+15\,d^2e^3x^{\frac{4}{3}}-20\,d^3e^2x+30\,d^4ex^{\frac{2}{3}}-60\,d^5x^{\frac{1}{3}}}{e^6}\right)+\frac{1}{2}\,bx^2\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] -1/120*b*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e^5*x^2 - 12*d*e^4*x^(5/3
) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^(2/3) - 60*d^5*x^(1/3))/
e^6) + 1/2*b*x^2*log((e*x^(1/3) + d)^n*c) + 1/2*a*x^2
```

mupad [B] time = 0.42, size = 111, normalized size = 0.82

$$\frac{a\,x^2}{2}-\frac{b\,n\,x^2}{12}+\frac{b\,x^2\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{2}+\frac{b\,d^3\,n\,x}{6\,e^3}+\frac{b\,d\,n\,x^{5/3}}{10\,e}-\frac{b\,d^6\,n\,\ln\left(d+e\,x^{1/3}\right)}{2\,e^6}-\frac{b\,d^2\,n\,x^{4/3}}{8\,e^2}-\frac{b\,d^4\,n\,x^{2/3}}{4\,e^4}+\frac{b\,d^5\,n}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(1/3))^n)),x)
```

```
[Out] (a*x^2)/2 - (b*n*x^2)/12 + (b*x^2*log(c*(d + e*x^(1/3))^n))/2 + (b*d^3*n*x)
/(6*e^3) + (b*d*n*x^(5/3))/(10*e) - (b*d^6*n*log(d + e*x^(1/3)))/(2*e^6) -
(b*d^2*n*x^(4/3))/(8*e^2) - (b*d^4*n*x^(2/3))/(4*e^4) + (b*d^5*n*x^(1/3))/(
2*e^5)
```

sympy [A] time = 5.80, size = 131, normalized size = 0.96

$$\frac{ax^2}{2}+b\left(-\frac{en\left(\frac{3d^6\left(\begin{cases}\frac{\sqrt[3]{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt[3]{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^6}-\frac{3d^5\sqrt[3]{x}}{e^6}+\frac{3d^4x^{\frac{2}{3}}}{2e^5}-\frac{d^3x}{e^4}+\frac{3d^2x^{\frac{4}{3}}}{4e^3}-\frac{3dx^{\frac{5}{3}}}{5e^2}+\frac{x^2}{2e}\right)}{6}+\frac{x^2\log\left(c\left(d+e\sqrt[3]{x}\right.\right.}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))**n)),x)
```

```
[Out] a*x**2/2 + b*(-e*n*(3*d**6*Piecewise((x**(1/3)/d, Eq(e, 0)), (log(d + e*x**
(1/3))/e, True))/e**6 - 3*d**5*x**(1/3)/e**6 + 3*d**4*x**(2/3)/(2*e**5) - d
**3*x/e**4 + 3*d**2*x**(4/3)/(4*e**3) - 3*d*x**(5/3)/(5*e**2) + x**2/(2*e))
/6 + x**2*log(c*(d + e*x**(1/3))**n)/2)
```

3.445 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$

Optimal. Leaf size=77

$$ax+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)+\frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3}-\frac{bd^2n\sqrt[3]{x}}{e^2}+\frac{bdnx^{2/3}}{2e}-\frac{bnx}{3}$$

```
[Out] -b*d^2*n*x^(1/3)/e^2+1/2*b*d*n*x^(2/3)/e+a*x-1/3*b*n*x+b*d^3*n*ln(d+e*x^(1/
3))/e^3+b*x*ln(c*(d+e*x^(1/3))^n)
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2448, 266, 43}

$$ax+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-\frac{bd^2n\sqrt[3]{x}}{e^2}+\frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3}+\frac{bdnx^{2/3}}{2e}-\frac{bnx}{3}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e*x^(1/3))^n],x]
```

```
[Out] -((b*d^2*n*x^(1/3))/e^2) + (b*d*n*x^(2/3))/(2*e) + a*x - (b*n*x)/3 + (b*d^3
*n*Log[d + e*x^(1/3)])/e^3 + b*x*Log[c*(d + e*x^(1/3))^n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx &= ax+b\int \log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\,dx\\
&= ax+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-\frac{1}{3}(ben)\int \frac{\sqrt[3]{x}}{d+e\sqrt[3]{x}}\,dx\\
&= ax+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-(ben)\,\text{Subst}\left(\int \frac{x^3}{d+ex}\,dx,x,\sqrt[3]{x}\right)\\
&= ax+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-(ben)\,\text{Subst}\left(\int \left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right)\,d\\
&= -\frac{bd^2n\sqrt[3]{x}}{e^2}+\frac{bdnx^{2/3}}{2e}+ax-\frac{bnx}{3}+\frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3}+bx\log\left(c\left(d+e\sqrt[3]{x}\right.\right.
\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 1.00

$$ax+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)+\frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3}-\frac{bd^2n\sqrt[3]{x}}{e^2}+\frac{bdnx^{2/3}}{2e}-\frac{bnx}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e*x^(1/3))^n],x]
```

```
[Out] -((b*d^2*n*x^(1/3))/e^2) + (b*d*n*x^(2/3))/(2*e) + a*x - (b*n*x)/3 + (b*d^3
*n*Log[d + e*x^(1/3)])/e^3 + b*x*Log[c*(d + e*x^(1/3))^n]
```

fricas [A] time = 0.46, size = 77, normalized size = 1.00

$$\frac{6\,be^3x\log(c)+3\,bde^2nx^{\frac{2}{3}}-6\,bd^2enx^{\frac{1}{3}}-2\left(be^3n-3\,ae^3\right)x+6\left(be^3nx+bd^3n\right)\log\left(ex^{\frac{1}{3}}+d\right)}{6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(1/3))^n),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b*e^3*x*log(c) + 3*b*d*e^2*n*x^(2/3) - 6*b*d^2*e*n*x^(1/3) - 2*(b*e^
3*n - 3*a*e^3)*x + 6*(b*e^3*n*x + b*d^3*n)*log(e*x^(1/3) + d))/e^3
```

giac [B] time = 0.17, size = 135, normalized size = 1.75

$$\frac{1}{6}\left(6\,xe\log(c)+\left(6\left(x^{\frac{1}{3}}e+d\right)^3e^{(-2)}\log\left(x^{\frac{1}{3}}e+d\right)-18\left(x^{\frac{1}{3}}e+d\right)^2de^{(-2)}\log\left(x^{\frac{1}{3}}e+d\right)+18\left(x^{\frac{1}{3}}e+d\right)d^2e^{(-2)}\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(1/3))^n),x, algorithm="giac")
```

```
[Out] 1/6*(6*x*e*log(c) + (6*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d) - 18*(x^
(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d) + 18*(x^(1/3)*e + d)*d^2*e^(-2)*
log(x^(1/3)*e + d) - 2*(x^(1/3)*e + d)^3*e^(-2) + 9*(x^(1/3)*e + d)^2*d*e^(
-2) - 18*(x^(1/3)*e + d)*d^2*e^(-2))*n)*b*e^(-1) + a*x
```

maple [A] time = 0.07, size = 66, normalized size = 0.86

$$\frac{b\,d^3n\ln\left(e\,x^{\frac{1}{3}}+d\right)}{e^3}-\frac{bnx}{3}+bx\ln\left(c\left(e\,x^{\frac{1}{3}}+d\right)^n\right)+\frac{bdn\,x^{\frac{2}{3}}}{2e}-\frac{b\,d^2n\,x^{\frac{1}{3}}}{e^2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*ln(c*(e*x^(1/3)+d)^n)+a,x)
```

```
[Out] -b*d^2*n*x^(1/3)/e^2+1/2*b*d*n*x^(2/3)/e+a*x-1/3*b*n*x+b*d^3*n*ln(e*x^(1/3)
+d)/e^3+b*x*ln(c*(e*x^(1/3)+d)^n)
```

maxima [A] time = 0.81, size = 70, normalized size = 0.91

$$\frac{1}{6}\left(en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-3\,dex^{\frac{2}{3}}+6\,d^2x^{\frac{1}{3}}}{e^3}\right)+6\,x\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(1/3))^n),x, algorithm="maxima")
```

```
[Out] 1/6*(e*n*(6*d^3*log(e*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*x
^(1/3))/e^3) + 6*x*log((e*x^(1/3) + d)^n*c))*b + a*x
```

mupad [B] time = 0.34, size = 65, normalized size = 0.84

$$a\,x+b\,x\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)-\frac{b\,n\,x}{3}+\frac{b\,d\,n\,x^{2/3}}{2\,e}+\frac{b\,d^3\,n\,\ln\left(d+e\,x^{1/3}\right)}{e^3}-\frac{b\,d^2\,n\,x^{1/3}}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*log(c*(d + e*x^(1/3))^n),x)
```

```
[Out] a*x + b*x*log(c*(d + e*x^(1/3))^n) - (b*n*x)/3 + (b*d*n*x^(2/3))/(2*e) + (b
*d^3*n*log(d + e*x^(1/3)))/e^3 - (b*d^2*n*x^(1/3))/e^2
```

sympy [A] time = 1.71, size = 82, normalized size = 1.06

$$ax+b\left(-\frac{en\left(-\frac{3d^3\left(\begin{cases}\frac{\sqrt[3]{x}}{d} & \text{for}\ e=0\\ \frac{\log\left(d+e\sqrt[3]{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^3}+\frac{3d^2\sqrt[3]{x}}{e^3}-\frac{3dx^{\frac{2}{3}}}{2e^2}+\frac{x}{e}\right)}{3}+x\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e*x**(1/3))**n),x)
```

```
[Out] a*x + b*(-e*n*(-3*d**3*Piecewise((x**(1/3)/d, Eq(e, 0)), (log(d + e*x**(1/3
))/e, True))/e**3 + 3*d**2*x**(1/3)/e**3 - 3*d*x**(2/3)/(2*e**2) + x/e)/3 +
 x*log(c*(d + e*x**(1/3))**n))
```

3.446 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=51

$$3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)+3bn\text{Li}_2\left(\frac{\sqrt[3]{x}\,e}{d}+1\right)$$

```
[Out] 3*(a+b*ln(c*(d+e*x^(1/3))^n))*ln(-e*x^(1/3)/d)+3*b*n*polylog(2,1+e*x^(1/3)/
d)
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2394, 2315}

$$3bn\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)+3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x,x]
```

```
[Out] 3*(a + b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)] + 3*b*n*PolyLog[2,
 1 + (e*x^(1/3))/d]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx &= 3\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx,x,\sqrt[3]{x}\right)\\
&= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right)-(3ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,
\right.\\
&= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right)+3bn\text{Li}_2\left(1+\frac{e\sqrt[3]{x}}{d}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 53, normalized size = 1.04

$$a\log(x)+3b\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)+3bn\text{Li}_2\left(\frac{d+e\sqrt[3]{x}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x,x]
[Out] 3*b*Log[c*(d + e*x^(1/3))^n]*Log[-((e*x^(1/3))/d)] + a*Log[x] + 3*b*n*PolyL
og[2, (d + e*x^(1/3))/d]
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x,x, algorithm="fricas")
[Out] integral((b*log((e*x^(1/3) + d)^n*c) + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)/x, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int\frac{b\ln\left(c\left(e\,x^{\frac{1}{3}}+d\right)^n\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/3)+d)^n)+a)/x,x)
[Out] int((b*ln(c*(e*x^(1/3)+d)^n)+a)/x,x)
```

maxima [B] time = 1.50, size = 166, normalized size = 3.25

$$-3\left(\log\left(\frac{ex^{\frac{1}{3}}}{d}+1\right)\log\left(x^{\frac{1}{3}}\right)+\text{Li}_2\left(-\frac{ex^{\frac{1}{3}}}{d}\right)\right)bn+\frac{4\,bd^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^n\right)\log(x)+4\left(bd^2\log(c)+ad^2\right)\log(x)+\frac{2\,be^2 n}{}}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x,x, algorithm="maxima")
[Out] -3*(log(e*x^(1/3)/d + 1)*log(x^(1/3)) + dilog(-e*x^(1/3)/d))*b*n + 1/4*(4*b
*d^2*log((e*x^(1/3) + d)^n)*log(x) + 4*(b*d^2*log(c) + a*d^2)*log(x) + (2*b
*e^2*n*x*log(x) - 3*b*e^2*n*x)/x^(1/3) - 4*(b*d*e*n*x*log(x) - 3*b*d*e*n*x)
```

```
/x^(2/3))/d^2 + 3/4*(b*e^2*n*x^(2/3) - 4*b*d*e*n*x^(1/3) - 2*(b*e^2*n*x^(2/
3) - 2*b*d*e*n*x^(1/3))*log(x^(1/3)))/d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b\,\ln\left(c\left(d + e\,x^{1/3}\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))/x, x)
```

3.447 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=87

$$-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}-\frac{be^3n\log\left(d+e\sqrt[3]{x}\right)}{d^3}+\frac{be^3n\log(x)}{3d^3}+\frac{be^2n}{d^2\sqrt[3]{x}}-\frac{ben}{2dx^{2/3}}$$

```
[Out] -1/2*b*e*n/d/x^(2/3)+b*e^2*n/d^2/x^(1/3)-b*e^3*n*ln(d+e*x^(1/3))/d^3+(-a-b*
ln(c*(d+e*x^(1/3))^n))/x+1/3*b*e^3*n*ln(x)/d^3
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}+\frac{be^2n}{d^2\sqrt[3]{x}}-\frac{be^3n\log\left(d+e\sqrt[3]{x}\right)}{d^3}+\frac{be^3n\log(x)}{3d^3}-\frac{ben}{2dx^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x^2,x]
```

```
[Out] -(b*e*n)/(2*d*x^(2/3)) + (b*e^2*n)/(d^2*x^(1/3)) - (b*e^3*n*Log[d + e*x^(1/
3)])/d^3 - (a + b*Log[c*(d + e*x^(1/3))^n])/x + (b*e^3*n*Log[x])/(3*d^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x} + (ben)\,\text{Subst}\left(\int \frac{1}{x^3(d+ex)}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x} + (ben)\,\text{Subst}\left(\int \left(\frac{1}{dx^3} - \frac{e}{d^2x^2} + \frac{e^2}{d^3x} - \frac{e^3}{d^3(d+e\right.\right.$$

$$= -\frac{ben}{2dx^{2/3}} + \frac{be^2n}{d^2\sqrt[3]{x}} - \frac{be^3n\log\left(d+e\sqrt[3]{x}\right)}{d^3} - \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x} + \frac{be^3n}{3}$$

Mathematica [A] time = 0.04, size = 84, normalized size = 0.97

$$-\frac{a}{x} - \frac{b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x} + ben\left(-\frac{e^2\log\left(d+e\sqrt[3]{x}\right)}{d^3} + \frac{e^2\log(x)}{3d^3} + \frac{e}{d^2\sqrt[3]{x}} - \frac{1}{2dx^{2/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x^2,x]
```

```
[Out] -(a/x) - (b*Log[c*(d + e*x^(1/3))^n])/x + b*e*n*(-1/2*1/(d*x^(2/3)) + e/(d^
2*x^(1/3)) - (e^2*Log[d + e*x^(1/3)])/d^3 + (e^2*Log[x])/(3*d^3))
```

fricas [A] time = 0.45, size = 81, normalized size = 0.93

$$\frac{2\,be^3nx\log\left(x^{\frac{1}{3}}\right) + 2\,bde^2nx^{\frac{2}{3}} - bd^2enx^{\frac{1}{3}} - 2\,bd^3\log(c) - 2\,ad^3 - 2\left(be^3nx + bd^3n\right)\log\left(ex^{\frac{1}{3}} + d\right)}{2\,d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*e^3*n*x*log(x^(1/3)) + 2*b*d*e^2*n*x^(2/3) - b*d^2*e*n*x^(1/3) - 2
*b*d^3*log(c) - 2*a*d^3 - 2*(b*e^3*n*x + b*d^3*n)*log(e*x^(1/3) + d))/(d^3*
x)
```

giac [B] time = 0.22, size = 280, normalized size = 3.22

$$-\frac{\left(2\left(x^{\frac{1}{3}}e+d\right)^3bne^4\log\left(x^{\frac{1}{3}}e+d\right) - 6\left(x^{\frac{1}{3}}e+d\right)^2bdne^4\log\left(x^{\frac{1}{3}}e+d\right) + 6\left(x^{\frac{1}{3}}e+d\right)bd^2ne^4\log\left(x^{\frac{1}{3}}e+d\right) - 2\left(x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(x^(1/3)*e + d)^3*b*n*e^4*log(x^(1/3)*e + d) - 6*(x^(1/3)*e + d)^2*
b*d*n*e^4*log(x^(1/3)*e + d) + 6*(x^(1/3)*e + d)*b*d^2*n*e^4*log(x^(1/3)*e
+ d) - 2*(x^(1/3)*e + d)^3*b*n*e^4*log(x^(1/3)*e) + 6*(x^(1/3)*e + d)^2*b*d
*n*e^4*log(x^(1/3)*e) - 6*(x^(1/3)*e + d)*b*d^2*n*e^4*log(x^(1/3)*e) + 2*b*
d^3*n*e^4*log(x^(1/3)*e) - 2*(x^(1/3)*e + d)^2*b*d*n*e^4 + 5*(x^(1/3)*e + d
)*b*d^2*n*e^4 - 3*b*d^3*n*e^4 + 2*b*d^3*e^4*log(c) + 2*a*d^3*e^4)*e^(-1)/((
```

```
x^(1/3)*e + d)^3*d^3 - 3*(x^(1/3)*e + d)^2*d^4 + 3*(x^(1/3)*e + d)*d^5 - d^
6)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{b \ln \left(c \left(e x^{\frac{1}{3}}+d\right)^{n}\right)+a}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/3)+d)^n)+a)/x^2,x)
[Out] int((b*ln(c*(e*x^(1/3)+d)^n)+a)/x^2,x)
```

maxima [A] time = 0.62, size = 75, normalized size = 0.86

$$-\frac{1}{6}\, ben\left(\frac{6\, e^{2} \log\left(ex^{\frac{1}{3}}+d\right)}{d^{3}}-\frac{2\, e^{2} \log(x)}{d^{3}}-\frac{3\left(2\, ex^{\frac{1}{3}}-d\right)}{d^{2} x^{\frac{2}{3}}}\right)-\frac{b \log\left(\left(ex^{\frac{1}{3}}+d\right)^{n} c\right)}{x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^2,x, algorithm="maxima")
[Out] -1/6*b*e*n*(6*e^2*log(e*x^(1/3) + d)/d^3 - 2*e^2*log(x)/d^3 - 3*(2*e*x^(1/3
) - d)/(d^2*x^(2/3))) - b*log((e*x^(1/3) + d)^n*c)/x - a/x
```

mupad [B] time = 0.59, size = 74, normalized size = 0.85

$$-\frac{\frac{b\,e\,n}{2\,d}-\frac{b\,e^{2}\,n\,x^{1/3}}{d^{2}}}{x^{2/3}}-\frac{a}{x}-\frac{b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^{n}\right)}{x}-\frac{2\,b\,e^{3}\,n\,\mathrm{atanh}\left(\frac{2\,e\,x^{1/3}}{d}+1\right)}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))/x^2,x)
[Out] - ((b*e*n)/(2*d) - (b*e^2*n*x^(1/3))/d^2)/x^(2/3) - a/x - (b*log(c*(d + e*x
^(1/3))^n))/x - (2*b*e^3*n*atanh((2*e*x^(1/3))/d + 1))/d^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x**2,x)
[Out] Timed out
```

3.448 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=143

$$-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}+\frac{be^6n\log\left(d+e\sqrt[3]{x}\right)}{2d^6}-\frac{be^6n\log(x)}{6d^6}-\frac{be^5n}{2d^5\sqrt[3]{x}}+\frac{be^4n}{4d^4x^{2/3}}-\frac{be^3n}{6d^3x}+\frac{be^2n}{8d^2x^{4/3}}-\frac{ben}{10dx^{5/3}}$$

```
[Out] -1/10*b*e*n/d/x^(5/3)+1/8*b*e^2*n/d^2/x^(4/3)-1/6*b*e^3*n/d^3/x+1/4*b*e^4*n
/d^4/x^(2/3)-1/2*b*e^5*n/d^5/x^(1/3)+1/2*b*e^6*n*ln(d+e*x^(1/3))/d^6+1/2*(-
a-b*ln(c*(d+e*x^(1/3))^n))/x^2-1/6*b*e^6*n*ln(x)/d^6
```

Rubi [A] time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}+\frac{be^4n}{4d^4x^{2/3}}+\frac{be^2n}{8d^2x^{4/3}}-\frac{be^5n}{2d^5\sqrt[3]{x}}-\frac{be^3n}{6d^3x}+\frac{be^6n\log\left(d+e\sqrt[3]{x}\right)}{2d^6}-\frac{be^6n\log(x)}{6d^6}-\frac{ben}{10dx^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x^3,x]
```

```
[Out] -(b*e*n)/(10*d*x^(5/3)) + (b*e^2*n)/(8*d^2*x^(4/3)) - (b*e^3*n)/(6*d^3*x) +
 (b*e^4*n)/(4*d^4*x^(2/3)) - (b*e^5*n)/(2*d^5*x^(1/3)) + (b*e^6*n*Log[d + e
*x^(1/3)])/(2*d^6) - (a + b*Log[c*(d + e*x^(1/3))^n])/(2*x^2) - (b*e^6*n*Lo
g[x])/(6*d^6)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx = 3\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^7}\,dx,x,\sqrt[3]{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\,\text{Subst}\left(\int \frac{1}{x^6(d+ex)}\,dx,x,\sqrt[3]{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\,\text{Subst}\left(\int \left(\frac{1}{dx^6} - \frac{e}{d^2x^5} + \frac{e^2}{d^3x^4} - \frac{e^3}{d^4x^3} + \right.\right.$$
$$= -\frac{ben}{10dx^{5/3}} + \frac{be^2n}{8d^2x^{4/3}} - \frac{be^3n}{6d^3x} + \frac{be^4n}{4d^4x^{2/3}} - \frac{be^5n}{2d^5\sqrt[3]{x}} + \frac{be^6n\log\left(d+e\sqrt[3]{x}\right)}{2d^6} - \frac{a+b}{}$$

Mathematica [A] time = 0.15, size = 134, normalized size = 0.94

$$-\frac{a}{2x^2} - \frac{b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2} + \frac{1}{2}ben\left(\frac{e^5\log\left(d+e\sqrt[3]{x}\right)}{d^6} - \frac{e^5\log(x)}{3d^6} - \frac{e^4}{d^5\sqrt[3]{x}} + \frac{e^3}{2d^4x^{2/3}} - \frac{e^2}{3d^3x} + \frac{e}{4d^2x^{4/3}} - \frac{1}{5dx^{5/3}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*Log[c*(d + e*x^(1/3))^n])/(2*x^2) + (b*e*n*(-1/5*1/(d*x^(5/
3)) + e/(4*d^2*x^(4/3)) - e^2/(3*d^3*x) + e^3/(2*d^4*x^(2/3)) - e^4/(d^5*x^
(1/3)) + (e^5*Log[d + e*x^(1/3)])/d^6 - (e^5*Log[x])/(3*d^6)))/2
```

fricas [A] time = 0.45, size = 125, normalized size = 0.87

$$-\frac{60\,be^6nx^2\log\left(x^{\frac{1}{3}}\right) + 20\,bd^3e^3nx + 60\,bd^6\log(c) + 60\,ad^6 - 60\left(be^6nx^2 - bd^6n\right)\log\left(ex^{\frac{1}{3}} + d\right) + 15\left(4\,bde^5nx - \right.}{120\,d^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^3,x, algorithm="fricas")
```

```
[Out] -1/120*(60*b*e^6*n*x^2*log(x^(1/3)) + 20*b*d^3*e^3*n*x + 60*b*d^6*log(c) +
60*a*d^6 - 60*(b*e^6*n*x^2 - b*d^6*n)*log(e*x^(1/3) + d) + 15*(4*b*d*e^5*n*
x - b*d^4*e^2*n)*x^(2/3) - 6*(5*b*d^2*e^4*n*x - 2*b*d^5*e*n)*x^(1/3))/(d^6*
x^2)
```

giac [B] time = 0.21, size = 542, normalized size = 3.79

$$\frac{\left(60\left(x^{\frac{1}{3}}e+d\right)^6 bne^7\log\left(x^{\frac{1}{3}}e+d\right) - 360\left(x^{\frac{1}{3}}e+d\right)^5 bdne^7\log\left(x^{\frac{1}{3}}e+d\right) + 900\left(x^{\frac{1}{3}}e+d\right)^4 bd^2ne^7\log\left(x^{\frac{1}{3}}e+d\right) - 1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/120*(60*(x^(1/3)*e + d)^6*b*n*e^7*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d
)^5*b*d*n*e^7*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^4*b*d^2*n*e^7*log(x^
(1/3)*e + d) - 1200*(x^(1/3)*e + d)^3*b*d^3*n*e^7*log(x^(1/3)*e + d) + 900*
(x^(1/3)*e + d)^2*b*d^4*n*e^7*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)*b*d^
5*n*e^7*log(x^(1/3)*e + d) - 60*(x^(1/3)*e + d)^6*b*n*e^7*log(x^(1/3)*e) +
```

```
360*(x^(1/3)*e + d)^5*b*d*n*e^7*log(x^(1/3)*e) - 900*(x^(1/3)*e + d)^4*b*d^
2*n*e^7*log(x^(1/3)*e) + 1200*(x^(1/3)*e + d)^3*b*d^3*n*e^7*log(x^(1/3)*e)
- 900*(x^(1/3)*e + d)^2*b*d^4*n*e^7*log(x^(1/3)*e) + 360*(x^(1/3)*e + d)*b*
d^5*n*e^7*log(x^(1/3)*e) - 60*b*d^6*n*e^7*log(x^(1/3)*e) - 60*(x^(1/3)*e +
d)^5*b*d*n*e^7 + 330*(x^(1/3)*e + d)^4*b*d^2*n*e^7 - 740*(x^(1/3)*e + d)^3*
b*d^3*n*e^7 + 855*(x^(1/3)*e + d)^2*b*d^4*n*e^7 - 522*(x^(1/3)*e + d)*b*d^5
*n*e^7 + 137*b*d^6*n*e^7 - 60*b*d^6*e^7*log(c) - 60*a*d^6*e^7)*e^(-1)/((x^(
1/3)*e + d)^6*d^6 - 6*(x^(1/3)*e + d)^5*d^7 + 15*(x^(1/3)*e + d)^4*d^8 - 20
*(x^(1/3)*e + d)^3*d^9 + 15*(x^(1/3)*e + d)^2*d^10 - 6*(x^(1/3)*e + d)*d^11
 + d^12)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{b \ln \left(c \left(e x^{\frac{1}{3}}+d\right)^{n}\right)+a}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/3)+d)^n)+a)/x^3,x)
```

```
[Out] int((b*ln(c*(e*x^(1/3)+d)^n)+a)/x^3,x)
```

maxima [A] time = 0.72, size = 106, normalized size = 0.74

$$\frac{1}{120}\, ben\left(\frac{60\, e^{5} \log\left(ex^{\frac{1}{3}}+d\right)}{d^{6}}-\frac{20\, e^{5} \log(x)}{d^{6}}-\frac{60\, e^{4} x^{\frac{4}{3}}-30\, de^{3} x+20\, d^{2} e^{2} x^{\frac{2}{3}}-15\, d^{3} ex^{\frac{1}{3}}+12\, d^{4}}{d^{5} x^{\frac{5}{3}}}\right)-\frac{b \log\left(\left(ex^{\frac{1}{3}}\right.\right.}{2\, x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^3,x, algorithm="maxima")
```

```
[Out] 1/120*b*e*n*(60*e^5*log(e*x^(1/3) + d)/d^6 - 20*e^5*log(x)/d^6 - (60*e^4*x^
(4/3) - 30*d*e^3*x + 20*d^2*e^2*x^(2/3) - 15*d^3*e*x^(1/3) + 12*d^4)/(d^5*x
^(5/3))) - 1/2*b*log((e*x^(1/3) + d)^n*c)/x^2 - 1/2*a/x^2
```

mupad [B] time = 0.66, size = 109, normalized size = 0.76

$$\frac{b\, e^{6}\, n\, \mathrm{atanh}\left(\frac{2\, e\, x^{1/3}}{d}+1\right)}{d^{6}}-\frac{\frac{b\, e\, n}{5\, d}-\frac{b\, e^{4}\, n\, x}{2\, d^{4}}-\frac{b\, e^{2}\, n\, x^{1/3}}{4\, d^{2}}+\frac{b\, e^{3}\, n\, x^{2/3}}{3\, d^{3}}+\frac{b\, e^{5}\, n\, x^{4/3}}{d^{5}}}{2\, x^{5/3}}-\frac{b\, \ln\left(c\, {\left(d+e\, x^{1/3}\right)}^{n}\right)}{2\, x^{2}}-\frac{a}{2\, x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))/x^3,x)
```

```
[Out] (b*e^6*n*atanh((2*e*x^(1/3))/d + 1))/d^6 - ((b*e*n)/(5*d) - (b*e^4*n*x)/(2*
d^4) - (b*e^2*n*x^(1/3))/(4*d^2) + (b*e^3*n*x^(2/3))/(3*d^3) + (b*e^5*n*x^(
4/3))/d^5)/(2*x^(5/3)) - (b*log(c*(d + e*x^(1/3))^n))/(2*x^2) - a/(2*x^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x**3,x)
```

```
[Out] Timed out
```

3.449 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=192

$$-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3}-\frac{be^9n\log\left(d+e\sqrt[3]{x}\right)}{3d^9}+\frac{be^9n\log(x)}{9d^9}+\frac{be^8n}{3d^8\sqrt[3]{x}}-\frac{be^7n}{6d^7x^{2/3}}+\frac{be^6n}{9d^6x}-\frac{be^5n}{12d^5x^{4/3}}+\frac{be^4n}{15d^4x^{5/3}}-\frac{be}{18d}$$

```
[Out] -1/24*b*e*n/d/x^(8/3)+1/21*b*e^2*n/d^2/x^(7/3)-1/18*b*e^3*n/d^3/x^2+1/15*b*
e^4*n/d^4/x^(5/3)-1/12*b*e^5*n/d^5/x^(4/3)+1/9*b*e^6*n/d^6/x-1/6*b*e^7*n/d^
7/x^(2/3)+1/3*b*e^8*n/d^8/x^(1/3)-1/3*b*e^9*n*ln(d+e*x^(1/3))/d^9+1/3*(-a-b
*ln(c*(d+e*x^(1/3))^n))/x^3+1/9*b*e^9*n*ln(x)/d^9
```

Rubi [A] time = 0.13, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3}-\frac{be^7n}{6d^7x^{2/3}}-\frac{be^5n}{12d^5x^{4/3}}+\frac{be^4n}{15d^4x^{5/3}}-\frac{be^3n}{18d^3x^2}+\frac{be^2n}{21d^2x^{7/3}}+\frac{be^8n}{3d^8\sqrt[3]{x}}+\frac{be^6n}{9d^6x}-\frac{be^9n\log\left(d+e\sqrt[3]{x}\right)}{3d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x^4,x]
```

```
[Out] -(b*e*n)/(24*d*x^(8/3)) + (b*e^2*n)/(21*d^2*x^(7/3)) - (b*e^3*n)/(18*d^3*x^
2) + (b*e^4*n)/(15*d^4*x^(5/3)) - (b*e^5*n)/(12*d^5*x^(4/3)) + (b*e^6*n)/(9
*d^6*x) - (b*e^7*n)/(6*d^7*x^(2/3)) + (b*e^8*n)/(3*d^8*x^(1/3)) - (b*e^9*n*
Log[d + e*x^(1/3)])/(3*d^9) - (a + b*Log[c*(d + e*x^(1/3))^n])/(3*x^3) + (b
*e^9*n*Log[x])/(9*d^9)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx = 3\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3} + \frac{1}{3}(ben)\,\text{Subst}\left(\int \frac{1}{x^9(d+ex)}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3} + \frac{1}{3}(ben)\,\text{Subst}\left(\int \left(\frac{1}{dx^9} - \frac{e}{d^2x^8} + \frac{e^2}{d^3x^7} - \frac{e^3}{d^4x^6}\right.\right.$$
$$= -\frac{ben}{24dx^{8/3}} + \frac{be^2n}{21d^2x^{7/3}} - \frac{be^3n}{18d^3x^2} + \frac{be^4n}{15d^4x^{5/3}} - \frac{be^5n}{12d^5x^{4/3}} + \frac{be^6n}{9d^6x} - \frac{be^7n}{6d^7x^{2/3}} +$$

Mathematica [A] time = 0.20, size = 177, normalized size = 0.92

$$-\frac{a}{3x^3} - \frac{b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3} + \frac{1}{3}ben\left(-\frac{e^8\log\left(d+e\sqrt[3]{x}\right)}{d^9} + \frac{e^8\log(x)}{3d^9} + \frac{e^7}{d^8\sqrt[3]{x}} - \frac{e^6}{2d^7x^{2/3}} + \frac{e^5}{3d^6x} - \frac{e^4}{4d^5x^{4/3}} + \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*Log[c*(d + e*x^(1/3))^n])/(3*x^3) + (b*e*n*(-1/8*1/(d*x^(8/
3)) + e/(7*d^2*x^(7/3)) - e^2/(6*d^3*x^2) + e^3/(5*d^4*x^(5/3)) - e^4/(4*d^
5*x^(4/3)) + e^5/(3*d^6*x) - e^6/(2*d^7*x^(2/3)) + e^7/(d^8*x^(1/3)) - (e^8
*Log[d + e*x^(1/3)])/d^9 + (e^8*Log[x])/(3*d^9)))/3
```

fricas [A] time = 0.48, size = 163, normalized size = 0.85

$$\frac{840\,be^9nx^3\log\left(x^{\frac{1}{3}}\right) + 280\,bd^3e^6nx^2 - 140\,bd^6e^3nx - 840\,bd^9\log(c) - 840\,ad^9 - 840\left(be^9nx^3 + bd^9n\right)\log\left(ex^{\frac{1}{3}}\right.}{2520\,d^9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^4,x, algorithm="fricas")
```

```
[Out] 1/2520*(840*b*e^9*n*x^3*log(x^(1/3)) + 280*b*d^3*e^6*n*x^2 - 140*b*d^6*e^3*
n*x - 840*b*d^9*log(c) - 840*a*d^9 - 840*(b*e^9*n*x^3 + b*d^9*n)*log(e*x^(1
/3) + d) + 30*(28*b*d*e^8*n*x^2 - 7*b*d^4*e^5*n*x + 4*b*d^7*e^2*n)*x^(2/3)
- 21*(20*b*d^2*e^7*n*x^2 - 8*b*d^5*e^4*n*x + 5*b*d^8*e*n)*x^(1/3))/(d^9*x^3
)
```

giac [B] time = 0.22, size = 808, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^4,x, algorithm="giac")
```

```
[Out] -1/2520*(840*(x^(1/3)*e + d)^9*b*n*e^10*log(x^(1/3)*e + d) - 7560*(x^(1/3)*
e + d)^8*b*d*n*e^10*log(x^(1/3)*e + d) + 30240*(x^(1/3)*e + d)^7*b*d^2*n*e^
10*log(x^(1/3)*e + d) - 70560*(x^(1/3)*e + d)^6*b*d^3*n*e^10*log(x^(1/3)*e
+ d) + 105840*(x^(1/3)*e + d)^5*b*d^4*n*e^10*log(x^(1/3)*e + d) - 105840*(x
^(1/3)*e + d)^4*b*d^5*n*e^10*log(x^(1/3)*e + d) + 70560*(x^(1/3)*e + d)^3*b
*d^6*n*e^10*log(x^(1/3)*e + d) - 30240*(x^(1/3)*e + d)^2*b*d^7*n*e^10*log(x
^(1/3)*e + d) + 7560*(x^(1/3)*e + d)*b*d^8*n*e^10*log(x^(1/3)*e + d) - 840*
```

```
(x^(1/3)*e + d)^9*b*n*e^10*log(x^(1/3)*e) + 7560*(x^(1/3)*e + d)^8*b*d*n*e^
10*log(x^(1/3)*e) - 30240*(x^(1/3)*e + d)^7*b*d^2*n*e^10*log(x^(1/3)*e) + 7
0560*(x^(1/3)*e + d)^6*b*d^3*n*e^10*log(x^(1/3)*e) - 105840*(x^(1/3)*e + d)
^5*b*d^4*n*e^10*log(x^(1/3)*e) + 105840*(x^(1/3)*e + d)^4*b*d^5*n*e^10*log(
x^(1/3)*e) - 70560*(x^(1/3)*e + d)^3*b*d^6*n*e^10*log(x^(1/3)*e) + 30240*(x
^(1/3)*e + d)^2*b*d^7*n*e^10*log(x^(1/3)*e) - 7560*(x^(1/3)*e + d)*b*d^8*n*
e^10*log(x^(1/3)*e) + 840*b*d^9*n*e^10*log(x^(1/3)*e) - 840*(x^(1/3)*e + d)
^8*b*d*n*e^10 + 7140*(x^(1/3)*e + d)^7*b*d^2*n*e^10 - 26740*(x^(1/3)*e + d)
^6*b*d^3*n*e^10 + 57750*(x^(1/3)*e + d)^5*b*d^4*n*e^10 - 78918*(x^(1/3)*e +
 d)^4*b*d^5*n*e^10 + 70252*(x^(1/3)*e + d)^3*b*d^6*n*e^10 - 40188*(x^(1/3)*
e + d)^2*b*d^7*n*e^10 + 13827*(x^(1/3)*e + d)*b*d^8*n*e^10 - 2283*b*d^9*n*e
^10 + 840*b*d^9*e^10*log(c) + 840*a*d^9*e^10)*e^(-1)/((x^(1/3)*e + d)^9*d^9
 - 9*(x^(1/3)*e + d)^8*d^10 + 36*(x^(1/3)*e + d)^7*d^11 - 84*(x^(1/3)*e + d
)^6*d^12 + 126*(x^(1/3)*e + d)^5*d^13 - 126*(x^(1/3)*e + d)^4*d^14 + 84*(x^
(1/3)*e + d)^3*d^15 - 36*(x^(1/3)*e + d)^2*d^16 + 9*(x^(1/3)*e + d)*d^17 -
d^18)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{b \ln\left(c \left(e x^{\frac{1}{3}} + d\right)^{n}\right) + a}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/3)+d)^n)+a)/x^4,x)
```

```
[Out] int((b*ln(c*(e*x^(1/3)+d)^n)+a)/x^4,x)
```

maxima [A] time = 0.97, size = 139, normalized size = 0.72

$$-\frac{1}{2520} ben \left(\frac{840 e^{8} \log\left(e x^{\frac{1}{3}} + d\right)}{d^{9}} - \frac{280 e^{8} \log(x)}{d^{9}} - \frac{840 e^{7} x^{\frac{7}{3}} - 420 d e^{6} x^{2} + 280 d^{2} e^{5} x^{\frac{5}{3}} - 210 d^{3} e^{4} x^{\frac{4}{3}} + 168 d^{4} e^{3} x}{d^{8} x^{\frac{8}{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^4,x, algorithm="maxima")
```

```
[Out] -1/2520*b*e*n*(840*e^8*log(e*x^(1/3) + d)/d^9 - 280*e^8*log(x)/d^9 - (840*e
^7*x^(7/3) - 420*d*e^6*x^2 + 280*d^2*e^5*x^(5/3) - 210*d^3*e^4*x^(4/3) + 16
8*d^4*e^3*x - 140*d^5*e^2*x^(2/3) + 120*d^6*e*x^(1/3) - 105*d^7)/(d^8*x^(8/
3))) - 1/3*b*log((e*x^(1/3) + d)^n*c)/x^3 - 1/3*a/x^3
```

mupad [B] time = 0.61, size = 154, normalized size = 0.80

$$-\frac{\frac{a d^{9}}{3} + \frac{b d^{9} \ln\left(c \left(d + e x^{1/3}\right)^{n}\right)}{3} + \frac{b d^{6} e^{3} n x}{18} + \frac{b d^{8} e n x^{1/3}}{24} - \frac{b d e^{8} n x^{8/3}}{3} - \frac{b d^{3} e^{6} n x^{2}}{9} - \frac{b d^{7} e^{2} n x^{2/3}}{21} - \frac{b d^{5} e^{4} n x^{4/3}}{15} + \frac{b d^{4} e^{5} n x^{5/3}}{12} + \frac{b d^{2}}{}}{d^{9} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))/x^4,x)
```

```
[Out] - ((a*d^9)/3 + (b*d^9*log(c*(d + e*x^(1/3))^n))/3 + (b*d^6*e^3*n*x)/18 + (b
*d^8*e*n*x^(1/3))/24 - (b*d*e^8*n*x^(8/3))/3 - (b*d^3*e^6*n*x^2)/9 - (b*d^7
*e^2*n*x^(2/3))/21 - (b*d^5*e^4*n*x^(4/3))/15 + (b*d^4*e^5*n*x^(5/3))/12 +
(b*d^2*e^7*n*x^(7/3))/6)/(d^9*x^3) - (2*b*e^9*n*atanh((2*e*x^(1/3))/d + 1))
/(3*d^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x**4,x)
```

```
[Out] Timed out
```

3.450 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=680

$$\frac{2bd^9n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9}-\frac{6bd^8n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9}+\frac{12bd^7n\left(d+e\sqrt[3]{x}\right.}{}$$

```
[Out] -6*b^2*d^7*n^2*(d+e*x^(1/3))^2/e^9+56/9*b^2*d^6*n^2*(d+e*x^(1/3))^3/e^9-21/
4*b^2*d^5*n^2*(d+e*x^(1/3))^4/e^9+84/25*b^2*d^4*n^2*(d+e*x^(1/3))^5/e^9-14/
9*b^2*d^3*n^2*(d+e*x^(1/3))^6/e^9+24/49*b^2*d^2*n^2*(d+e*x^(1/3))^7/e^9-3/3
2*b^2*d*n^2*(d+e*x^(1/3))^8/e^9+2/243*b^2*n^2*(d+e*x^(1/3))^9/e^9+6*b^2*d^8
*n^2*x^(1/3)/e^8-1/3*b^2*d^9*n^2*ln(d+e*x^(1/3))^2/e^9-6*b*d^8*n*(d+e*x^(1/
3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+12*b*d^7*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d
+e*x^(1/3))^n))/e^9-56/3*b*d^6*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n)
)/e^9+21*b*d^5*n*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-84/5*b*d^4
*n*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+28/3*b*d^3*n*(d+e*x^(1/3
))^6*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-24/7*b*d^2*n*(d+e*x^(1/3))^7*(a+b*ln(c
*(d+e*x^(1/3))^n))/e^9+3/4*b*d*n*(d+e*x^(1/3))^8*(a+b*ln(c*(d+e*x^(1/3))^n)
)/e^9-2/27*b*n*(d+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+2/3*b*d^9*n*
ln(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+1/3*x^3*(a+b*ln(c*(d+e*x^(1
/3))^n))^2
```

Rubi [A] time = 0.70, antiderivative size = 491, normalized size of antiderivative = 0.72, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$-\frac{bn\left(\frac{22680d^8\left(d+e\sqrt[3]{x}\right)}{e^9}-\frac{45360d^7\left(d+e\sqrt[3]{x}\right)^2}{e^9}+\frac{70560d^6\left(d+e\sqrt[3]{x}\right)^3}{e^9}-\frac{79380d^5\left(d+e\sqrt[3]{x}\right)^4}{e^9}+\frac{63504d^4\left(d+e\sqrt[3]{x}\right)^5}{e^9}-\frac{35280d^3\left(d+e\sqrt[3]{x}\right)^6}{e^9}+\frac{1296\ldots}{}\right.}{3780}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (-6*b^2*d^7*n^2*(d + e*x^(1/3))^2)/e^9 + (56*b^2*d^6*n^2*(d + e*x^(1/3))^3)
/(9*e^9) - (21*b^2*d^5*n^2*(d + e*x^(1/3))^4)/(4*e^9) + (84*b^2*d^4*n^2*(d
+ e*x^(1/3))^5)/(25*e^9) - (14*b^2*d^3*n^2*(d + e*x^(1/3))^6)/(9*e^9) + (24
*b^2*d^2*n^2*(d + e*x^(1/3))^7)/(49*e^9) - (3*b^2*d*n^2*(d + e*x^(1/3))^8)/
(32*e^9) + (2*b^2*n^2*(d + e*x^(1/3))^9)/(243*e^9) + (6*b^2*d^8*n^2*x^(1/3)
)/e^8 - (b^2*d^9*n^2*Log[d + e*x^(1/3)]^2)/(3*e^9) - (b*n*((22680*d^8*(d +
e*x^(1/3)))/e^9 - (45360*d^7*(d + e*x^(1/3))^2)/e^9 + (70560*d^6*(d + e*x^(
1/3))^3)/e^9 - (79380*d^5*(d + e*x^(1/3))^4)/e^9 + (63504*d^4*(d + e*x^(1/3
))^5)/e^9 - (35280*d^3*(d + e*x^(1/3))^6)/e^9 + (12960*d^2*(d + e*x^(1/3))^
7)/e^9 - (2835*d*(d + e*x^(1/3))^8)/e^9 + (280*(d + e*x^(1/3))^9)/e^9 - (25
20*d^9*Log[d + e*x^(1/3)])/e^9)*(a + b*Log[c*(d + e*x^(1/3))^n]))/3780 + (x
^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2dx = 3\,\text{Subst}\left(\int x^8\left(a+b\log\left(c(d+ex)^n\right)\right)^2dx,x,\sqrt[3]{x}\right)$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-\frac{1}{3}(2ben)\,\text{Subst}\left(\int\frac{x^9\left(a+b\log\left(c(d+\right.\right.}{d+ex}\right.$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-\frac{1}{3}(2bn)\,\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^9\left(a+b\log\right.}{x}\right.$$

$$=-\frac{bn\left(\frac{22680d^8\left(d+e\sqrt[3]{x}\right)}{e^9}-\frac{45360d^7\left(d+e\sqrt[3]{x}\right)^2}{e^9}+\frac{70560d^6\left(d+e\sqrt[3]{x}\right)^3}{e^9}-\frac{79380d^5\left(d+e\sqrt[3]{x}\right)^4}{e^9}-\right.}{}$$

$$=-\frac{bn\left(\frac{22680d^8\left(d+e\sqrt[3]{x}\right)}{e^9}-\frac{45360d^7\left(d+e\sqrt[3]{x}\right)^2}{e^9}+\frac{70560d^6\left(d+e\sqrt[3]{x}\right)^3}{e^9}-\frac{79380d^5\left(d+e\sqrt[3]{x}\right)^4}{e^9}-\right.}{}$$

$$=-\frac{bn\left(\frac{22680d^8\left(d+e\sqrt[3]{x}\right)}{e^9}-\frac{45360d^7\left(d+e\sqrt[3]{x}\right)^2}{e^9}+\frac{70560d^6\left(d+e\sqrt[3]{x}\right)^3}{e^9}-\frac{79380d^5\left(d+e\sqrt[3]{x}\right)^4}{e^9}-\right.}{}$$

$$=-\frac{6b^2d^7n^2\left(d+e\sqrt[3]{x}\right)^2}{e^9}+\frac{56b^2d^6n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^9}-\frac{21b^2d^5n^2\left(d+e\sqrt[3]{x}\right)^4}{4e^9}+$$

$$=-\frac{6b^2d^7n^2\left(d+e\sqrt[3]{x}\right)^2}{e^9}+\frac{56b^2d^6n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^9}-\frac{21b^2d^5n^2\left(d+e\sqrt[3]{x}\right)^4}{4e^9}+$$

Mathematica [A] time = 0.57, size = 411, normalized size = 0.60

$$\frac{e\sqrt[3]{x}\left(3175200a^2e^8x^{8/3}-2520abn\left(2520d^8-1260d^7e\sqrt[3]{x}+840d^6e^2x^{2/3}-630d^5e^3x+504d^4e^4x^{4/3}-420d^3e^5x^{5/3}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (e*x^(1/3)*(3175200*a^2*e^8*x^(8/3) - 2520*a*b*n*(2520*d^8 - 1260*d^7*e*x^(
1/3) + 840*d^6*e^2*x^(2/3) - 630*d^5*e^3*x + 504*d^4*e^4*x^(4/3) - 420*d^3*
e^5*x^(5/3) + 360*d^2*e^6*x^2 - 315*d*e^7*x^(7/3) + 280*e^8*x^(8/3)) + b^2*
n^2*(17965080*d^8 - 5807340*d^7*e*x^(1/3) + 2813160*d^6*e^2*x^(2/3) - 15806
70*d^5*e^3*x + 947016*d^4*e^4*x^(4/3) - 577500*d^3*e^5*x^(5/3) + 343800*d^2
*e^6*x^2 - 187425*d*e^7*x^(7/3) + 78400*e^8*x^(8/3))) + 2520*b*(2520*a*(d^9
 + e^9*x^3) - b*n*(7129*d^9 + 2520*d^8*e*x^(1/3) - 1260*d^7*e^2*x^(2/3) + 8
40*d^6*e^3*x - 630*d^5*e^4*x^(4/3) + 504*d^4*e^5*x^(5/3) - 420*d^3*e^6*x^2
+ 360*d^2*e^7*x^(7/3) - 315*d*e^8*x^(8/3) + 280*e^9*x^3))*Log[c*(d + e*x^(1
/3))^n] + 3175200*b^2*(d^9 + e^9*x^3)*Log[c*(d + e*x^(1/3))^n]^2)/(9525600*
e^9)
```

fricas [A] time = 0.52, size = 674, normalized size = 0.99

$$\frac{3175200\,b^2e^9x^3\log(c)^2+39200\left(2\,b^2e^9n^2-18\,abe^9n+81\,a^2e^9\right)x^3-2100\left(275\,b^2d^3e^6n^2-504\,abd^3e^6n\right)x^2+3175}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/9525600*(3175200*b^2*e^9*x^3*log(c)^2 + 39200*(2*b^2*e^9*n^2 - 18*a*b*e^9
*n + 81*a^2*e^9)*x^3 - 2100*(275*b^2*d^3*e^6*n^2 - 504*a*b*d^3*e^6*n)*x^2 +
 3175200*(b^2*e^9*n^2*x^3 + b^2*d^9*n^2)*log(e*x^(1/3) + d)^2 + 840*(3349*b
^2*d^6*e^3*n^2 - 2520*a*b*d^6*e^3*n)*x + 2520*(420*b^2*d^3*e^6*n^2*x^2 - 84
0*b^2*d^6*e^3*n^2*x - 7129*b^2*d^9*n^2 + 2520*a*b*d^9*n - 280*(b^2*e^9*n^2
- 9*a*b*e^9*n)*x^3 + 2520*(b^2*e^9*n*x^3 + b^2*d^9*n)*log(c) + 63*(5*b^2*d*
e^8*n^2*x^2 - 8*b^2*d^4*e^5*n^2*x + 20*b^2*d^7*e^2*n^2)*x^(2/3) - 90*(4*b^2
*d^2*e^7*n^2*x^2 - 7*b^2*d^5*e^4*n^2*x + 28*b^2*d^8*e*n^2)*x^(1/3))*log(e*x
^(1/3) + d) + 352800*(3*b^2*d^3*e^6*n*x^2 - 6*b^2*d^6*e^3*n*x - 2*(b^2*e^9*
n - 9*a*b*e^9)*x^3)*log(c) - 63*(92180*b^2*d^7*e^2*n^2 - 50400*a*b*d^7*e^2*
n + 175*(17*b^2*d*e^8*n^2 - 72*a*b*d*e^8*n)*x^2 - 8*(1879*b^2*d^4*e^5*n^2 -
 2520*a*b*d^4*e^5*n)*x - 2520*(5*b^2*d*e^8*n*x^2 - 8*b^2*d^4*e^5*n*x + 20*b
^2*d^7*e^2*n)*log(c))*x^(2/3) + 90*(199612*b^2*d^8*e*n^2 - 70560*a*b*d^8*e*
n + 20*(191*b^2*d^2*e^7*n^2 - 504*a*b*d^2*e^7*n)*x^2 - 7*(2509*b^2*d^5*e^4*
n^2 - 2520*a*b*d^5*e^4*n)*x - 2520*(4*b^2*d^2*e^7*n*x^2 - 7*b^2*d^5*e^4*n*x
 + 28*b^2*d^8*e*n)*log(c))*x^(1/3))/e^9
```

giac [B] time = 0.28, size = 1427, normalized size = 2.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/9525600*(3175200*b^2*x^3*e*log(c)^2 + 6350400*a*b*x^3*e*log(c) + 3175200*
a^2*x^3*e + (3175200*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d)^2 - 285768
00*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d)^2 + 114307200*(x^(1/3)*e +
 d)^7*d^2*e^(-8)*log(x^(1/3)*e + d)^2 - 266716800*(x^(1/3)*e + d)^6*d^3*e^(
-8)*log(x^(1/3)*e + d)^2 + 400075200*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/
3)*e + d)^2 - 400075200*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d)^2 +
 266716800*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d)^2 - 114307200*(x
^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d)^2 + 28576800*(x^(1/3)*e + d)*
d^8*e^(-8)*log(x^(1/3)*e + d)^2 - 705600*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/
3)*e + d) + 7144200*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d) - 3265920
0*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d) + 88905600*(x^(1/3)*e + d
)^6*d^3*e^(-8)*log(x^(1/3)*e + d) - 160030080*(x^(1/3)*e + d)^5*d^4*e^(-8)*
log(x^(1/3)*e + d) + 200037600*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e +
 d) - 177811200*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d) + 114307200
*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d) - 57153600*(x^(1/3)*e + d)
*d^8*e^(-8)*log(x^(1/3)*e + d) + 78400*(x^(1/3)*e + d)^9*e^(-8) - 893025*(x
^(1/3)*e + d)^8*d*e^(-8) + 4665600*(x^(1/3)*e + d)^7*d^2*e^(-8) - 14817600*
(x^(1/3)*e + d)^6*d^3*e^(-8) + 32006016*(x^(1/3)*e + d)^5*d^4*e^(-8) - 5000
9400*(x^(1/3)*e + d)^4*d^5*e^(-8) + 59270400*(x^(1/3)*e + d)^3*d^6*e^(-8) -
 57153600*(x^(1/3)*e + d)^2*d^7*e^(-8) + 57153600*(x^(1/3)*e + d)*d^8*e^(-8
))*b^2*n^2 + 2520*(2520*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d) - 22680
*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d) + 90720*(x^(1/3)*e + d)^7*d^
2*e^(-8)*log(x^(1/3)*e + d) - 211680*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/
3)*e + d) + 317520*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d) - 317520
*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d) + 211680*(x^(1/3)*e + d)^3
*d^6*e^(-8)*log(x^(1/3)*e + d) - 90720*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(
1/3)*e + d) + 22680*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e + d) - 280*(x^
(1/3)*e + d)^9*e^(-8) + 2835*(x^(1/3)*e + d)^8*d*e^(-8) - 12960*(x^(1/3)*e
+ d)^7*d^2*e^(-8) + 35280*(x^(1/3)*e + d)^6*d^3*e^(-8) - 63504*(x^(1/3)*e +
 d)^5*d^4*e^(-8) + 79380*(x^(1/3)*e + d)^4*d^5*e^(-8) - 70560*(x^(1/3)*e +
d)^3*d^6*e^(-8) + 45360*(x^(1/3)*e + d)^2*d^7*e^(-8) - 22680*(x^(1/3)*e + d
)*d^8*e^(-8))*b^2*n*log(c) + 2520*(2520*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3
)*e + d) - 22680*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d) + 90720*(x^(
1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d) - 211680*(x^(1/3)*e + d)^6*d^3*
```

```
e^(-8)*log(x^(1/3)*e + d) + 317520*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)
*e + d) - 317520*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d) + 211680*(
x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d) - 90720*(x^(1/3)*e + d)^2*d^
7*e^(-8)*log(x^(1/3)*e + d) + 22680*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*
e + d) - 280*(x^(1/3)*e + d)^9*e^(-8) + 2835*(x^(1/3)*e + d)^8*d*e^(-8) - 1
2960*(x^(1/3)*e + d)^7*d^2*e^(-8) + 35280*(x^(1/3)*e + d)^6*d^3*e^(-8) - 63
504*(x^(1/3)*e + d)^5*d^4*e^(-8) + 79380*(x^(1/3)*e + d)^4*d^5*e^(-8) - 705
60*(x^(1/3)*e + d)^3*d^6*e^(-8) + 45360*(x^(1/3)*e + d)^2*d^7*e^(-8) - 2268
0*(x^(1/3)*e + d)*d^8*e^(-8))*a*b*n)*e^(-1)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\left(e\,x^{\frac{1}{3}}+d\right)^{n}\right)+a\right)^{2} x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*(e*x^(1/3)+d)^n)+a)^2,x)
```

```
[Out] int(x^2*(b*ln(c*(e*x^(1/3)+d)^n)+a)^2,x)
```

maxima [A] time = 0.59, size = 424, normalized size = 0.62

$$\frac{1}{3}\,b^{2}x^{3}\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)^{2}+\frac{2}{3}\,abx^{3}\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+\frac{1}{3}\,a^{2}x^{3}+\frac{1}{3780}\,aben\left(\frac{2520\,d^{9}\log\left(ex^{\frac{1}{3}}+d\right)}{e^{10}}-\frac{280\,e^{8}x^{3}-31}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*log((e*x^(1/3) + d)^n*c)^2 + 2/3*a*b*x^3*log((e*x^(1/3) + d)^n*
c) + 1/3*a^2*x^3 + 1/3780*a*b*e*n*(2520*d^9*log(e*x^(1/3) + d)/e^10 - (280*
e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^2*e^6*x^(7/3) - 420*d^3*e^5*x^2 + 504*d
^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3) + 840*d^6*e^2*x - 1260*d^7*e*x^(2/3) +
 2520*d^8*x^(1/3))/e^9) + 1/9525600*(2520*e*n*(2520*d^9*log(e*x^(1/3) + d)/
e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^2*e^6*x^(7/3) - 420*d^3*e^5
*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3) + 840*d^6*e^2*x - 1260*d^7
*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9)*log((e*x^(1/3) + d)^n*c) + (78400*e^9*x
^3 - 187425*d*e^8*x^(8/3) + 343800*d^2*e^7*x^(7/3) - 577500*d^3*e^6*x^2 - 3
175200*d^9*log(e*x^(1/3) + d)^2 + 947016*d^4*e^5*x^(5/3) - 1580670*d^5*e^4*
x^(4/3) + 2813160*d^6*e^3*x - 17965080*d^9*log(e*x^(1/3) + d) - 5807340*d^7
*e^2*x^(2/3) + 17965080*d^8*e*x^(1/3))*n^2/e^9)*b^2
```

mupad [B] time = 4.70, size = 608, normalized size = 0.89

$$\frac{a^2\,x^3}{3}+\frac{b^2\,x^3\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)^2}{3}+\frac{2\,b^2\,n^2\,x^3}{243}+\frac{2\,a\,b\,x^3\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)}{3}+\frac{b^2\,d^9\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)^2}{3\,e^9}-\frac{2\,a\,b\,n\,x^3}{27}-\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/3))^n))^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^3*log(c*(d + e*x^(1/3))^n)^2)/3 + (2*b^2*n^2*x^3)/243
+ (2*a*b*x^3*log(c*(d + e*x^(1/3))^n))/3 + (b^2*d^9*log(c*(d + e*x^(1/3))^n
)^2)/(3*e^9) - (2*a*b*n*x^3)/27 - (2*b^2*n*x^3*log(c*(d + e*x^(1/3))^n))/27
 - (7129*b^2*d^9*n^2*log(d + e*x^(1/3)))/(3780*e^9) - (275*b^2*d^3*n^2*x^2)
/(4536*e^3) + (191*b^2*d^2*n^2*x^(7/3))/(5292*e^2) + (1879*b^2*d^4*n^2*x^(5
/3))/(18900*e^4) - (2509*b^2*d^5*n^2*x^(4/3))/(15120*e^5) - (4609*b^2*d^7*n
```

```
^2*x^(2/3))/(7560*e^7) + (7129*b^2*d^8*n^2*x^(1/3))/(3780*e^8) - (17*b^2*d*
n^2*x^(8/3))/(864*e) + (3349*b^2*d^6*n^2*x)/(11340*e^6) + (b^2*d^3*n*x^2*lo
g(c*(d + e*x^(1/3))^n))/(9*e^3) - (2*b^2*d^2*n*x^(7/3)*log(c*(d + e*x^(1/3)
)^n))/(21*e^2) - (2*b^2*d^4*n*x^(5/3)*log(c*(d + e*x^(1/3))^n))/(15*e^4) +
(b^2*d^5*n*x^(4/3)*log(c*(d + e*x^(1/3))^n))/(6*e^5) + (b^2*d^7*n*x^(2/3)*l
og(c*(d + e*x^(1/3))^n))/(3*e^7) - (2*b^2*d^8*n*x^(1/3)*log(c*(d + e*x^(1/3
))^n))/(3*e^8) + (a*b*d*n*x^(8/3))/(12*e) - (2*a*b*d^6*n*x)/(9*e^6) + (2*a*
b*d^9*n*log(d + e*x^(1/3)))/(3*e^9) + (b^2*d*n*x^(8/3)*log(c*(d + e*x^(1/3)
)^n))/(12*e) - (2*b^2*d^6*n*x*log(c*(d + e*x^(1/3))^n))/(9*e^6) + (a*b*d^3*
n*x^2)/(9*e^3) - (2*a*b*d^2*n*x^(7/3))/(21*e^2) - (2*a*b*d^4*n*x^(5/3))/(15
*e^4) + (a*b*d^5*n*x^(4/3))/(6*e^5) + (a*b*d^7*n*x^(2/3))/(3*e^7) - (2*a*b*
d^8*n*x^(1/3))/(3*e^8)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \log \left(c \left(d + e\sqrt[3]{x}\right)^n\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**n))**2,x)
```

```
[Out] Integral(x**2*(a + b*log(c*(d + e*x**(1/3))**n))**2, x)
```

3.451 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=480

$$-\frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6}+\frac{6bd^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6}-\frac{15bd^4n\left(d+e\sqrt[3]{\right.}}{}$$

```
[Out] 15/4*b^2*d^4*n^2*(d+e*x^(1/3))^2/e^6-20/9*b^2*d^3*n^2*(d+e*x^(1/3))^3/e^6+1
5/16*b^2*d^2*n^2*(d+e*x^(1/3))^4/e^6-6/25*b^2*d*n^2*(d+e*x^(1/3))^5/e^6+1/3
6*b^2*n^2*(d+e*x^(1/3))^6/e^6-6*b^2*d^5*n^2*x^(1/3)/e^5+1/2*b^2*d^6*n^2*ln(
d+e*x^(1/3))^2/e^6+6*b*d^5*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-
15/2*b*d^4*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6+20/3*b*d^3*n*(
d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-15/4*b*d^2*n*(d+e*x^(1/3))^4
*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6+6/5*b*d*n*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x
^(1/3))^n))/e^6-1/6*b*n*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-b*d
^6*n*ln(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6+1/2*x^2*(a+b*ln(c*(d+e
*x^(1/3))^n))^2
```

Rubi [A] time = 0.46, antiderivative size = 355, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$\frac{1}{60}bn\left(\frac{360d^5\left(d+e\sqrt[3]{x}\right)}{e^6}-\frac{450d^4\left(d+e\sqrt[3]{x}\right)^2}{e^6}+\frac{400d^3\left(d+e\sqrt[3]{x}\right)^3}{e^6}-\frac{225d^2\left(d+e\sqrt[3]{x}\right)^4}{e^6}-\frac{60d^6\log\left(d+e\sqrt[3]{x}\right)}{e^6}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (15*b^2*d^4*n^2*(d + e*x^(1/3))^2)/(4*e^6) - (20*b^2*d^3*n^2*(d + e*x^(1/3)
)^3)/(9*e^6) + (15*b^2*d^2*n^2*(d + e*x^(1/3))^4)/(16*e^6) - (6*b^2*d*n^2*(
d + e*x^(1/3))^5)/(25*e^6) + (b^2*n^2*(d + e*x^(1/3))^6)/(36*e^6) - (6*b^2*
d^5*n^2*x^(1/3))/e^5 + (b^2*d^6*n^2*Log[d + e*x^(1/3)]^2)/(2*e^6) + (b*n*((
360*d^5*(d + e*x^(1/3)))/e^6 - (450*d^4*(d + e*x^(1/3))^2)/e^6 + (400*d^3*(
d + e*x^(1/3))^3)/e^6 - (225*d^2*(d + e*x^(1/3))^4)/e^6 + (72*d*(d + e*x^(1
/3))^5)/e^6 - (10*(d + e*x^(1/3))^6)/e^6 - (60*d^6*Log[d + e*x^(1/3)])/e^6)
*(a + b*Log[c*(d + e*x^(1/3))^n]))/60 + (x^2*(a + b*Log[c*(d + e*x^(1/3))^n
])^2)/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 dx = 3\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^2 dx, x, \sqrt[3]{x}\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 - (ben)\,\text{Subst}\left(\int \frac{x^6\left(a+b\log\left(c(d+ex)\right.\right.}{d+ex}\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 - (bn)\,\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6\left(a+b\log\left(cx\right.\right.}{x}\right.$$

$$= \frac{1}{60}bn\left(\frac{360d^5\left(d+e\sqrt[3]{x}\right)}{e^6} - \frac{450d^4\left(d+e\sqrt[3]{x}\right)^2}{e^6} + \frac{400d^3\left(d+e\sqrt[3]{x}\right)^3}{e^6} - \frac{225d}{}\right.$$

$$= \frac{1}{60}bn\left(\frac{360d^5\left(d+e\sqrt[3]{x}\right)}{e^6} - \frac{450d^4\left(d+e\sqrt[3]{x}\right)^2}{e^6} + \frac{400d^3\left(d+e\sqrt[3]{x}\right)^3}{e^6} - \frac{225d}{}\right.$$

$$= \frac{1}{60}bn\left(\frac{360d^5\left(d+e\sqrt[3]{x}\right)}{e^6} - \frac{450d^4\left(d+e\sqrt[3]{x}\right)^2}{e^6} + \frac{400d^3\left(d+e\sqrt[3]{x}\right)^3}{e^6} - \frac{225d}{}\right.$$

$$= \frac{15b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^2}{4e^6} - \frac{20b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^6} + \frac{15b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^4}{16e^6} - \frac{6}{}$$

$$= \frac{15b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^2}{4e^6} - \frac{20b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^6} + \frac{15b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^4}{16e^6} - \frac{6}{}$$

Mathematica [A] time = 0.39, size = 301, normalized size = 0.63

$$\frac{e\sqrt[3]{x}\left(1800a^2e^5x^{5/3} + 60abn\left(60d^5 - 30d^4e\sqrt[3]{x} + 20d^3e^2x^{2/3} - 15d^2e^3x + 12de^4x^{4/3} - 10e^5x^{5/3}\right) + b^2n^2\left(-8820d^5 + \right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (e*x^(1/3)*(1800*a^2*e^5*x^(5/3) + 60*a*b*n*(60*d^5 - 30*d^4*e*x^(1/3) + 20
*d^3*e^2*x^(2/3) - 15*d^2*e^3*x + 12*d*e^4*x^(4/3) - 10*e^5*x^(5/3)) + b^2*
n^2*(-8820*d^5 + 2610*d^4*e*x^(1/3) - 1140*d^3*e^2*x^(2/3) + 555*d^2*e^3*x
- 264*d*e^4*x^(4/3) + 100*e^5*x^(5/3))) - 60*b*(60*a*(d^6 - e^6*x^2) + b*n*
(-147*d^6 - 60*d^5*e*x^(1/3) + 30*d^4*e^2*x^(2/3) - 20*d^3*e^3*x + 15*d^2*e
^4*x^(4/3) - 12*d*e^5*x^(5/3) + 10*e^6*x^2))*Log[c*(d + e*x^(1/3))^n] - 180
0*b^2*(d^6 - e^6*x^2)*Log[c*(d + e*x^(1/3))^n]^2)/(3600*e^6)
```

fricas [A] time = 0.52, size = 484, normalized size = 1.01

$$\frac{1800\,b^2e^6x^2\log(c)^2 + 100\left(b^2e^6n^2 - 6\,abe^6n + 18\,a^2e^6\right)x^2 + 1800\left(b^2e^6n^2x^2 - b^2d^6n^2\right)\log\left(ex^{\frac{1}{3}} + d\right)^2 - 60\left(19\,b^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/3600*(1800*b^2*e^6*x^2*log(c)^2 + 100*(b^2*e^6*n^2 - 6*a*b*e^6*n + 18*a^2
*e^6)*x^2 + 1800*(b^2*e^6*n^2*x^2 - b^2*d^6*n^2)*log(e*x^(1/3) + d)^2 - 60*
(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x + 60*(20*b^2*d^3*e^3*n^2*x + 147*
```

```
b^2*d^6*n^2 - 60*a*b*d^6*n - 10*(b^2*e^6*n^2 - 6*a*b*e^6*n)*x^2 + 60*(b^2*e
^6*n*x^2 - b^2*d^6*n)*log(c) + 6*(2*b^2*d*e^5*n^2*x - 5*b^2*d^4*e^2*n^2)*x^
(2/3) - 15*(b^2*d^2*e^4*n^2*x - 4*b^2*d^5*e*n^2)*x^(1/3))*log(e*x^(1/3) + d
) + 600*(2*b^2*d^3*e^3*n*x - (b^2*e^6*n - 6*a*b*e^6)*x^2)*log(c) + 6*(435*b
^2*d^4*e^2*n^2 - 300*a*b*d^4*e^2*n - 4*(11*b^2*d*e^5*n^2 - 30*a*b*d*e^5*n)*
x + 60*(2*b^2*d*e^5*n*x - 5*b^2*d^4*e^2*n)*log(c))*x^(2/3) - 15*(588*b^2*d^
5*e*n^2 - 240*a*b*d^5*e*n - (37*b^2*d^2*e^4*n^2 - 60*a*b*d^2*e^4*n)*x + 60*
(b^2*d^2*e^4*n*x - 4*b^2*d^5*e*n)*log(c))*x^(1/3))/e^6
```

giac [B] time = 0.27, size = 956, normalized size = 1.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/3600*(1800*b^2*x^2*e*log(c)^2 + 3600*a*b*x^2*e*log(c) + (1800*(x^(1/3)*e
+ d)^6*e^(-5)*log(x^(1/3)*e + d)^2 - 10800*(x^(1/3)*e + d)^5*d*e^(-5)*log(x
^(1/3)*e + d)^2 + 27000*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d)^2 -
 36000*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d)^2 + 27000*(x^(1/3)*e
 + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d)^2 - 10800*(x^(1/3)*e + d)*d^5*e^(-5)*
log(x^(1/3)*e + d)^2 - 600*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) + 43
20*(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/3)*e + d) - 13500*(x^(1/3)*e + d)^4*
d^2*e^(-5)*log(x^(1/3)*e + d) + 24000*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1
/3)*e + d) - 27000*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d) + 21600*
(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e + d) + 100*(x^(1/3)*e + d)^6*e^(-5
) - 864*(x^(1/3)*e + d)^5*d*e^(-5) + 3375*(x^(1/3)*e + d)^4*d^2*e^(-5) - 80
00*(x^(1/3)*e + d)^3*d^3*e^(-5) + 13500*(x^(1/3)*e + d)^2*d^4*e^(-5) - 2160
0*(x^(1/3)*e + d)*d^5*e^(-5))*b^2*n^2 + 1800*a^2*x^2*e + 60*(60*(x^(1/3)*e
+ d)^6*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/
3)*e + d) + 900*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d) - 1200*(x^(
1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^2*d^4*e^(
-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e + d)
- 10*(x^(1/3)*e + d)^6*e^(-5) + 72*(x^(1/3)*e + d)^5*d*e^(-5) - 225*(x^(1/3
)*e + d)^4*d^2*e^(-5) + 400*(x^(1/3)*e + d)^3*d^3*e^(-5) - 450*(x^(1/3)*e +
 d)^2*d^4*e^(-5) + 360*(x^(1/3)*e + d)*d^5*e^(-5))*b^2*n*log(c) + 60*(60*(x
^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)^5*d*e^(-5)*
log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d) -
1200*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^
2*d^4*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3
)*e + d) - 10*(x^(1/3)*e + d)^6*e^(-5) + 72*(x^(1/3)*e + d)^5*d*e^(-5) - 22
5*(x^(1/3)*e + d)^4*d^2*e^(-5) + 400*(x^(1/3)*e + d)^3*d^3*e^(-5) - 450*(x^
(1/3)*e + d)^2*d^4*e^(-5) + 360*(x^(1/3)*e + d)*d^5*e^(-5))*a*b*n)*e^(-1)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\left(e\,x^{\frac{1}{3}}+d\right)^{n}\right)+a\right)^{2} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(e*x^(1/3)+d)^n)+a)^2,x)
```

```
[Out] int(x*(b*ln(c*(e*x^(1/3)+d)^n)+a)^2,x)
```

maxima [A] time = 0.55, size = 323, normalized size = 0.67

$$\frac{1}{2}\,b^2x^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)^2-\frac{1}{60}\,aben\left(\frac{60\,d^6\log\left(ex^{\frac{1}{3}}+d\right)}{e^7}+\frac{10\,e^5x^2-12\,de^4x^{\frac{5}{3}}+15\,d^2e^3x^{\frac{4}{3}}-20\,d^3e^2x+30\,d^4e}{e^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log((e*x^(1/3) + d)^n*c)^2 - 1/60*a*b*e*n*(60*d^6*log(e*x^(1/3)
 + d)/e^7 + (10*e^5*x^2 - 12*d*e^4*x^(5/3) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^
2*x + 30*d^4*e*x^(2/3) - 60*d^5*x^(1/3))/e^6) + a*b*x^2*log((e*x^(1/3) + d)
^n*c) + 1/2*a^2*x^2 - 1/3600*(60*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e
^5*x^2 - 12*d*e^4*x^(5/3) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^
(2/3) - 60*d^5*x^(1/3))/e^6)*log((e*x^(1/3) + d)^n*c) - (100*e^6*x^2 + 1800
*d^6*log(e*x^(1/3) + d)^2 - 264*d*e^5*x^(5/3) + 555*d^2*e^4*x^(4/3) - 1140*
d^3*e^3*x + 8820*d^6*log(e*x^(1/3) + d) + 2610*d^4*e^2*x^(2/3) - 8820*d^5*e
*x^(1/3))*n^2/e^6)*b^2
```

mupad [B] time = 1.72, size = 431, normalized size = 0.90

$$\frac{a^2\,x^2}{2}+\frac{b^2\,x^2\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)^2}{2}+\frac{b^2\,n^2\,x^2}{36}+a\,b\,x^2\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)-\frac{b^2\,d^6\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)^2}{2\,e^6}-\frac{a\,b\,n\,x^2}{6}-\frac{b^2\,n\,x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(1/3))^n))^2,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*x^2*log(c*(d + e*x^(1/3))^n)^2)/2 + (b^2*n^2*x^2)/36 + a
*b*x^2*log(c*(d + e*x^(1/3))^n) - (b^2*d^6*log(c*(d + e*x^(1/3))^n)^2)/(2*e
^6) - (a*b*n*x^2)/6 - (b^2*n*x^2*log(c*(d + e*x^(1/3))^n))/6 + (49*b^2*d^6*
n^2*log(d + e*x^(1/3)))/(20*e^6) + (37*b^2*d^2*n^2*x^(4/3))/(240*e^2) + (29
*b^2*d^4*n^2*x^(2/3))/(40*e^4) - (49*b^2*d^5*n^2*x^(1/3))/(20*e^5) - (19*b^
2*d^3*n^2*x)/(60*e^3) - (11*b^2*d*n^2*x^(5/3))/(150*e) - (b^2*d^2*n*x^(4/3)
*log(c*(d + e*x^(1/3))^n))/(4*e^2) - (b^2*d^4*n*x^(2/3)*log(c*(d + e*x^(1/3
))^n))/(2*e^4) + (b^2*d^5*n*x^(1/3)*log(c*(d + e*x^(1/3))^n))/e^5 + (a*b*d^
3*n*x)/(3*e^3) + (a*b*d*n*x^(5/3))/(5*e) - (a*b*d^6*n*log(d + e*x^(1/3)))/e
^6 + (b^2*d^3*n*x*log(c*(d + e*x^(1/3))^n))/(3*e^3) + (b^2*d*n*x^(5/3)*log(
c*(d + e*x^(1/3))^n))/(5*e) - (a*b*d^2*n*x^(4/3))/(4*e^2) - (a*b*d^4*n*x^(2
/3))/(2*e^4) + (a*b*d^5*n*x^(1/3))/e^5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))**n))**2,x)
```

```
[Out] Integral(x*(a + b*log(c*(d + e*x**(1/3))**n))**2, x)
```

3.452 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=267

$$\frac{2bd^3n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}-\frac{6bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}+\frac{3bdn\left(d+e\sqrt[3]{x}\right.}{}$$

```
[Out] -3/2*b^2*d*n^2*(d+e*x^(1/3))^2/e^3+2/9*b^2*n^2*(d+e*x^(1/3))^3/e^3+6*b^2*d^
2*n^2*x^(1/3)/e^2-b^2*d^3*n^2*ln(d+e*x^(1/3))^2/e^3-6*b*d^2*n*(d+e*x^(1/3))
*(a+b*ln(c*(d+e*x^(1/3))^n))/e^3+3*b*d*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(
1/3))^n))/e^3-2/3*b*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^3+2*b*d
^3*n*ln(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^3+x*(a+b*ln(c*(d+e*x^(1/
3))^n))^2
```

Rubi [A] time = 0.29, antiderivative size = 210, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2451, 2398, 2411, 43, 2334, 12, 14, 2301}

$$-\frac{1}{3}bn\left(\frac{18d^2\left(d+e\sqrt[3]{x}\right)}{e^3}-\frac{6d^3\log\left(d+e\sqrt[3]{x}\right)}{e^3}-\frac{9d\left(d+e\sqrt[3]{x}\right)^2}{e^3}+\frac{2\left(d+e\sqrt[3]{x}\right)^3}{e^3}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)+x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (-3*b^2*d*n^2*(d + e*x^(1/3))^2)/(2*e^3) + (2*b^2*n^2*(d + e*x^(1/3))^3)/(9
*e^3) + (6*b^2*d^2*n^2*x^(1/3))/e^2 - (b^2*d^3*n^2*Log[d + e*x^(1/3)]^2)/e^
3 - (b*n*((18*d^2*(d + e*x^(1/3)))/e^3 - (9*d*(d + e*x^(1/3))^2)/e^3 + (2*(
d + e*x^(1/3))^3)/e^3 - (6*d^3*Log[d + e*x^(1/3)])/e^3)*(a + b*Log[c*(d + e
*x^(1/3))^n]))/3 + x*(a + b*Log[c*(d + e*x^(1/3))^n])^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
```

```
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx = 3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-(2ben)\,\text{Subst}\left(\int \frac{x^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}\right.$$

$$= x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-(2bn)\,\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3\left(a+b\log\left(cx^n\right)\right)}{x}\right.$$

$$= -\frac{1}{3}bn\left(\frac{18d^2\left(d+e\sqrt[3]{x}\right)}{e^3}-\frac{9d\left(d+e\sqrt[3]{x}\right)^2}{e^3}+\frac{2\left(d+e\sqrt[3]{x}\right)^3}{e^3}-\frac{6d^3\log\left(d+e\sqrt[3]{\right.}}{e^3}\right.$$

$$= -\frac{1}{3}bn\left(\frac{18d^2\left(d+e\sqrt[3]{x}\right)}{e^3}-\frac{9d\left(d+e\sqrt[3]{x}\right)^2}{e^3}+\frac{2\left(d+e\sqrt[3]{x}\right)^3}{e^3}-\frac{6d^3\log\left(d+e\sqrt[3]{\right.}}{e^3}\right.$$

$$= -\frac{1}{3}bn\left(\frac{18d^2\left(d+e\sqrt[3]{x}\right)}{e^3}-\frac{9d\left(d+e\sqrt[3]{x}\right)^2}{e^3}+\frac{2\left(d+e\sqrt[3]{x}\right)^3}{e^3}-\frac{6d^3\log\left(d+e\sqrt[3]{\right.}}{e^3}\right.$$

$$= -\frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^2}{2e^3}+\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^3}+\frac{6b^2d^2n^2\sqrt[3]{x}}{e^2}-\frac{1}{3}bn\left(\frac{18d^2\left(d+\right.}{e^3}\right.$$

$$= -\frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^2}{2e^3}+\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^3}+\frac{6b^2d^2n^2\sqrt[3]{x}}{e^2}-\frac{b^2d^3n^2\log^2\left(d\right.}{e^3}$$

Mathematica [A] time = 0.13, size = 197, normalized size = 0.74

$$\frac{18a^2\left(d^3+e^3x\right)+6b\left(6a\left(d^3+e^3x\right)-bn\left(11d^3+6d^2e\sqrt[3]{x}-3de^2x^{2/3}+2e^3x\right)\right)\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)+6abn\left(7d^3\right.}{18e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (b^2*e*n^2*(66*d^2 - 15*d*e*x^(1/3) + 4*e^2*x^(2/3))*x^(1/3) + 6*a*b*n*(7*d
^3 - 6*d^2*e*x^(1/3) + 3*d*e^2*x^(2/3) - 2*e^3*x) + 18*a^2*(d^3 + e^3*x) +
6*b*(6*a*(d^3 + e^3*x) - b*n*(11*d^3 + 6*d^2*e*x^(1/3) - 3*d*e^2*x^(2/3) +
2*e^3*x))*Log[c*(d + e*x^(1/3))^n] + 18*b^2*(d^3 + e^3*x)*Log[c*(d + e*x^(1
/3))^n]^2)/(18*e^3)
```

fricas [A] time = 0.46, size = 287, normalized size = 1.07

$$18\,b^2e^3x\log(c)^2+18\left(b^2e^3n^2x+b^2d^3n^2\right)\log\left(ex^{\frac{1}{3}}+d\right)^2-12\left(b^2e^3n-3\,abe^3\right)x\log(c)+2\left(2\,b^2e^3n^2-6\,abe^3n\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/18*(18*b^2*e^3*x*log(c)^2 + 18*(b^2*e^3*n^2*x + b^2*d^3*n^2)*log(e*x^(1/3
) + d)^2 - 12*(b^2*e^3*n - 3*a*b*e^3)*x*log(c) + 2*(2*b^2*e^3*n^2 - 6*a*b*e
^3*n + 9*a^2*e^3)*x + 6*(3*b^2*d*e^2*n^2*x^(2/3) - 6*b^2*d^2*e*n^2*x^(1/3)
- 11*b^2*d^3*n^2 + 6*a*b*d^3*n - 2*(b^2*e^3*n^2 - 3*a*b*e^3*n)*x + 6*(b^2*e
^3*n*x + b^2*d^3*n)*log(c))*log(e*x^(1/3) + d) - 3*(5*b^2*d*e^2*n^2 - 6*b^2
*d*e^2*n*log(c) - 6*a*b*d*e^2*n)*x^(2/3) + 6*(11*b^2*d^2*e*n^2 - 6*b^2*d^2*
e*n*log(c) - 6*a*b*d^2*e*n)*x^(1/3))/e^3
```

giac [B] time = 0.21, size = 479, normalized size = 1.79

$$\frac{1}{18}\left(18\,b^2xe\log(c)^2+\left(18\left(x^{\frac{1}{3}}e+d\right)^3e^{(-2)}\log\left(x^{\frac{1}{3}}e+d\right)^2-54\left(x^{\frac{1}{3}}e+d\right)^2de^{(-2)}\log\left(x^{\frac{1}{3}}e+d\right)^2+54\left(x^{\frac{1}{3}}e+d\right)d^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/18*(18*b^2*x*e*log(c)^2 + (18*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d)
^2 - 54*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d)^2 + 54*(x^(1/3)*e + d
)*d^2*e^(-2)*log(x^(1/3)*e + d)^2 - 12*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)
*e + d) + 54*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d) - 108*(x^(1/3)*e
 + d)*d^2*e^(-2)*log(x^(1/3)*e + d) + 4*(x^(1/3)*e + d)^3*e^(-2) - 27*(x^(1
/3)*e + d)^2*d*e^(-2) + 108*(x^(1/3)*e + d)*d^2*e^(-2))*b^2*n^2 + 6*(6*(x^(
1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d) - 18*(x^(1/3)*e + d)^2*d*e^(-2)*log
(x^(1/3)*e + d) + 18*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d) - 2*(x^(
1/3)*e + d)^3*e^(-2) + 9*(x^(1/3)*e + d)^2*d*e^(-2) - 18*(x^(1/3)*e + d)*d^
2*e^(-2))*b^2*n*log(c) + 36*a*b*x*e*log(c) + 6*(6*(x^(1/3)*e + d)^3*e^(-2)*
log(x^(1/3)*e + d) - 18*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d) + 18*
(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d) - 2*(x^(1/3)*e + d)^3*e^(-2)
+ 9*(x^(1/3)*e + d)^2*d*e^(-2) - 18*(x^(1/3)*e + d)*d^2*e^(-2))*a*b*n + 18*
a^2*x*e)*e^(-1)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(e\,x^{\frac{1}{3}}+d\right)^n\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/3)+d)^n)+a)^2,x)
```

```
[Out] int((b*ln(c*(e*x^(1/3)+d)^n)+a)^2,x)
```

maxima [A] time = 0.57, size = 217, normalized size = 0.81

$$\frac{1}{3}\left(en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-3\,dex^{\frac{2}{3}}+6\,d^2x^{\frac{1}{3}}}{e^3}\right)+6\,x\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)\right)ab+\frac{1}{18}\left(6\,en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(e*n*(6*d^3*log(e*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*x
^(1/3))/e^3) + 6*x*log((e*x^(1/3) + d)^n*c))*a*b + 1/18*(6*e*n*(6*d^3*log(e
*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*x^(1/3))/e^3)*log((e*x
^(1/3) + d)^n*c) + 18*x*log((e*x^(1/3) + d)^n*c)^2 - (18*d^3*log(e*x^(1/3)
+ d)^2 - 4*e^3*x + 66*d^3*log(e*x^(1/3) + d) + 15*d*e^2*x^(2/3) - 66*d^2*e*
x^(1/3))*n^2/e^3)*b^2 + a^2*x
```

mupad [B] time = 0.51, size = 290, normalized size = 1.09

$$\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)\left(\frac{2\,b\,x\,\left(3\,a-b\,n\right)}{3}-x^{2/3}\left(\frac{b\,d\,\left(3\,a-b\,n\right)}{e}-\frac{3\,a\,b\,d}{e}\right)+\frac{d\,x^{1/3}\,\left(\frac{2\,b\,d\,\left(3\,a-b\,n\right)}{e}-\frac{6\,a\,b\,d}{e}\right)}{e}\right)-x^{2/3}\left(\frac{d\,\left(3\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^2,x)
```

```
[Out] log(c*(d + e*x^(1/3))^n)*((2*b*x*(3*a - b*n))/3 - x^(2/3)*((b*d*(3*a - b*n)
)/e - (3*a*b*d)/e) + (d*x^(1/3)*((2*b*d*(3*a - b*n))/e - (6*a*b*d)/e))/e) -
 x^(2/3)*((d*(3*a^2 + (2*b^2*n^2)/3 - 2*a*b*n))/(2*e) - (d*(3*a^2 - b^2*n^2
))/(2*e)) + x^(1/3)*((d*((d*(3*a^2 + (2*b^2*n^2)/3 - 2*a*b*n))/e - (d*(3*a^
2 - b^2*n^2))/e))/e + (2*b^2*d^2*n^2)/e^2) + x*(a^2 + (2*b^2*n^2)/9 - (2*a*
b*n)/3) + log(c*(d + e*x^(1/3))^n)^2*(b^2*x + (b^2*d^3)/e^3) - (log(d + e*x
^(1/3))*(11*b^2*d^3*n^2 - 6*a*b*d^3*n))/(3*e^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**2, x)
```

3.453 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx$

Optimal. Leaf size=93

$$6bn\text{Li}_2\left(\frac{\sqrt[3]{x}e}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)+3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-6b^2n^2\text{Li}_3\left(\frac{\sqrt[3]{x}e}{d}+$$

```
[Out] 3*(a+b*ln(c*(d+e*x^(1/3))^n))^2*ln(-e*x^(1/3)/d)+6*b*n*(a+b*ln(c*(d+e*x^(1/
3))^n))*polylog(2,1+e*x^(1/3)/d)-6*b^2*n^2*polylog(3,1+e*x^(1/3)/d)
```

Rubi [A] time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2454, 2396, 2433, 2374, 6589}

$$6bn\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-6b^2n^2\text{PolyLog}\left(3,\frac{e\sqrt[3]{x}}{d}+1\right)+3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x,x]
```

```
[Out] 3*(a + b*Log[c*(d + e*x^(1/3))^n])^2*Log[-((e*x^(1/3))/d)] + 6*b*n*(a + b*L
og[c*(d + e*x^(1/3))^n])*PolyLog[2, 1 + (e*x^(1/3))/d] - 6*b^2*n^2*PolyLog[
3, 1 + (e*x^(1/3))/d]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x}\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 \log\left(-\frac{e\sqrt[3]{x}}{d}\right) - (6ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\left(a\right.}{}\right.$$

$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 \log\left(-\frac{e\sqrt[3]{x}}{d}\right) - (6bn)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(cx\right.\right.}{}\right.$$

$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 \log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right.$$

$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 \log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right.$$

Mathematica [B] time = 0.11, size = 195, normalized size = 2.10

$$2bn\left(\log(x)\left(\log\left(d+e\sqrt[3]{x}\right)-\log\left(\frac{e\sqrt[3]{x}}{d}+1\right)\right)-3\text{Li}_2\left(-\frac{e\sqrt[3]{x}}{d}\right)\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-bn\log\left(d+e\sqrt[3]{x}\right)\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])*((Log[d + e*x^(1/
3)] - Log[1 + (e*x^(1/3))/d])*Log[x] - 3*PolyLog[2, -((e*x^(1/3))/d)]) + 3*
b^2*n^2*(Log[d + e*x^(1/3)]^2*Log[-((e*x^(1/3))/d)] + 2*Log[d + e*x^(1/3)]*
PolyLog[2, 1 + (e*x^(1/3))/d] - 2*PolyLog[3, 1 + (e*x^(1/3))/d])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)^2+2\,ab\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(1/3) + d)^n*c)^2 + 2*a*b*log((e*x^(1/3) + d)^n*c) +
 a^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)^2/x, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(c \left(e x^{\frac{1}{3}}+d\right)^{n}\right)+a\right)^{2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/3)+d)^n)+a)^2/x,x)
```

```
[Out] int((b*ln(c*(e*x^(1/3)+d)^n)+a)^2/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2 \log\left(\left(ex^{\frac{1}{3}}+d\right)^n\right)^2 \log(x)+\int \frac{3\left(b^2e\log(c)^2+2\,abe\log(c)+a^2e\right)x-2\left(b^2enx\log(x)-3\left(b^2e\log(c)+abe\right)x\right.}{3\left(e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*log((e*x^(1/3) + d)^n)^2*log(x) + integrate(1/3*(3*(b^2*e*log(c)^2 + 2*
a*b*e*log(c) + a^2*e)*x - 2*(b^2*e*n*x*log(x) - 3*(b^2*e*log(c) + a*b*e)*x
- 3*(b^2*d*log(c) + a*b*d)*x^(2/3))*log((e*x^(1/3) + d)^n) + 3*(b^2*d*log(c
)^2 + 2*a*b*d*log(c) + a^2*d)*x^(2/3))/(e*x^2 + d*x^(5/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)\right)}^2}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^2/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \log{\left(c \left(d + e \sqrt[3]{x}\right)^{n} \right)}\right)^{2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**2/x, x)
```

3.454 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=231

$$\frac{2be^3n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}+\frac{2be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}}-\frac{ben\left(a+b\log\left(\right.\right.}{dx}$$

```
[Out] -b^2*e^2*n^2/d^2/x^(1/3)+b^2*e^3*n^2*ln(d+e*x^(1/3))/d^3-b*e*n*(a+b*ln(c*(d
+e*x^(1/3))^n))/d/x^(2/3)+2*b*e^2*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n
))/d^3/x^(1/3)+2*b*e^3*n*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*(d+e*x^(1/3))^n))/
d^3-(a+b*ln(c*(d+e*x^(1/3))^n))^2/x-b^2*e^3*n^2*ln(x)/d^3-2*b^2*e^3*n^2*pol
ylog(2,d/(d+e*x^(1/3)))/d^3
```

Rubi [A] time = 0.50, antiderivative size = 253, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$\frac{2b^2e^3n^2\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)}{d^3}-\frac{e^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{d^3}+\frac{2be^3n\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}+\frac{2}{}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^2,x]
```

```
[Out] -((b^2*e^2*n^2)/(d^2*x^(1/3))) + (b^2*e^3*n^2*Log[d + e*x^(1/3)])/d^3 - (b*
e*n*(a + b*Log[c*(d + e*x^(1/3))^n]))/(d*x^(2/3)) + (2*b*e^2*n*(d + e*x^(1/
3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/(d^3*x^(1/3)) - (e^3*(a + b*Log[c*(d
+ e*x^(1/3))^n])^2)/d^3 - (a + b*Log[c*(d + e*x^(1/3))^n])^2/x + (2*b*e^3*n
*(a + b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)])/d^3 - (b^2*e^3*n^2
*Log[x])/d^3 + (2*b^2*e^3*n^2*PolyLog[2, 1 + (e*x^(1/3))/d])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x} + (2ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3(d+ex)}\,dx,\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x} + (2bn)\,\text{Subst}\left(\int \frac{a+b\log\left(cx^n\right)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x} + \frac{(2bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)}{d}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x} - \frac{(2ben)\,\text{Su}}{}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}} + \frac{2be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.\right.}{d^3\sqrt[3]{x}}$$

$$= -\frac{b^2e^2n^2}{d^2\sqrt[3]{x}} + \frac{b^2e^3n^2\log\left(d+e\sqrt[3]{x}\right)}{d^3} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}} + \frac{2be^2n}{}$$

$$= -\frac{b^2e^2n^2}{d^2\sqrt[3]{x}} + \frac{b^2e^3n^2\log\left(d+e\sqrt[3]{x}\right)}{d^3} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}} + \frac{2be^2n}{}$$

Mathematica [A] time = 0.23, size = 274, normalized size = 1.19

$$3\left(\frac{2}{3}ben\left(-\frac{e^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2bd^3n} + \frac{e^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3} + \frac{e\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.\right.}{d^2\sqrt[3]{x}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^2,x]
```

```
[Out] 3*(-1/3*(a + b*Log[c*(d + e*x^(1/3))^n])^2/x + (2*b*e*n*(-1/2*(a + b*Log[c*
(d + e*x^(1/3))^n])/(d*x^(2/3)) + (e*(a + b*Log[c*(d + e*x^(1/3))^n]))/(d^2
*x^(1/3)) - (e^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*b*d^3*n) + (e^2*(a
+ b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)])/d^3 - (b*e^2*n*(-(Log[
d + e*x^(1/3)]/d) + Log[x]/(3*d)))/d^2 - (b*e*n*(1/(d*x^(1/3)) - (e*Log[d +
 e*x^(1/3)])/d^2 + (e*Log[x])/(3*d^2)))/(2*d) + (b*e^2*n*PolyLog[2, (d + e*
x^(1/3))/d])/d^3))/3)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)^2 + 2\,ab\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right) + a^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(1/3) + d)^n*c)^2 + 2*a*b*log((e*x^(1/3) + d)^n*c) +
 a^2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)^2/x^2, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(e\,x^{\frac{1}{3}}+d\right)^{n}\right)+a\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/3)+d)^n)+a)^2/x^2,x)
```

```
[Out] int((b*ln(c*(e*x^(1/3)+d)^n)+a)^2/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(\log\left(\frac{ex^{\frac{1}{3}}}{d}+1\right)\log\left(x^{\frac{1}{3}}\right)+\mathrm{Li}_2\left(-\frac{ex^{\frac{1}{3}}}{d}\right)\right)b^2e^3n^2}{d^3}-\frac{\left(2\,abe^3n-\left(3\,e^3n^2-2\,e^3n\log(c)\right)b^2\right)\log\left(ex^{\frac{1}{3}}+d\right)}{d^3}+\frac{2\left(b^2e^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -2*(log(e*x^(1/3)/d + 1)*log(x^(1/3)) + dilog(-e*x^(1/3)/d))*b^2*e^3*n^2/d^
3 - (2*a*b*e^3*n - (3*e^3*n^2 - 2*e^3*n*log(c))*b^2)*log(e*x^(1/3) + d)/d^3
 + 2*(b^2*e^3*n*log(c) + a*b*e^3*n)*log(x^(1/3))/d^3 + integrate((b^2*e^6*n
^2*x - b^2*d^3*e^3*n^2)/x, x)/d^6 - 1/20*(12*b^2*e^8*n^2*x^(5/3) - 15*b^2*d
*e^7*n^2*x^(4/3) + 20*b^2*d^2*e^6*n^2*x - 40*b^2*d^3*e^5*n^2*x^(2/3) + 100*
b^2*d^4*e^4*n^2*x^(1/3) + 20*(b^2*d^3*e^5*n^2*x^(2/3) - 2*b^2*d^4*e^4*n^2*x
^(1/3))*log(x^(1/3)))/d^8 + 1/60*(60*b^2*d^5*e^3*n^2*x^(5/3)*log(e*x^(1/3)
+ d)^2 - 45*b^2*d*e^7*n^2*x^3 - 40*b^2*d^4*e^4*n^2*x^2*log(x) + 300*b^2*d^4
*e^4*n^2*x^2 - 60*b^2*d^8*x^(2/3)*log((e*x^(1/3) + d)^n)^2 - 60*(b^2*d^7*e*
n*log(c) + a*b*d^7*e*n)*x - 20*(6*b^2*d^5*e^3*n*x^(5/3)*log(e*x^(1/3) + d)
- 6*b^2*d^6*e^2*n*x^(4/3) + 3*b^2*d^7*e*n*x - 2*(b^2*d^5*e^3*n*x*log(x) - 3
*b^2*d^8*log(c) - 3*a*b*d^8)*x^(2/3))*log((e*x^(1/3) + d)^n) - 60*(b^2*d^8*
log(c)^2 + 2*a*b*d^8*log(c) + a^2*d^8)*x^(2/3) + 4*(9*b^2*e^8*n^2*x^3 + 5*b
^2*d^3*e^5*n^2*x^2*log(x) - 15*b^2*d^3*e^5*n^2*x^2 + 30*(b^2*d^6*e^2*n*log(
c) + a*b*d^6*e^2*n)*x)*x^(1/3) - 60*(b^2*d^3*e^5*n^2*x^3 + b^2*d^6*e^2*n^2*
x^2)/x^(2/3))/(d^8*x^(5/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^2/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**2/x**2, x)
```

3.455 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=405

$$-\frac{be^6n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6}-\frac{be^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6\sqrt[3]{x}}+\frac{be^4n\left(a+b\log\right.}{2}$$

```
[Out] -1/20*b^2*e^2*n^2/d^2/x^(4/3)+3/20*b^2*e^3*n^2/d^3/x-47/120*b^2*e^4*n^2/d^4
/x^(2/3)+77/60*b^2*e^5*n^2/d^5/x^(1/3)-77/60*b^2*e^6*n^2*ln(d+e*x^(1/3))/d^
6-1/5*b*e*n*(a+b*ln(c*(d+e*x^(1/3))^n))/d/x^(5/3)+1/4*b*e^2*n*(a+b*ln(c*(d+
e*x^(1/3))^n))/d^2/x^(4/3)-1/3*b*e^3*n*(a+b*ln(c*(d+e*x^(1/3))^n))/d^3/x+1/
2*b*e^4*n*(a+b*ln(c*(d+e*x^(1/3))^n))/d^4/x^(2/3)-b*e^5*n*(d+e*x^(1/3))*(a+
b*ln(c*(d+e*x^(1/3))^n))/d^6/x^(1/3)-b*e^6*n*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(
c*(d+e*x^(1/3))^n))/d^6-1/2*(a+b*ln(c*(d+e*x^(1/3))^n))^2/x^2+137/180*b^2*e
^6*n^2*ln(x)/d^6+b^2*e^6*n^2*polylog(2,d/(d+e*x^(1/3)))/d^6
```

Rubi [A] time = 1.01, antiderivative size = 430, normalized size of antiderivative = 1.06, number of steps used = 26, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$-\frac{b^2e^6n^2\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)}{d^6}+\frac{be^4n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2d^4x^{2/3}}+\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4d^2x^{4/3}}+\frac{e^6\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^3,x]
```

```
[Out] -(b^2*e^2*n^2)/(20*d^2*x^(4/3)) + (3*b^2*e^3*n^2)/(20*d^3*x) - (47*b^2*e^4*
n^2)/(120*d^4*x^(2/3)) + (77*b^2*e^5*n^2)/(60*d^5*x^(1/3)) - (77*b^2*e^6*n^
2*Log[d + e*x^(1/3)])/(60*d^6) - (b*e*n*(a + b*Log[c*(d + e*x^(1/3))^n]))/(
5*d*x^(5/3)) + (b*e^2*n*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*d^2*x^(4/3)) -
 (b*e^3*n*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*d^3*x) + (b*e^4*n*(a + b*Log
[c*(d + e*x^(1/3))^n]))/(2*d^4*x^(2/3)) - (b*e^5*n*(d + e*x^(1/3))*(a + b*L
og[c*(d + e*x^(1/3))^n]))/(d^6*x^(1/3)) + (e^6*(a + b*Log[c*(d + e*x^(1/3))
^n])^2)/(2*d^6) - (a + b*Log[c*(d + e*x^(1/3))^n])^2/(2*x^2) - (b*e^6*n*(a
+ b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)])/d^6 + (137*b^2*e^6*n^2
*Log[x])/(180*d^6) - (b^2*e^6*n^2*PolyLog[2, 1 + (e*x^(1/3))/d])/d^6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^7}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} + (ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^6(d+ex)}\,dx\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} + (bn)\,\text{Subst}\left(\int \frac{a+b\log\left(cx^n\right)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} + \frac{(bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+e\sqrt[3]{x}\right.}{d}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5dx^{5/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} - \frac{(ben)\,\text{S}}{}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5dx^{5/3}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4d^2x^{4/3}} - \frac{(a}{}$$

$$= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{b^2e^3n^2}{15d^3x} - \frac{b^2e^4n^2}{10d^4x^{2/3}} + \frac{b^2e^5n^2}{5d^5\sqrt[3]{x}} - \frac{b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{5d^6} - \frac{ben}{}$$

$$= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{3b^2e^3n^2}{20d^3x} - \frac{9b^2e^4n^2}{40d^4x^{2/3}} + \frac{9b^2e^5n^2}{20d^5\sqrt[3]{x}} - \frac{9b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{20d^6} -$$

$$= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{3b^2e^3n^2}{20d^3x} - \frac{47b^2e^4n^2}{120d^4x^{2/3}} + \frac{47b^2e^5n^2}{60d^5\sqrt[3]{x}} - \frac{47b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{60d^6}$$

$$= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{3b^2e^3n^2}{20d^3x} - \frac{47b^2e^4n^2}{120d^4x^{2/3}} + \frac{77b^2e^5n^2}{60d^5\sqrt[3]{x}} - \frac{77b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{60d^6}$$

$$= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{3b^2e^3n^2}{20d^3x} - \frac{47b^2e^4n^2}{120d^4x^{2/3}} + \frac{77b^2e^5n^2}{60d^5\sqrt[3]{x}} - \frac{77b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{60d^6}$$

Mathematica [A] time = 0.29, size = 533, normalized size = 1.32

$$-\frac{180a^2d^6 + 360abd^6\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right) - 360abe^6x^2\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right) + 72abd^5en\sqrt[3]{x} - 90abd^4e^2nx^{2/3} + 1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^3,x]
```

```
[Out] -1/360*(180*a^2*d^6 + 72*a*b*d^5*e*n*x^(1/3) - 90*a*b*d^4*e^2*n*x^(2/3) + 1
8*b^2*d^4*e^2*n^2*x^(2/3) + 120*a*b*d^3*e^3*n*x - 54*b^2*d^3*e^3*n^2*x - 18
0*a*b*d^2*e^4*n*x^(4/3) + 141*b^2*d^2*e^4*n^2*x^(4/3) + 360*a*b*d*e^5*n*x^(
5/3) - 462*b^2*d*e^5*n^2*x^(5/3) + 822*b^2*e^6*n^2*x^2*Log[d + e*x^(1/3)] +
 360*a*b*d^6*Log[c*(d + e*x^(1/3))^n] + 72*b^2*d^5*e*n*x^(1/3)*Log[c*(d + e
*x^(1/3))^n] - 90*b^2*d^4*e^2*n*x^(2/3)*Log[c*(d + e*x^(1/3))^n] + 120*b^2*
d^3*e^3*n*x*Log[c*(d + e*x^(1/3))^n] - 180*b^2*d^2*e^4*n*x^(4/3)*Log[c*(d +
 e*x^(1/3))^n] + 360*b^2*d*e^5*n*x^(5/3)*Log[c*(d + e*x^(1/3))^n] - 360*a*b
*e^6*x^2*Log[c*(d + e*x^(1/3))^n] + 180*b^2*d^6*Log[c*(d + e*x^(1/3))^n]^2
- 180*b^2*e^6*x^2*Log[c*(d + e*x^(1/3))^n]^2 + 360*a*b*e^6*n*x^2*Log[-((e*x
^(1/3))/d)] + 360*b^2*e^6*n*x^2*Log[c*(d + e*x^(1/3))^n]*Log[-((e*x^(1/3))/
d)] - 274*b^2*e^6*n^2*x^2*Log[x] + 360*b^2*e^6*n^2*x^2*PolyLog[2, 1 + (e*x^
(1/3))/d])/(d^6*x^2)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)^2+2\,ab\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(1/3) + d)^n*c)^2 + 2*a*b*log((e*x^(1/3) + d)^n*c) +
 a^2)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)^2/x^3, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(e\,x^{\frac{1}{3}}+d\right)^n\right)+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/3)+d)^n)+a)^2/x^3,x)
```

```
[Out] int((b*ln(c*(e*x^(1/3)+d)^n)+a)^2/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^n\right)^2}{2\,x^2}+\int\frac{3\left(b^2e\log(c)^2+2\,abe\log(c)+a^2e\right)x+\left(b^2enx+6\left(b^2e\log(c)+abe\right)x+6\left(b^2d\log(c)\right.\right.}{3\left(ex^4+dx^{\frac{11}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log((e*x^(1/3) + d)^n)^2/x^2 + integrate(1/3*(3*(b^2*e*log(c)^2 +
2*a*b*e*log(c) + a^2*e)*x + (b^2*e*n*x + 6*(b^2*e*log(c) + a*b*e)*x + 6*(b^
2*d*log(c) + a*b*d)*x^(2/3))*log((e*x^(1/3) + d)^n) + 3*(b^2*d*log(c)^2 + 2
*a*b*d*log(c) + a^2*d)*x^(2/3))/(e*x^4 + d*x^(11/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^2/x^3,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))^2/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2/x**3,x)
```

```
[Out] Timed out
```

3.456 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3 dx$

Optimal. Leaf size=1835

result too large to display

```
[Out] 1/4*(d+e*x^(1/3))^12*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12-99/8*b^3*d^10*n^3*(
d+e*x^(1/3))^2/e^12+110/9*b^3*d^9*n^3*(d+e*x^(1/3))^3/e^12-1485/128*b^3*d^8
*n^3*(d+e*x^(1/3))^4/e^12+1188/125*b^3*d^7*n^3*(d+e*x^(1/3))^5/e^12-77/12*b
^3*d^6*n^3*(d+e*x^(1/3))^6/e^12+1188/343*b^3*d^5*n^3*(d+e*x^(1/3))^7/e^12-1
485/1024*b^3*d^4*n^3*(d+e*x^(1/3))^8/e^12+110/243*b^3*d^3*n^3*(d+e*x^(1/3))
^9/e^12-99/1000*b^3*d^2*n^3*(d+e*x^(1/3))^10/e^12+18/1331*b^3*d*n^3*(d+e*x^
(1/3))^11/e^12+18*b^3*d^11*n^3*x^(1/3)/e^11-18*a*b^2*d^11*n^2*x^(1/3)/e^11+
33/2*d^2*(d+e*x^(1/3))^10*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12-3*d*(d+e*x^(1/
3))^11*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12-3*d^11*(d+e*x^(1/3))*(a+b*ln(c*(d
+e*x^(1/3))^n))^3/e^12+33/2*d^10*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n)
)^3/e^12-55*d^9*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12+495/4*d^
8*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12-198*d^7*(d+e*x^(1/3))^
5*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12+231*d^6*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e
*x^(1/3))^n))^3/e^12-198*d^5*(d+e*x^(1/3))^7*(a+b*ln(c*(d+e*x^(1/3))^n))^3/
e^12+495/4*d^4*(d+e*x^(1/3))^8*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12-55*d^3*(d
+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12-1/1152*b^3*n^3*(d+e*x^(1/3
))^12/e^12+1/96*b^2*n^2*(d+e*x^(1/3))^12*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12-1
/16*b*n*(d+e*x^(1/3))^12*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12-1188/49*b^2*d^5
*n^2*(d+e*x^(1/3))^7*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12+1485/128*b^2*d^4*n^2*
(d+e*x^(1/3))^8*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12-110/27*b^2*d^3*n^2*(d+e*x^
(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12+99/100*b^2*d^2*n^2*(d+e*x^(1/3))^
10*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12-18/121*b^2*d*n^2*(d+e*x^(1/3))^11*(a+b*
ln(c*(d+e*x^(1/3))^n))/e^12+9*b*d^11*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3)
)^n))^2/e^12-99/4*b*d^10*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^
12+55*b*d^9*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12-1485/16*b*
d^8*n*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12+594/5*b*d^7*n*(d+e
*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12-231/2*b*d^6*n*(d+e*x^(1/3))^
6*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12+594/7*b*d^5*n*(d+e*x^(1/3))^7*(a+b*ln(
c*(d+e*x^(1/3))^n))^2/e^12-1485/32*b*d^4*n*(d+e*x^(1/3))^8*(a+b*ln(c*(d+e*x
^(1/3))^n))^2/e^12+55/3*b*d^3*n*(d+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))
^2/e^12-99/20*b*d^2*n*(d+e*x^(1/3))^10*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12+9
/11*b*d*n*(d+e*x^(1/3))^11*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12-18*b^3*d^11*n
^2*(d+e*x^(1/3))*ln(c*(d+e*x^(1/3))^n)/e^12+99/4*b^2*d^10*n^2*(d+e*x^(1/3))
^2*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12-110/3*b^2*d^9*n^2*(d+e*x^(1/3))^3*(a+b*
ln(c*(d+e*x^(1/3))^n))/e^12+1485/32*b^2*d^8*n^2*(d+e*x^(1/3))^4*(a+b*ln(c*(
d+e*x^(1/3))^n))/e^12-1188/25*b^2*d^7*n^2*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^
(1/3))^n))/e^12+77/2*b^2*d^6*n^2*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n)
)/e^12
```

Rubi [A] time = 2.27, antiderivative size = 1835, normalized size of antiderivative = 1.00, number of steps used = 52, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (-99*b^3*d^10*n^3*(d + e*x^(1/3))^2)/(8*e^12) + (110*b^3*d^9*n^3*(d + e*x^(
1/3))^3)/(9*e^12) - (1485*b^3*d^8*n^3*(d + e*x^(1/3))^4)/(128*e^12) + (1188
*b^3*d^7*n^3*(d + e*x^(1/3))^5)/(125*e^12) - (77*b^3*d^6*n^3*(d + e*x^(1/3)
)^6)/(12*e^12) + (1188*b^3*d^5*n^3*(d + e*x^(1/3))^7)/(343*e^12) - (1485*b^
3*d^4*n^3*(d + e*x^(1/3))^8)/(1024*e^12) + (110*b^3*d^3*n^3*(d + e*x^(1/3))
```

```
^9)/(243*e^12) - (99*b^3*d^2*n^3*(d + e*x^(1/3))^10)/(1000*e^12) + (18*b^3*
d*n^3*(d + e*x^(1/3))^11)/(1331*e^12) - (b^3*n^3*(d + e*x^(1/3))^12)/(1152*
e^12) - (18*a*b^2*d^11*n^2*x^(1/3))/e^11 + (18*b^3*d^11*n^3*x^(1/3))/e^11 -
 (18*b^3*d^11*n^2*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^12 + (99*b^2*
d^10*n^2*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*e^12) - (11
0*b^2*d^9*n^2*(d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^12)
+ (1485*b^2*d^8*n^2*(d + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n]))/(32
*e^12) - (1188*b^2*d^7*n^2*(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n
]))/(25*e^12) + (77*b^2*d^6*n^2*(d + e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/
3))^n]))/(2*e^12) - (1188*b^2*d^5*n^2*(d + e*x^(1/3))^7*(a + b*Log[c*(d + e
*x^(1/3))^n]))/(49*e^12) + (1485*b^2*d^4*n^2*(d + e*x^(1/3))^8*(a + b*Log[c
*(d + e*x^(1/3))^n]))/(128*e^12) - (110*b^2*d^3*n^2*(d + e*x^(1/3))^9*(a +
b*Log[c*(d + e*x^(1/3))^n]))/(27*e^12) + (99*b^2*d^2*n^2*(d + e*x^(1/3))^10
*(a + b*Log[c*(d + e*x^(1/3))^n]))/(100*e^12) - (18*b^2*d*n^2*(d + e*x^(1/3
))^11*(a + b*Log[c*(d + e*x^(1/3))^n]))/(121*e^12) + (b^2*n^2*(d + e*x^(1/3
))^12*(a + b*Log[c*(d + e*x^(1/3))^n]))/(96*e^12) + (9*b*d^11*n*(d + e*x^(1
/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^12 - (99*b*d^10*n*(d + e*x^(1/3)
)^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(4*e^12) + (55*b*d^9*n*(d + e*x^(1/
3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^12 - (1485*b*d^8*n*(d + e*x^(1/
3))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(16*e^12) + (594*b*d^7*n*(d + e*x
^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(5*e^12) - (231*b*d^6*n*(d +
e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*e^12) + (594*b*d^5*n*(d
 + e*x^(1/3))^7*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(7*e^12) - (1485*b*d^4*
n*(d + e*x^(1/3))^8*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(32*e^12) + (55*b*d
^3*n*(d + e*x^(1/3))^9*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(3*e^12) - (99*b
*d^2*n*(d + e*x^(1/3))^10*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(20*e^12) + (
9*b*d*n*(d + e*x^(1/3))^11*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(11*e^12) -
(b*n*(d + e*x^(1/3))^12*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(16*e^12) - (3*
d^11*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (33*d^10*(d
 + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/(2*e^12) - (55*d^9*(d +
 e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (495*d^8*(d + e*x^
(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/(4*e^12) - (198*d^7*(d + e*x^(
1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (231*d^6*(d + e*x^(1/3))
^6*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 - (198*d^5*(d + e*x^(1/3))^7*(a
 + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (495*d^4*(d + e*x^(1/3))^8*(a + b*
Log[c*(d + e*x^(1/3))^n])^3)/(4*e^12) - (55*d^3*(d + e*x^(1/3))^9*(a + b*Lo
g[c*(d + e*x^(1/3))^n])^3)/e^12 + (33*d^2*(d + e*x^(1/3))^10*(a + b*Log[c*(
d + e*x^(1/3))^n])^3)/(2*e^12) - (3*d*(d + e*x^(1/3))^11*(a + b*Log[c*(d +
e*x^(1/3))^n])^3)/e^12 + ((d + e*x^(1/3))^12*(a + b*Log[c*(d + e*x^(1/3))^n
])^3)/(4*e^12)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = 3\,\text{Subst}\left(\int x^{11}\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int\left(-\frac{d^{11}\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^{11}}+\frac{11d^{10}(d+ex)\left(a+b\log\right.}{e^{11}}\right.\right.$$

$$= \frac{3\,\text{Subst}\left(\int(d+ex)^{11}\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt[3]{x}\right)}{e^{11}} - \frac{(33d)\,\text{Sub}}{}$$

$$= \frac{3\,\text{Subst}\left(\int x^{11}\left(a+b\log\left(cx^n\right)\right)^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} - \frac{(33d)\,\text{Subst}\left(\int x^{1\ldots}\right.}{}$$

$$= -\frac{3d^{11}\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^{12}} + \frac{33d^{10}\left(d+e\sqrt[3]{x}\right)^2\left(a\right.}{2\ldots}$$

$$= \frac{9bd^{11}n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^{12}} - \frac{99bd^{10}n\left(d+e\sqrt[3]{x}\right)^2}{}$$

$$= -\frac{99b^3d^{10}n^3\left(d+e\sqrt[3]{x}\right)^2}{8e^{12}} + \frac{110b^3d^9n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^{12}} - \frac{1485b^3d^8n^3\left(d+\right.}{128e^{12}}$$

$$= -\frac{99b^3d^{10}n^3\left(d+e\sqrt[3]{x}\right)^2}{8e^{12}} + \frac{110b^3d^9n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^{12}} - \frac{1485b^3d^8n^3\left(d+\right.}{128e^{12}}$$

Mathematica [A] time = 1.31, size = 1009, normalized size = 0.55

$$\frac{-3550000608000b^3\left(d^{12}-e^{12}x^4\right)\log^3\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-384199200b^2\left(27720a\left(d^{12}-e^{12}x^4\right)+bn\left(-86021d^{12}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (e*x^(1/3)*(3550000608000*a^3*e^11*x^(11/3) + b^3*n^3*(119225632485960*d^11
 - 26563616859780*d^10*e*x^(1/3) + 10242678720120*d^9*e^2*x^(2/3) - 4836309
598890*d^8*e^3*x + 2516628075192*d^7*e^4*x^(4/3) - 1373077023780*d^6*e^5*x^
(5/3) + 761128152840*d^5*e^6*x^2 - 417533743935*d^4*e^7*x^(7/3) + 220161492
320*d^3*e^8*x^(8/3) - 106944990768*d^2*e^9*x^3 + 44119404000*d*e^10*x^(10/3
) - 12326391000*e^11*x^(11/3)) - 27720*a*b^2*n^2*(2384502120*d^11 - 8080518
60*d^10*e*x^(1/3) + 410634840*d^9*e^2*x^(2/3) - 243942930*d^8*e^3*x + 15673
4424*d^7*e^4*x^(4/3) - 104998740*d^6*e^5*x^(5/3) + 71703720*d^5*e^6*x^2 - 4
9019355*d^4*e^7*x^(7/3) + 32900560*d^3*e^8*x^(8/3) - 21072744*d^2*e^9*x^3 +
 12171600*d*e^10*x^(10/3) - 5336100*e^11*x^(11/3)) + 384199200*a^2*b*n*(277
20*d^11 - 13860*d^10*e*x^(1/3) + 9240*d^9*e^2*x^(2/3) - 6930*d^8*e^3*x + 55
44*d^7*e^4*x^(4/3) - 4620*d^6*e^5*x^(5/3) + 3960*d^5*e^6*x^2 - 3465*d^4*e^7
*x^(7/3) + 3080*d^3*e^8*x^(8/3) - 2772*d^2*e^9*x^3 + 2520*d*e^10*x^(10/3) -
 2310*e^11*x^(11/3))) - 27720*b*(b^2*n^2*(4301068993*d^12 + 2384502120*d^11
*e*x^(1/3) - 808051860*d^10*e^2*x^(2/3) + 410634840*d^9*e^3*x - 243942930*d
^8*e^4*x^(4/3) + 156734424*d^7*e^5*x^(5/3) - 104998740*d^6*e^6*x^2 + 717037
20*d^5*e^7*x^(7/3) - 49019355*d^4*e^8*x^(8/3) + 32900560*d^3*e^9*x^3 - 2107
2744*d^2*e^10*x^(10/3) + 12171600*d*e^11*x^(11/3) - 5336100*e^12*x^4) - 277
20*a*b*n*(86021*d^12 + 27720*d^11*e*x^(1/3) - 13860*d^10*e^2*x^(2/3) + 9240
*d^9*e^3*x - 6930*d^8*e^4*x^(4/3) + 5544*d^7*e^5*x^(5/3) - 4620*d^6*e^6*x^2
 + 3960*d^5*e^7*x^(7/3) - 3465*d^4*e^8*x^(8/3) + 3080*d^3*e^9*x^3 - 2772*d^
2*e^10*x^(10/3) + 2520*d*e^11*x^(11/3) - 2310*e^12*x^4) + 384199200*a^2*(d^
12 - e^12*x^4))*Log[c*(d + e*x^(1/3))^n] - 384199200*b^2*(27720*a*(d^12 - e
```

```
^12*x^4) + b*n*(-86021*d^12 - 27720*d^11*e*x^(1/3) + 13860*d^10*e^2*x^(2/3)
 - 9240*d^9*e^3*x + 6930*d^8*e^4*x^(4/3) - 5544*d^7*e^5*x^(5/3) + 4620*d^6*
e^6*x^2 - 3960*d^5*e^7*x^(7/3) + 3465*d^4*e^8*x^(8/3) - 3080*d^3*e^9*x^3 +
2772*d^2*e^10*x^(10/3) - 2520*d*e^11*x^(11/3) + 2310*e^12*x^4))*Log[c*(d +
e*x^(1/3))^n]^2 - 3550000608000*b^3*(d^12 - e^12*x^4)*Log[c*(d + e*x^(1/3))
^n]^3)/(14200002432000*e^12)
```

fricas [A] time = 0.78, size = 2183, normalized size = 1.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/14200002432000*(3550000608000*b^3*e^12*x^4*log(c)^3 - 12326391000*(b^3*e^
12*n^3 - 12*a*b^2*e^12*n^2 + 72*a^2*b*e^12*n - 288*a^3*e^12)*x^4 + 603680*(
364699*b^3*d^3*e^9*n^3 - 1510740*a*b^2*d^3*e^9*n^2 + 1960200*a^2*b*d^3*e^9*
n)*x^3 + 3550000608000*(b^3*e^12*n^3*x^4 - b^3*d^12*n^3)*log(e*x^(1/3) + d)
^3 - 4620*(297202819*b^3*d^6*e^6*n^3 - 629992440*a*b^2*d^6*e^6*n^2 + 384199
200*a^2*b*d^6*e^6*n)*x^2 + 384199200*(3080*b^3*d^3*e^9*n^3*x^3 - 4620*b^3*d
^6*e^6*n^3*x^2 + 9240*b^3*d^9*e^3*n^3*x + 86021*b^3*d^12*n^3 - 27720*a*b^2*
d^12*n^2 - 2310*(b^3*e^12*n^3 - 12*a*b^2*e^12*n^2)*x^4 + 27720*(b^3*e^12*n^
2*x^4 - b^3*d^12*n^2)*log(c) + 63*(40*b^3*d*e^11*n^3*x^3 - 55*b^3*d^4*e^8*n
^3*x^2 + 88*b^3*d^7*e^5*n^3*x - 220*b^3*d^10*e^2*n^3)*x^(2/3) - 198*(14*b^3
*d^2*e^10*n^3*x^3 - 20*b^3*d^5*e^7*n^3*x^2 + 35*b^3*d^8*e^4*n^3*x - 140*b^3
*d^11*e*n^3)*x^(1/3))*log(e*x^(1/3) + d)^2 + 295833384000*(4*b^3*d^3*e^9*n*
x^3 - 6*b^3*d^6*e^6*n*x^2 + 12*b^3*d^9*e^3*n*x - 3*(b^3*e^12*n - 12*a*b^2*e
^12)*x^4)*log(c)^2 + 9240*(1108515013*b^3*d^9*e^3*n^3 - 1231904520*a*b^2*d^
9*e^3*n^2 + 384199200*a^2*b*d^9*e^3*n)*x - 27720*(4301068993*b^3*d^12*n^3 -
 2384502120*a*b^2*d^12*n^2 + 384199200*a^2*b*d^12*n - 5336100*(b^3*e^12*n^3
 - 12*a*b^2*e^12*n^2 + 72*a^2*b*e^12*n)*x^4 + 43120*(763*b^3*d^3*e^9*n^3 -
1980*a*b^2*d^3*e^9*n^2)*x^3 - 4620*(22727*b^3*d^6*e^6*n^3 - 27720*a*b^2*d^6
*e^6*n^2)*x^2 - 384199200*(b^3*e^12*n*x^4 - b^3*d^12*n)*log(c)^2 + 9240*(44
441*b^3*d^9*e^3*n^3 - 27720*a*b^2*d^9*e^3*n^2)*x - 27720*(3080*b^3*d^3*e^9*
n^2*x^3 - 4620*b^3*d^6*e^6*n^2*x^2 + 9240*b^3*d^9*e^3*n^2*x + 86021*b^3*d^1
2*n^2 - 27720*a*b^2*d^12*n - 2310*(b^3*e^12*n^2 - 12*a*b^2*e^12*n)*x^4)*log
(c) - 63*(12826220*b^3*d^10*e^2*n^3 - 6098400*a*b^2*d^10*e^2*n^2 - 8400*(23
*b^3*d*e^11*n^3 - 132*a*b^2*d*e^11*n^2)*x^3 + 385*(2021*b^3*d^4*e^8*n^3 - 3
960*a*b^2*d^4*e^8*n^2)*x^2 - 88*(28271*b^3*d^7*e^5*n^3 - 27720*a*b^2*d^7*e^
5*n^2)*x + 27720*(40*b^3*d*e^11*n^2*x^3 - 55*b^3*d^4*e^8*n^2*x^2 + 88*b^3*d
^7*e^5*n^2*x - 220*b^3*d^10*e^2*n^2)*log(c))*x^(2/3) + 198*(12042940*b^3*d^
11*e*n^3 - 3880800*a*b^2*d^11*e*n^2 - 588*(181*b^3*d^2*e^10*n^3 - 660*a*b^2
*d^2*e^10*n^2)*x^3 + 20*(18107*b^3*d^5*e^7*n^3 - 27720*a*b^2*d^5*e^7*n^2)*x
^2 - 35*(35201*b^3*d^8*e^4*n^3 - 27720*a*b^2*d^8*e^4*n^2)*x + 27720*(14*b^3
*d^2*e^10*n^2*x^3 - 20*b^3*d^5*e^7*n^2*x^2 + 35*b^3*d^8*e^4*n^2*x - 140*b^3
*d^11*e*n^2)*log(c))*x^(1/3))*log(e*x^(1/3) + d) + 42688800*(3465*(b^3*e^12
*n^2 - 12*a*b^2*e^12*n + 72*a^2*b*e^12)*x^4 - 28*(763*b^3*d^3*e^9*n^2 - 198
0*a*b^2*d^3*e^9*n)*x^3 + 3*(22727*b^3*d^6*e^6*n^2 - 27720*a*b^2*d^6*e^6*n)*
x^2 - 6*(44441*b^3*d^9*e^3*n^2 - 27720*a*b^2*d^9*e^3*n)*x)*log(c) - 63*(421
644712060*b^3*d^10*e^2*n^3 - 355542818400*a*b^2*d^10*e^2*n^2 + 84523824000*
a^2*b*d^10*e^2*n - 1764000*(397*b^3*d*e^11*n^3 - 3036*a*b^2*d*e^11*n^2 + 87
12*a^2*b*d*e^11*n)*x^3 + 2695*(2459191*b^3*d^4*e^8*n^3 - 8003160*a*b^2*d^4*
e^8*n^2 + 7840800*a^2*b*d^4*e^8*n)*x^2 - 384199200*(40*b^3*d*e^11*n*x^3 - 5
5*b^3*d^4*e^8*n*x^2 + 88*b^3*d^7*e^5*n*x - 220*b^3*d^10*e^2*n)*log(c)^2 - 8
8*(453937243*b^3*d^7*e^5*n^3 - 783672120*a*b^2*d^7*e^5*n^2 + 384199200*a^2*
b*d^7*e^5*n)*x - 27720*(12826220*b^3*d^10*e^2*n^2 - 6098400*a*b^2*d^10*e^2*
n - 8400*(23*b^3*d*e^11*n^2 - 132*a*b^2*d*e^11*n)*x^3 + 385*(2021*b^3*d^4*e
^8*n^2 - 3960*a*b^2*d^4*e^8*n)*x^2 - 88*(28271*b^3*d^7*e^5*n^2 - 27720*a*b^
2*d^7*e^5*n)*x)*log(c))*x^(2/3) + 198*(602149659020*b^3*d^11*e*n^3 - 333830
296800*a*b^2*d^11*e*n^2 + 53787888000*a^2*b*d^11*e*n - 24696*(21871*b^3*d^2
```

```
*e^10*n^3 - 119460*a*b^2*d^2*e^10*n^2 + 217800*a^2*b*d^2*e^10*n)*x^3 + 20*(
192204079*b^3*d^5*e^7*n^3 - 501926040*a*b^2*d^5*e^7*n^2 + 384199200*a^2*b*d
^5*e^7*n)*x^2 - 384199200*(14*b^3*d^2*e^10*n*x^3 - 20*b^3*d^5*e^7*n*x^2 + 3
5*b^3*d^8*e^4*n*x - 140*b^3*d^11*e*n)*log(c)^2 - 35*(697880173*b^3*d^8*e^4*
n^3 - 975771720*a*b^2*d^8*e^4*n^2 + 384199200*a^2*b*d^8*e^4*n)*x - 27720*(1
2042940*b^3*d^11*e*n^2 - 3880800*a*b^2*d^11*e*n - 588*(181*b^3*d^2*e^10*n^2
 - 660*a*b^2*d^2*e^10*n)*x^3 + 20*(18107*b^3*d^5*e^7*n^2 - 27720*a*b^2*d^5*
e^7*n)*x^2 - 35*(35201*b^3*d^8*e^4*n^2 - 27720*a*b^2*d^8*e^4*n)*x)*log(c))*
x^(1/3))/e^12
```

giac [B] time = 0.51, size = 4443, normalized size = 2.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/14200002432000*(3550000608000*b^3*x^4*e*log(c)^3 + 10650001824000*a*b^2*x
^4*e*log(c)^2 + 10650001824000*a^2*b*x^4*e*log(c) + 3550000608000*a^3*x^4*e
 + (3550000608000*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d)^3 - 4260000
7296000*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d)^3 + 234300040128000
*(x^(1/3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d)^3 - 781000133760000*(x^(
1/3)*e + d)^9*d^3*e^(-11)*log(x^(1/3)*e + d)^3 + 1757250300960000*(x^(1/3)*
e + d)^8*d^4*e^(-11)*log(x^(1/3)*e + d)^3 - 2811600481536000*(x^(1/3)*e + d
)^7*d^5*e^(-11)*log(x^(1/3)*e + d)^3 + 3280200561792000*(x^(1/3)*e + d)^6*d
^6*e^(-11)*log(x^(1/3)*e + d)^3 - 2811600481536000*(x^(1/3)*e + d)^5*d^7*e^
(-11)*log(x^(1/3)*e + d)^3 + 1757250300960000*(x^(1/3)*e + d)^4*d^8*e^(-11)
*log(x^(1/3)*e + d)^3 - 781000133760000*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x
^(1/3)*e + d)^3 + 234300040128000*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3
)*e + d)^3 - 42600007296000*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d)
^3 - 887500152000*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d)^2 + 1161818
3808000*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d)^2 - 70290012038400*
(x^(1/3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d)^2 + 260333377920000*(x^(1
/3)*e + d)^9*d^3*e^(-11)*log(x^(1/3)*e + d)^2 - 658968862860000*(x^(1/3)*e
+ d)^8*d^4*e^(-11)*log(x^(1/3)*e + d)^2 + 1204971634944000*(x^(1/3)*e + d)^
7*d^5*e^(-11)*log(x^(1/3)*e + d)^2 - 1640100280896000*(x^(1/3)*e + d)^6*d^6
*e^(-11)*log(x^(1/3)*e + d)^2 + 1686960288921600*(x^(1/3)*e + d)^5*d^7*e^(-
11)*log(x^(1/3)*e + d)^2 - 1317937725720000*(x^(1/3)*e + d)^4*d^8*e^(-11)*l
og(x^(1/3)*e + d)^2 + 781000133760000*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x^(
1/3)*e + d)^2 - 351450060192000*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3)*
e + d)^2 + 127800021888000*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d)^
2 + 147916692000*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d) - 2112397056
000*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d) + 14058002407680*(x^(1/
3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d) - 57851861760000*(x^(1/3)*e + d
)^9*d^3*e^(-11)*log(x^(1/3)*e + d) + 164742215715000*(x^(1/3)*e + d)^8*d^4*
e^(-11)*log(x^(1/3)*e + d) - 344277609984000*(x^(1/3)*e + d)^7*d^5*e^(-11)*
log(x^(1/3)*e + d) + 546700093632000*(x^(1/3)*e + d)^6*d^6*e^(-11)*log(x^(1
/3)*e + d) - 674784115568640*(x^(1/3)*e + d)^5*d^7*e^(-11)*log(x^(1/3)*e +
d) + 658968862860000*(x^(1/3)*e + d)^4*d^8*e^(-11)*log(x^(1/3)*e + d) - 520
666755840000*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x^(1/3)*e + d) + 35145006019
2000*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3)*e + d) - 255600043776000*(x
^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d) - 12326391000*(x^(1/3)*e + d)
^12*e^(-11) + 192036096000*(x^(1/3)*e + d)^11*d*e^(-11) - 1405800240768*(x^
(1/3)*e + d)^10*d^2*e^(-11) + 6427984640000*(x^(1/3)*e + d)^9*d^3*e^(-11) -
 20592776964375*(x^(1/3)*e + d)^8*d^4*e^(-11) + 49182515712000*(x^(1/3)*e +
 d)^7*d^5*e^(-11) - 91116682272000*(x^(1/3)*e + d)^6*d^6*e^(-11) + 13495682
3113728*(x^(1/3)*e + d)^5*d^7*e^(-11) - 164742215715000*(x^(1/3)*e + d)^4*d
^8*e^(-11) + 173555585280000*(x^(1/3)*e + d)^3*d^9*e^(-11) - 17572503009600
0*(x^(1/3)*e + d)^2*d^10*e^(-11) + 255600043776000*(x^(1/3)*e + d)*d^11*e^(
-11))*b^3*n^3 + 27720*(384199200*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e +
```

```
d)^2 - 4610390400*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d)^2 + 2535
7147200*(x^(1/3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d)^2 - 84523824000*(
x^(1/3)*e + d)^9*d^3*e^(-11)*log(x^(1/3)*e + d)^2 + 190178604000*(x^(1/3)*e
 + d)^8*d^4*e^(-11)*log(x^(1/3)*e + d)^2 - 304285766400*(x^(1/3)*e + d)^7*d
^5*e^(-11)*log(x^(1/3)*e + d)^2 + 355000060800*(x^(1/3)*e + d)^6*d^6*e^(-11
)*log(x^(1/3)*e + d)^2 - 304285766400*(x^(1/3)*e + d)^5*d^7*e^(-11)*log(x^(
1/3)*e + d)^2 + 190178604000*(x^(1/3)*e + d)^4*d^8*e^(-11)*log(x^(1/3)*e +
d)^2 - 84523824000*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x^(1/3)*e + d)^2 + 253
57147200*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3)*e + d)^2 - 4610390400*(
x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d)^2 - 64033200*(x^(1/3)*e + d)
^12*e^(-11)*log(x^(1/3)*e + d) + 838252800*(x^(1/3)*e + d)^11*d*e^(-11)*log
(x^(1/3)*e + d) - 5071429440*(x^(1/3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e +
 d) + 18783072000*(x^(1/3)*e + d)^9*d^3*e^(-11)*log(x^(1/3)*e + d) - 475446
51000*(x^(1/3)*e + d)^8*d^4*e^(-11)*log(x^(1/3)*e + d) + 86938790400*(x^(1/
3)*e + d)^7*d^5*e^(-11)*log(x^(1/3)*e + d) - 118333353600*(x^(1/3)*e + d)^6
*d^6*e^(-11)*log(x^(1/3)*e + d) + 121714306560*(x^(1/3)*e + d)^5*d^7*e^(-11
)*log(x^(1/3)*e + d) - 95089302000*(x^(1/3)*e + d)^4*d^8*e^(-11)*log(x^(1/3
)*e + d) + 56349216000*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x^(1/3)*e + d) - 2
5357147200*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3)*e + d) + 9220780800*(
x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d) + 5336100*(x^(1/3)*e + d)^12
*e^(-11) - 76204800*(x^(1/3)*e + d)^11*d*e^(-11) + 507142944*(x^(1/3)*e + d
)^10*d^2*e^(-11) - 2087008000*(x^(1/3)*e + d)^9*d^3*e^(-11) + 5943081375*(x
^(1/3)*e + d)^8*d^4*e^(-11) - 12419827200*(x^(1/3)*e + d)^7*d^5*e^(-11) + 1
9722225600*(x^(1/3)*e + d)^6*d^6*e^(-11) - 24342861312*(x^(1/3)*e + d)^5*d^
7*e^(-11) + 23772325500*(x^(1/3)*e + d)^4*d^8*e^(-11) - 18783072000*(x^(1/3
)*e + d)^3*d^9*e^(-11) + 12678573600*(x^(1/3)*e + d)^2*d^10*e^(-11) - 92207
80800*(x^(1/3)*e + d)*d^11*e^(-11))*b^3*n^2*log(c) + 384199200*(27720*(x^(1
/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d) - 332640*(x^(1/3)*e + d)^11*d*e^(-
11)*log(x^(1/3)*e + d) + 1829520*(x^(1/3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)
*e + d) - 6098400*(x^(1/3)*e + d)^9*d^3*e^(-11)*log(x^(1/3)*e + d) + 137214
00*(x^(1/3)*e + d)^8*d^4*e^(-11)*log(x^(1/3)*e + d) - 21954240*(x^(1/3)*e +
 d)^7*d^5*e^(-11)*log(x^(1/3)*e + d) + 25613280*(x^(1/3)*e + d)^6*d^6*e^(-1
1)*log(x^(1/3)*e + d) - 21954240*(x^(1/3)*e + d)^5*d^7*e^(-11)*log(x^(1/3)*
e + d) + 13721400*(x^(1/3)*e + d)^4*d^8*e^(-11)*log(x^(1/3)*e + d) - 609840
0*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x^(1/3)*e + d) + 1829520*(x^(1/3)*e + d
)^2*d^10*e^(-11)*log(x^(1/3)*e + d) - 332640*(x^(1/3)*e + d)*d^11*e^(-11)*l
og(x^(1/3)*e + d) - 2310*(x^(1/3)*e + d)^12*e^(-11) + 30240*(x^(1/3)*e + d)
^11*d*e^(-11) - 182952*(x^(1/3)*e + d)^10*d^2*e^(-11) + 677600*(x^(1/3)*e +
 d)^9*d^3*e^(-11) - 1715175*(x^(1/3)*e + d)^8*d^4*e^(-11) + 3136320*(x^(1/3
)*e + d)^7*d^5*e^(-11) - 4268880*(x^(1/3)*e + d)^6*d^6*e^(-11) + 4390848*(x
^(1/3)*e + d)^5*d^7*e^(-11) - 3430350*(x^(1/3)*e + d)^4*d^8*e^(-11) + 20328
00*(x^(1/3)*e + d)^3*d^9*e^(-11) - 914760*(x^(1/3)*e + d)^2*d^10*e^(-11) +
332640*(x^(1/3)*e + d)*d^11*e^(-11))*b^3*n*log(c)^2 + 27720*(384199200*(x^(
1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d)^2 - 4610390400*(x^(1/3)*e + d)^11
*d*e^(-11)*log(x^(1/3)*e + d)^2 + 25357147200*(x^(1/3)*e + d)^10*d^2*e^(-11
)*log(x^(1/3)*e + d)^2 - 84523824000*(x^(1/3)*e + d)^9*d^3*e^(-11)*log(x^(1
/3)*e + d)^2 + 190178604000*(x^(1/3)*e + d)^8*d^4*e^(-11)*log(x^(1/3)*e + d
)^2 - 304285766400*(x^(1/3)*e + d)^7*d^5*e^(-11)*log(x^(1/3)*e + d)^2 + 355
000060800*(x^(1/3)*e + d)^6*d^6*e^(-11)*log(x^(1/3)*e + d)^2 - 304285766400
*(x^(1/3)*e + d)^5*d^7*e^(-11)*log(x^(1/3)*e + d)^2 + 190178604000*(x^(1/3)
*e + d)^4*d^8*e^(-11)*log(x^(1/3)*e + d)^2 - 84523824000*(x^(1/3)*e + d)^3*
d^9*e^(-11)*log(x^(1/3)*e + d)^2 + 25357147200*(x^(1/3)*e + d)^2*d^10*e^(-1
1)*log(x^(1/3)*e + d)^2 - 4610390400*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/
3)*e + d)^2 - 64033200*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d) + 8382
52800*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d) - 5071429440*(x^(1/3)
*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d) + 18783072000*(x^(1/3)*e + d)^9*d
^3*e^(-11)*log(x^(1/3)*e + d) - 47544651000*(x^(1/3)*e + d)^8*d^4*e^(-11)*l
og(x^(1/3)*e + d) + 86938790400*(x^(1/3)*e + d)^7*d^5*e^(-11)*log(x^(1/3)*e
 + d) - 118333353600*(x^(1/3)*e + d)^6*d^6*e^(-11)*log(x^(1/3)*e + d) + 121
```

```
714306560*(x^(1/3)*e + d)^5*d^7*e^(-11)*log(x^(1/3)*e + d) - 95089302000*(x
^(1/3)*e + d)^4*d^8*e^(-11)*log(x^(1/3)*e + d) + 56349216000*(x^(1/3)*e + d
)^3*d^9*e^(-11)*log(x^(1/3)*e + d) - 25357147200*(x^(1/3)*e + d)^2*d^10*e^(
-11)*log(x^(1/3)*e + d) + 9220780800*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/
3)*e + d) + 5336100*(x^(1/3)*e + d)^12*e^(-11) - 76204800*(x^(1/3)*e + d)^1
1*d*e^(-11) + 507142944*(x^(1/3)*e + d)^10*d^2*e^(-11) - 2087008000*(x^(1/3
)*e + d)^9*d^3*e^(-11) + 5943081375*(x^(1/3)*e + d)^8*d^4*e^(-11) - 1241982
7200*(x^(1/3)*e + d)^7*d^5*e^(-11) + 19722225600*(x^(1/3)*e + d)^6*d^6*e^(-
11) - 24342861312*(x^(1/3)*e + d)^5*d^7*e^(-11) + 23772325500*(x^(1/3)*e +
d)^4*d^8*e^(-11) - 18783072000*(x^(1/3)*e + d)^3*d^9*e^(-11) + 12678573600*
(x^(1/3)*e + d)^2*d^10*e^(-11) - 9220780800*(x^(1/3)*e + d)*d^11*e^(-11))*a
*b^2*n^2 + 768398400*(27720*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d) -
 332640*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d) + 1829520*(x^(1/3)*
e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d) - 6098400*(x^(1/3)*e + d)^9*d^3*e^
(-11)*log(x^(1/3)*e + d) + 13721400*(x^(1/3)*e + d)^8*d^4*e^(-11)*log(x^(1/
3)*e + d) - 21954240*(x^(1/3)*e + d)^7*d^5*e^(-11)*log(x^(1/3)*e + d) + 256
13280*(x^(1/3)*e + d)^6*d^6*e^(-11)*log(x^(1/3)*e + d) - 21954240*(x^(1/3)*
e + d)^5*d^7*e^(-11)*log(x^(1/3)*e + d) + 13721400*(x^(1/3)*e + d)^4*d^8*e^
(-11)*log(x^(1/3)*e + d) - 6098400*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x^(1/3
)*e + d) + 1829520*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3)*e + d) - 3326
40*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d) - 2310*(x^(1/3)*e + d)^1
2*e^(-11) + 30240*(x^(1/3)*e + d)^11*d*e^(-11) - 182952*(x^(1/3)*e + d)^10*
d^2*e^(-11) + 677600*(x^(1/3)*e + d)^9*d^3*e^(-11) - 1715175*(x^(1/3)*e + d
)^8*d^4*e^(-11) + 3136320*(x^(1/3)*e + d)^7*d^5*e^(-11) - 4268880*(x^(1/3)*
e + d)^6*d^6*e^(-11) + 4390848*(x^(1/3)*e + d)^5*d^7*e^(-11) - 3430350*(x^(
1/3)*e + d)^4*d^8*e^(-11) + 2032800*(x^(1/3)*e + d)^3*d^9*e^(-11) - 914760*
(x^(1/3)*e + d)^2*d^10*e^(-11) + 332640*(x^(1/3)*e + d)*d^11*e^(-11))*a*b^2
*n*log(c) + 384199200*(27720*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d)
- 332640*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d) + 1829520*(x^(1/3)
*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d) - 6098400*(x^(1/3)*e + d)^9*d^3*e
^(-11)*log(x^(1/3)*e + d) + 13721400*(x^(1/3)*e + d)^8*d^4*e^(-11)*log(x^(1
/3)*e + d) - 21954240*(x^(1/3)*e + d)^7*d^5*e^(-11)*log(x^(1/3)*e + d) + 25
613280*(x^(1/3)*e + d)^6*d^6*e^(-11)*log(x^(1/3)*e + d) - 21954240*(x^(1/3)
*e + d)^5*d^7*e^(-11)*log(x^(1/3)*e + d) + 13721400*(x^(1/3)*e + d)^4*d^8*e
^(-11)*log(x^(1/3)*e + d) - 6098400*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x^(1/
3)*e + d) + 1829520*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3)*e + d) - 332
640*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d) - 2310*(x^(1/3)*e + d)^
12*e^(-11) + 30240*(x^(1/3)*e + d)^11*d*e^(-11) - 182952*(x^(1/3)*e + d)^10
*d^2*e^(-11) + 677600*(x^(1/3)*e + d)^9*d^3*e^(-11) - 1715175*(x^(1/3)*e +
d)^8*d^4*e^(-11) + 3136320*(x^(1/3)*e + d)^7*d^5*e^(-11) - 4268880*(x^(1/3)
*e + d)^6*d^6*e^(-11) + 4390848*(x^(1/3)*e + d)^5*d^7*e^(-11) - 3430350*(x^
(1/3)*e + d)^4*d^8*e^(-11) + 2032800*(x^(1/3)*e + d)^3*d^9*e^(-11) - 914760
*(x^(1/3)*e + d)^2*d^10*e^(-11) + 332640*(x^(1/3)*e + d)*d^11*e^(-11))*a^2*
b*n)*e^(-1)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(b \ln \left(c \left(e x^{\frac{1}{3}}+d\right)^{n}\right)+a\right)^{3} x^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*ln(c*(e*x^(1/3)+d)^n)+a)^3,x)
[Out] int(x^3*(b*ln(c*(e*x^(1/3)+d)^n)+a)^3,x)
```

maxima [A] time = 0.68, size = 1064, normalized size = 0.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4*log((e*x^(1/3) + d)^n*c)^3 + 3/4*a*b^2*x^4*log((e*x^(1/3) + d)^
n*c)^2 + 3/4*a^2*b*x^4*log((e*x^(1/3) + d)^n*c) + 1/4*a^3*x^4 - 1/36960*a^2
*b*e*n*(27720*d^12*log(e*x^(1/3) + d)/e^13 + (2310*e^11*x^4 - 2520*d*e^10*x
^(11/3) + 2772*d^2*e^9*x^(10/3) - 3080*d^3*e^8*x^3 + 3465*d^4*e^7*x^(8/3) -
 3960*d^5*e^6*x^(7/3) + 4620*d^6*e^5*x^2 - 5544*d^7*e^4*x^(5/3) + 6930*d^8*
e^3*x^(4/3) - 9240*d^9*e^2*x + 13860*d^10*e*x^(2/3) - 27720*d^11*x^(1/3))/e
^12) - 1/512265600*(27720*e*n*(27720*d^12*log(e*x^(1/3) + d)/e^13 + (2310*e
^11*x^4 - 2520*d*e^10*x^(11/3) + 2772*d^2*e^9*x^(10/3) - 3080*d^3*e^8*x^3 +
 3465*d^4*e^7*x^(8/3) - 3960*d^5*e^6*x^(7/3) + 4620*d^6*e^5*x^2 - 5544*d^7*
e^4*x^(5/3) + 6930*d^8*e^3*x^(4/3) - 9240*d^9*e^2*x + 13860*d^10*e*x^(2/3)
- 27720*d^11*x^(1/3))/e^12)*log((e*x^(1/3) + d)^n*c) - (5336100*e^12*x^4 -
12171600*d*e^11*x^(11/3) + 21072744*d^2*e^10*x^(10/3) - 32900560*d^3*e^9*x^
3 + 49019355*d^4*e^8*x^(8/3) - 71703720*d^5*e^7*x^(7/3) + 104998740*d^6*e^6
*x^2 + 384199200*d^12*log(e*x^(1/3) + d)^2 - 156734424*d^7*e^5*x^(5/3) + 24
3942930*d^8*e^4*x^(4/3) - 410634840*d^9*e^3*x + 2384502120*d^12*log(e*x^(1/
3) + d) + 808051860*d^10*e^2*x^(2/3) - 2384502120*d^11*e*x^(1/3))*n^2/e^12)
*a*b^2 - 1/14200002432000*(384199200*e*n*(27720*d^12*log(e*x^(1/3) + d)/e^1
3 + (2310*e^11*x^4 - 2520*d*e^10*x^(11/3) + 2772*d^2*e^9*x^(10/3) - 3080*d^
3*e^8*x^3 + 3465*d^4*e^7*x^(8/3) - 3960*d^5*e^6*x^(7/3) + 4620*d^6*e^5*x^2
- 5544*d^7*e^4*x^(5/3) + 6930*d^8*e^3*x^(4/3) - 9240*d^9*e^2*x + 13860*d^10
*e*x^(2/3) - 27720*d^11*x^(1/3))/e^12)*log((e*x^(1/3) + d)^n*c)^2 + e*n*((1
2326391000*e^12*x^4 - 44119404000*d*e^11*x^(11/3) + 106944990768*d^2*e^10*x
^(10/3) - 220161492320*d^3*e^9*x^3 + 3550000608000*d^12*log(e*x^(1/3) + d)^
3 + 417533743935*d^4*e^8*x^(8/3) - 761128152840*d^5*e^7*x^(7/3) + 137307702
3780*d^6*e^6*x^2 + 33049199383200*d^12*log(e*x^(1/3) + d)^2 - 2516628075192
*d^7*e^5*x^(5/3) + 4836309598890*d^8*e^4*x^(4/3) - 10242678720120*d^9*e^3*x
 + 119225632485960*d^12*log(e*x^(1/3) + d) + 26563616859780*d^10*e^2*x^(2/3
) - 119225632485960*d^11*e*x^(1/3))*n^2/e^13 - 27720*(5336100*e^12*x^4 - 12
171600*d*e^11*x^(11/3) + 21072744*d^2*e^10*x^(10/3) - 32900560*d^3*e^9*x^3
+ 49019355*d^4*e^8*x^(8/3) - 71703720*d^5*e^7*x^(7/3) + 104998740*d^6*e^6*x
^2 + 384199200*d^12*log(e*x^(1/3) + d)^2 - 156734424*d^7*e^5*x^(5/3) + 2439
42930*d^8*e^4*x^(4/3) - 410634840*d^9*e^3*x + 2384502120*d^12*log(e*x^(1/3)
 + d) + 808051860*d^10*e^2*x^(2/3) - 2384502120*d^11*e*x^(1/3))*n*log((e*x^
(1/3) + d)^n*c)/e^13))*b^3
```

mupad [B] time = 8.47, size = 1802, normalized size = 0.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/3))^n))^3,x)
```

```
[Out] (a^3*x^4)/4 + (b^3*x^4*log(c*(d + e*x^(1/3))^n)^3)/4 - (b^3*n^3*x^4)/1152 +
 (3*a*b^2*x^4*log(c*(d + e*x^(1/3))^n)^2)/4 - (b^3*n*x^4*log(c*(d + e*x^(1/
3))^n)^2)/16 + (b^3*n^2*x^4*log(c*(d + e*x^(1/3))^n))/96 + (a*b^2*n^2*x^4)/
96 - (b^3*d^12*log(c*(d + e*x^(1/3))^n)^3)/(4*e^12) + (3*a^2*b*x^4*log(c*(d
 + e*x^(1/3))^n))/4 - (a^2*b*n*x^4)/16 - (a*b^2*n*x^4*log(c*(d + e*x^(1/3))
^n))/8 - (4301068993*b^3*d^12*n^3*log(d + e*x^(1/3)))/(512265600*e^12) + (3
64699*b^3*d^3*n^3*x^3)/(23522400*e^3) - (297202819*b^3*d^6*n^3*x^2)/(307359
3600*e^6) - (21871*b^3*d^2*n^3*x^(10/3))/(2904000*e^2) - (2459191*b^3*d^4*n
^3*x^(8/3))/(83635200*e^4) + (192204079*b^3*d^5*n^3*x^(7/3))/(3585859200*e^
5) + (453937243*b^3*d^7*n^3*x^(5/3))/(2561328000*e^7) - (697880173*b^3*d^8*
n^3*x^(4/3))/(2049062400*e^8) - (1916566873*b^3*d^10*n^3*x^(2/3))/(10245312
00*e^10) + (4301068993*b^3*d^11*n^3*x^(1/3))/(512265600*e^11) - (3*a*b^2*d^
12*log(c*(d + e*x^(1/3))^n)^2)/(4*e^12) + (86021*b^3*d^12*n*log(c*(d + e*x^
(1/3))^n)^2)/(36960*e^12) + (397*b^3*d*n^3*x^(11/3))/(127776*e) + (11085150
13*b^3*d^9*n^3*x)/(1536796800*e^9) - (3*a^2*b*d^12*n*log(d + e*x^(1/3)))/(4
*e^12) + (3*b^3*d*n*x^(11/3)*log(c*(d + e*x^(1/3))^n)^2)/(44*e) - (23*b^3*d
```

```
*n^2*x^(11/3)*log(c*(d + e*x^(1/3))^n))/(968*e) + (b^3*d^9*n*x*log(c*(d + e
*x^(1/3))^n)^2)/(4*e^9) - (44441*b^3*d^9*n^2*x*log(c*(d + e*x^(1/3))^n))/(5
5440*e^9) + (a^2*b*d^3*n*x^3)/(12*e^3) - (a^2*b*d^6*n*x^2)/(8*e^6) - (23*a*
b^2*d*n^2*x^(11/3))/(968*e) - (3*a^2*b*d^2*n*x^(10/3))/(40*e^2) - (3*a^2*b*
d^4*n*x^(8/3))/(32*e^4) - (44441*a*b^2*d^9*n^2*x)/(55440*e^9) + (3*a^2*b*d^
5*n*x^(7/3))/(28*e^5) + (3*a^2*b*d^7*n*x^(5/3))/(20*e^7) - (3*a^2*b*d^8*n*x
^(4/3))/(16*e^8) - (3*a^2*b*d^10*n*x^(2/3))/(8*e^10) + (3*a^2*b*d^11*n*x^(1
/3))/(4*e^11) + (86021*a*b^2*d^12*n^2*log(d + e*x^(1/3)))/(18480*e^12) + (b
^3*d^3*n*x^3*log(c*(d + e*x^(1/3))^n)^2)/(12*e^3) - (763*b^3*d^3*n^2*x^3*lo
g(c*(d + e*x^(1/3))^n))/(11880*e^3) - (b^3*d^6*n*x^2*log(c*(d + e*x^(1/3))^
n)^2)/(8*e^6) + (22727*b^3*d^6*n^2*x^2*log(c*(d + e*x^(1/3))^n))/(110880*e^
6) - (3*b^3*d^2*n*x^(10/3)*log(c*(d + e*x^(1/3))^n)^2)/(40*e^2) + (181*b^3*
d^2*n^2*x^(10/3)*log(c*(d + e*x^(1/3))^n))/(4400*e^2) - (3*b^3*d^4*n*x^(8/3
)*log(c*(d + e*x^(1/3))^n)^2)/(32*e^4) + (2021*b^3*d^4*n^2*x^(8/3)*log(c*(d
 + e*x^(1/3))^n))/(21120*e^4) + (3*b^3*d^5*n*x^(7/3)*log(c*(d + e*x^(1/3))^
n)^2)/(28*e^5) - (18107*b^3*d^5*n^2*x^(7/3)*log(c*(d + e*x^(1/3))^n))/(1293
60*e^5) + (3*b^3*d^7*n*x^(5/3)*log(c*(d + e*x^(1/3))^n)^2)/(20*e^7) - (2827
1*b^3*d^7*n^2*x^(5/3)*log(c*(d + e*x^(1/3))^n))/(92400*e^7) - (3*b^3*d^8*n*
x^(4/3)*log(c*(d + e*x^(1/3))^n)^2)/(16*e^8) + (35201*b^3*d^8*n^2*x^(4/3)*l
og(c*(d + e*x^(1/3))^n))/(73920*e^8) - (3*b^3*d^10*n*x^(2/3)*log(c*(d + e*x
^(1/3))^n)^2)/(8*e^10) + (58301*b^3*d^10*n^2*x^(2/3)*log(c*(d + e*x^(1/3))^
n))/(36960*e^10) + (3*b^3*d^11*n*x^(1/3)*log(c*(d + e*x^(1/3))^n)^2)/(4*e^1
1) - (86021*b^3*d^11*n^2*x^(1/3)*log(c*(d + e*x^(1/3))^n))/(18480*e^11) - (
763*a*b^2*d^3*n^2*x^3)/(11880*e^3) + (22727*a*b^2*d^6*n^2*x^2)/(110880*e^6)
 + (181*a*b^2*d^2*n^2*x^(10/3))/(4400*e^2) + (2021*a*b^2*d^4*n^2*x^(8/3))/(
21120*e^4) - (18107*a*b^2*d^5*n^2*x^(7/3))/(129360*e^5) - (28271*a*b^2*d^7*
n^2*x^(5/3))/(92400*e^7) + (35201*a*b^2*d^8*n^2*x^(4/3))/(73920*e^8) + (583
01*a*b^2*d^10*n^2*x^(2/3))/(36960*e^10) - (86021*a*b^2*d^11*n^2*x^(1/3))/(1
8480*e^11) + (3*a^2*b*d*n*x^(11/3))/(44*e) + (a^2*b*d^9*n*x)/(4*e^9) + (3*a
*b^2*d*n*x^(11/3)*log(c*(d + e*x^(1/3))^n))/(22*e) + (a*b^2*d^9*n*x*log(c*(
d + e*x^(1/3))^n))/(2*e^9) + (a*b^2*d^3*n*x^3*log(c*(d + e*x^(1/3))^n))/(6*
e^3) - (a*b^2*d^6*n*x^2*log(c*(d + e*x^(1/3))^n))/(4*e^6) - (3*a*b^2*d^2*n*
x^(10/3)*log(c*(d + e*x^(1/3))^n))/(20*e^2) - (3*a*b^2*d^4*n*x^(8/3)*log(c*
(d + e*x^(1/3))^n))/(16*e^4) + (3*a*b^2*d^5*n*x^(7/3)*log(c*(d + e*x^(1/3))
^n))/(14*e^5) + (3*a*b^2*d^7*n*x^(5/3)*log(c*(d + e*x^(1/3))^n))/(10*e^7) -
 (3*a*b^2*d^8*n*x^(4/3)*log(c*(d + e*x^(1/3))^n))/(8*e^8) - (3*a*b^2*d^10*n
*x^(2/3)*log(c*(d + e*x^(1/3))^n))/(4*e^10) + (3*a*b^2*d^11*n*x^(1/3)*log(c
*(d + e*x^(1/3))^n))/(2*e^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))**n))**3,x)
[Out] Timed out
```

3.457 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3 dx$

Optimal. Leaf size=1357

$$-\frac{2b^3n^3\left(d+e\sqrt[3]{x}\right)^9}{729e^9}+\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^9}{3e^9}-\frac{bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^9}{9e^9}+\frac{2b^2n}{}$$

```
[Out] 1/3*(d+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9+9*b^3*d^7*n^3*(d+e*x^
(1/3))^2/e^9-56/9*b^3*d^6*n^3*(d+e*x^(1/3))^3/e^9+63/16*b^3*d^5*n^3*(d+e*x^
(1/3))^4/e^9-252/125*b^3*d^4*n^3*(d+e*x^(1/3))^5/e^9+7/9*b^3*d^3*n^3*(d+e*x
^(1/3))^6/e^9-72/343*b^3*d^2*n^3*(d+e*x^(1/3))^7/e^9+9/256*b^3*d*n^3*(d+e*x
^(1/3))^8/e^9-18*b^3*d^8*n^3*x^(1/3)/e^8+18*a*b^2*d^8*n^2*x^(1/3)/e^8-9*b*d
^8*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9+18*b*d^7*n*(d+e*x^(1/3
))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9-28*b*d^6*n*(d+e*x^(1/3))^3*(a+b*ln(c
*(d+e*x^(1/3))^n))^2/e^9+63/2*b*d^5*n*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3
))^n))^2/e^9-126/5*b*d^4*n*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^
9+14*b*d^3*n*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9-36/7*b*d^2*n
*(d+e*x^(1/3))^7*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9+9/8*b*d*n*(d+e*x^(1/3))^
8*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9+3*d^8*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1
/3))^n))^3/e^9-12*d^7*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9+28*
d^6*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9-42*d^5*(d+e*x^(1/3))^
4*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9+42*d^4*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x
^(1/3))^n))^3/e^9-28*d^3*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9+
12*d^2*(d+e*x^(1/3))^7*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9-3*d*(d+e*x^(1/3))^
8*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9-2/729*b^3*n^3*(d+e*x^(1/3))^9/e^9+2/81*
b^2*n^2*(d+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-1/9*b*n*(d+e*x^(1/3
))^9*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9+18*b^3*d^8*n^2*(d+e*x^(1/3))*ln(c*(d
+e*x^(1/3))^n)/e^9-18*b^2*d^7*n^2*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n
))/e^9+56/3*b^2*d^6*n^2*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-63/
4*b^2*d^5*n^2*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+252/25*b^2*d^
4*n^2*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-14/3*b^2*d^3*n^2*(d+e
*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+72/49*b^2*d^2*n^2*(d+e*x^(1/3))
^7*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-9/32*b^2*d*n^2*(d+e*x^(1/3))^8*(a+b*ln(c
*(d+e*x^(1/3))^n))/e^9
```

Rubi [A] time = 1.57, antiderivative size = 1357, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (9*b^3*d^7*n^3*(d + e*x^(1/3))^2)/e^9 - (56*b^3*d^6*n^3*(d + e*x^(1/3))^3)/
(9*e^9) + (63*b^3*d^5*n^3*(d + e*x^(1/3))^4)/(16*e^9) - (252*b^3*d^4*n^3*(d
 + e*x^(1/3))^5)/(125*e^9) + (7*b^3*d^3*n^3*(d + e*x^(1/3))^6)/(9*e^9) - (7
2*b^3*d^2*n^3*(d + e*x^(1/3))^7)/(343*e^9) + (9*b^3*d*n^3*(d + e*x^(1/3))^8
)/(256*e^9) - (2*b^3*n^3*(d + e*x^(1/3))^9)/(729*e^9) + (18*a*b^2*d^8*n^2*x
^(1/3))/e^8 - (18*b^3*d^8*n^3*x^(1/3))/e^8 + (18*b^3*d^8*n^2*(d + e*x^(1/3)
)*Log[c*(d + e*x^(1/3))^n])/e^9 - (18*b^2*d^7*n^2*(d + e*x^(1/3))^2*(a + b*
Log[c*(d + e*x^(1/3))^n]))/e^9 + (56*b^2*d^6*n^2*(d + e*x^(1/3))^3*(a + b*L
og[c*(d + e*x^(1/3))^n]))/(3*e^9) - (63*b^2*d^5*n^2*(d + e*x^(1/3))^4*(a +
b*Log[c*(d + e*x^(1/3))^n]))/(4*e^9) + (252*b^2*d^4*n^2*(d + e*x^(1/3))^5*(
a + b*Log[c*(d + e*x^(1/3))^n]))/(25*e^9) - (14*b^2*d^3*n^2*(d + e*x^(1/3))
^6*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^9) + (72*b^2*d^2*n^2*(d + e*x^(1/
3))^7*(a + b*Log[c*(d + e*x^(1/3))^n]))/(49*e^9) - (9*b^2*d*n^2*(d + e*x^(1
```

```
/3))^8*(a + b*Log[c*(d + e*x^(1/3))^n]))/(32*e^9) + (2*b^2*n^2*(d + e*x^(1/
3))^9*(a + b*Log[c*(d + e*x^(1/3))^n]))/(81*e^9) - (9*b*d^8*n*(d + e*x^(1/3
))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 + (18*b*d^7*n*(d + e*x^(1/3))^2*
(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 - (28*b*d^6*n*(d + e*x^(1/3))^3*(a
+ b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 + (63*b*d^5*n*(d + e*x^(1/3))^4*(a + b
*Log[c*(d + e*x^(1/3))^n])^2)/(2*e^9) - (126*b*d^4*n*(d + e*x^(1/3))^5*(a +
 b*Log[c*(d + e*x^(1/3))^n])^2)/(5*e^9) + (14*b*d^3*n*(d + e*x^(1/3))^6*(a
+ b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 - (36*b*d^2*n*(d + e*x^(1/3))^7*(a + b
*Log[c*(d + e*x^(1/3))^n])^2)/(7*e^9) + (9*b*d*n*(d + e*x^(1/3))^8*(a + b*L
og[c*(d + e*x^(1/3))^n])^2)/(8*e^9) - (b*n*(d + e*x^(1/3))^9*(a + b*Log[c*(
d + e*x^(1/3))^n])^2)/(9*e^9) + (3*d^8*(d + e*x^(1/3))*(a + b*Log[c*(d + e*
x^(1/3))^n])^3)/e^9 - (12*d^7*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3)
)^n])^3)/e^9 + (28*d^6*(d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3
)/e^9 - (42*d^5*(d + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 +
 (42*d^4*(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 - (28*d^
3*(d + e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 + (12*d^2*(d +
e*x^(1/3))^7*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 - (3*d*(d + e*x^(1/3))
^8*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 + ((d + e*x^(1/3))^9*(a + b*Log[
c*(d + e*x^(1/3))^n])^3)/(3*e^9)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
```

```
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = 3\,\text{Subst}\left(\int x^8\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int\left(\frac{d^8\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^8}-\frac{8d^7(d+ex)\left(a+b\log\left(c(d+e\right.\right.}{e^8}\right.\right.$$

$$= \frac{3\,\text{Subst}\left(\int(d+ex)^8\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt[3]{x}\right)}{e^8}-\frac{(24d)\,\text{Subst}\left(\right.}{}$$

$$= \frac{3\,\text{Subst}\left(\int x^8\left(a+b\log\left(cx^n\right)\right)^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^9}-\frac{(24d)\,\text{Subst}\left(\int x^7\left(a+\right.\right.}{}$$

$$= \frac{3d^8\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^9}-\frac{12d^7\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\right.}{e^9}$$

$$= -\frac{9bd^8n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^9}+\frac{18bd^7n\left(d+e\sqrt[3]{x}\right)^2\left(a\right.}{}$$

$$= \frac{9b^3d^7n^3\left(d+e\sqrt[3]{x}\right)^2}{e^9}-\frac{56b^3d^6n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^9}+\frac{63b^3d^5n^3\left(d+e\sqrt[3]{x}\right)^4}{16e^9}-$$

$$= \frac{9b^3d^7n^3\left(d+e\sqrt[3]{x}\right)^2}{e^9}-\frac{56b^3d^6n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^9}+\frac{63b^3d^5n^3\left(d+e\sqrt[3]{x}\right)^4}{16e^9}-$$

Mathematica [A] time = 0.88, size = 808, normalized size = 0.60

$$2667168000\left(d^9+e^9x^3\right)a^3-3175200bn\left(7129d^9+2520e\sqrt[3]{x}\,d^8-1260e^2x^{2/3}d^7+840e^3xd^6-630e^4x^{4/3}d^5+504e\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (b^3*e*n^3*x^(1/3)*(-76356985320*d^8 + 15542491860*d^7*e*x^(1/3) - 54834956
40*d^6*e^2*x^(2/3) + 2340330930*d^5*e^3*x - 1075607064*d^4*e^4*x^(4/3) + 49
8592500*d^3*e^5*x^(5/3) - 219465000*d^2*e^6*x^2 + 83734875*d*e^7*x^(7/3) -
21952000*e^8*x^(8/3)) - 2520*a*b^2*n^2*(26853209*d^9 - 17965080*d^8*e*x^(1/
3) + 5807340*d^7*e^2*x^(2/3) - 2813160*d^6*e^3*x + 1580670*d^5*e^4*x^(4/3)
- 947016*d^4*e^5*x^(5/3) + 577500*d^3*e^6*x^2 - 343800*d^2*e^7*x^(7/3) + 18
7425*d*e^8*x^(8/3) - 78400*e^9*x^3) + 2667168000*a^3*(d^9 + e^9*x^3) - 3175
200*a^2*b*n*(7129*d^9 + 2520*d^8*e*x^(1/3) - 1260*d^7*e^2*x^(2/3) + 840*d^6
*e^3*x - 630*d^5*e^4*x^(4/3) + 504*d^4*e^5*x^(5/3) - 420*d^3*e^6*x^2 + 360*
```

```
d^2*e^7*x^(7/3) - 315*d*e^8*x^(8/3) + 280*e^9*x^3) + 2520*b*(3175200*a^2*(d
^9 + e^9*x^3) - 2520*a*b*n*(7129*d^9 + 2520*d^8*e*x^(1/3) - 1260*d^7*e^2*x^
(2/3) + 840*d^6*e^3*x - 630*d^5*e^4*x^(4/3) + 504*d^4*e^5*x^(5/3) - 420*d^3
*e^6*x^2 + 360*d^2*e^7*x^(7/3) - 315*d*e^8*x^(8/3) + 280*e^9*x^3) + b^2*n^2
*(30300391*d^9 + 17965080*d^8*e*x^(1/3) - 5807340*d^7*e^2*x^(2/3) + 2813160
*d^6*e^3*x - 1580670*d^5*e^4*x^(4/3) + 947016*d^4*e^5*x^(5/3) - 577500*d^3*
e^6*x^2 + 343800*d^2*e^7*x^(7/3) - 187425*d*e^8*x^(8/3) + 78400*e^9*x^3))*L
og[c*(d + e*x^(1/3))^n] + 3175200*b^2*(2520*a*(d^9 + e^9*x^3) - b*n*(7129*d
^9 + 2520*d^8*e*x^(1/3) - 1260*d^7*e^2*x^(2/3) + 840*d^6*e^3*x - 630*d^5*e^
4*x^(4/3) + 504*d^4*e^5*x^(5/3) - 420*d^3*e^6*x^2 + 360*d^2*e^7*x^(7/3) - 3
15*d*e^8*x^(8/3) + 280*e^9*x^3))*Log[c*(d + e*x^(1/3))^n]^2 + 2667168000*b^
3*(d^9 + e^9*x^3)*Log[c*(d + e*x^(1/3))^n]^3)/(8001504000*e^9)
```

fricas [A] time = 0.67, size = 1688, normalized size = 1.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/8001504000*(2667168000*b^3*e^9*x^3*log(c)^3 - 10976000*(2*b^3*e^9*n^3 - 1
8*a*b^2*e^9*n^2 + 81*a^2*b*e^9*n - 243*a^3*e^9)*x^3 + 2667168000*(b^3*e^9*n
^3*x^3 + b^3*d^9*n^3)*log(e*x^(1/3) + d)^3 + 10500*(47485*b^3*d^3*e^6*n^3 -
 138600*a*b^2*d^3*e^6*n^2 + 127008*a^2*b*d^3*e^6*n)*x^2 + 3175200*(420*b^3*
d^3*e^6*n^3*x^2 - 840*b^3*d^6*e^3*n^3*x - 7129*b^3*d^9*n^3 + 2520*a*b^2*d^9
*n^2 - 280*(b^3*e^9*n^3 - 9*a*b^2*e^9*n^2)*x^3 + 2520*(b^3*e^9*n^2*x^3 + b^
3*d^9*n^2)*log(c) + 63*(5*b^3*d*e^8*n^3*x^2 - 8*b^3*d^4*e^5*n^3*x + 20*b^3*
d^7*e^2*n^3)*x^(2/3) - 90*(4*b^3*d^2*e^7*n^3*x^2 - 7*b^3*d^5*e^4*n^3*x + 28
*b^3*d^8*e*n^3)*x^(1/3))*log(e*x^(1/3) + d)^2 + 444528000*(3*b^3*d^3*e^6*n*
x^2 - 6*b^3*d^6*e^3*n*x - 2*(b^3*e^9*n - 9*a*b^2*e^9)*x^3)*log(c)^2 - 840*(
6527971*b^3*d^6*e^3*n^3 - 8439480*a*b^2*d^6*e^3*n^2 + 3175200*a^2*b*d^6*e^3
*n)*x + 2520*(30300391*b^3*d^9*n^3 - 17965080*a*b^2*d^9*n^2 + 3175200*a^2*b
*d^9*n + 39200*(2*b^3*e^9*n^3 - 18*a*b^2*e^9*n^2 + 81*a^2*b*e^9*n)*x^3 - 21
00*(275*b^3*d^3*e^6*n^3 - 504*a*b^2*d^3*e^6*n^2)*x^2 + 3175200*(b^3*e^9*n*x
^3 + b^3*d^9*n)*log(c)^2 + 840*(3349*b^3*d^6*e^3*n^3 - 2520*a*b^2*d^6*e^3*n
^2)*x + 2520*(420*b^3*d^3*e^6*n^2*x^2 - 840*b^3*d^6*e^3*n^2*x - 7129*b^3*d^
9*n^2 + 2520*a*b^2*d^9*n - 280*(b^3*e^9*n^2 - 9*a*b^2*e^9*n)*x^3)*log(c) -
63*(92180*b^3*d^7*e^2*n^3 - 50400*a*b^2*d^7*e^2*n^2 + 175*(17*b^3*d*e^8*n^3
 - 72*a*b^2*d*e^8*n^2)*x^2 - 8*(1879*b^3*d^4*e^5*n^3 - 2520*a*b^2*d^4*e^5*n
^2)*x - 2520*(5*b^3*d*e^8*n^2*x^2 - 8*b^3*d^4*e^5*n^2*x + 20*b^3*d^7*e^2*n^
2)*log(c))*x^(2/3) + 90*(199612*b^3*d^8*e*n^3 - 70560*a*b^2*d^8*e*n^2 + 20*
(191*b^3*d^2*e^7*n^3 - 504*a*b^2*d^2*e^7*n^2)*x^2 - 7*(2509*b^3*d^5*e^4*n^3
 - 2520*a*b^2*d^5*e^4*n^2)*x - 2520*(4*b^3*d^2*e^7*n^2*x^2 - 7*b^3*d^5*e^4*
n^2*x + 28*b^3*d^8*e*n^2)*log(c))*x^(1/3))*log(e*x^(1/3) + d) + 352800*(280
*(2*b^3*e^9*n^2 - 18*a*b^2*e^9*n + 81*a^2*b*e^9)*x^3 - 15*(275*b^3*d^3*e^6*
n^2 - 504*a*b^2*d^3*e^6*n)*x^2 + 6*(3349*b^3*d^6*e^3*n^2 - 2520*a*b^2*d^6*e
^3*n)*x)*log(c) + 63*(246706220*b^3*d^7*e^2*n^3 - 232293600*a*b^2*d^7*e^2*n
^2 + 63504000*a^2*b*d^7*e^2*n + 6125*(217*b^3*d*e^8*n^3 - 1224*a*b^2*d*e^8*
n^2 + 2592*a^2*b*d*e^8*n)*x^2 + 3175200*(5*b^3*d*e^8*n*x^2 - 8*b^3*d^4*e^5*
n*x + 20*b^3*d^7*e^2*n)*log(c)^2 - 8*(2134141*b^3*d^4*e^5*n^3 - 4735080*a*b
^2*d^4*e^5*n^2 + 3175200*a^2*b*d^4*e^5*n)*x - 2520*(92180*b^3*d^7*e^2*n^2 -
 50400*a*b^2*d^7*e^2*n + 175*(17*b^3*d*e^8*n^2 - 72*a*b^2*d*e^8*n)*x^2 - 8*
(1879*b^3*d^4*e^5*n^2 - 2520*a*b^2*d^4*e^5*n)*x)*log(c))*x^(2/3) - 90*(8484
10948*b^3*d^8*e*n^3 - 503022240*a*b^2*d^8*e*n^2 + 88905600*a^2*b*d^8*e*n +
100*(24385*b^3*d^2*e^7*n^3 - 96264*a*b^2*d^2*e^7*n^2 + 127008*a^2*b*d^2*e^7
*n)*x^2 + 3175200*(4*b^3*d^2*e^7*n*x^2 - 7*b^3*d^5*e^4*n*x + 28*b^3*d^8*e*n
)*log(c)^2 - 7*(3714811*b^3*d^5*e^4*n^3 - 6322680*a*b^2*d^5*e^4*n^2 + 31752
00*a^2*b*d^5*e^4*n)*x - 2520*(199612*b^3*d^8*e*n^2 - 70560*a*b^2*d^8*e*n +
20*(191*b^3*d^2*e^7*n^2 - 504*a*b^2*d^2*e^7*n)*x^2 - 7*(2509*b^3*d^5*e^4*n^
2 - 2520*a*b^2*d^5*e^4*n)*x)*log(c))*x^(1/3))/e^9
```

giac [B] time = 0.43, size = 3333, normalized size = 2.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/8001504000*(2667168000*b^3*x^3*e*log(c)^3 + 8001504000*a*b^2*x^3*e*log(c)
^2 + 8001504000*a^2*b*x^3*e*log(c) + (2667168000*(x^(1/3)*e + d)^9*e^(-8)*l
og(x^(1/3)*e + d)^3 - 24004512000*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e
+ d)^3 + 96018048000*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d)^3 - 22
4042112000*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/3)*e + d)^3 + 336063168000
*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d)^3 - 336063168000*(x^(1/3)*
e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d)^3 + 224042112000*(x^(1/3)*e + d)^3*d
^6*e^(-8)*log(x^(1/3)*e + d)^3 - 96018048000*(x^(1/3)*e + d)^2*d^7*e^(-8)*l
og(x^(1/3)*e + d)^3 + 24004512000*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e
+ d)^3 - 889056000*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d)^2 + 90016920
00*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d)^2 - 41150592000*(x^(1/3)*e
 + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d)^2 + 112021056000*(x^(1/3)*e + d)^6*d^
3*e^(-8)*log(x^(1/3)*e + d)^2 - 201637900800*(x^(1/3)*e + d)^5*d^4*e^(-8)*l
og(x^(1/3)*e + d)^2 + 252047376000*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)
*e + d)^2 - 224042112000*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d)^2
+ 144027072000*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d)^2 - 72013536
000*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e + d)^2 + 197568000*(x^(1/3)*e
+ d)^9*e^(-8)*log(x^(1/3)*e + d) - 2250423000*(x^(1/3)*e + d)^8*d*e^(-8)*lo
g(x^(1/3)*e + d) + 11757312000*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e +
 d) - 37340352000*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/3)*e + d) + 8065516
0320*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d) - 126023688000*(x^(1/3
)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d) + 149361408000*(x^(1/3)*e + d)^3*d
^6*e^(-8)*log(x^(1/3)*e + d) - 144027072000*(x^(1/3)*e + d)^2*d^7*e^(-8)*lo
g(x^(1/3)*e + d) + 144027072000*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e +
d) - 21952000*(x^(1/3)*e + d)^9*e^(-8) + 281302875*(x^(1/3)*e + d)^8*d*e^(-
8) - 1679616000*(x^(1/3)*e + d)^7*d^2*e^(-8) + 6223392000*(x^(1/3)*e + d)^6
*d^3*e^(-8) - 16131032064*(x^(1/3)*e + d)^5*d^4*e^(-8) + 31505922000*(x^(1/
3)*e + d)^4*d^5*e^(-8) - 49787136000*(x^(1/3)*e + d)^3*d^6*e^(-8) + 7201353
6000*(x^(1/3)*e + d)^2*d^7*e^(-8) - 144027072000*(x^(1/3)*e + d)*d^8*e^(-8)
)*b^3*n^3 + 2667168000*a^3*x^3*e + 2520*(3175200*(x^(1/3)*e + d)^9*e^(-8)*l
og(x^(1/3)*e + d)^2 - 28576800*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d
)^2 + 114307200*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d)^2 - 2667168
00*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/3)*e + d)^2 + 400075200*(x^(1/3)*e
 + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d)^2 - 400075200*(x^(1/3)*e + d)^4*d^5*e
^(-8)*log(x^(1/3)*e + d)^2 + 266716800*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(
1/3)*e + d)^2 - 114307200*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d)^2
 + 28576800*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e + d)^2 - 705600*(x^(1/
3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d) + 7144200*(x^(1/3)*e + d)^8*d*e^(-8)*
log(x^(1/3)*e + d) - 32659200*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e +
d) + 88905600*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/3)*e + d) - 160030080*(
x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d) + 200037600*(x^(1/3)*e + d)^
4*d^5*e^(-8)*log(x^(1/3)*e + d) - 177811200*(x^(1/3)*e + d)^3*d^6*e^(-8)*lo
g(x^(1/3)*e + d) + 114307200*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d
) - 57153600*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e + d) + 78400*(x^(1/3)
*e + d)^9*e^(-8) - 893025*(x^(1/3)*e + d)^8*d*e^(-8) + 4665600*(x^(1/3)*e +
 d)^7*d^2*e^(-8) - 14817600*(x^(1/3)*e + d)^6*d^3*e^(-8) + 32006016*(x^(1/3
)*e + d)^5*d^4*e^(-8) - 50009400*(x^(1/3)*e + d)^4*d^5*e^(-8) + 59270400*(x
^(1/3)*e + d)^3*d^6*e^(-8) - 57153600*(x^(1/3)*e + d)^2*d^7*e^(-8) + 571536
00*(x^(1/3)*e + d)*d^8*e^(-8))*b^3*n^2*log(c) + 3175200*(2520*(x^(1/3)*e +
d)^9*e^(-8)*log(x^(1/3)*e + d) - 22680*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/
3)*e + d) + 90720*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d) - 211680*
(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/3)*e + d) + 317520*(x^(1/3)*e + d)^5*
```

```
d^4*e^(-8)*log(x^(1/3)*e + d) - 317520*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(
1/3)*e + d) + 211680*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d) - 9072
0*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d) + 22680*(x^(1/3)*e + d)*d
^8*e^(-8)*log(x^(1/3)*e + d) - 280*(x^(1/3)*e + d)^9*e^(-8) + 2835*(x^(1/3)
*e + d)^8*d*e^(-8) - 12960*(x^(1/3)*e + d)^7*d^2*e^(-8) + 35280*(x^(1/3)*e
+ d)^6*d^3*e^(-8) - 63504*(x^(1/3)*e + d)^5*d^4*e^(-8) + 79380*(x^(1/3)*e +
 d)^4*d^5*e^(-8) - 70560*(x^(1/3)*e + d)^3*d^6*e^(-8) + 45360*(x^(1/3)*e +
d)^2*d^7*e^(-8) - 22680*(x^(1/3)*e + d)*d^8*e^(-8))*b^3*n*log(c)^2 + 2520*(
3175200*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d)^2 - 28576800*(x^(1/3)*e
 + d)^8*d*e^(-8)*log(x^(1/3)*e + d)^2 + 114307200*(x^(1/3)*e + d)^7*d^2*e^(
-8)*log(x^(1/3)*e + d)^2 - 266716800*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/
3)*e + d)^2 + 400075200*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d)^2 -
 400075200*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d)^2 + 266716800*(x
^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d)^2 - 114307200*(x^(1/3)*e + d)
^2*d^7*e^(-8)*log(x^(1/3)*e + d)^2 + 28576800*(x^(1/3)*e + d)*d^8*e^(-8)*lo
g(x^(1/3)*e + d)^2 - 705600*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d) + 7
144200*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d) - 32659200*(x^(1/3)*e
+ d)^7*d^2*e^(-8)*log(x^(1/3)*e + d) + 88905600*(x^(1/3)*e + d)^6*d^3*e^(-8
)*log(x^(1/3)*e + d) - 160030080*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e
 + d) + 200037600*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d) - 1778112
00*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d) + 114307200*(x^(1/3)*e +
 d)^2*d^7*e^(-8)*log(x^(1/3)*e + d) - 57153600*(x^(1/3)*e + d)*d^8*e^(-8)*l
og(x^(1/3)*e + d) + 78400*(x^(1/3)*e + d)^9*e^(-8) - 893025*(x^(1/3)*e + d)
^8*d*e^(-8) + 4665600*(x^(1/3)*e + d)^7*d^2*e^(-8) - 14817600*(x^(1/3)*e +
d)^6*d^3*e^(-8) + 32006016*(x^(1/3)*e + d)^5*d^4*e^(-8) - 50009400*(x^(1/3)
*e + d)^4*d^5*e^(-8) + 59270400*(x^(1/3)*e + d)^3*d^6*e^(-8) - 57153600*(x^
(1/3)*e + d)^2*d^7*e^(-8) + 57153600*(x^(1/3)*e + d)*d^8*e^(-8))*a*b^2*n^2
+ 6350400*(2520*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d) - 22680*(x^(1/3
)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d) + 90720*(x^(1/3)*e + d)^7*d^2*e^(-8)
*log(x^(1/3)*e + d) - 211680*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/3)*e + d
) + 317520*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d) - 317520*(x^(1/3
)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d) + 211680*(x^(1/3)*e + d)^3*d^6*e^(
-8)*log(x^(1/3)*e + d) - 90720*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e +
 d) + 22680*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e + d) - 280*(x^(1/3)*e
+ d)^9*e^(-8) + 2835*(x^(1/3)*e + d)^8*d*e^(-8) - 12960*(x^(1/3)*e + d)^7*d
^2*e^(-8) + 35280*(x^(1/3)*e + d)^6*d^3*e^(-8) - 63504*(x^(1/3)*e + d)^5*d^
4*e^(-8) + 79380*(x^(1/3)*e + d)^4*d^5*e^(-8) - 70560*(x^(1/3)*e + d)^3*d^6
*e^(-8) + 45360*(x^(1/3)*e + d)^2*d^7*e^(-8) - 22680*(x^(1/3)*e + d)*d^8*e^
(-8))*a*b^2*n*log(c) + 3175200*(2520*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3)*e
 + d) - 22680*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d) + 90720*(x^(1/3
)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d) - 211680*(x^(1/3)*e + d)^6*d^3*e^(
-8)*log(x^(1/3)*e + d) + 317520*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e
+ d) - 317520*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d) + 211680*(x^(
1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d) - 90720*(x^(1/3)*e + d)^2*d^7*e
^(-8)*log(x^(1/3)*e + d) + 22680*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e +
 d) - 280*(x^(1/3)*e + d)^9*e^(-8) + 2835*(x^(1/3)*e + d)^8*d*e^(-8) - 1296
0*(x^(1/3)*e + d)^7*d^2*e^(-8) + 35280*(x^(1/3)*e + d)^6*d^3*e^(-8) - 63504
*(x^(1/3)*e + d)^5*d^4*e^(-8) + 79380*(x^(1/3)*e + d)^4*d^5*e^(-8) - 70560*
(x^(1/3)*e + d)^3*d^6*e^(-8) + 45360*(x^(1/3)*e + d)^2*d^7*e^(-8) - 22680*(
x^(1/3)*e + d)*d^8*e^(-8))*a^2*b*n)*e^(-1)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(b \ln\left(c \left(e x^{\frac{1}{3}} + d\right)^{n}\right) + a\right)^{3} x^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*(e*x^(1/3)+d)^n)+a)^3,x)
```

```
[Out] int(x^2*(b*ln(c*(e*x^(1/3)+d)^n)+a)^3,x)
```

maxima [A] time = 0.65, size = 867, normalized size = 0.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3*log((e*x^(1/3) + d)^n*c)^3 + a*b^2*x^3*log((e*x^(1/3) + d)^n*c)
^2 + a^2*b*x^3*log((e*x^(1/3) + d)^n*c) + 1/3*a^3*x^3 + 1/2520*a^2*b*e*n*(2
520*d^9*log(e*x^(1/3) + d)/e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^
2*e^6*x^(7/3) - 420*d^3*e^5*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3)
 + 840*d^6*e^2*x - 1260*d^7*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9) + 1/3175200*
(2520*e*n*(2520*d^9*log(e*x^(1/3) + d)/e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8
/3) + 360*d^2*e^6*x^(7/3) - 420*d^3*e^5*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5
*e^3*x^(4/3) + 840*d^6*e^2*x - 1260*d^7*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9)*
log((e*x^(1/3) + d)^n*c) + (78400*e^9*x^3 - 187425*d*e^8*x^(8/3) + 343800*d
^2*e^7*x^(7/3) - 577500*d^3*e^6*x^2 - 3175200*d^9*log(e*x^(1/3) + d)^2 + 94
7016*d^4*e^5*x^(5/3) - 1580670*d^5*e^4*x^(4/3) + 2813160*d^6*e^3*x - 179650
80*d^9*log(e*x^(1/3) + d) - 5807340*d^7*e^2*x^(2/3) + 17965080*d^8*e*x^(1/3
))*n^2/e^9)*a*b^2 + 1/8001504000*(3175200*e*n*(2520*d^9*log(e*x^(1/3) + d)/
e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^2*e^6*x^(7/3) - 420*d^3*e^5
*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3) + 840*d^6*e^2*x - 1260*d^7
*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9)*log((e*x^(1/3) + d)^n*c)^2 - e*n*((2195
2000*e^9*x^3 - 2667168000*d^9*log(e*x^(1/3) + d)^3 - 83734875*d*e^8*x^(8/3)
 + 219465000*d^2*e^7*x^(7/3) - 498592500*d^3*e^6*x^2 - 22636000800*d^9*log(
e*x^(1/3) + d)^2 + 1075607064*d^4*e^5*x^(5/3) - 2340330930*d^5*e^4*x^(4/3)
+ 5483495640*d^6*e^3*x - 76356985320*d^9*log(e*x^(1/3) + d) - 15542491860*d
^7*e^2*x^(2/3) + 76356985320*d^8*e*x^(1/3))*n^2/e^10 - 2520*(78400*e^9*x^3
- 187425*d*e^8*x^(8/3) + 343800*d^2*e^7*x^(7/3) - 577500*d^3*e^6*x^2 - 3175
200*d^9*log(e*x^(1/3) + d)^2 + 947016*d^4*e^5*x^(5/3) - 1580670*d^5*e^4*x^(
4/3) + 2813160*d^6*e^3*x - 17965080*d^9*log(e*x^(1/3) + d) - 5807340*d^7*e^
2*x^(2/3) + 17965080*d^8*e*x^(1/3))*n*log((e*x^(1/3) + d)^n*c)/e^10))*b^3
```

mupad [B] time = 8.25, size = 1386, normalized size = 1.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/3))^n))^3,x)
```

```
[Out] (a^3*x^3)/3 + (b^3*x^3*log(c*(d + e*x^(1/3))^n)^3)/3 - (2*b^3*n^3*x^3)/729
+ a*b^2*x^3*log(c*(d + e*x^(1/3))^n)^2 - (b^3*n*x^3*log(c*(d + e*x^(1/3))^n
)^2)/9 + (2*b^3*n^2*x^3*log(c*(d + e*x^(1/3))^n))/81 + (2*a*b^2*n^2*x^3)/81
 + (b^3*d^9*log(c*(d + e*x^(1/3))^n)^3)/(3*e^9) + a^2*b*x^3*log(c*(d + e*x^
(1/3))^n) - (a^2*b*n*x^3)/9 - (2*a*b^2*n*x^3*log(c*(d + e*x^(1/3))^n))/9 +
(30300391*b^3*d^9*n^3*log(d + e*x^(1/3)))/(3175200*e^9) + (47485*b^3*d^3*n^
3*x^2)/(762048*e^3) - (24385*b^3*d^2*n^3*x^(7/3))/(889056*e^2) - (2134141*b
^3*d^4*n^3*x^(5/3))/(15876000*e^4) + (3714811*b^3*d^5*n^3*x^(4/3))/(1270080
0*e^5) + (12335311*b^3*d^7*n^3*x^(2/3))/(6350400*e^7) - (30300391*b^3*d^8*n
^3*x^(1/3))/(3175200*e^8) + (a*b^2*d^9*log(c*(d + e*x^(1/3))^n)^2)/e^9 - (7
129*b^3*d^9*n*log(c*(d + e*x^(1/3))^n)^2)/(2520*e^9) + (217*b^3*d*n^3*x^(8/
3))/(20736*e) - (6527971*b^3*d^6*n^3*x)/(9525600*e^6) + (a^2*b*d^9*n*log(d
+ e*x^(1/3)))/e^9 + (b^3*d*n*x^(8/3)*log(c*(d + e*x^(1/3))^n)^2)/(8*e) - (1
7*b^3*d*n^2*x^(8/3)*log(c*(d + e*x^(1/3))^n))/(288*e) - (b^3*d^6*n*x*log(c*
(d + e*x^(1/3))^n)^2)/(3*e^6) + (3349*b^3*d^6*n^2*x*log(c*(d + e*x^(1/3))^n
))/(3780*e^6) + (a^2*b*d^3*n*x^2)/(6*e^3) - (17*a*b^2*d*n^2*x^(8/3))/(288*e
) + (3349*a*b^2*d^6*n^2*x)/(3780*e^6) - (a^2*b*d^2*n*x^(7/3))/(7*e^2) - (a^
2*b*d^4*n*x^(5/3))/(5*e^4) + (a^2*b*d^5*n*x^(4/3))/(4*e^5) + (a^2*b*d^7*n*x
^(2/3))/(2*e^7) - (a^2*b*d^8*n*x^(1/3))/e^8 - (7129*a*b^2*d^9*n^2*log(d + e
```

```
*x^(1/3)))/(1260*e^9) + (b^3*d^3*n*x^2*log(c*(d + e*x^(1/3))^n)^2)/(6*e^3)
- (275*b^3*d^3*n^2*x^2*log(c*(d + e*x^(1/3))^n))/(1512*e^3) - (b^3*d^2*n*x^
(7/3)*log(c*(d + e*x^(1/3))^n)^2)/(7*e^2) + (191*b^3*d^2*n^2*x^(7/3)*log(c*
(d + e*x^(1/3))^n))/(1764*e^2) - (b^3*d^4*n*x^(5/3)*log(c*(d + e*x^(1/3))^n
)^2)/(5*e^4) + (1879*b^3*d^4*n^2*x^(5/3)*log(c*(d + e*x^(1/3))^n))/(6300*e^
4) + (b^3*d^5*n*x^(4/3)*log(c*(d + e*x^(1/3))^n)^2)/(4*e^5) - (2509*b^3*d^5
*n^2*x^(4/3)*log(c*(d + e*x^(1/3))^n))/(5040*e^5) + (b^3*d^7*n*x^(2/3)*log(
c*(d + e*x^(1/3))^n)^2)/(2*e^7) - (4609*b^3*d^7*n^2*x^(2/3)*log(c*(d + e*x^
(1/3))^n))/(2520*e^7) - (b^3*d^8*n*x^(1/3)*log(c*(d + e*x^(1/3))^n)^2)/e^8
+ (7129*b^3*d^8*n^2*x^(1/3)*log(c*(d + e*x^(1/3))^n))/(1260*e^8) - (275*a*b
^2*d^3*n^2*x^2)/(1512*e^3) + (191*a*b^2*d^2*n^2*x^(7/3))/(1764*e^2) + (1879
*a*b^2*d^4*n^2*x^(5/3))/(6300*e^4) - (2509*a*b^2*d^5*n^2*x^(4/3))/(5040*e^5
) - (4609*a*b^2*d^7*n^2*x^(2/3))/(2520*e^7) + (7129*a*b^2*d^8*n^2*x^(1/3))/
(1260*e^8) + (a^2*b*d*n*x^(8/3))/(8*e) - (a^2*b*d^6*n*x)/(3*e^6) + (a*b^2*d
*n*x^(8/3)*log(c*(d + e*x^(1/3))^n))/(4*e) - (2*a*b^2*d^6*n*x*log(c*(d + e*
x^(1/3))^n))/(3*e^6) + (a*b^2*d^3*n*x^2*log(c*(d + e*x^(1/3))^n))/(3*e^3) -
 (2*a*b^2*d^2*n*x^(7/3)*log(c*(d + e*x^(1/3))^n))/(7*e^2) - (2*a*b^2*d^4*n*
x^(5/3)*log(c*(d + e*x^(1/3))^n))/(5*e^4) + (a*b^2*d^5*n*x^(4/3)*log(c*(d +
 e*x^(1/3))^n))/(2*e^5) + (a*b^2*d^7*n*x^(2/3)*log(c*(d + e*x^(1/3))^n))/e^
7 - (2*a*b^2*d^8*n*x^(1/3)*log(c*(d + e*x^(1/3))^n))/e^8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^n\right)\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**n))**3,x)
[Out] Integral(x**2*(a + b*log(c*(d + e*x**(1/3))**n))**3, x)
```

3.458 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=907

$$-\frac{b^3n^3\left(d+e\sqrt[3]{x}\right)^6}{72e^6}+\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^6}{2e^6}-\frac{bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^6}{4e^6}+\frac{b^2n^2\left(\right.}{}$$

```
[Out] 1/2*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^6-45/8*b^3*d^4*n^3*(d+e
*x^(1/3))^2/e^6+20/9*b^3*d^3*n^3*(d+e*x^(1/3))^3/e^6-45/64*b^3*d^2*n^3*(d+e
*x^(1/3))^4/e^6+18/125*b^3*d*n^3*(d+e*x^(1/3))^5/e^6+18*b^3*d^5*n^3*x^(1/3)
/e^5-18*a*b^2*d^5*n^2*x^(1/3)/e^5-18*b^3*d^5*n^2*(d+e*x^(1/3))*ln(c*(d+e*x^
(1/3))^n)/e^6+45/4*b^2*d^4*n^2*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))/
e^6-20/3*b^2*d^3*n^2*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6+45/16*
b^2*d^2*n^2*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-18/25*b^2*d*n^2
*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6+9*b*d^5*n*(d+e*x^(1/3))*(a
+b*ln(c*(d+e*x^(1/3))^n))^2/e^6-45/4*b*d^4*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e
*x^(1/3))^n))^2/e^6+10*b*d^3*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))^
2/e^6-45/8*b*d^2*n*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^6+9/5*b*
d*n*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^6-3*d^5*(d+e*x^(1/3))*(
a+b*ln(c*(d+e*x^(1/3))^n))^3/e^6+15/2*d^4*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^
(1/3))^n))^3/e^6-10*d^3*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^6+1
5/2*d^2*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^6-3*d*(d+e*x^(1/3))
^5*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^6-1/72*b^3*n^3*(d+e*x^(1/3))^6/e^6+1/12*
b^2*n^2*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-1/4*b*n*(d+e*x^(1/3
))^6*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^6
```

Rubi [A] time = 0.98, antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$-\frac{b^3n^3\left(d+e\sqrt[3]{x}\right)^6}{72e^6}+\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^6}{2e^6}-\frac{bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^6}{4e^6}+\frac{b^2n^2\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (-45*b^3*d^4*n^3*(d + e*x^(1/3))^2)/(8*e^6) + (20*b^3*d^3*n^3*(d + e*x^(1/3
))^3)/(9*e^6) - (45*b^3*d^2*n^3*(d + e*x^(1/3))^4)/(64*e^6) + (18*b^3*d*n^3
*(d + e*x^(1/3))^5)/(125*e^6) - (b^3*n^3*(d + e*x^(1/3))^6)/(72*e^6) - (18*
a*b^2*d^5*n^2*x^(1/3))/e^5 + (18*b^3*d^5*n^3*x^(1/3))/e^5 - (18*b^3*d^5*n^2
*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^6 + (45*b^2*d^4*n^2*(d + e*x^(
1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*e^6) - (20*b^2*d^3*n^2*(d + e*
x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^6) + (45*b^2*d^2*n^2*(d +
 e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n]))/(16*e^6) - (18*b^2*d*n^2*(d
 + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n]))/(25*e^6) + (b^2*n^2*(d +
e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n]))/(12*e^6) + (9*b*d^5*n*(d + e
*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^6 - (45*b*d^4*n*(d + e*x^(1
/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(4*e^6) + (10*b*d^3*n*(d + e*x^(
1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^6 - (45*b*d^2*n*(d + e*x^(1/3
))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(8*e^6) + (9*b*d*n*(d + e*x^(1/3))
^5*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(5*e^6) - (b*n*(d + e*x^(1/3))^6*(a
+ b*Log[c*(d + e*x^(1/3))^n])^2)/(4*e^6) - (3*d^5*(d + e*x^(1/3))*(a + b*Lo
g[c*(d + e*x^(1/3))^n])^3)/e^6 + (15*d^4*(d + e*x^(1/3))^2*(a + b*Log[c*(d
+ e*x^(1/3))^n])^3)/(2*e^6) - (10*d^3*(d + e*x^(1/3))^3*(a + b*Log[c*(d + e
*x^(1/3))^n])^3)/e^6 + (15*d^2*(d + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3
```

```
))^n])^3)/(2*e^6) - (3*d*(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])
^3)/e^6 + ((d + e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/(2*e^6)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = 3\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^5}+\frac{5d^4(d+ex)\left(a+b\log(c(d+\right.}{e^5}\right.\right.$$

$$= \frac{3\,\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt[3]{x}\right)}{e^5}-\frac{(15d)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{3\,\text{Subst}\left(\int x^5\left(a+b\log\left(cx^n\right)\right)^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^6}-\frac{(15d)\,\text{Subst}\left(\int x^4\left(a+\right.\right.}{}$$

$$= -\frac{3d^5\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^6}+\frac{15d^4\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\right.}{2e^6}$$

$$= \frac{9bd^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^6}-\frac{45bd^4n\left(d+e\sqrt[3]{x}\right)^2\left(a+\right.}{4e^6}$$

$$= -\frac{45b^3d^4n^3\left(d+e\sqrt[3]{x}\right)^2}{8e^6}+\frac{20b^3d^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^6}-\frac{45b^3d^2n^3\left(d+e\sqrt[3]{x}\right)^4}{64e^6}+$$

$$= -\frac{45b^3d^4n^3\left(d+e\sqrt[3]{x}\right)^2}{8e^6}+\frac{20b^3d^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^6}-\frac{45b^3d^2n^3\left(d+e\sqrt[3]{x}\right)^4}{64e^6}+$$

Mathematica [A] time = 0.51, size = 589, normalized size = 0.65

$$\frac{-36000a^3\left(d^6-e^6x^2\right)-60b\left(1800a^2\left(d^6-e^6x^2\right)-60abn\left(147d^6+60d^5e\sqrt[3]{x}-30d^4e^2x^{2/3}+20d^3e^3x-15d^2e^4x^{4/3}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (b^3*e*n^3*x^(1/3)*(809340*d^5 - 140070*d^4*e*x^(1/3) + 41180*d^3*e^2*x^(2/
3) - 13785*d^2*e^3*x + 4368*d*e^4*x^(4/3) - 1000*e^5*x^(5/3)) + 1800*a^2*b*
n*(147*d^6 + 60*d^5*e*x^(1/3) - 30*d^4*e^2*x^(2/3) + 20*d^3*e^3*x - 15*d^2*
e^4*x^(4/3) + 12*d*e^5*x^(5/3) - 10*e^6*x^2) - 36000*a^3*(d^6 - e^6*x^2) +
60*a*b^2*n^2*(8111*d^6 - 8820*d^5*e*x^(1/3) + 2610*d^4*e^2*x^(2/3) - 1140*d
^3*e^3*x + 555*d^2*e^4*x^(4/3) - 264*d*e^5*x^(5/3) + 100*e^6*x^2) - 60*b*(b
^2*n^2*(13489*d^6 + 8820*d^5*e*x^(1/3) - 2610*d^4*e^2*x^(2/3) + 1140*d^3*e^
3*x - 555*d^2*e^4*x^(4/3) + 264*d*e^5*x^(5/3) - 100*e^6*x^2) - 60*a*b*n*(14
7*d^6 + 60*d^5*e*x^(1/3) - 30*d^4*e^2*x^(2/3) + 20*d^3*e^3*x - 15*d^2*e^4*x
^(4/3) + 12*d*e^5*x^(5/3) - 10*e^6*x^2) + 1800*a^2*(d^6 - e^6*x^2))*Log[c*(
d + e*x^(1/3))^n] - 1800*b^2*(60*a*(d^6 - e^6*x^2) + b*n*(-147*d^6 - 60*d^5
*e*x^(1/3) + 30*d^4*e^2*x^(2/3) - 20*d^3*e^3*x + 15*d^2*e^4*x^(4/3) - 12*d*
e^5*x^(5/3) + 10*e^6*x^2))*Log[c*(d + e*x^(1/3))^n]^2 - 36000*b^3*(d^6 - e^
6*x^2)*Log[c*(d + e*x^(1/3))^n]^3)/(72000*e^6)
```

fricas [A] time = 0.56, size = 1190, normalized size = 1.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/72000*(36000*b^3*e^6*x^2*log(c)^3 + 36000*(b^3*e^6*n^3*x^2 - b^3*d^6*n^3)
*log(e*x^(1/3) + d)^3 - 1000*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2 + 18*a^2*b*e^6*
```

```
n - 36*a^3*e^6)*x^2 + 1800*(20*b^3*d^3*e^3*n^3*x + 147*b^3*d^6*n^3 - 60*a*b
^2*d^6*n^2 - 10*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2)*x^2 + 60*(b^3*e^6*n^2*x^2 -
 b^3*d^6*n^2)*log(c) + 6*(2*b^3*d*e^5*n^3*x - 5*b^3*d^4*e^2*n^3)*x^(2/3) -
15*(b^3*d^2*e^4*n^3*x - 4*b^3*d^5*e*n^3)*x^(1/3))*log(e*x^(1/3) + d)^2 + 18
000*(2*b^3*d^3*e^3*n*x - (b^3*e^6*n - 6*a*b^2*e^6)*x^2)*log(c)^2 + 20*(2059
*b^3*d^3*e^3*n^3 - 3420*a*b^2*d^3*e^3*n^2 + 1800*a^2*b*d^3*e^3*n)*x - 60*(1
3489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a^2*b*d^6*n - 100*(b^3*e^6*n^3
 - 6*a*b^2*e^6*n^2 + 18*a^2*b*e^6*n)*x^2 - 1800*(b^3*e^6*n*x^2 - b^3*d^6*n)
*log(c)^2 + 60*(19*b^3*d^3*e^3*n^3 - 20*a*b^2*d^3*e^3*n^2)*x - 60*(20*b^3*d
^3*e^3*n^2*x + 147*b^3*d^6*n^2 - 60*a*b^2*d^6*n - 10*(b^3*e^6*n^2 - 6*a*b^2
*e^6*n)*x^2)*log(c) - 6*(435*b^3*d^4*e^2*n^3 - 300*a*b^2*d^4*e^2*n^2 - 4*(1
1*b^3*d*e^5*n^3 - 30*a*b^2*d*e^5*n^2)*x + 60*(2*b^3*d*e^5*n^2*x - 5*b^3*d^4
*e^2*n^2)*log(c))*x^(2/3) + 15*(588*b^3*d^5*e*n^3 - 240*a*b^2*d^5*e*n^2 - (
37*b^3*d^2*e^4*n^3 - 60*a*b^2*d^2*e^4*n^2)*x + 60*(b^3*d^2*e^4*n^2*x - 4*b^
3*d^5*e*n^2)*log(c))*x^(1/3))*log(e*x^(1/3) + d) + 1200*(5*(b^3*e^6*n^2 - 6
*a*b^2*e^6*n + 18*a^2*b*e^6)*x^2 - 3*(19*b^3*d^3*e^3*n^2 - 20*a*b^2*d^3*e^3
*n)*x)*log(c) - 6*(23345*b^3*d^4*e^2*n^3 - 26100*a*b^2*d^4*e^2*n^2 + 9000*a
^2*b*d^4*e^2*n - 1800*(2*b^3*d*e^5*n*x - 5*b^3*d^4*e^2*n)*log(c)^2 - 8*(91*
b^3*d*e^5*n^3 - 330*a*b^2*d*e^5*n^2 + 450*a^2*b*d*e^5*n)*x - 60*(435*b^3*d^
4*e^2*n^2 - 300*a*b^2*d^4*e^2*n - 4*(11*b^3*d*e^5*n^2 - 30*a*b^2*d*e^5*n)*x
)*log(c))*x^(2/3) + 15*(53956*b^3*d^5*e*n^3 - 35280*a*b^2*d^5*e*n^2 + 7200*
a^2*b*d^5*e*n - 1800*(b^3*d^2*e^4*n*x - 4*b^3*d^5*e*n)*log(c)^2 - (919*b^3*
d^2*e^4*n^3 - 2220*a*b^2*d^2*e^4*n^2 + 1800*a^2*b*d^2*e^4*n)*x - 60*(588*b^
3*d^5*e*n^2 - 240*a*b^2*d^5*e*n - (37*b^3*d^2*e^4*n^2 - 60*a*b^2*d^2*e^4*n)
*x)*log(c))*x^(1/3))/e^6
```

giac [B] time = 0.36, size = 2223, normalized size = 2.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/72000*(36000*b^3*x^2*e*log(c)^3 + 108000*a*b^2*x^2*e*log(c)^2 + (36000*(x
^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d)^3 - 216000*(x^(1/3)*e + d)^5*d*e^
(-5)*log(x^(1/3)*e + d)^3 + 540000*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)
*e + d)^3 - 720000*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d)^3 + 5400
00*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d)^3 - 216000*(x^(1/3)*e +
d)*d^5*e^(-5)*log(x^(1/3)*e + d)^3 - 18000*(x^(1/3)*e + d)^6*e^(-5)*log(x^(
1/3)*e + d)^2 + 129600*(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/3)*e + d)^2 - 40
5000*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d)^2 + 720000*(x^(1/3)*e
+ d)^3*d^3*e^(-5)*log(x^(1/3)*e + d)^2 - 810000*(x^(1/3)*e + d)^2*d^4*e^(-5
)*log(x^(1/3)*e + d)^2 + 648000*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e +
d)^2 + 6000*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) - 51840*(x^(1/3)*e
+ d)^5*d*e^(-5)*log(x^(1/3)*e + d) + 202500*(x^(1/3)*e + d)^4*d^2*e^(-5)*lo
g(x^(1/3)*e + d) - 480000*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) +
 810000*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d) - 1296000*(x^(1/3)*
e + d)*d^5*e^(-5)*log(x^(1/3)*e + d) - 1000*(x^(1/3)*e + d)^6*e^(-5) + 1036
8*(x^(1/3)*e + d)^5*d*e^(-5) - 50625*(x^(1/3)*e + d)^4*d^2*e^(-5) + 160000*
(x^(1/3)*e + d)^3*d^3*e^(-5) - 405000*(x^(1/3)*e + d)^2*d^4*e^(-5) + 129600
0*(x^(1/3)*e + d)*d^5*e^(-5))*b^3*n^3 + 60*(1800*(x^(1/3)*e + d)^6*e^(-5)*l
og(x^(1/3)*e + d)^2 - 10800*(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/3)*e + d)^2
 + 27000*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d)^2 - 36000*(x^(1/3)
*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d)^2 + 27000*(x^(1/3)*e + d)^2*d^4*e^(
-5)*log(x^(1/3)*e + d)^2 - 10800*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e +
 d)^2 - 600*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) + 4320*(x^(1/3)*e +
 d)^5*d*e^(-5)*log(x^(1/3)*e + d) - 13500*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(
x^(1/3)*e + d) + 24000*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) - 27
000*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d) + 21600*(x^(1/3)*e + d)
*d^5*e^(-5)*log(x^(1/3)*e + d) + 100*(x^(1/3)*e + d)^6*e^(-5) - 864*(x^(1/3
```

```
)*e + d)^5*d*e^(-5) + 3375*(x^(1/3)*e + d)^4*d^2*e^(-5) - 8000*(x^(1/3)*e +
 d)^3*d^3*e^(-5) + 13500*(x^(1/3)*e + d)^2*d^4*e^(-5) - 21600*(x^(1/3)*e +
d)*d^5*e^(-5))*b^3*n^2*log(c) + 108000*a^2*b*x^2*e*log(c) + 1800*(60*(x^(1/
3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)^5*d*e^(-5)*log(
x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d) - 1200
*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^2*d^
4*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e
+ d) - 10*(x^(1/3)*e + d)^6*e^(-5) + 72*(x^(1/3)*e + d)^5*d*e^(-5) - 225*(x
^(1/3)*e + d)^4*d^2*e^(-5) + 400*(x^(1/3)*e + d)^3*d^3*e^(-5) - 450*(x^(1/3
)*e + d)^2*d^4*e^(-5) + 360*(x^(1/3)*e + d)*d^5*e^(-5))*b^3*n*log(c)^2 + 60
*(1800*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d)^2 - 10800*(x^(1/3)*e + d
)^5*d*e^(-5)*log(x^(1/3)*e + d)^2 + 27000*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(
x^(1/3)*e + d)^2 - 36000*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d)^2
+ 27000*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d)^2 - 10800*(x^(1/3)*
e + d)*d^5*e^(-5)*log(x^(1/3)*e + d)^2 - 600*(x^(1/3)*e + d)^6*e^(-5)*log(x
^(1/3)*e + d) + 4320*(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/3)*e + d) - 13500*
(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d) + 24000*(x^(1/3)*e + d)^3*d
^3*e^(-5)*log(x^(1/3)*e + d) - 27000*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/
3)*e + d) + 21600*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e + d) + 100*(x^(1
/3)*e + d)^6*e^(-5) - 864*(x^(1/3)*e + d)^5*d*e^(-5) + 3375*(x^(1/3)*e + d)
^4*d^2*e^(-5) - 8000*(x^(1/3)*e + d)^3*d^3*e^(-5) + 13500*(x^(1/3)*e + d)^2
*d^4*e^(-5) - 21600*(x^(1/3)*e + d)*d^5*e^(-5))*a*b^2*n^2 + 36000*a^3*x^2*e
 + 3600*(60*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e +
d)^5*d*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(
1/3)*e + d) - 1200*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) + 900*(x
^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)*d^5*e^(
-5)*log(x^(1/3)*e + d) - 10*(x^(1/3)*e + d)^6*e^(-5) + 72*(x^(1/3)*e + d)^5
*d*e^(-5) - 225*(x^(1/3)*e + d)^4*d^2*e^(-5) + 400*(x^(1/3)*e + d)^3*d^3*e^
(-5) - 450*(x^(1/3)*e + d)^2*d^4*e^(-5) + 360*(x^(1/3)*e + d)*d^5*e^(-5))*a
*b^2*n*log(c) + 1800*(60*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) - 360*
(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^4*d^2*e
^(-5)*log(x^(1/3)*e + d) - 1200*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e
+ d) + 900*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e
 + d)*d^5*e^(-5)*log(x^(1/3)*e + d) - 10*(x^(1/3)*e + d)^6*e^(-5) + 72*(x^(
1/3)*e + d)^5*d*e^(-5) - 225*(x^(1/3)*e + d)^4*d^2*e^(-5) + 400*(x^(1/3)*e
+ d)^3*d^3*e^(-5) - 450*(x^(1/3)*e + d)^2*d^4*e^(-5) + 360*(x^(1/3)*e + d)*
d^5*e^(-5))*a^2*b*n)*e^(-1)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\left(e\,x^{\frac{1}{3}}+d\right)^{n}\right)+a\right)^{3} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(e*x^(1/3)+d)^n)+a)^3,x)
```

```
[Out] int(x*(b*ln(c*(e*x^(1/3)+d)^n)+a)^3,x)
```

maxima [A] time = 0.64, size = 668, normalized size = 0.74

$$\frac{1}{2}\,b^3x^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)^3+\frac{3}{2}\,ab^2x^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)^2-\frac{1}{40}\,a^2ben\left(\frac{60\,d^6\log\left(ex^{\frac{1}{3}}+d\right)}{e^7}+\frac{10\,e^5x^2-12\,de^4x^{\frac{5}{3}}+15}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*log((e*x^(1/3) + d)^n*c)^3 + 3/2*a*b^2*x^2*log((e*x^(1/3) + d)^
n*c)^2 - 1/40*a^2*b*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e^5*x^2 - 12*d
*e^4*x^(5/3) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^(2/3) - 60*d^
5*x^(1/3))/e^6) + 3/2*a^2*b*x^2*log((e*x^(1/3) + d)^n*c) + 1/2*a^3*x^2 - 1/
1200*(60*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e^5*x^2 - 12*d*e^4*x^(5/3
) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^(2/3) - 60*d^5*x^(1/3))/
e^6)*log((e*x^(1/3) + d)^n*c) - (100*e^6*x^2 + 1800*d^6*log(e*x^(1/3) + d)^
2 - 264*d*e^5*x^(5/3) + 555*d^2*e^4*x^(4/3) - 1140*d^3*e^3*x + 8820*d^6*log
(e*x^(1/3) + d) + 2610*d^4*e^2*x^(2/3) - 8820*d^5*e*x^(1/3))*n^2/e^6)*a*b^2
 - 1/72000*(1800*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e^5*x^2 - 12*d*e^
4*x^(5/3) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^(2/3) - 60*d^5*x
^(1/3))/e^6)*log((e*x^(1/3) + d)^n*c)^2 + e*n*((36000*d^6*log(e*x^(1/3) + d
)^3 + 1000*e^6*x^2 + 264600*d^6*log(e*x^(1/3) + d)^2 - 4368*d*e^5*x^(5/3) +
 13785*d^2*e^4*x^(4/3) - 41180*d^3*e^3*x + 809340*d^6*log(e*x^(1/3) + d) +
140070*d^4*e^2*x^(2/3) - 809340*d^5*e*x^(1/3))*n^2/e^7 - 60*(100*e^6*x^2 +
1800*d^6*log(e*x^(1/3) + d)^2 - 264*d*e^5*x^(5/3) + 555*d^2*e^4*x^(4/3) - 1
140*d^3*e^3*x + 8820*d^6*log(e*x^(1/3) + d) + 2610*d^4*e^2*x^(2/3) - 8820*d
^5*e*x^(1/3))*n*log((e*x^(1/3) + d)^n*c)/e^7))*b^3
```

mupad [B] time = 8.06, size = 979, normalized size = 1.08

$$\frac{a^3\,x^2}{2}+\frac{b^3\,x^2\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)^3}{2}-\frac{b^3\,n^3\,x^2}{72}+\frac{3\,a\,b^2\,x^2\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)^2}{2}-\frac{b^3\,n\,x^2\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)^2}{4}+\frac{b^3\,n^2\,x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(1/3))^n))^3,x)
```

```
[Out] (a^3*x^2)/2 + (b^3*x^2*log(c*(d + e*x^(1/3))^n)^3)/2 - (b^3*n^3*x^2)/72 + (
3*a*b^2*x^2*log(c*(d + e*x^(1/3))^n)^2)/2 - (b^3*n*x^2*log(c*(d + e*x^(1/3)
)^n)^2)/4 + (b^3*n^2*x^2*log(c*(d + e*x^(1/3))^n))/12 + (a*b^2*n^2*x^2)/12
- (b^3*d^6*log(c*(d + e*x^(1/3))^n)^3)/(2*e^6) + (3*a^2*b*x^2*log(c*(d + e*
x^(1/3))^n))/2 - (a^2*b*n*x^2)/4 - (a*b^2*n*x^2*log(c*(d + e*x^(1/3))^n))/2
 - (13489*b^3*d^6*n^3*log(d + e*x^(1/3)))/(1200*e^6) - (919*b^3*d^2*n^3*x^(
4/3))/(4800*e^2) - (4669*b^3*d^4*n^3*x^(2/3))/(2400*e^4) + (13489*b^3*d^5*n
^3*x^(1/3))/(1200*e^5) - (3*a*b^2*d^6*log(c*(d + e*x^(1/3))^n)^2)/(2*e^6) +
 (147*b^3*d^6*n*log(c*(d + e*x^(1/3))^n)^2)/(40*e^6) + (2059*b^3*d^3*n^3*x)
/(3600*e^3) + (91*b^3*d*n^3*x^(5/3))/(1500*e) - (3*a^2*b*d^6*n*log(d + e*x^
(1/3)))/(2*e^6) + (b^3*d^3*n*x*log(c*(d + e*x^(1/3))^n)^2)/(2*e^3) - (19*b^
3*d^3*n^2*x*log(c*(d + e*x^(1/3))^n))/(20*e^3) + (3*b^3*d*n*x^(5/3)*log(c*(
d + e*x^(1/3))^n)^2)/(10*e) - (11*b^3*d*n^2*x^(5/3)*log(c*(d + e*x^(1/3))^n
))/(50*e) - (19*a*b^2*d^3*n^2*x)/(20*e^3) - (11*a*b^2*d*n^2*x^(5/3))/(50*e)
 - (3*a^2*b*d^2*n*x^(4/3))/(8*e^2) - (3*a^2*b*d^4*n*x^(2/3))/(4*e^4) + (3*a
^2*b*d^5*n*x^(1/3))/(2*e^5) + (147*a*b^2*d^6*n^2*log(d + e*x^(1/3)))/(20*e^
6) - (3*b^3*d^2*n*x^(4/3)*log(c*(d + e*x^(1/3))^n)^2)/(8*e^2) + (37*b^3*d^2
*n^2*x^(4/3)*log(c*(d + e*x^(1/3))^n))/(80*e^2) - (3*b^3*d^4*n*x^(2/3)*log(
c*(d + e*x^(1/3))^n)^2)/(4*e^4) + (87*b^3*d^4*n^2*x^(2/3)*log(c*(d + e*x^(1
/3))^n))/(40*e^4) + (3*b^3*d^5*n*x^(1/3)*log(c*(d + e*x^(1/3))^n)^2)/(2*e^5
) - (147*b^3*d^5*n^2*x^(1/3)*log(c*(d + e*x^(1/3))^n))/(20*e^5) + (37*a*b^2
*d^2*n^2*x^(4/3))/(80*e^2) + (87*a*b^2*d^4*n^2*x^(2/3))/(40*e^4) - (147*a*b
^2*d^5*n^2*x^(1/3))/(20*e^5) + (a^2*b*d^3*n*x)/(2*e^3) + (3*a^2*b*d*n*x^(5/
3))/(10*e) + (a*b^2*d^3*n*x*log(c*(d + e*x^(1/3))^n))/e^3 + (3*a*b^2*d*n*x^
(5/3)*log(c*(d + e*x^(1/3))^n))/(5*e) - (3*a*b^2*d^2*n*x^(4/3)*log(c*(d + e
*x^(1/3))^n))/(4*e^2) - (3*a*b^2*d^4*n*x^(2/3)*log(c*(d + e*x^(1/3))^n))/(2
*e^4) + (3*a*b^2*d^5*n*x^(1/3)*log(c*(d + e*x^(1/3))^n))/e^5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(a + b \log{\left(c \left(d + e \sqrt[3]{x}\right)^{n} \right)}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))**n))**3,x)
```

```
[Out] Integral(x*(a + b*log(c*(d + e*x**(1/3))**n))**3, x)
```

3.459 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=438

$$\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3}-\frac{9b^2dn^2\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^3}+\frac{18ab^2d^2n^2\sqrt[3]{x}}{e^2}$$

```
[Out] 9/4*b^3*d*n^3*(d+e*x^(1/3))^2/e^3-2/9*b^3*n^3*(d+e*x^(1/3))^3/e^3+18*a*b^2*
d^2*n^2*x^(1/3)/e^2-18*b^3*d^2*n^3*x^(1/3)/e^2+18*b^3*d^2*n^2*(d+e*x^(1/3))
*ln(c*(d+e*x^(1/3))^n)/e^3-9/2*b^2*d*n^2*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(
1/3))^n))/e^3+2/3*b^2*n^2*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^3-9
*b*d^2*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^3+9/2*b*d*n*(d+e*x^(
1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^3-b*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+
e*x^(1/3))^n))^2/e^3+3*d^2*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^3-
3*d*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^3+(d+e*x^(1/3))^3*(a+b*
ln(c*(d+e*x^(1/3))^n))^3/e^3
```

Rubi [A] time = 0.44, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2451, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3}-\frac{9b^2dn^2\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^3}+\frac{18ab^2d^2n^2\sqrt[3]{x}}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (9*b^3*d*n^3*(d + e*x^(1/3))^2)/(4*e^3) - (2*b^3*n^3*(d + e*x^(1/3))^3)/(9*
e^3) + (18*a*b^2*d^2*n^2*x^(1/3))/e^2 - (18*b^3*d^2*n^3*x^(1/3))/e^2 + (18*
b^3*d^2*n^2*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^3 - (9*b^2*d*n^2*(d
 + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(2*e^3) + (2*b^2*n^2*(d +
 e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^3) - (9*b*d^2*n*(d + e
*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^3 + (9*b*d*n*(d + e*x^(1/3)
)^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*e^3) - (b*n*(d + e*x^(1/3))^3*(a
 + b*Log[c*(d + e*x^(1/3))^n])^2)/e^3 + (3*d^2*(d + e*x^(1/3))*(a + b*Log[c
*(d + e*x^(1/3))^n])^3)/e^3 - (3*d*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^
(1/3))^n])^3)/e^3 + ((d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/
e^3
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3 dx = 3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3 dx, x, \sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int \left(\frac{d^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2} - \frac{2d(d+ex)\left(a+b\log\left(c(d+e\right.\right.}{e^2}\right.\right.$$

$$= \frac{3\,\text{Subst}\left(\int (d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3 dx, x, \sqrt[3]{x}\right)}{e^2} - \frac{(6d)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{3\,\text{Subst}\left(\int x^2\left(a+b\log\left(cx^n\right)\right)^3 dx, x, d+e\sqrt[3]{x}\right)}{e^3} - \frac{(6d)\,\text{Subst}\left(\int x\left(a+b\right.\right.}{}$$

$$= \frac{3d^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3} - \frac{3d\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(\right.\right.}{e^3}$$

$$= -\frac{9bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^3} + \frac{9bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+\right.}{2}$$

$$= \frac{9b^3dn^3\left(d+e\sqrt[3]{x}\right)^2}{4e^3} - \frac{2b^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^3} + \frac{18ab^2d^2n^2\sqrt[3]{x}}{e^2} - \frac{9b^2dn^2\left(d+\right.}{}$$

$$= \frac{9b^3dn^3\left(d+e\sqrt[3]{x}\right)^2}{4e^3} - \frac{2b^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^3} + \frac{18ab^2d^2n^2\sqrt[3]{x}}{e^2} - \frac{18b^3d^2n^3\sqrt[3]{x}}{e^2}$$

Mathematica [A] time = 0.23, size = 362, normalized size = 0.83

$$\frac{36a^3\left(d^3+e^3x\right)+6b\left(d+e\sqrt[3]{x}\right)\left(18a^2\left(d^2-de\sqrt[3]{x}+e^2x^{2/3}\right)-6abn\left(11d^2-5de\sqrt[3]{x}+2e^2x^{2/3}\right)+b^2n^2\left(85d^2-1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (b^3*e*n^3*(-510*d^2 + 57*d*e*x^(1/3) - 8*e^2*x^(2/3))*x^(1/3) - 6*a*b^2*n^
2*(23*d^3 - 66*d^2*e*x^(1/3) + 15*d*e^2*x^(2/3) - 4*e^3*x) + 36*a^3*(d^3 +
e^3*x) - 18*a^2*b*n*(11*d^3 + 6*d^2*e*x^(1/3) - 3*d*e^2*x^(2/3) + 2*e^3*x)
+ 6*b*(18*a^2*(d^2 - d*e*x^(1/3) + e^2*x^(2/3)) - 6*a*b*n*(11*d^2 - 5*d*e*x
^(1/3) + 2*e^2*x^(2/3)) + b^2*n^2*(85*d^2 - 19*d*e*x^(1/3) + 4*e^2*x^(2/3))
)*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n] + 18*b^2*(6*a*(d^3 + e^3*x) - b*
n*(11*d^3 + 6*d^2*e*x^(1/3) - 3*d*e^2*x^(2/3) + 2*e^3*x))*Log[c*(d + e*x^(1
/3))^n]^2 + 36*b^3*(d^3 + e^3*x)*Log[c*(d + e*x^(1/3))^n]^3)/(36*e^3)
```

fricas [A] time = 0.48, size = 690, normalized size = 1.58

$$\frac{36\,b^3e^3x\log(c)^3+36\left(b^3e^3n^3x+b^3d^3n^3\right)\log\left(ex^{\frac{1}{3}}+d\right)^3-36\left(b^3e^3n-3\,ab^2e^3\right)x\log(c)^2+18\left(3\,b^3de^2n^3x^{\frac{2}{3}}-6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/36*(36*b^3*e^3*x*log(c)^3 + 36*(b^3*e^3*n^3*x + b^3*d^3*n^3)*log(e*x^(1/3
) + d)^3 - 36*(b^3*e^3*n - 3*a*b^2*e^3)*x*log(c)^2 + 18*(3*b^3*d*e^2*n^3*x^
(2/3) - 6*b^3*d^2*e*n^3*x^(1/3) - 11*b^3*d^3*n^3 + 6*a*b^2*d^3*n^2 - 2*(b^3
*e^3*n^3 - 3*a*b^2*e^3*n^2)*x + 6*(b^3*e^3*n^2*x + b^3*d^3*n^2)*log(c))*log
(e*x^(1/3) + d)^2 + 12*(2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + 9*a^2*b*e^3)*x*log(
```

```
c) - 4*(2*b^3*e^3*n^3 - 6*a*b^2*e^3*n^2 + 9*a^2*b*e^3*n - 9*a^3*e^3)*x + 6*
(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n + 18*(b^3*e^3*n*x + b^3
*d^3*n)*log(c)^2 + 2*(2*b^3*e^3*n^3 - 6*a*b^2*e^3*n^2 + 9*a^2*b*e^3*n)*x -
6*(11*b^3*d^3*n^2 - 6*a*b^2*d^3*n + 2*(b^3*e^3*n^2 - 3*a*b^2*e^3*n)*x)*log(
c) - 3*(5*b^3*d*e^2*n^3 - 6*b^3*d*e^2*n^2*log(c) - 6*a*b^2*d*e^2*n^2)*x^(2/
3) + 6*(11*b^3*d^2*e*n^3 - 6*b^3*d^2*e*n^2*log(c) - 6*a*b^2*d^2*e*n^2)*x^(1
/3))*log(e*x^(1/3) + d) + 3*(19*b^3*d*e^2*n^3 + 18*b^3*d*e^2*n*log(c)^2 - 3
0*a*b^2*d*e^2*n^2 + 18*a^2*b*d*e^2*n - 6*(5*b^3*d*e^2*n^2 - 6*a*b^2*d*e^2*n
)*log(c))*x^(2/3) - 6*(85*b^3*d^2*e*n^3 + 18*b^3*d^2*e*n*log(c)^2 - 66*a*b^
2*d^2*e*n^2 + 18*a^2*b*d^2*e*n - 6*(11*b^3*d^2*e*n^2 - 6*a*b^2*d^2*e*n)*log
(c))*x^(1/3))/e^3
```

giac [B] time = 0.28, size = 1105, normalized size = 2.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/36*(36*b^3*x*e*log(c)^3 + (36*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d)
^3 - 108*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d)^3 + 108*(x^(1/3)*e +
 d)*d^2*e^(-2)*log(x^(1/3)*e + d)^3 - 36*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/
3)*e + d)^2 + 162*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d)^2 - 324*(x^
(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d)^2 + 24*(x^(1/3)*e + d)^3*e^(-2)*
log(x^(1/3)*e + d) - 162*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d) + 64
8*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d) - 8*(x^(1/3)*e + d)^3*e^(-2
) + 81*(x^(1/3)*e + d)^2*d*e^(-2) - 648*(x^(1/3)*e + d)*d^2*e^(-2))*b^3*n^3
 + 6*(18*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d)^2 - 54*(x^(1/3)*e + d)
^2*d*e^(-2)*log(x^(1/3)*e + d)^2 + 54*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3
)*e + d)^2 - 12*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d) + 54*(x^(1/3)*e
 + d)^2*d*e^(-2)*log(x^(1/3)*e + d) - 108*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^
(1/3)*e + d) + 4*(x^(1/3)*e + d)^3*e^(-2) - 27*(x^(1/3)*e + d)^2*d*e^(-2) +
 108*(x^(1/3)*e + d)*d^2*e^(-2))*b^3*n^2*log(c) + 18*(6*(x^(1/3)*e + d)^3*e
^(-2)*log(x^(1/3)*e + d) - 18*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d)
 + 18*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d) - 2*(x^(1/3)*e + d)^3*e
^(-2) + 9*(x^(1/3)*e + d)^2*d*e^(-2) - 18*(x^(1/3)*e + d)*d^2*e^(-2))*b^3*n
*log(c)^2 + 108*a*b^2*x*e*log(c)^2 + 6*(18*(x^(1/3)*e + d)^3*e^(-2)*log(x^(
1/3)*e + d)^2 - 54*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d)^2 + 54*(x^
(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d)^2 - 12*(x^(1/3)*e + d)^3*e^(-2)*
log(x^(1/3)*e + d) + 54*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d) - 108
*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d) + 4*(x^(1/3)*e + d)^3*e^(-2)
 - 27*(x^(1/3)*e + d)^2*d*e^(-2) + 108*(x^(1/3)*e + d)*d^2*e^(-2))*a*b^2*n^
2 + 36*(6*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d) - 18*(x^(1/3)*e + d)^
2*d*e^(-2)*log(x^(1/3)*e + d) + 18*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e
 + d) - 2*(x^(1/3)*e + d)^3*e^(-2) + 9*(x^(1/3)*e + d)^2*d*e^(-2) - 18*(x^(
1/3)*e + d)*d^2*e^(-2))*a*b^2*n*log(c) + 108*a^2*b*x*e*log(c) + 18*(6*(x^(1
/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d) - 18*(x^(1/3)*e + d)^2*d*e^(-2)*log(
x^(1/3)*e + d) + 18*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d) - 2*(x^(1
/3)*e + d)^3*e^(-2) + 9*(x^(1/3)*e + d)^2*d*e^(-2) - 18*(x^(1/3)*e + d)*d^2
*e^(-2))*a^2*b*n + 36*a^3*x*e)*e^(-1)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(b \ln \left(c \left(e x^{\frac{1}{3}}+d \right)^{n}\right)+a \right)^{3}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/3)+d)^n)+a)^3,x)
```

```
[Out] int((b*ln(c*(e*x^(1/3)+d)^n)+a)^3,x)
```

maxima [A] time = 0.61, size = 455, normalized size = 1.04

$$\frac{1}{2}\left(en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-3\,dex^{\frac{2}{3}}+6\,d^2x^{\frac{1}{3}}}{e^3}\right)+6\,x\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)\right)a^2b+\frac{1}{6}\left(6\,en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(e*n*(6*d^3*log(e*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*x
^(1/3))/e^3) + 6*x*log((e*x^(1/3) + d)^n*c))*a^2*b + 1/6*(6*e*n*(6*d^3*log(
e*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*x^(1/3))/e^3)*log((e*
x^(1/3) + d)^n*c) + 18*x*log((e*x^(1/3) + d)^n*c)^2 - (18*d^3*log(e*x^(1/3)
 + d)^2 - 4*e^3*x + 66*d^3*log(e*x^(1/3) + d) + 15*d*e^2*x^(2/3) - 66*d^2*e
*x^(1/3))*n^2/e^3)*a*b^2 + 1/36*(18*e*n*(6*d^3*log(e*x^(1/3) + d)/e^4 - (2*
e^2*x - 3*d*e*x^(2/3) + 6*d^2*x^(1/3))/e^3)*log((e*x^(1/3) + d)^n*c)^2 + 36
*x*log((e*x^(1/3) + d)^n*c)^3 + e*n*((36*d^3*log(e*x^(1/3) + d)^3 + 198*d^3
*log(e*x^(1/3) + d)^2 - 8*e^3*x + 510*d^3*log(e*x^(1/3) + d) + 57*d*e^2*x^(
2/3) - 510*d^2*e*x^(1/3))*n^2/e^4 - 6*(18*d^3*log(e*x^(1/3) + d)^2 - 4*e^3*
x + 66*d^3*log(e*x^(1/3) + d) + 15*d*e^2*x^(2/3) - 66*d^2*e*x^(1/3))*n*log(
(e*x^(1/3) + d)^n*c)/e^4))*b^3 + a^3*x
```

mupad [B] time = 0.69, size = 558, normalized size = 1.27

$$x\left(a^3-a^2\,b\,n+\frac{2\,a\,b^2\,n^2}{3}-\frac{2\,b^3\,n^3}{9}\right)-x^{2/3}\left(\frac{d\,\left(3\,a^3-3\,a^2\,b\,n+2\,a\,b^2\,n^2-\frac{2\,b^3\,n^3}{3}\right)}{2\,e}-\frac{d\,\left(6\,a^3-6\,a\,b^2\,n^2+5\,b^3\,n\right.}{4\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^3,x)
```

```
[Out] x*(a^3 - (2*b^3*n^3)/9 + (2*a*b^2*n^2)/3 - a^2*b*n) - x^(2/3)*((d*(3*a^3 -
(2*b^3*n^3)/3 + 2*a*b^2*n^2 - 3*a^2*b*n))/(2*e) - (d*(6*a^3 + 5*b^3*n^3 - 6
*a*b^2*n^2))/(4*e)) + log(c*(d + e*x^(1/3))^n)^3*(b^3*x + (b^3*d^3)/e^3) +
log(c*(d + e*x^(1/3))^n)^2*((d*(6*a*b^2*d^2 - 11*b^3*d^2*n))/(2*e^3) - x^(2
/3)*((3*b^2*d*(3*a - b*n))/(2*e) - (9*a*b^2*d)/(2*e)) + b^2*x*(3*a - b*n) +
 (d*x^(1/3)*((3*b^2*d*(3*a - b*n))/e - (9*a*b^2*d)/e))/e) + x^(1/3)*((d*((d
*(3*a^3 - (2*b^3*n^3)/3 + 2*a*b^2*n^2 - 3*a^2*b*n))/e - (d*(6*a^3 + 5*b^3*n
^3 - 6*a*b^2*n^2))/(2*e)))/e + (b^2*d^2*n^2*(6*a - 11*b*n))/e^2) + (log(d +
 e*x^(1/3))*(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n))/(6*e^3) +
 (log(c*(d + e*x^(1/3))^n)*((x^(1/3)*((d*(b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*
n) - 3*b*d*e*(3*a^2 - b^2*n^2)))/e + 6*b^3*d^2*n^2))/e - (x^(2/3)*(b*d*e*(9
*a^2 + 2*b^2*n^2 - 6*a*b*n) - 3*b*d*e*(3*a^2 - b^2*n^2)))/(2*e) + (b*e*x*(9
*a^2 + 2*b^2*n^2 - 6*a*b*n))/3))/e
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**3, x)
```

3.460 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx$

Optimal. Leaf size=135

$$-18b^2n^2\text{Li}_3\left(\frac{\sqrt[3]{x}e}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)+9bn\text{Li}_2\left(\frac{\sqrt[3]{x}e}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2+3\log\left(-\frac{e\sqrt[3]{x}}{d}\right.$$

```
[Out] 3*(a+b*ln(c*(d+e*x^(1/3))^n))^3*ln(-e*x^(1/3)/d)+9*b*n*(a+b*ln(c*(d+e*x^(1/
3))^n))^2*polylog(2,1+e*x^(1/3)/d)-18*b^2*n^2*(a+b*ln(c*(d+e*x^(1/3))^n))*p
olylog(3,1+e*x^(1/3)/d)+18*b^3*n^3*polylog(4,1+e*x^(1/3)/d)
```

Rubi [A] time = 0.19, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2454, 2396, 2433, 2374, 2383, 6589}

$$-18b^2n^2\text{PolyLog}\left(3,\frac{e\sqrt[3]{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)+9bn\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x,x]
```

```
[Out] 3*(a + b*Log[c*(d + e*x^(1/3))^n])^3*Log[-((e*x^(1/3))/d)] + 9*b*n*(a + b*L
og[c*(d + e*x^(1/3))^n])^2*PolyLog[2, 1 + (e*x^(1/3))/d] - 18*b^2*n^2*(a +
b*Log[c*(d + e*x^(1/3))^n])*PolyLog[3, 1 + (e*x^(1/3))/d] + 18*b^3*n^3*Poly
Log[4, 1 + (e*x^(1/3))/d]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx = 3\,\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x}\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right) - (9ben)\,\mathrm{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{\cdots}\right.$$

$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right) - (9bn)\,\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(\cdots\right.\right.}{\cdots}\right.$$

$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 9bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.\right.$$

$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 9bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.\right.$$

$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 9bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.\right.$$

Mathematica [B] time = 0.18, size = 333, normalized size = 2.47

$$9b^2n^2\left(-2\mathrm{Li}_3\left(\frac{\sqrt[3]{x}\,e}{d}+1\right)+2\mathrm{Li}_2\left(\frac{\sqrt[3]{x}\,e}{d}+1\right)\log\left(d+e\sqrt[3]{x}\right)+\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\log^2\left(d+e\sqrt[3]{x}\right)\right)\left(a+b\log\left(c\left(d+\cdots\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2*((Log[d + e*x^(
1/3)] - Log[1 + (e*x^(1/3))/d])*Log[x] - 3*PolyLog[2, -((e*x^(1/3))/d)]) +
9*b^2*n^2*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])*(Log[d
+ e*x^(1/3)]^2*Log[-((e*x^(1/3))/d)] + 2*Log[d + e*x^(1/3)]*PolyLog[2, 1 +
(e*x^(1/3))/d] - 2*PolyLog[3, 1 + (e*x^(1/3))/d]) + 3*b^3*n^3*(Log[d + e*x^
(1/3)]^3*Log[-((e*x^(1/3))/d)] + 3*Log[d + e*x^(1/3)]^2*PolyLog[2, 1 + (e*x
^(1/3))/d] - 6*Log[d + e*x^(1/3)]*PolyLog[3, 1 + (e*x^(1/3))/d] + 6*PolyLog
[4, 1 + (e*x^(1/3))/d])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)^3+3\,ab^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)^2+3\,a^2b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(1/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(1/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(1/3) + d)^n*c) + a^3)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)^3/x, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\left(e\,x^{\frac{1}{3}}+d\right)^n\right)+a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/3)+d)^n)+a)^3/x,x)
```

```
[Out] int((b*ln(c*(e*x^(1/3)+d)^n)+a)^3/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^3\log\left(\left(ex^{\frac{1}{3}}+d\right)^n\right)^3\log(x)+\int-\frac{\left(b^3enx\log(x)-3\left(b^3e\log(c)+ab^2e\right)x-3\left(b^3d\log(c)+ab^2d\right)x^{\frac{2}{3}}\right)\log\left(\left(ex^{\frac{1}{3}}+d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*log((e*x^(1/3) + d)^n)^3*log(x) + integrate(-((b^3*e*n*x*log(x) - 3*(b^
3*e*log(c) + a*b^2*e)*x - 3*(b^3*d*log(c) + a*b^2*d)*x^(2/3))*log((e*x^(1/3
) + d)^n)^2 - (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3
*e)*x - 3*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*log(c)^
2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^(2/3))*log((e*x^(1/3) + d)^n) - (b^3*d*lo
g(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(2/3))/(e*x^2 + d
*x^(5/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)\right)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^3/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \log \left(c \left(d + e \sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**3/x, x)
```

3.461 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=439

$$-\frac{6b^2e^3n^2\text{Li}_2\left(\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}-\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}-\frac{6b^2e^3n^2\ }{}$$

```
[Out] -3*b^2*e^2*n^2*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/d^3/x^(1/3)-3*b^2*
e^3*n^2*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*(d+e*x^(1/3))^n))/d^3-3/2*b*e*n*(a+
b*ln(c*(d+e*x^(1/3))^n))^2/d/x^(2/3)+3*b*e^2*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e
*x^(1/3))^n))^2/d^3/x^(1/3)+3*b*e^3*n*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*(d+e*
x^(1/3))^n))^2/d^3-(a+b*ln(c*(d+e*x^(1/3))^n))^3/x-6*b^2*e^3*n^2*(a+b*ln(c*
(d+e*x^(1/3))^n))*ln(-e*x^(1/3)/d)/d^3+b^3*e^3*n^3*ln(x)/d^3+3*b^3*e^3*n^3*
polylog(2,d/(d+e*x^(1/3)))/d^3-6*b^2*e^3*n^2*(a+b*ln(c*(d+e*x^(1/3))^n))*po
lylog(2,d/(d+e*x^(1/3)))/d^3-6*b^3*e^3*n^3*polylog(2,1+e*x^(1/3)/d)/d^3-6*b
^3*e^3*n^3*polylog(3,d/(d+e*x^(1/3)))/d^3
```

Rubi [A] time = 1.01, antiderivative size = 414, normalized size of antiderivative = 0.94, number of steps used = 22, number of rules used = 16, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31}

$$\frac{6b^2e^3n^2\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}-\frac{9b^3e^3n^3\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)}{d^3}-\frac{6b^3e^3n^3\text{PolyLog}\left(3,\frac{e}{}\right.}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^2,x]
```

```
[Out] (-3*b^2*e^2*n^2*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/(d^3*x^(1
/3)) + (3*b*e^3*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*d^3) - (3*b*e*n*(a
 + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*d*x^(2/3)) + (3*b*e^2*n*(d + e*x^(1/3)
)*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(d^3*x^(1/3)) - (e^3*(a + b*Log[c*(d
+ e*x^(1/3))^n])^3)/d^3 - (a + b*Log[c*(d + e*x^(1/3))^n])^3/x - (9*b^2*e^3
*n^2*(a + b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)])/d^3 + (3*b*e^3
*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2*Log[-((e*x^(1/3))/d)])/d^3 + (b^3*e^3
*n^3*Log[x])/d^3 - (9*b^3*e^3*n^3*PolyLog[2, 1 + (e*x^(1/3))/d])/d^3 + (6*b
^2*e^3*n^2*(a + b*Log[c*(d + e*x^(1/3))^n])*PolyLog[2, 1 + (e*x^(1/3))/d])/
d^3 - (6*b^3*e^3*n^3*PolyLog[3, 1 + (e*x^(1/3))/d])/d^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
```

```
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x} + (3ben)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)}{x^3(d+ex)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x} + (3bn)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(cx^n\right)\right)^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x,\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x} + \frac{(3bn)\,\text{Subst}\left(\int \frac{\left(a+b\log(cx^n)\right)^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\right.}{d}$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2dx^{2/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x} - \frac{(3be}{}$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2dx^{2/3}} + \frac{3be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+\right.\right.\right.}{d^3\sqrt[3]{x}}$$

$$= -\frac{3b^2e^2n^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}} - \frac{3ben\left(a+b\log\left(c\left(d\right.\right.\right.}{2dx^{2/3}}$$

$$= -\frac{3b^2e^2n^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}} + \frac{3be^3n\left(a+b\log\left(c\left(d\right.\right.\right.}{2d^3}$$

$$= -\frac{3b^2e^2n^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}} + \frac{3be^3n\left(a+b\log\left(c\left(d\right.\right.\right.}{2d^3}$$

Mathematica [A] time = 0.75, size = 733, normalized size = 1.67

$$\frac{-6b^2n^2\left(\left(d^3+e^3x\right)\log^2\left(d+e\sqrt[3]{x}\right)+\log\left(d+e\sqrt[3]{x}\right)\left(d^2e\sqrt[3]{x}-2e^3x\log\left(-\frac{e\sqrt[3]{x}}{d}\right)-2de^2x^{2/3}-3e^3x\right)-2e^3x\text{Li}_2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^2,x]
```

```
[Out] (-3*b*d^2*e*n*x^(1/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))
^n])^2 + 6*b*d*e^2*n*x^(2/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x
^(1/3))^n])^2 - 6*b*d^3*n*Log[d + e*x^(1/3)]*(a - b*n*Log[d + e*x^(1/3)] +
b*Log[c*(d + e*x^(1/3))^n])^2 - 6*b*e^3*n*x*Log[d + e*x^(1/3)]*(a - b*n*Log
[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 - 2*d^3*(a - b*n*Log[d + e*
x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^3 + 2*b*e^3*n*x*(a - b*n*Log[d + e*x
^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2*Log[x] - 6*b^2*n^2*(a - b*n*Log[d +
 e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])*(d*e^2*x^(2/3) + (d^3 + e^3*x)*Lo
g[d + e*x^(1/3)]^2 + 3*e^3*x*Log[-((e*x^(1/3))/d)] + Log[d + e*x^(1/3)]*(d^
2*e*x^(1/3) - 2*d*e^2*x^(2/3) - 3*e^3*x - 2*e^3*x*Log[-((e*x^(1/3))/d)]) -
2*e^3*x*PolyLog[2, 1 + (e*x^(1/3))/d]) + b^3*n^3*(-6*d*e^2*x^(2/3)*Log[d +
```

```
e*x^(1/3)] - 6*e^3*x*Log[d + e*x^(1/3)] - 3*d^2*e*x^(1/3)*Log[d + e*x^(1/3)
]^2 + 6*d*e^2*x^(2/3)*Log[d + e*x^(1/3)]^2 + 9*e^3*x*Log[d + e*x^(1/3)]^2 -
 2*d^3*Log[d + e*x^(1/3)]^3 - 2*e^3*x*Log[d + e*x^(1/3)]^3 + 6*e^3*x*Log[-(
(e*x^(1/3))/d)] - 18*e^3*x*Log[d + e*x^(1/3)]*Log[-((e*x^(1/3))/d)] + 6*e^3
*x*Log[d + e*x^(1/3)]^2*Log[-((e*x^(1/3))/d)] + 6*e^3*x*(-3 + 2*Log[d + e*x
^(1/3)])*PolyLog[2, 1 + (e*x^(1/3))/d] - 12*e^3*x*PolyLog[3, 1 + (e*x^(1/3)
)/d]))/(2*d^3*x)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)^3+3\,ab^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)^2+3\,a^2b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(1/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(1/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(1/3) + d)^n*c) + a^3)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)^3/x^2, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(e\,x^{\frac{1}{3}}+d\right)^n\right)+a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/3)+d)^n)+a)^3/x^2,x)
```

```
[Out] int((b*ln(c*(e*x^(1/3)+d)^n)+a)^3/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,b^3d^3x^{\frac{2}{3}}\log\left(\left(ex^{\frac{1}{3}}+d\right)^n\right)^3+\left(6\,b^3e^3nx^{\frac{5}{3}}\log\left(ex^{\frac{1}{3}}+d\right)-6\,b^3de^2nx^{\frac{4}{3}}+3\,b^3d^2enx-2\left(b^3e^3nx\log(x)-3\,b^3d^3\log\right.\right.}{2\,d^3x^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^3*d^3*x^(2/3)*log((e*x^(1/3) + d)^n)^3 + (6*b^3*e^3*n*x^(5/3)*log
(e*x^(1/3) + d) - 6*b^3*d*e^2*n*x^(4/3) + 3*b^3*d^2*e*n*x - 2*(b^3*e^3*n*x*
log(x) - 3*b^3*d^3*log(c) - 3*a*b^2*d^3)*x^(2/3))*log((e*x^(1/3) + d)^n)^2)
/(d^3*x^(5/3)) + integrate(1/3*(3*(b^3*d^3*e*log(c)^3 + 3*a*b^2*d^3*e*log(c
```

```
)^2 + 3*a^2*b*d^3*e*log(c) + a^3*d^3*e)*x^(5/3) + 3*(b^3*d^4*log(c)^3 + 3*a
*b^2*d^4*log(c)^2 + 3*a^2*b*d^4*log(c) + a^3*d^4)*x^(4/3) + (6*b^3*e^4*n^2*
x^(8/3)*log(e*x^(1/3) + d) - 6*b^3*d*e^3*n^2*x^(7/3) + 3*b^3*d^2*e^2*n^2*x^
2 + 9*(b^3*d^3*e*log(c)^2 + 2*a*b^2*d^3*e*log(c) + a^2*b*d^3*e)*x^(5/3) + 9
*(b^3*d^4*log(c)^2 + 2*a*b^2*d^4*log(c) + a^2*b*d^4)*x^(4/3) - 2*(b^3*e^4*n
^2*x^2*log(x) - 3*(b^3*d^3*e*n*log(c) + a*b^2*d^3*e*n)*x)*x^(2/3))*log((e*x
^(1/3) + d)^n))/(d^3*e*x^(11/3) + d^4*x^(10/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)\right)^3}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^3/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))^3/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3/x**2,x)
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**3/x**2, x)
```

3.462 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=765

$$\frac{3b^2e^6n^2\text{Li}_2\left(\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6}+\frac{77b^2e^6n^2\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^6}+\frac{3b^2e^6n^2}{}$$

```
[Out] -1/20*b^3*e^3*n^3/d^3/x+3/10*b^3*e^4*n^3/d^4/x^(2/3)-71/40*b^3*e^5*n^3/d^5/
x^(1/3)+71/40*b^3*e^6*n^3*ln(d+e*x^(1/3))/d^6-3/20*b^2*e^2*n^2*(a+b*ln(c*(d
+e*x^(1/3))^n))/d^2/x^(4/3)+9/20*b^2*e^3*n^2*(a+b*ln(c*(d+e*x^(1/3))^n))/d^
3/x-47/40*b^2*e^4*n^2*(a+b*ln(c*(d+e*x^(1/3))^n))/d^4/x^(2/3)+77/20*b^2*e^5
*n^2*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/d^6/x^(1/3)+77/20*b^2*e^6*n^
2*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*(d+e*x^(1/3))^n))/d^6-3/10*b*e*n*(a+b*ln(
c*(d+e*x^(1/3))^n))^2/d/x^(5/3)+3/8*b*e^2*n*(a+b*ln(c*(d+e*x^(1/3))^n))^2/d
^2/x^(4/3)-1/2*b*e^3*n*(a+b*ln(c*(d+e*x^(1/3))^n))^2/d^3/x+3/4*b*e^4*n*(a+b
*ln(c*(d+e*x^(1/3))^n))^2/d^4/x^(2/3)-3/2*b*e^5*n*(d+e*x^(1/3))*(a+b*ln(c*(
d+e*x^(1/3))^n))^2/d^6/x^(1/3)-3/2*b*e^6*n*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*
(d+e*x^(1/3))^n))^2/d^6-1/2*(a+b*ln(c*(d+e*x^(1/3))^n))^3/x^2+3*b^2*e^6*n^2
*(a+b*ln(c*(d+e*x^(1/3))^n))*ln(-e*x^(1/3)/d)/d^6-15/8*b^3*e^6*n^3*ln(x)/d^
6-77/20*b^3*e^6*n^3*polylog(2,d/(d+e*x^(1/3)))/d^6+3*b^2*e^6*n^2*(a+b*ln(c*
(d+e*x^(1/3))^n))*polylog(2,d/(d+e*x^(1/3)))/d^6+3*b^3*e^6*n^3*polylog(2,1+
e*x^(1/3)/d)/d^6+3*b^3*e^6*n^3*polylog(3,d/(d+e*x^(1/3)))/d^6
```

Rubi [A] time = 3.05, antiderivative size = 742, normalized size of antiderivative = 0.97, number of steps used = 73, number of rules used = 17, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.708, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31, 44}

$$-\frac{3b^2e^6n^2\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6}+\frac{137b^3e^6n^3\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)}{20d^6}+\frac{3b^3e^6n^3\text{PolyLog}(}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^3,x]
```

```
[Out] -(b^3*e^3*n^3)/(20*d^3*x) + (3*b^3*e^4*n^3)/(10*d^4*x^(2/3)) - (71*b^3*e^5*
n^3)/(40*d^5*x^(1/3)) + (71*b^3*e^6*n^3*Log[d + e*x^(1/3)])/(40*d^6) - (3*b
^2*e^2*n^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(20*d^2*x^(4/3)) + (9*b^2*e^3*
n^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(20*d^3*x) - (47*b^2*e^4*n^2*(a + b*L
og[c*(d + e*x^(1/3))^n]))/(40*d^4*x^(2/3)) + (77*b^2*e^5*n^2*(d + e*x^(1/3)
)*(a + b*Log[c*(d + e*x^(1/3))^n]))/(20*d^6*x^(1/3)) - (77*b*e^6*n*(a + b*L
og[c*(d + e*x^(1/3))^n])^2)/(40*d^6) - (3*b*e*n*(a + b*Log[c*(d + e*x^(1/3)
)^n])^2)/(10*d*x^(5/3)) + (3*b*e^2*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(8
*d^2*x^(4/3)) - (b*e^3*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*d^3*x) + (3
*b*e^4*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(4*d^4*x^(2/3)) - (3*b*e^5*n*(
d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*d^6*x^(1/3)) + (e^6*(
a + b*Log[c*(d + e*x^(1/3))^n])^3)/(2*d^6) - (a + b*Log[c*(d + e*x^(1/3))^n
])^3/(2*x^2) + (137*b^2*e^6*n^2*(a + b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x
^(1/3))/d)])/(20*d^6) - (3*b*e^6*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2*Log[-
((e*x^(1/3))/d)])/(2*d^6) - (15*b^3*e^6*n^3*Log[x])/(8*d^6) + (137*b^3*e^6*
n^3*PolyLog[2, 1 + (e*x^(1/3))/d])/(20*d^6) - (3*b^2*e^6*n^2*(a + b*Log[c*(
d + e*x^(1/3))^n])*PolyLog[2, 1 + (e*x^(1/3))/d])/d^6 + (3*b^3*e^6*n^3*Poly
Log[3, 1 + (e*x^(1/3))/d])/d^6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x^7}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2} + \frac{1}{2}(3ben)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right.}{x^6(d+ex)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2} + \frac{1}{2}(3bn)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(cx^n\right)\right)^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx,\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2} + \frac{(3bn)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(cx^n\right)\right)^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+e\right.}{2d}$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{10dx^{5/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2} - \frac{(3be}{}$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{10dx^{5/3}} + \frac{3be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{8d^2x^{4/3}}$$

$$= -\frac{3b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^2x^{4/3}} - \frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{10dx^{5/3}}$$

$$= -\frac{3b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^2x^{4/3}} + \frac{9b^2e^3n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right.\right.}{20d^3x}$$

$$= -\frac{b^3e^3n^3}{20d^3x} + \frac{3b^3e^4n^3}{40d^4x^{2/3}} - \frac{3b^3e^5n^3}{20d^5\sqrt[3]{x}} + \frac{3b^3e^6n^3\log\left(d+e\sqrt[3]{x}\right)}{20d^6} - \frac{3b^2e^2n^2\left(a+\right.}{}$$

$$= -\frac{b^3e^3n^3}{20d^3x} + \frac{3b^3e^4n^3}{10d^4x^{2/3}} - \frac{3b^3e^5n^3}{5d^5\sqrt[3]{x}} + \frac{3b^3e^6n^3\log\left(d+e\sqrt[3]{x}\right)}{5d^6} - \frac{3b^2e^2n^2\left(a+\right.}{}$$

$$= -\frac{b^3e^3n^3}{20d^3x} + \frac{3b^3e^4n^3}{10d^4x^{2/3}} - \frac{71b^3e^5n^3}{40d^5\sqrt[3]{x}} + \frac{71b^3e^6n^3\log\left(d+e\sqrt[3]{x}\right)}{40d^6} - \frac{3b^2e^2n^2\left(a\right.}{}$$

$$= -\frac{b^3e^3n^3}{20d^3x} + \frac{3b^3e^4n^3}{10d^4x^{2/3}} - \frac{71b^3e^5n^3}{40d^5\sqrt[3]{x}} + \frac{71b^3e^6n^3\log\left(d+e\sqrt[3]{x}\right)}{40d^6} - \frac{3b^2e^2n^2\left(a\right.}{}$$

Mathematica [A] time = 1.87, size = 1074, normalized size = 1.40

$$-\frac{20\left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3d^6+60bn\log\left(d+e\sqrt[3]{x}\right)\left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^3,x]
```

```
[Out] -1/40*(12*b*d^5*e*n*x^(1/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^
(1/3))^n])^2 - 15*b*d^4*e^2*n*x^(2/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c
*(d + e*x^(1/3))^n])^2 + 20*b*d^3*e^3*n*x*(a - b*n*Log[d + e*x^(1/3)] + b*L
og[c*(d + e*x^(1/3))^n])^2 - 30*b*d^2*e^4*n*x^(4/3)*(a - b*n*Log[d + e*x^(1
/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 + 60*b*d*e^5*n*x^(5/3)*(a - b*n*Log[d
+ e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 + 60*b*d^6*n*Log[d + e*x^(1/3)
]*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 - 60*b*e^6*n*
x^2*Log[d + e*x^(1/3)]*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3)
)^n])^2 + 20*d^6*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^
3 + 20*b*e^6*n*x^2*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n]
)^2*Log[x] + b^2*n^2*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^
n])*(6*d^4*e^2*x^(2/3) - 18*d^3*e^3*x + 47*d^2*e^4*x^(4/3) - 154*d*e^5*x^(5
/3) + 60*(d^6 - e^6*x^2)*Log[d + e*x^(1/3)]^2 - 274*e^6*x^2*Log[-((e*x^(1/3
))/d)] + 2*Log[d + e*x^(1/3)]*(12*d^5*e*x^(1/3) - 15*d^4*e^2*x^(2/3) + 20*d
^3*e^3*x - 30*d^2*e^4*x^(4/3) + 60*d*e^5*x^(5/3) + 137*e^6*x^2 + 60*e^6*x^2
*Log[-((e*x^(1/3))/d)]) + 120*e^6*x^2*PolyLog[2, 1 + (e*x^(1/3))/d]) + b^3*
n^3*(3*d^4*e^2*x^(2/3)*(2 - 5*Log[d + e*x^(1/3)])*Log[d + e*x^(1/3)] + 12*d
^5*e*x^(1/3)*Log[d + e*x^(1/3)]^2 + 20*d^6*Log[d + e*x^(1/3)]^3 + 2*d^3*e^3
*x*(1 - 9*Log[d + e*x^(1/3)] + 10*Log[d + e*x^(1/3)]^2) - d^2*e^4*x^(4/3)*(
12 - 47*Log[d + e*x^(1/3)] + 30*Log[d + e*x^(1/3)]^2) + d*e^5*x^(5/3)*(71 -
 154*Log[d + e*x^(1/3)] + 60*Log[d + e*x^(1/3)]^2) + 225*e^6*x^2*(-Log[d +
e*x^(1/3)] + Log[-((e*x^(1/3))/d)]) + 137*e^6*x^2*(Log[d + e*x^(1/3)]*(Log[
d + e*x^(1/3)] - 2*Log[-((e*x^(1/3))/d)]) - 2*PolyLog[2, 1 + (e*x^(1/3))/d]
) - 20*e^6*x^2*(Log[d + e*x^(1/3)]^2*(Log[d + e*x^(1/3)] - 3*Log[-((e*x^(1/
3))/d)]) - 6*Log[d + e*x^(1/3)]*PolyLog[2, 1 + (e*x^(1/3))/d] + 6*PolyLog[3
, 1 + (e*x^(1/3))/d])))/(d^6*x^2)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)^3 + 3\,ab^2 \log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)^2 + 3\,a^2 b \log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right) + a^3}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(1/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(1/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(1/3) + d)^n*c) + a^3)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right) + a\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)^3/x^3, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c \left(e\,x^{\frac{1}{3}}+d\right)^n\right) + a\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(1/3)+d)^n)+a)^3/x^3,x)
```

```
[Out] int((b*ln(c*(e*x^(1/3)+d)^n)+a)^3/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3\log\left(\left(ex^{\frac{1}{3}}+d\right)^n\right)^3}{2\,x^2}+\int\frac{\left(b^3enx+6\left(b^3e\log(c)+ab^2e\right)x+6\left(b^3d\log(c)+ab^2d\right)x^{\frac{2}{3}}\right)\log\left(\left(ex^{\frac{1}{3}}+d\right)^n\right)^2+2\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log((e*x^(1/3) + d)^n)^3/x^2 + integrate(1/2*((b^3*e*n*x + 6*(b^3*
e*log(c) + a*b^2*e)*x + 6*(b^3*d*log(c) + a*b^2*d)*x^(2/3))*log((e*x^(1/3)
+ d)^n)^2 + 2*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3
*e)*x + 6*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*log(c)^
2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^(2/3))*log((e*x^(1/3) + d)^n) + 2*(b^3*d*
log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(2/3))/(e*x^4 +
 d*x^(11/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^3/x^3,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))^3/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3/x**3,x)
```

```
[Out] Timed out
```

3.463 $\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx$

Optimal. Leaf size=138

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{bd^6n\log\left(d+ex^{2/3}\right)}{4e^6}+\frac{bd^5nx^{2/3}}{4e^5}-\frac{bd^4nx^{4/3}}{8e^4}+\frac{bd^3nx^2}{12e^3}-\frac{bd^2nx^{8/3}}{16e^2}+\frac{bdnx^{10/3}}{20e}-\frac{1}{24}bnx$$

```
[Out] 1/4*b*d^5*n*x^(2/3)/e^5-1/8*b*d^4*n*x^(4/3)/e^4+1/12*b*d^3*n*x^2/e^3-1/16*b
*d^2*n*x^(8/3)/e^2+1/20*b*d*n*x^(10/3)/e-1/24*b*n*x^4-1/4*b*d^6*n*ln(d+e*x^
(2/3))/e^6+1/4*x^4*(a+b*ln(c*(d+e*x^(2/3))^n))
```

Rubi [A] time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{bd^5nx^{2/3}}{4e^5}-\frac{bd^4nx^{4/3}}{8e^4}+\frac{bd^3nx^2}{12e^3}-\frac{bd^2nx^{8/3}}{16e^2}-\frac{bd^6n\log\left(d+ex^{2/3}\right)}{4e^6}+\frac{bdnx^{10/3}}{20e}-\frac{1}{24}bnx$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] (b*d^5*n*x^(2/3))/(4*e^5) - (b*d^4*n*x^(4/3))/(8*e^4) + (b*d^3*n*x^2)/(12*e
^3) - (b*d^2*n*x^(8/3))/(16*e^2) + (b*d*n*x^(10/3))/(20*e) - (b*n*x^4)/24 -
 (b*d^6*n*Log[d + e*x^(2/3)])/(4*e^6) + (x^4*(a + b*Log[c*(d + e*x^(2/3))^n
]))/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)dx = \frac{3}{2}\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)dx,x,x^{2/3}\right)$$
$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int \frac{x^6}{d+ex}dx,x,x^{2/3}\right)$$
$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int\left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3x^2}{e^4}\right.\right.$$
$$= \frac{bd^5nx^{2/3}}{4e^5}-\frac{bd^4nx^{4/3}}{8e^4}+\frac{bd^3nx^2}{12e^3}-\frac{bd^2nx^{8/3}}{16e^2}+\frac{bdnx^{10/3}}{20e}-\frac{1}{24}bnx^4-\frac{bd^6}{}$$

Mathematica [A] time = 0.11, size = 135, normalized size = 0.98

$$\frac{ax^4}{4}+\frac{1}{4}bx^4\log\left(c\left(d+ex^{2/3}\right)^n\right)-\frac{1}{4}ben\left(\frac{d^6\log\left(d+ex^{2/3}\right)}{e^7}-\frac{d^5x^{2/3}}{e^6}+\frac{d^4x^{4/3}}{2e^5}-\frac{d^3x^2}{3e^4}+\frac{d^2x^{8/3}}{4e^3}-\frac{dx^{10/3}}{5e^2}+\frac{x^4}{6e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] (a*x^4)/4 - (b*e*n*(-((d^5*x^(2/3))/e^6) + (d^4*x^(4/3))/(2*e^5) - (d^3*x^2
)/(3*e^4) + (d^2*x^(8/3))/(4*e^3) - (d*x^(10/3))/(5*e^2) + x^4/(6*e) + (d^6
*Log[d + e*x^(2/3)])/e^7))/4 + (b*x^4*Log[c*(d + e*x^(2/3))^n])/4
```

fricas [A] time = 0.46, size = 129, normalized size = 0.93

$$\frac{60\,be^6x^4\log(c)+20\,bd^3e^3nx^2-10\left(be^6n-6\,ae^6\right)x^4+60\left(be^6nx^4-bd^6n\right)\log\left(ex^{\frac{2}{3}}+d\right)-15\left(bd^2e^4nx^2-4\,bd^5\right.}{240\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/240*(60*b*e^6*x^4*log(c) + 20*b*d^3*e^3*n*x^2 - 10*(b*e^6*n - 6*a*e^6)*x^
4 + 60*(b*e^6*n*x^4 - b*d^6*n)*log(e*x^(2/3) + d) - 15*(b*d^2*e^4*n*x^2 - 4
*b*d^5*e*n)*x^(2/3) + 6*(2*b*d*e^5*n*x^3 - 5*b*d^4*e^2*n*x)*x^(1/3))/e^6
```

giac [B] time = 0.34, size = 266, normalized size = 1.93

$$\frac{1}{4}bx^4\log(c)+\frac{1}{4}ax^4+\frac{1}{240}\left(60\left(x^{\frac{2}{3}}e+d\right)^6e^{(-5)}\log\left(x^{\frac{2}{3}}e+d\right)-360\left(x^{\frac{2}{3}}e+d\right)^5de^{(-5)}\log\left(x^{\frac{2}{3}}e+d\right)+900\left(x^{\frac{2}{3}}e+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*log(c) + 1/4*a*x^4 + 1/240*(60*(x^(2/3)*e + d)^6*e^(-5)*log(x^(2/
3)*e + d) - 360*(x^(2/3)*e + d)^5*d*e^(-5)*log(x^(2/3)*e + d) + 900*(x^(2/3
)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e + d) - 1200*(x^(2/3)*e + d)^3*d^3*e^(-5
)*log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d)
- 360*(x^(2/3)*e + d)*d^5*e^(-5)*log(x^(2/3)*e + d) - 10*(x^(2/3)*e + d)^6*
e^(-5) + 72*(x^(2/3)*e + d)^5*d*e^(-5) - 225*(x^(2/3)*e + d)^4*d^2*e^(-5) +
 400*(x^(2/3)*e + d)^3*d^3*e^(-5) - 450*(x^(2/3)*e + d)^2*d^4*e^(-5) + 360*
(x^(2/3)*e + d)*d^5*e^(-5))*b*n*e^(-1)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^n\right)+a\right)x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

maxima [A] time = 0.49, size = 108, normalized size = 0.78

$$\frac{1}{4}bx^4\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+\frac{1}{4}ax^4-\frac{1}{240}ben\left(\frac{60\,d^6\log\left(ex^{\frac{2}{3}}+d\right)}{e^7}+\frac{10\,e^5x^4-12\,de^4x^{\frac{10}{3}}+15\,d^2e^3x^{\frac{8}{3}}-20\,d^3e^2x^2+30$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log((e*x^(2/3) + d)^n*c) + 1/4*a*x^4 - 1/240*b*e*n*(60*d^6*log(e*
x^(2/3) + d)/e^7 + (10*e^5*x^4 - 12*d*e^4*x^(10/3) + 15*d^2*e^3*x^(8/3) - 2
0*d^3*e^2*x^2 + 30*d^4*e*x^(4/3) - 60*d^5*x^(2/3))/e^6)
```

mupad [B] time = 0.45, size = 113, normalized size = 0.82

$$\frac{a\,x^4}{4}-\frac{b\,n\,x^4}{24}+\frac{b\,x^4\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{4}+\frac{b\,d\,n\,x^{10/3}}{20\,e}-\frac{b\,d^6\,n\,\ln\left(d+e\,x^{2/3}\right)}{4\,e^6}+\frac{b\,d^3\,n\,x^2}{12\,e^3}-\frac{b\,d^2\,n\,x^{8/3}}{16\,e^2}-\frac{b\,d^4\,n\,x^{4/3}}{8\,e^4}+\frac{b\,d^5\,n}{4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(2/3))^n)),x)
```

```
[Out] (a*x^4)/4 - (b*n*x^4)/24 + (b*x^4*log(c*(d + e*x^(2/3))^n))/4 + (b*d*n*x^(1
0/3))/(20*e) - (b*d^6*n*log(d + e*x^(2/3)))/(4*e^6) + (b*d^3*n*x^2)/(12*e^3
) - (b*d^2*n*x^(8/3))/(16*e^2) - (b*d^4*n*x^(4/3))/(8*e^4) + (b*d^5*n*x^(2/
3))/(4*e^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**n)),x)
```

```
[Out] Timed out
```

3.464 $\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) dx$

Optimal. Leaf size=130

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{9/2}}-\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}-\frac{2bd^2nx^{5/3}}{15e^2}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3$$

```
[Out] -2/3*b*d^4*n*x^(1/3)/e^4+2/9*b*d^3*n*x/e^3-2/15*b*d^2*n*x^(5/3)/e^2+2/21*b*
d*n*x^(7/3)/e-2/27*b*n*x^3+2/3*b*d^(9/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2))/
e^(9/2)+1/3*x^3*(a+b*ln(c*(d+e*x^(2/3))^n))
```

Rubi [A] time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2455, 341, 302, 205}

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{2bd^2nx^{5/3}}{15e^2}-\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}+\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{9/2}}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] (-2*b*d^4*n*x^(1/3))/(3*e^4) + (2*b*d^3*n*x)/(9*e^3) - (2*b*d^2*n*x^(5/3))/
(15*e^2) + (2*b*d*n*x^(7/3))/(21*e) - (2*b*n*x^3)/27 + (2*b*d^(9/2)*n*ArcTa
n[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(9/2)) + (x^3*(a + b*Log[c*(d + e*x^(2/3
))^n]))/3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)dx=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{9}(2ben)\int\frac{x^{8/3}}{d+ex^{2/3}}dx$$
$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{3}(2ben)\,\text{Subst}\left(\int\frac{x^{10}}{d+ex^2}dx,x,\sqrt[3]{x}\right)$$
$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{3}(2ben)\,\text{Subst}\left(\int\left(\frac{d^4}{e^5}-\frac{d^3x^2}{e^4}+\frac{d^2x^4}{e^3}-\right.\right.$$
$$=-\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}-\frac{2bd^2nx^{5/3}}{15e^2}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3+\frac{1}{3}x^3\left(a+b\log\right.$$
$$=-\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}-\frac{2bd^2nx^{5/3}}{15e^2}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3+\frac{2bd^{9/2}n\tan^{-1}}{3e^{9/2}}$$

Mathematica [A] time = 0.10, size = 135, normalized size = 1.04

$$\frac{ax^3}{3}+\frac{1}{3}bx^3\log\left(c\left(d+ex^{2/3}\right)^n\right)+\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{9/2}}-\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}-\frac{2bd^2nx^{5/3}}{15e^2}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] (-2*b*d^4*n*x^(1/3))/(3*e^4) + (2*b*d^3*n*x)/(9*e^3) - (2*b*d^2*n*x^(5/3))/
(15*e^2) + (2*b*d*n*x^(7/3))/(21*e) + (a*x^3)/3 - (2*b*n*x^3)/27 + (2*b*d^(
9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(9/2)) + (b*x^3*Log[c*(d + e
*x^(2/3))^n])/3
```

fricas [A] time = 0.47, size = 337, normalized size = 2.59

$$\left[\frac{315\,be^4nx^3\log\left(ex^{\frac{2}{3}}+d\right)+315\,be^4x^3\log(c)-126\,bd^2e^2nx^{\frac{5}{3}}+315\,bd^4n\sqrt{-\frac{d}{e}}\log\left(\frac{e^3x^2-2\,de^2x\sqrt{-\frac{d}{e}}-d^3+2\left(e^3x\sqrt{-\frac{d}{e}}+d^2e\right.}{e^3x^2+d^3}\right.}{945\,e^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] [1/945*(315*b*e^4*n*x^3*log(e*x^(2/3) + d) + 315*b*e^4*x^3*log(c) - 126*b*d
^2*e^2*n*x^(5/3) + 315*b*d^4*n*sqrt(-d/e)*log((e^3*x^2 - 2*d*e^2*x*sqrt(-d/
e) - d^3 + 2*(e^3*x*sqrt(-d/e) + d^2*e)*x^(2/3) - 2*(d*e^2*x - d^2*e*sqrt(-
d/e))*x^(1/3))/(e^3*x^2 + d^3)) + 210*b*d^3*e*n*x - 35*(2*b*e^4*n - 9*a*e^4
)*x^3 + 90*(b*d*e^3*n*x^2 - 7*b*d^4*n)*x^(1/3))/e^4, 1/945*(315*b*e^4*n*x^3
*log(e*x^(2/3) + d) + 315*b*e^4*x^3*log(c) - 126*b*d^2*e^2*n*x^(5/3) + 630*
b*d^4*n*sqrt(d/e)*arctan(e*x^(1/3)*sqrt(d/e)/d) + 210*b*d^3*e*n*x - 35*(2*b
*e^4*n - 9*a*e^4)*x^3 + 90*(b*d*e^3*n*x^2 - 7*b*d^4*n)*x^(1/3))/e^4]
```

giac [A] time = 0.32, size = 104, normalized size = 0.80

$$\frac{1}{3}\,bx^3\log(c)+\frac{1}{3}\,ax^3+\frac{1}{945}\left(315\,x^3\log\left(x^{\frac{2}{3}}e+d\right)+2\left(315\,d^{\frac{9}{2}}\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{11}{2}\right)}-\left(315\,d^4x^{\frac{1}{3}}e^4-105\,d^3xe^5+63\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*log(c) + 1/3*a*x^3 + 1/945*(315*x^3*log(x^(2/3)*e + d) + 2*(315*d
^(9/2)*arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(-11/2) - (315*d^4*x^(1/3)*e^4 - 1
05*d^3*x*e^5 + 63*d^2*x^(5/3)*e^6 - 45*d*x^(7/3)*e^7 + 35*x^3*e^8)*e^(-9))*
e)*b*n
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^{n}\right)+a\right)x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*(e*x^(2/3)+d)^n)+a),x)
```

```
[Out] int(x^2*(b*ln(c*(e*x^(2/3)+d)^n)+a),x)
```

maxima [A] time = 1.00, size = 104, normalized size = 0.80

$$\frac{1}{3}\,bx^{3}\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+\frac{1}{3}\,ax^{3}+\frac{2}{945}\,ben\left(\frac{315\,d^{5}\arctan\left(\frac{ex^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}\,e^{5}}-\frac{35\,e^{4}x^{3}-45\,de^{3}x^{\frac{7}{3}}+63\,d^{2}e^{2}x^{\frac{5}{3}}-105\,d^{3}ex+}{e^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3*log((e*x^(2/3) + d)^n*c) + 1/3*a*x^3 + 2/945*b*e*n*(315*d^5*arcta
n(e*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*e^5) - (35*e^4*x^3 - 45*d*e^3*x^(7/3) + 6
3*d^2*e^2*x^(5/3) - 105*d^3*e*x + 315*d^4*x^(1/3))/e^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(2/3))^n)),x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e*x^(2/3))^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**n)),x)
```

```
[Out] Timed out
```

3.465 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx$

Optimal. Leaf size=89

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{bd^3n\log\left(d+ex^{2/3}\right)}{2e^3}-\frac{bd^2nx^{2/3}}{2e^2}+\frac{bdnx^{4/3}}{4e}-\frac{1}{6}bnx^2$$

```
[Out] -1/2*b*d^2*n*x^(2/3)/e^2+1/4*b*d*n*x^(4/3)/e-1/6*b*n*x^2+1/2*b*d^3*n*ln(d+e
*x^(2/3))/e^3+1/2*x^2*(a+b*ln(c*(d+e*x^(2/3))^n))
```

Rubi [A] time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2454, 2395, 43}

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{bd^2nx^{2/3}}{2e^2}+\frac{bd^3n\log\left(d+ex^{2/3}\right)}{2e^3}+\frac{bdnx^{4/3}}{4e}-\frac{1}{6}bnx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] -(b*d^2*n*x^(2/3))/(2*e^2) + (b*d*n*x^(4/3))/(4*e) - (b*n*x^2)/6 + (b*d^3*n
*Log[d + e*x^(2/3)])/(2*e^3) + (x^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx &= \frac{3}{2}\,\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx,x,x^{2/3}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{2}(ben)\,\text{Subst}\left(\int \frac{x^3}{d+ex}\,dx,x,x^{2/3}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{2}(ben)\,\text{Subst}\left(\int\left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+\dots}\right.\right.\\
&= -\frac{bd^2nx^{2/3}}{2e^2}+\frac{bdnx^{4/3}}{4e}-\frac{1}{6}bnx^2+\frac{bd^3n\log\left(d+ex^{2/3}\right)}{2e^3}+\frac{1}{2}x^2\left(a+b\log\left(c\left(\dots\right.\right.\right.
\end{aligned}$$

Mathematica [A] time = 0.03, size = 94, normalized size = 1.06

$$\frac{ax^2}{2}+\frac{1}{2}bx^2\log\left(c\left(d+ex^{2/3}\right)^n\right)+\frac{bd^3n\log\left(d+ex^{2/3}\right)}{2e^3}-\frac{bd^2nx^{2/3}}{2e^2}+\frac{bdnx^{4/3}}{4e}-\frac{1}{6}bnx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] -1/2*(b*d^2*n*x^(2/3))/e^2 + (b*d*n*x^(4/3))/(4*e) + (a*x^2)/2 - (b*n*x^2)/
6 + (b*d^3*n*Log[d + e*x^(2/3)])/(2*e^3) + (b*x^2*Log[c*(d + e*x^(2/3))^n])
/2
```

fricas [A] time = 0.45, size = 83, normalized size = 0.93

$$\frac{6\,be^3x^2\log(c)+3\,bde^2nx^{\frac{4}{3}}-6\,bd^2enx^{\frac{2}{3}}-2\left(be^3n-3\,ae^3\right)x^2+6\left(be^3nx^2+bd^3n\right)\log\left(ex^{\frac{2}{3}}+d\right)}{12\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*b*e^3*x^2*log(c) + 3*b*d*e^2*n*x^(4/3) - 6*b*d^2*e*n*x^(2/3) - 2*(b
*e^3*n - 3*a*e^3)*x^2 + 6*(b*e^3*n*x^2 + b*d^3*n)*log(e*x^(2/3) + d))/e^3
```

giac [A] time = 0.38, size = 82, normalized size = 0.92

$$\frac{1}{2}\,bx^2\log(c)+\frac{1}{12}\left(6\,x^2\log\left(x^{\frac{2}{3}}e+d\right)+\left(6\,d^3e^{(-4)}\log\left(\left|x^{\frac{2}{3}}e+d\right|\right)+\left(3\,dx^{\frac{4}{3}}e-2\,x^2e^2-6\,d^2x^{\frac{2}{3}}\right)e^{(-3)}\right)e\right)bn+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*log(c) + 1/12*(6*x^2*log(x^(2/3)*e + d) + (6*d^3*e^(-4)*log(abs(x
^(2/3)*e + d)) + (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*e)*b*n
 + 1/2*a*x^2
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^n\right)+a\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(e*x^(2/3)+d)^n)+a),x)
```

```
[Out] int(x*(b*ln(c*(e*x^(2/3)+d)^n)+a),x)
```

maxima [A] time = 0.48, size = 76, normalized size = 0.85

$$\frac{1}{12}\,ben\left(\frac{6\,d^3\log\left(ex^{\frac{2}{3}}+d\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)+\frac{1}{2}\,bx^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/12*b*e*n*(6*d^3*log(e*x^(2/3) + d)/e^4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d
^2*x^(2/3))/e^3) + 1/2*b*x^2*log((e*x^(2/3) + d)^n*c) + 1/2*a*x^2
```

mupad [B] time = 0.39, size = 74, normalized size = 0.83

$$\frac{a\,x^2}{2}-\frac{b\,n\,x^2}{6}+\frac{b\,x^2\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{2}+\frac{b\,d\,n\,x^{4/3}}{4\,e}+\frac{b\,d^3\,n\,\ln\left(d+e\,x^{2/3}\right)}{2\,e^3}-\frac{b\,d^2\,n\,x^{2/3}}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(2/3))^n)),x)
```

```
[Out] (a*x^2)/2 - (b*n*x^2)/6 + (b*x^2*log(c*(d + e*x^(2/3))^n))/2 + (b*d*n*x^(4/
3))/(4*e) + (b*d^3*n*log(d + e*x^(2/3)))/(2*e^3) - (b*d^2*n*x^(2/3))/(2*e^2
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))**n)),x)
```

```
[Out] Timed out
```

3.466 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx$

Optimal. Leaf size=72

$$ax+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+\frac{2bdn\sqrt[3]{x}}{e}-\frac{2bnx}{3}$$

```
[Out] 2*b*d*n*x^(1/3)/e+a*x-2/3*b*n*x-2*b*d^(3/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2
))/e^(3/2)+b*x*ln(c*(d+e*x^(2/3))^n)
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2448, 341, 302, 205}

$$ax+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+\frac{2bdn\sqrt[3]{x}}{e}-\frac{2bnx}{3}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e*x^(2/3))^n],x]
```

```
[Out] (2*b*d*n*x^(1/3))/e + a*x - (2*b*n*x)/3 - (2*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x^
(1/3))/Sqrt[d]])/e^(3/2) + b*x*Log[c*(d + e*x^(2/3))^n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)dx = ax+b\int \log\left(c\left(d+ex^{2/3}\right)^n\right)dx$$
$$= ax+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-\frac{1}{3}(2ben)\int \frac{x^{2/3}}{d+ex^{2/3}}dx$$
$$= ax+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-(2ben)\,\text{Subst}\left(\int \frac{x^4}{d+ex^2}dx,x,\sqrt[3]{x}\right)$$
$$= ax+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-(2ben)\,\text{Subst}\left(\int \left(-\frac{d}{e^2}+\frac{x^2}{e}+\frac{d^2}{e^2\left(d+ex^2\right)}\right)dx$$
$$= \frac{2bdn\sqrt[3]{x}}{e}+ax-\frac{2bnx}{3}+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-\frac{\left(2bd^2n\right)\text{Subst}\left(\int \frac{1}{d+ex^2}dx\right.}{e}$$
$$= \frac{2bdn\sqrt[3]{x}}{e}+ax-\frac{2bnx}{3}-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)$$

Mathematica [A] time = 0.03, size = 72, normalized size = 1.00

$$ax+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+\frac{2bdn\sqrt[3]{x}}{e}-\frac{2bnx}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e*x^(2/3))^n],x]
```

```
[Out] (2*b*d*n*x^(1/3))/e + a*x - (2*b*n*x)/3 - (2*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x^
(1/3))/Sqrt[d]])/e^(3/2) + b*x*Log[c*(d + e*x^(2/3))^n]
```

fricas [A] time = 0.47, size = 231, normalized size = 3.21

$$\left[\frac{3\,benx\log\left(ex^{\frac{2}{3}}+d\right)+3\,bdn\sqrt{-\frac{d}{e}}\,\log\left(\frac{e^3x^2+2\,de^2x\sqrt{-\frac{d}{e}}-d^3-2\left(e^3x\sqrt{-\frac{d}{e}}-d^2e\right)x^{\frac{2}{3}}-2\left(de^2x+d^2e\sqrt{-\frac{d}{e}}\right)x^{\frac{1}{3}}}{e^3x^2+d^3}\right)+3\,bex\log(c)+6\,b}{3\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(2/3))^n),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*e*n*x*log(e*x^(2/3) + d) + 3*b*d*n*sqrt(-d/e)*log((e^3*x^2 + 2*d*
e^2*x*sqrt(-d/e) - d^3 - 2*(e^3*x*sqrt(-d/e) - d^2*e)*x^(2/3) - 2*(d*e^2*x
+ d^2*e*sqrt(-d/e))*x^(1/3))/(e^3*x^2 + d^3)) + 3*b*e*x*log(c) + 6*b*d*n*x^
(1/3) - (2*b*e*n - 3*a*e)*x)/e, 1/3*(3*b*e*n*x*log(e*x^(2/3) + d) - 6*b*d*n
*sqrt(d/e)*arctan(e*x^(1/3)*sqrt(d/e)/d) + 3*b*e*x*log(c) + 6*b*d*n*x^(1/3)
 - (2*b*e*n - 3*a*e)*x)/e]
```

giac [A] time = 0.26, size = 68, normalized size = 0.94

$$-\frac{1}{3}\left(\left(2\left(3\,d^{\frac{3}{2}}\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}-\left(3\,dx^{\frac{1}{3}}e-xe^2\right)e^{(-3)}\right)e-3\,x\log\left(x^{\frac{2}{3}}e+d\right)\right)n-3\,x\log(c)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(2/3))^n),x, algorithm="giac")
```

```
[Out] -1/3*((2*(3*d^(3/2)*arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(-5/2) - (3*d*x^(1/3)
*e - x*e^2)*e^(-3))*e - 3*x*log(x^(2/3)*e + d))*n - 3*x*log(c))*b + a*x
```

maple [A] time = 0.08, size = 62, normalized size = 0.86

$$-\frac{2b\,d^{2}n\arctan\left(\frac{e\,x^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}\,e}-\frac{2bnx}{3}+bx\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^{n}\right)+\frac{2bdn\,x^{\frac{1}{3}}}{e}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*ln(c*(e*x^(2/3)+d)^n)+a,x)
```

```
[Out] a*x+b*x*ln(c*(e*x^(2/3)+d)^n)-2/3*b*n*x+2*b*d*n*x^(1/3)/e-2*b/e*n*d^2/(d*e)
^(1/2)*arctan(x^(1/3)*e/(d*e)^(1/2))
```

maxima [A] time = 1.00, size = 66, normalized size = 0.92

$$-\frac{1}{3}\left(2\,en\left(\frac{3\,d^{2}\arctan\left(\frac{ex^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}\,e^{2}}+\frac{ex-3\,dx^{\frac{1}{3}}}{e^{2}}\right)-3\,x\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(2/3))^n),x, algorithm="maxima")
```

```
[Out] -1/3*(2*e*n*(3*d^2*arctan(e*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*e^2) + (e*x - 3*d
*x^(1/3))/e^2) - 3*x*log((e*x^(2/3) + d)^n*c))*b + a*x
```

mupad [B] time = 0.39, size = 56, normalized size = 0.78

$$a\,x+b\,x\,\ln\left(c\left(d+e\,x^{2/3}\right)^{n}\right)-\frac{2\,b\,n\,x}{3}+\frac{2\,b\,d\,n\,x^{1/3}}{e}-\frac{2\,b\,d^{3/2}\,n\,\mathrm{atan}\left(\frac{\sqrt{e}\,x^{1/3}}{\sqrt{d}}\right)}{e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*log(c*(d + e*x^(2/3))^n),x)
```

```
[Out] a*x + b*x*log(c*(d + e*x^(2/3))^n) - (2*b*n*x)/3 + (2*b*d*n*x^(1/3))/e - (2
*b*d^(3/2)*n*atan((e^(1/2)*x^(1/3))/d^(1/2)))/e^(3/2)
```

sympy [A] time = 7.61, size = 133, normalized size = 1.85

$$ax+b\left(-\frac{2en\left(\begin{cases}-\frac{3id^{\frac{3}{2}}\log\left(-i\sqrt{d}\sqrt{\frac{1}{e}}+\sqrt[3]{x}\right)}{2e^{3}\sqrt{\frac{1}{e}}}+\frac{3id^{\frac{3}{2}}\log\left(i\sqrt{d}\sqrt{\frac{1}{e}}+\sqrt[3]{x}\right)}{2e^{3}\sqrt{\frac{1}{e}}}-\frac{3d\sqrt[3]{x}}{e^{2}}+\frac{x}{e} & \text{for}\ e\neq 0\\ \frac{3x^{\frac{5}{3}}}{5d} & \text{otherwise}\end{cases}\right)}{3}+x\log\left(c\left(d+ex^{\frac{2}{3}}\right)^{n}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e*x**(2/3))**n),x)
```

```
[Out] a*x + b*(-2*e*n*Piecewise((-3*I*d**(3/2)*log(-I*sqrt(d)*sqrt(1/e) + x**(1/3
))/(2*e**3*sqrt(1/e)) + 3*I*d**(3/2)*log(I*sqrt(d)*sqrt(1/e) + x**(1/3))/(2
*e**3*sqrt(1/e)) - 3*d*x**(1/3)/e**2 + x/e, Ne(e, 0)), (3*x**(5/3)/(5*d), T
rue))/3 + x*log(c*(d + e*x**(2/3))**n))
```

3.467 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=55

$$\frac{3}{2}\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{3}{2}bn\text{Li}_2\left(\frac{x^{2/3}e}{d}+1\right)$$

```
[Out] 3/2*(a+b*ln(c*(d+e*x^(2/3))^n))*ln(-e*x^(2/3)/d)+3/2*b*n*polylog(2,1+e*x^(2
/3)/d)
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2394, 2315}

$$\frac{3}{2}bn\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)+\frac{3}{2}\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x,x]
```

```
[Out] (3*(a + b*Log[c*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)])/2 + (3*b*n*PolyL
og[2, 1 + (e*x^(2/3))/d])/2
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx &= \frac{3}{2}\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx,x,x^{2/3}\right)\\ &= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\log\left(-\frac{ex^{2/3}}{d}\right)-\frac{1}{2}(3ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,d\right.\\ &= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\log\left(-\frac{ex^{2/3}}{d}\right)+\frac{3}{2}bn\text{Li}_2\left(1+\frac{ex^{2/3}}{d}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.00

$$a\log(x)+\frac{3}{2}b\left(\log\left(-\frac{ex^{2/3}}{d}\right)\log\left(c\left(d+ex^{2/3}\right)^n\right)+n\text{Li}_2\left(\frac{d+ex^{2/3}}{d}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x,x]
```

```
[Out] a*Log[x] + (3*b*(Log[c*(d + e*x^(2/3))^n]*Log[-((e*x^(2/3))/d)] + n*PolyLog
[2, (d + e*x^(2/3))/d]))/2
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x,x, algorithm="fricas")
```

```
[Out] integral((b*log((e*x^(2/3) + d)^n*c) + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)/x, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^n\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(2/3)+d)^n)+a)/x,x)
```

```
[Out] int((b*ln(c*(e*x^(2/3)+d)^n)+a)/x,x)
```

maxima [B] time = 1.01, size = 113, normalized size = 2.05

$$-\frac{3}{2}\left(2\,\log\left(\frac{ex^{\frac{2}{3}}}{d}+1\right)\log\left(x^{\frac{1}{3}}\right)+\text{Li}_2\left(-\frac{ex^{\frac{2}{3}}}{d}\right)\right)bn+\frac{2\,bdn\log\left(ex^{\frac{2}{3}}+d\right)\log(x)+2\left(bd\log(c)+ad\right)\log(x)-\frac{2\,benx\log}{\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x,x, algorithm="maxima")
```

```
[Out] -3/2*(2*log(e*x^(2/3)/d + 1)*log(x^(1/3)) + dilog(-e*x^(2/3)/d))*b*n + 1/2*
(2*b*d*n*log(e*x^(2/3) + d)*log(x) + 2*(b*d*log(c) + a*d)*log(x) - (2*b*e*n
*x*log(x) - 3*b*e*n*x)/x^(1/3))/d + 3/2*(2*b*e*n*x^(2/3)*log(x^(1/3)) - b*e
*n*x^(2/3))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))/x,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x,x)
[Out] Timed out
```

3.468 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=68

$$-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}-\frac{2be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2ben}{d\sqrt[3]{x}}$$

```
[Out] -2*b*e*n/d/x^(1/3)-2*b*e^(3/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2))/d^(3/2)+(-
a-b*ln(c*(d+e*x^(2/3))^n))/x
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2455, 341, 325, 205}

$$-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}-\frac{2be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2ben}{d\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x^2,x]
```

```
[Out] (-2*b*e*n)/(d*x^(1/3)) - (2*b*e^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/
d^(3/2) - (a + b*Log[c*(d + e*x^(2/3))^n])/x
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx = -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x} + \frac{1}{3}(2ben)\int \frac{1}{\left(d+ex^{2/3}\right)x^{4/3}}\,dx$$
$$= -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x} + (2ben)\,\text{Subst}\left(\int \frac{1}{x^2\left(d+ex^2\right)}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{2ben}{d\sqrt[3]{x}} - \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x} - \frac{\left(2be^2n\right)\text{Subst}\left(\int \frac{1}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{d}$$
$$= -\frac{2ben}{d\sqrt[3]{x}} - \frac{2be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}$$

Mathematica [C] time = 0.02, size = 59, normalized size = 0.87

$$-\frac{a}{x} - \frac{b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x} - \frac{2ben\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{ex^{2/3}}{d}\right)}{d\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x^2,x]
```

```
[Out] -(a/x) - (2*b*e*n*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^(2/3))/d)])/(d*x^(
1/3)) - (b*Log[c*(d + e*x^(2/3))^n])/x
```

fricas [A] time = 0.48, size = 208, normalized size = 3.06

$$\left[\frac{benx\sqrt{-\frac{e}{d}}\,\log\left(\frac{e^3x^2+2\,d^2ex\sqrt{-\frac{e}{d}}-d^3-2\left(de^2x\sqrt{-\frac{e}{d}}-d^2e\right)x^{\frac{2}{3}}-2\left(de^2x+d^3\sqrt{-\frac{e}{d}}\right)x^{\frac{1}{3}}}{e^3x^2+d^3}\right) - bdn\log\left(ex^{\frac{2}{3}}+d\right) - 2\,benx^{\frac{2}{3}} - bd\log(c\right.}{dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^2,x, algorithm="fricas")
```

```
[Out] [(b*e*n*x*sqrt(-e/d)*log((e^3*x^2 + 2*d^2*e*x*sqrt(-e/d) - d^3 - 2*(d*e^2*x
*sqrt(-e/d) - d^2*e)*x^(2/3) - 2*(d*e^2*x + d^3*sqrt(-e/d))*x^(1/3))/(e^3*x
^2 + d^3)) - b*d*n*log(e*x^(2/3) + d) - 2*b*e*n*x^(2/3) - b*d*log(c) - a*d)
/(d*x), -(2*b*e*n*x*sqrt(e/d)*arctan(x^(1/3)*sqrt(e/d)) + b*d*n*log(e*x^(2/
3) + d) + 2*b*e*n*x^(2/3) + b*d*log(c) + a*d)/(d*x)]
```

giac [A] time = 0.28, size = 61, normalized size = 0.90

$$-\left(2\left(\frac{\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{d^{\frac{3}{2}}} + \frac{1}{dx^{\frac{1}{3}}}\right)e + \frac{\log\left(x^{\frac{2}{3}}e+d\right)}{x}\right)bn - \frac{b\log(c)}{x} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^2,x, algorithm="giac")
```

```
[Out] -(2*(arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(1/2)/d^(3/2) + 1/(d*x^(1/3)))*e + l
og(x^(2/3)*e + d)/x)*b*n - b*log(c)/x - a/x
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^{n}\right)+a}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(2/3)+d)^n)+a)/x^2,x)
[Out] int((b*ln(c*(e*x^(2/3)+d)^n)+a)/x^2,x)
```

maxima [A] time = 1.01, size = 59, normalized size = 0.87

$$-2\,ben\left(\frac{e\arctan\left(\frac{ex^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}\,d}+\frac{1}{dx^{\frac{1}{3}}}\right)-\frac{b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)}{x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^2,x, algorithm="maxima")
[Out] -2*b*e*n*(e*arctan(e*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*d) + 1/(d*x^(1/3))) - b*
log((e*x^(2/3) + d)^n*c)/x - a/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^{n}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x**2,x)
[Out] Timed out
```

3.469 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=94

$$-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}-\frac{be^3n\log\left(d+ex^{2/3}\right)}{2d^3}+\frac{be^3n\log(x)}{3d^3}+\frac{be^2n}{2d^2x^{2/3}}-\frac{ben}{4dx^{4/3}}$$

```
[Out] -1/4*b*e*n/d/x^(4/3)+1/2*b*e^2*n/d^2/x^(2/3)-1/2*b*e^3*n*ln(d+e*x^(2/3))/d^
3+1/2*(-a-b*ln(c*(d+e*x^(2/3))^n))/x^2+1/3*b*e^3*n*ln(x)/d^3
```

Rubi [A] time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}+\frac{be^2n}{2d^2x^{2/3}}-\frac{be^3n\log\left(d+ex^{2/3}\right)}{2d^3}+\frac{be^3n\log(x)}{3d^3}-\frac{ben}{4dx^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x^3,x]
```

```
[Out] -(b*e*n)/(4*d*x^(4/3)) + (b*e^2*n)/(2*d^2*x^(2/3)) - (b*e^3*n*Log[d + e*x^(
2/3)])/(2*d^3) - (a + b*Log[c*(d + e*x^(2/3))^n])/(2*x^2) + (b*e^3*n*Log[x]
)/(3*d^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx = \frac{3}{2}\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4}\,dx, x, x^{2/3}\right)$$
$$= -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\,\text{Subst}\left(\int \frac{1}{x^3(d+ex)}\,dx, x, x^{2/3}\right)$$
$$= -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\,\text{Subst}\left(\int \left(\frac{1}{dx^3} - \frac{e}{d^2x^2} + \frac{e^2}{d^3x} - \frac{e^3}{d^3(d+ex}\right.\right.$$
$$= -\frac{ben}{4dx^{4/3}} + \frac{be^2n}{2d^2x^{2/3}} - \frac{be^3n\log\left(d+ex^{2/3}\right)}{2d^3} - \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2} + \frac{be^3n\,\text{lo}}{3d}$$

Mathematica [A] time = 0.03, size = 91, normalized size = 0.97

$$-\frac{a}{2x^2} - \frac{b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2} + \frac{1}{2}ben\left(-\frac{e^2\log\left(d+ex^{2/3}\right)}{d^3} + \frac{2e^2\log(x)}{3d^3} + \frac{e}{d^2x^{2/3}} - \frac{1}{2dx^{4/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*Log[c*(d + e*x^(2/3))^n])/(2*x^2) + (b*e*n*(-1/2*1/(d*x^(4/
3)) + e/(d^2*x^(2/3)) - (e^2*Log[d + e*x^(2/3)])/d^3 + (2*e^2*Log[x])/(3*d^
3)))/2
```

fricas [A] time = 0.46, size = 85, normalized size = 0.90

$$\frac{4\,be^3nx^2\log\left(x^{\frac{1}{3}}\right) + 2\,bde^2nx^{\frac{4}{3}} - bd^2enx^{\frac{2}{3}} - 2\,bd^3\log(c) - 2\,ad^3 - 2\left(be^3nx^2 + bd^3n\right)\log\left(ex^{\frac{2}{3}} + d\right)}{4\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*e^3*n*x^2*log(x^(1/3)) + 2*b*d*e^2*n*x^(4/3) - b*d^2*e*n*x^(2/3) -
 2*b*d^3*log(c) - 2*a*d^3 - 2*(b*e^3*n*x^2 + b*d^3*n)*log(e*x^(2/3) + d))/(
d^3*x^2)
```

giac [A] time = 0.28, size = 95, normalized size = 1.01

$$\frac{1}{4}\left(\left(\left(\frac{2\,\log\left(x^{\frac{2}{3}}e\right)}{d^3} - \frac{2\,\log\left(\left|x^{\frac{2}{3}}e + d\right|\right)}{d^3} + \frac{\left(2\left(x^{\frac{2}{3}}e + d\right)d - 3\,d^2\right)e^{(-2)}}{d^3x^{\frac{4}{3}}}\right)e^4 - \frac{2\,e\log\left(x^{\frac{2}{3}}e + d\right)}{x^2}\right)bne^{(-1)} - \frac{b\log(c)}{2\,x^2} - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/4*((2*log(x^(2/3)*e)/d^3 - 2*log(abs(x^(2/3)*e + d))/d^3 + (2*(x^(2/3)*e
+ d)*d - 3*d^2)*e^(-2)/(d^3*x^(4/3)))*e^4 - 2*e*log(x^(2/3)*e + d)/x^2)*b*n
*e^(-1) - 1/2*b*log(c)/x^2 - 1/2*a/x^2
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{b\ln\left(c\left(e\,x^{\frac{2}{3}} + d\right)^n\right) + a}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(2/3)+d)^n)+a)/x^3,x)
```

```
[Out] int((b*ln(c*(e*x^(2/3)+d)^n)+a)/x^3,x)
```

maxima [A] time = 0.48, size = 77, normalized size = 0.82

$$-\frac{1}{4}\,ben\left(\frac{2\,e^2\log\left(ex^{\frac{2}{3}}+d\right)}{d^3}-\frac{2\,e^2\log\left(x^{\frac{2}{3}}\right)}{d^3}-\frac{2\,ex^{\frac{2}{3}}-d}{d^2x^{\frac{4}{3}}}\right)-\frac{b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*e*n*(2*e^2*log(e*x^(2/3) + d)/d^3 - 2*e^2*log(x^(2/3))/d^3 - (2*e*x^
(2/3) - d)/(d^2*x^(4/3))) - 1/2*b*log((e*x^(2/3) + d)^n*c)/x^2 - 1/2*a/x^2
```

mupad [B] time = 0.61, size = 74, normalized size = 0.79

$$-\frac{\frac{b\,e\,n}{2\,d}-\frac{b\,e^2\,n\,x^{2/3}}{d^2}}{2\,x^{4/3}}-\frac{a}{2\,x^2}-\frac{b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{2\,x^2}-\frac{b\,e^3\,n\,\mathrm{atanh}\left(\frac{2\,e\,x^{2/3}}{d}+1\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))/x^3,x)
```

```
[Out] - ((b*e*n)/(2*d) - (b*e^2*n*x^(2/3))/d^2)/(2*x^(4/3)) - a/(2*x^2) - (b*log(
c*(d + e*x^(2/3))^n))/(2*x^2) - (b*e^3*n*atanh((2*e*x^(2/3))/d + 1))/d^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x**3,x)
```

```
[Out] Timed out
```

3.470 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=123

$$-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}+\frac{2be^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3d^{9/2}}+\frac{2be^4n}{3d^4\sqrt[3]{x}}-\frac{2be^3n}{9d^3x}+\frac{2be^2n}{15d^2x^{5/3}}-\frac{2ben}{21dx^{7/3}}$$

```
[Out] -2/21*b*e*n/d/x^(7/3)+2/15*b*e^2*n/d^2/x^(5/3)-2/9*b*e^3*n/d^3/x+2/3*b*e^4*
n/d^4/x^(1/3)+2/3*b*e^(9/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2))/d^(9/2)+1/3*(
-a-b*ln(c*(d+e*x^(2/3))^n))/x^3
```

Rubi [A] time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2455, 341, 325, 205}

$$-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}+\frac{2be^2n}{15d^2x^{5/3}}+\frac{2be^4n}{3d^4\sqrt[3]{x}}-\frac{2be^3n}{9d^3x}+\frac{2be^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3d^{9/2}}-\frac{2ben}{21dx^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x^4,x]
```

```
[Out] (-2*b*e*n)/(21*d*x^(7/3)) + (2*b*e^2*n)/(15*d^2*x^(5/3)) - (2*b*e^3*n)/(9*d
^3*x) + (2*b*e^4*n)/(3*d^4*x^(1/3)) + (2*b*e^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3
))/Sqrt[d]])/(3*d^(9/2)) - (a + b*Log[c*(d + e*x^(2/3))^n])/(3*x^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx = -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3} + \frac{1}{9}(2ben)\int \frac{1}{\left(d+ex^{2/3}\right)x^{10/3}}\,dx$$

$$= -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3} + \frac{1}{3}(2ben)\,\text{Subst}\left(\int \frac{1}{x^8\left(d+ex^2\right)}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2ben}{21dx^{7/3}} - \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3} - \frac{\left(2be^2n\right)\text{Subst}\left(\int \frac{1}{x^6\left(d+ex^2\right)}\,dx, x, \sqrt[3]{x}\right.}{3d}$$

$$= -\frac{2ben}{21dx^{7/3}} + \frac{2be^2n}{15d^2x^{5/3}} - \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3} + \frac{\left(2be^3n\right)\text{Subst}\left(\int \frac{1}{x^4\left(d+e\right.}\right.}{3d^2}$$

$$= -\frac{2ben}{21dx^{7/3}} + \frac{2be^2n}{15d^2x^{5/3}} - \frac{2be^3n}{9d^3x} - \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3} - \frac{\left(2be^4n\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{2ben}{21dx^{7/3}} + \frac{2be^2n}{15d^2x^{5/3}} - \frac{2be^3n}{9d^3x} + \frac{2be^4n}{3d^4\sqrt[3]{x}} - \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3} + \frac{\left(2be\right.}{}$$

$$= -\frac{2ben}{21dx^{7/3}} + \frac{2be^2n}{15d^2x^{5/3}} - \frac{2be^3n}{9d^3x} + \frac{2be^4n}{3d^4\sqrt[3]{x}} + \frac{2be^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3d^{9/2}} - \frac{a+b\log}{}$$

Mathematica [C] time = 0.01, size = 65, normalized size = 0.53

$$-\frac{a}{3x^3} - \frac{b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3} - \frac{2ben\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};-\frac{ex^{2/3}}{d}\right)}{21dx^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (2*b*e*n*Hypergeometric2F1[-7/2, 1, -5/2, -((e*x^(2/3))/d)])/(
21*d*x^(7/3)) - (b*Log[c*(d + e*x^(2/3))^n])/(3*x^3)
```

fricas [A] time = 0.49, size = 313, normalized size = 2.54

$$\left[\frac{105\,be^4nx^3\sqrt{-\frac{e}{d}}\,\log\left(\frac{e^3x^2-2\,d^2ex\sqrt{-\frac{e}{d}}-d^3+2\left(de^2x\sqrt{-\frac{e}{d}}+d^2e\right)x^{\frac{2}{3}}-2\left(de^2x-d^3\sqrt{-\frac{e}{d}}\right)x^{\frac{1}{3}}}{e^3x^2+d^3}\right)-70\,bde^3nx^2+42\,bd^2e^2nx^{\frac{4}{3}}-105}{315\,d^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^4,x, algorithm="fricas")
```

```
[Out] [1/315*(105*b*e^4*n*x^3*sqrt(-e/d)*log((e^3*x^2 - 2*d^2*e*x*sqrt(-e/d) - d^
3 + 2*(d*e^2*x*sqrt(-e/d) + d^2*e)*x^(2/3) - 2*(d*e^2*x - d^3*sqrt(-e/d))*x
^(1/3))/(e^3*x^2 + d^3)) - 70*b*d*e^3*n*x^2 + 42*b*d^2*e^2*n*x^(4/3) - 105*
b*d^4*n*log(e*x^(2/3) + d) - 105*b*d^4*log(c) - 105*a*d^4 + 30*(7*b*e^4*n*x
^2 - b*d^3*e*n)*x^(2/3))/(d^4*x^3), 1/315*(210*b*e^4*n*x^3*sqrt(e/d)*arctan
(x^(1/3)*sqrt(e/d)) - 70*b*d*e^3*n*x^2 + 42*b*d^2*e^2*n*x^(4/3) - 105*b*d^4
*n*log(e*x^(2/3) + d) - 105*b*d^4*log(c) - 105*a*d^4 + 30*(7*b*e^4*n*x^2 -
b*d^3*e*n)*x^(2/3))/(d^4*x^3)]
```

giac [A] time = 0.33, size = 94, normalized size = 0.76

$$\frac{1}{315}\left(2\left(\frac{105\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{7}{2}}}{d^{\frac{9}{2}}}+\frac{21\,d^2x^{\frac{2}{3}}e-35\,dx^{\frac{4}{3}}e^2-15\,d^3+105\,x^2e^3}{d^4x^{\frac{7}{3}}}\right)e-\frac{105\,\log\left(x^{\frac{2}{3}}e+d\right)}{x^3}\right)bn-\frac{b\log(c)}{3\,x^3}-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^4,x, algorithm="giac")
```

```
[Out] 1/315*(2*(105*arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(7/2)/d^(9/2) + (21*d^2*x^(
2/3)*e - 35*d*x^(4/3)*e^2 - 15*d^3 + 105*x^2*e^3)/(d^4*x^(7/3)))*e - 105*lo
g(x^(2/3)*e + d)/x^3)*b*n - 1/3*b*log(c)/x^3 - 1/3*a/x^3
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int\frac{b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^{n}\right)+a}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(2/3)+d)^n)+a)/x^4,x)
```

```
[Out] int((b*ln(c*(e*x^(2/3)+d)^n)+a)/x^4,x)
```

maxima [A] time = 1.02, size = 94, normalized size = 0.76

$$\frac{2}{315}\,ben\left(\frac{105\,e^4\arctan\left(\frac{ex^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}\,d^4}+\frac{105\,e^3x^2-35\,de^2x^{\frac{4}{3}}+21\,d^2ex^{\frac{2}{3}}-15\,d^3}{d^4x^{\frac{7}{3}}}\right)-\frac{b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)}{3\,x^3}-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^4,x, algorithm="maxima")
```

```
[Out] 2/315*b*e*n*(105*e^4*arctan(e*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*d^4) + (105*e^3
*x^2 - 35*d*e^2*x^(4/3) + 21*d^2*e*x^(2/3) - 15*d^3)/(d^4*x^(7/3))) - 1/3*b
*log((e*x^(2/3) + d)^n*c)/x^3 - 1/3*a/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))/x^4,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x**4,x)
```

```
[Out] Timed out
```

3.471 $\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=482

$$-\frac{bd^6n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^6}+\frac{3bd^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^6}-\frac{15bd^4n\left(d+\right.}{}$$

```
[Out] 15/8*b^2*d^4*n^2*(d+e*x^(2/3))^2/e^6-10/9*b^2*d^3*n^2*(d+e*x^(2/3))^3/e^6+1
5/32*b^2*d^2*n^2*(d+e*x^(2/3))^4/e^6-3/25*b^2*d*n^2*(d+e*x^(2/3))^5/e^6+1/7
2*b^2*n^2*(d+e*x^(2/3))^6/e^6-3*b^2*d^5*n^2*x^(2/3)/e^5+1/4*b^2*d^6*n^2*ln(
d+e*x^(2/3))^2/e^6+3*b*d^5*n*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6-
15/4*b*d^4*n*(d+e*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6+10/3*b*d^3*n*(
d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6-15/8*b*d^2*n*(d+e*x^(2/3))^4
*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6+3/5*b*d*n*(d+e*x^(2/3))^5*(a+b*ln(c*(d+e*x
^(2/3))^n))/e^6-1/12*b*n*(d+e*x^(2/3))^6*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6-1/
2*b*d^6*n*ln(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6+1/4*x^4*(a+b*ln(c
*(d+e*x^(2/3))^n))^2
```

Rubi [A] time = 0.49, antiderivative size = 355, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$\frac{1}{120}bn\left(\frac{360d^5\left(d+ex^{2/3}\right)}{e^6}-\frac{450d^4\left(d+ex^{2/3}\right)^2}{e^6}+\frac{400d^3\left(d+ex^{2/3}\right)^3}{e^6}-\frac{225d^2\left(d+ex^{2/3}\right)^4}{e^6}-\frac{60d^6\log\left(d+ex^{2/3}\right)}{e^6}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (15*b^2*d^4*n^2*(d + e*x^(2/3))^2)/(8*e^6) - (10*b^2*d^3*n^2*(d + e*x^(2/3)
)^3)/(9*e^6) + (15*b^2*d^2*n^2*(d + e*x^(2/3))^4)/(32*e^6) - (3*b^2*d*n^2*(
d + e*x^(2/3))^5)/(25*e^6) + (b^2*n^2*(d + e*x^(2/3))^6)/(72*e^6) - (3*b^2*
d^5*n^2*x^(2/3))/e^5 + (b^2*d^6*n^2*Log[d + e*x^(2/3)]^2)/(4*e^6) + (b*n*((
360*d^5*(d + e*x^(2/3)))/e^6 - (450*d^4*(d + e*x^(2/3))^2)/e^6 + (400*d^3*(
d + e*x^(2/3))^3)/e^6 - (225*d^2*(d + e*x^(2/3))^4)/e^6 + (72*d*(d + e*x^(2
/3))^5)/e^6 - (10*(d + e*x^(2/3))^6)/e^6 - (60*d^6*Log[d + e*x^(2/3)])/e^6)
*(a + b*Log[c*(d + e*x^(2/3))^n]))/120 + (x^4*(a + b*Log[c*(d + e*x^(2/3))^
n])^2)/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2dx=\frac{3}{2}\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^2dx,x,x^{2/3}\right)$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{1}{2}(ben)\text{Subst}\left(\int\frac{x^6\left(a+b\log(c(d\right.}{d+ex}\right.$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{1}{2}(bn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6\left(a+b\right.}{x}\right.$$
$$=\frac{1}{120}bn\left(\frac{360d^5\left(d+ex^{2/3}\right)}{e^6}-\frac{450d^4\left(d+ex^{2/3}\right)^2}{e^6}+\frac{400d^3\left(d+ex^{2/3}\right)^3}{e^6}-\right.$$
$$=\frac{1}{120}bn\left(\frac{360d^5\left(d+ex^{2/3}\right)}{e^6}-\frac{450d^4\left(d+ex^{2/3}\right)^2}{e^6}+\frac{400d^3\left(d+ex^{2/3}\right)^3}{e^6}-\right.$$
$$=\frac{1}{120}bn\left(\frac{360d^5\left(d+ex^{2/3}\right)}{e^6}-\frac{450d^4\left(d+ex^{2/3}\right)^2}{e^6}+\frac{400d^3\left(d+ex^{2/3}\right)^3}{e^6}-\right.$$
$$=\frac{15b^2d^4n^2\left(d+ex^{2/3}\right)^2}{8e^6}-\frac{10b^2d^3n^2\left(d+ex^{2/3}\right)^3}{9e^6}+\frac{15b^2d^2n^2\left(d+ex^{2/3}\right)^4}{32e^6}$$
$$=\frac{15b^2d^4n^2\left(d+ex^{2/3}\right)^2}{8e^6}-\frac{10b^2d^3n^2\left(d+ex^{2/3}\right)^3}{9e^6}+\frac{15b^2d^2n^2\left(d+ex^{2/3}\right)^4}{32e^6}$$

Mathematica [A] time = 0.38, size = 328, normalized size = 0.68

$$ex^{2/3}\left(1800a^2e^5x^{10/3}+60abn\left(60d^5-30d^4ex^{2/3}+20d^3e^2x^{4/3}-15d^2e^3x^2+12de^4x^{8/3}-10e^5x^{10/3}\right)+b^2n^2\left(-882\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (e*x^(2/3)*(1800*a^2*e^5*x^(10/3) + 60*a*b*n*(60*d^5 - 30*d^4*e*x^(2/3) + 2
0*d^3*e^2*x^(4/3) - 15*d^2*e^3*x^2 + 12*d*e^4*x^(8/3) - 10*e^5*x^(10/3)) +
b^2*n^2*(-8820*d^5 + 2610*d^4*e*x^(2/3) - 1140*d^3*e^2*x^(4/3) + 555*d^2*e^
3*x^2 - 264*d*e^4*x^(8/3) + 100*e^5*x^(10/3))) + 5220*b^2*d^6*n^2*Log[d + e
*x^(2/3)] + 60*b*(b*n*(60*d^6 + 60*d^5*e*x^(2/3) - 30*d^4*e^2*x^(4/3) + 20*
d^3*e^3*x^2 - 15*d^2*e^4*x^(8/3) + 12*d*e^5*x^(10/3) - 10*e^6*x^4) - 60*a*(
d^6 - e^6*x^4))*Log[c*(d + e*x^(2/3))^n] - 1800*b^2*(d^6 - e^6*x^4)*Log[c*(
d + e*x^(2/3))^n]^2)/(7200*e^6)
```

fricas [A] time = 0.53, size = 508, normalized size = 1.05

$$1800\,b^2e^6x^4\log(c)^2+100\left(b^2e^6n^2-6\,abe^6n+18\,a^2e^6\right)x^4-60\left(19\,b^2d^3e^3n^2-20\,abd^3e^3n\right)x^2+1800\left(b^2e^6n^2x^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/7200*(1800*b^2*e^6*x^4*log(c)^2 + 100*(b^2*e^6*n^2 - 6*a*b*e^6*n + 18*a^2
*e^6)*x^4 - 60*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x^2 + 1800*(b^2*e^6*
```

```
n^2*x^4 - b^2*d^6*n^2)*log(e*x^(2/3) + d)^2 + 60*(20*b^2*d^3*e^3*n^2*x^2 +
147*b^2*d^6*n^2 - 60*a*b*d^6*n - 10*(b^2*e^6*n^2 - 6*a*b*e^6*n)*x^4 + 60*(b
^2*e^6*n*x^4 - b^2*d^6*n)*log(c) - 15*(b^2*d^2*e^4*n^2*x^2 - 4*b^2*d^5*e*n^
2)*x^(2/3) + 6*(2*b^2*d*e^5*n^2*x^3 - 5*b^2*d^4*e^2*n^2*x)*x^(1/3))*log(e*x
^(2/3) + d) + 600*(2*b^2*d^3*e^3*n*x^2 - (b^2*e^6*n - 6*a*b*e^6)*x^4)*log(c
) - 15*(588*b^2*d^5*e*n^2 - 240*a*b*d^5*e*n - (37*b^2*d^2*e^4*n^2 - 60*a*b*
d^2*e^4*n)*x^2 + 60*(b^2*d^2*e^4*n*x^2 - 4*b^2*d^5*e*n)*log(c))*x^(2/3) - 6
*(4*(11*b^2*d*e^5*n^2 - 30*a*b*d*e^5*n)*x^3 - 15*(29*b^2*d^4*e^2*n^2 - 20*a
*b*d^4*e^2*n)*x - 60*(2*b^2*d*e^5*n*x^3 - 5*b^2*d^4*e^2*n*x)*log(c))*x^(1/3
))/e^6
```

giac [B] time = 0.71, size = 953, normalized size = 1.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4*log(c)^2 + 1/2*a*b*x^4*log(c) + 1/4*a^2*x^4 + 1/7200*(1800*(x^(
2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e + d)^2 - 10800*(x^(2/3)*e + d)^5*d*e^(-5
)*log(x^(2/3)*e + d)^2 + 27000*(x^(2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e +
 d)^2 - 36000*(x^(2/3)*e + d)^3*d^3*e^(-5)*log(x^(2/3)*e + d)^2 + 27000*(x^
(2/3)*e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d)^2 - 10800*(x^(2/3)*e + d)*d^5*
e^(-5)*log(x^(2/3)*e + d)^2 - 600*(x^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e +
d) + 4320*(x^(2/3)*e + d)^5*d*e^(-5)*log(x^(2/3)*e + d) - 13500*(x^(2/3)*e
+ d)^4*d^2*e^(-5)*log(x^(2/3)*e + d) + 24000*(x^(2/3)*e + d)^3*d^3*e^(-5)*l
og(x^(2/3)*e + d) - 27000*(x^(2/3)*e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d) +
 21600*(x^(2/3)*e + d)*d^5*e^(-5)*log(x^(2/3)*e + d) + 100*(x^(2/3)*e + d)^
6*e^(-5) - 864*(x^(2/3)*e + d)^5*d*e^(-5) + 3375*(x^(2/3)*e + d)^4*d^2*e^(-
5) - 8000*(x^(2/3)*e + d)^3*d^3*e^(-5) + 13500*(x^(2/3)*e + d)^2*d^4*e^(-5)
 - 21600*(x^(2/3)*e + d)*d^5*e^(-5))*b^2*n^2*e^(-1) + 1/120*(60*(x^(2/3)*e
+ d)^6*e^(-5)*log(x^(2/3)*e + d) - 360*(x^(2/3)*e + d)^5*d*e^(-5)*log(x^(2/
3)*e + d) + 900*(x^(2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e + d) - 1200*(x^(
2/3)*e + d)^3*d^3*e^(-5)*log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^2*d^4*e^(
-5)*log(x^(2/3)*e + d) - 360*(x^(2/3)*e + d)*d^5*e^(-5)*log(x^(2/3)*e + d)
- 10*(x^(2/3)*e + d)^6*e^(-5) + 72*(x^(2/3)*e + d)^5*d*e^(-5) - 225*(x^(2/3
)*e + d)^4*d^2*e^(-5) + 400*(x^(2/3)*e + d)^3*d^3*e^(-5) - 450*(x^(2/3)*e +
 d)^2*d^4*e^(-5) + 360*(x^(2/3)*e + d)*d^5*e^(-5))*b^2*n*e^(-1)*log(c) + 1/
120*(60*(x^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e + d) - 360*(x^(2/3)*e + d)^5
*d*e^(-5)*log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)
*e + d) - 1200*(x^(2/3)*e + d)^3*d^3*e^(-5)*log(x^(2/3)*e + d) + 900*(x^(2/
3)*e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d) - 360*(x^(2/3)*e + d)*d^5*e^(-5)*
log(x^(2/3)*e + d) - 10*(x^(2/3)*e + d)^6*e^(-5) + 72*(x^(2/3)*e + d)^5*d*e
^(-5) - 225*(x^(2/3)*e + d)^4*d^2*e^(-5) + 400*(x^(2/3)*e + d)^3*d^3*e^(-5)
 - 450*(x^(2/3)*e + d)^2*d^4*e^(-5) + 360*(x^(2/3)*e + d)*d^5*e^(-5))*a*b*n
*e^(-1)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \left(b \ln \left(c \left(e \,x^{\frac{2}{3}}+d \right)^{n}\right)+a \right)^{2} x^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*ln(c*(e*x^(2/3)+d)^n)+a)^2,x)
```

```
[Out] int(x^3*(b*ln(c*(e*x^(2/3)+d)^n)+a)^2,x)
```

maxima [A] time = 0.52, size = 330, normalized size = 0.68

$$\frac{1}{4}b^2x^4\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+\frac{1}{2}abx^4\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+\frac{1}{4}a^2x^4-\frac{1}{120}aben\left(\frac{60\,d^6\log\left(ex^{\frac{2}{3}}+d\right)}{e^7}+\frac{10\,e^5x^4-12\,de\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*log((e*x^(2/3) + d)^n*c)^2 + 1/2*a*b*x^4*log((e*x^(2/3) + d)^n*
c) + 1/4*a^2*x^4 - 1/120*a*b*e*n*(60*d^6*log(e*x^(2/3) + d)/e^7 + (10*e^5*x
^4 - 12*d*e^4*x^(10/3) + 15*d^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 + 30*d^4*e*x^(
4/3) - 60*d^5*x^(2/3))/e^6) - 1/7200*(60*e*n*(60*d^6*log(e*x^(2/3) + d)/e^7
 + (10*e^5*x^4 - 12*d*e^4*x^(10/3) + 15*d^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 +
30*d^4*e*x^(4/3) - 60*d^5*x^(2/3))/e^6)*log((e*x^(2/3) + d)^n*c) - (100*e^6
*x^4 - 264*d*e^5*x^(10/3) + 555*d^2*e^4*x^(8/3) - 1140*d^3*e^3*x^2 + 1800*d
^6*log(e*x^(2/3) + d)^2 + 2610*d^4*e^2*x^(4/3) + 8820*d^6*log(e*x^(2/3) + d
) - 8820*d^5*e*x^(2/3))*n^2/e^6)*b^2
```

mupad [B] time = 1.75, size = 440, normalized size = 0.91

$$\frac{a^2\,x^4}{4}+\frac{b^2\,x^4\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)^2}{4}+\frac{b^2\,n^2\,x^4}{72}+\frac{a\,b\,x^4\,\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)}{2}-\frac{b^2\,d^6\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)^2}{4\,e^6}-\frac{a\,b\,n\,x^4}{12}-\frac{b^2\,n\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(2/3))^n))^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*x^4*log(c*(d + e*x^(2/3))^n)^2)/4 + (b^2*n^2*x^4)/72 + (
a*b*x^4*log(c*(d + e*x^(2/3))^n))/2 - (b^2*d^6*log(c*(d + e*x^(2/3))^n)^2)/
(4*e^6) - (a*b*n*x^4)/12 - (b^2*n*x^4*log(c*(d + e*x^(2/3))^n))/12 + (49*b^
2*d^6*n^2*log(d + e*x^(2/3)))/(40*e^6) - (19*b^2*d^3*n^2*x^2)/(120*e^3) + (
37*b^2*d^2*n^2*x^(8/3))/(480*e^2) + (29*b^2*d^4*n^2*x^(4/3))/(80*e^4) - (49
*b^2*d^5*n^2*x^(2/3))/(40*e^5) - (11*b^2*d*n^2*x^(10/3))/(300*e) + (b^2*d^3
*n*x^2*log(c*(d + e*x^(2/3))^n))/(6*e^3) - (b^2*d^2*n*x^(8/3)*log(c*(d + e*
x^(2/3))^n))/(8*e^2) - (b^2*d^4*n*x^(4/3)*log(c*(d + e*x^(2/3))^n))/(4*e^4)
 + (b^2*d^5*n*x^(2/3)*log(c*(d + e*x^(2/3))^n))/(2*e^5) + (a*b*d*n*x^(10/3)
)/(10*e) - (a*b*d^6*n*log(d + e*x^(2/3)))/(2*e^6) + (b^2*d*n*x^(10/3)*log(c
*(d + e*x^(2/3))^n))/(10*e) + (a*b*d^3*n*x^2)/(6*e^3) - (a*b*d^2*n*x^(8/3))
/(8*e^2) - (a*b*d^4*n*x^(4/3))/(4*e^4) + (a*b*d^5*n*x^(2/3))/(2*e^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
```

```
[Out] Timed out
```

3.472 $\int x \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right)^2 \, dx$

Optimal. Leaf size=275

$$\frac{bd^3 n \log \left(d+ex^{2/3}\right) \left(a+b \log \left(c \left(d+ex^{2/3}\right)^n\right)\right)}{e^3}-\frac{3bd^2 n \left(d+ex^{2/3}\right) \left(a+b \log \left(c \left(d+ex^{2/3}\right)^n\right)\right)}{e^3}+\frac{3bdn \left(d+ex^{2/3}\right)^2}{}$$

```
[Out] -3/4*b^2*d*n^2*(d+e*x^(2/3))^2/e^3+1/9*b^2*n^2*(d+e*x^(2/3))^3/e^3+3*b^2*d^
2*n^2*x^(2/3)/e^2-1/2*b^2*d^3*n^2*ln(d+e*x^(2/3))^2/e^3-3*b*d^2*n*(d+e*x^(2
/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3+3/2*b*d*n*(d+e*x^(2/3))^2*(a+b*ln(c*(d
+e*x^(2/3))^n))/e^3-1/3*b*n*(d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3
+b*d^3*n*ln(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3+1/2*x^2*(a+b*ln(c*
(d+e*x^(2/3))^n))^2
```

Rubi [A] time = 0.31, antiderivative size = 217, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$-\frac{1}{6}bn\left(\frac{18d^2\left(d+ex^{2/3}\right)}{e^3}-\frac{6d^3\log\left(d+ex^{2/3}\right)}{e^3}-\frac{9d\left(d+ex^{2/3}\right)^2}{e^3}+\frac{2\left(d+ex^{2/3}\right)^3}{e^3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{1}{2}x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (-3*b^2*d*n^2*(d + e*x^(2/3))^2)/(4*e^3) + (b^2*n^2*(d + e*x^(2/3))^3)/(9*e
^3) + (3*b^2*d^2*n^2*x^(2/3))/e^2 - (b^2*d^3*n^2*Log[d + e*x^(2/3)]^2)/(2*e
^3) - (b*n*((18*d^2*(d + e*x^(2/3)))/e^3 - (9*d*(d + e*x^(2/3))^2)/e^3 + (2
*(d + e*x^(2/3))^3)/e^3 - (6*d^3*Log[d + e*x^(2/3)])/e^3)*(a + b*Log[c*(d +
 e*x^(2/3))^n]))/6 + (x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
```

```
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx = \frac{3}{2}\,\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, x^{2/3}\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 - (ben)\,\text{Subst}\left(\int \frac{x^3\left(a+b\log(c(d+\right.}{d+ex}\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 - (bn)\,\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3\left(a+b\log\right.}{x}\right.$$

$$= -\frac{1}{6}bn\left(\frac{18d^2\left(d+ex^{2/3}\right)}{e^3} - \frac{9d\left(d+ex^{2/3}\right)^2}{e^3} + \frac{2\left(d+ex^{2/3}\right)^3}{e^3} - \frac{6d^3\log\left(d\right.}{e^3}\right.$$

$$= -\frac{1}{6}bn\left(\frac{18d^2\left(d+ex^{2/3}\right)}{e^3} - \frac{9d\left(d+ex^{2/3}\right)^2}{e^3} + \frac{2\left(d+ex^{2/3}\right)^3}{e^3} - \frac{6d^3\log\left(d\right.}{e^3}\right.$$

$$= -\frac{1}{6}bn\left(\frac{18d^2\left(d+ex^{2/3}\right)}{e^3} - \frac{9d\left(d+ex^{2/3}\right)^2}{e^3} + \frac{2\left(d+ex^{2/3}\right)^3}{e^3} - \frac{6d^3\log\left(d\right.}{e^3}\right.$$

$$= -\frac{3b^2dn^2\left(d+ex^{2/3}\right)^2}{4e^3} + \frac{b^2n^2\left(d+ex^{2/3}\right)^3}{9e^3} + \frac{3b^2d^2n^2x^{2/3}}{e^2} - \frac{1}{6}bn\left(\frac{18d^2\left(\right.}{}\right.$$

$$= -\frac{3b^2dn^2\left(d+ex^{2/3}\right)^2}{4e^3} + \frac{b^2n^2\left(d+ex^{2/3}\right)^3}{9e^3} + \frac{3b^2d^2n^2x^{2/3}}{e^2} - \frac{b^2d^3n^2\log^2}{2e}$$

Mathematica [A] time = 0.16, size = 239, normalized size = 0.87

$$\frac{18a^2d^3+18a^2e^3x^2+6b\left(6a\left(d^3+e^3x^2\right)-bn\left(6d^3+6d^2ex^{2/3}-3de^2x^{4/3}+2e^3x^2\right)\right)\log\left(c\left(d+ex^{2/3}\right)^n\right)-36abd^2en}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (18*a^2*d^3 - 36*a*b*d^2*e*n*x^(2/3) + 66*b^2*d^2*e*n^2*x^(2/3) + 18*a*b*d*
e^2*n*x^(4/3) - 15*b^2*d*e^2*n^2*x^(4/3) + 18*a^2*e^3*x^2 - 12*a*b*e^3*n*x^
2 + 4*b^2*e^3*n^2*x^2 - 30*b^2*d^3*n^2*Log[d + e*x^(2/3)] + 6*b*(6*a*(d^3 +
 e^3*x^2) - b*n*(6*d^3 + 6*d^2*e*x^(2/3) - 3*d*e^2*x^(4/3) + 2*e^3*x^2))*Lo
g[c*(d + e*x^(2/3))^n] + 18*b^2*(d^3 + e^3*x^2)*Log[c*(d + e*x^(2/3))^n]^2)
/(36*e^3)
```

fricas [A] time = 0.47, size = 304, normalized size = 1.11

$$\frac{18\,b^2e^3x^2\log(c)^2-12\left(b^2e^3n-3\,abe^3\right)x^2\log(c)+2\left(2\,b^2e^3n^2-6\,abe^3n+9\,a^2e^3\right)x^2+18\left(b^2e^3n^2x^2+b^2d^3n^2\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/36*(18*b^2*e^3*x^2*log(c)^2 - 12*(b^2*e^3*n - 3*a*b*e^3)*x^2*log(c) + 2*(
2*b^2*e^3*n^2 - 6*a*b*e^3*n + 9*a^2*e^3)*x^2 + 18*(b^2*e^3*n^2*x^2 + b^2*d^
3*n^2)*log(e*x^(2/3) + d)^2 + 6*(3*b^2*d*e^2*n^2*x^(4/3) - 6*b^2*d^2*e*n^2*
x^(2/3) - 11*b^2*d^3*n^2 + 6*a*b*d^3*n - 2*(b^2*e^3*n^2 - 3*a*b*e^3*n)*x^2
+ 6*(b^2*e^3*n*x^2 + b^2*d^3*n)*log(c))*log(e*x^(2/3) + d) + 6*(11*b^2*d^2*
e*n^2 - 6*b^2*d^2*e*n*log(c) - 6*a*b*d^2*e*n)*x^(2/3) + 3*(6*b^2*d*e^2*n*x*
log(c) - (5*b^2*d*e^2*n^2 - 6*a*b*d*e^2*n)*x)*x^(1/3))/e^3
```

giac [A] time = 0.95, size = 316, normalized size = 1.15

$$\frac{1}{2}\,b^2x^2\log(c)^2+\frac{1}{36}\left(18\,x^2\log\left(x^{\frac{2}{3}}e+d\right)^2+\left(18\,d^3\log\left(x^{\frac{2}{3}}e+d\right)^2-12\left(x^{\frac{2}{3}}e+d\right)^3\log\left(x^{\frac{2}{3}}e+d\right)+54\left(x^{\frac{2}{3}}e+d\right)^2d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2*log(c)^2 + 1/36*(18*x^2*log(x^(2/3)*e + d)^2 + (18*d^3*log(x^(2
/3)*e + d)^2 - 12*(x^(2/3)*e + d)^3*log(x^(2/3)*e + d) + 54*(x^(2/3)*e + d)
^2*d*log(x^(2/3)*e + d) - 108*(x^(2/3)*e + d)*d^2*log(x^(2/3)*e + d) + 4*(x
^(2/3)*e + d)^3 - 27*(x^(2/3)*e + d)^2*d + 108*(x^(2/3)*e + d)*d^2)*e^(-3))
*b^2*n^2 + 1/6*(6*x^2*log(x^(2/3)*e + d) + (6*d^3*e^(-4)*log(abs(x^(2/3)*e
+ d)) + (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*e)*b^2*n*log(c)
 + a*b*x^2*log(c) + 1/6*(6*x^2*log(x^(2/3)*e + d) + (6*d^3*e^(-4)*log(abs(x
^(2/3)*e + d)) + (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*e)*a*b
*n + 1/2*a^2*x^2
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^{n}\right)+a\right)^{2}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(e*x^(2/3)+d)^n)+a)^2,x)
```

```
[Out] int(x*(b*ln(c*(e*x^(2/3)+d)^n)+a)^2,x)
```

maxima [A] time = 0.51, size = 231, normalized size = 0.84

$$\frac{1}{2}\,b^2x^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+\frac{1}{6}\,aben\left(\frac{6\,d^3\log\left(ex^{\frac{2}{3}}+d\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)+abx^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log((e*x^(2/3) + d)^n*c)^2 + 1/6*a*b*e*n*(6*d^3*log(e*x^(2/3) +
 d)/e^4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3) + a*b*x^2*log((e
*x^(2/3) + d)^n*c) + 1/2*a^2*x^2 + 1/36*(6*e*n*(6*d^3*log(e*x^(2/3) + d)/e^
4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3)*log((e*x^(2/3) + d)^n*
c) + (4*e^3*x^2 - 18*d^3*log(e*x^(2/3) + d)^2 - 15*d*e^2*x^(4/3) - 66*d^3*l
og(e*x^(2/3) + d) + 66*d^2*e*x^(2/3))*n^2/e^3)*b^2
```

mupad [B] time = 0.53, size = 299, normalized size = 1.09

$$\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)^2\left(\frac{b^2\,x^2}{2}+\frac{b^2\,d^3}{2\,e^3}\right)-x^{4/3}\left(\frac{d\left(\frac{3\,a^2}{2}-a\,b\,n+\frac{b^2\,n^2}{3}\right)}{2\,e}-\frac{d\left(3\,a^2-b^2\,n^2\right)}{4\,e}\right)+x^2\left(\frac{a^2}{2}-\frac{a\,b\,n}{3}+\frac{b^2\,n^2}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(2/3))^n))^2,x)
```

```
[Out] log(c*(d + e*x^(2/3))^n)^2*((b^2*x^2)/2 + (b^2*d^3)/(2*e^3)) - x^(4/3)*((d*
((3*a^2)/2 + (b^2*n^2)/3 - a*b*n))/(2*e) - (d*(3*a^2 - b^2*n^2))/(4*e)) + x
^2*(a^2/2 + (b^2*n^2)/9 - (a*b*n)/3) + log(c*(d + e*x^(2/3))^n)*((b*x^2*(3*
a - b*n))/3 - x^(4/3)*((b*d*(3*a - b*n))/(2*e) - (3*a*b*d)/(2*e)) + (d*x^(2
/3)*((b*d*(3*a - b*n))/e - (3*a*b*d)/e))/e) + x^(2/3)*((d*((d*((3*a^2)/2 +
(b^2*n^2)/3 - a*b*n))/e - (d*(3*a^2 - b^2*n^2))/(2*e)))/e + (b^2*d^2*n^2)/e
^2) - (log(d + e*x^(2/3))*(11*b^2*d^3*n^2 - 6*a*b*d^3*n))/(6*e^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
```

```
[Out] Timed out
```

3.473 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x} dx$

Optimal. Leaf size=95

$$3bn\text{Li}_2\left(\frac{x^{2/3}e}{d}+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{3}{2}\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-3b^2n^2\text{Li}_3\left(\frac{x^{2/3}e}{d}+1\right.$$

```
[Out] 3/2*(a+b*ln(c*(d+e*x^(2/3))^n))^2*ln(-e*x^(2/3)/d)+3*b*n*(a+b*ln(c*(d+e*x^(
2/3))^n))*polylog(2,1+e*x^(2/3)/d)-3*b^2*n^2*polylog(3,1+e*x^(2/3)/d)
```

Rubi [A] time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2454, 2396, 2433, 2374, 6589}

$$3bn\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-3b^2n^2\text{PolyLog}\left(3,\frac{ex^{2/3}}{d}+1\right)+\frac{3}{2}\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x,x]
```

```
[Out] (3*(a + b*Log[c*(d + e*x^(2/3))^n])^2*Log[-((e*x^(2/3))/d)])/2 + 3*b*n*(a +
 b*Log[c*(d + e*x^(2/3))^n])*PolyLog[2, 1 + (e*x^(2/3))/d] - 3*b^2*n^2*Poly
Log[3, 1 + (e*x^(2/3))/d]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\,dx = \frac{3}{2}\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x}\,dx, x, x^{2/3}\right)$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log\left(-\frac{ex^{2/3}}{d}\right) - (3ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{\ldots}\right.$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log\left(-\frac{ex^{2/3}}{d}\right) - (3bn)\,\text{Subst}\left(\int \frac{(a+b\log}{\ldots}\right.$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log\left(-\frac{ex^{2/3}}{d}\right) + 3bn\left(a+b\log\left(c\left(d+ex^{2/3}\ldots\right.\right.\right.$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log\left(-\frac{ex^{2/3}}{d}\right) + 3bn\left(a+b\log\left(c\left(d+ex^{2/3}\ldots\right.\right.\right.$$

Mathematica [B] time = 0.12, size = 199, normalized size = 2.09

$$2bn\left(\log(x)\left(\log\left(d+ex^{2/3}\right)-\log\left(\frac{ex^{2/3}}{d}+1\right)\right)-\frac{3}{2}\text{Li}_2\left(-\frac{ex^{2/3}}{d}\right)\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)-bn\log\left(d+ex^{2/3}\ldots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*((Log[d + e*x^(2/
3)] - Log[1 + (e*x^(2/3))/d])*Log[x] - (3*PolyLog[2, -((e*x^(2/3))/d)])/2)
+ (3*b^2*n^2*(Log[d + e*x^(2/3)]^2*Log[-((e*x^(2/3))/d)] + 2*Log[d + e*x^(2
/3)]*PolyLog[2, 1 + (e*x^(2/3))/d] - 2*PolyLog[3, 1 + (e*x^(2/3))/d]))/2
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+2\,ab\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
 a^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^{n}\right)+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^2/x,x)
```

```
[Out] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^2/x,x)
```

maxima [A] time = 0.79, size = 148, normalized size = 1.56

$$\frac{3}{2}\left(\log\left(ex^{\frac{2}{3}}+d\right)^{2}\log\left(-\frac{ex^{\frac{2}{3}}+d}{d}+1\right)+2\,\mathrm{Li}_2\left(\frac{ex^{\frac{2}{3}}+d}{d}\right)\log\left(ex^{\frac{2}{3}}+d\right)-2\,\mathrm{Li}_3\left(\frac{ex^{\frac{2}{3}}+d}{d}\right)\right)b^2n^2+a^2\log(x)+3\left(b^2n\,\mathrm{l}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x,x, algorithm="maxima")
```

```
[Out] 3/2*(log(e*x^(2/3) + d)^2*log(-(e*x^(2/3) + d)/d + 1) + 2*dilog((e*x^(2/3)
+ d)/d)*log(e*x^(2/3) + d) - 2*polylog(3, (e*x^(2/3) + d)/d))*b^2*n^2 + a^2
*log(x) + 3*(b^2*n*log(c) + a*b*n)*(log(e*x^(2/3) + d)*log(-(e*x^(2/3) + d)
/d + 1) + dilog((e*x^(2/3) + d)/d)) + (b^2*log(c)^2 + 2*a*b*log(c))*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x,x)
```

```
[Out] Timed out
```

3.474 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=238

$$\frac{be^3n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3}+\frac{be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3x^{2/3}}-\frac{ben\left(a+b\log\right.}{2d}$$

```
[Out] -1/2*b^2*e^2*n^2/d^2/x^(2/3)+1/2*b^2*e^3*n^2*ln(d+e*x^(2/3))/d^3-1/2*b*e*n*
(a+b*ln(c*(d+e*x^(2/3))^n))/d/x^(4/3)+b*e^2*n*(d+e*x^(2/3))*(a+b*ln(c*(d+e*
x^(2/3))^n))/d^3/x^(2/3)+b*e^3*n*ln(1-d/(d+e*x^(2/3)))*(a+b*ln(c*(d+e*x^(2/
3))^n))/d^3-1/2*(a+b*ln(c*(d+e*x^(2/3))^n))^2/x^2-b^2*e^3*n^2*ln(x)/d^3-b^2
*e^3*n^2*polylog(2,d/(d+e*x^(2/3)))/d^3
```

Rubi [A] time = 0.50, antiderivative size = 261, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$\frac{b^2e^3n^2\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)}{d^3}-\frac{e^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2d^3}+\frac{be^3n\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^3,x]
```

```
[Out] -(b^2*e^2*n^2)/(2*d^2*x^(2/3)) + (b^2*e^3*n^2*Log[d + e*x^(2/3)])/(2*d^3) -
 (b*e*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*d*x^(4/3)) + (b*e^2*n*(d + e*x
^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n]))/(d^3*x^(2/3)) - (e^3*(a + b*Log[c
*(d + e*x^(2/3))^n])^2)/(2*d^3) - (a + b*Log[c*(d + e*x^(2/3))^n])^2/(2*x^2
) + (b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)])/d^3 -
(b^2*e^3*n^2*Log[x])/d^3 + (b^2*e^3*n^2*PolyLog[2, 1 + (e*x^(2/3))/d])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}dx = \frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4}dx, x, x^{2/3}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2} + (ben)\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^3(d+ex)}dx\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2} + (bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}dx, x, d+\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2} + \frac{(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}dx, x, d+ex^{2/3}\right.}{d}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2} - \frac{(ben)}{}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}} + \frac{be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex\right.\right.\right.}{d^3x^{2/3}}$$

$$= -\frac{b^2e^2n^2}{2d^2x^{2/3}} + \frac{b^2e^3n^2\log\left(d+ex^{2/3}\right)}{2d^3} - \frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}} + \frac{b\ldots}{}$$

$$= -\frac{b^2e^2n^2}{2d^2x^{2/3}} + \frac{b^2e^3n^2\log\left(d+ex^{2/3}\right)}{2d^3} - \frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}} + \frac{b\ldots}{}$$

Mathematica [A] time = 0.31, size = 264, normalized size = 1.11

$$-\frac{\frac{ex^{2/3}\left(3bd^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-6be^2nx^{4/3}\left(\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+bn\text{Li}_2\left(\frac{x^{2/3}e}{d}+1\right)\right)+3e^2x^{4/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-6bdenx\ldots}{d^3}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^3,x]
```

```
[Out] -1/6*(3*(a + b*Log[c*(d + e*x^(2/3))^n])^2 + (e*x^(2/3)*(3*b*d^2*n*(a + b*L
og[c*(d + e*x^(2/3))^n]) - 6*b*d*e*n*x^(2/3)*(a + b*Log[c*(d + e*x^(2/3))^n
]) + 3*e^2*x^(4/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2 - 2*b^2*e^2*n^2*x^(4/
3)*(3*Log[d + e*x^(2/3)] - 2*Log[x]) + b^2*e*n^2*x^(2/3)*(3*d - 3*e*x^(2/3)
*Log[d + e*x^(2/3)] + 2*e*x^(2/3)*Log[x]) - 6*b*e^2*n*x^(4/3)*((a + b*Log[c
*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)] + b*n*PolyLog[2, 1 + (e*x^(2/3))
/d])))/d^3)/x^2
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2 + 2\,ab\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right) + a^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^3,x, algorithm="fricas")

[Out] integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) + a^2)/x^3, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(\left(ex^{\frac{2}{3}} + d\right)^{n} c\right) + a\right)^{2}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^3,x, algorithm="giac")

[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^3, x)

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c \left(e x^{\frac{2}{3}} + d\right)^{n}\right) + a\right)^{2}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^2/x^3,x)

[Out] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^2/x^3,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^{2} n^{2} \log\left(ex^{\frac{2}{3}} + d\right)^{2}}{2\,x^{2}} + \int \frac{2\left(b^{2} enx + 3\left(b^{2} e \log(c) + abe\right)x + 3\left(b^{2} d \log(c) + abd\right)x^{\frac{1}{3}}\right)n \log\left(ex^{\frac{2}{3}} + d\right) + 3\left(b^{2} e \log\right.}{3\left(ex^{4} + dx^{\frac{10}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^3,x, algorithm="maxima")

[Out] -1/2*b^2*n^2*log(e*x^(2/3) + d)^2/x^2 + integrate(1/3*(2*(b^2*e*n*x + 3*(b^2*e*log(c) + a*b*e)*x + 3*(b^2*d*log(c) + a*b*d)*x^(1/3))*n*log(e*x^(2/3) + d) + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x + 3*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x^(1/3))/(e*x^4 + d*x^(10/3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b\,\ln\left(c\,\left(d + e\,x^{2/3}\right)^{n}\right)\right)^{2}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^3,x)

[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**3,x)

[Out] Timed out

3.475 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx$

Optimal. Leaf size=412

$$-\frac{be^6n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^6}-\frac{be^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^6x^{2/3}}+\frac{be^4n\left(a+b\log\right.}{4}$$

```
[Out] -1/40*b^2*e^2*n^2/d^2/x^(8/3)+3/40*b^2*e^3*n^2/d^3/x^2-47/240*b^2*e^4*n^2/d
^4/x^(4/3)+77/120*b^2*e^5*n^2/d^5/x^(2/3)-77/120*b^2*e^6*n^2*ln(d+e*x^(2/3)
)/d^6-1/10*b*e*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d/x^(10/3)+1/8*b*e^2*n*(a+b*ln
(c*(d+e*x^(2/3))^n))/d^2/x^(8/3)-1/6*b*e^3*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^
3/x^2+1/4*b*e^4*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^4/x^(4/3)-1/2*b*e^5*n*(d+e*
x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^6/x^(2/3)-1/2*b*e^6*n*ln(1-d/(d+e*x^
(2/3)))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^6-1/4*(a+b*ln(c*(d+e*x^(2/3))^n))^2/x
^4+137/180*b^2*e^6*n^2*ln(x)/d^6+1/2*b^2*e^6*n^2*polylog(2,d/(d+e*x^(2/3)))
/d^6
```

Rubi [A] time = 1.02, antiderivative size = 436, normalized size of antiderivative = 1.06, number of steps used = 26, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$-\frac{b^2e^6n^2\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)}{2d^6}+\frac{e^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4d^6}-\frac{be^6n\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^6}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^5,x]
```

```
[Out] -(b^2*e^2*n^2)/(40*d^2*x^(8/3)) + (3*b^2*e^3*n^2)/(40*d^3*x^2) - (47*b^2*e^
4*n^2)/(240*d^4*x^(4/3)) + (77*b^2*e^5*n^2)/(120*d^5*x^(2/3)) - (77*b^2*e^6
*n^2*Log[d + e*x^(2/3)])/(120*d^6) - (b*e*n*(a + b*Log[c*(d + e*x^(2/3))^n]
))/(10*d*x^(10/3)) + (b*e^2*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(8*d^2*x^(8
/3)) - (b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(6*d^3*x^2) + (b*e^4*n*(a
 + b*Log[c*(d + e*x^(2/3))^n]))/(4*d^4*x^(4/3)) - (b*e^5*n*(d + e*x^(2/3))*
(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*d^6*x^(2/3)) + (e^6*(a + b*Log[c*(d +
e*x^(2/3))^n])^2)/(4*d^6) - (a + b*Log[c*(d + e*x^(2/3))^n])^2/(4*x^4) - (b
*e^6*n*(a + b*Log[c*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)])/(2*d^6) + (1
37*b^2*e^6*n^2*Log[x])/(180*d^6) - (b^2*e^6*n^2*PolyLog[2, 1 + (e*x^(2/3))/
d])/(2*d^6)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx = \frac{3}{2}\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^7}\,dx, x, x^{2/3}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} + \frac{1}{2}(ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^6(d+ex)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} + \frac{1}{2}(bn)\,\text{Subst}\left(\int \frac{a+b\log\left(cx^n\right)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} + \frac{(bn)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+ex^{2/3}\right.}{2d}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{10dx^{10/3}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} - \frac{(ben)}{}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{10dx^{10/3}} + \frac{be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8d^2x^{8/3}} - $$

$$= -\frac{b^2e^2n^2}{40d^2x^{8/3}} + \frac{b^2e^3n^2}{30d^3x^2} - \frac{b^2e^4n^2}{20d^4x^{4/3}} + \frac{b^2e^5n^2}{10d^5x^{2/3}} - \frac{b^2e^6n^2\log\left(d+ex^{2/3}\right)}{10d^6} - $$

$$= -\frac{b^2e^2n^2}{40d^2x^{8/3}} + \frac{3b^2e^3n^2}{40d^3x^2} - \frac{9b^2e^4n^2}{80d^4x^{4/3}} + \frac{9b^2e^5n^2}{40d^5x^{2/3}} - \frac{9b^2e^6n^2\log\left(d+ex^{2/3}\right)}{40d^6} - $$

$$= -\frac{b^2e^2n^2}{40d^2x^{8/3}} + \frac{3b^2e^3n^2}{40d^3x^2} - \frac{47b^2e^4n^2}{240d^4x^{4/3}} + \frac{47b^2e^5n^2}{120d^5x^{2/3}} - \frac{47b^2e^6n^2\log\left(d+ex^{2/3}\right.}{120d^6}$$

$$= -\frac{b^2e^2n^2}{40d^2x^{8/3}} + \frac{3b^2e^3n^2}{40d^3x^2} - \frac{47b^2e^4n^2}{240d^4x^{4/3}} + \frac{77b^2e^5n^2}{120d^5x^{2/3}} - \frac{77b^2e^6n^2\log\left(d+ex^{2/3}\right.}{120d^6}$$

$$= -\frac{b^2e^2n^2}{40d^2x^{8/3}} + \frac{3b^2e^3n^2}{40d^3x^2} - \frac{47b^2e^4n^2}{240d^4x^{4/3}} + \frac{77b^2e^5n^2}{120d^5x^{2/3}} - \frac{77b^2e^6n^2\log\left(d+ex^{2/3}\right.}{120d^6}$$

Mathematica [A] time = 0.35, size = 539, normalized size = 1.31

$$-\frac{180a^2d^6 + 360abd^6\log\left(c\left(d+ex^{2/3}\right)^n\right) - 360abe^6x^4\log\left(c\left(d+ex^{2/3}\right)^n\right) + 72abd^5enx^{2/3} - 90abd^4e^2nx^{4/3} + 1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^5,x]
```

```
[Out] -1/720*(180*a^2*d^6 + 72*a*b*d^5*e*n*x^(2/3) - 90*a*b*d^4*e^2*n*x^(4/3) + 1
8*b^2*d^4*e^2*n^2*x^(4/3) + 120*a*b*d^3*e^3*n*x^2 - 54*b^2*d^3*e^3*n^2*x^2
- 180*a*b*d^2*e^4*n*x^(8/3) + 141*b^2*d^2*e^4*n^2*x^(8/3) + 360*a*b*d*e^5*n
*x^(10/3) - 462*b^2*d*e^5*n^2*x^(10/3) + 822*b^2*e^6*n^2*x^4*Log[d + e*x^(2
/3)] + 360*a*b*d^6*Log[c*(d + e*x^(2/3))^n] + 72*b^2*d^5*e*n*x^(2/3)*Log[c*
(d + e*x^(2/3))^n] - 90*b^2*d^4*e^2*n*x^(4/3)*Log[c*(d + e*x^(2/3))^n] + 12
0*b^2*d^3*e^3*n*x^2*Log[c*(d + e*x^(2/3))^n] - 180*b^2*d^2*e^4*n*x^(8/3)*Lo
g[c*(d + e*x^(2/3))^n] + 360*b^2*d*e^5*n*x^(10/3)*Log[c*(d + e*x^(2/3))^n]
- 360*a*b*e^6*x^4*Log[c*(d + e*x^(2/3))^n] + 180*b^2*d^6*Log[c*(d + e*x^(2/
3))^n]^2 - 180*b^2*e^6*x^4*Log[c*(d + e*x^(2/3))^n]^2 + 360*a*b*e^6*n*x^4*L
og[-((e*x^(2/3))/d)] + 360*b^2*e^6*n*x^4*Log[c*(d + e*x^(2/3))^n]*Log[-((e*
x^(2/3))/d)] - 548*b^2*e^6*n^2*x^4*Log[x] + 360*b^2*e^6*n^2*x^4*PolyLog[2,
1 + (e*x^(2/3))/d])/(d^6*x^4)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \log\left(\left(ex^{\frac{2}{3}} + d\right)^n c\right)^2 + 2\,ab\log\left(\left(ex^{\frac{2}{3}} + d\right)^n c\right) + a^2}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
 a^2)/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}} + d\right)^n c\right) + a\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^5, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(e\,x^{\frac{2}{3}} + d\right)^n\right) + a\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^2/x^5,x)
```

```
[Out] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^2/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2n^2\log\left(ex^{\frac{2}{3}} + d\right)^2}{4\,x^4} + \int \frac{\left(b^2enx + 6\left(b^2e\log(c) + abe\right)x + 6\left(b^2d\log(c) + abd\right)x^{\frac{1}{3}}\right)n\log\left(ex^{\frac{2}{3}} + d\right) + 3\left(b^2e\log(c\right.}{3\left(ex^6 + dx^{\frac{16}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*n^2*log(e*x^(2/3) + d)^2/x^4 + integrate(1/3*((b^2*e*n*x + 6*(b^2*
e*log(c) + a*b*e)*x + 6*(b^2*d*log(c) + a*b*d)*x^(1/3))*n*log(e*x^(2/3) + d
) + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x + 3*(b^2*d*log(c)^2 + 2*a
*b*d*log(c) + a^2*d)*x^(1/3))/(e*x^6 + d*x^(16/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^5,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**5,x)
```

```
[Out] Timed out
```

3.476 $\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=547

$$\frac{4bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^{9/2}}+\frac{4bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9e^3}-\frac{4bd^2nx^{5/3}\left(a+b\log\left(c\left(\right.\right.\right.}{15e^2}$$

```
[Out] -4/3*a*b*d^4*n*x^(1/3)/e^4+4504/945*b^2*d^4*n^2*x^(1/3)/e^4-1984/2835*b^2*d
^3*n^2*x/e^3+1144/4725*b^2*d^2*n^2*x^(5/3)/e^2-128/1323*b^2*d*n^2*x^(7/3)/e
+8/243*b^2*n^2*x^3-4504/945*b^2*d^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))
/e^(9/2)+4/3*I*b^2*d^(9/2)*n^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(
1/2)))/e^(9/2)-4/3*b^2*d^4*n*x^(1/3)*ln(c*(d+e*x^(2/3))^n)/e^4+4/9*b*d^3*n*
x*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3-4/15*b*d^2*n*x^(5/3)*(a+b*ln(c*(d+e*x^(2/
3))^n))/e^2+4/21*b*d*n*x^(7/3)*(a+b*ln(c*(d+e*x^(2/3))^n))/e-4/27*b*n*x^3*(
a+b*ln(c*(d+e*x^(2/3))^n))+4/3*b*d^(9/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2))*
(a+b*ln(c*(d+e*x^(2/3))^n))/e^(9/2)+1/3*x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^2+8
/3*b^2*d^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*
x^(1/3)*e^(1/2)))/e^(9/2)+4/3*I*b^2*d^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1
/2))^2/e^(9/2)
```

Rubi [A] time = 0.77, antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {2458, 2457, 2476, 2448, 321, 205, 2455, 302, 2470, 12, 4920, 4854, 2402, 2315}

$$\frac{4ib^2d^{9/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3e^{9/2}}+\frac{4bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9e^3}-\frac{4bd^2nx^{5/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right.}{15e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (-4*a*b*d^4*n*x^(1/3))/(3*e^4) + (4504*b^2*d^4*n^2*x^(1/3))/(945*e^4) - (19
84*b^2*d^3*n^2*x)/(2835*e^3) + (1144*b^2*d^2*n^2*x^(5/3))/(4725*e^2) - (128
*b^2*d*n^2*x^(7/3))/(1323*e) + (8*b^2*n^2*x^3)/243 - (4504*b^2*d^(9/2)*n^2*
ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(945*e^(9/2)) + (((4*I)/3)*b^2*d^(9/2)*n
^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e^(9/2) + (8*b^2*d^(9/2)*n^2*ArcTan
[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])
/(3*e^(9/2)) - (4*b^2*d^4*n*x^(1/3)*Log[c*(d + e*x^(2/3))^n])/(3*e^4) + (4*
b*d^3*n*x*(a + b*Log[c*(d + e*x^(2/3))^n]))/(9*e^3) - (4*b*d^2*n*x^(5/3)*(a
 + b*Log[c*(d + e*x^(2/3))^n]))/(15*e^2) + (4*b*d*n*x^(7/3)*(a + b*Log[c*(d
 + e*x^(2/3))^n]))/(21*e) - (4*b*n*x^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/27
 + (4*b*d^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(
2/3))^n]))/(3*e^(9/2)) + (x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/3 + (((4*
I)/3)*b^2*d^(9/2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/
3))])/e^(9/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2dx = 3\,\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2dx, x, \sqrt[3]{x}\right)$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 - \frac{1}{3}(4ben)\,\text{Subst}\left(\int \frac{x^{10}\left(a+b\log\left(c\right.\right.}{d+e}\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 - \frac{1}{3}(4ben)\,\text{Subst}\left(\int\left(\frac{d^4\left(a+b\log\left(c\right.\right.}{e^5}\right.\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 - \frac{1}{3}(4bn)\,\text{Subst}\left(\int x^8\left(a+b\log\left(c\right.\right.\right.$$

$$= -\frac{4abd^4n\sqrt[3]{x}}{3e^4} + \frac{4bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9e^3} - \frac{4bd^2nx^{5/3}\left(a+b\right.}{}$$

$$= -\frac{4abd^4n\sqrt[3]{x}}{3e^4} - \frac{4b^2d^4n\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3e^4} + \frac{4bd^3nx\left(a+b\log\left(c\right.\right.}{9e^3}$$

$$= -\frac{4abd^4n\sqrt[3]{x}}{3e^4} + \frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4} - \frac{1984b^2d^3n^2x}{2835e^3} + \frac{1144b^2d^2n^2x^{5/3}}{4725e^2} - 1$$

$$= -\frac{4abd^4n\sqrt[3]{x}}{3e^4} + \frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4} - \frac{1984b^2d^3n^2x}{2835e^3} + \frac{1144b^2d^2n^2x^{5/3}}{4725e^2} - 1$$

$$= -\frac{4abd^4n\sqrt[3]{x}}{3e^4} + \frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4} - \frac{1984b^2d^3n^2x}{2835e^3} + \frac{1144b^2d^2n^2x^{5/3}}{4725e^2} - 1$$

$$= -\frac{4abd^4n\sqrt[3]{x}}{3e^4} + \frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4} - \frac{1984b^2d^3n^2x}{2835e^3} + \frac{1144b^2d^2n^2x^{5/3}}{4725e^2} - 1$$

Mathematica [A] time = 0.53, size = 438, normalized size = 0.80

$$\frac{\sqrt{e}\sqrt[3]{x}\left(99225a^2e^4x^{8/3} - 630b\left(2bn\left(315d^4 - 105d^3ex^{2/3} + 63d^2e^2x^{4/3} - 45de^3x^2 + 35e^4x^{8/3}\right) - 315ae^4x^{8/3}\right)\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] ((396900*I)*b^2*d^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2 + 1260*b*d^
(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(315*a - 1126*b*n + 630*b*n*Log[(
2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))] + 315*b*Log[c*(d + e*x^(2/3))^n])
 + Sqrt[e]*x^(1/3)*(99225*a^2*e^4*x^(8/3) - 1260*a*b*n*(315*d^4 - 105*d^3*e
```

```
*x^(2/3) + 63*d^2*e^2*x^(4/3) - 45*d*e^3*x^2 + 35*e^4*x^(8/3)) + 8*b^2*n^2*
(177345*d^4 - 26040*d^3*e*x^(2/3) + 9009*d^2*e^2*x^(4/3) - 3600*d*e^3*x^2 +
 1225*e^4*x^(8/3)) - 630*b*(-315*a*e^4*x^(8/3) + 2*b*n*(315*d^4 - 105*d^3*e
*x^(2/3) + 63*d^2*e^2*x^(4/3) - 45*d*e^3*x^2 + 35*e^4*x^(8/3)))*Log[c*(d +
e*x^(2/3))^n] + 99225*b^2*e^4*x^(8/3)*Log[c*(d + e*x^(2/3))^n]^2) + (396900
*I)*b^2*d^(9/2)*n^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/((-I)*Sqrt[d]
+ Sqrt[e]*x^(1/3))])/(297675*e^(9/2))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^2x^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+2\,abx^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a^2x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*x^2*log((e*x^(2/3) + d)
^n*c) + a^2*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2*x^2, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^n\right)+a\right)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*(e*x^(2/3)+d)^n)+a)^2,x)
```

```
[Out] int(x^2*(b*ln(c*(e*x^(2/3)+d)^n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,b^2n^2x^3\log\left(ex^{\frac{2}{3}}+d\right)^2+\int\frac{9\left(b^2e\log(c)^2+2\,abe\log(c)+a^2e\right)x^3+9\left(b^2d\log(c)^2+2\,abd\log(c)+a^2d\right)x^{\frac{7}{3}}-2\left(2\right.}{9\left(ex+dx^{\frac{1}{3}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*n^2*x^3*log(e*x^(2/3) + d)^2 + integrate(1/9*(9*(b^2*e*log(c)^2 + 2
*a*b*e*log(c) + a^2*e)*x^3 + 9*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x^
(7/3) - 2*(2*b^2*e*n*x^3 - 9*(b^2*e*log(c) + a*b*e)*x^3 - 9*(b^2*d*log(c) +
 a*b*d)*x^(7/3))*n*log(e*x^(2/3) + d))/(e*x + d*x^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(a+b\,\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(2/3))^n))^2,x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e*x^(2/3))^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
```

```
[Out] Timed out
```

3.477 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=364

$$-\frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}-\frac{4}{3}bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right.\right.$$

```
[Out] 4*a*b*d*n*x^(1/3)/e-32/3*b^2*d*n^2*x^(1/3)/e+8/9*b^2*n^2*x+32/3*b^2*d^(3/2)
*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))/e^(3/2)-4*I*b^2*d^(3/2)*n^2*arctan(x^(
1/3)*e^(1/2)/d^(1/2))^2/e^(3/2)+4*b^2*d*n*x^(1/3)*ln(c*(d+e*x^(2/3))^n)/e-4
/3*b*n*x*(a+b*ln(c*(d+e*x^(2/3))^n))-4*b*d^(3/2)*n*arctan(x^(1/3)*e^(1/2)/d
^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^(3/2)+x*(a+b*ln(c*(d+e*x^(2/3))^n))^2
-8*b^2*d^(3/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*
x^(1/3)*e^(1/2)))/e^(3/2)-4*I*b^2*d^(3/2)*n^2*polylog(2,1-2*d^(1/2)/(d^(1/2
)+I*x^(1/3)*e^(1/2)))/e^(3/2)
```

Rubi [A] time = 0.45, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {2451, 2457, 2476, 2448, 321, 205, 2455, 302, 2470, 12, 4920, 4854, 2402, 2315}

$$-\frac{4ib^2d^{3/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{e^{3/2}}-\frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}-\frac{4}{3}bnx\left(a+b\log\left(c\left(\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (4*a*b*d*n*x^(1/3))/e - (32*b^2*d*n^2*x^(1/3))/(3*e) + (8*b^2*n^2*x)/9 + (3
2*b^2*d^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(3/2)) - ((4*I)*b
^2*d^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e^(3/2) - (8*b^2*d^(3/2
)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e
]*x^(1/3))])/e^(3/2) + (4*b^2*d*n*x^(1/3)*Log[c*(d + e*x^(2/3))^n])/e - (4*
b*n*x*(a + b*Log[c*(d + e*x^(2/3))^n]))/3 - (4*b*d^(3/2)*n*ArcTan[(Sqrt[e]*
x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/e^(3/2) + x*(a + b*Log[
c*(d + e*x^(2/3))^n])^2 - ((4*I)*b^2*d^(3/2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])
/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/e^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 dx = 3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2 dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 - (4ben)\,\text{Subst}\left(\int \frac{x^4\left(a+b\log\left(c\left(d+ex^2\right)^n\right.\right.}{d+ex^2}\right.$$

$$= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 - (4ben)\,\text{Subst}\left(\int\left(-\frac{d\left(a+b\log\left(c\left(d+ex^2\right.\right.\right.}{e^2}\right.\right.$$

$$= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 - (4bn)\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right.\right.$$

$$= \frac{4abdn\sqrt[3]{x}}{e} - \frac{4}{3}bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) - \frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+\right.}{e^{3/2}}$$

$$= \frac{4abdn\sqrt[3]{x}}{e} + \frac{4b^2dn\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e} - \frac{4}{3}bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{\prime}\right.\right.$$

$$= \frac{4abdn\sqrt[3]{x}}{e} - \frac{32b^2dn^2\sqrt[3]{x}}{3e} + \frac{8}{9}b^2n^2x - \frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}} + \frac{4b^2dn\sqrt[3]{}}{}$$

$$= \frac{4abdn\sqrt[3]{x}}{e} - \frac{32b^2dn^2\sqrt[3]{x}}{3e} + \frac{8}{9}b^2n^2x + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}} - \frac{4ib^2d^{3/2}r}{}$$

$$= \frac{4abdn\sqrt[3]{x}}{e} - \frac{32b^2dn^2\sqrt[3]{x}}{3e} + \frac{8}{9}b^2n^2x + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}} - \frac{4ib^2d^{3/2}r}{}$$

$$= \frac{4abdn\sqrt[3]{x}}{e} - \frac{32b^2dn^2\sqrt[3]{x}}{3e} + \frac{8}{9}b^2n^2x + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}} - \frac{4ib^2d^{3/2}r}{}$$

Mathematica [A] time = 0.23, size = 319, normalized size = 0.88

$$\frac{\sqrt{e}\sqrt[3]{x}\left(9a^2ex^{2/3}+6b\left(3aex^{2/3}+6bdn-2benx^{2/3}\right)\log\left(c\left(d+ex^{2/3}\right)^n\right)+12abn\left(3d-ex^{2/3}\right)+9b^2ex^{2/3}\log^2\left(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] ((-36*I)*b^2*d^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2 - 12*b*d^(3/2)
*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(3*a - 8*b*n + 6*b*n*Log[(2*Sqrt[d])/(
Sqrt[d] + I*Sqrt[e]*x^(1/3))] + 3*b*Log[c*(d + e*x^(2/3))^n]) + Sqrt[e]*x^(
1/3)*(12*a*b*n*(3*d - e*x^(2/3)) + 8*b^2*n^2*(-12*d + e*x^(2/3)) + 9*a^2*e*
x^(2/3) + 6*b*(6*b*d*n + 3*a*e*x^(2/3) - 2*b*e*n*x^(2/3))*Log[c*(d + e*x^(2
/3))^n] + 9*b^2*e*x^(2/3)*Log[c*(d + e*x^(2/3))^n]^2) - (36*I)*b^2*d^(3/2)*
n^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/((-I)*Sqrt[d] + Sqrt[e]*x^(1/3
))])/(9*e^(3/2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(b^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+2\,ab\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
a^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^n\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^2,x)
```

```
[Out] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{3}\left(2\,en\left(\frac{3\,d^2\arctan\left(\frac{ex^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}\,e^2}+\frac{ex-3\,dx^{\frac{1}{3}}}{e^2}\right)-3\,x\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)\right)ab+\left(n^2x\log\left(ex^{\frac{2}{3}}+d\right)^2+\int\frac{3\,ex\log(c)^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(2*e*n*(3*d^2*arctan(e*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*e^2) + (e*x - 3*d
*x^(1/3))/e^2) - 3*x*log((e*x^(2/3) + d)^n*c))*a*b + (n^2*x*log(e*x^(2/3) +
 d)^2 + integrate(1/3*(3*e*x*log(c)^2 + 3*d*x^(1/3)*log(c)^2 - 2*(2*e*n*x -
 3*e*x*log(c) - 3*d*x^(1/3)*log(c))*n*log(e*x^(2/3) + d))/(e*x + d*x^(1/3))
, x))*b^2 + a^2*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b\,\ln\left(c\left(d + e\,x^{2/3}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\log\left(c\left(d + ex^{\frac{2}{3}}\right)^{n}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(2/3))**n))**2, x)
```

3.478 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=298

$$-\frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}}-\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\sqrt[3]{x}}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right.}{x}$$

```
[Out] 8*b^2*e^(3/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))/d^(3/2)-4*I*b^2*e^(3/2)*n
^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))^2/d^(3/2)-4*b*e*n*(a+b*ln(c*(d+e*x^(2/3)
)^n))/d/x^(1/3)-4*b*e^(3/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2))*(a+b*ln(c*(d+
e*x^(2/3))^n))/d^(3/2)-(a+b*ln(c*(d+e*x^(2/3))^n))^2/x-8*b^2*e^(3/2)*n^2*ar
ctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(
3/2)-4*I*b^2*e^(3/2)*n^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))
/d^(3/2)
```

Rubi [A] time = 0.41, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {2458, 2457, 2476, 2455, 205, 2470, 12, 4920, 4854, 2402, 2315}

$$-\frac{4ib^2e^{3/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}}-\frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}}-\frac{4ben\left(a+b\log\left(\right.\right.}{d\sqrt[3]{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^2,x]
```

```
[Out] (8*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/d^(3/2) - ((4*I)*b^2*
e^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/d^(3/2) - (8*b^2*e^(3/2)*n
^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x
^(1/3))])/d^(3/2) - (4*b*e*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(d*x^(1/3))
- (4*b*e^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2
/3))^n]))/d^(3/2) - (a + b*Log[c*(d + e*x^(2/3))^n])^2/x - ((4*I)*b^2*e^(3/
2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/d^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x} + (4ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^2\left(d+ex^2\right)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x} + (4ben)\,\text{Subst}\left(\int\left(\frac{a+b\log\left(c\left(d+ex^2\right)\right.}{dx^2}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x} + \frac{(4ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^2}\,dx, x, \sqrt[3]{\right.}{d}$$

$$= -\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\sqrt[3]{x}} - \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\right.\right.}{d^{3/2}}$$

$$= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\sqrt[3]{x}} - \frac{4be^{3/2}n\tan}{}$$

$$= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{4ben\left(a+b\log\left(\right.\right.}{d\sqrt[3]{}}$$

$$= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\right.}{}$$

$$= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\right.}{}$$

$$= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\right.}{}$$

Mathematica [A] time = 0.19, size = 247, normalized size = 0.83

$$\frac{-4be^{3/2}nx\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)+2bn\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)-2bn\right)-\sqrt{d}\left(a+b\log\left(c\left(d+ex^{2/\right.\right.\right.}{d^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^2,x]
```

```
[Out] ((-4*I)*b^2*e^(3/2)*n^2*x*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2 - 4*b*e^(3/2)
*n*x*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - 2*b*n + 2*b*n*Log[(2*Sqrt[d])/(
```

```
Sqrt[d] + I*Sqrt[e]*x^(1/3))] + b*Log[c*(d + e*x^(2/3))^n]) - Sqrt[d]*(a +
b*Log[c*(d + e*x^(2/3))^n])*(a*d + 4*b*e*n*x^(2/3) + b*d*Log[c*(d + e*x^(2/
3))^n]) - (4*I)*b^2*e^(3/2)*n^2*x*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/
((-I)*Sqrt[d] + Sqrt[e]*x^(1/3))])/(d^(3/2)*x)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \log\left(\left(ex^{\frac{2}{3}} + d\right)^n c\right)^2 + 2\,ab\log\left(\left(ex^{\frac{2}{3}} + d\right)^n c\right) + a^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
 a^2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}} + d\right)^n c\right) + a\right)^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^2, x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(e\,x^{\frac{2}{3}} + d\right)^n\right) + a\right)^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^2/x^2,x)
```

```
[Out] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^2/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2n^2\log\left(ex^{\frac{2}{3}} + d\right)^2}{x} + \int \frac{2\left(2\,b^2enx + 3\left(b^2e\log(c) + abe\right)x + 3\left(b^2d\log(c) + abd\right)x^{\frac{1}{3}}\right)n\log\left(ex^{\frac{2}{3}} + d\right) + 3\left(b^2e\,\text{lo}\right.}{3\left(ex^3 + dx^{\frac{7}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -b^2*n^2*log(e*x^(2/3) + d)^2/x + integrate(1/3*(2*(2*b^2*e*n*x + 3*(b^2*e*
log(c) + a*b*e)*x + 3*(b^2*d*log(c) + a*b*d)*x^(1/3))*n*log(e*x^(2/3) + d)
+ 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x + 3*(b^2*d*log(c)^2 + 2*a*b
*d*log(c) + a^2*d)*x^(1/3))/(e*x^3 + d*x^(7/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b\,\ln\left(c\left(d + e\,x^{2/3}\right)^n\right)\right)^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^2, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**2,x)
```

```
[Out] Timed out
```

3.479 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=476

$$\frac{4be^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^{9/2}}+\frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^4\sqrt[3]{x}}-\frac{4be^3n\left(a+b\log\left(c\left(d+ex\right.\right.\right.}{9d^3x}$$

```
[Out] -8/105*b^2*e^2*n^2/d^2/x^(5/3)+32/105*b^2*e^3*n^2/d^3/x-568/315*b^2*e^4*n^2
/d^4/x^(1/3)-1408/315*b^2*e^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))/d^(9/
2)+4/3*I*b^2*e^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))^2/d^(9/2)-4/21*b*e
*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d/x^(7/3)+4/15*b*e^2*n*(a+b*ln(c*(d+e*x^(2/3
))^n))/d^2/x^(5/3)-4/9*b*e^3*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^3/x+4/3*b*e^4*
n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^4/x^(1/3)+4/3*b*e^(9/2)*n*arctan(x^(1/3)*e^
(1/2)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^(9/2)-1/3*(a+b*ln(c*(d+e*x^(2/
3))^n))^2/x^3+8/3*b^2*e^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1
/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(9/2)+4/3*I*b^2*e^(9/2)*n^2*polylog(2,1-
2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(9/2)
```

Rubi [A] time = 0.62, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2458, 2457, 2476, 2455, 325, 205, 2470, 12, 4920, 4854, 2402, 2315}

$$\frac{4ib^2e^{9/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3d^{9/2}}+\frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^4\sqrt[3]{x}}-\frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9d^3x}+\frac{4b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^4,x]
```

```
[Out] (-8*b^2*e^2*n^2)/(105*d^2*x^(5/3)) + (32*b^2*e^3*n^2)/(105*d^3*x) - (568*b^
2*e^4*n^2)/(315*d^4*x^(1/3)) - (1408*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3
))/Sqrt[d]])/(315*d^(9/2)) + (((4*I)/3)*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(
1/3))/Sqrt[d]]^2)/d^(9/2) + (8*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqr
t[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/(3*d^(9/2)) - (4*b*e*
n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(21*d*x^(7/3)) + (4*b*e^2*n*(a + b*Log[
c*(d + e*x^(2/3))^n]))/(15*d^2*x^(5/3)) - (4*b*e^3*n*(a + b*Log[c*(d + e*x^
(2/3))^n]))/(9*d^3*x) + (4*b*e^4*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*d^4
*x^(1/3)) + (4*b*e^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(
d + e*x^(2/3))^n]))/(3*d^(9/2)) - (a + b*Log[c*(d + e*x^(2/3))^n])^2/(3*x^3
) + (((4*I)/3)*b^2*e^(9/2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt
[e]*x^(1/3))])/d^(9/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(4ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^8\left(d+ex^2\right)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(4ben)\,\text{Subst}\left(\int\left(\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{dx^8}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} + \frac{(4ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^8}\,dx, x, \sqrt[3]{x}\right)}{3d}$$

$$= -\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21dx^{7/3}} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^2x^{5/3}} - \frac{4b}{}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{8b^2e^3n^2}{45d^3x} - \frac{8b^2e^4n^2}{9d^4\sqrt[3]{x}} - \frac{8b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3d^{9/2}} - \frac{4ben\left(a+b\right.}{}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{32b^2e^3n^2}{105d^3x} - \frac{64b^2e^4n^2}{45d^4\sqrt[3]{x}} - \frac{32b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{9d^{9/2}} + \frac{4ib^2e^{9/2}n}{}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{32b^2e^3n^2}{105d^3x} - \frac{568b^2e^4n^2}{315d^4\sqrt[3]{x}} - \frac{184b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{45d^{9/2}} + \frac{4ib^2e^{9}}{}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{32b^2e^3n^2}{105d^3x} - \frac{568b^2e^4n^2}{315d^4\sqrt[3]{x}} - \frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{315d^{9/2}} + \frac{4ib^2e}{}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{32b^2e^3n^2}{105d^3x} - \frac{568b^2e^4n^2}{315d^4\sqrt[3]{x}} - \frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{315d^{9/2}} + \frac{4ib^2e}{}$$

Mathematica [C] time = 0.55, size = 473, normalized size = 0.99

$$-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3}+\frac{4}{3}ben\left(\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{9/2}}+\frac{e^3\left(a+b\log\left(c\left(d+ex\right.\right.\right.}{d^4\sqrt[3]{x}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^4,x]
```

```
[Out] -1/3*(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^3 + (4*b*e*n*((-2*b*e^(7/2)*n*Arc
Tan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/d^(9/2) - (2*b*e*n*Hypergeometric2F1[-5/2,
1, -3/2, -((e*x^(2/3))/d)])/(35*d^2*x^(5/3)) + (2*b*e^2*n*Hypergeometric2F1
[-3/2, 1, -1/2, -((e*x^(2/3))/d)])/(15*d^3*x) - (2*b*e^3*n*Hypergeometric2F
1[-1/2, 1, 1/2, -((e*x^(2/3))/d)])/(3*d^4*x^(1/3)) - (a + b*Log[c*(d + e*x^
(2/3))^n])/(7*d*x^(7/3)) + (e*(a + b*Log[c*(d + e*x^(2/3))^n]))/(5*d^2*x^(5
/3)) - (e^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*d^3*x) + (e^3*(a + b*Log[c
*(d + e*x^(2/3))^n]))/(d^4*x^(1/3)) + (e^(7/2)*ArcTan[(Sqrt[e]*x^(1/3))/Sqr
t[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/d^(9/2) + (I*b*e^(7/2)*n*(ArcTan[(S
qrt[e]*x^(1/3))/Sqrt[d]]*(ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]] - (2*I)*Log[(2*
Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))]) + PolyLog[2, (I*Sqrt[d] + Sqrt[e]*
x^(1/3))/((-I)*Sqrt[d] + Sqrt[e]*x^(1/3))]))/d^(9/2)))/3
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+2\,ab\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
 a^2)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^4, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^n\right)+a\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^2/x^4,x)
```

```
[Out] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^2/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2n^2\log\left(ex^{\frac{2}{3}}+d\right)^2}{3\,x^3}+\int\frac{2\left(2\,b^2enx+9\left(b^2e\log(c)+abe\right)x+9\left(b^2d\log(c)+abd\right)x^{\frac{1}{3}}\right)n\log\left(ex^{\frac{2}{3}}+d\right)+9\left(b^2e\log\cdots\right.}{9\left(ex^5+dx^{\frac{13}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*n^2*log(e*x^(2/3) + d)^2/x^3 + integrate(1/9*(2*(2*b^2*e*n*x + 9*(
b^2*e*log(c) + a*b*e)*x + 9*(b^2*d*log(c) + a*b*d)*x^(1/3))*n*log(e*x^(2/3)
 + d) + 9*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x + 9*(b^2*d*log(c)^2 +
 2*a*b*d*log(c) + a^2*d)*x^(1/3))/(e*x^5 + d*x^(13/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+b\,\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^4,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**4,x)
```

```
[Out] Timed out
```

3.480 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx$

Optimal. Leaf size=640

$$-\frac{4be^{15/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^{15/2}}-\frac{4be^7n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^7\sqrt[3]{x}}+\frac{4be^6n\left(a+b\log\left(c\left(d\right.\right.\right.}{15d^6x}$$

```
[Out] -8/715*b^2*e^2*n^2/d^2/x^(11/3)+64/2145*b^2*e^3*n^2/d^3/x^3-2872/45045*b^2*
e^4*n^2/d^4/x^(7/3)+1216/9009*b^2*e^5*n^2/d^5/x^(5/3)-224072/675675*b^2*e^6
*n^2/d^6/x+344192/225225*b^2*e^7*n^2/d^7/x^(1/3)+704552/225225*b^2*e^(15/2)
*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))/d^(15/2)-4/5*I*b^2*e^(15/2)*n^2*polylo
g(2,1-2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(15/2)-4/65*b*e*n*(a+b*ln(c*
(d+e*x^(2/3))^n))/d/x^(13/3)+4/55*b*e^2*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^2/x
^(11/3)-4/45*b*e^3*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^3/x^3+4/35*b*e^4*n*(a+b*
ln(c*(d+e*x^(2/3))^n))/d^4/x^(7/3)-4/25*b*e^5*n*(a+b*ln(c*(d+e*x^(2/3))^n))
/d^5/x^(5/3)+4/15*b*e^6*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^6/x-4/5*b*e^7*n*(a+
b*ln(c*(d+e*x^(2/3))^n))/d^7/x^(1/3)-4/5*b*e^(15/2)*n*arctan(x^(1/3)*e^(1/2
)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^(15/2)-1/5*(a+b*ln(c*(d+e*x^(2/3))
^n))^2/x^5-8/5*b^2*e^(15/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1/2
)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(15/2)-4/5*I*b^2*e^(15/2)*n^2*arctan(x^(1/
3)*e^(1/2)/d^(1/2))^2/d^(15/2)
```

Rubi [A] time = 0.94, antiderivative size = 640, normalized size of antiderivative = 1.00, number of steps used = 45, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2458, 2457, 2476, 2455, 325, 205, 2470, 12, 4920, 4854, 2402, 2315}

$$-\frac{4ib^2e^{15/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{5d^{15/2}}-\frac{4be^7n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^7\sqrt[3]{x}}+\frac{4be^6n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right.}{15d^6x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^6,x]
```

```
[Out] (-8*b^2*e^2*n^2)/(715*d^2*x^(11/3)) + (64*b^2*e^3*n^2)/(2145*d^3*x^3) - (28
72*b^2*e^4*n^2)/(45045*d^4*x^(7/3)) + (1216*b^2*e^5*n^2)/(9009*d^5*x^(5/3))
 - (224072*b^2*e^6*n^2)/(675675*d^6*x) + (344192*b^2*e^7*n^2)/(225225*d^7*x
^(1/3)) + (704552*b^2*e^(15/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(2252
25*d^(15/2)) - (((4*I)/5)*b^2*e^(15/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]
]^2)/d^(15/2) - (8*b^2*e^(15/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(
2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/(5*d^(15/2)) - (4*b*e*n*(a + b*L
og[c*(d + e*x^(2/3))^n]))/(65*d*x^(13/3)) + (4*b*e^2*n*(a + b*Log[c*(d + e*
x^(2/3))^n]))/(55*d^2*x^(11/3)) - (4*b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n
]))/(45*d^3*x^3) + (4*b*e^4*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(35*d^4*x^(
7/3)) - (4*b*e^5*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(25*d^5*x^(5/3)) + (4*
b*e^6*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(15*d^6*x) - (4*b*e^7*n*(a + b*Lo
g[c*(d + e*x^(2/3))^n]))/(5*d^7*x^(1/3)) - (4*b*e^(15/2)*n*ArcTan[(Sqrt[e]*
x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/(5*d^(15/2)) - (a + b*L
og[c*(d + e*x^(2/3))^n])^2/(5*x^5) - (((4*I)/5)*b^2*e^(15/2)*n^2*PolyLog[2,
 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/d^(15/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
```

```
& IntegerQ[s]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^{16}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} + \frac{1}{5}(4ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^{14}\left(d+ex^2\right)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} + \frac{1}{5}(4ben)\,\text{Subst}\left(\int \left(\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{dx^{14}}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} + \frac{(4ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^{14}}\,dx, x, \sqrt[3]{x}\right)}{5d}$$

$$= -\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{65dx^{13/3}} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{55d^2x^{11/3}} - \frac{4b}{}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{8b^2e^3n^2}{495d^3x^3} - \frac{8b^2e^4n^2}{315d^4x^{7/3}} + \frac{8b^2e^5n^2}{175d^5x^{5/3}} - \frac{8b^2e^6n^2}{75d^6x} + \frac{8b^2e^7n^2}{15d^7\sqrt[3]{x}} + \frac{8}{}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{32b^2e^4n^2}{693d^4x^{7/3}} + \frac{128b^2e^5n^2}{1575d^5x^{5/3}} - \frac{32b^2e^6n^2}{175d^6x} + \frac{64b^2e^7n^2}{75d^7\sqrt[3]{x}}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{2872b^2e^4n^2}{45045d^4x^{7/3}} + \frac{1912b^2e^5n^2}{17325d^5x^{5/3}} - \frac{1144b^2e^6n^2}{4725d^6x} + \frac{56}{52}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{2872b^2e^4n^2}{45045d^4x^{7/3}} + \frac{1216b^2e^5n^2}{9009d^5x^{5/3}} - \frac{15104b^2e^6n^2}{51975d^6x} + \frac{19}{15}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{2872b^2e^4n^2}{45045d^4x^{7/3}} + \frac{1216b^2e^5n^2}{9009d^5x^{5/3}} - \frac{224072b^2e^6n^2}{675675d^6x} + \frac{2}{}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{2872b^2e^4n^2}{45045d^4x^{7/3}} + \frac{1216b^2e^5n^2}{9009d^5x^{5/3}} - \frac{224072b^2e^6n^2}{675675d^6x} + \frac{3}{}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{2872b^2e^4n^2}{45045d^4x^{7/3}} + \frac{1216b^2e^5n^2}{9009d^5x^{5/3}} - \frac{224072b^2e^6n^2}{675675d^6x} + \frac{3}{}$$

Mathematica [C] time = 1.24, size = 678, normalized size = 1.06

$$-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5}+\frac{4}{5}ben\left(-\frac{e^{13/2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{15/2}}-\frac{e^6\left(a+b\log\left(c\left(d+\right.\right.\right.}{d^7\sqrt[3]{x}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^6,x]
```

```
[Out] -1/5*(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^5 + (4*b*e*n*((2*b*e^(13/2)*n*Arc
Tan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/d^(15/2) - (2*b*e*n*Hypergeometric2F1[-11/2
, 1, -9/2, -((e*x^(2/3))/d)])/(143*d^2*x^(11/3)) + (2*b*e^2*n*Hypergeometri
c2F1[-9/2, 1, -7/2, -((e*x^(2/3))/d)])/(99*d^3*x^3) - (2*b*e^3*n*Hypergeome
tric2F1[-7/2, 1, -5/2, -((e*x^(2/3))/d)])/(63*d^4*x^(7/3)) + (2*b*e^4*n*Hyp
ergeometric2F1[-5/2, 1, -3/2, -((e*x^(2/3))/d)])/(35*d^5*x^(5/3)) - (2*b*e^
5*n*Hypergeometric2F1[-3/2, 1, -1/2, -((e*x^(2/3))/d)])/(15*d^6*x) + (2*b*e
^6*n*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^(2/3))/d)])/(3*d^7*x^(1/3)) - (
a + b*Log[c*(d + e*x^(2/3))^n])/(13*d*x^(13/3)) + (e*(a + b*Log[c*(d + e*x^
(2/3))^n]))/(11*d^2*x^(11/3)) - (e^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(9*d
^3*x^3) + (e^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(7*d^4*x^(7/3)) - (e^4*(a
+ b*Log[c*(d + e*x^(2/3))^n]))/(5*d^5*x^(5/3)) + (e^5*(a + b*Log[c*(d + e*x
^(2/3))^n]))/(3*d^6*x) - (e^6*(a + b*Log[c*(d + e*x^(2/3))^n]))/(d^7*x^(1/3
)) - (e^(13/2)*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3
))^n]))/d^(15/2) - (I*b*e^(13/2)*n*(ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(ArcT
an[(Sqrt[e]*x^(1/3))/Sqrt[d]] - (2*I)*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*
x^(1/3))]) + PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/((-I)*Sqrt[d] + Sqrt[
e]*x^(1/3))]))/d^(15/2)))/5
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+2\,ab\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a^2}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^6,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
 a^2)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^6, x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^n\right)+a\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^2/x^6,x)
```

```
[Out] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^2/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2n^2\log\left(ex^{\frac{2}{3}}+d\right)^2}{5\,x^5}+\int\frac{2\left(2\,b^2enx+15\left(b^2e\log(c)+abe\right)x+15\left(b^2d\log(c)+abd\right)x^{\frac{1}{3}}\right)n\log\left(ex^{\frac{2}{3}}+d\right)+15\left(b^2\right.}{15\left(ex^7+dx^{\frac{19}{3}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*b^2*n^2*log(e*x^(2/3) + d)^2/x^5 + integrate(1/15*(2*(2*b^2*e*n*x + 15
*(b^2*e*log(c) + a*b*e)*x + 15*(b^2*d*log(c) + a*b*d)*x^(1/3))*n*log(e*x^(2
/3) + d) + 15*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x + 15*(b^2*d*log(c
)^2 + 2*a*b*d*log(c) + a^2*d)*x^(1/3))/(e*x^7 + d*x^(19/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^6,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**6,x)
```

```
[Out] Timed out
```

3.481 $\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 dx$

Optimal. Leaf size=913

$$-\frac{b^3n^3\left(d+ex^{2/3}\right)^6}{144e^6}+\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\left(d+ex^{2/3}\right)^6}{4e^6}-\frac{bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\left(d+ex^{2/3}\right)^6}{8e^6}+\frac{b^2}{}$$

```
[Out] 1/4*(d+e*x^(2/3))^6*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^6-9*b^3*d^5*n^2*(d+e*x^
(2/3))*ln(c*(d+e*x^(2/3))^n)/e^6+45/8*b^2*d^4*n^2*(d+e*x^(2/3))^2*(a+b*ln(c
*(d+e*x^(2/3))^n))/e^6-10/3*b^2*d^3*n^2*(d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2
/3))^n))/e^6+45/32*b^2*d^2*n^2*(d+e*x^(2/3))^4*(a+b*ln(c*(d+e*x^(2/3))^n))/
e^6-9/25*b^2*d*n^2*(d+e*x^(2/3))^5*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6+9/2*b*d^
5*n*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^6-45/8*b*d^4*n*(d+e*x^(2/
3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^6+5*b*d^3*n*(d+e*x^(2/3))^3*(a+b*ln(c
*(d+e*x^(2/3))^n))^2/e^6-45/16*b*d^2*n*(d+e*x^(2/3))^4*(a+b*ln(c*(d+e*x^(2/
3))^n))^2/e^6+9/10*b*d*n*(d+e*x^(2/3))^5*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^6-
45/16*b^3*d^4*n^3*(d+e*x^(2/3))^2/e^6+10/9*b^3*d^3*n^3*(d+e*x^(2/3))^3/e^6-
45/128*b^3*d^2*n^3*(d+e*x^(2/3))^4/e^6+9/125*b^3*d*n^3*(d+e*x^(2/3))^5/e^6+
9*b^3*d^5*n^3*x^(2/3)/e^5-3/2*d^5*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))
^3/e^6+15/4*d^4*(d+e*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^6-5*d^3*(d+
e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^6+15/4*d^2*(d+e*x^(2/3))^4*(a+
b*ln(c*(d+e*x^(2/3))^n))^3/e^6-3/2*d*(d+e*x^(2/3))^5*(a+b*ln(c*(d+e*x^(2/3)
)^n))^3/e^6-1/144*b^3*n^3*(d+e*x^(2/3))^6/e^6+1/24*b^2*n^2*(d+e*x^(2/3))^6*
(a+b*ln(c*(d+e*x^(2/3))^n))/e^6-1/8*b*n*(d+e*x^(2/3))^6*(a+b*ln(c*(d+e*x^(2
/3))^n))^2/e^6-9*a*b^2*d^5*n^2*x^(2/3)/e^5
```

Rubi [A] time = 1.03, antiderivative size = 913, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$-\frac{b^3n^3\left(d+ex^{2/3}\right)^6}{144e^6}+\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\left(d+ex^{2/3}\right)^6}{4e^6}-\frac{bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\left(d+ex^{2/3}\right)^6}{8e^6}+\frac{b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (-45*b^3*d^4*n^3*(d + e*x^(2/3))^2)/(16*e^6) + (10*b^3*d^3*n^3*(d + e*x^(2/
3))^3)/(9*e^6) - (45*b^3*d^2*n^3*(d + e*x^(2/3))^4)/(128*e^6) + (9*b^3*d*n^
3*(d + e*x^(2/3))^5)/(125*e^6) - (b^3*n^3*(d + e*x^(2/3))^6)/(144*e^6) - (9
*a*b^2*d^5*n^2*x^(2/3))/e^5 + (9*b^3*d^5*n^3*x^(2/3))/e^5 - (9*b^3*d^5*n^2*
(d + e*x^(2/3))*Log[c*(d + e*x^(2/3))^n])/e^6 + (45*b^2*d^4*n^2*(d + e*x^(2
/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(8*e^6) - (10*b^2*d^3*n^2*(d + e*x
^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*e^6) + (45*b^2*d^2*n^2*(d +
e*x^(2/3))^4*(a + b*Log[c*(d + e*x^(2/3))^n]))/(32*e^6) - (9*b^2*d*n^2*(d +
 e*x^(2/3))^5*(a + b*Log[c*(d + e*x^(2/3))^n]))/(25*e^6) + (b^2*n^2*(d + e*
x^(2/3))^6*(a + b*Log[c*(d + e*x^(2/3))^n]))/(24*e^6) + (9*b*d^5*n*(d + e*x
^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(2*e^6) - (45*b*d^4*n*(d + e*x^
(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(8*e^6) + (5*b*d^3*n*(d + e*x^
(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/e^6 - (45*b*d^2*n*(d + e*x^(2/
3))^4*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(16*e^6) + (9*b*d*n*(d + e*x^(2/3
))^5*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(10*e^6) - (b*n*(d + e*x^(2/3))^6*
(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(8*e^6) - (3*d^5*(d + e*x^(2/3))*(a + b
*Log[c*(d + e*x^(2/3))^n])^3)/(2*e^6) + (15*d^4*(d + e*x^(2/3))^2*(a + b*Lo
g[c*(d + e*x^(2/3))^n])^3)/(4*e^6) - (5*d^3*(d + e*x^(2/3))^3*(a + b*Log[c*
(d + e*x^(2/3))^n])^3)/e^6 + (15*d^2*(d + e*x^(2/3))^4*(a + b*Log[c*(d + e*
x^(2/3))^n])^3)/(4*e^6) - (3*d*(d + e*x^(2/3))^5*(a + b*Log[c*(d + e*x^(2/3
```

```
))^n])^3)/(2*e^6) + ((d + e*x^(2/3))^6*(a + b*Log[c*(d + e*x^(2/3))^n])^3)/
(4*e^6)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 dx = \frac{3}{2}\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3 dx, x, x^{2/3}\right)$$

$$= \frac{3}{2}\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\left(c\right.\right.}{e^5}\right.\right.$$

$$= \frac{3\,\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3 dx, x, x^{2/3}\right)}{2e^5} - \frac{(15d)\,\text{Subs}}{}$$

$$= \frac{3\,\text{Subst}\left(\int x^5\left(a+b\log\left(cx^n\right)\right)^3 dx, x, d+ex^{2/3}\right)}{2e^6} - \frac{(15d)\,\text{Subst}\left(\int x^4\right.}{}$$

$$= -\frac{3d^5\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^6}+\frac{15d^4\left(d+ex^{2/3}\right)^2\left(a+\right.}{4}$$

$$= \frac{9bd^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^6}-\frac{45bd^4n\left(d+ex^{2/3}\right)^2\left(\right.}{}$$

$$= -\frac{45b^3d^4n^3\left(d+ex^{2/3}\right)^2}{16e^6}+\frac{10b^3d^3n^3\left(d+ex^{2/3}\right)^3}{9e^6}-\frac{45b^3d^2n^3\left(d+ex^{2/3}\right.}{128e^6}$$

$$= -\frac{45b^3d^4n^3\left(d+ex^{2/3}\right)^2}{16e^6}+\frac{10b^3d^3n^3\left(d+ex^{2/3}\right)^3}{9e^6}-\frac{45b^3d^2n^3\left(d+ex^{2/3}\right.}{128e^6}$$

Mathematica [A] time = 1.07, size = 598, normalized size = 0.65

$$\frac{-60b\left(1800a^2\left(d^6-e^6x^4\right)-60abn\left(147d^6+60d^5ex^{2/3}-30d^4e^2x^{4/3}+20d^3e^3x^2-15d^2e^4x^{8/3}+12de^5x^{10/3}-10e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (e*x^(2/3)*(36000*a^3*e^5*x^(10/3) + b^3*n^3*(809340*d^5 - 140070*d^4*e*x^(
2/3) + 41180*d^3*e^2*x^(4/3) - 13785*d^2*e^3*x^2 + 4368*d*e^4*x^(8/3) - 100
0*e^5*x^(10/3)) - 60*a*b^2*n^2*(8820*d^5 - 2610*d^4*e*x^(2/3) + 1140*d^3*e^
2*x^(4/3) - 555*d^2*e^3*x^2 + 264*d*e^4*x^(8/3) - 100*e^5*x^(10/3)) + 1800*
a^2*b*n*(60*d^5 - 30*d^4*e*x^(2/3) + 20*d^3*e^2*x^(4/3) - 15*d^2*e^3*x^2 +
12*d*e^4*x^(8/3) - 10*e^5*x^(10/3))) - 280140*b^3*d^6*n^3*Log[d + e*x^(2/3)
] - 60*b*(b^2*n^2*(8820*d^6 + 8820*d^5*e*x^(2/3) - 2610*d^4*e^2*x^(4/3) + 1
140*d^3*e^3*x^2 - 555*d^2*e^4*x^(8/3) + 264*d*e^5*x^(10/3) - 100*e^6*x^4) -
 60*a*b*n*(147*d^6 + 60*d^5*e*x^(2/3) - 30*d^4*e^2*x^(4/3) + 20*d^3*e^3*x^2
 - 15*d^2*e^4*x^(8/3) + 12*d*e^5*x^(10/3) - 10*e^6*x^4) + 1800*a^2*(d^6 - e
^6*x^4))*Log[c*(d + e*x^(2/3))^n] + 1800*b^2*(b*n*(147*d^6 + 60*d^5*e*x^(2/
3) - 30*d^4*e^2*x^(4/3) + 20*d^3*e^3*x^2 - 15*d^2*e^4*x^(8/3) + 12*d*e^5*x^
(10/3) - 10*e^6*x^4) - 60*a*(d^6 - e^6*x^4))*Log[c*(d + e*x^(2/3))^n]^2 - 3
6000*b^3*(d^6 - e^6*x^4)*Log[c*(d + e*x^(2/3))^n]^3)/(144000*e^6)
```

fricas [A] time = 0.63, size = 1241, normalized size = 1.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/144000*(36000*b^3*e^6*x^4*log(c)^3 - 1000*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2
+ 18*a^2*b*e^6*n - 36*a^3*e^6)*x^4 + 36000*(b^3*e^6*n^3*x^4 - b^3*d^6*n^3)*
```

```
log(e*x^(2/3) + d)^3 + 20*(2059*b^3*d^3*e^3*n^3 - 3420*a*b^2*d^3*e^3*n^2 +
1800*a^2*b*d^3*e^3*n)*x^2 + 1800*(20*b^3*d^3*e^3*n^3*x^2 + 147*b^3*d^6*n^3
- 60*a*b^2*d^6*n^2 - 10*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2)*x^4 + 60*(b^3*e^6*n
^2*x^4 - b^3*d^6*n^2)*log(c) - 15*(b^3*d^2*e^4*n^3*x^2 - 4*b^3*d^5*e*n^3)*x
^(2/3) + 6*(2*b^3*d*e^5*n^3*x^3 - 5*b^3*d^4*e^2*n^3*x)*x^(1/3))*log(e*x^(2/
3) + d)^2 + 18000*(2*b^3*d^3*e^3*n*x^2 - (b^3*e^6*n - 6*a*b^2*e^6)*x^4)*log
(c)^2 - 60*(13489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a^2*b*d^6*n - 100
*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2 + 18*a^2*b*e^6*n)*x^4 + 60*(19*b^3*d^3*e^3*
n^3 - 20*a*b^2*d^3*e^3*n^2)*x^2 - 1800*(b^3*e^6*n*x^4 - b^3*d^6*n)*log(c)^2
 - 60*(20*b^3*d^3*e^3*n^2*x^2 + 147*b^3*d^6*n^2 - 60*a*b^2*d^6*n - 10*(b^3*
e^6*n^2 - 6*a*b^2*e^6*n)*x^4)*log(c) + 15*(588*b^3*d^5*e*n^3 - 240*a*b^2*d^
5*e*n^2 - (37*b^3*d^2*e^4*n^3 - 60*a*b^2*d^2*e^4*n^2)*x^2 + 60*(b^3*d^2*e^4
*n^2*x^2 - 4*b^3*d^5*e*n^2)*log(c))*x^(2/3) + 6*(4*(11*b^3*d*e^5*n^3 - 30*a
*b^2*d*e^5*n^2)*x^3 - 15*(29*b^3*d^4*e^2*n^3 - 20*a*b^2*d^4*e^2*n^2)*x - 60
*(2*b^3*d*e^5*n^2*x^3 - 5*b^3*d^4*e^2*n^2*x)*log(c))*x^(1/3))*log(e*x^(2/3)
 + d) + 1200*(5*(b^3*e^6*n^2 - 6*a*b^2*e^6*n + 18*a^2*b*e^6)*x^4 - 3*(19*b^
3*d^3*e^3*n^2 - 20*a*b^2*d^3*e^3*n)*x^2)*log(c) + 15*(53956*b^3*d^5*e*n^3 -
 35280*a*b^2*d^5*e*n^2 + 7200*a^2*b*d^5*e*n - (919*b^3*d^2*e^4*n^3 - 2220*a
*b^2*d^2*e^4*n^2 + 1800*a^2*b*d^2*e^4*n)*x^2 - 1800*(b^3*d^2*e^4*n*x^2 - 4*
b^3*d^5*e*n)*log(c)^2 - 60*(588*b^3*d^5*e*n^2 - 240*a*b^2*d^5*e*n - (37*b^3
*d^2*e^4*n^2 - 60*a*b^2*d^2*e^4*n)*x^2)*log(c))*x^(2/3) + 6*(8*(91*b^3*d*e^
5*n^3 - 330*a*b^2*d*e^5*n^2 + 450*a^2*b*d*e^5*n)*x^3 + 1800*(2*b^3*d*e^5*n*
x^3 - 5*b^3*d^4*e^2*n*x)*log(c)^2 - 5*(4669*b^3*d^4*e^2*n^3 - 5220*a*b^2*d^
4*e^2*n^2 + 1800*a^2*b*d^4*e^2*n)*x - 60*(4*(11*b^3*d*e^5*n^2 - 30*a*b^2*d*
e^5*n)*x^3 - 15*(29*b^3*d^4*e^2*n^2 - 20*a*b^2*d^4*e^2*n)*x)*log(c))*x^(1/3
))/e^6
```

giac [B] time = 1.35, size = 2224, normalized size = 2.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4*log(c)^3 + 3/4*a*b^2*x^4*log(c)^2 + 1/144000*(36000*(x^(2/3)*e
+ d)^6*e^(-5)*log(x^(2/3)*e + d)^3 - 216000*(x^(2/3)*e + d)^5*d*e^(-5)*log(
x^(2/3)*e + d)^3 + 540000*(x^(2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e + d)^3
 - 720000*(x^(2/3)*e + d)^3*d^3*e^(-5)*log(x^(2/3)*e + d)^3 + 540000*(x^(2/
3)*e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d)^3 - 216000*(x^(2/3)*e + d)*d^5*e^
(-5)*log(x^(2/3)*e + d)^3 - 18000*(x^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e +
d)^2 + 129600*(x^(2/3)*e + d)^5*d*e^(-5)*log(x^(2/3)*e + d)^2 - 405000*(x^(
2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e + d)^2 + 720000*(x^(2/3)*e + d)^3*d^
3*e^(-5)*log(x^(2/3)*e + d)^2 - 810000*(x^(2/3)*e + d)^2*d^4*e^(-5)*log(x^(
2/3)*e + d)^2 + 648000*(x^(2/3)*e + d)*d^5*e^(-5)*log(x^(2/3)*e + d)^2 + 60
00*(x^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e + d) - 51840*(x^(2/3)*e + d)^5*d*
e^(-5)*log(x^(2/3)*e + d) + 202500*(x^(2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)
*e + d) - 480000*(x^(2/3)*e + d)^3*d^3*e^(-5)*log(x^(2/3)*e + d) + 810000*(
x^(2/3)*e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d) - 1296000*(x^(2/3)*e + d)*d^
5*e^(-5)*log(x^(2/3)*e + d) - 1000*(x^(2/3)*e + d)^6*e^(-5) + 10368*(x^(2/3
)*e + d)^5*d*e^(-5) - 50625*(x^(2/3)*e + d)^4*d^2*e^(-5) + 160000*(x^(2/3)*
e + d)^3*d^3*e^(-5) - 405000*(x^(2/3)*e + d)^2*d^4*e^(-5) + 1296000*(x^(2/3
)*e + d)*d^5*e^(-5))*b^3*n^3*e^(-1) + 3/4*a^2*b*x^4*log(c) + 1/2400*(1800*(
x^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e + d)^2 - 10800*(x^(2/3)*e + d)^5*d*e^
(-5)*log(x^(2/3)*e + d)^2 + 27000*(x^(2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*
e + d)^2 - 36000*(x^(2/3)*e + d)^3*d^3*e^(-5)*log(x^(2/3)*e + d)^2 + 27000*
(x^(2/3)*e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d)^2 - 10800*(x^(2/3)*e + d)*d
^5*e^(-5)*log(x^(2/3)*e + d)^2 - 600*(x^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e
 + d) + 4320*(x^(2/3)*e + d)^5*d*e^(-5)*log(x^(2/3)*e + d) - 13500*(x^(2/3)
*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e + d) + 24000*(x^(2/3)*e + d)^3*d^3*e^(-5
)*log(x^(2/3)*e + d) - 27000*(x^(2/3)*e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d
```

```
) + 21600*(x^(2/3)*e + d)*d^5*e^(-5)*log(x^(2/3)*e + d) + 100*(x^(2/3)*e +
d)^6*e^(-5) - 864*(x^(2/3)*e + d)^5*d*e^(-5) + 3375*(x^(2/3)*e + d)^4*d^2*e
^(-5) - 8000*(x^(2/3)*e + d)^3*d^3*e^(-5) + 13500*(x^(2/3)*e + d)^2*d^4*e^(
-5) - 21600*(x^(2/3)*e + d)*d^5*e^(-5))*b^3*n^2*e^(-1)*log(c) + 1/80*(60*(x
^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e + d) - 360*(x^(2/3)*e + d)^5*d*e^(-5)*
log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e + d) -
1200*(x^(2/3)*e + d)^3*d^3*e^(-5)*log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^
2*d^4*e^(-5)*log(x^(2/3)*e + d) - 360*(x^(2/3)*e + d)*d^5*e^(-5)*log(x^(2/3
)*e + d) - 10*(x^(2/3)*e + d)^6*e^(-5) + 72*(x^(2/3)*e + d)^5*d*e^(-5) - 22
5*(x^(2/3)*e + d)^4*d^2*e^(-5) + 400*(x^(2/3)*e + d)^3*d^3*e^(-5) - 450*(x^
(2/3)*e + d)^2*d^4*e^(-5) + 360*(x^(2/3)*e + d)*d^5*e^(-5))*b^3*n*e^(-1)*lo
g(c)^2 + 1/4*a^3*x^4 + 1/2400*(1800*(x^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e
+ d)^2 - 10800*(x^(2/3)*e + d)^5*d*e^(-5)*log(x^(2/3)*e + d)^2 + 27000*(x^(
2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e + d)^2 - 36000*(x^(2/3)*e + d)^3*d^3
*e^(-5)*log(x^(2/3)*e + d)^2 + 27000*(x^(2/3)*e + d)^2*d^4*e^(-5)*log(x^(2/
3)*e + d)^2 - 10800*(x^(2/3)*e + d)*d^5*e^(-5)*log(x^(2/3)*e + d)^2 - 600*(
x^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e + d) + 4320*(x^(2/3)*e + d)^5*d*e^(-5
)*log(x^(2/3)*e + d) - 13500*(x^(2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e + d
) + 24000*(x^(2/3)*e + d)^3*d^3*e^(-5)*log(x^(2/3)*e + d) - 27000*(x^(2/3)*
e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d) + 21600*(x^(2/3)*e + d)*d^5*e^(-5)*l
og(x^(2/3)*e + d) + 100*(x^(2/3)*e + d)^6*e^(-5) - 864*(x^(2/3)*e + d)^5*d*
e^(-5) + 3375*(x^(2/3)*e + d)^4*d^2*e^(-5) - 8000*(x^(2/3)*e + d)^3*d^3*e^(
-5) + 13500*(x^(2/3)*e + d)^2*d^4*e^(-5) - 21600*(x^(2/3)*e + d)*d^5*e^(-5)
)*a*b^2*n^2*e^(-1) + 1/40*(60*(x^(2/3)*e + d)^6*e^(-5)*log(x^(2/3)*e + d) -
 360*(x^(2/3)*e + d)^5*d*e^(-5)*log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^4*
d^2*e^(-5)*log(x^(2/3)*e + d) - 1200*(x^(2/3)*e + d)^3*d^3*e^(-5)*log(x^(2/
3)*e + d) + 900*(x^(2/3)*e + d)^2*d^4*e^(-5)*log(x^(2/3)*e + d) - 360*(x^(2
/3)*e + d)*d^5*e^(-5)*log(x^(2/3)*e + d) - 10*(x^(2/3)*e + d)^6*e^(-5) + 72
*(x^(2/3)*e + d)^5*d*e^(-5) - 225*(x^(2/3)*e + d)^4*d^2*e^(-5) + 400*(x^(2/
3)*e + d)^3*d^3*e^(-5) - 450*(x^(2/3)*e + d)^2*d^4*e^(-5) + 360*(x^(2/3)*e
+ d)*d^5*e^(-5))*a*b^2*n*e^(-1)*log(c) + 1/80*(60*(x^(2/3)*e + d)^6*e^(-5)*
log(x^(2/3)*e + d) - 360*(x^(2/3)*e + d)^5*d*e^(-5)*log(x^(2/3)*e + d) + 90
0*(x^(2/3)*e + d)^4*d^2*e^(-5)*log(x^(2/3)*e + d) - 1200*(x^(2/3)*e + d)^3*
d^3*e^(-5)*log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^2*d^4*e^(-5)*log(x^(2/3
)*e + d) - 360*(x^(2/3)*e + d)*d^5*e^(-5)*log(x^(2/3)*e + d) - 10*(x^(2/3)*
e + d)^6*e^(-5) + 72*(x^(2/3)*e + d)^5*d*e^(-5) - 225*(x^(2/3)*e + d)^4*d^2
*e^(-5) + 400*(x^(2/3)*e + d)^3*d^3*e^(-5) - 450*(x^(2/3)*e + d)^2*d^4*e^(-
5) + 360*(x^(2/3)*e + d)*d^5*e^(-5))*a^2*b*n*e^(-1)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^{n}\right)+a\right)^{3} x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*ln(c*(e*x^(2/3)+d)^n)+a)^3,x)
```

```
[Out] int(x^3*(b*ln(c*(e*x^(2/3)+d)^n)+a)^3,x)
```

maxima [A] time = 0.59, size = 680, normalized size = 0.74

$$\frac{1}{4}\,b^3x^4\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)^{3}+\frac{3}{4}\,ab^2x^4\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)^{2}+\frac{3}{4}\,a^2bx^4\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+\frac{1}{4}\,a^3x^4-\frac{1}{80}\,a^2ben\left(\frac{60\,d^6\,\mathrm{lo}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4*log((e*x^(2/3) + d)^n*c)^3 + 3/4*a*b^2*x^4*log((e*x^(2/3) + d)^
n*c)^2 + 3/4*a^2*b*x^4*log((e*x^(2/3) + d)^n*c) + 1/4*a^3*x^4 - 1/80*a^2*b*
e*n*(60*d^6*log(e*x^(2/3) + d)/e^7 + (10*e^5*x^4 - 12*d*e^4*x^(10/3) + 15*d
^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 + 30*d^4*e*x^(4/3) - 60*d^5*x^(2/3))/e^6) -
 1/2400*(60*e*n*(60*d^6*log(e*x^(2/3) + d)/e^7 + (10*e^5*x^4 - 12*d*e^4*x^(
10/3) + 15*d^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 + 30*d^4*e*x^(4/3) - 60*d^5*x^(
2/3))/e^6)*log((e*x^(2/3) + d)^n*c) - (100*e^6*x^4 - 264*d*e^5*x^(10/3) + 5
55*d^2*e^4*x^(8/3) - 1140*d^3*e^3*x^2 + 1800*d^6*log(e*x^(2/3) + d)^2 + 261
0*d^4*e^2*x^(4/3) + 8820*d^6*log(e*x^(2/3) + d) - 8820*d^5*e*x^(2/3))*n^2/e
^6)*a*b^2 - 1/144000*(1800*e*n*(60*d^6*log(e*x^(2/3) + d)/e^7 + (10*e^5*x^4
 - 12*d*e^4*x^(10/3) + 15*d^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 + 30*d^4*e*x^(4/
3) - 60*d^5*x^(2/3))/e^6)*log((e*x^(2/3) + d)^n*c)^2 + e*n*((1000*e^6*x^4 -
 4368*d*e^5*x^(10/3) + 36000*d^6*log(e*x^(2/3) + d)^3 + 13785*d^2*e^4*x^(8/
3) - 41180*d^3*e^3*x^2 + 264600*d^6*log(e*x^(2/3) + d)^2 + 140070*d^4*e^2*x
^(4/3) + 809340*d^6*log(e*x^(2/3) + d) - 809340*d^5*e*x^(2/3))*n^2/e^7 - 60
*(100*e^6*x^4 - 264*d*e^5*x^(10/3) + 555*d^2*e^4*x^(8/3) - 1140*d^3*e^3*x^2
 + 1800*d^6*log(e*x^(2/3) + d)^2 + 2610*d^4*e^2*x^(4/3) + 8820*d^6*log(e*x^
(2/3) + d) - 8820*d^5*e*x^(2/3))*n*log((e*x^(2/3) + d)^n*c)/e^7))*b^3
```

mupad [B] time = 8.10, size = 992, normalized size = 1.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(2/3))^n))^3,x)
```

```
[Out] (a^3*x^4)/4 + (b^3*x^4*log(c*(d + e*x^(2/3))^n)^3)/4 - (b^3*n^3*x^4)/144 +
(3*a*b^2*x^4*log(c*(d + e*x^(2/3))^n)^2)/4 - (b^3*n*x^4*log(c*(d + e*x^(2/3
))^n)^2)/8 + (b^3*n^2*x^4*log(c*(d + e*x^(2/3))^n))/24 + (a*b^2*n^2*x^4)/24
 - (b^3*d^6*log(c*(d + e*x^(2/3))^n)^3)/(4*e^6) + (3*a^2*b*x^4*log(c*(d + e
*x^(2/3))^n))/4 - (a^2*b*n*x^4)/8 - (a*b^2*n*x^4*log(c*(d + e*x^(2/3))^n))/
4 - (13489*b^3*d^6*n^3*log(d + e*x^(2/3)))/(2400*e^6) + (2059*b^3*d^3*n^3*x
^2)/(7200*e^3) - (919*b^3*d^2*n^3*x^(8/3))/(9600*e^2) - (4669*b^3*d^4*n^3*x
^(4/3))/(4800*e^4) + (13489*b^3*d^5*n^3*x^(2/3))/(2400*e^5) - (3*a*b^2*d^6*
log(c*(d + e*x^(2/3))^n)^2)/(4*e^6) + (147*b^3*d^6*n*log(c*(d + e*x^(2/3))^
n)^2)/(80*e^6) + (91*b^3*d*n^3*x^(10/3))/(3000*e) - (3*a^2*b*d^6*n*log(d +
e*x^(2/3)))/(4*e^6) + (3*b^3*d*n*x^(10/3)*log(c*(d + e*x^(2/3))^n)^2)/(20*e
) - (11*b^3*d*n^2*x^(10/3)*log(c*(d + e*x^(2/3))^n))/(100*e) + (a^2*b*d^3*n
*x^2)/(4*e^3) - (3*a^2*b*d^2*n*x^(8/3))/(16*e^2) - (3*a^2*b*d^4*n*x^(4/3))/
(8*e^4) + (3*a^2*b*d^5*n*x^(2/3))/(4*e^5) - (11*a*b^2*d*n^2*x^(10/3))/(100*
e) + (147*a*b^2*d^6*n^2*log(d + e*x^(2/3)))/(40*e^6) + (b^3*d^3*n*x^2*log(c
*(d + e*x^(2/3))^n)^2)/(4*e^3) - (19*b^3*d^3*n^2*x^2*log(c*(d + e*x^(2/3))^
n))/(40*e^3) - (3*b^3*d^2*n*x^(8/3)*log(c*(d + e*x^(2/3))^n)^2)/(16*e^2) +
(37*b^3*d^2*n^2*x^(8/3)*log(c*(d + e*x^(2/3))^n))/(160*e^2) - (3*b^3*d^4*n*
x^(4/3)*log(c*(d + e*x^(2/3))^n)^2)/(8*e^4) + (87*b^3*d^4*n^2*x^(4/3)*log(c
*(d + e*x^(2/3))^n))/(80*e^4) + (3*b^3*d^5*n*x^(2/3)*log(c*(d + e*x^(2/3))^
n)^2)/(4*e^5) - (147*b^3*d^5*n^2*x^(2/3)*log(c*(d + e*x^(2/3))^n))/(40*e^5)
 - (19*a*b^2*d^3*n^2*x^2)/(40*e^3) + (37*a*b^2*d^2*n^2*x^(8/3))/(160*e^2) +
 (87*a*b^2*d^4*n^2*x^(4/3))/(80*e^4) - (147*a*b^2*d^5*n^2*x^(2/3))/(40*e^5)
 + (3*a^2*b*d*n*x^(10/3))/(20*e) + (3*a*b^2*d*n*x^(10/3)*log(c*(d + e*x^(2/
3))^n))/(10*e) + (a*b^2*d^3*n*x^2*log(c*(d + e*x^(2/3))^n))/(2*e^3) - (3*a*
b^2*d^2*n*x^(8/3)*log(c*(d + e*x^(2/3))^n))/(8*e^2) - (3*a*b^2*d^4*n*x^(4/3
)*log(c*(d + e*x^(2/3))^n))/(4*e^4) + (3*a*b^2*d^5*n*x^(2/3)*log(c*(d + e*x
^(2/3))^n))/(2*e^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
[Out] Timed out
```

3.482 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 dx$

Optimal. Leaf size=449

$$\frac{b^2n^2\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3}-\frac{9b^2dn^2\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4e^3}+\frac{9ab^2d^2n^2x^{2/3}}{e^2}-\frac{9bc}{}$$

```
[Out] 9/8*b^3*d*n^3*(d+e*x^(2/3))^2/e^3-1/9*b^3*n^3*(d+e*x^(2/3))^3/e^3+9*a*b^2*d
^2*n^2*x^(2/3)/e^2-9*b^3*d^2*n^3*x^(2/3)/e^2+9*b^3*d^2*n^2*(d+e*x^(2/3))*ln
(c*(d+e*x^(2/3))^n)/e^3-9/4*b^2*d*n^2*(d+e*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3
))^n))/e^3+1/3*b^2*n^2*(d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3-9/2*
b*d^2*n*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^3+9/4*b*d*n*(d+e*x^(2
/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^3-1/2*b*n*(d+e*x^(2/3))^3*(a+b*ln(c*
(d+e*x^(2/3))^n))^2/e^3+3/2*d^2*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))^3
/e^3-3/2*d*(d+e*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^3+1/2*(d+e*x^(2/
3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^3
```

Rubi [A] time = 0.46, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{b^2n^2\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3}-\frac{9b^2dn^2\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4e^3}+\frac{9ab^2d^2n^2x^{2/3}}{e^2}-\frac{9bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (9*b^3*d*n^3*(d + e*x^(2/3))^2)/(8*e^3) - (b^3*n^3*(d + e*x^(2/3))^3)/(9*e^
3) + (9*a*b^2*d^2*n^2*x^(2/3))/e^2 - (9*b^3*d^2*n^3*x^(2/3))/e^2 + (9*b^3*d
^2*n^2*(d + e*x^(2/3))*Log[c*(d + e*x^(2/3))^n])/e^3 - (9*b^2*d*n^2*(d + e*
x^(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(4*e^3) + (b^2*n^2*(d + e*x^(2
/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*e^3) - (9*b*d^2*n*(d + e*x^(2/3
))*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(2*e^3) + (9*b*d*n*(d + e*x^(2/3))^2
*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(4*e^3) - (b*n*(d + e*x^(2/3))^3*(a +
b*Log[c*(d + e*x^(2/3))^n])^2)/(2*e^3) + (3*d^2*(d + e*x^(2/3))*(a + b*Log[
c*(d + e*x^(2/3))^n])^3)/(2*e^3) - (3*d*(d + e*x^(2/3))^2*(a + b*Log[c*(d +
 e*x^(2/3))^n])^3)/(2*e^3) + ((d + e*x^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3)
)^n])^3)/(2*e^3)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 dx = \frac{3}{2}\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3 dx, x, x^{2/3}\right)$$
$$= \frac{3}{2}\text{Subst}\left(\int\left(\frac{d^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2} - \frac{2d(d+ex)\left(a+b\log\left(c(d+ex\right.\right.}{e^2}\right.\right.$$
$$= \frac{3\,\text{Subst}\left(\int(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3 dx, x, x^{2/3}\right)}{2e^2} - \frac{(3d)\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{3\,\text{Subst}\left(\int x^2\left(a+b\log\left(cx^n\right)\right)^3 dx, x, d+ex^{2/3}\right)}{2e^3} - \frac{(3d)\,\text{Subst}\left(\int x\left(a+b\right.\right.}{}$$
$$= \frac{3d^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3} - \frac{3d\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\right.\right.}{2e^3}$$
$$= -\frac{9bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^3} + \frac{9bdn\left(d+ex^{2/3}\right)^2\left(a+b\right.}{4e}$$
$$= \frac{9b^3dn^3\left(d+ex^{2/3}\right)^2}{8e^3} - \frac{b^3n^3\left(d+ex^{2/3}\right)^3}{9e^3} + \frac{9ab^2d^2n^2x^{2/3}}{e^2} - \frac{9b^2dn^2\left(d+ex^{2\right.}}{}$$
$$= \frac{9b^3dn^3\left(d+ex^{2/3}\right)^2}{8e^3} - \frac{b^3n^3\left(d+ex^{2/3}\right)^3}{9e^3} + \frac{9ab^2d^2n^2x^{2/3}}{e^2} - \frac{9b^3d^2n^3x^{2/3}}{e^2} + \cdots$$

Mathematica [A] time = 0.43, size = 428, normalized size = 0.95

$$\frac{36a^3d^3 + 36a^3e^3x^2 + 6b\left(18a^2\left(d^3+e^3x^2\right) - 6abn\left(11d^3+6d^2ex^{2/3}-3de^2x^{4/3}+2e^3x^2\right) + b^2n^2\left(66d^3+66d^2ex^{2/3}\cdots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (36*a^3*d^3 - 198*a^2*b*d^3*n - 108*a^2*b*d^2*e*n*x^(2/3) + 396*a*b^2*d^2*e
*n^2*x^(2/3) - 510*b^3*d^2*e*n^3*x^(2/3) + 54*a^2*b*d*e^2*n*x^(4/3) - 90*a*
b^2*d*e^2*n^2*x^(4/3) + 57*b^3*d*e^2*n^3*x^(4/3) + 36*a^3*e^3*x^2 - 36*a^2*
b*e^3*n*x^2 + 24*a*b^2*e^3*n^2*x^2 - 8*b^3*e^3*n^3*x^2 + 114*b^3*d^3*n^3*Lo
g[d + e*x^(2/3)] + 6*b*(18*a^2*(d^3 + e^3*x^2) - 6*a*b*n*(11*d^3 + 6*d^2*e*
x^(2/3) - 3*d*e^2*x^(4/3) + 2*e^3*x^2) + b^2*n^2*(66*d^3 + 66*d^2*e*x^(2/3)
 - 15*d*e^2*x^(4/3) + 4*e^3*x^2))*Log[c*(d + e*x^(2/3))^n] + 18*b^2*(6*a*(d
^3 + e^3*x^2) - b*n*(11*d^3 + 6*d^2*e*x^(2/3) - 3*d*e^2*x^(4/3) + 2*e^3*x^2
))*Log[c*(d + e*x^(2/3))^n]^2 + 36*b^3*(d^3 + e^3*x^2)*Log[c*(d + e*x^(2/3)
)^n]^3)/(72*e^3)
```

fricas [A] time = 0.52, size = 720, normalized size = 1.60

$$\frac{36\,b^3e^3x^2\log(c)^3 - 36\left(b^3e^3n - 3\,ab^2e^3\right)x^2\log(c)^2 + 36\left(b^3e^3n^3x^2 + b^3d^3n^3\right)\log\left(ex^{\frac{2}{3}}+d\right)^3 + 12\left(2\,b^3e^3n^2 - 6\,ab^2\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/72*(36*b^3*e^3*x^2*log(c)^3 - 36*(b^3*e^3*n - 3*a*b^2*e^3)*x^2*log(c)^2 +
 36*(b^3*e^3*n^3*x^2 + b^3*d^3*n^3)*log(e*x^(2/3) + d)^3 + 12*(2*b^3*e^3*n^
2 - 6*a*b^2*e^3*n + 9*a^2*b*e^3)*x^2*log(c) - 4*(2*b^3*e^3*n^3 - 6*a*b^2*e^
```

```
3*n^2 + 9*a^2*b*e^3*n - 9*a^3*e^3)*x^2 + 18*(3*b^3*d*e^2*n^3*x^(4/3) - 6*b^
3*d^2*e*n^3*x^(2/3) - 11*b^3*d^3*n^3 + 6*a*b^2*d^3*n^2 - 2*(b^3*e^3*n^3 - 3
*a*b^2*e^3*n^2)*x^2 + 6*(b^3*e^3*n^2*x^2 + b^3*d^3*n^2)*log(c))*log(e*x^(2/
3) + d)^2 + 6*(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n + 2*(2*b^
3*e^3*n^3 - 6*a*b^2*e^3*n^2 + 9*a^2*b*e^3*n)*x^2 + 18*(b^3*e^3*n*x^2 + b^3*
d^3*n)*log(c)^2 - 6*(11*b^3*d^3*n^2 - 6*a*b^2*d^3*n + 2*(b^3*e^3*n^2 - 3*a*
b^2*e^3*n)*x^2)*log(c) + 6*(11*b^3*d^2*e*n^3 - 6*b^3*d^2*e*n^2*log(c) - 6*a
*b^2*d^2*e*n^2)*x^(2/3) + 3*(6*b^3*d*e^2*n^2*x*log(c) - (5*b^3*d*e^2*n^3 -
6*a*b^2*d*e^2*n^2)*x)*x^(1/3))*log(e*x^(2/3) + d) - 6*(85*b^3*d^2*e*n^3 + 1
8*b^3*d^2*e*n*log(c)^2 - 66*a*b^2*d^2*e*n^2 + 18*a^2*b*d^2*e*n - 6*(11*b^3*
d^2*e*n^2 - 6*a*b^2*d^2*e*n)*log(c))*x^(2/3) + 3*(18*b^3*d*e^2*n*x*log(c)^2
 - 6*(5*b^3*d*e^2*n^2 - 6*a*b^2*d*e^2*n)*x*log(c) + (19*b^3*d*e^2*n^3 - 30*
a*b^2*d*e^2*n^2 + 18*a^2*b*d*e^2*n)*x)*x^(1/3))/e^3
```

giac [B] time = 1.87, size = 778, normalized size = 1.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*log(c)^3 + 1/72*(36*x^2*log(x^(2/3)*e + d)^3 + (36*d^3*log(x^(2
/3)*e + d)^3 - 36*(x^(2/3)*e + d)^3*log(x^(2/3)*e + d)^2 + 162*(x^(2/3)*e +
 d)^2*d*log(x^(2/3)*e + d)^2 - 324*(x^(2/3)*e + d)*d^2*log(x^(2/3)*e + d)^2
 + 24*(x^(2/3)*e + d)^3*log(x^(2/3)*e + d) - 162*(x^(2/3)*e + d)^2*d*log(x^
(2/3)*e + d) + 648*(x^(2/3)*e + d)*d^2*log(x^(2/3)*e + d) - 8*(x^(2/3)*e +
d)^3 + 81*(x^(2/3)*e + d)^2*d - 648*(x^(2/3)*e + d)*d^2)*e^(-3))*b^3*n^3 +
1/12*(18*x^2*log(x^(2/3)*e + d)^2 + (18*d^3*log(x^(2/3)*e + d)^2 - 12*(x^(2
/3)*e + d)^3*log(x^(2/3)*e + d) + 54*(x^(2/3)*e + d)^2*d*log(x^(2/3)*e + d)
 - 108*(x^(2/3)*e + d)*d^2*log(x^(2/3)*e + d) + 4*(x^(2/3)*e + d)^3 - 27*(x
^(2/3)*e + d)^2*d + 108*(x^(2/3)*e + d)*d^2)*e^(-3))*b^3*n^2*log(c) + 1/4*(
6*x^2*log(x^(2/3)*e + d) + (6*d^3*e^(-4)*log(abs(x^(2/3)*e + d)) + (3*d*x^(
4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*e)*b^3*n*log(c)^2 + 3/2*a*b^2*x
^2*log(c)^2 + 1/12*(18*x^2*log(x^(2/3)*e + d)^2 + (18*d^3*log(x^(2/3)*e + d
)^2 - 12*(x^(2/3)*e + d)^3*log(x^(2/3)*e + d) + 54*(x^(2/3)*e + d)^2*d*log(
x^(2/3)*e + d) - 108*(x^(2/3)*e + d)*d^2*log(x^(2/3)*e + d) + 4*(x^(2/3)*e
+ d)^3 - 27*(x^(2/3)*e + d)^2*d + 108*(x^(2/3)*e + d)*d^2)*e^(-3))*a*b^2*n^
2 + 1/2*(6*x^2*log(x^(2/3)*e + d) + (6*d^3*e^(-4)*log(abs(x^(2/3)*e + d)) +
 (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*e)*a*b^2*n*log(c) + 3/
2*a^2*b*x^2*log(c) + 1/4*(6*x^2*log(x^(2/3)*e + d) + (6*d^3*e^(-4)*log(abs(
x^(2/3)*e + d)) + (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*e)*a^
2*b*n + 1/2*a^3*x^2
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(b \ln \left(c \left(e\, x^{\frac{2}{3}}+d\right)^{n}\right)+a\right)^{3} x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(e*x^(2/3)+d)^n)+a)^3,x)
```

```
[Out] int(x*(b*ln(c*(e*x^(2/3)+d)^n)+a)^3,x)
```

maxima [A] time = 0.57, size = 484, normalized size = 1.08

$$\frac{1}{2}\, b^3x^2 \log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^3 + \frac{3}{2}\, ab^2x^2 \log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2 + \frac{1}{4}\, a^2ben\left(\frac{6\, d^3 \log\left(ex^{\frac{2}{3}}+d\right)}{e^4} - \frac{2\, e^2x^2 - 3\, dex^{\frac{4}{3}} + 6\, d^2x}{e^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*log((e*x^(2/3) + d)^n*c)^3 + 3/2*a*b^2*x^2*log((e*x^(2/3) + d)^
n*c)^2 + 1/4*a^2*b*e*n*(6*d^3*log(e*x^(2/3) + d)/e^4 - (2*e^2*x^2 - 3*d*e*x
^(4/3) + 6*d^2*x^(2/3))/e^3) + 3/2*a^2*b*x^2*log((e*x^(2/3) + d)^n*c) + 1/2
*a^3*x^2 + 1/12*(6*e*n*(6*d^3*log(e*x^(2/3) + d)/e^4 - (2*e^2*x^2 - 3*d*e*x
^(4/3) + 6*d^2*x^(2/3))/e^3)*log((e*x^(2/3) + d)^n*c) + (4*e^3*x^2 - 18*d^3
*log(e*x^(2/3) + d)^2 - 15*d*e^2*x^(4/3) - 66*d^3*log(e*x^(2/3) + d) + 66*d
^2*e*x^(2/3))*n^2/e^3)*a*b^2 + 1/72*(18*e*n*(6*d^3*log(e*x^(2/3) + d)/e^4 -
 (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3)*log((e*x^(2/3) + d)^n*c)^
2 + e*n*((36*d^3*log(e*x^(2/3) + d)^3 - 8*e^3*x^2 + 198*d^3*log(e*x^(2/3) +
 d)^2 + 57*d*e^2*x^(4/3) + 510*d^3*log(e*x^(2/3) + d) - 510*d^2*e*x^(2/3))*
n^2/e^4 + 6*(4*e^3*x^2 - 18*d^3*log(e*x^(2/3) + d)^2 - 15*d*e^2*x^(4/3) - 6
6*d^3*log(e*x^(2/3) + d) + 66*d^2*e*x^(2/3))*n*log((e*x^(2/3) + d)^n*c)/e^4
))*b^3
```

mupad [B] time = 0.72, size = 575, normalized size = 1.28

$$\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)^3\,\left(\frac{b^3\,x^2}{2}+\frac{b^3\,d^3}{2\,e^3}\right)-x^{4/3}\,\left(\frac{d\,\left(\frac{3\,a^3}{2}-\frac{3\,a^2\,b\,n}{2}+a\,b^2\,n^2-\frac{b^3\,n^3}{3}\right)}{2\,e}-\frac{d\,\left(6\,a^3-6\,a\,b^2\,n^2+5\,b^3\,n^3\right)}{8\,e}\right)+\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(2/3))^n))^3,x)
```

```
[Out] log(c*(d + e*x^(2/3))^n)^3*((b^3*x^2)/2 + (b^3*d^3)/(2*e^3)) - x^(4/3)*((d*
((3*a^3)/2 - (b^3*n^3)/3 + a*b^2*n^2 - (3*a^2*b*n)/2))/(2*e) - (d*(6*a^3 +
5*b^3*n^3 - 6*a*b^2*n^2))/(8*e)) + log(c*(d + e*x^(2/3))^n)^2*((b^2*x^2*(3*
a - b*n))/2 - (x^(4/3)*((3*b^2*d*(3*a - b*n))/(2*e) - (9*a*b^2*d)/(2*e)))/2
 + (d*(6*a*b^2*d^2 - 11*b^3*d^2*n))/(4*e^3) + (d*x^(2/3)*((6*b^2*d*(3*a - b
*n))/e - (18*a*b^2*d)/e))/(4*e)) + x^(2/3)*((d*((d*((3*a^3)/2 - (b^3*n^3)/3
 + a*b^2*n^2 - (3*a^2*b*n)/2))/e - (d*(6*a^3 + 5*b^3*n^3 - 6*a*b^2*n^2))/(4
*e)))/e + (b^2*d^2*n^2*(6*a - 11*b*n))/(2*e^2)) + x^2*(a^3/2 - (b^3*n^3)/9
+ (a*b^2*n^2)/3 - (a^2*b*n)/2) + (log(c*(d + e*x^(2/3))^n)*((x^(2/3)*((d*(2
*b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 6*b*d*e*(3*a^2 - b^2*n^2)))/e + 12*b
^3*d^2*n^2))/(2*e) - (x^(4/3)*(2*b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 6*b*
d*e*(3*a^2 - b^2*n^2)))/(4*e) + (b*e*x^2*(9*a^2 + 2*b^2*n^2 - 6*a*b*n))/3))
/(2*e) + (log(d + e*x^(2/3))*(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*
d^3*n))/(12*e^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

3.483 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx$

Optimal. Leaf size=139

$$-9b^2n^2\text{Li}_3\left(\frac{x^{2/3}e}{d}+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{9}{2}bn\text{Li}_2\left(\frac{x^{2/3}e}{d}+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+\frac{3}{2}\log\left(-\right.$$

```
[Out] 3/2*(a+b*ln(c*(d+e*x^(2/3))^n))^3*ln(-e*x^(2/3)/d)+9/2*b*n*(a+b*ln(c*(d+e*x
^(2/3))^n))^2*polylog(2,1+e*x^(2/3)/d)-9*b^2*n^2*(a+b*ln(c*(d+e*x^(2/3))^n)
)*polylog(3,1+e*x^(2/3)/d)+9*b^3*n^3*polylog(4,1+e*x^(2/3)/d)
```

Rubi [A] time = 0.20, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2454, 2396, 2433, 2374, 2383, 6589}

$$-9b^2n^2\text{PolyLog}\left(3,\frac{ex^{2/3}}{d}+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{9}{2}bn\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x,x]
```

```
[Out] (3*(a + b*Log[c*(d + e*x^(2/3))^n])^3*Log[-((e*x^(2/3))/d)])/2 + (9*b*n*(a
+ b*Log[c*(d + e*x^(2/3))^n])^2*PolyLog[2, 1 + (e*x^(2/3))/d])/2 - 9*b^2*n^
2*(a + b*Log[c*(d + e*x^(2/3))^n])*PolyLog[3, 1 + (e*x^(2/3))/d] + 9*b^3*n^
3*PolyLog[4, 1 + (e*x^(2/3))/d]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx = \frac{3}{2}\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x}\,dx, x, x^{2/3}\right)$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right) - \frac{1}{2}(9ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\left(\right.}{}\right.$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right) - \frac{1}{2}(9bn)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(\right.\right.}{}\right.$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right) + \frac{9}{2}bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right.\right.$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right) + \frac{9}{2}bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right.\right.$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right) + \frac{9}{2}bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right.\right.$$

Mathematica [B] time = 0.26, size = 339, normalized size = 2.44

$$\frac{9}{2}b^2n^2\left(-2\text{Li}_3\left(\frac{x^{2/3}e}{d}+1\right)+2\text{Li}_2\left(\frac{x^{2/3}e}{d}+1\right)\log\left(d+ex^{2/3}\right)+\log\left(-\frac{ex^{2/3}}{d}\right)\log^2\left(d+ex^{2/3}\right)\right)\left(a+b\log\left(c\left(d+ex\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*((Log[d + e*x^(
2/3)] - Log[1 + (e*x^(2/3))/d])*Log[x] - (3*PolyLog[2, -((e*x^(2/3))/d)])/2
) + (9*b^2*n^2*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*(L
og[d + e*x^(2/3)]^2*Log[-((e*x^(2/3))/d)] + 2*Log[d + e*x^(2/3)]*PolyLog[2,
 1 + (e*x^(2/3))/d] - 2*PolyLog[3, 1 + (e*x^(2/3))/d]))/2 + (3*b^3*n^3*(Log
[d + e*x^(2/3)]^3*Log[-((e*x^(2/3))/d)] + 3*Log[d + e*x^(2/3)]^2*PolyLog[2,
 1 + (e*x^(2/3))/d] - 6*Log[d + e*x^(2/3)]*PolyLog[3, 1 + (e*x^(2/3))/d] +
6*PolyLog[4, 1 + (e*x^(2/3))/d]))/2
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^3+3\,ab^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+3\,a^2b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3/x, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^n\right)+a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^3/x,x)
```

```
[Out] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^3/x,x)
```

maxima [B] time = 0.80, size = 282, normalized size = 2.03

$$\frac{3}{2}\left(\log\left(ex^{\frac{2}{3}}+d\right)^3\log\left(-\frac{ex^{\frac{2}{3}}+d}{d}+1\right)+3\,\mathrm{Li}_2\left(\frac{ex^{\frac{2}{3}}+d}{d}\right)\log\left(ex^{\frac{2}{3}}+d\right)^2-6\,\log\left(ex^{\frac{2}{3}}+d\right)\mathrm{Li}_3\left(\frac{ex^{\frac{2}{3}}+d}{d}\right)+6\,\mathrm{Li}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x,x, algorithm="maxima")
```

```
[Out] 3/2*(log(e*x^(2/3) + d)^3*log(-(e*x^(2/3) + d)/d + 1) + 3*dilog((e*x^(2/3)
+ d)/d)*log(e*x^(2/3) + d)^2 - 6*log(e*x^(2/3) + d)*polylog(3, (e*x^(2/3) +
 d)/d) + 6*polylog(4, (e*x^(2/3) + d)/d))*b^3*n^3 + a^3*log(x) + 9/2*(b^3*n
^2*log(c) + a*b^2*n^2)*(log(e*x^(2/3) + d)^2*log(-(e*x^(2/3) + d)/d + 1) +
2*dilog((e*x^(2/3) + d)/d)*log(e*x^(2/3) + d) - 2*polylog(3, (e*x^(2/3) + d
)/d)) + 9/2*(b^3*n*log(c)^2 + 2*a*b^2*n*log(c) + a^2*b*n)*(log(e*x^(2/3) +
d)*log(-(e*x^(2/3) + d)/d + 1) + dilog((e*x^(2/3) + d)/d)) + (b^3*log(c)^3
+ 3*a*b^2*log(c)^2 + 3*a^2*b*log(c))*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x,x)
[Out] Timed out
```

3.484 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=451

$$-\frac{3b^2e^3n^2\text{Li}_2\left(\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3}-\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3}-\frac{3b^2e^3}{}$$

```
[Out] -3/2*b^2*e^2*n^2*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^3/x^(2/3)-3/2*
b^2*e^3*n^2*ln(1-d/(d+e*x^(2/3)))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^3-3/4*b*e*n
*(a+b*ln(c*(d+e*x^(2/3))^n))^2/d/x^(4/3)+3/2*b*e^2*n*(d+e*x^(2/3))*(a+b*ln(
c*(d+e*x^(2/3))^n))^2/d^3/x^(2/3)+3/2*b*e^3*n*ln(1-d/(d+e*x^(2/3)))*(a+b*ln
(c*(d+e*x^(2/3))^n))^2/d^3-1/2*(a+b*ln(c*(d+e*x^(2/3))^n))^3/x^2-3*b^2*e^3*
n^2*(a+b*ln(c*(d+e*x^(2/3))^n))*ln(-e*x^(2/3)/d)/d^3+b^3*e^3*n^3*ln(x)/d^3+
3/2*b^3*e^3*n^3*polylog(2,d/(d+e*x^(2/3)))/d^3-3*b^2*e^3*n^2*(a+b*ln(c*(d+e
*x^(2/3))^n))*polylog(2,d/(d+e*x^(2/3)))/d^3-3*b^3*e^3*n^3*polylog(2,1+e*x^
(2/3)/d)/d^3-3*b^3*e^3*n^3*polylog(3,d/(d+e*x^(2/3)))/d^3
```

Rubi [A] time = 1.01, antiderivative size = 428, normalized size of antiderivative = 0.95, number of steps used = 22, number of rules used = 16, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31}

$$\frac{3b^2e^3n^2\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3}-\frac{9b^3e^3n^3\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)}{2d^3}-\frac{3b^3e^3n^3\text{PolyLog}(}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^3,x]
```

```
[Out] (-3*b^2*e^2*n^2*(d + e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*d^3*x^
(2/3)) + (3*b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(4*d^3) - (3*b*e*n*
(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(4*d*x^(4/3)) + (3*b*e^2*n*(d + e*x^(2/
3))*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(2*d^3*x^(2/3)) - (e^3*(a + b*Log[c
*(d + e*x^(2/3))^n])^3)/(2*d^3) - (a + b*Log[c*(d + e*x^(2/3))^n])^3/(2*x^2
) - (9*b^2*e^3*n^2*(a + b*Log[c*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)])/
(2*d^3) + (3*b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n])^2*Log[-((e*x^(2/3))/d
)])/(2*d^3) + (b^3*e^3*n^3*Log[x])/d^3 - (9*b^3*e^3*n^3*PolyLog[2, 1 + (e*x
^(2/3))/d])/(2*d^3) + (3*b^2*e^3*n^2*(a + b*Log[c*(d + e*x^(2/3))^n])*PolyL
og[2, 1 + (e*x^(2/3))/d])/d^3 - (3*b^3*e^3*n^3*PolyLog[3, 1 + (e*x^(2/3))/d
])/d^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
```

```
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}dx = \frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^4}dx, x, x^{2/3}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2} + \frac{1}{2}(3ben)\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(d+ex)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2} + \frac{1}{2}(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}dx, x, d\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2} + \frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}dx, x, d+ex^{2/3}\right.}{2d}$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4dx^{4/3}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2} - \frac{(3ben)}{}$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4dx^{4/3}} + \frac{3be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex\right.\right.\right.}{2d^3x^{2/3}}$$

$$= -\frac{3b^2e^2n^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3x^{2/3}} - \frac{3ben\left(a+b\log\left(c\left(d+e\right.\right.\right.}{4dx^{4/3}}$$

$$= -\frac{3b^2e^2n^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3x^{2/3}} + \frac{3be^3n\left(a+b\log\left(c\left(d+\right.\right.\right.}{4d^3}$$

$$= -\frac{3b^2e^2n^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3x^{2/3}} + \frac{3be^3n\left(a+b\log\left(c\left(d+\right.\right.\right.}{4d^3}$$

Mathematica [A] time = 0.87, size = 764, normalized size = 1.69

$$\frac{-6b^2n^2\left(\left(d^3+e^3x^2\right)\log^2\left(d+ex^{2/3}\right)+\log\left(d+ex^{2/3}\right)\left(d^2ex^{2/3}-2e^3x^2\log\left(-\frac{ex^{2/3}}{d}\right)-2de^2x^{4/3}-3e^3x^2\right)-2e^3x^2\text{Li}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^3,x]
```

```
[Out] (-3*b*d^2*e*n*x^(2/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))
^n])^2 + 6*b*d*e^2*n*x^(4/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x
^(2/3))^n])^2 - 6*b*d^3*n*Log[d + e*x^(2/3)]*(a - b*n*Log[d + e*x^(2/3)] +
b*Log[c*(d + e*x^(2/3))^n])^2 - 6*b*e^3*n*x^2*Log[d + e*x^(2/3)]*(a - b*n*L
og[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 - 2*d^3*(a - b*n*Log[d +
e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3 + 4*b*e^3*n*x^2*(a - b*n*Log[d +
 e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*Log[x] - 6*b^2*n^2*(a - b*n*Log
[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*((d^3 + e^3*x^2)*Log[d + e*x^
(2/3)]^2 + e^2*x^(4/3)*(d + 3*e*x^(2/3)*Log[-((e*x^(2/3))/d)]) + Log[d + e*
x^(2/3)]*(d^2*e*x^(2/3) - 2*d*e^2*x^(4/3) - 3*e^3*x^2 - 2*e^3*x^2*Log[-((e*
x^(2/3))/d)]) - 2*e^3*x^2*PolyLog[2, 1 + (e*x^(2/3))/d]) + b^3*n^3*(-6*d*e^
2*x^(4/3)*Log[d + e*x^(2/3)] - 6*e^3*x^2*Log[d + e*x^(2/3)] - 3*d^2*e*x^(2/
```

```
3)*Log[d + e*x^(2/3)]^2 + 6*d*e^2*x^(4/3)*Log[d + e*x^(2/3)]^2 + 9*e^3*x^2*
Log[d + e*x^(2/3)]^2 - 2*d^3*Log[d + e*x^(2/3)]^3 - 2*e^3*x^2*Log[d + e*x^(
2/3)]^3 + 6*e^3*x^2*Log[-((e*x^(2/3))/d)] - 18*e^3*x^2*Log[d + e*x^(2/3)]*L
og[-((e*x^(2/3))/d)] + 6*e^3*x^2*Log[d + e*x^(2/3)]^2*Log[-((e*x^(2/3))/d)]
 + 6*e^3*x^2*(-3 + 2*Log[d + e*x^(2/3)])*PolyLog[2, 1 + (e*x^(2/3))/d] - 12
*e^3*x^2*PolyLog[3, 1 + (e*x^(2/3))/d]))/(4*d^3*x^2)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^3+3\,ab^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+3\,a^2b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3/x^3, x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^n\right)+a\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^3/x^3,x)
```

```
[Out] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^3/x^3,x)
```

maxima [A] time = 1.01, size = 740, normalized size = 1.64

$$\frac{3\left(\log\left(ex^{\frac{2}{3}}+d\right)^2\log\left(-\frac{ex^{\frac{2}{3}}+d}{d}+1\right)+2\,\mathrm{Li}_2\left(\frac{ex^{\frac{2}{3}}+d}{d}\right)\log\left(ex^{\frac{2}{3}}+d\right)-2\,\mathrm{Li}_3\left(\frac{ex^{\frac{2}{3}}+d}{d}\right)\right)b^3e^3n^3}{2\,d^3}-\frac{3}{4}\,a^2ben\left(\frac{2\,e^2\log\left(ex^{\frac{2}{3}}\right.}{d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/2*(log(e*x^(2/3) + d)^2*log(-(e*x^(2/3) + d)/d + 1) + 2*dilog((e*x^(2/3)
+ d)/d)*log(e*x^(2/3) + d) - 2*polylog(3, (e*x^(2/3) + d)/d))*b^3*e^3*n^3/d
^3 - 3/4*a^2*b*e*n*(2*e^2*log(e*x^(2/3) + d)/d^3 - 2*e^2*log(x^(2/3))/d^3 -
 (2*e*x^(2/3) - d)/(d^2*x^(4/3))) - 3/2*a^2*b*log((e*x^(2/3) + d)^n*c)/x^2
- 1/2*a^3/x^2 + 3/2*(2*a*b^2*e^3*n^2 - (3*e^3*n^3 - 2*e^3*n^2*log(c))*b^3)*
```

```
(log(e*x^(2/3) + d)*log(-(e*x^(2/3) + d)/d + 1) + dilog((e*x^(2/3) + d)/d))
/d^3 - ((3*e^3*n^2 - 2*e^3*n*log(c))*a*b^2 - (e^3*n^3 - 3*e^3*n^2*log(c) +
e^3*n*log(c)^2)*b^3)*log(x)/d^3 - 1/4*(2*b^3*d^3*log(c)^3 + 6*a*b^2*d^3*log
(c)^2 + 2*(b^3*e^3*n^3*x^2 + b^3*d^3*n^3)*log(e*x^(2/3) + d)^3 - 3*(2*b^3*d
*e^2*n^3*x^(4/3) - b^3*d^2*e*n^3*x^(2/3) - 2*b^3*d^3*n^2*log(c) - 2*a*b^2*d
^3*n^2 - (2*a*b^2*e^3*n^2 - (3*e^3*n^3 - 2*e^3*n^2*log(c))*b^3)*x^2)*log(e*
x^(2/3) + d)^2 + 6*((d*e^2*n^2 - 2*d*e^2*n*log(c))*a*b^2 + (d*e^2*n^2*log(c
) - d*e^2*n*log(c)^2)*b^3)*x^(4/3) + 6*(b^3*d^3*n*log(c)^2 + 2*a*b^2*d^3*n*
log(c) - ((3*e^3*n^2 - 2*e^3*n*log(c))*a*b^2 - (e^3*n^3 - 3*e^3*n^2*log(c)
+ e^3*n*log(c)^2)*b^3)*x^2 - (2*a*b^2*d*e^2*n^2 - (d*e^2*n^3 - 2*d*e^2*n^2*
log(c))*b^3)*x^(4/3) + (b^3*d^2*e*n^2*log(c) + a*b^2*d^2*e*n^2)*x^(2/3))*lo
g(e*x^(2/3) + d) + 3*(b^3*d^2*e*n*log(c)^2 + 2*a*b^2*d^2*e*n*log(c))*x^(2/3
))/(d^3*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{2/3}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^3,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x**3,x)
```

```
[Out] Timed out
```

3.485 $\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=794

$$\frac{2bd^5n\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^{2/3}\left(d+ex^{2/3}\right)},x\right)}{3e^4}-\frac{4504b^2d^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{315e^{9/2}}-\frac{1984b^2d^3n^2x\left(a+\right.}{}$$

```
[Out] 4504/315*a*b^2*d^4*n^2*x^(1/3)/e^4-3475504/99225*b^3*d^4*n^3*x^(1/3)/e^4+63
7984/297675*b^3*d^3*n^3*x/e^3-221344/496125*b^3*d^2*n^3*x^(5/3)/e^2+3088/27
783*b^3*d*n^3*x^(7/3)/e-16/729*b^3*n^3*x^3+3475504/99225*b^3*d^(9/2)*n^3*ar
ctan(x^(1/3)*e^(1/2)/d^(1/2))/e^(9/2)-4504/315*I*b^3*d^(9/2)*n^3*polylog(2,
1-2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/e^(9/2)+4504/315*b^3*d^4*n^2*x^(1/
3)*ln(c*(d+e*x^(2/3))^n)/e^4-1984/945*b^2*d^3*n^2*x*(a+b*ln(c*(d+e*x^(2/3))
^n))/e^3+1144/1575*b^2*d^2*n^2*x^(5/3)*(a+b*ln(c*(d+e*x^(2/3))^n))/e^2-128/
441*b^2*d*n^2*x^(7/3)*(a+b*ln(c*(d+e*x^(2/3))^n))/e+8/81*b^2*n^2*x^3*(a+b*l
n(c*(d+e*x^(2/3))^n))-4504/315*b^2*d^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/
2))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^(9/2)-2*b*d^4*n*x^(1/3)*(a+b*ln(c*(d+e*x^
(2/3))^n))^2/e^4+2/3*b*d^3*n*x*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^3-2/5*b*d^2*
n*x^(5/3)*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^2+2/7*b*d*n*x^(7/3)*(a+b*ln(c*(d+
e*x^(2/3))^n))^2/e-2/9*b*n*x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^2+1/3*x^3*(a+b*l
n(c*(d+e*x^(2/3))^n))^3-9008/315*b^3*d^(9/2)*n^3*arctan(x^(1/3)*e^(1/2)/d^(
1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/e^(9/2)-4504/315*I*b^3*d^(9
/2)*n^3*arctan(x^(1/3)*e^(1/2)/d^(1/2))^2/e^(9/2)+2/3*b*d^5*n*Unintegrable(
(a+b*ln(c*(d+e*x^(2/3))^n))^2/(d+e*x^(2/3))/x^(2/3),x)/e^4
```

Rubi [A] time = 3.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (4504*a*b^2*d^4*n^2*x^(1/3))/(315*e^4) - (3475504*b^3*d^4*n^3*x^(1/3))/(992
25*e^4) + (637984*b^3*d^3*n^3*x)/(297675*e^3) - (221344*b^3*d^2*n^3*x^(5/3)
)/(496125*e^2) + (3088*b^3*d*n^3*x^(7/3))/(27783*e) - (16*b^3*n^3*x^3)/729
+ (3475504*b^3*d^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(99225*e^(9/2
)) - (((4504*I)/315)*b^3*d^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e
^(9/2) - (9008*b^3*d^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqr
t[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/(315*e^(9/2)) + (4504*b^3*d^4*n^2*x^(
1/3)*Log[c*(d + e*x^(2/3))^n])/(315*e^4) - (1984*b^2*d^3*n^2*x*(a + b*Log[c
*(d + e*x^(2/3))^n]))/(945*e^3) + (1144*b^2*d^2*n^2*x^(5/3)*(a + b*Log[c*(d
 + e*x^(2/3))^n]))/(1575*e^2) - (128*b^2*d*n^2*x^(7/3)*(a + b*Log[c*(d + e*
x^(2/3))^n]))/(441*e) + (8*b^2*n^2*x^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/81
 - (4504*b^2*d^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d
+ e*x^(2/3))^n]))/(315*e^(9/2)) - (2*b*d^4*n*x^(1/3)*(a + b*Log[c*(d + e*x^
(2/3))^n])^2)/e^4 + (2*b*d^3*n*x*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(3*e^3
) - (2*b*d^2*n*x^(5/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(5*e^2) + (2*b*d
*n*x^(7/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(7*e) - (2*b*n*x^3*(a + b*Lo
g[c*(d + e*x^(2/3))^n])^2)/9 + (x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^3)/3 -
 (((4504*I)/315)*b^3*d^(9/2)*n^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x^(1/3))])/e^(9/2) + (2*b*d^5*n*Defer[Subst][Defer[Int][(a + b*Log[c*
(d + e*x^2)^n])^2/(d + e*x^2), x], x, x^(1/3)])/e^4
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx = 3\,\mathrm{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^3\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - (2ben)\,\mathrm{Subst}\left(\int \frac{x^{10}\left(a+b\log\left(c\left(\right.\right.\right.}{d+ex}\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - (2ben)\,\mathrm{Subst}\left(\int\left(\frac{d^4\left(a+b\log\left(c\left(\right.\right.\right.}{e^5}\right.\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - (2bn)\,\mathrm{Subst}\left(\int x^8\left(a+b\log\left(c\left(d\right.\right.\right.\right.$$

$$= -\frac{2bd^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^4} + \frac{2bd^3nx\left(a+b\log\left(c\left(d+e\right.\right.\right.}{3e^3}$$

$$= -\frac{2bd^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^4} + \frac{2bd^3nx\left(a+b\log\left(c\left(d+e\right.\right.\right.}{3e^3}$$

$$= -\frac{2bd^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^4} + \frac{2bd^3nx\left(a+b\log\left(c\left(d+e\right.\right.\right.}{3e^3}$$

$$= \frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4} - \frac{1984b^2d^3n^2x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{945e^3} + \frac{1144b^2}{}$$

$$= \frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4} + \frac{4504b^3d^4n^2\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{315e^4} - \frac{1984b^2d^3n^2}{}$$

$$= \frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4} - \frac{3475504b^3d^4n^3\sqrt[3]{x}}{99225e^4} + \frac{637984b^3d^3n^3x}{297675e^3} - \frac{221344b^3}{4961}$$

$$= \frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4} - \frac{3475504b^3d^4n^3\sqrt[3]{x}}{99225e^4} + \frac{637984b^3d^3n^3x}{297675e^3} - \frac{221344b^3}{4961}$$

$$= \frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4} - \frac{3475504b^3d^4n^3\sqrt[3]{x}}{99225e^4} + \frac{637984b^3d^3n^3x}{297675e^3} - \frac{221344b^3}{4961}$$

$$= \frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4} - \frac{3475504b^3d^4n^3\sqrt[3]{x}}{99225e^4} + \frac{637984b^3d^3n^3x}{297675e^3} - \frac{221344b^3}{4961}$$

Mathematica [A] time = 9.25, size = 3146, normalized size = 3.96

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (b^3*n^3*x^(1/3)*(32*d^4 - 32*d^4*Sqrt[1 - (d + e*x^(2/3))/d] + 128*d^3*Sqr
t[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3)) - 192*d^2*Sqrt[1 - (d + e*x^(2/3))
/d]*(d + e*x^(2/3))^2 + 128*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3
 - 32*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4 + 1584*d^3*(d + e*x^(2/
3))*HypergeometricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d] - 4536
*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, (d + e*x
^(2/3))/d] + 3780*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2
, 2, 2}, (d + e*x^(2/3))/d] - 864*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-7
/2, 1, 1, 1, 1}, {2, 2, 2, 2}, (d + e*x^(2/3))/d] + 3024*d^3*(d + e*x^(2/3)
)*HypergeometricPFQ[{-5/2, 1, 1, 1, 1}, {2, 2, 2, 2}, (d + e*x^(2/3))/d] -
3780*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1, 1, 1}, {2, 2, 2, 2}
, (d + e*x^(2/3))/d] + 1890*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1,
 1, 1, 1}, {2, 2, 2, 2}, (d + e*x^(2/3))/d] - 240*d^4*Log[d + e*x^(2/3)] +
240*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)] - 672*d^3*Sqrt[1 - (
d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d + e*x^(2/3)] + 576*d^2*Sqrt[1 - (d
+ e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d + e*x^(2/3)] - 96*d*Sqrt[1 - (d + e
*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d + e*x^(2/3)] - 48*Sqrt[1 - (d + e*x^(2
/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(2/3)] - 3780*d^3*(d + e*x^(2/3))*Hyp
ergeometricPFQ[{-3/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)]
 + 864*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, (d
 + e*x^(2/3))/d]*Log[d + e*x^(2/3)] - 3024*d^3*(d + e*x^(2/3))*Hypergeometri
cPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)] + 37
80*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, (d + e
*x^(2/3))/d]*Log[d + e*x^(2/3)] - 1890*d^3*(d + e*x^(2/3))*HypergeometricPF
Q[{-1/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)] + 284*d
^4*Log[d + e*x^(2/3)]^2 - 284*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[d + e*x^(
2/3)]^2 + 668*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d + e*x^(
2/3)]^2 - 552*d^2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d + e*x
^(2/3)]^2 + 236*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d + e*x
^(2/3)]^2 - 68*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(2
/3)]^2 - 1890*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, (
d + e*x^(2/3))/d]*Log[d + e*x^(2/3)]^2 + 945*d^3*(d + e*x^(2/3))*Hypergeome
tricPFQ[{-1/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)]^2 - 70*
d^4*Log[d + e*x^(2/3)]^3 + 70*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[d + e*x^(
2/3)]^3 - 280*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d + e*x^(
2/3)]^3 + 420*d^2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d + e*x
^(2/3)]^3 - 280*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d + e*x
^(2/3)]^3 + 70*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(2
/3)]^3 + 1512*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-5/2, 1, 1}, {2, 2}, (
d + e*x^(2/3))/d]*(1 + 3*Log[d + e*x^(2/3)] + Log[d + e*x^(2/3)]^2) - 144*d
^3*(d + e*x^(2/3))*HypergeometricPFQ[{-7/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/
d]*(6 + 11*Log[d + e*x^(2/3)] + 3*Log[d + e*x^(2/3)]^2)))/(210*e^4*Sqrt[1 -
 (d + e*x^(2/3))/d]) + (b^2*n^2*x^(1/3)*(-120*d^4 + 120*d^4*Sqrt[1 - (d + e
*x^(2/3))/d] - 336*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3)) + 288*d^
2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2 - 48*d*Sqrt[1 - (d + e*x^(2
/3))/d]*(d + e*x^(2/3))^3 - 24*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^
4 - 1890*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, (d + e
*x^(2/3))/d] + 432*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-7/2, 1, 1, 1}, {
2, 2, 2}, (d + e*x^(2/3))/d] - 1512*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{
-5/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d] + 1890*d^3*(d + e*x^(2/3))*Hy
pergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d] - 945*d^3*(d
 + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))
```

```
/d] + 284*d^4*Log[d + e*x^(2/3)] - 284*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[
d + e*x^(2/3)] + 668*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d
+ e*x^(2/3)] - 552*d^2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d
+ e*x^(2/3)] + 236*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d +
e*x^(2/3)] - 68*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(
2/3)] - 1890*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, (d
 + e*x^(2/3))/d]*Log[d + e*x^(2/3)] + 945*d^3*(d + e*x^(2/3))*Hypergeometri
cPFQ[{-1/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)] - 105*d^4*
Log[d + e*x^(2/3)]^2 + 105*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[d + e*x^(2/3
)]^2 - 420*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d + e*x^(2/3
)]^2 + 630*d^2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d + e*x^(2
/3)]^2 - 420*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d + e*x^(2
/3)]^2 + 105*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(2/3
)]^2 + 756*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-5/2, 1, 1}, {2, 2}, (d +
 e*x^(2/3))/d]*(3 + 2*Log[d + e*x^(2/3)]) - 72*d^3*(d + e*x^(2/3))*Hypergeo
metricPFQ[{-7/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/d]*(11 + 6*Log[d + e*x^(2/3
)]))*(a + b*(-(n*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n])))/(105*e^4
*Sqrt[1 - (d + e*x^(2/3))/d]) - (2*b*d^4*n*x^(1/3)*(a + b*(-(n*Log[d + e*x^
(2/3)]) + Log[c*(d + e*x^(2/3))^n]))^2)/e^4 + (2*b*d^3*n*x*(a + b*(-(n*Log[
d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n]))^2)/(3*e^3) - (2*b*d^2*n*x^(5/3
)*(a + b*(-(n*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n]))^2)/(5*e^2) +
 (2*b*d*n*x^(7/3)*(a + b*(-(n*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n
]))^2)/(7*e) + (2*b*d^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*(-(n
*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n]))^2)/e^(9/2) + b*n*x^3*Log[
d + e*x^(2/3)]*(a + b*(-(n*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n]))
^2 + (x^3*(a + b*(-(n*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n]))^2*(3
*a - 2*b*n + 3*b*(-(n*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n])))/9
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(b^{3}x^{2}\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)^{3}+3\,ab^{2}x^{2}\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)^{2}+3\,a^{2}bx^{2}\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a^{3}x^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*x^2*log((e*x^(2/3) +
d)^n*c)^2 + 3*a^2*b*x^2*log((e*x^(2/3) + d)^n*c) + a^3*x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{3}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3*x^2, x)
```

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^{n}\right)+a\right)^{3}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*(e*x^(2/3)+d)^n)+a)^3,x)
```

```
[Out] int(x^2*(b*ln(c*(e*x^(2/3)+d)^n)+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}b^3n^3x^3\log\left(ex^{\frac{2}{3}}+d\right)^3+\int-\frac{\left(2\,b^3enx^3-9\left(b^3e\log(c)+ab^2e\right)x^3-9\left(b^3d\log(c)+ab^2d\right)x^{\frac{7}{3}}\right)n^2\log\left(ex^{\frac{2}{3}}+d\right)^2-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*n^3*x^3*log(e*x^(2/3) + d)^3 + integrate(-1/3*((2*b^3*e*n*x^3 - 9*(
b^3*e*log(c) + a*b^2*e)*x^3 - 9*(b^3*d*log(c) + a*b^2*d)*x^(7/3))*n^2*log(e
*x^(2/3) + d)^2 - 3*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c)
 + a^3*e)*x^3 - 3*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) +
 a^3*d)*x^(7/3) - 9*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^3 + (b
^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^(7/3))*n*log(e*x^(2/3) + d))/
(e*x + d*x^(1/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(a+b\,\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(2/3))^n))^3,x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e*x^(2/3))^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

3.486 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 dx$

Optimal. Leaf size=486

$$-\frac{2bd^2n\mathrm{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^{2/3}\left(d+ex^{2/3}\right)},x\right)}{e}+\frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}+\frac{8}{3}b^2n^2x\left(a+b\log\right.$$

```
[Out] -32*a*b^2*d*n^2*x^(1/3)/e+208/3*b^3*d*n^3*x^(1/3)/e-16/9*b^3*n^3*x-208/3*b^
3*d^(3/2)*n^3*arctan(x^(1/3)*e^(1/2)/d^(1/2))/e^(3/2)+32*I*b^3*d^(3/2)*n^3*
arctan(x^(1/3)*e^(1/2)/d^(1/2))^2/e^(3/2)-32*b^3*d*n^2*x^(1/3)*ln(c*(d+e*x^
(2/3))^n)/e+8/3*b^2*n^2*x*(a+b*ln(c*(d+e*x^(2/3))^n))+32*b^2*d^(3/2)*n^2*ar
ctan(x^(1/3)*e^(1/2)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^(3/2)+6*b*d*n*x
^(1/3)*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e-2*b*n*x*(a+b*ln(c*(d+e*x^(2/3))^n))^
2+x*(a+b*ln(c*(d+e*x^(2/3))^n))^3+64*b^3*d^(3/2)*n^3*arctan(x^(1/3)*e^(1/2)
/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/e^(3/2)+32*I*b^3*d^(3/2
)*n^3*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/e^(3/2)-2*b*d^2*n*
Unintegrable((a+b*ln(c*(d+e*x^(2/3))^n))^2/(d+e*x^(2/3))/x^(2/3),x)/e
```

Rubi [A] time = 1.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (-32*a*b^2*d*n^2*x^(1/3))/e + (208*b^3*d*n^3*x^(1/3))/(3*e) - (16*b^3*n^3*x
)/9 - (208*b^3*d^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(3/2)) +
 ((32*I)*b^3*d^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e^(3/2) + (64
*b^3*d^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
 + I*Sqrt[e]*x^(1/3))])/e^(3/2) - (32*b^3*d*n^2*x^(1/3)*Log[c*(d + e*x^(2/3
))^n])/e + (8*b^2*n^2*x*(a + b*Log[c*(d + e*x^(2/3))^n]))/3 + (32*b^2*d^(3/
2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/
e^(3/2) + (6*b*d*n*x^(1/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/e - 2*b*n*x*
(a + b*Log[c*(d + e*x^(2/3))^n])^2 + x*(a + b*Log[c*(d + e*x^(2/3))^n])^3 +
 ((32*I)*b^3*d^(3/2)*n^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^
(1/3))])/e^(3/2) - (6*b*d^2*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x
^2)^n])^2/(d + e*x^2), x], x, x^(1/3)])/e
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx = 3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^3\,dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - (6ben)\,\text{Subst}\left(\int \frac{x^4\left(a+b\log\left(c\left(d+ex^2\right)^n\right.\right.}{d+ex^2}\right.$$

$$= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - (6ben)\,\text{Subst}\left(\int\left(-\frac{d\left(a+b\log\left(c\left(d+ex^2\right.\right.\right.}{e^2}\right.\right.$$

$$= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - (6bn)\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right.\right.$$

$$= \frac{6bdn\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e} - 2bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 +$$

$$= \frac{6bdn\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e} - 2bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 +$$

$$= \frac{6bdn\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e} - 2bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 +$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} + \frac{8}{3}b^2n^2x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\right.}{}$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} - \frac{32b^3dn^2\sqrt[3]{x}\,\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e} + \frac{8}{3}b^2n^2x\left(a+b\log\left(c\left(d\right.\right.\right.$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} + \frac{208b^3dn^3\sqrt[3]{x}}{3e} - \frac{16}{9}b^3n^3x + \frac{32ib^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}} -$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} + \frac{208b^3dn^3\sqrt[3]{x}}{3e} - \frac{16}{9}b^3n^3x - \frac{208b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}} +$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} + \frac{208b^3dn^3\sqrt[3]{x}}{3e} - \frac{16}{9}b^3n^3x - \frac{208b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}} +$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} + \frac{208b^3dn^3\sqrt[3]{x}}{3e} - \frac{16}{9}b^3n^3x - \frac{208b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}} +$$

Mathematica [A] time = 1.28, size = 598, normalized size = 1.23

$$\frac{3b^2n^2x\left(-a-b\log\left(c\left(d+ex^{2/3}\right)^n\right)+bn\log\left(d+ex^{2/3}\right)\right)\left(3\left(d+ex^{2/3}\right){}_4F_3\left(-\frac{1}{2},1,1,1;2,2,2;\frac{x^{2/3}e}{d}+1\right)+\log(\right.}{d\left(-\frac{ex^{2/3}}{d}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] -1/2*(b^3*n^3*x*(-18*(d + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1, 1, 1},
{2, 2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e*x^(2/3)]*(18*(d + e*x^(2/3))*H
ypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e
*x^(2/3)]*(-9*(d + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1}, {2, 2}, 1 + (
e*x^(2/3))/d] + 2*(d - d*(-((e*x^(2/3))/d))^(3/2))*Log[d + e*x^(2/3)]))))/(
d*(-((e*x^(2/3))/d))^(3/2)) + (3*b^2*n^2*x*(3*(d + e*x^(2/3))*Hypergeometri
cPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e*x^(2/3)]*(-
3*(d + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1}, {2, 2}, 1 + (e*x^(2/3))/d
] + (d - d*(-((e*x^(2/3))/d))^(3/2))*Log[d + e*x^(2/3)]))*(-a + b*n*Log[d +
 e*x^(2/3)] - b*Log[c*(d + e*x^(2/3))^n]))/(d*(-((e*x^(2/3))/d))^(3/2)) + (
6*b*d*n*x^(1/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2
)/e - (6*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*Log[d + e*x
^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/e^(3/2) + 3*b*n*x*Log[d + e*x^(2/3
)]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + x*(a - b*n
*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*(a - 2*b*n - b*n*Log[d
+ e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^3+3\,ab^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2+3\,a^2b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c)^
2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3, x)
```

maple [A] time = 0.13, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^n\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^3,x)
```

```
[Out] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$b^3n^3x\log\left(ex^{\frac{2}{3}}+d\right)^3-\left(2\,en\left(\frac{3\,d^2\arctan\left(\frac{ex^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}\,e^2}+\frac{ex-3\,dx^{\frac{1}{3}}}{e^2}\right)-3\,x\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)\right)a^2b+a^3x+\int-\frac{\left(2\,b^3enx-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] b^3*n^3*x*log(e*x^(2/3) + d)^3 - (2*e*n*(3*d^2*arctan(e*x^(1/3)/sqrt(d*e))/
(sqrt(d*e)*e^2) + (e*x - 3*d*x^(1/3))/e^2) - 3*x*log((e*x^(2/3) + d)^n*c))*
a^2*b + a^3*x + integrate(-((2*b^3*e*n*x - 3*(b^3*e*log(c) + a*b^2*e)*x - 3
*(b^3*d*log(c) + a*b^2*d)*x^(1/3))*n^2*log(e*x^(2/3) + d)^2 - 3*((b^3*e*log
(c)^2 + 2*a*b^2*e*log(c))*x + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c))*x^(1/3))*
n*log(e*x^(2/3) + d) - (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2)*x - (b^3*d*log
(c)^3 + 3*a*b^2*d*log(c)^2)*x^(1/3))/(e*x + d*x^(1/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^3,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

3.487 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=319

$$-\frac{2be^2n\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^{2/3}\left(d+ex^{2/3}\right)},x\right)}{d}+\frac{24b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}}-\frac{6ben\left(a+b\log\left(c\left(\right.\right.\right.}{d\sqrt[3]{x}}$$

```
[Out] 24*I*b^3*e^(3/2)*n^3*arctan(x^(1/3)*e^(1/2)/d^(1/2))^2/d^(3/2)+24*b^2*e^(3/
2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^(3/2)-
6*b*e*n*(a+b*ln(c*(d+e*x^(2/3))^n))^2/d/x^(1/3)-(a+b*ln(c*(d+e*x^(2/3))^n))
^3/x+48*b^3*e^(3/2)*n^3*arctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/
2)+I*x^(1/3)*e^(1/2)))/d^(3/2)+24*I*b^3*e^(3/2)*n^3*polylog(2,1-2*d^(1/2)/(
d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(3/2)-2*b*e^2*n*Unintegrable((a+b*ln(c*(d+e*x
^(2/3))^n))^2/(d+e*x^(2/3))/x^(2/3),x)/d
```

Rubi [A] time = 0.49, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^2,x]
```

```
[Out] ((24*I)*b^3*e^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/d^(3/2) + (48*
b^3*e^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
+ I*Sqrt[e]*x^(1/3))])/d^(3/2) + (24*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3
))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/d^(3/2) - (6*b*e*n*(a + b*Log
[c*(d + e*x^(2/3))^n])^2)/(d*x^(1/3)) - (a + b*Log[c*(d + e*x^(2/3))^n])^3/
x + ((24*I)*b^3*e^(3/2)*n^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]
*x^(1/3))])/d^(3/2) - (6*b*e^2*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d +
e*x^2)^n])^2/(d + e*x^2), x], x, x^(1/3)])/d
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^3}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x} + (6ben)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)}{x^2\left(d+ex^2\right)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x} + (6ben)\,\text{Subst}\left(\int\left(\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right.}{dx^2}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x} + \frac{(6ben)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^2}\,dx, x, \sqrt[3]{\right.}{d}$$

$$= -\frac{6ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d\sqrt[3]{x}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x} - \frac{\left(6be^2n\right.}{}$$

$$= \frac{24b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}} - \frac{6ben\left(a+b\log\left(c\left(\right.\right.\right.}{d\sqrt[3]{x}}$$

$$= \frac{24b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}} - \frac{6ben\left(a+b\log\left(c\left(\right.\right.\right.}{d\sqrt[3]{x}}$$

$$= \frac{24ib^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{24b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex\right.\right.\right.}{d^{3/2}}$$

$$= \frac{24ib^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{48b^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}} + \cdots$$

$$= \frac{24ib^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{48b^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}} + \cdots$$

$$= \frac{24ib^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{48b^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}} + \cdots$$

Mathematica [A] time = 2.66, size = 646, normalized size = 2.03

$$-\frac{-3b^2n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)-bn\log\left(d+ex^{2/3}\right)\right)\left(-6d\left(d+ex^{2/3}\right)\left(-\frac{ex^{2/3}}{d}\right)^{3/2}{}_4F_3\left(1,1,1,\frac{5}{2};2,2,2;\frac{x^{2/3}e}{d}+1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^2,x]
```

```
[Out] -1/2*(2*b^3*d*n^3*(-9*(d + e*x^(2/3))*(-((e*x^(2/3))/d))^(3/2)*Hypergeometr
icPFQ[{1, 1, 1, 1, 5/2}, {2, 2, 2, 2}, 1 + (e*x^(2/3))/d] + 9*(d + e*x^(2/3
))*(-((e*x^(2/3))/d))^(3/2)*HypergeometricPFQ[{1, 1, 1, 5/2}, {2, 2, 2}, 1
+ (e*x^(2/3))/d]*Log[d + e*x^(2/3)] + Log[d + e*x^(2/3)]^2*(-6*e*(-1 + Sqrt
[-((e*x^(2/3))/d)])*x^(2/3) + 6*d*(-((e*x^(2/3))/d))^(3/2)*Log[(1 + Sqrt[-(
(e*x^(2/3))/d)])/2] + (d - d*(-((e*x^(2/3))/d))^(3/2))*Log[d + e*x^(2/3)]))
 - 3*b^2*n^2*(-6*d*(d + e*x^(2/3))*(-((e*x^(2/3))/d))^(3/2)*HypergeometricP
FQ[{1, 1, 1, 5/2}, {2, 2, 2}, 1 + (e*x^(2/3))/d] - 2*d*Log[d + e*x^(2/3)]*(
-4*e*(-1 + Sqrt[-((e*x^(2/3))/d)])*x^(2/3) + 4*d*(-((e*x^(2/3))/d))^(3/2)*L
og[(1 + Sqrt[-((e*x^(2/3))/d)])/2] + (d - d*(-((e*x^(2/3))/d))^(3/2))*Log[d
 + e*x^(2/3)]))*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n]) +
 12*b*d*e*n*x^(2/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n
])^2 + 12*b*Sqrt[d]*e^(3/2)*n*x*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*
Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + 6*b*d^2*n*Log[d + e*x^
(2/3)]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + 2*d^2*
(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3)/(d^2*x)
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^{3}\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)^{3}+3\,ab^{2}\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)^{2}+3\,a^{2}b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a^{3}}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3/x^2, x)
```

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^{n}\right)+a\right)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^3/x^2,x)
```

```
[Out] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^3/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^{3}n^{3}\log\left(ex^{\frac{2}{3}}+d\right)^{3}}{x}+\int\frac{\left(2\,b^{3}enx+3\left(b^{3}e\log(c)+ab^{2}e\right)x+3\left(b^{3}d\log(c)+ab^{2}d\right)x^{\frac{1}{3}}\right)n^{2}\log\left(ex^{\frac{2}{3}}+d\right)^{2}+3\Big(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] -b^3*n^3*log(e*x^(2/3) + d)^3/x + integrate(((2*b^3*e*n*x + 3*(b^3*e*log(c)
 + a*b^2*e)*x + 3*(b^3*d*log(c) + a*b^2*d)*x^(1/3))*n^2*log(e*x^(2/3) + d)^
2 + 3*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*log(c)^2 +
2*a*b^2*d*log(c) + a^2*b*d)*x^(1/3))*n*log(e*x^(2/3) + d) + (b^3*e*log(c)^3
 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x + (b^3*d*log(c)^3 + 3*a
*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(1/3))/(e*x^3 + d*x^(7/3)), x
)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x**2,x)
```

```
[Out] Timed out
```

3.488 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx$

Optimal. Leaf size=632

$$\frac{2be^5n\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^{2/3}\left(d+ex^{2/3}\right)},x\right)}{3d^4}-\frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{105d^{9/2}}-\frac{568b^2e^4n^2\left(a+b\log\ldots\right.}{105\ldots}$$

```
[Out] -16/105*b^3*e^3*n^3/d^3/x+16/7*b^3*e^4*n^3/d^4/x^(1/3)+1376/105*b^3*e^(9/2)
*n^3*arctan(x^(1/3)*e^(1/2)/d^(1/2))/d^(9/2)-1408/105*I*b^3*e^(9/2)*n^3*pol
ylog(2,1-2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(9/2)-8/35*b^2*e^2*n^2*(a
+b*ln(c*(d+e*x^(2/3))^n))/d^2/x^(5/3)+32/35*b^2*e^3*n^2*(a+b*ln(c*(d+e*x^(2
/3))^n))/d^3/x-568/105*b^2*e^4*n^2*(a+b*ln(c*(d+e*x^(2/3))^n))/d^4/x^(1/3)-
1408/105*b^2*e^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*(a+b*ln(c*(d+e*x^(
2/3))^n))/d^(9/2)-2/7*b*e*n*(a+b*ln(c*(d+e*x^(2/3))^n))^2/d/x^(7/3)+2/5*b*e
^2*n*(a+b*ln(c*(d+e*x^(2/3))^n))^2/d^2/x^(5/3)-2/3*b*e^3*n*(a+b*ln(c*(d+e*x
^(2/3))^n))^2/d^3/x+2*b*e^4*n*(a+b*ln(c*(d+e*x^(2/3))^n))^2/d^4/x^(1/3)-1/3
*(a+b*ln(c*(d+e*x^(2/3))^n))^3/x^3-2816/105*b^3*e^(9/2)*n^3*arctan(x^(1/3)*
e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(9/2)-1408/105
*I*b^3*e^(9/2)*n^3*arctan(x^(1/3)*e^(1/2)/d^(1/2))^2/d^(9/2)+2/3*b*e^5*n*Un
integrable((a+b*ln(c*(d+e*x^(2/3))^n))^2/(d+e*x^(2/3))/x^(2/3),x)/d^4
```

Rubi [A] time = 1.87, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^4,x]
```

```
[Out] (-16*b^3*e^3*n^3)/(105*d^3*x) + (16*b^3*e^4*n^3)/(7*d^4*x^(1/3)) + (1376*b^
3*e^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(105*d^(9/2)) - (((1408*I)
/105)*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/d^(9/2) - (2816*
b^3*e^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
+ I*Sqrt[e]*x^(1/3))])/(105*d^(9/2)) - (8*b^2*e^2*n^2*(a + b*Log[c*(d + e*x
^(2/3))^n]))/(35*d^2*x^(5/3)) + (32*b^2*e^3*n^2*(a + b*Log[c*(d + e*x^(2/3)
)^n]))/(35*d^3*x) - (568*b^2*e^4*n^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(105
*d^4*x^(1/3)) - (1408*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a
+ b*Log[c*(d + e*x^(2/3))^n]))/(105*d^(9/2)) - (2*b*e*n*(a + b*Log[c*(d + e
*x^(2/3))^n])^2)/(7*d*x^(7/3)) + (2*b*e^2*n*(a + b*Log[c*(d + e*x^(2/3))^n]
)^2)/(5*d^2*x^(5/3)) - (2*b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(3*d^
3*x) + (2*b*e^4*n*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(d^4*x^(1/3)) - (a +
b*Log[c*(d + e*x^(2/3))^n])^3/(3*x^3) - (((1408*I)/105)*b^3*e^(9/2)*n^3*Pol
yLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/d^(9/2) + (2*b*e^5*
n*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)^n])^2/(d + e*x^2), x], x
, x^(1/3)])/d^4
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^3}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3} + (2ben)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)}{x^8\left(d+ex^2\right)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3} + (2ben)\,\text{Subst}\left(\int\left(\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right.}{dx^8}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3} + \frac{(2ben)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^8}\,dx, x, \sqrt[3]{\right.}{d}$$

$$= -\frac{2ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{7dx^{7/3}} + \frac{2be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5d^2x^{5/3}} -$$

$$= -\frac{8b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{9/2}} - \frac{2ben\left(a+b\log\left(c\left(\right.\right.\right.}{7dx^{7/3}}$$

$$= -\frac{8b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{9/2}} - \frac{2ben\left(a+b\log\left(c\left(\right.\right.\right.}{7dx^{7/3}}$$

$$= -\frac{8ib^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{9/2}} - \frac{8b^2e^2n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^2x^{5/3}} + \frac{32b^2e^3}{}$$

$$= -\frac{16b^3e^3n^3}{105d^3x} + \frac{64b^3e^4n^3}{35d^4\sqrt[3]{x}} + \frac{1136b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{8ib^3e^{9/2}n^3\tan^{-1}\left(\right.}{d^{9/2}}$$

$$= -\frac{16b^3e^3n^3}{105d^3x} + \frac{16b^3e^4n^3}{7d^4\sqrt[3]{x}} + \frac{1328b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{1408ib^3e^{9/2}n^3\tan^{-1}}{105d^{9/2}}$$

$$= -\frac{16b^3e^3n^3}{105d^3x} + \frac{16b^3e^4n^3}{7d^4\sqrt[3]{x}} + \frac{1376b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{1408ib^3e^{9/2}n^3\tan^{-1}}{105d^{9/2}}$$

$$= -\frac{16b^3e^3n^3}{105d^3x} + \frac{16b^3e^4n^3}{7d^4\sqrt[3]{x}} + \frac{1376b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{1408ib^3e^{9/2}n^3\tan^{-1}}{105d^{9/2}}$$

$$= -\frac{16b^3e^3n^3}{105d^3x} + \frac{16b^3e^4n^3}{7d^4\sqrt[3]{x}} + \frac{1376b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{1408ib^3e^{9/2}n^3\tan^{-1}}{105d^{9/2}}$$

Mathematica [A] time = 2.94, size = 803, normalized size = 1.27

$$-70\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3d^5-210bn\log\left(d+ex^{2/3}\right)\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^4,x]
```

```
[Out] (35*b^3*n^3*(54*e^4*(d + e*x^(2/3))*Sqrt[-((e*x^(2/3))/d)]*x^(8/3)*Hypergeo
metricPFQ[{1, 1, 1, 1, 11/2}, {2, 2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e*
x^(2/3)]*(54*d*e^3*(d + e*x^(2/3))*(-((e*x^(2/3))/d))^(3/2)*x^2*Hypergeomet
ricPFQ[{1, 1, 1, 11/2}, {2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e*x^(2/3)]*
(27*e^4*(d + e*x^(2/3))*Sqrt[-((e*x^(2/3))/d)]*x^(8/3)*HypergeometricPFQ[{1
, 1, 11/2}, {2, 2}, 1 + (e*x^(2/3))/d] - 2*d*(d^4 + d*e^3*(-((e*x^(2/3))/d)
)^(3/2)*x^2)*Log[d + e*x^(2/3)]))) + (210*b^2*n^2*(-9*e^5*(d + e*x^(2/3))*x
^(10/3)*HypergeometricPFQ[{1, 1, 1, 11/2}, {2, 2, 2}, 1 + (e*x^(2/3))/d] +
Log[d + e*x^(2/3)]*(9*e^5*(d + e*x^(2/3))*x^(10/3)*HypergeometricPFQ[{1, 1,
 11/2}, {2, 2}, 1 + (e*x^(2/3))/d] + d*(d^5*Sqrt[-((e*x^(2/3))/d)] + e^5*x^
(10/3))*Log[d + e*x^(2/3)]))*(-a + b*n*Log[d + e*x^(2/3)] - b*Log[c*(d + e*
x^(2/3))^n]))/(d*Sqrt[-((e*x^(2/3))/d)]) - 60*b*d^4*e*n*x^(2/3)*(a - b*n*Lo
g[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + 84*b*d^3*e^2*n*x^(4/3)*(
a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 - 140*b*d^2*e^3*
n*x^2*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + 420*b*d
*e^4*n*x^(8/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2
 + 420*b*Sqrt[d]*e^(9/2)*n*x^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*Lo
g[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 - 210*b*d^5*n*Log[d + e*x^
(2/3)]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 - 70*d^5
*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3)/(210*d^5*x^3)
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)^3+3\,ab^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)^2+3\,a^2b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a^3}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3)/x^4, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3/x^4, x)
```

maple [A] time = 0.13, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\left(e\,x^{\frac{2}{3}}+d\right)^n\right)+a\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^3/x^4,x)

[Out] int((b*ln(c*(e*x^(2/3)+d)^n)+a)^3/x^4,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3n^3\log\left(ex^{\frac{2}{3}}+d\right)^3}{3\,x^3}+\int\frac{\left(2\,b^3enx+9\left(b^3e\log(c)+ab^2e\right)x+9\left(b^3d\log(c)+ab^2d\right)x^{\frac{1}{3}}\right)n^2\log\left(ex^{\frac{2}{3}}+d\right)^2+9\left(\left(b^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^4,x, algorithm="maxima")

[Out] -1/3*b^3*n^3*log(e*x^(2/3) + d)^3/x^3 + integrate(1/3*((2*b^3*e*n*x + 9*(b^3*e*log(c) + a*b^2*e)*x + 9*(b^3*d*log(c) + a*b^2*d)*x^(1/3))*n^2*log(e*x^(2/3) + d)^2 + 9*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^(1/3))*n*log(e*x^(2/3) + d) + 3*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x + 3*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(1/3))/(e*x^5 + d*x^(13/3)), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^4,x)

[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^4, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x**4,x)

[Out] Timed out

3.489 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$

Optimal. Leaf size=239

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{be^{12}n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{4d^{12}}-\frac{be^{12}n\log(x)}{12d^{12}}+\frac{be^{11}n\sqrt[3]{x}}{4d^{11}}-\frac{be^{10}nx^{2/3}}{8d^{10}}+\frac{be^9nx}{12d^9}-\frac{be^8nx^{4/3}}{16d^8}+\cdots$$

```
[Out] 1/4*b*e^11*n*x^(1/3)/d^11-1/8*b*e^10*n*x^(2/3)/d^10+1/12*b*e^9*n*x/d^9-1/16
*b*e^8*n*x^(4/3)/d^8+1/20*b*e^7*n*x^(5/3)/d^7-1/24*b*e^6*n*x^2/d^6+1/28*b*e
^5*n*x^(7/3)/d^5-1/32*b*e^4*n*x^(8/3)/d^4+1/36*b*e^3*n*x^3/d^3-1/40*b*e^2*n
*x^(10/3)/d^2+1/44*b*e*n*x^(11/3)/d-1/4*b*e^12*n*ln(d+e/x^(1/3))/d^12+1/4*x
^4*(a+b*ln(c*(d+e/x^(1/3))^n))-1/12*b*e^12*n*ln(x)/d^12
```

Rubi [A] time = 0.17, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2454, 2395, 44}

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{be^{10}nx^{2/3}}{8d^{10}}-\frac{be^8nx^{4/3}}{16d^8}+\frac{be^7nx^{5/3}}{20d^7}-\frac{be^6nx^2}{24d^6}+\frac{be^5nx^{7/3}}{28d^5}-\frac{be^4nx^{8/3}}{32d^4}+\frac{be^3nx^3}{36d^3}-\frac{be^2nx^{10/3}}{40d^2}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] (b*e^11*n*x^(1/3))/(4*d^11) - (b*e^10*n*x^(2/3))/(8*d^10) + (b*e^9*n*x)/(12
*d^9) - (b*e^8*n*x^(4/3))/(16*d^8) + (b*e^7*n*x^(5/3))/(20*d^7) - (b*e^6*n*
x^2)/(24*d^6) + (b*e^5*n*x^(7/3))/(28*d^5) - (b*e^4*n*x^(8/3))/(32*d^4) + (
b*e^3*n*x^3)/(36*d^3) - (b*e^2*n*x^(10/3))/(40*d^2) + (b*e*n*x^(11/3))/(44*
d) - (b*e^12*n*Log[d + e/x^(1/3)])/(4*d^12) + (x^4*(a + b*Log[c*(d + e/x^(1
/3))^n]))/4 - (b*e^12*n*Log[x])/(12*d^12)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx=-\left(3\,\text{Subst}\left(\int\frac{a+b\log(c(d+ex)^n)}{x^{13}}dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{4}(ben)\,\text{Subst}\left(\int\frac{1}{x^{12}(d+ex)}dx,x,\frac{1}{\sqrt[3]{x}}\right)$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{4}(ben)\,\text{Subst}\left(\int\left(\frac{1}{dx^{12}}-\frac{e}{d^2x^{11}}+\frac{e^2}{d^3x^{10}}\right.\right.$$
$$=\frac{be^{11}n\sqrt[3]{x}}{4d^{11}}-\frac{be^{10}nx^{2/3}}{8d^{10}}+\frac{be^9nx}{12d^9}-\frac{be^8nx^{4/3}}{16d^8}+\frac{be^7nx^{5/3}}{20d^7}-\frac{be^6nx^2}{24d^6}+\frac{be^5nx^{7/3}}{28d^5}$$

Mathematica [A] time = 0.24, size = 218, normalized size = 0.91

$$\frac{ax^4}{4}+\frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-\frac{1}{4}ben\left(\frac{e^{11}\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^{12}}+\frac{e^{11}\log(x)}{3d^{12}}-\frac{e^{10}\sqrt[3]{x}}{d^{11}}+\frac{e^9x^{2/3}}{2d^{10}}-\frac{e^8x}{3d^9}+\frac{e^7x^{4/3}}{4d^8}-\frac{e^6x^{5/3}}{5d^7}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] (a*x^4)/4 + (b*x^4*Log[c*(d + e/x^(1/3))^n])/4 - (b*e*n*(-((e^10*x^(1/3))/d
^11) + (e^9*x^(2/3))/(2*d^10) - (e^8*x)/(3*d^9) + (e^7*x^(4/3))/(4*d^8) - (
e^6*x^(5/3))/(5*d^7) + (e^5*x^2)/(6*d^6) - (e^4*x^(7/3))/(7*d^5) + (e^3*x^(
8/3))/(8*d^4) - (e^2*x^3)/(9*d^3) + (e*x^(10/3))/(10*d^2) - x^(11/3)/(11*d)
 + (e^11*Log[d + e/x^(1/3)])/d^12 + (e^11*Log[x])/(3*d^12)))/4
```

fricas [A] time = 0.48, size = 232, normalized size = 0.97

$$\frac{27720\,bd^{12}x^4\log(c)+3080\,bd^9e^3nx^3+27720\,ad^{12}x^4-4620\,bd^6e^6nx^2+9240\,bd^3e^9nx-27720\,bd^{12}n\log\left(x^{\frac{1}{3}}\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/110880*(27720*b*d^12*x^4*log(c) + 3080*b*d^9*e^3*n*x^3 + 27720*a*d^12*x^4
 - 4620*b*d^6*e^6*n*x^2 + 9240*b*d^3*e^9*n*x - 27720*b*d^12*n*log(x^(1/3))
+ 27720*(b*d^12 - b*e^12)*n*log(d*x^(1/3) + e) + 27720*(b*d^12*n*x^4 - b*d^
12*n)*log((d*x + e*x^(2/3))/x) + 63*(40*b*d^11*e*n*x^3 - 55*b*d^8*e^4*n*x^2
 + 88*b*d^5*e^7*n*x - 220*b*d^2*e^10*n)*x^(2/3) - 198*(14*b*d^10*e^2*n*x^3
- 20*b*d^7*e^5*n*x^2 + 35*b*d^4*e^8*n*x - 140*b*d*e^11*n)*x^(1/3))/d^12
```

giac [A] time = 0.44, size = 161, normalized size = 0.67

$$\frac{1}{4}\,bx^4\log(c)+\frac{1}{4}\,ax^4+\frac{1}{110880}\left(27720\,x^4\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)+\left(\frac{2520\,d^{10}x^{\frac{11}{3}}-2772\,d^9x^{\frac{10}{3}}e+3080\,d^8x^3e^2-3465\,d^7x^{\frac{8}{3}}e^3}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*log(c) + 1/4*a*x^4 + 1/110880*(27720*x^4*log(d + e/x^(1/3)) + ((2
520*d^10*x^(11/3) - 2772*d^9*x^(10/3)*e + 3080*d^8*x^3*e^2 - 3465*d^7*x^(8/
```

```
3)*e^3 + 3960*d^6*x^(7/3)*e^4 - 4620*d^5*x^2*e^5 + 5544*d^4*x^(5/3)*e^6 - 6
930*d^3*x^(4/3)*e^7 + 9240*d^2*x*e^8 - 13860*d*x^(2/3)*e^9 + 27720*x^(1/3)*
e^10)/d^11 - 27720*e^11*log(abs(d*x^(1/3) + e))/d^12)*e)*b*n
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

maxima [A] time = 0.47, size = 162, normalized size = 0.68

$$\frac{1}{4}\,bx^{4}\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+\frac{1}{4}\,ax^{4}-\frac{1}{110880}\,ben\left(\frac{27720\,e^{11}\log\left(dx^{\frac{1}{3}}+e\right)}{d^{12}}-\frac{2520\,d^{10}x^{\frac{11}{3}}-2772\,d^{9}ex^{\frac{10}{3}}+3080\,d^{8}e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log(c*(d + e/x^(1/3))^n) + 1/4*a*x^4 - 1/110880*b*e*n*(27720*e^11
*log(d*x^(1/3) + e)/d^12 - (2520*d^10*x^(11/3) - 2772*d^9*e*x^(10/3) + 3080
*d^8*e^2*x^3 - 3465*d^7*e^3*x^(8/3) + 3960*d^6*e^4*x^(7/3) - 4620*d^5*e^5*x
^2 + 5544*d^4*e^6*x^(5/3) - 6930*d^3*e^7*x^(4/3) + 9240*d^2*e^8*x - 13860*d
*e^9*x^(2/3) + 27720*e^10*x^(1/3))/d^11)
```

mupad [B] time = 0.80, size = 191, normalized size = 0.80

$$\frac{\frac{a\,d^{12}\,x^4}{4}-\frac{b\,e^{12}\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{1/3}}+1\right)}{2}+\frac{b\,d^{12}\,x^4\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{4}+\frac{b\,d^3\,e^9\,n\,x}{12}+\frac{b\,d\,e^{11}\,n\,x^{1/3}}{4}+\frac{b\,d^{11}\,e\,n\,x^{11/3}}{44}-\frac{b\,d^6\,e^6\,n\,x^2}{24}+\frac{b\,d^9\,e^3\,n\,x^3}{36}-}{d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e/x^(1/3))^n)),x)
```

```
[Out] ((a*d^12*x^4)/4 - (b*e^12*n*atanh((2*e)/(d*x^(1/3)) + 1))/2 + (b*d^12*x^4*l
og(c*(d + e/x^(1/3))^n))/4 + (b*d^3*e^9*n*x)/12 + (b*d*e^11*n*x^(1/3))/4 +
(b*d^11*e*n*x^(11/3))/44 - (b*d^6*e^6*n*x^2)/24 + (b*d^9*e^3*n*x^3)/36 - (b
*d^2*e^10*n*x^(2/3))/8 - (b*d^4*e^8*n*x^(4/3))/16 + (b*d^5*e^7*n*x^(5/3))/2
0 + (b*d^7*e^5*n*x^(7/3))/28 - (b*d^8*e^4*n*x^(8/3))/32 - (b*d^10*e^2*n*x^(
10/3))/40)/d^12
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(1/3))**n)),x)
```

```
[Out] Timed out
```

3.490 $\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right) \, dx$

Optimal. Leaf size=190

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)+\frac{be^9n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3d^9}+\frac{be^9n\log(x)}{9d^9}-\frac{be^8n\sqrt[3]{x}}{3d^8}+\frac{be^7nx^{2/3}}{6d^7}-\frac{be^6nx}{9d^6}+\frac{be^5nx^{4/3}}{12d^5}-\frac{be^4nx^{5/}}{15d^4}$$

```
[Out] -1/3*b*e^8*n*x^(1/3)/d^8+1/6*b*e^7*n*x^(2/3)/d^7-1/9*b*e^6*n*x/d^6+1/12*b*e
^5*n*x^(4/3)/d^5-1/15*b*e^4*n*x^(5/3)/d^4+1/18*b*e^3*n*x^2/d^3-1/21*b*e^2*n
*x^(7/3)/d^2+1/24*b*e*n*x^(8/3)/d+1/3*b*e^9*n*ln(d+e/x^(1/3))/d^9+1/3*x^3*(
a+b*ln(c*(d+e/x^(1/3))^n))+1/9*b*e^9*n*ln(x)/d^9
```

Rubi [A] time = 0.13, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2454, 2395, 44}

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)+\frac{be^7nx^{2/3}}{6d^7}+\frac{be^5nx^{4/3}}{12d^5}-\frac{be^4nx^{5/3}}{15d^4}+\frac{be^3nx^2}{18d^3}-\frac{be^2nx^{7/3}}{21d^2}-\frac{be^8n\sqrt[3]{x}}{3d^8}-\frac{be^6nx}{9d^6}+\frac{be^9n\log\left(d\right.}{3d^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] -(b*e^8*n*x^(1/3))/(3*d^8) + (b*e^7*n*x^(2/3))/(6*d^7) - (b*e^6*n*x)/(9*d^6
) + (b*e^5*n*x^(4/3))/(12*d^5) - (b*e^4*n*x^(5/3))/(15*d^4) + (b*e^3*n*x^2)
/(18*d^3) - (b*e^2*n*x^(7/3))/(21*d^2) + (b*e*n*x^(8/3))/(24*d) + (b*e^9*n*
Log[d + e/x^(1/3)])/(3*d^9) + (x^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/3 + (b
*e^9*n*Log[x])/(9*d^9)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx=-\left(3\,\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^{10}}dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{3}(ben)\,\text{Subst}\left(\int\frac{1}{x^9(d+ex)}dx,x,\frac{1}{\sqrt[3]{x}}\right.$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{3}(ben)\,\text{Subst}\left(\int\left(\frac{1}{dx^9}-\frac{e}{d^2x^8}+\frac{e^2}{d^3x^7}\right.\right.$$

$$=-\frac{be^8n\sqrt[3]{x}}{3d^8}+\frac{be^7nx^{2/3}}{6d^7}-\frac{be^6nx}{9d^6}+\frac{be^5nx^{4/3}}{12d^5}-\frac{be^4nx^{5/3}}{15d^4}+\frac{be^3nx^2}{18d^3}-\frac{be^2n}{21d}$$

Mathematica [A] time = 0.13, size = 175, normalized size = 0.92

$$\frac{ax^3}{3}+\frac{1}{3}bx^3\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-\frac{1}{3}ben\left(-\frac{e^8\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^9}-\frac{e^8\log(x)}{3d^9}+\frac{e^7\sqrt[3]{x}}{d^8}-\frac{e^6x^{2/3}}{2d^7}+\frac{e^5x}{3d^6}-\frac{e^4x^{4/3}}{4d^5}+\frac{e^3x^{5/3}}{5d^4}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] (a*x^3)/3 + (b*x^3*Log[c*(d + e/x^(1/3))^n])/3 - (b*e*n*((e^7*x^(1/3))/d^8
- (e^6*x^(2/3))/(2*d^7) + (e^5*x)/(3*d^6) - (e^4*x^(4/3))/(4*d^5) + (e^3*x^
(5/3))/(5*d^4) - (e^2*x^2)/(6*d^3) + (e*x^(7/3))/(7*d^2) - x^(8/3)/(8*d) -
(e^8*Log[d + e/x^(1/3)])/d^9 - (e^8*Log[x])/(3*d^9)))/3
```

fricas [A] time = 0.46, size = 192, normalized size = 1.01

$$\frac{840\,bd^9x^3\log(c)+140\,bd^6e^3nx^2+840\,ad^9x^3-280\,bd^3e^6nx-840\,bd^9n\log\left(x^{\frac{1}{3}}\right)+840\left(bd^9+be^9\right)n\log\left(dx^{\frac{1}{3}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/2520*(840*b*d^9*x^3*log(c) + 140*b*d^6*e^3*n*x^2 + 840*a*d^9*x^3 - 280*b*
d^3*e^6*n*x - 840*b*d^9*n*log(x^(1/3)) + 840*(b*d^9 + b*e^9)*n*log(d*x^(1/3
) + e) + 840*(b*d^9*n*x^3 - b*d^9*n)*log((d*x + e*x^(2/3))/x) + 21*(5*b*d^8
*e*n*x^2 - 8*b*d^5*e^4*n*x + 20*b*d^2*e^7*n)*x^(2/3) - 30*(4*b*d^7*e^2*n*x^
2 - 7*b*d^4*e^5*n*x + 28*b*d*e^8*n)*x^(1/3))/d^9
```

giac [A] time = 0.38, size = 131, normalized size = 0.69

$$\frac{1}{3}bx^3\log(c)+\frac{1}{3}ax^3+\frac{1}{2520}\left(840\,x^3\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)+\left(\frac{105\,d^7x^{\frac{8}{3}}-120\,d^6x^{\frac{7}{3}}e+140\,d^5x^2e^2-168\,d^4x^{\frac{5}{3}}e^3+210\,d^3x}{d^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*log(c) + 1/3*a*x^3 + 1/2520*(840*x^3*log(d + e/x^(1/3)) + ((105*d
^7*x^(8/3) - 120*d^6*x^(7/3)*e + 140*d^5*x^2*e^2 - 168*d^4*x^(5/3)*e^3 + 21
0*d^3*x^(4/3)*e^4 - 280*d^2*x*e^5 + 420*d*x^(2/3)*e^6 - 840*x^(1/3)*e^7)/d^
8 + 840*e^8*log(abs(d*x^(1/3) + e))/d^9)*e)*b*n
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(b\ln \left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*(d+e/x^(1/3))^n)+a),x)
```

```
[Out] int(x^2*(b*ln(c*(d+e/x^(1/3))^n)+a),x)
```

maxima [A] time = 0.47, size = 128, normalized size = 0.67

$$\frac{1}{3}\,bx^{3}\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+\frac{1}{3}\,ax^{3}+\frac{1}{2520}\,ben\left(\frac{840\,e^{8}\log\left(dx^{\frac{1}{3}}+e\right)}{d^{9}}+\frac{105\,d^{7}x^{\frac{8}{3}}-120\,d^{6}ex^{\frac{7}{3}}+140\,d^{5}e^{2}x^{2}-168\,d^{4}e^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3*log(c*(d + e/x^(1/3))^n) + 1/3*a*x^3 + 1/2520*b*e*n*(840*e^8*log(
d*x^(1/3) + e)/d^9 + (105*d^7*x^(8/3) - 120*d^6*e*x^(7/3) + 140*d^5*e^2*x^2
 - 168*d^4*e^3*x^(5/3) + 210*d^3*e^4*x^(4/3) - 280*d^2*e^5*x + 420*d*e^6*x^
(2/3) - 840*e^7*x^(1/3))/d^8)
```

mupad [B] time = 0.68, size = 153, normalized size = 0.81

$$\frac{840\,a\,d^{9}\,x^{3}+1680\,b\,e^{9}\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{1/3}}+1\right)+840\,b\,d^{9}\,x^{3}\,\ln\left(c\,\left(d+\frac{e}{x^{1/3}}\right)^{n}\right)-280\,b\,d^{3}\,e^{6}\,n\,x-840\,b\,d\,e^{8}\,n\,x^{1/3}+10}{2520\,d^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e/x^(1/3))^n)),x)
```

```
[Out] (840*a*d^9*x^3 + 1680*b*e^9*n*atanh((2*e)/(d*x^(1/3)) + 1) + 840*b*d^9*x^3*
log(c*(d + e/x^(1/3))^n) - 280*b*d^3*e^6*n*x - 840*b*d*e^8*n*x^(1/3) + 105*
b*d^8*e*n*x^(8/3) + 140*b*d^6*e^3*n*x^2 + 420*b*d^2*e^7*n*x^(2/3) + 210*b*d
^4*e^5*n*x^(4/3) - 168*b*d^5*e^4*n*x^(5/3) - 120*b*d^7*e^2*n*x^(7/3))/(2520
*d^9)
```

sympy [A] time = 102.01, size = 180, normalized size = 0.95

$$\frac{ax^3}{3}+b\left(\frac{en\left(\frac{3x^{\frac{8}{3}}}{8d} - \frac{3ex^{\frac{7}{3}}}{7d^2} + \frac{e^2x^2}{2d^3} - \frac{3e^3x^{\frac{5}{3}}}{5d^4} + \frac{3e^4x^{\frac{4}{3}}}{4d^5} - \frac{e^5x}{d^6} + \frac{3e^6x^{\frac{2}{3}}}{2d^7} - \frac{3e^7\sqrt[3]{x}}{d^8} + \frac{3e^9\left(\begin{cases}\frac{1}{d\sqrt[3]{x}} & \text{for } e=0\\ \frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e} & \text{otherwise}\end{cases}\right)}{d^9} - \frac{3e^8\log\left(\frac{1}{\sqrt[3]{x}}\right)}{d^9}\right)}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(1/3))**n)),x)
```

```
[Out] a*x**3/3 + b*(e*n*(3*x**(8/3)/(8*d) - 3*e*x**(7/3)/(7*d**2) + e**2*x**2/(2*
d**3) - 3*e**3*x**(5/3)/(5*d**4) + 3*e**4*x**(4/3)/(4*d**5) - e**5*x/d**6 +
 3*e**6*x**(2/3)/(2*d**7) - 3*e**7*x**(1/3)/d**8 + 3*e**9*Piecewise((1/(d*x
**(1/3)), Eq(e, 0)), (log(d + e/x**(1/3))/e, True))/d**9 - 3*e**8*log(x**(-
1/3))/d**9)/9 + x**3*log(c*(d + e/x**(1/3))**n)/3)
```

3.491 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx$

Optimal. Leaf size=141

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{be^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2d^6}-\frac{be^6n\log(x)}{6d^6}+\frac{be^5n\sqrt[3]{x}}{2d^5}-\frac{be^4nx^{2/3}}{4d^4}+\frac{be^3nx}{6d^3}-\frac{be^2nx^{4/3}}{8d^2}+\frac{benx^{5/3}}{10d}$$

```
[Out] 1/2*b*e^5*n*x^(1/3)/d^5-1/4*b*e^4*n*x^(2/3)/d^4+1/6*b*e^3*n*x/d^3-1/8*b*e^2
*n*x^(4/3)/d^2+1/10*b*e*n*x^(5/3)/d-1/2*b*e^6*n*ln(d+e/x^(1/3))/d^6+1/2*x^2
*(a+b*ln(c*(d+e/x^(1/3))^n))-1/6*b*e^6*n*ln(x)/d^6
```

Rubi [A] time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2454, 2395, 44}

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{be^4nx^{2/3}}{4d^4}-\frac{be^2nx^{4/3}}{8d^2}+\frac{be^5n\sqrt[3]{x}}{2d^5}+\frac{be^3nx}{6d^3}-\frac{be^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2d^6}-\frac{be^6n\log(x)}{6d^6}+\frac{benx^{5/3}}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] (b*e^5*n*x^(1/3))/(2*d^5) - (b*e^4*n*x^(2/3))/(4*d^4) + (b*e^3*n*x)/(6*d^3)
 - (b*e^2*n*x^(4/3))/(8*d^2) + (b*e*n*x^(5/3))/(10*d) - (b*e^6*n*Log[d + e/
x^(1/3)])/(2*d^6) + (x^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/2 - (b*e^6*n*Log
[x])/(6*d^6)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx = -\left(3\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^7}dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$
$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right) - \frac{1}{2}(ben)\,\text{Subst}\left(\int \frac{1}{x^6(d+ex)}dx, x, \frac{1}{\sqrt[3]{x}}\right.$$
$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right) - \frac{1}{2}(ben)\,\text{Subst}\left(\int\left(\frac{1}{dx^6} - \frac{e}{d^2x^5} + \frac{e^2}{d^3x^4}\right.\right.$$
$$= \frac{be^5n\sqrt[3]{x}}{2d^5} - \frac{be^4nx^{2/3}}{4d^4} + \frac{be^3nx}{6d^3} - \frac{be^2nx^{4/3}}{8d^2} + \frac{benx^{5/3}}{10d} - \frac{be^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2d^6}$$

Mathematica [A] time = 0.09, size = 132, normalized size = 0.94

$$\frac{ax^2}{2}+\frac{1}{2}bx^2\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-\frac{1}{2}ben\left(\frac{e^5\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^6}+\frac{e^5\log(x)}{3d^6}-\frac{e^4\sqrt[3]{x}}{d^5}+\frac{e^3x^{2/3}}{2d^4}-\frac{e^2x}{3d^3}+\frac{ex^{4/3}}{4d^2}-\frac{x^{5/3}}{5d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*Log[c*(d + e/x^(1/3))^n])/2 - (b*e*n*(-((e^4*x^(1/3))/d^
5) + (e^3*x^(2/3))/(2*d^4) - (e^2*x)/(3*d^3) + (e*x^(4/3))/(4*d^2) - x^(5/3
)/(5*d) + (e^5*Log[d + e/x^(1/3)])/d^6 + (e^5*Log[x])/(3*d^6)))/2
```

fricas [A] time = 0.48, size = 153, normalized size = 1.09

$$\frac{60\,bd^6x^2\log(c) + 20\,bd^3e^3nx + 60\,ad^6x^2 - 60\,bd^6n\log\left(x^{\frac{1}{3}}\right) + 60\left(bd^6 - be^6\right)n\log\left(dx^{\frac{1}{3}} + e\right) + 60\left(bd^6nx^2 - b\right.}{120\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/120*(60*b*d^6*x^2*log(c) + 20*b*d^3*e^3*n*x + 60*a*d^6*x^2 - 60*b*d^6*n*l
og(x^(1/3)) + 60*(b*d^6 - b*e^6)*n*log(d*x^(1/3) + e) + 60*(b*d^6*n*x^2 - b
*d^6*n)*log((d*x + e*x^(2/3))/x) + 6*(2*b*d^5*e*n*x - 5*b*d^2*e^4*n)*x^(2/3
) - 15*(b*d^4*e^2*n*x - 4*b*d*e^5*n)*x^(1/3))/d^6
```

giac [A] time = 0.36, size = 101, normalized size = 0.72

$$\frac{1}{2}bx^2\log(c)+\frac{1}{120}\left(60\,x^2\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)+\left(\frac{12\,d^4x^{\frac{5}{3}} - 15\,d^3x^{\frac{4}{3}}e + 20\,d^2xe^2 - 30\,dx^{\frac{2}{3}}e^3 + 60\,x^{\frac{1}{3}}e^4}{d^5} - \frac{60\,e^5\log\left(\left|dx\right.\right.}{d^6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*log(c) + 1/120*(60*x^2*log(d + e/x^(1/3)) + ((12*d^4*x^(5/3) - 15
*d^3*x^(4/3)*e + 20*d^2*x*e^2 - 30*d*x^(2/3)*e^3 + 60*x^(1/3)*e^4)/d^5 - 60
*e^5*log(abs(d*x^(1/3) + e))/d^6)*e)*b*n + 1/2*a*x^2
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(d+e/x^(1/3))^n)+a),x)
```

```
[Out] int(x*(b*ln(c*(d+e/x^(1/3))^n)+a),x)
```

maxima [A] time = 0.47, size = 96, normalized size = 0.68

$$-\frac{1}{120}\,ben\left(\frac{60\,e^5\log\left(dx^{\frac{1}{3}}+e\right)}{d^6}-\frac{12\,d^4x^{\frac{5}{3}}-15\,d^3ex^{\frac{4}{3}}+20\,d^2e^2x-30\,de^3x^{\frac{2}{3}}+60\,e^4x^{\frac{1}{3}}}{d^5}\right)+\frac{1}{2}\,bx^2\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] -1/120*b*e*n*(60*e^5*log(d*x^(1/3) + e)/d^6 - (12*d^4*x^(5/3) - 15*d^3*e*x^
(4/3) + 20*d^2*e^2*x - 30*d*e^3*x^(2/3) + 60*e^4*x^(1/3))/d^5) + 1/2*b*x^2*
log(c*(d + e/x^(1/3))^n) + 1/2*a*x^2
```

mupad [B] time = 0.79, size = 112, normalized size = 0.79

$$\frac{x^{5/3}\left(\frac{b\,e\,n}{5\,d}-\frac{b\,e^2\,n}{4\,d^2\,x^{1/3}}-\frac{b\,e^4\,n}{2\,d^4\,x}+\frac{b\,e^3\,n}{3\,d^3\,x^{2/3}}+\frac{b\,e^5\,n}{d^5\,x^{4/3}}\right)}{2}+\frac{a\,x^2}{2}+\frac{b\,x^2\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{2}-\frac{b\,e^6\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{1/3}}+1\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/3))^n)),x)
```

```
[Out] (x^(5/3)*((b*e*n)/(5*d) - (b*e^2*n)/(4*d^2*x^(1/3)) - (b*e^4*n)/(2*d^4*x) +
 (b*e^3*n)/(3*d^3*x^(2/3)) + (b*e^5*n)/(d^5*x^(4/3))))/2 + (a*x^2)/2 + (b*x
^2*log(c*(d + e/x^(1/3))^n))/2 - (b*e^6*n*atanh((2*e)/(d*x^(1/3)) + 1))/d^6
```

sympy [A] time = 27.06, size = 119, normalized size = 0.84

$$\frac{ax^2}{2}+b\left(\frac{en\left(\frac{3x^{\frac{5}{3}}}{5d}-\frac{3ex^{\frac{4}{3}}}{4d^2}+\frac{e^2x}{d^3}-\frac{3e^3x^{\frac{2}{3}}}{2d^4}-\frac{3e^5\left(\begin{cases}\frac{\sqrt[3]{x}}{e} & \text{for}\ d=0\\ \frac{\log\left(d\sqrt[3]{x}+e\right)}{d} & \text{otherwise}\end{cases}\right)}{d^5}+\frac{3e^4\sqrt[3]{x}}{d^5}\right)}{6}+\frac{x^2\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))**n)),x)
```

```
[Out] a*x**2/2 + b*(e*n*(3*x**(5/3)/(5*d) - 3*e*x**(4/3)/(4*d**2) + e**2*x/d**3 -
 3*e**3*x**(2/3)/(2*d**4) - 3*e**5*Piecewise((x**(1/3)/e, Eq(d, 0)), (log(d
*x**(1/3) + e)/d, True))/d**5 + 3*e**4*x**(1/3)/d**5)/6 + x**2*log(c*(d + e
/x**(1/3))**n)/2)
```

3.492 $\int \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right) \, dx$

Optimal. Leaf size=70

$$ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{be^3n\log\left(d\sqrt[3]{x}+e\right)}{d^3}-\frac{be^2n\sqrt[3]{x}}{d^2}+\frac{benx^{2/3}}{2d}$$

```
[Out] -b*e^2*n*x^(1/3)/d^2+1/2*b*e*n*x^(2/3)/d+a*x+b*x*ln(c*(d+e/x^(1/3))^n)+b*e^
3*n*ln(e+d*x^(1/3))/d^3
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2448, 263, 190, 43}

$$ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-\frac{be^2n\sqrt[3]{x}}{d^2}+\frac{be^3n\log\left(d\sqrt[3]{x}+e\right)}{d^3}+\frac{benx^{2/3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e/x^(1/3))^n],x]
```

```
[Out] -((b*e^2*n*x^(1/3))/d^2) + (b*e*n*x^(2/3))/(2*d) + a*x + b*x*Log[c*(d + e/x
^(1/3))^n] + (b*e^3*n*Log[e + d*x^(1/3)])/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx = ax+b\int \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)dx$$
$$= ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{1}{3}(ben)\int \frac{1}{\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}}dx$$
$$= ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{1}{3}(ben)\int \frac{1}{e+d\sqrt[3]{x}}dx$$
$$= ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+(ben)\,\mathrm{Subst}\left(\int \frac{x^2}{e+dx}dx,x,\sqrt[3]{x}\right)$$
$$= ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+(ben)\,\mathrm{Subst}\left(\int \left(-\frac{e}{d^2}+\frac{x}{d}+\frac{e^2}{d^2(e+dx)}\right)dx,x,\right.$$
$$= -\frac{be^2n\sqrt[3]{x}}{d^2}+\frac{benx^{2/3}}{2d}+ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{be^3n\log\left(e+d\sqrt[3]{x}\right)}{d^3}$$

Mathematica [A] time = 0.05, size = 79, normalized size = 1.13

$$ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-ben\left(-\frac{e^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}-\frac{e^2\log(x)}{3d^3}+\frac{e\sqrt[3]{x}}{d^2}-\frac{x^{2/3}}{2d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e/x^(1/3))^n],x]
```

```
[Out] a*x + b*x*Log[c*(d + e/x^(1/3))^n] - b*e*n*((e*x^(1/3))/d^2 - x^(2/3)/(2*d)
 - (e^2*Log[d + e/x^(1/3)])/d^3 - (e^2*Log[x])/(3*d^3))
```

fricas [A] time = 0.44, size = 107, normalized size = 1.53

$$\frac{2\,bd^3x\log(c)-2\,bd^3n\log\left(x^{\frac{1}{3}}\right)+bd^2enx^{\frac{2}{3}}-2\,bde^2nx^{\frac{1}{3}}+2\,ad^3x+2\left(bd^3+be^3\right)n\log\left(dx^{\frac{1}{3}}+e\right)+2\left(bd^3nx-bd^3\right.}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(1/3))^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d^3*x*log(c) - 2*b*d^3*n*log(x^(1/3)) + b*d^2*e*n*x^(2/3) - 2*b*d*
e^2*n*x^(1/3) + 2*a*d^3*x + 2*(b*d^3 + b*e^3)*n*log(d*x^(1/3) + e) + 2*(b*d
^3*n*x - b*d^3*n)*log((d*x + e*x^(2/3))/x))/d^3
```

giac [A] time = 0.28, size = 66, normalized size = 0.94

$$\frac{1}{2}\left(\left(\left(\left(\frac{dx^{\frac{2}{3}}-2\,x^{\frac{1}{3}}e}{d^2}+\frac{2\,e^2\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)}{d^3}\right)e+2\,x\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)n+2\,x\log(c)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(1/3))^n),x, algorithm="giac")
```

```
[Out] 1/2*((((d*x^(2/3) - 2*x^(1/3)*e)/d^2 + 2*e^2*log(abs(d*x^(1/3) + e))/d^3)*e
 + 2*x*log(d + e/x^(1/3)))*n + 2*x*log(c))*b + a*x
```

maple [A] time = 0.09, size = 115, normalized size = 1.64

$$\frac{2b\,e^{3}n\ln\left(d\,x^{\frac{1}{3}}+e\right)}{3d^{3}}-\frac{b\,e^{3}n\ln\left(d^{2}x^{\frac{2}{3}}-de\,x^{\frac{1}{3}}+e^{2}\right)}{3d^{3}}+\frac{b\,e^{3}n\ln\left(d^{3}x+e^{3}\right)}{3d^{3}}+bx\ln\left(c\left(\frac{d\,x^{\frac{1}{3}}+e}{x^{\frac{1}{3}}}\right)^{n}\right)+\frac{ben\,x^{\frac{2}{3}}}{2d}-\frac{b\,e^{2}n\,x^{\frac{1}{3}}}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*ln(c*(d+e/x^(1/3))^n)+a,x)
```

```
[Out] a*x+x*b*ln(c*((e+d*x^(1/3))/x^(1/3))^n)+1/3*b*e^3*n*ln(d^3*x+e^3)/d^3+1/2*b
*e*n*x^(2/3)/d-1/3*b*e^3*n/d^3*ln(d^2*x^(2/3)-e*d*x^(1/3)+e^2)+2/3*b*e^3*n*
ln(e+d*x^(1/3))/d^3-b*e^2*n*x^(1/3)/d^2
```

maxima [A] time = 0.45, size = 59, normalized size = 0.84

$$\frac{1}{2}\left(en\left(\frac{2\,e^{2}\log\left(dx^{\frac{1}{3}}+e\right)}{d^{3}}+\frac{dx^{\frac{2}{3}}-2\,ex^{\frac{1}{3}}}{d^{2}}\right)+2\,x\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(1/3))^n),x, algorithm="maxima")
```

```
[Out] 1/2*(e*n*(2*e^2*log(d*x^(1/3) + e)/d^3 + (d*x^(2/3) - 2*e*x^(1/3))/d^2) + 2
*x*log(c*(d + e/x^(1/3))^n))*b + a*x
```

mupad [B] time = 0.47, size = 59, normalized size = 0.84

$$a\,x+b\,x\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^{n}\right)+\frac{b\,\left(2\,e^{3}\,n\,\ln\left(e+d\,x^{1/3}\right)-2\,d\,e^{2}\,n\,x^{1/3}+d^{2}\,e\,n\,x^{2/3}\right)}{2\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*log(c*(d + e/x^(1/3))^n),x)
```

```
[Out] a*x + b*x*log(c*(d + e/x^(1/3))^n) + (b*(2*e^3*n*log(e + d*x^(1/3)) - 2*d*e
^2*n*x^(1/3) + d^2*e*n*x^(2/3)))/(2*d^3)
```

sympy [A] time = 7.10, size = 92, normalized size = 1.31

$$ax+b\left(\frac{en\left(\frac{3x^{\frac{2}{3}}}{2d}-\frac{3e\sqrt[3]{x}}{d^{2}}+\frac{3e^{3}\left(\begin{cases}\frac{1}{d\sqrt[3]{x}} & \text{for}\ e=0\\ \frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e} & \text{otherwise}\end{cases}\right)}{d^{3}}-\frac{3e^{2}\log\left(\frac{1}{\sqrt[3]{x}}\right)}{d^{3}}\right)}{3}+x\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e/x**(1/3))**n),x)
```

```
[Out] a*x + b*(e*n*(3*x**(2/3)/(2*d) - 3*e*x**(1/3)/d**2 + 3*e**3*Piecewise((1/(d
*x**(1/3)), Eq(e, 0)), (log(d + e/x**(1/3))/e, True))/d**3 - 3*e**2*log(x**
(-1/3))/d**3)/3 + x*log(c*(d + e/x**(1/3))**n))
```

3.493 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=51

$$-3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-3bn\text{Li}_2\left(\frac{e}{d\sqrt[3]{x}}+1\right)$$

```
[Out] -3*(a+b*ln(c*(d+e/x^(1/3))^n))*ln(-e/d/x^(1/3))-3*b*n*polylog(2,1+e/d/x^(1/
3))
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2394, 2315}

$$-3bn\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)-3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x,x]
```

```
[Out] -3*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))] - 3*b*n*PolyLog[2
, 1 + e/(d*x^(1/3))]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx &= -\left(3\,\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)\\ &= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right)+(3ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,\right.\\ &= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right)-3bn\text{Li}_2\left(1+\frac{e}{d\sqrt[3]{x}}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 53, normalized size = 1.04

$$a\log(x)-3b\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-3bn\text{Li}_2\left(\frac{d+\frac{e}{\sqrt[3]{x}}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x,x]
```

```
[Out] -3*b*Log[c*(d + e/x^(1/3))^n]*Log[-(e/(d*x^(1/3)))] + a*Log[x] - 3*b*n*Poly
Log[2, (d + e/x^(1/3))/d]
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*((d*x + e*x^(2/3))/x)^n) + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)/x, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int\frac{b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/3))^n)+a)/x,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/3))^n)+a)/x,x)
```

maxima [B] time = 1.92, size = 185, normalized size = 3.63

$$-3\left(\log\left(\frac{dx^{\frac{1}{3}}}{e}+1\right)\log\left(x^{\frac{1}{3}}\right)+\text{Li}_2\left(-\frac{dx^{\frac{1}{3}}}{e}\right)\right)bn+\frac{2\,be^2n\log(x)^2+12\,be^2\log\left(\left(dx^{\frac{1}{3}}+e\right)^n\right)\log(x)-12\,be^2\log(x)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x,x, algorithm="maxima")
```

```
[Out] -3*(log(d*x^(1/3)/e + 1)*log(x^(1/3)) + dilog(-d*x^(1/3)/e))*b*n + 1/12*(2*
b*e^2*n*log(x)^2 + 12*b*e^2*log((d*x^(1/3) + e)^n)*log(x) - 12*b*e^2*log(x)
*log(x^(1/3*n)) + 9*b*d^2*n*x^(2/3) - 36*b*d*e*n*x^(1/3) - 6*(b*d^2*n*x^(2/
3) - 2*b*d*e*n*x^(1/3))*log(x) + 12*(b*e^2*log(c) + a*e^2)*log(x) + 3*(2*b*
d^2*n*x*log(x) - 3*b*d^2*n*x)/x^(1/3) - 12*(b*d*e*n*x*log(x) - 3*b*d*e*n*x)
/x^(2/3))/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))/x,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/3))^n))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))/x, x)
```

3.494 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=82

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}-\frac{bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}+\frac{bd^2n}{e^2\sqrt[3]{x}}-\frac{bdn}{2ex^{2/3}}+\frac{bn}{3x}$$

```
[Out] 1/3*b*n/x-1/2*b*d*n/e/x^(2/3)+b*d^2*n/e^2/x^(1/3)-b*d^3*n*ln(d+e/x^(1/3))/e
^3+(-a-b*ln(c*(d+e/x^(1/3))^n))/x
```

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}+\frac{bd^2n}{e^2\sqrt[3]{x}}-\frac{bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}-\frac{bdn}{2ex^{2/3}}+\frac{bn}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x^2,x]
```

```
[Out] (b*n)/(3*x) - (b*d*n)/(2*e*x^(2/3)) + (b*d^2*n)/(e^2*x^(1/3)) - (b*d^3*n*Lo
g[d + e/x^(1/3)])/e^3 - (a + b*Log[c*(d + e/x^(1/3))^n])/x
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx = -\left(3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x} + (ben)\,\text{Subst}\left(\int \frac{x^3}{d+ex}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x} + (ben)\,\text{Subst}\left(\int \left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right)dx$$

$$= \frac{bn}{3x} - \frac{bdn}{2ex^{2/3}} + \frac{bd^2n}{e^2\sqrt[3]{x}} - \frac{bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} - \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}$$

Mathematica [A] time = 0.04, size = 85, normalized size = 1.04

$$-\frac{a}{x} - \frac{b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x} - \frac{bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} + \frac{bd^2n}{e^2\sqrt[3]{x}} - \frac{bdn}{2ex^{2/3}} + \frac{bn}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x^2,x]
```

```
[Out] -(a/x) + (b*n)/(3*x) - (b*d*n)/(2*e*x^(2/3)) + (b*d^2*n)/(e^2*x^(1/3)) - (b
*d^3*n*Log[d + e/x^(1/3)])/e^3 - (b*Log[c*(d + e/x^(1/3))^n])/x
```

fricas [A] time = 0.49, size = 107, normalized size = 1.30

$$\frac{6\,bd^2enx^{\frac{2}{3}} - 3\,bde^2nx^{\frac{1}{3}} + 2\,be^3n - 6\,ae^3 - 2\left(be^3n - 3\,ae^3\right)x + 6\left(be^3x - be^3\right)\log(c) - 6\left(bd^3nx + be^3n\right)\log\left(\frac{dx+}{}\right.}{6\,e^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(6*b*d^2*e*n*x^(2/3) - 3*b*d*e^2*n*x^(1/3) + 2*b*e^3*n - 6*a*e^3 - 2*(b
*e^3*n - 3*a*e^3)*x + 6*(b*e^3*x - b*e^3)*log(c) - 6*(b*d^3*n*x + b*e^3*n)*
log((d*x + e*x^(2/3))/x))/(e^3*x)
```

giac [A] time = 0.38, size = 95, normalized size = 1.16

$$-\frac{1}{6}\left(\left(6\,d^3e^{(-4)}\log\left(\left|dx^{\frac{1}{3}}+e\right|\right) - 2\,d^3e^{(-4)}\log\left(|x|\right) - \frac{\left(6\,d^2x^{\frac{2}{3}}e - 3\,dx^{\frac{1}{3}}e^2 + 2\,e^3\right)e^{(-4)}}{x}\right)e + \frac{6\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)}{x}\right)bn - \frac{b\,l}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^2,x, algorithm="giac")
```

```
[Out] -1/6*((6*d^3*e^(-4)*log(abs(d*x^(1/3) + e)) - 2*d^3*e^(-4)*log(abs(x)) - (6
*d^2*x^(2/3)*e - 3*d*x^(1/3)*e^2 + 2*e^3)*e^(-4)/x)*e + 6*log(d + e/x^(1/3)
)/x)*b*n - b*log(c)/x - a/x
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a}{x^{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/3))^n)+a)/x^2,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/3))^n)+a)/x^2,x)
```

maxima [A] time = 0.47, size = 86, normalized size = 1.05

$$-\frac{1}{6}\,ben\left(\frac{6\,d^{3}\log\left(dx^{\frac{1}{3}}+e\right)}{e^{4}}-\frac{2\,d^{3}\log\left(x\right)}{e^{4}}-\frac{6\,d^{2}x^{\frac{2}{3}}-3\,dex^{\frac{1}{3}}+2\,e^{2}}{e^{3}x}\right)-\frac{b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)}{x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*b*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^(2/3
) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x)) - b*log(c*(d + e/x^(1/3))^n)/x - a/x
```

mupad [B] time = 0.43, size = 73, normalized size = 0.89

$$\frac{b\,n}{3\,x}-\frac{a}{x}-\frac{b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{x}-\frac{b\,d\,n}{2\,e\,x^{2/3}}-\frac{b\,d^3\,n\,\ln\left(d+\frac{e}{x^{1/3}}\right)}{e^3}+\frac{b\,d^2\,n}{e^2\,x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))/x^2,x)
```

```
[Out] (b*n)/(3*x) - a/x - (b*log(c*(d + e/x^(1/3))^n))/x - (b*d*n)/(2*e*x^(2/3))
- (b*d^3*n*log(d + e/x^(1/3)))/e^3 + (b*d^2*n)/(e^2*x^(1/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x**2,x)
```

```
[Out] Timed out
```

3.495 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=138

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6}-\frac{bd^5n}{2e^5\sqrt[3]{x}}+\frac{bd^4n}{4e^4x^{2/3}}-\frac{bd^3n}{6e^3x}+\frac{bd^2n}{8e^2x^{4/3}}-\frac{bdn}{10ex^{5/3}}+\frac{bn}{12x^2}$$

```
[Out] 1/12*b*n/x^2-1/10*b*d*n/e/x^(5/3)+1/8*b*d^2*n/e^2/x^(4/3)-1/6*b*d^3*n/e^3/x
+1/4*b*d^4*n/e^4/x^(2/3)-1/2*b*d^5*n/e^5/x^(1/3)+1/2*b*d^6*n*ln(d+e/x^(1/3)
)/e^6+1/2*(-a-b*ln(c*(d+e/x^(1/3))^n))/x^2
```

Rubi [A] time = 0.10, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2454, 2395, 43}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2}+\frac{bd^4n}{4e^4x^{2/3}}+\frac{bd^2n}{8e^2x^{4/3}}-\frac{bd^5n}{2e^5\sqrt[3]{x}}-\frac{bd^3n}{6e^3x}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6}-\frac{bdn}{10ex^{5/3}}+\frac{bn}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x^3,x]
```

```
[Out] (b*n)/(12*x^2) - (b*d*n)/(10*e*x^(5/3)) + (b*d^2*n)/(8*e^2*x^(4/3)) - (b*d^
3*n)/(6*e^3*x) + (b*d^4*n)/(4*e^4*x^(2/3)) - (b*d^5*n)/(2*e^5*x^(1/3)) + (b
*d^6*n*Log[d + e/x^(1/3)])/(2*e^6) - (a + b*Log[c*(d + e/x^(1/3))^n])/(2*x^
2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx = -\left(3\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\,\text{Subst}\left(\int \frac{x^6}{d+ex}dx, x, \frac{1}{\sqrt[3]{x}}\right)$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\,\text{Subst}\left(\int\left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3x^2}{e^4}+\frac{d^2x^3}{e^3}-\frac{dx^4}{e^2}\right.\right.$$
$$= \frac{bn}{12x^2}-\frac{bdn}{10ex^{5/3}}+\frac{bd^2n}{8e^2x^{4/3}}-\frac{bd^3n}{6e^3x}+\frac{bd^4n}{4e^4x^{2/3}}-\frac{bd^5n}{2e^5\sqrt[3]{x}}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6} - \frac{a}{}$$

Mathematica [A] time = 0.09, size = 135, normalized size = 0.98

$$-\frac{a}{2x^2}-\frac{b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2}+\frac{1}{2}ben\left(\frac{d^6\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^7}-\frac{d^5}{e^6\sqrt[3]{x}}+\frac{d^4}{2e^5x^{2/3}}-\frac{d^3}{3e^4x}+\frac{d^2}{4e^3x^{4/3}}-\frac{d}{5e^2x^{5/3}}+\frac{1}{6ex^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x^3,x]
```

```
[Out] -1/2*a/x^2 + (b*e*n*(1/(6*e*x^2) - d/(5*e^2*x^(5/3)) + d^2/(4*e^3*x^(4/3))
- d^3/(3*e^4*x) + d^4/(2*e^5*x^(2/3)) - d^5/(e^6*x^(1/3)) + (d^6*Log[d + e/
x^(1/3)])/e^7))/2 - (b*Log[c*(d + e/x^(1/3))^n])/(2*x^2)
```

fricas [A] time = 0.47, size = 165, normalized size = 1.20

$$-\frac{20\,bd^3e^3nx - 10\,be^6n + 60\,ae^6 - 10\left(6\,ae^6 + \left(2\,bd^3e^3 - be^6\right)n\right)x^2 - 60\left(be^6x^2 - be^6\right)\log(c) - 60\left(bd^6nx^2 - be^6n\right)\text{l}}{120\,e^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^3,x, algorithm="fricas")
```

```
[Out] -1/120*(20*b*d^3*e^3*n*x - 10*b*e^6*n + 60*a*e^6 - 10*(6*a*e^6 + (2*b*d^3*e
^3 - b*e^6)*n)*x^2 - 60*(b*e^6*x^2 - b*e^6)*log(c) - 60*(b*d^6*n*x^2 - b*e^
6*n)*log((d*x + e*x^(2/3))/x) + 15*(4*b*d^5*e*n*x - b*d^2*e^4*n)*x^(2/3) -
6*(5*b*d^4*e^2*n*x - 2*b*d*e^5*n)*x^(1/3))/(e^6*x^2)
```

giac [A] time = 0.39, size = 123, normalized size = 0.89

$$\frac{1}{120}\left(\left(60\,d^6e^{(-7)}\log\left(\left|dx^{\frac{1}{3}}+e\right|\right) - 20\,d^6e^{(-7)}\log\left(|x|\right) - \frac{\left(60\,d^5x^{\frac{5}{3}}e - 30\,d^4x^{\frac{4}{3}}e^2 + 20\,d^3xe^3 - 15\,d^2x^{\frac{2}{3}}e^4 + 12\,dx^{\frac{1}{3}}e^5\right.}{x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/120*((60*d^6*e^(-7)*log(abs(d*x^(1/3) + e)) - 20*d^6*e^(-7)*log(abs(x)) -
 (60*d^5*x^(5/3)*e - 30*d^4*x^(4/3)*e^2 + 20*d^3*x*e^3 - 15*d^2*x^(2/3)*e^4
```

```
 + 12*d*x^(1/3)*e^5 - 10*e^6)*e^(-7)/x^2)*e - 60*log(d + e/x^(1/3))/x^2)*b*
n - 1/2*b*log(c)/x^2 - 1/2*a/x^2
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{b \ln \left(c \left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/3))^n)+a)/x^3,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/3))^n)+a)/x^3,x)
```

maxima [A] time = 0.47, size = 117, normalized size = 0.85

$$\frac{1}{120}\,ben\left(\frac{60\,d^{6}\log\left(dx^{\frac{1}{3}}+e\right)}{e^{7}}-\frac{20\,d^{6}\log(x)}{e^{7}}-\frac{60\,d^{5}x^{\frac{5}{3}}-30\,d^{4}ex^{\frac{4}{3}}+20\,d^{3}e^{2}x-15\,d^{2}e^{3}x^{\frac{2}{3}}+12\,de^{4}x^{\frac{1}{3}}-10\,e^{5}}{e^{6}x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^3,x, algorithm="maxima")
```

```
[Out] 1/120*b*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7 - 20*d^6*log(x)/e^7 - (60*d^5*x^
(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 15*d^2*e^3*x^(2/3) + 12*d*e^4*x^(
1/3) - 10*e^5)/(e^6*x^2)) - 1/2*b*log(c*(d + e/x^(1/3))^n)/x^2 - 1/2*a/x^2
```

mupad [B] time = 0.46, size = 113, normalized size = 0.82

$$\frac{b\,n}{12\,x^2}-\frac{a}{2\,x^2}-\frac{b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{2\,x^2}-\frac{b\,d\,n}{10\,e\,x^{5/3}}+\frac{b\,d^6\,n\,\ln\left(d+\frac{e}{x^{1/3}}\right)}{2\,e^6}-\frac{b\,d^3\,n}{6\,e^3\,x}+\frac{b\,d^2\,n}{8\,e^2\,x^{4/3}}+\frac{b\,d^4\,n}{4\,e^4\,x^{2/3}}-\frac{b\,d^5\,n}{2\,e^5\,x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))/x^3,x)
```

```
[Out] (b*n)/(12*x^2) - a/(2*x^2) - (b*log(c*(d + e/x^(1/3))^n))/(2*x^2) - (b*d*n)
/(10*e*x^(5/3)) + (b*d^6*n*log(d + e/x^(1/3)))/(2*e^6) - (b*d^3*n)/(6*e^3*x
) + (b*d^2*n)/(8*e^2*x^(4/3)) + (b*d^4*n)/(4*e^4*x^(2/3)) - (b*d^5*n)/(2*e^
5*x^(1/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x**3,x)
```

```
[Out] Timed out
```

3.496 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=187

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3}-\frac{bd^9n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3e^9}+\frac{bd^8n}{3e^8\sqrt[3]{x}}-\frac{bd^7n}{6e^7x^{2/3}}+\frac{bd^6n}{9e^6x}-\frac{bd^5n}{12e^5x^{4/3}}+\frac{bd^4n}{15e^4x^{5/3}}-\frac{bd^3n}{18e^3x^2}+\frac{bd^2n}{21e^2x^{7/3}}-\cdots$$

```
[Out] 1/27*b*n/x^3-1/24*b*d*n/e/x^(8/3)+1/21*b*d^2*n/e^2/x^(7/3)-1/18*b*d^3*n/e^3
/x^2+1/15*b*d^4*n/e^4/x^(5/3)-1/12*b*d^5*n/e^5/x^(4/3)+1/9*b*d^6*n/e^6/x-1/
6*b*d^7*n/e^7/x^(2/3)+1/3*b*d^8*n/e^8/x^(1/3)-1/3*b*d^9*n*ln(d+e/x^(1/3))/e
^9+1/3*(-a-b*ln(c*(d+e/x^(1/3))^n))/x^3
```

Rubi [A] time = 0.13, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2454, 2395, 43}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3}-\frac{bd^7n}{6e^7x^{2/3}}-\frac{bd^5n}{12e^5x^{4/3}}+\frac{bd^4n}{15e^4x^{5/3}}-\frac{bd^3n}{18e^3x^2}+\frac{bd^2n}{21e^2x^{7/3}}+\frac{bd^8n}{3e^8\sqrt[3]{x}}+\frac{bd^6n}{9e^6x}-\frac{bd^9n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3e^9}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x^4,x]
```

```
[Out] (b*n)/(27*x^3) - (b*d*n)/(24*e*x^(8/3)) + (b*d^2*n)/(21*e^2*x^(7/3)) - (b*d
^3*n)/(18*e^3*x^2) + (b*d^4*n)/(15*e^4*x^(5/3)) - (b*d^5*n)/(12*e^5*x^(4/3)
) + (b*d^6*n)/(9*e^6*x) - (b*d^7*n)/(6*e^7*x^(2/3)) + (b*d^8*n)/(3*e^8*x^(1
/3)) - (b*d^9*n*Log[d + e/x^(1/3)])/(3*e^9) - (a + b*Log[c*(d + e/x^(1/3))^
n])/(3*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx = -\left(3\,\text{Subst}\left(\int x^8\left(a+b\log\left(c(d+ex)^n\right)\right)dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3} + \frac{1}{3}(ben)\,\text{Subst}\left(\int \frac{x^9}{d+ex}dx, x, \frac{1}{\sqrt[3]{x}}\right)$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3} + \frac{1}{3}(ben)\,\text{Subst}\left(\int\left(\frac{d^8}{e^9}-\frac{d^7x}{e^8}+\frac{d^6x^2}{e^7}-\frac{d^5x^3}{e^6}+\frac{d^4}{}\right.\right.$$
$$= \frac{bn}{27x^3} - \frac{bdn}{24ex^{8/3}} + \frac{bd^2n}{21e^2x^{7/3}} - \frac{bd^3n}{18e^3x^2} + \frac{bd^4n}{15e^4x^{5/3}} - \frac{bd^5n}{12e^5x^{4/3}} + \frac{bd^6n}{9e^6x} - \frac{bd^7n}{6e^7x^{2/3}}$$

Mathematica [A] time = 0.16, size = 178, normalized size = 0.95

$$-\frac{a}{3x^3}-\frac{b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3}+\frac{1}{3}ben\left(-\frac{d^9\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^{10}}+\frac{d^8}{e^9\sqrt[3]{x}}-\frac{d^7}{2e^8x^{2/3}}+\frac{d^6}{3e^7x}-\frac{d^5}{4e^6x^{4/3}}+\frac{d^4}{5e^5x^{5/3}}-\frac{d^3}{6e^4x^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x^4,x]
```

```
[Out] -1/3*a/x^3 + (b*e*n*(1/(9*e*x^3) - d/(8*e^2*x^(8/3)) + d^2/(7*e^3*x^(7/3))
- d^3/(6*e^4*x^2) + d^4/(5*e^5*x^(5/3)) - d^5/(4*e^6*x^(4/3)) + d^6/(3*e^7*
x) - d^7/(2*e^8*x^(2/3)) + d^8/(e^9*x^(1/3)) - (d^9*Log[d + e/x^(1/3)])/e^1
0))/3 - (b*Log[c*(d + e/x^(1/3))^n])/(3*x^3)
```

fricas [A] time = 0.45, size = 213, normalized size = 1.14

$$\frac{840\,bd^6e^3nx^2 - 420\,bd^3e^6nx + 280\,be^9n - 2520\,ae^9 + 140\left(18\,ae^9 - \left(6\,bd^6e^3 - 3\,bd^3e^6 + 2\,be^9\right)n\right)x^3 + 2520\left(be^9\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^4,x, algorithm="fricas")
```

```
[Out] 1/7560*(840*b*d^6*e^3*n*x^2 - 420*b*d^3*e^6*n*x + 280*b*e^9*n - 2520*a*e^9
+ 140*(18*a*e^9 - (6*b*d^6*e^3 - 3*b*d^3*e^6 + 2*b*e^9)*n)*x^3 + 2520*(b*e^
9*x^3 - b*e^9)*log(c) - 2520*(b*d^9*n*x^3 + b*e^9*n)*log((d*x + e*x^(2/3))/
x) + 90*(28*b*d^8*e*n*x^2 - 7*b*d^5*e^4*n*x + 4*b*d^2*e^7*n)*x^(2/3) - 63*(
20*b*d^7*e^2*n*x^2 - 8*b*d^4*e^5*n*x + 5*b*d*e^8*n)*x^(1/3))/(e^9*x^3)
```

giac [A] time = 0.41, size = 153, normalized size = 0.82

$$-\frac{1}{7560}\left(\left(\left(2520\,d^9e^{(-10)}\log\left(\left|dx^{\frac{1}{3}}+e\right|\right) - 840\,d^9e^{(-10)}\log\left(|x|\right) - \frac{\left(2520\,d^8x^{\frac{8}{3}}e - 1260\,d^7x^{\frac{7}{3}}e^2 + 840\,d^6x^2e^3 - 630\right.}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^4,x, algorithm="giac")
```

```
[Out] -1/7560*((2520*d^9*e^(-10)*log(abs(d*x^(1/3) + e)) - 840*d^9*e^(-10)*log(ab
s(x)) - (2520*d^8*x^(8/3)*e - 1260*d^7*x^(7/3)*e^2 + 840*d^6*x^2*e^3 - 630*
d^5*x^(5/3)*e^4 + 504*d^4*x^(4/3)*e^5 - 420*d^3*x*e^6 + 360*d^2*x^(2/3)*e^7
 - 315*d*x^(1/3)*e^8 + 280*e^9)*e^(-10)/x^3)*e + 2520*log(d + e/x^(1/3))/x^
3)*b*n - 1/3*b*log(c)/x^3 - 1/3*a/x^3
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{b \ln \left(c \left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/3))^n)+a)/x^4,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/3))^n)+a)/x^4,x)
```

maxima [A] time = 0.48, size = 150, normalized size = 0.80

$$-\frac{1}{7560}\, ben\left(\frac{2520\, d^{9} \log\left(dx^{\frac{1}{3}}+e\right)}{e^{10}}-\frac{840\, d^{9} \log(x)}{e^{10}}-\frac{2520\, d^{8} x^{\frac{8}{3}}-1260\, d^{7} e x^{\frac{7}{3}}+840\, d^{6} e^{2} x^{2}-630\, d^{5} e^{3} x^{\frac{5}{3}}+504\, d^{4}}{e^{9} x^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^4,x, algorithm="maxima")
```

```
[Out] -1/7560*b*e*n*(2520*d^9*log(d*x^(1/3) + e)/e^10 - 840*d^9*log(x)/e^10 - (25
20*d^8*x^(8/3) - 1260*d^7*e*x^(7/3) + 840*d^6*e^2*x^2 - 630*d^5*e^3*x^(5/3)
 + 504*d^4*e^4*x^(4/3) - 420*d^3*e^5*x + 360*d^2*e^6*x^(2/3) - 315*d*e^7*x^
(1/3) + 280*e^8)/(e^9*x^3)) - 1/3*b*log(c*(d + e/x^(1/3))^n)/x^3 - 1/3*a/x^
3
```

mupad [B] time = 0.53, size = 152, normalized size = 0.81

$$\frac{b\, n}{27\, x^{3}}-\frac{a}{3\, x^{3}}-\frac{b\, \ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^{n}\right)}{3\, x^{3}}-\frac{b\, d\, n}{24\, e\, x^{8/3}}-\frac{b\, d^{9}\, n\, \ln\left(d+\frac{e}{x^{1/3}}\right)}{3\, e^{9}}-\frac{b\, d^{3}\, n}{18\, e^{3}\, x^{2}}+\frac{b\, d^{6}\, n}{9\, e^{6}\, x}+\frac{b\, d^{2}\, n}{21\, e^{2}\, x^{7/3}}+\frac{b\, d^{4}\, n}{15\, e^{4}\, x^{5/3}}-\frac{b\, d^{5}\, n}{12\, e^{5}\, x^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))/x^4,x)
```

```
[Out] (b*n)/(27*x^3) - a/(3*x^3) - (b*log(c*(d + e/x^(1/3))^n))/(3*x^3) - (b*d*n)
/(24*e*x^(8/3)) - (b*d^9*n*log(d + e/x^(1/3)))/(3*e^9) - (b*d^3*n)/(18*e^3*
x^2) + (b*d^6*n)/(9*e^6*x) + (b*d^2*n)/(21*e^2*x^(7/3)) + (b*d^4*n)/(15*e^4
*x^(5/3)) - (b*d^5*n)/(12*e^5*x^(4/3)) - (b*d^7*n)/(6*e^7*x^(2/3)) + (b*d^8
*n)/(3*e^8*x^(1/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x**4,x)
```

```
[Out] Timed out
```

3.497 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=572

$$-\frac{2be^9n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^9}-\frac{2be^8n\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^9}+\frac{be^7nx^{2/3}\left(a\right.}{}$$

```
[Out] 481/420*b^2*e^8*n^2*x^(1/3)/d^8-341/840*b^2*e^7*n^2*x^(2/3)/d^7+743/3780*b^
2*e^6*n^2*x/d^6-533/5040*b^2*e^5*n^2*x^(4/3)/d^5+73/1260*b^2*e^4*n^2*x^(5/3
)/d^4-5/168*b^2*e^3*n^2*x^2/d^3+1/84*b^2*e^2*n^2*x^(7/3)/d^2-481/420*b^2*e^
9*n^2*ln(d+e/x^(1/3))/d^9-2/3*b*e^8*n*(d+e/x^(1/3))*x^(1/3)*(a+b*ln(c*(d+e/
x^(1/3))^n))/d^9+1/3*b*e^7*n*x^(2/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^7-2/9*b*
e^6*n*x*(a+b*ln(c*(d+e/x^(1/3))^n))/d^6+1/6*b*e^5*n*x^(4/3)*(a+b*ln(c*(d+e/
x^(1/3))^n))/d^5-2/15*b*e^4*n*x^(5/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^4+1/9*b
*e^3*n*x^2*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3-2/21*b*e^2*n*x^(7/3)*(a+b*ln(c*(
d+e/x^(1/3))^n))/d^2+1/12*b*e*n*x^(8/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d-2/3*b
*e^9*n*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x^(1/3))^n))/d^9+1/3*x^3*(a+b*l
n(c*(d+e/x^(1/3))^n))^2-761/1260*b^2*e^9*n^2*ln(x)/d^9+2/3*b^2*e^9*n^2*poly
log(2,d/(d+e/x^(1/3)))/d^9
```

Rubi [A] time = 1.72, antiderivative size = 596, normalized size of antiderivative = 1.04, number of steps used = 38, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$-\frac{2b^2e^9n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)}{3d^9}+\frac{be^7nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^7}+\frac{be^5nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6d^5}-\frac{2be}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (481*b^2*e^8*n^2*x^(1/3))/(420*d^8) - (341*b^2*e^7*n^2*x^(2/3))/(840*d^7) +
 (743*b^2*e^6*n^2*x)/(3780*d^6) - (533*b^2*e^5*n^2*x^(4/3))/(5040*d^5) + (7
3*b^2*e^4*n^2*x^(5/3))/(1260*d^4) - (5*b^2*e^3*n^2*x^2)/(168*d^3) + (b^2*e^
2*n^2*x^(7/3))/(84*d^2) - (481*b^2*e^9*n^2*Log[d + e/x^(1/3)])/(420*d^9) -
(2*b*e^8*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*d^9
) + (b*e^7*n*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*d^7) - (2*b*e^6*n
*x*(a + b*Log[c*(d + e/x^(1/3))^n]))/(9*d^6) + (b*e^5*n*x^(4/3)*(a + b*Log[
c*(d + e/x^(1/3))^n]))/(6*d^5) - (2*b*e^4*n*x^(5/3)*(a + b*Log[c*(d + e/x^(
1/3))^n]))/(15*d^4) + (b*e^3*n*x^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(9*d^3
) - (2*b*e^2*n*x^(7/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(21*d^2) + (b*e*n*
x^(8/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(12*d) + (e^9*(a + b*Log[c*(d + e
/x^(1/3))^n])^2)/(3*d^9) + (x^3*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/3 - (2*
b*e^9*n*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))])/(3*d^9) - (
761*b^2*e^9*n^2*Log[x])/(1260*d^9) - (2*b^2*e^9*n^2*PolyLog[2, 1 + e/(d*x^(
1/3))])/(3*d^9)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2dx = -\left(3\,\text{Subst}\left(\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^{10}}dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-\frac{1}{3}(2ben)\,\text{Subst}\left(\int\frac{a+b\log(c(d+ex)}{x^9(d+ex)}\right.$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-\frac{1}{3}(2bn)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^9}dx,x\right.$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-\frac{(2bn)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^9}dx,x,d+\frac{e}{\sqrt[3]{}}\right.}{3d}$$

$$=\frac{benx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12d}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2+-$$

$$=-\frac{2be^2nx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{21d^2}+\frac{benx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right.}{12d}$$

$$=\frac{b^2e^8n^2\sqrt[3]{x}}{12d^8}-\frac{b^2e^7n^2x^{2/3}}{24d^7}+\frac{b^2e^6n^2x}{36d^6}-\frac{b^2e^5n^2x^{4/3}}{48d^5}+\frac{b^2e^4n^2x^{5/3}}{60d^4}-\frac{b^2e^3n^2x^2}{72d^3}$$

$$=\frac{5b^2e^8n^2\sqrt[3]{x}}{28d^8}-\frac{5b^2e^7n^2x^{2/3}}{56d^7}+\frac{5b^2e^6n^2x}{84d^6}-\frac{5b^2e^5n^2x^{4/3}}{112d^5}+\frac{b^2e^4n^2x^{5/3}}{28d^4}-\frac{5b^2}{1}$$

$$=\frac{73b^2e^8n^2\sqrt[3]{x}}{252d^8}-\frac{73b^2e^7n^2x^{2/3}}{504d^7}+\frac{73b^2e^6n^2x}{756d^6}-\frac{73b^2e^5n^2x^{4/3}}{1008d^5}+\frac{73b^2e^4n^2x^{5/}}{1260d^4}$$

$$=\frac{533b^2e^8n^2\sqrt[3]{x}}{1260d^8}-\frac{533b^2e^7n^2x^{2/3}}{2520d^7}+\frac{533b^2e^6n^2x}{3780d^6}-\frac{533b^2e^5n^2x^{4/3}}{5040d^5}+\frac{73b^2e^4}{126}$$

$$=\frac{743b^2e^8n^2\sqrt[3]{x}}{1260d^8}-\frac{743b^2e^7n^2x^{2/3}}{2520d^7}+\frac{743b^2e^6n^2x}{3780d^6}-\frac{533b^2e^5n^2x^{4/3}}{5040d^5}+\frac{73b^2e^4}{126}$$

$$=\frac{341b^2e^8n^2\sqrt[3]{x}}{420d^8}-\frac{341b^2e^7n^2x^{2/3}}{840d^7}+\frac{743b^2e^6n^2x}{3780d^6}-\frac{533b^2e^5n^2x^{4/3}}{5040d^5}+\frac{73b^2e^4}{126}$$

$$=\frac{481b^2e^8n^2\sqrt[3]{x}}{420d^8}-\frac{341b^2e^7n^2x^{2/3}}{840d^7}+\frac{743b^2e^6n^2x}{3780d^6}-\frac{533b^2e^5n^2x^{4/3}}{5040d^5}+\frac{73b^2e^4}{126}$$

$$=\frac{481b^2e^8n^2\sqrt[3]{x}}{420d^8}-\frac{341b^2e^7n^2x^{2/3}}{840d^7}+\frac{743b^2e^6n^2x}{3780d^6}-\frac{533b^2e^5n^2x^{4/3}}{5040d^5}+\frac{73b^2e^4}{126}$$

Mathematica [A] time = 0.44, size = 738, normalized size = 1.29

$$\frac{5040a^2d^9x^3 + 10080abd^9x^3\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right) + 1260abd^8enx^{8/3} - 1440abd^7e^2nx^{7/3} + 1680abd^6e^3nx^2 - 2016a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (-10080*a*b*d*e^8*n*x^(1/3) + 17316*b^2*d*e^8*n^2*x^(1/3) + 5040*a*b*d^2*e^
7*n*x^(2/3) - 6138*b^2*d^2*e^7*n^2*x^(2/3) - 3360*a*b*d^3*e^6*n*x + 2972*b^
2*d^3*e^6*n^2*x + 2520*a*b*d^4*e^5*n*x^(4/3) - 1599*b^2*d^4*e^5*n^2*x^(4/3)
 - 2016*a*b*d^5*e^4*n*x^(5/3) + 876*b^2*d^5*e^4*n^2*x^(5/3) + 1680*a*b*d^6*
e^3*n*x^2 - 450*b^2*d^6*e^3*n^2*x^2 - 1440*a*b*d^7*e^2*n*x^(7/3) + 180*b^2*
d^7*e^2*n^2*x^(7/3) + 1260*a*b*d^8*e*n*x^(8/3) + 5040*a^2*d^9*x^3 - 22356*b
^2*e^9*n^2*Log[d + e/x^(1/3)] - 10080*b^2*d*e^8*n*x^(1/3)*Log[c*(d + e/x^(1
/3))^n] + 5040*b^2*d^2*e^7*n*x^(2/3)*Log[c*(d + e/x^(1/3))^n] - 3360*b^2*d^
3*e^6*n*x*Log[c*(d + e/x^(1/3))^n] + 2520*b^2*d^4*e^5*n*x^(4/3)*Log[c*(d +
e/x^(1/3))^n] - 2016*b^2*d^5*e^4*n*x^(5/3)*Log[c*(d + e/x^(1/3))^n] + 1680*
b^2*d^6*e^3*n*x^2*Log[c*(d + e/x^(1/3))^n] - 1440*b^2*d^7*e^2*n*x^(7/3)*Log
[c*(d + e/x^(1/3))^n] + 1260*b^2*d^8*e*n*x^(8/3)*Log[c*(d + e/x^(1/3))^n] +
 10080*a*b*d^9*x^3*Log[c*(d + e/x^(1/3))^n] + 5040*b^2*d^9*x^3*Log[c*(d + e
/x^(1/3))^n]^2 + 10080*a*b*e^9*n*Log[e + d*x^(1/3)] - 5040*b^2*e^9*n^2*Log[
e + d*x^(1/3)] + 10080*b^2*e^9*n*Log[c*(d + e/x^(1/3))^n]*Log[e + d*x^(1/3)
] - 5040*b^2*e^9*n^2*Log[e + d*x^(1/3)]^2 + 10080*b^2*e^9*n^2*Log[e + d*x^(
1/3)]*Log[-((d*x^(1/3))/e)] - 7452*b^2*e^9*n^2*Log[x] + 10080*b^2*e^9*n^2*P
olyLog[2, 1 + (d*x^(1/3))/e])/(15120*d^9)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^2x^2\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)^2 + 2\,abx^2\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right) + a^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*log(c*((d*x + e*x^(2/3))/x)^n)^2 + 2*a*b*x^2*log(c*((d*x +
 e*x^(2/3))/x)^n) + a^2*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2 x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2*x^2, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2 x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*(d+e/x^(1/3))^n)+a)^2,x)
```

```
[Out] int(x^2*(b*ln(c*(d+e/x^(1/3))^n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,b^2x^3\log\left(\left(dx^{\frac{1}{3}}+e\right)^n\right)^2-\int-\frac{9\left(b^2d\log(c)^2+2\,abd\log(c)+a^2d\right)x^3+9\left(b^2e\log(c)^2+2\,abe\log(c)+a^2e\right)x^{\frac{8}{3}}+9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*log((d*x^(1/3) + e)^n)^2 - integrate(-1/9*(9*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^3 + 9*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x
^(8/3) + 9*(b^2*d*x^3 + b^2*e*x^(8/3))*log(x^(1/3*n))^2 - 2*(b^2*d*n*x^3 -
9*(b^2*d*log(c) + a*b*d)*x^3 - 9*(b^2*e*log(c) + a*b*e)*x^(8/3) + 9*(b^2*d*
x^3 + b^2*e*x^(8/3))*log(x^(1/3*n)))*log((d*x^(1/3) + e)^n) - 18*((b^2*d*lo
g(c) + a*b*d)*x^3 + (b^2*e*log(c) + a*b*e)*x^(8/3))*log(x^(1/3*n)))/(d*x +
e*x^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e/x^(1/3))^n))^2,x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e/x^(1/3))^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(1/3))**n))**2,x)
```

```
[Out] Timed out
```

3.498 $\int x \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=400

$$\frac{be^6n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6}+\frac{be^5n\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6}-\frac{be^4nx^{2/3}\left(a+b\right.}{}$$

```
[Out] -77/60*b^2*e^5*n^2*x^(1/3)/d^5+47/120*b^2*e^4*n^2*x^(2/3)/d^4-3/20*b^2*e^3*
n^2*x/d^3+1/20*b^2*e^2*n^2*x^(4/3)/d^2+77/60*b^2*e^6*n^2*ln(d+e/x^(1/3))/d^
6+b*e^5*n*(d+e/x^(1/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^6-1/2*b*e^4*n
*x^(2/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^4+1/3*b*e^3*n*x*(a+b*ln(c*(d+e/x^(1/
3))^n))/d^3-1/4*b*e^2*n*x^(4/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^2+1/5*b*e*n*x
^(5/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d+b*e^6*n*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(
c*(d+e/x^(1/3))^n))/d^6+1/2*x^2*(a+b*ln(c*(d+e/x^(1/3))^n))^2+137/180*b^2*e
^6*n^2*ln(x)/d^6-b^2*e^6*n^2*polylog(2,d/(d+e/x^(1/3)))/d^6
```

Rubi [A] time = 1.02, antiderivative size = 423, normalized size of antiderivative = 1.06, number of steps used = 26, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$\frac{b^2e^6n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)}{d^6}-\frac{be^4nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2d^4}-\frac{be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4d^2}-\frac{e^6\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (-77*b^2*e^5*n^2*x^(1/3))/(60*d^5) + (47*b^2*e^4*n^2*x^(2/3))/(120*d^4) - (
3*b^2*e^3*n^2*x)/(20*d^3) + (b^2*e^2*n^2*x^(4/3))/(20*d^2) + (77*b^2*e^6*n^
2*Log[d + e/x^(1/3)])/(60*d^6) + (b*e^5*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Lo
g[c*(d + e/x^(1/3))^n]))/d^6 - (b*e^4*n*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3)
)^n]))/(2*d^4) + (b*e^3*n*x*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*d^3) - (b*
e^2*n*x^(4/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(4*d^2) + (b*e*n*x^(5/3)*(a
 + b*Log[c*(d + e/x^(1/3))^n]))/(5*d) - (e^6*(a + b*Log[c*(d + e/x^(1/3))^n
])^2)/(2*d^6) + (x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/2 + (b*e^6*n*(a +
b*Log[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))])/d^6 + (137*b^2*e^6*n^2*L
og[x])/(180*d^6) + (b^2*e^6*n^2*PolyLog[2, 1 + e/(d*x^(1/3))])/d^6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx = -\left(3\,\text{Subst}\left(\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^7}dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - (ben)\,\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^6(d+ex)}\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - (bn)\,\text{Subst}\left(\int\frac{a+b\log\left(cx^n\right)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}dx, x,\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{(bn)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}dx, x, d+\right.}{d}$$

$$= \frac{benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5d} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 +$$

$$= -\frac{be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4d^2} + \frac{benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right.\right.}{5d}$$

$$= -\frac{b^2e^5n^2\sqrt[3]{x}}{5d^5} + \frac{b^2e^4n^2x^{2/3}}{10d^4} - \frac{b^2e^3n^2x}{15d^3} + \frac{b^2e^2n^2x^{4/3}}{20d^2} + \frac{b^2e^6n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right.}{5d^6}$$

$$= -\frac{9b^2e^5n^2\sqrt[3]{x}}{20d^5} + \frac{9b^2e^4n^2x^{2/3}}{40d^4} - \frac{3b^2e^3n^2x}{20d^3} + \frac{b^2e^2n^2x^{4/3}}{20d^2} + \frac{9b^2e^6n^2\log\left(d\right.}{20d^6}$$

$$= -\frac{47b^2e^5n^2\sqrt[3]{x}}{60d^5} + \frac{47b^2e^4n^2x^{2/3}}{120d^4} - \frac{3b^2e^3n^2x}{20d^3} + \frac{b^2e^2n^2x^{4/3}}{20d^2} + \frac{47b^2e^6n^2\log}{60d}$$

$$= -\frac{77b^2e^5n^2\sqrt[3]{x}}{60d^5} + \frac{47b^2e^4n^2x^{2/3}}{120d^4} - \frac{3b^2e^3n^2x}{20d^3} + \frac{b^2e^2n^2x^{4/3}}{20d^2} + \frac{77b^2e^6n^2\log}{60d}$$

$$= -\frac{77b^2e^5n^2\sqrt[3]{x}}{60d^5} + \frac{47b^2e^4n^2x^{2/3}}{120d^4} - \frac{3b^2e^3n^2x}{20d^3} + \frac{b^2e^2n^2x^{4/3}}{20d^2} + \frac{77b^2e^6n^2\log}{60d}$$

Mathematica [A] time = 0.25, size = 546, normalized size = 1.36

$$180a^2d^6x^2 + 360abd^6x^2\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right) + 72abd^5enx^{5/3} - 90abd^4e^2nx^{4/3} + 120abd^3e^3nx - 180abd^2e^4nx^{2/3} - 3$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (360*a*b*d*e^5*n*x^(1/3) - 462*b^2*d*e^5*n^2*x^(1/3) - 180*a*b*d^2*e^4*n*x^
(2/3) + 141*b^2*d^2*e^4*n^2*x^(2/3) + 120*a*b*d^3*e^3*n*x - 54*b^2*d^3*e^3*
n^2*x - 90*a*b*d^4*e^2*n*x^(4/3) + 18*b^2*d^4*e^2*n^2*x^(4/3) + 72*a*b*d^5*
e*n*x^(5/3) + 180*a^2*d^6*x^2 + 642*b^2*e^6*n^2*Log[d + e/x^(1/3)] + 360*b^
2*d*e^5*n*x^(1/3)*Log[c*(d + e/x^(1/3))^n] - 180*b^2*d^2*e^4*n*x^(2/3)*Log[
c*(d + e/x^(1/3))^n] + 120*b^2*d^3*e^3*n*x*Log[c*(d + e/x^(1/3))^n] - 90*b^
2*d^4*e^2*n*x^(4/3)*Log[c*(d + e/x^(1/3))^n] + 72*b^2*d^5*e*n*x^(5/3)*Log[c
*(d + e/x^(1/3))^n] + 360*a*b*d^6*x^2*Log[c*(d + e/x^(1/3))^n] + 180*b^2*d^
6*x^2*Log[c*(d + e/x^(1/3))^n]^2 - 360*a*b*e^6*n*Log[e + d*x^(1/3)] + 180*b
^2*e^6*n^2*Log[e + d*x^(1/3)] - 360*b^2*e^6*n*Log[c*(d + e/x^(1/3))^n]*Log[
e + d*x^(1/3)] + 180*b^2*e^6*n^2*Log[e + d*x^(1/3)]^2 - 360*b^2*e^6*n^2*Log
[e + d*x^(1/3)]*Log[-((d*x^(1/3))/e)] + 214*b^2*e^6*n^2*Log[x] - 360*b^2*e^
6*n^2*PolyLog[2, 1 + (d*x^(1/3))/e])/(360*d^6)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^2x\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)^2 + 2\,abx\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right) + a^2x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x*log(c*((d*x + e*x^(2/3))/x)^n)^2 + 2*a*b*x*log(c*((d*x + e*x
^(2/3))/x)^n) + a^2*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2*x, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(d+e/x^(1/3))^n)+a)^2,x)
```

```
[Out] int(x*(b*ln(c*(d+e/x^(1/3))^n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}b^2x^2\log\left(\left(dx^{\frac{1}{3}}+e\right)^n\right)^2-\int-\frac{3\left(b^2d\log(c)^2+2\,abd\log(c)+a^2d\right)x^2+3\left(b^2dx^2+b^2ex^{\frac{5}{3}}\right)\log\left(x^{\frac{1}{3}n}\right)^2+3\left(b^2e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log((d*x^(1/3) + e)^n)^2 - integrate(-1/3*(3*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^2 + 3*(b^2*d*x^2 + b^2*e*x^(5/3))*log(x^(1/3*n))^
2 + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(5/3) - (b^2*d*n*x^2 - 6*
(b^2*d*log(c) + a*b*d)*x^2 - 6*(b^2*e*log(c) + a*b*e)*x^(5/3) + 6*(b^2*d*x^
2 + b^2*e*x^(5/3))*log(x^(1/3*n)))*log((d*x^(1/3) + e)^n) - 6*((b^2*d*log(c
) + a*b*d)*x^2 + (b^2*e*log(c) + a*b*e)*x^(5/3))*log(x^(1/3*n)))/(d*x + e*x
^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/3))^n))^2,x)
```

```
[Out] int(x*(a + b*log(c*(d + e/x^(1/3))^n))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))**n))**2,x)
```

```
[Out] Integral(x*(a + b*log(c*(d + e/x**(1/3))**n))**2, x)
```

3.499 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=227

$$-\frac{2be^3n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}-\frac{2be^2n\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}+\frac{benx^{2/3}\left(a+b\right.}{}$$

```
[Out] b^2*e^2*n^2*x^(1/3)/d^2-b^2*e^3*n^2*ln(d+e/x^(1/3))/d^3-2*b*e^2*n*(d+e/x^(1
/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3+b*e*n*x^(2/3)*(a+b*ln(c*(d+e/x
^(1/3))^n))/d-2*b*e^3*n*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x^(1/3))^n))/d
^3+x*(a+b*ln(c*(d+e/x^(1/3))^n))^2-b^2*e^3*n^2*ln(x)/d^3+2*b^2*e^3*n^2*poly
log(2,d/(d+e/x^(1/3)))/d^3
```

Rubi [A] time = 0.53, antiderivative size = 248, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.650, Rules used = {2451, 2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$-\frac{2b^2e^3n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)}{d^3}+\frac{e^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3}-\frac{2be^3n\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}-\frac{2\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (b^2*e^2*n^2*x^(1/3))/d^2 - (b^2*e^3*n^2*Log[d + e/x^(1/3)])/d^3 - (2*b*e^2
*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/d^3 + (b*e*n*x
^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/d + (e^3*(a + b*Log[c*(d + e/x^(1/
3))^n])^2)/d^3 + x*(a + b*Log[c*(d + e/x^(1/3))^n])^2 - (2*b*e^3*n*(a + b*L
og[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))])/d^3 - (b^2*e^3*n^2*Log[x])/
d^3 - (2*b^2*e^3*n^2*PolyLog[2, 1 + e/(d*x^(1/3))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx = 3\,\mathrm{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^n\right)\right)^2 dx, x, \sqrt[3]{x}\right)$$

$$= -\left(3\,\mathrm{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4}dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - (2ben)\,\mathrm{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^3(d+ex)}dx, x\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - (2bn)\,\mathrm{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}dx, x, d+\frac{e}{\sqrt[3]{}}\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{(2bn)\,\mathrm{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d} + $$

$$= \frac{benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 + \frac{(2ben)}{}$$

$$= -\frac{2be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} + \frac{benx^{2/3}\left(a+b\log\left(c\left(d+\right.\right.\right.}{d}$$

$$= \frac{b^2e^2n^2\sqrt[3]{x}}{d^2} - \frac{b^2e^3n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3} - \frac{2be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\right.\right.\right.}{d^3}$$

$$= \frac{b^2e^2n^2\sqrt[3]{x}}{d^2} - \frac{b^2e^3n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3} - \frac{2be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\right.\right.\right.}{d^3}$$

Mathematica [A] time = 0.19, size = 237, normalized size = 1.04

$$x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{ben\left(-3d^2x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right) - 6e^2\log\left(d\sqrt[3]{x}+e\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{}}\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] x*(a + b*Log[c*(d + e/x^(1/3))^n])^2 - (b*e*n*(6*a*d*e*x^(1/3) + 6*b*d*e*x^
(1/3)*Log[c*(d + e/x^(1/3))^n] - 3*d^2*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3))
^n]) - 6*e^2*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[e + d*x^(1/3)] + 3*b*e*n*
```

```
(-(d*x^(1/3)) + e*Log[e + d*x^(1/3)]) + 2*b*e^2*n*(3*Log[d + e/x^(1/3)] + L
og[x]) + 3*b*e^2*n*(Log[e + d*x^(1/3)]*(Log[e + d*x^(1/3)] - 2*Log[-((d*x^(
1/3))/e)]) - 2*PolyLog[2, 1 + (d*x^(1/3))/e])))/(3*d^3)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)^2+2\,ab\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)+a^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*log(c*((d*x + e*x^(2/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(2/
3))/x)^n) + a^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2, x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/3))^n)+a)^2,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/3))^n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(en\left(\frac{2\,e^2\log\left(dx^{\frac{1}{3}}+e\right)}{d^3}+\frac{dx^{\frac{2}{3}}-2\,ex^{\frac{1}{3}}}{d^2}\right)+2\,x\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)\right)ab+\left(x\log\left(\left(dx^{\frac{1}{3}}+e\right)^n\right)^2-\int-\frac{3\,dx\log(c)^2+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] (e*n*(2*e^2*log(d*x^(1/3) + e)/d^3 + (d*x^(2/3) - 2*e*x^(1/3))/d^2) + 2*x*l
og(c*(d + e/x^(1/3))^n))*a*b + (x*log((d*x^(1/3) + e)^n)^2 - integrate(-1/3
*(3*d*x*log(c)^2 + 3*e*x^(2/3)*log(c)^2 + 3*(d*x + e*x^(2/3))*log(x^(1/3*n)
)^2 - 2*(d*n*x - 3*d*x*log(c) - 3*e*x^(2/3)*log(c) + 3*(d*x + e*x^(2/3))*lo
g(x^(1/3*n)))*log((d*x^(1/3) + e)^n) - 6*(d*x*log(c) + e*x^(2/3)*log(c))*lo
g(x^(1/3*n)))/(d*x + e*x^(2/3)), x))*b^2 + a^2*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{1/3}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^2,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/3))^n))^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \log \left(c \left(d + \frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))**2, x)
```

3.500 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx$

Optimal. Leaf size=93

$$-6bn\text{Li}_2\left(\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2+6b^2n^2\text{Li}_3\left(\frac{e}{d\sqrt[3]{x}}\right.$$

```
[Out] -3*(a+b*ln(c*(d+e/x^(1/3))^n))^2*ln(-e/d/x^(1/3))-6*b*n*(a+b*ln(c*(d+e/x^(1
/3))^n))*polylog(2,1+e/d/x^(1/3))+6*b^2*n^2*polylog(3,1+e/d/x^(1/3))
```

Rubi [A] time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2454, 2396, 2433, 2374, 6589}

$$-6bn\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)+6b^2n^2\text{PolyLog}\left(3,\frac{e}{d\sqrt[3]{x}}+1\right)-3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x,x]
```

```
[Out] -3*(a + b*Log[c*(d + e/x^(1/3))^n])^2*Log[-(e/(d*x^(1/3)))] - 6*b*n*(a + b*
Log[c*(d + e/x^(1/3))^n])*PolyLog[2, 1 + e/(d*x^(1/3))] + 6*b^2*n^2*PolyLog
[3, 1 + e/(d*x^(1/3))]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx = -\left(3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)+(6ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a\right.$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)+(6bn)\,\text{Subst}\left(\int \frac{\left(a+b\log(cx\right.}{}\right.$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)-6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right.$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)-6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right.$$

Mathematica [B] time = 0.22, size = 389, normalized size = 4.18

$$2bn\left(3\text{Li}_2\left(-\frac{e}{d\sqrt[3]{x}}\right)+\log(x)\left(\log\left(d+\frac{e}{\sqrt[3]{x}}\right)-\log\left(\frac{e}{d\sqrt[3]{x}}+1\right)\right)\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-bn\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])*((Log[d + e/x^(1/
3)] - Log[1 + e/(d*x^(1/3))])*Log[x] + 3*PolyLog[2, -(e/(d*x^(1/3)))]) + 3*
b^2*n^2*(2*Log[e/d + x^(1/3)]*PolyLog[2, 1 + (d*x^(1/3))/e] - 2*(Log[d + e/
x^(1/3)] - Log[e/d + x^(1/3)])*PolyLog[2, -((d*x^(1/3))/e)] + (81*Log[e/d +
 x^(1/3)]^2*Log[-((d*x^(1/3))/e)] + 27*Log[d + e/x^(1/3)]^2*Log[x] - 27*Log
[e/d + x^(1/3)]^2*Log[x] - 54*Log[d + e/x^(1/3)]*Log[1 + (d*x^(1/3))/e]*Log
[x] + 54*Log[e/d + x^(1/3)]*Log[1 + (d*x^(1/3))/e]*Log[x] + 9*Log[d + e/x^(
1/3)]*Log[x]^2 - 9*Log[1 + (d*x^(1/3))/e]*Log[x]^2 + Log[x]^3 - 162*PolyLog
[3, 1 + (d*x^(1/3))/e] - 162*PolyLog[3, -((d*x^(1/3))/e)])/81)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)^2+2\,ab\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)+a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*((d*x + e*x^(2/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(2
/3))/x)^n) + a^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2/x, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/3))^n)+a)^2/x,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/3))^n)+a)^2/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^{2}\log\left(\left(dx^{\frac{1}{3}}+e\right)^{n}\right)^{2}\log(x)-\int -\frac{3\left(b^{2}dx+b^{2}ex^{\frac{2}{3}}\right)\log\left(x^{\frac{1}{3}n}\right)^{2}+3\left(b^{2}d\log(c)^{2}+2\,abd\log(c)+a^{2}d\right)x-2\left(b^{2}d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*log((d*x^(1/3) + e)^n)^2*log(x) - integrate(-1/3*(3*(b^2*d*x + b^2*e*x^
(2/3))*log(x^(1/3*n))^2 + 3*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x - 2
*(b^2*d*n*x*log(x) - 3*(b^2*d*log(c) + a*b*d)*x + 3*(b^2*d*x + b^2*e*x^(2/3
))*log(x^(1/3*n)) - 3*(b^2*e*log(c) + a*b*e)*x^(2/3))*log((d*x^(1/3) + e)^n
) - 6*((b^2*d*log(c) + a*b*d)*x + (b^2*e*log(c) + a*b*e)*x^(2/3))*log(x^(1/
3*n)) + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(2/3))/(d*x^2 + e*x^(
5/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{1/3}}\right)^{n}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^2/x,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/3))^n))^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))**2/x, x)
```

3.501 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=269

$$-\frac{2bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3}+\frac{6bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3}-\frac{3bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{}$$

```
[Out] 3/2*b^2*d*n^2*(d+e/x^(1/3))^2/e^3-2/9*b^2*n^2*(d+e/x^(1/3))^3/e^3-6*b^2*d^2
*n^2/e^2/x^(1/3)+b^2*d^3*n^2*ln(d+e/x^(1/3))^2/e^3+6*b*d^2*n*(d+e/x^(1/3))*
(a+b*ln(c*(d+e/x^(1/3))^n))/e^3-3*b*d*n*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^(1
/3))^n))/e^3+2/3*b*n*(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))/e^3-2*b*d^
3*n*ln(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))/e^3-(a+b*ln(c*(d+e/x^(1/3))
^n))^2/x
```

Rubi [A] time = 0.31, antiderivative size = 212, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$\frac{1}{3}bn\left(\frac{18d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}-\frac{6d^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}-\frac{9d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^3}+\frac{2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^3}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^2,x]
```

```
[Out] (3*b^2*d*n^2*(d + e/x^(1/3))^2)/(2*e^3) - (2*b^2*n^2*(d + e/x^(1/3))^3)/(9*
e^3) - (6*b^2*d^2*n^2)/(e^2*x^(1/3)) + (b^2*d^3*n^2*Log[d + e/x^(1/3)]^2)/e
^3 + (b*n*((18*d^2*(d + e/x^(1/3)))/e^3 - (9*d*(d + e/x^(1/3))^2)/e^3 + (2*
(d + e/x^(1/3))^3)/e^3 - (6*d^3*Log[d + e/x^(1/3)])/e^3)*(a + b*Log[c*(d +
e/x^(1/3))^n]))/3 - (a + b*Log[c*(d + e/x^(1/3))^n])^2/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx = -\left(3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x} + (2ben)\,\text{Subst}\left(\int \frac{x^3\left(a+b\log\left(c(d+ex)^n\right)\right.}{d+ex}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x} + (2bn)\,\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3\left(a+b\log\left(cx^n\right.\right.}{x}\right.$$

$$= \frac{1}{3}bn\left(\frac{18d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^3} + \frac{2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^3} - \frac{6d^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}\right.$$

$$= \frac{1}{3}bn\left(\frac{18d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^3} + \frac{2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^3} - \frac{6d^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}\right.$$

$$= \frac{1}{3}bn\left(\frac{18d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^3} + \frac{2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^3} - \frac{6d^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}\right.$$

$$= \frac{3b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{2e^3} - \frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3} - \frac{6b^2d^2n^2}{e^2\sqrt[3]{x}} + \frac{1}{3}bn\left(\frac{18d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}\right.$$

$$= \frac{3b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{2e^3} - \frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3} - \frac{6b^2d^2n^2}{e^2\sqrt[3]{x}} + \frac{b^2d^3n^2\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}$$

Mathematica [C] time = 0.38, size = 374, normalized size = 1.39

$$\frac{bn\left(-36d^3x\log\left(d\sqrt[3]{x}+e\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-36d^3x\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)+12e^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-18de^2\sqrt[3]{x}\left(a+b\log\left(c\left(d+\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^2,x]
```

```
[Out] (-18*(a + b*Log[c*(d + e/x^(1/3))^n])^2 + (b*n*(-2*b*e*n*(2*e^2 - 3*d*e*x^(
1/3) + 6*d^2*x^(2/3)) + 9*b*d*e*n*(e - 2*d*x^(1/3))*x^(1/3) + 36*a*d^2*e*x^
(2/3) - 36*b*d^2*e*n*x^(2/3) + 30*b*d^3*n*x*Log[d + e/x^(1/3)] + 36*b*d^2*(
e + d*x^(1/3))*x^(2/3)*Log[c*(d + e/x^(1/3))^n] + 12*e^3*(a + b*Log[c*(d +
e/x^(1/3))^n]) - 18*d*e^2*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]) - 36*d^3
*x*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[e + d*x^(1/3)] - 36*d^3*x*(a + b*Lo
g[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))] + 18*b*d^3*n*x*Log[e + d*x^(1
/3)]*(Log[e + d*x^(1/3)] - 2*Log[-((d*x^(1/3))/e)]) - 36*b*d^3*n*x*PolyLog[
2, 1 + e/(d*x^(1/3))] - 36*b*d^3*n*x*PolyLog[2, 1 + (d*x^(1/3))/e]))/e^3)/(
18*x)
```

fricas [A] time = 0.48, size = 357, normalized size = 1.33

$$-\frac{4\,b^2e^3n^2 - 12\,abe^3n + 18\,a^2e^3 - 18\left(b^2e^3x - b^2e^3\right)\log(c)^2 + 18\left(b^2d^3n^2x + b^2e^3n^2\right)\log\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^2 - 2\left(2\,b^2e^3n^2 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/18*(4*b^2*e^3*n^2 - 12*a*b*e^3*n + 18*a^2*e^3 - 18*(b^2*e^3*x - b^2*e^3)
*log(c)^2 + 18*(b^2*d^3*n^2*x + b^2*e^3*n^2)*log((d*x + e*x^(2/3))/x)^2 - 2
*(2*b^2*e^3*n^2 - 6*a*b*e^3*n + 9*a^2*e^3)*x - 12*(b^2*e^3*n - 3*a*b*e^3 -
(b^2*e^3*n - 3*a*b*e^3)*x)*log(c) - 6*(6*b^2*d^2*e*n^2*x^(2/3) - 3*b^2*d*e^
2*n^2*x^(1/3) + 2*b^2*e^3*n^2 - 6*a*b*e^3*n + (11*b^2*d^3*n^2 - 6*a*b*d^3*n
)*x - 6*(b^2*d^3*n*x + b^2*e^3*n)*log(c))*log((d*x + e*x^(2/3))/x) + 6*(11*
b^2*d^2*e*n^2 - 6*b^2*d^2*e*n*log(c) - 6*a*b*d^2*e*n)*x^(2/3) - 3*(5*b^2*d*
e^2*n^2 - 6*b^2*d*e^2*n*log(c) - 6*a*b*d*e^2*n)*x^(1/3))/(e^3*x)
```

giac [B] time = 0.40, size = 787, normalized size = 2.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^2,x, algorithm="giac")
```

```
[Out] -1/18*(54*(d*x^(1/3) + e)*b^2*d^2*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^(1/3
) - 54*(d*x^(1/3) + e)^2*b^2*d*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^(2/3) +
 18*(d*x^(1/3) + e)^3*b^2*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x - 108*(d*x^(
1/3) + e)*b^2*d^2*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 108*(d*x^(1/3)
 + e)*b^2*d^2*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 54*(d*x^(1/3)
 + e)^2*b^2*d*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) - 108*(d*x^(1/3) + e
)^2*b^2*d*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) - 12*(d*x^(1/3) + e
)^3*b^2*n^2*log((d*x^(1/3) + e)/x^(1/3))/x + 36*(d*x^(1/3) + e)^3*b^2*n*log
(c)*log((d*x^(1/3) + e)/x^(1/3))/x + 108*(d*x^(1/3) + e)*b^2*d^2*n^2/x^(1/3
) - 108*(d*x^(1/3) + e)*b^2*d^2*n*log(c)/x^(1/3) + 54*(d*x^(1/3) + e)*b^2*d
^2*log(c)^2/x^(1/3) + 108*(d*x^(1/3) + e)*a*b*d^2*n*log((d*x^(1/3) + e)/x^(
1/3))/x^(1/3) - 27*(d*x^(1/3) + e)^2*b^2*d*n^2/x^(2/3) + 54*(d*x^(1/3) + e)
^2*b^2*d*n*log(c)/x^(2/3) - 54*(d*x^(1/3) + e)^2*b^2*d*log(c)^2/x^(2/3) - 1
08*(d*x^(1/3) + e)^2*a*b*d*n*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) + 4*(d*x^
(1/3) + e)^3*b^2*n^2/x - 12*(d*x^(1/3) + e)^3*b^2*n*log(c)/x + 18*(d*x^(1/3
) + e)^3*b^2*log(c)^2/x + 36*(d*x^(1/3) + e)^3*a*b*n*log((d*x^(1/3) + e)/x^
(1/3))/x - 108*(d*x^(1/3) + e)*a*b*d^2*n/x^(1/3) + 108*(d*x^(1/3) + e)*a*b*
d^2*log(c)/x^(1/3) + 54*(d*x^(1/3) + e)^2*a*b*d*n/x^(2/3) - 108*(d*x^(1/3)
+ e)^2*a*b*d*log(c)/x^(2/3) - 12*(d*x^(1/3) + e)^3*a*b*n/x + 36*(d*x^(1/3)
+ e)^3*a*b*log(c)/x + 54*(d*x^(1/3) + e)*a^2*d^2/x^(1/3) - 54*(d*x^(1/3) +
e)^2*a^2*d/x^(2/3) + 18*(d*x^(1/3) + e)^3*a^2/x)*e^(-3)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{2}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/3))^n)+a)^2/x^2,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/3))^n)+a)^2/x^2,x)
```

maxima [A] time = 0.52, size = 284, normalized size = 1.06

$$-\frac{1}{3}\,aben\left(\frac{6\,d^3\log\left(dx^{\frac{1}{3}}+e\right)}{e^4}-\frac{2\,d^3\log(x)}{e^4}-\frac{6\,d^2x^{\frac{2}{3}}-3\,dex^{\frac{1}{3}}+2\,e^2}{e^3x}\right)-\frac{1}{18}\left(6\,en\left(\frac{6\,d^3\log\left(dx^{\frac{1}{3}}+e\right)}{e^4}-\frac{2\,d^3\log(x}{e^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/3*a*b*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^(2
/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x)) - 1/18*(6*e*n*(6*d^3*log(d*x^(1/3) +
e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^(2/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x)
)*log(c*(d + e/x^(1/3))^n) - (18*d^3*x*log(d*x^(1/3) + e)^2 + 2*d^3*x*log(x
)^2 - 22*d^3*x*log(x) - 66*d^2*e*x^(2/3) + 15*d*e^2*x^(1/3) - 4*e^3 - 6*(2*
d^3*x*log(x) - 11*d^3*x)*log(d*x^(1/3) + e))*n^2/(e^3*x))*b^2 - b^2*log(c*(
d + e/x^(1/3))^n)^2/x - 2*a*b*log(c*(d + e/x^(1/3))^n)/x - a^2/x
```

mupad [B] time = 0.56, size = 299, normalized size = 1.11

$$\frac{\frac{d\left(3\,a^2-2\,a\,b\,n+\frac{2\,b^2\,n^2}{3}\right)}{2\,e}-\frac{d\left(3\,a^2-b^2\,n^2\right)}{2\,e}}{x^{2/3}}-\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)^2\left(\frac{b^2}{x}+\frac{b^2\,d^3}{e^3}\right)-\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)\left(\frac{2\,b\,(3\,a-b\,n)}{3\,x}-\frac{b\,d\,(3\,a}{e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^2/x^2,x)
```

```
[Out] ((d*(3*a^2 + (2*b^2*n^2)/3 - 2*a*b*n))/(2*e) - (d*(3*a^2 - b^2*n^2))/(2*e))
/x^(2/3) - log(c*(d + e/x^(1/3))^n)^2*(b^2/x + (b^2*d^3)/e^3) - log(c*(d +
e/x^(1/3))^n)*((2*b*(3*a - b*n))/(3*x) - ((b*d*(3*a - b*n))/e - (3*a*b*d)/e
)/x^(2/3) + (d*((2*b*d*(3*a - b*n))/e - (6*a*b*d)/e))/(e*x^(1/3))) - ((d*((
d*(3*a^2 + (2*b^2*n^2)/3 - 2*a*b*n))/e - (d*(3*a^2 - b^2*n^2))/e))/e + (2*b
^2*d^2*n^2)/e^2)/x^(1/3) - (a^2 + (2*b^2*n^2)/9 - (2*a*b*n)/3)/x + (log(d +
 e/x^(1/3))*(11*b^2*d^3*n^2 - 6*a*b*d^3*n))/(3*e^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))**2/x**2, x)
```

3.502 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=479

$$\frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6}-\frac{6bd^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6}+\frac{15bd^4n\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a\right.}{2}$$

```
[Out] -15/4*b^2*d^4*n^2*(d+e/x^(1/3))^2/e^6+20/9*b^2*d^3*n^2*(d+e/x^(1/3))^3/e^6-
15/16*b^2*d^2*n^2*(d+e/x^(1/3))^4/e^6+6/25*b^2*d*n^2*(d+e/x^(1/3))^5/e^6-1/
36*b^2*n^2*(d+e/x^(1/3))^6/e^6+6*b^2*d^5*n^2/e^5/x^(1/3)-1/2*b^2*d^6*n^2*ln
(d+e/x^(1/3))^2/e^6-6*b*d^5*n*(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6
+15/2*b*d^4*n*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6-20/3*b*d^3*n*
(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6+15/4*b*d^2*n*(d+e/x^(1/3))^
4*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6-6/5*b*d*n*(d+e/x^(1/3))^5*(a+b*ln(c*(d+e/
x^(1/3))^n))/e^6+1/6*b*n*(d+e/x^(1/3))^6*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6+b*
d^6*n*ln(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6-1/2*(a+b*ln(c*(d+e/x^
(1/3))^n))^2/x^2
```

Rubi [A] time = 0.48, antiderivative size = 355, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$-\frac{1}{60}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^6}-\frac{450d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^6}+\frac{400d^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^6}-\frac{225d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{e^6}-\frac{60d^6\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^6}+\frac{72d}{}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^3,x]
```

```
[Out] (-15*b^2*d^4*n^2*(d + e/x^(1/3))^2)/(4*e^6) + (20*b^2*d^3*n^2*(d + e/x^(1/3
))^3)/(9*e^6) - (15*b^2*d^2*n^2*(d + e/x^(1/3))^4)/(16*e^6) + (6*b^2*d*n^2*
(d + e/x^(1/3))^5)/(25*e^6) - (b^2*n^2*(d + e/x^(1/3))^6)/(36*e^6) + (6*b^2
*d^5*n^2)/(e^5*x^(1/3)) - (b^2*d^6*n^2*Log[d + e/x^(1/3)]^2)/(2*e^6) - (b*n
*((360*d^5*(d + e/x^(1/3)))/e^6 - (450*d^4*(d + e/x^(1/3))^2)/e^6 + (400*d^
3*(d + e/x^(1/3))^3)/e^6 - (225*d^2*(d + e/x^(1/3))^4)/e^6 + (72*d*(d + e/x
^(1/3))^5)/e^6 - (10*(d + e/x^(1/3))^6)/e^6 - (60*d^6*Log[d + e/x^(1/3)])/e
^6)*(a + b*Log[c*(d + e/x^(1/3))^n]))/60 - (a + b*Log[c*(d + e/x^(1/3))^n])
^2/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx = -\left(3\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2x^2} + (ben)\,\text{Subst}\left(\int \frac{x^6\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}\,dx\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2x^2} + (bn)\,\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6\left(a+b\log\left(cx^n\right)\right)}{x}\,dx\right.$$

$$= -\frac{1}{60}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^6} - \frac{225d^2\left(d\right.}{e}\right.$$

$$= -\frac{1}{60}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^6} - \frac{225d^2\left(d\right.}{e}\right.$$

$$= -\frac{1}{60}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^6} - \frac{225d^2\left(d\right.}{e}\right.$$

$$= -\frac{15b^2d^4n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^6} + \frac{20b^2d^3n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6} - \frac{15b^2d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{16e^6} + \frac{6b^2d}{}$$

$$= -\frac{15b^2d^4n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^6} + \frac{20b^2d^3n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6} - \frac{15b^2d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{16e^6} + \frac{6b^2d}{}$$

Mathematica [C] time = 0.35, size = 698, normalized size = 1.46

$$\frac{-1800a^2e^6 - 3600abe^6\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right) + 3600abd^6nx^2\log\left(d\sqrt[3]{x}+e\right) + 3600abd^6nx^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right) - 3600abd^5}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^3,x]
```

```
[Out] (-1800*a^2*e^6 + 600*a*b*e^6*n - 100*b^2*e^6*n^2 - 720*a*b*d*e^5*n*x^(1/3)
+ 264*b^2*d*e^5*n^2*x^(1/3) + 900*a*b*d^2*e^4*n*x^(2/3) - 555*b^2*d^2*e^4*n
^2*x^(2/3) - 1200*a*b*d^3*e^3*n*x + 1140*b^2*d^3*e^3*n^2*x + 1800*a*b*d^4*e
^2*n*x^(4/3) - 2610*b^2*d^4*e^2*n^2*x^(4/3) - 3600*a*b*d^5*e*n*x^(5/3) + 88
20*b^2*d^5*e*n^2*x^(5/3) - 5220*b^2*d^6*n^2*x^2*Log[d + e/x^(1/3)] - 3600*a
*b*e^6*Log[c*(d + e/x^(1/3))^n] + 600*b^2*e^6*n*Log[c*(d + e/x^(1/3))^n] -
720*b^2*d*e^5*n*x^(1/3)*Log[c*(d + e/x^(1/3))^n] + 900*b^2*d^2*e^4*n*x^(2/3
)*Log[c*(d + e/x^(1/3))^n] - 1200*b^2*d^3*e^3*n*x*Log[c*(d + e/x^(1/3))^n]
+ 1800*b^2*d^4*e^2*n*x^(4/3)*Log[c*(d + e/x^(1/3))^n] - 3600*b^2*d^5*e*n*x^
(5/3)*Log[c*(d + e/x^(1/3))^n] - 3600*b^2*d^6*n*x^2*Log[c*(d + e/x^(1/3))^n
```

```
] - 1800*b^2*e^6*Log[c*(d + e/x^(1/3))^n]^2 + 3600*a*b*d^6*n*x^2*Log[e + d*
x^(1/3)] + 3600*b^2*d^6*n*x^2*Log[c*(d + e/x^(1/3))^n]*Log[e + d*x^(1/3)] -
 1800*b^2*d^6*n^2*x^2*Log[e + d*x^(1/3)]^2 + 3600*a*b*d^6*n*x^2*Log[-(e/(d*
x^(1/3)))] + 3600*b^2*d^6*n*x^2*Log[c*(d + e/x^(1/3))^n]*Log[-(e/(d*x^(1/3)
))] + 3600*b^2*d^6*n^2*x^2*Log[e + d*x^(1/3)]*Log[-((d*x^(1/3))/e)] + 3600*
b^2*d^6*n^2*x^2*PolyLog[2, 1 + e/(d*x^(1/3))] + 3600*b^2*d^6*n^2*x^2*PolyLo
g[2, 1 + (d*x^(1/3))/e])/(3600*e^6*x^2)
```

fricas [A] time = 0.51, size = 597, normalized size = 1.25

$$-\frac{100\,b^2e^6n^2-600\,abe^6n+1800\,a^2e^6-20\left(90\,a^2e^6-\left(57\,b^2d^3e^3-5\,b^2e^6\right)n^2+30\left(2\,abd^3e^3-abe^6\right)n\right)x^2-1800}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/3600*(100*b^2*e^6*n^2 - 600*a*b*e^6*n + 1800*a^2*e^6 - 20*(90*a^2*e^6 -
(57*b^2*d^3*e^3 - 5*b^2*e^6)*n^2 + 30*(2*a*b*d^3*e^3 - a*b*e^6)*n)*x^2 - 18
00*(b^2*e^6*x^2 - b^2*e^6)*log(c)^2 - 1800*(b^2*d^6*n^2*x^2 - b^2*e^6*n^2)*
log((d*x + e*x^(2/3))/x)^2 - 60*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x +
 600*(2*b^2*d^3*e^3*n*x - b^2*e^6*n + 6*a*b*e^6 - (6*a*b*e^6 + (2*b^2*d^3*e
^3 - b^2*e^6)*n)*x^2)*log(c) + 60*(20*b^2*d^3*e^3*n^2*x - 10*b^2*e^6*n^2 +
60*a*b*e^6*n + 3*(49*b^2*d^6*n^2 - 20*a*b*d^6*n)*x^2 - 60*(b^2*d^6*n*x^2 -
b^2*e^6*n)*log(c) + 15*(4*b^2*d^5*e*n^2*x - b^2*d^2*e^4*n^2)*x^(2/3) - 6*(5
*b^2*d^4*e^2*n^2*x - 2*b^2*d*e^5*n^2)*x^(1/3))*log((d*x + e*x^(2/3))/x) + 1
5*(37*b^2*d^2*e^4*n^2 - 60*a*b*d^2*e^4*n - 12*(49*b^2*d^5*e*n^2 - 20*a*b*d^
5*e*n)*x + 60*(4*b^2*d^5*e*n*x - b^2*d^2*e^4*n)*log(c))*x^(2/3) - 6*(44*b^2
*d*e^5*n^2 - 120*a*b*d*e^5*n - 15*(29*b^2*d^4*e^2*n^2 - 20*a*b*d^4*e^2*n)*x
 + 60*(5*b^2*d^4*e^2*n*x - 2*b^2*d*e^5*n)*log(c))*x^(1/3))/(e^6*x^2)
```

giac [B] time = 0.59, size = 1639, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] 1/3600*(10800*(d*x^(1/3) + e)*b^2*d^5*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^
(1/3) - 27000*(d*x^(1/3) + e)^2*b^2*d^4*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/
x^(2/3) + 36000*(d*x^(1/3) + e)^3*b^2*d^3*n^2*log((d*x^(1/3) + e)/x^(1/3))^
2/x - 21600*(d*x^(1/3) + e)*b^2*d^5*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3
) + 21600*(d*x^(1/3) + e)*b^2*d^5*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(
1/3) - 27000*(d*x^(1/3) + e)^4*b^2*d^2*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x
^(4/3) + 27000*(d*x^(1/3) + e)^2*b^2*d^4*n^2*log((d*x^(1/3) + e)/x^(1/3))/x
^(2/3) - 54000*(d*x^(1/3) + e)^2*b^2*d^4*n*log(c)*log((d*x^(1/3) + e)/x^(1/
3))/x^(2/3) + 10800*(d*x^(1/3) + e)^5*b^2*d*n^2*log((d*x^(1/3) + e)/x^(1/3)
)^2/x^(5/3) - 24000*(d*x^(1/3) + e)^3*b^2*d^3*n^2*log((d*x^(1/3) + e)/x^(1/
3))/x + 72000*(d*x^(1/3) + e)^3*b^2*d^3*n*log(c)*log((d*x^(1/3) + e)/x^(1/3
))/x - 1800*(d*x^(1/3) + e)^6*b^2*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^2 +
21600*(d*x^(1/3) + e)*b^2*d^5*n^2/x^(1/3) - 21600*(d*x^(1/3) + e)*b^2*d^5*n
*log(c)/x^(1/3) + 10800*(d*x^(1/3) + e)*b^2*d^5*log(c)^2/x^(1/3) + 13500*(d
*x^(1/3) + e)^4*b^2*d^2*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^(4/3) + 21600*(d
*x^(1/3) + e)*a*b*d^5*n*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) - 54000*(d*x^(
1/3) + e)^4*b^2*d^2*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(4/3) - 13500*(
d*x^(1/3) + e)^2*b^2*d^4*n^2/x^(2/3) + 27000*(d*x^(1/3) + e)^2*b^2*d^4*n*lo
g(c)/x^(2/3) - 27000*(d*x^(1/3) + e)^2*b^2*d^4*log(c)^2/x^(2/3) - 4320*(d*x
^(1/3) + e)^5*b^2*d*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^(5/3) - 54000*(d*x^(
1/3) + e)^2*a*b*d^4*n*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) + 21600*(d*x^(1/
```

```
3) + e)^5*b^2*d*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(5/3) + 8000*(d*x^(
1/3) + e)^3*b^2*d^3*n^2/x - 24000*(d*x^(1/3) + e)^3*b^2*d^3*n*log(c)/x + 36
000*(d*x^(1/3) + e)^3*b^2*d^3*log(c)^2/x + 600*(d*x^(1/3) + e)^6*b^2*n^2*lo
g((d*x^(1/3) + e)/x^(1/3))/x^2 + 72000*(d*x^(1/3) + e)^3*a*b*d^3*n*log((d*x
^(1/3) + e)/x^(1/3))/x - 3600*(d*x^(1/3) + e)^6*b^2*n*log(c)*log((d*x^(1/3)
 + e)/x^(1/3))/x^2 - 3375*(d*x^(1/3) + e)^4*b^2*d^2*n^2/x^(4/3) - 21600*(d*
x^(1/3) + e)*a*b*d^5*n/x^(1/3) + 13500*(d*x^(1/3) + e)^4*b^2*d^2*n*log(c)/x
^(4/3) + 21600*(d*x^(1/3) + e)*a*b*d^5*log(c)/x^(1/3) - 27000*(d*x^(1/3) +
e)^4*b^2*d^2*log(c)^2/x^(4/3) - 54000*(d*x^(1/3) + e)^4*a*b*d^2*n*log((d*x^
(1/3) + e)/x^(1/3))/x^(4/3) + 864*(d*x^(1/3) + e)^5*b^2*d*n^2/x^(5/3) + 270
00*(d*x^(1/3) + e)^2*a*b*d^4*n/x^(2/3) - 4320*(d*x^(1/3) + e)^5*b^2*d*n*log
(c)/x^(5/3) - 54000*(d*x^(1/3) + e)^2*a*b*d^4*log(c)/x^(2/3) + 10800*(d*x^(
1/3) + e)^5*b^2*d*log(c)^2/x^(5/3) + 21600*(d*x^(1/3) + e)^5*a*b*d*n*log((d
*x^(1/3) + e)/x^(1/3))/x^(5/3) - 100*(d*x^(1/3) + e)^6*b^2*n^2/x^2 - 24000*
(d*x^(1/3) + e)^3*a*b*d^3*n/x + 600*(d*x^(1/3) + e)^6*b^2*n*log(c)/x^2 + 72
000*(d*x^(1/3) + e)^3*a*b*d^3*log(c)/x - 1800*(d*x^(1/3) + e)^6*b^2*log(c)^
2/x^2 - 3600*(d*x^(1/3) + e)^6*a*b*n*log((d*x^(1/3) + e)/x^(1/3))/x^2 + 135
00*(d*x^(1/3) + e)^4*a*b*d^2*n/x^(4/3) + 10800*(d*x^(1/3) + e)*a^2*d^5/x^(1
/3) - 54000*(d*x^(1/3) + e)^4*a*b*d^2*log(c)/x^(4/3) - 4320*(d*x^(1/3) + e)
^5*a*b*d*n/x^(5/3) - 27000*(d*x^(1/3) + e)^2*a^2*d^4/x^(2/3) + 21600*(d*x^(
1/3) + e)^5*a*b*d*log(c)/x^(5/3) + 600*(d*x^(1/3) + e)^6*a*b*n/x^2 + 36000*
(d*x^(1/3) + e)^3*a^2*d^3/x - 3600*(d*x^(1/3) + e)^6*a*b*log(c)/x^2 - 27000
*(d*x^(1/3) + e)^4*a^2*d^2/x^(4/3) + 10800*(d*x^(1/3) + e)^5*a^2*d/x^(5/3)
- 1800*(d*x^(1/3) + e)^6*a^2/x^2)*e^(-6)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/3))^n)+a)^2/x^3,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/3))^n)+a)^2/x^3,x)
```

maxima [A] time = 0.53, size = 387, normalized size = 0.81

$$\frac{1}{60}aben\left(\frac{60\,d^6\log\left(dx^{\frac{1}{3}}+e\right)}{e^7}-\frac{20\,d^6\log(x)}{e^7}-\frac{60\,d^5x^{\frac{5}{3}}-30\,d^4ex^{\frac{4}{3}}+20\,d^3e^2x-15\,d^2e^3x^{\frac{2}{3}}+12\,de^4x^{\frac{1}{3}}-10\,e^5}{e^6x^2}\right)+\frac{}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/60*a*b*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7 - 20*d^6*log(x)/e^7 - (60*d^5*x
^(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 15*d^2*e^3*x^(2/3) + 12*d*e^4*x^
(1/3) - 10*e^5)/(e^6*x^2)) + 1/3600*(60*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7
- 20*d^6*log(x)/e^7 - (60*d^5*x^(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 1
5*d^2*e^3*x^(2/3) + 12*d*e^4*x^(1/3) - 10*e^5)/(e^6*x^2))*log(c*(d + e/x^(1
/3))^n) - (1800*d^6*x^2*log(d*x^(1/3) + e)^2 + 200*d^6*x^2*log(x)^2 - 2940*
d^6*x^2*log(x) - 8820*d^5*e*x^(5/3) + 2610*d^4*e^2*x^(4/3) - 1140*d^3*e^3*x
 + 555*d^2*e^4*x^(2/3) - 264*d*e^5*x^(1/3) + 100*e^6 - 60*(20*d^6*x^2*log(x
) - 147*d^6*x^2)*log(d*x^(1/3) + e))*n^2/(e^6*x^2))*b^2 - 1/2*b^2*log(c*(d
+ e/x^(1/3))^n)^2/x^2 - a*b*log(c*(d + e/x^(1/3))^n)/x^2 - 1/2*a^2/x^2
```

mupad [B] time = 1.76, size = 439, normalized size = 0.92

$$\frac{b^2 d^6 \ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)^2}{2 e^6}-\frac{b^2 \ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)^2}{2 x^2}-\frac{b^2 n^2}{36 x^2}-\frac{a b \ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{x^2}-\frac{a^2}{2 x^2}+\frac{a b n}{6 x^2}+\frac{b^2 n \ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{6 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^2/x^3,x)
```

```
[Out] (b^2*d^6*log(c*(d + e/x^(1/3))^n)^2)/(2*e^6) - (b^2*log(c*(d + e/x^(1/3))^n
)^2)/(2*x^2) - (b^2*n^2)/(36*x^2) - (a*b*log(c*(d + e/x^(1/3))^n))/x^2 - a^
2/(2*x^2) + (a*b*n)/(6*x^2) + (b^2*n*log(c*(d + e/x^(1/3))^n))/(6*x^2) - (4
9*b^2*d^6*n^2*log(d + e/x^(1/3)))/(20*e^6) + (19*b^2*d^3*n^2)/(60*e^3*x) -
(37*b^2*d^2*n^2)/(240*e^2*x^(4/3)) - (29*b^2*d^4*n^2)/(40*e^4*x^(2/3)) + (4
9*b^2*d^5*n^2)/(20*e^5*x^(1/3)) + (11*b^2*d*n^2)/(150*e*x^(5/3)) - (b^2*d^3
*n*log(c*(d + e/x^(1/3))^n))/(3*e^3*x) + (b^2*d^2*n*log(c*(d + e/x^(1/3))^n
))/(4*e^2*x^(4/3)) + (b^2*d^4*n*log(c*(d + e/x^(1/3))^n))/(2*e^4*x^(2/3)) -
 (b^2*d^5*n*log(c*(d + e/x^(1/3))^n))/(e^5*x^(1/3)) - (a*b*d*n)/(5*e*x^(5/3
)) + (a*b*d^6*n*log(d + e/x^(1/3)))/e^6 - (b^2*d*n*log(c*(d + e/x^(1/3))^n)
)/(5*e*x^(5/3)) - (a*b*d^3*n)/(3*e^3*x) + (a*b*d^2*n)/(4*e^2*x^(4/3)) + (a*
b*d^4*n)/(2*e^4*x^(2/3)) - (a*b*d^5*n)/(e^5*x^(1/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2/x**3,x)
```

```
[Out] Timed out
```

3.503 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=759

$$-\frac{3b^2e^6n^2\text{Li}_2\left(\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6}-\frac{77b^2e^6n^2\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^6}-\frac{3b^2e^6n^2\log}{}$$

```
[Out] 71/40*b^3*e^5*n^3*x^(1/3)/d^5-3/10*b^3*e^4*n^3*x^(2/3)/d^4+1/20*b^3*e^3*n^3
*x/d^3-71/40*b^3*e^6*n^3*ln(d+e/x^(1/3))/d^6-77/20*b^2*e^5*n^2*(d+e/x^(1/3)
)*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^6+47/40*b^2*e^4*n^2*x^(2/3)*(a+b*ln
(c*(d+e/x^(1/3))^n))/d^4-9/20*b^2*e^3*n^2*x*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3
+3/20*b^2*e^2*n^2*x^(4/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^2-77/20*b^2*e^6*n^2
*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x^(1/3))^n))/d^6+3/2*b*e^5*n*(d+e/x^(
1/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))^2/d^6-3/4*b*e^4*n*x^(2/3)*(a+b*ln
(c*(d+e/x^(1/3))^n))^2/d^4+1/2*b*e^3*n*x*(a+b*ln(c*(d+e/x^(1/3))^n))^2/d^3-
3/8*b*e^2*n*x^(4/3)*(a+b*ln(c*(d+e/x^(1/3))^n))^2/d^2+3/10*b*e*n*x^(5/3)*(a
+b*ln(c*(d+e/x^(1/3))^n))^2/d+3/2*b*e^6*n*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(
d+e/x^(1/3))^n))^2/d^6+1/2*x^2*(a+b*ln(c*(d+e/x^(1/3))^n))^3-3*b^2*e^6*n^2*
(a+b*ln(c*(d+e/x^(1/3))^n))*ln(-e/d/x^(1/3))/d^6-15/8*b^3*e^6*n^3*ln(x)/d^6
+77/20*b^3*e^6*n^3*polylog(2,d/(d+e/x^(1/3)))/d^6-3*b^2*e^6*n^2*(a+b*ln(c*(
d+e/x^(1/3))^n))*polylog(2,d/(d+e/x^(1/3)))/d^6-3*b^3*e^6*n^3*polylog(2,1+e
/d/x^(1/3))/d^6-3*b^3*e^6*n^3*polylog(3,d/(d+e/x^(1/3)))/d^6
```

Rubi [A] time = 2.97, antiderivative size = 736, normalized size of antiderivative = 0.97, number of steps used = 73, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31, 44}

$$\frac{3b^2e^6n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6}-\frac{137b^3e^6n^3\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)}{20d^6}-\frac{3b^3e^6n^3\text{PolyLog}\left(3,\right.}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
```

```
[Out] (71*b^3*e^5*n^3*x^(1/3))/(40*d^5) - (3*b^3*e^4*n^3*x^(2/3))/(10*d^4) + (b^3
*e^3*n^3*x)/(20*d^3) - (71*b^3*e^6*n^3*Log[d + e/x^(1/3)])/(40*d^6) - (77*b
^2*e^5*n^2*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(20*d^
6) + (47*b^2*e^4*n^2*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(40*d^4) - (
9*b^2*e^3*n^2*x*(a + b*Log[c*(d + e/x^(1/3))^n]))/(20*d^3) + (3*b^2*e^2*n^2
*x^(4/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(20*d^2) + (77*b*e^6*n*(a + b*Lo
g[c*(d + e/x^(1/3))^n])^2)/(40*d^6) + (3*b*e^5*n*(d + e/x^(1/3))*x^(1/3)*(a
 + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*d^6) - (3*b*e^4*n*x^(2/3)*(a + b*Log[c
*(d + e/x^(1/3))^n])^2)/(4*d^4) + (b*e^3*n*x*(a + b*Log[c*(d + e/x^(1/3))^n
])^2)/(2*d^3) - (3*b*e^2*n*x^(4/3)*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(8*d
^2) + (3*b*e*n*x^(5/3)*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(10*d) - (e^6*(a
 + b*Log[c*(d + e/x^(1/3))^n])^3)/(2*d^6) + (x^2*(a + b*Log[c*(d + e/x^(1/3
))^n])^3)/2 - (137*b^2*e^6*n^2*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[-(e/(d*
x^(1/3)))])/(20*d^6) + (3*b*e^6*n*(a + b*Log[c*(d + e/x^(1/3))^n])^2*Log[-(
e/(d*x^(1/3)))])/(2*d^6) - (15*b^3*e^6*n^3*Log[x])/(8*d^6) - (137*b^3*e^6*n
^3*PolyLog[2, 1 + e/(d*x^(1/3))])/(20*d^6) + (3*b^2*e^6*n^2*(a + b*Log[c*(d
 + e/x^(1/3))^n])*PolyLog[2, 1 + e/(d*x^(1/3))])/d^6 - (3*b^3*e^6*n^3*PolyL
og[3, 1 + e/(d*x^(1/3))])/d^6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3dx=-\left(3\,\text{Subst}\left(\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x^7}dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-\frac{1}{2}(3ben)\,\text{Subst}\left(\int\frac{\left(a+b\log\left(c(d+\right.\right.}{x^6(d+ex)}\right.$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-\frac{1}{2}(3bn)\,\text{Subst}\left(\int\frac{\left(a+b\log\left(cx^n\right)\right)^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\right.$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-\frac{(3bn)\,\text{Subst}\left(\int\frac{\left(a+b\log(cx^n)\right)^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}dx,x,d\right.}{2d}$$

$$=\frac{3benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{10d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3$$

$$=-\frac{3be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{8d^2}+\frac{3benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right.\right.\right.}{10d}$$

$$=\frac{3b^2e^2n^2x^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^2}+\frac{be^3nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right.\right.}{2d^3}$$

$$=-\frac{9b^2e^3n^2x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^3}+\frac{3b^2e^2n^2x^{4/3}\left(a+b\log\left(c\left(d+\right.\right.\right.}{20d^2}$$

$$=\frac{3b^3e^5n^3\sqrt[3]{x}}{20d^5}-\frac{3b^3e^4n^3x^{2/3}}{40d^4}+\frac{b^3e^3n^3x}{20d^3}-\frac{3b^3e^6n^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{20d^6}+\frac{47b^2e}{}$$

$$=\frac{3b^3e^5n^3\sqrt[3]{x}}{5d^5}-\frac{3b^3e^4n^3x^{2/3}}{10d^4}+\frac{b^3e^3n^3x}{20d^3}-\frac{3b^3e^6n^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{5d^6}-\frac{77b^2e}{}$$

$$=\frac{71b^3e^5n^3\sqrt[3]{x}}{40d^5}-\frac{3b^3e^4n^3x^{2/3}}{10d^4}+\frac{b^3e^3n^3x}{20d^3}-\frac{71b^3e^6n^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{40d^6}-\frac{77b}{}$$

$$=\frac{71b^3e^5n^3\sqrt[3]{x}}{40d^5}-\frac{3b^3e^4n^3x^{2/3}}{10d^4}+\frac{b^3e^3n^3x}{20d^3}-\frac{71b^3e^6n^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{40d^6}-\frac{77b}{}$$

Mathematica [A] time = 1.66, size = 1006, normalized size = 1.33

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
```

```
[Out] (60*b*d*e^5*n*x^(1/3)*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))
^n])^2 - 30*b*d^2*e^4*n*x^(2/3)*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d +
e/x^(1/3))^n])^2 + 20*b*d^3*e^3*n*x*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(
d + e/x^(1/3))^n])^2 - 15*b*d^4*e^2*n*x^(4/3)*(a - b*n*Log[d + e/x^(1/3)] +
 b*Log[c*(d + e/x^(1/3))^n])^2 + 12*b*d^5*e*n*x^(5/3)*(a - b*n*Log[d + e/x^
(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^2 + 60*b*d^6*n*x^2*Log[d + e/x^(1/3)]*
(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^2 + 20*d^6*x^2*(a
 - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^3 - 60*b*e^6*n*(a -
 b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^2*Log[e + d*x^(1/3)]
+ b^2*n^2*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])*(d*e^2*
x^(1/3)*(-154*e^3 + 47*d*e^2*x^(1/3) - 18*d^2*e*x^(2/3) + 6*d^3*x) - 60*(e^
6 - d^6*x^2)*Log[d + e/x^(1/3)]^2 - 274*e^6*Log[-(e/(d*x^(1/3)))] + 2*e*Log
[d + e/x^(1/3)]*(137*e^5 + 60*d*e^4*x^(1/3) - 30*d^2*e^3*x^(2/3) + 20*d^3*e
^2*x - 15*d^4*e*x^(4/3) + 12*d^5*x^(5/3) + 60*e^5*Log[-(e/(d*x^(1/3)))]) +
120*e^6*PolyLog[2, 1 + e/(d*x^(1/3))]) + b^3*n^3*(3*d^4*e^2*x^(4/3)*(2 - 5*
Log[d + e/x^(1/3)])*Log[d + e/x^(1/3)] + 12*d^5*e*x^(5/3)*Log[d + e/x^(1/3)
]^2 + 20*d^6*x^2*Log[d + e/x^(1/3)]^3 + 2*d^3*e^3*x*(1 - 9*Log[d + e/x^(1/3
)] + 10*Log[d + e/x^(1/3)]^2) - d^2*e^4*x^(2/3)*(12 - 47*Log[d + e/x^(1/3)]
 + 30*Log[d + e/x^(1/3)]^2) + d*e^5*x^(1/3)*(71 - 154*Log[d + e/x^(1/3)] +
60*Log[d + e/x^(1/3)]^2) + 225*e^6*(-Log[d + e/x^(1/3)] + Log[-(e/(d*x^(1/3
)))]) + 137*e^6*(Log[d + e/x^(1/3)]*(Log[d + e/x^(1/3)] - 2*Log[-(e/(d*x^(1
/3)))]) - 2*PolyLog[2, 1 + e/(d*x^(1/3))]) - 20*e^6*(Log[d + e/x^(1/3)]^2*(
Log[d + e/x^(1/3)] - 3*Log[-(e/(d*x^(1/3)))]) - 6*Log[d + e/x^(1/3)]*PolyLo
g[2, 1 + e/(d*x^(1/3))] + 6*PolyLog[3, 1 + e/(d*x^(1/3))])))/(40*d^6)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)^3+3\,ab^2x\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)^2+3\,a^2bx\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)+a^3x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*log(c*((d*x + e*x^(2/3))/x)^n)^3 + 3*a*b^2*x*log(c*((d*x + e
*x^(2/3))/x)^n)^2 + 3*a^2*b*x*log(c*((d*x + e*x^(2/3))/x)^n) + a^3*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^3x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^3*x, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^3x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(d+e/x^(1/3))^n)+a)^3,x)
```

```
[Out] int(x*(b*ln(c*(d+e/x^(1/3))^n)+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,b^{3}x^{2}\log\left(\left(dx^{\frac{1}{3}}+e\right)^{n}\right)^{3}-\int\frac{2\left(b^{3}dx^{2}+b^{3}ex^{\frac{5}{3}}\right)\log\left(x^{\frac{1}{3}\,n}\right)^{3}-2\left(b^{3}d\log(c)^{3}+3\,ab^{2}d\log(c)^{2}+3\,a^{2}bd\log(c)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*log((d*x^(1/3) + e)^n)^3 - integrate(1/2*(2*(b^3*d*x^2 + b^3*e*
x^(5/3))*log(x^(1/3*n))^3 - 2*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*
b*d*log(c) + a^3*d)*x^2 + (b^3*d*n*x^2 - 6*(b^3*d*log(c) + a*b^2*d)*x^2 - 6
*(b^3*e*log(c) + a*b^2*e)*x^(5/3) + 6*(b^3*d*x^2 + b^3*e*x^(5/3))*log(x^(1/
3*n)))*log((d*x^(1/3) + e)^n)^2 - 6*((b^3*d*log(c) + a*b^2*d)*x^2 + (b^3*e*
log(c) + a*b^2*e)*x^(5/3))*log(x^(1/3*n))^2 - 2*(b^3*e*log(c)^3 + 3*a*b^2*e
*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(5/3) - 6*((b^3*d*log(c)^2 + 2*a*b^
2*d*log(c) + a^2*b*d)*x^2 + (b^3*d*x^2 + b^3*e*x^(5/3))*log(x^(1/3*n))^2 +
(b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(5/3) - 2*((b^3*d*log(c) +
a*b^2*d)*x^2 + (b^3*e*log(c) + a*b^2*e)*x^(5/3))*log(x^(1/3*n)))*log((d*x^(
1/3) + e)^n) + 6*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^2 + (b^3*
e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(5/3))*log(x^(1/3*n)))/(d*x + e*
x^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^{n}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/3))^n))^3,x)
```

```
[Out] int(x*(a + b*log(c*(d + e/x^(1/3))^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))**n))**3,x)
```

```
[Out] Timed out
```

3.504 $\int \left(a+b\log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 \, dx$

Optimal. Leaf size=436

$$\frac{6b^2e^3n^2\text{Li}_2\left(\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}+\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}+\frac{6b^2e^3n^2\log\left(\right.}{}$$

```
[Out] 3*b^2*e^2*n^2*(d+e/x^(1/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3+3*b^2*e
^3*n^2*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3-3*b*e^2*n*(d+e
/x^(1/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))^2/d^3+3/2*b*e*n*x^(2/3)*(a+b*
ln(c*(d+e/x^(1/3))^n))^2/d-3*b*e^3*n*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x
^(1/3))^n))^2/d^3+x*(a+b*ln(c*(d+e/x^(1/3))^n))^3+6*b^2*e^3*n^2*(a+b*ln(c*(
d+e/x^(1/3))^n))*ln(-e/d/x^(1/3))/d^3+b^3*e^3*n^3*ln(x)/d^3-3*b^3*e^3*n^3*p
olylog(2,d/(d+e/x^(1/3)))/d^3+6*b^2*e^3*n^2*(a+b*ln(c*(d+e/x^(1/3))^n))*pol
ylog(2,d/(d+e/x^(1/3)))/d^3+6*b^3*e^3*n^3*polylog(2,1+e/d/x^(1/3))/d^3+6*b^
3*e^3*n^3*polylog(3,d/(d+e/x^(1/3)))/d^3
```

Rubi [A] time = 1.04, antiderivative size = 410, normalized size of antiderivative = 0.94, number of steps used = 23, number of rules used = 17, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.850, Rules used = {2451, 2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31}

$$-\frac{6b^2e^3n^2\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}+\frac{9b^3e^3n^3\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)}{d^3}+\frac{6b^3e^3n^3\text{PolyLog}\left(3,\right.}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
```

```
[Out] (3*b^2*e^2*n^2*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/d^
3 - (3*b*e^3*n*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*d^3) - (3*b*e^2*n*(d
+ e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/d^3 + (3*b*e*n*x^(
2/3)*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*d) + (e^3*(a + b*Log[c*(d + e/x
^(1/3))^n])^3)/d^3 + x*(a + b*Log[c*(d + e/x^(1/3))^n])^3 + (9*b^2*e^3*n^2*
(a + b*Log[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))])/d^3 - (3*b*e^3*n*(a
 + b*Log[c*(d + e/x^(1/3))^n])^2*Log[-(e/(d*x^(1/3)))])/d^3 + (b^3*e^3*n^3*
Log[x])/d^3 + (9*b^3*e^3*n^3*PolyLog[2, 1 + e/(d*x^(1/3))])/d^3 - (6*b^2*e^
3*n^2*(a + b*Log[c*(d + e/x^(1/3))^n])*PolyLog[2, 1 + e/(d*x^(1/3))])/d^3 +
 (6*b^3*e^3*n^3*PolyLog[3, 1 + e/(d*x^(1/3))])/d^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
```

```
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx = 3\,\mathrm{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^n\right)\right)^3 dx, x, \sqrt[3]{x}\right)$$

$$= -\left(3\,\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x^4} dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 - (3ben)\,\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^{\cdots}}{x^3(d+ex)} \cdots\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 - (3bn)\,\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(cx^n\right)\right)^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3} dx, x, \cdots\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 - \frac{(3bn)\,\mathrm{Subst}\left(\int \frac{\left(a+b\log(cx^n)\right)^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3} dx, x, d+\frac{e}{\sqrt[3]{x}} \cdots\right.}{d}$$

$$= \frac{3benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 + \cdots$$

$$= -\frac{3be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3} + \frac{3benx^{2/3}\left(a+b\log\left(\cdots\right.\right.}{2d}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} - \frac{3be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(\cdots\right.}{\cdots}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} - \frac{3be^3n\left(a+b\log\left(c\left(d\cdots\right.\right.\right.}{2d^3}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} - \frac{3be^3n\left(a+b\log\left(c\left(d\cdots\right.\right.\right.}{2d^3}$$

Mathematica [A] time = 0.78, size = 675, normalized size = 1.55

$$\frac{6b^2n^2\left(\left(d^3x+e^3\right)\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)+e\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(d^2x^{2/3}-2e^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)-2de\sqrt[3]{x}-3e^2\right)-2e^3\mathrm{Li}_2\left(\frac{e}{d\sqrt[3]{x}}+\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
```

```
[Out] (-6*b*d*e^2*n*x^(1/3)*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))
^n])^2 + 3*b*d^2*e*n*x^(2/3)*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x
^(1/3))^n])^2 + 6*b*d^3*n*x*Log[d + e/x^(1/3)]*(a - b*n*Log[d + e/x^(1/3)]
+ b*Log[c*(d + e/x^(1/3))^n])^2 + 2*d^3*x*(a - b*n*Log[d + e/x^(1/3)] + b*L
og[c*(d + e/x^(1/3))^n])^3 + 6*b*e^3*n*(a - b*n*Log[d + e/x^(1/3)] + b*Log[
c*(d + e/x^(1/3))^n])^2*Log[e + d*x^(1/3)] + 6*b^2*n^2*(a - b*n*Log[d + e/x
^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])*((e^3 + d^3*x)*Log[d + e/x^(1/3)]^2 +
```

```
e^2*(d*x^(1/3) + 3*e*Log[-(e/(d*x^(1/3)))]) + e*Log[d + e/x^(1/3)]*(-3*e^2
 - 2*d*e*x^(1/3) + d^2*x^(2/3) - 2*e^2*Log[-(e/(d*x^(1/3)))]) - 2*e^3*PolyL
og[2, 1 + e/(d*x^(1/3))]) - b^3*n^3*(-6*e^3*Log[d + e/x^(1/3)] - 6*d*e^2*x^
(1/3)*Log[d + e/x^(1/3)] + 9*e^3*Log[d + e/x^(1/3)]^2 + 6*d*e^2*x^(1/3)*Log
[d + e/x^(1/3)]^2 - 3*d^2*e*x^(2/3)*Log[d + e/x^(1/3)]^2 - 2*e^3*Log[d + e/
x^(1/3)]^3 - 2*d^3*x*Log[d + e/x^(1/3)]^3 + 6*e^3*Log[-(e/(d*x^(1/3)))] - 1
8*e^3*Log[d + e/x^(1/3)]*Log[-(e/(d*x^(1/3)))] + 6*e^3*Log[d + e/x^(1/3)]^2
*Log[-(e/(d*x^(1/3)))] + 6*e^3*(-3 + 2*Log[d + e/x^(1/3)])*PolyLog[2, 1 + e
/(d*x^(1/3))] - 12*e^3*PolyLog[3, 1 + e/(d*x^(1/3))]))/(2*d^3)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^{3}\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^{n}\right)^{3}+3\,ab^{2}\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^{n}\right)^{2}+3\,a^{2}b\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^{n}\right)+a^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*log(c*((d*x + e*x^(2/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^(
2/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(2/3))/x)^n) + a^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^3, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/3))^n)+a)^3,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/3))^n)+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^{3}x\log\left(\left(dx^{\frac{1}{3}}+e\right)^{n}\right)^{3}+\frac{3}{2}\left(en\left(\frac{2\,e^{2}\log\left(dx^{\frac{1}{3}}+e\right)}{d^{3}}+\frac{dx^{\frac{2}{3}}-2\,ex^{\frac{1}{3}}}{d^{2}}\right)+2\,x\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)a^{2}b+a^{3}x-\int\frac{\left(b^{3}dx+b^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log((d*x^(1/3) + e)^n)^3 + 3/2*(e*n*(2*e^2*log(d*x^(1/3) + e)/d^3 + (
d*x^(2/3) - 2*e*x^(1/3))/d^2) + 2*x*log(c*(d + e/x^(1/3))^n))*a^2*b + a^3*x
 - integrate(((b^3*d*x + b^3*e*x^(2/3))*log(x^(1/3*n))^3 + (b^3*d*n*x - 3*(
b^3*d*log(c) + a*b^2*d)*x + 3*(b^3*d*x + b^3*e*x^(2/3))*log(x^(1/3*n)) - 3*
(b^3*e*log(c) + a*b^2*e)*x^(2/3))*log((d*x^(1/3) + e)^n)^2 - 3*((b^3*d*log(
```

```
c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(2/3))*log(x^(1/3*n))^2 - (b^3
*d*log(c)^3 + 3*a*b^2*d*log(c)^2)*x - 3*((b^3*d*x + b^3*e*x^(2/3))*log(x^(1
/3*n))^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c))*x - 2*((b^3*d*log(c) + a*b^2
*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(2/3))*log(x^(1/3*n)) + (b^3*e*log(c)^2
+ 2*a*b^2*e*log(c))*x^(2/3))*log((d*x^(1/3) + e)^n) + 3*((b^3*d*log(c)^2 +
2*a*b^2*d*log(c))*x + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c))*x^(2/3))*log(x^(1
/3*n)) - (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2)*x^(2/3))/(d*x + e*x^(2/3)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^3,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/3))^n))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))**3, x)
```

3.505 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx$

Optimal. Leaf size=135

$$18b^2n^2\text{Li}_3\left(\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-9bn\text{Li}_2\left(\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-3\log\left(-\frac{e}{d\sqrt[3]{x}}\right.$$

```
[Out] -3*(a+b*ln(c*(d+e/x^(1/3))^n))^3*ln(-e/d/x^(1/3))-9*b*n*(a+b*ln(c*(d+e/x^(1
/3))^n))^2*polylog(2,1+e/d/x^(1/3))+18*b^2*n^2*(a+b*ln(c*(d+e/x^(1/3))^n))*
polylog(3,1+e/d/x^(1/3))-18*b^3*n^3*polylog(4,1+e/d/x^(1/3))
```

Rubi [A] time = 0.20, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2454, 2396, 2433, 2374, 2383, 6589}

$$18b^2n^2\text{PolyLog}\left(3,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-9bn\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x,x]
```

```
[Out] -3*(a + b*Log[c*(d + e/x^(1/3))^n])^3*Log[-(e/(d*x^(1/3)))] - 9*b*n*(a + b*
Log[c*(d + e/x^(1/3))^n])^2*PolyLog[2, 1 + e/(d*x^(1/3))] + 18*b^2*n^2*(a +
 b*Log[c*(d + e/x^(1/3))^n])*PolyLog[3, 1 + e/(d*x^(1/3))] - 18*b^3*n^3*Pol
yLog[4, 1 + e/(d*x^(1/3))]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx = -\left(3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right) + (9ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right.}{}\right.$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right) + (9bn)\,\text{Subst}\left(\int \frac{\left(a+b\log\right.}{}\right.$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right) - 9bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right.\right.\right.$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right) - 9bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right.\right.\right.$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right) - 9bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right.\right.\right.$$

Mathematica [B] time = 0.32, size = 527, normalized size = 3.90

$$9b^2n^2\left(\frac{1}{81}\left(-162\text{Li}_3\left(\frac{\sqrt[3]{x}\,d}{e}+1\right)-162\text{Li}_3\left(-\frac{d\sqrt[3]{x}}{e}\right)+9\log^2(x)\log\left(d+\frac{e}{\sqrt[3]{x}}\right)-9\log^2(x)\log\left(\frac{d\sqrt[3]{x}}{e}+1\right)+27\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^2*((Log[d + e/x^(
1/3)] - Log[1 + e/(d*x^(1/3))])*Log[x] + 3*PolyLog[2, -(e/(d*x^(1/3)))]) +
9*b^2*n^2*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])*(2*Log[
e/d + x^(1/3)]*PolyLog[2, 1 + (d*x^(1/3))/e] - 2*(Log[d + e/x^(1/3)] - Log[
e/d + x^(1/3)])*PolyLog[2, -((d*x^(1/3))/e)] + (81*Log[e/d + x^(1/3)]^2*Log
[-((d*x^(1/3))/e)] + 27*Log[d + e/x^(1/3)]^2*Log[x] - 27*Log[e/d + x^(1/3)]
^2*Log[x] - 54*Log[d + e/x^(1/3)]*Log[1 + (d*x^(1/3))/e]*Log[x] + 54*Log[e/
d + x^(1/3)]*Log[1 + (d*x^(1/3))/e]*Log[x] + 9*Log[d + e/x^(1/3)]*Log[x]^2
```

```
- 9*Log[1 + (d*x^(1/3))/e]*Log[x]^2 + Log[x]^3 - 162*PolyLog[3, 1 + (d*x^(1
/3))/e] - 162*PolyLog[3, -((d*x^(1/3))/e)])/81) - 3*b^3*n^3*(Log[d + e/x^(1
/3)]^3*Log[-(e/(d*x^(1/3)))] + 3*Log[d + e/x^(1/3)]^2*PolyLog[2, 1 + e/(d*x
^(1/3))] - 6*Log[d + e/x^(1/3)]*PolyLog[3, 1 + e/(d*x^(1/3))] + 6*PolyLog[4
, 1 + e/(d*x^(1/3))])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)^3+3\,ab^2\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)^2+3\,a^2b\log\left(c\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^n\right)+a^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*((d*x + e*x^(2/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^
(2/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(2/3))/x)^n) + a^3)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^3/x, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/3))^n)+a)^3/x,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/3))^n)+a)^3/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^3\log\left(\left(dx^{\frac{1}{3}}+e\right)^n\right)^3\log(x)-\int\frac{\left(b^3dx+b^3ex^{\frac{2}{3}}\right)\log\left(x^{\frac{1}{3}n}\right)^3+\left(b^3dnx\log(x)-3\left(b^3d\log(c)+ab^2d\right)x+3\left(b^3dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*log((d*x^(1/3) + e)^n)^3*log(x) - integrate(((b^3*d*x + b^3*e*x^(2/3))*
log(x^(1/3*n))^3 + (b^3*d*n*x*log(x) - 3*(b^3*d*log(c) + a*b^2*d)*x + 3*(b^
3*d*x + b^3*e*x^(2/3))*log(x^(1/3*n)) - 3*(b^3*e*log(c) + a*b^2*e)*x^(2/3))
*log((d*x^(1/3) + e)^n)^2 - 3*((b^3*d*log(c) + a*b^2*d)*x + (b^3*e*log(c) +
 a*b^2*e)*x^(2/3))*log(x^(1/3*n))^2 - (b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2
```

```
+ 3*a^2*b*d*log(c) + a^3*d)*x - 3*((b^3*d*x + b^3*e*x^(2/3))*log(x^(1/3*n))
^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x - 2*((b^3*d*log(c) + a
*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(2/3))*log(x^(1/3*n)) + (b^3*e*log(c
)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(2/3))*log((d*x^(1/3) + e)^n) + 3*((b^3
*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x + (b^3*e*log(c)^2 + 2*a*b^2*e*l
og(c) + a^2*b*e)*x^(2/3))*log(x^(1/3*n)) - (b^3*e*log(c)^3 + 3*a*b^2*e*log(
c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(2/3))/(d*x^2 + e*x^(5/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\ln\left(c\left(d + \frac{e}{x^{1/3}}\right)^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^3/x,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/3))^n))^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))**3/x, x)
```

3.506 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=438

$$-\frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3}+\frac{9b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^3}-\frac{18ab^2d^2n^2}{e^2\sqrt[3]{x}}+\frac{9bd^2n\,(}{}$$

```
[Out] -9/4*b^3*d*n^3*(d+e/x^(1/3))^2/e^3+2/9*b^3*n^3*(d+e/x^(1/3))^3/e^3-18*a*b^2
*d^2*n^2/e^2/x^(1/3)+18*b^3*d^2*n^3/e^2/x^(1/3)-18*b^3*d^2*n^2*(d+e/x^(1/3)
)*ln(c*(d+e/x^(1/3))^n)/e^3+9/2*b^2*d*n^2*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^
(1/3))^n))/e^3-2/3*b^2*n^2*(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))/e^3+
9*b*d^2*n*(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^3-9/2*b*d*n*(d+e/x^
(1/3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^3+b*n*(d+e/x^(1/3))^3*(a+b*ln(c*(d
+e/x^(1/3))^n))^2/e^3-3*d^2*(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^3
+3*d*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^3-(d+e/x^(1/3))^3*(a+b
*ln(c*(d+e/x^(1/3))^n))^3/e^3
```

Rubi [A] time = 0.45, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$-\frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3}+\frac{9b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^3}-\frac{18ab^2d^2n^2}{e^2\sqrt[3]{x}}+\frac{9bd^2n\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^2,x]
```

```
[Out] (-9*b^3*d*n^3*(d + e/x^(1/3))^2)/(4*e^3) + (2*b^3*n^3*(d + e/x^(1/3))^3)/(9
*e^3) - (18*a*b^2*d^2*n^2)/(e^2*x^(1/3)) + (18*b^3*d^2*n^3)/(e^2*x^(1/3)) -
 (18*b^3*d^2*n^2*(d + e/x^(1/3))*Log[c*(d + e/x^(1/3))^n])/e^3 + (9*b^2*d*n
^2*(d + e/x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(2*e^3) - (2*b^2*n^2
*(d + e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*e^3) + (9*b*d^2*n*(
d + e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^3 - (9*b*d*n*(d + e/x^
(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*e^3) + (b*n*(d + e/x^(1/3))
^3*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^3 - (3*d^2*(d + e/x^(1/3))*(a + b*
Log[c*(d + e/x^(1/3))^n])^3)/e^3 + (3*d*(d + e/x^(1/3))^2*(a + b*Log[c*(d +
 e/x^(1/3))^n])^3)/e^3 - ((d + e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]
)^3)/e^3
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
```

```
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx = -\left(3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\,\text{Subst}\left(\int\left(\frac{d^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2} - \frac{2d(d+ex)\left(a+b\log\left(c(d+ex\right.\right.}{e^2}\right.\right.\right.$$

$$= -\frac{3\,\text{Subst}\left(\int(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\frac{1}{\sqrt[3]{x}}\right)}{e^2} + \frac{(6d)\,\text{Subst}\left(\int(d\right.}{}$$

$$= -\frac{3\,\text{Subst}\left(\int x^2\left(a+b\log\left(cx^n\right)\right)^3\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} + \frac{(6d)\,\text{Subst}\left(\int x\left(a+b\log\right.\right.}{e^3}$$

$$= -\frac{3d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3} + \frac{3d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\right.\right.\right.}{e^3}$$

$$= \frac{9bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^3} - \frac{9bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\right.\right.}{2e^3}$$

$$= -\frac{9b^3dn^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^3} + \frac{2b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3} - \frac{18ab^2d^2n^2}{e^2\sqrt[3]{x}} + \frac{9b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(\right.}{}$$

$$= -\frac{9b^3dn^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^3} + \frac{2b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3} - \frac{18ab^2d^2n^2}{e^2\sqrt[3]{x}} + \frac{18b^3d^2n^3}{e^2\sqrt[3]{x}} - \frac{18b^3d^2n}{}$$

Mathematica [A] time = 0.90, size = 666, normalized size = 1.52

$$\frac{-36a^3e^3 - 6b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\left(18a^2e^3 + 6bd^3nx(6a-11bn)\log\left(d\sqrt[3]{x}+e\right) + 2bd^3nx\log(x)(11bn-6a) - 6aben\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^2,x]
```

```
[Out] (-36*a^3*e^3 + 36*a^2*b*e^3*n - 24*a*b^2*e^3*n^2 + 8*b^3*e^3*n^3 - 54*a^2*b
*d*e^2*n*x^(1/3) + 90*a*b^2*d*e^2*n^2*x^(1/3) - 57*b^3*d*e^2*n^3*x^(1/3) +
108*a^2*b*d^2*e*n*x^(2/3) - 396*a*b^2*d^2*e*n^2*x^(2/3) + 510*b^3*d^2*e*n^3
*x^(2/3) + 72*b^3*d^3*n^3*x*Log[d + e/x^(1/3)]^3 - 36*b^3*e^3*Log[c*(d + e/
x^(1/3))^n]^3 - 108*a^2*b*d^3*n*x*Log[e + d*x^(1/3)] + 396*a*b^2*d^3*n^2*x*
Log[e + d*x^(1/3)] - 510*b^3*d^3*n^3*x*Log[e + d*x^(1/3)] + 12*b^2*d^3*n^2*
x*Log[d + e/x^(1/3)]*(6*a - 11*b*n + 6*b*Log[c*(d + e/x^(1/3))^n])*(3*Log[e
 + d*x^(1/3)] - Log[x]) + 36*a^2*b*d^3*n*x*Log[x] - 132*a*b^2*d^3*n^2*x*Log
[x] + 170*b^3*d^3*n^3*x*Log[x] - 18*b^2*d^3*n^2*x*Log[d + e/x^(1/3)]^2*(6*a
 - 11*b*n + 6*b*Log[c*(d + e/x^(1/3))^n] + 6*b*n*Log[e + d*x^(1/3)] - 2*b*n
*Log[x]) + 18*b^2*Log[c*(d + e/x^(1/3))^n]^2*(e*(-6*a*e^2 + 2*b*e^2*n - 3*b
*d*e*n*x^(1/3) + 6*b*d^2*n*x^(2/3)) - 6*b*d^3*n*x*Log[e + d*x^(1/3)] + 2*b*
d^3*n*x*Log[x]) - 6*b*Log[c*(d + e/x^(1/3))^n]*(18*a^2*e^3 - 6*a*b*e*n*(2*e
^2 - 3*d*e*x^(1/3) + 6*d^2*x^(2/3)) + b^2*e*n^2*(4*e^2 - 15*d*e*x^(1/3) + 6
6*d^2*x^(2/3)) + 6*b*d^3*n*(6*a - 11*b*n)*x*Log[e + d*x^(1/3)] + 2*b*d^3*n*
(-6*a + 11*b*n)*x*Log[x]))/(36*e^3*x)
```

fricas [B] time = 0.52, size = 814, normalized size = 1.86

$$\frac{8\,b^3e^3n^3-24\,ab^2e^3n^2+36\,a^2be^3n-36\,a^3e^3+36\left(b^3e^3x-b^3e^3\right)\log(c)^3-36\left(b^3d^3n^3x+b^3e^3n^3\right)\log\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/36*(8*b^3*e^3*n^3 - 24*a*b^2*e^3*n^2 + 36*a^2*b*e^3*n - 36*a^3*e^3 + 36*(
b^3*e^3*x - b^3*e^3)*log(c)^3 - 36*(b^3*d^3*n^3*x + b^3*e^3*n^3)*log((d*x +
 e*x^(2/3))/x)^3 + 36*(b^3*e^3*n - 3*a*b^2*e^3 - (b^3*e^3*n - 3*a*b^2*e^3)*
x)*log(c)^2 + 18*(6*b^3*d^2*e*n^3*x^(2/3) - 3*b^3*d*e^2*n^3*x^(1/3) + 2*b^3
*e^3*n^3 - 6*a*b^2*e^3*n^2 + (11*b^3*d^3*n^3 - 6*a*b^2*d^3*n^2)*x - 6*(b^3*
d^3*n^2*x + b^3*e^3*n^2)*log(c))*log((d*x + e*x^(2/3))/x)^2 - 4*(2*b^3*e^3*
n^3 - 6*a*b^2*e^3*n^2 + 9*a^2*b*e^3*n - 9*a^3*e^3)*x - 12*(2*b^3*e^3*n^2 -
6*a*b^2*e^3*n + 9*a^2*b*e^3 - (2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + 9*a^2*b*e^3)
*x)*log(c) - 6*(4*b^3*e^3*n^3 - 12*a*b^2*e^3*n^2 + 18*a^2*b*e^3*n + 18*(b^3
*d^3*n*x + b^3*e^3*n)*log(c)^2 + (85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^
2*b*d^3*n)*x - 6*(2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + (11*b^3*d^3*n^2 - 6*a*b^2
*d^3*n)*x)*log(c) + 6*(11*b^3*d^2*e*n^3 - 6*b^3*d^2*e*n^2*log(c) - 6*a*b^2*
d^2*e*n^2)*x^(2/3) - 3*(5*b^3*d*e^2*n^3 - 6*b^3*d*e^2*n^2*log(c) - 6*a*b^2*
d*e^2*n^2)*x^(1/3))*log((d*x + e*x^(2/3))/x) + 6*(85*b^3*d^2*e*n^3 + 18*b^3
*d^2*e*n*log(c)^2 - 66*a*b^2*d^2*e*n^2 + 18*a^2*b*d^2*e*n - 6*(11*b^3*d^2*e
*n^2 - 6*a*b^2*d^2*e*n)*log(c))*x^(2/3) - 3*(19*b^3*d*e^2*n^3 + 18*b^3*d*e^
2*n*log(c)^2 - 30*a*b^2*d*e^2*n^2 + 18*a^2*b*d*e^2*n - 6*(5*b^3*d*e^2*n^2 -
 6*a*b^2*d*e^2*n)*log(c))*x^(1/3))/(e^3*x)
```

giac [B] time = 0.70, size = 1758, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] -1/36*(108*(d*x^(1/3) + e)*b^3*d^2*n^3*log((d*x^(1/3) + e)/x^(1/3))^3/x^(1/
3) - 108*(d*x^(1/3) + e)^2*b^3*d*n^3*log((d*x^(1/3) + e)/x^(1/3))^3/x^(2/3)
 + 36*(d*x^(1/3) + e)^3*b^3*n^3*log((d*x^(1/3) + e)/x^(1/3))^3/x - 324*(d*x
^(1/3) + e)*b^3*d^2*n^3*log((d*x^(1/3) + e)/x^(1/3))^2/x^(1/3) + 324*(d*x^(
1/3) + e)*b^3*d^2*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))^2/x^(1/3) + 162*(
d*x^(1/3) + e)^2*b^3*d*n^3*log((d*x^(1/3) + e)/x^(1/3))^2/x^(2/3) - 324*(d*
x^(1/3) + e)^2*b^3*d*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))^2/x^(2/3) - 36
*(d*x^(1/3) + e)^3*b^3*n^3*log((d*x^(1/3) + e)/x^(1/3))^2/x + 108*(d*x^(1/3
) + e)^3*b^3*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))^2/x + 648*(d*x^(1/3) +
 e)*b^3*d^2*n^3*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) - 648*(d*x^(1/3) + e)*
b^3*d^2*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 324*(d*x^(1/3) +
e)*b^3*d^2*n*log(c)^2*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 324*(d*x^(1/3)
 + e)*a*b^2*d^2*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^(1/3) - 162*(d*x^(1/3)
 + e)^2*b^3*d*n^3*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) + 324*(d*x^(1/3) + e
)^2*b^3*d*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) - 324*(d*x^(1/3)
+ e)^2*b^3*d*n*log(c)^2*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) - 324*(d*x^(1/
3) + e)^2*a*b^2*d*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^(2/3) + 24*(d*x^(1/3
) + e)^3*b^3*n^3*log((d*x^(1/3) + e)/x^(1/3))/x - 72*(d*x^(1/3) + e)^3*b^3*
n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x + 108*(d*x^(1/3) + e)^3*b^3*n*log
(c)^2*log((d*x^(1/3) + e)/x^(1/3))/x + 108*(d*x^(1/3) + e)^3*a*b^2*n^2*log(
(d*x^(1/3) + e)/x^(1/3))^2/x - 648*(d*x^(1/3) + e)*b^3*d^2*n^3/x^(1/3) + 64
8*(d*x^(1/3) + e)*b^3*d^2*n^2*log(c)/x^(1/3) - 324*(d*x^(1/3) + e)*b^3*d^2*
n*log(c)^2/x^(1/3) + 108*(d*x^(1/3) + e)*b^3*d^2*log(c)^3/x^(1/3) - 648*(d*
```

```
x^(1/3) + e)*a*b^2*d^2*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 648*(d*x^
(1/3) + e)*a*b^2*d^2*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 81*(d*
x^(1/3) + e)^2*b^3*d*n^3/x^(2/3) - 162*(d*x^(1/3) + e)^2*b^3*d*n^2*log(c)/x
^(2/3) + 162*(d*x^(1/3) + e)^2*b^3*d*n*log(c)^2/x^(2/3) - 108*(d*x^(1/3) +
e)^2*b^3*d*log(c)^3/x^(2/3) + 324*(d*x^(1/3) + e)^2*a*b^2*d*n^2*log((d*x^(1
/3) + e)/x^(1/3))/x^(2/3) - 648*(d*x^(1/3) + e)^2*a*b^2*d*n*log(c)*log((d*x
^(1/3) + e)/x^(1/3))/x^(2/3) - 8*(d*x^(1/3) + e)^3*b^3*n^3/x + 24*(d*x^(1/3
) + e)^3*b^3*n^2*log(c)/x - 36*(d*x^(1/3) + e)^3*b^3*n*log(c)^2/x + 36*(d*x
^(1/3) + e)^3*b^3*log(c)^3/x - 72*(d*x^(1/3) + e)^3*a*b^2*n^2*log((d*x^(1/3
) + e)/x^(1/3))/x + 216*(d*x^(1/3) + e)^3*a*b^2*n*log(c)*log((d*x^(1/3) + e
)/x^(1/3))/x + 648*(d*x^(1/3) + e)*a*b^2*d^2*n^2/x^(1/3) - 648*(d*x^(1/3) +
 e)*a*b^2*d^2*n*log(c)/x^(1/3) + 324*(d*x^(1/3) + e)*a*b^2*d^2*log(c)^2/x^(
1/3) + 324*(d*x^(1/3) + e)*a^2*b*d^2*n*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3)
 - 162*(d*x^(1/3) + e)^2*a*b^2*d*n^2/x^(2/3) + 324*(d*x^(1/3) + e)^2*a*b^2*
d*n*log(c)/x^(2/3) - 324*(d*x^(1/3) + e)^2*a*b^2*d*log(c)^2/x^(2/3) - 324*(
d*x^(1/3) + e)^2*a^2*b*d*n*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) + 24*(d*x^(
1/3) + e)^3*a*b^2*n^2/x - 72*(d*x^(1/3) + e)^3*a*b^2*n*log(c)/x + 108*(d*x^
(1/3) + e)^3*a*b^2*log(c)^2/x + 108*(d*x^(1/3) + e)^3*a^2*b*n*log((d*x^(1/3
) + e)/x^(1/3))/x - 324*(d*x^(1/3) + e)*a^2*b*d^2*n/x^(1/3) + 324*(d*x^(1/3
) + e)*a^2*b*d^2*log(c)/x^(1/3) + 162*(d*x^(1/3) + e)^2*a^2*b*d*n/x^(2/3) -
 324*(d*x^(1/3) + e)^2*a^2*b*d*log(c)/x^(2/3) - 36*(d*x^(1/3) + e)^3*a^2*b*
n/x + 108*(d*x^(1/3) + e)^3*a^2*b*log(c)/x + 108*(d*x^(1/3) + e)*a^3*d^2/x^
(1/3) - 108*(d*x^(1/3) + e)^2*a^3*d/x^(2/3) + 36*(d*x^(1/3) + e)^3*a^3/x)*e
^(-3)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(c \left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{3}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/3))^n)+a)^3/x^2,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/3))^n)+a)^3/x^2,x)
```

maxima [A] time = 0.59, size = 640, normalized size = 1.46

$$-\frac{1}{2}a^2ben\left(\frac{6\,d^3\log\left(dx^{\frac{1}{3}}+e\right)}{e^4}-\frac{2\,d^3\log(x)}{e^4}-\frac{6\,d^2x^{\frac{2}{3}}-3\,dex^{\frac{1}{3}}+2\,e^2}{e^3x}\right)-\frac{b^3\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)^3}{x}-\frac{1}{6}\left(6\,en\left(\frac{6\,d^3\log\left(d\right.}{e^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*b*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^
(2/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x)) - b^3*log(c*(d + e/x^(1/3))^n)^3/x
- 1/6*(6*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^(2
/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x))*log(c*(d + e/x^(1/3))^n) - (18*d^3*x*
log(d*x^(1/3) + e)^2 + 2*d^3*x*log(x)^2 - 22*d^3*x*log(x) - 66*d^2*e*x^(2/3
) + 15*d*e^2*x^(1/3) - 4*e^3 - 6*(2*d^3*x*log(x) - 11*d^3*x)*log(d*x^(1/3)
+ e))*n^2/(e^3*x))*a*b^2 - 1/108*(54*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*
d^3*log(x)/e^4 - (6*d^2*x^(2/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x))*log(c*(d
+ e/x^(1/3))^n)^2 + e*n*((108*d^3*x*log(d*x^(1/3) + e)^3 - 4*d^3*x*log(x)^3
 + 66*d^3*x*log(x)^2 - 510*d^3*x*log(x) - 1530*d^2*e*x^(2/3) + 171*d*e^2*x^
(1/3) - 24*e^3 - 54*(2*d^3*x*log(x) - 11*d^3*x)*log(d*x^(1/3) + e)^2 + 18*(
2*d^3*x*log(x)^2 - 22*d^3*x*log(x) + 85*d^3*x)*log(d*x^(1/3) + e))*n^2/(e^4
```

```
*x) - 18*(18*d^3*x*log(d*x^(1/3) + e)^2 + 2*d^3*x*log(x)^2 - 22*d^3*x*log(x
) - 66*d^2*e*x^(2/3) + 15*d*e^2*x^(1/3) - 4*e^3 - 6*(2*d^3*x*log(x) - 11*d^
3*x)*log(d*x^(1/3) + e))*n*log(c*(d + e/x^(1/3))^n)/(e^4*x)))*b^3 - 3*a*b^2
*log(c*(d + e/x^(1/3))^n)^2/x - 3*a^2*b*log(c*(d + e/x^(1/3))^n)/x - a^3/x
```

mupad [B] time = 0.74, size = 570, normalized size = 1.30

$$\frac{\frac{d\left(3a^3-3a^2bn+2ab^2n^2-\frac{2b^3n^3}{3}\right)}{2e}-\frac{d\left(6a^3-6ab^2n^2+5b^3n^3\right)}{4e}}{x^{2/3}}-\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)^3\left(\frac{b^3}{x}+\frac{b^3d^3}{e^3}\right)-\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)^2\left(\frac{b^2\,(3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^3/x^2,x)
```

```
[Out] ((d*(3*a^3 - (2*b^3*n^3)/3 + 2*a*b^2*n^2 - 3*a^2*b*n))/(2*e) - (d*(6*a^3 +
5*b^3*n^3 - 6*a*b^2*n^2))/(4*e))/x^(2/3) - log(c*(d + e/x^(1/3))^n)^3*(b^3/
x + (b^3*d^3)/e^3) - log(c*(d + e/x^(1/3))^n)^2*((b^2*(3*a - b*n))/x - ((3*
b^2*d*(3*a - b*n))/(2*e) - (9*a*b^2*d)/(2*e))/x^(2/3) + (d*(6*a*b^2*d^2 - 1
1*b^3*d^2*n))/(2*e^3) + (d*((3*b^2*d*(3*a - b*n))/e - (9*a*b^2*d)/e))/(e*x^
(1/3))) - (a^3 - (2*b^3*n^3)/9 + (2*a*b^2*n^2)/3 - a^2*b*n)/x - ((d*((d*(3*
a^3 - (2*b^3*n^3)/3 + 2*a*b^2*n^2 - 3*a^2*b*n))/e - (d*(6*a^3 + 5*b^3*n^3 -
 6*a*b^2*n^2))/(2*e)))/e + (b^2*d^2*n^2*(6*a - 11*b*n))/e^2)/x^(1/3) - (log
(c*(d + e/x^(1/3))^n)*(((d*(b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 3*b*d*e*(
3*a^2 - b^2*n^2)))/e + 6*b^3*d^2*n^2)/(e*x^(1/3)) - (b*d*e*(9*a^2 + 2*b^2*n
^2 - 6*a*b*n) - 3*b*d*e*(3*a^2 - b^2*n^2))/(2*e*x^(2/3)) + (b*e*(9*a^2 + 2*
b^2*n^2 - 6*a*b*n))/(3*x)))/e - (log(d + e/x^(1/3))*(85*b^3*d^3*n^3 - 66*a*
b^2*d^3*n^2 + 18*a^2*b*d^3*n))/(6*e^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3/x**2,x)
```

```
[Out] Timed out
```

3.507 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=907

$$\frac{b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{72e^6}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{2e^6}+\frac{bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{4e^6}-\frac{b^2n^2\left(a+b\,l\right.}{}$$

```
[Out] -1/2*(d+e/x^(1/3))^6*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^6+18*a*b^2*d^5*n^2/e^5
/x^(1/3)+45/8*b^3*d^4*n^3*(d+e/x^(1/3))^2/e^6-20/9*b^3*d^3*n^3*(d+e/x^(1/3)
)^3/e^6+45/64*b^3*d^2*n^3*(d+e/x^(1/3))^4/e^6-18/125*b^3*d*n^3*(d+e/x^(1/3)
)^5/e^6-18*b^3*d^5*n^3/e^5/x^(1/3)+45/4*b*d^4*n*(d+e/x^(1/3))^2*(a+b*ln(c*(
d+e/x^(1/3))^n))^2/e^6-10*b*d^3*n*(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n
))^2/e^6+45/8*b*d^2*n*(d+e/x^(1/3))^4*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^6-9/5
*b*d*n*(d+e/x^(1/3))^5*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^6+10*d^3*(d+e/x^(1/3
))^3*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^6-15/2*d^2*(d+e/x^(1/3))^4*(a+b*ln(c*(
d+e/x^(1/3))^n))^3/e^6+3*d*(d+e/x^(1/3))^5*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^
6+3*d^5*(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^6-15/2*d^4*(d+e/x^(1/
3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^6+1/72*b^3*n^3*(d+e/x^(1/3))^6/e^6-1/
12*b^2*n^2*(d+e/x^(1/3))^6*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6+1/4*b*n*(d+e/x^(
1/3))^6*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^6+18*b^3*d^5*n^2*(d+e/x^(1/3))*ln(c
*(d+e/x^(1/3))^n)/e^6-45/4*b^2*d^4*n^2*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^(1/
3))^n))/e^6+20/3*b^2*d^3*n^2*(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))/e^
6-45/16*b^2*d^2*n^2*(d+e/x^(1/3))^4*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6+18/25*b
^2*d*n^2*(d+e/x^(1/3))^5*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6-9*b*d^5*n*(d+e/x^(
1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^6
```

Rubi [A] time = 1.00, antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$\frac{b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{72e^6}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{2e^6}+\frac{bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{4e^6}-\frac{b^2n^2\left(a+b\,l\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^3,x]
```

```
[Out] (45*b^3*d^4*n^3*(d + e/x^(1/3))^2)/(8*e^6) - (20*b^3*d^3*n^3*(d + e/x^(1/3)
)^3)/(9*e^6) + (45*b^3*d^2*n^3*(d + e/x^(1/3))^4)/(64*e^6) - (18*b^3*d*n^3*
(d + e/x^(1/3))^5)/(125*e^6) + (b^3*n^3*(d + e/x^(1/3))^6)/(72*e^6) + (18*a
*b^2*d^5*n^2)/(e^5*x^(1/3)) - (18*b^3*d^5*n^3)/(e^5*x^(1/3)) + (18*b^3*d^5*
n^2*(d + e/x^(1/3))*Log[c*(d + e/x^(1/3))^n])/e^6 - (45*b^2*d^4*n^2*(d + e/
x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(4*e^6) + (20*b^2*d^3*n^2*(d +
 e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*e^6) - (45*b^2*d^2*n^2*(
d + e/x^(1/3))^4*(a + b*Log[c*(d + e/x^(1/3))^n]))/(16*e^6) + (18*b^2*d*n^2
*(d + e/x^(1/3))^5*(a + b*Log[c*(d + e/x^(1/3))^n]))/(25*e^6) - (b^2*n^2*(d
 + e/x^(1/3))^6*(a + b*Log[c*(d + e/x^(1/3))^n]))/(12*e^6) - (9*b*d^5*n*(d
+ e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^6 + (45*b*d^4*n*(d + e/x
^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(4*e^6) - (10*b*d^3*n*(d + e/
x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^6 + (45*b*d^2*n*(d + e/x^(
1/3))^4*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(8*e^6) - (9*b*d*n*(d + e/x^(1/
3))^5*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(5*e^6) + (b*n*(d + e/x^(1/3))^6*
(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(4*e^6) + (3*d^5*(d + e/x^(1/3))*(a + b
*Log[c*(d + e/x^(1/3))^n])^3)/e^6 - (15*d^4*(d + e/x^(1/3))^2*(a + b*Log[c*
```

```
(d + e/x^(1/3))^n])^3)/(2*e^6) + (10*d^3*(d + e/x^(1/3))^3*(a + b*Log[c*(d
+ e/x^(1/3))^n])^3)/e^6 - (15*d^2*(d + e/x^(1/3))^4*(a + b*Log[c*(d + e/x^(
1/3))^n])^3)/(2*e^6) + (3*d*(d + e/x^(1/3))^5*(a + b*Log[c*(d + e/x^(1/3))^
n])^3)/e^6 - ((d + e/x^(1/3))^6*(a + b*Log[c*(d + e/x^(1/3))^n])^3)/(2*e^6)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx = -\left(3\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3 dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\,\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^5} + \frac{5d^4(d+ex)\left(a+b\log\left(c(d+\right.\right.}{e^5}\right.\right.\right.$$

$$= -\frac{3\,\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3 dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5} + \frac{(15d)\,\text{Subst}\left(\int(\right.}{}$$

$$= -\frac{3\,\text{Subst}\left(\int x^5\left(a+b\log\left(cx^n\right)\right)^3 dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} + \frac{(15d)\,\text{Subst}\left(\int x^4\left(a+b\right.\right.}{e}$$

$$= \frac{3d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^6} - \frac{15d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d\right.\right.\right.}{2e^6}$$

$$= -\frac{9bd^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^6} + \frac{45bd^4n\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\right.}{4e^6}$$

$$= \frac{45b^3d^4n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{8e^6} - \frac{20b^3d^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6} + \frac{45b^3d^2n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{64e^6} - \frac{18b^3d}{}$$

$$= \frac{45b^3d^4n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{8e^6} - \frac{20b^3d^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6} + \frac{45b^3d^2n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{64e^6} - \frac{18b^3d}{}$$

Mathematica [A] time = 1.87, size = 962, normalized size = 1.06

$$\frac{-72000b^3n^3x^2\log^3\left(d+\frac{e}{\sqrt[3]{x}}\right)d^6 + 809340b^3n^3x^2\log\left(\sqrt[3]{x}\,d+e\right)d^6 - 529200ab^2n^2x^2\log\left(\sqrt[3]{x}\,d+e\right)d^6 + 108000}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^3,x]
```

```
[Out] (-36000*a^3*e^6 + 18000*a^2*b*e^6*n - 6000*a*b^2*e^6*n^2 + 1000*b^3*e^6*n^3
 - 21600*a^2*b*d*e^5*n*x^(1/3) + 15840*a*b^2*d*e^5*n^2*x^(1/3) - 4368*b^3*d
*e^5*n^3*x^(1/3) + 27000*a^2*b*d^2*e^4*n*x^(2/3) - 33300*a*b^2*d^2*e^4*n^2*
x^(2/3) + 13785*b^3*d^2*e^4*n^3*x^(2/3) - 36000*a^2*b*d^3*e^3*n*x + 68400*a
*b^2*d^3*e^3*n^2*x - 41180*b^3*d^3*e^3*n^3*x + 54000*a^2*b*d^4*e^2*n*x^(4/3
) - 156600*a*b^2*d^4*e^2*n^2*x^(4/3) + 140070*b^3*d^4*e^2*n^3*x^(4/3) - 108
000*a^2*b*d^5*e*n*x^(5/3) + 529200*a*b^2*d^5*e*n^2*x^(5/3) - 809340*b^3*d^5
*e*n^3*x^(5/3) - 72000*b^3*d^6*n^3*x^2*Log[d + e/x^(1/3)]^3 - 36000*b^3*e^6
*Log[c*(d + e/x^(1/3))^n]^3 + 108000*a^2*b*d^6*n*x^2*Log[e + d*x^(1/3)] - 5
29200*a*b^2*d^6*n^2*x^2*Log[e + d*x^(1/3)] + 809340*b^3*d^6*n^3*x^2*Log[e +
 d*x^(1/3)] + 3600*b^2*d^6*n^2*x^2*Log[d + e/x^(1/3)]*(-20*a + 49*b*n - 20*
b*Log[c*(d + e/x^(1/3))^n])*(3*Log[e + d*x^(1/3)] - Log[x]) - 36000*a^2*b*d
^6*n*x^2*Log[x] + 176400*a*b^2*d^6*n^2*x^2*Log[x] - 269780*b^3*d^6*n^3*x^2*
Log[x] + 1800*b^2*d^6*n^2*x^2*Log[d + e/x^(1/3)]^2*(60*a - 147*b*n + 60*b*L
og[c*(d + e/x^(1/3))^n] + 60*b*n*Log[e + d*x^(1/3)] - 20*b*n*Log[x]) + 1800
*b^2*Log[c*(d + e/x^(1/3))^n]^2*(e*(-60*a*e^5 + 10*b*e^5*n - 12*b*d*e^4*n*x
^(1/3) + 15*b*d^2*e^3*n*x^(2/3) - 20*b*d^3*e^2*n*x + 30*b*d^4*e*n*x^(4/3) -
 60*b*d^5*n*x^(5/3)) + 60*b*d^6*n*x^2*Log[e + d*x^(1/3)] - 20*b*d^6*n*x^2*L
```

og[x]) - 60*b*Log[c*(d + e/x^(1/3))^n]*(1800*a^2*e^6 + b^2*e*n^2*(100*e^5 - 264*d*e^4*x^(1/3) + 555*d^2*e^3*x^(2/3) - 1140*d^3*e^2*x + 2610*d^4*e*x^(4/3) - 8820*d^5*x^(5/3)) - 60*a*b*e*n*(10*e^5 - 12*d*e^4*x^(1/3) + 15*d^2*e^3*x^(2/3) - 20*d^3*e^2*x + 30*d^4*e*x^(4/3) - 60*d^5*x^(5/3)) + 180*b*d^6*n*(-20*a + 49*b*n)*x^2*Log[e + d*x^(1/3)] + 60*b*d^6*n*(20*a - 49*b*n)*x^2*Log[x]))/(72000*e^6*x^2)

fricas [A] time = 0.58, size = 1404, normalized size = 1.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^3,x, algorithm="fricas")

[Out] 1/72000*(1000*b^3*e^6*n^3 - 6000*a*b^2*e^6*n^2 + 18000*a^2*b*e^6*n - 36000*a^3*e^6 + 36000*(b^3*e^6*x^2 - b^3*e^6)*log(c)^3 + 36000*(b^3*d^6*n^3*x^2 - b^3*e^6*n^3)*log((d*x + e*x^(2/3))/x)^3 + 20*(1800*a^3*e^6 + (2059*b^3*d^3*e^3 - 50*b^3*e^6)*n^3 - 60*(57*a*b^2*d^3*e^3 - 5*a*b^2*e^6)*n^2 + 900*(2*a^2*b*d^3*e^3 - a^2*b*e^6)*n)*x^2 - 18000*(2*b^3*d^3*e^3*n*x - b^3*e^6*n + 6*a*b^2*e^6 - (6*a*b^2*e^6 + (2*b^3*d^3*e^3 - b^3*e^6)*n)*x^2)*log(c)^2 - 1800*(20*b^3*d^3*e^3*n^3*x - 10*b^3*e^6*n^3 + 60*a*b^2*e^6*n^2 + 3*(49*b^3*d^6*n^3 - 20*a*b^2*d^6*n^2)*x^2 - 60*(b^3*d^6*n^2*x^2 - b^3*e^6*n^2)*log(c) + 15*(4*b^3*d^5*e*n^3*x - b^3*d^2*e^4*n^3)*x^(2/3) - 6*(5*b^3*d^4*e^2*n^3*x - 2*b^3*d*e^5*n^3)*x^(1/3))*log((d*x + e*x^(2/3))/x)^2 - 20*(2059*b^3*d^3*e^3*n^3 - 3420*a*b^2*d^3*e^3*n^2 + 1800*a^2*b*d^3*e^3*n)*x - 1200*(5*b^3*e^6*n^2 - 30*a*b^2*e^6*n + 90*a^2*b*e^6 - (90*a^2*b*e^6 - (57*b^3*d^3*e^3 - 5*b^3*e^6)*n^2 + 30*(2*a*b^2*d^3*e^3 - a*b^2*e^6)*n)*x^2 - 3*(19*b^3*d^3*e^3*n^2 - 20*a*b^2*d^3*e^3*n)*x)*log(c) - 60*(100*b^3*e^6*n^3 - 600*a*b^2*e^6*n^2 + 1800*a^2*b*e^6*n - (13489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a^2*b*d^6*n)*x^2 - 1800*(b^3*d^6*n*x^2 - b^3*e^6*n)*log(c)^2 - 60*(19*b^3*d^3*e^3*n^3 - 20*a*b^2*d^3*e^3*n^2)*x + 60*(20*b^3*d^3*e^3*n^2*x - 10*b^3*e^6*n^2 + 60*a*b^2*e^6*n + 3*(49*b^3*d^6*n^2 - 20*a*b^2*d^6*n)*x^2)*log(c) + 15*(37*b^3*d^2*e^4*n^3 - 60*a*b^2*d^2*e^4*n^2 - 12*(49*b^3*d^5*e*n^3 - 20*a*b^2*d^5*e*n^2)*x + 60*(4*b^3*d^5*e*n^2*x - b^3*d^2*e^4*n^2)*log(c))*x^(2/3) - 6*(44*b^3*d*e^5*n^3 - 120*a*b^2*d*e^5*n^2 - 15*(29*b^3*d^4*e^2*n^3 - 20*a*b^2*d^4*e^2*n^2)*x + 60*(5*b^3*d^4*e^2*n^2*x - 2*b^3*d*e^5*n^2)*log(c))*x^(1/3))*log((d*x + e*x^(2/3))/x) + 15*(919*b^3*d^2*e^4*n^3 - 2220*a*b^2*d^2*e^4*n^2 + 1800*a^2*b*d^2*e^4*n - 1800*(4*b^3*d^5*e*n*x - b^3*d^2*e^4*n)*log(c)^2 - 4*(13489*b^3*d^5*e*n^3 - 8820*a*b^2*d^5*e*n^2 + 1800*a^2*b*d^5*e*n)*x - 60*(37*b^3*d^2*e^4*n^2 - 60*a*b^2*d^2*e^4*n - 12*(49*b^3*d^5*e*n^2 - 20*a*b^2*d^5*e*n)*x)*log(c))*x^(2/3) - 6*(728*b^3*d*e^5*n^3 - 2640*a*b^2*d*e^5*n^2 + 3600*a^2*b*d*e^5*n - 1800*(5*b^3*d^4*e^2*n*x - 2*b^3*d*e^5*n)*log(c)^2 - 5*(4669*b^3*d^4*e^2*n^3 - 5220*a*b^2*d^4*e^2*n^2 + 1800*a^2*b*d^4*e^2*n)*x - 60*(44*b^3*d*e^5*n^2 - 120*a*b^2*d*e^5*n - 15*(29*b^3*d^4*e^2*n^2 - 20*a*b^2*d^4*e^2*n)*x)*log(c))*x^(1/3))/(e^6*x^2)

giac [B] time = 0.89, size = 3651, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^3,x, algorithm="giac")

[Out] 1/72000*(216000*(d*x^(1/3) + e)*b^3*d^5*n^3*log((d*x^(1/3) + e)/x^(1/3))^3/x^(1/3) - 540000*(d*x^(1/3) + e)^2*b^3*d^4*n^3*log((d*x^(1/3) + e)/x^(1/3))^3/x^(2/3) + 720000*(d*x^(1/3) + e)^3*b^3*d^3*n^3*log((d*x^(1/3) + e)/x^(1/3))^3/x - 648000*(d*x^(1/3) + e)*b^3*d^5*n^3*log((d*x^(1/3) + e)/x^(1/3))^2/x^(1/3) + 648000*(d*x^(1/3) + e)*b^3*d^5*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))^2/x^(1/3) - 540000*(d*x^(1/3) + e)^4*b^3*d^2*n^3*log((d*x^(1/3) + e)/x^(1/3))^3/x^(4/3) + 810000*(d*x^(1/3) + e)^2*b^3*d^4*n^3*log((d*x^(1/3) +

e)/x^(1/3))^2/x^(2/3) - 1620000*(d*x^(1/3) + e)^2*b^3*d^4*n^2*log(c)*log((
d*x^(1/3) + e)/x^(1/3))^2/x^(2/3) + 216000*(d*x^(1/3) + e)^5*b^3*d*n^3*log(
(d*x^(1/3) + e)/x^(1/3))^3/x^(5/3) - 720000*(d*x^(1/3) + e)^3*b^3*d^3*n^3*l
og((d*x^(1/3) + e)/x^(1/3))^2/x + 2160000*(d*x^(1/3) + e)^3*b^3*d^3*n^2*log
(c)*log((d*x^(1/3) + e)/x^(1/3))^2/x - 36000*(d*x^(1/3) + e)^6*b^3*n^3*log(
(d*x^(1/3) + e)/x^(1/3))^3/x^2 + 1296000*(d*x^(1/3) + e)*b^3*d^5*n^3*log((d
x^(1/3) + e)/x^(1/3))/x^(1/3) - 1296000(d*x^(1/3) + e)*b^3*d^5*n^2*log(c)
*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 648000*(d*x^(1/3) + e)*b^3*d^5*n*lo
g(c)^2*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 405000*(d*x^(1/3) + e)^4*b^3*
d^2*n^3*log((d*x^(1/3) + e)/x^(1/3))^2/x^(4/3) + 648000*(d*x^(1/3) + e)*a*b
^2*d^5*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^(1/3) - 1620000*(d*x^(1/3) + e)
^4*b^3*d^2*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))^2/x^(4/3) - 810000*(d*x^
(1/3) + e)^2*b^3*d^4*n^3*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) + 1620000*(d*
x^(1/3) + e)^2*b^3*d^4*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) - 16
20000*(d*x^(1/3) + e)^2*b^3*d^4*n*log(c)^2*log((d*x^(1/3) + e)/x^(1/3))/x^(
2/3) - 129600*(d*x^(1/3) + e)^5*b^3*d*n^3*log((d*x^(1/3) + e)/x^(1/3))^2/x^
(5/3) - 1620000*(d*x^(1/3) + e)^2*a*b^2*d^4*n^2*log((d*x^(1/3) + e)/x^(1/3)
)^2/x^(2/3) + 648000*(d*x^(1/3) + e)^5*b^3*d*n^2*log(c)*log((d*x^(1/3) + e)
/x^(1/3))^2/x^(5/3) + 480000*(d*x^(1/3) + e)^3*b^3*d^3*n^3*log((d*x^(1/3) +
e)/x^(1/3))/x - 1440000*(d*x^(1/3) + e)^3*b^3*d^3*n^2*log(c)*log((d*x^(1/3
) + e)/x^(1/3))/x + 2160000*(d*x^(1/3) + e)^3*b^3*d^3*n*log(c)^2*log((d*x^(
1/3) + e)/x^(1/3))/x + 18000*(d*x^(1/3) + e)^6*b^3*n^3*log((d*x^(1/3) + e)/
x^(1/3))^2/x^2 + 2160000*(d*x^(1/3) + e)^3*a*b^2*d^3*n^2*log((d*x^(1/3) + e
)/x^(1/3))^2/x - 108000*(d*x^(1/3) + e)^6*b^3*n^2*log(c)*log((d*x^(1/3) + e
)/x^(1/3))^2/x^2 - 1296000*(d*x^(1/3) + e)*b^3*d^5*n^3/x^(1/3) + 1296000*(d
*x^(1/3) + e)*b^3*d^5*n^2*log(c)/x^(1/3) - 648000*(d*x^(1/3) + e)*b^3*d^5*n
log(c)^2/x^(1/3) + 216000(d*x^(1/3) + e)*b^3*d^5*log(c)^3/x^(1/3) - 20250
0*(d*x^(1/3) + e)^4*b^3*d^2*n^3*log((d*x^(1/3) + e)/x^(1/3))/x^(4/3) - 1296
000*(d*x^(1/3) + e)*a*b^2*d^5*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 81
0000*(d*x^(1/3) + e)^4*b^3*d^2*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(4
/3) + 1296000*(d*x^(1/3) + e)*a*b^2*d^5*n*log(c)*log((d*x^(1/3) + e)/x^(1/3
))/x^(1/3) - 1620000*(d*x^(1/3) + e)^4*b^3*d^2*n*log(c)^2*log((d*x^(1/3) +
e)/x^(1/3))/x^(4/3) - 1620000*(d*x^(1/3) + e)^4*a*b^2*d^2*n^2*log((d*x^(1/3
) + e)/x^(1/3))^2/x^(4/3) + 405000*(d*x^(1/3) + e)^2*b^3*d^4*n^3/x^(2/3) -
810000*(d*x^(1/3) + e)^2*b^3*d^4*n^2*log(c)/x^(2/3) + 810000*(d*x^(1/3) + e
)^2*b^3*d^4*n*log(c)^2/x^(2/3) - 540000*(d*x^(1/3) + e)^2*b^3*d^4*log(c)^3/
x^(2/3) + 51840*(d*x^(1/3) + e)^5*b^3*d*n^3*log((d*x^(1/3) + e)/x^(1/3))/x^
(5/3) + 1620000*(d*x^(1/3) + e)^2*a*b^2*d^4*n^2*log((d*x^(1/3) + e)/x^(1/3)
)/x^(2/3) - 259200*(d*x^(1/3) + e)^5*b^3*d*n^2*log(c)*log((d*x^(1/3) + e)/x
^(1/3))/x^(5/3) - 3240000*(d*x^(1/3) + e)^2*a*b^2*d^4*n*log(c)*log((d*x^(1/
3) + e)/x^(1/3))/x^(2/3) + 648000*(d*x^(1/3) + e)^5*b^3*d*n*log(c)^2*log((d
x^(1/3) + e)/x^(1/3))/x^(5/3) + 648000(d*x^(1/3) + e)^5*a*b^2*d*n^2*log((
d*x^(1/3) + e)/x^(1/3))^2/x^(5/3) - 160000*(d*x^(1/3) + e)^3*b^3*d^3*n^3/x
+ 480000*(d*x^(1/3) + e)^3*b^3*d^3*n^2*log(c)/x - 720000*(d*x^(1/3) + e)^3*
b^3*d^3*n*log(c)^2/x + 720000*(d*x^(1/3) + e)^3*b^3*d^3*log(c)^3/x - 6000*(
d*x^(1/3) + e)^6*b^3*n^3*log((d*x^(1/3) + e)/x^(1/3))/x^2 - 1440000*(d*x^(1
/3) + e)^3*a*b^2*d^3*n^2*log((d*x^(1/3) + e)/x^(1/3))/x + 36000*(d*x^(1/3)
+ e)^6*b^3*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^2 + 4320000*(d*x^(1/3)
+ e)^3*a*b^2*d^3*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x - 108000*(d*x^(1/
3) + e)^6*b^3*n*log(c)^2*log((d*x^(1/3) + e)/x^(1/3))/x^2 - 108000*(d*x^(1/
3) + e)^6*a*b^2*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^2 + 50625*(d*x^(1/3) +
e)^4*b^3*d^2*n^3/x^(4/3) + 1296000*(d*x^(1/3) + e)*a*b^2*d^5*n^2/x^(1/3) -
202500*(d*x^(1/3) + e)^4*b^3*d^2*n^2*log(c)/x^(4/3) - 1296000*(d*x^(1/3) +
e)*a*b^2*d^5*n*log(c)/x^(1/3) + 405000*(d*x^(1/3) + e)^4*b^3*d^2*n*log(c)^
2/x^(4/3) + 648000*(d*x^(1/3) + e)*a*b^2*d^5*log(c)^2/x^(1/3) - 540000*(d*x
^(1/3) + e)^4*b^3*d^2*log(c)^3/x^(4/3) + 810000*(d*x^(1/3) + e)^4*a*b^2*d^2
*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^(4/3) + 648000*(d*x^(1/3) + e)*a^2*b*d^
5*n*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) - 3240000*(d*x^(1/3) + e)^4*a*b^2*
d^2*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(4/3) - 10368*(d*x^(1/3) + e)^5

```
*b^3*d*n^3/x^(5/3) - 810000*(d*x^(1/3) + e)^2*a*b^2*d^4*n^2/x^(2/3) + 51840
*(d*x^(1/3) + e)^5*b^3*d*n^2*log(c)/x^(5/3) + 1620000*(d*x^(1/3) + e)^2*a*b
^2*d^4*n*log(c)/x^(2/3) - 129600*(d*x^(1/3) + e)^5*b^3*d*n*log(c)^2/x^(5/3)
 - 1620000*(d*x^(1/3) + e)^2*a*b^2*d^4*log(c)^2/x^(2/3) + 216000*(d*x^(1/3)
 + e)^5*b^3*d*log(c)^3/x^(5/3) - 259200*(d*x^(1/3) + e)^5*a*b^2*d*n^2*log((
d*x^(1/3) + e)/x^(1/3))/x^(5/3) - 1620000*(d*x^(1/3) + e)^2*a^2*b*d^4*n*log
((d*x^(1/3) + e)/x^(1/3))/x^(2/3) + 1296000*(d*x^(1/3) + e)^5*a*b^2*d*n*log
(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(5/3) + 1000*(d*x^(1/3) + e)^6*b^3*n^3/x
^2 + 480000*(d*x^(1/3) + e)^3*a*b^2*d^3*n^2/x - 6000*(d*x^(1/3) + e)^6*b^3*
n^2*log(c)/x^2 - 1440000*(d*x^(1/3) + e)^3*a*b^2*d^3*n*log(c)/x + 18000*(d*
x^(1/3) + e)^6*b^3*n*log(c)^2/x^2 + 2160000*(d*x^(1/3) + e)^3*a*b^2*d^3*log
(c)^2/x - 36000*(d*x^(1/3) + e)^6*b^3*log(c)^3/x^2 + 36000*(d*x^(1/3) + e)^
6*a*b^2*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^2 + 2160000*(d*x^(1/3) + e)^3*a^
2*b*d^3*n*log((d*x^(1/3) + e)/x^(1/3))/x - 216000*(d*x^(1/3) + e)^6*a*b^2*n
*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^2 - 202500*(d*x^(1/3) + e)^4*a*b^2*d
^2*n^2/x^(4/3) - 648000*(d*x^(1/3) + e)*a^2*b*d^5*n/x^(1/3) + 810000*(d*x^(
1/3) + e)^4*a*b^2*d^2*n*log(c)/x^(4/3) + 648000*(d*x^(1/3) + e)*a^2*b*d^5*l
og(c)/x^(1/3) - 1620000*(d*x^(1/3) + e)^4*a*b^2*d^2*log(c)^2/x^(4/3) - 1620
000*(d*x^(1/3) + e)^4*a^2*b*d^2*n*log((d*x^(1/3) + e)/x^(1/3))/x^(4/3) + 51
840*(d*x^(1/3) + e)^5*a*b^2*d*n^2/x^(5/3) + 810000*(d*x^(1/3) + e)^2*a^2*b*
d^4*n/x^(2/3) - 259200*(d*x^(1/3) + e)^5*a*b^2*d*n*log(c)/x^(5/3) - 1620000
*(d*x^(1/3) + e)^2*a^2*b*d^4*log(c)/x^(2/3) + 648000*(d*x^(1/3) + e)^5*a*b^
2*d*log(c)^2/x^(5/3) + 648000*(d*x^(1/3) + e)^5*a^2*b*d*n*log((d*x^(1/3) +
e)/x^(1/3))/x^(5/3) - 6000*(d*x^(1/3) + e)^6*a*b^2*n^2/x^2 - 720000*(d*x^(1
/3) + e)^3*a^2*b*d^3*n/x + 36000*(d*x^(1/3) + e)^6*a*b^2*n*log(c)/x^2 + 216
0000*(d*x^(1/3) + e)^3*a^2*b*d^3*log(c)/x - 108000*(d*x^(1/3) + e)^6*a*b^2*
log(c)^2/x^2 - 108000*(d*x^(1/3) + e)^6*a^2*b*n*log((d*x^(1/3) + e)/x^(1/3)
)/x^2 + 405000*(d*x^(1/3) + e)^4*a^2*b*d^2*n/x^(4/3) + 216000*(d*x^(1/3) +
e)*a^3*d^5/x^(1/3) - 1620000*(d*x^(1/3) + e)^4*a^2*b*d^2*log(c)/x^(4/3) - 1
29600*(d*x^(1/3) + e)^5*a^2*b*d*n/x^(5/3) - 540000*(d*x^(1/3) + e)^2*a^3*d^
4/x^(2/3) + 648000*(d*x^(1/3) + e)^5*a^2*b*d*log(c)/x^(5/3) + 18000*(d*x^(1
/3) + e)^6*a^2*b*n/x^2 + 720000*(d*x^(1/3) + e)^3*a^3*d^3/x - 108000*(d*x^(
1/3) + e)^6*a^2*b*log(c)/x^2 - 540000*(d*x^(1/3) + e)^4*a^3*d^2/x^(4/3) + 2
16000*(d*x^(1/3) + e)^5*a^3*d/x^(5/3) - 36000*(d*x^(1/3) + e)^6*a^3/x^2)*e^
(-6)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(c \left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{3}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(1/3))^n)+a)^3/x^3,x)
```

```
[Out] int((b*ln(c*(d+e/x^(1/3))^n)+a)^3/x^3,x)
```

maxima [A] time = 0.64, size = 864, normalized size = 0.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/40*a^2*b*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7 - 20*d^6*log(x)/e^7 - (60*d^5
*x^(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 15*d^2*e^3*x^(2/3) + 12*d*e^4*
x^(1/3) - 10*e^5)/(e^6*x^2)) + 1/1200*(60*e*n*(60*d^6*log(d*x^(1/3) + e)/e^
7 - 20*d^6*log(x)/e^7 - (60*d^5*x^(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x -
 15*d^2*e^3*x^(2/3) + 12*d*e^4*x^(1/3) - 10*e^5)/(e^6*x^2))*log(c*(d + e/x^
```

```
(1/3))^n) - (1800*d^6*x^2*log(d*x^(1/3) + e)^2 + 200*d^6*x^2*log(x)^2 - 294
0*d^6*x^2*log(x) - 8820*d^5*e*x^(5/3) + 2610*d^4*e^2*x^(4/3) - 1140*d^3*e^3
*x + 555*d^2*e^4*x^(2/3) - 264*d*e^5*x^(1/3) + 100*e^6 - 60*(20*d^6*x^2*log
(x) - 147*d^6*x^2)*log(d*x^(1/3) + e))*n^2/(e^6*x^2))*a*b^2 + 1/216000*(540
0*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7 - 20*d^6*log(x)/e^7 - (60*d^5*x^(5/3)
- 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 15*d^2*e^3*x^(2/3) + 12*d*e^4*x^(1/3) -
 10*e^5)/(e^6*x^2))*log(c*(d + e/x^(1/3))^n)^2 + e*n*((108000*d^6*x^2*log(d
*x^(1/3) + e)^3 - 4000*d^6*x^2*log(x)^3 + 88200*d^6*x^2*log(x)^2 - 809340*d
^6*x^2*log(x) - 2428020*d^5*e*x^(5/3) + 420210*d^4*e^2*x^(4/3) - 123540*d^3
*e^3*x + 41355*d^2*e^4*x^(2/3) - 13104*d*e^5*x^(1/3) + 3000*e^6 - 5400*(20*
d^6*x^2*log(x) - 147*d^6*x^2)*log(d*x^(1/3) + e)^2 + 180*(200*d^6*x^2*log(x
)^2 - 2940*d^6*x^2*log(x) + 13489*d^6*x^2)*log(d*x^(1/3) + e))*n^2/(e^7*x^2
) - 180*(1800*d^6*x^2*log(d*x^(1/3) + e)^2 + 200*d^6*x^2*log(x)^2 - 2940*d^
6*x^2*log(x) - 8820*d^5*e*x^(5/3) + 2610*d^4*e^2*x^(4/3) - 1140*d^3*e^3*x +
 555*d^2*e^4*x^(2/3) - 264*d*e^5*x^(1/3) + 100*e^6 - 60*(20*d^6*x^2*log(x)
- 147*d^6*x^2)*log(d*x^(1/3) + e))*n*log(c*(d + e/x^(1/3))^n)/(e^7*x^2)))*b
^3 - 1/2*b^3*log(c*(d + e/x^(1/3))^n)^3/x^2 - 3/2*a*b^2*log(c*(d + e/x^(1/3
))^n)^2/x^2 - 3/2*a^2*b*log(c*(d + e/x^(1/3))^n)/x^2 - 1/2*a^3/x^2
```

mupad [B] time = 8.20, size = 992, normalized size = 1.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^3/x^3,x)
```

```
[Out] (b^3*n^3)/(72*x^2) - (b^3*log(c*(d + e/x^(1/3))^n)^3)/(2*x^2) - a^3/(2*x^2)
 - (3*a*b^2*log(c*(d + e/x^(1/3))^n)^2)/(2*x^2) + (b^3*n*log(c*(d + e/x^(1/
3))^n)^2)/(4*x^2) - (b^3*n^2*log(c*(d + e/x^(1/3))^n))/(12*x^2) - (a*b^2*n^
2)/(12*x^2) + (b^3*d^6*log(c*(d + e/x^(1/3))^n)^3)/(2*e^6) - (3*a^2*b*log(c
*(d + e/x^(1/3))^n))/(2*x^2) + (a^2*b*n)/(4*x^2) + (a*b^2*n*log(c*(d + e/x^
(1/3))^n))/(2*x^2) + (13489*b^3*d^6*n^3*log(d + e/x^(1/3)))/(1200*e^6) - (2
059*b^3*d^3*n^3)/(3600*e^3*x) + (919*b^3*d^2*n^3)/(4800*e^2*x^(4/3)) + (466
9*b^3*d^4*n^3)/(2400*e^4*x^(2/3)) - (13489*b^3*d^5*n^3)/(1200*e^5*x^(1/3))
 + (3*a*b^2*d^6*log(c*(d + e/x^(1/3))^n)^2)/(2*e^6) - (147*b^3*d^6*n*log(c*(
d + e/x^(1/3))^n)^2)/(40*e^6) - (91*b^3*d*n^3)/(1500*e*x^(5/3)) + (3*a^2*b*
d^6*n*log(d + e/x^(1/3)))/(2*e^6) - (3*b^3*d*n*log(c*(d + e/x^(1/3))^n)^2)/
(10*e*x^(5/3)) + (11*b^3*d*n^2*log(c*(d + e/x^(1/3))^n))/(50*e*x^(5/3)) - (
a^2*b*d^3*n)/(2*e^3*x) + (11*a*b^2*d*n^2)/(50*e*x^(5/3)) + (3*a^2*b*d^2*n)/
(8*e^2*x^(4/3)) + (3*a^2*b*d^4*n)/(4*e^4*x^(2/3)) - (3*a^2*b*d^5*n)/(2*e^5*
x^(1/3)) - (147*a*b^2*d^6*n^2*log(d + e/x^(1/3)))/(20*e^6) - (b^3*d^3*n*log
(c*(d + e/x^(1/3))^n)^2)/(2*e^3*x) + (19*b^3*d^3*n^2*log(c*(d + e/x^(1/3))^
n))/(20*e^3*x) + (3*b^3*d^2*n*log(c*(d + e/x^(1/3))^n)^2)/(8*e^2*x^(4/3)) -
 (37*b^3*d^2*n^2*log(c*(d + e/x^(1/3))^n))/(80*e^2*x^(4/3)) + (3*b^3*d^4*n*
log(c*(d + e/x^(1/3))^n)^2)/(4*e^4*x^(2/3)) - (87*b^3*d^4*n^2*log(c*(d + e/
x^(1/3))^n))/(40*e^4*x^(2/3)) - (3*b^3*d^5*n*log(c*(d + e/x^(1/3))^n)^2)/(2
*e^5*x^(1/3)) + (147*b^3*d^5*n^2*log(c*(d + e/x^(1/3))^n))/(20*e^5*x^(1/3))
 + (19*a*b^2*d^3*n^2)/(20*e^3*x) - (37*a*b^2*d^2*n^2)/(80*e^2*x^(4/3)) - (8
7*a*b^2*d^4*n^2)/(40*e^4*x^(2/3)) + (147*a*b^2*d^5*n^2)/(20*e^5*x^(1/3)) -
(3*a^2*b*d*n)/(10*e*x^(5/3)) - (3*a*b^2*d*n*log(c*(d + e/x^(1/3))^n))/(5*e*
x^(5/3)) - (a*b^2*d^3*n*log(c*(d + e/x^(1/3))^n))/(e^3*x) + (3*a*b^2*d^2*n*
log(c*(d + e/x^(1/3))^n))/(4*e^2*x^(4/3)) + (3*a*b^2*d^4*n*log(c*(d + e/x^(
1/3))^n))/(2*e^4*x^(2/3)) - (3*a*b^2*d^5*n*log(c*(d + e/x^(1/3))^n))/(e^5*x
^(1/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3/x**3,x)
```

```
[Out] Timed out
```

3.508 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx$

Optimal. Leaf size=143

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{be^6n\log\left(d+\frac{e}{x^{2/3}}\right)}{4d^6}-\frac{be^6n\log(x)}{6d^6}+\frac{be^5nx^{2/3}}{4d^5}-\frac{be^4nx^{4/3}}{8d^4}+\frac{be^3nx^2}{12d^3}-\frac{be^2nx^{8/3}}{16d^2}+\frac{benx^{10/3}}{20d}$$

```
[Out] 1/4*b*e^5*n*x^(2/3)/d^5-1/8*b*e^4*n*x^(4/3)/d^4+1/12*b*e^3*n*x^2/d^3-1/16*b
*e^2*n*x^(8/3)/d^2+1/20*b*e*n*x^(10/3)/d-1/4*b*e^6*n*ln(d+e/x^(2/3))/d^6+1/
4*x^4*(a+b*ln(c*(d+e/x^(2/3))^n))-1/6*b*e^6*n*ln(x)/d^6
```

Rubi [A] time = 0.11, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 44}

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{be^5nx^{2/3}}{4d^5}-\frac{be^4nx^{4/3}}{8d^4}+\frac{be^3nx^2}{12d^3}-\frac{be^2nx^{8/3}}{16d^2}-\frac{be^6n\log\left(d+\frac{e}{x^{2/3}}\right)}{4d^6}-\frac{be^6n\log(x)}{6d^6}+\frac{benx^{10/3}}{20d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] (b*e^5*n*x^(2/3))/(4*d^5) - (b*e^4*n*x^(4/3))/(8*d^4) + (b*e^3*n*x^2)/(12*d
^3) - (b*e^2*n*x^(8/3))/(16*d^2) + (b*e*n*x^(10/3))/(20*d) - (b*e^6*n*Log[d
 + e/x^(2/3)])/(4*d^6) + (x^4*(a + b*Log[c*(d + e/x^(2/3))^n]))/4 - (b*e^6*
n*Log[x])/(6*d^6)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx=-\left(\frac{3}{2}\operatorname{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^7}dx,x,\frac{1}{x^{2/3}}\right)\right)$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{1}{4}(ben)\operatorname{Subst}\left(\int\frac{1}{x^6(d+ex)}dx,x,\frac{1}{x^{2/}}\right.$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{1}{4}(ben)\operatorname{Subst}\left(\int\left(\frac{1}{dx^6}-\frac{e}{d^2x^5}+\frac{e^2}{d^3x^4}\right.\right.$$
$$=\frac{be^5nx^{2/3}}{4d^5}-\frac{be^4nx^{4/3}}{8d^4}+\frac{be^3nx^2}{12d^3}-\frac{be^2nx^{8/3}}{16d^2}+\frac{benx^{10/3}}{20d}-\frac{be^6n\log\left(d+\frac{e}{x^2}\right.}{4d^6}$$

Mathematica [A] time = 0.12, size = 134, normalized size = 0.94

$$\frac{ax^4}{4}+\frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)-\frac{1}{4}ben\left(\frac{e^5\log\left(d+\frac{e}{x^{2/3}}\right)}{d^6}+\frac{2e^5\log(x)}{3d^6}-\frac{e^4x^{2/3}}{d^5}+\frac{e^3x^{4/3}}{2d^4}-\frac{e^2x^2}{3d^3}+\frac{ex^{8/3}}{4d^2}-\frac{x^{10/3}}{5d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] (a*x^4)/4 + (b*x^4*Log[c*(d + e/x^(2/3))^n])/4 - (b*e*n*(-((e^4*x^(2/3))/d^
5) + (e^3*x^(4/3))/(2*d^4) - (e^2*x^2)/(3*d^3) + (e*x^(8/3))/(4*d^2) - x^(1
0/3)/(5*d) + (e^5*Log[d + e/x^(2/3)])/d^6 + (2*e^5*Log[x])/(3*d^6)))/4
```

fricas [A] time = 0.50, size = 160, normalized size = 1.12

$$\frac{60\,bd^6x^4\log(c)+60\,ad^6x^4+20\,bd^3e^3nx^2-120\,bd^6n\log\left(x^{\frac{1}{3}}\right)+60\left(bd^6-be^6\right)n\log\left(dx^{\frac{2}{3}}+e\right)+60\left(bd^6nx^4-\right.}{240\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/240*(60*b*d^6*x^4*log(c) + 60*a*d^6*x^4 + 20*b*d^3*e^3*n*x^2 - 120*b*d^6*
n*log(x^(1/3)) + 60*(b*d^6 - b*e^6)*n*log(d*x^(2/3) + e) + 60*(b*d^6*n*x^4
- b*d^6*n)*log((d*x + e*x^(1/3))/x) - 15*(b*d^4*e^2*n*x^2 - 4*b*d*e^5*n)*x^
(2/3) + 6*(2*b*d^5*e*n*x^3 - 5*b*d^2*e^4*n*x)*x^(1/3))/d^6
```

giac [A] time = 0.50, size = 103, normalized size = 0.72

$$\frac{1}{4}bx^4\log(c)+\frac{1}{4}ax^4+\frac{1}{240}\left(60\,x^4\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)+\left(\frac{12\,d^4x^{\frac{10}{3}}-15\,d^3x^{\frac{8}{3}}e+20\,d^2x^2e^2-30\,dx^{\frac{4}{3}}e^3+60\,x^{\frac{2}{3}}e^4}{d^5}-\frac{60\,e}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*log(c) + 1/4*a*x^4 + 1/240*(60*x^4*log(d + e/x^(2/3)) + ((12*d^4*
x^(10/3) - 15*d^3*x^(8/3)*e + 20*d^2*x^2*e^2 - 30*d*x^(4/3)*e^3 + 60*x^(2/3
)*e^4)/d^5 - 60*e^5*log(abs(d*x^(2/3) + e))/d^6)*e)*b*n
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

maxima [A] time = 0.47, size = 98, normalized size = 0.69

$$\frac{1}{4}\,bx^4\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+\frac{1}{4}\,ax^4-\frac{1}{240}\,ben\left(\frac{60\,e^5\log\left(dx^{\frac{2}{3}}+e\right)}{d^6}-\frac{12\,d^4x^{\frac{10}{3}}-15\,d^3ex^{\frac{8}{3}}+20\,d^2e^2x^2-30\,de^3x^{\frac{4}{3}}+60\,e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log(c*(d + e/x^(2/3))^n) + 1/4*a*x^4 - 1/240*b*e*n*(60*e^5*log(d*
x^(2/3) + e)/d^6 - (12*d^4*x^(10/3) - 15*d^3*e*x^(8/3) + 20*d^2*e^2*x^2 - 3
0*d*e^3*x^(4/3) + 60*e^4*x^(2/3))/d^5)
```

mupad [B] time = 0.70, size = 112, normalized size = 0.78

$$\frac{x^{10/3}\left(\frac{b\,e\,n}{5\,d}-\frac{b\,e^2\,n}{4\,d^2\,x^{2/3}}-\frac{b\,e^4\,n}{2\,d^4\,x^2}+\frac{b\,e^3\,n}{3\,d^3\,x^{4/3}}+\frac{b\,e^5\,n}{d^5\,x^{8/3}}\right)}{4}+\frac{a\,x^4}{4}+\frac{b\,x^4\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)}{4}-\frac{b\,e^6\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{2/3}}+1\right)}{2\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e/x^(2/3))^n)),x)
```

```
[Out] (x^(10/3)*((b*e*n)/(5*d) - (b*e^2*n)/(4*d^2*x^(2/3)) - (b*e^4*n)/(2*d^4*x^2
) + (b*e^3*n)/(3*d^3*x^(4/3)) + (b*e^5*n)/(d^5*x^(8/3))))/4 + (a*x^4)/4 + (
b*x^4*log(c*(d + e/x^(2/3))^n))/4 - (b*e^6*n*atanh((2*e)/(d*x^(2/3)) + 1))/
(2*d^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**n)),x)
```

```
[Out] Timed out
```

3.509 $\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx$

Optimal. Leaf size=121

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{2be^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3d^{9/2}}-\frac{2be^4n\sqrt[3]{x}}{3d^4}+\frac{2be^3nx}{9d^3}-\frac{2be^2nx^{5/3}}{15d^2}+\frac{2benx^{7/3}}{21d}$$

```
[Out] -2/3*b*e^4*n*x^(1/3)/d^4+2/9*b*e^3*n*x/d^3-2/15*b*e^2*n*x^(5/3)/d^2+2/21*b*
e*n*x^(7/3)/d+2/3*b*e^(9/2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2))/d^(9/2)+1/3*x
^3*(a+b*ln(c*(d+e/x^(2/3))^n))
```

Rubi [A] time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2455, 263, 341, 302, 205}

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{2be^2nx^{5/3}}{15d^2}-\frac{2be^4n\sqrt[3]{x}}{3d^4}+\frac{2be^3nx}{9d^3}+\frac{2be^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3d^{9/2}}+\frac{2benx^{7/3}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] (-2*b*e^4*n*x^(1/3))/(3*d^4) + (2*b*e^3*n*x)/(9*d^3) - (2*b*e^2*n*x^(5/3))/
(15*d^2) + (2*b*e*n*x^(7/3))/(21*d) + (2*b*e^(9/2)*n*ArcTan[(Sqrt[d]*x^(1/3
))/Sqrt[e]])/(3*d^(9/2)) + (x^3*(a + b*Log[c*(d + e/x^(2/3))^n]))/3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{1}{9}(2ben)\int\frac{x^{4/3}}{d+\frac{e}{x^{2/3}}}dx$$
$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{1}{9}(2ben)\int\frac{x^2}{e+dx^{2/3}}dx$$
$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{1}{3}(2ben)\,\text{Subst}\left(\int\frac{x^8}{e+dx^2}dx,x,\sqrt[3]{x}\right)$$
$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{1}{3}(2ben)\,\text{Subst}\left(\int\left(-\frac{e^3}{d^4}+\frac{e^2x^2}{d^3}-\frac{ex^4}{d^2}+\right.\right.$$
$$=-\frac{2be^4n\sqrt[3]{x}}{3d^4}+\frac{2be^3nx}{9d^3}-\frac{2be^2nx^{5/3}}{15d^2}+\frac{2benx^{7/3}}{21d}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/}}\right.\right.\right.$$
$$=-\frac{2be^4n\sqrt[3]{x}}{3d^4}+\frac{2be^3nx}{9d^3}-\frac{2be^2nx^{5/3}}{15d^2}+\frac{2benx^{7/3}}{21d}+\frac{2be^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3d^{9/2}}+$$

Mathematica [C] time = 0.02, size = 65, normalized size = 0.54

$$\frac{ax^3}{3}+\frac{1}{3}bx^3\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{2benx^{7/3}\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};-\frac{e}{dx^{2/3}}\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] (a*x^3)/3 + (2*b*e*n*x^(7/3)*Hypergeometric2F1[-7/2, 1, -5/2, -(e/(d*x^(2/3
)))]])/(21*d) + (b*x^3*Log[c*(d + e/x^(2/3))^n])/3
```

fricas [A] time = 0.50, size = 399, normalized size = 3.30

$$\left[\frac{105\,bd^4x^3\log(c)+105\,ad^4x^3-42\,bd^2e^2nx^{\frac{5}{3}}+105\,be^4n\sqrt{-\frac{e}{d}}\,\log\left(\frac{d^3x^2-2\,d^2ex\sqrt{-\frac{e}{d}}-e^3+2\left(d^3x\sqrt{-\frac{e}{d}}+de^2\right)x^{\frac{2}{3}}-2\left(d^2ex-de^2\sqrt{-\right.\right.}{d^3x^2+e^3}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] [1/315*(105*b*d^4*x^3*log(c) + 105*a*d^4*x^3 - 42*b*d^2*e^2*n*x^(5/3) + 105
*b*e^4*n*sqrt(-e/d)*log((d^3*x^2 - 2*d^2*e*x*sqrt(-e/d) - e^3 + 2*(d^3*x*sq
rt(-e/d) + d*e^2)*x^(2/3) - 2*(d^2*e*x - d*e^2*sqrt(-e/d))*x^(1/3))/(d^3*x^
2 + e^3)) + 70*b*d*e^3*n*x + 105*b*d^4*n*log(d*x^(2/3) + e) - 210*b*d^4*n*l
og(x^(1/3)) + 105*(b*d^4*n*x^3 - b*d^4*n)*log((d*x + e*x^(1/3))/x) + 30*(b*
d^3*e*n*x^2 - 7*b*e^4*n)*x^(1/3))/d^4, 1/315*(105*b*d^4*x^3*log(c) + 105*a*
d^4*x^3 - 42*b*d^2*e^2*n*x^(5/3) + 210*b*e^4*n*sqrt(e/d)*arctan(d*x^(1/3)*s
qrt(e/d)/e) + 70*b*d*e^3*n*x + 105*b*d^4*n*log(d*x^(2/3) + e) - 210*b*d^4*n
*log(x^(1/3)) + 105*(b*d^4*n*x^3 - b*d^4*n)*log((d*x + e*x^(1/3))/x) + 30*(
b*d^3*e*n*x^2 - 7*b*e^4*n)*x^(1/3))/d^4]
```

giac [A] time = 0.52, size = 97, normalized size = 0.80

$$\frac{1}{3}\,bx^3\log(c)+\frac{1}{3}\,ax^3+\frac{1}{315}\left(105\,x^3\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)+2\left(\frac{105\,\arctan\left(\sqrt{d}\,x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\frac{7}{2}}}{d^{\frac{9}{2}}}+\frac{15\,d^6x^{\frac{7}{3}}-21\,d^5x^{\frac{5}{3}}e+35\,d^4xe^2}{d^7}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*log(c) + 1/3*a*x^3 + 1/315*(105*x^3*log(d + e/x^(2/3)) + 2*(105*a
rctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(7/2)/d^(9/2) + (15*d^6*x^(7/3) - 21*d^5*
x^(5/3)*e + 35*d^4*x*e^2 - 105*d^3*x^(1/3)*e^3)/d^7)*e)*b*n
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(b \ln \left(c \left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right) x^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*(d+e/x^(2/3))^n)+a),x)
```

```
[Out] int(x^2*(b*ln(c*(d+e/x^(2/3))^n)+a),x)
```

maxima [A] time = 1.00, size = 92, normalized size = 0.76

$$\frac{1}{3}\, b x^{3} \log \left(c \left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+\frac{1}{3}\, a x^{3}+\frac{2}{315}\, b e n \left(\frac{105\, e^{4} \arctan \left(\frac{d x^{\frac{1}{3}}}{\sqrt{d e}}\right)}{\sqrt{d e}\, d^{4}}+\frac{15\, d^{3} x^{\frac{7}{3}}-21\, d^{2} e x^{\frac{5}{3}}+35\, d e^{2} x-105\, e^{3} x^{\frac{1}{3}}}{d^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3*log(c*(d + e/x^(2/3))^n) + 1/3*a*x^3 + 2/315*b*e*n*(105*e^4*arcta
n(d*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*d^4) + (15*d^3*x^(7/3) - 21*d^2*e*x^(5/3)
 + 35*d*e^2*x - 105*e^3*x^(1/3))/d^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\, \left(a+b\, \ln\left(c\, {\left(d+\frac{e}{x^{2/3}}\right)}^n\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e/x^(2/3))^n)),x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e/x^(2/3))^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**n)),x)
```

```
[Out] Timed out
```

3.510 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx$

Optimal. Leaf size=94

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{be^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}+\frac{be^3n\log(x)}{3d^3}-\frac{be^2nx^{2/3}}{2d^2}+\frac{benx^{4/3}}{4d}$$

```
[Out] -1/2*b*e^2*n*x^(2/3)/d^2+1/4*b*e*n*x^(4/3)/d+1/2*b*e^3*n*ln(d+e/x^(2/3))/d^
3+1/2*x^2*(a+b*ln(c*(d+e/x^(2/3))^n))+1/3*b*e^3*n*ln(x)/d^3
```

Rubi [A] time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2454, 2395, 44}

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{be^2nx^{2/3}}{2d^2}+\frac{be^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}+\frac{be^3n\log(x)}{3d^3}+\frac{benx^{4/3}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] -(b*e^2*n*x^(2/3))/(2*d^2) + (b*e*n*x^(4/3))/(4*d) + (b*e^3*n*Log[d + e/x^(
2/3)])/(2*d^3) + (x^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/2 + (b*e^3*n*Log[x]
)/(3*d^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx=-\left(\frac{3}{2}\text{Subst}\left(\int\frac{a+b\log(c(d+ex)^n)}{x^4}dx,x,\frac{1}{x^{2/3}}\right)\right)$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int\frac{1}{x^3(d+ex)}dx,x,\frac{1}{x^{2/3}}\right.$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int\left(\frac{1}{dx^3}-\frac{e}{d^2x^2}+\frac{e^2}{d^3x}\right.\right.$$
$$=-\frac{be^2nx^{2/3}}{2d^2}+\frac{benx^{4/3}}{4d}+\frac{be^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.$$

Mathematica [A] time = 0.02, size = 91, normalized size = 0.97

$$\frac{ax^2}{2}+\frac{1}{2}bx^2\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)-\frac{1}{2}ben\left(-\frac{e^2\log\left(d+\frac{e}{x^{2/3}}\right)}{d^3}-\frac{2e^2\log(x)}{3d^3}+\frac{ex^{2/3}}{d^2}-\frac{x^{4/3}}{2d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*Log[c*(d + e/x^(2/3))^n])/2 - (b*e*n*((e*x^(2/3))/d^2 -
x^(4/3)/(2*d) - (e^2*Log[d + e/x^(2/3)])/d^3 - (2*e^2*Log[x])/(3*d^3)))/2
```

fricas [A] time = 0.48, size = 113, normalized size = 1.20

$$\frac{2\,bd^3x^2\log(c)+bd^2enx^{\frac{4}{3}}+2\,ad^3x^2-4\,bd^3n\log\left(x^{\frac{1}{3}}\right)-2\,bde^2nx^{\frac{2}{3}}+2\left(bd^3+be^3\right)n\log\left(dx^{\frac{2}{3}}+e\right)+2\left(bd^3nx^2\right.}{4\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*d^3*x^2*log(c) + b*d^2*e*n*x^(4/3) + 2*a*d^3*x^2 - 4*b*d^3*n*log(x
^(1/3)) - 2*b*d*e^2*n*x^(2/3) + 2*(b*d^3 + b*e^3)*n*log(d*x^(2/3) + e) + 2*
(b*d^3*n*x^2 - b*d^3*n)*log((d*x + e*x^(1/3))/x))/d^3
```

giac [A] time = 0.44, size = 72, normalized size = 0.77

$$\frac{1}{2}\,bx^2\log(c)+\frac{1}{4}\left(2\,x^2\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)+\left(\frac{dx^{\frac{4}{3}}-2\,x^{\frac{2}{3}}e}{d^2}+\frac{2\,e^2\log\left(\left|dx^{\frac{2}{3}}+e\right|\right)}{d^3}\right)e\right)bn+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*log(c) + 1/4*(2*x^2*log(d + e/x^(2/3)) + ((d*x^(4/3) - 2*x^(2/3)*
e)/d^2 + 2*e^2*log(abs(d*x^(2/3) + e))/d^3)*e)*b*n + 1/2*a*x^2
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(d+e/x^(2/3))^n)+a),x)
[Out] int(x*(b*ln(c*(d+e/x^(2/3))^n)+a),x)
```

maxima [A] time = 0.47, size = 63, normalized size = 0.67

$$\frac{1}{4}\,ben\left(\frac{2\,e^2\log\left(dx^{\frac{2}{3}}+e\right)}{d^3}+\frac{dx^{\frac{4}{3}}-2\,ex^{\frac{2}{3}}}{d^2}\right)+\frac{1}{2}\,bx^2\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="maxima")
[Out] 1/4*b*e*n*(2*e^2*log(d*x^(2/3) + e)/d^3 + (d*x^(4/3) - 2*e*x^(2/3))/d^2) +
1/2*b*x^2*log(c*(d + e/x^(2/3))^n) + 1/2*a*x^2
```

mupad [B] time = 0.59, size = 73, normalized size = 0.78

$$\frac{x^{4/3}\left(\frac{b\,e\,n}{2\,d}-\frac{b\,e^2\,n}{d^2\,x^{2/3}}\right)}{2}+\frac{a\,x^2}{2}+\frac{b\,x^2\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2}+\frac{b\,e^3\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{2/3}}+1\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(2/3))^n)),x)
[Out] (x^(4/3)*((b*e*n)/(2*d) - (b*e^2*n)/(d^2*x^(2/3))))/2 + (a*x^2)/2 + (b*x^2*
log(c*(d + e/x^(2/3))^n))/2 + (b*e^3*n*atanh((2*e)/(d*x^(2/3)) + 1))/d^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))**n)),x)
[Out] Timed out
```

3.511 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx$

Optimal. Leaf size=65

$$ax+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)-\frac{2be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}+\frac{2ben\sqrt[3]{x}}{d}$$

```
[Out] 2*b*e*n*x^(1/3)/d+a*x-2*b*e^(3/2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2))/d^(3/2)
+b*x*ln(c*(d+e/x^(2/3))^n)
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2448, 263, 243, 321, 205}

$$ax+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)-\frac{2be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}+\frac{2ben\sqrt[3]{x}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e/x^(2/3))^n],x]
```

```
[Out] (2*b*e*n*x^(1/3))/d + a*x - (2*b*e^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]
])/d^(3/2) + b*x*Log[c*(d + e/x^(2/3))^n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Free
Q[{a, b, p}, x] && FractionQ[n]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx = ax+b\int \log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)dx$$
$$= ax+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{1}{3}(2ben)\int \frac{1}{\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}}dx$$
$$= ax+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{1}{3}(2ben)\int \frac{1}{e+dx^{2/3}}dx$$
$$= ax+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+(2ben)\,\text{Subst}\left(\int \frac{x^2}{e+dx^2}dx,x,\sqrt[3]{x}\right)$$
$$= \frac{2ben\sqrt[3]{x}}{d}+ax+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)-\frac{\left(2be^2n\right)\text{Subst}\left(\int \frac{1}{e+dx^2}dx,x,\sqrt[3]{x}\right)}{d}$$
$$= \frac{2ben\sqrt[3]{x}}{d}+ax-\frac{2be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)$$

Mathematica [C] time = 0.01, size = 53, normalized size = 0.82

$$ax+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{2ben\sqrt[3]{x}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{e}{dx^{2/3}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e/x^(2/3))^n],x]
```

```
[Out] a*x + (2*b*e*n*x^(1/3)*Hypergeometric2F1[-1/2, 1, 1/2, -(e/(d*x^(2/3)))])/d
 + b*x*Log[c*(d + e/x^(2/3))^n]
```

fricas [B] time = 0.51, size = 279, normalized size = 4.29

$$\left[\frac{ben\sqrt{-\frac{e}{d}}\log\left(\frac{d^3x^2+2d^2ex\sqrt{-\frac{e}{d}}-e^3-2\left(d^3x\sqrt{-\frac{e}{d}}-de^2\right)x^{\frac{2}{3}}-2\left(d^2ex+de^2\sqrt{-\frac{e}{d}}\right)x^{\frac{1}{3}}}{d^3x^2+e^3}\right)+bdn\log\left(dx^{\frac{2}{3}}+e\right)+bdx\log(c)-2\,bdn\log}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(2/3))^n),x, algorithm="fricas")
```

```
[Out] [(b*e*n*sqrt(-e/d)*log((d^3*x^2 + 2*d^2*e*x*sqrt(-e/d) - e^3 - 2*(d^3*x*sqr
t(-e/d) - d*e^2)*x^(2/3) - 2*(d^2*e*x + d*e^2*sqrt(-e/d))*x^(1/3))/(d^3*x^2
 + e^3)) + b*d*n*log(d*x^(2/3) + e) + b*d*x*log(c) - 2*b*d*n*log(x^(1/3)) +
 2*b*e*n*x^(1/3) + a*d*x + (b*d*n*x - b*d*n)*log((d*x + e*x^(1/3))/x))/d, -
(2*b*e*n*sqrt(e/d)*arctan(d*x^(1/3)*sqrt(e/d)/e) - b*d*n*log(d*x^(2/3) + e)
 - b*d*x*log(c) + 2*b*d*n*log(x^(1/3)) - 2*b*e*n*x^(1/3) - a*d*x - (b*d*n*x
 - b*d*n)*log((d*x + e*x^(1/3))/x))/d]
```

giac [A] time = 0.33, size = 57, normalized size = 0.88

$$-\left(\left(2\left(\frac{\arctan\left(\sqrt{d}\,x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\frac{1}{2}}}{d^{\frac{3}{2}}}-\frac{x^{\frac{1}{3}}}{d}\right)e-x\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)n-x\log(c)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(2/3))^n),x, algorithm="giac")
```

```
[Out] -((2*(arctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(1/2)/d^(3/2) - x^(1/3)/d)*e - x*l
og(d + e/x^(2/3)))*n - x*log(c))*b + a*x
```

maple [B] time = 0.28, size = 168, normalized size = 2.58

$$-\frac{2b e^{2} n \arctan\left(\frac{2d x^{\frac{1}{3}}+\sqrt{3}\,\sqrt{d}\,\sqrt{e}}{\sqrt{de}}\right)}{3\sqrt{de}\,d}+\frac{2b e^{2} n \arctan\left(\frac{-2d x^{\frac{1}{3}}+\sqrt{3}\,\sqrt{d}\,\sqrt{e}}{\sqrt{de}}\right)}{3\sqrt{de}\,d}-\frac{4b e^{2} n \arctan\left(\frac{d x^{\frac{1}{3}}}{\sqrt{de}}\right)}{3\sqrt{de}\,d}+\frac{2b e^{2} n \arctan\left(\frac{d^{2} x}{\sqrt{de}\,e}\right.}{3\sqrt{de}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*ln(c*(d+e/x^(2/3))^n)+a,x)
```

```
[Out] a*x+x*b*ln(c*((e+d*x^(2/3))/x^(2/3))^n)+2/3*b*e^2*n/d/(d*e)^(1/2)*arctan(x*
d^2/e/(d*e)^(1/2))+2*b*e*n*x^(1/3)/d-4/3*b*e^2*n/d/(d*e)^(1/2)*arctan(x^(1/
3)*d/(d*e)^(1/2))+2/3*b*e^2*n/d/(d*e)^(1/2)*arctan((3^(1/2)*d^(1/2)*e^(1/2)
-2*d*x^(1/3))/(d*e)^(1/2))-2/3*b*e^2*n/d/(d*e)^(1/2)*arctan((2*d*x^(1/3)+3^
(1/2)*d^(1/2)*e^(1/2))/(d*e)^(1/2))
```

maxima [A] time = 0.99, size = 57, normalized size = 0.88

$$-\left(2\,en\left(\frac{e\arctan\left(\frac{dx^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}\,d}-\frac{x^{\frac{1}{3}}}{d}\right)-x\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(2/3))^n),x, algorithm="maxima")
```

```
[Out] -(2*e*n*(e*arctan(d*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*d) - x^(1/3)/d) - x*log(c
*(d + e/x^(2/3))^n))*b + a*x
```

mupad [B] time = 0.43, size = 51, normalized size = 0.78

$$a\,x+b\,x\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^{n}\right)+\frac{2\,b\,e\,n\,x^{1/3}}{d}-\frac{2\,b\,e^{3/2}\,n\,\mathrm{atan}\left(\frac{\sqrt{d}\,x^{1/3}}{\sqrt{e}}\right)}{d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*log(c*(d + e/x^(2/3))^n),x)
```

```
[Out] a*x + b*x*log(c*(d + e/x^(2/3))^n) + (2*b*e*n*x^(1/3))/d - (2*b*e^(3/2)*n*a
tan((d^(1/2)*x^(1/3))/e^(1/2)))/d^(3/2)
```

sympy [A] time = 52.16, size = 61, normalized size = 0.94

$$ax+b\left(\frac{2en\left(\frac{3\sqrt[3]{x}}{d}-\frac{3e\,\mathrm{atan}\left(\frac{\sqrt[3]{x}}{\sqrt{\frac{e}{d}}}\right)}{d^{2}\sqrt{\frac{e}{d}}}\right)}{3}+x\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e/x**(2/3))**n),x)
```

```
[Out] a*x + b*(2*e*n*(3*x**(1/3)/d - 3*e*atan(x**(1/3)/sqrt(e/d))/(d**2*sqrt(e/d)
))/3 + x*log(c*(d + e/x**(2/3))**n))
```

3.512 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=55

$$-\frac{3}{2}\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{3}{2}bn\text{Li}_2\left(\frac{e}{dx^{2/3}}+1\right)$$

```
[Out] -3/2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(-e/d/x^(2/3))-3/2*b*n*polylog(2,1+e/d/x
^(2/3))
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2394, 2315}

$$-\frac{3}{2}bn\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)-\frac{3}{2}\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x,x]
```

```
[Out] (-3*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))])/2 - (3*b*n*Poly
Log[2, 1 + e/(d*x^(2/3))])/2
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx &= -\left(\frac{3}{2}\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx,x,\frac{1}{x^{2/3}}\right)\right)\\
&= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(-\frac{e}{dx^{2/3}}\right)+\frac{1}{2}(3ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,d\right.\\
&= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{3}{2}bn\text{Li}_2\left(1+\frac{e}{dx^{2/3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.00

$$a\log(x)-\frac{3}{2}b\left(\log\left(-\frac{e}{dx^{2/3}}\right)\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+n\text{Li}_2\left(\frac{d+\frac{e}{x^{2/3}}}{d}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x,x]
```

```
[Out] a*Log[x] - (3*b*(Log[c*(d + e/x^(2/3))^n]*Log[-(e/(d*x^(2/3)))] + n*PolyLog
[2, (d + e/x^(2/3))/d]))/2
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*((d*x + e*x^(1/3))/x)^n) + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)/x, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int\frac{b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(2/3))^n)+a)/x,x)
```

```
[Out] int((b*ln(c*(d+e/x^(2/3))^n)+a)/x,x)
```

maxima [B] time = 1.74, size = 126, normalized size = 2.29

$$-\frac{3}{2}\left(2\log\left(\frac{dx^{\frac{2}{3}}}{e}+1\right)\log\left(x^{\frac{1}{3}}\right)+\text{Li}_2\left(-\frac{dx^{\frac{2}{3}}}{e}\right)\right)bn+\frac{6\,ben\log\left(dx^{\frac{2}{3}}+e\right)\log(x)+2\,ben\log(x)^2+6\,bdnx^{\frac{2}{3}}\log(x)-1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x,x, algorithm="maxima")
```

```
[Out] -3/2*(2*log(d*x^(2/3)/e + 1)*log(x^(1/3)) + dilog(-d*x^(2/3)/e))*b*n + 1/6*
(6*b*e*n*log(d*x^(2/3) + e)*log(x) + 2*b*e*n*log(x)^2 + 6*b*d*n*x^(2/3)*log
```

```
(x) - 12*b*e*log(x)*log(x^(1/3*n)) - 9*b*d*n*x^(2/3) + 6*(b*e*log(c) + a*e)
*log(x) - 3*(2*b*d*n*x*log(x) - 3*b*d*n*x)/x^(1/3))/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))/x,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x,x)
```

```
[Out] Timed out
```

3.513 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=77

$$-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}}-\frac{2bdn}{e\sqrt[3]{x}}+\frac{2bn}{3x}$$

```
[Out] 2/3*b*n/x-2*b*d*n/e/x^(1/3)-2*b*d^(3/2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2))/e
^(3/2)+(-a-b*ln(c*(d+e/x^(2/3))^n))/x
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2455, 263, 341, 325, 205}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}}-\frac{2bdn}{e\sqrt[3]{x}}+\frac{2bn}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x^2,x]
```

```
[Out] (2*b*n)/(3*x) - (2*b*d*n)/(e*x^(1/3)) - (2*b*d^(3/2)*n*ArcTan[(Sqrt[d]*x^(1
/3))/Sqrt[e]])/e^(3/2) - (a + b*Log[c*(d + e/x^(2/3))^n])/x
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx = -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x} - \frac{1}{3}(2ben)\int \frac{1}{\left(d+\frac{e}{x^{2/3}}\right)x^{8/3}}\,dx$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x} - \frac{1}{3}(2ben)\int \frac{1}{\left(e+dx^{2/3}\right)x^2}\,dx$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x} - (2ben)\,\text{Subst}\left(\int \frac{1}{x^4\left(e+dx^2\right)}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{2bn}{3x} - \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x} + (2bdn)\,\text{Subst}\left(\int \frac{1}{x^2\left(e+dx^2\right)}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{2bn}{3x} - \frac{2bdn}{e\sqrt[3]{x}} - \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x} - \frac{\left(2bd^2n\right)\text{Subst}\left(\int \frac{1}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{e}$$
$$= \frac{2bn}{3x} - \frac{2bdn}{e\sqrt[3]{x}} - \frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}} - \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}$$

Mathematica [A] time = 0.05, size = 80, normalized size = 1.04

$$-\frac{a}{x} - \frac{b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x} + \frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}} - \frac{2bdn}{e\sqrt[3]{x}} + \frac{2bn}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x^2,x]
```

```
[Out] -(a/x) + (2*b*n)/(3*x) - (2*b*d*n)/(e*x^(1/3)) + (2*b*d^(3/2)*n*ArcTan[Sqrt
[e]/(Sqrt[d]*x^(1/3))])/e^(3/2) - (b*Log[c*(d + e/x^(2/3))^n])/x
```

fricas [A] time = 0.45, size = 235, normalized size = 3.05

$$\left[\frac{3\,bdnx\sqrt{-\frac{d}{e}}\,\log\left(\frac{d^3x^2+2\,de^2x\sqrt{-\frac{d}{e}}-e^3-2\left(d^2ex\sqrt{-\frac{d}{e}}-de^2\right)x^{\frac{2}{3}}-2\left(d^2ex+e^3\sqrt{-\frac{d}{e}}\right)x^{\frac{1}{3}}}{d^3x^2+e^3}\right)-3\,ben\log\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)-6\,bdnx^{\frac{2}{3}}+2\,ben}{3\,ex}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^2,x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*d*n*x*sqrt(-d/e)*log((d^3*x^2 + 2*d*e^2*x*sqrt(-d/e) - e^3 - 2*(d
^2*e*x*sqrt(-d/e) - d*e^2)*x^(2/3) - 2*(d^2*e*x + e^3*sqrt(-d/e))*x^(1/3))/
(d^3*x^2 + e^3)) - 3*b*e*n*log((d*x + e*x^(1/3))/x) - 6*b*d*n*x^(2/3) + 2*b
*e*n - 3*b*e*log(c) - 3*a*e)/(e*x), -1/3*(6*b*d*n*x*sqrt(d/e)*arctan(x^(1/3
)*sqrt(d/e)) + 3*b*e*n*log((d*x + e*x^(1/3))/x) + 6*b*d*n*x^(2/3) - 2*b*e*n
 + 3*b*e*log(c) + 3*a*e)/(e*x)]
```

giac [A] time = 0.46, size = 73, normalized size = 0.95

$$-\frac{1}{3}\left(2\left(3\,d^{\frac{3}{2}}\arctan\left(\sqrt{d}\,x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{5}{2}\right)}+\frac{\left(3\,dx^{\frac{2}{3}}-e\right)e^{(-2)}}{x}\right)e+\frac{3\,\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)}{x}\right)bn-\frac{b\log(c)}{x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^2,x, algorithm="giac")
```

```
[Out] -1/3*(2*(3*d^(3/2)*arctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(-5/2) + (3*d*x^(2/3)
 - e)*e^(-2)/x)*e + 3*log(d + e/x^(2/3))/x)*b*n - b*log(c)/x - a/x
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{b \ln \left(c \left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(2/3))^n)+a)/x^2,x)
```

```
[Out] int((b*ln(c*(d+e/x^(2/3))^n)+a)/x^2,x)
```

maxima [A] time = 0.99, size = 72, normalized size = 0.94

$$-\frac{2}{3}\, ben\left(\frac{3\, d^{2} \arctan\left(\frac{d x^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}\, e^{2}}+\frac{3\, d x^{\frac{2}{3}}-e}{e^{2} x}\right)-\frac{b \log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)}{x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^2,x, algorithm="maxima")
```

```
[Out] -2/3*b*e*n*(3*d^2*arctan(d*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*e^2) + (3*d*x^(2/3
) - e)/(e^2*x)) - b*log(c*(d + e/x^(2/3))^n)/x - a/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x**2,x)
```

```
[Out] Timed out
```

3.514 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=89

$$-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}-\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}+\frac{bd^2n}{2e^2x^{2/3}}-\frac{bdn}{4ex^{4/3}}+\frac{bn}{6x^2}$$

```
[Out] 1/6*b*n/x^2-1/4*b*d*n/e/x^(4/3)+1/2*b*d^2*n/e^2/x^(2/3)-1/2*b*d^3*n*ln(d+e/
x^(2/3))/e^3+1/2*(-a-b*ln(c*(d+e/x^(2/3))^n))/x^2
```

Rubi [A] time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2454, 2395, 43}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}+\frac{bd^2n}{2e^2x^{2/3}}-\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}-\frac{bdn}{4ex^{4/3}}+\frac{bn}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x^3,x]
```

```
[Out] (b*n)/(6*x^2) - (b*d*n)/(4*e*x^(4/3)) + (b*d^2*n)/(2*e^2*x^(2/3)) - (b*d^3*
n*Log[d + e/x^(2/3)])/(2*e^3) - (a + b*Log[c*(d + e/x^(2/3))^n])/(2*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx = -\left(\frac{3}{2}\operatorname{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)dx, x, \frac{1}{x^{2/3}}\right)\right)$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\operatorname{Subst}\left(\int \frac{x^3}{d+ex}dx, x, \frac{1}{x^{2/3}}\right)$$
$$= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\operatorname{Subst}\left(\int \left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right)dx, x\right.$$
$$= \frac{bn}{6x^2} - \frac{bdn}{4ex^{4/3}} + \frac{bd^2n}{2e^2x^{2/3}} - \frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2e^3} - \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}$$

Mathematica [A] time = 0.04, size = 94, normalized size = 1.06

$$-\frac{a}{2x^2} - \frac{b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2} - \frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2e^3} + \frac{bd^2n}{2e^2x^{2/3}} - \frac{bdn}{4ex^{4/3}} + \frac{bn}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x^3,x]
```

```
[Out] -1/2*a/x^2 + (b*n)/(6*x^2) - (b*d*n)/(4*e*x^(4/3)) + (b*d^2*n)/(2*e^2*x^(2/
3)) - (b*d^3*n*Log[d + e/x^(2/3)])/(2*e^3) - (b*Log[c*(d + e/x^(2/3))^n])/(
2*x^2)
```

fricas [A] time = 0.45, size = 84, normalized size = 0.94

$$\frac{6\,bd^2enx^{\frac{4}{3}} - 3\,bde^2nx^{\frac{2}{3}} + 2\,be^3n - 6\,be^3\log(c) - 6\,ae^3 - 6\left(bd^3nx^2 + be^3n\right)\log\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)}{12\,e^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(6*b*d^2*e*n*x^(4/3) - 3*b*d*e^2*n*x^(2/3) + 2*b*e^3*n - 6*b*e^3*log(c
) - 6*a*e^3 - 6*(b*d^3*n*x^2 + b*e^3*n)*log((d*x + e*x^(1/3))/x))/(e^3*x^2)
```

giac [A] time = 0.52, size = 104, normalized size = 1.17

$$\frac{1}{12}\left(\left(12\,d^3e^{(-4)}\log\left(x^{\frac{1}{3}}\right) - 6\,d^3e^{(-4)}\log\left(\left|dx^{\frac{2}{3}}+e\right|\right) - \frac{\left(11\,d^3x^2 - 6\,d^2x^{\frac{4}{3}}e + 3\,dx^{\frac{2}{3}}e^2 - 2\,e^3\right)e^{(-4)}}{x^2}\right)e - \frac{6\,\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/12*((12*d^3*e^(-4)*log(x^(1/3)) - 6*d^3*e^(-4)*log(abs(d*x^(2/3) + e)) -
(11*d^3*x^2 - 6*d^2*x^(4/3)*e + 3*d*x^(2/3)*e^2 - 2*e^3)*e^(-4)/x^2)*e - 6*
log(d + e/x^(2/3))/x^2)*b*n - 1/2*b*log(c)/x^2 - 1/2*a/x^2
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(2/3))^n)+a)/x^3,x)
```

```
[Out] int((b*ln(c*(d+e/x^(2/3))^n)+a)/x^3,x)
```

maxima [A] time = 0.47, size = 88, normalized size = 0.99

$$-\frac{1}{12}\,ben\left(\frac{6\,d^3\log\left(dx^{\frac{2}{3}}+e\right)}{e^4}-\frac{6\,d^3\log\left(x^{\frac{2}{3}}\right)}{e^4}-\frac{6\,d^2x^{\frac{4}{3}}-3\,dex^{\frac{2}{3}}+2\,e^2}{e^3x^2}\right)-\frac{b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^3,x, algorithm="maxima")
```

```
[Out] -1/12*b*e*n*(6*d^3*log(d*x^(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*d^2
*x^(4/3) - 3*d*e*x^(2/3) + 2*e^2)/(e^3*x^2)) - 1/2*b*log(c*(d + e/x^(2/3))^
n)/x^2 - 1/2*a/x^2
```

mupad [B] time = 0.44, size = 74, normalized size = 0.83

$$\frac{b\,n}{6\,x^2}-\frac{a}{2\,x^2}-\frac{b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2\,x^2}-\frac{b\,d\,n}{4\,e\,x^{4/3}}-\frac{b\,d^3\,n\,\ln\left(d+\frac{e}{x^{2/3}}\right)}{2\,e^3}+\frac{b\,d^2\,n}{2\,e^2\,x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))/x^3,x)
```

```
[Out] (b*n)/(6*x^2) - a/(2*x^2) - (b*log(c*(d + e/x^(2/3))^n))/(2*x^2) - (b*d*n)/
(4*e*x^(4/3)) - (b*d^3*n*log(d + e/x^(2/3)))/(2*e^3) + (b*d^2*n)/(2*e^2*x^(
2/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x**3,x)
```

```
[Out] Timed out
```

3.515 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=132

$$-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}+\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{9/2}}+\frac{2bd^4n}{3e^4\sqrt[3]{x}}-\frac{2bd^3n}{9e^3x}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{2bdn}{21ex^{7/3}}+\frac{2bn}{27x^3}$$

```
[Out] 2/27*b*n/x^3-2/21*b*d*n/e/x^(7/3)+2/15*b*d^2*n/e^2/x^(5/3)-2/9*b*d^3*n/e^3/
x+2/3*b*d^4*n/e^4/x^(1/3)+2/3*b*d^(9/2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2))/e
^(9/2)+1/3*(-a-b*ln(c*(d+e/x^(2/3))^n))/x^3
```

Rubi [A] time = 0.09, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2455, 263, 341, 325, 205}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}+\frac{2bd^2n}{15e^2x^{5/3}}+\frac{2bd^4n}{3e^4\sqrt[3]{x}}-\frac{2bd^3n}{9e^3x}+\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{9/2}}-\frac{2bdn}{21ex^{7/3}}+\frac{2bn}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x^4,x]
```

```
[Out] (2*b*n)/(27*x^3) - (2*b*d*n)/(21*e*x^(7/3)) + (2*b*d^2*n)/(15*e^2*x^(5/3))
- (2*b*d^3*n)/(9*e^3*x) + (2*b*d^4*n)/(3*e^4*x^(1/3)) + (2*b*d^(9/2)*n*ArcT
an[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(3*e^(9/2)) - (a + b*Log[c*(d + e/x^(2/3))^n
])/(3*x^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
```

```
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx = -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3} - \frac{1}{9}(2ben)\int \frac{1}{\left(d+\frac{e}{x^{2/3}}\right)x^{14/3}}\,dx$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3} - \frac{1}{9}(2ben)\int \frac{1}{\left(e+dx^{2/3}\right)x^4}\,dx$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3} - \frac{1}{3}(2ben)\,\text{Subst}\left(\int \frac{1}{x^{10}\left(e+dx^2\right)}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2bn}{27x^3} - \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3} + \frac{1}{3}(2bdn)\,\text{Subst}\left(\int \frac{1}{x^8\left(e+dx^2\right)}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2bn}{27x^3} - \frac{2bdn}{21ex^{7/3}} - \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3} - \frac{\left(2bd^2n\right)\text{Subst}\left(\int \frac{1}{x^6\left(e+dx^2\right)}\,dx, x, \right.}{3e}$$

$$= \frac{2bn}{27x^3} - \frac{2bdn}{21ex^{7/3}} + \frac{2bd^2n}{15e^2x^{5/3}} - \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3} + \frac{\left(2bd^3n\right)\text{Subst}\left(\int \frac{}{x^4(}\right.}{3e^2}$$

$$= \frac{2bn}{27x^3} - \frac{2bdn}{21ex^{7/3}} + \frac{2bd^2n}{15e^2x^{5/3}} - \frac{2bd^3n}{9e^3x} - \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3} - \frac{\left(2bd^4n\right)\text{Sub}}{}$$

$$= \frac{2bn}{27x^3} - \frac{2bdn}{21ex^{7/3}} + \frac{2bd^2n}{15e^2x^{5/3}} - \frac{2bd^3n}{9e^3x} + \frac{2bd^4n}{3e^4\sqrt[3]{x}} - \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3} + \frac{(}{}$$

$$= \frac{2bn}{27x^3} - \frac{2bdn}{21ex^{7/3}} + \frac{2bd^2n}{15e^2x^{5/3}} - \frac{2bd^3n}{9e^3x} + \frac{2bd^4n}{3e^4\sqrt[3]{x}} + \frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{9/2}} - \frac{a+}{}$$

Mathematica [A] time = 0.07, size = 137, normalized size = 1.04

$$-\frac{a}{3x^3}-\frac{b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}-\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{d}\sqrt[3]{x}}\right)}{3e^{9/2}}+\frac{2bd^4n}{3e^4\sqrt[3]{x}}-\frac{2bd^3n}{9e^3x}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{2bdn}{21ex^{7/3}}+\frac{2bn}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x^4,x]
```

```
[Out] -1/3*a/x^3 + (2*b*n)/(27*x^3) - (2*b*d*n)/(21*e*x^(7/3)) + (2*b*d^2*n)/(15*
e^2*x^(5/3)) - (2*b*d^3*n)/(9*e^3*x) + (2*b*d^4*n)/(3*e^4*x^(1/3)) - (2*b*d
^(9/2)*n*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))])/(3*e^(9/2)) - (b*Log[c*(d + e/x
^(2/3))^n])/(3*x^3)
```

fricas [A] time = 0.48, size = 339, normalized size = 2.57

$$\left[\frac{315\,bd^4nx^3\sqrt{-\frac{d}{e}}\,\log\left(\frac{d^3x^2-2\,de^2x\sqrt{-\frac{d}{e}}-e^3+2\left(d^2ex\sqrt{-\frac{d}{e}}+de^2\right)x^{\frac{2}{3}}-2\left(d^2ex-e^3\sqrt{-\frac{d}{e}}\right)x^{\frac{1}{3}}}{d^3x^2+e^3}\right)-210\,bd^3enx^2+126\,bd^2e^2nx^{\frac{4}{3}}-3}{945\,e^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^4,x, algorithm="fricas")
```

```
[Out] [1/945*(315*b*d^4*n*x^3*sqrt(-d/e)*log((d^3*x^2 - 2*d*e^2*x*sqrt(-d/e) - e^
3 + 2*(d^2*e*x*sqrt(-d/e) + d*e^2)*x^(2/3) - 2*(d^2*e*x - e^3*sqrt(-d/e))*x
^(1/3))/(d^3*x^2 + e^3)) - 210*b*d^3*e*n*x^2 + 126*b*d^2*e^2*n*x^(4/3) - 31
5*b*e^4*n*log((d*x + e*x^(1/3))/x) + 70*b*e^4*n - 315*b*e^4*log(c) - 315*a*
e^4 + 90*(7*b*d^4*n*x^2 - b*d*e^3*n)*x^(2/3))/(e^4*x^3), 1/945*(630*b*d^4*n
*x^3*sqrt(d/e)*arctan(x^(1/3)*sqrt(d/e)) - 210*b*d^3*e*n*x^2 + 126*b*d^2*e^
2*n*x^(4/3) - 315*b*e^4*n*log((d*x + e*x^(1/3))/x) + 70*b*e^4*n - 315*b*e^4
*log(c) - 315*a*e^4 + 90*(7*b*d^4*n*x^2 - b*d*e^3*n)*x^(2/3))/(e^4*x^3)]
```

giac [A] time = 0.46, size = 103, normalized size = 0.78

$$\frac{1}{945}\left(2\left(315\,d^{\frac{9}{2}}\arctan\left(\sqrt{d}\,x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{11}{2}\right)}+\frac{\left(315\,d^4x^{\frac{8}{3}}-105\,d^3x^2e+63\,d^2x^{\frac{4}{3}}e^2-45\,dx^{\frac{2}{3}}e^3+35\,e^4\right)e^{(-5)}}{x^3}\right)e-\frac{315}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^4,x, algorithm="giac")
```

```
[Out] 1/945*(2*(315*d^(9/2)*arctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(-11/2) + (315*d^4
*x^(8/3) - 105*d^3*x^2*e + 63*d^2*x^(4/3)*e^2 - 45*d*x^(2/3)*e^3 + 35*e^4)*
e^(-5)/x^3)*e - 315*log(d + e/x^(2/3))/x^3)*b*n - 1/3*b*log(c)/x^3 - 1/3*a/
x^3
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\frac{b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(2/3))^n)+a)/x^4,x)
```

```
[Out] int((b*ln(c*(d+e/x^(2/3))^n)+a)/x^4,x)
```

maxima [A] time = 1.00, size = 105, normalized size = 0.80

$$\frac{2}{945}\,ben\left(\frac{315\,d^5\arctan\left(\frac{dx^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}\,e^5}+\frac{315\,d^4x^{\frac{8}{3}}-105\,d^3ex^2+63\,d^2e^2x^{\frac{4}{3}}-45\,de^3x^{\frac{2}{3}}+35\,e^4}{e^5x^3}\right)-\frac{b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)}{3\,x^3}-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^4,x, algorithm="maxima")
```

```
[Out] 2/945*b*e*n*(315*d^5*arctan(d*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*e^5) + (315*d^4
*x^(8/3) - 105*d^3*e*x^2 + 63*d^2*e^2*x^(4/3) - 45*d*e^3*x^(2/3) + 35*e^4)/
(e^5*x^3)) - 1/3*b*log(c*(d + e/x^(2/3))^n)/x^3 - 1/3*a/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))/x^4,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))/x^4, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x**4,x)
[Out] Timed out
```

3.516 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=412

$$\frac{be^6n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6}+\frac{be^5nx^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6}-\frac{be^4nx^{4/3}\left(a+b\log\right.}{4d}$$

```
[Out] -77/120*b^2*e^5*n^2*x^(2/3)/d^5+47/240*b^2*e^4*n^2*x^(4/3)/d^4-3/40*b^2*e^3
*n^2*x^2/d^3+1/40*b^2*e^2*n^2*x^(8/3)/d^2+77/120*b^2*e^6*n^2*ln(d+e/x^(2/3)
)/d^6+1/2*b*e^5*n*(d+e/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^6-1/4
*b*e^4*n*x^(4/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^4+1/6*b*e^3*n*x^2*(a+b*ln(c*
(d+e/x^(2/3))^n))/d^3-1/8*b*e^2*n*x^(8/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^2+1
/10*b*e*n*x^(10/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d+1/2*b*e^6*n*ln(1-d/(d+e/x^
(2/3)))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^6+1/4*x^4*(a+b*ln(c*(d+e/x^(2/3))^n))
^2+137/180*b^2*e^6*n^2*ln(x)/d^6-1/2*b^2*e^6*n^2*polylog(2,d/(d+e/x^(2/3)))
/d^6
```

Rubi [A] time = 1.02, antiderivative size = 436, normalized size of antiderivative = 1.06, number of steps used = 26, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$\frac{b^2e^6n^2\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)}{2d^6}-\frac{e^6\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d^6}+\frac{be^6n\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6}+\frac{be^5nx}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (-77*b^2*e^5*n^2*x^(2/3))/(120*d^5) + (47*b^2*e^4*n^2*x^(4/3))/(240*d^4) -
(3*b^2*e^3*n^2*x^2)/(40*d^3) + (b^2*e^2*n^2*x^(8/3))/(40*d^2) + (77*b^2*e^6
*n^2*Log[d + e/x^(2/3)])/(120*d^6) + (b*e^5*n*(d + e/x^(2/3))*x^(2/3)*(a +
b*Log[c*(d + e/x^(2/3))^n]))/(2*d^6) - (b*e^4*n*x^(4/3)*(a + b*Log[c*(d + e
/x^(2/3))^n]))/(4*d^4) + (b*e^3*n*x^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(6*
d^3) - (b*e^2*n*x^(8/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(8*d^2) + (b*e*n*
x^(10/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(10*d) - (e^6*(a + b*Log[c*(d +
e/x^(2/3))^n])^2)/(4*d^6) + (x^4*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/4 + (b
*e^6*n*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))])/(2*d^6) + (1
37*b^2*e^6*n^2*Log[x])/(180*d^6) + (b^2*e^6*n^2*PolyLog[2, 1 + e/(d*x^(2/3)
)])/(2*d^6)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2dx=-\left(\frac{3}{2}\text{Subst}\left(\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^7}dx,x,\frac{1}{x^{2/3}}\right)\right)$$

$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{1}{2}(ben)\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right.}{x^6(d+ex)}\right.$$

$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{1}{2}(bn)\text{Subst}\left(\int\frac{a+b\log\left(cx^n\right)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}dx,x,\right.$$

$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{(bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}dx,x,d+\frac{e}{x^{2/3}}\right.}{2d}$$

$$=\frac{benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{10d}+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+$$

$$=-\frac{be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8d^2}+\frac{benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{10d}$$

$$=-\frac{b^2e^5n^2x^{2/3}}{10d^5}+\frac{b^2e^4n^2x^{4/3}}{20d^4}-\frac{b^2e^3n^2x^2}{30d^3}+\frac{b^2e^2n^2x^{8/3}}{40d^2}+\frac{b^2e^6n^2\log\left(d+\frac{e}{x^{2/3}}\right.}{10d^6}$$

$$=-\frac{9b^2e^5n^2x^{2/3}}{40d^5}+\frac{9b^2e^4n^2x^{4/3}}{80d^4}-\frac{3b^2e^3n^2x^2}{40d^3}+\frac{b^2e^2n^2x^{8/3}}{40d^2}+\frac{9b^2e^6n^2\log\left(d\right.}{40d^6}$$

$$=-\frac{47b^2e^5n^2x^{2/3}}{120d^5}+\frac{47b^2e^4n^2x^{4/3}}{240d^4}-\frac{3b^2e^3n^2x^2}{40d^3}+\frac{b^2e^2n^2x^{8/3}}{40d^2}+\frac{47b^2e^6n^2\log}{120d}$$

$$=-\frac{77b^2e^5n^2x^{2/3}}{120d^5}+\frac{47b^2e^4n^2x^{4/3}}{240d^4}-\frac{3b^2e^3n^2x^2}{40d^3}+\frac{b^2e^2n^2x^{8/3}}{40d^2}+\frac{77b^2e^6n^2\log}{120d}$$

$$=-\frac{77b^2e^5n^2x^{2/3}}{120d^5}+\frac{47b^2e^4n^2x^{4/3}}{240d^4}-\frac{3b^2e^3n^2x^2}{40d^3}+\frac{b^2e^2n^2x^{8/3}}{40d^2}+\frac{77b^2e^6n^2\log}{120d}$$

Mathematica [B] time = 0.45, size = 968, normalized size = 2.35

$$180a^2x^4d^6+180b^2x^4\log^2\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)d^6+360abx^4\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)d^6+72abenx^{10/3}d^5+72b^2enx^{10/3}\log\left(c\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (360*a*b*d*e^5*n*x^(2/3) - 462*b^2*d*e^5*n^2*x^(2/3) - 180*a*b*d^2*e^4*n*x^
(4/3) + 141*b^2*d^2*e^4*n^2*x^(4/3) + 120*a*b*d^3*e^3*n*x^2 - 54*b^2*d^3*e^
3*n^2*x^2 - 90*a*b*d^4*e^2*n*x^(8/3) + 18*b^2*d^4*e^2*n^2*x^(8/3) + 72*a*b*
d^5*e*n*x^(10/3) + 180*a^2*d^6*x^4 + 822*b^2*e^6*n^2*Log[d + e/x^(2/3)] + 3
60*b^2*d*e^5*n*x^(2/3)*Log[c*(d + e/x^(2/3))^n] - 180*b^2*d^2*e^4*n*x^(4/3)
*Log[c*(d + e/x^(2/3))^n] + 120*b^2*d^3*e^3*n*x^2*Log[c*(d + e/x^(2/3))^n]
- 90*b^2*d^4*e^2*n*x^(8/3)*Log[c*(d + e/x^(2/3))^n] + 72*b^2*d^5*e*n*x^(10/
3)*Log[c*(d + e/x^(2/3))^n] + 360*a*b*d^6*x^4*Log[c*(d + e/x^(2/3))^n] + 18
0*b^2*d^6*x^4*Log[c*(d + e/x^(2/3))^n]^2 - 360*a*b*e^6*n*Log[Sqrt[e] - Sqrt
[-d]*x^(1/3)] - 360*b^2*e^6*n*Log[c*(d + e/x^(2/3))^n]*Log[Sqrt[e] - Sqrt[-
d]*x^(1/3)] + 180*b^2*e^6*n^2*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]^2 - 360*a*b*e
^6*n*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] - 360*b^2*e^6*n*Log[c*(d + e/x^(2/3))^
n]*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 180*b^2*e^6*n^2*Log[Sqrt[e] + Sqrt[-d]
*x^(1/3)]^2 + 360*b^2*e^6*n^2*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[1/2 - (Sq
rt[-d]*x^(1/3))/(2*Sqrt[e])] + 360*b^2*e^6*n^2*Log[Sqrt[e] - Sqrt[-d]*x^(1/
3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 720*b^2*e^6*n^2*Log[Sqrt[e] +
 Sqrt[-d]*x^(1/3)]*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])] - 720*b^2*e^6*n^2*Log
[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]] + 548*b^2*e^6*
n^2*Log[x] - 720*b^2*e^6*n^2*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]] + 3
60*b^2*e^6*n^2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] + 360*b^2*e
^6*n^2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 720*b^2*e^6*n^2*Pol
yLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]])/(720*d^6)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^{2}x^{3}\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^{n}\right)^{2}+2\,abx^{3}\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^{n}\right)+a^{2}x^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^3*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*x^3*log(c*((d*x +
 e*x^(1/3))/x)^n) + a^2*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{2}x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2*x^3, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{2}x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*ln(c*(d+e/x^(2/3))^n)+a)^2,x)
```

```
[Out] int(x^3*(b*ln(c*(d+e/x^(2/3))^n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,b^2n^2x^4\log\left(dx^{\frac{2}{3}}+e\right)^2-\int-\frac{3\left(b^2d\log(c)^2+2\,abd\log(c)+a^2d\right)x^4+3\left(b^2e\log(c)^2+2\,abe\log(c)+a^2e\right)x^{\frac{10}{3}}-\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*n^2*x^4*log(d*x^(2/3) + e)^2 - integrate(-1/3*(3*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^4 + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x
^(10/3) - (b^2*d*n*x^4 - 6*(b^2*d*log(c) + a*b*d)*x^4 - 6*(b^2*e*log(c) + a
*b*e)*x^(10/3) + 12*(b^2*d*x^4 + b^2*e*x^(10/3))*log(x^(1/3*n)))*n*log(d*x^
(2/3) + e) + 12*(b^2*d*x^4 + b^2*e*x^(10/3))*log(x^(1/3*n))^2 - 12*((b^2*d*
log(c) + a*b*d)*x^4 + (b^2*e*log(c) + a*b*e)*x^(10/3))*log(x^(1/3*n)))/(d*x
 + e*x^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e/x^(2/3))^n))^2,x)
```

```
[Out] int(x^3*(a + b*log(c*(d + e/x^(2/3))^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**n))**2,x)
```

```
[Out] Timed out
```

3.517 $\int x \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 \, dx$

Optimal. Leaf size=239

$$-\frac{be^3 n \log \left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right) \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}-\frac{be^2 n x^{2/3} \left(d+\frac{e}{x^{2/3}}\right) \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}+\frac{benx^{4/3} \left(a+ \right.}{}$$

```
[Out] 1/2*b^2*e^2*n^2*x^(2/3)/d^2-1/2*b^2*e^3*n^2*ln(d+e/x^(2/3))/d^3-b*e^2*n*(d+
e/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3+1/2*b*e*n*x^(4/3)*(a+b*l
n(c*(d+e/x^(2/3))^n))/d-b*e^3*n*ln(1-d/(d+e/x^(2/3)))*(a+b*ln(c*(d+e/x^(2/3
))^n))/d^3+1/2*x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^2-b^2*e^3*n^2*ln(x)/d^3+b^2*
e^3*n^2*polylog(2,d/(d+e/x^(2/3)))/d^3
```

Rubi [A] time = 0.48, antiderivative size = 264, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2454, 2398, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44}

$$-\frac{b^2 e^3 n^2 \text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)}{d^3}+\frac{e^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3}-\frac{be^3 n \log \left(-\frac{e}{dx^{2/3}}\right) \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}-\frac{b\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (b^2*e^2*n^2*x^(2/3))/(2*d^2) - (b^2*e^3*n^2*Log[d + e/x^(2/3)])/(2*d^3) -
(b*e^2*n*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/d^3 + (b
*e*n*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2*d) + (e^3*(a + b*Log[c*(d
 + e/x^(2/3))^n])^2)/(2*d^3) + (x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/2 -
 (b*e^3*n*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))])/d^3 - (b^
2*e^3*n^2*Log[x])/d^3 - (b^2*e^3*n^2*PolyLog[2, 1 + e/(d*x^(2/3))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=-\left(\frac{3}{2}\text{Subst}\left(\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^4}\,dx,x,\frac{1}{x^{2/3}}\right)\right)$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-(ben)\,\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^3(d+ex)}\right.$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-(bn)\,\text{Subst}\left(\int\frac{a+b\log\left(cx^n\right)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx,x,\right.$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{(bn)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx,x,d+\frac{e}{x^{2/3}}\right.}{d}$$

$$=\frac{benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+$$

$$=-\frac{be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}+\frac{benx^{4/3}\left(a+b\log\left(c\left(\right.\right.\right.}{2d}$$

$$=\frac{b^2e^2n^2x^{2/3}}{2d^2}-\frac{b^2e^3n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}-\frac{be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d\right.\right.\right.}{d^3}$$

$$=\frac{b^2e^2n^2x^{2/3}}{2d^2}-\frac{b^2e^3n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}-\frac{be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d\right.\right.\right.}{d^3}$$

Mathematica [B] time = 0.46, size = 542, normalized size = 2.27

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{ben\left(-3d^2x^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-6e^2\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)\left(a+b\log\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/2 - (b*e*n*(6*d*e*x^(2/3)*(a + b*L
og[c*(d + e/x^(2/3))^n]) - 3*d^2*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n]) -
 6*e^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] - 6
*e^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 2*b
*e^2*n*(3*Log[d + e/x^(2/3)] + 2*Log[x]) + b*e*n*(-3*d*x^(2/3) + 3*e*Log[d
+ e/x^(2/3)] + 2*e*Log[x]) + 3*b*e^2*n*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*(Lo
g[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] -
 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/S
qrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])]) + 3*b*e^2*n*(
Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 2*Log[1/
2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 4*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])])
 + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*PolyLog[2, 1 + (Sqr
t[-d]*x^(1/3))/Sqrt[e]])))/(6*d^3)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^2+2\,abx\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a^2x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*x*log(c*((d*x + e*x
^(1/3))/x)^n) + a^2*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{n}\right) + a\right)^{2} x \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2*x, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(b \ln \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{n}\right) + a\right)^{2} x \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(d+e/x^(2/3))^n)+a)^2,x)
```

```
[Out] int(x*(b*ln(c*(d+e/x^(2/3))^n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} b^{2} n^{2} x^{2} \log \left(d x^{\frac{2}{3}} + e\right)^{2} - \int -\frac{3 \left(b^{2} d \log(c)^{2} + 2 \, a b d \log(c) + a^{2} d\right) x^{2} - 2 \left(b^{2} d n x^{2} - 3 \left(b^{2} d \log(c) + a b d\right) x^{2} - 3 \left(b^{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*n^2*x^2*log(d*x^(2/3) + e)^2 - integrate(-1/3*(3*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^2 - 2*(b^2*d*n*x^2 - 3*(b^2*d*log(c) + a*b*d)*x^2
 - 3*(b^2*e*log(c) + a*b*e)*x^(4/3) + 6*(b^2*d*x^2 + b^2*e*x^(4/3))*log(x^(
1/3*n)))*n*log(d*x^(2/3) + e) + 12*(b^2*d*x^2 + b^2*e*x^(4/3))*log(x^(1/3*n
))^2 + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(4/3) - 12*((b^2*d*log
(c) + a*b*d)*x^2 + (b^2*e*log(c) + a*b*e)*x^(4/3))*log(x^(1/3*n)))/(d*x + e
*x^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \left(a + b \, \ln \left(c \left(d + \frac{e}{x^{2/3}}\right)^{n}\right)\right)^{2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(2/3))^n))^2,x)
```

```
[Out] int(x*(a + b*log(c*(d + e/x^(2/3))^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))**n))**2,x)
```

```
[Out] Timed out
```

3.518 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx$

Optimal. Leaf size=95

$$-3bn\text{Li}_2\left(\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{3}{2}\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+3b^2n^2\text{Li}_3\left(\frac{e}{dx^{2/3}}+\right.$$

```
[Out] -3/2*(a+b*ln(c*(d+e/x^(2/3))^n))^2*ln(-e/d/x^(2/3))-3*b*n*(a+b*ln(c*(d+e/x^
(2/3))^n))*polylog(2,1+e/d/x^(2/3))+3*b^2*n^2*polylog(3,1+e/d/x^(2/3))
```

Rubi [A] time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2454, 2396, 2433, 2374, 6589}

$$-3bn\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+3b^2n^2\text{PolyLog}\left(3,\frac{e}{dx^{2/3}}+1\right)-\frac{3}{2}\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x,x]
```

```
[Out] (-3*(a + b*Log[c*(d + e/x^(2/3))^n])^2*Log[-(e/(d*x^(2/3)))])/2 - 3*b*n*(a
+ b*Log[c*(d + e/x^(2/3))^n])*PolyLog[2, 1 + e/(d*x^(2/3))] + 3*b^2*n^2*Pol
yLog[3, 1 + e/(d*x^(2/3))]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx = -\left(\frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log\left(-\frac{e}{dx^{2/3}}\right) + (3ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right.}{}\right.$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log\left(-\frac{e}{dx^{2/3}}\right) + (3bn)\,\text{Subst}\left(\int \frac{(a+b\log}{}\right.$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log\left(-\frac{e}{dx^{2/3}}\right) - 3bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log\left(-\frac{e}{dx^{2/3}}\right) - 3bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.$$

Mathematica [B] time = 0.15, size = 199, normalized size = 2.09

$$2bn\left(\frac{3}{2}\text{Li}_2\left(-\frac{e}{dx^{2/3}}\right)+\log(x)\left(\log\left(d+\frac{e}{x^{2/3}}\right)-\log\left(\frac{e}{dx^{2/3}}+1\right)\right)\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)-bn\log\left(d+\frac{e}{x^{2/3}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])*((Log[d + e/x^(2/
3)] - Log[1 + e/(d*x^(2/3))])*Log[x] + (3*PolyLog[2, -(e/(d*x^(2/3)))])/2)
- (3*b^2*n^2*(Log[d + e/x^(2/3)]^2*Log[-(e/(d*x^(2/3)))] + 2*Log[d + e/x^(2
/3)]*PolyLog[2, 1 + e/(d*x^(2/3))] - 2*PolyLog[3, 1 + e/(d*x^(2/3))]))/2
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^2+2\,ab\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(1
/3))/x)^n) + a^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(2/3))^n)+a)^2/x,x)
[Out] int((b*ln(c*(d+e/x^(2/3))^n)+a)^2/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^{2}n^{2}\log\left(dx^{\frac{2}{3}}+e\right)^{2}\log(x)-\int \frac{2\left(2\,b^{2}dnx\log(x)-3\left(b^{2}d\log(c)+abd\right)x+6\left(b^{2}dx+b^{2}ex^{\frac{1}{3}}\right)\log\left(x^{\frac{1}{3}n}\right)-3\left(b^{2}e\log\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x,x, algorithm="maxima")
[Out] b^2*n^2*log(d*x^(2/3) + e)^2*log(x) - integrate(1/3*(2*(2*b^2*d*n*x*log(x)
- 3*(b^2*d*log(c) + a*b*d)*x + 6*(b^2*d*x + b^2*e*x^(1/3))*log(x^(1/3*n)) -
 3*(b^2*e*log(c) + a*b*e)*x^(1/3))*n*log(d*x^(2/3) + e) - 12*(b^2*d*x + b^2
*e*x^(1/3))*log(x^(1/3*n))^2 - 3*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*
x + 12*((b^2*d*log(c) + a*b*d)*x + (b^2*e*log(c) + a*b*e)*x^(1/3))*log(x^(1
/3*n)) - 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(1/3))/(d*x^2 + e*x^
(4/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^{n}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^2/x,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^2/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x,x)
[Out] Timed out
```

3.519 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=276

$$-\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3}+\frac{3bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3}-\frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(\right.}{}$$

```
[Out] 3/4*b^2*d*n^2*(d+e/x^(2/3))^2/e^3-1/9*b^2*n^2*(d+e/x^(2/3))^3/e^3-3*b^2*d^2
*n^2/e^2/x^(2/3)+1/2*b^2*d^3*n^2*ln(d+e/x^(2/3))^2/e^3+3*b*d^2*n*(d+e/x^(2/
3))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^3-3/2*b*d*n*(d+e/x^(2/3))^2*(a+b*ln(c*(d+
e/x^(2/3))^n))/e^3+1/3*b*n*(d+e/x^(2/3))^3*(a+b*ln(c*(d+e/x^(2/3))^n))/e^3-
b*d^3*n*ln(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^3-1/2*(a+b*ln(c*(d+e/
x^(2/3))^n))^2/x^2
```

Rubi [A] time = 0.30, antiderivative size = 217, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$\frac{1}{6}bn\left(\frac{18d^2\left(d+\frac{e}{x^{2/3}}\right)}{e^3}-\frac{6d^3\log\left(d+\frac{e}{x^{2/3}}\right)}{e^3}-\frac{9d\left(d+\frac{e}{x^{2/3}}\right)^2}{e^3}+\frac{2\left(d+\frac{e}{x^{2/3}}\right)^3}{e^3}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^3,x]
```

```
[Out] (3*b^2*d*n^2*(d + e/x^(2/3))^2)/(4*e^3) - (b^2*n^2*(d + e/x^(2/3))^3)/(9*e^
3) - (3*b^2*d^2*n^2)/(e^2*x^(2/3)) + (b^2*d^3*n^2*Log[d + e/x^(2/3)]^2)/(2*
e^3) + (b*n*((18*d^2*(d + e/x^(2/3)))/e^3 - (9*d*(d + e/x^(2/3))^2)/e^3 + (
2*(d + e/x^(2/3))^3)/e^3 - (6*d^3*Log[d + e/x^(2/3)])/e^3)*(a + b*Log[c*(d
+ e/x^(2/3))^n]))/6 - (a + b*Log[c*(d + e/x^(2/3))^n])^2/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx = -\left(\frac{3}{2}\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2x^2} + (ben)\,\text{Subst}\left(\int \frac{x^3\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2x^2} + (bn)\,\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3\left(a+b\log\left(cx^n\right)\right)}{x}\right.$$

$$= \frac{1}{6}bn\left(\frac{18d^2\left(d+\frac{e}{x^{2/3}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{x^{2/3}}\right)^2}{e^3} + \frac{2\left(d+\frac{e}{x^{2/3}}\right)^3}{e^3} - \frac{6d^3\log\left(d+\frac{e}{x^{2/3}}\right)}{e^3}\right.$$

$$= \frac{1}{6}bn\left(\frac{18d^2\left(d+\frac{e}{x^{2/3}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{x^{2/3}}\right)^2}{e^3} + \frac{2\left(d+\frac{e}{x^{2/3}}\right)^3}{e^3} - \frac{6d^3\log\left(d+\frac{e}{x^{2/3}}\right)}{e^3}\right.$$

$$= \frac{1}{6}bn\left(\frac{18d^2\left(d+\frac{e}{x^{2/3}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{x^{2/3}}\right)^2}{e^3} + \frac{2\left(d+\frac{e}{x^{2/3}}\right)^3}{e^3} - \frac{6d^3\log\left(d+\frac{e}{x^{2/3}}\right)}{e^3}\right.$$

$$= \frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2}{4e^3} - \frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3} - \frac{3b^2d^2n^2}{e^2x^{2/3}} + \frac{1}{6}bn\left(\frac{18d^2\left(d+\frac{e}{x^{2/3}}\right)}{e^3} -\right.$$

$$= \frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2}{4e^3} - \frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3} - \frac{3b^2d^2n^2}{e^2x^{2/3}} + \frac{b^2d^3n^2\log^2\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}$$

Mathematica [C] time = 0.54, size = 691, normalized size = 2.50

$$\frac{bn\left(-36d^3x^2\left(\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+bn\text{Li}_2\left(\frac{e}{dx^{2/3}}+1\right)\right)-36d^3x^2\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)\left(a+b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^3,x]
```

```
[Out] (-18*e^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + b*n*(9*b*d*n*x^(2/3)*(e*(e -
2*d*x^(2/3)) + 2*d^2*x^(4/3)*Log[d + e/x^(2/3)]) - 2*b*n*(e*(2*e^2 - 3*d*e*
x^(2/3) + 6*d^2*x^(4/3)) - 6*d^3*x^2*Log[d + e/x^(2/3)]) + 12*e^3*(a + b*Lo
g[c*(d + e/x^(2/3))^n]) - 18*d*e^2*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n])
 + 36*d^2*x^(4/3)*(e*(a - b*n) + b*(e + d*x^(2/3))*Log[c*(d + e/x^(2/3))^n]
) - 36*d^3*x^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1
/3)] - 36*d^3*x^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x
^(1/3)] - 36*d^3*x^2*((a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))
] + b*n*PolyLog[2, 1 + e/(d*x^(2/3))]) + 18*b*d^3*n*x^2*(Log[Sqrt[e] - Sqrt
[-d]*x^(1/3)]*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x^(1/
3))/Sqrt[e])/2] - 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 - (Sq
rt[-d]*x^(1/3))/Sqrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e]
)]) + 18*b*d^3*n*x^2*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] + Sqrt[-
d]*x^(1/3)] + 2*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 4*Log[-((Sqrt[-
d]*x^(1/3))/Sqrt[e])]) + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] -
 4*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]])))/(36*e^3*x^2)
```

fricas [A] time = 0.48, size = 307, normalized size = 1.11

$$-\frac{4\,b^2e^3n^2+18\,b^2e^3\log(c)^2-12\,abe^3n+18\,a^2e^3+18\left(b^2d^3n^2x^2+b^2e^3n^2\right)\log\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^2-12\left(b^2e^3n-3\,abe^3\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/36*(4*b^2*e^3*n^2 + 18*b^2*e^3*log(c)^2 - 12*a*b*e^3*n + 18*a^2*e^3 + 18
*(b^2*d^3*n^2*x^2 + b^2*e^3*n^2)*log((d*x + e*x^(1/3))/x)^2 - 12*(b^2*e^3*n
 - 3*a*b*e^3)*log(c) - 6*(6*b^2*d^2*e*n^2*x^(4/3) - 3*b^2*d*e^2*n^2*x^(2/3)
 + 2*b^2*e^3*n^2 - 6*a*b*e^3*n + (11*b^2*d^3*n^2 - 6*a*b*d^3*n)*x^2 - 6*(b^
2*d^3*n*x^2 + b^2*e^3*n)*log(c))*log((d*x + e*x^(1/3))/x) - 3*(5*b^2*d*e^2*
n^2 - 6*b^2*d*e^2*n*log(c) - 6*a*b*d*e^2*n)*x^(2/3) - 6*(6*b^2*d^2*e*n*x*lo
g(c) - (11*b^2*d^2*e*n^2 - 6*a*b*d^2*e*n)*x)*x^(1/3))/(e^3*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x^3, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(2/3))^n)+a)^2/x^3,x)
```

```
[Out] int((b*ln(c*(d+e/x^(2/3))^n)+a)^2/x^3,x)
```

maxima [A] time = 0.52, size = 298, normalized size = 1.08

$$-\frac{1}{6}\,aben\left(\frac{6\,d^3\log\left(dx^{\frac{2}{3}}+e\right)}{e^4}-\frac{6\,d^3\log\left(x^{\frac{2}{3}}\right)}{e^4}-\frac{6\,d^2x^{\frac{4}{3}}-3\,dex^{\frac{2}{3}}+2\,e^2}{e^3x^2}\right)-\frac{1}{36}\left(6\,en\left(\frac{6\,d^3\log\left(dx^{\frac{2}{3}}+e\right)}{e^4}-\frac{6\,d^3\log\left(x\right.}{e^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/6*a*b*e*n*(6*d^3*log(d*x^(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*d^
2*x^(4/3) - 3*d*e*x^(2/3) + 2*e^2)/(e^3*x^2)) - 1/36*(6*e*n*(6*d^3*log(d*x^
(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*d^2*x^(4/3) - 3*d*e*x^(2/3) +
2*e^2)/(e^3*x^2))*log(c*(d + e/x^(2/3))^n) - (18*d^3*x^2*log(d*x^(2/3) + e)
^2 + 8*d^3*x^2*log(x)^2 - 44*d^3*x^2*log(x) - 66*d^2*e*x^(4/3) + 15*d*e^2*x
^(2/3) - 4*e^3 - 6*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2/3) + e))*n^2/
```

```
(e^3*x^2))*b^2 - 1/2*b^2*log(c*(d + e/x^(2/3))^n)^2/x^2 - a*b*log(c*(d + e/
x^(2/3))^n)/x^2 - 1/2*a^2/x^2
```

mupad [B] time = 0.57, size = 302, normalized size = 1.09

$$\frac{\frac{d\left(\frac{3a^2}{2}-a\,b\,n+\frac{b^2n^2}{3}\right)}{2e}-\frac{d\left(3a^2-b^2n^2\right)}{4e}}{x^{4/3}}-\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)^2\left(\frac{b^2}{2x^2}+\frac{b^2d^3}{2e^3}\right)-\frac{\frac{a^2}{2}-\frac{a\,b\,n}{3}+\frac{b^2n^2}{9}}{x^2}-\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\left(\frac{b\,(3a}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^2/x^3,x)
```

```
[Out] ((d*((3*a^2)/2 + (b^2*n^2)/3 - a*b*n))/(2*e) - (d*(3*a^2 - b^2*n^2))/(4*e))
/x^(4/3) - log(c*(d + e/x^(2/3))^n)^2*(b^2/(2*x^2) + (b^2*d^3)/(2*e^3)) - (
a^2/2 + (b^2*n^2)/9 - (a*b*n)/3)/x^2 - log(c*(d + e/x^(2/3))^n)*((b*(3*a -
b*n))/(3*x^2) - ((b*d*(3*a - b*n))/(2*e) - (3*a*b*d)/(2*e))/x^(4/3) + (d*((
b*d*(3*a - b*n))/e - (3*a*b*d)/e))/(e*x^(2/3))) - ((d*((d*((3*a^2)/2 + (b^2
*n^2)/3 - a*b*n))/e - (d*(3*a^2 - b^2*n^2))/(2*e)))/e + (b^2*d^2*n^2)/e^2)/
x^(2/3) + (log(d + e/x^(2/3))*(11*b^2*d^3*n^2 - 6*a*b*d^3*n))/(6*e^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x**3,x)
```

```
[Out] Timed out
```

3.520 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx$

Optimal. Leaf size=482

$$\frac{bd^6n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^6}-\frac{3bd^5n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^6}+\frac{15bd^4n\left(d+\frac{e}{x^{2/3}}\right)^2\left(a\cdots\right.}{4\cdots}$$

```
[Out] -15/8*b^2*d^4*n^2*(d+e/x^(2/3))^2/e^6+10/9*b^2*d^3*n^2*(d+e/x^(2/3))^3/e^6-
15/32*b^2*d^2*n^2*(d+e/x^(2/3))^4/e^6+3/25*b^2*d*n^2*(d+e/x^(2/3))^5/e^6-1/
72*b^2*n^2*(d+e/x^(2/3))^6/e^6+3*b^2*d^5*n^2/e^5/x^(2/3)-1/4*b^2*d^6*n^2*ln
(d+e/x^(2/3))^2/e^6-3*b*d^5*n*(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6
+15/4*b*d^4*n*(d+e/x^(2/3))^2*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6-10/3*b*d^3*n*
(d+e/x^(2/3))^3*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6+15/8*b*d^2*n*(d+e/x^(2/3))^
4*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6-3/5*b*d*n*(d+e/x^(2/3))^5*(a+b*ln(c*(d+e/
x^(2/3))^n))/e^6+1/12*b*n*(d+e/x^(2/3))^6*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6+1
/2*b*d^6*n*ln(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6-1/4*(a+b*ln(c*(d
+e/x^(2/3))^n))^2/x^4
```

Rubi [A] time = 0.48, antiderivative size = 355, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2454, 2398, 2411, 43, 2334, 12, 14, 2301}

$$-\frac{1}{120}bn\left(\frac{360d^5\left(d+\frac{e}{x^{2/3}}\right)}{e^6}-\frac{450d^4\left(d+\frac{e}{x^{2/3}}\right)^2}{e^6}+\frac{400d^3\left(d+\frac{e}{x^{2/3}}\right)^3}{e^6}-\frac{225d^2\left(d+\frac{e}{x^{2/3}}\right)^4}{e^6}-\frac{60d^6\log\left(d+\frac{e}{x^{2/3}}\right)}{e^6}+\frac{72\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^5,x]
```

```
[Out] (-15*b^2*d^4*n^2*(d + e/x^(2/3))^2)/(8*e^6) + (10*b^2*d^3*n^2*(d + e/x^(2/3
))^3)/(9*e^6) - (15*b^2*d^2*n^2*(d + e/x^(2/3))^4)/(32*e^6) + (3*b^2*d*n^2*
(d + e/x^(2/3))^5)/(25*e^6) - (b^2*n^2*(d + e/x^(2/3))^6)/(72*e^6) + (3*b^2
*d^5*n^2)/(e^5*x^(2/3)) - (b^2*d^6*n^2*Log[d + e/x^(2/3)]^2)/(4*e^6) - (b*n
*((360*d^5*(d + e/x^(2/3)))/e^6 - (450*d^4*(d + e/x^(2/3))^2)/e^6 + (400*d^
3*(d + e/x^(2/3))^3)/e^6 - (225*d^2*(d + e/x^(2/3))^4)/e^6 + (72*d*(d + e/x
^(2/3))^5)/e^6 - (10*(d + e/x^(2/3))^6)/e^6 - (60*d^6*Log[d + e/x^(2/3)])/e
^6)*(a + b*Log[c*(d + e/x^(2/3))^n]))/120 - (a + b*Log[c*(d + e/x^(2/3))^n]
)^2/(4*x^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx = -\left(\frac{3}{2}\operatorname{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4x^4} + \frac{1}{2}(ben)\operatorname{Subst}\left(\int \frac{x^6\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4x^4} + \frac{1}{2}(bn)\operatorname{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6\left(a+b\log\left(cx^n\right)\right)}{x}\right.$$

$$= -\frac{1}{120}bn\left(\frac{360d^5\left(d+\frac{e}{x^{2/3}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{x^{2/3}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{x^{2/3}}\right)^3}{e^6} - \frac{225d^2\left(\right.}{}\right.$$

$$= -\frac{1}{120}bn\left(\frac{360d^5\left(d+\frac{e}{x^{2/3}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{x^{2/3}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{x^{2/3}}\right)^3}{e^6} - \frac{225d^2\left(\right.}{}\right.$$

$$= -\frac{1}{120}bn\left(\frac{360d^5\left(d+\frac{e}{x^{2/3}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{x^{2/3}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{x^{2/3}}\right)^3}{e^6} - \frac{225d^2\left(\right.}{}\right.$$

$$= -\frac{15b^2d^4n^2\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^6} + \frac{10b^2d^3n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^6} - \frac{15b^2d^2n^2\left(d+\frac{e}{x^{2/3}}\right)^4}{32e^6} + \frac{3b^2dn}{}$$

$$= -\frac{15b^2d^4n^2\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^6} + \frac{10b^2d^3n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^6} - \frac{15b^2d^2n^2\left(d+\frac{e}{x^{2/3}}\right)^4}{32e^6} + \frac{3b^2dn}{}$$

Mathematica [C] time = 0.86, size = 1021, normalized size = 2.12

$$\frac{bn\left(-1800bnx^4\log^2\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)d^6-1800bnx^4\log^2\left(\sqrt[3]{x}\sqrt{-d}+\sqrt{e}\right)d^6-5220bnx^4\log\left(d+\frac{e}{x^{2/3}}\right)d^6-3600bx^4\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)d^6+3600ax^4\log\left(\sqrt{e}-\sqrt{-}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^5,x]
```

```
[Out] (-1800*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + (b*n*(600*a*e^6 - 100*b*e^6*n -
 720*a*d*e^5*x^(2/3) + 264*b*d*e^5*n*x^(2/3) + 900*a*d^2*e^4*x^(4/3) - 555*
b*d^2*e^4*n*x^(4/3) - 1200*a*d^3*e^3*x^2 + 1140*b*d^3*e^3*n*x^2 + 1800*a*d^
4*e^2*x^(8/3) - 2610*b*d^4*e^2*n*x^(8/3) - 3600*a*d^5*e*x^(10/3) + 8820*b*d
^5*e*n*x^(10/3) - 5220*b*d^6*n*x^4*Log[d + e/x^(2/3)] + 600*b*e^6*Log[c*(d
+ e/x^(2/3))^n] - 720*b*d*e^5*x^(2/3)*Log[c*(d + e/x^(2/3))^n] + 900*b*d^2*
e^4*x^(4/3)*Log[c*(d + e/x^(2/3))^n] - 1200*b*d^3*e^3*x^2*Log[c*(d + e/x^(2
/3))^n] + 1800*b*d^4*e^2*x^(8/3)*Log[c*(d + e/x^(2/3))^n] - 3600*b*d^5*e*x^
(10/3)*Log[c*(d + e/x^(2/3))^n] - 3600*b*d^6*x^4*Log[c*(d + e/x^(2/3))^n] +
 3600*a*d^6*x^4*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 3600*b*d^6*x^4*Log[c*(d +
 e/x^(2/3))^n]*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] - 1800*b*d^6*n*x^4*Log[Sqrt[
e] - Sqrt[-d]*x^(1/3)]^2 + 3600*a*d^6*x^4*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] +
 3600*b*d^6*x^4*Log[c*(d + e/x^(2/3))^n]*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] -
1800*b*d^6*n*x^4*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]^2 - 3600*b*d^6*n*x^4*Log[S
qrt[e] + Sqrt[-d]*x^(1/3)]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 3600
*b*d^6*n*x^4*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/Sq
```

```
rt[e])/2] + 3600*a*d^6*x^4*Log[-(e/(d*x^(2/3)))] + 3600*b*d^6*x^4*Log[c*(d
+ e/x^(2/3))^n]*Log[-(e/(d*x^(2/3)))] + 7200*b*d^6*n*x^4*Log[Sqrt[e] + Sqrt
[-d]*x^(1/3)]*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])] + 7200*b*d^6*n*x^4*Log[Sqr
t[e] - Sqrt[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]] + 3600*b*d^6*n*x^4
*PolyLog[2, 1 + e/(d*x^(2/3))] + 7200*b*d^6*n*x^4*PolyLog[2, 1 - (Sqrt[-d]*
x^(1/3))/Sqrt[e]] - 3600*b*d^6*n*x^4*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2
*Sqrt[e])] - 3600*b*d^6*n*x^4*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2
] + 7200*b*d^6*n*x^4*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]]))/e^6)/(720
0*x^4)
```

fricas [A] time = 0.46, size = 513, normalized size = 1.06

$$-\frac{100\,b^2e^6n^2+1800\,b^2e^6\log(c)^2-600\,abe^6n+1800\,a^2e^6-60\left(19\,b^2d^3e^3n^2-20\,abd^3e^3n\right)x^2-1800\left(b^2d^6n^2x^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/7200*(100*b^2*e^6*n^2 + 1800*b^2*e^6*log(c)^2 - 600*a*b*e^6*n + 1800*a^2
*e^6 - 60*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x^2 - 1800*(b^2*d^6*n^2*x
^4 - b^2*e^6*n^2)*log((d*x + e*x^(1/3))/x)^2 + 600*(2*b^2*d^3*e^3*n*x^2 - b
^2*e^6*n + 6*a*b*e^6)*log(c) + 60*(20*b^2*d^3*e^3*n^2*x^2 - 10*b^2*e^6*n^2
+ 60*a*b*e^6*n + 3*(49*b^2*d^6*n^2 - 20*a*b*d^6*n)*x^4 - 60*(b^2*d^6*n*x^4
- b^2*e^6*n)*log(c) - 6*(5*b^2*d^4*e^2*n^2*x^2 - 2*b^2*d*e^5*n^2)*x^(2/3) +
 15*(4*b^2*d^5*e*n^2*x^3 - b^2*d^2*e^4*n^2*x)*x^(1/3))*log((d*x + e*x^(1/3)
)/x) - 6*(44*b^2*d*e^5*n^2 - 120*a*b*d*e^5*n - 15*(29*b^2*d^4*e^2*n^2 - 20*
a*b*d^4*e^2*n)*x^2 + 60*(5*b^2*d^4*e^2*n*x^2 - 2*b^2*d*e^5*n)*log(c))*x^(2/
3) - 15*(12*(49*b^2*d^5*e*n^2 - 20*a*b*d^5*e*n)*x^3 - (37*b^2*d^2*e^4*n^2 -
 60*a*b*d^2*e^4*n)*x - 60*(4*b^2*d^5*e*n*x^3 - b^2*d^2*e^4*n*x)*log(c))*x^(
1/3))/(e^6*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x^5, x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(2/3))^n)+a)^2/x^5,x)
```

```
[Out] int((b*ln(c*(d+e/x^(2/3))^n)+a)^2/x^5,x)
```

maxima [A] time = 0.54, size = 397, normalized size = 0.82

$$\frac{1}{120}\,aben\left(\frac{60\,d^6\log\left(dx^{\frac{2}{3}}+e\right)}{e^7}-\frac{60\,d^6\log\left(x^{\frac{2}{3}}\right)}{e^7}-\frac{60\,d^5x^{\frac{10}{3}}-30\,d^4ex^{\frac{8}{3}}+20\,d^3e^2x^2-15\,d^2e^3x^{\frac{4}{3}}+12\,de^4x^{\frac{2}{3}}-1\right.}{e^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/120*a*b*e*n*(60*d^6*log(d*x^(2/3) + e)/e^7 - 60*d^6*log(x^(2/3))/e^7 - (6
0*d^5*x^(10/3) - 30*d^4*e*x^(8/3) + 20*d^3*e^2*x^2 - 15*d^2*e^3*x^(4/3) + 1
2*d*e^4*x^(2/3) - 10*e^5)/(e^6*x^4)) + 1/7200*(60*e*n*(60*d^6*log(d*x^(2/3)
 + e)/e^7 - 60*d^6*log(x^(2/3))/e^7 - (60*d^5*x^(10/3) - 30*d^4*e*x^(8/3) +
 20*d^3*e^2*x^2 - 15*d^2*e^3*x^(4/3) + 12*d*e^4*x^(2/3) - 10*e^5)/(e^6*x^4)
)*log(c*(d + e/x^(2/3))^n) - (1800*d^6*x^4*log(d*x^(2/3) + e)^2 + 800*d^6*x
^4*log(x)^2 - 5880*d^6*x^4*log(x) - 8820*d^5*e*x^(10/3) + 2610*d^4*e^2*x^(8
/3) - 1140*d^3*e^3*x^2 + 555*d^2*e^4*x^(4/3) - 264*d*e^5*x^(2/3) + 100*e^6
- 60*(40*d^6*x^4*log(x) - 147*d^6*x^4)*log(d*x^(2/3) + e))*n^2/(e^6*x^4))*b
^2 - 1/4*b^2*log(c*(d + e/x^(2/3))^n)^2/x^4 - 1/2*a*b*log(c*(d + e/x^(2/3))
^n)/x^4 - 1/4*a^2/x^4
```

mupad [B] time = 1.81, size = 440, normalized size = 0.91

$$\frac{b^2\,d^6\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)^2}{4\,e^6}-\frac{b^2\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)^2}{4\,x^4}-\frac{b^2\,n^2}{72\,x^4}-\frac{a\,b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2\,x^4}-\frac{a^2}{4\,x^4}+\frac{a\,b\,n}{12\,x^4}+\frac{b^2\,n\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{12\,x^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^2/x^5,x)
```

```
[Out] (b^2*d^6*log(c*(d + e/x^(2/3))^n)^2)/(4*e^6) - (b^2*log(c*(d + e/x^(2/3))^n
)^2)/(4*x^4) - (b^2*n^2)/(72*x^4) - (a*b*log(c*(d + e/x^(2/3))^n))/(2*x^4)
- a^2/(4*x^4) + (a*b*n)/(12*x^4) + (b^2*n*log(c*(d + e/x^(2/3))^n))/(12*x^4
) - (49*b^2*d^6*n^2*log(d + e/x^(2/3)))/(40*e^6) + (19*b^2*d^3*n^2)/(120*e^
3*x^2) - (37*b^2*d^2*n^2)/(480*e^2*x^(8/3)) - (29*b^2*d^4*n^2)/(80*e^4*x^(4
/3)) + (49*b^2*d^5*n^2)/(40*e^5*x^(2/3)) + (11*b^2*d*n^2)/(300*e*x^(10/3))
- (b^2*d^3*n*log(c*(d + e/x^(2/3))^n))/(6*e^3*x^2) + (b^2*d^2*n*log(c*(d +
e/x^(2/3))^n))/(8*e^2*x^(8/3)) + (b^2*d^4*n*log(c*(d + e/x^(2/3))^n))/(4*e^
4*x^(4/3)) - (b^2*d^5*n*log(c*(d + e/x^(2/3))^n))/(2*e^5*x^(2/3)) - (a*b*d*
n)/(10*e*x^(10/3)) + (a*b*d^6*n*log(d + e/x^(2/3)))/(2*e^6) - (b^2*d*n*log(
c*(d + e/x^(2/3))^n))/(10*e*x^(10/3)) - (a*b*d^3*n)/(6*e^3*x^2) + (a*b*d^2*
n)/(8*e^2*x^(8/3)) + (a*b*d^4*n)/(4*e^4*x^(4/3)) - (a*b*d^5*n)/(2*e^5*x^(2/
3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x**5,x)
```

```
[Out] Timed out
```

3.521 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=490

$$\frac{4be^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3d^{9/2}}+\frac{4be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9d^3}-\frac{4be^2nx^{5/3}\left(a+b\log\left(c\left(d\right.\right.\right.}{15d^2}$$

```
[Out] -4/3*a*b*e^4*n*x^(1/3)/d^4+568/315*b^2*e^4*n^2*x^(1/3)/d^4-32/105*b^2*e^3*n
^2*x/d^3+8/105*b^2*e^2*n^2*x^(5/3)/d^2-1408/315*b^2*e^(9/2)*n^2*arctan(x^(1
/3)*d^(1/2)/e^(1/2))/d^(9/2)-4/3*I*b^2*e^(9/2)*n^2*arctan(x^(1/3)*d^(1/2)/e
^(1/2))^2/d^(9/2)-4/3*b^2*e^4*n*x^(1/3)*ln(c*(d+e/x^(2/3))^n)/d^4+4/9*b*e^3
*n*x*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3-4/15*b*e^2*n*x^(5/3)*(a+b*ln(c*(d+e/x^
(2/3))^n))/d^2+4/21*b*e*n*x^(7/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d+4/3*b*e^(9/
2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^(9/2)+1/
3*x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^2+8/3*b^2*e^(9/2)*n^2*arctan(x^(1/3)*d^(1
/2)/e^(1/2))*ln(2-2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^(1/2)))/d^(9/2)-4/3*I*b^2
*e^(9/2)*n^2*polylog(2,-1+2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^(1/2)))/d^(9/2)
```

Rubi [A] time = 0.81, antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 17, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.708, Rules used = {2458, 2457, 2476, 2448, 263, 205, 2455, 193, 321, 302, 2470, 12, 260, 6688, 4924, 4868, 2447}

$$-\frac{4ib^2e^{9/2}n^2\text{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{3d^{9/2}}+\frac{4be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9d^3}-\frac{4be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.}{15d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (-4*a*b*e^4*n*x^(1/3))/(3*d^4) + (568*b^2*e^4*n^2*x^(1/3))/(315*d^4) - (32*
b^2*e^3*n^2*x)/(105*d^3) + (8*b^2*e^2*n^2*x^(5/3))/(105*d^2) - (1408*b^2*e^
(9/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(315*d^(9/2)) - (((4*I)/3)*b^2
*e^(9/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/d^(9/2) + (8*b^2*e^(9/2)*
n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt
[d]*x^(1/3))])/(3*d^(9/2)) - (4*b^2*e^4*n*x^(1/3)*Log[c*(d + e/x^(2/3))^n])
/(3*d^4) + (4*b*e^3*n*x*(a + b*Log[c*(d + e/x^(2/3))^n]))/(9*d^3) - (4*b*e^
2*n*x^(5/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(15*d^2) + (4*b*e*n*x^(7/3)*(
a + b*Log[c*(d + e/x^(2/3))^n]))/(21*d) + (4*b*e^(9/2)*n*ArcTan[(Sqrt[d]*x^
(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*d^(9/2)) + (x^3*(a + b
*Log[c*(d + e/x^(2/3))^n])^2)/3 - (((4*I)/3)*b^2*e^(9/2)*n^2*PolyLog[2, -1
+ (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/d^(9/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2dx = 3\,\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2dx, x, \sqrt[3]{x}\right)$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{1}{3}(4ben)\,\text{Subst}\left(\int \frac{x^6\left(a+b\log\left(c\left(d\right.\right.\right.}{d+\frac{e}{x^2}}\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{1}{3}(4ben)\,\text{Subst}\left(\int\left(-\frac{e^3\left(a+b\log\left(c\right.\right.}{d^4}\right.\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{(4ben)\,\text{Subst}\left(\int x^6\left(a+b\log\left(c\left(d+\right.\right.\right.\right.}{3d}$$

$$= -\frac{4abe^4n\sqrt[3]{x}}{3d^4} + \frac{4be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9d^3} - \frac{4be^2nx^{5/3}\left(a+b\log\left(\right.\right.}{15d^2}$$

$$= -\frac{4abe^4n\sqrt[3]{x}}{3d^4} - \frac{4b^2e^4n\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3d^4} + \frac{4be^3nx\left(a+b\log\left(c\left(d+\right.\right.\right.}{9d^3}$$

$$= -\frac{4abe^4n\sqrt[3]{x}}{3d^4} + \frac{8b^2e^4n^2\sqrt[3]{x}}{9d^4} - \frac{4b^2e^4n\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3d^4} + \frac{4be^3nx\left(a\right.}{}$$

$$= -\frac{4abe^4n\sqrt[3]{x}}{3d^4} + \frac{568b^2e^4n^2\sqrt[3]{x}}{315d^4} - \frac{32b^2e^3n^2x}{105d^3} + \frac{8b^2e^2n^2x^{5/3}}{105d^2} - \frac{32b^2e^{9/2}n^2\,\text{ta}}{9d}$$

$$= -\frac{4abe^4n\sqrt[3]{x}}{3d^4} + \frac{568b^2e^4n^2\sqrt[3]{x}}{315d^4} - \frac{32b^2e^3n^2x}{105d^3} + \frac{8b^2e^2n^2x^{5/3}}{105d^2} - \frac{1408b^2e^{9/2}n^2}{31}$$

$$= -\frac{4abe^4n\sqrt[3]{x}}{3d^4} + \frac{568b^2e^4n^2\sqrt[3]{x}}{315d^4} - \frac{32b^2e^3n^2x}{105d^3} + \frac{8b^2e^2n^2x^{5/3}}{105d^2} - \frac{1408b^2e^{9/2}n^2}{31}$$

Mathematica [C] time = 2.19, size = 735, normalized size = 1.50

$$\frac{1}{3}\left(x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 - 4ben\left(-\frac{e^2x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3d^3} + \frac{ex^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{5d^2} + \frac{e^7}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2 - 4*b*e*n*((a*e^3*x^(1/3))/d^4 - (2
*b*e^(7/2)*n*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))])/d^(9/2) - (2*b*e*n*x^(5/3)*
```

```
Hypergeometric2F1[-5/2, 1, -3/2, -(e/(d*x^(2/3)))])/(35*d^2) + (2*b*e^2*n*x
*Hypergeometric2F1[-3/2, 1, -1/2, -(e/(d*x^(2/3)))])/(15*d^3) - (2*b*e^3*n*
x^(1/3)*Hypergeometric2F1[-1/2, 1, 1/2, -(e/(d*x^(2/3)))])/(3*d^4) + (b*e^3
*x^(1/3)*Log[c*(d + e/x^(2/3))^n])/d^4 - (e^2*x*(a + b*Log[c*(d + e/x^(2/3)
)^n]))/(3*d^3) + (e*x^(5/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(5*d^2) - (x^
(7/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(7*d) + (e^(7/2)*(a + b*Log[c*(d +
e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)])/(2*(-d)^(9/2)) - (e^(7/2)*(
a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)])/(2*(-d)^(9
/2)) - (b*e^(7/2)*n*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] - Sqrt[-d
]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*Log[(Sqrt[-d]*x^
(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]] + 2*PolyLog
[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])]))/(4*(-d)^(9/2)) + (b*e^(7/2)*n*(
Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 2*Log[1/
2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 4*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])])
 + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*PolyLog[2, 1 + (Sqr
t[-d]*x^(1/3))/Sqrt[e]]))/(4*(-d)^(9/2))))/3
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^2x^2\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^2+2\,abx^2\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a^2x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*x^2*log(c*((d*x +
 e*x^(1/3))/x)^n) + a^2*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2*x^2, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*(d+e/x^(2/3))^n)+a)^2,x)
```

```
[Out] int(x^2*(b*ln(c*(d+e/x^(2/3))^n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,b^2n^2x^3\log\left(dx^{\frac{2}{3}}+e\right)^2-\int-\frac{9\left(b^2d\log(c)^2+2\,abd\log(c)+a^2d\right)x^3+9\left(b^2e\log(c)^2+2\,abe\log(c)+a^2e\right)x^{\frac{7}{3}}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*n^2*x^3*log(d*x^(2/3) + e)^2 - integrate(-1/9*(9*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^3 + 9*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x
^(7/3) - 2*(2*b^2*d*n*x^3 - 9*(b^2*d*log(c) + a*b*d)*x^3 - 9*(b^2*e*log(c)
+ a*b*e)*x^(7/3) + 18*(b^2*d*x^3 + b^2*e*x^(7/3))*log(x^(1/3*n)))*n*log(d*x
^(2/3) + e) + 36*(b^2*d*x^3 + b^2*e*x^(7/3))*log(x^(1/3*n))^2 - 36*((b^2*d*
log(c) + a*b*d)*x^3 + (b^2*e*log(c) + a*b*e)*x^(7/3))*log(x^(1/3*n)))/(d*x
+ e*x^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e/x^(2/3))^n))^2,x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e/x^(2/3))^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**n))**2,x)
```

```
[Out] Timed out
```

3.522 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=309

$$-\frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}}+x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{4aben\sqrt[3]{x}}{d}+\frac{4b^2en\sqrt[3]{x}\log\left(c\right.}{d}$$

```
[Out] 4*a*b*e*n*x^(1/3)/d+8*b^2*e^(3/2)*n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2))/d^(3/
2)+4*I*b^2*e^(3/2)*n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2))^2/d^(3/2)+4*b^2*e*n*
x^(1/3)*ln(c*(d+e/x^(2/3))^n)/d-4*b*e^(3/2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2
))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^(3/2)+x*(a+b*ln(c*(d+e/x^(2/3))^n))^2-8*b^
2*e^(3/2)*n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2))*ln(2-2*e^(1/2)/(-I*x^(1/3)*d^
(1/2)+e^(1/2)))/d^(3/2)+4*I*b^2*e^(3/2)*n^2*polylog(2,-1+2*e^(1/2)/(-I*x^(1
/3)*d^(1/2)+e^(1/2)))/d^(3/2)
```

Rubi [A] time = 0.44, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.650, Rules used = {2451, 2457, 2471, 2448, 263, 205, 2470, 12, 260, 6688, 4924, 4868, 2447}

$$\frac{4ib^2e^{3/2}n^2\text{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{d^{3/2}}-\frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}}+x\left(a+b\log\left(c\left(d\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (4*a*b*e*n*x^(1/3))/d + (8*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]
])/d^(3/2) + ((4*I)*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/d^
(3/2) - (8*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqr
t[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/d^(3/2) + (4*b^2*e*n*x^(1/3)*Log[c*(d
 + e/x^(2/3))^n])/d - (4*b*e^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a +
 b*Log[c*(d + e/x^(2/3))^n]))/d^(3/2) + x*(a + b*Log[c*(d + e/x^(2/3))^n])^
2 + ((4*I)*b^2*e^(3/2)*n^2*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]
*x^(1/3))])/d^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = 3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2 dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + (4ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{d+\frac{e}{x^2}}\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + (4ben)\,\text{Subst}\left(\int\left(\frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{d}\right.\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{(4ben)\,\text{Subst}\left(\int\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)\right.}{d}$$

$$= \frac{4aben\sqrt[3]{x}}{d} - \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}} + x\left(a+b\right.$$

$$= \frac{4aben\sqrt[3]{x}}{d} + \frac{4b^2en\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d} - \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\right.}{d^{3/2}}$$

$$= \frac{4aben\sqrt[3]{x}}{d} + \frac{4b^2en\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d} - \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\right.}{d^{3/2}}$$

$$= \frac{4aben\sqrt[3]{x}}{d} + \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}} + \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{d^{3/2}} + \frac{4b^2}{}$$

$$= \frac{4aben\sqrt[3]{x}}{d} + \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}} + \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{d^{3/2}} - \frac{8b^2}{}$$

$$= \frac{4aben\sqrt[3]{x}}{d} + \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}} + \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{d^{3/2}} - \frac{8b^2}{}$$

Mathematica [A] time = 1.07, size = 523, normalized size = 1.69

$$ben\left(-\frac{2\sqrt{e}\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{(-d)^{3/2}} + \frac{2\sqrt{e}\log\left(\sqrt{-d}\sqrt[3]{x}+\sqrt{e}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right.\right.}{(-d)^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] x*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + b*e*n*((4*a*x^(1/3))/d - (8*b*Sqrt[e
]*n*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))])/d^(3/2) + (4*b*x^(1/3)*Log[c*(d + e/
x^(2/3))^n])/d - (2*Sqrt[e]*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] -
Sqrt[-d]*x^(1/3)])/(-d)^(3/2) + (2*Sqrt[e]*(a + b*Log[c*(d + e/x^(2/3))^n])
*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)])/(-d)^(3/2) + (b*Sqrt[e]*n*(Log[Sqrt[e] -
Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x
^(1/3))/Sqrt[e])/2] - 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 -
 (Sqrt[-d]*x^(1/3))/Sqrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqr
t[e])]))/(-d)^(3/2) + (b*d*Sqrt[e]*n*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[
Sqrt[e] + Sqrt[-d]*x^(1/3)] + 2*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] -
 4*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])]) + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3
))/Sqrt[e])/2] - 4*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]]))/(-d)^(5/2))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^{2}\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^{n}\right)^{2}+2\,ab\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^{n}\right)+a^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(1/
3))/x)^n) + a^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(2/3))^n)+a)^2,x)
```

```
[Out] int((b*ln(c*(d+e/x^(2/3))^n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-2\left(2\,en\left(\frac{e\arctan\left(\frac{dx^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}\,d}-\frac{x^{\frac{1}{3}}}{d}\right)-x\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)ab+\left(n^{2}x\log\left(dx^{\frac{2}{3}}+e\right)^{2}-\int-\frac{3\,dx\log(c)^{2}+3\,ex^{\frac{1}{3}}\log(c)^{2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] -2*(2*e*n*(e*arctan(d*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*d) - x^(1/3)/d) - x*log
(c*(d + e/x^(2/3))^n))*a*b + (n^2*x*log(d*x^(2/3) + e)^2 - integrate(-1/3*(
3*d*x*log(c)^2 + 3*e*x^(1/3)*log(c)^2 - 2*(2*d*n*x - 3*d*x*log(c) - 3*e*x^(
1/3)*log(c) + 6*(d*x + e*x^(1/3))*log(x^(1/3*n)))*n*log(d*x^(2/3) + e) + 12
*(d*x + e*x^(1/3))*log(x^(1/3*n))^2 - 12*(d*x*log(c) + e*x^(1/3)*log(c))*lo
g(x^(1/3*n)))/(d*x + e*x^(1/3)), x))*b^2 + a^2*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^2,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2,x)
```

```
[Out] Timed out
```

3.523 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=361

$$-\frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}}-\frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}}+\frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right.\right.}{3x}$$

```
[Out] -8/9*b^2*n^2/x+32/3*b^2*d*n^2/e/x^(1/3)+32/3*b^2*d^(3/2)*n^2*arctan(x^(1/3)
*d^(1/2)/e^(1/2))/e^(3/2)+4*I*b^2*d^(3/2)*n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2
))^2/e^(3/2)+4/3*b*n*(a+b*ln(c*(d+e/x^(2/3))^n))/x-4*b*d*n*(a+b*ln(c*(d+e/x
^(2/3))^n))/e/x^(1/3)-4*b*d^(3/2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2))*(a+b*ln
(c*(d+e/x^(2/3))^n))/e^(3/2)-(a+b*ln(c*(d+e/x^(2/3))^n))^2/x-8*b^2*d^(3/2)*
n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2))*ln(2-2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^(1
/2)))/e^(3/2)+4*I*b^2*d^(3/2)*n^2*polylog(2,-1+2*e^(1/2)/(-I*x^(1/3)*d^(1/2
)+e^(1/2)))/e^(3/2)
```

Rubi [A] time = 0.59, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {2458, 2457, 2476, 2455, 263, 325, 205, 2470, 12, 260, 6688, 4924, 4868, 2447}

$$\frac{4ib^2d^{3/2}n^2\text{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}}-\frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}}-\frac{4bdn\left(a+b\log\left(c\left(d\right.\right.\right.}{e\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^2,x]
```

```
[Out] (-8*b^2*n^2)/(9*x) + (32*b^2*d*n^2)/(3*e*x^(1/3)) + (32*b^2*d^(3/2)*n^2*Arc
Tan[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(3*e^(3/2)) + ((4*I)*b^2*d^(3/2)*n^2*ArcTan
[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/e^(3/2) - (8*b^2*d^(3/2)*n^2*ArcTan[(Sqrt[d]
*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/e^(3
/2) + (4*b*n*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*x) - (4*b*d*n*(a + b*Log[
c*(d + e/x^(2/3))^n]))/(e*x^(1/3)) - (4*b*d^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3)
)/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^(3/2) - (a + b*Log[c*(d + e/
x^(2/3))^n])^2/x + ((4*I)*b^2*d^(3/2)*n^2*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt
[e] - I*Sqrt[d]*x^(1/3))])/e^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
```

```
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{x^4}dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - (4ben)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{\left(d+\frac{e}{x^2}\right)x^6}dx$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - (4ben)\,\text{Subst}\left(\int\left(\frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{ex^4} -\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - (4bn)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{x^4}dx,\right.$$

$$= \frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} - \frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} - \frac{4bd^{3/2}}{}$$

$$= \frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} - \frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} - \frac{4bd^{3/2}}{}$$

$$= -\frac{8b^2n^2}{9x} + \frac{8b^2dn^2}{e\sqrt[3]{x}} + \frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} - \frac{4bdn\left(a+b\log\left(c\left(\right.\right.\right.}{e\sqrt[3]{x}}$$

$$= -\frac{8b^2n^2}{9x} + \frac{32b^2dn^2}{3e\sqrt[3]{x}} + \frac{8b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}} + \frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right.}{e^{3/2}}$$

$$= -\frac{8b^2n^2}{9x} + \frac{32b^2dn^2}{3e\sqrt[3]{x}} + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}} + \frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{}}{\sqrt{e}}\right.}{e^{3/2}}$$

$$= -\frac{8b^2n^2}{9x} + \frac{32b^2dn^2}{3e\sqrt[3]{x}} + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}} + \frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{}}{\sqrt{e}}\right.}{e^{3/2}}$$

Mathematica [A] time = 1.27, size = 598, normalized size = 1.66

$$\frac{bn\left(12e^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+18(-d)^{3/2}x\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+18d\sqrt{-d}\,x\log\left(\sqrt{-d}\sqrt[3]{x}+\sqrt{e}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-36\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^2,x]
```

```
[Out] (-9*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + (b*n*(72*b*d*Sqrt[e]*n*x^(2/3) - 7
2*b*d^(3/2)*n*x*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))] - 8*b*n*(Sqrt[e]*(e - 3*d
*x^(2/3)) + 3*d^(3/2)*x*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))]) + 12*e^(3/2)*(a
+ b*Log[c*(d + e/x^(2/3))^n]) - 36*d*Sqrt[e]*x^(2/3)*(a + b*Log[c*(d + e/x^
(2/3))^n]) + 18*(-d)^(3/2)*x*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] -
 Sqrt[-d]*x^(1/3)] + 18*Sqrt[-d]*d*x*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[S
qrt[e] + Sqrt[-d]*x^(1/3)] + 9*b*Sqrt[-d]*d*n*x*(Log[Sqrt[e] - Sqrt[-d]*x^(
1/3)]*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt
[e])/2] - 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 - (Sqrt[-d]*x
^(1/3))/Sqrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])]) + 9*
b*(-d)^(3/2)*n*x*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] + Sqrt[-d]*x
^(1/3)] + 2*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 4*Log[-((Sqrt[-d]*x
^(1/3))/Sqrt[e])]) + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*P
olyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]])))/e^(3/2))/(9*x)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^2+2\,ab\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(1
/3))/x)^n) + a^2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x^2, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(2/3))^n)+a)^2/x^2,x)
```

```
[Out] int((b*ln(c*(d+e/x^(2/3))^n)+a)^2/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2n^2\log\left(dx^{\frac{2}{3}}+e\right)^2}{x}-\int-\frac{2\left(2\,b^2dnx+3\left(b^2d\log(c)+abd\right)x-6\left(b^2dx+b^2ex^{\frac{1}{3}}\right)\log\left(x^{\frac{1}{3}n}\right)+3\left(b^2e\log(c)+abe\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -b^2*n^2*log(d*x^(2/3) + e)^2/x - integrate(-1/3*(2*(2*b^2*d*n*x + 3*(b^2*d
*log(c) + a*b*d)*x - 6*(b^2*d*x + b^2*e*x^(1/3))*log(x^(1/3*n)) + 3*(b^2*e*
log(c) + a*b*e)*x^(1/3))*n*log(d*x^(2/3) + e) + 12*(b^2*d*x + b^2*e*x^(1/3)
)*log(x^(1/3*n))^2 + 3*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x - 12*((b
^2*d*log(c) + a*b*d)*x + (b^2*e*log(c) + a*b*e)*x^(1/3))*log(x^(1/3*n)) + 3
*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(1/3))/(d*x^3 + e*x^(7/3)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^2/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^2/x^2, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x**2,x)
```

```
[Out] Timed out
```

3.524 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$

Optimal. Leaf size=773

$$-\frac{3b^2e^6n^2\text{Li}_2\left(\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6}-\frac{77b^2e^6n^2\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^6}-\frac{3b^2e^6n^2\log}{}$$

```
[Out] 71/80*b^3*e^5*n^3*x^(2/3)/d^5-3/20*b^3*e^4*n^3*x^(4/3)/d^4+1/40*b^3*e^3*n^3
*x^2/d^3-71/80*b^3*e^6*n^3*ln(d+e/x^(2/3))/d^6-77/40*b^2*e^5*n^2*(d+e/x^(2/
3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^6+47/80*b^2*e^4*n^2*x^(4/3)*(a+b*
ln(c*(d+e/x^(2/3))^n))/d^4-9/40*b^2*e^3*n^2*x^2*(a+b*ln(c*(d+e/x^(2/3))^n))
/d^3+3/40*b^2*e^2*n^2*x^(8/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^2-77/40*b^2*e^6
*n^2*ln(1-d/(d+e/x^(2/3)))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^6+3/4*b*e^5*n*(d+e
/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^6-3/8*b*e^4*n*x^(4/3)*(a+
b*ln(c*(d+e/x^(2/3))^n))^2/d^4+1/4*b*e^3*n*x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^
2/d^3-3/16*b*e^2*n*x^(8/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^2+3/20*b*e*n*x^(
10/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d+3/4*b*e^6*n*ln(1-d/(d+e/x^(2/3)))*(a+
b*ln(c*(d+e/x^(2/3))^n))^2/d^6+1/4*x^4*(a+b*ln(c*(d+e/x^(2/3))^n))^3-3/2*b^
2*e^6*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(-e/d/x^(2/3))/d^6-15/8*b^3*e^6*n^3
*ln(x)/d^6+77/40*b^3*e^6*n^3*polylog(2,d/(d+e/x^(2/3)))/d^6-3/2*b^2*e^6*n^2
*(a+b*ln(c*(d+e/x^(2/3))^n))*polylog(2,d/(d+e/x^(2/3)))/d^6-3/2*b^3*e^6*n^3
*polylog(2,1+e/d/x^(2/3))/d^6-3/2*b^3*e^6*n^3*polylog(3,d/(d+e/x^(2/3)))/d^
6
```

Rubi [A] time = 3.02, antiderivative size = 746, normalized size of antiderivative = 0.97, number of steps used = 73, number of rules used = 17, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.708, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31, 44}

$$\frac{3b^2e^6n^2\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6}-\frac{137b^3e^6n^3\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)}{40d^6}-\frac{3b^3e^6n^3\text{PolyLog}\left(3,\right.}{2d^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (71*b^3*e^5*n^3*x^(2/3))/(80*d^5) - (3*b^3*e^4*n^3*x^(4/3))/(20*d^4) + (b^3
*e^3*n^3*x^2)/(40*d^3) - (71*b^3*e^6*n^3*Log[d + e/x^(2/3)])/(80*d^6) - (77
*b^2*e^5*n^2*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(40*
d^6) + (47*b^2*e^4*n^2*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(80*d^4) -
 (9*b^2*e^3*n^2*x^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(40*d^3) + (3*b^2*e^2
*n^2*x^(8/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(40*d^2) + (77*b*e^6*n*(a +
b*Log[c*(d + e/x^(2/3))^n])^2)/(80*d^6) + (3*b*e^5*n*(d + e/x^(2/3))*x^(2/3
)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(4*d^6) - (3*b*e^4*n*x^(4/3)*(a + b*L
og[c*(d + e/x^(2/3))^n])^2)/(8*d^4) + (b*e^3*n*x^2*(a + b*Log[c*(d + e/x^(2
/3))^n])^2)/(4*d^3) - (3*b*e^2*n*x^(8/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2
)/(16*d^2) + (3*b*e*n*x^(10/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(20*d) -
 (e^6*(a + b*Log[c*(d + e/x^(2/3))^n])^3)/(4*d^6) + (x^4*(a + b*Log[c*(d +
e/x^(2/3))^n])^3)/4 - (137*b^2*e^6*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log
[-(e/(d*x^(2/3)))])/(40*d^6) + (3*b*e^6*n*(a + b*Log[c*(d + e/x^(2/3))^n])^
2*Log[-(e/(d*x^(2/3)))])/(4*d^6) - (15*b^3*e^6*n^3*Log[x])/(8*d^6) - (137*b
^3*e^6*n^3*PolyLog[2, 1 + e/(d*x^(2/3))])/(40*d^6) + (3*b^2*e^6*n^2*(a + b*
Log[c*(d + e/x^(2/3))^n])*PolyLog[2, 1 + e/(d*x^(2/3))])/(2*d^6) - (3*b^3*e
^6*n^3*PolyLog[3, 1 + e/(d*x^(2/3))])/(2*d^6)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = -\left(\frac{3}{2}\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x^7}dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{1}{4}(3ben)\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+\right.\right.}{x^6(d+ex}\right.$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{1}{4}(3bn)\text{Subst}\left(\int \frac{\left(a+b\log\left(cx^n\right)\right)^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\right.$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{(3bn)\text{Subst}\left(\int \frac{\left(a+b\log\left(cx^n\right)\right)^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}dx, x, d\right.}{4d}$$

$$= \frac{3benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{20d} + \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right.$$

$$= -\frac{3be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{16d^2} + \frac{3benx^{10/3}\left(a+b\log\left(c\left(d+\right.\right.\right.}{20d}$$

$$= \frac{3b^2e^2n^2x^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^2} + \frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.}{4d^3}$$

$$= -\frac{9b^2e^3n^2x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^3} + \frac{3b^2e^2n^2x^{8/3}\left(a+b\log\left(c\left(d+\right.\right.\right.}{40d^2}$$

$$= \frac{3b^3e^5n^3x^{2/3}}{40d^5} - \frac{3b^3e^4n^3x^{4/3}}{80d^4} + \frac{b^3e^3n^3x^2}{40d^3} - \frac{3b^3e^6n^3\log\left(d+\frac{e}{x^{2/3}}\right)}{40d^6} + \frac{47b}{}$$

$$= \frac{3b^3e^5n^3x^{2/3}}{10d^5} - \frac{3b^3e^4n^3x^{4/3}}{20d^4} + \frac{b^3e^3n^3x^2}{40d^3} - \frac{3b^3e^6n^3\log\left(d+\frac{e}{x^{2/3}}\right)}{10d^6} - \frac{77b}{}$$

$$= \frac{71b^3e^5n^3x^{2/3}}{80d^5} - \frac{3b^3e^4n^3x^{4/3}}{20d^4} + \frac{b^3e^3n^3x^2}{40d^3} - \frac{71b^3e^6n^3\log\left(d+\frac{e}{x^{2/3}}\right)}{80d^6} - \frac{7}{}$$

$$= \frac{71b^3e^5n^3x^{2/3}}{80d^5} - \frac{3b^3e^4n^3x^{4/3}}{20d^4} + \frac{b^3e^3n^3x^2}{40d^3} - \frac{71b^3e^6n^3\log\left(d+\frac{e}{x^{2/3}}\right)}{80d^6} - \frac{7}{}$$

Mathematica [A] time = 2.59, size = 1014, normalized size = 1.31

$$\frac{20x^4\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 d^6 + 60bnx^4\log\left(d+\frac{e}{x^{2/3}}\right)\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (60*b*d*e^5*n*x^(2/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))
^n])^2 - 30*b*d^2*e^4*n*x^(4/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d +
e/x^(2/3))^n])^2 + 20*b*d^3*e^3*n*x^2*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c
*(d + e/x^(2/3))^n])^2 - 15*b*d^4*e^2*n*x^(8/3)*(a - b*n*Log[d + e/x^(2/3)]
 + b*Log[c*(d + e/x^(2/3))^n])^2 + 12*b*d^5*e*n*x^(10/3)*(a - b*n*Log[d + e
/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2 + 60*b*d^6*n*x^4*Log[d + e/x^(2/3
)]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2 + 20*d^6*x^4
*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^3 - 60*b*e^6*n*(
a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2*Log[e + d*x^(2/3
)] + b^2*n^2*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])*(d*e
^2*x^(2/3)*(-154*e^3 + 47*d*e^2*x^(2/3) - 18*d^2*e*x^(4/3) + 6*d^3*x^2) - 6
0*(e^6 - d^6*x^4)*Log[d + e/x^(2/3)]^2 - 274*e^6*Log[-(e/(d*x^(2/3)))] + 2*
e*Log[d + e/x^(2/3)]*(137*e^5 + 60*d*e^4*x^(2/3) - 30*d^2*e^3*x^(4/3) + 20*
d^3*e^2*x^2 - 15*d^4*e*x^(8/3) + 12*d^5*x^(10/3) + 60*e^5*Log[-(e/(d*x^(2/3
)))])) + 120*e^6*PolyLog[2, 1 + e/(d*x^(2/3))]) + b^3*n^3*(3*d^4*e^2*x^(8/3)
*(2 - 5*Log[d + e/x^(2/3)])*Log[d + e/x^(2/3)] + 12*d^5*e*x^(10/3)*Log[d +
e/x^(2/3)]^2 + 20*d^6*x^4*Log[d + e/x^(2/3)]^3 + 2*d^3*e^3*x^2*(1 - 9*Log[d
 + e/x^(2/3)] + 10*Log[d + e/x^(2/3)]^2) - d^2*e^4*x^(4/3)*(12 - 47*Log[d +
 e/x^(2/3)] + 30*Log[d + e/x^(2/3)]^2) + d*e^5*x^(2/3)*(71 - 154*Log[d + e/
x^(2/3)] + 60*Log[d + e/x^(2/3)]^2) + 225*e^6*(-Log[d + e/x^(2/3)] + Log[-(
e/(d*x^(2/3)))]) + 137*e^6*(Log[d + e/x^(2/3)]*(Log[d + e/x^(2/3)] - 2*Log[
-(e/(d*x^(2/3)))]) - 2*PolyLog[2, 1 + e/(d*x^(2/3))]) - 20*e^6*(Log[d + e/x
^(2/3)]^2*(Log[d + e/x^(2/3)] - 3*Log[-(e/(d*x^(2/3)))]) - 6*Log[d + e/x^(2
/3)]*PolyLog[2, 1 + e/(d*x^(2/3))] + 6*PolyLog[3, 1 + e/(d*x^(2/3))])))/(80
*d^6)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^3\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^3+3\,ab^2x^3\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^2+3\,a^2bx^3\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a^3x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*x^3*log(c*((d*x
 + e*x^(1/3))/x)^n)^2 + 3*a^2*b*x^3*log(c*((d*x + e*x^(1/3))/x)^n) + a^3*x^
3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3*x^3, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*ln(c*(d+e/x^(2/3))^n)+a)^3,x)
```

```
[Out] int(x^3*(b*ln(c*(d+e/x^(2/3))^n)+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,b^3n^3x^4\log\left(dx^{\frac{2}{3}}+e\right)^3-\int-\frac{2\left(b^3d\log(c)^3+3\,ab^2d\log(c)^2+3\,a^2bd\log(c)+a^3d\right)x^4-\left(b^3dnx^4-6\left(b^3d\log(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*n^3*x^4*log(d*x^(2/3) + e)^3 - integrate(-1/2*(2*(b^3*d*log(c)^3 +
3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^4 - (b^3*d*n*x^4 - 6*(b^3*
d*log(c) + a*b^2*d)*x^4 - 6*(b^3*e*log(c) + a*b^2*e)*x^(10/3) + 12*(b^3*d*x
^4 + b^3*e*x^(10/3))*log(x^(1/3*n)))*n^2*log(d*x^(2/3) + e)^2 + 2*(b^3*e*lo
g(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(10/3) - 16*(b^3*
d*x^4 + b^3*e*x^(10/3))*log(x^(1/3*n))^3 + 6*((b^3*d*log(c)^2 + 2*a*b^2*d*l
og(c) + a^2*b*d)*x^4 + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(10/
3) + 4*(b^3*d*x^4 + b^3*e*x^(10/3))*log(x^(1/3*n))^2 - 4*((b^3*d*log(c) + a
*b^2*d)*x^4 + (b^3*e*log(c) + a*b^2*e)*x^(10/3))*log(x^(1/3*n)))*n*log(d*x^
(2/3) + e) + 24*((b^3*d*log(c) + a*b^2*d)*x^4 + (b^3*e*log(c) + a*b^2*e)*x^
(10/3))*log(x^(1/3*n))^2 - 12*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d
)*x^4 + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(10/3))*log(x^(1/3*
n)))/(d*x + e*x^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e/x^(2/3))^n))^3,x)
```

```
[Out] int(x^3*(a + b*log(c*(d + e/x^(2/3))^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

3.525 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=451

$$\frac{3b^2e^3n^2\text{Li}_2\left(\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}+\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3}+\frac{3b^2e^3n^2\log(}{}$$

```
[Out] 3/2*b^2*e^2*n^2*(d+e/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3+3/2*b
^2*e^3*n^2*ln(1-d/(d+e/x^(2/3)))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3-3/2*b*e^2*
n*(d+e/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^3+3/4*b*e*n*x^(4/3)
*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d-3/2*b*e^3*n*ln(1-d/(d+e/x^(2/3)))*(a+b*ln(
c*(d+e/x^(2/3))^n))^2/d^3+1/2*x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^3+3*b^2*e^3*n
^2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(-e/d/x^(2/3))/d^3+b^3*e^3*n^3*ln(x)/d^3-3
/2*b^3*e^3*n^3*polylog(2,d/(d+e/x^(2/3)))/d^3+3*b^2*e^3*n^2*(a+b*ln(c*(d+e/
x^(2/3))^n))*polylog(2,d/(d+e/x^(2/3)))/d^3+3*b^3*e^3*n^3*polylog(2,1+e/d/x
^(2/3))/d^3+3*b^3*e^3*n^3*polylog(3,d/(d+e/x^(2/3)))/d^3
```

Rubi [A] time = 1.00, antiderivative size = 428, normalized size of antiderivative = 0.95, number of steps used = 22, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2454, 2398, 2411, 2347, 2344, 2302, 30, 2317, 2374, 6589, 2318, 2391, 2319, 2301, 2314, 31}

$$-\frac{3b^2e^3n^2\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}+\frac{9b^3e^3n^3\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)}{2d^3}+\frac{3b^3e^3n^3\text{PolyLog}\left(3,\right.}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (3*b^2*e^2*n^2*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2
*d^3) - (3*b*e^3*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(4*d^3) - (3*b*e^2*n
*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(2*d^3) + (3*b
*e*n*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(4*d) + (e^3*(a + b*Log[c*
(d + e/x^(2/3))^n])^3)/(2*d^3) + (x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^3)/2
 + (9*b^2*e^3*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))])/(
2*d^3) - (3*b*e^3*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2*Log[-(e/(d*x^(2/3)))
])/(2*d^3) + (b^3*e^3*n^3*Log[x])/d^3 + (9*b^3*e^3*n^3*PolyLog[2, 1 + e/(d*
x^(2/3))])/(2*d^3) - (3*b^2*e^3*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*PolyLo
g[2, 1 + e/(d*x^(2/3))])/d^3 + (3*b^3*e^3*n^3*PolyLog[3, 1 + e/(d*x^(2/3))]
)/d^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2318

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[(x*(a + b*Log[c*x^n])^p)/(d*(d + e*x)), x] - Dist[(b*n*p)/d,
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
```

```
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = -\left(\frac{3}{2}\operatorname{Subst}\left(\int\frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x^4}dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{1}{2}(3ben)\operatorname{Subst}\left(\int\frac{\left(a+b\log\left(c(d+\right.\right.}{x^3(d+ex)}\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{1}{2}(3bn)\operatorname{Subst}\left(\int\frac{\left(a+b\log\left(cx^n\right)\right)^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{(3bn)\operatorname{Subst}\left(\int\frac{\left(a+b\log(cx^n)\right)^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}dx, x, d\right.}{2d}$$

$$= \frac{3benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3$$

$$= -\frac{3be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3} + \frac{3benx^{4/3}\left(a+b\log\right.}{4d}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3} - \frac{3be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}}{}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3} - \frac{3be^3n\left(a+b\log\left(c\right.\right.}{4d^3}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3} - \frac{3be^3n\left(a+b\log\left(c\right.\right.}{4d^3}$$

Mathematica [A] time = 1.42, size = 683, normalized size = 1.51

$$\frac{6b^2n^2\left(\left(d^3x^2+e^3\right)\log^2\left(d+\frac{e}{x^{2/3}}\right)+e\log\left(d+\frac{e}{x^{2/3}}\right)\left(d^2x^{4/3}-2e^2\log\left(-\frac{e}{dx^{2/3}}\right)-2dex^{2/3}-3e^2\right)-2e^3\operatorname{Li}_2\left(\frac{e}{dx^{2/3}}+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (-6*b*d*e^2*n*x^(2/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))
^n])^2 + 3*b*d^2*e*n*x^(4/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x
^(2/3))^n])^2 + 6*b*d^3*n*x^2*Log[d + e/x^(2/3)]*(a - b*n*Log[d + e/x^(2/3)
] + b*Log[c*(d + e/x^(2/3))^n])^2 + 2*d^3*x^2*(a - b*n*Log[d + e/x^(2/3)] +
 b*Log[c*(d + e/x^(2/3))^n])^3 + 6*b*e^3*n*(a - b*n*Log[d + e/x^(2/3)] + b*
Log[c*(d + e/x^(2/3))^n])^2*Log[e + d*x^(2/3)] + 6*b^2*n^2*(a - b*n*Log[d +
 e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])*((e^3 + d^3*x^2)*Log[d + e/x^(2/3
)]^2 + e^2*(d*x^(2/3) + 3*e*Log[-(e/(d*x^(2/3)))]) + e*Log[d + e/x^(2/3)]*(
-3*e^2 - 2*d*e*x^(2/3) + d^2*x^(4/3) - 2*e^2*Log[-(e/(d*x^(2/3)))]) - 2*e^3
*PolyLog[2, 1 + e/(d*x^(2/3))]) - b^3*n^3*(-6*e^3*Log[d + e/x^(2/3)] - 6*d*
e^2*x^(2/3)*Log[d + e/x^(2/3)] + 9*e^3*Log[d + e/x^(2/3)]^2 + 6*d*e^2*x^(2/
3)*Log[d + e/x^(2/3)]^2 - 3*d^2*e*x^(4/3)*Log[d + e/x^(2/3)]^2 - 2*e^3*Log[
```

```
d + e/x^(2/3)]^3 - 2*d^3*x^2*Log[d + e/x^(2/3)]^3 + 6*e^3*Log[-(e/(d*x^(2/3
)))] - 18*e^3*Log[d + e/x^(2/3)]*Log[-(e/(d*x^(2/3)))] + 6*e^3*Log[d + e/x^
(2/3)]^2*Log[-(e/(d*x^(2/3)))] + 6*e^3*(-3 + 2*Log[d + e/x^(2/3)])*PolyLog[
2, 1 + e/(d*x^(2/3))] - 12*e^3*PolyLog[3, 1 + e/(d*x^(2/3))]))/(4*d^3)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^3+3\,ab^2x\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^2+3\,a^2bx\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a^3x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*x*log(c*((d*x + e
*x^(1/3))/x)^n)^2 + 3*a^2*b*x*log(c*((d*x + e*x^(1/3))/x)^n) + a^3*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3*x, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln(c*(d+e/x^(2/3))^n)+a)^3,x)
```

```
[Out] int(x*(b*ln(c*(d+e/x^(2/3))^n)+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,b^3n^3x^2\log\left(dx^{\frac{2}{3}}+e\right)^3-\int\frac{\left(b^3dnx^2-3\left(b^3d\log(c)+ab^2d\right)x^2-3\left(b^3e\log(c)+ab^2e\right)x^{\frac{4}{3}}+6\left(b^3dx^2+b^3ex^{\frac{4}{3}}\right)\log\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*n^3*x^2*log(d*x^(2/3) + e)^3 - integrate(((b^3*d*n*x^2 - 3*(b^3*d*l
og(c) + a*b^2*d)*x^2 - 3*(b^3*e*log(c) + a*b^2*e)*x^(4/3) + 6*(b^3*d*x^2 +
b^3*e*x^(4/3))*log(x^(1/3*n)))*n^2*log(d*x^(2/3) + e)^2 + 8*(b^3*d*x^2 + b^
3*e*x^(4/3))*log(x^(1/3*n))^3 - (b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^
2*b*d*log(c) + a^3*d)*x^2 - 3*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d
)*x^2 + 4*(b^3*d*x^2 + b^3*e*x^(4/3))*log(x^(1/3*n))^2 + (b^3*e*log(c)^2 +
2*a*b^2*e*log(c) + a^2*b*e)*x^(4/3) - 4*((b^3*d*log(c) + a*b^2*d)*x^2 + (b^
3*e*log(c) + a*b^2*e)*x^(4/3))*log(x^(1/3*n)))*n*log(d*x^(2/3) + e) - 12*((
b^3*d*log(c) + a*b^2*d)*x^2 + (b^3*e*log(c) + a*b^2*e)*x^(4/3))*log(x^(1/3*
n))^2 - (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^
```

```
(4/3) + 6*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^2 + (b^3*e*log(c
)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(4/3))*log(x^(1/3*n)))/(d*x + e*x^(1/3)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(2/3))^n))^3,x)
```

```
[Out] int(x*(a + b*log(c*(d + e/x^(2/3))^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

3.526 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx$

Optimal. Leaf size=139

$$9b^2n^2\text{Li}_3\left(\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{9}{2}bn\text{Li}_2\left(\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{3}{2}\log\left(-\frac{e}{dx^{2/3}}\right.$$

```
[Out] -3/2*(a+b*ln(c*(d+e/x^(2/3))^n))^3*ln(-e/d/x^(2/3))-9/2*b*n*(a+b*ln(c*(d+e/
x^(2/3))^n))^2*polylog(2,1+e/d/x^(2/3))+9*b^2*n^2*(a+b*ln(c*(d+e/x^(2/3))^n
))*polylog(3,1+e/d/x^(2/3))-9*b^3*n^3*polylog(4,1+e/d/x^(2/3))
```

Rubi [A] time = 0.20, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2454, 2396, 2433, 2374, 2383, 6589}

$$9b^2n^2\text{PolyLog}\left(3,\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{9}{2}bn\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x,x]
```

```
[Out] (-3*(a + b*Log[c*(d + e/x^(2/3))^n])^3*Log[-(e/(d*x^(2/3)))])/2 - (9*b*n*(a
 + b*Log[c*(d + e/x^(2/3))^n])^2*PolyLog[2, 1 + e/(d*x^(2/3))])/2 + 9*b^2*n
^2*(a + b*Log[c*(d + e/x^(2/3))^n])*PolyLog[3, 1 + e/(d*x^(2/3))] - 9*b^3*n
^3*PolyLog[4, 1 + e/(d*x^(2/3))]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx = -\left(\frac{3}{2}\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x}\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right)+\frac{1}{2}(9ben)\,\text{Subst}\left(\int \frac{\log\left(-\right.}{}\right.$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right)+\frac{1}{2}(9bn)\,\text{Subst}\left(\int \frac{\left(a+b\,\mathrm{lo}\right.}{}\right.$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{9}{2}bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{9}{2}bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{9}{2}bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.$$

Mathematica [B] time = 0.26, size = 341, normalized size = 2.45

$$\frac{9}{2}b^2n^2\left(-2\text{Li}_3\left(\frac{e}{dx^{2/3}}+1\right)+2\text{Li}_2\left(\frac{e}{dx^{2/3}}+1\right)\log\left(d+\frac{e}{x^{2/3}}\right)+\log\left(-\frac{e}{dx^{2/3}}\right)\log^2\left(d+\frac{e}{x^{2/3}}\right)\right)\left(-a-b\log\left(c\left(d\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2*((Log[d + e/x^(
2/3)] - Log[1 + e/(d*x^(2/3))])*Log[x] + (3*PolyLog[2, -(e/(d*x^(2/3)))])/2
) + (9*b^2*n^2*(-a + b*n*Log[d + e/x^(2/3)] - b*Log[c*(d + e/x^(2/3))^n])*(
Log[d + e/x^(2/3)]^2*Log[-(e/(d*x^(2/3)))] + 2*Log[d + e/x^(2/3)]*PolyLog[2
, 1 + e/(d*x^(2/3))] - 2*PolyLog[3, 1 + e/(d*x^(2/3))]))/2 - (3*b^3*n^3*(Lo
g[d + e/x^(2/3)]^3*Log[-(e/(d*x^(2/3)))] + 3*Log[d + e/x^(2/3)]^2*PolyLog[2
, 1 + e/(d*x^(2/3))] - 6*Log[d + e/x^(2/3)]*PolyLog[3, 1 + e/(d*x^(2/3))] +
 6*PolyLog[4, 1 + e/(d*x^(2/3))]))/2
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^3\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^3+3\,ab^2\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^2+3\,a^2b\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^
(1/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(1/3))/x)^n) + a^3)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(2/3))^n)+a)^3/x,x)
```

```
[Out] int((b*ln(c*(d+e/x^(2/3))^n)+a)^3/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^3n^3\log\left(dx^{\frac{2}{3}}+e\right)^3\log(x)-\int\frac{\left(2\,b^3dnx\log(x)-3\left(b^3d\log(c)+ab^2d\right)x+6\left(b^3dx+b^3ex^{\frac{1}{3}}\right)\log\left(x^{\frac{1}{3}n}\right)-3\left(b^3e\log\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*n^3*log(d*x^(2/3) + e)^3*log(x) - integrate(((2*b^3*d*n*x*log(x) - 3*(b
^3*d*log(c) + a*b^2*d)*x + 6*(b^3*d*x + b^3*e*x^(1/3))*log(x^(1/3*n)) - 3*(
b^3*e*log(c) + a*b^2*e)*x^(1/3))*n^2*log(d*x^(2/3) + e)^2 + 8*(b^3*d*x + b^
3*e*x^(1/3))*log(x^(1/3*n))^3 - 3*(4*(b^3*d*x + b^3*e*x^(1/3))*log(x^(1/3*n
))^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x - 4*((b^3*d*log(c) +
 a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(1/3))*log(x^(1/3*n)) + (b^3*e*log
(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(1/3))*n*log(d*x^(2/3) + e) - 12*((b^
3*d*log(c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(1/3))*log(x^(1/3*n))^
2 - (b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x + 6*
((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x + (b^3*e*log(c)^2 + 2*a*b^
```

```
2*e*log(c) + a^2*b*e)*x^(1/3))*log(x^(1/3*n)) - (b^3*e*log(c)^3 + 3*a*b^2*e
*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(1/3))/(d*x^2 + e*x^(4/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x,x)
[Out] Timed out
```

3.527 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=449

$$-\frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^3}+\frac{9b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4e^3}-\frac{9ab^2d^2n^2}{e^2x^{2/3}}+\frac{9bd^2n\left(d-\right.}{}$$

```
[Out] -9/8*b^3*d*n^3*(d+e/x^(2/3))^2/e^3+1/9*b^3*n^3*(d+e/x^(2/3))^3/e^3-9*a*b^2*
d^2*n^2/e^2/x^(2/3)+9*b^3*d^2*n^3/e^2/x^(2/3)-9*b^3*d^2*n^2*(d+e/x^(2/3))*l
n(c*(d+e/x^(2/3))^n)/e^3+9/4*b^2*d*n^2*(d+e/x^(2/3))^2*(a+b*ln(c*(d+e/x^(2/
3))^n))/e^3-1/3*b^2*n^2*(d+e/x^(2/3))^3*(a+b*ln(c*(d+e/x^(2/3))^n))/e^3+9/2
*b*d^2*n*(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e^3-9/4*b*d*n*(d+e/x^(
2/3))^2*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e^3+1/2*b*n*(d+e/x^(2/3))^3*(a+b*ln(c
*(d+e/x^(2/3))^n))^2/e^3-3/2*d^2*(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))^
3/e^3+3/2*d*(d+e/x^(2/3))^2*(a+b*ln(c*(d+e/x^(2/3))^n))^3/e^3-1/2*(d+e/x^(2
/3))^3*(a+b*ln(c*(d+e/x^(2/3))^n))^3/e^3
```

Rubi [A] time = 0.46, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304}

$$-\frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^3}+\frac{9b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4e^3}-\frac{9ab^2d^2n^2}{e^2x^{2/3}}+\frac{9bd^2n\left(d-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^3,x]
```

```
[Out] (-9*b^3*d*n^3*(d + e/x^(2/3))^2)/(8*e^3) + (b^3*n^3*(d + e/x^(2/3))^3)/(9*e
^3) - (9*a*b^2*d^2*n^2)/(e^2*x^(2/3)) + (9*b^3*d^2*n^3)/(e^2*x^(2/3)) - (9*
b^3*d^2*n^2*(d + e/x^(2/3))*Log[c*(d + e/x^(2/3))^n])/e^3 + (9*b^2*d*n^2*(d
 + e/x^(2/3))^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(4*e^3) - (b^2*n^2*(d + e
/x^(2/3))^3*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*e^3) + (9*b*d^2*n*(d + e/x
^(2/3))*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(2*e^3) - (9*b*d*n*(d + e/x^(2/
3))^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(4*e^3) + (b*n*(d + e/x^(2/3))^3*
(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(2*e^3) - (3*d^2*(d + e/x^(2/3))*(a + b
*Log[c*(d + e/x^(2/3))^n])^3)/(2*e^3) + (3*d*(d + e/x^(2/3))^2*(a + b*Log[c
*(d + e/x^(2/3))^n])^3)/(2*e^3) - ((d + e/x^(2/3))^3*(a + b*Log[c*(d + e/x^
(2/3))^n])^3)/(2*e^3)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx = -\left(\frac{3}{2}\operatorname{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= -\left(\frac{3}{2}\operatorname{Subst}\left(\int\left(\frac{d^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^2} - \frac{2d(d+ex)\left(a+b\log\left(c(d+ex\right.\right.}{e^2}\right.\right.\right.$$

$$= -\frac{3\operatorname{Subst}\left(\int(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \frac{1}{x^{2/3}}\right)}{2e^2} + \frac{(3d)\operatorname{Subst}\left(\int(d\right.}{}$$

$$= -\frac{3\operatorname{Subst}\left(\int x^2\left(a+b\log\left(cx^n\right)\right)^3\,dx, x, d+\frac{e}{x^{2/3}}\right)}{2e^3} + \frac{(3d)\operatorname{Subst}\left(\int x\left(a+b\log\right.\right.}{e^3}$$

$$= -\frac{3d^2\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3} + \frac{3d\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\right.\right.\right.}{2e^3}$$

$$= \frac{9bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2e^3} - \frac{9bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(\right.\right.\right.}{4e^3}$$

$$= -\frac{9b^3dn^3\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^3} + \frac{b^3n^3\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3} - \frac{9ab^2d^2n^2}{e^2x^{2/3}} + \frac{9b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+\right.}{4e}$$

$$= -\frac{9b^3dn^3\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^3} + \frac{b^3n^3\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3} - \frac{9ab^2d^2n^2}{e^2x^{2/3}} + \frac{9b^3d^2n^3}{e^2x^{2/3}} - \frac{9b^3d^2n^2\left(d\right.}{}$$

Mathematica [A] time = 1.53, size = 692, normalized size = 1.54

$$\frac{-36a^3e^3 - 6b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\left(18a^2e^3 + 6bd^3nx^2(6a-11bn)\log\left(dx^{2/3}+e\right) + 4bd^3nx^2\log(x)(11bn-6a) - 6ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^3,x]
```

```
[Out] (-36*a^3*e^3 + 36*a^2*b*e^3*n - 24*a*b^2*e^3*n^2 + 8*b^3*e^3*n^3 - 54*a^2*b
*d*e^2*n*x^(2/3) + 90*a*b^2*d*e^2*n^2*x^(2/3) - 57*b^3*d*e^2*n^3*x^(2/3) +
108*a^2*b*d^2*e*n*x^(4/3) - 396*a*b^2*d^2*e*n^2*x^(4/3) + 510*b^3*d^2*e*n^3
*x^(4/3) + 72*b^3*d^3*n^3*x^2*Log[d + e/x^(2/3)]^3 - 36*b^3*e^3*Log[c*(d +
e/x^(2/3))^n]^3 - 108*a^2*b*d^3*n*x^2*Log[e + d*x^(2/3)] + 396*a*b^2*d^3*n^
2*x^2*Log[e + d*x^(2/3)] - 510*b^3*d^3*n^3*x^2*Log[e + d*x^(2/3)] + 12*b^2*
d^3*n^2*x^2*Log[d + e/x^(2/3)]*(6*a - 11*b*n + 6*b*Log[c*(d + e/x^(2/3))^n]
)*(3*Log[e + d*x^(2/3)] - 2*Log[x]) + 72*a^2*b*d^3*n*x^2*Log[x] - 264*a*b^2
*d^3*n^2*x^2*Log[x] + 340*b^3*d^3*n^3*x^2*Log[x] - 18*b^2*d^3*n^2*x^2*Log[d
 + e/x^(2/3)]^2*(6*a - 11*b*n + 6*b*Log[c*(d + e/x^(2/3))^n] + 6*b*n*Log[e
+ d*x^(2/3)] - 4*b*n*Log[x]) + 18*b^2*Log[c*(d + e/x^(2/3))^n]^2*(e*(-6*a*e
^2 + 2*b*e^2*n - 3*b*d*e*n*x^(2/3) + 6*b*d^2*n*x^(4/3)) - 6*b*d^3*n*x^2*Log
[e + d*x^(2/3)] + 4*b*d^3*n*x^2*Log[x]) - 6*b*Log[c*(d + e/x^(2/3))^n]*(18*
a^2*e^3 - 6*a*b*e*n*(2*e^2 - 3*d*e*x^(2/3) + 6*d^2*x^(4/3)) + b^2*e*n^2*(4*
e^2 - 15*d*e*x^(2/3) + 66*d^2*x^(4/3)) + 6*b*d^3*n*(6*a - 11*b*n)*x^2*Log[e
 + d*x^(2/3)] + 4*b*d^3*n*(-6*a + 11*b*n)*x^2*Log[x]))/(72*e^3*x^2)
```

fricas [A] time = 0.45, size = 725, normalized size = 1.61

$$\frac{8b^3e^3n^3 - 36b^3e^3\log(c)^3 - 24ab^2e^3n^2 + 36a^2be^3n - 36a^3e^3 - 36\left(b^3d^3n^3x^2 + b^3e^3n^3\right)\log\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^3 + 36\left(b^3e^3n\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/72*(8*b^3*e^3*n^3 - 36*b^3*e^3*log(c)^3 - 24*a*b^2*e^3*n^2 + 36*a^2*b*e^3
*n - 36*a^3*e^3 - 36*(b^3*d^3*n^3*x^2 + b^3*e^3*n^3)*log((d*x + e*x^(1/3))/
x)^3 + 36*(b^3*e^3*n - 3*a*b^2*e^3)*log(c)^2 + 18*(6*b^3*d^2*e*n^3*x^(4/3)
- 3*b^3*d*e^2*n^3*x^(2/3) + 2*b^3*e^3*n^3 - 6*a*b^2*e^3*n^2 + (11*b^3*d^3*n
^3 - 6*a*b^2*d^3*n^2)*x^2 - 6*(b^3*d^3*n^2*x^2 + b^3*e^3*n^2)*log(c))*log((
d*x + e*x^(1/3))/x)^2 - 12*(2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + 9*a^2*b*e^3)*lo
g(c) - 6*(4*b^3*e^3*n^3 - 12*a*b^2*e^3*n^2 + 18*a^2*b*e^3*n + (85*b^3*d^3*n
^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n)*x^2 + 18*(b^3*d^3*n*x^2 + b^3*e^3*n
)*log(c)^2 - 6*(2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + (11*b^3*d^3*n^2 - 6*a*b^2*d
^3*n)*x^2)*log(c) - 3*(5*b^3*d*e^2*n^3 - 6*b^3*d*e^2*n^2*log(c) - 6*a*b^2*d
*e^2*n^2)*x^(2/3) - 6*(6*b^3*d^2*e*n^2*x*log(c) - (11*b^3*d^2*e*n^3 - 6*a*b
^2*d^2*e*n^2)*x)*x^(1/3))*log((d*x + e*x^(1/3))/x) - 3*(19*b^3*d*e^2*n^3 +
18*b^3*d*e^2*n*log(c)^2 - 30*a*b^2*d*e^2*n^2 + 18*a^2*b*d*e^2*n - 6*(5*b^3*
d*e^2*n^2 - 6*a*b^2*d*e^2*n)*log(c))*x^(2/3) + 6*(18*b^3*d^2*e*n*x*log(c)^2
 - 6*(11*b^3*d^2*e*n^2 - 6*a*b^2*d^2*e*n)*x*log(c) + (85*b^3*d^2*e*n^3 - 66
*a*b^2*d^2*e*n^2 + 18*a^2*b*d^2*e*n)*x)*x^(1/3))/(e^3*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{n}\right) + a\right)^{3}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x^3, x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{n}\right) + a\right)^{3}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(2/3))^n)+a)^3/x^3,x)
```

```
[Out] int((b*ln(c*(d+e/x^(2/3))^n)+a)^3/x^3,x)
```

maxima [A] time = 0.62, size = 684, normalized size = 1.52

$$-\frac{1}{4} a^{2} b e n \left(\frac{6 d^{3} \log \left(d x^{\frac{2}{3}} + e\right)}{e^{4}} - \frac{6 d^{3} \log \left(x^{\frac{2}{3}}\right)}{e^{4}} - \frac{6 d^{2} x^{\frac{4}{3}} - 3 d e x^{\frac{2}{3}} + 2 e^{2}}{e^{3} x^{2}}\right) - \frac{1}{12} \left(6 e n \left(\frac{6 d^{3} \log \left(d x^{\frac{2}{3}} + e\right)}{e^{4}} - \frac{6 d^{3} \log}{e}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*b*e*n*(6*d^3*log(d*x^(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*
d^2*x^(4/3) - 3*d*e*x^(2/3) + 2*e^2)/(e^3*x^2)) - 1/12*(6*e*n*(6*d^3*log(d*
x^(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*d^2*x^(4/3) - 3*d*e*x^(2/3)
+ 2*e^2)/(e^3*x^2))*log(c*(d + e/x^(2/3))^n) - (18*d^3*x^2*log(d*x^(2/3) +
e)^2 + 8*d^3*x^2*log(x)^2 - 44*d^3*x^2*log(x) - 66*d^2*e*x^(4/3) + 15*d*e^2
```

```
*x^(2/3) - 4*e^3 - 6*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2/3) + e))*n^
2/(e^3*x^2))*a*b^2 - 1/216*(54*e*n*(6*d^3*log(d*x^(2/3) + e)/e^4 - 6*d^3*lo
g(x^(2/3))/e^4 - (6*d^2*x^(4/3) - 3*d*e*x^(2/3) + 2*e^2)/(e^3*x^2))*log(c*(
d + e/x^(2/3))^n)^2 + e*n*((108*d^3*x^2*log(d*x^(2/3) + e)^3 - 32*d^3*x^2*l
og(x)^3 + 264*d^3*x^2*log(x)^2 - 1020*d^3*x^2*log(x) - 1530*d^2*e*x^(4/3) +
 171*d*e^2*x^(2/3) - 24*e^3 - 54*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2
/3) + e)^2 + 18*(8*d^3*x^2*log(x)^2 - 44*d^3*x^2*log(x) + 85*d^3*x^2)*log(d
*x^(2/3) + e))*n^2/(e^4*x^2) - 18*(18*d^3*x^2*log(d*x^(2/3) + e)^2 + 8*d^3*
x^2*log(x)^2 - 44*d^3*x^2*log(x) - 66*d^2*e*x^(4/3) + 15*d*e^2*x^(2/3) - 4*
e^3 - 6*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2/3) + e))*n*log(c*(d + e/
x^(2/3))^n)/(e^4*x^2)))*b^3 - 1/2*b^3*log(c*(d + e/x^(2/3))^n)^3/x^2 - 3/2*
a*b^2*log(c*(d + e/x^(2/3))^n)^2/x^2 - 3/2*a^2*b*log(c*(d + e/x^(2/3))^n)/x
^2 - 1/2*a^3/x^2
```

mupad [B] time = 0.75, size = 578, normalized size = 1.29

$$\frac{\frac{d\left(\frac{3a^3}{2}-\frac{3a^2bn}{2}+ab^2n^2-\frac{b^3n^3}{3}\right)}{2e}-\frac{d\left(6a^3-6ab^2n^2+5b^3n^3\right)}{8e}}{x^{4/3}}-\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)^3\left(\frac{b^3}{2x^2}+\frac{b^3d^3}{2e^3}\right)-\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)^2\left(\frac{b^2\,(3a-}{2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^3,x)
```

```
[Out] ((d*((3*a^3)/2 - (b^3*n^3)/3 + a*b^2*n^2 - (3*a^2*b*n)/2))/(2*e) - (d*(6*a^
3 + 5*b^3*n^3 - 6*a*b^2*n^2))/(8*e))/x^(4/3) - log(c*(d + e/x^(2/3))^n)^3*(
b^3/(2*x^2) + (b^3*d^3)/(2*e^3)) - log(c*(d + e/x^(2/3))^n)^2*((b^2*(3*a -
b*n))/(2*x^2) - ((3*b^2*d*(3*a - b*n))/(2*e) - (9*a*b^2*d)/(2*e))/(2*x^(4/3
)) + (d*(6*a*b^2*d^2 - 11*b^3*d^2*n))/(4*e^3) + (d*((6*b^2*d*(3*a - b*n))/e
 - (18*a*b^2*d)/e))/(4*e*x^(2/3))) - ((d*((d*((3*a^3)/2 - (b^3*n^3)/3 + a*b
^2*n^2 - (3*a^2*b*n)/2))/e - (d*(6*a^3 + 5*b^3*n^3 - 6*a*b^2*n^2))/(4*e)))/
e + (b^2*d^2*n^2*(6*a - 11*b*n))/(2*e^2))/x^(2/3) - (a^3/2 - (b^3*n^3)/9 +
(a*b^2*n^2)/3 - (a^2*b*n)/2)/x^2 - (log(c*(d + e/x^(2/3))^n)*(((d*(2*b*d*e*
(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 6*b*d*e*(3*a^2 - b^2*n^2)))/e + 12*b^3*d^2*
n^2)/(2*e*x^(2/3)) - (2*b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 6*b*d*e*(3*a^
2 - b^2*n^2))/(4*e*x^(4/3)) + (b*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n))/(3*x^2)))
/(2*e) - (log(d + e/x^(2/3))*(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*
d^3*n))/(12*e^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x**3,x)
```

```
[Out] Timed out
```

3.528 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$

Optimal. Leaf size=1278

$$\frac{2bn\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{\left(x^{2/3}d+e\right)x^{2/3}},x\right)e^5}{3d^4}+\frac{568ib^3n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2e^{9/2}}{105d^{9/2}}-\frac{2b^3n^3\log^2\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)e^{9/2}}{(-d)^{9/2}}+\frac{2b^3n^3\log^2}{}$$

```
[Out] 568/105*a*b^2*e^4*n^2*x^(1/3)/d^4+1/3*x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^3-16/
7*b^3*e^4*n^3*x^(1/3)/d^4+16/105*b^3*e^3*n^3*x/d^3+1376/105*b^3*e^(9/2)*n^3
*arctan(x^(1/3)*d^(1/2)/e^(1/2))/d^(9/2)-2*b^3*e^(9/2)*n^3*ln(-x^(1/3)*(-d)
^(1/2)+e^(1/2))^2/(-d)^(9/2)+2*b^3*e^(9/2)*n^3*ln(x^(1/3)*(-d)^(1/2)+e^(1/2
))^2/(-d)^(9/2)+8*b^3*e^(9/2)*n^3*polylog(2,1-x^(1/3)*(-d)^(1/2)/e^(1/2))/(
-d)^(9/2)-4*b^3*e^(9/2)*n^3*polylog(2,1/2-1/2*x^(1/3)*(-d)^(1/2)/e^(1/2))/(
-d)^(9/2)+4*b^3*e^(9/2)*n^3*polylog(2,1/2+1/2*x^(1/3)*(-d)^(1/2)/e^(1/2))/(
-d)^(9/2)-8*b^3*e^(9/2)*n^3*polylog(2,1+x^(1/3)*(-d)^(1/2)/e^(1/2))/(-d)^(9
/2)+2/3*b*e^5*n*Unintegrable((a+b*ln(c*(d+e/x^(2/3))^n))^2/(e+d*x^(2/3))/x^
(2/3),x)/d^4+568/105*b^3*e^4*n^2*x^(1/3)*ln(c*(d+e/x^(2/3))^n)/d^4-32/35*b^
2*e^3*n^2*x*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3+8/35*b^2*e^2*n^2*x^(5/3)*(a+b*l
n(c*(d+e/x^(2/3))^n))/d^2-568/105*b^2*e^(9/2)*n^2*arctan(x^(1/3)*d^(1/2)/e^
(1/2))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^(9/2)-2*b*e^4*n*x^(1/3)*(a+b*ln(c*(d+e
/x^(2/3))^n))^2/d^4+2/3*b*e^3*n*x*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^3-2/5*b*e
^2*n*x^(5/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^2+2/7*b*e*n*x^(7/3)*(a+b*ln(c*
(d+e/x^(2/3))^n))^2/d+4*b^2*e^(9/2)*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(-x^(
1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(9/2)-4*b^3*e^(9/2)*n^3*ln(1/2+1/2*x^(1/3)*(-
d)^(1/2)/e^(1/2))*ln(-x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(9/2)+8*b^3*e^(9/2)*
n^3*ln(x^(1/3)*(-d)^(1/2)/e^(1/2))*ln(-x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(9/
2)-4*b^2*e^(9/2)*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(x^(1/3)*(-d)^(1/2)+e^(1
/2))/(-d)^(9/2)+4*b^3*e^(9/2)*n^3*ln(1/2-1/2*x^(1/3)*(-d)^(1/2)/e^(1/2))*ln
(x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(9/2)-8*b^3*e^(9/2)*n^3*ln(-x^(1/3)*(-d)^
(1/2)/e^(1/2))*ln(x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(9/2)-1136/105*b^3*e^(9/
2)*n^3*arctan(x^(1/3)*d^(1/2)/e^(1/2))*ln(2-2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e
^(1/2)))/d^(9/2)+568/105*I*b^3*e^(9/2)*n^3*polylog(2,-1+2*e^(1/2)/(-I*x^(1/
3)*d^(1/2)+e^(1/2)))/d^(9/2)+568/105*I*b^3*e^(9/2)*n^3*arctan(x^(1/3)*d^(1/
2)/e^(1/2))^2/d^(9/2)
```

Rubi [A] time = 3.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (568*a*b^2*e^4*n^2*x^(1/3))/(105*d^4) - (16*b^3*e^4*n^3*x^(1/3))/(7*d^4) +
(16*b^3*e^3*n^3*x)/(105*d^3) + (1376*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[d]*x^(1/3
))/Sqrt[e]])/(105*d^(9/2)) + (((568*I)/105)*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[d]
*x^(1/3))/Sqrt[e]]^2)/d^(9/2) - (1136*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[d]*x^(1/
3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/(105*d^(9/
2)) + (568*b^3*e^4*n^2*x^(1/3)*Log[c*(d + e/x^(2/3))^n])/(105*d^4) - (32*b^
2*e^3*n^2*x*(a + b*Log[c*(d + e/x^(2/3))^n]))/(35*d^3) + (8*b^2*e^2*n^2*x^(
5/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(35*d^2) - (568*b^2*e^(9/2)*n^2*ArcT
an[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(105*d^(9/2
```

```
)) - (2*b*e^4*n*x^(1/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/d^4 + (2*b*e^3*
n*x*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(3*d^3) - (2*b*e^2*n*x^(5/3)*(a + b
*Log[c*(d + e/x^(2/3))^n])^2)/(5*d^2) + (2*b*e*n*x^(7/3)*(a + b*Log[c*(d +
e/x^(2/3))^n])^2)/(7*d) + (x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^3)/3 + (4*b
^2*e^(9/2)*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1
/3)])/(-d)^(9/2) - (2*b^3*e^(9/2)*n^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]^2)/(-
d)^(9/2) - (4*b^2*e^(9/2)*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e]
+ Sqrt[-d]*x^(1/3)])/(-d)^(9/2) + (2*b^3*e^(9/2)*n^3*Log[Sqrt[e] + Sqrt[-d]
*x^(1/3)]^2)/(-d)^(9/2) + (4*b^3*e^(9/2)*n^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)
]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])])/(-d)^(9/2) - (4*b^3*e^(9/2)*n^
3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2])/
(-d)^(9/2) - (8*b^3*e^(9/2)*n^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[-((Sqrt
[-d]*x^(1/3))/Sqrt[e])])/(-d)^(9/2) + (8*b^3*e^(9/2)*n^3*Log[Sqrt[e] - Sqrt
[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]])/(-d)^(9/2) + (((568*I)/105)*
b^3*e^(9/2)*n^3*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])
/d^(9/2) + (8*b^3*e^(9/2)*n^3*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]])/(
-d)^(9/2) - (4*b^3*e^(9/2)*n^3*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[
e])])/(-d)^(9/2) + (4*b^3*e^(9/2)*n^3*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sq
rt[e])/2])/(-d)^(9/2) - (8*b^3*e^(9/2)*n^3*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3)
)/Sqrt[e]])/(-d)^(9/2) + (2*b*e^5*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d
 + e/x^2)^n])^2/(e + d*x^2), x], x, x^(1/3)])/d^4
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = 3\,\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^3 dx, x, \sqrt[3]{x}\right)$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + (2ben)\,\text{Subst}\left(\int \frac{x^6\left(a+b\log\left(c\left(d\right.\right.\right.}{d+\frac{e}{x^2}}\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + (2ben)\,\text{Subst}\left(\int\left(-\frac{e^3\left(a+b\log\left(c\right.\right.}{d^4}\right.\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{(2ben)\,\text{Subst}\left(\int x^6\left(a+b\log\left(c\left(d\right.\right.\right.\right.}{d}$$

$$= -\frac{2be^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d^4} + \frac{2be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.}{3d^3}$$

$$= -\frac{2be^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d^4} + \frac{2be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.}{3d^3}$$

$$= -\frac{2be^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d^4} + \frac{2be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.}{3d^3}$$

$$= \frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4} - \frac{32b^2e^3n^2x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{35d^3} + \frac{8b^2e^2n^2x^{5/3}}{}$$

$$= \frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4} + \frac{568b^3e^4n^2\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{105d^4} - \frac{32b^2e^3n^2x\left(a+\right.}{}$$

$$= \frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4} - \frac{64b^3e^4n^3\sqrt[3]{x}}{35d^4} + \frac{568b^3e^4n^2\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{105d^4} -$$

$$= \frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4} - \frac{16b^3e^4n^3\sqrt[3]{x}}{7d^4} + \frac{16b^3e^3n^3x}{105d^3} + \frac{1328b^3e^{9/2}n^3\tan^{-1}\left(\right.}{105d^{9/2}}$$

$$= \frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4} - \frac{16b^3e^4n^3\sqrt[3]{x}}{7d^4} + \frac{16b^3e^3n^3x}{105d^3} + \frac{1376b^3e^{9/2}n^3\tan^{-1}\left(\right.}{105d^{9/2}}$$

$$= \frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4} - \frac{16b^3e^4n^3\sqrt[3]{x}}{7d^4} + \frac{16b^3e^3n^3x}{105d^3} + \frac{1376b^3e^{9/2}n^3\tan^{-1}\left(\right.}{105d^{9/2}}$$

Mathematica [A] time = 4.95, size = 764, normalized size = 0.60

$$-\frac{b^2n^2\left(-a-b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+bn\log\left(d+\frac{e}{x^{2/3}}\right)\right)\left(\log\left(d+\frac{e}{x^{2/3}}\right)\left(9e^5\left(dx^{2/3}+e\right){}_3F_2\left(1,1,\frac{11}{2};2,2;\frac{e}{dx^{2/3}}+1\right)+\right.\right.}{d^6x\sqrt{-\frac{e}{dx^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (b^3*n^3*(54*e^5*(e + d*x^(2/3))*HypergeometricPFQ[{1, 1, 1, 1, 11/2}, {2,
2, 2, 2}, 1 + e/(d*x^(2/3))] + Log[d + e/x^(2/3)]*(-54*e^5*(e + d*x^(2/3))*
HypergeometricPFQ[{1, 1, 1, 11/2}, {2, 2, 2}, 1 + e/(d*x^(2/3))] + Log[d +
e/x^(2/3)]*(27*e^5*(e + d*x^(2/3))*HypergeometricPFQ[{1, 1, 11/2}, {2, 2},
1 + e/(d*x^(2/3))] + 2*d*x^(2/3)*(e^5 + d^5*Sqrt[-(e/(d*x^(2/3)))]*x^(10/3)
)*Log[d + e/x^(2/3)]))))/(6*d^6*Sqrt[-(e/(d*x^(2/3)))]*x) - (b^2*n^2*(-9*e^
5*(e + d*x^(2/3))*HypergeometricPFQ[{1, 1, 1, 11/2}, {2, 2, 2}, 1 + e/(d*x^
(2/3))] + Log[d + e/x^(2/3)]*(9*e^5*(e + d*x^(2/3))*HypergeometricPFQ[{1, 1
, 11/2}, {2, 2}, 1 + e/(d*x^(2/3))] + d*x^(2/3)*(e^5 + d^5*Sqrt[-(e/(d*x^(2
/3)))]*x^(10/3))*Log[d + e/x^(2/3)]))*(-a + b*n*Log[d + e/x^(2/3)] - b*Log[
c*(d + e/x^(2/3))^n]))/(d^6*Sqrt[-(e/(d*x^(2/3)))]*x) - (2*b*e^4*n*x^(1/3)*
(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/d^4 + (2*b*e^3
*n*x*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/(3*d^3) -
 (2*b*e^2*n*x^(5/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n
])^2)/(5*d^2) + (2*b*e*n*x^(7/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d +
 e/x^(2/3))^n])^2)/(7*d) + (2*b*e^(9/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]
*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/d^(9/2) + b*n
*x^3*Log[d + e/x^(2/3)]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3
))^n])^2 + (x^3*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^3
)/3
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^3x^2\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^3+3\,ab^2x^2\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^2+3\,a^2bx^2\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a^3x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*x^2*log(c*((d*x
 + e*x^(1/3))/x)^n)^2 + 3*a^2*b*x^2*log(c*((d*x + e*x^(1/3))/x)^n) + a^3*x^
2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3*x^2, x)
```

maple [A] time = 0.12, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln(c*(d+e/x^(2/3))^n)+a)^3,x)
```

```
[Out] int(x^2*(b*ln(c*(d+e/x^(2/3))^n)+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,b^3n^3x^3\log\left(dx^{\frac{2}{3}}+e\right)^3-\int\frac{\left(2\,b^3dnx^3-9\left(b^3d\log(c)+ab^2d\right)x^3-9\left(b^3e\log(c)+ab^2e\right)x^{\frac{7}{3}}+18\left(b^3dx^3+b^3ex\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*n^3*x^3*log(d*x^(2/3) + e)^3 - integrate(1/3*((2*b^3*d*n*x^3 - 9*(b
^3*d*log(c) + a*b^2*d)*x^3 - 9*(b^3*e*log(c) + a*b^2*e)*x^(7/3) + 18*(b^3*d
*x^3 + b^3*e*x^(7/3))*log(x^(1/3*n)))*n^2*log(d*x^(2/3) + e)^2 - 3*(b^3*d*l
og(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^3 + 24*(b^3*d*x^
3 + b^3*e*x^(7/3))*log(x^(1/3*n))^3 - 3*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^
2 + 3*a^2*b*e*log(c) + a^3*e)*x^(7/3) - 9*((b^3*d*log(c)^2 + 2*a*b^2*d*log(
c) + a^2*b*d)*x^3 + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(7/3) +
 4*(b^3*d*x^3 + b^3*e*x^(7/3))*log(x^(1/3*n))^2 - 4*((b^3*d*log(c) + a*b^2*
d)*x^3 + (b^3*e*log(c) + a*b^2*e)*x^(7/3))*log(x^(1/3*n)))*n*log(d*x^(2/3)
+ e) - 36*((b^3*d*log(c) + a*b^2*d)*x^3 + (b^3*e*log(c) + a*b^2*e)*x^(7/3))
*log(x^(1/3*n))^2 + 18*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^3 +
 (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(7/3))*log(x^(1/3*n)))/(d*
x + e*x^(1/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e/x^(2/3))^n))^3,x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e/x^(2/3))^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

3.529 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=738

$$-\frac{2be^2n\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^{2/3}\left(dx^{2/3}+e\right)},x\right)}{d}+\frac{12b^2e^{3/2}n^2\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{(-d)^{3/2}}-\frac{12b^2e^{3/2}n^2\log\left(\sqrt{\right.}\right.}{}$$

```
[Out] 6*b*e*n*x^(1/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d+x*(a+b*ln(c*(d+e/x^(2/3))^n
))^3+12*b^2*e^(3/2)*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(-x^(1/3)*(-d)^(1/2)+
e^(1/2))/(-d)^(3/2)-12*b^3*e^(3/2)*n^3*ln(1/2+1/2*x^(1/3)*(-d)^(1/2)/e^(1/2
))*ln(-x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(3/2)+24*b^3*e^(3/2)*n^3*ln(x^(1/3)
*(-d)^(1/2)/e^(1/2))*ln(-x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(3/2)-6*b^3*e^(3/
2)*n^3*ln(-x^(1/3)*(-d)^(1/2)+e^(1/2))^2/(-d)^(3/2)-12*b^2*e^(3/2)*n^2*(a+b
*ln(c*(d+e/x^(2/3))^n))*ln(x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(3/2)+12*b^3*e^
(3/2)*n^3*ln(1/2-1/2*x^(1/3)*(-d)^(1/2)/e^(1/2))*ln(x^(1/3)*(-d)^(1/2)+e^(1
/2))/(-d)^(3/2)-24*b^3*e^(3/2)*n^3*ln(-x^(1/3)*(-d)^(1/2)/e^(1/2))*ln(x^(1/
3)*(-d)^(1/2)+e^(1/2))/(-d)^(3/2)+6*b^3*e^(3/2)*n^3*ln(x^(1/3)*(-d)^(1/2)+e
^(1/2))^2/(-d)^(3/2)+24*b^3*e^(3/2)*n^3*polylog(2,1-x^(1/3)*(-d)^(1/2)/e^(1
/2))/(-d)^(3/2)-12*b^3*e^(3/2)*n^3*polylog(2,1/2-1/2*x^(1/3)*(-d)^(1/2)/e^(
1/2))/(-d)^(3/2)+12*b^3*e^(3/2)*n^3*polylog(2,1/2+1/2*x^(1/3)*(-d)^(1/2)/e^
(1/2))/(-d)^(3/2)-24*b^3*e^(3/2)*n^3*polylog(2,1+x^(1/3)*(-d)^(1/2)/e^(1/2)
)/(-d)^(3/2)-2*b*e^2*n*Unintegrable((a+b*ln(c*(d+e/x^(2/3))^n))^2/(e+d*x^(2
/3))/x^(2/3),x)/d
```

Rubi [A] time = 1.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (6*b*e*n*x^(1/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/d + x*(a + b*Log[c*(d
+ e/x^(2/3))^n])^3 + (12*b^2*e^(3/2)*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*L
og[Sqrt[e] - Sqrt[-d]*x^(1/3)])/(-d)^(3/2) - (6*b^3*e^(3/2)*n^3*Log[Sqrt[e]
 - Sqrt[-d]*x^(1/3)]^2)/(-d)^(3/2) - (12*b^2*e^(3/2)*n^2*(a + b*Log[c*(d +
e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)])/(-d)^(3/2) + (6*b^3*e^(3/2)
*n^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]^2)/(-d)^(3/2) + (12*b^3*e^(3/2)*n^3*Lo
g[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])]])/(-
d)^(3/2) - (12*b^3*e^(3/2)*n^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sq
rt[-d]*x^(1/3))/Sqrt[e])/2])/(-d)^(3/2) - (24*b^3*e^(3/2)*n^3*Log[Sqrt[e] +
 Sqrt[-d]*x^(1/3)]*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])])/(-d)^(3/2) + (24*b^3
*e^(3/2)*n^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]
])/(-d)^(3/2) + (24*b^3*e^(3/2)*n^3*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[
e]])/(-d)^(3/2) - (12*b^3*e^(3/2)*n^3*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(
2*Sqrt[e])])/(-d)^(3/2) + (12*b^3*e^(3/2)*n^3*PolyLog[2, (1 + (Sqrt[-d]*x^(
1/3))/Sqrt[e])/2])/(-d)^(3/2) - (24*b^3*e^(3/2)*n^3*PolyLog[2, 1 + (Sqrt[-d
]*x^(1/3))/Sqrt[e]])/(-d)^(3/2) - (6*b*e^2*n*Defer[Subst][Defer[Int][(a + b
*Log[c*(d + e/x^2)^n])^2/(e + d*x^2), x], x, x^(1/3)])/d
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = 3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^3 dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + (6ben)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right.}{d+\frac{e}{x^2}}\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + (6ben)\,\text{Subst}\left(\int\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right.\right.}{d}\right.\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{(6ben)\,\text{Subst}\left(\int\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right.\right.\right.}{d}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{(6}{}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{(6}{}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{(6}{}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{12}{}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{12}{}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{12}{}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{12}{}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{12}{}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{12}{}$$

Mathematica [A] time = 3.22, size = 475, normalized size = 0.64

$$-9b^2en^2\left(dx^{2/3}+e\right)\sqrt{-\frac{e}{dx^{2/3}}}\,{}_4F_3\left(1,1,1,\frac{5}{2};2,2,2;\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+9b^3en^3\left(dx^{2/3}+e\right)\sqrt{-\frac{e}{dx^{2/}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (9*b^3*e*n^3*(e + d*x^(2/3))*Sqrt[-(e/(d*x^(2/3)))]*HypergeometricPFQ[{1, 1
, 1, 1, 5/2}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] - 9*b^2*e*n^2*(e + d*x^(2/3)
)*Sqrt[-(e/(d*x^(2/3)))]*HypergeometricPFQ[{1, 1, 1, 5/2}, {2, 2, 2}, 1 + e
/(d*x^(2/3))]*(a + b*Log[c*(d + e/x^(2/3))^n]) - 6*b*Sqrt[d]*e^(3/2)*n*x^(1
/3)*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c
*(d + e/x^(2/3))^n])^2 + d*x^(2/3)*(-2*b^3*e*n^3*Sqrt[-(e/(d*x^(2/3)))]*Log
[d + e/x^(2/3)]^3 - 12*b^2*e*n^2*Sqrt[-(e/(d*x^(2/3)))]*(1 + Log[(1 + Sqrt[
-(e/(d*x^(2/3)))])/2])*Log[d + e/x^(2/3)]*(a + b*Log[c*(d + e/x^(2/3))^n])
+ 3*b^2*e*n^2*Sqrt[-(e/(d*x^(2/3)))]*Log[d + e/x^(2/3)]^2*(a + 2*b*n + 2*b*
n*Log[(1 + Sqrt[-(e/(d*x^(2/3)))])/2] + b*Log[c*(d + e/x^(2/3))^n]) + (a +
b*Log[c*(d + e/x^(2/3))^n])^2*(6*b*e*n + a*d*x^(2/3) + b*d*x^(2/3)*Log[c*(d
 + e/x^(2/3))^n])))/(d^2*x^(1/3))
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^3\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^3+3\,ab^2\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^2+3\,a^2b\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^(
1/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(1/3))/x)^n) + a^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3, x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(2/3))^n)+a)^3,x)
```

```
[Out] int((b*ln(c*(d+e/x^(2/3))^n)+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$b^3n^3x\log\left(dx^{\frac{2}{3}}+e\right)^3-3\left(2\,en\left(\frac{e\arctan\left(\frac{dx^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}\,d}-\frac{x^{\frac{1}{3}}}{d}\right)-x\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)a^2b+a^3x-\int\frac{\left(2\,b^3dnx-3\left(b^3d\log(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] b^3*n^3*x*log(d*x^(2/3) + e)^3 - 3*(2*e*n*(e*arctan(d*x^(1/3)/sqrt(d*e))/(s
qrt(d*e)*d) - x^(1/3)/d) - x*log(c*(d + e/x^(2/3))^n))*a^2*b + a^3*x - inte
grate(((2*b^3*d*n*x - 3*(b^3*d*log(c) + a*b^2*d)*x + 6*(b^3*d*x + b^3*e*x^(
1/3))*log(x^(1/3*n)) - 3*(b^3*e*log(c) + a*b^2*e)*x^(1/3))*n^2*log(d*x^(2/3
) + e)^2 + 8*(b^3*d*x + b^3*e*x^(1/3))*log(x^(1/3*n))^3 - 3*(4*(b^3*d*x + b
^3*e*x^(1/3))*log(x^(1/3*n))^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c))*x - 4*
((b^3*d*log(c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(1/3))*log(x^(1/3*
n)) + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c))*x^(1/3))*n*log(d*x^(2/3) + e) - 1
2*((b^3*d*log(c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(1/3))*log(x^(1/
3*n))^2 - (b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2)*x + 6*((b^3*d*log(c)^2 + 2*
a*b^2*d*log(c))*x + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c))*x^(1/3))*log(x^(1/3
*n)) - (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2)*x^(1/3))/(d*x + e*x^(1/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^3,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

3.530 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=483

$$-\frac{2bd^2n\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^{2/3}\left(dx^{2/3}+e\right)},x\right)}{e}+\frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}}+\frac{32b^2dn^2\left(a+b\log\left(c\left(\right.\right.\right.}{e\sqrt[3]{x}}$$

```
[Out] 16/9*b^3*n^3/x-208/3*b^3*d*n^3/e/x^(1/3)-208/3*b^3*d^(3/2)*n^3*arctan(x^(1/
3)*d^(1/2)/e^(1/2))/e^(3/2)-32*I*b^3*d^(3/2)*n^3*arctan(x^(1/3)*d^(1/2)/e^(
1/2))^2/e^(3/2)-8/3*b^2*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))/x+32*b^2*d*n^2*(a+b
*ln(c*(d+e/x^(2/3))^n))/e/x^(1/3)+32*b^2*d^(3/2)*n^2*arctan(x^(1/3)*d^(1/2)
/e^(1/2))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^(3/2)+2*b*n*(a+b*ln(c*(d+e/x^(2/3))
^n))^2/x-6*b*d*n*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e/x^(1/3)-(a+b*ln(c*(d+e/x^(
2/3))^n))^3/x+64*b^3*d^(3/2)*n^3*arctan(x^(1/3)*d^(1/2)/e^(1/2))*ln(2-2*e^(
1/2)/(-I*x^(1/3)*d^(1/2)+e^(1/2)))/e^(3/2)-32*I*b^3*d^(3/2)*n^3*polylog(2,-
1+2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^(1/2)))/e^(3/2)-2*b*d^2*n*Unintegrable((a
+b*ln(c*(d+e/x^(2/3))^n))^2/(e+d*x^(2/3))/x^(2/3),x)/e
```

Rubi [A] time = 1.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^2,x]
```

```
[Out] (16*b^3*n^3)/(9*x) - (208*b^3*d*n^3)/(3*e*x^(1/3)) - (208*b^3*d^(3/2)*n^3*A
rcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(3*e^(3/2)) - ((32*I)*b^3*d^(3/2)*n^3*Arc
Tan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/e^(3/2) + (64*b^3*d^(3/2)*n^3*ArcTan[(Sqr
t[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/
e^(3/2) - (8*b^2*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*x) + (32*b^2*d*n^
2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(e*x^(1/3)) + (32*b^2*d^(3/2)*n^2*ArcTa
n[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^(3/2) + (2
*b*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/x - (6*b*d*n*(a + b*Log[c*(d + e/x
^(2/3))^n])^2)/(e*x^(1/3)) - (a + b*Log[c*(d + e/x^(2/3))^n])^3/x - ((32*I)
*b^3*d^(3/2)*n^3*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))]
)/e^(3/2) - (6*b*d^2*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)^n])
^2/(e + d*x^2), x], x, x^(1/3)])/e
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^3}{x^4}dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x} - (6ben)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{\left(d+\frac{e}{x^2}\right)x^6} \right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x} - (6ben)\,\text{Subst}\left(\int\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)}{ex^4}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x} - (6bn)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{x^4}\right.$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}} - \frac{\left(a+\right.}{}$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}} - \frac{\left(a+\right.}{}$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}} - \frac{\left(a+\right.}{}$$

$$= -\frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} + \frac{32b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} +$$

$$= -\frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} + \frac{32b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} +$$

$$= \frac{16b^3n^3}{9x} - \frac{64b^3dn^3}{e\sqrt[3]{x}} - \frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} + \frac{32b^2dn^2\left(a+b\,\text{lo}\right.}{e\sqrt[3]{}}$$

$$= \frac{16b^3n^3}{9x} - \frac{208b^3dn^3}{3e\sqrt[3]{x}} - \frac{64b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}} - \frac{32ib^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}}{\sqrt{}}\right.}{e^{3/2}}$$

$$= \frac{16b^3n^3}{} - \frac{208b^3dn^3}{} - \frac{208b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt[3]{x}}{\sqrt{e}}\right)}{} - \frac{32ib^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}}{}\right.}{}$$

Mathematica [A] time = 2.31, size = 1097, normalized size = 2.27

$$\frac{b^3\left(18\left(x^{2/3}d+e\right)\,_5F_4\left(-\frac{1}{2},1,1,1,1;2,2,2,2;\frac{e}{dx^{2/3}}+1\right)-\log\left(d+\frac{e}{x^{2/3}}\right)\left(18\left(x^{2/3}d+e\right)\,_4F_3\left(-\frac{1}{2},1,1,1;2,2,2;\frac{e}{dx^{2/3}}\right.\right.\right.}{2e\sqrt{-\frac{e}{dx^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^2,x]
```

```
[Out] (b^3*n^3*(18*(e + d*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1, 1, 1}, {2, 2, 2
, 2}, 1 + e/(d*x^(2/3))] - Log[d + e/x^(2/3)]*(18*(e + d*x^(2/3))*Hypergeom
etricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))] + Log[d + e/x^(2/3)
]*(-9*(e + d*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2
/3))] + 2*(e*Sqrt[-(e/(d*x^(2/3)))] + d*x^(2/3))*Log[d + e/x^(2/3)]))))/(2*
e*Sqrt[-(e/(d*x^(2/3)))]*x) - (6*b*d*n*(a - b*n*Log[d + e/x^(2/3)] + b*Log[
c*(d + e/x^(2/3))^n])^2)/(e*x^(1/3)) - (6*b*d^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/
3))/Sqrt[e]]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/e
^(3/2) - (3*b*n*Log[d + e/x^(2/3)]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d
 + e/x^(2/3))^n])^2)/x - ((a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2
/3))^n])^2*(a - 2*b*n - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n]
))/x + (b^2*n^2*(-a + b*n*Log[d + e/x^(2/3)] - b*Log[c*(d + e/x^(2/3))^n])*
(8*e^(3/2) - 96*d*Sqrt[e]*x^(2/3) + 96*d^(3/2)*x*ArcTan[Sqrt[e]/(Sqrt[d]*x^
(1/3))] - 12*e^(3/2)*Log[d + e/x^(2/3)] + 36*d*Sqrt[e]*x^(2/3)*Log[d + e/x^
(2/3)] + 9*e^(3/2)*Log[d + e/x^(2/3)]^2 + 18*Sqrt[-d]*d*x*Log[d + e/x^(2/3)
]*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 9*(-d)^(3/2)*x*Log[Sqrt[e] - Sqrt[-d]*x
^(1/3)]^2 + 18*(-d)^(3/2)*x*Log[d + e/x^(2/3)]*Log[Sqrt[e] + Sqrt[-d]*x^(1/
3)] + 9*Sqrt[-d]*d*x*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]^2 + 18*Sqrt[-d]*d*x*Lo
g[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] + 1
8*(-d)^(3/2)*x*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/
Sqrt[e])/2] + 36*(-d)^(3/2)*x*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[-((Sqrt[-
d]*x^(1/3))/Sqrt[e])] + 36*Sqrt[-d]*d*x*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log
[(Sqrt[-d]*x^(1/3))/Sqrt[e]] + 36*Sqrt[-d]*d*x*PolyLog[2, 1 - (Sqrt[-d]*x^(
1/3))/Sqrt[e]] + 18*(-d)^(3/2)*x*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqr
t[e])] + 18*Sqrt[-d]*d*x*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] + 3
6*(-d)^(3/2)*x*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]]))/(3*e^(3/2)*x)
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^3\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^3+3\,ab^2\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^2+3\,a^2b\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^
(1/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(1/3))/x)^n) + a^3)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x^2, x)
```

maple [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(c \left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a \right)^{3}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(2/3))^n)+a)^3/x^2,x)
```

```
[Out] int((b*ln(c*(d+e/x^(2/3))^n)+a)^3/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^{3} n^{3} \log \left(d x^{\frac{2}{3}}+e\right)^{3}}{x}-\int -\frac{\left(2\, b^{3} d n x+3\left(b^{3} d \log(c)+a b^{2} d\right) x-6\left(b^{3} d x+b^{3} e x^{\frac{1}{3}}\right) \log\left(x^{\frac{1}{3} n}\right)+3\left(b^{3} e \log(c)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] -b^3*n^3*log(d*x^(2/3) + e)^3/x - integrate(-((2*b^3*d*n*x + 3*(b^3*d*log(c
) + a*b^2*d)*x - 6*(b^3*d*x + b^3*e*x^(1/3))*log(x^(1/3*n)) + 3*(b^3*e*log(
c) + a*b^2*e)*x^(1/3))*n^2*log(d*x^(2/3) + e)^2 - 8*(b^3*d*x + b^3*e*x^(1/3
))*log(x^(1/3*n))^3 + 3*(4*(b^3*d*x + b^3*e*x^(1/3))*log(x^(1/3*n))^2 + (b^
3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x - 4*((b^3*d*log(c) + a*b^2*d)*
x + (b^3*e*log(c) + a*b^2*e)*x^(1/3))*log(x^(1/3*n)) + (b^3*e*log(c)^2 + 2*
a*b^2*e*log(c) + a^2*b*e)*x^(1/3))*n*log(d*x^(2/3) + e) + 12*((b^3*d*log(c)
 + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(1/3))*log(x^(1/3*n))^2 + (b^3*d
*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x - 6*((b^3*d*lo
g(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c)
 + a^2*b*e)*x^(1/3))*log(x^(1/3*n)) + (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2
+ 3*a^2*b*e*log(c) + a^3*e)*x^(1/3))/(d*x^3 + e*x^(7/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)\right)}^3}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x**2,x)
```

```
[Out] Timed out
```

3.531 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx$

Optimal. Leaf size=784

$$\frac{2bd^5n\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^{2/3}\left(dx^{2/3}+e\right)},x\right)}{3e^4}-\frac{4504b^2d^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{315e^{9/2}}-\frac{4504b^2d^4n^2\left(a+b\log\right.}{315e^4\sqrt[3]{}}$$

[Out] 16/729*b^3*n^3/x^3-3088/27783*b^3*d*n^3/e/x^(7/3)+221344/496125*b^3*d^2*n^3/e^2/x^(5/3)-637984/297675*b^3*d^3*n^3/e^3/x+3475504/99225*b^3*d^4*n^3/e^4/x^(1/3)+3475504/99225*b^3*d^(9/2)*n^3*arctan(x^(1/3)*d^(1/2)/e^(1/2))/e^(9/2)+4504/315*I*b^3*d^(9/2)*n^3*polylog(2,-1+2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^(1/2)))/e^(9/2)-8/81*b^2*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))/x^3+128/441*b^2*d*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))/e/x^(7/3)-1144/1575*b^2*d^2*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))/e^2/x^(5/3)+1984/945*b^2*d^3*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))/e^3/x-4504/315*b^2*d^4*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))/e^4/x^(1/3)-4504/315*b^2*d^(9/2)*n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^(9/2)+2/9*b*n*(a+b*ln(c*(d+e/x^(2/3))^n))^2/x^3-2/7*b*d*n*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e/x^(7/3)+2/5*b*d^2*n*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e^2/x^(5/3)-2/3*b*d^3*n*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e^3/x+2*b*d^4*n*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e^4/x^(1/3)-1/3*(a+b*ln(c*(d+e/x^(2/3))^n))^3/x^3-9008/315*b^3*d^(9/2)*n^3*arctan(x^(1/3)*d^(1/2)/e^(1/2))*ln(2-2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^(1/2)))/e^(9/2)+4504/315*I*b^3*d^(9/2)*n^3*arctan(x^(1/3)*d^(1/2)/e^(1/2))^2/e^(9/2)+2/3*b*d^5*n*Unintegrable((a+b*ln(c*(d+e/x^(2/3))^n))^2/(e+d*x^(2/3))/x^(2/3),x)/e^4

Rubi [A] time = 3.62, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^4,x]

[Out] (16*b^3*n^3)/(729*x^3) - (3088*b^3*d*n^3)/(27783*e*x^(7/3)) + (221344*b^3*d^2*n^3)/(496125*e^2*x^(5/3)) - (637984*b^3*d^3*n^3)/(297675*e^3*x) + (3475504*b^3*d^4*n^3)/(99225*e^4*x^(1/3)) + (3475504*b^3*d^(9/2)*n^3*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(99225*e^(9/2)) + (((4504*I)/315)*b^3*d^(9/2)*n^3*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/e^(9/2) - (9008*b^3*d^(9/2)*n^3*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/(315*e^(9/2)) - (8*b^2*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(81*x^3) + (128*b^2*d*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(441*e*x^(7/3)) - (1144*b^2*d^2*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(1575*e^2*x^(5/3)) + (1984*b^2*d^3*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(945*e^3*x) - (4504*b^2*d^4*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(315*e^4*x^(1/3)) - (4504*b^2*d^(9/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(315*e^(9/2)) + (2*b*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(9*x^3) - (2*b*d*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(7*e*x^(7/3)) + (2*b*d^2*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(5*e^2*x^(5/3)) - (2*b*d^3*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(3*e^3*x) + (2*b*d^4*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(e^4*x^(1/3)) - (a + b*Log[c*(d + e/x^(2/3))^n])^3/(3*x^3) + (((4504*I)/315)*b^3*d^(9/2)*n^3*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/e^(

```
9/2) + (2*b*d^5*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)^n])^2/(e
 + d*x^2), x], x, x^(1/3)])/e^4
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^3}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{3x^3} - (2ben)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{\left(d+\frac{e}{x^2}\right)x^{12}}\,dx \ldots\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{3x^3} - (2ben)\,\text{Subst}\left(\int\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{ex^{10}} - \ldots\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{3x^3} - (2bn)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{x^{10}}\,dx, \ldots\right.$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{9x^3} - \frac{2bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{7ex^{7/3}} + \frac{2bd^2n\,(\ldots}{\ldots}$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{9x^3} - \frac{2bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{7ex^{7/3}} + \frac{2bd^2n\,(\ldots}{\ldots}$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{9x^3} - \frac{2bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{7ex^{7/3}} + \frac{2bd^2n\,(\ldots}{\ldots}$$

$$= -\frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{81x^3} + \frac{128b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{441ex^{7/3}} - \ldots$$

$$= -\frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{81x^3} + \frac{128b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{441ex^{7/3}} - \ldots$$

$$= \frac{16b^3n^3}{729x^3} - \frac{256b^3dn^3}{3087ex^{7/3}} + \frac{2288b^3d^2n^3}{7875e^2x^{5/3}} - \frac{3968b^3d^3n^3}{2835e^3x} + \frac{9008b^3d^4n^3}{315e^4\sqrt[3]{x}} - \frac{8b^2n^2\,(a \ldots}{\ldots}$$

$$= \frac{16b^3n^3}{729x^3} - \frac{3088b^3dn^3}{27783ex^{7/3}} + \frac{7472b^3d^2n^3}{18375e^2x^{5/3}} - \frac{26704b^3d^3n^3}{14175e^3x} + \frac{30992b^3d^4n^3}{945e^4\sqrt[3]{x}} + \frac{9008\ldots}{\ldots}$$

$$= \frac{16b^3n^3}{729x^3} - \frac{3088b^3dn^3}{27783ex^{7/3}} + \frac{221344b^3d^2n^3}{496125e^2x^{5/3}} - \frac{206128b^3d^3n^3}{99225e^3x} + \frac{161824b^3d^4n^3}{4725e^4\sqrt[3]{x}} + \ldots$$

Mathematica [A] time = 8.71, size = 2726, normalized size = 3.48

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^4,x]
```

```
[Out] (b^3*n^3*(32*e^4*Sqrt[-(e/(d*x^(2/3)))] - 32*d^4*x^(8/3) - 1584*d^3*e*x^2*H
ypergeometricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))] - 1584*d^4*
x^(8/3)*HypergeometricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))] +
4536*d^3*e*x^2*HypergeometricPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/
3))] + 4536*d^4*x^(8/3)*HypergeometricPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, 1 + e
/(d*x^(2/3))] - 3780*d^3*e*x^2*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}
, 1 + e/(d*x^(2/3))] - 3780*d^4*x^(8/3)*HypergeometricPFQ[{-3/2, 1, 1, 1},
{2, 2, 2}, 1 + e/(d*x^(2/3))] + 864*d^3*e*x^2*HypergeometricPFQ[{-7/2, 1, 1
, 1, 1}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] + 864*d^4*x^(8/3)*HypergeometricP
FQ[{-7/2, 1, 1, 1, 1}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] - 3024*d^3*e*x^2*Hy
pergeometricPFQ[{-5/2, 1, 1, 1, 1}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] - 3024
*d^4*x^(8/3)*HypergeometricPFQ[{-5/2, 1, 1, 1, 1}, {2, 2, 2, 2}, 1 + e/(d*x
^(2/3))] + 3780*d^3*e*x^2*HypergeometricPFQ[{-3/2, 1, 1, 1, 1}, {2, 2, 2, 2
}, 1 + e/(d*x^(2/3))] + 3780*d^4*x^(8/3)*HypergeometricPFQ[{-3/2, 1, 1, 1,
1}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] - 1890*d^3*e*x^2*HypergeometricPFQ[{-1
/2, 1, 1, 1, 1}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] - 1890*d^4*x^(8/3)*Hyperg
eometricPFQ[{-1/2, 1, 1, 1, 1}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] - (288*e^4
*Log[d + e/x^(2/3)])/Sqrt[-(e/(d*x^(2/3)))] + 48*e^4*Sqrt[-(e/(d*x^(2/3)))]
*Log[d + e/x^(2/3)] + 240*d^4*x^(8/3)*Log[d + e/x^(2/3)] + 3780*d^3*e*x^2*H
ypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)
] + 3780*d^4*x^(8/3)*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2/
3))]*Log[d + e/x^(2/3)] - 864*d^3*e*x^2*HypergeometricPFQ[{-7/2, 1, 1, 1},
{2, 2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)] - 864*d^4*x^(8/3)*Hypergeo
metricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)]
 + 3024*d^3*e*x^2*HypergeometricPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^
(2/3))]*Log[d + e/x^(2/3)] + 3024*d^4*x^(8/3)*HypergeometricPFQ[{-5/2, 1, 1
, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)] - 3780*d^3*e*x^2*Hyp
ergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(
2/3)] - 3780*d^4*x^(8/3)*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, 1 +
e/(d*x^(2/3))]*Log[d + e/x^(2/3)] + 1890*d^3*e*x^2*HypergeometricPFQ[{-1/2,
 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)] + 1890*d^4*x^(8
/3)*HypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))]*Log[d
+ e/x^(2/3)] + (252*e^4*Log[d + e/x^(2/3)]^2)/(-(e/(d*x^(2/3))))^(3/2) - (3
6*e^4*Log[d + e/x^(2/3)]^2)/Sqrt[-(e/(d*x^(2/3)))] + 68*e^4*Sqrt[-(e/(d*x^(
2/3)))]*Log[d + e/x^(2/3)]^2 - 284*d^4*x^(8/3)*Log[d + e/x^(2/3)]^2 + 1890*
d^3*e*x^2*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2/3))]*Log[d
+ e/x^(2/3)]^2 + 1890*d^4*x^(8/3)*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, 1
 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)]^2 - 945*d^3*e*x^2*HypergeometricPFQ[{-
1/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)]^2 - 945*d^4*x^(8/
3)*HypergeometricPFQ[{-1/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(
2/3)]^2 - 70*e^4*Sqrt[-(e/(d*x^(2/3)))]*Log[d + e/x^(2/3)]^3 + 70*d^4*x^(8/
3)*Log[d + e/x^(2/3)]^3 - 1512*d^3*(e + d*x^(2/3))*x^2*HypergeometricPFQ[{-
5/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2/3))]*(1 + 3*Log[d + e/x^(2/3)] + Log[d +
e/x^(2/3)]^2) + 144*d^3*(e + d*x^(2/3))*x^2*HypergeometricPFQ[{-7/2, 1, 1},
 {2, 2}, 1 + e/(d*x^(2/3))]*(6 + 11*Log[d + e/x^(2/3)] + 3*Log[d + e/x^(2/3
)]^2)))/(210*e^4*Sqrt[-(e/(d*x^(2/3)))]*x^3) - (2*b*d*n*(a - b*n*Log[d + e/
x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/(7*e*x^(7/3)) + (2*b*d^2*n*(a - b
*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/(5*e^2*x^(5/3)) - (2
*b*d^3*n*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/(3*e^
3*x) + (2*b*d^4*n*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])
^2)/(e^4*x^(1/3)) + (2*b*d^(9/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a - b
*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/e^(9/2) - (b*n*Log[d
```

```
 + e/x^(2/3)]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/
x^3 - ((a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2*(3*a - 2
*b*n - 3*b*n*Log[d + e/x^(2/3)] + 3*b*Log[c*(d + e/x^(2/3))^n]))/(9*x^3) +
(b^2*n^2*(-a + b*n*Log[d + e/x^(2/3)] - b*Log[c*(d + e/x^(2/3))^n])*(9800*e
^(9/2) - 28800*d*e^(7/2)*x^(2/3) + 72072*d^2*e^(5/2)*x^(4/3) - 208320*d^3*e
^(3/2)*x^2 + 1418760*d^4*Sqrt[e]*x^(8/3) - 1418760*d^(9/2)*x^3*ArcTan[Sqrt[
e]/(Sqrt[d]*x^(1/3))] - 44100*e^(9/2)*Log[d + e/x^(2/3)] + 56700*d*e^(7/2)*
x^(2/3)*Log[d + e/x^(2/3)] - 79380*d^2*e^(5/2)*x^(4/3)*Log[d + e/x^(2/3)] +
 132300*d^3*e^(3/2)*x^2*Log[d + e/x^(2/3)] - 396900*d^4*Sqrt[e]*x^(8/3)*Log
[d + e/x^(2/3)] + 99225*e^(9/2)*Log[d + e/x^(2/3)]^2 - 198450*(-d)^(9/2)*x^
3*Log[d + e/x^(2/3)]*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 99225*(-d)^(9/2)*x^3
*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]^2 + 198450*(-d)^(9/2)*x^3*Log[d + e/x^(2/3
)]*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] - 99225*(-d)^(9/2)*x^3*Log[Sqrt[e] + Sqr
t[-d]*x^(1/3)]^2 - 198450*(-d)^(9/2)*x^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Lo
g[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] + 198450*(-d)^(9/2)*x^3*Log[Sqrt[e]
 - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] + 396900*(-d)^
(9/2)*x^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])
] - 396900*(-d)^(9/2)*x^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(
1/3))/Sqrt[e]] - 396900*(-d)^(9/2)*x^3*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sq
rt[e]] + 198450*(-d)^(9/2)*x^3*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[
e])] - 198450*(-d)^(9/2)*x^3*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2]
 + 396900*(-d)^(9/2)*x^3*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]]))/(9922
5*e^(9/2)*x^3)
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^3\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^3+3\,ab^2\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)^2+3\,a^2b\log\left(c\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^n\right)+a^3}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^4,x, algorithm="fricas")
[Out] integral((b^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^
(1/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(1/3))/x)^n) + a^3)/x^4, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x^4, x)
```

maple [A] time = 0.21, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+e/x^(2/3))^n)+a)^3/x^4,x)
```

[Out] int((b*ln(c*(d+e/x^(2/3))^n)+a)^3/x^4,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3n^3\log\left(dx^{\frac{2}{3}}+e\right)^3}{3\,x^3}-\int-\frac{\left(2\,b^3dnx+9\left(b^3d\log(c)+ab^2d\right)x-18\left(b^3dx+b^3ex^{\frac{1}{3}}\right)\log\left(x^{\frac{1}{3}n}\right)+9\left(b^3e\log(c)+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^3*n^3*log(d*x^(2/3) + e)^3/x^3 - integrate(-1/3*((2*b^3*d*n*x + 9*(b
^3*d*log(c) + a*b^2*d)*x - 18*(b^3*d*x + b^3*e*x^(1/3))*log(x^(1/3*n)) + 9*
(b^3*e*log(c) + a*b^2*e)*x^(1/3))*n^2*log(d*x^(2/3) + e)^2 - 24*(b^3*d*x +
b^3*e*x^(1/3))*log(x^(1/3*n))^3 + 9*(4*(b^3*d*x + b^3*e*x^(1/3))*log(x^(1/3
*n))^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x - 4*((b^3*d*log(c)
 + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(1/3))*log(x^(1/3*n)) + (b^3*e*l
og(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(1/3))*n*log(d*x^(2/3) + e) + 36*((
b^3*d*log(c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(1/3))*log(x^(1/3*n)
)^2 + 3*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x
- 18*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x + (b^3*e*log(c)^2 + 2
*a*b^2*e*log(c) + a^2*b*e)*x^(1/3))*log(x^(1/3*n)) + 3*(b^3*e*log(c)^3 + 3*
a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(1/3))/(d*x^5 + e*x^(13/3)),
 x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^4,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x**4,x)
```

```
[Out] Timed out
```

3.532 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=730

$$\frac{2^{-3p-2}e^{-\frac{8a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{8\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^8e^8}-\frac{2d7^{-p}e^{-\frac{7a}{b}}\left(a+b\log\left(\right.\right.}{}$$

```
[Out] 2^(-2-3*p)*GAMMA(1+p,-8*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2)
)))^p/c^8/e^8/exp(8*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d*GAMMA(1+p,-
7*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(7^p)/c^7/e^8/ex
p(7*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)+7*d^2*GAMMA(1+p,-6*(a+b*ln(c*(d
+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(6^p)/c^6/e^8/exp(6*a/b)/(((-a
-b*ln(c*(d+e*x^(1/2))))/b)^p)-14*d^3*GAMMA(1+p,-5*(a+b*ln(c*(d+e*x^(1/2))))
/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(5^p)/c^5/e^8/exp(5*a/b)/(((-a-b*ln(c*(d+e*
x^(1/2))))/b)^p)+35*2^(-1-2*p)*d^4*GAMMA(1+p,-4*(a+b*ln(c*(d+e*x^(1/2))))/b
)*(a+b*ln(c*(d+e*x^(1/2))))^p/c^4/e^8/exp(4*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))
))/b)^p)-14*d^5*GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^
(1/2))))^p/(3^p)/c^3/e^8/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)+7*d^
6*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(2^
p)/c^2/e^8/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d^7*GAMMA(1+p,(-
a-b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/c/e^8/exp(a/b)/(((-
a-b*ln(c*(d+e*x^(1/2))))/b)^p)
```

Rubi [A] time = 1.34, antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{7d^26^{-p}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^6e^8}-\frac{14d^35^{-p}e^{-\frac{5a}{b}}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (2^(-2 - 3*p)*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])])^p)/(c^8*e^8*E^((8*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x
])])/b))^p) - (2*d*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a +
 b*Log[c*(d + e*Sqrt[x])])^p)/(7^p*c^7*e^8*E^((7*a)/b)*(-((a + b*Log[c*(d +
 e*Sqrt[x])])/b))^p) + (7*d^2*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*Sqrt[x])
]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(6^p*c^6*e^8*E^((6*a)/b)*(-((a + b
*Log[c*(d + e*Sqrt[x])])/b))^p) - (14*d^3*Gamma[1 + p, (-5*(a + b*Log[c*(d
+ e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(5^p*c^5*e^8*E^((5*a)/
b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) + (35*2^(-1 - 2*p)*d^4*Gamma[1
+ p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^
p)/(c^4*e^8*E^((4*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) - (14*d^5*
Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqr
t[x])])^p)/(3^p*c^3*e^8*E^((3*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p
) + (7*d^6*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c
*(d + e*Sqrt[x])])^p)/(2^p*c^2*e^8*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[
x])])/b))^p) - (2*d^7*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)]*(a
+ b*Log[c*(d + e*Sqrt[x])])^p)/(c*e^8*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x
])])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
```

```
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p dx = 2\,\mathrm{Subst}\left(\int x^7(a+b\log(c(d+ex)))^p\,dx, x, \sqrt{x}\right)$$
$$= 2\,\mathrm{Subst}\left(\int \left(-\frac{d^7(a+b\log(c(d+ex)))^p}{e^7}+\frac{7d^6(d+ex)(a+b\log(c(d+ex))}{e^7}\right.\right.$$
$$= \frac{2\,\mathrm{Subst}\left(\int (d+ex)^7(a+b\log(c(d+ex)))^p\,dx, x, \sqrt{x}\right)}{e^7} - \frac{(14d)\,\mathrm{Subst}\left(\int (d\right.}{}$$
$$= \frac{2\,\mathrm{Subst}\left(\int x^7(a+b\log(cx))^p\,dx, x, d+e\sqrt{x}\right)}{e^8} - \frac{(14d)\,\mathrm{Subst}\left(\int x^6(a+b\log\right.}{e^8}$$
$$= \frac{2\,\mathrm{Subst}\left(\int e^{8x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^8e^8} - \frac{(14d)\,\mathrm{Subst}\left(\int e^{7x}(a+\right.}{}$$
$$= \frac{2^{-2-3p}e^{-\frac{8a}{b}}\Gamma\left(1+p,-\frac{8\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log}{}\right.}{c^8e^8}$$

Mathematica [A] time = 1.02, size = 435, normalized size = 0.60

$$\frac{2^{-3p-2}105^{-p}e^{-\frac{8a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\left(c^7d^7\left(-8^{p+1}\right)105^pe^{\frac{7a}{b}}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (2^(-2 - 3*p)*(105^p*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*Sqrt[x])]))/b] -
8^(1 + p)*15^p*c*d*E^(a/b)*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*Sqrt[x])]))
/b] + 5^p*28^(1 + p)*c^2*d^2*E^((2*a)/b)*Gamma[1 + p, (-6*(a + b*Log[c*(d +
 e*Sqrt[x])]))/b] - 3^p*56^(1 + p)*c^3*d^3*E^((3*a)/b)*Gamma[1 + p, (-5*(a
+ b*Log[c*(d + e*Sqrt[x])]))/b] + 3^p*70^(1 + p)*c^4*d^4*E^((4*a)/b)*Gamma[
1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b] - 5^p*56^(1 + p)*c^5*d^5*E^((
5*a)/b)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])]))/b] + 15^p*28^(1 +
p)*c^6*d^6*E^((6*a)/b)*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]
- 8^(1 + p)*105^p*c^7*d^7*E^((7*a)/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*Sq
rt[x])])/b)])*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(105^p*c^8*e^8*E^((8*a)/b)*
(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\log\left(ce\sqrt{x}+cd\right)+a\right)^p x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*sqrt(x) + c*d) + a)^p*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x^3, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x^{3} \left(b \ln \left(\left(e \sqrt{x}+d \right) c \right)+a \right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(e*x^(1/2)+d)))^p,x)
[Out] int(x^3*(a+b*ln(c*(e*x^(1/2)+d)))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(\left(e \sqrt{x}+d\right)c\right)+a\right)^{p} x^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/2))))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e*x^(1/2))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/2))))**p,x)
[Out] Timed out
```

3.533 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=551

$$\frac{2^{-p}3^{-p-1}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^6e^6}-\frac{2d5^{-p}e^{-\frac{5a}{b}}\left(a+b\log\right.}{}$$

```
[Out] 3^(-1-p)*GAMMA(1+p,-6*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2)))
)^p/(2^p)/c^6/e^6/exp(6*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d*GAMMA(1
+p,-5*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(5^p)/c^5/e^
6/exp(5*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)+5*d^2*GAMMA(1+p,-4*(a+b*ln(
c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(4^p)/c^4/e^6/exp(4*a/b)/(
((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)-20*3^(-1-p)*d^3*GAMMA(1+p,-3*(a+b*ln(c*(d
+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/c^3/e^6/exp(3*a/b)/(((-a-b*ln(
c*(d+e*x^(1/2))))/b)^p)+5*d^4*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+
b*ln(c*(d+e*x^(1/2))))^p/(2^p)/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2)
)))/b)^p)-2*d^5*GAMMA(1+p,(-a-b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1
/2))))^p/c/e^6/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)
```

Rubi [A] time = 0.86, antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{5d^2 4^{-p}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^4e^6}-\frac{20d^3 3^{-p-1}e^{-\frac{3a}{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (3^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[
c*(d + e*Sqrt[x])])^p)/(2^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*Sqrt
[x])])/b))^p) - (2*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a
 + b*Log[c*(d + e*Sqrt[x])])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*(d
 + e*Sqrt[x])])/b))^p) + (5*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x
])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(4^p*c^4*e^6*E^((4*a)/b)*(-((a +
 b*Log[c*(d + e*Sqrt[x])])/b))^p) - (20*3^(-1 - p)*d^3*Gamma[1 + p, (-3*(a
+ b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(c^3*e^6*
E^((3*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) + (5*d^4*Gamma[1 + p,
(-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(2
^p*c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) - (2*d^5*Ga
mma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)]*(a + b*Log[c*(d + e*Sqrt[x]
)])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx = 2\,\text{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d+e$$

$$= \frac{2\,\text{Subst}\left(\int(d+ex)^5(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e^5}-\frac{(10d)\,\text{Subst}\left(\int(\right.}{}$$

$$= \frac{2\,\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^6}-\frac{(10d)\,\text{Subst}\left(\int x^4(a+b\right.}{}$$

$$= \frac{2\,\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^6e^6}-\frac{(10d)\,\text{Subst}\left(\int e^{5x}(a\right.}{}$$

$$= \frac{2^{-p}3^{-1-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(\right.}{c^6e^6}$$

Mathematica [A] time = 0.90, size = 325, normalized size = 0.59

$$3^{-p-1}20^{-p}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\left(10^p\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)-cde^{a/b}\left(2^{2p}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (3^(-1 - p)*(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*Sqrt[x])]))/b] - c*d
*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*Sqrt[x
])]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c*(d +
 e*Sqrt[x])]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*(a + b*Lo
g[c*(d + e*Sqrt[x])]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p, (-2*(a +
 b*Log[c*(d + e*Sqrt[x])]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a +
b*Log[c*(d + e*Sqrt[x])])/b)])))))*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(20^p*
c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(ce\sqrt{x}+cd\right)+a\right)^{p}x^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*sqrt(x) + c*d) + a)^p*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x^2, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x^{2}\left(b\ln\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln((e*x^(1/2)+d)*c)+a)^p,x)
```

```
[Out] int(x^2*(b*ln((e*x^(1/2)+d)*c)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)\right)\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/2))))^p,x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e*x^(1/2))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))))**p,x)
[Out] Timed out
```

3.534 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=360

$$\frac{2^{-2p-1}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^4e^4}-\frac{2d3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(\right.\right.}{}$$

```
[Out] 2^(-1-2*p)*GAMMA(1+p,-4*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2)
)))^p/c^4/e^4/exp(4*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d*GAMMA(1+p,-
3*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(3^p)/c^3/e^4/ex
p(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)+3*d^2*GAMMA(1+p,-2*(a+b*ln(c*(d
+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(2^p)/c^2/e^4/exp(2*a/b)/(((-a
-b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d^3*GAMMA(1+p,(-a-b*ln(c*(d+e*x^(1/2))))/b)
*(a+b*ln(c*(d+e*x^(1/2))))^p/c/e^4/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)
^p)
```

Rubi [A] time = 0.54, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{3d^2 2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^2e^4}+\frac{2^{-2p-1}e^{-\frac{4a}{b}}\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (2^(-1 - 2*p)*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])])^p)/(c^4*e^4*E^((4*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x
])])/b))^p) - (2*d*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a +
 b*Log[c*(d + e*Sqrt[x])])^p)/(3^p*c^3*e^4*E^((3*a)/b)*(-((a + b*Log[c*(d +
 e*Sqrt[x])])/b))^p) + (3*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])
]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(2^p*c^2*e^4*E^((2*a)/b)*(-((a + b
*Log[c*(d + e*Sqrt[x])])/b))^p) - (2*d^3*Gamma[1 + p, -((a + b*Log[c*(d + e
*Sqrt[x])])/b)]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(c*e^4*E^(a/b)*(-((a + b*
Log[c*(d + e*Sqrt[x])])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx = 2\,\text{Subst}\left(\int x^3(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int\left(-\frac{d^3(a+b\log(c(d+ex)))^p}{e^3}+\frac{3d^2(d+ex)(a+b\log(c(d+ex$$
$$= \frac{2\,\text{Subst}\left(\int(d+ex)^3(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e^3}-\frac{(6d)\,\text{Subst}\left(\int(d}{}$$
$$= \frac{2\,\text{Subst}\left(\int x^3(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^4}-\frac{(6d)\,\text{Subst}\left(\int x^2(a+b\log}{e}$$
$$= \frac{2\,\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^4e^4}-\frac{(6d)\,\text{Subst}\left(\int e^{3x}(a+}{}$$
$$= \frac{2^{-1-2p}e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+}{}\right.}{c^4e^4}$$

Mathematica [A] time = 0.43, size = 229, normalized size = 0.64

$$2^{-2p-1}3^{-p}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\left(3^p\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)-cd2^{p+1}e^{a/b}\right($$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (2^(-1 - 2*p)*(3^p*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x]])]))/b] - 2^
(1 + p)*c*d*E^(a/b)*(2^(1 + p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x]
)]))/b] + 3^p*c*d*E^(a/b)*(-3*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x]])
```

```
]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])]
)/b)])))*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(3^p*c^4*e^4*E^((4*a)/b)*(-((a +
 b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(ce\sqrt{x}+cd\right)+a\right)^{p}x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e*sqrt(x) + c*d) + a)^p*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x\left(b\ln\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln((e*x^(1/2)+d)*c)+a)^p,x)
[Out] int(x*(b*ln((e*x^(1/2)+d)*c)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(1/2))))^p,x)
[Out] int(x*(a + b*log(c*(d + e*x^(1/2))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))))**p,x)
[Out] Timed out
```

3.535 $\int \left(a+b\log \left(c \left(d+e\sqrt{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=174

$$\frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^2e^2}-\frac{2de^{-\frac{a}{b}}\left(a+b\log\left(c\left(d\right.\right.\right.}{}$$

```
[Out] GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(2^p)
/c^2/e^2/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d*GAMMA(1+p,(-a-b*
ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/c/e^2/exp(a/b)/(((-a-b*
ln(c*(d+e*x^(1/2))))/b)^p)
```

Rubi [A] time = 0.22, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2451, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^2e^2}-\frac{2de^{-\frac{a}{b}}\left(a+b\,\mathrm{lo}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sq
rt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^
p) - (2*d*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)]*(a + b*Log[c*(d
 + e*Sqrt[x])])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx = 2\,\text{Subst}\left(\int x(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int\left(-\frac{d(a+b\log(c(d+ex)))^p}{e}+\frac{(d+ex)(a+b\log(c(d+ex)))^p}{e}\right)dx,$$
$$= \frac{2\,\text{Subst}\left(\int(d+ex)(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e} - \frac{(2d)\,\text{Subst}\left(\int(a+b\,\text{lo}\right.}{}$$
$$= \frac{2\,\text{Subst}\left(\int x(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^2} - \frac{(2d)\,\text{Subst}\left(\int(a+b\log(cx))^p\right.}{e^2}$$
$$= \frac{2\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^2e^2} - \frac{(2d)\,\text{Subst}\left(\int e^x(a+bx)^p\right.}{ce}$$
$$= \frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\right.\right.}{b}\right.}{c^2e^2}$$

Mathematica [A] time = 0.13, size = 130, normalized size = 0.75

$$\frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\left(\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)-cd2^{p+1}e^{a/b}\Gamma\left(p+1,-\frac{a+}{}\right.\right.}{c^2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] ((Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b] - 2^(1 + p)*c*d*E^(a/
b)*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)])*(a + b*Log[c*(d + e*S
qrt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))
^p)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(ce\sqrt{x}+cd\right)+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*sqrt(x) + c*d) + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(1/2)+d)*c)+a)^p,x)
[Out] int((b*ln((e*x^(1/2)+d)*c)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)\right)\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))))^p,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))))**p,x)
[Out] Timed out
```

3.536 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/2))))^p/x,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])])^p/x,x]
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)])^p/x, x], x, Sqrt[x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx = 2\,\mathrm{Subst}\left(\int \frac{(a+b\log(c(d+ex)))^p}{x}\,dx,x,\sqrt{x}\right)$$

Mathematica [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x, x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b\log\left(ce\sqrt{x}+cd\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x,x, algorithm="fricas")
[Out] integral((b*log(c*e*sqrt(x) + c*d) + a)^p/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p/x, x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(1/2)+d)*c)+a)^p/x,x)
[Out] int((b*ln((e*x^(1/2)+d)*c)+a)^p/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log \left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)\right)\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))))^p/x,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))))^p/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))))**p/x,x)
[Out] Timed out
```

3.537 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/2))))^p/x^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])])^p/x^2,x]
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)])^p/x^3, x], x, Sqrt[x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx = 2\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)))^p}{x^3}\,dx,x,\sqrt{x}\right)$$

Mathematica [A] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x^2, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(ce\sqrt{x}+cd\right)+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log(c*e*sqrt(x) + c*d) + a)^p/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p/x^2, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(1/2)+d)*c)+a)^p/x^2,x)
[Out] int((b*ln((e*x^(1/2)+d)*c)+a)^p/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)\right)\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))))^p/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))))^p/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))))**p/x**2,x)
[Out] Timed out
```

3.538 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p \, dx$

Optimal. Leaf size=907

$$\frac{2^{-2(p+1)}e^{-\frac{4a}{b}}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^4e^8}-\frac{2^{p+1}7^{-p}de^{-\frac{7a}{2b}}\left(d+\right.}{}$$

```
[Out] GAMMA(1+p,-4*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/(
2^(2+2*p))/c^4/e^8/exp(4*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)+7*d^2*GA
MMA(1+p,-3*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/(3^
p)/c^3/e^8/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)+35*2^(-1-p)*d^4*
GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/c
^2/e^8/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)+7*d^6*GAMMA(1+p,(-a-
b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/c/e^8/exp(a/b)/((
(-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)-2^(1+p)*d*GAMMA(1+p,-7/2*(a+b*ln(c*(d+e*
x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))^7/(7^p)/e^8/exp
(7/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(7/2)-7*
2^(1+p)*d^3*GAMMA(1+p,-5/2*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^
(1/2))^2))^p*(d+e*x^(1/2))^5/(5^p)/e^8/exp(5/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/
2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(5/2)-7*2^(1+p)*d^5*GAMMA(1+p,-3/2*(a+b*l
n(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))^3/(3^p
)/e^8/exp(3/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)
^(3/2)-2^(1+p)*d^7*GAMMA(1+p,1/2*(-a-b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*
(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))/e^8/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2
))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(1/2)
```

Rubi [A] time = 1.40, antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$\frac{2^{-2(p+1)}e^{-\frac{4a}{b}}\text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^4e^8}-\frac{2^{p+1}7^{-p}de}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

```
[Out] (Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e*
Sqrt[x])^2])^p)/(2^(2*(1 + p))*c^4*e^8*E^((4*a)/b)*(-((a + b*Log[c*(d + e*S
qrt[x])^2])/b))^p) - (2^(1 + p)*d*(d + e*Sqrt[x])^7*Gamma[1 + p, (-7*(a + b
*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(7^p
*e^8*E^((7*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(7/2)*(-((a + b*Log[c*(d + e*Sqr
t[x])^2])/b))^p) + (7*d^2*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2])
)/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(3^p*c^3*e^8*E^((3*a)/b)*(-((a + b
*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (7*2^(1 + p)*d^3*(d + e*Sqrt[x])^5*Gamm
a[1 + p, (-5*(a + b*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*S
qrt[x])^2])^p)/(5^p*e^8*E^((5*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(5/2)*(-((a +
 b*Log[c*(d + e*Sqrt[x])^2])/b))^p) + (35*2^(-1 - p)*d^4*Gamma[1 + p, (-2*(
a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(c^
2*e^8*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (7*2^(1 + p)
*d^5*(d + e*Sqrt[x])^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2]))/(
2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(3^p*e^8*E^((3*a)/(2*b))*(c*(d +
e*Sqrt[x])^2)^(3/2)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) + (7*d^6*Gam
```

```
ma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e*Sqrt[x
])^2])^p)/(c*e^8*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (2^(1
 + p)*d^7*(d + e*Sqrt[x])*Gamma[1 + p, -(a + b*Log[c*(d + e*Sqrt[x])^2])/(2
*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^8*E^(a/(2*b))*Sqrt[c*(d + e*Sqr
t[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx = 2\,\text{Subst}\left(\int x^7\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int\left(-\frac{d^7\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^7}+\frac{7d^6(d+ex)\left(a+b\log\left(c(d+\right.\right.}{e^7}\right.\right.$$
$$= \frac{2\,\text{Subst}\left(\int(d+ex)^7\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt{x}\right)}{e^7} - \frac{(14d)\,\text{Subst}\left(\right.}{}$$
$$= \frac{2\,\text{Subst}\left(\int x^7\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+e\sqrt{x}\right)}{e^8} - \frac{(14d)\,\text{Subst}\left(\int x^6\left(a+\right.\right.}{}$$
$$= \frac{\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{c^4e^8} + \frac{\left(21d^2\right)\text{Subst}\left(\int e^{3x}(a\right.}{}$$
$$= \frac{4^{-1-p}e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+}{}\right.}{c^4e^8}$$

Mathematica [F] time = 0.57, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
[Out] Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])^2])^p, x]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(ce^2x+2\,cde\sqrt{x}+cd^2\right)+a\right)^p x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x^3, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int x^3\left(b\ln\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(e*x^(1/2)+d)^2))^p,x)
[Out] int(x^3*(a+b*ln(c*(e*x^(1/2)+d)^2))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(\left(e\sqrt{x} + d\right)^2 c\right) + a\right)^p x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(a + b\, \ln\left(c \left(d + e\, \sqrt{x}\right)^2\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/2))^2))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e*x^(1/2))^2))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
[Out] Timed out
```

3.539 $\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=677

$$\frac{3^{-p-1}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)}{c^3e^6}+\frac{5d^22^{-p}e^{-\frac{2a}{b}}\left(a+b\log\ldots\right.}{\ldots}$$

```
[Out] 3^(-1-p)*GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2)
)^2))^p/c^3/e^6/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)+5*d^2*GAMMA
(1+p,-2*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/(2^p)/
c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)+5*d^4*GAMMA(1+p,(-a
-b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/c/e^6/exp(a/b)/(
((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)-2^(1+p)*d*GAMMA(1+p,-5/2*(a+b*ln(c*(d+e
*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))^5/(5^p)/e^6/ex
p(5/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(5/2)-5
*2^(2+p)*3^(-1-p)*d^3*GAMMA(1+p,-3/2*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln
(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))^3/e^6/exp(3/2*a/b)/(((-a-b*ln(c*(d+e*x
^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(3/2)-2^(1+p)*d^5*GAMMA(1+p,1/2*(-a-b
*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))/e^6/
exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.98, antiderivative size = 677, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$\frac{5d^22^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)}{c^2e^6}+\frac{3^{-p-1}e^{-\frac{3a}{b}}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

```
[Out] (3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])^2])^p)/(c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*Sqrt
[x])^2])/b))^p) - (2^(1 + p)*d*(d + e*Sqrt[x])^5*Gamma[1 + p, (-5*(a + b*Lo
g[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(5^p*e^
6*E^((5*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(5/2)*(-((a + b*Log[c*(d + e*Sqrt[x
])^2])/b))^p) + (5*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b
]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(2^p*c^2*e^6*E^((2*a)/b)*(-((a + b*Lo
g[c*(d + e*Sqrt[x])^2])/b))^p) - (5*2^(2 + p)*3^(-1 - p)*d^3*(d + e*Sqrt[x]
)^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*
(d + e*Sqrt[x])^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(3/2)*(-(
(a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) + (5*d^4*Gamma[1 + p, -((a + b*Log[
c*(d + e*Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(c*e^6*E^(a/b
)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (2^(1 + p)*d^5*(d + e*Sqrt[x
])*Gamma[1 + p, -(a + b*Log[c*(d + e*Sqrt[x])^2])/(2*b)]*(a + b*Log[c*(d +
e*Sqrt[x])^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e*Sqrt[x])^2]*(-((a + b*Log[
c*(d + e*Sqrt[x])^2])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
```

```
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx = 2\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\left(c(d+\right.\right.}{e^5}\right.\right.$$

$$= \frac{2\,\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt{x}\right)}{e^5} - \frac{(10d)\,\text{Subst}\left(\right.}{}$$

$$= \frac{2\,\text{Subst}\left(\int x^5\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+e\sqrt{x}\right)}{e^6} - \frac{(10d)\,\text{Subst}\left(\int x^4\left(a+\right.\right.}{}$$

$$= \frac{\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{c^3e^6} + \frac{\left(10d^2\right)\text{Subst}\left(\int e^{2x}(a\right.}{}$$

$$= \frac{3^{-1-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+}{}\right.}{c^3e^6}$$

Mathematica [F] time = 0.39, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
[Out] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^2])^p, x]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(ce^2x+2\,cde\sqrt{x}+cd^2\right)+a\right)^px^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^px^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x^2, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^2\left(b\ln\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln((e*x^(1/2)+d)^2*c)+a)^p,x)
[Out] int(x^2*(b*ln((e*x^(1/2)+d)^2*c)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(\left(e\sqrt{x} + d\right)^{2} c\right) + a\right)^{p} x^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b \ln\left(c \left(d + e \sqrt{x}\right)^2\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/2))^2))^p,x)
[Out] int(x^2*(a + b*log(c*(d + e*x^(1/2))^2))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
[Out] Timed out
```

3.540 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=445

$$\frac{2^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)}{c^2e^4}-\frac{d^3 2^{p+1}e^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right.}{}$$

```
[Out] 2^(-1-p)*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2)
)^2))^p/c^2/e^4/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)+3*d^2*GAMMA
(1+p,(-a-b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/c/e^4/ex
p(a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)-2^(1+p)*d*GAMMA(1+p,-3/2*(a+b*l
n(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))^3/(3^p
)/e^4/exp(3/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)
^(3/2)-2^(1+p)*d^3*GAMMA(1+p,1/2*(-a-b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*
(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))/e^4/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2
))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 445, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$\frac{2^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)}{c^2e^4}+\frac{3d^2e^{-\frac{a}{b}}\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

```
[Out] (2^(-1 - p)*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])^2])^p)/(c^2*e^4*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt
[x])^2])/b))^p) - (2^(1 + p)*d*(d + e*Sqrt[x])^3*Gamma[1 + p, (-3*(a + b*Lo
g[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(3^p*e^
4*E^((3*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(3/2)*(-((a + b*Log[c*(d + e*Sqrt[x
])^2])/b))^p) + (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b)]*
(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(c*e^4*E^(a/b)*(-((a + b*Log[c*(d + e*S
qrt[x])^2])/b))^p) - (2^(1 + p)*d^3*(d + e*Sqrt[x])*Gamma[1 + p, -(a + b*Lo
g[c*(d + e*Sqrt[x])^2])/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^4*E^(
a/(2*b))*Sqrt[c*(d + e*Sqrt[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^
p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p dx = 2\,\text{Subst}\left(\int x^3\left(a+b\log\left(c(d+ex)^2\right)\right)^p dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int\left(-\frac{d^3\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^3}+\frac{3d^2(d+ex)\left(a+b\log\left(c(d\right.\right.}{e^3}\right.\right.$$

$$= \frac{2\,\text{Subst}\left(\int(d+ex)^3\left(a+b\log\left(c(d+ex)^2\right)\right)^p dx, x, \sqrt{x}\right)}{e^3} - \frac{(6d)\,\text{Subst}\left(\right.}{}$$

$$= \frac{2\,\text{Subst}\left(\int x^3\left(a+b\log\left(cx^2\right)\right)^p dx, x, d+e\sqrt{x}\right)}{e^4} - \frac{(6d)\,\text{Subst}\left(\int x^2\left(a\right.\right.}{}$$

$$= \frac{\text{Subst}\left(\int e^{2x}(a+bx)^p dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{c^2e^4} + \frac{\left(3d^2\right)\text{Subst}\left(\int e^x(a\right.}{}$$

$$= \frac{2^{-1-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\right.}{c^2e^4}$$

Mathematica **[F]** time = 0.25, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^2])^p, x]
```

fricas **[F]** time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(ce^2x+2\,cde\sqrt{x}+cd^2\right)+a\right)^p x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p*x, x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x, x)
```

maple **[F]** time = 0.07, size = 0, normalized size = 0.00

$$\int x\left(b\ln\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln((e*x^(1/2)+d)^2*c)+a)^p,x)
[Out] int(x*(b*ln((e*x^(1/2)+d)^2*c)+a)^p,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\,\ln\left(c\left(d+e\,\sqrt{x}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(1/2))^2))^p,x)
[Out] int(x*(a + b*log(c*(d + e*x^(1/2))^2))^p, x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
```

```
[Out] Timed out
```

3.541 $\int \left(a + b\log\left(c\left(d + e\sqrt{x}\right)^2\right)\right)^p \, dx$

Optimal. Leaf size=213

$$\frac{e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)}{ce^2}-\frac{d2^{p+1}e^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\left(a+b\right.}{}$$

```
[Out] GAMMA(1+p,(-a-b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/c/e
^2/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)-2^(1+p)*d*GAMMA(1+p,1/2*(-
a-b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))/e
^2/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(1
/2)
```

Rubi [A] time = 0.26, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2451, 2401, 2389, 2300, 2181, 2390, 2310}

$$\frac{e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)}{ce^2}-\frac{d2^{p+1}e^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

```
[Out] (Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e*Sq
rt[x])^2])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (
2^(1 + p)*d*(d + e*Sqrt[x])*Gamma[1 + p, -(a + b*Log[c*(d + e*Sqrt[x])^2])/
(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^2*E^(a/(2*b))*Sqrt[c*(d + e*S
qrt[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx = 2\,\text{Subst}\left(\int x\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int\left(-\frac{d\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e}+\frac{(d+ex)\left(a+b\log\left(c(d+ex)\right.\right.}{e}\right.\right.$$
$$= \frac{2\,\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)}{e}-\frac{(2d)\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{2\,\text{Subst}\left(\int x\left(a+b\log\left(cx^2\right)\right)^p\,dx,x,d+e\sqrt{x}\right)}{e^2}-\frac{(2d)\,\text{Subst}\left(\int\left(a+b\log\right.\right.}{}$$
$$= \frac{\text{Subst}\left(\int e^x(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{ce^2}-\frac{\left(d\left(d+e\sqrt{x}\right)\right)\text{Subst}\left(\right.}{}$$
$$= \frac{e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\right.\right.}{b}\right.}{ce^2}$$

Mathematica [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p, x]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(ce^2x + 2\,cde\sqrt{x} + cd^2\right) + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(\left(e\sqrt{x} + d\right)^2 c\right) + a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(\left(e\sqrt{x} + d\right)^2 c\right) + a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(1/2)+d)^2*c)+a)^p,x)
[Out] int((b*ln((e*x^(1/2)+d)^2*c)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(\left(e\sqrt{x} + d\right)^2 c\right) + a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b\,\ln\left(c\left(d + e\,\sqrt{x}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^2))^p,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))^2))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
[Out] Timed out
```

3.542 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/2))^2))^p/x,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x,x]
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)^2])^p/x, x], x, Sqrt[x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx = 2\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{x}\,dx,x,\sqrt{x}\right)$$

Mathematica [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x, x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(ce^2x+2\,cde\sqrt{x}+cd^2\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p/x, x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(\left(e\sqrt{x}+d\right)^{2} c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(1/2)+d)^2*c)+a)^p/x,x)
[Out] int((b*ln((e*x^(1/2)+d)^2*c)+a)^p/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log \left(\left(e\sqrt{x}+d\right)^{2} c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^2\right)\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^2))^p/x,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))^2))^p/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**2))**p/x,x)
[Out] Timed out
```

3.543 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/2))^2))^p/x^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x^2,x]
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)^2])^p/x^3, x], x, Sqrt[x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx = 2\,\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{x^3}\,dx,x,\sqrt{x}\right)$$

Mathematica [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x^2, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b\log\left(ce^2x+2\,cde\sqrt{x}+cd^2\right)+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p/x^2, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(\left(e\sqrt{x}+d\right)^{2} c\right)+a\right)^{p}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(1/2)+d)^2*c)+a)^p/x^2,x)
```

```
[Out] int((b*ln((e*x^(1/2)+d)^2*c)+a)^p/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log \left(\left(e\sqrt{x}+d\right)^{2} c\right)+a\right)^{p}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^2\right)\right)^p}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^2))^p/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/2))^2))^p/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**2))**p/x**2,x)
```

```
[Out] Timed out
```

3.544 $\int x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p \, dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
[Out] 2*Defer[Subst][Defer[Int][x^3*(a + b*Log[c*(d + e/x)])^p, x], x, Sqrt[x]]
```

Rubi steps

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p \, dx = 2 \operatorname{Subst}\left(\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x}\right)\right)\right)^p \, dx,x,\sqrt{x}\right)$$

Mathematica [A] time = 1.16, size = 0, normalized size = 0.00

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])])^p, x]
```

fricas [A] time = 0.75, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b \log \left(\frac{c d x + c e \sqrt{x}}{x}\right) + a\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p*x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(c \left(d + \frac{e}{\sqrt{x}}\right)\right) + a\right)^p x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p*x, x)
```

maple [A] time = 0.12, size = 0, normalized size = 0.00

$$\int x\left(b\ln\left(\left(d+\frac{e}{\sqrt{x}}\right)c\right)+a\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/2))))^p,x)
[Out] int(x*(a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^{p}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p*x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/2))))^p,x)
[Out] int(x*(a + b*log(c*(d + e/x^(1/2))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))))**p,x)
[Out] Timed out
```

3.545 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p dx$

Optimal. Leaf size=21

$$\text{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
[Out] 2*Defer[Subst][Defer[Int][x*(a + b*Log[c*(d + e/x)])^p, x], x, Sqrt[x]]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p dx = 2\,\text{Subst}\left(\int x\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p dx, x, \sqrt{x}\right)$$

Mathematica [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p, x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(\frac{cdx+ce\sqrt{x}}{x}\right)+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(\left(d+\frac{e}{\sqrt{x}}\right)c\right)+a\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/2))*c)+a)^p,x)
[Out] int((b*ln((d+e/x^(1/2))*c)+a)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))))^p,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p,x)
[Out] Timed out
```

3.546 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/2))))^p/x,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x,x]
```

```
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x)])^p/x, x], x, Sqrt[x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx = 2\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p}{x}\,dx,x,\sqrt{x}\right)$$

Mathematica [A] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x, x]
```

fricas [A] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cdx+ce\sqrt{x}}{x}\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\left(d+\frac{e}{\sqrt{x}}\right)c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/2))*c)+a)^p/x,x)
[Out] int((b*ln((d+e/x^(1/2))*c)+a)^p/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))))^p/x,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))))^p/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x,x)
[Out] Timed out
```

3.547 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=175

$$\frac{2de^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)}{ce^2}-\frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+\right.\right.\right.}{}$$

```
[Out] -GAMMA(1+p,-2*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(2^p
)/c^2/e^2/exp(2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)+2*d*GAMMA(1+p,(-a-b
*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/c/e^2/exp(a/b)/(((-a-b
*ln(c*(d+e/x^(1/2))))/b)^p)
```

Rubi [A] time = 0.25, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{2de^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)}{ce^2}-\frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^2,x]
```

```
[Out] -((Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/
Sqrt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b)
)^p)) + (2*d*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)]*(a + b*Log[c
*(d + e/Sqrt[x])])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^
p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx = -\left(2\,\text{Subst}\left(\int x(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\,\text{Subst}\left(\int\left(-\frac{d(a+b\log(c(d+ex)))^p}{e}+\frac{(d+ex)(a+b\log(c(d+ex)))^p}{e}\right) d\right.\right.$$

$$= -\frac{2\,\text{Subst}\left(\int(d+ex)(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e}+\frac{(2d)\,\text{Subst}\left(\int(a+b\,\text{l}\right.}{}$$

$$= -\frac{2\,\text{Subst}\left(\int x(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^2}+\frac{(2d)\,\text{Subst}\left(\int(a+b\log(cx))^p\right.}{e^2}$$

$$= -\frac{2\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^2e^2}+\frac{(2d)\,\text{Subst}\left(\int e^{x}(a+bx)^p\right.}{ce^2}$$

$$= -\frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\right.\right.}{b}\right.}{c^2e^2}$$

Mathematica [A] time = 0.18, size = 131, normalized size = 0.75

$$\frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\left(cd2^{p+1}e^{a/b}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)-\Gamma\left(p+1,-\frac{2\left(a+b\log\right.}{}\right.\right.}{c^2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^2,x]
```

```
[Out] ((-Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b] + 2^(1 + p)*c*d*E^(a
/b)*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)])*(a + b*Log[c*(d + e/
Sqrt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b)
)^p)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cdx+ce\sqrt{x}}{x}\right)+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^2, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(\left(d+\frac{e}{\sqrt{x}}\right)c\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/2))*c)+a)^p/x^2,x)
[Out] int((b*ln((d+e/x^(1/2))*c)+a)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))))^p/x^2,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))))^p/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x**2,x)
[Out] Timed out
```

3.548 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx$

Optimal. Leaf size=552

$$-\frac{2^{-p}3^{-p-1}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)}{c^6e^6}+\frac{2d5^{-p}e^{-\frac{5a}{b}}\left(a+b\log\right.}{}$$

```
[Out] -3^(-1-p)*GAMMA(1+p,-6*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))
))^p/(2^p)/c^6/e^6/exp(6*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)+2*d*GAMMA(
1+p,-5*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(5^p)/c^5/e
^6/exp(5*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)-5*d^2*GAMMA(1+p,-4*(a+b*ln
(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(4^p)/c^4/e^6/exp(4*a/b)/
(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)+20*3^(-1-p)*d^3*GAMMA(1+p,-3*(a+b*ln(c*(
d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/c^3/e^6/exp(3*a/b)/(((-a-b*ln
(c*(d+e/x^(1/2))))/b)^p)-5*d^4*GAMMA(1+p,-2*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a
+b*ln(c*(d+e/x^(1/2))))^p/(2^p)/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2
))))/b)^p)+2*d^5*GAMMA(1+p,(-a-b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(
1/2))))^p/c/e^6/exp(a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)
```

Rubi [A] time = 0.85, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$-\frac{5d^24^{-p}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)}{c^4e^6}+\frac{20d^33^{-p-1}e^{-\frac{3a}{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^4,x]
```

```
[Out] -((3^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Lo
g[c*(d + e/Sqrt[x])])^p)/(2^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/Sq
rt[x])])/b))^p)) + (2*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])]))/b]
*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c
*(d + e/Sqrt[x])])/b))^p) - (5*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/Sqr
t[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(4^p*c^4*e^6*E^((4*a)/b)*(-((
a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (20*3^(-1 - p)*d^3*Gamma[1 + p, (-3*
(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^3*e
^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) - (5*d^4*Gamma[1 +
p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)
/(2^p*c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (2*d^5
*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)]*(a + b*Log[c*(d + e/Sqrt
[x])])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
```

```
c, p}, x] && IntegerQ[1/n]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx = -\left(2\,\text{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\,\text{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d+e\right.\right.\right.$$

$$= -\frac{2\,\text{Subst}\left(\int (d+ex)^5(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^5}+\frac{(10d)\,\text{Subst}\left(\int (d\right.}{}$$

$$= -\frac{2\,\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6}+\frac{(10d)\,\text{Subst}\left(\int x^4(a+b\,\text{lo}\right.}{e^6}$$

$$= -\frac{2\,\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^6e^6}+\frac{(10d)\,\text{Subst}\left(\int e^{5x}(a+\right.}{}$$

$$= -\frac{2^{-p}3^{-1-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+}{}\right.}{c^6e^6}$$

Mathematica [A] time = 0.79, size = 325, normalized size = 0.59

$$3^{-p-1}20^{-p}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\left(cde^{a/b}\left(2^{2p+1}3^{p+1}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^4,x]
```

```
[Out] (3^(-1 - p)*(-(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b]) +
c*d*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqr
t[x])]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c*(
d + e/Sqrt[x])]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*(a + b
*Log[c*(d + e/Sqrt[x])]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p, (-2*(
a + b*Log[c*(d + e/Sqrt[x])]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a
 + b*Log[c*(d + e/Sqrt[x])])/b)])))))*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(20
^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cdx+ce\sqrt{x}}{x}\right)+a\right)^p}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^4, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(\left(d+\frac{e}{\sqrt{x}}\right)c\right)+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/2))*c)+a)^p/x^4,x)
```

```
[Out] int((b*ln((d+e/x^(1/2))*c)+a)^p/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))))^p/x^4,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/2))))^p/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x**4,x)
```

```
[Out] Timed out
```

3.549 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx$

Optimal. Leaf size=926

$$-\frac{2^{-p}5^{-p-1}e^{-\frac{10a}{b}}\Gamma\left(p+1,-\frac{10\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^{10}e^{10}}+\frac{2\,9^{-p}de^{-\frac{9a}{b}}\Gamma\left(p+1\right.}{}$$

```
[Out] -5^(-1-p)*GAMMA(1+p,-10*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2)
)))^p/(2^p)/c^10/e^10/exp(10*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)+2*d*GA
MMA(1+p,-9*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(9^p)/c
^9/e^10/exp(9*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)-9*d^2*GAMMA(1+p,-8*(a
+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(8^p)/c^8/e^10/exp(8
*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)+24*d^3*GAMMA(1+p,-7*(a+b*ln(c*(d+e
/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(7^p)/c^7/e^10/exp(7*a/b)/(((-a-
b*ln(c*(d+e/x^(1/2))))/b)^p)-7*6^(1-p)*d^4*GAMMA(1+p,-6*(a+b*ln(c*(d+e/x^(1
/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/c^6/e^10/exp(6*a/b)/(((-a-b*ln(c*(d+e
/x^(1/2))))/b)^p)+252*5^(-1-p)*d^5*GAMMA(1+p,-5*(a+b*ln(c*(d+e/x^(1/2))))/b
)*(a+b*ln(c*(d+e/x^(1/2))))^p/c^5/e^10/exp(5*a/b)/(((-a-b*ln(c*(d+e/x^(1/2)
)))/b)^p)-21*2^(1-2*p)*d^6*GAMMA(1+p,-4*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*l
n(c*(d+e/x^(1/2))))^p/c^4/e^10/exp(4*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p
)+8*3^(1-p)*d^7*GAMMA(1+p,-3*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^
(1/2))))^p/c^3/e^10/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)-9*d^8*GAM
MA(1+p,-2*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(2^p)/c^
2/e^10/exp(2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)+2*d^9*GAMMA(1+p,(-a-b*
ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/c/e^10/exp(a/b)/(((-a-b
*ln(c*(d+e/x^(1/2))))/b)^p)
```

Rubi [A] time = 1.56, antiderivative size = 926, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^6,x]
```

```
[Out] -((5^(-1 - p)*Gamma[1 + p, (-10*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*L
og[c*(d + e/Sqrt[x])])^p)/(2^p*c^10*e^10*E^((10*a)/b)*(-((a + b*Log[c*(d +
e/Sqrt[x])])/b))^p)) + (2*d*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/Sqrt[x])])
)/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(9^p*c^9*e^10*E^((9*a)/b)*(-((a + b*
Log[c*(d + e/Sqrt[x])])/b))^p) - (9*d^2*Gamma[1 + p, (-8*(a + b*Log[c*(d +
e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(8^p*c^8*e^10*E^((8*a)/b
)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (24*d^3*Gamma[1 + p, (-7*(a +
b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(7^p*c^7*e^
10*E^((7*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) - (7*6^(1 - p)*d^4*
Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqr
t[x])])^p)/(c^6*e^10*E^((6*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) +
 (252*5^(-1 - p)*d^5*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a
 + b*Log[c*(d + e/Sqrt[x])])^p)/(c^5*e^10*E^((5*a)/b)*(-((a + b*Log[c*(d +
e/Sqrt[x])])/b))^p) - (21*2^(1 - 2*p)*d^6*Gamma[1 + p, (-4*(a + b*Log[c*(d
+ e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^4*e^10*E^((4*a)/b)*
(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (8*3^(1 - p)*d^7*Gamma[1 + p, (-
3*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^3
*e^10*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) - (9*d^8*Gamma[1
 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])
```

```
^p)/(2^p*c^2*e^10*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (2
*d^9*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)]*(a + b*Log[c*(d + e/
Sqrt[x])])^p)/(c*e^10*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx = -\left(2\,\mathrm{Subst}\left(\int x^9(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\,\mathrm{Subst}\left(\int \left(-\frac{d^9(a+b\log(c(d+ex)))^p}{e^9} + \frac{9d^8(d+ex)(a+b\log(c(d+ex)))}{e^9}\right.\right.\right.$$

$$= -\frac{2\,\mathrm{Subst}\left(\int (d+ex)^9(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^9} + \frac{(18d)\,\mathrm{Subst}\left(\int (d+e\right.}{}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int x^9(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^{10}} + \frac{(18d)\,\mathrm{Subst}\left(\int x^8(a+b\log(c\right.}{e^{10}}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int e^{10x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^{10}e^{10}} + \frac{(18d)\,\mathrm{Subst}\left(\int e^{9x}(a+b\right.}{c^9}$$

$$= -\frac{2^{-p}5^{-1-p}e^{-\frac{10a}{b}}\Gamma\left(1+p, -\frac{10\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b}{}\right.}{c^{10}e^{10}}$$

Mathematica [A] time = 3.31, size = 525, normalized size = 0.57

$$\frac{5^{-p-1}504^{-p}e^{-\frac{10a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\left(cde^{a/b}\left(2^{3p+1}5^{p+1}7^p\Gamma\left(p+1, -\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^6,x]
```

```
[Out] (5^(-1 - p)*(-(252^p*Gamma[1 + p, (-10*(a + b*Log[c*(d + e/Sqrt[x])]))/b])
+ c*d*E^(a/b)*(2^(1 + 3*p)*5^(1 + p)*7^p*Gamma[1 + p, (-9*(a + b*Log[c*(d +
 e/Sqrt[x])]))/b] + c*d*E^(a/b)*(-(7^p*45^(1 + p)*Gamma[1 + p, (-8*(a + b*L
og[c*(d + e/Sqrt[x])]))/b]) + 2^p*c*d*E^(a/b)*(2^(3 + 2*p)*3^(1 + 2*p)*5^(1
 + p)*Gamma[1 + p, (-7*(a + b*Log[c*(d + e/Sqrt[x])]))/b] + 7^p*c*d*E^(a/b)
*(-7*30^(1 + p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b] + c*d*E
^(a/b)*(7*36^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])]))/b] +
3^p*5^(1 + p)*c*d*E^(a/b)*(-14*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c*(d +
 e/Sqrt[x])]))/b] + 2^p*c*d*E^(a/b)*(3*2^(3 + p)*Gamma[1 + p, (-3*(a + b*Lo
g[c*(d + e/Sqrt[x])]))/b] + 3^p*c*d*E^(a/b)*(-9*Gamma[1 + p, (-2*(a + b*Log
[c*(d + e/Sqrt[x])]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a + b*Log[
c*(d + e/Sqrt[x])])/b)])))))))))*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(504^p*c
^10*e^10*E^((10*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b\log\left(\frac{cdx+ce\sqrt{x}}{x}\right)+a\right)^p}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^{p}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^6,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^6, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\left(d+\frac{e}{\sqrt{x}}\right)c\right)+a\right)^{p}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/2))*c)+a)^p/x^6,x)
[Out] int((b*ln((d+e/x^(1/2))*c)+a)^p/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^{p}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^6,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^{p}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))))^p/x^6,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))))^p/x^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x**6,x)
[Out] Timed out
```

3.550 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
[Out] 2*Defer[Subst][Defer[Int][x^3*(a + b*Log[c*(d + e/x)^2])^p, x], x, Sqrt[x]]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=2\operatorname{Subst}\left(\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p\,dx,x,\sqrt{x}\right)$$

Mathematica [A] time = 0.23, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^2])^p, x]
```

fricas [A] time = 0.81, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b\log\left(\frac{cd^2x+2\,cde\sqrt{x}\,+ce^2}{x}\right)+a\right)^p x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p*x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p*x, x)
```

maple [A] time = 0.23, size = 0, normalized size = 0.00

$$\int x \left(b \ln \left(\left(d+\frac{e}{\sqrt{x}}\right)^{2} c\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
[Out] int(x*(a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)+a\right)^{p} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p*x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x \left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/2))^2))^p,x)
[Out] int(x*(a + b*log(c*(d + e/x^(1/2))^2))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))**2))**p,x)
[Out] Timed out
```

3.551 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
[Out] 2*Defer[Subst][Defer[Int][x*(a + b*Log[c*(d + e/x)^2])^p, x], x, Sqrt[x]]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx = 2\,\text{Subst}\left(\int x\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p\,dx,x,\sqrt{x}\right)$$

Mathematica [A] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p, x]
```

fricas [A] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(\frac{cd^2x+2\,cde\sqrt{x}+ce^2}{x}\right)+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \left(b \ln \left(\left(d + \frac{e}{\sqrt{x}}\right)^2 c\right) + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/2))^2*c)+a)^p,x)
```

```
[Out] int((b*ln((d+e/x^(1/2))^2*c)+a)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(c \left(d + \frac{e}{\sqrt{x}}\right)^2\right) + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a + b \ln \left(c \left(d + \frac{e}{\sqrt{x}}\right)^2\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^2))^p,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/2))^2))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p,x)
```

```
[Out] Timed out
```

3.552 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/2))^2))^p/x,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x,x]
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x)^2])^p/x, x], x, Sqrt[x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx = 2\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p}{x}\,dx, x, \sqrt{x}\right)$$

Mathematica [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x, x]
```

fricas [A] time = 0.98, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(b\log\left(\frac{cd^2x+2\,cde\sqrt{x}+ce^2}{x}\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p/x, x)
```

giac [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x, x)
```

maple [**A**] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\left(d+\frac{e}{\sqrt{x}}\right)^{2}c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/2))^2*c)+a)^p/x,x)
[Out] int((b*ln((d+e/x^(1/2))^2*c)+a)^p/x,x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x, x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x,x)
[Out] Timed out
```

3.553 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=216

$$\frac{d2^{p+1}e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,\frac{-a-b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{2b}\right)}{e^2\sqrt{c\left(d+\frac{e}{\sqrt{x}}\right)^2}}-\frac{e^{-\frac{a}{b}}\left(a+b\log\right.}{}$$

```
[Out] -GAMMA(1+p,(-a-b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/c/
e^2/exp(a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)+2^(1+p)*d*GAMMA(1+p,1/2*(
-a-b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2))/
e^2/exp(1/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(
1/2)
```

Rubi [A] time = 0.29, antiderivative size = 213, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$\frac{d2^{p+1}e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{2b}\right)}{e^2\sqrt{c\left(d+\frac{e}{\sqrt{x}}\right)^2}}-\frac{e^{-\frac{a}{b}}\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^2,x]
```

```
[Out] -((Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e/
Sqrt[x])^2])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p))
+ (2^(1 + p)*d*(d + e/Sqrt[x])*Gamma[1 + p, -(a + b*Log[c*(d + e/Sqrt[x])^2
])/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(e^2*E^(a/(2*b))*Sqrt[c*(d +
e/Sqrt[x])^2]*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx = -\left(2\,\text{Subst}\left(\int x\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\,\text{Subst}\left(\int\left(-\frac{d\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e}+\frac{(d+ex)\left(a+b\log\left(c(d+e\right.\right.}{e}\right.\right.\right.$$

$$= -\frac{2\,\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e}+\frac{(2d)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2\,\text{Subst}\left(\int x\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^2}+\frac{(2d)\,\text{Subst}\left(\int\left(a+b\log\right.\right.}{e}$$

$$= -\frac{\text{Subst}\left(\int e^x(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{ce^2}+\frac{\left(d\left(d+\frac{e}{\sqrt{x}}\right)\right)\text{Subst}\left(\int\right.}{e}$$

$$= -\frac{e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\right.\right.}{b}\right.}{ce^2}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^2, x]
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b\log\left(\frac{cd^2x+2\,cde\sqrt{x}+ce^2}{x}\right)+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^2, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\left(d+\frac{e}{\sqrt{x}}\right)^2 c\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/2))^2*c)+a)^p/x^2,x)
[Out] int((b*ln((d+e/x^(1/2))^2*c)+a)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x**2,x)
```

```
[Out] Timed out
```

3.554 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx$

Optimal. Leaf size=676

$$-\frac{3^{-p-1}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)}{c^3e^6}-\frac{5d^2 2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\right.}{}$$

```
[Out] -3^(-1-p)*GAMMA(1+p,-3*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2
))^2))^p/c^3/e^6/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)-5*d^2*GAMM
A(1+p,-2*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/(2^p)
/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)-5*d^4*GAMMA(1+p,(-
a-b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/c/e^6/exp(a/b)/
(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)+2^(1+p)*d*GAMMA(1+p,-5/2*(a+b*ln(c*(d+
e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2))^5/(5^p)/e^6/e
xp(5/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(5/2)+
5*2^(2+p)*3^(-1-p)*d^3*GAMMA(1+p,-3/2*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*l
n(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2))^3/e^6/exp(3/2*a/b)/(((-a-b*ln(c*(d+e/
x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(3/2)+2^(1+p)*d^5*GAMMA(1+p,1/2*(-a-
b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2))/e^6
/exp(1/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(1/2
)
```

Rubi [A] time = 1.00, antiderivative size = 676, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$-\frac{5d^2 2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)}{c^2e^6}-\frac{3^{-p-1}e^{-\frac{3a}{b}}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^4,x]
```

```
[Out] -((3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a + b*
Log[c*(d + e/Sqrt[x])^2])^p)/(c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sq
rt[x])^2])/b))^p)) + (2^(1 + p)*d*(d + e/Sqrt[x])^5*Gamma[1 + p, (-5*(a + b
*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(5^p
*e^6*E^((5*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(5/2)*(-((a + b*Log[c*(d + e/Sqr
t[x])^2])/b))^p) - (5*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])^2])
)/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(2^p*c^2*e^6*E^((2*a)/b)*(-((a + b
*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (5*2^(2 + p)*3^(-1 - p)*d^3*(d + e/Sqrt
[x])^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log
[c*(d + e/Sqrt[x])^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(3/2)*
(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) - (5*d^4*Gamma[1 + p, -((a + b*L
og[c*(d + e/Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c*e^6*E^(
a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (2^(1 + p)*d^5*(d + e/Sqr
t[x])*Gamma[1 + p, -(a + b*Log[c*(d + e/Sqrt[x])^2])/(2*b)]*(a + b*Log[c*(d
 + e/Sqrt[x])^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e/Sqrt[x])^2]*(-((a + b*L
og[c*(d + e/Sqrt[x])^2])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx = -\left(2\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\,\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5} + \frac{5d^4(d+ex)\left(a+b\log\left(c(d+\right.\right.}{e^5}\right.\right.\right.$$

$$= -\frac{2\,\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^5} + \frac{(10d)\,\text{Subst}\left(\int(\right.}{}$$

$$= -\frac{2\,\text{Subst}\left(\int x^5\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} + \frac{(10d)\,\text{Subst}\left(\int x^4\left(a+b\right.\right.}{e}$$

$$= -\frac{\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{c^3e^6} - \frac{\left(10d^2\right)\text{Subst}\left(\int e^{2x}(a+b\right.}{}$$

$$= -\frac{3^{-1-p}e^{-\frac{3a}{b}}\Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{c^3e^6}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^4,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^4, x]
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cd^2x+2\,cde\sqrt{x}+ce^2}{x}\right)+a\right)^p}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^4,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^4, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\left(d+\frac{e}{\sqrt{x}}\right)^{2}c\right)+a\right)^{p}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/2))^2*c)+a)^p/x^4,x)
[Out] int((b*ln((d+e/x^(1/2))^2*c)+a)^p/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)+a\right)^{p}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^4,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)\right)^{p}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^4,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x**4,x)
[Out] Timed out
```

3.555 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx$

Optimal. Leaf size=1141

$$-\frac{5^{-p-1}e^{-\frac{5a}{b}}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{c^5e^{10}}+\frac{2^{p+1}9^{-p}de^{-\frac{9a}{2b}}\left(d+\right.}{}$$

```
[Out] -5^(-1-p)*GAMMA(1+p,-5*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2
))^2))^p/c^5/e^10/exp(5*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)-9*d^2*GAM
MA(1+p,-4*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/(4^p
)/c^4/e^10/exp(4*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)-14*3^(1-p)*d^4*G
AMMA(1+p,-3*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/c^
3/e^10/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)-21*2^(1-p)*d^6*GAMMA
(1+p,-2*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/c^2/e^
10/exp(2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)-9*d^8*GAMMA(1+p,(-a-b*ln
(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/c/e^10/exp(a/b)/(((-a
-b*ln(c*(d+e/x^(1/2))^2))/b)^p)+2^(1+p)*d*GAMMA(1+p,-9/2*(a+b*ln(c*(d+e/x^(
1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2))^9/(9^p)/e^10/exp(9
/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(9/2)+3*2^
(3+p)*d^3*GAMMA(1+p,-7/2*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1
/2))^2))^p*(d+e/x^(1/2))^7/(7^p)/e^10/exp(7/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2
))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(7/2)+63*2^(2+p)*5^(-1-p)*d^5*GAMMA(1+p,-5
/2*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2
))^5/e^10/exp(5/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2)
)^2)^(5/2)+2^(3+p)*3^(1-p)*d^7*GAMMA(1+p,-3/2*(a+b*ln(c*(d+e/x^(1/2))^2))/b
)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2))^3/e^10/exp(3/2*a/b)/(((-a-b*l
n(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(3/2)+2^(1+p)*d^9*GAMMA(1+p
,1/2*(-a-b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(
1/2))/e^10/exp(1/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2
))^2)^(1/2)
```

Rubi [A] time = 1.74, antiderivative size = 1141, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^6,x]
```

```
[Out] -((5^(-1 - p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a + b*
Log[c*(d + e/Sqrt[x])^2])^p)/(c^5*e^10*E^((5*a)/b)*(-((a + b*Log[c*(d + e/S
qrt[x])^2])/b))^p)) + (2^(1 + p)*d*(d + e/Sqrt[x])^9*Gamma[1 + p, (-9*(a +
b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(9^
p*e^10*E^((9*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(9/2)*(-((a + b*Log[c*(d + e/S
qrt[x])^2])/b))^p) - (9*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/Sqrt[x])^2
]))/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(4^p*c^4*e^10*E^((4*a)/b)*(-((a
+ b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (3*2^(3 + p)*d^3*(d + e/Sqrt[x])^7*G
amma[1 + p, (-7*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d +
e/Sqrt[x])^2])^p)/(7^p*e^10*E^((7*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(7/2)*(-(
(a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) - (14*3^(1 - p)*d^4*Gamma[1 + p, (-
3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/
```

```
(c^3*e^10*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (63*2^(2
 + p)*5^(-1 - p)*d^5*(d + e/Sqrt[x])^5*Gamma[1 + p, (-5*(a + b*Log[c*(d + e
/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(e^10*E^((5*a)/(2
*b))*(c*(d + e/Sqrt[x])^2)^(5/2)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p)
 - (21*2^(1 - p)*d^6*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*
(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c^2*e^10*E^((2*a)/b)*(-((a + b*Log[c*(
d + e/Sqrt[x])^2])/b))^p) + (2^(3 + p)*3^(1 - p)*d^7*(d + e/Sqrt[x])^3*Gamm
a[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/S
qrt[x])^2])^p)/(e^10*E^((3*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(3/2)*(-((a + b*
Log[c*(d + e/Sqrt[x])^2])/b))^p) - (9*d^8*Gamma[1 + p, -((a + b*Log[c*(d +
e/Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c*e^10*E^(a/b)*(-((
a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (2^(1 + p)*d^9*(d + e/Sqrt[x])*Gam
ma[1 + p, -(a + b*Log[c*(d + e/Sqrt[x])^2])/(2*b)]*(a + b*Log[c*(d + e/Sqrt
[x])^2])^p)/(e^10*E^(a/(2*b))*Sqrt[c*(d + e/Sqrt[x])^2]*(-((a + b*Log[c*(d
+ e/Sqrt[x])^2])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx = -\left(2\,\text{Subst}\left(\int x^9\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\,\text{Subst}\left(\int\left(-\frac{d^9\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^9}+\frac{9d^8(d+ex)\left(a+b\log\left(c(d+\right.\right.}{e^9}\right.\right.\right.$$

$$= -\frac{2\,\text{Subst}\left(\int(d+ex)^9\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\frac{1}{\sqrt{x}}\right)}{e^9}+\frac{(18d)\,\text{Subst}\left(\int(\right.}{}$$

$$= -\frac{2\,\text{Subst}\left(\int x^9\left(a+b\log\left(cx^2\right)\right)^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}}+\frac{(18d)\,\text{Subst}\left(\int x^8\left(a+b\,l\right.\right.}{e}$$

$$= -\frac{\text{Subst}\left(\int e^{5x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{c^5e^{10}}-\frac{\left(36d^2\right)\text{Subst}\left(\int e^{4x}(a+b\right.}{c}$$

$$= -\frac{5^{-1-p}e^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{c^5e^{10}}$$

Mathematica [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^6,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^6, x]
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cd^2x+2\,cde\sqrt{x}+ce^2}{x}\right)+a\right)^p}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^6,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^6,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^6, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\left(d+\frac{e}{\sqrt{x}}\right)^{2} c\right)+a\right)^{p}}{x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/2))^2*c)+a)^p/x^6,x)
[Out] int((b*ln((d+e/x^(1/2))^2*c)+a)^p/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)+a\right)^{p}}{x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^6,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)\right)^{p}}{x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^6,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x**6,x)
[Out] Timed out
```

3.556 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=1121

$$\frac{3^{-p}4^{-p-1}e^{-\frac{12a}{b}}\Gamma\left(p+1,-\frac{12\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^{12}e^{12}}-\frac{3\,11^{-p}de^{-\frac{11a}{b}}\Gamma\left(p\right.}{}$$

```
[Out] 4^(-1-p)*GAMMA(1+p,-12*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))
))^p/(3^p)/c^12/e^12/exp(12*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d*GAM
MA(1+p,-11*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(11^p)/
c^11/e^12/exp(11*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+33*2^(-1-p)*d^2*GA
MMA(1+p,-10*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(5^p)/
c^10/e^12/exp(10*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-55*d^3*GAMMA(1+p,-
9*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(9^p)/c^9/e^12/e
xp(9*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+495*2^(-2-3*p)*d^4*GAMMA(1+p,-
8*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c^8/e^12/exp(8*a
/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-198*d^5*GAMMA(1+p,-7*(a+b*ln(c*(d+e*
x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(7^p)/c^7/e^12/exp(7*a/b)/(((-a-b
*ln(c*(d+e*x^(1/3))))/b)^p)+77*3^(1-p)*d^6*GAMMA(1+p,-6*(a+b*ln(c*(d+e*x^(1
/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(2^p)/c^6/e^12/exp(6*a/b)/(((-a-b*ln(
c*(d+e*x^(1/3))))/b)^p)-198*d^7*GAMMA(1+p,-5*(a+b*ln(c*(d+e*x^(1/3))))/b)*(
a+b*ln(c*(d+e*x^(1/3))))^p/(5^p)/c^5/e^12/exp(5*a/b)/(((-a-b*ln(c*(d+e*x^(1
/3))))/b)^p)+495*4^(-1-p)*d^8*GAMMA(1+p,-4*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+
b*ln(c*(d+e*x^(1/3))))^p/c^4/e^12/exp(4*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b
)^p)-55*d^9*GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3
))))^p/(3^p)/c^3/e^12/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+33*2^(-
1-p)*d^10*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))
))^p/c^2/e^12/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d^11*GAMMA(1+
p,(-a-b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c/e^12/exp(a/b)
/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)
```

Rubi [A] time = 1.87, antiderivative size = 1121, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (4^(-1 - p)*Gamma[1 + p, (-12*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log
[c*(d + e*x^(1/3))])^p)/(3^p*c^12*e^12*E^((12*a)/b)*(-((a + b*Log[c*(d + e*
x^(1/3))])/b))^p) - (3*d*Gamma[1 + p, (-11*(a + b*Log[c*(d + e*x^(1/3))]))/
b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(11^p*c^11*e^12*E^((11*a)/b)*(-((a + b
*Log[c*(d + e*x^(1/3))])/b))^p) + (33*2^(-1 - p)*d^2*Gamma[1 + p, (-10*(a +
 b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(5^p*c^10*
e^12*E^((10*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (55*d^3*Gamma[
1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))]
)^p)/(9^p*c^9*e^12*E^((9*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (
495*2^(-2 - 3*p)*d^4*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))])^p)/(c^8*e^12*E^((8*a)/b)*(-((a + b*Log[c*(d +
e*x^(1/3))])/b))^p) - (198*d^5*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*x^(1/3)
)]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(7^p*c^7*e^12*E^((7*a)/b)*(-((a +
 b*Log[c*(d + e*x^(1/3))])/b))^p) + (77*3^(1 - p)*d^6*Gamma[1 + p, (-6*(a +
 b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(2^p*c^6*e
^12*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (198*d^7*Gamma[1
 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])
```

```
^p)/(5^p*c^5*e^12*E^((5*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (4
95*4^(-1 - p)*d^8*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a +
b*Log[c*(d + e*x^(1/3))])^p)/(c^4*e^12*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x
^(1/3))])/b))^p) - (55*d^9*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))
/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^3*e^12*E^((3*a)/b)*(-((a + b*L
og[c*(d + e*x^(1/3))])/b))^p) + (33*2^(-1 - p)*d^10*Gamma[1 + p, (-2*(a + b
*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^2*e^12*E^
((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (3*d^11*Gamma[1 + p, -
((a + b*Log[c*(d + e*x^(1/3))])/b)]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c*e^
12*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx = 3\,\text{Subst}\left(\int x^{11}(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)$$
$$= 3\,\text{Subst}\left(\int\left(-\frac{d^{11}(a+b\log(c(d+ex)))^p}{e^{11}}+\frac{11d^{10}(d+ex)(a+b\log(c(d+e\ldots}{e^{11}}\right.\right.$$
$$= \frac{3\,\text{Subst}\left(\int(d+ex)^{11}(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^{11}} - \frac{(33d)\,\text{Subst}\left(\int(d\ldots\right.}{}$$
$$= \frac{3\,\text{Subst}\left(\int x^{11}(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^{12}} - \frac{(33d)\,\text{Subst}\left(\int x^{10}(a+b\ldots\right.}{e^{1\ldots}}$$
$$= \frac{3\,\text{Subst}\left(\int e^{12x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^{12}e^{12}} - \frac{(33d)\,\text{Subst}\left(\int e^{11x}(a\ldots\right.}{}$$
$$= \frac{3^{-p}4^{-1-p}e^{-\frac{12a}{b}}\Gamma\left(1+p,-\frac{12\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\ldots\right.}{c^{12}e^{12}}$$

Mathematica [A] time = 2.71, size = 670, normalized size = 0.60

$$-\frac{2^{-3p-2}3465^{-p}e^{-\frac{12a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\left(c^2d^2e^{\frac{2a}{b}}\left(c^2d^2e^{\frac{2a}{b}}\left(c^7d^72^{3p+2}3^{2p+1}385^pe^{\frac{7a}{b}}\Gamma\ldots\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] -((2^(-2 - 3*p)*(-(2310^p*Gamma[1 + p, (-12*(a + b*Log[c*(d + e*x^(1/3))]))
/b]) + 2^(2 + 3*p)*3^(1 + 2*p)*35^p*c*d*E^(a/b)*Gamma[1 + p, (-11*(a + b*Lo
g[c*(d + e*x^(1/3))]))/b] + c^2*d^2*E^((2*a)/b)*(-(6^(1 + 2*p)*7^p*11^(1 +
p)*Gamma[1 + p, (-10*(a + b*Log[c*(d + e*x^(1/3))]))/b]) + 2^(2 + 3*p)*7^p*
55^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))]))/b]
+ c^2*d^2*E^((2*a)/b)*(-(7^p*495^(1 + p)*Gamma[1 + p, (-8*(a + b*Log[c*(d +
 e*x^(1/3))]))/b]) + 5^p*792^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, (-7*(a + b*Lo
g[c*(d + e*x^(1/3))]))/b] - 5^p*924^(1 + p)*c^2*d^2*E^((2*a)/b)*Gamma[1 + p
, (-6*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 7^p*792^(1 + p)*c^3*d^3*E^((3*a)
/b)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b] - 14^p*495^(1 + p)*
c^4*d^4*E^((4*a)/b)*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 2
^(2 + 3*p)*21^p*55^(1 + p)*c^5*d^5*E^((5*a)/b)*Gamma[1 + p, (-3*(a + b*Log[
c*(d + e*x^(1/3))]))/b] - 6^(1 + 2*p)*11^(1 + p)*35^p*c^6*d^6*E^((6*a)/b)*G
amma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 2^(2 + 3*p)*3^(1 + 2*p
)*385^p*c^7*d^7*E^((7*a)/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b
)])))*(a + b*Log[c*(d + e*x^(1/3))])^p)/(3465^p*c^12*e^12*E^((12*a)/b)*(-((
a + b*Log[c*(d + e*x^(1/3))])/b))^p))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(cex^{\frac{1}{3}}+cd\right)+a\right)^p x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(1/3) + c*d) + a)^p*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x^3, x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int x^{3} \left(b \ln \left(\left(e \,x^{\frac{1}{3}}+d \right) c \right)+a \right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(e*x^(1/3)+d)))^p,x)
[Out] int(x^3*(a+b*ln(c*(e*x^(1/3)+d)))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(b \log\left({\left(e x^{\frac{1}{3}} + d\right)} c\right) + a\right)}^{p} x^{3}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/3))))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e*x^(1/3))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))))**p,x)
[Out] Timed out
```

3.557 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=831

$$\frac{3^{-2p-1}e^{-\frac{9a}{b}}\Gamma\left(p+1,-\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^9e^9}-\frac{3\,8^{-p}de^{-\frac{8a}{b}}\Gamma\left(p+1,-\right.}{}$$

```
[Out] 3^(-1-2*p)*GAMMA(1+p,-9*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3)
)))^p/c^9/e^9/exp(9*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d*GAMMA(1+p,-
8*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(8^p)/c^8/e^9/ex
p(8*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+12*d^2*GAMMA(1+p,-7*(a+b*ln(c*(
d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(7^p)/c^7/e^9/exp(7*a/b)/(((-
a-b*ln(c*(d+e*x^(1/3))))/b)^p)-7*2^(2-p)*d^3*GAMMA(1+p,-6*(a+b*ln(c*(d+e*x^
(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(3^p)/c^6/e^9/exp(6*a/b)/(((-a-b*ln
(c*(d+e*x^(1/3))))/b)^p)+42*d^4*GAMMA(1+p,-5*(a+b*ln(c*(d+e*x^(1/3))))/b)*(
a+b*ln(c*(d+e*x^(1/3))))^p/(5^p)/c^5/e^9/exp(5*a/b)/(((-a-b*ln(c*(d+e*x^(1/
3))))/b)^p)-21*2^(1-2*p)*d^5*GAMMA(1+p,-4*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b
*ln(c*(d+e*x^(1/3))))^p/c^4/e^9/exp(4*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^
p)+28*d^6*GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))
))^p/(3^p)/c^3/e^9/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-3*2^(2-p)*
d^7*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c
^2/e^9/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+3*d^8*GAMMA(1+p,(-a-b*
ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c/e^9/exp(a/b)/(((-a-b*
ln(c*(d+e*x^(1/3))))/b)^p)
```

Rubi [A] time = 1.35, antiderivative size = 831, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{3^{-2p-1}e^{-\frac{9a}{b}}\text{Gamma}\left(p+1,-\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^9e^9}-\frac{3\,8^{-p}de^{-\frac{8a}{b}}\text{Gar}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (3^(-1 - 2*p)*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Lo
g[c*(d + e*x^(1/3))])^p)/(c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3
))])/b))^p) - (3*d*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a +
 b*Log[c*(d + e*x^(1/3))])^p)/(8^p*c^8*e^9*E^((8*a)/b)*(-((a + b*Log[c*(d +
 e*x^(1/3))])/b))^p) + (12*d^2*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*x^(1/3)
)]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(7^p*c^7*e^9*E^((7*a)/b)*(-((a +
b*Log[c*(d + e*x^(1/3))])/b))^p) - (7*2^(2 - p)*d^3*Gamma[1 + p, (-6*(a + b
*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^6*e^9
*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (42*d^4*Gamma[1 + p
, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/
(5^p*c^5*e^9*E^((5*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (21*2^(
1 - 2*p)*d^5*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log
[c*(d + e*x^(1/3))])^p)/(c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3)
)])/b))^p) + (28*d^6*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a + b*Log[c*(d
 + e*x^(1/3))])/b))^p) - (3*2^(2 - p)*d^7*Gamma[1 + p, (-2*(a + b*Log[c*(d
+ e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^2*e^9*E^((2*a)/b)*(
-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (3*d^8*Gamma[1 + p, -((a + b*Log[
c*(d + e*x^(1/3))])/b)]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c*e^9*E^(a/b)*(-
((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p dx = 3\,\text{Subst}\left(\int x^8(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)$$
$$= 3\,\text{Subst}\left(\int \left(\frac{d^8(a+b\log(c(d+ex)))^p}{e^8} - \frac{8d^7(d+ex)(a+b\log(c(d+ex)))^p}{e^8}\right.\right.$$
$$= \frac{3\,\text{Subst}\left(\int (d+ex)^8(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^8} - \frac{(24d)\,\text{Subst}\left(\int (d\right.}{}$$
$$= \frac{3\,\text{Subst}\left(\int x^8(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} - \frac{(24d)\,\text{Subst}\left(\int x^7(a+b\log\right.}{e^9}$$
$$= \frac{3\,\text{Subst}\left(\int e^{9x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^9e^9} - \frac{(24d)\,\text{Subst}\left(\int e^{8x}(a+\right.}{}$$
$$= \frac{3^{-1-2p}e^{-\frac{9a}{b}}\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log}{}\right.}{c^9e^9}$$

Mathematica [A] time = 1.00, size = 501, normalized size = 0.60

$$\frac{3^{-2p-1}280^{-p}e^{-\frac{9a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\left(c^8d^89^{p+1}280^pe^{\frac{8a}{b}}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (3^(-1 - 2*p)*(280^p*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))]))/b] -
9^(1 + p)*35^p*c*d*E^(a/b)*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*x^(1/3))]))
/b] + 2^(2 + 3*p)*5^p*9^(1 + p)*c^2*d^2*E^((2*a)/b)*Gamma[1 + p, (-7*(a + b
*Log[c*(d + e*x^(1/3))]))/b] - 5^p*84^(1 + p)*c^3*d^3*E^((3*a)/b)*Gamma[1 +
 p, (-6*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 2^(1 + 3*p)*63^(1 + p)*c^4*d^4
*E^((4*a)/b)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b] - 5^p*126^
(1 + p)*c^5*d^5*E^((5*a)/b)*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))])
)/b] + 2^(2 + 3*p)*5^p*21^(1 + p)*c^6*d^6*E^((6*a)/b)*Gamma[1 + p, (-3*(a +
 b*Log[c*(d + e*x^(1/3))]))/b] - 35^p*36^(1 + p)*c^7*d^7*E^((7*a)/b)*Gamma[
1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 9^(1 + p)*280^p*c^8*d^8*E^(
(8*a)/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)])*(a + b*Log[c*(d
 + e*x^(1/3))])^p)/(280^p*c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3
))])/b))^p)
```

fricas [F] time = 1.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(cex^{\frac{1}{3}}+cd\right)+a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(1/3) + c*d) + a)^p*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x^2, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^{2}\left(b\ln\left(\left(e\,x^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln((e*x^(1/3)+d)*c)+a)^p,x)
[Out] int(x^2*(b*ln((e*x^(1/3)+d)*c)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{2}\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/3))))^p,x)
[Out] int(x^2*(a + b*log(c*(d + e*x^(1/3))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))))**p,x)
[Out] Timed out
```

3.558 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=553

$$\frac{2^{-p-1}3^{-p}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^6e^6}-\frac{3d5^{-p}e^{-\frac{5a}{b}}\left(a+b\log\right.}{}$$

```
[Out] 2^(-1-p)*GAMMA(1+p,-6*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3)))
)^p/(3^p)/c^6/e^6/exp(6*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d*GAMMA(1
+p,-5*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(5^p)/c^5/e^
6/exp(5*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+15*2^(-1-2*p)*d^2*GAMMA(1+p
,-4*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c^4/e^6/exp(4*
a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-10*d^3*GAMMA(1+p,-3*(a+b*ln(c*(d+e*
x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(3^p)/c^3/e^6/exp(3*a/b)/(((-a-b*
ln(c*(d+e*x^(1/3))))/b)^p)+15*2^(-1-p)*d^4*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1
/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e*
x^(1/3))))/b)^p)-3*d^5*GAMMA(1+p,(-a-b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d
+e*x^(1/3))))^p/c/e^6/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)
```

Rubi [A] time = 0.85, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{15d^2 2^{-2p-1}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^4e^6}-\frac{10d^3 3^{-p}e^{\cdots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (2^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[
c*(d + e*x^(1/3))])^p)/(3^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1
/3))])/b))^p) - (3*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*(d
 + e*x^(1/3))])/b))^p) + (15*2^(-1 - 2*p)*d^2*Gamma[1 + p, (-4*(a + b*Log[c
*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^4*e^6*E^((4*a)/
b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (10*d^3*Gamma[1 + p, (-3*(a +
 b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^3*e
^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (15*2^(-1 - p)*d^
4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x
^(1/3))])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
- (3*d^5*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)]*(a + b*Log[c*(d
+ e*x^(1/3))])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx = 3\,\text{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)$$
$$= 3\,\text{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d+e\ldots$$
$$= \frac{3\,\text{Subst}\left(\int(d+ex)^5(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^5}-\frac{(15d)\,\text{Subst}\left(\int(\ldots\right.}{\ldots}$$
$$= \frac{3\,\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^6}-\frac{(15d)\,\text{Subst}\left(\int x^4(a+b\ldots\right.}{\ldots}$$
$$= \frac{3\,\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^6e^6}-\frac{(15d)\,\text{Subst}\left(\int e^{5x}(a\ldots\right.}{\ldots}$$
$$= \frac{2^{-1-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\ldots\right.}{c^6e^6}$$

Mathematica [A] time = 0.95, size = 325, normalized size = 0.59

$$2^{-2p-1}15^{-p}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\left(10^p\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)-cde^{a/b}\left(2^{\ldots}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (2^(-1 - 2*p)*(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(1/3))]))/b] - c
*d*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1
/3))]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c*(d
 + e*x^(1/3))]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*(a + b*
Log[c*(d + e*x^(1/3))]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p, (-2*(a
 + b*Log[c*(d + e*x^(1/3))]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a
+ b*Log[c*(d + e*x^(1/3))])/b)])))))*(a + b*Log[c*(d + e*x^(1/3))])^p)/(15^
p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(cex^{\frac{1}{3}}+cd\right)+a\right)^{p}x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(1/3) + c*d) + a)^p*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x\left(b\ln\left(\left(e\,x^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln((e*x^(1/3)+d)*c)+a)^p,x)
```

```
[Out] int(x*(b*ln((e*x^(1/3)+d)*c)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(1/3))))^p,x)
```

```
[Out] int(x*(a + b*log(c*(d + e*x^(1/3))))^p, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))))**p,x)
```

```
[Out] Timed out
```

3.559 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=266

$$\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^3e^3}-\frac{3d2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(\right.\right.\right.}{}$$

```
[Out] GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(3^p)
/c^3/e^3/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d*GAMMA(1+p,-2*(a+
b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(2^p)/c^2/e^3/exp(2*a
/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+3*d^2*GAMMA(1+p,(-a-b*ln(c*(d+e*x^(1
/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c/e^3/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1
/3))))/b)^p)
```

Rubi [A] time = 0.39, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2451, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^3e^3}-\frac{3d2^{-p}e^{-\frac{2a}{b}}\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^
(1/3))])^p)/(3^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^
p) - (3*d*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*
(d + e*x^(1/3))])^p)/(2^p*c^2*e^3*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3
))])/b))^p) + (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)]*(a +
 b*Log[c*(d + e*x^(1/3))])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3)
)])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx &= 3\,\text{Subst}\left(\int x^2(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)\\
&= 3\,\text{Subst}\left(\int\left(\frac{d^2(a+b\log(c(d+ex)))^p}{e^2}-\frac{2d(d+ex)(a+b\log(c(d+ex)))^p}{e^2}\right.\right.\\
&= \frac{3\,\text{Subst}\left(\int(d+ex)^2(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)}{e^2}-\frac{(6d)\,\text{Subst}\left(\int(d+\right.}{}\\
&= \frac{3\,\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^3}-\frac{(6d)\,\text{Subst}\left(\int x(a+b\log(\right.}{e^3}\\
&= \frac{3\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^3e^3}-\frac{(6d)\,\text{Subst}\left(\int e^{2x}(a+b\right.}{}\\
&= \frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log(}{}\right.}{c^3e^3}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 174, normalized size = 0.65

$$\frac{6^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\left(2^p\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)+cd3^{p+1}e^{a/b}\left(cd2^pe\right.\right.}{c^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] ((2^p*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 3^(1 + p)*c*d*E
^(a/b)*(-Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 2^p*c*d*E^(a
/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)]))*(a + b*Log[c*(d + e
*x^(1/3))])^p)/(6^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b
))^p)
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(cex^{\frac{1}{3}}+cd\right)+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(1/3) + c*d) + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(\left(ex^{\frac{1}{3}} + d\right)c\right) + a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(b \ln\left(\left(e\, x^{\frac{1}{3}} + d\right) c\right) + a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(1/3)+d)*c)+a)^p,x)
```

```
[Out] int((b*ln((e*x^(1/3)+d)*c)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(\left(ex^{\frac{1}{3}} + d\right)c\right) + a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b\, \ln\left(c\, \left(d + e\, x^{1/3}\right)\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))))^p,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/3))))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \log\left(c \left(d + e \sqrt[3]{x}\right)\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))))**p,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))))**p, x)
```

3.560 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/3))))^p/x,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx = 3\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)))^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x, x]
```

fricas [A] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cex^{\frac{1}{3}}+cd\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(1/3) + c*d) + a)^p/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p/x, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(\left(e\,x^{\frac{1}{3}}+d\right) c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(1/3)+d)*c)+a)^p/x,x)
[Out] int((b*ln((e*x^(1/3)+d)*c)+a)^p/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log \left(\left(ex^{\frac{1}{3}}+d\right) c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)\right)\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))))^p/x,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))))^p/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))))**p/x,x)
[Out] Timed out
```

3.561 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/3))))^p/x^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))])^p/x^2,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)])^p/x^4, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)))^p}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x^2, x]
```

fricas [A] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cex^{\frac{1}{3}}+cd\right)+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(1/3) + c*d) + a)^p/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p/x^2, x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\left(e\,x^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(1/3)+d)*c)+a)^p/x^2,x)
```

```
[Out] int((b*ln((e*x^(1/3)+d)*c)+a)^p/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)\right)\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))))^p/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/3))))^p/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))))**p/x**2,x)
```

```
[Out] Timed out
```

3.562 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=1363

result too large to display

```
[Out] 2^(-2-p)*GAMMA(1+p,-6*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3)
)^2))^p/(3^p)/c^6/e^12/exp(6*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*(2
/11)^p*d*(d+e*x^(1/3))^11*GAMMA(1+p,-11/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a
+b*ln(c*(d+e*x^(1/3))^2))^p/e^12/exp(11/2*a/b)/(c*(d+e*x^(1/3))^2)^(11/2)/(
((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+33/2*d^2*GAMMA(1+p,-5*(a+b*ln(c*(d+e*x^
(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(5^p)/c^5/e^12/exp(5*a/b)/(((-a
-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-55*(2/9)^p*d^3*(d+e*x^(1/3))^9*GAMMA(1+p,-9
/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^12/exp(9/
2*a/b)/(c*(d+e*x^(1/3))^2)^(9/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+495*d
^4*GAMMA(1+p,-4*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^
p/(2^(2+2*p))/c^4/e^12/exp(4*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-99*2
^(1+p)*d^5*(d+e*x^(1/3))^7*GAMMA(1+p,-7/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a
+b*ln(c*(d+e*x^(1/3))^2))^p/(7^p)/e^12/exp(7/2*a/b)/(c*(d+e*x^(1/3))^2)^(7/
2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+77*3^(1-p)*d^6*GAMMA(1+p,-3*(a+b*ln
(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c^3/e^12/exp(3*a/b)/(
((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-99*2^(1+p)*d^7*(d+e*x^(1/3))^5*GAMMA(1+
p,-5/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(5^p)/e
^12/exp(5/2*a/b)/(c*(d+e*x^(1/3))^2)^(5/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b
)^p)+495*2^(-2-p)*d^8*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c
*(d+e*x^(1/3))^2))^p/c^2/e^12/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^
p)-55*(2/3)^p*d^9*(d+e*x^(1/3))^3*GAMMA(1+p,-3/2*(a+b*ln(c*(d+e*x^(1/3))^2)
)/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^12/exp(3/2*a/b)/(c*(d+e*x^(1/3))^2)^(3
/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+33/2*d^10*GAMMA(1+p,(-a-b*ln(c*(d+
e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c/e^12/exp(a/b)/(((-a-b*ln(
c*(d+e*x^(1/3))^2))/b)^p)-3*2^p*d^11*(d+e*x^(1/3))*GAMMA(1+p,1/2*(-a-b*ln(c
*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^12/exp(1/2*a/b)/(((-a
-b*ln(c*(d+e*x^(1/3))^2))/b)^p)/(c*(d+e*x^(1/3))^2)^(1/2)
```

Rubi [A] time = 2.13, antiderivative size = 1363, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

```
[Out] (2^(-2 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Lo
g[c*(d + e*x^(1/3))^2])^p)/(3^p*c^6*e^12*E^((6*a)/b)*(-((a + b*Log[c*(d + e
*x^(1/3))^2])/b))^p) - (3*(2/11)^p*d*(d + e*x^(1/3))^11*Gamma[1 + p, (-11*(
a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)
/(e^12*E^((11*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(11/2)*(-((a + b*Log[c*(d + e
*x^(1/3))^2])/b))^p) + (33*d^2*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3)
)^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(2*5^p*c^5*e^12*E^((5*a)/b)*(
-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (55*(2/9)^p*d^3*(d + e*x^(1/3))
^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(
d + e*x^(1/3))^2])^p)/(e^12*E^((9*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(9/2)*(-(
(a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (495*d^4*Gamma[1 + p, (-4*(a + b*
Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(2^(2*(1
+ p))*c^4*e^12*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (99
*2^(1 + p)*d^5*(d + e*x^(1/3))^7*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*x^(1/
3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(7^p*e^12*E^((7*a)/(2*b
))*(c*(d + e*x^(1/3))^2)^(7/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) +
```

```
(77*3^(1 - p)*d^6*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^3*e^12*E^((3*a)/b)*(-((a + b*Log[c*(d
+ e*x^(1/3))^2])/b))^p) - (99*2^(1 + p)*d^7*(d + e*x^(1/3))^5*Gamma[1 + p,
(-5*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2
])^p)/(5^p*e^12*E^((5*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(5/2)*(-((a + b*Log[c
*(d + e*x^(1/3))^2])/b))^p) + (495*2^(-2 - p)*d^8*Gamma[1 + p, (-2*(a + b*L
og[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^2*e^12*
E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (55*(2/3)^p*d^9*(d
 + e*x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(
a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^12*E^((3*a)/(2*b))*(c*(d + e*x^(1/3))
^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (33*d^10*Gamma[1 + p
, -((a + b*Log[c*(d + e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p
)/(2*c*e^12*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (3*2^p*d^1
1*(d + e*x^(1/3))*Gamma[1 + p, -(a + b*Log[c*(d + e*x^(1/3))^2])/(2*b)]*(a
+ b*Log[c*(d + e*x^(1/3))^2])^p)/(e^12*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))^2
]*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
```

```
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=3\,\text{Subst}\left(\int x^{11}\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$
$$=3\,\text{Subst}\left(\int\left(-\frac{d^{11}\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^{11}}+\frac{11d^{10}(d+ex)\left(a+b\log\right.}{e^{11}}\right.\right.$$
$$=\frac{3\,\text{Subst}\left(\int(d+ex)^{11}\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)}{e^{11}}-\frac{(33d)\,\text{Sub}}{}$$
$$=\frac{3\,\text{Subst}\left(\int x^{11}\left(a+b\log\left(cx^2\right)\right)^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}}-\frac{(33d)\,\text{Subst}\left(\int x^{1}\right.}{}$$
$$=\frac{3\,\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2c^6e^{12}}+\frac{\left(165d^2\right)\text{Subst}\left(\int\right.}{}$$
$$=\frac{2^{-2-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right.}{c^6e^{12}}$$

Mathematica [F] time = 0.71, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
[Out] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(ce^2x^{\frac{2}{3}}+2\,cdex^{\frac{1}{3}}+cd^2\right)+a\right)^px^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^px^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x^3, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int x^{3} \left(b \ln \left(\left(e \,x^{\frac{1}{3}}+d \right)^{2} c \right)+a \right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(e*x^(1/3)+d)^2))^p,x)
[Out] int(x^3*(a+b*ln(c*(e*x^(1/3)+d)^2))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(b \log\left({\left(e x^{\frac{1}{3}} + d\right)}^{2} c\right) + a\right)}^{p} x^{3}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^2\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/3))^2))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e*x^(1/3))^2))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
[Out] Timed out
```

3.563 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=1035

$$\frac{2^p 3^{-2p-1} e^{-\frac{9a}{2b}} \left(d+e\sqrt[3]{x}\right)^9 \Gamma\left(p+1,-\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}}{e^9\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{9/2}}\ 3$$

```
[Out] 2^p*3^(-1-2*p)*(d+e*x^(1/3))^9*GAMMA(1+p,-9/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b
)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^9/exp(9/2*a/b)/(c*(d+e*x^(1/3))^2)^(9/2)/
(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*d*GAMMA(1+p,-4*(a+b*ln(c*(d+e*x^(1/3
))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(4^p)/c^4/e^9/exp(4*a/b)/(((-a-b*ln
(c*(d+e*x^(1/3))^2))/b)^p)+3*2^(2+p)*d^2*(d+e*x^(1/3))^7*GAMMA(1+p,-7/2*(a+
b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(7^p)/e^9/exp(7/2
*a/b)/(c*(d+e*x^(1/3))^2)^(7/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-28*d^3
*GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/
(3^p)/c^3/e^9/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+21*2^(1+p)*d^
4*(d+e*x^(1/3))^5*GAMMA(1+p,-5/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(
d+e*x^(1/3))^2))^p/(5^p)/e^9/exp(5/2*a/b)/(c*(d+e*x^(1/3))^2)^(5/2)/(((-a-b
*ln(c*(d+e*x^(1/3))^2))/b)^p)-21*2^(1-p)*d^5*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^
(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c^2/e^9/exp(2*a/b)/(((-a-b*ln(c
*(d+e*x^(1/3))^2))/b)^p)+7*2^(2+p)*d^6*(d+e*x^(1/3))^3*GAMMA(1+p,-3/2*(a+b*
ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(3^p)/e^9/exp(3/2*a
/b)/(c*(d+e*x^(1/3))^2)^(3/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-12*d^7*G
AMMA(1+p,(-a-b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c/e^
9/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+3*2^p*d^8*(d+e*x^(1/3))*GAM
MA(1+p,1/2*(-a-b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^
9/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)/(c*(d+e*x^(1/3))^2)^(1/
2)
```

Rubi [A] time = 1.57, antiderivative size = 1035, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

```
[Out] (2^p*3^(-1 - 2*p)*(d + e*x^(1/3))^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^
(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^9*E^((9*a)/(2*b))
*(c*(d + e*x^(1/3))^2)^(9/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (
3*d*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d +
 e*x^(1/3))^2])^p)/(4^p*c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))
^2])/b))^p) + (3*2^(2 + p)*d^2*(d + e*x^(1/3))^7*Gamma[1 + p, (-7*(a + b*Lo
g[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(7^p*e^
9*E^((7*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(7/2)*(-((a + b*Log[c*(d + e*x^(1/3
))^2])/b))^p) - (28*d^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/
b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a + b*L
og[c*(d + e*x^(1/3))^2])/b))^p) + (21*2^(1 + p)*d^4*(d + e*x^(1/3))^5*Gamma
[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^
(1/3))^2])^p)/(5^p*e^9*E^((5*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(5/2)*(-((a +
b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (21*2^(1 - p)*d^5*Gamma[1 + p, (-2*(a
+ b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^2*
e^9*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (7*2^(2 + p)*d
^6*(d + e*x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*
```

```
b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(3^p*e^9*E^((3*a)/(2*b))*(c*(d + e*
x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (12*d^7*Gamm
a[1 + p, -((a + b*Log[c*(d + e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3)
)^2])^p)/(c*e^9*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (3*2^p
*d^8*(d + e*x^(1/3))*Gamma[1 + p, -(a + b*Log[c*(d + e*x^(1/3))^2])/(2*b)]*
(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^9*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))
^2]*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx = 3\,\text{Subst}\left(\int x^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)$$
$$= 3\,\text{Subst}\left(\int\left(\frac{d^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^8} - \frac{8d^7(d+ex)\left(a+b\log\left(c(d\right.\right.}{e^8}\right.\right.$$
$$= \frac{3\,\text{Subst}\left(\int (d+ex)^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)}{e^8} - \frac{(24d)\,\text{Subs}}{}$$
$$= \frac{3\,\text{Subst}\left(\int x^8\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} - \frac{(24d)\,\text{Subst}\left(\int x^7\right.}{}$$
$$= -\frac{(12d)\,\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{c^4e^9} - \frac{\left(84d^3\right)\text{Sub}}{}$$
$$= \frac{2^p 3^{-1-2p} e^{-\frac{9a}{2b}}\left(d+e\sqrt[3]{x}\right)^9\Gamma\left(1+p, -\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(\right.\right.\right.}{e^9\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{9/2}}$$

Mathematica [F] time = 0.50, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
[Out] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(ce^2x^{\frac{2}{3}} + 2\,cdex^{\frac{1}{3}} + cd^2\right) + a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(ex^{\frac{1}{3}} + d\right)^2 c\right) + a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x^2, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^2\left(b\ln\left(\left(e\,x^{\frac{1}{3}} + d\right)^2 c\right) + a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln((e*x^(1/3)+d)^2*c)+a)^p,x)
[Out] int(x^2*(b*ln((e*x^(1/3)+d)^2*c)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(\left(e x^{\frac{1}{3}} + d\right)^{2} c\right) + a\right)^{p} x^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b \ln\left(c \left(d + e\, x^{1/3}\right)^2\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/3))^2))^p,x)
[Out] int(x^2*(a + b*log(c*(d + e*x^(1/3))^2))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
[Out] Timed out
```

3.564 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=673

$$\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)}{2c^3e^6}+\frac{15d^22^{-p-1}e^{-\frac{2a}{b}}\left(a-\right.}{}$$

```
[Out] 1/2*GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))
^p/(3^p)/c^3/e^6/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*(2/5)^p*
d*(d+e*x^(1/3))^5*GAMMA(1+p,-5/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(
d+e*x^(1/3))^2))^p/e^6/exp(5/2*a/b)/(c*(d+e*x^(1/3))^2)^(5/2)/(((-a-b*ln(c*
(d+e*x^(1/3))^2))/b)^p)+15*2^(-1-p)*d^2*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1/3)
)^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e
*x^(1/3))^2))/b)^p)-5*2^(1+p)*d^3*(d+e*x^(1/3))^3*GAMMA(1+p,-3/2*(a+b*ln(c*
(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(3^p)/e^6/exp(3/2*a/b)/(
c*(d+e*x^(1/3))^2)^(3/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+15/2*d^4*GAMM
A(1+p,(-a-b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c/e^6/e
xp(a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*2^p*d^5*(d+e*x^(1/3))*GAMMA(
1+p,1/2*(-a-b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^6/e
xp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)/(c*(d+e*x^(1/3))^2)^(1/2)
```

Rubi [A] time = 0.97, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$\frac{15d^22^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)}{c^2e^6}+\frac{3^{-p}e}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*
x^(1/3))^2])^p)/(2*3^p*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^
2])/b))^p) - (3*(2/5)^p*d*(d + e*x^(1/3))^5*Gamma[1 + p, (-5*(a + b*Log[c*(
d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^6*E^((5*a
)/(2*b))*(c*(d + e*x^(1/3))^2)^(5/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b)
)^p) + (15*2^(-1 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))^2])
)/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log
[c*(d + e*x^(1/3))^2])/b))^p) - (5*2^(1 + p)*d^3*(d + e*x^(1/3))^3*Gamma[1
+ p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/
3))^2])^p)/(3^p*e^6*E^((3*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(3/2)*(-((a + b*L
og[c*(d + e*x^(1/3))^2])/b))^p) + (15*d^4*Gamma[1 + p, -((a + b*Log[c*(d +
e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(2*c*e^6*E^(a/b)*(-(
(a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (3*2^p*d^5*(d + e*x^(1/3))*Gamma[
1 + p, -(a + b*Log[c*(d + e*x^(1/3))^2])/(2*b)]*(a + b*Log[c*(d + e*x^(1/3)
)^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))^2]*(-((a + b*Log[c*(d + e*
x^(1/3))^2])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
```

```
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx = 3\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$
$$= 3\,\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\left(c(d\right.\right.}{e^5}\right.\right.$$
$$= \frac{3\,\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)}{e^5} - \frac{(15d)\,\text{Subst}}{}$$
$$= \frac{3\,\text{Subst}\left(\int x^5\left(a+b\log\left(cx^2\right)\right)^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^6} - \frac{(15d)\,\text{Subst}\left(\int x^4\left(\right.\right.}{}$$
$$= \frac{3\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2c^3e^6} + \frac{\left(15d^2\right)\text{Subst}\left(\int e\right.}{}$$
$$= \frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a}{}\right.}{2c^3e^6}$$

Mathematica [F] time = 0.37, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(ce^2x^{\frac{2}{3}}+2\,cdex^{\frac{1}{3}}+cd^2\right)+a\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x\left(b\ln\left(\left(e\,x^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln((e*x^(1/3)+d)^2*c)+a)^p,x)
[Out] int(x*(b*ln((e*x^(1/3)+d)^2*c)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(\left(ex^{\frac{1}{3}} + d\right)^2 c\right) + a\right)^p x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \left(a + b \ln\left(c \left(d + e\, x^{1/3}\right)^2\right)\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(1/3))^2))^p,x)
[Out] int(x*(a + b*log(c*(d + e*x^(1/3))^2))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
[Out] Timed out
```

3.565 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=338

$$\frac{3d^2 2^p e^{-\frac{a}{2b}}\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{2b}\right)}{e^3\sqrt{c\left(d+e\sqrt[3]{x}\right)^2}}+\left(\frac{2}{3}\right)^p e^-$$

```
[Out] (2/3)^p*(d+e*x^(1/3))^3*GAMMA(1+p,-3/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*
ln(c*(d+e*x^(1/3))^2))^p/e^3/exp(3/2*a/b)/(c*(d+e*x^(1/3))^2)^(3/2)/(((-a-b
*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*d*GAMMA(1+p,(-a-b*ln(c*(d+e*x^(1/3))^2))/b)
*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c/e^3/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2)
)/b)^p)+3*2^p*d^2*(d+e*x^(1/3))*GAMMA(1+p,1/2*(-a-b*ln(c*(d+e*x^(1/3))^2))/
b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^3/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3)
)^2))/b)^p)/(c*(d+e*x^(1/3))^2)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2451, 2401, 2389, 2300, 2181, 2390, 2310}

$$\frac{3d^2 2^p e^{-\frac{a}{2b}}\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{2b}\right)}{e^3\sqrt{c\left(d+e\sqrt[3]{x}\right)^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

```
[Out] ((2/3)^p*(d + e*x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]
))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^3*E^((3*a)/(2*b))*(c*(d +
e*x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (3*d*Gamma
[1 + p, -((a + b*Log[c*(d + e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3))
^2])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (3*2^p*
d^2*(d + e*x^(1/3))*Gamma[1 + p, -(a + b*Log[c*(d + e*x^(1/3))^2])/(2*b)]*(
a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^3*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))^
2]*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
```

```
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx = 3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int\left(\frac{d^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2}-\frac{2d(d+ex)\left(a+b\log\left(c(d+ex)^2\right.\right.}{e^2}\right.\right.$$

$$=\frac{3\,\text{Subst}\left(\int(d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)}{e^2}-\frac{(6d)\,\text{Subst}\left(\int(d\right.}{}$$

$$=\frac{3\,\text{Subst}\left(\int x^2\left(a+b\log\left(cx^2\right)\right)^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^3}-\frac{(6d)\,\text{Subst}\left(\int x\left(a+b\,\text{lo}\right.\right.}{e}$$

$$=-\frac{(3d)\,\text{Subst}\left(\int e^x(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{ce^3}+\frac{\left(3\left(d+e\sqrt[3]{x}\right)^3\right)\text{Sul}}{}$$

$$=\frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(d+e\sqrt[3]{x}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.\right.}{e^3\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{3/2}}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(ce^2x^{\frac{2}{3}}+2\,cdex^{\frac{1}{3}}+cd^2\right)+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(\left(e\,x^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(1/3)+d)^2*c)+a)^p,x)
[Out] int((b*ln((e*x^(1/3)+d)^2*c)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^2\right)\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^2))^p,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))^2))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
```

```
[Out] Timed out
```

3.566 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/3))^2))^p/x,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)^2])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x, x]
```

fricas [A] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(ce^2x^{\frac{2}{3}}+2\,cdex^{\frac{1}{3}}+cd^2\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p/x, x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(\left(e\,x^{\frac{1}{3}}+d\right)^{2} c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(1/3)+d)^2*c)+a)^p/x,x)
[Out] int((b*ln((e*x^(1/3)+d)^2*c)+a)^p/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(\left(ex^{\frac{1}{3}}+d\right)^{2} c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^2\right)\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^2))^p/x,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))^2))^p/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**2))**p/x,x)
[Out] Timed out
```

3.567 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/3))^2))^p/x^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x^2,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)^2])^p/x^4, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx = 3\,\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x^2, x]
```

fricas [A] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b\log\left(ce^2x^{\frac{2}{3}}+2\,cdex^{\frac{1}{3}}+cd^2\right)+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p/x^2, x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(\left(e\,x^{\frac{1}{3}}+d\right)^{2} c\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(1/3)+d)^2*c)+a)^p/x^2,x)
```

```
[Out] int((b*ln((e*x^(1/3)+d)^2*c)+a)^p/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log \left(\left(e x^{\frac{1}{3}}+d\right)^{2} c\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^2))^p/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/3))^2))^p/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**2))**p/x**2,x)
```

```
[Out] Timed out
```

3.568 $\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$

Optimal. Leaf size=557

$$\frac{2^{-p-2}3^{-p}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)}{c^6e^6}-\frac{3d5^{-p}e^{-\frac{5a}{b}}\left(a+b\right.}{}$$

```
[Out] 2^(-2-p)*GAMMA(1+p,-6*(a+b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3)))
)^p/(3^p)/c^6/e^6/exp(6*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))))/b)^p)-3/2*d*GAMMA
(1+p,-5*(a+b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/(5^p)/c^5/
e^6/exp(5*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))))/b)^p)+15*d^2*GAMMA(1+p,-4*(a+b*
ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/(2^(2+2*p))/c^4/e^6/exp
(4*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))))/b)^p)-5*d^3*GAMMA(1+p,-3*(a+b*ln(c*(d+
e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/(3^p)/c^3/e^6/exp(3*a/b)/(((-a-
b*ln(c*(d+e*x^(2/3))))/b)^p)+15*2^(-2-p)*d^4*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^
(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+
e*x^(2/3))))/b)^p)-3/2*d^5*GAMMA(1+p,(-a-b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(
c*(d+e*x^(2/3))))^p/c/e^6/exp(a/b)/(((-a-b*ln(c*(d+e*x^(2/3))))/b)^p)
```

Rubi [A] time = 0.87, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{15d^2 2^{-2(p+1)}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)}{c^4e^6}-\frac{5d^3 3}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

```
[Out] (2^(-2 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[
c*(d + e*x^(2/3))])^p)/(3^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(2
/3))])/b))^p) - (3*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a
 + b*Log[c*(d + e*x^(2/3))])^p)/(2*5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*
(d + e*x^(2/3))])/b))^p) + (15*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(
2/3))]))/b]*(a + b*Log[c*(d + e*x^(2/3))])^p)/(2^(2*(1 + p))*c^4*e^6*E^((4*
a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p) - (5*d^3*Gamma[1 + p, (-3*(a
 + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[c*(d + e*x^(2/3))])^p)/(3^p*c^3
*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p) + (15*2^(-2 - p)*
d^4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[c*(d + e
*x^(2/3))])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p
) - (3*d^5*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))])/b)]*(a + b*Log[c*(
d + e*x^(2/3))])^p)/(2*c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^
p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx = \frac{3}{2}\operatorname{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx, x, x^{2/3}\right)$$
$$= \frac{3}{2}\operatorname{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d+ex)\right.\right.$$
$$= \frac{3\operatorname{Subst}\left(\int(d+ex)^5(a+b\log(c(d+ex)))^p\,dx, x, x^{2/3}\right)}{2e^5}-\frac{(15d)\operatorname{Subst}\left(\int(d\right.}{}$$
$$= \frac{3\operatorname{Subst}\left(\int x^5(a+b\log(cx))^p\,dx, x, d+ex^{2/3}\right)}{2e^6}-\frac{(15d)\operatorname{Subst}\left(\int x^4(a+b\log\right.}{2e^6}$$
$$= \frac{3\operatorname{Subst}\left(\int e^{6x}(a+bx)^p\,dx, x, \log\left(c\left(d+ex^{2/3}\right)\right)\right)}{2c^6e^6}-\frac{(15d)\operatorname{Subst}\left(\int e^{5x}(a+\right.}{}$$
$$= \frac{2^{-2-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a}{}\right.}{c^6e^6}$$

Mathematica [A] time = 0.94, size = 325, normalized size = 0.58

$$4^{-p-1}15^{-p}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\left(10^p\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)-cde^{a/b}\left(2^{2p+1}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

```
[Out] (4^(-1 - p)*(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(2/3))]))/b] - c*d
*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(2/3
))]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c*(d +
 e*x^(2/3))]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*(a + b*Lo
g[c*(d + e*x^(2/3))]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p, (-2*(a +
 b*Log[c*(d + e*x^(2/3))]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a +
b*Log[c*(d + e*x^(2/3))])/b)])))))*(a + b*Log[c*(d + e*x^(2/3))])^p)/(15^p*
c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\log\left(cex^{\frac{2}{3}}+cd\right)+a\right)^{p}x^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x^3, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x^{3}\left(b\ln\left(\left(e\,x^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(e*x^(2/3)+d)))^p,x)
```

```
[Out] int(x^3*(a+b*ln(c*(e*x^(2/3)+d)))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)\right)\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(2/3))))^p,x)
```

```
[Out] int(x^3*(a + b*log(c*(d + e*x^(2/3))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))))**p,x)
```

```
[Out] Timed out
```

3.569 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$

Optimal. Leaf size=273

$$\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)}{2c^3e^3}-\frac{3d2^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\right.}{}$$

```
[Out] 1/2*GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/(
3^p)/c^3/e^3/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))))/b)^p)-3*2^(-1-p)*d*GAM
MA(1+p,-2*(a+b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/c^2/e^3/
exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))))/b)^p)+3/2*d^2*GAMMA(1+p,(-a-b*ln(c*
(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/c/e^3/exp(a/b)/(((-a-b*ln(c*
(d+e*x^(2/3))))/b)^p)
```

Rubi [A] time = 0.38, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)}{2c^3e^3}-\frac{3d2^{-p-1}e^{-\frac{2a}{b}}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[c*(d + e*x^
(2/3))])^p)/(2*3^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b)
)^p) - (3*2^(-1 - p)*d*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/3))]))/b]*
(a + b*Log[c*(d + e*x^(2/3))])^p)/(c^2*e^3*E^((2*a)/b)*(-((a + b*Log[c*(d +
 e*x^(2/3))])/b))^p) + (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))])
/b)]*(a + b*Log[c*(d + e*x^(2/3))])^p)/(2*c*e^3*E^(a/b)*(-((a + b*Log[c*(d
+ e*x^(2/3))])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx &= \frac{3}{2}\text{Subst}\left(\int x^2(a+b\log(c(d+ex)))^p\,dx,x,x^{2/3}\right)\\ &= \frac{3}{2}\text{Subst}\left(\int\left(\frac{d^2(a+b\log(c(d+ex)))^p}{e^2}-\frac{2d(d+ex)(a+b\log(c(d+ex)))^p}{e^2}\right.\right.\\ &= \frac{3\,\text{Subst}\left(\int(d+ex)^2(a+b\log(c(d+ex)))^p\,dx,x,x^{2/3}\right)}{2e^2}-\frac{(3d)\,\text{Subst}\left(\int(d+\right.}{}\\ &= \frac{3\,\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx,x,d+ex^{2/3}\right)}{2e^3}-\frac{(3d)\,\text{Subst}\left(\int x(a+b\log(c\right.}{e^3}\\ &= \frac{3\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{2c^3e^3}-\frac{(3d)\,\text{Subst}\left(\int e^{2x}(a+bx\right.}{}\\ &= \frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log(c(}{b}\right.}{2c^3e^3}\end{aligned}$$

Mathematica [A] time = 0.22, size = 181, normalized size = 0.66

$$\frac{2^{-p-1}3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\left(2^p\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)+cd3^{p+1}e^{a/b}\left(cd2\right.\right.}{c^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

```
[Out] (2^(-1 - p)*(2^p*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))]))/b] + 3^(1
 + p)*c*d*E^(a/b)*(-Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/3))]))/b] + 2
^p*c*d*E^(a/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))])/b)]))*(a + b*L
og[c*(d + e*x^(2/3))])^p)/(3^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x
^(2/3))])/b))^p)
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(cex^{\frac{2}{3}}+cd\right)+a\right)^px,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(\left(ex^{\frac{2}{3}} + d\right)c\right) + a\right)^{p} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x \left(b \ln \left(\left(e\, x^{\frac{2}{3}} + d\right) c\right) + a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln((e*x^(2/3)+d)*c)+a)^p,x)
```

```
[Out] int(x*(b*ln((e*x^(2/3)+d)*c)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(\left(ex^{\frac{2}{3}} + d\right)c\right) + a\right)^{p} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \left(a + b\, \ln\left(c\, \left(d + e\, x^{2/3}\right)\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(2/3))))^p,x)
```

```
[Out] int(x*(a + b*log(c*(d + e*x^(2/3))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))))**p,x)
```

```
[Out] Timed out
```

3.570 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))))^p/x,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p}{x}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x, x]
```

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cex^{\frac{2}{3}}+cd\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x, x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(\left(e x^{\frac{2}{3}}+d\right) c\right)+a\right)^{p}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(2/3)+d)*c)+a)^p/x,x)
[Out] int((b*ln((e*x^(2/3)+d)*c)+a)^p/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log \left(\left(e x^{\frac{2}{3}}+d\right) c\right)+a\right)^{p}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)\right)\right)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))))^p/x,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))))^p/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))))**p/x,x)
[Out] Timed out
```

3.571 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))))^p/x^3,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))])^p/x^3,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)])^p/x^7, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p}{x^7}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^3,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^3, x]
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(cex^{\frac{2}{3}}+cd\right)+a\right)^p}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^3,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p/x^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^3,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x^3, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\left(e\,x^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(2/3)+d)*c)+a)^p/x^3,x)
[Out] int((b*ln((e*x^(2/3)+d)*c)+a)^p/x^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^3,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)\right)\right)^{p}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))))^p/x^3,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))))^p/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))))**p/x**3,x)
[Out] Timed out
```

3.572 $\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p, x\right)$$

```
[Out] Unintegrable(x^2*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^8*(a + b*Log[c*(d + e*x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p \, dx = 3\,\text{Subst}\left(\int x^8 \left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p \, dx, x, \sqrt[3]{x}\right)$$

Mathematica [A] time = 0.74, size = 0, normalized size = 0.00

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
[Out] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))])^p, x]
```

fricas [A] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(cex^{\frac{2}{3}}+cd\right)+a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p*x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x^2, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int x^2 \left(b\ln\left(\left(e\,x^{\frac{2}{3}}+d\right)c\right)+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln((e*x^(2/3)+d)*c)+a)^p,x)
```

```
[Out] int(x^2*(b*ln((e*x^(2/3)+d)*c)+a)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(\left(e x^{\frac{2}{3}} + d\right) c\right) + a\right)^{p} x^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x^2\,{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(2/3))))^p,x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e*x^(2/3))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))))**p,x)
```

```
[Out] Timed out
```

3.573 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e*x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p, x]
```

fricas [A] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(cex^{\frac{2}{3}}+cd\right)+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p, x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(\left(e\,x^{\frac{2}{3}}+d\right)c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(2/3)+d)*c)+a)^p,x)
```

```
[Out] int((b*ln((e*x^(2/3)+d)*c)+a)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(\left(ex^{\frac{2}{3}} + d\right)c\right) + a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \left(a + b\ \ln\left(c\ \left(d + e\,x^{2/3}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))))^p,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))))**p,x)
```

```
[Out] Timed out
```

3.574 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))))^p/x^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))])^p/x^2,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)])^p/x^4, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx=3\,\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^2, x]
```

fricas [A] time = 1.31, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b\log\left(cex^{\frac{2}{3}}+cd\right)+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x^2, x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(\left(e x^{\frac{2}{3}}+d\right) c\right)+a\right)^{p}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(2/3)+d)*c)+a)^p/x^2,x)
[Out] int((b*ln((e*x^(2/3)+d)*c)+a)^p/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log \left(\left(e x^{\frac{2}{3}}+d\right) c\right)+a\right)^{p}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)\right)\right)^p}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))))^p/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))))^p/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))))**p/x**2,x)
[Out] Timed out
```

3.575 $\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=678

$$\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{b}\right)}{4c^3e^6}+\frac{15d^22^{-p-2}e^{-\frac{2a}{b}}\left(a+b\right.}{}$$

```
[Out] 1/4*GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))
^p/(3^p)/c^3/e^6/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))^2))/b)^p)-3*2^(-1+p)
*d*(d+e*x^(2/3))^5*GAMMA(1+p,-5/2*(a+b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*
(d+e*x^(2/3))^2))^p/(5^p)/e^6/exp(5/2*a/b)/(c*(d+e*x^(2/3))^2)^(5/2)/(((-a-
b*ln(c*(d+e*x^(2/3))^2))/b)^p)+15*2^(-2-p)*d^2*GAMMA(1+p,-2*(a+b*ln(c*(d+e*
x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/c^2/e^6/exp(2*a/b)/(((-a-b*ln
(c*(d+e*x^(2/3))^2))/b)^p)-5*(2/3)^p*d^3*(d+e*x^(2/3))^3*GAMMA(1+p,-3/2*(a+
b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/e^6/exp(3/2*a/b)/
(c*(d+e*x^(2/3))^2)^(3/2)/(((-a-b*ln(c*(d+e*x^(2/3))^2))/b)^p)+15/4*d^4*GAM
MA(1+p,(-a-b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/c/e^6/
exp(a/b)/(((-a-b*ln(c*(d+e*x^(2/3))^2))/b)^p)-3*2^(-1+p)*d^5*(d+e*x^(2/3))*
GAMMA(1+p,1/2*(-a-b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p
/e^6/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))^2))/b)^p)/(c*(d+e*x^(2/3))^2)^
(1/2)
```

Rubi [A] time = 0.97, antiderivative size = 675, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$\frac{15d^22^{-p-2}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{b}\right)}{c^2e^6}+\frac{3^{-p}e^{-\frac{3a}{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))^2]))/b]*(a + b*Log[c*(d + e*
x^(2/3))^2])^p)/(4*3^p*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))^
2])/b))^p) - (3*2^(-1 + p)*d*(d + e*x^(2/3))^5*Gamma[1 + p, (-5*(a + b*Log[
c*(d + e*x^(2/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(5^p*e^6*
E^((5*a)/(2*b))*(c*(d + e*x^(2/3))^2)^(5/2)*(-((a + b*Log[c*(d + e*x^(2/3))
^2])/b))^p) + (15*2^(-2 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/
3))^2]))/b]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(c^2*e^6*E^((2*a)/b)*(-((a
+ b*Log[c*(d + e*x^(2/3))^2])/b))^p) - (5*(2/3)^p*d^3*(d + e*x^(2/3))^3*Gam
ma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*
x^(2/3))^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e*x^(2/3))^2)^(3/2)*(-((a + b*
Log[c*(d + e*x^(2/3))^2])/b))^p) + (15*d^4*Gamma[1 + p, -((a + b*Log[c*(d +
 e*x^(2/3))^2])/b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(4*c*e^6*E^(a/b)*(-
((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p) - (3*2^(-1 + p)*d^5*(d + e*x^(2/3)
)*Gamma[1 + p, -(a + b*Log[c*(d + e*x^(2/3))^2])/(2*b)]*(a + b*Log[c*(d + e
*x^(2/3))^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e*x^(2/3))^2]*(-((a + b*Log[c
*(d + e*x^(2/3))^2])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
```

```
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx = \frac{3}{2}\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,x^{2/3}\right)$$
$$= \frac{3}{2}\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\left(c(d\right.\right.}{e^5}\right.\right.$$
$$= \frac{3\,\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,x^{2/3}\right)}{2e^5}-\frac{(15d)\,\text{Subst}\left(\right.}{}$$
$$= \frac{3\,\text{Subst}\left(\int x^5\left(a+b\log\left(cx^2\right)\right)^p\,dx,x,d+ex^{2/3}\right)}{2e^6}-\frac{(15d)\,\text{Subst}\left(\int x^4\left(a\right.\right.}{}$$
$$= \frac{3\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{4c^3e^6}+\frac{\left(15d^2\right)\text{Subst}\left(\int e^{2x}\right.}{}$$
$$= \frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b}{}\right.}{4c^3e^6}$$

Mathematica [F] time = 0.52, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
[Out] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(ce^2x^{\frac{4}{3}}+2\,cdex^{\frac{2}{3}}+cd^2\right)+a\right)^p x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x^3, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int x^3\left(b\ln\left(\left(e\,x^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(e*x^(2/3)+d)^2))^p,x)
[Out] int(x^3*(a+b*ln(c*(e*x^(2/3)+d)^2))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(\left(ex^{\frac{2}{3}} + d\right)^{2} c\right) + a\right)^{p} x^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(a + b \ln\left(c \left(d + e\, x^{2/3}\right)^2\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(2/3))^2))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e*x^(2/3))^2))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
[Out] Timed out
```

3.576 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=350

$$\frac{3d^2 2^{p-1}e^{-\frac{a}{2b}}\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,\frac{-a-b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{2b}\right)}{e^3\sqrt{c\left(d+ex^{2/3}\right)^2}}+\frac{2^{p-1}3^{-p}e\ldots}{}$$

```
[Out] 2^(-1+p)*(d+e*x^(2/3))^3*GAMMA(1+p,-3/2*(a+b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b
*ln(c*(d+e*x^(2/3))^2))^p/(3^p)/e^3/exp(3/2*a/b)/(c*(d+e*x^(2/3))^2)^(3/2)/
(((-a-b*ln(c*(d+e*x^(2/3))^2))/b)^p)-3/2*d*GAMMA(1+p,(-a-b*ln(c*(d+e*x^(2/3
))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/c/e^3/exp(a/b)/(((-a-b*ln(c*(d+e*x^
(2/3))^2))/b)^p)+3*2^(-1+p)*d^2*(d+e*x^(2/3))*GAMMA(1+p,1/2*(-a-b*ln(c*(d+e
*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/e^3/exp(1/2*a/b)/(((-a-b*ln(
c*(d+e*x^(2/3))^2))/b)^p)/(c*(d+e*x^(2/3))^2)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 347, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$\frac{3d^2 2^{p-1}e^{-\frac{a}{2b}}\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{2b}\right)}{e^3\sqrt{c\left(d+ex^{2/3}\right)^2}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] (2^(-1 + p)*(d + e*x^(2/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))
^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(3^p*e^3*E^((3*a)/(2*b))*(
c*(d + e*x^(2/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p) - (3*
d*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))^2])/b)]*(a + b*Log[c*(d + e*x
^(2/3))^2])^p)/(2*c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p)
+ (3*2^(-1 + p)*d^2*(d + e*x^(2/3))*Gamma[1 + p, -(a + b*Log[c*(d + e*x^(2/
3))^2])/(2*b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(e^3*E^(a/(2*b))*Sqrt[c*
(d + e*x^(2/3))^2]*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
```

```
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\frac{3}{2}\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,x^{2/3}\right)$$

$$=\frac{3}{2}\text{Subst}\left(\int\left(\frac{d^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2}-\frac{2d(d+ex)\left(a+b\log\left(c(d+\right.\right.}{e^2}\right.\right.$$

$$=\frac{3\,\text{Subst}\left(\int(d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,x^{2/3}\right)}{2e^2}-\frac{(3d)\,\text{Subst}\left(\right.}{}$$

$$=\frac{3\,\text{Subst}\left(\int x^2\left(a+b\log\left(cx^2\right)\right)^p\,dx,x,d+ex^{2/3}\right)}{2e^3}-\frac{(3d)\,\text{Subst}\left(\int x\left(a+\right.\right.}{}$$

$$=-\frac{(3d)\,\text{Subst}\left(\int e^x(a+bx)^p\,dx,x,\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2ce^3}+\frac{\left(3\left(d+ex^{2/3}\right)^3\right.}{}$$

$$=\frac{2^{-1+p}3^{-p}e^{-\frac{3a}{2b}}\left(d+ex^{2/3}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(\right.\right.\right.}{e^3\left(c\left(d+ex^{2/3}\right)^2\right)^{3/2}}$$

Mathematica [F] time = 0.30, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

fricas [F] time = 1.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(ce^2x^{\frac{4}{3}} + 2\,cdex^{\frac{2}{3}} + cd^2\right) + a\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}} + d\right)^2 c\right) + a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x\left(b\ln\left(\left(e\,x^{\frac{2}{3}} + d\right)^2 c\right) + a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln((e*x^(2/3)+d)^2*c)+a)^p,x)
[Out] int(x*(b*ln((e*x^(2/3)+d)^2*c)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}} + d\right)^2 c\right) + a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a + b\,\ln\left(c\left(d + e\,x^{2/3}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(2/3))^2))^p,x)
[Out] int(x*(a + b*log(c*(d + e*x^(2/3))^2))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
```

```
[Out] Timed out
```

3.577 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))^2))^p/x,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)^2])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^2\right)\right)^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x, x]
```

fricas [A] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(ce^2x^{\frac{4}{3}}+2\,cdex^{\frac{2}{3}}+cd^2\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x, x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(\left(e\,x^{\frac{2}{3}}+d\right)^{2} c\right)+a\right)^{p}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(2/3)+d)^2*c)+a)^p/x,x)
[Out] int((b*ln((e*x^(2/3)+d)^2*c)+a)^p/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log \left(\left(ex^{\frac{2}{3}}+d\right)^{2} c\right)+a\right)^{p}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln \left(c\,{\left(d+e\,x^{2/3}\right)}^2\right)\right)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^2))^p/x,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^2))^p/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p/x,x)
[Out] Timed out
```

3.578 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^3,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^3,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)^2])^p/x^7, x], x, x^(1/3
)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^2\right)\right)^p}{x^7}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^3,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^3, x]
```

fricas [A] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(ce^2x^{\frac{4}{3}}+2\,cdex^{\frac{2}{3}}+cd^2\right)+a\right)^p}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p/x^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^3,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x^3, x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\left(e\,x^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(2/3)+d)^2*c)+a)^p/x^3,x)
[Out] int((b*ln((e*x^(2/3)+d)^2*c)+a)^p/x^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^3,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^2\right)\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^2))^p/x^3,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^2))^p/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p/x**3,x)
[Out] Timed out
```

3.579 $\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x^2*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^8*(a + b*Log[c*(d + e*x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=3\,\mathrm{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+ex^2\right)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.18, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
[Out] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\log\left(ce^2x^{\frac{4}{3}}+2\,cdex^{\frac{2}{3}}+cd^2\right)+a\right)^p x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p*x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x^2, x)
```

maple [A]   time = 0.07, size = 0, normalized size = 0.00

$$\int x^{2} \left(b \ln \left(\left(e x^{\frac{2}{3}}+d \right)^{2} c \right)+a \right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln((e*x^(2/3)+d)^2*c)+a)^p,x)
[Out] int(x^2*(b*ln((e*x^(2/3)+d)^2*c)+a)^p,x)
```

maxima [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(\left(e x^{\frac{2}{3}} + d\right)^{2} c\right) + a\right)^{p} x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x^2, x)
```

mupad [A]   time = 0.00, size = -1, normalized size = -0.04

$$\int x^2\,{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^2\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(2/3))^2))^p,x)
[Out] int(x^2*(a + b*log(c*(d + e*x^(2/3))^2))^p, x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
[Out] Timed out
```

3.580 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e*x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx = 3\,\mathrm{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

fricas [A] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\log\left(ce^2x^{\frac{4}{3}}+2\,cdex^{\frac{2}{3}}+cd^2\right)+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p, x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(\left(e\,x^{\frac{2}{3}}+d\right)^{2} c\right)+a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(2/3)+d)^2*c)+a)^p,x)
[Out] int((b*ln((e*x^(2/3)+d)^2*c)+a)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{2} c\right)+a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^2\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^2))^p,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^2))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
[Out] Timed out
```

3.581 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^2,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^2,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)^2])^p/x^4, x], x, x^(1/3
)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx = 3\,\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^2\right)\right)^p}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^2,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^2, x]
```

fricas [A] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b\log\left(ce^2x^{\frac{4}{3}}+2\,cdex^{\frac{2}{3}}+cd^2\right)+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{2}c\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x^2, x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\left(e\,x^{\frac{2}{3}}+d\right)^{2}c\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((e*x^(2/3)+d)^2*c)+a)^p/x^2,x)
[Out] int((b*ln((e*x^(2/3)+d)^2*c)+a)^p/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{2}c\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^{2}\right)\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^2))^p/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^2))^p/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p/x**2,x)
[Out] Timed out
```

3.582 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^5*(a + b*Log[c*(d + e/x)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=3\,\text{Subst}\left(\int x^5\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 1.71, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))])^p,x]
```

```
[Out] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))])^p, x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(\frac{cdx+cex^{\frac{2}{3}}}{x}\right)+a\right)^p x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p*x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p*x, x)
```

maple [A] time = 0.11, size = 0, normalized size = 0.00

$$\int x \left(b \ln \left(\left(d + \frac{e}{x^{\frac{1}{3}}}\right) c\right) + a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/3))))^p,x)
[Out] int(x*(a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{1}{3}}}\right)\right) + a\right)^{p} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p*x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x \left(a + b \ln\left(c \left(d + \frac{e}{x^{1/3}}\right)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/3))))^p,x)
[Out] int(x*(a + b*log(c*(d + e/x^(1/3))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))))**p,x)
[Out] Timed out
```

3.583 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e/x)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.29, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p, x]
```

fricas [A] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(\frac{cdx+cex^{\frac{2}{3}}}{x}\right)+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p, x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(b \ln \left(\left(d+\frac{e}{x^{\frac{1}{3}}}\right) c\right)+a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/3))*c)+a)^p,x)
[Out] int((b*ln((d+e/x^(1/3))*c)+a)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(c \left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))))^p,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p,x)
[Out] Timed out
```

3.584 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/3))))^p/x,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x)])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p}{x}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x, x]
```

fricas [A] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cdx+cex^{\frac{2}{3}}}{x}\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(c\left(d + \frac{e}{x^{\frac{1}{3}}}\right)\right) + a\right)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(\left(d + \frac{e}{x^{\frac{1}{3}}}\right) c\right) + a\right)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/3))*c)+a)^p/x,x)
[Out] int((b*ln((d+e/x^(1/3))*c)+a)^p/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(c\left(d + \frac{e}{x^{\frac{1}{3}}}\right)\right) + a\right)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b\,\ln\left(c\,\left(d + \frac{e}{x^{1/3}}\right)\right)\right)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))))^p/x,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))))^p/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x,x)
[Out] Timed out
```

3.585 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=267

$$-\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^3e^3}+\frac{3d2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(\right.\right.\right.}{}$$

```
[Out] -GAMMA(1+p,-3*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(3^p
)/c^3/e^3/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)+3*d*GAMMA(1+p,-2*(a
+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(2^p)/c^2/e^3/exp(2*
a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)-3*d^2*GAMMA(1+p,(-a-b*ln(c*(d+e/x^(
1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/c/e^3/exp(a/b)/(((-a-b*ln(c*(d+e/x^(
1/3))))/b)^p)
```

Rubi [A] time = 0.39, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$-\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^3e^3}+\frac{3d2^{-p}e^{-\frac{2a}{b}}\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x^2,x]
```

```
[Out] -((Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/
x^(1/3))])^p)/(3^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b)
)^p)) + (3*d*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log
[c*(d + e/x^(1/3))])^p)/(2^p*c^2*e^3*E^((2*a)/b)*(-((a + b*Log[c*(d + e/x^(
1/3))])/b))^p) - (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)]*(
a + b*Log[c*(d + e/x^(1/3))])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx = -\left(3\,\text{Subst}\left(\int x^2(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\,\text{Subst}\left(\int \left(\frac{d^2(a+b\log(c(d+ex)))^p}{e^2} - \frac{2d(d+ex)(a+b\log(c(d+ex))}{e^2}\right.\right.\right.$$

$$= -\frac{3\,\text{Subst}\left(\int (d+ex)^2(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^2} + \frac{(6d)\,\text{Subst}\left(\int (d\right.}{}$$

$$= -\frac{3\,\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} + \frac{(6d)\,\text{Subst}\left(\int x(a+b\log(\right.}{e^3}$$

$$= -\frac{3\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^3e^3} + \frac{(6d)\,\text{Subst}\left(\int e^{2x}(a+\right.}{}$$

$$= -\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log(}{}\right.}{c^3e^3}$$

Mathematica [A] time = 0.25, size = 175, normalized size = 0.66

$$-\frac{6^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\left(2^p\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+cd3^{p+1}e^{a/b}\left(cd2^pe\right.\right.}{c^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x^2,x]
```

```
[Out] -(((2^p*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 3^(1 + p)*c*d
*E^(a/b)*(-Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 2^p*c*d*E^
(a/b)*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)]))*(a + b*Log[c*(d +
 e/x^(1/3))])^p)/(6^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])
/b))^p))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cdx+cex^{\frac{2}{3}}}{x}\right)+a\right)^{p}}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^2, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int\frac{\left(b\ln\left(\left(d+\frac{e}{x^{\frac{1}{3}}}\right)c\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/3))*c)+a)^p/x^2,x)
```

```
[Out] int((b*ln((d+e/x^(1/3))*c)+a)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{1/3}}\right)\right)\right)}^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))))^p/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/3))))^p/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x**2,x)
```

```
[Out] Timed out
```

3.586 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx$

Optimal. Leaf size=554

$$-\frac{2^{-p-1}3^{-p}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^6e^6}+\frac{3d5^{-p}e^{-\frac{5a}{b}}\left(a+b\log\right.}{}$$

```
[Out] -2^(-1-p)*GAMMA(1+p,-6*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))
))^p/(3^p)/c^6/e^6/exp(6*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)+3*d*GAMMA(
1+p,-5*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(5^p)/c^5/e
^6/exp(5*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)-15*2^(-1-2*p)*d^2*GAMMA(1+
p,-4*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/c^4/e^6/exp(4
*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)+10*d^3*GAMMA(1+p,-3*(a+b*ln(c*(d+e
/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(3^p)/c^3/e^6/exp(3*a/b)/(((-a-b
*ln(c*(d+e/x^(1/3))))/b)^p)-15*2^(-1-p)*d^4*GAMMA(1+p,-2*(a+b*ln(c*(d+e/x^(
1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e
/x^(1/3))))/b)^p)+3*d^5*GAMMA(1+p,(-a-b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(
d+e/x^(1/3))))^p/c/e^6/exp(a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)
```

Rubi [A] time = 0.83, antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$-\frac{15d^2 2^{-2p-1}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^4e^6}+\frac{10d^3 3^{-p}e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x^3,x]
```

```
[Out] -((2^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Lo
g[c*(d + e/x^(1/3))])^p)/(3^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/x^
(1/3))])/b))^p)) + (3*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/x^(1/3))]))/b]
*(a + b*Log[c*(d + e/x^(1/3))])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c
*(d + e/x^(1/3))])/b))^p) - (15*2^(-1 - 2*p)*d^2*Gamma[1 + p, (-4*(a + b*Lo
g[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c^4*e^6*E^((4*
a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) + (10*d^3*Gamma[1 + p, (-3*(
a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(3^p*c^
3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) - (15*2^(-1 - p)
*d^4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d +
e/x^(1/3))])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^
p) + (3*d^5*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)]*(a + b*Log[c*
(d + e/x^(1/3))])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p
)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx = -\left(3\,\mathrm{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$
$$= -\left(3\,\mathrm{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d+ex)))}{e^5}\right.\right.\right.$$
$$= -\frac{3\,\mathrm{Subst}\left(\int(d+ex)^5(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5}+\frac{(15d)\,\mathrm{Subst}\left(\int(d+\right.}{}$$
$$= -\frac{3\,\mathrm{Subst}\left(\int x^5(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6}+\frac{(15d)\,\mathrm{Subst}\left(\int x^4(a+b\log(\right.}{e^6}$$
$$= -\frac{3\,\mathrm{Subst}\left(\int e^{6x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^6e^6}+\frac{(15d)\,\mathrm{Subst}\left(\int e^{5x}(a+bx\right.}{c^5}$$
$$= -\frac{2^{-1-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log}{}\right.}{c^6e^6}$$

Mathematica [A] time = 0.79, size = 325, normalized size = 0.59

$$\frac{2^{-2p-1}15^{-p}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\left(cde^{a/b}\left(2^{2p+1}3^{p+1}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x^3,x]
```

```
[Out] (2^(-1 - 2*p)*(-(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/x^(1/3))]))/b])
+ c*d*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/x
^(1/3))]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c
*(d + e/x^(1/3))]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*(a +
 b*Log[c*(d + e/x^(1/3))]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p, (-2
*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -(
(a + b*Log[c*(d + e/x^(1/3))])/b)])))))*(a + b*Log[c*(d + e/x^(1/3))])^p)/(
15^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b\log\left(\frac{cdx+cex^{\frac{2}{3}}}{x}\right)+a\right)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^3, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(\left(d+\frac{e}{x^{\frac{1}{3}}}\right) c\right)+a\right)^{p}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/3))*c)+a)^p/x^3,x)
```

```
[Out] int((b*ln((d+e/x^(1/3))*c)+a)^p/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log \left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^{p}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln \left(c\,\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))))^p/x^3,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/3))))^p/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x**3,x)
```

```
[Out] Timed out
```

3.587 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx$

Optimal. Leaf size=832

$$-\frac{3^{-2p-1}e^{-\frac{9a}{b}}\Gamma\left(p+1,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{c^9e^9}+\frac{3\,8^{-p}de^{-\frac{8a}{b}}\Gamma\left(p+1,-\right.}{}$$

```
[Out] -3^(-1-2*p)*GAMMA(1+p,-9*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3
))))^p/c^9/e^9/exp(9*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)+3*d*GAMMA(1+p,
-8*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(8^p)/c^8/e^9/e
xp(8*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)-12*d^2*GAMMA(1+p,-7*(a+b*ln(c*
(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(7^p)/c^7/e^9/exp(7*a/b)/(((
-a-b*ln(c*(d+e/x^(1/3))))/b)^p)+7*2^(2-p)*d^3*GAMMA(1+p,-6*(a+b*ln(c*(d+e/x
^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(3^p)/c^6/e^9/exp(6*a/b)/(((-a-b*l
n(c*(d+e/x^(1/3))))/b)^p)-42*d^4*GAMMA(1+p,-5*(a+b*ln(c*(d+e/x^(1/3))))/b)*
(a+b*ln(c*(d+e/x^(1/3))))^p/(5^p)/c^5/e^9/exp(5*a/b)/(((-a-b*ln(c*(d+e/x^(1
/3))))/b)^p)+21*2^(1-2*p)*d^5*GAMMA(1+p,-4*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+
b*ln(c*(d+e/x^(1/3))))^p/c^4/e^9/exp(4*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)
^p)-28*d^6*GAMMA(1+p,-3*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3)
)))^p/(3^p)/c^3/e^9/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)+3*2^(2-p)
*d^7*GAMMA(1+p,-2*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/
c^2/e^9/exp(2*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)-3*d^8*GAMMA(1+p,(-a-b
*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/c/e^9/exp(a/b)/(((-a-b
*ln(c*(d+e/x^(1/3))))/b)^p)
```

Rubi [A] time = 1.30, antiderivative size = 832, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2454, 2401, 2389, 2299, 2181, 2390, 2309}

$$-\frac{3^{-2p-1}e^{-\frac{9a}{b}}\text{Gamma}\left(p+1,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{c^9e^9}+\frac{3\,8^{-p}de^{-\frac{8a}{b}}\text{Gar}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x^4,x]
```

```
[Out] -((3^(-1 - 2*p)*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*
Log[c*(d + e/x^(1/3))])^p)/(c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p)) + (3*d*Gamma[1 + p, (-8*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(
a + b*Log[c*(d + e/x^(1/3))])^p)/(8^p*c^8*e^9*E^((8*a)/b)*(-((a + b*Log[c*(
d + e/x^(1/3))])/b))^p) - (12*d^2*Gamma[1 + p, (-7*(a + b*Log[c*(d + e/x^(1
/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(7^p*c^7*e^9*E^((7*a)/b)*(-((a
 + b*Log[c*(d + e/x^(1/3))])/b))^p) + (7*2^(2 - p)*d^3*Gamma[1 + p, (-6*(a
+ b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(3^p*c^6*
e^9*E^((6*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) - (42*d^4*Gamma[1
+ p, (-5*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^
p)/(5^p*c^5*e^9*E^((5*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) + (21*
2^(1 - 2*p)*d^5*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*
Log[c*(d + e/x^(1/3))])^p)/(c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p) - (28*d^6*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))]))/b]
*(a + b*Log[c*(d + e/x^(1/3))])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a + b*Log[c
*(d + e/x^(1/3))])/b))^p) + (3*2^(2 - p)*d^7*Gamma[1 + p, (-2*(a + b*Log[c*
(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c^2*e^9*E^((2*a)/b
```

```
)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) - (3*d^8*Gamma[1 + p, -((a + b*L
og[c*(d + e/x^(1/3))])/b)]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c*e^9*E^(a/b)
*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx = -\left(3\,\text{Subst}\left(\int x^8(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\,\text{Subst}\left(\int \left(\frac{d^8(a+b\log(c(d+ex)))^p}{e^8} - \frac{8d^7(d+ex)(a+b\log(c(d+ex)))^p}{e^8}\right.\right.\right.$$

$$= -\frac{3\,\text{Subst}\left(\int (d+ex)^8(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} + \frac{(24d)\,\text{Subst}\left(\int (d+\right.}{}$$

$$= -\frac{3\,\text{Subst}\left(\int x^8(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^9} + \frac{(24d)\,\text{Subst}\left(\int x^7(a+b\log(\right.}{e^9}$$

$$= -\frac{3\,\text{Subst}\left(\int e^{9x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^9e^9} + \frac{(24d)\,\text{Subst}\left(\int e^{8x}(a+bx\right.}{c^8}$$

$$= -\frac{3^{-1-2p}e^{-\frac{9a}{b}}\Gamma\left(1+p, -\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(\right.}{}\right.}{c^9e^9}$$

Mathematica [A] time = 0.87, size = 502, normalized size = 0.60

$$-\frac{3^{-2p-1}280^{-p}e^{-\frac{9a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\left(c^8d^89^{p+1}280^pe^{\frac{8a}{b}}\Gamma\left(p+1, -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right) - \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x^4,x]
```

```
[Out] -((3^(-1 - 2*p)*(280^p*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/x^(1/3))]))/b]
- 9^(1 + p)*35^p*c*d*E^(a/b)*Gamma[1 + p, (-8*(a + b*Log[c*(d + e/x^(1/3))]
))/b] + 2^(2 + 3*p)*5^p*9^(1 + p)*c^2*d^2*E^((2*a)/b)*Gamma[1 + p, (-7*(a +
 b*Log[c*(d + e/x^(1/3))]))/b] - 5^p*84^(1 + p)*c^3*d^3*E^((3*a)/b)*Gamma[1
 + p, (-6*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 2^(1 + 3*p)*63^(1 + p)*c^4*d
^4*E^((4*a)/b)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/x^(1/3))]))/b] - 5^p*12
6^(1 + p)*c^5*d^5*E^((5*a)/b)*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/x^(1/3))
]))/b] + 2^(2 + 3*p)*5^p*21^(1 + p)*c^6*d^6*E^((6*a)/b)*Gamma[1 + p, (-3*(a
 + b*Log[c*(d + e/x^(1/3))]))/b] - 35^p*36^(1 + p)*c^7*d^7*E^((7*a)/b)*Gamm
a[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 9^(1 + p)*280^p*c^8*d^8*E
^((8*a)/b)*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)])*(a + b*Log[c*
(d + e/x^(1/3))])^p)/(280^p*c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cdx+cex^{\frac{2}{3}}}{x}\right)+a\right)^p}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^{p}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^4, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\left(d+\frac{e}{x^{\frac{1}{3}}}\right)c\right)+a\right)^{p}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/3))*c)+a)^p/x^4,x)
[Out] int((b*ln((d+e/x^(1/3))*c)+a)^p/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^{p}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^4,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))))^p/x^4,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))))^p/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x**4,x)
[Out] Timed out
```

3.588 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p,x\right)$$

[Out] Unintegrable(x*(a+b*ln(c*(d+e/x^(1/3))^2))^p,x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]

[Out] 3*Defer[Subst][Defer[Int][x^5*(a + b*Log[c*(d + e/x)^2])^p, x], x, x^(1/3)]

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx=3\,\text{Subst}\left(\int x^5\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.24, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]

[Out] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^2])^p, x]

fricas [A] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(\frac{cd^2x+2\,cdex^{\frac{2}{3}}+ce^2x^{\frac{1}{3}}}{x}\right)+a\right)^p x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="fricas")

[Out] integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p*x, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p*x, x)
```

maple [A] time = 0.20, size = 0, normalized size = 0.00

$$\int x \left(b \ln\left(\left(d + \frac{e}{x^{\frac{1}{3}}}\right)^2 c\right) + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
[Out] int(x*(a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(c \left(d + \frac{e}{x^{\frac{1}{3}}}\right)^2\right) + a\right)^p x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p*x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x \left(a + b \ln\left(c \left(d + \frac{e}{x^{1/3}}\right)^2\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/3))^2))^p,x)
[Out] int(x*(a + b*log(c*(d + e/x^(1/3))^2))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))**2))**p,x)
[Out] Timed out
```

3.589 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e/x)^2])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx = 3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p, x]
```

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(\frac{cd^2x+2\,cdex^{\frac{2}{3}}+ce^2x^{\frac{1}{3}}}{x}\right)+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p, x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2} c\right)+a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/3))^2*c)+a)^p,x)
[Out] int((b*ln((d+e/x^(1/3))^2*c)+a)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)+a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^2\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^2))^p,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))^2))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p,x)
[Out] Timed out
```

3.590 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/3))^2))^p/x,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x)^2])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x, x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cd^2x+2\,cdex^{\frac{2}{3}}+ce^2x^{\frac{1}{3}}}{x}\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/3))^2*c)+a)^p/x,x)
[Out] int((b*ln((d+e/x^(1/3))^2*c)+a)^p/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^{2}\right)\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x,x)
[Out] Timed out
```

3.591 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=342

$$-\frac{3d^2 2^p e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,\frac{-a-b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{2b}\right)}{e^3\sqrt{c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}}-\frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(d\right.}{}$$

```
[Out] -(2/3)^p*(d+e/x^(1/3))^3*GAMMA(1+p,-3/2*(a+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b
*ln(c*(d+e/x^(1/3))^2))^p/e^3/exp(3/2*a/b)/(c*(d+e/x^(1/3))^2)^(3/2)/(((-a-
b*ln(c*(d+e/x^(1/3))^2))/b)^p)+3*d*GAMMA(1+p,(-a-b*ln(c*(d+e/x^(1/3))^2))/b
)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/c/e^3/exp(a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2
))/b)^p)-3*2^p*d^2*(d+e/x^(1/3))*GAMMA(1+p,1/2*(-a-b*ln(c*(d+e/x^(1/3))^2))
/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/e^3/exp(1/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/3
))^2))/b)^p)/(c*(d+e/x^(1/3))^2)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 339, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$-\frac{3d^2 2^p e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{2b}\right)}{e^3\sqrt{c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}}-\frac{\left(\frac{2}{3}\right)^p}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^2,x]
```

```
[Out] -(((2/3)^p*(d + e/x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^
2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^3*E^((3*a)/(2*b))*(c*(d
+ e/x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p)) + (3*d*Ga
mma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))^2])/b)]*(a + b*Log[c*(d + e/x^(1/
3))^2])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (3*2
^p*d^2*(d + e/x^(1/3))*Gamma[1 + p, -(a + b*Log[c*(d + e/x^(1/3))^2])/(2*b)
]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^3*E^(a/(2*b))*Sqrt[c*(d + e/x^(1/3
))^2]*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx = -\left(3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\,\text{Subst}\left(\int\left(\frac{d^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2} - \frac{2d(d+ex)\left(a+b\log\left(c(d+ex\right.\right.}{e^2}\right.\right.\right.$$

$$= -\frac{3\,\text{Subst}\left(\int(d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^2} + \frac{(6d)\,\text{Subst}\left(\int(d\right.}{}$$

$$= -\frac{3\,\text{Subst}\left(\int x^2\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} + \frac{(6d)\,\text{Subst}\left(\int x\left(a+b\log\right.\right.}{e^3}$$

$$= \frac{(3d)\,\text{Subst}\left(\int e^x(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{ce^3} - \frac{\left(3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\right)\text{Subst}\left(\right.}{2}$$

$$= -\frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right.\right.}{e^3\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{3/2}}$$

Mathematica [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^2, x]
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cd^2x+2\,cdex^{\frac{2}{3}}+ce^2x^{\frac{1}{3}}}{x}\right)+a\right)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p/x^2,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^2, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2} c\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/3))^2*c)+a)^p/x^2,x)
[Out] int((b*ln((d+e/x^(1/3))^2*c)+a)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log \left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^2\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^2,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x**2,x)
[Out] Timed out
```

3.592 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx$

Optimal. Leaf size=673

$$-\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{b}\right)}{2c^3e^6}-\frac{15d^2 2^{-p-1}e^{-\frac{2a}{b}}\left(a+b\right.}{}$$

```
[Out] -1/2*GAMMA(1+p,-3*(a+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2)
)^p/(3^p)/c^3/e^6/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)+3*(2/5)^p
*d*(d+e/x^(1/3))^5*GAMMA(1+p,-5/2*(a+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*
(d+e/x^(1/3))^2))^p/e^6/exp(5/2*a/b)/(c*(d+e/x^(1/3))^2)^(5/2)/(((-a-b*ln(c
*(d+e/x^(1/3))^2))/b)^p)-15*2^(-1-p)*d^2*GAMMA(1+p,-2*(a+b*ln(c*(d+e/x^(1/3
))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+
e/x^(1/3))^2))/b)^p)+5*2^(1+p)*d^3*(d+e/x^(1/3))^3*GAMMA(1+p,-3/2*(a+b*ln(c
*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/(3^p)/e^6/exp(3/2*a/b)/
(c*(d+e/x^(1/3))^2)^(3/2)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)-15/2*d^4*GAM
MA(1+p,(-a-b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/c/e^6/
exp(a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)+3*2^p*d^5*(d+e/x^(1/3))*GAMMA
(1+p,1/2*(-a-b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/e^6/
exp(1/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)/(c*(d+e/x^(1/3))^2)^(1/2)
```

Rubi [A] time = 0.97, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

$$-\frac{15d^2 2^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{b}\right)}{c^2e^6}-\frac{3^{-p}e^{-\frac{3a}{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^3,x]
```

```
[Out] -(Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]))/b]*(a + b*Log[c*(d + e
/x^(1/3))^2])^p)/(2*3^p*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))
^2])/b))^p) + (3*(2/5)^p*d*(d + e/x^(1/3))^5*Gamma[1 + p, (-5*(a + b*Log[c*
(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^6*E^((5*
a)/(2*b))*(c*(d + e/x^(1/3))^2)^(5/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b
))^p) - (15*2^(-1 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))^2]
))/b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Lo
g[c*(d + e/x^(1/3))^2])/b))^p) + (5*2^(1 + p)*d^3*(d + e/x^(1/3))^3*Gamma[1
 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1
/3))^2])^p)/(3^p*e^6*E^((3*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(3/2)*(-((a + b*
Log[c*(d + e/x^(1/3))^2])/b))^p) - (15*d^4*Gamma[1 + p, -((a + b*Log[c*(d +
 e/x^(1/3))^2])/b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(2*c*e^6*E^(a/b)*(-
((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) + (3*2^p*d^5*(d + e/x^(1/3))*Gamma
[1 + p, -(a + b*Log[c*(d + e/x^(1/3))^2])/(2*b)]*(a + b*Log[c*(d + e/x^(1/3
))^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e/x^(1/3))^2]*(-((a + b*Log[c*(d + e
/x^(1/3))^2])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx = -\left(3\,\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\,\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5} + \frac{5d^4(d+ex)\left(a+b\log\left(c(d+\right.\right.}{e^5}\right.\right.\right.$$

$$= -\frac{3\,\text{Subst}\left(\int (d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5} + \frac{(15d)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{3\,\text{Subst}\left(\int x^5\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} + \frac{(15d)\,\text{Subst}\left(\int x^4\left(a+b\right.\right.}{e}$$

$$= -\frac{3\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2c^3e^6} - \frac{\left(15d^2\right)\text{Subst}\left(\int e^{2x}(a+\right.}{}$$

$$= -\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(\right.}{}\right.}{2c^3e^6}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^3,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^3, x]
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cd^2x+2\,cdex^{\frac{2}{3}}+ce^2x^{\frac{1}{3}}}{x}\right)+a\right)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^3,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p/x^3,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^3,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^3, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2} c\right)+a\right)^{p}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/3))^2*c)+a)^p/x^3,x)
[Out] int((b*ln((d+e/x^(1/3))^2*c)+a)^p/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)+a\right)^{p}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^3,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^{2}\right)\right)^{p}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^3,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x**3,x)
[Out] Timed out
```

3.593 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx$

Optimal. Leaf size=1036

$$-\frac{2^p3^{-2p-1}e^{-\frac{9a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^9\Gamma\left(p+1,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}}{e^9\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{9/2}}+\frac{3\,4^{-p}\cdots}{\cdots}$$

```
[Out] -2^p*3^(-1-2*p)*(d+e/x^(1/3))^9*GAMMA(1+p,-9/2*(a+b*ln(c*(d+e/x^(1/3))^2))/
b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/e^9/exp(9/2*a/b)/(c*(d+e/x^(1/3))^2)^(9/2)
/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)+3*d*GAMMA(1+p,-4*(a+b*ln(c*(d+e/x^(1/
3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/(4^p)/c^4/e^9/exp(4*a/b)/(((-a-b*l
n(c*(d+e/x^(1/3))^2))/b)^p)-3*2^(2+p)*d^2*(d+e/x^(1/3))^7*GAMMA(1+p,-7/2*(a
+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/(7^p)/e^9/exp(7/
2*a/b)/(c*(d+e/x^(1/3))^2)^(7/2)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)+28*d^
3*GAMMA(1+p,-3*(a+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p
/(3^p)/c^3/e^9/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)-21*2^(1+p)*d
^4*(d+e/x^(1/3))^5*GAMMA(1+p,-5/2*(a+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*
(d+e/x^(1/3))^2))^p/(5^p)/e^9/exp(5/2*a/b)/(c*(d+e/x^(1/3))^2)^(5/2)/(((-a-
b*ln(c*(d+e/x^(1/3))^2))/b)^p)+21*2^(1-p)*d^5*GAMMA(1+p,-2*(a+b*ln(c*(d+e/x
^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/c^2/e^9/exp(2*a/b)/(((-a-b*ln(
c*(d+e/x^(1/3))^2))/b)^p)-7*2^(2+p)*d^6*(d+e/x^(1/3))^3*GAMMA(1+p,-3/2*(a+b
*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/(3^p)/e^9/exp(3/2*
a/b)/(c*(d+e/x^(1/3))^2)^(3/2)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)+12*d^7*
GAMMA(1+p,(-a-b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/c/e
^9/exp(a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)-3*2^p*d^8*(d+e/x^(1/3))*GA
MMA(1+p,1/2*(-a-b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/e
^9/exp(1/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)/(c*(d+e/x^(1/3))^2)^(1
/2)
```

Rubi [A] time = 1.52, antiderivative size = 1036, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2454, 2401, 2389, 2300, 2181, 2390, 2310}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^4,x]
```

```
[Out] -((2^p*3^(-1 - 2*p)*(d + e/x^(1/3))^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/
x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^9*E^((9*a)/(2*b
))*(c*(d + e/x^(1/3))^2)^(9/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p))
+ (3*d*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/x^(1/3))^2]))/b]*(a + b*Log[c*(
d + e/x^(1/3))^2])^p)/(4^p*c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e/x^(1/
3))^2])/b))^p) - (3*2^(2 + p)*d^2*(d + e/x^(1/3))^7*Gamma[1 + p, (-7*(a + b
*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(7^p
*e^9*E^((7*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(7/2)*(-((a + b*Log[c*(d + e/x^(
1/3))^2])/b))^p) + (28*d^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]
))/b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a +
b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (21*2^(1 + p)*d^4*(d + e/x^(1/3))^5*Ga
mma[1 + p, (-5*(a + b*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e
/x^(1/3))^2])^p)/(5^p*e^9*E^((5*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(5/2)*(-((a
 + b*Log[c*(d + e/x^(1/3))^2])/b))^p) + (21*2^(1 - p)*d^5*Gamma[1 + p, (-2*
```

```
(a + b*Log[c*(d + e/x^(1/3))^2]))/b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(c
^2*e^9*E^((2*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (7*2^(2 + p
)*d^6*(d + e/x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]))/
(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(3^p*e^9*E^((3*a)/(2*b))*(c*(d +
 e/x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) + (12*d^7*G
amma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))^2])/b)]*(a + b*Log[c*(d + e/x^(1
/3))^2])^p)/(c*e^9*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (3*
2^p*d^8*(d + e/x^(1/3))*Gamma[1 + p, -(a + b*Log[c*(d + e/x^(1/3))^2])/(2*b
)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^9*E^(a/(2*b))*Sqrt[c*(d + e/x^(1/
3))^2]*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx = -\left(3\,\text{Subst}\left(\int x^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\,\text{Subst}\left(\int\left(\frac{d^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^8} - \frac{8d^7(d+ex)\left(a+b\log\left(c(d+e\right.\right.}{e^8}\right.\right.\right.$$

$$= -\frac{3\,\text{Subst}\left(\int (d+ex)^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} + \frac{(24d)\,\text{Subst}\left(\int(\right.}{}$$

$$= -\frac{3\,\text{Subst}\left(\int x^8\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^9} + \frac{(24d)\,\text{Subst}\left(\int x^7\left(a+b\right.\right.}{e}$$

$$= \frac{(12d)\,\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{c^4e^9} + \frac{\left(84d^3\right)\text{Subst}\left(\int e^{3x}(\right.}{}$$

$$= -\frac{2^p3^{-1-2p}e^{-\frac{9a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^9\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{\cdot}}\right.\right.\right.}{e^9\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{9/2}}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^4,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^4, x]
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cd^2x+2\,cdex^{\frac{2}{3}}+ce^2x^{\frac{1}{3}}}{x}\right)+a\right)^p}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^4,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p/x^4,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^4, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2} c\right)+a\right)^{p}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(1/3))^2*c)+a)^p/x^4,x)
```

```
[Out] int((b*ln((d+e/x^(1/3))^2*c)+a)^p/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)+a\right)^{p}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^2\right)\right)^p}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^4,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x**4,x)
```

```
[Out] Timed out
```

3.594 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

[Out] Unintegrable(x^3*(a+b*ln(c*(d+e/x^(2/3))))^p,x)

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))])^p,x]

[Out] 3*Defer[Subst][Defer[Int][x^11*(a + b*Log[c*(d + e/x^2)])^p, x], x, x^(1/3)
]

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=3\,\text{Subst}\left(\int x^{11}\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 1.50, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))])^p,x]

[Out] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))])^p, x]

fricas [A] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(\frac{cdx+cex^{\frac{1}{3}}}{x}\right)+a\right)^p x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")

[Out] integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p*x^3, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^3, x)
```

maple [A] time = 0.11, size = 0, normalized size = 0.00

$$\int x^{3} \left(b \ln \left(\left(d+\frac{e}{x^{\frac{2}{3}}}\right) c\right)+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
[Out] int(x^3*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(b \log \left(c {\left(d + \frac{e}{x^{\frac{2}{3}}}\right)}\right) + a\right)}^{p} x^{3}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x^3\,{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e/x^(2/3))))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e/x^(2/3))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))))**p,x)
[Out] Timed out
```

3.595 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

[Out] Unintegrable(x^2*(a+b*ln(c*(d+e/x^(2/3))))^p,x)

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))])^p,x]

[Out] 3*Defer[Subst][Defer[Int][x^8*(a + b*Log[c*(d + e/x^2)])^p, x], x, x^(1/3)]

Rubi steps

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx = 3\,\text{Subst}\left(\int x^8 \left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.75, size = 0, normalized size = 0.00

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))])^p,x]

[Out] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))])^p, x]

fricas [A] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(\frac{cdx+cex^{\frac{1}{3}}}{x}\right)+a\right)^p x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")

[Out] integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p*x^2, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")

[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^2, x)

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int x^{2}\left(b\ln\left(\left(d+\frac{e}{x^{\frac{2}{3}}}\right)c\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln((d+e/x^(2/3))*c)+a)^p,x)
```

```
[Out] int(x^2*(b*ln((d+e/x^(2/3))*c)+a)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^{p}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x^{2}\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e/x^(2/3))))^p,x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e/x^(2/3))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))))**p,x)
```

```
[Out] Timed out
```

3.596 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^5*(a + b*Log[c*(d + e/x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=3\,\mathrm{Subst}\left(\int x^5\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.82, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\log\left(\frac{cdx+cex^{\frac{1}{3}}}{x}\right)+a\right)^p x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p*x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int x\left(b\ln\left(\left(d+\frac{e}{x^{\frac{2}{3}}}\right)c\right)+a\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln((d+e/x^(2/3))*c)+a)^p,x)
[Out] int(x*(b*ln((d+e/x^(2/3))*c)+a)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^{p}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(2/3))))^p,x)
[Out] int(x*(a + b*log(c*(d + e/x^(2/3))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))))**p,x)
[Out] Timed out
```

3.597 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e/x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=3\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

fricas [A] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(\frac{cdx+cex^{\frac{1}{3}}}{x}\right)+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \left(b\ln \left(\left(d+\frac{e}{x^{\frac{2}{3}}}\right)c\right)+a\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(2/3))*c)+a)^p,x)
```

```
[Out] int((b*ln((d+e/x^(2/3))*c)+a)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log \left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int {\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))))^p,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(2/3))))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))))**p,x)
```

```
[Out] Timed out
```

3.598 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(2/3))))^p/x,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx = 3\,\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x, x]
```

fricas [A] time = 0.72, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b\log\left(\frac{cdx+cex^{\frac{1}{3}}}{x}\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\left(d+\frac{e}{x^{\frac{2}{3}}}\right)c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(2/3))*c)+a)^p/x,x)
[Out] int((b*ln((d+e/x^(2/3))*c)+a)^p/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))))^p/x,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))))^p/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))))**p/x,x)
[Out] Timed out
```

3.599 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(2/3))))^p/x^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))])^p/x^2,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)])^p/x^4, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x^2, x]
```

fricas [A] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\log\left(\frac{cdx+cex^{\frac{1}{3}}}{x}\right)+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x^2, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\left(d+\frac{e}{x^{\frac{2}{3}}}\right)c\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(2/3))*c)+a)^p/x^2,x)
```

```
[Out] int((b*ln((d+e/x^(2/3))*c)+a)^p/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))))^p/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(2/3))))^p/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))))**p/x**2,x)
```

```
[Out] Timed out
```

3.600 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x^3*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^11*(a + b*Log[c*(d + e/x^2)^2])^p, x], x, x^(1/
3)]
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx = 3\,\text{Subst}\left(\int x^{11}\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A] time = 0.31, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

fricas [A] time = 1.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(\frac{cd^2x^2+2\,cdex^{\frac{4}{3}}+ce^2x^{\frac{2}{3}}}{x^2}\right)+a\right)^p x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p*x
^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^3, x)
```

maple [A] time = 0.22, size = 0, normalized size = 0.00

$$\int x^{3}\left(b\ln\left(\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2} c\right)+a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
[Out] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2}\right)+a\right)^{p} x^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x^{3}\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^{2}\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e/x^(2/3))^2))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e/x^(2/3))^2))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
[Out] Timed out
```

3.601 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$

Optimal. Leaf size=27

$$\text{Int}\left(x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x^2*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^8*(a + b*Log[c*(d + e/x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx=3\,\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.21, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

fricas [A] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\log\left(\frac{cd^2x^2+2\,cdex^{\frac{4}{3}}+ce^2x^{\frac{2}{3}}}{x^2}\right)+a\right)^p x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p*x
^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^2, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int x^{2} \left(b \ln \left(\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2} c\right)+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*ln((d+e/x^(2/3))^2*c)+a)^p,x)
```

```
[Out] int(x^2*(b*ln((d+e/x^(2/3))^2*c)+a)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(c \left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2}\right)+a\right)^{p} x^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x^2\,{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^2\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e/x^(2/3))^2))^p,x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e/x^(2/3))^2))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
```

```
[Out] Timed out
```

3.602 $\int x \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(x \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^5*(a + b*Log[c*(d + e/x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p \, dx=3 \text{Subst}\left(\int x^5 \left(a+b \log \left(c \left(d+\frac{e}{x^2}\right)^2\right)\right)^p \, dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.25, size = 0, normalized size = 0.00

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

fricas [A] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b \log\left(\frac{c d^2 x^2 + 2 \, c d e x^{\frac{4}{3}} + c e^2 x^{\frac{2}{3}}}{x^2}\right) + a\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p*x
, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^2\right) + a\right)^p x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int x\left(b\ln\left(\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2} c\right)+a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*ln((d+e/x^(2/3))^2*c)+a)^p,x)
[Out] int(x*(b*ln((d+e/x^(2/3))^2*c)+a)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2}\right)+a\right)^{p} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^{2}\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(2/3))^2))^p,x)
[Out] int(x*(a + b*log(c*(d + e/x^(2/3))^2))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
[Out] Timed out
```

3.603 $\int \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$

Optimal. Leaf size=23

$$\text{Int}\left(\left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e/x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\int \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx = 3\,\text{Subst}\left(\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^2}\right)^2\right)\right)^p dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

fricas [A] time = 1.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b \log \left(\frac{cd^2x^2+2\,cdex^{\frac{4}{3}}+ce^2x^{\frac{2}{3}}}{x^2}\right)+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p,
x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p, x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(b\ln\left(\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2} c\right)+a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(2/3))^2*c)+a)^p,x)
[Out] int((b*ln((d+e/x^(2/3))^2*c)+a)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2}\right)+a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^{2}\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^2))^p,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^2))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
[Out] Timed out
```

3.604 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(2/3))^2))^p/x,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)^2])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx=3\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x, x]
```

fricas [A] time = 0.84, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(b\log\left(\frac{cd^2x^2+2\,cdex^{\frac{4}{3}}+ce^2x^{\frac{2}{3}}}{x^2}\right)+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p/x
, x)
```

giac **[A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2}\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x, x)
```

maple **[A]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2}c\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(2/3))^2*c)+a)^p/x,x)
[Out] int((b*ln((d+e/x^(2/3))^2*c)+a)^p/x,x)
```

maxima **[A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2}\right)+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x, x)
```

mupad **[A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^{2}\right)\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^2))^p/x,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^2))^p/x, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**2))**p/x,x)
[Out] Timed out
```

3.605 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(2/3))^2))^p/x^2,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x^2,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)^2])^p/x^4, x], x, x^(1/3
)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx = 3\,\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x^2,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x^2, x]
```

fricas [A] time = 1.11, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b\log\left(\frac{cd^2x^2+2\,cdex^{\frac{4}{3}}+ce^2x^{\frac{2}{3}}}{x^2}\right)+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p/x
^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(c\left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{2}\right) + a\right)^{p}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x^2, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(\left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{2} c\right) + a\right)^{p}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln((d+e/x^(2/3))^2*c)+a)^p/x^2,x)
[Out] int((b*ln((d+e/x^(2/3))^2*c)+a)^p/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left(c\left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{2}\right) + a\right)^{p}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b\, \ln\left(c\left(d + \frac{e}{x^{2/3}}\right)^{2}\right)\right)^{p}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^2))^p/x^2,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^2))^p/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**2))**p/x**2,x)
[Out] Timed out
```

3.606 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}\,dx$

Optimal. Leaf size=631

$$\frac{2g(hx)^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2}+\frac{2af\sqrt{hx}}{h}+\frac{2bf\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{h}+\frac{\sqrt{2}\,bd^{3/4}gp\log\left(-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}\right. +}{3e^{3/4}\sqrt{h}}$$

```
[Out] -8/9*b*g*p*(h*x)^(3/2)/h^2+2/3*g*(h*x)^(3/2)*(a+b*ln(c*(e*x^2+d)^p))/h^2-2*
b*d^(1/4)*f*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)
/e^(1/4)/h^(1/2)-2/3*b*d^(3/4)*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(
1/4)/h^(1/2))*2^(1/2)/e^(3/4)/h^(1/2)+2*b*d^(1/4)*f*p*arctan(1+e^(1/4)*2^(1
/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(1/4)/h^(1/2)+2/3*b*d^(3/4)*g*p*
arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(3/4)/h^(1/
2)-b*d^(1/4)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/
2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(1/2)+1/3*b*d^(3/4)*g*p*ln(d^(1/2)*h^(1/2
)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(3/4)/h^
(1/2)+b*d^(1/4)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^
(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(1/2)-1/3*b*d^(3/4)*g*p*ln(d^(1/2)*h^(
1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(3/4)
/h^(1/2)+2*a*f*(h*x)^(1/2)/h-8*b*f*p*(h*x)^(1/2)/h+2*b*f*ln(c*(e*x^2+d)^p)*
(h*x)^(1/2)/h
```

Rubi [A] time = 0.90, antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {2467, 2471, 2448, 321, 211, 1165, 628, 1162, 617, 204, 2455, 297}

$$\frac{2g(hx)^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2}+\frac{2af\sqrt{hx}}{h}+\frac{2bf\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{h}+\frac{\sqrt{2}\,bd^{3/4}gp\log\left(-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}\right. +}{3e^{3/4}\sqrt{h}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]
```

```
[Out] (2*a*f*Sqrt[h*x])/h - (8*b*f*p*Sqrt[h*x])/h - (8*b*g*p*(h*x)^(3/2))/(9*h^2)
 - (2*Sqrt[2]*b*d^(1/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)
*Sqrt[h])])/(e^(1/4)*Sqrt[h]) - (2*Sqrt[2]*b*d^(3/4)*g*p*ArcTan[1 - (Sqrt[2
]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*Sqrt[h]) + (2*Sqrt[2]*b
*d^(1/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^
(1/4)*Sqrt[h]) + (2*Sqrt[2]*b*d^(3/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*Sqrt[h]) + (2*b*f*Sqrt[h*x]*Log[c*(d +
 e*x^2)^p])/h + (2*g*(h*x)^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^2) - (S
qrt[2]*b*d^(1/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1
/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) + (Sqrt[2]*b*d^(3/4)*g*p*Log[Sqrt
[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^
(3/4)*Sqrt[h]) + (Sqrt[2]*b*d^(1/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[
h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) - (Sqrt[2]*b*d
^(3/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(3*e^(3/4)*Sqrt[h])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
```

```
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}\,dx = \frac{2\,\text{Subst}\left(\int\left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx,x,\sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int\left(f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)+\frac{gx^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h}\right)dx\right.}{h}$$

$$= \frac{(2g)\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx,x,\sqrt{hx}\right)}{h^2} + \frac{(2f)\,\text{Sul}}{}$$

$$= \frac{2af\sqrt{hx}}{h} + \frac{2g(hx)^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2} + \frac{(2bf)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{2af\sqrt{hx}}{h} - \frac{8bgp(hx)^{3/2}}{9h^2} + \frac{2bf\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{h} + \frac{2g(hx)^{3/2}}{}$$

$$= \frac{2af\sqrt{hx}}{h} - \frac{8bfp\sqrt{hx}}{h} - \frac{8bgp(hx)^{3/2}}{9h^2} + \frac{2bf\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{h}$$

$$= \frac{2af\sqrt{hx}}{h} - \frac{8bfp\sqrt{hx}}{h} - \frac{8bgp(hx)^{3/2}}{9h^2} + \frac{2bf\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{h}$$

$$= \frac{2af\sqrt{hx}}{h} - \frac{8bfp\sqrt{hx}}{h} - \frac{8bgp(hx)^{3/2}}{9h^2} + \frac{2bf\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{h}$$

$$= \frac{2af\sqrt{hx}}{h} - \frac{8bfp\sqrt{hx}}{h} - \frac{8bgp(hx)^{3/2}}{9h^2} - \frac{2\sqrt{2}\,bd^{3/4}gp\tan^{-1}\left(1-\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{3e^{3/4}\sqrt{h}}$$

$$= \frac{2af\sqrt{hx}}{h} - \frac{8bfp\sqrt{hx}}{h} - \frac{8bgp(hx)^{3/2}}{9h^2} - \frac{2\sqrt{2}\,b\sqrt[4]{d}\,fp\tan^{-1}\left(1-\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{\sqrt[4]{e}\sqrt{h}}$$

Mathematica [A] time = 0.47, size = 344, normalized size = 0.55

$$\frac{2\sqrt{x}\left(\frac{1}{3}gx^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)+af\sqrt{x}+bf\sqrt{x}\,\log\left(c\left(d+ex^2\right)^p\right)-\frac{2bgp\left(2\sqrt[4]{-d}\,e^{3/4}x^{3/2}-3d\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{-d}}\right)+3d\right.}{9\sqrt[4]{-d}\,e^{3/4}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]
```

```
[Out] (2*Sqrt[x]*(a*f*Sqrt[x] - (2*b*g*p*(2*(-d)^(1/4)*e^(3/4)*x^(3/2) - 3*d*ArcT
an[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + 3*d*ArcTanh[(e^(1/4)*Sqrt[x])/(-d)^(1/4)
]))/(9*(-d)^(1/4)*e^(3/4)) - (b*f*p*(8*e^(1/4)*Sqrt[x] + 2*Sqrt[2]*d^(1/4)*
ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*Sqrt[2]*d^(1/4)*ArcTan[1
```

```
+ (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Sqrt[2]*d^(1/4)*Log[Sqrt[d] - Sqrt[2
]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Sqrt[2]*d^(1/4)*Log[Sqrt[d] + Sqrt
[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(2*e^(1/4)) + b*f*Sqrt[x]*Log[c*
(d + e*x^2)^p] + (g*x^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/3))/Sqrt[h*x]
```

fricas [B] time = 1.14, size = 1196, normalized size = 1.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="fricas
")
```

```
[Out] -2/9*(3*h*sqrt(-(6*b^2*d*f*g*p^2 + e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2
*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))*log(-32*(81*b^3*e^2*f^4 -
b^3*d^2*g^4)*sqrt(h*x)*p^3 + 32*(e^2*g*h^2*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4
*d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)) + 3*(9*b^2*e^2*f^3 - b^2*d*e*f
*g^2)*h*p^2)*sqrt(-(6*b^2*d*f*g*p^2 + e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*
d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))) - 3*h*sqrt(-(6*b^2*d*f
*g*p^2 + e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^4)*
p^4/(e^3*h^2)))/(e*h))*log(-32*(81*b^3*e^2*f^4 - b^3*d^2*g^4)*sqrt(h*x)*p^3
 - 32*(e^2*g*h^2*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g
^4)*p^4/(e^3*h^2)) + 3*(9*b^2*e^2*f^3 - b^2*d*e*f*g^2)*h*p^2)*sqrt(-(6*b^2*
d*f*g*p^2 + e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^
4)*p^4/(e^3*h^2)))/(e*h))) - 3*h*sqrt(-(6*b^2*d*f*g*p^2 - e*h*sqrt(-(81*b^4
*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))*log
(-32*(81*b^3*e^2*f^4 - b^3*d^2*g^4)*sqrt(h*x)*p^3 + 32*(e^2*g*h^2*sqrt(-(81
*b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)) - 3*(9*
b^2*e^2*f^3 - b^2*d*e*f*g^2)*h*p^2)*sqrt(-(6*b^2*d*f*g*p^2 - e*h*sqrt(-(81*
b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h)))
 + 3*h*sqrt(-(6*b^2*d*f*g*p^2 - e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2*e*
f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))*log(-32*(81*b^3*e^2*f^4 - b^3
*d^2*g^4)*sqrt(h*x)*p^3 - 32*(e^2*g*h^2*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^
2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)) - 3*(9*b^2*e^2*f^3 - b^2*d*e*f*g^
2)*h*p^2)*sqrt(-(6*b^2*d*f*g*p^2 - e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2
*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))) + (36*b*f*p - 9*a*f + (4*
b*g*p - 3*a*g)*x - 3*(b*g*p*x + 3*b*f*p)*log(e*x^2 + d) - 3*(b*g*x + 3*b*f)
*log(c))*sqrt(h*x))/h
```

giac [A] time = 0.30, size = 514, normalized size = 0.81

$$\frac{6\sqrt{hx}\,bgx\log(c) + 9\left(\left(\left(2\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^2\right)^{\frac{1}{4}}}\right)e^{\left(-\frac{5}{4}\right)} + 2\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)\right.}{2\left(\right.}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/9*(6*sqrt(h*x)*b*g*x*log(c) + 9*((2*sqrt(2)*(d*h^2)^(1/4)*arctan(1/2*sqrt
(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e
^(-5/4) + 2*sqrt(2)*(d*h^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4
)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-5/4) + sqrt(2)*(d*h^2)
^(1/4)*e^(-5/4)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d
*h^2)*e^(-1/2)) - sqrt(2)*(d*h^2)^(1/4)*e^(-5/4)*log(-sqrt(2)*(d*h^2)^(1/4)
*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2)) - 8*sqrt(h*x)*e^(-1))*e +
 2*sqrt(h*x)*log(x^2*e + d))*b*f*p + 6*sqrt(h*x)*a*g*x + 18*sqrt(h*x)*b*f*l
```

```
og(c) + (6*sqrt(h*x)*h*x*log(x^2*e + d) - (8*sqrt(h*x)*h*x*e^(-1) - 6*sqrt(
2)*(d*h^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqr
t(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-7/4) - 6*sqrt(2)*(d*h^2)^(3/4)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(
1/4))*e^(-7/4) + 3*sqrt(2)*(d*h^2)^(3/4)*e^(-7/4)*log(sqrt(2)*(d*h^2)^(1/4)
*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2)) - 3*sqrt(2)*(d*h^2)^(3/4)
*e^(-7/4)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)
*e^(-1/2)))*e)*b*g*p/h + 18*sqrt(h*x)*a*f)/h
```

maple [F] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)\left(b\ln\left(c\left(e\,x^2+d\right)^p\right)+a\right)}{\sqrt{hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2),x)
```

```
[Out] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2),x)
```

maxima [A] time = 1.09, size = 572, normalized size = 0.91

$$\frac{2\,bgx^2\log\left(\left(ex^2+d\right)^p c\right)}{3\sqrt{hx}}+\frac{2\,agx^2}{3\sqrt{hx}}+\frac{2\sqrt{hx}\,bf\log\left(\left(ex^2+d\right)^p c\right)}{h}-\frac{\left(\frac{8\sqrt{hx}\,h^2}{e}-\frac{\left(\frac{\sqrt{2}h^4\log\left(\sqrt{e}hx+\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\sqrt{hx}\,e^{\frac{1}{4}}+\sqrt{d}h\right)}{\left(dh^2\right)^{\frac{3}{4}}e^{\frac{1}{4}}}-\sqrt{2}h^4\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="maxima
")
```

```
[Out] 2/3*b*g*x^2*log((e*x^2 + d)^p*c)/sqrt(h*x) + 2/3*a*g*x^2/sqrt(h*x) + 2*sqrt
(h*x)*b*f*log((e*x^2 + d)^p*c)/h - (8*sqrt(h*x)*h^2/e - (sqrt(2)*h^4*log(sq
rt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(
3/4)*e^(1/4)) - sqrt(2)*h^4*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*
x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + 2*sqrt(2)*h^3*arctan(1/2*
sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*
sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)) + 2*sqrt(2)*h^3*arctan(-1/2*s
qrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*s
qrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)))*d/e)*b*e*f*p/h^3 + 2*sqrt(h*x
)*a*f/h + 1/9*(3*d*h^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)
*e^(1/4) + 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt
(e)*h)*sqrt(e)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1
/4) - 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h
)*sqrt(e)) - sqrt(2)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1
/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) + sqrt(2)*log(sqrt(e)*h*x - sqrt(2
)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)))/e -
 8*(h*x)^(3/2)*h^2/e)*b*e*g*p/h^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{\sqrt{h\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(1/2),x)
[Out] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(1/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(1/2),x)
[Out] Exception raised: TypeError
```

3.607 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{3/2}}\,dx$

Optimal. Leaf size=603

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h\sqrt{hx}}+\frac{2ag\sqrt{hx}}{h^2}+\frac{2bg\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{h^2}+\frac{\sqrt{2}\,b\sqrt[4]{e}\,fp\log\left(-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\right.}{\sqrt[4]{d}\,h^{3/2}}$$

```
[Out] -2*b*e^(1/4)*f*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1
/2)/d^(1/4)/h^(3/2)-2*b*d^(1/4)*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^
(1/4)/h^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)+2*b*e^(1/4)*f*p*arctan(1+e^(1/4)*2^(
1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(1/4)/h^(3/2)+2*b*d^(1/4)*g*p*a
rctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(1/4)/h^(3/2
)+b*e^(1/4)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2
)*(h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(3/2)-b*d^(1/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e
^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)
-b*e^(1/4)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)
*(h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(3/2)+b*d^(1/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^
(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)-
2*f*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(1/2)+2*a*g*(h*x)^(1/2)/h^2-8*b*g*p*(h*
x)^(1/2)/h^2+2*b*g*ln(c*(e*x^2+d)^p)*(h*x)^(1/2)/h^2
```

Rubi [A] time = 0.79, antiderivative size = 603, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {2467, 2476, 2448, 321, 211, 1165, 628, 1162, 617, 204, 2455, 297}

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h\sqrt{hx}}+\frac{2ag\sqrt{hx}}{h^2}+\frac{2bg\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{h^2}+\frac{\sqrt{2}\,b\sqrt[4]{e}\,fp\log\left(-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\right.}{\sqrt[4]{d}\,h^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

```
[Out] (2*a*g*Sqrt[h*x])/h^2 - (8*b*g*p*Sqrt[h*x])/h^2 - (2*Sqrt[2]*b*e^(1/4)*f*p*
ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(3/2)
) - (2*Sqrt[2]*b*d^(1/4)*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4
)*Sqrt[h])])/(e^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*e^(1/4)*f*p*ArcTan[1 + (Sqrt[
2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*
d^(1/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(
1/4)*h^(3/2)) + (2*b*g*Sqrt[h*x]*Log[c*(d + e*x^2)^p])/h^2 - (2*f*(a + b*Lo
g[c*(d + e*x^2)^p]))/(h*Sqrt[h*x]) + (Sqrt[2]*b*e^(1/4)*f*p*Log[Sqrt[d]*Sqr
t[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(
3/2)) - (Sqrt[2]*b*d^(1/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sq
rt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*h^(3/2)) - (Sqrt[2]*b*e^(1/4)*f*
p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*
x]])/(d^(1/4)*h^(3/2)) + (Sqrt[2]*b*d^(1/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[
e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*h^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
```

```
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^2}dx,x,\sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h}+\frac{f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^2}\right)dx,x,\sqrt{hx}\right)}{h}$$

$$= \frac{(2g)\,\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx,x,\sqrt{hx}\right)}{h^2} + \frac{(2f)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h\sqrt{hx}} + \frac{(2bg)\,\text{Subst}\left(\int \log\left(c\left(d+\right.\right.\right.}{h^2}$$

$$= \frac{2ag\sqrt{hx}}{h^2} + \frac{2bg\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{h^2} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right.}{h\sqrt{hx}}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} + \frac{2bg\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{h^2} - \frac{2f\left(a+b\log\left(\right.\right.}{h\sqrt{}}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} + \frac{2bg\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{h^2} - \frac{2f\left(a+b\log\left(\right.\right.}{h\sqrt{}}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} - \frac{2\sqrt{2}\,b\sqrt[4]{e}\,fp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{d}\,h^{3/2}} + \frac{2\sqrt{2}\,b\sqrt[4]{}}{}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} - \frac{2\sqrt{2}\,b\sqrt[4]{e}\,fp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{d}\,h^{3/2}} + \frac{2\sqrt{2}\,b\sqrt[4]{}}{}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} - \frac{2\sqrt{2}\,b\sqrt[4]{e}\,fp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{d}\,h^{3/2}} - \frac{2\sqrt{2}\,b\sqrt[4]{}}{}$$

Mathematica [A] time = 0.48, size = 316, normalized size = 0.52

$$\frac{2x^{3/2}\left(-\frac{f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{x}} + ag\sqrt{x} + bg\sqrt{x}\,\log\left(c\left(d+ex^2\right)^p\right) + \frac{2b\sqrt[4]{e}\,fp\left(\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{-d}}\right)+\tanh^{-1}\left(\frac{d\sqrt[4]{e}\,\sqrt{x}}{(-d)^{5/4}}\right)\right)}{\sqrt[4]{-d}} - \frac{bgp\left(\sqrt{2}\,\sqrt[4]{d}\,\log\right.}{}\right.}{(hx)^{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

```
[Out] (2*x^(3/2)*(a*g*Sqrt[x] + (2*b*e^(1/4)*f*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(
1/4)] + ArcTanh[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (b*g*p*(8*e^
```

```
(1/4)*Sqrt[x] + 2*Sqrt[2]*d^(1/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1
/4)] - 2*Sqrt[2]*d^(1/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Sq
rt[2]*d^(1/4)*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] -
Sqrt[2]*d^(1/4)*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])
)/(2*e^(1/4)) + b*g*Sqrt[x]*Log[c*(d + e*x^2)^p] - (f*(a + b*Log[c*(d + e*x
^2)^p]))/Sqrt[x]))/(h*x)^(3/2)
```

fricas [B] time = 0.76, size = 1162, normalized size = 1.93

$$\frac{2\left(h^2x\sqrt{-\frac{2\,b^2fgp^2+h^3\sqrt{-\frac{\left(b^4e^2f^4-2\,b^4def^2g^2+b^4d^2g^4\right)p^4}{deh^6}}}{h^3}}\,\log\left(-32\left(b^3e^2f^4-b^3d^2g^4\right)\sqrt{hx}\,p^3+32\left(defh^5\sqrt{-\frac{\left(b^4e^2f^4-2\,b^4def\right.}{deh}}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="fricas
")
```

```
[Out] 2*(h^2*x*sqrt(-(2*b^2*f*g*p^2 + h^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2
+ b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)*log(-32*(b^3*e^2*f^4 - b^3*d^2*g^4)*sqr
t(h*x)*p^3 + 32*(d*e*f*h^5*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2
*g^4)*p^4/(d*e*h^6)) - (b^2*d*e*f^2*g - b^2*d^2*g^3)*h^2*p^2)*sqrt(-(2*b^2*
f*g*p^2 + h^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*
e*h^6)))/h^3)) - h^2*x*sqrt(-(2*b^2*f*g*p^2 + h^3*sqrt(-(b^4*e^2*f^4 - 2*b^
4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)*log(-32*(b^3*e^2*f^4 - b^
3*d^2*g^4)*sqrt(h*x)*p^3 - 32*(d*e*f*h^5*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2
*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)) - (b^2*d*e*f^2*g - b^2*d^2*g^3)*h^2*p^2)
*sqrt(-(2*b^2*f*g*p^2 + h^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^
2*g^4)*p^4/(d*e*h^6)))/h^3)) - h^2*x*sqrt(-(2*b^2*f*g*p^2 - h^3*sqrt(-(b^4*
e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)*log(-32*(b^
3*e^2*f^4 - b^3*d^2*g^4)*sqrt(h*x)*p^3 + 32*(d*e*f*h^5*sqrt(-(b^4*e^2*f^4 -
 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)) + (b^2*d*e*f^2*g - b^2*d^2
*g^3)*h^2*p^2)*sqrt(-(2*b^2*f*g*p^2 - h^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^
2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)) + h^2*x*sqrt(-(2*b^2*f*g*p^2 - h
^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^
3)*log(-32*(b^3*e^2*f^4 - b^3*d^2*g^4)*sqrt(h*x)*p^3 - 32*(d*e*f*h^5*sqrt(-
(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)) + (b^2*d*e*f
^2*g - b^2*d^2*g^3)*h^2*p^2)*sqrt(-(2*b^2*f*g*p^2 - h^3*sqrt(-(b^4*e^2*f^4
- 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)) - (a*f + (4*b*g*p
- a*g)*x - (b*g*p*x - b*f*p)*log(e*x^2 + d) - (b*g*x - b*f)*log(c))*sqrt(h*
x))/(h^2*x)
```

giac [A] time = 0.36, size = 431, normalized size = 0.71

$$\frac{2\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bdghpe^{\frac{7}{4}}+\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bfpe^{\frac{9}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^2\right)^{\frac{1}{4}}}\right)e^{(-2)}}{dh^2}+\frac{2\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bdghpe^{\frac{7}{4}}+\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bfpe^{\frac{9}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\right.}{}\right.}{dh^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="giac")
```

```
[Out] (2*(sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*p*e^(7/4) + sqrt(2)*(d*h^2)^(3/4)*b*f*p*e
^(9/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e
^(1/4)/(d*h^2)^(1/4))*e^(-2)/(d*h^2) + 2*(sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*p*e
^(7/4) + sqrt(2)*(d*h^2)^(3/4)*b*f*p*e^(9/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*
(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-2)/(d*h^2)
```

```
 + (sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*p*e^(7/4) - sqrt(2)*(d*h^2)^(3/4)*b*f*p*e
^(9/4))*e^(-2)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*
h^2)*e^(-1/2))/(d*h^2) - (sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*p*e^(7/4) - sqrt(2)
*(d*h^2)^(3/4)*b*f*p*e^(9/4))*e^(-2)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e
^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d*h^2) + 2*(b*g*h*p*x*log(h^2*x^2*e
+ d*h^2) - b*g*h*p*x*log(h^2) - 4*b*g*h*p*x - b*f*h*p*log(h^2*x^2*e + d*h^2
) + b*f*h*p*log(h^2) + b*g*h*x*log(c) + a*g*h*x - b*f*h*log(c) - a*f*h)/(sq
rt(h*x)*h))/h
```

maple [F] time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)\left(b\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)+a\right)}{\left(hx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(3/2),x)
```

```
[Out] int((g*x+f)*(b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(3/2),x)
```

maxima [A] time = 1.08, size = 548, normalized size = 0.91

$$\frac{befp\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{hx}\sqrt{e}\right)}{2\sqrt{\sqrt{d}\sqrt{e}h}}\right)}{\sqrt{\sqrt{d}\sqrt{e}h}\sqrt{e}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{hx}\sqrt{e}\right)}{2\sqrt{\sqrt{d}\sqrt{e}h}}\right)}{\sqrt{\sqrt{d}\sqrt{e}h}\sqrt{e}}-\frac{\sqrt{2}\log\left(\sqrt{e}hx+\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{\left(dh^{2}\right)^{\frac{1}{4}}e^{\frac{3}{4}}}+\cdots\right)}{h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="maxima
")
```

```
[Out] b*e*f*p*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sq
rt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(e))
 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*
x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(e)) - sq
rt(2)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h
)/((d*h^2)^(1/4)*e^(3/4)) + sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)
*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)))/h + 2*b*g*x^2*log(
(e*x^2 + d)^p*c)/(h*x)^(3/2) + 2*a*g*x^2/(h*x)^(3/2) - 2*b*f*log((e*x^2 + d
)^p*c)/(sqrt(h*x)*h) - (8*sqrt(h*x)*h^2/e - (sqrt(2)*h^4*log(sqrt(e)*h*x +
sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)
) - sqrt(2)*h^4*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) +
 sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + 2*sqrt(2)*h^3*arctan(1/2*sqrt(2)*(sqr
t(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/
(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)) + 2*sqrt(2)*h^3*arctan(-1/2*sqrt(2)*(sqrt
(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(
sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)))*d/e)*b*e*g*p/h^4 - 2*a*f/(sqrt(h*x)*h)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(3/2),x)
[Out] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(3/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(3/2),x)
[Out] Exception raised: TypeError
```

3.608 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{5/2}}\,dx$

Optimal. Leaf size=588

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h(hx)^{3/2}}-\frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h^2\sqrt{hx}}-\frac{\sqrt{2}be^{3/4}fp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{}\right.}{3d^{3/4}h^{5/2}}$$

```
[Out] -2/3*f*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(3/2)-2/3*b*e^(3/4)*f*p*arctan(1-e^(
1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(3/4)/h^(5/2)-2*b*e^(1/
4)*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(1/4
)/h^(5/2)+2/3*b*e^(3/4)*f*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^
(1/2))*2^(1/2)/d^(3/4)/h^(5/2)+2*b*e^(1/4)*g*p*arctan(1+e^(1/4)*2^(1/2)*(h*
x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(1/4)/h^(5/2)-1/3*b*e^(3/4)*f*p*ln(d^(1
/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/
d^(3/4)/h^(5/2)+b*e^(1/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*
e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(5/2)+1/3*b*e^(3/4)*f*p*ln(d
^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/
2)/d^(3/4)/h^(5/2)-b*e^(1/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/
4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(5/2)-2*g*(a+b*ln(c*(e*x^
2+d)^p))/h^2/(h*x)^(1/2)
```

Rubi [A] time = 0.74, antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2467, 2476, 2455, 211, 1165, 628, 1162, 617, 204, 297}

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h(hx)^{3/2}}-\frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h^2\sqrt{hx}}-\frac{\sqrt{2}be^{3/4}fp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{}\right.}{3d^{3/4}h^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[2]*b*e^(3/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*S
qrt[h])]])/(3*d^(3/4)*h^(5/2)) - (2*Sqrt[2]*b*e^(1/4)*g*p*ArcTan[1 - (Sqrt[2
]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])]])/(d^(1/4)*h^(5/2)) + (2*Sqrt[2]*b*e
^(3/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])]])/(3*d^
(3/4)*h^(5/2)) + (2*Sqrt[2]*b*e^(1/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
h*x])/(d^(1/4)*Sqrt[h])]])/(d^(1/4)*h^(5/2)) - (2*f*(a + b*Log[c*(d + e*x^2)
^p]))/(3*h*(h*x)^(3/2)) - (2*g*(a + b*Log[c*(d + e*x^2)^p]))/(h^2*Sqrt[h*x]
) - (Sqrt[2]*b*e^(3/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2
]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2)) + (Sqrt[2]*b*e^(1/4)*g*p*
Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]
])/(d^(1/4)*h^(5/2)) + (Sqrt[2]*b*e^(3/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]
*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2)) - (Sqr
t[2]*b*e^(1/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4
)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(5/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
```

```
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{5/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^4}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^4}+\frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^2}\right)dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{(2g)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2}\,dx, x, \sqrt{hx}\right)}{h^2} + \frac{(2f)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d\right.\right.}{x^4}\right.}{h}$$

$$= -\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h(hx)^{3/2}} - \frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h^2\sqrt{hx}} + \frac{(8be}{}$$

$$= -\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h(hx)^{3/2}} - \frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h^2\sqrt{hx}} + \frac{(4be}{}$$

$$= -\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h(hx)^{3/2}} - \frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h^2\sqrt{hx}} - \frac{(\sqrt{2}}{}$$

$$= -\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h(hx)^{3/2}} - \frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h^2\sqrt{hx}} - \sqrt{2}\,b$$

$$= -\frac{2\sqrt{2}\,be^{3/4}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} - \frac{2\sqrt{2}\,b\sqrt[4]{e}\,gp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{}}{\sqrt[4]{d}\sqrt{h}}\right.}{\sqrt[4]{d}\,h^{5/2}}$$

Mathematica [A] time = 0.44, size = 271, normalized size = 0.46

$$\frac{2x^{5/2}\left(-\frac{f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3x^{3/2}} - \frac{g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{x}} - \frac{be^{3/4}fp\left(\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right)-\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right)+2\tan^{-1}\left(\right.\right.}{3\sqrt{2}\,d^{3/4}}\right.}{(hx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```

```
[Out] (2*x^(5/2)*((2*b*e^(1/4)*g*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + ArcTan
h[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (b*e^(3/4)*f*p*(2*ArcTan[1
 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] -
 Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(3*Sqrt[2]*d^
```

(3/4)) - (f*(a + b*Log[c*(d + e*x^2)^p]))/(3*x^(3/2)) - (g*(a + b*Log[c*(d
+ e*x^2)^p]))/Sqrt[x]))/(h*x)^(5/2)

fricas [B] time = 0.60, size = 1236, normalized size = 2.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="fricas
")
```

```
[Out] -2/3*(h^3*x^2*sqrt(-(6*b^2*e*f*g*p^2 + d*h^5*sqrt(-(b^4*e^3*f^4 - 18*b^4*d*
e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))*log(-32*(b^3*e^3*
f^4 - 81*b^3*d^2*e*g^4)*sqrt(h*x)*p^3 + 32*(3*d^3*g*h^8*sqrt(-(b^4*e^3*f^4
- 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)) + (b^2*d*e^2*f^3
 - 9*b^2*d^2*e*f*g^2)*h^3*p^2)*sqrt(-(6*b^2*e*f*g*p^2 + d*h^5*sqrt(-(b^4*e^
3*f^4 - 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5)))
 - h^3*x^2*sqrt(-(6*b^2*e*f*g*p^2 + d*h^5*sqrt(-(b^4*e^3*f^4 - 18*b^4*d*e^2
*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))*log(-32*(b^3*e^3*f^4
 - 81*b^3*d^2*e*g^4)*sqrt(h*x)*p^3 - 32*(3*d^3*g*h^8*sqrt(-(b^4*e^3*f^4 - 1
8*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)) + (b^2*d*e^2*f^3 -
9*b^2*d^2*e*f*g^2)*h^3*p^2)*sqrt(-(6*b^2*e*f*g*p^2 + d*h^5*sqrt(-(b^4*e^3*f
^4 - 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))) -
h^3*x^2*sqrt(-(6*b^2*e*f*g*p^2 - d*h^5*sqrt(-(b^4*e^3*f^4 - 18*b^4*d*e^2*f^
2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))*log(-32*(b^3*e^3*f^4 -
81*b^3*d^2*e*g^4)*sqrt(h*x)*p^3 + 32*(3*d^3*g*h^8*sqrt(-(b^4*e^3*f^4 - 18*b
^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)) - (b^2*d*e^2*f^3 - 9*b
^2*d^2*e*f*g^2)*h^3*p^2)*sqrt(-(6*b^2*e*f*g*p^2 - d*h^5*sqrt(-(b^4*e^3*f^4
- 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))) + h^3
*x^2*sqrt(-(6*b^2*e*f*g*p^2 - d*h^5*sqrt(-(b^4*e^3*f^4 - 18*b^4*d*e^2*f^2*g
^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))*log(-32*(b^3*e^3*f^4 - 81*
b^3*d^2*e*g^4)*sqrt(h*x)*p^3 - 32*(3*d^3*g*h^8*sqrt(-(b^4*e^3*f^4 - 18*b^4*
d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)) - (b^2*d*e^2*f^3 - 9*b^2*
d^2*e*f*g^2)*h^3*p^2)*sqrt(-(6*b^2*e*f*g*p^2 - d*h^5*sqrt(-(b^4*e^3*f^4 - 1
8*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))) + (3*a*g
*x + a*f + (3*b*g*p*x + b*f*p)*log(e*x^2 + d) + (3*b*g*x + b*f)*log(c))*sqr
t(h*x))/(h^3*x^2)
```

giac [A] time = 0.37, size = 443, normalized size = 0.75

$$\frac{2\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bfhpe^{\frac{11}{4}}+3\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bgpe^{\frac{9}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^2\right)^{\frac{1}{4}}}\right)e^{(-2)}}{dh}+\frac{2\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bfhpe^{\frac{11}{4}}+3\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bgpe^{\frac{9}{4}}\right)\arctan\left(\cdots\right.}{dh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*(sqrt(2)*(d*h^2)^(1/4)*b*f*h*p*e^(11/4) + 3*sqrt(2)*(d*h^2)^(3/4)*b*
g*p*e^(9/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*
x))*e^(1/4)/(d*h^2)^(1/4))*e^(-2)/(d*h) + 2*(sqrt(2)*(d*h^2)^(1/4)*b*f*h*p*
e^(11/4) + 3*sqrt(2)*(d*h^2)^(3/4)*b*g*p*e^(9/4))*arctan(-1/2*sqrt(2)*(sqrt
(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-2)/(d*
h) + (sqrt(2)*(d*h^2)^(1/4)*b*f*h*p*e^(11/4) - 3*sqrt(2)*(d*h^2)^(3/4)*b*g*
p*e^(9/4))*e^(-2)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt
(d*h^2)*e^(-1/2))/(d*h) - (sqrt(2)*(d*h^2)^(1/4)*b*f*h*p*e^(11/4) - 3*sqrt(
2)*(d*h^2)^(3/4)*b*g*p*e^(9/4))*e^(-2)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)
*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d*h) - 2*(3*b*g*h^2*p*x*log(h^2*x^
```

```
2*e + d*h^2) - 3*b*g*h^2*p*x*log(h^2) + b*f*h^2*p*log(h^2*x^2*e + d*h^2) -
b*f*h^2*p*log(h^2) + 3*b*g*h^2*x*log(c) + 3*a*g*h^2*x + b*f*h^2*log(c) + a*
f*h^2)/(sqrt(h*x)*h*x))/h^3
```

maple [F]   time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)\left(b\ln\left(c\left(ex^2+d\right)^p\right)+a\right)}{(hx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(5/2),x)
```

```
[Out] int((g*x+f)*(b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(5/2),x)
```

maxima [A]   time = 1.08, size = 524, normalized size = 0.89

$$\frac{begp\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{hx}\sqrt{e}\right)}{2\sqrt{\sqrt{d}\sqrt{e}h}}\right)}{\sqrt{\sqrt{d}\sqrt{e}h}\sqrt{e}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{hx}\sqrt{e}\right)}{2\sqrt{\sqrt{d}\sqrt{e}h}}\right)}{\sqrt{\sqrt{d}\sqrt{e}h}\sqrt{e}}-\frac{\sqrt{2}\log\left(\sqrt{e}hx+\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{\left(dh^2\right)^{\frac{1}{4}}e^{\frac{3}{4}}}+\frac{\sqrt{2}\cdots}{\cdots}\right)}{h^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="maxima
")
```

```
[Out] b*e*g*p*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sq
rt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(e))
 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*
x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(e)) - sq
rt(2)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h
)/((d*h^2)^(1/4)*e^(3/4)) + sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)
*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)))/h^2 - 2*b*g*x^2*lo
g((e*x^2 + d)^p*c)/(h*x)^(5/2) + 1/3*(sqrt(2)*h^2*log(sqrt(e)*h*x + sqrt(2)
*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) - sqr
t(2)*h^2*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d
)*h)/((d*h^2)^(3/4)*e^(1/4)) + 2*sqrt(2)*h*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h
^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqr
t(d)*sqrt(e)*h)*sqrt(d)) + 2*sqrt(2)*h*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)
^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d
)*sqrt(e)*h)*sqrt(d)))*b*e*f*p/h^3 - 2*a*g*x^2/(h*x)^(5/2) - 2/3*b*f*log((e
*x^2 + d)^p*c)/((h*x)^(3/2)*h) - 2/3*a*f/((h*x)^(3/2)*h)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)\,\left(a+b\,\ln\left(c\,\left(e\,x^2+d\right)^p\right)\right)}{(h\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(5/2),x)
```

```
[Out] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(5/2),x)
[Out] Timed out
```

3.609 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{7/2}}\,dx$

Optimal. Leaf size=620

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}}-\frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2(hx)^{3/2}}-\frac{\sqrt{2}\,be^{5/4}fp\log\left(-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\,\sqrt{h}+\sqrt{e}\,\sqrt{}\right.}{5d^{5/4}h^{7/2}}$$

```
[Out] -2/5*f*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(5/2)-2/3*g*(a+b*ln(c*(e*x^2+d)^p))/
h^2/(h*x)^(3/2)+2/5*b*e^(5/4)*f*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1
/4)/h^(1/2))*2^(1/2)/d^(5/4)/h^(7/2)-2/3*b*e^(3/4)*g*p*arctan(1-e^(1/4)*2^(
1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(3/4)/h^(7/2)-2/5*b*e^(5/4)*f*p
*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(5/4)/h^(7
/2)+2/3*b*e^(3/4)*g*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))
*2^(1/2)/d^(3/4)/h^(7/2)-1/5*b*e^(5/4)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(
1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(5/4)/h^(7/2)-1/3*b*e^(
3/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)
^(1/2))*2^(1/2)/d^(3/4)/h^(7/2)+1/5*b*e^(5/4)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1
/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(5/4)/h^(7/2)+1/
3*b*e^(3/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2
)*(h*x)^(1/2))*2^(1/2)/d^(3/4)/h^(7/2)-8/5*b*e*f*p/d/h^3/(h*x)^(1/2)
```

Rubi [A] time = 0.80, antiderivative size = 620, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2467, 2476, 2455, 325, 297, 1162, 617, 204, 1165, 628, 211}

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}}-\frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2(hx)^{3/2}}-\frac{\sqrt{2}\,be^{5/4}fp\log\left(-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\,\sqrt{h}+\sqrt{e}\,\sqrt{}\right.}{5d^{5/4}h^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

```
[Out] (-8*b*e*f*p)/(5*d*h^3*Sqrt[h*x]) + (2*Sqrt[2]*b*e^(5/4)*f*p*ArcTan[1 - (Sqr
t[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(7/2)) - (2*Sqrt[2
]*b*e^(3/4)*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/
(3*d^(3/4)*h^(7/2)) - (2*Sqrt[2]*b*e^(5/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*
Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(7/2)) + (2*Sqrt[2]*b*e^(3/4)*g
*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)*h^
(7/2)) - (2*f*(a + b*Log[c*(d + e*x^2)^p]))/(5*h*(h*x)^(5/2)) - (2*g*(a + b
*Log[c*(d + e*x^2)^p]))/(3*h^2*(h*x)^(3/2)) - (Sqrt[2]*b*e^(5/4)*f*p*Log[Sq
rt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*
d^(5/4)*h^(7/2)) - (Sqrt[2]*b*e^(3/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqr
t[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(7/2)) + (Sqrt[2]
*b*e^(5/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^
(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(7/2)) + (Sqrt[2]*b*e^(3/4)*g*p*Log[Sqrt[d]*
Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4
)*h^(7/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
```

```
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{7/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^6}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^6} + \frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^4}\right) dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{(2g)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^4}\,dx, x, \sqrt{hx}\right)}{h^2} + \frac{(2f)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d\right.\right.}{x^6}\right.}{h}$$

$$= -\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}} - \frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2(hx)^{3/2}} + \frac{(8be}{}$$

$$= -\frac{8befp}{5dh^3\sqrt{hx}} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}} - \frac{2g\left(a+b\log\left(c\left(d+ex\right.\right.\right.}{3h^2(hx)^{3/2}}$$

$$= -\frac{8befp}{5dh^3\sqrt{hx}} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}} - \frac{2g\left(a+b\log\left(c\left(d+ex\right.\right.\right.}{3h^2(hx)^{3/2}}$$

$$= -\frac{8befp}{5dh^3\sqrt{hx}} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}} - \frac{2g\left(a+b\log\left(c\left(d+ex\right.\right.\right.}{3h^2(hx)^{3/2}}$$

$$= -\frac{8befp}{5dh^3\sqrt{hx}} - \frac{2\sqrt{2}\,be^{3/4}gp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{3d^{3/4}h^{7/2}} + \frac{2\sqrt{2}\,be^{3/4}gp\tan^{-1}}{3d^{3/4}}$$

$$= -\frac{8befp}{5dh^3\sqrt{hx}} + \frac{2\sqrt{2}\,be^{5/4}fp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{5d^{5/4}h^{7/2}} - \frac{2\sqrt{2}\,be^{3/4}gp\tan^{-1}}{3d^{3/4}}$$

Mathematica [C] time = 0.19, size = 309, normalized size = 0.50

$$\frac{2x^{7/2}\left(-\frac{f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5x^{5/2}}-\frac{g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3x^{3/2}}-\frac{1}{6}bgp\left(\frac{\frac{\sqrt{2}e^{3/4}\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right)}{\sqrt[4]{d}}-\frac{\sqrt{2}e^{3/4}\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right)}{\sqrt[4]{d}}}{\sqrt{d}}\right.\right.}{(hx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

```
[Out] (2*x^(7/2)*((-4*b*e*f*p*Hypergeometric2F1[-1/4, 1, 3/4, -((e*x^2)/d)])/(5*d
*Sqrt[x]) - (b*g*p*((2*((Sqrt[2]*e^(3/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x
])/d^(1/4)])/d^(1/4) - (Sqrt[2]*e^(3/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x]
)/d^(1/4)])/d^(1/4)))/Sqrt[d] + ((Sqrt[2]*e^(3/4)*Log[Sqrt[d] - Sqrt[2]*d^(
1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/d^(1/4) - (Sqrt[2]*e^(3/4)*Log[Sqrt[d] +
 Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/d^(1/4))/Sqrt[d]))/6 - (f*(a
 + b*Log[c*(d + e*x^2)^p]))/(5*x^(5/2)) - (g*(a + b*Log[c*(d + e*x^2)^p]))/
(3*x^(3/2))))/(h*x)^(7/2)
```

fricas [B] time = 1.33, size = 1348, normalized size = 2.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="fricas
")
```

```
[Out] 2/15*(d*h^4*x^3*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2
 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) + 30*b^2*e^2*f*g*p^2)/(d^2*h^7))*lo
g(-32*(81*b^3*e^4*f^4 - 625*b^3*d^2*e^2*g^4)*sqrt(h*x)*p^3 + 32*(3*d^4*f*h^
11*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4
/(d^5*h^14)) - 5*(9*b^2*d^2*e^2*f^2*g - 25*b^2*d^3*e*g^3)*h^4*p^2)*sqrt((d^
2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*
p^4/(d^5*h^14)) + 30*b^2*e^2*f*g*p^2)/(d^2*h^7))) - d*h^4*x^3*sqrt((d^2*h^7
*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(
d^5*h^14)) + 30*b^2*e^2*f*g*p^2)/(d^2*h^7))*log(-32*(81*b^3*e^4*f^4 - 625*b
^3*d^2*e^2*g^4)*sqrt(h*x)*p^3 - 32*(3*d^4*f*h^11*sqrt(-(81*b^4*e^5*f^4 - 45
0*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) - 5*(9*b^2*d^2*e
^2*f^2*g - 25*b^2*d^3*e*g^3)*h^4*p^2)*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f^4 -
 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) + 30*b^2*e^2*
f*g*p^2)/(d^2*h^7))) - d*h^4*x^3*sqrt(-(d^2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450
*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) - 30*b^2*e^2*f*g*
p^2)/(d^2*h^7))*log(-32*(81*b^3*e^4*f^4 - 625*b^3*d^2*e^2*g^4)*sqrt(h*x)*p^
3 + 32*(3*d^4*f*h^11*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^
4*d^2*e^3*g^4)*p^4/(d^5*h^14)) + 5*(9*b^2*d^2*e^2*f^2*g - 25*b^2*d^3*e*g^3)
*h^4*p^2)*sqrt(-(d^2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 62
5*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) - 30*b^2*e^2*f*g*p^2)/(d^2*h^7))) + d*h^
4*x^3*sqrt(-(d^2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^
4*d^2*e^3*g^4)*p^4/(d^5*h^14)) - 30*b^2*e^2*f*g*p^2)/(d^2*h^7))*log(-32*(81
*b^3*e^4*f^4 - 625*b^3*d^2*e^2*g^4)*sqrt(h*x)*p^3 - 32*(3*d^4*f*h^11*sqrt(-
(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^1
4)) + 5*(9*b^2*d^2*e^2*f^2*g - 25*b^2*d^3*e*g^3)*h^4*p^2)*sqrt(-(d^2*h^7*sq
rt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5
*h^14)) - 30*b^2*e^2*f*g*p^2)/(d^2*h^7))) - (12*b*e*f*p*x^2 + 5*a*d*g*x + 3
*a*d*f + (5*b*d*g*p*x + 3*b*d*f*p)*log(e*x^2 + d) + (5*b*d*g*x + 3*b*d*f)*l
og(c))*sqrt(h*x))/(d*h^4*x^3)
```

giac [A] time = 0.41, size = 478, normalized size = 0.77

$$\frac{2\left(5\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}bdghpe^{\frac{7}{4}}-3\sqrt{2}\left(dh^{2}\right)^{\frac{3}{4}}bfpe^{\frac{9}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^{2}\right)^{\frac{1}{4}}}\right)e^{(-1)}}{d^{2}h}+\frac{2\left(5\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}bdghpe^{\frac{7}{4}}-3\sqrt{2}\left(dh^{2}\right)^{\frac{3}{4}}bfpe^{\frac{9}{4}}\right)\arctan\left(-\cdots\right.}{d^{2}h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(2*(5*sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*p*e^(7/4) - 3*sqrt(2)*(d*h^2)^(3/4
)*b*f*p*e^(9/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqr
t(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-1)/(d^2*h) + 2*(5*sqrt(2)*(d*h^2)^(1/4)*
b*d*g*h*p*e^(7/4) - 3*sqrt(2)*(d*h^2)^(3/4)*b*f*p*e^(9/4))*arctan(-1/2*sqrt
(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e
^(-1)/(d^2*h) + (5*sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*p*e^(7/4) + 3*sqrt(2)*(d*h
^2)^(3/4)*b*f*p*e^(9/4))*e^(-1)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4
) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h) - (5*sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*
p*e^(7/4) + 3*sqrt(2)*(d*h^2)^(3/4)*b*f*p*e^(9/4))*e^(-1)*log(-sqrt(2)*(d*h
^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h) - 2*(12*
b*f*h^3*p*x^2*e + 5*b*d*g*h^3*p*x*log(h^2*x^2*e + d*h^2) - 5*b*d*g*h^3*p*x*
log(h^2) + 3*b*d*f*h^3*p*log(h^2*x^2*e + d*h^2) - 3*b*d*f*h^3*p*log(h^2) +
5*b*d*g*h^3*x*log(c) + 5*a*d*g*h^3*x + 3*b*d*f*h^3*log(c) + 3*a*d*f*h^3)/(s
qrt(h*x)*d*h^2*x^2))/h^4
```

maple [F] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)\left(b\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)+a\right)}{\left(hx\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(7/2),x)
```

```
[Out] int((g*x+f)*(b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(7/2),x)
```

maxima [A] time = 1.08, size = 541, normalized size = 0.87

$$-\frac{befp\left(\frac{e\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{hx}\sqrt{e}\right)}{2\sqrt{\sqrt{d}\sqrt{e}h}}\right)}{\sqrt{\sqrt{d}\sqrt{e}h}\sqrt{e}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{hx}\sqrt{e}\right)}{2\sqrt{\sqrt{d}\sqrt{e}h}}\right)}{\sqrt{\sqrt{d}\sqrt{e}h}\sqrt{e}}-\frac{\sqrt{2}\log\left(\sqrt{e}hx+\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{\left(dh^{2}\right)^{\frac{1}{4}}e^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{e}hx-\cdots\right.}{\cdots}\right)}{d}\cdots\right)}{5h^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="maxima
")
```

```
[Out] -1/5*b*e*f*p*(e*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4
) + 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*
sqrt(e)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2
*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(
e)) - sqrt(2)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + s
qrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) + sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^
2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)))/d + 8/(sqr
t(h*x)*d))/h^3 - 2/3*b*g*x^2*log((e*x^2 + d)^p*c)/(h*x)^(7/2) + 1/3*(sqrt(2
)*h^2*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h
)/((d*h^2)^(3/4)*e^(1/4)) - sqrt(2)*h^2*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(
1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + 2*sqrt(2)*h*a
rctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e))/sqr
t(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)) + 2*sqrt(2)*h*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/sqrt(s
qrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)))*b*e*g*p/h^4 - 2/3*a*g
*x^2/(h*x)^(7/2) - 2/5*b*f*log((e*x^2 + d)^p*c)/((h*x)^(5/2)*h) - 2/5*a*f/(
(h*x)^(5/2)*h)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(7/2),x)
```

```
[Out] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(7/2),x)
```

```
[Out] Timed out
```

3.610 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{9/2}}\,dx$

Optimal. Leaf size=641

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}}-\frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h^2(hx)^{5/2}}+\frac{\sqrt{2}\,be^{7/4}fp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{\ldots}\right.}{7d^{7/4}h^{9/2}}$$

```
[Out] -8/21*b*e*f*p/d/h^3/(h*x)^(3/2)-2/7*f*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(7/2)
-2/5*g*(a+b*ln(c*(e*x^2+d)^p))/h^2/(h*x)^(5/2)+2/7*b*e^(7/4)*f*p*arctan(1-e
^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(7/4)/h^(9/2)+2/5*b*e
^(5/4)*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^
(5/4)/h^(9/2)-2/7*b*e^(7/4)*f*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4
)/h^(1/2))*2^(1/2)/d^(7/4)/h^(9/2)-2/5*b*e^(5/4)*g*p*arctan(1+e^(1/4)*2^(1/
2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(5/4)/h^(9/2)+1/7*b*e^(7/4)*f*p*l
n(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^
(1/2)/d^(7/4)/h^(9/2)-1/5*b*e^(5/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2
)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(5/4)/h^(9/2)-1/7*b*e^(7/4
)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1
/2))*2^(1/2)/d^(7/4)/h^(9/2)+1/5*b*e^(5/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)
*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(5/4)/h^(9/2)-8/5*b
*e*g*p/d/h^4/(h*x)^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 641, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2467, 2476, 2455, 325, 211, 1165, 628, 1162, 617, 204, 297}

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}}-\frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h^2(hx)^{5/2}}+\frac{\sqrt{2}\,be^{7/4}fp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{\ldots}\right.}{7d^{7/4}h^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```

```
[Out] (-8*b*e*f*p)/(21*d*h^3*(h*x)^(3/2)) - (8*b*e*g*p)/(5*d*h^4*Sqrt[h*x]) + (2*
Sqrt[2]*b*e^(7/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[
h])])/(7*d^(7/4)*h^(9/2)) + (2*Sqrt[2]*b*e^(5/4)*g*p*ArcTan[1 - (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(
7/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(7*d^(7
/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(5/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*
x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(9/2)) - (2*f*(a + b*Log[c*(d + e*x^2)
^p]))/(7*h*(h*x)^(7/2)) - (2*g*(a + b*Log[c*(d + e*x^2)^p]))/(5*h^2*(h*x)^(
5/2)) + (Sqrt[2]*b*e^(7/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sq
rt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2)) - (Sqrt[2]*b*e^(5/4)*
g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(5*d^(5/4)*h^(9/2)) - (Sqrt[2]*b*e^(7/4)*f*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2))
+ (Sqrt[2]*b*e^(5/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*
d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(9/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
```

```
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{9/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^8}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^8} + \frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^6}\right) dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{(2g)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^6}\,dx, x, \sqrt{hx}\right)}{h^2} + \frac{(2f)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d\right.\right.\right.}{x^8}\right.}{h}$$

$$= -\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h^2(hx)^{5/2}} + \frac{(8be\ldots}{\ldots}$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g\left(a+\ldots\right.}{\ldots}$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g\left(a+\ldots\right.}{\ldots}$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g\left(a+\ldots\right.}{\ldots}$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} - \frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g\left(a+\ldots\right.}{\ldots}$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} + \frac{2\sqrt{2}\,be^{7/4}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}} + \frac{2\ldots}{\ldots}$$

Mathematica [C] time = 0.07, size = 100, normalized size = 0.16

$$-\frac{2\sqrt{hx}\left(3d(5f+7gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)+20befpx^2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{ex^2}{d}\right)+84begpx^3\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{ex^2}{d}\right)\right)}{105dh^5x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```

```
[Out] (-2*Sqrt[h*x]*(20*b*e*f*p*x^2*Hypergeometric2F1[-3/4, 1, 1/4, -((e*x^2)/d)]
 + 84*b*e*g*p*x^3*Hypergeometric2F1[-1/4, 1, 3/4, -((e*x^2)/d)] + 3*d*(5*f
+ 7*g*x)*(a + b*Log[c*(d + e*x^2)^p])))/(105*d*h^5*x^4)
```

fricas [B] time = 1.27, size = 1369, normalized size = 2.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="fricas
")
```

```
[Out] 2/105*(3*d*h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f
^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) + 70*b^2*e^3*f*g*p^2)/(d^3*h
^9))*log(-32*(625*b^3*e^6*f^4 - 2401*b^3*d^2*e^4*g^4)*sqrt(h*x)*p^3 + 32*(7
*d^6*g*h^14*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*
e^5*g^4)*p^4/(d^7*h^18)) + 5*(25*b^2*d^2*e^4*f^3 - 49*b^2*d^3*e^3*f*g^2)*h^
5*p^2)*sqrt(-(d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 240
1*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) + 70*b^2*e^3*f*g*p^2)/(d^3*h^9))) - 3*d*
h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 24
01*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) + 70*b^2*e^3*f*g*p^2)/(d^3*h^9))*log(-3
2*(625*b^3*e^6*f^4 - 2401*b^3*d^2*e^4*g^4)*sqrt(h*x)*p^3 - 32*(7*d^6*g*h^14
*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^
4/(d^7*h^18)) + 5*(25*b^2*d^2*e^4*f^3 - 49*b^2*d^3*e^3*f*g^2)*h^5*p^2)*sqrt
(-(d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e
^5*g^4)*p^4/(d^7*h^18)) + 70*b^2*e^3*f*g*p^2)/(d^3*h^9))) - 3*d*h^5*x^4*sqr
t((d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e
^5*g^4)*p^4/(d^7*h^18)) - 70*b^2*e^3*f*g*p^2)/(d^3*h^9))*log(-32*(625*b^3*e
^6*f^4 - 2401*b^3*d^2*e^4*g^4)*sqrt(h*x)*p^3 + 32*(7*d^6*g*h^14*sqrt(-(625*
b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)
) - 5*(25*b^2*d^2*e^4*f^3 - 49*b^2*d^3*e^3*f*g^2)*h^5*p^2)*sqrt((d^3*h^9*sq
rt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(
d^7*h^18)) - 70*b^2*e^3*f*g*p^2)/(d^3*h^9))) + 3*d*h^5*x^4*sqrt((d^3*h^9*sq
rt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(
d^7*h^18)) - 70*b^2*e^3*f*g*p^2)/(d^3*h^9))*log(-32*(625*b^3*e^6*f^4 - 2401
*b^3*d^2*e^4*g^4)*sqrt(h*x)*p^3 - 32*(7*d^6*g*h^14*sqrt(-(625*b^4*e^7*f^4 -
 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) - 5*(25*b^2
*d^2*e^4*f^3 - 49*b^2*d^3*e^3*f*g^2)*h^5*p^2)*sqrt((d^3*h^9*sqrt(-(625*b^4*
e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) -
70*b^2*e^3*f*g*p^2)/(d^3*h^9))) - (84*b*e*g*p*x^3 + 20*b*e*f*p*x^2 + 21*a*d
*g*x + 15*a*d*f + 3*(7*b*d*g*p*x + 5*b*d*f*p)*log(e*x^2 + d) + 3*(7*b*d*g*x
 + 5*b*d*f)*log(c))*sqrt(h*x))/(d*h^5*x^4)
```

giac [A] time = 0.38, size = 488, normalized size = 0.76

$$-\frac{6\left(5\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bfhpe^{\frac{11}{4}}+7\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bgpe^{\frac{9}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^2\right)^{\frac{1}{4}}}\right)e^{(-1)}}{d^2h}+\frac{6\left(5\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bfhpe^{\frac{11}{4}}+7\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bgpe^{\frac{9}{4}}\right)\text{arcta}}{d^2h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(6*(5*sqrt(2)*(d*h^2)^(1/4)*b*f*h*p*e^(11/4) + 7*sqrt(2)*(d*h^2)^(3/
4)*b*g*p*e^(9/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sq
rt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-1)/(d^2*h) + 6*(5*sqrt(2)*(d*h^2)^(1/4)
*b*f*h*p*e^(11/4) + 7*sqrt(2)*(d*h^2)^(3/4)*b*g*p*e^(9/4))*arctan(-1/2*sqrt
(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e
^(-1)/(d^2*h) + 3*(5*sqrt(2)*(d*h^2)^(1/4)*b*f*h*p*e^(11/4) - 7*sqrt(2)*(d*
h^2)^(3/4)*b*g*p*e^(9/4))*e^(-1)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/
4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h) - 3*(5*sqrt(2)*(d*h^2)^(1/4)*b*f*h
*p*e^(11/4) - 7*sqrt(2)*(d*h^2)^(3/4)*b*g*p*e^(9/4))*e^(-1)*log(-sqrt(2)*(d
*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h) + 2*(8
4*b*g*h^4*p*x^3*e + 20*b*f*h^4*p*x^2*e + 21*b*d*g*h^4*p*x*log(h^2*x^2*e + d
*h^2) - 21*b*d*g*h^4*p*x*log(h^2) + 15*b*d*f*h^4*p*log(h^2*x^2*e + d*h^2) -
 15*b*d*f*h^4*p*log(h^2) + 21*b*d*g*h^4*x*log(c) + 21*a*d*g*h^4*x + 15*b*d*
f*h^4*log(c) + 15*a*d*f*h^4)/(sqrt(h*x)*d*h^3*x^3))/h^5
```

maple [F] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)\left(b\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)+a\right)}{\left(hx\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(9/2),x)
```

```
[Out] int((g*x+f)*(b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(9/2),x)
```

maxima [A] time = 1.08, size = 557, normalized size = 0.87

$$-\frac{bef p\left(\frac{3\left(\frac{\sqrt{2}\,e^{\frac{3}{4}}\log\left(\sqrt{e}\,hx+\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}\sqrt{hx}\,e^{\frac{1}{4}}+\sqrt{d}\,h\right)}{\left(dh^{2}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\,e^{\frac{3}{4}}\log\left(\sqrt{e}\,hx-\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}\sqrt{hx}\,e^{\frac{1}{4}}+\sqrt{d}\,h\right)}{\left(dh^{2}\right)^{\frac{3}{4}}}+\frac{2\sqrt{2}\,e\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{hx}\sqrt{e}\right)}{2\sqrt{\sqrt{d}\sqrt{e}\,h}}\right)}{\sqrt{\sqrt{d}\sqrt{e}\,h}\,\sqrt{d}\,h}+\frac{2\sqrt{2}\,e\arctan\left(-\cdots\right.}{\cdots}\right.}{d}\cdots}{21\,h^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="maxima
")
```

```
[Out] -1/21*b*e*f*p*(3*(sqrt(2)*e^(3/4)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*s
qrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) - sqrt(2)*e^(3/4)*log(sqrt(e)*h
*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) + 2
*sqrt(2)*e*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*
sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)*h) + 2*s
qrt(2)*e*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*s
```

```
qrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)*h))/d + 8
/((h*x)^(3/2)*d))/h^3 - 1/5*b*e*g*p*(e*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(
2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(s
qrt(sqrt(d)*sqrt(e)*h)*sqrt(e)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d
*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(s
qrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4
)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) + sqrt(2)*log(sqrt
(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/
4)*e^(3/4)))/d + 8/(sqrt(h*x)*d))/h^4 - 2/5*b*g*x^2*log((e*x^2 + d)^p*c)/(h
*x)^(9/2) - 2/5*a*g*x^2/(h*x)^(9/2) - 2/7*b*f*log((e*x^2 + d)^p*c)/((h*x)^(
7/2)*h) - 2/7*a*f/((h*x)^(7/2)*h)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(9/2),x)
[Out] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(9/2),x)
[Out] Timed out
```

3.611 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}\,dx$

Optimal. Leaf size=1002

$$-\frac{8bg^2p(hx)^{5/2}}{25h^3}+\frac{2g^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)(hx)^{5/2}}{5h^3}-\frac{16bfgp(hx)^{3/2}}{9h^2}+\frac{4fg\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)(hx)^{3/2}}{3h^2}-8bf$$

```
[Out] -16/9*b*f*g*p*(h*x)^(3/2)/h^2-8/25*b*g^2*p*(h*x)^(5/2)/h^3+4/3*f*g*(h*x)^(3
/2)*(a+b*ln(c*(e*x^2+d)^p))/h^2+2/5*g^2*(h*x)^(5/2)*(a+b*ln(c*(e*x^2+d)^p))
/h^3-2*b*d^(1/4)*f^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2)
)*2^(1/2)/e^(1/4)/h^(1/2)-4/3*b*d^(3/4)*f*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x
)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(3/4)/h^(1/2)+2/5*b*d^(5/4)*g^2*p*arctan
(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(5/4)/h^(1/2)+2*b
*d^(1/4)*f^2*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2
)/e^(1/4)/h^(1/2)+4/3*b*d^(3/4)*f*g*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/
d^(1/4)/h^(1/2))*2^(1/2)/e^(3/4)/h^(1/2)-2/5*b*d^(5/4)*g^2*p*arctan(1+e^(1/
4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(5/4)/h^(1/2)-b*d^(1/4)*f
^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/
2))*2^(1/2)/e^(1/4)/h^(1/2)+2/3*b*d^(3/4)*f*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2
)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(3/4)/h^(1/2)+1/5*
b*d^(5/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2
)*(h*x)^(1/2))*2^(1/2)/e^(5/4)/h^(1/2)+b*d^(1/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x
*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(1/
2)-2/3*b*d^(3/4)*f*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)
*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(3/4)/h^(1/2)-1/5*b*d^(5/4)*g^2*p*ln(d^(1/2
)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^
(5/4)/h^(1/2)+2*a*f^2*(h*x)^(1/2)/h-8*b*f^2*p*(h*x)^(1/2)/h+8/5*b*d*g^2*p*(
h*x)^(1/2)/e/h+2*b*f^2*ln(c*(e*x^2+d)^p)*(h*x)^(1/2)/h
```

Rubi [A] time = 1.31, antiderivative size = 1002, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules used = {2467, 2471, 2448, 321, 211, 1165, 628, 1162, 617, 204, 2455, 297, 302}

$$-\frac{8bg^2p(hx)^{5/2}}{25h^3}+\frac{2g^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)(hx)^{5/2}}{5h^3}-\frac{16bfgp(hx)^{3/2}}{9h^2}+\frac{4fg\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)(hx)^{3/2}}{3h^2}-8bf$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]
```

```
[Out] (2*a*f^2*Sqrt[h*x])/h - (8*b*f^2*p*Sqrt[h*x])/h + (8*b*d*g^2*p*Sqrt[h*x])/(
5*e*h) - (16*b*f*g*p*(h*x)^(3/2))/(9*h^2) - (8*b*g^2*p*(h*x)^(5/2))/(25*h^3
) - (2*Sqrt[2]*b*d^(1/4)*f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1
/4)*Sqrt[h])])/(e^(1/4)*Sqrt[h]) - (4*Sqrt[2]*b*d^(3/4)*f*g*p*ArcTan[1 - (S
qrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*Sqrt[h]) + (2*Sqrt
[2]*b*d^(5/4)*g^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h]
)])/(5*e^(5/4)*Sqrt[h]) + (2*Sqrt[2]*b*d^(1/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*Sqrt[h]) + (4*Sqrt[2]*b*d^(3/
4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3
/4)*Sqrt[h]) - (2*Sqrt[2]*b*d^(5/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
h*x])/(d^(1/4)*Sqrt[h])])/(5*e^(5/4)*Sqrt[h]) + (2*b*f^2*Sqrt[h*x]*Log[c*(d
 + e*x^2)^p])/h + (4*f*g*(h*x)^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^2)
+ (2*g^2*(h*x)^(5/2)*(a + b*Log[c*(d + e*x^2)^p]))/(5*h^3) - (Sqrt[2]*b*d^(
1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) + (2*Sqrt[2]*b*d^(3/4)*f*g*p*Log[Sqrt[d]*Sq
```

```
rt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*
Sqrt[h]) + (Sqrt[2]*b*d^(5/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x
 - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*e^(5/4)*Sqrt[h]) + (Sqrt[2]*b*d^(
1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) - (2*Sqrt[2]*b*d^(3/4)*f*g*p*Log[Sqrt[d]*Sq
rt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*
Sqrt[h]) - (Sqrt[2]*b*d^(5/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x
 + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*e^(5/4)*Sqrt[h])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}\,dx = \frac{2\,\text{Subst}\left(\int\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx,x,\sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int\left(f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)+\frac{2fgx^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h}\right.\right.}{h}$$

$$= \frac{\left(2g^2\right)\text{Subst}\left(\int x^4\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx,x,\sqrt{hx}\right)}{h^3} + \frac{(4fg)}{}$$

$$= \frac{2af^2\sqrt{hx}}{h} + \frac{4fg(hx)^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2} + \frac{2g^2(hx)^{5/2}\left(a\right.}{}$$

$$= \frac{2af^2\sqrt{hx}}{h} - \frac{16bfgp(hx)^{3/2}}{9h^2} + \frac{2bf^2\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h} + \frac{4fg}{}$$

$$= \frac{2af^2\sqrt{hx}}{h} - \frac{8bf^2p\sqrt{hx}}{h} + \frac{8bdg^2p\sqrt{hx}}{5eh} - \frac{16bfgp(hx)^{3/2}}{9h^2} - \frac{8bg^2p}{25}$$

$$= \frac{2af^2\sqrt{hx}}{h} - \frac{8bf^2p\sqrt{hx}}{h} + \frac{8bdg^2p\sqrt{hx}}{5eh} - \frac{16bfgp(hx)^{3/2}}{9h^2} - \frac{8bg^2p}{25}$$

$$= \frac{2af^2\sqrt{hx}}{h} - \frac{8bf^2p\sqrt{hx}}{h} + \frac{8bdg^2p\sqrt{hx}}{5eh} - \frac{16bfgp(hx)^{3/2}}{9h^2} - \frac{8bg^2p}{25}$$

$$= \frac{2af^2\sqrt{hx}}{h} - \frac{8bf^2p\sqrt{hx}}{h} + \frac{8bdg^2p\sqrt{hx}}{5eh} - \frac{16bfgp(hx)^{3/2}}{9h^2} - \frac{8bg^2p}{25}$$

$$= \frac{2af^2\sqrt{hx}}{h} - \frac{8bf^2p\sqrt{hx}}{h} + \frac{8bdg^2p\sqrt{hx}}{5eh} - \frac{16bfgp(hx)^{3/2}}{9h^2} - \frac{8bg^2p}{25}$$

Mathematica [A] time = 1.48, size = 588, normalized size = 0.59

$$\frac{2\sqrt{x}\left(\frac{2}{3}fgx^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)+\frac{1}{5}g^2x^{5/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)+af^2\sqrt{x}+bf^2\sqrt{x}\log\left(c\left(d+ex\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]
```

```
[Out] (2*Sqrt[x]*(a*f^2*Sqrt[x] - (4*b*f*g*p*(2*(-d)^(1/4)*e^(3/4)*x^(3/2) - 3*d*
ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + 3*d*ArcTanh[(e^(1/4)*Sqrt[x])/(-d)^(
1/4)]))/(9*(-d)^(1/4)*e^(3/4)) - (b*f^2*p*(8*e^(1/4)*Sqrt[x] + 2*Sqrt[2]*d^
```

```
(1/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*Sqrt[2]*d^(1/4)*Arc
Tan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Sqrt[2]*d^(1/4)*Log[Sqrt[d] -
Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Sqrt[2]*d^(1/4)*Log[Sqrt[d]
+ Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(2*e^(1/4)) - (b*g^2*p*(-4
0*d*e^(1/4)*Sqrt[x] + 8*e^(5/4)*x^(5/2) - 10*Sqrt[2]*d^(5/4)*ArcTan[1 - (Sq
rt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + 10*Sqrt[2]*d^(5/4)*ArcTan[1 + (Sqrt[2]*e^
(1/4)*Sqrt[x])/d^(1/4)] - 5*Sqrt[2]*d^(5/4)*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e
^(1/4)*Sqrt[x] + Sqrt[e]*x] + 5*Sqrt[2]*d^(5/4)*Log[Sqrt[d] + Sqrt[2]*d^(1/
4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(50*e^(5/4)) + b*f^2*Sqrt[x]*Log[c*(d + e
*x^2)^p] + (2*f*g*x^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/3 + (g^2*x^(5/2)*(a
 + b*Log[c*(d + e*x^2)^p]))/5))/Sqrt[h*x]
```

<u>**fricas**</u> **[B]** time = 1.04, size = 2178, normalized size = 2.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="fric
as")
```

```
[Out] 2/225*(15*e*h*sqrt(-(e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f
^6*g^2 + 40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^
8)*p^4/(e^5*h^2)) + 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))*log(
16*(50625*b^3*e^4*f^8 - 40500*b^3*d*e^3*f^6*g^2 + 2150*b^3*d^2*e^2*f^4*g^4
- 1620*b^3*d^3*e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 + 16*(10*e^4*f*g*h
^2*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 40150*b^4*d^3*e
^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*h^2)) + 3*(1
125*b^2*e^4*f^6 - 1175*b^2*d*e^3*f^4*g^2 + 235*b^2*d^2*e^2*f^2*g^4 - 9*b^2*
d^3*e*g^6)*h*p^2)*sqrt(-(e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e
^3*f^6*g^2 + 40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^
5*g^8)*p^4/(e^5*h^2)) + 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h)))
 - 15*e*h*sqrt(-(e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g
^2 + 40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p
^4/(e^5*h^2)) + 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))*log(16*(
50625*b^3*e^4*f^8 - 40500*b^3*d*e^3*f^6*g^2 + 2150*b^3*d^2*e^2*f^4*g^4 - 16
20*b^3*d^3*e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 - 16*(10*e^4*f*g*h^2*s
qrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 40150*b^4*d^3*e^2*f
^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*h^2)) + 3*(1125*
b^2*e^4*f^6 - 1175*b^2*d*e^3*f^4*g^2 + 235*b^2*d^2*e^2*f^2*g^4 - 9*b^2*d^3*
e*g^6)*h*p^2)*sqrt(-(e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f
^6*g^2 + 40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^
8)*p^4/(e^5*h^2)) + 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))) - 1
5*e*h*sqrt((e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 +
40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e
^5*h^2)) - 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))*log(16*(50625
*b^3*e^4*f^8 - 40500*b^3*d*e^3*f^6*g^2 + 2150*b^3*d^2*e^2*f^4*g^4 - 1620*b^
3*d^3*e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 + 16*(10*e^4*f*g*h^2*sqrt(-
(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 40150*b^4*d^3*e^2*f^4*g^
4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*h^2)) - 3*(1125*b^2*e
^4*f^6 - 1175*b^2*d*e^3*f^4*g^2 + 235*b^2*d^2*e^2*f^2*g^4 - 9*b^2*d^3*e*g^6
)*h*p^2)*sqrt((e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2
 + 40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4
/(e^5*h^2)) - 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))) + 15*e*h*
sqrt((e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 40150*
b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*h^2
)) - 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))*log(16*(50625*b^3*e
^4*f^8 - 40500*b^3*d*e^3*f^6*g^2 + 2150*b^3*d^2*e^2*f^4*g^4 - 1620*b^3*d^3*
e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 - 16*(10*e^4*f*g*h^2*sqrt(-(50625
*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 40150*b^4*d^3*e^2*f^4*g^4 - 34
20*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*h^2)) - 3*(1125*b^2*e^4*f^6
```

```
 - 1175*b^2*d*e^3*f^4*g^2 + 235*b^2*d^2*e^2*f^2*g^4 - 9*b^2*d^3*e*g^6)*h*p^
2)*sqrt((e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 401
50*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*
h^2)) - 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))) + (225*a*e*f^2
- 9*(4*b*e*g^2*p - 5*a*e*g^2)*x^2 - 180*(5*b*e*f^2 - b*d*g^2)*p - 50*(4*b*e
*f*g*p - 3*a*e*f*g)*x + 15*(3*b*e*g^2*p*x^2 + 10*b*e*f*g*p*x + 15*b*e*f^2*p
)*log(e*x^2 + d) + 15*(3*b*e*g^2*x^2 + 10*b*e*f*g*x + 15*b*e*f^2)*log(c))*s
qrt(h*x))/(e*h)
```

giac [A] time = 0.41, size = 820, normalized size = 0.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="giac
")
```

```
[Out] 1/225*(90*sqrt(h*x)*b*g^2*x^2*log(c) + 90*sqrt(h*x)*a*g^2*x^2 + 300*sqrt(h*
x)*b*f*g*x*log(c) + 225*((2*sqrt(2)*(d*h^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(
2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-5/4) +
2*sqrt(2)*(d*h^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4)
 - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-5/4) + sqrt(2)*(d*h^2)^(1/4)*e^(
-5/4)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-
1/2)) - sqrt(2)*(d*h^2)^(1/4)*e^(-5/4)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)
*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2)) - 8*sqrt(h*x)*e^(-1))*e + 2*sqrt(h*
x)*log(x^2*e + d))*b*f^2*p + 9*(10*sqrt(h*x)*x^2*log(x^2*e + d) - (10*sqrt(
2)*(d*h^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*s
qrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-9/4) + 10*sqrt(2)*(d*h^2)^(1/4)*d*arct
an(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h
^2)^(1/4))*e^(-9/4) + 5*sqrt(2)*(d*h^2)^(1/4)*d*e^(-9/4)*log(sqrt(2)*(d*h^2
)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2)) - 5*sqrt(2)*(d*h^2
)^(1/4)*d*e^(-9/4)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sq
rt(d*h^2)*e^(-1/2)) + 8*(sqrt(h*x)*h^10*x^2*e^4 - 5*sqrt(h*x)*d*h^10*e^3)*e
^(-5)/h^10)*e)*b*g^2*p + 300*sqrt(h*x)*a*f*g*x + 450*sqrt(h*x)*b*f^2*log(c)
 + 50*(6*sqrt(h*x)*h*x*log(x^2*e + d) - (8*sqrt(h*x)*h*x*e^(-1) - 6*sqrt(2)
*(d*h^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(
h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-7/4) - 6*sqrt(2)*(d*h^2)^(3/4)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/
4))*e^(-7/4) + 3*sqrt(2)*(d*h^2)^(3/4)*e^(-7/4)*log(sqrt(2)*(d*h^2)^(1/4)*s
qrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2)) - 3*sqrt(2)*(d*h^2)^(3/4)*e
^(-7/4)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e
^(-1/2)))*e)*b*f*g*p/h + 450*sqrt(h*x)*a*f^2)/h
```

maple [F] time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^{2} \left(b\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)+a\right)}{\sqrt{hx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(1/2),x)
```

```
[Out] int((g*x+f)^2*(b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(1/2),x)
```

maxima [A] time = 1.15, size = 893, normalized size = 0.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 2/5*b*g^2*x^3*log((e*x^2 + d)^p*c)/sqrt(h*x) + 2/5*a*g^2*x^3/sqrt(h*x) + 4/
3*b*f*g*x^2*log((e*x^2 + d)^p*c)/sqrt(h*x) + 4/3*a*f*g*x^2/sqrt(h*x) + 2*sq
rt(h*x)*b*f^2*log((e*x^2 + d)^p*c)/h - (8*sqrt(h*x)*h^2/e - (sqrt(2)*h^4*lo
g(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^
2)^(3/4)*e^(1/4)) - sqrt(2)*h^4*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqr
t(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + 2*sqrt(2)*h^3*arctan(
1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt
(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)) + 2*sqrt(2)*h^3*arctan(-1
/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(
d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)))*d/e)*b*e*f^2*p/h^3 + 2*sq
rt(h*x)*a*f^2/h + 2/9*(3*d*h^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^
2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt
(d)*sqrt(e)*h)*sqrt(e)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1
/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*s
qrt(e)*h)*sqrt(e)) - sqrt(2)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h
*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) + sqrt(2)*log(sqrt(e)*h*x
- sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/
4)))/e - 8*(h*x)^(3/2)*h^2/e)*b*e*f*g*p/h^4 - 1/25*b*(5*(sqrt(2)*h^6*log(sq
rt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(
3/4)*e^(1/4)) - sqrt(2)*h^6*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*
x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + 2*sqrt(2)*h^5*arctan(1/2*
sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*
sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)) + 2*sqrt(2)*h^5*arctan(-1/2*s
qrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*s
qrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)))*d^2/e^2 + 8*((h*x)^(5/2)*e*h^
2 - 5*sqrt(h*x)*d*h^4)/e^2)*e*g^2*p/h^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^2\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{\sqrt{h\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(1/2),x)
```

```
[Out] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(1/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

3.612 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{3/2}}\,dx$

Optimal. Leaf size=949

$$-\frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{\sqrt[4]{d}\,h^{3/2}}+\frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{\sqrt[4]{d}\,h^{3/2}}-\frac{2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f^2}{h\sqrt{hx}}+\ldots$$

```
[Out] -8/9*b*g^2*p*(h*x)^(3/2)/h^3+2/3*g^2*(h*x)^(3/2)*(a+b*ln(c*(e*x^2+d)^p))/h^
3-2*b*e^(1/4)*f^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2
^(1/2)/d^(1/4)/h^(3/2)-4*b*d^(1/4)*f*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/
2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)-2/3*b*d^(3/4)*g^2*p*arctan(1-e^
(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(3/4)/h^(3/2)+2*b*e^(1
/4)*f^2*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(
1/4)/h^(3/2)+4*b*d^(1/4)*f*g*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)
/h^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)+2/3*b*d^(3/4)*g^2*p*arctan(1+e^(1/4)*2^(1
/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(3/4)/h^(3/2)+b*e^(1/4)*f^2*p*ln
(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(
1/2)/d^(1/4)/h^(3/2)-2*b*d^(1/4)*f*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)
-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)+1/3*b*d^(3/4)
*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(
1/2))*2^(1/2)/e^(3/4)/h^(3/2)-b*e^(1/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*
h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(3/2)+2*b*d^
(1/4)*f*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h
*x)^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)-1/3*b*d^(3/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x
*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(3/4)/h^(3/
2)-2*f^2*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(1/2)+4*a*f*g*(h*x)^(1/2)/h^2-16*b
*f*g*p*(h*x)^(1/2)/h^2+4*b*f*g*ln(c*(e*x^2+d)^p)*(h*x)^(1/2)/h^2
```

Rubi [A] time = 1.26, antiderivative size = 949, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {2467, 2476, 2448, 321, 211, 1165, 628, 1162, 617, 204, 2455, 297}

$$-\frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{\sqrt[4]{d}\,h^{3/2}}+\frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{\sqrt[4]{d}\,h^{3/2}}-\frac{2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f^2}{h\sqrt{hx}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

```
[Out] (4*a*f*g*Sqrt[h*x])/h^2 - (16*b*f*g*p*Sqrt[h*x])/h^2 - (8*b*g^2*p*(h*x)^(3/
2))/(9*h^3) - (2*Sqrt[2]*b*e^(1/4)*f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h
*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(3/2)) - (4*Sqrt[2]*b*d^(1/4)*f*g*p*Arc
Tan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*h^(3/2)) -
 (2*Sqrt[2]*b*d^(3/4)*g^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)
*Sqrt[h])])/(3*e^(3/4)*h^(3/2)) + (2*Sqrt[2]*b*e^(1/4)*f^2*p*ArcTan[1 + (Sq
rt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(3/2)) + (4*Sqrt[2]
*b*d^(1/4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])
/(e^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*d^(3/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)
*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*h^(3/2)) + (4*b*f*g*Sqrt[h*x]*Lo
g[c*(d + e*x^2)^p])/h^2 - (2*f^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*Sqrt[h*x]
) + (2*g^2*(h*x)^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^3) + (Sqrt[2]*b*e
^(1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1
/4)*Sqrt[h*x]])/(d^(1/4)*h^(3/2)) - (2*Sqrt[2]*b*d^(1/4)*f*g*p*Log[Sqrt[d]*
Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*
h^(3/2)) + (Sqrt[2]*b*d^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x
```

```
 - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*h^(3/2)) - (Sqrt[2]*b*e^(
1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(d^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*d^(1/4)*f*g*p*Log[Sqrt[d]*Sq
rt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*h^
(3/2)) - (Sqrt[2]*b*d^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x +
 Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*h^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{3/2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^2}dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{2fg\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h} + \frac{f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^2} + \frac{g^2x^2\left(a+b\log\left(\right.\right.}{h^2}\right.\right.}{h}$$

$$= \frac{(2g^2)\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx, x, \sqrt{hx}\right)}{h^3} + \frac{(4fg)\,\text{Su}}{}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h\sqrt{hx}} + \frac{2g^2(hx)^{3/2}\left(a+b\log\left(\right.\right.}{3h^3}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} + \frac{4bfg\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{h^2} - \frac{2f^2\left(a+\right.}{}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} + \frac{4bfg\sqrt{hx}\,\log\left(c\left(d+ex\right.\right.}{h^2}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} + \frac{4bfg\sqrt{hx}\,\log\left(c\left(d+ex\right.\right.}{h^2}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} - \frac{2\sqrt{2}\,b\sqrt[4]{e}\,f^2p\tan^{-1}\left(1-\right.}{\sqrt[4]{d}\,h^{3/2}}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} - \frac{2\sqrt{2}\,b\sqrt[4]{e}\,f^2p\tan^{-1}\left(1-\right.}{\sqrt[4]{d}\,h^{3/2}}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} - \frac{2\sqrt{2}\,b\sqrt[4]{e}\,f^2p\tan^{-1}\left(1-\right.}{\sqrt[4]{d}\,h^{3/2}}$$

Mathematica [A] time = 0.87, size = 436, normalized size = 0.46

$$\frac{2x^{3/2}\left(-\frac{f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{x}} + \frac{1}{3}g^2x^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right) + 2afg\sqrt{x} + 2bfg\sqrt{x}\,\log\left(c\left(d+ex^2\right)^p\right) - \frac{2bg^2p\left(2\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

```
[Out] (2*x^(3/2)*(2*a*f*g*Sqrt[x] - (2*b*g^2*p*(2*(-d)^(1/4)*e^(3/4)*x^(3/2) - 3*
d*ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + 3*d*ArcTanh[(e^(1/4)*Sqrt[x])/(-d)
^(1/4)]))/(9*(-d)^(1/4)*e^(3/4)) + (2*b*e^(1/4)*f^2*p*(ArcTan[(e^(1/4)*Sqrt
[x])/(-d)^(1/4)] + ArcTanh[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (
b*f*g*p*(8*e^(1/4)*Sqrt[x] + 2*Sqrt[2]*d^(1/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*
Sqrt[x])/d^(1/4)] - 2*Sqrt[2]*d^(1/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/
d^(1/4)] + Sqrt[2]*d^(1/4)*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] +
Sqrt[e]*x] - Sqrt[2]*d^(1/4)*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x]
+ Sqrt[e]*x]))/e^(1/4) + 2*b*f*g*Sqrt[x]*Log[c*(d + e*x^2)^p] - (f^2*(a + b
*Log[c*(d + e*x^2)^p]))/Sqrt[x] + (g^2*x^(3/2)*(a + b*Log[c*(d + e*x^2)^p])
)/3))/(h*x)^(3/2)
```

fricas [B] time = 1.79, size = 2118, normalized size = 2.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="fric
as")
```

```
[Out] -2/9*(3*h^2*x*sqrt(-(e*h^3*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 +
918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^
6)) + 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)*p^2)/(e*h^3))*log(32*(81*b^3*e^4*f^8
 + 108*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 12*b^3*d^3*e*f^2*g^6
+ b^3*d^4*g^8)*sqrt(h*x)*p^3 + 32*((3*d*e^3*f^2 + d^2*e^2*g^2)*h^5*sqrt(-(8
1*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^
3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) - 6*(9*b^2*d*e^3*f^5*g - 30*b^2
*d^2*e^2*f^3*g^3 + b^2*d^3*e*f*g^5)*h^2*p^2)*sqrt(-(e*h^3*sqrt(-(81*b^4*e^4
*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e*f^2*g
^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) + 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)*p^2)/
(e*h^3))) - 3*h^2*x*sqrt(-(e*h^3*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*
g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*
e^3*h^6)) + 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)*p^2)/(e*h^3))*log(32*(81*b^3*e
^4*f^8 + 108*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 12*b^3*d^3*e*f^
2*g^6 + b^3*d^4*g^8)*sqrt(h*x)*p^3 - 32*((3*d*e^3*f^2 + d^2*e^2*g^2)*h^5*sq
rt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*
b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) - 6*(9*b^2*d*e^3*f^5*g -
30*b^2*d^2*e^2*f^3*g^3 + b^2*d^3*e*f*g^5)*h^2*p^2)*sqrt(-(e*h^3*sqrt(-(81*b
^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e
*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) + 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)
*p^2)/(e*h^3))) - 3*h^2*x*sqrt((e*h^3*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3
*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^
4/(d*e^3*h^6)) - 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)*p^2)/(e*h^3))*log(32*(81*
b^3*e^4*f^8 + 108*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 12*b^3*d^3
*e*f^2*g^6 + b^3*d^4*g^8)*sqrt(h*x)*p^3 + 32*((3*d*e^3*f^2 + d^2*e^2*g^2)*h
^5*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4
- 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) + 6*(9*b^2*d*e^3*f^5
*g - 30*b^2*d^2*e^2*f^3*g^3 + b^2*d^3*e*f*g^5)*h^2*p^2)*sqrt((e*h^3*sqrt(-(
81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d
^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) - 12*(3*b^2*e*f^3*g + b^2*d*f*
g^3)*p^2)/(e*h^3))) + 3*h^2*x*sqrt((e*h^3*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d
*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8
)*p^4/(d*e^3*h^6)) - 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)*p^2)/(e*h^3))*log(32*
(81*b^3*e^4*f^8 + 108*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 12*b^3
*d^3*e*f^2*g^6 + b^3*d^4*g^8)*sqrt(h*x)*p^3 - 32*((3*d*e^3*f^2 + d^2*e^2*g^
2)*h^5*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*
g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) + 6*(9*b^2*d*e^3
*f^5*g - 30*b^2*d^2*e^2*f^3*g^3 + b^2*d^3*e*f*g^5)*h^2*p^2)*sqrt((e*h^3*sqr
t(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b
^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) - 12*(3*b^2*e*f^3*g + b^2*
```

```
d*f*g^3)*p^2)/(e*h^3))) + (9*a*f^2 + (4*b*g^2*p - 3*a*g^2)*x^2 + 18*(4*b*f*
g*p - a*f*g)*x - 3*(b*g^2*p*x^2 + 6*b*f*g*p*x - 3*b*f^2*p)*log(e*x^2 + d) -
 3*(b*g^2*x^2 + 6*b*f*g*x - 3*b*f^2)*log(c))*sqrt(h*x))/(h^2*x)
```

giac [A] time = 0.58, size = 649, normalized size = 0.68

$$\frac{\frac{6\left(6\,\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bdfghpe^{\frac{11}{4}}+\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bdg^2pe^{\frac{9}{4}}+3\,\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bf^2pe^{\frac{13}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\,\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^2\right)^{\frac{1}{4}}}\right)e^{(-3)}}{dh^2}+\frac{6\left(6\,\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bdfghpe^{\frac{11}{4}}+\sqrt{2}\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="giac
")
```

```
[Out] 1/9*(6*(6*sqrt(2)*(d*h^2)^(1/4)*b*d*f*g*h*p*e^(11/4) + sqrt(2)*(d*h^2)^(3/4
)*b*d*g^2*p*e^(9/4) + 3*sqrt(2)*(d*h^2)^(3/4)*b*f^2*p*e^(13/4))*arctan(1/2*
sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4
))*e^(-3)/(d*h^2) + 6*(6*sqrt(2)*(d*h^2)^(1/4)*b*d*f*g*h*p*e^(11/4) + sqrt(
2)*(d*h^2)^(3/4)*b*d*g^2*p*e^(9/4) + 3*sqrt(2)*(d*h^2)^(3/4)*b*f^2*p*e^(13/
4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1
/4)/(d*h^2)^(1/4))*e^(-3)/(d*h^2) + 3*(6*sqrt(2)*(d*h^2)^(1/4)*b*d*f*g*h*p*
e^(11/4) - sqrt(2)*(d*h^2)^(3/4)*b*d*g^2*p*e^(9/4) - 3*sqrt(2)*(d*h^2)^(3/4
)*b*f^2*p*e^(13/4))*e^(-3)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h
*x + sqrt(d*h^2)*e^(-1/2))/(d*h^2) - 3*(6*sqrt(2)*(d*h^2)^(1/4)*b*d*f*g*h*p
*e^(11/4) - sqrt(2)*(d*h^2)^(3/4)*b*d*g^2*p*e^(9/4) - 3*sqrt(2)*(d*h^2)^(3/
4)*b*f^2*p*e^(13/4))*e^(-3)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) +
 h*x + sqrt(d*h^2)*e^(-1/2))/(d*h^2) + 2*(3*b*g^2*h^2*p*x^2*log(h^2*x^2*e +
 d*h^2) - 3*b*g^2*h^2*p*x^2*log(h^2) - 4*b*g^2*h^2*p*x^2 + 18*b*f*g*h^2*p*x
*log(h^2*x^2*e + d*h^2) - 18*b*f*g*h^2*p*x*log(h^2) + 3*b*g^2*h^2*x^2*log(c
) - 72*b*f*g*h^2*p*x + 3*a*g^2*h^2*x^2 - 9*b*f^2*h^2*p*log(h^2*x^2*e + d*h^
2) + 9*b*f^2*h^2*p*log(h^2) + 18*b*f*g*h^2*x*log(c) + 18*a*f*g*h^2*x - 9*b*
f^2*h^2*log(c) - 9*a*f^2*h^2)/(sqrt(h*x)*h^2))/h
```

maple [F] time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^{2}\left(b\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)+a\right)}{\left(hx\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(3/2),x)
```

```
[Out] int((g*x+f)^2*(b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(3/2),x)
```

maxima [A] time = 1.15, size = 844, normalized size = 0.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] 2/3*b*g^2*x^3*log((e*x^2 + d)^p*c)/(h*x)^(3/2) + b*e*f^2*p*(2*sqrt(2)*arcta
n(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e))/sqrt(sq
rt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(e)) + 2*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)
```

```
*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(sqrt(e)*h*x +
sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)
) + sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqr
t(d)*h)/((d*h^2)^(1/4)*e^(3/4)))/h + 2/3*a*g^2*x^3/(h*x)^(3/2) + 4*b*f*g*x^
2*log((e*x^2 + d)^p*c)/(h*x)^(3/2) + 4*a*f*g*x^2/(h*x)^(3/2) - 2*b*f^2*log(
(e*x^2 + d)^p*c)/(sqrt(h*x)*h) - 2*(8*sqrt(h*x)*h^2/e - (sqrt(2)*h^4*log(sq
rt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(
3/4)*e^(1/4)) - sqrt(2)*h^4*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*
x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + 2*sqrt(2)*h^3*arctan(1/2*
sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*
sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)) + 2*sqrt(2)*h^3*arctan(-1/2*s
qrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*s
qrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)))*d/e)*b*e*f*g*p/h^4 - 2*a*f^2/
(sqrt(h*x)*h) + 1/9*(3*d*h^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)
^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d
)*sqrt(e)*h)*sqrt(e)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4
)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqr
t(e)*h)*sqrt(e)) - sqrt(2)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x
)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) + sqrt(2)*log(sqrt(e)*h*x -
sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)
))/e - 8*(h*x)^(3/2)*h^2/e)*b*e*g^2*p/h^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f+g\,x\right)}^2\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(3/2),x)
[Out] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(3/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(3/2),x)
[Out] Exception raised: TypeError
```

3.613 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{5/2}}\,dx$

Optimal. Leaf size=932

$$-\frac{2\sqrt{2}\,be^{3/4}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{3d^{3/4}h^{5/2}}+\frac{2\sqrt{2}\,be^{3/4}p\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{3d^{3/4}h^{5/2}}-\frac{2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f^2}{3h(hx)^{3/2}}-\frac{\sqrt{2}\,b}{}$$

```
[Out] -2/3*f^2*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(3/2)-2/3*b*e^(3/4)*f^2*p*arctan(1
-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(3/4)/h^(5/2)-4*b*e
^(1/4)*f*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/
d^(1/4)/h^(5/2)-2*b*d^(1/4)*g^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1
/4)/h^(1/2))*2^(1/2)/e^(1/4)/h^(5/2)+2/3*b*e^(3/4)*f^2*p*arctan(1+e^(1/4)*2
^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(3/4)/h^(5/2)+4*b*e^(1/4)*f*g
*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(1/4)/h^
(5/2)+2*b*d^(1/4)*g^2*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2
))*2^(1/2)/e^(1/4)/h^(5/2)-1/3*b*e^(3/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)
*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(3/4)/h^(5/2)+2*b*e
^(1/4)*f*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(
h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(5/2)-b*d^(1/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^
(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(5/2)+
1/3*b*e^(3/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^
(1/2)*(h*x)^(1/2))*2^(1/2)/d^(3/4)/h^(5/2)-2*b*e^(1/4)*f*g*p*ln(d^(1/2)*h^(
1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(1/4)
/h^(5/2)+b*d^(1/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/
4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(5/2)-4*f*g*(a+b*ln(c*(e*x^2+d)^p
))/h^2/(h*x)^(1/2)+2*a*g^2*(h*x)^(1/2)/h^3-8*b*g^2*p*(h*x)^(1/2)/h^3+2*b*g^
2*ln(c*(e*x^2+d)^p)*(h*x)^(1/2)/h^3
```

Rubi [A] time = 1.22, antiderivative size = 932, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {2467, 2476, 2448, 321, 211, 1165, 628, 1162, 617, 204, 2455, 297}

$$-\frac{2\sqrt{2}\,be^{3/4}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{3d^{3/4}h^{5/2}}+\frac{2\sqrt{2}\,be^{3/4}p\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{3d^{3/4}h^{5/2}}-\frac{2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f^2}{3h(hx)^{3/2}}-\frac{\sqrt{2}\,b}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```

```
[Out] (2*a*g^2*Sqrt[h*x])/h^3 - (8*b*g^2*p*Sqrt[h*x])/h^3 - (2*Sqrt[2]*b*e^(3/4)*
f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)
*h^(5/2)) - (4*Sqrt[2]*b*e^(1/4)*f*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x
])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(5/2)) - (2*Sqrt[2]*b*d^(1/4)*g^2*p*ArcTa
n[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*h^(5/2)) + (
2*Sqrt[2]*b*e^(3/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*S
qrt[h])])/(3*d^(3/4)*h^(5/2)) + (4*Sqrt[2]*b*e^(1/4)*f*g*p*ArcTan[1 + (Sqrt
[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(5/2)) + (2*Sqrt[2]*b
*d^(1/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(
e^(1/4)*h^(5/2)) + (2*b*g^2*Sqrt[h*x]*Log[c*(d + e*x^2)^p])/h^3 - (2*f^2*(a
 + b*Log[c*(d + e*x^2)^p]))/(3*h*(h*x)^(3/2)) - (4*f*g*(a + b*Log[c*(d + e*
x^2)^p]))/(h^2*Sqrt[h*x]) - (Sqrt[2]*b*e^(3/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] +
Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2))
 + (2*Sqrt[2]*b*e^(1/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqr
t[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(5/2)) - (Sqrt[2]*b*d^(1/4)*g^2
*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h
```

```
*x]])/(e^(1/4)*h^(5/2)) + (Sqrt[2]*b*e^(3/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sq
rt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2)) -
 (2*Sqrt[2]*b*e^(1/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[
2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(5/2)) + (Sqrt[2]*b*d^(1/4)*g^2*p
*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x
]])/(e^(1/4)*h^(5/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{5/2}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^4}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \left(\frac{g^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h^2} + \frac{f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^4} + \frac{2fg\left(a+b\log\right.}{}\right.\right.}{h}$$

$$= \frac{\left(2g^2\right)\mathrm{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx, x, \sqrt{hx}\right)}{h^3} + \frac{(4fg)\,\mathrm{Su}}{}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3} - \frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h(hx)^{3/2}} - \frac{4fg\left(a+b\log\left(c\left(d\right.\right.\right.}{h^2\sqrt{hx}}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3} + \frac{2bg^2\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{h^3} - \frac{2f^2\left(a+b\log\left(c\left(d+\right.\right.\right.}{3h(hx)^{3/2}}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3} - \frac{8bg^2p\sqrt{hx}}{h^3} + \frac{2bg^2\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{h^3} - \frac{2f^2\left(a\right.}{}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3} - \frac{8bg^2p\sqrt{hx}}{h^3} + \frac{2bg^2\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{h^3} - \frac{2f^2\left(a\right.}{}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3} - \frac{8bg^2p\sqrt{hx}}{h^3} - \frac{2\sqrt{2}\,be^{3/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} - $$

$$= \frac{2ag^2\sqrt{hx}}{h^3} - \frac{8bg^2p\sqrt{hx}}{h^3} - \frac{2\sqrt{2}\,be^{3/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} - $$

$$= \frac{2ag^2\sqrt{hx}}{h^3} - \frac{8bg^2p\sqrt{hx}}{h^3} - \frac{2\sqrt{2}\,be^{3/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} - $$

Mathematica [A] time = 0.98, size = 503, normalized size = 0.54

$$2x^{5/2}\left(-\frac{f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3x^{3/2}} - \frac{2fg\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{x}} + ag^2\sqrt{x} + bg^2\sqrt{x}\,\log\left(c\left(d+ex^2\right)^p\right) - \frac{be^{3/4}f^2p\left(\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```

```
[Out] (2*x^(5/2)*(a*g^2*Sqrt[x] + (4*b*e^(1/4)*f*g*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-
d)^(1/4)] + ArcTanh[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (b*e^(3/
```

```
4)*f^2*p*(2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*ArcTan[1 + (S
qrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sq
rt[x] + Sqrt[e]*x] - Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e
]*x]))/(3*Sqrt[2]*d^(3/4)) - (b*g^2*p*(8*e^(1/4)*Sqrt[x] + 2*Sqrt[2]*d^(1/4
)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*Sqrt[2]*d^(1/4)*ArcTan[
1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Sqrt[2]*d^(1/4)*Log[Sqrt[d] - Sqrt
[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Sqrt[2]*d^(1/4)*Log[Sqrt[d] + Sq
rt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(2*e^(1/4)) + b*g^2*Sqrt[x]*Lo
g[c*(d + e*x^2)^p] - (f^2*(a + b*Log[c*(d + e*x^2)^p]))/(3*x^(3/2)) - (2*f*
g*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[x]))/(h*x)^(5/2)
```

fricas [B] time = 1.75, size = 2112, normalized size = 2.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="fric
as")
```

```
[Out] 2/3*(h^3*x^2*sqrt(-(d*h^5*sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b
^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^1
0)) + 12*(b^2*e*f^3*g + 3*b^2*d*f*g^3)*p^2)/(d*h^5))*log(16*(b^3*e^4*f^8 +
12*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 108*b^3*d^3*e*f^2*g^6 + 8
1*b^3*d^4*g^8)*sqrt(h*x)*p^3 + 16*(6*d^3*e*f*g*h^8*sqrt(-(b^4*e^4*f^8 - 60*
b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6 + 81*b^
4*d^4*g^8)*p^4/(d^3*e*h^10)) + (b^2*d*e^3*f^6 - 27*b^2*d^2*e^2*f^4*g^2 - 81
*b^2*d^3*e*f^2*g^4 + 27*b^2*d^4*g^6)*h^3*p^2)*sqrt(-(d*h^5*sqrt(-(b^4*e^4*f
^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6
 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) + 12*(b^2*e*f^3*g + 3*b^2*d*f*g^3)*p^2
)/(d*h^5))) - h^3*x^2*sqrt(-(d*h^5*sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^
2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(
d^3*e*h^10)) + 12*(b^2*e*f^3*g + 3*b^2*d*f*g^3)*p^2)/(d*h^5))*log(16*(b^3*e
^4*f^8 + 12*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 108*b^3*d^3*e*f^
2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 - 16*(6*d^3*e*f*g*h^8*sqrt(-(b^4*e^4*
f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^
6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) + (b^2*d*e^3*f^6 - 27*b^2*d^2*e^2*f^4
*g^2 - 81*b^2*d^3*e*f^2*g^4 + 27*b^2*d^4*g^6)*h^3*p^2)*sqrt(-(d*h^5*sqrt(-(
b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*
e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) + 12*(b^2*e*f^3*g + 3*b^2*d*f
*g^3)*p^2)/(d*h^5))) - h^3*x^2*sqrt((d*h^5*sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^
3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^
8)*p^4/(d^3*e*h^10)) - 12*(b^2*e*f^3*g + 3*b^2*d*f*g^3)*p^2)/(d*h^5))*log(1
6*(b^3*e^4*f^8 + 12*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 108*b^3*
d^3*e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 + 16*(6*d^3*e*f*g*h^8*sqrt(-(
b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*
e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) - (b^2*d*e^3*f^6 - 27*b^2*d^2
*e^2*f^4*g^2 - 81*b^2*d^3*e*f^2*g^4 + 27*b^2*d^4*g^6)*h^3*p^2)*sqrt((d*h^5*
sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b
^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) - 12*(b^2*e*f^3*g + 3*
b^2*d*f*g^3)*p^2)/(d*h^5))) + h^3*x^2*sqrt((d*h^5*sqrt(-(b^4*e^4*f^8 - 60*b
^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6 + 81*b^4
*d^4*g^8)*p^4/(d^3*e*h^10)) - 12*(b^2*e*f^3*g + 3*b^2*d*f*g^3)*p^2)/(d*h^5)
)*log(16*(b^3*e^4*f^8 + 12*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 1
08*b^3*d^3*e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 - 16*(6*d^3*e*f*g*h^8*
sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b
^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) - (b^2*d*e^3*f^6 - 27*
b^2*d^2*e^2*f^4*g^2 - 81*b^2*d^3*e*f^2*g^4 + 27*b^2*d^4*g^6)*h^3*p^2)*sqrt(
(d*h^5*sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4
- 540*b^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) - 12*(b^2*e*f^3
*g + 3*b^2*d*f*g^3)*p^2)/(d*h^5))) - (6*a*f*g*x + a*f^2 + 3*(4*b*g^2*p - a*
```

```
g^2)*x^2 - (3*b*g^2*p*x^2 - 6*b*f*g*p*x - b*f^2*p)*log(e*x^2 + d) - (3*b*g^
2*x^2 - 6*b*f*g*x - b*f^2)*log(c))*sqrt(h*x))/(h^3*x^2)
```

giac [A] time = 0.49, size = 638, normalized size = 0.68

$$\frac{\frac{2\left(3\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bdg^2hpe^{\frac{7}{4}}+\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bf^2hpe^{\frac{11}{4}}+6\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bfgpe^{\frac{9}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^2\right)^{\frac{1}{4}}}\right)e^{(-2)}}{dh}+\frac{2\left(3\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bdg^2hpe^{\frac{7}{4}}+\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="giac
")
```

```
[Out] 1/3*(2*(3*sqrt(2)*(d*h^2)^(1/4)*b*d*g^2*h*p*e^(7/4) + sqrt(2)*(d*h^2)^(1/4)
*b*f^2*h*p*e^(11/4) + 6*sqrt(2)*(d*h^2)^(3/4)*b*f*g*p*e^(9/4))*arctan(1/2*s
qrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4)
)*e^(-2)/(d*h) + 2*(3*sqrt(2)*(d*h^2)^(1/4)*b*d*g^2*h*p*e^(7/4) + sqrt(2)*(
d*h^2)^(1/4)*b*f^2*h*p*e^(11/4) + 6*sqrt(2)*(d*h^2)^(3/4)*b*f*g*p*e^(9/4))*
arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/
(d*h^2)^(1/4))*e^(-2)/(d*h) + (3*sqrt(2)*(d*h^2)^(1/4)*b*d*g^2*h*p*e^(7/4)
+ sqrt(2)*(d*h^2)^(1/4)*b*f^2*h*p*e^(11/4) - 6*sqrt(2)*(d*h^2)^(3/4)*b*f*g*
p*e^(9/4))*e^(-2)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt
(d*h^2)*e^(-1/2))/(d*h) - (3*sqrt(2)*(d*h^2)^(1/4)*b*d*g^2*h*p*e^(7/4) + sq
rt(2)*(d*h^2)^(1/4)*b*f^2*h*p*e^(11/4) - 6*sqrt(2)*(d*h^2)^(3/4)*b*f*g*p*e^
(9/4))*e^(-2)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*
h^2)*e^(-1/2))/(d*h) + 2*(3*b*g^2*h^2*p*x^2*log(h^2*x^2*e + d*h^2) - 3*b*g^
2*h^2*p*x^2*log(h^2) - 12*b*g^2*h^2*p*x^2 - 6*b*f*g*h^2*p*x*log(h^2*x^2*e +
 d*h^2) + 6*b*f*g*h^2*p*x*log(h^2) + 3*b*g^2*h^2*x^2*log(c) + 3*a*g^2*h^2*x
^2 - b*f^2*h^2*p*log(h^2*x^2*e + d*h^2) + b*f^2*h^2*p*log(h^2) - 6*b*f*g*h^
2*x*log(c) - 6*a*f*g*h^2*x - b*f^2*h^2*log(c) - a*f^2*h^2)/(sqrt(h*x)*h*x))
/h^3
```

maple [F] time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^{2}\left(b\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)+a\right)}{\left(hx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(5/2),x)
```

```
[Out] int((g*x+f)^2*(b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(5/2),x)
```

maxima [A] time = 1.15, size = 830, normalized size = 0.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] 2*b*g^2*x^3*log((e*x^2 + d)^p*c)/(h*x)^(5/2) + 2*b*e*f*g*p*(2*sqrt(2)*arcta
n(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e))/sqrt(sq
rt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(e)) + 2*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)
*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(sqrt(e)*h*x +
```

```
sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)
) + sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqr
t(d)*h)/((d*h^2)^(1/4)*e^(3/4)))/h^2 + 2*a*g^2*x^3/(h*x)^(5/2) - 4*b*f*g*x^
2*log((e*x^2 + d)^p*c)/(h*x)^(5/2) + 1/3*(sqrt(2)*h^2*log(sqrt(e)*h*x + sqr
t(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) -
 sqrt(2)*h^2*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sq
rt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + 2*sqrt(2)*h*arctan(1/2*sqrt(2)*(sqrt(2)*
(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt
(sqrt(d)*sqrt(e)*h)*sqrt(d)) + 2*sqrt(2)*h*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*
h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sq
rt(d)*sqrt(e)*h)*sqrt(d)))*b*e*f^2*p/h^3 - 4*a*f*g*x^2/(h*x)^(5/2) - 2/3*b*
f^2*log((e*x^2 + d)^p*c)/((h*x)^(3/2)*h) - (8*sqrt(h*x)*h^2/e - (sqrt(2)*h^
4*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((
d*h^2)^(3/4)*e^(1/4)) - sqrt(2)*h^4*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)
*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + 2*sqrt(2)*h^3*arc
tan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e))/sqrt(
sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)) + 2*sqrt(2)*h^3*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/sqrt(s
qrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)))*d/e)*b*e*g^2*p/h^5 -
2/3*a*f^2/((h*x)^(3/2)*h)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^2\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(5/2),x)
[Out] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(5/2),x)
[Out] Timed out
```

3.614 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{7/2}}\,dx$

Optimal. Leaf size=935

$$\frac{2\sqrt{2}\,be^{5/4}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{5d^{5/4}h^{7/2}}-\frac{2\sqrt{2}\,be^{5/4}p\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{5d^{5/4}h^{7/2}}-\frac{2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f^2}{5h(hx)^{5/2}}-\sqrt{2}$$

```
[Out] -2/5*f^2*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(5/2)-4/3*f*g*(a+b*ln(c*(e*x^2+d)^
p))/h^2/(h*x)^(3/2)+2/5*b*e^(5/4)*f^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2
)/d^(1/4)/h^(1/2))*2^(1/2)/d^(5/4)/h^(7/2)-4/3*b*e^(3/4)*f*g*p*arctan(1-e^(
1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(3/4)/h^(7/2)-2*b*e^(1/
4)*g^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(1
/4)/h^(7/2)-2/5*b*e^(5/4)*f^2*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4
)/h^(1/2))*2^(1/2)/d^(5/4)/h^(7/2)+4/3*b*e^(3/4)*f*g*p*arctan(1+e^(1/4)*2^(
1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(3/4)/h^(7/2)+2*b*e^(1/4)*g^2*p
*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(1/4)/h^(7
/2)-1/5*b*e^(5/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4
)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(5/4)/h^(7/2)-2/3*b*e^(3/4)*f*g*p*ln(d^(1/
2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d
^(3/4)/h^(7/2)+b*e^(1/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)
*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(7/2)+1/5*b*e^(5/4)*f^2*p*l
n(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^
(1/2)/d^(5/4)/h^(7/2)+2/3*b*e^(3/4)*f*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1
/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(3/4)/h^(7/2)-b*e^(1/4)*
g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1
/2))*2^(1/2)/d^(1/4)/h^(7/2)-8/5*b*e*f^2*p/d/h^3/(h*x)^(1/2)-2*g^2*(a+b*ln(
c*(e*x^2+d)^p))/h^3/(h*x)^(1/2)
```

Rubi [A] time = 1.18, antiderivative size = 935, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2467, 2476, 2455, 325, 297, 1162, 617, 204, 1165, 628, 211}

$$\frac{2\sqrt{2}\,be^{5/4}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{5d^{5/4}h^{7/2}}-\frac{2\sqrt{2}\,be^{5/4}p\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{5d^{5/4}h^{7/2}}-\frac{2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f^2}{5h(hx)^{5/2}}-\sqrt{2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

```
[Out] (-8*b*e*f^2*p)/(5*d*h^3*Sqrt[h*x]) + (2*Sqrt[2]*b*e^(5/4)*f^2*p*ArcTan[1 -
(Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(7/2)) - (4*Sq
rt[2]*b*e^(3/4)*f*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[
h])])/(3*d^(3/4)*h^(7/2)) - (2*Sqrt[2]*b*e^(1/4)*g^2*p*ArcTan[1 - (Sqrt[2]*
e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(7/2)) - (2*Sqrt[2]*b*e^(
5/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^
(5/4)*h^(7/2)) + (4*Sqrt[2]*b*e^(3/4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqr
t[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)*h^(7/2)) + (2*Sqrt[2]*b*e^(1/4)*g^2*
p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(7/
2)) - (2*f^2*(a + b*Log[c*(d + e*x^2)^p]))/(5*h*(h*x)^(5/2)) - (4*f*g*(a +
b*Log[c*(d + e*x^2)^p]))/(3*h^2*(h*x)^(3/2)) - (2*g^2*(a + b*Log[c*(d + e*x
^2)^p]))/(h^3*Sqrt[h*x]) - (Sqrt[2]*b*e^(5/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(7/2))
- (2*Sqrt[2]*b*e^(3/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt
[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(7/2)) + (Sqrt[2]*b*e^(1/4)*g^
2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
```

```
h*x]])/(d^(1/4)*h^(7/2)) + (Sqrt[2]*b*e^(5/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(7/2))
+ (2*Sqrt[2]*b*e^(3/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt
[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(7/2)) - (Sqrt[2]*b*e^(1/4)*g^
2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(d^(1/4)*h^(7/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{7/2}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^6}dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \left(\frac{f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^6} + \frac{2fg\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^4} + \frac{g^2\left(a+b\log\left(c\left(\right.\right.\right.}{h^2x^2}\right.\right.}{h}$$

$$= \frac{\left(2g^2\right)\mathrm{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2}dx, x, \sqrt{hx}\right)}{h^3} + \frac{(4fg)\,\mathrm{Subst}\left(\int \frac{a+b\log}{\ }\right.}{h}$$

$$= -\frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}} - \frac{4fg\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2(hx)^{3/2}} - \frac{2}{\ }$$

$$= -\frac{8bef^2p}{5dh^3\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}} - \frac{4fg\left(a+b\log\left(c\left(d+\right.\right.\right.}{3h^2(hx)^{3/2}}$$

$$= -\frac{8bef^2p}{5dh^3\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}} - \frac{4fg\left(a+b\log\left(c\left(d+\right.\right.\right.}{3h^2(hx)^{3/2}}$$

$$= -\frac{8bef^2p}{5dh^3\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}} - \frac{4fg\left(a+b\log\left(c\left(d+\right.\right.\right.}{3h^2(hx)^{3/2}}$$

$$= -\frac{8bef^2p}{5dh^3\sqrt{hx}} - \frac{4\sqrt{2}be^{3/4}fgp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}} - \frac{2\sqrt{2}b\sqrt[4]{e}g^2p\tan}{\sqrt[4]{\ }}$$

$$= -\frac{8bef^2p}{5dh^3\sqrt{hx}} + \frac{2\sqrt{2}be^{5/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}} - \frac{4\sqrt{2}be^{3/4}fgp\tan}{3d}$$

Mathematica [C] time = 1.08, size = 340, normalized size = 0.36

$$\frac{2x^{7/2}\left(-\frac{f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5x^{5/2}} - \frac{2fg\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3x^{3/2}} - \frac{g^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{x}} - \frac{\sqrt{2}be^{3/4}fgp\left(\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right)-\log\left(\right.\right.}{\ }\right.}{(}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

```
[Out] (2*x^(7/2)*((2*b*e^(1/4)*g^2*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + ArcT
anh[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (4*b*e*f^2*p*Hypergeomet
ric2F1[-1/4, 1, 3/4, -((e*x^2)/d)])/(5*d*Sqrt[x]) - (Sqrt[2]*b*e^(3/4)*f*g*
p*(2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*
e^(1/4)*Sqrt[x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] +
```

```
Sqrt[e]*x] - Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/
(3*d^(3/4)) - (f^2*(a + b*Log[c*(d + e*x^2)^p]))/(5*x^(5/2)) - (2*f*g*(a +
b*Log[c*(d + e*x^2)^p]))/(3*x^(3/2)) - (g^2*(a + b*Log[c*(d + e*x^2)^p]))/S
qrt[x]))/(h*x)^(7/2)
```

fricas [B] time = 1.11, size = 2205, normalized size = 2.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="fric
as")

[Out] -2/15*(d*h^4*x^3*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*g
^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^4*
e*g^8)*p^4/(d^5*h^14)) + 60*(b^2*e^2*f^3*g - 5*b^2*d*e*f*g^3)*p^2)/(d^2*h^7
))*log(32*(81*b^3*e^5*f^8 - 1620*b^3*d*e^4*f^6*g^2 + 2150*b^3*d^2*e^3*f^4*g
^4 - 40500*b^3*d^3*e^2*f^2*g^6 + 50625*b^3*d^4*e*g^8)*sqrt(h*x)*p^3 + 32*(3
*(d^4*e*f^2 - 5*d^5*g^2)*h^11*sqrt(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*g^
2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^4*e
*g^8)*p^4/(d^5*h^14)) - 10*(9*b^2*d^2*e^3*f^5*g - 190*b^2*d^3*e^2*f^3*g^3 +
 225*b^2*d^4*e*f*g^5)*h^4*p^2)*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f^8 - 3420*b
^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 +
50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) + 60*(b^2*e^2*f^3*g - 5*b^2*d*e*f*g^3)
*p^2)/(d^2*h^7))) - d*h^4*x^3*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f^8 - 3420*b^
4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 + 5
0625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) + 60*(b^2*e^2*f^3*g - 5*b^2*d*e*f*g^3)*
p^2)/(d^2*h^7))*log(32*(81*b^3*e^5*f^8 - 1620*b^3*d*e^4*f^6*g^2 + 2150*b^3*
d^2*e^3*f^4*g^4 - 40500*b^3*d^3*e^2*f^2*g^6 + 50625*b^3*d^4*e*g^8)*sqrt(h*x
)*p^3 - 32*(3*(d^4*e*f^2 - 5*d^5*g^2)*h^11*sqrt(-(81*b^4*e^5*f^8 - 3420*b^4
*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 + 50
625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) - 10*(9*b^2*d^2*e^3*f^5*g - 190*b^2*d^3*
e^2*f^3*g^3 + 225*b^2*d^4*e*f*g^5)*h^4*p^2)*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5
*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e
^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) + 60*(b^2*e^2*f^3*g - 5*b
^2*d*e*f*g^3)*p^2)/(d^2*h^7))) - d*h^4*x^3*sqrt(-(d^2*h^7*sqrt(-(81*b^4*e^5
*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e
^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) - 60*(b^2*e^2*f^3*g - 5*b
^2*d*e*f*g^3)*p^2)/(d^2*h^7))*log(32*(81*b^3*e^5*f^8 - 1620*b^3*d*e^4*f^6*g
^2 + 2150*b^3*d^2*e^3*f^4*g^4 - 40500*b^3*d^3*e^2*f^2*g^6 + 50625*b^3*d^4*e
*g^8)*sqrt(h*x)*p^3 + 32*(3*(d^4*e*f^2 - 5*d^5*g^2)*h^11*sqrt(-(81*b^4*e^5*
f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^
2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) + 10*(9*b^2*d^2*e^3*f^5*g
- 190*b^2*d^3*e^2*f^3*g^3 + 225*b^2*d^4*e*f*g^5)*h^4*p^2)*sqrt(-(d^2*h^7*sq
rt(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 -
85500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) - 60*(b^2*
e^2*f^3*g - 5*b^2*d*e*f*g^3)*p^2)/(d^2*h^7))) + d*h^4*x^3*sqrt(-(d^2*h^7*sq
rt(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 -
85500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) - 60*(b^2*
e^2*f^3*g - 5*b^2*d*e*f*g^3)*p^2)/(d^2*h^7))*log(32*(81*b^3*e^5*f^8 - 1620*
b^3*d*e^4*f^6*g^2 + 2150*b^3*d^2*e^3*f^4*g^4 - 40500*b^3*d^3*e^2*f^2*g^6 +
50625*b^3*d^4*e*g^8)*sqrt(h*x)*p^3 - 32*(3*(d^4*e*f^2 - 5*d^5*g^2)*h^11*sqr
t(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 8
5500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) + 10*(9*b^2
*d^2*e^3*f^5*g - 190*b^2*d^3*e^2*f^3*g^3 + 225*b^2*d^4*e*f*g^5)*h^4*p^2)*sq
rt(-(d^2*h^7*sqrt(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2
*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^
14)) - 60*(b^2*e^2*f^3*g - 5*b^2*d*e*f*g^3)*p^2)/(d^2*h^7))) + (10*a*d*f*g*
x + 3*a*d*f^2 + 3*(4*b*e*f^2*p + 5*a*d*g^2)*x^2 + (15*b*d*g^2*p*x^2 + 10*b*
```

```
d*f*g*p*x + 3*b*d*f^2*p)*log(e*x^2 + d) + (15*b*d*g^2*x^2 + 10*b*d*f*g*x +
3*b*d*f^2)*log(c))*sqrt(h*x))/(d*h^4*x^3)
```

giac [A] time = 0.56, size = 660, normalized size = 0.71

$$\frac{\frac{2\left(10\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bdfghpe^{\frac{11}{4}}+15\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bdg^2pe^{\frac{9}{4}}-3\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bf^2pe^{\frac{13}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^2\right)^{\frac{1}{4}}}\right)e^{(-2)}}{d^2h}+\frac{2\left(10\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bdfghpe^{\frac{11}{4}}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="giac
")
```

```
[Out] 1/15*(2*(10*sqrt(2)*(d*h^2)^(1/4)*b*d*f*g*h*p*e^(11/4) + 15*sqrt(2)*(d*h^2)
^(3/4)*b*d*g^2*p*e^(9/4) - 3*sqrt(2)*(d*h^2)^(3/4)*b*f^2*p*e^(13/4))*arctan
(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2)
^(1/4))*e^(-2)/(d^2*h) + 2*(10*sqrt(2)*(d*h^2)^(1/4)*b*d*f*g*h*p*e^(11/4) +
 15*sqrt(2)*(d*h^2)^(3/4)*b*d*g^2*p*e^(9/4) - 3*sqrt(2)*(d*h^2)^(3/4)*b*f^2
*p*e^(13/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h
*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-2)/(d^2*h) + (10*sqrt(2)*(d*h^2)^(1/4)*b*d*
f*g*h*p*e^(11/4) - 15*sqrt(2)*(d*h^2)^(3/4)*b*d*g^2*p*e^(9/4) + 3*sqrt(2)*(
d*h^2)^(3/4)*b*f^2*p*e^(13/4))*e^(-2)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e
^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h) - (10*sqrt(2)*(d*h^2)^(1/4)*b
*d*f*g*h*p*e^(11/4) - 15*sqrt(2)*(d*h^2)^(3/4)*b*d*g^2*p*e^(9/4) + 3*sqrt(2
)*(d*h^2)^(3/4)*b*f^2*p*e^(13/4))*e^(-2)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*
x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h) - 2*(15*b*d*g^2*h^3*p*x^2
*log(h^2*x^2*e + d*h^2) - 15*b*d*g^2*h^3*p*x^2*log(h^2) + 12*b*f^2*h^3*p*x^
2*e + 10*b*d*f*g*h^3*p*x*log(h^2*x^2*e + d*h^2) - 10*b*d*f*g*h^3*p*x*log(h^
2) + 15*b*d*g^2*h^3*x^2*log(c) + 15*a*d*g^2*h^3*x^2 + 3*b*d*f^2*h^3*p*log(h
^2*x^2*e + d*h^2) - 3*b*d*f^2*h^3*p*log(h^2) + 10*b*d*f*g*h^3*x*log(c) + 10
*a*d*f*g*h^3*x + 3*b*d*f^2*h^3*log(c) + 3*a*d*f^2*h^3)/(sqrt(h*x)*d*h^2*x^2
))/h^4
```

maple [F] time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^{2}\left(b\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)+a\right)}{\left(hx\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(7/2),x)
```

```
[Out] int((g*x+f)^2*(b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(7/2),x)
```

maxima [A] time = 1.15, size = 813, normalized size = 0.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] -2*b*g^2*x^3*log((e*x^2 + d)^p*c)/(h*x)^(7/2) - 1/5*b*e*f^2*p*(e*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e))/s
qrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(e)) + 2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/sqrt(s
```

```
qrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(sqrt(e)*
h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e
^(3/4)) + sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4)
 + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)))/d + 8/(sqrt(h*x)*d))/h^3 + b*e*g^2*p
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)
*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(e)) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt
(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*l
og(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h
^2)^(1/4)*e^(3/4)) + sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h
*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)))/h^3 - 2*a*g^2*x^3/(h*x)^(
7/2) - 4/3*b*f*g*x^2*log((e*x^2 + d)^p*c)/(h*x)^(7/2) + 2/3*(sqrt(2)*h^2*lo
g(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^
2)^(3/4)*e^(1/4)) - sqrt(2)*h^2*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqr
t(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + 2*sqrt(2)*h*arctan(1/
2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d
)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)) + 2*sqrt(2)*h*arctan(-1/2*s
qrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*s
qrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)))*b*e*f*g*p/h^4 - 4/3*a*f*g*x^2
/(h*x)^(7/2) - 2/5*b*f^2*log((e*x^2 + d)^p*c)/((h*x)^(5/2)*h) - 2/5*a*f^2/(
(h*x)^(5/2)*h)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^2\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(7/2),x)
[Out] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(7/2),x)
[Out] Timed out
```

3.615 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{9/2}}\,dx$

Optimal. Leaf size=968

$$\frac{2\sqrt{2}\,be^{7/4}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{7d^{7/4}h^{9/2}}-\frac{2\sqrt{2}\,be^{7/4}p\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{7d^{7/4}h^{9/2}}-\frac{2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f^2}{7h(hx)^{7/2}}+\frac{\sqrt{2}\,be}{}$$

```
[Out] -8/21*b*e*f^2*p/d/h^3/(h*x)^(3/2)-2/7*f^2*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(
7/2)-4/5*f*g*(a+b*ln(c*(e*x^2+d)^p))/h^2/(h*x)^(5/2)-2/3*g^2*(a+b*ln(c*(e*x
^2+d)^p))/h^3/(h*x)^(3/2)+2/7*b*e^(7/4)*f^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x
)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(7/4)/h^(9/2)+4/5*b*e^(5/4)*f*g*p*arctan
(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(5/4)/h^(9/2)-2/3
*b*e^(3/4)*g^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1
/2)/d^(3/4)/h^(9/2)-2/7*b*e^(7/4)*f^2*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2
)/d^(1/4)/h^(1/2))*2^(1/2)/d^(7/4)/h^(9/2)-4/5*b*e^(5/4)*f*g*p*arctan(1+e^(
1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(5/4)/h^(9/2)+2/3*b*e^(
3/4)*g^2*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^
(3/4)/h^(9/2)+1/7*b*e^(7/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1
/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(7/4)/h^(9/2)-2/5*b*e^(5/4)*f*g*
p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))
*2^(1/2)/d^(5/4)/h^(9/2)-1/3*b*e^(3/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h
^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(3/4)/h^(9/2)-1/7*b*e
^(7/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(
h*x)^(1/2))*2^(1/2)/d^(7/4)/h^(9/2)+2/5*b*e^(5/4)*f*g*p*ln(d^(1/2)*h^(1/2)+
x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(5/4)/h^(9
/2)+1/3*b*e^(3/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4
)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(3/4)/h^(9/2)-16/5*b*e*f*g*p/d/h^4/(h*x)^(
1/2)
```

Rubi [A] time = 1.23, antiderivative size = 968, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2467, 2476, 2455, 325, 211, 1165, 628, 1162, 617, 204, 297}

$$\frac{2\sqrt{2}\,be^{7/4}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{7d^{7/4}h^{9/2}}-\frac{2\sqrt{2}\,be^{7/4}p\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{7d^{7/4}h^{9/2}}-\frac{2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f^2}{7h(hx)^{7/2}}+\frac{\sqrt{2}\,be}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```

```
[Out] (-8*b*e*f^2*p)/(21*d*h^3*(h*x)^(3/2)) - (16*b*e*f*g*p)/(5*d*h^4*Sqrt[h*x])
+ (2*Sqrt[2]*b*e^(7/4)*f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4
)*Sqrt[h])])/(7*d^(7/4)*h^(9/2)) + (4*Sqrt[2]*b*e^(5/4)*f*g*p*ArcTan[1 - (S
qrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(9/2)) - (2*Sqrt
[2]*b*e^(3/4)*g^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h]
)])/(3*d^(3/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(7/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(7*d^(7/4)*h^(9/2)) - (4*Sqrt[2]*b*e^(
5/4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^
(5/4)*h^(9/2)) + (2*Sqrt[2]*b*e^(3/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqr
t[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)*h^(9/2)) - (2*f^2*(a + b*Log[c*(d +
e*x^2)^p]))/(7*h*(h*x)^(7/2)) - (4*f*g*(a + b*Log[c*(d + e*x^2)^p]))/(5*h^2
*(h*x)^(5/2)) - (2*g^2*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^3*(h*x)^(3/2)) +
(Sqrt[2]*b*e^(7/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*
d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(5/4)*f*g*
p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*
```

```
x]])/(5*d^(5/4)*h^(9/2)) - (Sqrt[2]*b*e^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(9/2))
- (Sqrt[2]*b*e^(7/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2
]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2)) + (2*Sqrt[2]*b*e^(5/4)*f*
g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(5*d^(5/4)*h^(9/2)) + (Sqrt[2]*b*e^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] +
 Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(9/2)
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{9/2}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^8}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \left(\frac{f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^8} + \frac{2fg\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^6} + \frac{g^2\left(a+b\log\right.}{}\right.\right.}{h}$$

$$= \frac{\left(2g^2\right)\mathrm{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^4}\,dx, x, \sqrt{hx}\right)}{h^3} + \frac{(4fg)\,\mathrm{Subst}\left(\int \frac{a+b}{}\right.}{}$$

$$= -\frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}} - \frac{4fg\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h^2(hx)^{5/2}}$$

$$= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}} - \frac{4f}{}$$

$$= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}} - \frac{4f}{}$$

$$= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}} - \frac{4f}{}$$

$$= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} - \frac{2\sqrt{2}\,be^{3/4}g^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{9/2}}$$

$$= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} + \frac{2\sqrt{2}\,be^{7/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}}$$

Mathematica [C] time = 0.27, size = 294, normalized size = 0.30

$$\frac{x\left(-30df^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right) - 84dfgx\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right) - 70dg^2x^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```

```
[Out] (x*(-40*b*e*f^2*p*x^2*Hypergeometric2F1[-3/4, 1, 1/4, -((e*x^2)/d)] - 336*b
*e*f*g*p*x^3*Hypergeometric2F1[-1/4, 1, 3/4, -((e*x^2)/d)] - 35*Sqrt[2]*b*d
^(1/4)*e^(3/4)*g^2*p*x^(7/2)*(2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4
)] - 2*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2
]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1
/4)*Sqrt[x] + Sqrt[e]*x]) - 30*d*f^2*(a + b*Log[c*(d + e*x^2)^p]) - 84*d*f*
```

```
g*x*(a + b*Log[c*(d + e*x^2)^p]) - 70*d*g^2*x^2*(a + b*Log[c*(d + e*x^2)^p]
)))/(105*d*(h*x)^(9/2))
```

fricas [B] time = 1.05, size = 2283, normalized size = 2.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="fric
as")

[Out] -2/105*(d*h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e
^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 15
00625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) + 420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2
*f*g^3)*p^2)/(d^3*h^9))*log(16*(50625*b^3*e^6*f^8 - 472500*b^3*d*e^5*f^6*g^
2 - 1457946*b^3*d^2*e^4*f^4*g^4 - 2572500*b^3*d^3*e^3*f^2*g^6 + 1500625*b^3
*d^4*e^2*g^8)*sqrt(h*x)*p^3 + 16*(42*d^6*f*g*h^14*sqrt(-(50625*b^4*e^7*f^8
- 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3
*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) + 5*(675*b^2*d^2*e^
4*f^6 - 10017*b^2*d^3*e^3*f^4*g^2 + 23373*b^2*d^4*e^2*f^2*g^4 - 8575*b^2*d^
5*e*g^6)*h^5*p^2)*sqrt(-(d^3*h^9*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e
^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 15
00625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) + 420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2
*f*g^3)*p^2)/(d^3*h^9))) - d*h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(50625*b^4*e^7*f^
8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d
^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) + 420*(3*b^2*e^3*
f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9))*log(16*(50625*b^3*e^6*f^8 - 4725
00*b^3*d*e^5*f^6*g^2 - 1457946*b^3*d^2*e^4*f^4*g^4 - 2572500*b^3*d^3*e^3*f^
2*g^6 + 1500625*b^3*d^4*e^2*g^8)*sqrt(h*x)*p^3 - 16*(42*d^6*f*g*h^14*sqrt(-
(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^
4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18))
+ 5*(675*b^2*d^2*e^4*f^6 - 10017*b^2*d^3*e^3*f^4*g^2 + 23373*b^2*d^4*e^2*f^
2*g^4 - 8575*b^2*d^5*e*g^6)*h^5*p^2)*sqrt(-(d^3*h^9*sqrt(-(50625*b^4*e^7*f^
8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d
^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) + 420*(3*b^2*e^3*
f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9))) - d*h^5*x^4*sqrt((d^3*h^9*sqrt(
-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g
^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18))
 - 420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9))*log(16*(50625*
b^3*e^6*f^8 - 472500*b^3*d*e^5*f^6*g^2 - 1457946*b^3*d^2*e^4*f^4*g^4 - 2572
500*b^3*d^3*e^3*f^2*g^6 + 1500625*b^3*d^4*e^2*g^8)*sqrt(h*x)*p^3 + 16*(42*d
^6*f*g*h^14*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*
b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8
)*p^4/(d^7*h^18)) - 5*(675*b^2*d^2*e^4*f^6 - 10017*b^2*d^3*e^3*f^4*g^2 + 23
373*b^2*d^4*e^2*f^2*g^4 - 8575*b^2*d^5*e*g^6)*h^5*p^2)*sqrt((d^3*h^9*sqrt(-
(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^
4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18))
- 420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9))) + d*h^5*x^4*sq
rt((d^3*h^9*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*
b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8
)*p^4/(d^7*h^18)) - 420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9
))*log(16*(50625*b^3*e^6*f^8 - 472500*b^3*d*e^5*f^6*g^2 - 1457946*b^3*d^2*e
^4*f^4*g^4 - 2572500*b^3*d^3*e^3*f^2*g^6 + 1500625*b^3*d^4*e^2*g^8)*sqrt(h*
x)*p^3 - 16*(42*d^6*f*g*h^14*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f
^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 150062
5*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) - 5*(675*b^2*d^2*e^4*f^6 - 10017*b^2*d^3
*e^3*f^4*g^2 + 23373*b^2*d^4*e^2*f^2*g^4 - 8575*b^2*d^5*e*g^6)*h^5*p^2)*sqr
t((d^3*h^9*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b
^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)
*p^4/(d^7*h^18)) - 420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9)
```

```
)) + (168*b*e*f*g*p*x^3 + 42*a*d*f*g*x + 15*a*d*f^2 + 5*(4*b*e*f^2*p + 7*a*
d*g^2)*x^2 + (35*b*d*g^2*p*x^2 + 42*b*d*f*g*p*x + 15*b*d*f^2*p)*log(e*x^2 +
 d) + (35*b*d*g^2*x^2 + 42*b*d*f*g*x + 15*b*d*f^2)*log(c))*sqrt(h*x))/(d*h^
5*x^4)
```

giac [A] time = 0.49, size = 674, normalized size = 0.70

$$\frac{2\left(35\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bdg^2hpe^{\frac{7}{4}}-15\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bf^2hpe^{\frac{11}{4}}-42\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bfgpe^{\frac{9}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^2\right)^{\frac{1}{4}}}\right)e^{(-1)}}{d^2h}+\frac{2\left(35\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bdg^2hp\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="giac
")
```

```
[Out] 1/105*(2*(35*sqrt(2)*(d*h^2)^(1/4)*b*d*g^2*h*p*e^(7/4) - 15*sqrt(2)*(d*h^2)
^(1/4)*b*f^2*h*p*e^(11/4) - 42*sqrt(2)*(d*h^2)^(3/4)*b*f*g*p*e^(9/4))*arcta
n(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2
)^(1/4))*e^(-1)/(d^2*h) + 2*(35*sqrt(2)*(d*h^2)^(1/4)*b*d*g^2*h*p*e^(7/4) -
 15*sqrt(2)*(d*h^2)^(1/4)*b*f^2*h*p*e^(11/4) - 42*sqrt(2)*(d*h^2)^(3/4)*b*f
*g*p*e^(9/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(
h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-1)/(d^2*h) + (35*sqrt(2)*(d*h^2)^(1/4)*b*d
*g^2*h*p*e^(7/4) - 15*sqrt(2)*(d*h^2)^(1/4)*b*f^2*h*p*e^(11/4) + 42*sqrt(2)
*(d*h^2)^(3/4)*b*f*g*p*e^(9/4))*e^(-1)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*
e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h) - (35*sqrt(2)*(d*h^2)^(1/4)*
b*d*g^2*h*p*e^(7/4) - 15*sqrt(2)*(d*h^2)^(1/4)*b*f^2*h*p*e^(11/4) + 42*sqrt
(2)*(d*h^2)^(3/4)*b*f*g*p*e^(9/4))*e^(-1)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h
*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h) - 2*(168*b*f*g*h^4*p*x^3
*e + 35*b*d*g^2*h^4*p*x^2*log(h^2*x^2*e + d*h^2) - 35*b*d*g^2*h^4*p*x^2*log
(h^2) + 20*b*f^2*h^4*p*x^2*e + 42*b*d*f*g*h^4*p*x*log(h^2*x^2*e + d*h^2) -
42*b*d*f*g*h^4*p*x*log(h^2) + 35*b*d*g^2*h^4*x^2*log(c) + 35*a*d*g^2*h^4*x^
2 + 15*b*d*f^2*h^4*p*log(h^2*x^2*e + d*h^2) - 15*b*d*f^2*h^4*p*log(h^2) + 4
2*b*d*f*g*h^4*x*log(c) + 42*a*d*f*g*h^4*x + 15*b*d*f^2*h^4*log(c) + 15*a*d*
f^2*h^4)/(sqrt(h*x)*d*h^3*x^3))/h^5
```

maple [F] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^{2}\left(b\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)+a\right)}{\left(hx\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(9/2),x)
```

```
[Out] int((g*x+f)^2*(b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(9/2),x)
```

maxima [A] time = 1.16, size = 838, normalized size = 0.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="maxi
ma")
```

```
[Out] -1/21*b*e*f^2*p*(3*(sqrt(2)*e^(3/4)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)
*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) - sqrt(2)*e^(3/4)*log(sqrt(e)
```

```
*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) +
 2*sqrt(2)*e*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x
)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)*h) + 2
*sqrt(2)*e*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)
*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt(d)*h))/d +
 8/((h*x)^(3/2)*d))/h^3 - 2/3*b*g^2*x^3*log((e*x^2 + d)^p*c)/(h*x)^(9/2) -
2/5*b*e*f*g*p*(e*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/
4) + 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)
*sqrt(e)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) -
2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*sqrt
(e)) - sqrt(2)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) +
sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) + sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h
^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)))/d + 8/(sq
rt(h*x)*d))/h^4 - 2/3*a*g^2*x^3/(h*x)^(9/2) - 4/5*b*f*g*x^2*log((e*x^2 + d)
^p*c)/(h*x)^(9/2) + 1/3*(sqrt(2)*h^2*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4
)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) - sqrt(2)*h^2*log(
sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)
^(3/4)*e^(1/4)) + 2*sqrt(2)*h*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(
1/4) + 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*
h)*sqrt(d)) + 2*sqrt(2)*h*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4
) - 2*sqrt(h*x)*sqrt(e))/sqrt(sqrt(d)*sqrt(e)*h))/(sqrt(sqrt(d)*sqrt(e)*h)*
sqrt(d)))*b*e*g^2*p/h^5 - 4/5*a*f*g*x^2/(h*x)^(9/2) - 2/7*b*f^2*log((e*x^2
+ d)^p*c)/((h*x)^(7/2)*h) - 2/7*a*f^2/((h*x)^(7/2)*h)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f+g\,x\right)}^2\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(9/2),x)
[Out] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(9/2),x)
[Out] Timed out
```

3.616 $\int \frac{\sqrt{hx}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f+gx}\,dx$

Optimal. Leaf size=1680

$$\frac{2\sqrt{hx}\,a}{g}-\frac{2\sqrt{2}\,b\sqrt[4]{d}\,\sqrt{h}\,p\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{e}\,g}+\frac{2\sqrt{2}\,b\sqrt[4]{d}\,\sqrt{h}\,p\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}+1\right)}{\sqrt[4]{e}\,g}+\frac{2b\sqrt{hx}\,\log\left(c\left(ex^2+d\right)\right.}{g}$$

```
[Out] -2*b*d^(1/4)*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2
)*h^(1/2)/e^(1/4)/g+2*b*d^(1/4)*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1
/4)/h^(1/2))*2^(1/2)*h^(1/2)/e^(1/4)/g-b*d^(1/4)*p*ln(d^(1/2)*h^(1/2)+x*e^(
1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)*h^(1/2)/e^(1/4)/g
+b*d^(1/4)*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(
h*x)^(1/2))*2^(1/2)*h^(1/2)/e^(1/4)/g-2*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/
h^(1/2))*(a+b*ln(c*(e*x^2+d)^p))*f^(1/2)*h^(1/2)/g^(3/2)-8*b*p*arctan(g^(1/
2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*h^(1/2)/(f^(1/2)*h^(1/2)-I*g^(
1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)+2*b*p*arctan(g^(1/2)*(h*x)^(1/2)
/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/
4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(
f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)+2*b*p*arcta
n(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^
(1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)-(-d)^(1/4)*g^(1/2))/(f^(1/2)*
h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)+2*b*p*arctan(g^(1/2
)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h
)^(1/2)+e^(1/4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/
2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2
)+2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*((
-d)^(1/4)*h^(1/2)+e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)+(-d)^(1/4)*g^(1/2
))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)-I*b*p*p
olylog(2,1-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)+e^(1/4)*(h*x)^(1/2))/(I*e^
(1/4)*f^(1/2)+(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*
f^(1/2)*h^(1/2)/g^(3/2)-I*b*p*polylog(2,1+2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(
1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)-(-d)^(1/4)*g^(1/2))/(f^(1/2)*h
^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)-I*b*p*polylog(2,1-2*
f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)+e^(1/4)*(h*x)^(1/2))/((-d)^(
1/4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/
2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)+4*I*b*p*polylog(2,1-2*f^(1/2)*h^(1
/2)/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)-I*b*p*
polylog(2,1-2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/4)*(h*x)^
(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h
^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)+2*a*(h*x)^(1/2)/g-8*
b*p*(h*x)^(1/2)/g+2*b*ln(c*(e*x^2+d)^p)*(h*x)^(1/2)/g
```

Rubi [A] time = 3.08, antiderivative size = 1680, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 20, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.645, Rules used = {2467, 2476, 2448, 321, 211, 1165, 628, 1162, 617, 204, 205, 2470, 12, 260, 6725, 4928, 4856, 2402, 2315, 2447}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[h*x]*(a + b*Log[c*(d + e*x^2)^p]))/(f + g*x),x]
```

```
[Out] (2*a*Sqrt[h*x])/g - (8*b*p*Sqrt[h*x])/g - (2*Sqrt[2]*b*d^(1/4)*Sqrt[h]*p*Ar
cTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*g) + (2*S
qrt[2]*b*d^(1/4)*Sqrt[h]*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*
```

```
Sqrt[h])])/(e^(1/4)*g) + (2*b*Sqrt[h*x]*Log[c*(d + e*x^2)^p])/g - (2*Sqrt[f
]*Sqrt[h]*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*(a + b*Log[c*(d + e
*x^2)^p]))/g^(3/2) - (Sqrt[2]*b*d^(1/4)*Sqrt[h]*p*Log[Sqrt[d]*Sqrt[h] + Sqr
t[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*g) + (Sqrt[2]
*b*d^(1/4)*Sqrt[h]*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1
/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*g) - (8*b*Sqrt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g
]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] -
I*Sqrt[g]*Sqrt[h*x])])/g^(3/2) + (2*b*Sqrt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqr
t[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[
-h] - e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]
*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/g^(3/2) + (2*b*Sqrt[f]
*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(-2*Sqrt[f]*Sq
rt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^
(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/g^(3/2) + (2*b*Sq
rt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f
]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*S
qrt[g]*Sqrt[-h] + I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*S
qrt[h*x]))])/g^(3/2) + (2*b*Sqrt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(S
qrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[
h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[
g]*Sqrt[h*x]))])/g^(3/2) + ((4*I)*b*Sqrt[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqr
t[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])/g^(3/2) - (I*b*Sqrt
[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h
] - e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*S
qrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/g^(3/2) - (I*b*Sqrt[f]*S
qrt[h]*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sq
rt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sq
rt[g]*Sqrt[h*x]))])/g^(3/2) - (I*b*Sqrt[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqrt
[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)
*Sqrt[g]*Sqrt[-h] + I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]
*Sqrt[h*x]))])/g^(3/2) - (I*b*Sqrt[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqrt[f]*S
qrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)
^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/g^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{hx}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f+gx}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{f+\frac{gx^2}{h}}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{h\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{g} - \frac{fh\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{g\left(f+\frac{gx^2}{h}\right)}\right) dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right) dx, x, \sqrt{hx}\right)}{g} - \frac{(2f)\,\text{Subst}\left(\int \frac{a+b \cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{2\sqrt{f}\,\sqrt{h}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{hx}}{\sqrt{f}\,\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{g^{3/2}} + \frac{(2b)\,\text{S}\cdots}{\cdots}$$

$$= \frac{2a\sqrt{hx}}{g} + \frac{2b\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{2\sqrt{f}\,\sqrt{h}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{hx}}{\sqrt{f}\,\sqrt{h}}\right)\left(a+b \cdots\right.}{g^{3/2}}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} + \frac{2b\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{2\sqrt{f}\,\sqrt{h}\,\tan^{-1}\left(\frac{\sqrt{g} \cdots}{\sqrt{f} \cdots}\right.}{\cdots}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} + \frac{2b\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{2\sqrt{f}\,\sqrt{h}\,\tan^{-1}\left(\frac{\sqrt{g} \cdots}{\sqrt{f} \cdots}\right.}{\cdots}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} + \frac{2b\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{2\sqrt{f}\,\sqrt{h}\,\tan^{-1}\left(\frac{\sqrt{g} \cdots}{\sqrt{f} \cdots}\right.}{\cdots}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} + \frac{2b\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{g} - \frac{2\sqrt{f}\,\sqrt{h}\,\tan^{-1}\left(\frac{\sqrt{g} \cdots}{\sqrt{f} \cdots}\right.}{\cdots}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} - \frac{2\sqrt{2}\,b\sqrt[4]{d}\,\sqrt{h}\,p\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{e}\,g} + \frac{2\sqrt{2}\,b\sqrt[4]{\cdots}}{\cdots}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} - \frac{2\sqrt{2}\,b\sqrt[4]{d}\,\sqrt{h}\,p\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{e}\,g} + \frac{2\sqrt{2}\,b\sqrt[4]{\cdots}}{\cdots}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} - \frac{2\sqrt{2}\,b\sqrt[4]{d}\,\sqrt{h}\,p\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{e}\,g} + \frac{2\sqrt{2}\,b\sqrt[4]{\cdots}}{\cdots}$$

Mathematica [A] time = 1.41, size = 1471, normalized size = 0.88

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[h*x]*(a + b*Log[c*(d + e*x^2)^p]))/(f + g*x),x]
```

```
[Out] (Sqrt[h*x]*(2*a*Sqrt[g]*Sqrt[x] - (b*Sqrt[g]*p*(8*e^(1/4)*Sqrt[x] + 2*Sqrt[
2]*d^(1/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*Sqrt[2]*d^(1/4
)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Sqrt[2]*d^(1/4)*Log[Sqrt[
d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Sqrt[2]*d^(1/4)*Log[Sqr
t[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/e^(1/4) + 2*b*Sqrt[g]
*Sqrt[x]*Log[c*(d + e*x^2)^p] + Sqrt[-f]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]]*(a
 + b*Log[c*(d + e*x^2)^p]) - Sqrt[-f]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]]*(a +
b*Log[c*(d + e*x^2)^p]) - b*Sqrt[-f]*p*(Log[(Sqrt[g]*((-d)^(1/4) - e^(1/4)*
Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g
]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt
[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*
(I*(-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])
]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt
[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[
x]] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] -
 (-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(
e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] -
 Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2,
(e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[
g])]) + b*Sqrt[-f]*p*(Log[(Sqrt[g]*((-d)^(1/4) - e^(1/4)*Sqrt[x]))/(e^(1/4)
*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqr
t[g]*((-d)^(1/4) - I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqr
t[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/
4)*Sqrt[x]))/((-I)*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + S
qrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt[x]))/(-(e^(1/4)*S
qrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + PolyLog[2
, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - (-d)^(1/4)*Sqr
t[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f
] - I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x
]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[
-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])])))/(g^(3/2
)*Sqrt[x])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{hx}\, b \log\left(\left(ex^2+d\right)^p c\right) + \sqrt{hx}\, a}{gx+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^(1/2)*(a+b*log(c*(e*x^2+d)^p))/(g*x+f),x, algorithm="fricas
")
```

```
[Out] integral((sqrt(h*x)*b*log((e*x^2 + d)^p*c) + sqrt(h*x)*a)/(g*x + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{hx}\left(b \log\left(\left(ex^2+d\right)^p c\right) + a\right)}{gx+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^(1/2)*(a+b*log(c*(e*x^2+d)^p))/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate(sqrt(h*x)*(b*log((e*x^2 + d)^p*c) + a)/(g*x + f), x)
```

maple [F] time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{hx}\,\left(b\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)+a\right)}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x)^(1/2)*(b*ln(c*(e*x^2+d)^p)+a)/(g*x+f),x)
```

```
[Out] int((h*x)^(1/2)*(b*ln(c*(e*x^2+d)^p)+a)/(g*x+f),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{\sqrt{h}\,p\sqrt{x}\,\log\left(ex^{2}+d\right)+\sqrt{h}\,\sqrt{x}\,\log(c)}{gx+f}\,dx-\frac{2\left(\frac{fh^{2}\arctan\left(\frac{\sqrt{hx}\,g}{\sqrt{fgh}}\right)}{\sqrt{fgh}\,g}-\frac{\sqrt{hx}\,h}{g}\right)a}{h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^(1/2)*(a+b*log(c*(e*x^2+d)^p))/(g*x+f),x, algorithm="maxima
")
```

```
[Out] b*integrate((sqrt(h)*p*sqrt(x)*log(e*x^2 + d) + sqrt(h)*sqrt(x)*log(c))/(g*
x + f), x) - 2*(f*h^2*arctan(sqrt(h*x)*g/sqrt(f*g*h))/(sqrt(f*g*h)*g) - sqr
t(h*x)*h/g)*a/h
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{h\,x}\,\left(a+b\,\ln\left(c\,{\left(e\,x^{2}+d\right)}^{p}\right)\right)}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((h*x)^(1/2)*(a + b*log(c*(d + e*x^2)^p)))/(f + g*x),x)
```

```
[Out] int(((h*x)^(1/2)*(a + b*log(c*(d + e*x^2)^p)))/(f + g*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)**(1/2)*(a+b*ln(c*(e*x**2+d)**p))/(g*x+f),x)
```

```
[Out] Timed out
```

3.617 $\int \frac{a+b\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{hx}\,(f+gx)}\,dx$

Optimal. Leaf size=1361

$$\frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}+\frac{8bp\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}-\frac{2bp\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\right.}{}$$

```
[Out] 2*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*(a+b*ln(c*(e*x^2+d)^p))/f^(1/
2)/g^(1/2)/h^(1/2)+8*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f
^(1/2)*h^(1/2)/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(
1/2)-2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)
*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-
h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2))
)/f^(1/2)/g^(1/2)/h^(1/2)-2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))
*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*
f^(1/2)-(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2
)/g^(1/2)/h^(1/2)-2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^
(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)+e^(1/4)*(h*x)^(1/2))/((-d)^(1/
4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)
*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)-2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(
1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)+e^(1/4)*(h*x)^(1/2))
/(I*e^(1/4)*f^(1/2)+(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1
/2)))/f^(1/2)/g^(1/2)/h^(1/2)-4*I*b*p*polylog(2,1-2*f^(1/2)*h^(1/2)/(f^(1/2
)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)+I*b*p*polylog(2,1
-2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/4)*(h*x)^(1/2))/((-d
)^(1/4)*g^(1/2)*(-h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^
(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)+I*b*p*polylog(2,1+2*f^(1/2)*g^(
1/2)*((-d)^(1/4)*h^(1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)-(-d)^(1/4)
*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)+
I*b*p*polylog(2,1-2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)+e^(1/4)*
(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(
1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)+I*b*p*polylog(
2,1-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)+e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f
^(1/2)+(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)
/g^(1/2)/h^(1/2)
```

Rubi [A] time = 1.83, antiderivative size = 1361, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2467, 205, 2470, 12, 260, 6725, 4928, 4856, 2402, 2315, 2447}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}+\frac{8bp\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}-\frac{2bp\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^2)^p])/(Sqrt[h*x]*(f + g*x)),x]
```

```
[Out] (2*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*(a + b*Log[c*(d + e*x^2)^p
]))/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (8*b*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*
Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])
/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqr
t[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4)*Sqrt[h
*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*S
```

```
qrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*ArcTan[
(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(-2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*
Sqrt[h] - e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sq
rt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*
ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h
]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h]
+ I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sq
rt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h]
)]*Log[(2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(
1/4)*Sqrt[f] + (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))
])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - ((4*I)*b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[h]
)/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*
b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4
)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(S
qrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*b*p
*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sqrt[h*x])
)/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sq
rt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sq
rt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[
g]*Sqrt[-h] + I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[
h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[
g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)^(1/
4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqr
t[h])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{hx}\,(f+gx)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{f+\frac{gx^2}{h}}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{(8bep)\,\text{Subst}\left(\int \frac{\sqrt{h}\,x^3\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}\sqrt{h}}\right.}{\sqrt{f}\sqrt{g}\left(d+\frac{ex^4}{h^2}\right)}\right.}{h^3}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{(8bep)\,\text{Subst}\left(\int \frac{x^3\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}\sqrt{h}}\right)}{d+\frac{ex^4}{h^2}}\,d\right.}{\sqrt{f}\sqrt{g}\,h^{5/2}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{(8bep)\,\text{Subst}\left(\int\left(\frac{h^2x\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}\sqrt{h}}\right.}{2\left(-\sqrt{-d}\sqrt{e}\,h+ex^2\right.}\right.\right.}{\sqrt{f}\ldots}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{(4bep)\,\text{Subst}\left(\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}\sqrt{h}}\right)}{-\sqrt{-d}\sqrt{e}\,h+ex^2}\,d\right.}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{(4bep)\,\text{Subst}\left(\int\left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}\ldots}{\sqrt{f}\ldots}\right.}{2e^{3/4}\left(\sqrt[4]{-d}\sqrt{-h}\right.}\right.\right.}{\ldots}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} + \frac{\left(2b\sqrt[4]{e}\,p\right)\text{Subst}\left(\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}\ldots}\right.}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} + \frac{8bp\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{\ldots}}{\sqrt{f}\sqrt{h}-\ldots}\right.}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} + \frac{8bp\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{\ldots}}{\sqrt{f}\sqrt{h}-\ldots}\right.}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} + \frac{8bp\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{\ldots}}{\sqrt{f}\sqrt{h}-\ldots}\right.}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

Mathematica [A] time = 0.39, size = 1297, normalized size = 0.95

$$\sqrt{x}\left(a\log\left(\sqrt{-f}-\sqrt{g}\sqrt{x}\right)-bp\log\left(\frac{\sqrt{g}\left(\sqrt[4]{-d}-\sqrt[4]{e}\sqrt{x}\right)}{\sqrt[4]{-d}\sqrt{g}-\sqrt[4]{e}\sqrt{-f}}\right)\log\left(\sqrt{-f}-\sqrt{g}\sqrt{x}\right)-bp\log\left(\frac{\sqrt{g}\left(i\sqrt[4]{e}\sqrt{x}+\sqrt[4]{-d}\right)}{i\sqrt[4]{e}\sqrt{-f}+\sqrt[4]{-d}\sqrt{g}}\right)\log\left(\sqrt{-f}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^2)^p])/(Sqrt[h*x]*(f + g*x)),x]
```

```
[Out] (Sqrt[x]*(a*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] - b*p*Log[(Sqrt[g]*((-d)^(1/4)
- e^(1/4)*Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f
] - Sqrt[g]*Sqrt[x]] - b*p*Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/(
I*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] -
 b*p*Log[(Sqrt[g]*(I*(-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(
-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] - b*p*Log[(Sqrt[g]*((-d
)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sq
rt[-f] - Sqrt[g]*Sqrt[x]] - a*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*p*Log[(Sq
rt[g]*((-d)^(1/4) - e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g
])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*p*Log[(Sqrt[g]*((-d)^(1/4) - I*e^(1
/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqr
t[g]*Sqrt[x]] + b*p*Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/((-I)*e^
(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*p
*Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(
1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*Log[Sqrt[-f] - Sqrt[g]*S
qrt[x]]*Log[c*(d + e*x^2)^p] - b*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]]*Log[c*(d +
 e*x^2)^p] - b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)
*Sqrt[-f] - (-d)^(1/4)*Sqrt[g])] - b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt
[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] - b*p*PolyLog[2, (
e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt
[g])] - b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt
[-f] + (-d)^(1/4)*Sqrt[g])] + b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*S
qrt[x]))/(e^(1/4)*Sqrt[-f] - (-d)^(1/4)*Sqrt[g])] + b*p*PolyLog[2, (e^(1/4)
*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] +
 b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] +
I*(-d)^(1/4)*Sqrt[g])] + b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x
]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]))/(Sqrt[-f]*Sqrt[g]*Sqrt[h*x])
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{hx}\,b\log\left(\left(ex^2+d\right)^p c\right)+\sqrt{hx}\,a}{ghx^2+fhx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x, algorithm="fricas
")
```

```
[Out] integral((sqrt(h*x)*b*log((e*x^2 + d)^p*c) + sqrt(h*x)*a)/(g*h*x^2 + f*h*x)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\log\left(\left(ex^2+d\right)^p c\right)+a}{\left(gx+f\right)\sqrt{hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^2 + d)^p*c) + a)/((g*x + f)*sqrt(h*x)), x)
```

maple [F] time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{b\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)+a}{\sqrt{hx}\,\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(1/2)/(g*x+f),x)
```

```
[Out] int((b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(1/2)/(g*x+f),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{\sqrt{h}\,p\log\left(ex^{2}+d\right)+\sqrt{h}\,\log(c)}{ghx^{\frac{3}{2}}+fh\sqrt{x}}\,dx+\frac{2\,a\arctan\left(\frac{\sqrt{hx}\,g}{\sqrt{fgh}}\right)}{\sqrt{fgh}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x, algorithm="maxima
")
```

```
[Out] b*integrate((sqrt(h)*p*log(e*x^2 + d) + sqrt(h)*log(c))/(g*h*x^(3/2) + f*h*
sqrt(x)), x) + 2*a*arctan(sqrt(h*x)*g/sqrt(f*g*h))/sqrt(f*g*h)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{\left(f+g\,x\right)\,\sqrt{h\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^2)^p))/((f + g*x)*(h*x)^(1/2)),x)
```

```
[Out] int((a + b*log(c*(d + e*x^2)^p))/((f + g*x)*(h*x)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(e*x**2+d)**p))/(h*x)**(1/2)/(g*x+f),x)
```

```
[Out] Timed out
```

3.618 $\int \frac{a+b\log\left(c\left(d+ex^2\right)^p\right)}{(hx)^{3/2}(f+gx)}\,dx$

Optimal. Leaf size=1659

$$-\frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}\,fh^{3/2}}+\frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{\sqrt[4]{d}\,fh^{3/2}}-\frac{2\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(ex^2+d\right.\right.\right.}{f^{3/2}h^{3/2}}$$

```
[Out] -2*b*e^(1/4)*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2
)/d^(1/4)/f/h^(3/2)+2*b*e^(1/4)*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1
/4)/h^(1/2))*2^(1/2)/d^(1/4)/f/h^(3/2)+b*e^(1/4)*p*ln(d^(1/2)*h^(1/2)+x*e^(
1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(1/4)/f/h^(3/2)
-b*e^(1/4)*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(
h*x)^(1/2))*2^(1/2)/d^(1/4)/f/h^(3/2)-2*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/
h^(1/2))*(a+b*ln(c*(e*x^2+d)^p))*g^(1/2)/f^(3/2)/h^(3/2)-8*b*p*arctan(g^(1/
2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*h^(1/2)/(f^(1/2)*h^(1/2)-I*g^(
1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)+2*b*p*arctan(g^(1/2)*(h*x)^(1/2)
/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/
4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(
f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)+2*b*p*arcta
n(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^
(1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)-(-d)^(1/4)*g^(1/2))/(f^(1/2)*
h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)+2*b*p*arctan(g^(1/2
)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h
)^(1/2)+e^(1/4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/
2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2
)+2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*((
-d)^(1/4)*h^(1/2)+e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)+(-d)^(1/4)*g^(1/2
))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)-I*b*p*p
olylog(2,1-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)+e^(1/4)*(h*x)^(1/2))/(I*e^
(1/4)*f^(1/2)+(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*
g^(1/2)/f^(3/2)/h^(3/2)-I*b*p*polylog(2,1-2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(
1/4)*(-h)^(1/2)-e^(1/4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)-I*e^(1/
4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2
)/h^(3/2)-I*b*p*polylog(2,1-2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2
)+e^(1/4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/2)*h^(
1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)+4*I*
b*p*polylog(2,1-2*f^(1/2)*h^(1/2)/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*
g^(1/2)/f^(3/2)/h^(3/2)-I*b*p*polylog(2,1+2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(
1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)-(-d)^(1/4)*g^(1/2))/(f^(1/2)*h
^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)-2*(a+b*ln(c*(e*x^2+d
)^p))/f/h/(h*x)^(1/2)
```

Rubi [A] time = 2.53, antiderivative size = 1659, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 19, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.613$, Rules used = {2467, 2476, 2455, 297, 1162, 617, 204, 1165, 628, 205, 2470, 12, 260, 6725, 4928, 4856, 2402, 2315, 2447}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^2)^p])/((h*x)^(3/2)*(f + g*x)),x]
```

```
[Out] (-2*Sqrt[2]*b*e^(1/4)*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqr
t[h])])/(d^(1/4)*f*h^(3/2)) + (2*Sqrt[2]*b*e^(1/4)*p*ArcTan[1 + (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*f*h^(3/2)) - (2*(a + b*Log[c*
```

```
(d + e*x^2)^p]))/(f*h*Sqrt[h*x]) - (2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[h*x])/(S
qrt[f]*Sqrt[h])]*(a + b*Log[c*(d + e*x^2)^p]))/(f^(3/2)*h^(3/2)) + (Sqrt[2]
*b*e^(1/4)*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1
/4)*Sqrt[h*x]])/(d^(1/4)*f*h^(3/2)) - (Sqrt[2]*b*e^(1/4)*p*Log[Sqrt[d]*Sqrt
[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*f*h^
(3/2)) - (8*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(
2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])/(f^(3/2)*h^(3/
2)) + (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*S
qrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4)*Sqrt[h*x]))/(((-d)^(1
/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt
[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2)) + (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h
*x])/(Sqrt[f]*Sqrt[h])]*Log[(-2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/
4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] -
 I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2)) + (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]
*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*S
qrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] + I*e^(1/4)*Sqr
t[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2))
+ (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[
f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] +
(-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(
3/2)) + ((4*I)*b*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt
[h] - I*Sqrt[g]*Sqrt[h*x])])/(f^(3/2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2,
1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4)*Sqrt[h*x]))/(
((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h]
- I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2, 1 +
 (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*S
qrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^
(3/2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*(
(-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] + I
*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/
2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*
Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)^(1/4)*Sqrt[g])*(Sq
rt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2467

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
```

```
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + (g*x^k)/h)^r*(a + b*Log[c*(
d + (e*x^(k*n))/h^n)^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+ex^2\right)^p\right)}{(hx)^{3/2}(f+gx)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2\left(f+\frac{gx^2}{h}\right)}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{fx^2} - \frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{f\left(fh+gx^2\right)}\right)dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2}\,dx, x, \sqrt{hx}\right)}{fh} - \frac{(2g)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{fh+gx^2}\,dx, x, \sqrt{\ldots}\right.}{fh}$$

$$= -\frac{2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{fh\sqrt{hx}} - \frac{2\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f^{3/2}h^{3/2}} + \ldots$$

$$= -\frac{2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{fh\sqrt{hx}} - \frac{2\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f^{3/2}h^{3/2}} + \ldots$$

$$= -\frac{2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{fh\sqrt{hx}} - \frac{2\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f^{3/2}h^{3/2}} + \ldots$$

$$= -\frac{2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{fh\sqrt{hx}} - \frac{2\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f^{3/2}h^{3/2}} + \ldots$$

$$= -\frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}\,fh^{3/2}} + \frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}\,fh^{3/2}} - \frac{2\left(a+b\ldots\right.}{\ldots}$$

$$= -\frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}\,fh^{3/2}} + \frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}\,fh^{3/2}} - \frac{2\left(a+b\ldots\right.}{\ldots}$$

$$= -\frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}\,fh^{3/2}} + \frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}\,fh^{3/2}} - \frac{2\left(a+b\ldots\right.}{\ldots}$$

$$= -\frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}\,fh^{3/2}} + \frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}\,fh^{3/2}} - \frac{2\left(a+b\ldots\right.}{\ldots}$$

$$= -\frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}\,fh^{3/2}} + \frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}\,fh^{3/2}} - \frac{2\left(a+b\ldots\right.}{\ldots}$$

Mathematica [A] time = 1.62, size = 1336, normalized size = 0.81

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^2)^p])/((h*x)^(3/2)*(f + g*x)),x]
```

```
[Out] (x^(3/2)*((4*b*e^(1/4)*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + ArcTanh[(d
*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (2*(a + b*Log[c*(d + e*x^2)^p]
))/Sqrt[x] + (f*Sqrt[g]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]]*(a + b*Log[c*(d + e
*x^2)^p]))/(-f)^(3/2) + (Sqrt[g]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]]*(a + b*Log
[c*(d + e*x^2)^p]))/Sqrt[-f] + (b*Sqrt[g]*p*(Log[(Sqrt[g]*((-d)^(1/4) - e^(
1/4)*Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - S
qrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)
*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqr
t[g]*(I*(-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqr
t[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)
*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*
Sqrt[x]] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[
-f] - (-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x
]))/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[
-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] + PolyLo
g[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*
Sqrt[g])]))/Sqrt[-f] + (b*f*Sqrt[g]*p*(Log[(Sqrt[g]*((-d)^(1/4) - e^(1/4)*S
qrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sq
rt[x]] + Log[(Sqrt[g]*((-d)^(1/4) - I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt[-f]
 + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d
)^(1/4) + I*e^(1/4)*Sqrt[x]))/((-I)*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]
*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt[
x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqr
t[x]] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f]
 - (-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))
/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f]
 + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2
, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqr
t[g])]))/(-f)^(3/2)))/(f*(h*x)^(3/2))
```

fricas [F] time = 1.33, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{hx}\,b\log\left(\left(ex^2+d\right)^p c\right)+\sqrt{hx}\,a}{gh^2x^3+fh^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x, algorithm="fricas
")
```

```
[Out] integral((sqrt(h*x)*b*log((e*x^2 + d)^p*c) + sqrt(h*x)*a)/(g*h^2*x^3 + f*h^
2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\log\left(\left(ex^2+d\right)^p c\right)+a}{\left(gx+f\right)\left(hx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^2 + d)^p*c) + a)/((g*x + f)*(h*x)^(3/2)), x)
```

maple [F] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{b\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)+a}{\left(hx\right)^{\frac{3}{2}}\left(gx+f\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(3/2)/(g*x+f),x)
[Out] int((b*ln(c*(e*x^2+d)^p)+a)/(h*x)^(3/2)/(g*x+f),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{\sqrt{h}\,p\log\left(ex^{2}+d\right)+\sqrt{h}\,\log\left(c\right)}{gh^{2}x^{\frac{5}{2}}+fh^{2}x^{\frac{3}{2}}}\,dx-\frac{2\,a\left(\frac{g\arctan\left(\frac{\sqrt{hx}\,g}{\sqrt{fgh}}\right)}{\sqrt{fgh}\,f}+\frac{1}{\sqrt{hx}\,f}\right)}{h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x, algorithm="maxima
")
[Out] b*integrate((sqrt(h)*p*log(e*x^2 + d) + sqrt(h)*log(c))/(g*h^2*x^(5/2) + f*
h^2*x^(3/2)), x) - 2*a*(g*arctan(sqrt(h*x)*g/sqrt(f*g*h))/(sqrt(f*g*h)*f) +
 1/(sqrt(h*x)*f))/h
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{\left(f+g\,x\right)\,{\left(h\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^2)^p))/((f + g*x)*(h*x)^(3/2)),x)
[Out] int((a + b*log(c*(d + e*x^2)^p))/((f + g*x)*(h*x)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(e*x**2+d)**p))/(h*x)**(3/2)/(g*x+f),x)
[Out] Timed out
```

3.619 $\int \frac{\log(fx^p)\log(1+ex^m)}{x}\,dx$

Optimal. Leaf size=33

$$\frac{p\text{Li}_3(-ex^m)}{m^2}-\frac{\text{Li}_2(-ex^m)\log(fx^p)}{m}$$

```
[Out] -ln(f*x^p)*polylog(2,-e*x^m)/m+p*polylog(3,-e*x^m)/m^2
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2374, 6589}

$$\frac{p\text{PolyLog}(3,-ex^m)}{m^2}-\frac{\log(fx^p)\text{PolyLog}(2,-ex^m)}{m}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[f*x^p]*Log[1 + e*x^m])/x,x]
```

```
[Out] -((Log[f*x^p]*PolyLog[2, -(e*x^m)])/m) + (p*PolyLog[3, -(e*x^m)])/m^2
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int\frac{\log(fx^p)\log(1+ex^m)}{x}\,dx&=-\frac{\log(fx^p)\text{Li}_2(-ex^m)}{m}+\frac{p\int\frac{\text{Li}_2(-ex^m)}{x}\,dx}{m}\\&=-\frac{\log(fx^p)\text{Li}_2(-ex^m)}{m}+\frac{p\text{Li}_3(-ex^m)}{m^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{p\text{Li}_3(-ex^m)}{m^2}-\frac{\text{Li}_2(-ex^m)\log(fx^p)}{m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[f*x^p]*Log[1 + e*x^m])/x,x]
```

```
[Out] -((Log[f*x^p]*PolyLog[2, -(e*x^m)])/m) + (p*PolyLog[3, -(e*x^m)])/m^2
```

fricas [C] time = 0.70, size = 35, normalized size = 1.06

$$-\frac{(mp\log(x)+m\log(f))\text{Li}_2(-ex^m)-p\text{polylog}(3,-ex^m)}{m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(f*x^p)*log(1+e*x^m)/x,x, algorithm="fricas")

[Out] -((m*p*log(x) + m*log(f))*dilog(-e*x^m) - p*polylog(3, -e*x^m))/m^2

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(ex^m+1)\log\left(fx^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(f*x^p)*log(1+e*x^m)/x,x, algorithm="giac")

[Out] integrate(log(e*x^m + 1)*log(f*x^p)/x, x)

maple [C] time = 2.06, size = 191, normalized size = 5.79

$$\frac{i\pi\,\mathrm{dilog}(e\,x^m+1)\,\mathrm{csgn}\left(if\right)\mathrm{csgn}\left(ix^p\right)\mathrm{csgn}\left(if\,x^p\right)}{2m}-\frac{i\pi\,\mathrm{dilog}(e\,x^m+1)\,\mathrm{csgn}\left(if\right)\mathrm{csgn}\left(if\,x^p\right)^2}{2m}-\frac{i\pi\,\mathrm{dilog}(e\,x^m$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(f*x^p)*ln(1+e*x^m)/x,x)

[Out] -p/m*ln(x)*polylog(2,-e*x^m)+p*polylog(3,-e*x^m)/m^2-(ln(x^p)-p*ln(x))/m*dilog(1+e*x^m)+1/2*I/m*dilog(1+e*x^m)*Pi*csgn(I*f)*csgn(I*x^p)*csgn(I*f*x^p)-1/2*I/m*dilog(1+e*x^m)*Pi*csgn(I*f)*csgn(I*f*x^p)^2-1/2*I/m*dilog(1+e*x^m)*Pi*csgn(I*x^p)*csgn(I*f*x^p)^2+1/2*I/m*dilog(1+e*x^m)*Pi*csgn(I*f*x^p)^3-1/m*dilog(1+e*x^m)*ln(f)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(p\log(x)^2-2\,\log(f)\log(x)-2\,\log(x)\log\left(x^p\right)\right)\log\left(ex^m+1\right)-\int\frac{2\,emx^m\log(x)\log\left(x^p\right)-\left(emp\log(x)^2-2\right.}{2\left(exx^m+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(f*x^p)*log(1+e*x^m)/x,x, algorithm="maxima")

[Out] -1/2*(p*log(x)^2 - 2*log(f)*log(x) - 2*log(x)*log(x^p))*log(e*x^m + 1) - integrate(1/2*(2*e*m*x^m*log(x)*log(x^p) - (e*m*p*log(x)^2 - 2*e*m*log(f)*log(x))*x^m)/(e*x*x^m + x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(f\,x^p\right)\,\ln\left(e\,x^m+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((log(f*x^p)*log(e*x^m + 1))/x,x)

[Out] int((log(f*x^p)*log(e*x^m + 1))/x, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(f*x**p)*ln(1+e*x**m)/x,x)

[Out] Timed out

3.620 $\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx$

Optimal. Leaf size=75

$$\frac{2p\log\left(fx^p\right)\text{Li}_2\left(-\frac{ex^m}{d}\right)}{em^2}+\frac{\log^2\left(fx^p\right)\log\left(\frac{ex^m}{d}+1\right)}{em}-\frac{2p^2\text{Li}_3\left(-\frac{ex^m}{d}\right)}{em^3}$$

```
[Out] ln(f*x^p)^2*ln(1+e*x^m/d)/e/m+2*p*ln(f*x^p)*polylog(2,-e*x^m/d)/e/m^2-2*p^2
*polylog(3,-e*x^m/d)/e/m^3
```

Rubi [A] time = 0.12, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2337, 2374, 6589}

$$\frac{2p\log\left(fx^p\right)\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{em^2}-\frac{2p^2\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{em^3}+\frac{\log^2\left(fx^p\right)\log\left(\frac{ex^m}{d}+1\right)}{em}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + m)*Log[f*x^p]^2)/(d + e*x^m),x]
```

```
[Out] (Log[f*x^p]^2*Log[1 + (e*x^m)/d])/(e*m) + (2*p*Log[f*x^p]*PolyLog[2, -((e*x
^m)/d)])/(e*m^2) - (2*p^2*PolyLog[3, -((e*x^m)/d)])/(e*m^3)
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+m}\log^2\left(fx^p\right)}{d+ex^m}\,dx &= \frac{\log^2\left(fx^p\right)\log\left(1+\frac{ex^m}{d}\right)}{em}-\frac{(2p)\int \frac{\log\left(fx^p\right)\log\left(1+\frac{ex^m}{d}\right)}{x}\,dx}{em}\\ &= \frac{\log^2\left(fx^p\right)\log\left(1+\frac{ex^m}{d}\right)}{em}+\frac{2p\log\left(fx^p\right)\text{Li}_2\left(-\frac{ex^m}{d}\right)}{em^2}-\frac{\left(2p^2\right)\int \frac{\text{Li}_2\left(-\frac{ex^m}{d}\right)}{x}\,dx}{em^2}\\ &= \frac{\log^2\left(fx^p\right)\log\left(1+\frac{ex^m}{d}\right)}{em}+\frac{2p\log\left(fx^p\right)\text{Li}_2\left(-\frac{ex^m}{d}\right)}{em^2}-\frac{2p^2\text{Li}_3\left(-\frac{ex^m}{d}\right)}{em^3}\end{aligned}$$

Mathematica [B] time = 0.14, size = 210, normalized size = 2.80

$$\frac{3m^2\log^2\left(fx^p\right)\log\left(d+ex^m\right)+6mp\left(p\log(x)-\log\left(fx^p\right)\right)\text{Li}_2\left(\frac{ex^m}{d}+1\right)-6mp\log\left(fx^p\right)\log\left(-\frac{ex^m}{d}\right)\log\left(d+ex^m\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + m)*Log[f*x^p]^2)/(d + e*x^m),x]
```

```
[Out] (m^3*p^2*Log[x]^3 + 3*m^2*p^2*Log[x]^2*Log[1 + d/(e*x^m)] - 3*m^2*p^2*Log[x
]^2*Log[d + e*x^m] + 6*m*p^2*Log[x]*Log[-((e*x^m)/d)]*Log[d + e*x^m] - 6*m*
p*Log[-((e*x^m)/d)]*Log[f*x^p]*Log[d + e*x^m] + 3*m^2*Log[f*x^p]^2*Log[d +
e*x^m] - 6*m*p^2*Log[x]*PolyLog[2, -(d/(e*x^m))] + 6*m*p*(p*Log[x] - Log[f*
x^p])*PolyLog[2, 1 + (e*x^m)/d] - 6*p^2*PolyLog[3, -(d/(e*x^m))])/(3*e*m^3)
```

fricas [C] time = 0.82, size = 105, normalized size = 1.40

$$\frac{m^2\log\left(ex^m+d\right)\log(f)^2-2\,p^2\text{polylog}\left(3,-\frac{ex^m}{d}\right)+2\left(mp^2\log(x)+mp\log(f)\right)\text{Li}_2\left(-\frac{ex^m+d}{d}+1\right)+\left(m^2p^2\log(x)^2\right.}{em^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*log(f*x^p)^2/(d+e*x^m),x, algorithm="fricas")
```

```
[Out] (m^2*log(e*x^m + d)*log(f)^2 - 2*p^2*polylog(3, -e*x^m/d) + 2*(m*p^2*log(x)
 + m*p*log(f))*dilog(-(e*x^m + d)/d + 1) + (m^2*p^2*log(x)^2 + 2*m^2*p*log(
f)*log(x))*log((e*x^m + d)/d))/(e*m^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m-1}\log\left(fx^p\right)^2}{ex^m+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*log(f*x^p)^2/(d+e*x^m),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)*log(f*x^p)^2/(e*x^m + d), x)
```

maple [C] time = 0.48, size = 1373, normalized size = 18.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-1)*ln(f*x^p)^2/(e*x^m+d),x)
```

```
[Out] 1/m*ln(e*x^m+d)/e*ln(f)^2+1/m*(-p*ln(x)+ln(x^p))^2*ln(e*x^m+d)/e+I/m*ln(e*x
^m+d)/e*p*ln(x)*Pi*csgn(I*f)*csgn(I*x^p)*csgn(I*f*x^p)+I/m*ln(e*x^m+d)/e*ln
(f)*Pi*csgn(I*f)*csgn(I*f*x^p)^2+I/m*ln(e*x^m+d)/e*ln(x^p)*Pi*csgn(I*f)*csg
n(I*f*x^p)^2+I/m*ln(e*x^m+d)/e*ln(f)*Pi*csgn(I*x^p)*csgn(I*f*x^p)^2-I/m*p*l
n(x)*ln((e*x^m+d)/d)/e*Pi*csgn(I*f*x^p)^3-I/m*ln(e*x^m+d)/e*ln(x^p)*Pi*csgn
(I*f*x^p)^3+I/m^2*p*dilog((e*x^m+d)/d)/e*Pi*csgn(I*f)*csgn(I*f*x^p)^2+I/m*l
n(e*x^m+d)/e*p*ln(x)*Pi*csgn(I*f*x^p)^3+2/m*p*ln(x)*ln((e*x^m+d)/d)/e*ln(f)
-2/m*ln(e*x^m+d)/e*p*ln(x)*ln(f)+1/2/m*ln(e*x^m+d)/e*Pi^2*csgn(I*x^p)*csgn(
I*f*x^p)^5+I/m^2*p*dilog((e*x^m+d)/d)/e*Pi*csgn(I*x^p)*csgn(I*f*x^p)^2-1/4/
m*ln(e*x^m+d)/e*Pi^2*csgn(I*f)^2*csgn(I*f*x^p)^4-1/4/m*ln(e*x^m+d)/e*Pi^2*c
sgn(I*x^p)^2*csgn(I*f*x^p)^4+1/2/m*ln(e*x^m+d)/e*Pi^2*csgn(I*f)*csgn(I*f*x^
p)^5+I/m*ln(e*x^m+d)/e*ln(x^p)*Pi*csgn(I*x^p)*csgn(I*f*x^p)^2-2*p^2*polylog
(3,-1/d*e*x^m)/e/m^3+2/m*p*(-p*ln(x)+ln(x^p))*ln(x)*ln((e*x^m+d)/d)/e-I/m*p
*ln(x)*ln((e*x^m+d)/d)/e*Pi*csgn(I*f)*csgn(I*x^p)*csgn(I*f*x^p)+I/m*p*ln(x)
```

```
*ln((e*x^m+d)/d)/e*Pi*csgn(I*x^p)*csgn(I*f*x^p)^2-I/m^2*p*dilog((e*x^m+d)/d
)/e*Pi*csgn(I*f)*csgn(I*x^p)*csgn(I*f*x^p)-I/m*ln(e*x^m+d)/e*ln(x^p)*Pi*csg
n(I*f)*csgn(I*x^p)*csgn(I*f*x^p)+I/m*p*ln(x)*ln((e*x^m+d)/d)/e*Pi*csgn(I*f)
*csgn(I*f*x^p)^2-I/m*ln(e*x^m+d)/e*p*ln(x)*Pi*csgn(I*x^p)*csgn(I*f*x^p)^2-I
/m^2*p*dilog((e*x^m+d)/d)/e*Pi*csgn(I*f*x^p)^3-I/m*ln(e*x^m+d)/e*ln(f)*Pi*c
sgn(I*f*x^p)^3+1/2/m*ln(e*x^m+d)/e*Pi^2*csgn(I*f)^2*csgn(I*x^p)*csgn(I*f*x^
p)^3-1/m*ln(e*x^m+d)/e*Pi^2*csgn(I*f)*csgn(I*x^p)*csgn(I*f*x^p)^4+1/2/m*ln(
e*x^m+d)/e*Pi^2*csgn(I*f)*csgn(I*x^p)^2*csgn(I*f*x^p)^3-1/4/m*ln(e*x^m+d)/e
*Pi^2*csgn(I*f)^2*csgn(I*x^p)^2*csgn(I*f*x^p)^2-I/m*ln(e*x^m+d)/e*ln(f)*Pi*
csgn(I*f)*csgn(I*x^p)*csgn(I*f*x^p)-I/m*ln(e*x^m+d)/e*p*ln(x)*Pi*csgn(I*f)*
csgn(I*f*x^p)^2+2/m^2*p^2/e*ln(x)*polylog(2,-1/d*e*x^m)+2/m^2*p*(-p*ln(x)+l
n(x^p))*dilog((e*x^m+d)/d)/e+2/m*ln(e*x^m+d)/e*ln(x^p)*ln(f)+2/m^2*p*dilog(
(e*x^m+d)/d)/e*ln(f)+1/m*p^2/e*ln(x)^2*ln(1/d*e*x^m+1)-1/4/m*ln(e*x^m+d)/e*
Pi^2*csgn(I*f*x^p)^6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m-1}\log\left(fx^p\right)^2}{ex^m + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*log(f*x^p)^2/(d+e*x^m),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 1)*log(f*x^p)^2/(e*x^m + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{m-1}\ln\left(f\,x^p\right)^2}{d + e\,x^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(m - 1)*log(f*x^p)^2)/(d + e*x^m),x)
```

```
[Out] int((x^(m - 1)*log(f*x^p)^2)/(d + e*x^m), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m-1}\log\left(fx^p\right)^2}{d + ex^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*ln(f*x**p)**2/(d+e*x**m),x)
```

```
[Out] Integral(x**(m - 1)*log(f*x**p)**2/(d + e*x**m), x)
```

3.621 $\int \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx$

Optimal. Leaf size=161

$$\frac{\log^4(fx^p)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{4p}-\frac{6bnp^2\log(fx^p)\,\mathrm{Li}_4\left(-\frac{ex^m}{d}\right)}{m^3}+\frac{3bnp\log^2(fx^p)\,\mathrm{Li}_3\left(-\frac{ex^m}{d}\right)}{m^2}-\frac{bn\log^3(fx^p)\,\mathrm{L}}{m}$$

```
[Out] 1/4*ln(f*x^p)^4*(a+b*ln(c*(d+e*x^m)^n))/p-1/4*b*n*ln(f*x^p)^4*ln(1+e*x^m/d)
/p-b*n*ln(f*x^p)^3*polylog(2,-e*x^m/d)/m+3*b*n*p*ln(f*x^p)^2*polylog(3,-e*x
^m/d)/m^2-6*b*n*p^2*ln(f*x^p)*polylog(4,-e*x^m/d)/m^3+6*b*n*p^3*polylog(5,-
e*x^m/d)/m^4
```

Rubi [A] time = 0.22, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2481, 2337, 2374, 2383, 6589}

$$-\frac{6bnp^2\log(fx^p)\,\mathrm{PolyLog}\left(4,-\frac{ex^m}{d}\right)}{m^3}+\frac{3bnp\log^2(fx^p)\,\mathrm{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}-\frac{bn\log^3(fx^p)\,\mathrm{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}+\frac{6}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[f*x^p]^3*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] (Log[f*x^p]^4*(a + b*Log[c*(d + e*x^m)^n]))/(4*p) - (b*n*Log[f*x^p]^4*Log[1
 + (e*x^m)/d])/(4*p) - (b*n*Log[f*x^p]^3*PolyLog[2, -((e*x^m)/d)])/m + (3*b
*n*p*Log[f*x^p]^2*PolyLog[3, -((e*x^m)/d)])/m^2 - (6*b*n*p^2*Log[f*x^p]*Pol
yLog[4, -((e*x^m)/d)])/m^3 + (6*b*n*p^3*PolyLog[5, -((e*x^m)/d)])/m^4
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2481

```
Int[(Log[(f_.)*(x_)^(q_.)]^(m_.)*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_)
)^(p_.)]*(b_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^q]^(m + 1)*(a + b*Log[c*(
d + e*x^n)^p]))/(q*(m + 1)), x] - Dist[(b*e*n*p)/(q*(m + 1)), Int[(x^(n - 1
)*Log[f*x^q]^(m + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n,
 p, q}, x] && NeQ[m, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^3\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{x}\,dx = \frac{\log^4\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{4p} - \frac{(bemn)\int \frac{x^{-1+m}\log^4\left(fx^p\right)}{d+ex^m}\,d}{4p}$$

$$= \frac{\log^4\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{4p} - \frac{bn\log^4\left(fx^p\right)\log\left(1+\frac{e}{}\right.}{4p}$$

$$= \frac{\log^4\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{4p} - \frac{bn\log^4\left(fx^p\right)\log\left(1+\frac{e}{}\right.}{4p}$$

$$= \frac{\log^4\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{4p} - \frac{bn\log^4\left(fx^p\right)\log\left(1+\frac{e}{}\right.}{4p}$$

$$= \frac{\log^4\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{4p} - \frac{bn\log^4\left(fx^p\right)\log\left(1+\frac{e}{}\right.}{4p}$$

$$= \frac{\log^4\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{4p} - \frac{bn\log^4\left(fx^p\right)\log\left(1+\frac{e}{}\right.}{4p}$$

Mathematica [B] time = 0.28, size = 659, normalized size = 4.09

$$\frac{a\log^4\left(fx^p\right)}{4p}+bp^2\log^3(x)\log\left(fx^p\right)\log\left(c\left(d+ex^m\right)^n\right)+b\log(x)\log^3\left(fx^p\right)\log\left(c\left(d+ex^m\right)^n\right)-\frac{3}{2}bp\log^2(x)\log$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[f*x^p]^3*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] (-3*b*m*n*p^3*Log[x]^5)/10 + (3*b*m*n*p^2*Log[x]^4*Log[f*x^p])/4 - (b*m*n*p
*Log[x]^3*Log[f*x^p]^2)/2 + (a*Log[f*x^p]^4)/(4*p) - (3*b*n*p^3*Log[x]^4*Lo
g[1 + d/(e*x^m)])/4 + 2*b*n*p^2*Log[x]^3*Log[f*x^p]*Log[1 + d/(e*x^m)] - (3
*b*n*p*Log[x]^2*Log[f*x^p]^2*Log[1 + d/(e*x^m)])/2 + b*n*p^3*Log[x]^4*Log[d
 + e*x^m] - (b*n*p^3*Log[x]^3*Log[-((e*x^m)/d)]*Log[d + e*x^m])/m - 3*b*n*p
^2*Log[x]^3*Log[f*x^p]*Log[d + e*x^m] + (3*b*n*p^2*Log[x]^2*Log[-((e*x^m)/d
)]*Log[f*x^p]*Log[d + e*x^m])/m + 3*b*n*p*Log[x]^2*Log[f*x^p]^2*Log[d + e*x
^m] - (3*b*n*p*Log[x]*Log[-((e*x^m)/d)]*Log[f*x^p]^2*Log[d + e*x^m])/m - b*
n*Log[x]*Log[f*x^p]^3*Log[d + e*x^m] + (b*n*Log[-((e*x^m)/d)]*Log[f*x^p]^3*
Log[d + e*x^m])/m - (b*p^3*Log[x]^4*Log[c*(d + e*x^m)^n])/4 + b*p^2*Log[x]^
3*Log[f*x^p]*Log[c*(d + e*x^m)^n] - (3*b*p*Log[x]^2*Log[f*x^p]^2*Log[c*(d +
 e*x^m)^n])/2 + b*Log[x]*Log[f*x^p]^3*Log[c*(d + e*x^m)^n] + (b*n*p*Log[x]*
(p^2*Log[x]^2 - 3*p*Log[x]*Log[f*x^p] + 3*Log[f*x^p]^2)*PolyLog[2, -(d/(e*x
^m))])/m - (b*n*(p*Log[x] - Log[f*x^p])^3*PolyLog[2, 1 + (e*x^m)/d])/m + (3
*b*n*p*Log[f*x^p]^2*PolyLog[3, -(d/(e*x^m))])/m^2 + (6*b*n*p^2*Log[f*x^p]*P
olyLog[4, -(d/(e*x^m))])/m^3 + (6*b*n*p^3*PolyLog[5, -(d/(e*x^m))])/m^4
```

fricas [C] time = 1.29, size = 417, normalized size = 2.59

$$24\,bnp^3\mathrm{polylog}\left(5,-\frac{ex^m}{d}\right)+4\left(bm^4\log(c)+am^4\right)\log(f)^3\log(x)+6\left(bm^4p\log(c)+am^4p\right)\log(f)^2\log(x)^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)^3*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")
```

```
[Out] 1/4*(24*b*n*p^3*polylog(5, -e*x^m/d) + 4*(b*m^4*log(c) + a*m^4)*log(f)^3*lo
g(x) + 6*(b*m^4*p*log(c) + a*m^4*p)*log(f)^2*log(x)^2 + 4*(b*m^4*p^2*log(c)
 + a*m^4*p^2)*log(f)*log(x)^3 + (b*m^4*p^3*log(c) + a*m^4*p^3)*log(x)^4 - 4
*(b*m^3*n*p^3*log(x)^3 + 3*b*m^3*n*p^2*log(f)*log(x)^2 + 3*b*m^3*n*p*log(f)
^2*log(x) + b*m^3*n*log(f)^3)*dilog(-(e*x^m + d)/d + 1) + (b*m^4*n*p^3*log(
x)^4 + 4*b*m^4*n*p^2*log(f)*log(x)^3 + 6*b*m^4*n*p*log(f)^2*log(x)^2 + 4*b*
m^4*n*log(f)^3*log(x))*log(e*x^m + d) - (b*m^4*n*p^3*log(x)^4 + 4*b*m^4*n*p
^2*log(f)*log(x)^3 + 6*b*m^4*n*p*log(f)^2*log(x)^2 + 4*b*m^4*n*log(f)^3*log
(x))*log((e*x^m + d)/d) - 24*(b*m*n*p^3*log(x) + b*m*n*p^2*log(f))*polylog(
4, -e*x^m/d) + 12*(b*m^2*n*p^3*log(x)^2 + 2*b*m^2*n*p^2*log(f)*log(x) + b*m
^2*n*p*log(f)^2)*polylog(3, -e*x^m/d))/m^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\log\left((ex^m+d)^n c\right)+a\right)\log\left(fx^p\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)^3*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^m + d)^n*c) + a)*log(f*x^p)^3/x, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\,(e\,x^m+d)^n\right)+a\right)\ln\left(f\,x^p\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(f*x^p)^3*(b*ln(c*(e*x^m+d)^n)+a)/x,x)
```

```
[Out] int(ln(f*x^p)^3*(b*ln(c*(e*x^m+d)^n)+a)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}\left(bp^3\log(x)^4-4\,bp^2\log(f)\log(x)^3+6\,bp\log(f)^2\log(x)^2-4\,b\log(f)^3\log(x)-4\,b\log(x)\log\left(x^p\right)^3+6\left(bp\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)^3*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
```

```
[Out] -1/4*(b*p^3*log(x)^4 - 4*b*p^2*log(f)*log(x)^3 + 6*b*p*log(f)^2*log(x)^2 -
4*b*log(f)^3*log(x) - 4*b*log(x)*log(x^p)^3 + 6*(b*p*log(x)^2 - 2*b*log(f)*
log(x))*log(x^p)^2 - 4*(b*p^2*log(x)^3 - 3*b*p*log(f)*log(x)^2 + 3*b*log(f)
^2*log(x))*log(x^p))*log((e*x^m + d)^n) - integrate(-1/4*(4*b*d*log(c)*log(
f)^3 + 4*a*d*log(f)^3 + 4*(b*d*log(c) + a*d - (b*e*m*n*log(x) - b*e*log(c)
- a*e)*x^m)*log(x^p)^3 + 6*(2*b*d*log(c)*log(f) + 2*a*d*log(f) + (b*e*m*n*p
*log(x)^2 - 2*b*e*m*n*log(f)*log(x) + 2*b*e*log(c)*log(f) + 2*a*e*log(f))*x
^m)*log(x^p)^2 + (b*e*m*n*p^3*log(x)^4 - 4*b*e*m*n*p^2*log(f)*log(x)^3 + 6*
b*e*m*n*p*log(f)^2*log(x)^2 - 4*b*e*m*n*log(f)^3*log(x) + 4*b*e*log(c)*log(
f)^3 + 4*a*e*log(f)^3)*x^m + 4*(3*b*d*log(c)*log(f)^2 + 3*a*d*log(f)^2 - (b
*e*m*n*p^2*log(x)^3 - 3*b*e*m*n*p*log(f)*log(x)^2 + 3*b*e*m*n*log(f)^2*log(
x) - 3*b*e*log(c)*log(f)^2 - 3*a*e*log(f)^2)*x^m)*log(x^p))/(e*x*x^m + d*x)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(f\,x^p\right)^3\,\left(a+b\,\ln\left(c\,{\left(d+e\,x^m\right)}^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(f*x^p)^3*(a + b*log(c*(d + e*x^m)^n)))/x,x)
[Out] int((log(f*x^p)^3*(a + b*log(c*(d + e*x^m)^n)))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(f*x**p)**3*(a+b*ln(c*(d+e*x**m)**n))/x,x)
[Out] Timed out
```

3.622 $\int \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx$

Optimal. Leaf size=132

$$\frac{\log^3(fx^p)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{3p}+\frac{2bnp\log(fx^p)\operatorname{Li}_3\left(-\frac{ex^m}{d}\right)}{m^2}-\frac{bn\log^2(fx^p)\operatorname{Li}_2\left(-\frac{ex^m}{d}\right)}{m}-\frac{bn\log^3(fx^p)\log(\cdots}{3p}$$

```
[Out] 1/3*ln(f*x^p)^3*(a+b*ln(c*(d+e*x^m)^n))/p-1/3*b*n*ln(f*x^p)^3*ln(1+e*x^m/d)
/p-b*n*ln(f*x^p)^2*polylog(2,-e*x^m/d)/m+2*b*n*p*ln(f*x^p)*polylog(3,-e*x^m
/d)/m^2-2*b*n*p^2*polylog(4,-e*x^m/d)/m^3
```

Rubi [A] time = 0.19, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2481, 2337, 2374, 2383, 6589}

$$\frac{2bnp\log(fx^p)\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}-\frac{bn\log^2(fx^p)\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}-\frac{2bnp^2\operatorname{PolyLog}\left(4,-\frac{ex^m}{d}\right)}{m^3}+\frac{\log^3(fx^p)(a+\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[f*x^p]^2*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] (Log[f*x^p]^3*(a + b*Log[c*(d + e*x^m)^n]))/(3*p) - (b*n*Log[f*x^p]^3*Log[1
 + (e*x^m)/d])/(3*p) - (b*n*Log[f*x^p]^2*PolyLog[2, -((e*x^m)/d)])/m + (2*b
*n*p*Log[f*x^p]*PolyLog[3, -((e*x^m)/d)])/m^2 - (2*b*n*p^2*PolyLog[4, -((e*
x^m)/d)])/m^3
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2481

```
Int[(Log[(f_.)*(x_)^(q_.)]^(m_.)*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_)
)^(p_.)]*(b_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^q]^(m + 1)*(a + b*Log[c*(
d + e*x^n)^p]))/(q*(m + 1)), x] - Dist[(b*e*n*p)/(q*(m + 1)), Int[(x^(n - 1
)*Log[f*x^q]^(m + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n,
 p, q}, x] && NeQ[m, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^2\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{x}\,dx = \frac{\log^3\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{3p} - \frac{(bemn)\int \frac{x^{-1+m}\log^3\left(fx^p\right)}{d+ex^m}\,d}{3p}$$

$$= \frac{\log^3\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{3p} - \frac{bn\log^3\left(fx^p\right)\log\left(1+\frac{ex}{d}\right.}{3p}$$

$$= \frac{\log^3\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{3p} - \frac{bn\log^3\left(fx^p\right)\log\left(1+\frac{ex}{d}\right.}{3p}$$

$$= \frac{\log^3\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{3p} - \frac{bn\log^3\left(fx^p\right)\log\left(1+\frac{ex}{d}\right.}{3p}$$

$$= \frac{\log^3\left(fx^p\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{3p} - \frac{bn\log^3\left(fx^p\right)\log\left(1+\frac{ex}{d}\right.}{3p}$$

Mathematica [B] time = 0.25, size = 456, normalized size = 3.45

$$\frac{a\log^3\left(fx^p\right)}{3p} - bp\log^2(x)\log\left(fx^p\right)\log\left(c\left(d+ex^m\right)^n\right) + b\log(x)\log^2\left(fx^p\right)\log\left(c\left(d+ex^m\right)^n\right) + \frac{1}{3}bp^2\log^3(x)\log$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[f*x^p]^2*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] (b*m*n*p^2*Log[x]^4)/4 - (b*m*n*p*Log[x]^3*Log[f*x^p])/3 + (a*Log[f*x^p]^3)
/(3*p) + (2*b*n*p^2*Log[x]^3*Log[1 + d/(e*x^m)])/3 - b*n*p*Log[x]^2*Log[f*x
^p]*Log[1 + d/(e*x^m)] - b*n*p^2*Log[x]^3*Log[d + e*x^m] + (b*n*p^2*Log[x]^
2*Log[-((e*x^m)/d)]*Log[d + e*x^m])/m + 2*b*n*p*Log[x]^2*Log[f*x^p]*Log[d +
 e*x^m] - (2*b*n*p*Log[x]*Log[-((e*x^m)/d)]*Log[f*x^p]*Log[d + e*x^m])/m -
b*n*Log[x]*Log[f*x^p]^2*Log[d + e*x^m] + (b*n*Log[-((e*x^m)/d)]*Log[f*x^p]^
2*Log[d + e*x^m])/m + (b*p^2*Log[x]^3*Log[c*(d + e*x^m)^n])/3 - b*p*Log[x]^
2*Log[f*x^p]*Log[c*(d + e*x^m)^n] + b*Log[x]*Log[f*x^p]^2*Log[c*(d + e*x^m)
^n] - (b*n*p*Log[x]*(p*Log[x] - 2*Log[f*x^p])*PolyLog[2, -(d/(e*x^m))])/m +
 (b*n*(-(p*Log[x]) + Log[f*x^p])^2*PolyLog[2, 1 + (e*x^m)/d])/m + (2*b*n*p*
Log[f*x^p]*PolyLog[3, -(d/(e*x^m))])/m^2 + (2*b*n*p^2*PolyLog[4, -(d/(e*x^m
))])/m^3
```

fricas [C] time = 0.72, size = 281, normalized size = 2.13

$$-\frac{6\,bnp^2\mathrm{polylog}\left(4,-\frac{ex^m}{d}\right) - 3\left(bm^3\log(c)+am^3\right)\log(f)^2\log(x) - 3\left(bm^3p\log(c)+am^3p\right)\log(f)\log(x)^2 - \left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)^2*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")
```

```
[Out] -1/3*(6*b*n*p^2*polylog(4, -e*x^m/d) - 3*(b*m^3*log(c) + a*m^3)*log(f)^2*lo
g(x) - 3*(b*m^3*p*log(c) + a*m^3*p)*log(f)*log(x)^2 - (b*m^3*p^2*log(c) + a
```

```
*m^3*p^2)*log(x)^3 + 3*(b*m^2*n*p^2*log(x)^2 + 2*b*m^2*n*p*log(f)*log(x) +
b*m^2*n*log(f)^2)*dilog(-(e*x^m + d)/d + 1) - (b*m^3*n*p^2*log(x)^3 + 3*b*m
^3*n*p*log(f)*log(x)^2 + 3*b*m^3*n*log(f)^2*log(x))*log(e*x^m + d) + (b*m^3
*n*p^2*log(x)^3 + 3*b*m^3*n*p*log(f)*log(x)^2 + 3*b*m^3*n*log(f)^2*log(x))*
log((e*x^m + d)/d) - 6*(b*m*n*p^2*log(x) + b*m*n*p*log(f))*polylog(3, -e*x^
m/d))/m^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log\left((ex^m + d)^n c\right) + a\right) \log\left(fx^p\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)^2*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^m + d)^n*c) + a)*log(f*x^p)^2/x, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln\left(c\,(e\,x^m + d)^n\right) + a\right) \ln\left(f\,x^p\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(f*x^p)^2*(b*ln(c*(e*x^m+d)^n)+a)/x,x)
```

```
[Out] int(ln(f*x^p)^2*(b*ln(c*(e*x^m+d)^n)+a)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(bp^2 \log(x)^3 - 3\,bp \log(f) \log(x)^2 + 3\,b \log(f)^2 \log(x) + 3\,b \log(x) \log\left(x^p\right)^2 - 3\left(bp \log(x)^2 - 2\,b \log(f) \log(x)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)^2*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
```

```
[Out] 1/3*(b*p^2*log(x)^3 - 3*b*p*log(f)*log(x)^2 + 3*b*log(f)^2*log(x) + 3*b*log
(x)*log(x^p)^2 - 3*(b*p*log(x)^2 - 2*b*log(f)*log(x))*log(x^p))*log((e*x^m
+ d)^n) - integrate(-1/3*(3*b*d*log(c)*log(f)^2 + 3*a*d*log(f)^2 + 3*(b*d*l
og(c) + a*d - (b*e*m*n*log(x) - b*e*log(c) - a*e)*x^m)*log(x^p)^2 - (b*e*m*
n*p^2*log(x)^3 - 3*b*e*m*n*p*log(f)*log(x)^2 + 3*b*e*m*n*log(f)^2*log(x) -
3*b*e*log(c)*log(f)^2 - 3*a*e*log(f)^2)*x^m + 3*(2*b*d*log(c)*log(f) + 2*a*
d*log(f) + (b*e*m*n*p*log(x)^2 - 2*b*e*m*n*log(f)*log(x) + 2*b*e*log(c)*log
(f) + 2*a*e*log(f))*x^m)*log(x^p))/(e*x*x^m + d*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(f\,x^p\right)^2\,\left(a + b\,\ln\left(c\,(d + e\,x^m)^n\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(f*x^p)^2*(a + b*log(c*(d + e*x^m)^n)))/x,x)
```

```
[Out] int((log(f*x^p)^2*(a + b*log(c*(d + e*x^m)^n)))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \log\left(c\left(d + ex^m\right)^n\right)\right) \log\left(fx^p\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(f*x**p)**2*(a+b*ln(c*(d+e*x**m)**n))/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**m)**n))*log(f*x**p)**2/x, x)
```

3.623 $\int \frac{\log(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{x}\,dx$

Optimal. Leaf size=102

$$\frac{\log^2(fx^p)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{2p}-\frac{bn\log(fx^p)\,\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{m}-\frac{bn\log^2(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{2p}+\frac{bnp\,\mathrm{Li}_3\left(-\frac{ex^m}{d}\right)}{m^2}$$

```
[Out] 1/2*ln(f*x^p)^2*(a+b*ln(c*(d+e*x^m)^n))/p-1/2*b*n*ln(f*x^p)^2*ln(1+e*x^m/d)
/p-b*n*ln(f*x^p)*polylog(2,-e*x^m/d)/m+b*n*p*polylog(3,-e*x^m/d)/m^2
```

Rubi [A] time = 0.14, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2481, 2337, 2374, 6589}

$$-\frac{bn\log(fx^p)\,\mathrm{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}+\frac{bnp\,\mathrm{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}+\frac{\log^2(fx^p)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{2p}-\frac{bn\log^2(fx^p)}{2p}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[f*x^p]*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] (Log[f*x^p]^2*(a + b*Log[c*(d + e*x^m)^n]))/(2*p) - (b*n*Log[f*x^p]^2*Log[1
 + (e*x^m)/d])/(2*p) - (b*n*Log[f*x^p]*PolyLog[2, -((e*x^m)/d)])/m + (b*n*p
*PolyLog[3, -((e*x^m)/d)])/m^2
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2481

```
Int[(Log[(f_.)*(x_)^(q_.)]^(m_.)*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_)
)^(p_.)]*(b_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^q]^(m + 1)*(a + b*Log[c*(
d + e*x^n)^p]))/(q*(m + 1)), x] - Dist[(b*e*n*p)/(q*(m + 1)), Int[(x^(n - 1
)*Log[f*x^q]^(m + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n,
 p, q}, x] && NeQ[m, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p} - \frac{(bemn)\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx}{2p}$$

$$= \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p} - \frac{bn\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{2p}$$

$$= \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p} - \frac{bn\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{2p}$$

$$= \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p} - \frac{bn\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{2p}$$

Mathematica [B] time = 0.19, size = 265, normalized size = 2.60

$$\frac{a\log^2(fx^p)}{2p}+b\log(x)\log(fx^p)\log(c(d+ex^m)^n)-\frac{1}{2}bp\log^2(x)\log(c(d+ex^m)^n)-\frac{bn(p\log(x)-\log(fx^p))}{m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[f*x^p]*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] -1/6*(b*m*n*p*Log[x]^3) + (a*Log[f*x^p]^2)/(2*p) - (b*n*p*Log[x]^2*Log[1 +
d/(e*x^m)])/2 + b*n*p*Log[x]^2*Log[d + e*x^m] - (b*n*p*Log[x]*Log[-((e*x^m)
/d)]*Log[d + e*x^m])/m - b*n*Log[x]*Log[f*x^p]*Log[d + e*x^m] + (b*n*Log[-(
(e*x^m)/d)]*Log[f*x^p]*Log[d + e*x^m])/m - (b*p*Log[x]^2*Log[c*(d + e*x^m)^
n])/2 + b*Log[x]*Log[f*x^p]*Log[c*(d + e*x^m)^n] + (b*n*p*Log[x]*PolyLog[2,
 -(d/(e*x^m))])/m - (b*n*(p*Log[x] - Log[f*x^p])*PolyLog[2, 1 + (e*x^m)/d])
/m + (b*n*p*PolyLog[3, -(d/(e*x^m))])/m^2
```

fricas [C] time = 0.74, size = 161, normalized size = 1.58

$$2\,bnp\text{polylog}\left(3,-\frac{ex^m}{d}\right)+2\left(bm^2\log(c)+am^2\right)\log(f)\log(x)+\left(bm^2p\log(c)+am^2p\right)\log(x)^2-2\left(bmnp\log(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*n*p*polylog(3, -e*x^m/d) + 2*(b*m^2*log(c) + a*m^2)*log(f)*log(x)
+ (b*m^2*p*log(c) + a*m^2*p)*log(x)^2 - 2*(b*m*n*p*log(x) + b*m*n*log(f))*d
ilog(-(e*x^m + d)/d + 1) + (b*m^2*n*p*log(x)^2 + 2*b*m^2*n*log(f)*log(x))*l
og(e*x^m + d) - (b*m^2*n*p*log(x)^2 + 2*b*m^2*n*log(f)*log(x))*log((e*x^m +
 d)/d))/m^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\log((ex^m+d)^nc)+a)\log(fx^p)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^m + d)^n*c) + a)*log(f*x^p)/x, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\left(b\ln\left(c\left(e\,x^{m}+d\right)^{n}\right)+a\right)\ln\left(f\,x^{p}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(f*x^p)*(b*ln(c*(e*x^m+d)^n)+a)/x,x)
```

```
[Out] int(ln(f*x^p)*(b*ln(c*(e*x^m+d)^n)+a)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(bp\log(x)^{2}-2\,b\log(f)\log(x)-2\,b\log(x)\log\left(x^{p}\right)\right)\log\left(\left(ex^{m}+d\right)^{n}\right)-\int -\frac{2\,bd\log(c)\log(f)+2\,ad\log(f)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*p*log(x)^2 - 2*b*log(f)*log(x) - 2*b*log(x)*log(x^p))*log((e*x^m +
d)^n) - integrate(-1/2*(2*b*d*log(c)*log(f) + 2*a*d*log(f) + (b*e*m*n*p*log
(x)^2 - 2*b*e*m*n*log(f)*log(x) + 2*b*e*log(c)*log(f) + 2*a*e*log(f))*x^m +
 2*(b*d*log(c) + a*d - (b*e*m*n*log(x) - b*e*log(c) - a*e)*x^m)*log(x^p))/(
e*x*x^m + d*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(f\,x^{p}\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x^{m}\right)}^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(f*x^p)*(a + b*log(c*(d + e*x^m)^n)))/x,x)
```

```
[Out] int((log(f*x^p)*(a + b*log(c*(d + e*x^m)^n)))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(f*x**p)*(a+b*ln(c*(d+e*x**m)**n))/x,x)
```

```
[Out] Timed out
```

3.624 $\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x}\,dx$

Optimal. Leaf size=49

$$\frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{m}+\frac{bn\text{Li}_2\left(\frac{ex^m}{d}+1\right)}{m}$$

[Out] ln(-e*x^m/d)*(a+b*ln(c*(d+e*x^m)^n))/m+b*n*polylog(2,1+e*x^m/d)/m

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2454, 2394, 2315}

$$\frac{bn\text{PolyLog}\left(2,\frac{ex^m}{d}+1\right)}{m}+\frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{m}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^m)^n])/x,x]
```

```
[Out] (Log[-((e*x^m)/d)]*(a + b*Log[c*(d + e*x^m)^n]))/m + (b*n*PolyLog[2, 1 + (e
*x^m)/d])/m
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x}\,dx,x,x^m\right)}{m}\\
&= \frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{m}-\frac{(ben)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,x^m\right)}{m}\\
&= \frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c\left(d+ex^m\right)^n\right)\right)}{m}+\frac{bn\text{Li}_2\left(1+\frac{ex^m}{d}\right)}{m}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.00

$$a\log(x)+\frac{b\left(\log\left(-\frac{ex^m}{d}\right)\log\left(c\left(d+ex^m\right)^n\right)+n\text{Li}_2\left(\frac{ex^m+d}{d}\right)\right)}{m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^m)^n])/x,x]
[Out] a*Log[x] + (b*(Log[-((e*x^m)/d)]*Log[c*(d + e*x^m)^n] + n*PolyLog[2, (d + e
*x^m)/d]))/m
```

fricas [A] time = 0.77, size = 69, normalized size = 1.41

$$\frac{bmn\log\left(ex^m+d\right)\log(x)-bmn\log(x)\log\left(\frac{ex^m+d}{d}\right)-bn\text{Li}_2\left(-\frac{ex^m+d}{d}+1\right)+\left(bm\log(c)+am\right)\log(x)}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")
[Out] (b*m*n*log(e*x^m + d)*log(x) - b*m*n*log(x)*log((e*x^m + d)/d) - b*n*dilog(
-(e*x^m + d)/d + 1) + (b*m*log(c) + a*m)*log(x))/m
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\log\left(\left(ex^m+d\right)^nc\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
[Out] integrate((b*log((e*x^m + d)^n*c) + a)/x, x)
```

maple [C] time = 3.19, size = 189, normalized size = 3.86

$$-\frac{i\pi b\,\text{csgn}\left(ic\right)\text{csgn}\left(i\left(e\,x^m+d\right)^n\right)\text{csgn}\left(ic\left(e\,x^m+d\right)^n\right)\ln(x)}{2}+\frac{i\pi b\,\text{csgn}\left(ic\right)\text{csgn}\left(ic\left(e\,x^m+d\right)^n\right)^2\ln(x)}{2}+\frac{i\pi b\,\text{csg}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^m+d)^n)+a)/x,x)
[Out] b*ln(x)*ln((e*x^m+d)^n)+1/2*I*ln(x)*b*Pi*csgn(I*(e*x^m+d)^n)*csgn(I*c*(e*x^
m+d)^n)^2-1/2*I*ln(x)*b*Pi*csgn(I*(e*x^m+d)^n)*csgn(I*c*(e*x^m+d)^n)*csgn(I
*c)-1/2*I*ln(x)*b*Pi*csgn(I*c*(e*x^m+d)^n)^3+1/2*I*ln(x)*b*Pi*csgn(I*c*(e*x
^m+d)^n)^2*csgn(I*c)+b*ln(c)*ln(x)+a*ln(x)-b*n/m*dilog((e*x^m+d)/d)-b*n*ln(
x)*ln((e*x^m+d)/d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(2\,dmn\int\frac{\log(x)}{exx^m+dx}\,dx-mn\log(x)^2+2\,\log\left(\left(ex^m+d\right)^n\right)\log(x)+2\,\log(c)\log(x)\right)b+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
[Out] 1/2*(2*d*m*n*integrate(log(x)/(e*x*x^m + d*x), x) - m*n*log(x)^2 + 2*log((e
*x^m + d)^n)*log(x) + 2*log(c)*log(x))*b + a*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x^m\right)}^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^m)^n))/x,x)
[Out] int((a + b*log(c*(d + e*x^m)^n))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**m)**n))/x,x)
[Out] Integral((a + b*log(c*(d + e*x**m)**n))/x, x)
```

3.625 $\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log\left(fx^p\right)}\,dx$

Optimal. Leaf size=42

$$b\text{Int}\left(\frac{\log\left(c\left(d+ex^m\right)^n\right)}{x\log\left(fx^p\right)},x\right)+\frac{a\log\left(\log\left(fx^p\right)\right)}{p}$$

```
[Out] a*ln(ln(f*x^p))/p+b*Unintegrable(ln(c*(d+e*x^m)^n)/x/ln(f*x^p),x)
```

Rubi [A] time = 0.29, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log\left(fx^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]),x]
```

```
[Out] (a*Log[Log[f*x^p]])/p + b*Defer[Int][Log[c*(d + e*x^m)^n]/(x*Log[f*x^p]), x
]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log\left(fx^p\right)}\,dx &= \int \left(\frac{a}{x\log\left(fx^p\right)}+\frac{b\log\left(c\left(d+ex^m\right)^n\right)}{x\log\left(fx^p\right)}\right)dx\\
&= a\int \frac{1}{x\log\left(fx^p\right)}\,dx+b\int \frac{\log\left(c\left(d+ex^m\right)^n\right)}{x\log\left(fx^p\right)}\,dx\\
&= b\int \frac{\log\left(c\left(d+ex^m\right)^n\right)}{x\log\left(fx^p\right)}\,dx+\frac{a\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,\log\left(fx^p\right)\right)}{p}\\
&= \frac{a\log\left(\log\left(fx^p\right)\right)}{p}+b\int \frac{\log\left(c\left(d+ex^m\right)^n\right)}{x\log\left(fx^p\right)}\,dx
\end{aligned}$$

Mathematica [A] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log\left(fx^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]),x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]), x]
```

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\log\left(\left(ex^m+d\right)^n c\right)+a}{x\log\left(fx^p\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p),x, algorithm="fricas")
[Out] integral((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\log\left((ex^m+d)^n c\right)+a}{x\log\left(fx^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p),x, algorithm="giac")
[Out] integrate((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)), x)
```

maple [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{b\ln\left(c\,(e\,x^m+d)^n\right)+a}{x\ln\left(f\,x^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^m+d)^n)+a)/x/ln(f*x^p),x)
[Out] int((b*ln(c*(e*x^m+d)^n)+a)/x/ln(f*x^p),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{\log\left((ex^m+d)^n\right)+\log(c)}{x\log(f)+x\log\left(x^p\right)}\,dx+\frac{a\log\left(\log\left(fx^p\right)\right)}{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p),x, algorithm="maxima")
[Out] b*integrate((log((e*x^m + d)^n) + log(c))/(x*log(f) + x*log(x^p)), x) + a*l
og(log(f*x^p))/p
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x^m\right)}^n\right)}{x\,\ln\left(f\,x^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)),x)
[Out] int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log\left(fx^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**m)**n))/x/ln(f*x**p),x)
[Out] Integral((a + b*log(c*(d + e*x**m)**n))/(x*log(f*x**p)), x)
```

3.626 $\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2(fx^p)}\,dx$

Optimal. Leaf size=64

$$\frac{bemn\mathrm{Int}\left(\frac{x^{m-1}}{(d+ex^m)\log(fx^p)},x\right)}{p}-\frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{p\log\left(fx^p\right)}$$

```
[Out] (-a-b*ln(c*(d+e*x^m)^n))/p/ln(f*x^p)+b*e*m*n*Unintegrable(x^(-1+m)/(d+e*x^m
)/ln(f*x^p),x)/p
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log^2\left(fx^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^2),x]
```

```
[Out] -((a + b*Log[c*(d + e*x^m)^n])/(p*Log[f*x^p])) + (b*e*m*n*Defer[Int][x^(-1
+ m)/((d + e*x^m)*Log[f*x^p]), x])/p
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log^2\left(fx^p\right)}\,dx=-\frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{p\log\left(fx^p\right)}+\frac{(bemn)\int \frac{x^{-1+m}}{(d+ex^m)\log(fx^p)}\,dx}{p}$$

Mathematica [A] time = 2.16, size = 0, normalized size = 0.00

$$\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log^2\left(fx^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^2),x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^2), x]
```

fricas [A] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\log\left(\left(ex^m+d\right)^n c\right)+a}{x\log\left(fx^p\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\log\left(\left(ex^m+d\right)^n c\right)+a}{x\log\left(fx^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)^2), x)
```

maple [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{b \ln \left(c \left(e \,x^{m}+d \right)^{n}\right)+a}{x \ln \left(f \,x^{p}\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^m+d)^n)+a)/x/ln(f*x^p)^2,x)
```

```
[Out] int((b*ln(c*(e*x^m+d)^n)+a)/x/ln(f*x^p)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\left(emn \int \frac{x^{m}}{epxx^{m} \log(f) + dpx \log(f) + \left(epxx^{m} + dpx\right) \log\left(x^{p}\right)}\,dx - \frac{\log\left(\left(ex^{m} + d\right)^{n}\right) + \log(c)}{p \log(f) + p \log\left(x^{p}\right)}\right)b - \frac{a}{p \log\left(fx^{p}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^2,x, algorithm="maxima")
```

```
[Out] (e*m*n*integrate(x^m/(e*p*x*x^m*log(f) + d*p*x*log(f) + (e*p*x*x^m + d*p*x)
*log(x^p)), x) - (log((e*x^m + d)^n) + log(c))/(p*log(f) + p*log(x^p)))*b -
 a/(p*log(f*x^p))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x^m\right)}^n\right)}{x\,{\ln\left(f\,x^p\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)^2),x)
```

```
[Out] int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**m)**n))/x/ln(f*x**p)**2,x)
```

```
[Out] Timed out
```

3.627 $\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3\left(fx^p\right)}\,dx$

Optimal. Leaf size=69

$$\frac{bemn\text{Int}\left(\frac{x^{m-1}}{(d+ex^m)\log^2(fx^p)},x\right)}{2p}-\frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{2p\log^2\left(fx^p\right)}$$

```
[Out] 1/2*(-a-b*ln(c*(d+e*x^m)^n))/p/ln(f*x^p)^2+1/2*b*e*m*n*Unintegrable(x^(-1+m
)/(d+e*x^m)/ln(f*x^p)^2,x)/p
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log^3\left(fx^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^3),x]
```

```
[Out] -(a + b*Log[c*(d + e*x^m)^n])/(2*p*Log[f*x^p]^2) + (b*e*m*n*Defer[Int][x^(-
1 + m)/((d + e*x^m)*Log[f*x^p]^2), x])/(2*p)
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log^3\left(fx^p\right)}\,dx=-\frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{2p\log^2\left(fx^p\right)}+\frac{(bemn)\int \frac{x^{-1+m}}{(d+ex^m)\log^2(fx^p)}\,dx}{2p}$$

Mathematica [A] time = 11.06, size = 0, normalized size = 0.00

$$\int \frac{a+b\log\left(c\left(d+ex^m\right)^n\right)}{x\log^3\left(fx^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^3),x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^3), x]
```

fricas [A] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\log\left(\left(ex^m+d\right)^n c\right)+a}{x\log\left(fx^p\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^3,x, algorithm="fricas")
```

```
[Out] integral((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\log\left(\left(ex^m+d\right)^n c\right)+a}{x\log\left(fx^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)^3), x)
```

maple [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{b\ln\left(c\,(e\,x^{m}+d)^{n}\right)+a}{x\ln\left(f\,x^{p}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(e*x^m+d)^n)+a)/x/ln(f*x^p)^3,x)
```

```
[Out] int((b*ln(c*(e*x^m+d)^n)+a)/x/ln(f*x^p)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(2\,dem^{2}n\int\frac{x^{m}}{2\left(e^{2}p^{2}xx^{2\,m}\log(f)+2\,dep^{2}xx^{m}\log(f)+d^{2}p^{2}x\log(f)+\left(e^{2}p^{2}xx^{2\,m}+2\,dep^{2}xx^{m}+d^{2}p^{2}x\right)\log\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*d*e*m^2*n*integrate(1/2*x^m/(e^2*p^2*x*x^(2*m)*log(f) + 2*d*e*p^2*x*
x^m*log(f) + d^2*p^2*x*log(f) + (e^2*p^2*x*x^(2*m) + 2*d*e*p^2*x*x^m + d^2*
p^2*x)*log(x^p)), x) - (e*m*n*x^m*log(x^p) + d*p*log(c) + (e*m*n*log(f) + e
*p*log(c))*x^m + (e*p*x^m + d*p)*log((e*x^m + d)^n))/(e*p^2*x^m*log(f)^2 +
d*p^2*log(f)^2 + (e*p^2*x^m + d*p^2)*log(x^p)^2 + 2*(e*p^2*x^m*log(f) + d*p
^2*log(f))*log(x^p)))*b - 1/2*a/(p*log(f*x^p)^2)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x^m\right)}^n\right)}{x\,{\ln\left(f\,x^p\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)^3),x)
```

```
[Out] int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**m)**n))/x/ln(f*x**p)**3,x)
```

```
[Out] Timed out
```

3.628 $\int \log\left(c\left(d+e(f+gx)^p\right)^q\right)dx$

Optimal. Leaf size=76

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^p\right)^q\right)}{g}-\frac{epq(f+gx)^{p+1}\,{}_2F_1\left(1,1+\frac{1}{p};2+\frac{1}{p};-\frac{e(f+gx)^p}{d}\right)}{dg(p+1)}$$

```
[Out] -e*p*q*(g*x+f)^(1+p)*hypergeom([1, 1+1/p],[2+1/p],-e*(g*x+f)^p/d)/d/g/(1+p)
+(g*x+f)*ln(c*(d+e*(g*x+f)^p)^q)/g
```

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2483, 2448, 364}

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^p\right)^q\right)}{g}-\frac{epq(f+gx)^{p+1}\,{}_2F_1\left(1,1+\frac{1}{p};2+\frac{1}{p};-\frac{e(f+gx)^p}{d}\right)}{dg(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*(f + g*x)^p)^q],x]
```

```
[Out] -((e*p*q*(f + g*x)^(1 + p)*Hypergeometric2F1[1, 1 + p^(-1), 2 + p^(-1), -((
e*(f + g*x)^p)/d)])/(d*g*(1 + p))) + ((f + g*x)*Log[c*(d + e*(f + g*x)^p)^q
])/g
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int \log\left(c\left(d+e(f+gx)^p\right)^q\right)dx &= \frac{\text{Subst}\left(\int \log\left(c\left(d+ex^p\right)^q\right)dx,x,f+gx\right)}{g}\\ &= \frac{(f+gx)\log\left(c\left(d+e(f+gx)^p\right)^q\right)}{g}-\frac{(epq)\,\text{Subst}\left(\int \frac{x^p}{d+ex^p}dx,x,f+gx\right)}{g}\\ &= -\frac{epq(f+gx)^{1+p}\,{}_2F_1\left(1,1+\frac{1}{p};2+\frac{1}{p};-\frac{e(f+gx)^p}{d}\right)}{dg(1+p)}+\frac{(f+gx)\log\left(c\left(d+e(f+gx\right.\right.}{g}\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 0.86

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^p\right)^q\right)}{g}+\frac{pq(f+gx)\,{}_2F_1\left(1,\frac{1}{p};1+\frac{1}{p};-\frac{e(f+gx)^p}{d}\right)}{g}-pqx$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*(f + g*x)^p)^q],x]
```

```
[Out] -(p*q*x) + (p*q*(f + g*x)*Hypergeometric2F1[1, p^(-1), 1 + p^(-1), -((e*(f
+ g*x)^p)/d)])/g + ((f + g*x)*Log[c*(d + e*(f + g*x)^p)^q])/g
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\log\left(\left(\left(gx+f\right)^p e+d\right)^q c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^p)^q),x, algorithm="fricas")
```

```
[Out] integral(log(((g*x + f)^p*e + d)^q*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\left(\left(gx+f\right)^p e+d\right)^q c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^p)^q),x, algorithm="giac")
```

```
[Out] integrate(log(((g*x + f)^p*e + d)^q*c), x)
```

maple [F] time = 2.06, size = 0, normalized size = 0.00

$$\int \ln\left(c\left(e\left(gx+f\right)^p+d\right)^q\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*(g*x+f)^p)^q),x)
```

```
[Out] int(ln(c*(d+e*(g*x+f)^p)^q),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$dgpq\int \frac{x}{dgx+\left(egx+ef\right)\left(gx+f\right)^p+df}\,dx+\frac{fpq\log\left(gx+f\right)+gx\log\left(\left(\left(gx+f\right)^p e+d\right)^q\right)-\left(gpq-g\log(c)\right)}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^p)^q),x, algorithm="maxima")
```

```
[Out] d*g*p*q*integrate(x/(d*g*x + (e*g*x + e*f)*(g*x + f)^p + d*f), x) + (f*p*q*
log(g*x + f) + g*x*log(((g*x + f)^p*e + d)^q) - (g*p*q - g*log(c))*x)/g
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(d+e\left(f+g\,x\right)^p\right)^q\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*(f + g*x)^p)^q),x)
```

```
[Out] int(log(c*(d + e*(f + g*x)^p)^q), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log \left(c\left(d+e\left(f+gx\right)^{p}\right)^{q}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*(g*x+f)**p)**q),x)
```

```
[Out] Integral(log(c*(d + e*(f + g*x)**p)**q), x)
```

3.629 $\int \log \left(c \left(d+e(f+gx)^3\right)^q\right) \, dx$

Optimal. Leaf size=169

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g}-\frac{\sqrt[3]{d}\,q\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}(f+gx)^2\right)}{2\sqrt[3]{e}g}+\frac{\sqrt[3]{d}\,q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+\right.}{\sqrt[3]{e}g}$$

```
[Out] -3*q*x+d^(1/3)*q*ln(d^(1/3)+e^(1/3)*(g*x+f))/e^(1/3)/g-1/2*d^(1/3)*q*ln(d^(
2/3)-d^(1/3)*e^(1/3)*(g*x+f)+e^(2/3)*(g*x+f)^2)/e^(1/3)/g+(g*x+f)*ln(c*(d+e
*(g*x+f)^3)^q)/g-d^(1/3)*q*arctan(1/3*(d^(1/3)-2*e^(1/3)*(g*x+f))/d^(1/3)*3
^(1/2))*3^(1/2)/e^(1/3)/g
```

Rubi [A] time = 0.21, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {2483, 2448, 321, 200, 31, 634, 617, 204, 628}

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g}-\frac{\sqrt[3]{d}\,q\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}(f+gx)^2\right)}{2\sqrt[3]{e}g}+\frac{\sqrt[3]{d}\,q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+\right.}{\sqrt[3]{e}g}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*(f + g*x)^3)^q],x]
```

```
[Out] -3*q*x - (Sqrt[3]*d^(1/3)*q*ArcTan[(d^(1/3) - 2*e^(1/3)*(f + g*x))/(Sqrt[3]
*d^(1/3))])/(e^(1/3)*g) + (d^(1/3)*q*Log[d^(1/3) + e^(1/3)*(f + g*x)])/(e^(
1/3)*g) - (d^(1/3)*q*Log[d^(2/3) - d^(1/3)*e^(1/3)*(f + g*x) + e^(2/3)*(f +
 g*x)^2])/(2*e^(1/3)*g) + ((f + g*x)*Log[c*(d + e*(f + g*x)^3)^q])/g
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\int \log\left(c\left(d+e(f+gx)^3\right)^q\right)dx = \frac{\text{Subst}\left(\int \log\left(c\left(d+ex^3\right)^q\right)dx,x,f+gx\right)}{g}$$

$$= \frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g} - \frac{(3eq)\,\text{Subst}\left(\int \frac{x^3}{d+ex^3}dx,x,f+gx\right)}{g}$$

$$= -3qx + \frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g} + \frac{(3dq)\,\text{Subst}\left(\int \frac{1}{d+ex^3}dx,x,f+gx\right)}{g}$$

$$= -3qx + \frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g} + \frac{\left(\sqrt[3]{d}\,q\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}\,x}dx,x,f\ldots\right.}{g}$$

$$= -3qx + \frac{\sqrt[3]{d}\,q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)}{\sqrt[3]{e}\,g} + \frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g} + \ldots$$

$$= -3qx + \frac{\sqrt[3]{d}\,q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)}{\sqrt[3]{e}\,g} - \frac{\sqrt[3]{d}\,q\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}\ldots\right.}{2\sqrt[3]{e}\,g}$$

$$= -3qx - \frac{\sqrt{3}\sqrt[3]{d}\,q\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}(f+gx)}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}\,g} + \frac{\sqrt[3]{d}\,q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)}{\sqrt[3]{e}\,g} - \frac{\sqrt[3]{d}\,q\log\ldots}{}$$

Mathematica [A] time = 0.10, size = 147, normalized size = 0.87

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g}+\frac{\sqrt[3]{d}\,q\left(-\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}(f+gx)^2\right)+2\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f\right.\right.}{2\sqrt[3]{e}g}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*(f + g*x)^3)^q],x]
```

```
[Out] -3*q*x + (d^(1/3)*q*(2*Sqrt[3]*ArcTan[(-d^(1/3) + 2*e^(1/3)*(f + g*x))/(Sqr
t[3]*d^(1/3))] + 2*Log[d^(1/3) + e^(1/3)*(f + g*x)] - Log[d^(2/3) - d^(1/3)
*e^(1/3)*(f + g*x) + e^(2/3)*(f + g*x)^2]))/(2*e^(1/3)*g) + ((f + g*x)*Log[
c*(d + e*(f + g*x)^3)^q])/g
```

fricas [C] time = 6.18, size = 1357, normalized size = 8.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^3)^q),x, algorithm="fricas")
```

```
[Out] 1/4*(4*g*q*x*log(e*g^3*x^3 + 3*e*f*g^2*x^2 + 3*e*f^2*g*x + e*f^3 + d) - 12*
g*q*x - 4*sqrt(3)*g*sqrt((((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f
^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*
f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I
*sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^2*q^2)/g^2)*arctan(-1/24*(2*sqrt(3)*sq
rt(4*g^2*q^2*x^2 + 12*f*g*q^2*x + ((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) +
1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 1
2*f^2*q^2 + 2*(g^2*q*x + 3*f*g*q)*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) +
1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g))*(((-1/2*
f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I
*sqrt(3) + 1) - 2*f*q/g)*e*g^2 + 2*e*f*g*q)*sqrt((((-1/2*f^3*q^3/g^3 + 1/2*
d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*
f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 +
d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^2*q^2)/g^2) -
sqrt(3)*(8*e*f*g^2*q^2*x + ((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*
f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*e*g^3 + 12*e*f
^2*g*q^2 + 4*(e*g^3*q*x + 2*e*f*g^2*q)*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^
3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g))*sqr
t((((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3)
)^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3
/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g
)*f*g*q + 4*f^2*q^2)/g^2))/(d*q^3)) - 2*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g
^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g*l
og(q*x - 1/2*(-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3
)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) + f*q/g) + 4*g*x*log(c) + (((-1/2*f^3*q^3/
g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3)
 + 1) - 2*f*q/g)*g + 6*f*q)*log(4*g^2*q^2*x^2 + 12*f*g*q^2*x + ((-1/2*f^3*q
^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt
(3) + 1) - 2*f*q/g)^2*g^2 + 12*f^2*q^2 + 2*(g^2*q*x + 3*f*g*q)*((-1/2*f^3*q
^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt
(3) + 1) - 2*f*q/g)))/g
```

giac [A] time = 0.46, size = 265, normalized size = 1.57

$$qx\log\left(g^3x^3e+3fg^2x^2e+3f^2gxe+f^3e+d\right)-3qx+x\log(c)+\frac{fq\log\left(\left|g^3x^3e+3fg^2x^2e+3f^2gxe+f^3e+d\right|\right.}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^3)^q),x, algorithm="giac")
```

```
[Out] q*x*log(g^3*x^3*e + 3*f*g^2*x^2*e + 3*f^2*g*x*e + f^3*e + d) - 3*q*x + x*lo
g(c) + f*q*log(abs(g^3*x^3*e + 3*f*g^2*x^2*e + 3*f^2*g*x*e + f^3*e + d))/g
+ 1/2*(2*sqrt(3)*(d*g^6*q^3)^(1/3)*arctan(-(g*x*e + f*e + d^(1/3)*e^(2/3))/
(sqrt(3)*g*x*e + sqrt(3)*f*e - sqrt(3)*d^(1/3)*e^(2/3)))*e^(2/3) - (d*g^6*q
^3)^(1/3)*e^(2/3)*log(4*(sqrt(3)*g*x*e + sqrt(3)*f*e - sqrt(3)*d^(1/3)*e^(2
/3))^2 + 4*(g*x*e + f*e + d^(1/3)*e^(2/3))^2) + 2*(d*g^6*q^3)^(1/3)*e^(2/3)
*log(abs(g*x*e + f*e + d^(1/3)*e^(2/3))))*e^(-1)/g^3
```

maple [C] time = 0.48, size = 145, normalized size = 0.86

$$-3qx+x\ln\left(c\left(e g^3x^3+3ef g^2x^2+3e f^2gx+e f^3+d\right)^q\right)-\frac{q\left(-\mathrm{RootOf}\left(e g^3\_Z^3+3ef g^2\_Z^2+3e f^2g\_Z+e f^3+\right.\right.}{eg\left(g^2\,\mathrm{RootOf}\left(e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*(g*x+f)^3)^q),x)
```

```
[Out] x*ln(c*(e*g^3*x^3+3*e*f*g^2*x^2+3*e*f^2*g*x+e*f^3+d)^q)-3*q*x-1/g/e*q*sum((
-_R^2*e*f*g^2-2*_R*e*f^2*g-e*f^3-d)/(_R^2*g^2+2*_R*f*g+f^2)*ln(-_R+x),_R=Ro
otOf(_Z^3*e*g^3+3*_Z^2*e*f*g^2+3*_Z*e*f^2*g+e*f^3+d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(3\,q-\log(c)\right)x+3\,q\int\frac{efg^2x^2+2\,ef^2gx+ef^3+d}{eg^3x^3+3\,efg^2x^2+3\,ef^2gx+ef^3+d}\,dx+x\log\left(\left(eg^3x^3+3\,efg^2x^2+3\,ef^2gx+ef^3+d\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^3)^q),x, algorithm="maxima")
```

```
[Out] -(3*q - log(c))*x + 3*q*integrate((e*f*g^2*x^2 + 2*e*f^2*g*x + e*f^3 + d)/(
e*g^3*x^3 + 3*e*f*g^2*x^2 + 3*e*f^2*g*x + e*f^3 + d), x) + x*log((e*g^3*x^3
 + 3*e*f*g^2*x^2 + 3*e*f^2*g*x + e*f^3 + d)^q)
```

mupad [B] time = 0.28, size = 192, normalized size = 1.14

$$x\,\ln\left(c\,{\left(d+e\,{\left(f+g\,x\right)}^3\right)}^q\right)-\left(\sum_{k=1}^{3}\ln\left(d\,e^2\,g^5\,\left(\mathrm{root}\left(b^3\,e\,g^3+3\,b^2\,e\,f\,g^2\,q+3\,b\,e\,f^2\,g\,q^2+e\,f^3\,q^3+d\,q^3,b,k\right)\,g+f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*(f + g*x)^3)^q),x)
```

```
[Out] x*log(c*(d + e*(f + g*x)^3)^q) - symsum(log(9*d*e^2*g^5*(root(b^3*e*g^3 + 3
*b^2*e*f*g^2*q + 3*b*e*f^2*g*q^2 + e*f^3*q^3 + d*q^3, b, k)*g + f*q)*(root(
b^3*e*g^3 + 3*b^2*e*f*g^2*q + 3*b*e*f^2*g*q^2 + e*f^3*q^3 + d*q^3, b, k) -
q*x))*root(b^3*e*g^3 + 3*b^2*e*f*g^2*q + 3*b*e*f^2*g*q^2 + e*f^3*q^3 + d*q^
3, b, k), k, 1, 3) - 3*q*x
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*(g*x+f)**3)**q),x)
```

```
[Out] Timed out
```

3.630 $\int \log \left(c \left(d+e(f+gx)^2\right)^q\right) dx$

Optimal. Leaf size=63

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g}+\frac{2\sqrt{d}\,q\tan^{-1}\left(\frac{\sqrt{e}(f+gx)}{\sqrt{d}}\right)}{\sqrt{e}\,g}-2qx$$

```
[Out] -2*q*x+(g*x+f)*ln(c*(d+e*(g*x+f)^2)^q)/g+2*q*arctan((g*x+f)*e^(1/2)/d^(1/2)
)*d^(1/2)/g/e^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2483, 2448, 321, 205}

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g}+\frac{2\sqrt{d}\,q\tan^{-1}\left(\frac{\sqrt{e}(f+gx)}{\sqrt{d}}\right)}{\sqrt{e}\,g}-2qx$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*(f + g*x)^2)^q],x]
```

```
[Out] -2*q*x + (2*Sqrt[d]*q*ArcTan[(Sqrt[e]*(f + g*x))/Sqrt[d]])/(Sqrt[e]*g) + ((
f + g*x)*Log[c*(d + e*(f + g*x)^2)^q])/g
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\int \log\left(c\left(d+e(f+gx)^2\right)^q\right)dx = \frac{\text{Subst}\left(\int \log\left(c\left(d+ex^2\right)^q\right)dx, x, f+gx\right)}{g}$$
$$= \frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g} - \frac{(2eq)\,\text{Subst}\left(\int \frac{x^2}{d+ex^2}dx, x, f+gx\right)}{g}$$
$$= -2qx + \frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g} + \frac{(2dq)\,\text{Subst}\left(\int \frac{1}{d+ex^2}dx, x, f+gx\right)}{g}$$
$$= -2qx + \frac{2\sqrt{d}\,q\tan^{-1}\left(\frac{\sqrt{e}(f+gx)}{\sqrt{d}}\right)}{\sqrt{e}\,g} + \frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 1.00

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g} + \frac{2\sqrt{d}\,q\tan^{-1}\left(\frac{\sqrt{e}(f+gx)}{\sqrt{d}}\right)}{\sqrt{e}\,g} - 2qx$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*(f + g*x)^2)^q],x]
```

```
[Out] -2*q*x + (2*Sqrt[d]*q*ArcTan[(Sqrt[e]*(f + g*x))/Sqrt[d]])/(Sqrt[e]*g) + ((
f + g*x)*Log[c*(d + e*(f + g*x)^2)^q])/g
```

fricas [A] time = 0.77, size = 206, normalized size = 3.27

$$\left[-\frac{2gqx - gx\log(c) - q\sqrt{-\frac{d}{e}}\log\left(\frac{eg^2x^2+2efgx+ef^2+2\left(egx+ef\right)\sqrt{-\frac{d}{e}}-d}{eg^2x^2+2efgx+ef^2+d}\right) - \left(gqx+fq\right)\log\left(eg^2x^2+2efgx+ef^2+d\right)}{g}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^2)^q),x, algorithm="fricas")
```

```
[Out] [-(2*g*q*x - g*x*log(c) - q*sqrt(-d/e)*log((e*g^2*x^2 + 2*e*f*g*x + e*f^2 +
 2*(e*g*x + e*f)*sqrt(-d/e) - d)/(e*g^2*x^2 + 2*e*f*g*x + e*f^2 + d)) - (g*
q*x + f*q)*log(e*g^2*x^2 + 2*e*f*g*x + e*f^2 + d))/g, -(2*g*q*x - g*x*log(c
) - 2*q*sqrt(d/e)*arctan((e*g*x + e*f)*sqrt(d/e)/d) - (g*q*x + f*q)*log(e*g
^2*x^2 + 2*e*f*g*x + e*f^2 + d))/g]
```

giac [A] time = 0.20, size = 96, normalized size = 1.52

$$qx\log\left(g^2x^2e+2fgxe+f^2e+d\right)+\frac{2\sqrt{d}\,q\arctan\left(\frac{\left(gxe+fe\right)e^{\left(-\frac{1}{2}\right)}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{g}-2qx+\frac{fq\log\left(g^2x^2e+2fgxe+f^2e+d\right)}{g}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^2)^q),x, algorithm="giac")
```

```
[Out] q*x*log(g^2*x^2*e + 2*f*g*x*e + f^2*e + d) + 2*sqrt(d)*q*arctan((g*x*e + f*
e)*e^(-1/2)/sqrt(d))*e^(-1/2)/g - 2*q*x + f*q*log(g^2*x^2*e + 2*f*g*x*e + f
^2*e + d)/g + x*log(c)
```

maple [A] time = 0.17, size = 98, normalized size = 1.56

$$\frac{2dq\arctan\left(\frac{2eg^2x+2efg}{2\sqrt{de}\,g}\right)}{\sqrt{de}\,g}+\frac{fq\ln\left(eg^2x^2+2efgx+ef^2+d\right)}{g}-2qx+x\ln\left(c\left(eg^2x^2+2efgx+ef^2+d\right)^q\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*(g*x+f)^2)^q),x)
```

```
[Out] x*ln(c*(e*g^2*x^2+2*e*f*g*x+e*f^2+d)^q)-2*q*x+q/g*f*ln(e*g^2*x^2+2*e*f*g*x+
e*f^2+d)+2*q/g*d/(d*e)^(1/2)*arctan(1/2*(2*e*g^2*x+2*e*f*g)/g/(d*e)^(1/2))
```

maxima [A] time = 1.48, size = 100, normalized size = 1.59

$$-egq\left(\frac{2x}{eg}-\frac{f\log\left(eg^2x^2+2efgx+ef^2+d\right)}{eg^2}-\frac{2d\arctan\left(\frac{eg^2x+efg}{\sqrt{de}\,g}\right)}{\sqrt{de}\,eg^2}\right)+x\log\left(\left(\left(gx+f\right)^2e+d\right)^qc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^2)^q),x, algorithm="maxima")
```

```
[Out] -e*g*q*(2*x/(e*g) - f*log(e*g^2*x^2 + 2*e*f*g*x + e*f^2 + d)/(e*g^2) - 2*d*
arctan((e*g^2*x + e*f*g)/(sqrt(d*e)*g))/(sqrt(d*e)*e*g^2)) + x*log(((g*x +
f)^2*e + d)^q*c)
```

mupad [B] time = 0.13, size = 82, normalized size = 1.30

$$x\ln\left(c\left(d+e\left(f+gx\right)^2\right)^q\right)-2qx+\frac{fq\ln\left(ef^2+2efgx+eg^2x^2+d\right)}{g}+\frac{2\sqrt{d}\,q\,\mathrm{atan}\left(\frac{\sqrt{e}\,f}{\sqrt{d}}+\frac{\sqrt{e}\,gx}{\sqrt{d}}\right)}{\sqrt{e}\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*(f + g*x)^2)^q),x)
```

```
[Out] x*log(c*(d + e*(f + g*x)^2)^q) - 2*q*x + (f*q*log(d + e*f^2 + e*g^2*x^2 + 2
*e*f*g*x))/g + (2*d^(1/2)*q*atan((e^(1/2)*f)/d^(1/2) + (e^(1/2)*g*x)/d^(1/2
)))/(e^(1/2)*g)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*(g*x+f)**2)**q),x)
```

```
[Out] Timed out
```

3.631 $\int \log \left(c(d+e(f+gx))^q\right) \, dx$

Optimal. Leaf size=35

$$\frac{(d+ef+egx)\log\left(c(d+e(f+gx))^q\right)}{eg}-qx$$

```
[Out] -q*x+(e*g*x+e*f+d)*ln(c*(d+e*(g*x+f))^q)/e/g
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2444, 2389, 2295}

$$\frac{(d+ef+egx)\log\left(c(d+e(f+gx))^q\right)}{eg}-qx$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*(f + g*x))^q],x]
```

```
[Out] -(q*x) + ((d + e*f + e*g*x)*Log[c*(d + e*(f + g*x))^q])/(e*g)
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] && !LinearMatchQ[v, x] && !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rubi steps

$$\begin{aligned}\int \log\left(c(d+e(f+gx))^q\right)\,dx &= \int \log\left(c(d+ef+egx)^q\right)\,dx\\ &= \frac{\text{Subst}\left(\int \log\left(cx^q\right)\,dx,x,d+ef+egx\right)}{eg}\\ &= -qx+\frac{(d+ef+egx)\log\left(c(d+e(f+gx))^q\right)}{eg}\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.34

$$\frac{(f+gx)\log\left(c(d+e(f+gx))^q\right)}{g}+\frac{dq\log(d+ef+egx)}{eg}-qx$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*(f + g*x))^q],x]
```

```
[Out] -(q*x) + (d*q*Log[d + e*f + e*g*x])/(e*g) + ((f + g*x)*Log[c*(d + e*(f + g*
x))^q])/g
```

fricas [A] time = 0.70, size = 46, normalized size = 1.31

$$-\frac{egqx - egx\log(c) - \left(egqx + \left(ef + d\right)q\right)\log\left(egx + ef + d\right)}{eg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f))^q),x, algorithm="fricas")
```

```
[Out] -(e*g*q*x - e*g*x*log(c) - (e*g*q*x + (e*f + d)*q)*log(e*g*x + e*f + d))/(e
*g)
```

giac [A] time = 0.16, size = 69, normalized size = 1.97

$$\frac{\left(gxe + fe + d\right)qe^{(-1)}\log\left(gxe + fe + d\right)}{g} - \frac{\left(gxe + fe + d\right)qe^{(-1)}}{g} + \frac{\left(gxe + fe + d\right)e^{(-1)}\log(c)}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f))^q),x, algorithm="giac")
```

```
[Out] (g*x*e + f*e + d)*q*e^(-1)*log(g*x*e + f*e + d)/g - (g*x*e + f*e + d)*q*e^(
-1)/g + (g*x*e + f*e + d)*e^(-1)*log(c)/g
```

maple [A] time = 0.12, size = 57, normalized size = 1.63

$$\frac{fq\ln\left(egx + ef + d\right)}{g} - qx + x\ln\left(c\left(egx + ef + d\right)^{q}\right) + \frac{dq\ln\left(egx + ef + d\right)}{eg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+(g*x+f)*e)^q),x)
```

```
[Out] x*ln(c*(e*g*x+e*f+d)^q)-q*x+q/g*ln(e*g*x+e*f+d)*f+q/e/g*ln(e*g*x+e*f+d)*d
```

maxima [A] time = 0.51, size = 54, normalized size = 1.54

$$-egq\left(\frac{x}{eg} - \frac{\left(ef + d\right)\log\left(egx + ef + d\right)}{e^2g^2}\right) + x\log\left(\left(\left(gx + f\right)e + d\right)^{q}c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f))^q),x, algorithm="maxima")
```

```
[Out] -e*g*q*(x/(e*g) - (e*f + d)*log(e*g*x + e*f + d)/(e^2*g^2)) + x*log(((g*x +
 f)*e + d)^q*c)
```

mupad [B] time = 0.31, size = 46, normalized size = 1.31

$$x\,\ln\left(c\left(d + e\,\left(f + g\,x\right)\right)^{q}\right) - q\,x + \frac{\ln\left(d + e\,f + e\,g\,x\right)\,\left(d\,q + e\,f\,q\right)}{e\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*(f + g*x))^q),x)
```

```
[Out] x*log(c*(d + e*(f + g*x))^q) - q*x + (log(d + e*f + e*g*x)*(d*q + e*f*q))/(
e*g)
```

sympy [A] time = 1.52, size = 80, normalized size = 2.29

$$\begin{cases} x\log\left(cd^{q}\right) & \text{for } e=0 \wedge \left(e=0 \vee g=0\right) \\ x\log\left(c\left(d+ef\right)^{q}\right) & \text{for } g=0 \\ \frac{dq\log\left(d+ef+egx\right)}{eg}+\frac{fq\log\left(d+ef+egx\right)}{g}+qx\log\left(d+ef+egx\right)-qx+x\log\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*(g*x+f))**q),x)
```

```
[Out] Piecewise((x*log(c*d**q), Eq(e, 0) & (Eq(e, 0) | Eq(g, 0))), (x*log(c*(d +
e*f)**q), Eq(g, 0)), (d*q*log(d + e*f + e*g*x)/(e*g) + f*q*log(d + e*f + e*
g*x)/g + q*x*log(d + e*f + e*g*x) - q*x + x*log(c), True))
```

3.632 $\int \log \left(c \left(d+\frac{e}{f+gx}\right)^q\right) dx$

Optimal. Leaf size=45

$$\frac{(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g}+\frac{eq\log(d(f+gx)+e)}{dg}$$

```
[Out] (g*x+f)*ln(c*(d+e/(g*x+f))^q)/g+e*q*ln(e+d*(g*x+f))/d/g
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2483, 2448, 263, 31}

$$\frac{(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g}+\frac{eq\log(d(f+gx)+e)}{dg}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e/(f + g*x))^q],x]
```

```
[Out] ((f + g*x)*Log[c*(d + e/(f + g*x))^q])/g + (e*q*Log[e + d*(f + g*x)])/(d*g)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\int \log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)dx = \frac{\text{Subst}\left(\int \log\left(c\left(d+\frac{e}{x}\right)^q\right)dx, x, f+gx\right)}{g}$$

$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g} + \frac{(eq)\,\text{Subst}\left(\int \frac{1}{\left(d+\frac{e}{x}\right)x}dx, x, f+gx\right)}{g}$$

$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g} + \frac{(eq)\,\text{Subst}\left(\int \frac{1}{e+dx}dx, x, f+gx\right)}{g}$$

$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g} + \frac{eq\log(e+d(f+gx))}{dg}$$

Mathematica [A] time = 0.05, size = 56, normalized size = 1.24

$$\frac{dgx\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right) + q(df+e)\log(df+dgx+e) - dfq\log(f+gx)}{dg}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/(f + g*x))^q],x]
```

```
[Out] (-(d*f*q*Log[f + g*x]) + (e + d*f)*q*Log[e + d*f + d*g*x] + d*g*x*Log[c*(d
+ e/(f + g*x))^q])/(d*g)
```

fricas [A] time = 0.84, size = 65, normalized size = 1.44

$$\frac{dgqx\log\left(\frac{dgx+df+e}{gx+f}\right) - dfq\log\left(gx+f\right) + dgx\log(c) + \left(df+e\right)q\log\left(dgx+df+e\right)}{dg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f))^q),x, algorithm="fricas")
```

```
[Out] (d*g*q*x*log((d*g*x + d*f + e)/(g*x + f)) - d*f*q*log(g*x + f) + d*g*x*log(
c) + (d*f + e)*q*log(d*g*x + d*f + e))/(d*g)
```

giac [B] time = 0.21, size = 172, normalized size = 3.82

$$\frac{\left(dfge^{(-2)} - \left(df+e\right)ge^{(-2)}\right)\left(dqe^2\log\left(-d+\frac{dgx+df+e}{gx+f}\right) + de^2\log(c) - \frac{\left(dgx+df+e\right)qe^2\log\left(-d+\frac{dgx+df+e}{gx+f}\right)}{gx+f} + \frac{\left(dgx+df+e\right)qe^2\log(}{gx+f}\right.}{d^2g^2 - \frac{\left(dgx+df+e\right)dg^2}{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f))^q),x, algorithm="giac")
```

```
[Out] (d*f*g*e^(-2) - (d*f + e)*g*e^(-2))*(d*q*e^2*log(-d + (d*g*x + d*f + e)/(g*
x + f)) + d*e^2*log(c) - (d*g*x + d*f + e)*q*e^2*log(-d + (d*g*x + d*f + e)
/(g*x + f))/(g*x + f) + (d*g*x + d*f + e)*q*e^2*log((d*g*x + d*f + e)/(g*x
+ f))/(g*x + f))/(d^2*g^2 - (d*g*x + d*f + e)*d*g^2/(g*x + f))
```

maple [A] time = 0.13, size = 74, normalized size = 1.64

$$-\frac{fq\ln\left(gx+f\right)}{g} + \frac{fq\ln\left(dgx+df+e\right)}{g} + x\ln\left(c\left(\frac{dgx+df+e}{gx+f}\right)^q\right) + \frac{eq\ln\left(dgx+df+e\right)}{dg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/(g*x+f))^q),x)
```

```
[Out] x*ln(c*((d*g*x+d*f+e)/(g*x+f))^q)+1/g*q*ln(d*g*x+d*f+e)*f+1/g*e*q/d*ln(d*g*
x+d*f+e)-1/g*q*f*ln(g*x+f)
```

maxima [A] time = 0.64, size = 65, normalized size = 1.44

$$-egq\left(\frac{f\log\left(gx+f\right)}{eg^2}-\frac{\left(df+e\right)\log\left(dgx+df+e\right)}{deg^2}\right)+x\log\left(c\left(d+\frac{e}{gx+f}\right)^q\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f))^q),x, algorithm="maxima")
```

```
[Out] -e*g*q*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d*e*g^2))
+ x*log(c*(d + e/(g*x + f))^q)
```

mupad [B] time = 0.18, size = 67, normalized size = 1.49

$$x\,\ln\left(c\left(d+\frac{e}{f+g\,x}\right)^q\right)-\frac{f\,q\,\ln\left(f+g\,x\right)}{g}+\frac{f\,q\,\ln\left(e+d\,f+d\,g\,x\right)}{g}+\frac{e\,q\,\ln\left(e+d\,f+d\,g\,x\right)}{d\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/(f + g*x))^q),x)
```

```
[Out] x*log(c*(d + e/(f + g*x))^q) - (f*q*log(f + g*x))/g + (f*q*log(e + d*f + d*
g*x))/g + (e*q*log(e + d*f + d*g*x))/(d*g)
```

sympy [A] time = 2.08, size = 109, normalized size = 2.42

$$\begin{cases} x\log\left(c\left(\frac{e}{f}\right)^q\right) & \text{for}\ d=0\wedge g=0 \\ x\log\left(c\left(d+\frac{e}{f}\right)^q\right) & \text{for}\ g=0 \\ -\frac{fq\log\left(f+gx\right)}{g}+qx\log\left(e\right)-qx\log\left(f+gx\right)+qx+x\log\left(c\right) & \text{for}\ d=0 \\ \frac{fq\log\left(d+\frac{e}{f+gx}\right)}{g}+qx\log\left(d+\frac{e}{f+gx}\right)+x\log\left(c\right)+\frac{eq\log\left(df+dgx+e\right)}{dg} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/(g*x+f))**q),x)
```

```
[Out] Piecewise((x*log(c*(e/f)**q), Eq(d, 0) & Eq(g, 0)), (x*log(c*(d + e/f)**q),
 Eq(g, 0)), (-f*q*log(f + g*x)/g + q*x*log(e) - q*x*log(f + g*x) + q*x + x*
log(c), Eq(d, 0)), (f*q*log(d + e/(f + g*x))/g + q*x*log(d + e/(f + g*x)) +
 x*log(c) + e*q*log(d*f + d*g*x + e)/(d*g), True))
```

3.633 $\int \log \left(c \left(d+\frac{e}{(f+gx)^2}\right)^q\right) \, dx$

Optimal. Leaf size=59

$$\frac{(f+gx)\log \left(c \left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}+\frac{2\sqrt{e}\, q \tan^{-1}\left(\frac{\sqrt{d}\,(f+gx)}{\sqrt{e}}\right)}{\sqrt{d}\, g}$$

```
[Out] (g*x+f)*ln(c*(d+e/(g*x+f)^2)^q)/g+2*q*arctan((g*x+f)*d^(1/2)/e^(1/2))*e^(1/
2)/g/d^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2483, 2448, 263, 205}

$$\frac{(f+gx)\log \left(c \left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}+\frac{2\sqrt{e}\, q \tan^{-1}\left(\frac{\sqrt{d}\,(f+gx)}{\sqrt{e}}\right)}{\sqrt{d}\, g}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e/(f + g*x)^2)^q],x]
```

```
[Out] (2*Sqrt[e]*q*ArcTan[(Sqrt[d]*(f + g*x))/Sqrt[e]])/(Sqrt[d]*g) + ((f + g*x)*
Log[c*(d + e/(f + g*x)^2)^q])/g
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\int \log \left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)dx=\frac{\text{Subst}\left(\int \log \left(c\left(d+\frac{e}{x^2}\right)^q\right)dx,x,f+gx\right)}{g}$$
$$=\frac{(f+gx)\log \left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}+\frac{(2eq)\,\text{Subst}\left(\int \frac{1}{\left(d+\frac{e}{x^2}\right)x^2}dx,x,f+gx\right)}{g}$$
$$=\frac{(f+gx)\log \left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}+\frac{(2eq)\,\text{Subst}\left(\int \frac{1}{e+dx^2}dx,x,f+gx\right)}{g}$$
$$=\frac{2\sqrt{e}\,q\tan^{-1}\left(\frac{\sqrt{d}\,(f+gx)}{\sqrt{e}}\right)}{\sqrt{d}\,g}+\frac{(f+gx)\log \left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 1.03

$$\frac{(f+gx)\log \left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}-\frac{2\sqrt{e}\,q\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{d}\,(f+gx)}\right)}{\sqrt{d}\,g}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/(f + g*x)^2)^q],x]
```

```
[Out] (-2*Sqrt[e]*q*ArcTan[Sqrt[e]/(Sqrt[d]*(f + g*x))])/(Sqrt[d]*g) + ((f + g*x)
*Log[c*(d + e/(f + g*x)^2)^q])/g
```

fricas [B] time = 0.85, size = 287, normalized size = 4.86

$$\left[\frac{gqx\log \left(\frac{dg^2x^2+2\,dfgx+df^2+e}{g^2x^2+2\,fgx+f^2}\right)+fq\log \left(dg^2x^2+2\,dfgx+df^2+e\right)-2\,fq\log \left(gx+f\right)+gx\log(c)+q\sqrt{-\frac{e}{d}}\,\log}{g}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f)^2)^q),x, algorithm="fricas")
```

```
[Out] [(g*q*x*log((d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e)/(g^2*x^2 + 2*f*g*x + f^2))
+ f*q*log(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e) - 2*f*q*log(g*x + f) + g*x*log
(c) + q*sqrt(-e/d)*log((d*g^2*x^2 + 2*d*f*g*x + d*f^2 + 2*(d*g*x + d*f)*sqr
t(-e/d) - e)/(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e)))/g, (g*q*x*log((d*g^2*x^2
 + 2*d*f*g*x + d*f^2 + e)/(g^2*x^2 + 2*f*g*x + f^2)) + f*q*log(d*g^2*x^2 +
2*d*f*g*x + d*f^2 + e) - 2*f*q*log(g*x + f) + g*x*log(c) + 2*q*sqrt(e/d)*ar
ctan((d*g*x + d*f)*sqrt(e/d)/e))/g]
```

giac [B] time = 0.47, size = 137, normalized size = 2.32

$$dg^4q\left(\frac{fe^{(-1)}\log \left(dg^2x^2+2\,dfgx+df^2+e\right)}{dg^5}-\frac{2\,fe^{(-1)}\log \left(\left|gx+f\right|\right)}{dg^5}+\frac{2\,\arctan \left(\frac{(dgx+df)e^{\left(-\frac{1}{2}\right)}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{d^{\frac{3}{2}}g^5}\right)e+qx\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f)^2)^q),x, algorithm="giac")
```

```
[Out] d*g^4*q*(f*e^(-1)*log(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e)/(d*g^5) - 2*f*e^(-
1)*log(abs(g*x + f))/(d*g^5) + 2*arctan((d*g*x + d*f)*e^(-1/2)/sqrt(d))*e^(
-1/2)/(d^(3/2)*g^5))*e + q*x*log(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e) - q*x*l
og(g^2*x^2 + 2*f*g*x + f^2) + x*log(c)
```

maple [B] time = 0.13, size = 115, normalized size = 1.95

$$\frac{2eq\arctan\left(\frac{2dg^2x+2dfg}{2\sqrt{de}\,g}\right)}{\sqrt{de}\,g}-\frac{2fq\ln\left(gx+f\right)}{g}+\frac{fq\ln\left(dg^2x^2+2dfgx+df^2+e\right)}{g}+x\ln\left(c\left(\frac{dg^2x^2+2dfgx+df^2+e}{\left(gx+f\right)^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/(g*x+f)^2)^q),x)
```

```
[Out] x*ln(c*((d*g^2*x^2+2*d*f*g*x+d*f^2+e)/(g*x+f)^2)^q)+1/g*q*f*ln(d*g^2*x^2+2*
d*f*g*x+d*f^2+e)+2/g*e*q/(d*e)^(1/2)*arctan(1/2*(2*d*g^2*x+2*d*f*g)/g/(d*e)
^(1/2))-2*f/g*q*ln(g*x+f)
```

maxima [A] time = 1.64, size = 100, normalized size = 1.69

$$egq\left(\frac{f\log\left(dg^2x^2+2\,dfgx+df^2+e\right)}{eg^2}-\frac{2\,f\log\left(gx+f\right)}{eg^2}+\frac{2\,\arctan\left(\frac{dg^2x+dfg}{\sqrt{de}\,g}\right)}{\sqrt{de}\,g^2}\right)+x\log\left(c\left(d+\frac{e}{\left(gx+f\right)^2}\right)^q\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f)^2)^q),x, algorithm="maxima")
```

```
[Out] e*g*q*(f*log(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e)/(e*g^2) - 2*f*log(g*x + f)/
(e*g^2) + 2*arctan((d*g^2*x + d*f*g)/(sqrt(d*e)*g))/(sqrt(d*e)*g^2)) + x*lo
g(c*(d + e/(g*x + f)^2)^q)
```

mupad [B] time = 0.52, size = 163, normalized size = 2.76

$$x\,\ln\left(c\,\left(d+\frac{e}{\left(f+g\,x\right)^2}\right)^q\right)-\frac{2\,f\,q\,\ln\left(f+g\,x\right)}{g}+\frac{\ln\left(e\,\sqrt{-d\,e}-3\,d\,f^2\,\sqrt{-d\,e}+4\,d\,e\,f+d\,e\,g\,x-3\,d\,f\,g\,x\,\sqrt{-d\,e}\right)\,\left(q\right.}{d\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/(f + g*x)^2)^q),x)
```

```
[Out] x*log(c*(d + e/(f + g*x)^2)^q) - (2*f*q*log(f + g*x))/g + (log(e*(-d*e)^(1/
2) - 3*d*f^2*(-d*e)^(1/2) + 4*d*e*f + d*e*g*x - 3*d*f*g*x*(-d*e)^(1/2))*(q*
(-d*e)^(1/2) + d*f*q))/(d*g) - (log(3*d*f^2*(-d*e)^(1/2) - e*(-d*e)^(1/2) +
 4*d*e*f + d*e*g*x + 3*d*f*g*x*(-d*e)^(1/2))*(q*(-d*e)^(1/2) - d*f*q))/(d*g
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/(g*x+f)**2)**q),x)
```

```
[Out] Timed out
```

3.634 $\int \log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)\,dx$

Optimal. Leaf size=165

$$\frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}-\frac{\sqrt[3]{e}\,q\log\left(d^{2/3}(f+gx)^2-\sqrt[3]{d}\,\sqrt[3]{e}\,(f+gx)+e^{2/3}\right)}{2\sqrt[3]{d}\,g}+\frac{\sqrt[3]{e}\,q\log\left(\sqrt[3]{d}\,(f+gx)+\sqrt[3]{e}\right.}{\sqrt[3]{d}\,g}$$

```
[Out] (g*x+f)*ln(c*(d+e/(g*x+f)^3)^q)/g+e^(1/3)*q*ln(e^(1/3)+d^(1/3)*(g*x+f))/d^(
1/3)/g-1/2*e^(1/3)*q*ln(e^(2/3)-d^(1/3)*e^(1/3)*(g*x+f)+d^(2/3)*(g*x+f)^2)/
d^(1/3)/g-e^(1/3)*q*arctan(1/3*(e^(1/3)-2*d^(1/3)*(g*x+f))/e^(1/3)*3^(1/2))
*3^(1/2)/d^(1/3)/g
```

Rubi [A] time = 0.17, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {2483, 2448, 263, 200, 31, 634, 617, 204, 628}

$$\frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}-\frac{\sqrt[3]{e}\,q\log\left(d^{2/3}(f+gx)^2-\sqrt[3]{d}\,\sqrt[3]{e}\,(f+gx)+e^{2/3}\right)}{2\sqrt[3]{d}\,g}+\frac{\sqrt[3]{e}\,q\log\left(\sqrt[3]{d}\,(f+gx)+\sqrt[3]{e}\right.}{\sqrt[3]{d}\,g}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e/(f + g*x)^3)^q],x]
```

```
[Out] -((Sqrt[3]*e^(1/3)*q*ArcTan[(e^(1/3) - 2*d^(1/3)*(f + g*x))/(Sqrt[3]*e^(1/3
))])/(d^(1/3)*g)) + ((f + g*x)*Log[c*(d + e/(f + g*x)^3)^q])/g + (e^(1/3)*q
*Log[e^(1/3) + d^(1/3)*(f + g*x)])/(d^(1/3)*g) - (e^(1/3)*q*Log[e^(2/3) - d
^(1/3)*e^(1/3)*(f + g*x) + d^(2/3)*(f + g*x)^2])/(2*d^(1/3)*g)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)dx = \frac{\text{Subst}\left(\int \log\left(c\left(d+\frac{e}{x^3}\right)^q\right)dx, x, f+gx\right)}{g}$$

$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} + \frac{(3eq)\,\text{Subst}\left(\int \frac{1}{\left(d+\frac{e}{x^3}\right)x^3}dx, x, f+gx\right)}{g}$$

$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} + \frac{(3eq)\,\text{Subst}\left(\int \frac{1}{e+dx^3}dx, x, f+gx\right)}{g}$$

$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} + \frac{\left(\sqrt[3]{e}\,q\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{e}+\sqrt[3]{d}\,x}dx, x, f+gx\right)}{g} + \frac{\left(\sqrt[3]{e}\right.}{}$$

$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} + \frac{\sqrt[3]{e}\,q\log\left(\sqrt[3]{e}+\sqrt[3]{d}(f+gx)\right)}{\sqrt[3]{d}\,g} - \frac{\left(\sqrt[3]{e}\,q\right)\text{Subst}}{}$$

$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} + \frac{\sqrt[3]{e}\,q\log\left(\sqrt[3]{e}+\sqrt[3]{d}(f+gx)\right)}{\sqrt[3]{d}\,g} - \frac{\sqrt[3]{e}\,q\log\left(e^{2/3}\right.}{}$$

$$= -\frac{\sqrt{3}\,\sqrt[3]{e}\,q\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}(f+gx)}{\sqrt[3]{e}}}{\sqrt{3}}\right)}{\sqrt[3]{d}\,g} + \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} + \frac{\sqrt[3]{e}\,q\log\left(\sqrt[3]{e}\right.}{}$$

Mathematica [C] time = 0.33, size = 66, normalized size = 0.40

$$\frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}-\frac{3eq\,_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{e}{d(f+gx)^3}\right)}{2dg(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/(f + g*x)^3)^q],x]
```

```
[Out] (-3*e*q*Hypergeometric2F1[2/3, 1, 5/3, -(e/(d*(f + g*x)^3))])/(2*d*g*(f + g
*x)^2) + ((f + g*x)*Log[c*(d + e/(f + g*x)^3)^q])/g
```

fricas [C] time = 3.90, size = 1392, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f)^3)^q),x, algorithm="fricas")
```

```
[Out] 1/4*(4*g*q*x*log((d*g^3*x^3 + 3*d*f*g^2*x^2 + 3*d*f^2*g*x + d*f^3 + e)/(g^3
*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)) - 4*sqrt(3)*g*sqrt((((-1/2*f^3*q^3/g
^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3)
+ 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f
^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^2*q^2
)/g^2)*arctan(-1/24*(2*sqrt(3)*sqrt(4*g^2*q^2*x^2 + 12*f*g*q^2*x + ((-1/2*f
^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*
sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 12*f^2*q^2 + 2*(g^2*q*x + 3*f*g*q)*((-1/2*f
^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*
sqrt(3) + 1) - 2*f*q/g))*(((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f
^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*d*g^2 + 2*d*f*g*q
)*sqrt((((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d
*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*
e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*
f*q/g)*f*g*q + 4*f^2*q^2)/g^2) - sqrt(3)*(8*d*f*g^2*q^2*x + ((-1/2*f^3*q^3/
g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3)
 + 1) - 2*f*q/g)^2*d*g^3 + 12*d*f^2*g*q^2 + 4*(d*g^3*q*x + 2*d*f*g^2*q)*((-
1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3
)*(I*sqrt(3) + 1) - 2*f*q/g))*sqrt((((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3)
+ 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 +
 4*((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3)
)^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^2*q^2)/g^2))/(e*q^3)) - 12*f
*q*log(g*x + f) - 2*((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3
 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g*log(q*x - 1/2*(-1/2*f
^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*
sqrt(3) + 1) + f*q/g) + 4*g*x*log(c) + (((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g
^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g +
 6*f*q)*log(4*g^2*q^2*x^2 + 12*f*g*q^2*x + ((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(
d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^
2*g^2 + 12*f^2*q^2 + 2*(g^2*q*x + 3*f*g*q)*((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(
d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g))
)/g
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f)^3)^q),x, algorithm="giac")
```

[Out] Timed out

maple [C] time = 0.48, size = 157, normalized size = 0.95

$$-\frac{3fq\ln\left(gx+f\right)}{g}+x\ln\left(c\left(\frac{d\,g^{3}x^{3}+3df\,g^{2}x^{2}+3d\,f^{2}gx+d\,f^{3}+e}{\left(gx+f\right)^{3}}\right)^{q}\right)+\frac{q\left(\mathrm{RootOf}\left(d\,g^{3}\_Z^{3}+3df\,g^{2}\_Z^{2}+3d\,f^{2}\right.\right.}{dg\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/(g*x+f)^3)^q),x)
```

```
[Out] x*ln(c*((d*g^3*x^3+3*d*f*g^2*x^2+3*d*f^2*g*x+d*f^3+e)/(g*x+f)^3)^q)+1/g*q/d
*sum((_R^2*d*f*g^2+2*_R*d*f^2*g+d*f^3+e)/(_R^2*g^2+2*_R*f*g+f^2)*ln(-_R+x),
_R=RootOf(_Z^3*d*g^3+3*_Z^2*d*f*g^2+3*_Z*d*f^2*g+d*f^3+e))-3*f/g*q*ln(g*x+f
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$3\,q\int\frac{d\,f\,g^{2}x^{2}+2\,d\,f^{2}gx+d\,f^{3}+e}{dg^{3}x^{3}+3\,d\,f\,g^{2}x^{2}+3\,d\,f^{2}gx+d\,f^{3}+e}\,dx-\frac{3\,f\,q\log\left(gx+f\right)-gx\log\left(\left(dg^{3}x^{3}+3\,d\,f\,g^{2}x^{2}+3\,d\,f^{2}gx+d\,f^{3}\right.\right.}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f)^3)^q),x, algorithm="maxima")
```

```
[Out] 3*q*integrate((d*f*g^2*x^2 + 2*d*f^2*g*x + d*f^3 + e)/(d*g^3*x^3 + 3*d*f*g^
2*x^2 + 3*d*f^2*g*x + d*f^3 + e), x) - (3*f*q*log(g*x + f) - g*x*log((d*g^3
*x^3 + 3*d*f*g^2*x^2 + 3*d*f^2*g*x + d*f^3 + e)^q) + 3*g*x*log((g*x + f)^q)
 - g*x*log(c))/g
```

mupad [B] time = 0.63, size = 499, normalized size = 3.02

$$x\,\ln\left(c\left(d+\frac{e}{\left(f+g\,x\right)^{3}}\right)^{q}\right)-\left(\sum_{k=1}^{3}\ln\left(-d^{2}\,e^{2}\,g^{11}\,\left(3\,e\,q^{3}\,x+\mathrm{root}\left(d\,g^{3}\,z^{3}+3\,d\,f\,g^{2}\,q\,z^{2}+3\,d\,f^{2}\,g\,q^{2}\,z+d\,f^{3}\,q^{3}+e\,q\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/(f + g*x)^3)^q),x)
```

```
[Out] x*log(c*(d + e/(f + g*x)^3)^q) - symsum(log(-9*d^2*e^2*g^11*(3*e*q^3*x + ro
ot(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^3, z, k)
*e*q^2 + 4*root(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 +
 e*q^3, z, k)^3*d*f*g^2 + 4*root(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^
2*z + d*f^3*q^3 + e*q^3, z, k)*d*f^3*q^2 + 4*root(d*g^3*z^3 + 3*d*f*g^2*q*z
^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^3, z, k)^3*d*g^3*x + 8*root(d*g^3*z^
3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^3, z, k)^2*d*f^2*g*
q + 4*root(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^
3, z, k)*d*f^2*g*q^2*x + 8*root(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2
*z + d*f^3*q^3 + e*q^3, z, k)^2*d*f*g^2*q*x))*root(d*g^3*z^3 + 3*d*f*g^2*q*
z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^3, z, k), k, 1, 3) - (3*f*q*log(f +
 g*x))/g
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/(g*x+f)**3)**q),x)
```

```
[Out] Timed out
```

3.635 $\int \left(a+b \log \left(c \left(d+\frac{e}{f+gx}\right)^p\right)\right)^n \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\left(a+b \log \left(c \left(d+\frac{e}{f+gx}\right)^p\right)\right)^n,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/(g*x+f))^p))^n,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b \log \left(c \left(d+\frac{e}{f+gx}\right)^p\right)\right)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^n,x]
[Out] Defer[Int][(a + b*Log[c*(d + e/(f + g*x))^p])^n, x]
```

Rubi steps

$$\int \left(a+b \log \left(c \left(d+\frac{e}{f+gx}\right)^p\right)\right)^n \, dx = \int \left(a+b \log \left(c \left(d+\frac{e}{f+gx}\right)^p\right)\right)^n \, dx$$

Mathematica [A] time = 0.42, size = 0, normalized size = 0.00

$$\int \left(a+b \log \left(c \left(d+\frac{e}{f+gx}\right)^p\right)\right)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^n,x]
[Out] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^n, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b \log \left(c \left(\frac{dgx+df+e}{gx+f}\right)^p\right)+a\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^n,x, algorithm="fricas")
[Out] integral((b*log(c*((d*g*x + d*f + e)/(g*x + f))^p) + a)^n, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log \left(c \left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^n,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^n, x)
```

maple [A] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(b \ln\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/(g*x+f))^p))^n,x)
[Out] int((a+b*ln(c*(d+e/(g*x+f))^p))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^n,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^n, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+b\,\ln\left(c\,\left(d+\frac{e}{f+g\,x}\right)^{p}\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/(f + g*x))^p))^n,x)
[Out] int((a + b*log(c*(d + e/(f + g*x))^p))^n, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b \log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/(g*x+f))**p))**n,x)
[Out] Integral((a + b*log(c*(d + e/(f + g*x))**p))**n, x)
```

3.636 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4 dx$

Optimal. Leaf size=221

$$\frac{24b^3ep^3\text{Li}_3\left(\frac{e}{d(f+gx)}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}-\frac{12b^2ep^2\text{Li}_2\left(\frac{e}{d(f+gx)}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg}-4$$

```
[Out] -4*b*e*p*ln(-e/d/(g*x+f))*(a+b*ln(c*(d+e/(g*x+f))^p))^3/d/g+(e+d*(g*x+f))*(
a+b*ln(c*(d+e/(g*x+f))^p))^4/d/g-12*b^2*e*p^2*(a+b*ln(c*(d+e/(g*x+f))^p))^2
*polylog(2,1+e/d/(g*x+f))/d/g+24*b^3*e*p^3*(a+b*ln(c*(d+e/(g*x+f))^p))*poly
log(3,1+e/d/(g*x+f))/d/g-24*b^4*e*p^4*polylog(4,1+e/d/(g*x+f))/d/g
```

Rubi [A] time = 0.28, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2483, 2449, 2454, 2396, 2433, 2374, 2383, 6589}

$$\frac{24b^3ep^3\text{PolyLog}\left(3,\frac{e}{d(f+gx)}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}-\frac{12b^2ep^2\text{PolyLog}\left(2,\frac{e}{d(f+gx)}+1\right)\left(a+b\log\left(c\left(d\right.\right.\right.}{dg}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^4,x]
```

```
[Out] (-4*b*e*p*Log[-(e/(d*(f + g*x)))]*(a + b*Log[c*(d + e/(f + g*x))^p])^3)/(d*
g) + ((e + d*(f + g*x))*(a + b*Log[c*(d + e/(f + g*x))^p])^4)/(d*g) - (12*b
^2*e*p^2*(a + b*Log[c*(d + e/(f + g*x))^p])^2*PolyLog[2, 1 + e/(d*(f + g*x)
)])/(d*g) + (24*b^3*e*p^3*(a + b*Log[c*(d + e/(f + g*x))^p])*PolyLog[3, 1 +
 e/(d*(f + g*x))])/(d*g) - (24*b^4*e*p^4*PolyLog[4, 1 + e/(d*(f + g*x))])/(
d*g)
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
```

```
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2449

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[((e + d*x)*(a + b*Log[c*(d + e/x)^p])^q)/d, x] + Dist[(b*e*p*q)/d, In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4\,dx = \frac{\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{e}{x}\right)^p\right)\right)^4\,dx,x,f+gx\right)}{g}$$

$$= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{dg} + \frac{(4bep)\,\text{Subst}\left(\int \frac{\left(a+b\log\right.}{}\right.}{d}$$

$$= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{dg} - \frac{(4bep)\,\text{Subst}\left(\int \frac{\left(a+b\log\right.}{}\right.}{dg}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+gx))\left(\right.}{}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+gx))\left(\right.}{}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+gx))\left(\right.}{}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+gx))\left(\right.}{}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+gx))\left(\right.}{}$$

Mathematica [B] time = 1.42, size = 739, normalized size = 3.34

$$\frac{4b^3p^3\left(6e\text{Li}_3\left(\frac{e}{df+dgx}+1\right)-6e\text{Li}_2\left(\frac{e}{df+dgx}+1\right)\log\left(d+\frac{e}{f+gx}\right)+\left((df+dgx+e)\log\left(d+\frac{e}{f+gx}\right)-3e\log\left(-\frac{}{df+}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^4,x]
```

```
[Out] (-4*b*p*(d*f*Log[f + g*x] - (e + d*f)*Log[e + d*f + d*g*x] - d*g*x*Log[(e +
 d*f + d*g*x)/(f + g*x)])*(a - b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f
 + g*x))^p])^3 + d*g*x*(a - b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f +
g*x))^p])^4 + 6*b^2*p^2*(a - b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f +
 g*x))^p])^2*(2*d*f*Log[-(e/(d*f + d*g*x))]*Log[(e + d*f + d*g*x)/(f + g*x)
] + 2*(e + d*f)*Log[e + d*f + d*g*x]*Log[(e + d*f + d*g*x)/(f + g*x)] + d*g
*x*Log[(e + d*f + d*g*x)/(f + g*x)]^2 - d*f*(Log[-(e/(d*f + d*g*x))]*(Log[-
(e/(d*f + d*g*x))] + 2*Log[(e + d*f + d*g*x)/e]) - 2*PolyLog[2, -((d*(f + g
*x))/e)]) + (e + d*f)*((2*Log[-((d*(f + g*x))/e)] - Log[e + d*f + d*g*x])*L
og[e + d*f + d*g*x] + 2*PolyLog[2, (e + d*f + d*g*x)/e])) + 4*b^3*p^3*(a -
b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f + g*x))^p])*(Log[d + e/(f + g*
x)]^2*(-3*e*Log[-(e/(d*f + d*g*x))] + (e + d*f + d*g*x)*Log[d + e/(f + g*x)
]) - 6*e*Log[d + e/(f + g*x)]*PolyLog[2, 1 + e/(d*f + d*g*x)] + 6*e*PolyLog
```

```
[3, 1 + e/(d*f + d*g*x)]) - b^4*p^4*(4*e*Log[-(e/(d*f + d*g*x))]*Log[d + e/
(f + g*x)]^3 - e*Log[d + e/(f + g*x)]^4 - d*f*Log[d + e/(f + g*x)]^4 - d*g*
x*Log[d + e/(f + g*x)]^4 + 12*e*Log[d + e/(f + g*x)]^2*PolyLog[2, 1 + e/(d*
f + d*g*x)] - 24*e*Log[d + e/(f + g*x)]*PolyLog[3, 1 + e/(d*f + d*g*x)] + 2
4*e*PolyLog[4, 1 + e/(d*f + d*g*x)]))/(d*g)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(b^4\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)^4+4\,ab^3\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)^3+6\,a^2b^2\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)^2+4\,a^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^4,x, algorithm="fricas")
```

```
[Out] integral(b^4*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^4 + 4*a*b^3*log(c*((d*g
*x + d*f + e)/(g*x + f))^p)^3 + 6*a^2*b^2*log(c*((d*g*x + d*f + e)/(g*x + f
))^p)^2 + 4*a^3*b*log(c*((d*g*x + d*f + e)/(g*x + f))^p) + a^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^4, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+1/(g*x+f)*e)^p)+a)^4,x)
```

```
[Out] int((b*ln(c*(d+1/(g*x+f)*e)^p)+a)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^4,x, algorithm="maxima")
```

```
[Out] -4*a^3*b*e*g*p*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d*
e*g^2)) + 4*a^3*b*x*log(c*(d + e/(g*x + f))^p) + a^4*x + (b^4*d*g*x*log((d*
g*x + d*f + e)^p)^4 - 4*(b^4*d*f*p*log(g*x + f) + b^4*d*g*x*log((g*x + f)^p
) - (d*f*p + e*p)*b^4*log(d*g*x + d*f + e) - (b^4*d*g*log(c) + a*b^3*d*g)*x
)*log((d*g*x + d*f + e)^p)^3)/(d*g) + integrate(((d*f + e)*b^4*log(c)^4 + 4
*(d*f + e)*a*b^3*log(c)^3 + 6*(d*f + e)*a^2*b^2*log(c)^2 + (b^4*d*g*x + (d*
f + e)*b^4)*log((g*x + f)^p)^4 - 4*((d*f + e)*b^4*log(c) + (d*f + e)*a*b^3
+ (b^4*d*g*log(c) + a*b^3*d*g)*x)*log((g*x + f)^p)^3 + 6*(2*b^4*d*f*p^2*log
(g*x + f) + (d*f + e)*b^4*log(c)^2 - 2*(d*f*p^2 + e*p^2)*b^4*log(d*g*x + d*
f + e) + 2*(d*f + e)*a*b^3*log(c) + (d*f + e)*a^2*b^2 + (b^4*d*g*x + (d*f +
 e)*b^4)*log((g*x + f)^p)^2 + (a^2*b^2*d*g - 2*(d*g*p - d*g*log(c))*a*b^3 -
 (2*d*g*p*log(c) - d*g*log(c)^2)*b^4)*x - 2*((d*f + e)*b^4*log(c) + (d*f +
```

```
e)*a*b^3 + (a*b^3*d*g - (d*g*p - d*g*log(c))*b^4)*x)*log((g*x + f)^p))*log(
(d*g*x + d*f + e)^p)^2 + 6*((d*f + e)*b^4*log(c)^2 + 2*(d*f + e)*a*b^3*log(
c) + (d*f + e)*a^2*b^2 + (b^4*d*g*log(c)^2 + 2*a*b^3*d*g*log(c) + a^2*b^2*d
*g)*x)*log((g*x + f)^p)^2 + (b^4*d*g*log(c)^4 + 4*a*b^3*d*g*log(c)^3 + 6*a^
2*b^2*d*g*log(c)^2)*x + 4*((d*f + e)*b^4*log(c)^3 + 3*(d*f + e)*a*b^3*log(c
)^2 + 3*(d*f + e)*a^2*b^2*log(c) - (b^4*d*g*x + (d*f + e)*b^4)*log((g*x + f
)^p)^3 + 3*((d*f + e)*b^4*log(c) + (d*f + e)*a*b^3 + (b^4*d*g*log(c) + a*b^
3*d*g)*x)*log((g*x + f)^p)^2 + (b^4*d*g*log(c)^3 + 3*a*b^3*d*g*log(c)^2 + 3
*a^2*b^2*d*g*log(c))*x - 3*((d*f + e)*b^4*log(c)^2 + 2*(d*f + e)*a*b^3*log(
c) + (d*f + e)*a^2*b^2 + (b^4*d*g*log(c)^2 + 2*a*b^3*d*g*log(c) + a^2*b^2*d
*g)*x)*log((g*x + f)^p))*log((d*g*x + d*f + e)^p) - 4*((d*f + e)*b^4*log(c)
^3 + 3*(d*f + e)*a*b^3*log(c)^2 + 3*(d*f + e)*a^2*b^2*log(c) + (b^4*d*g*log
(c)^3 + 3*a*b^3*d*g*log(c)^2 + 3*a^2*b^2*d*g*log(c))*x)*log((g*x + f)^p))/(
d*g*x + d*f + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\ln\left(c\,\left(d+\frac{e}{f+g\,x}\right)^p\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/(f + g*x))^p))^4,x)
[Out] int((a + b*log(c*(d + e/(f + g*x))^p))^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\log\left(c\left(d + \frac{e}{f + gx}\right)^{p}\right)\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/(g*x+f))**p))**4,x)
[Out] Integral((a + b*log(c*(d + e/(f + g*x))**p))**4, x)
```

3.637 $\int \left(a+b \log \left(c \left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx$

Optimal. Leaf size=168

$$-\frac{6b^2ep^2\text{Li}_2\left(\frac{e}{d(f+gx)}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}-\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg}+\frac{(d(f+g$$

```
[Out] -3*b*e*p*ln(-e/d/(g*x+f))*(a+b*ln(c*(d+e/(g*x+f))^p))^2/d/g+(e+d*(g*x+f))*(
a+b*ln(c*(d+e/(g*x+f))^p))^3/d/g-6*b^2*e*p^2*(a+b*ln(c*(d+e/(g*x+f))^p))*po
lylog(2,1+e/d/(g*x+f))/d/g+6*b^3*e*p^3*polylog(3,1+e/d/(g*x+f))/d/g
```

Rubi [A] time = 0.18, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2483, 2449, 2454, 2396, 2433, 2374, 6589}

$$-\frac{6b^2ep^2\text{PolyLog}\left(2,\frac{e}{d(f+gx)}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}+\frac{6b^3ep^3\text{PolyLog}\left(3,\frac{e}{d(f+gx)}+1\right)}{dg}-\frac{3bep\log\left(-\frac{e}{d(f+gx}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^3,x]
```

```
[Out] (-3*b*e*p*Log[-(e/(d*(f + g*x)))]*(a + b*Log[c*(d + e/(f + g*x))^p])^2)/(d*
g) + ((e + d*(f + g*x))*(a + b*Log[c*(d + e/(f + g*x))^p])^3)/(d*g) - (6*b^
2*e*p^2*(a + b*Log[c*(d + e/(f + g*x))^p])*PolyLog[2, 1 + e/(d*(f + g*x))])
/(d*g) + (6*b^3*e*p^3*PolyLog[3, 1 + e/(d*(f + g*x))])/(d*g)
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2449

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[((e + d*x)*(a + b*Log[c*(d + e/x)^p])^q)/d, x] + Dist[(b*e*p*q)/d, In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
```

```
x] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx = \frac{\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{e}{x}\right)^p\right)\right)^3 dx, x, f+gx\right)}{g}$$

$$= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(3bep)\,\text{Subst}\left(\int \frac{\left(a+b\log \cdots\right.}{\cdots}\right.}{d\cdots}$$

$$= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} - \frac{(3bep)\,\text{Subst}\left(\int \frac{\left(a+b\log \cdots\right.}{\cdots}\right.}{dg\cdots}$$

$$= -\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(e+d(f+gx))\left(\cdots\right.}{\cdots}$$

$$= -\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(e+d(f+gx))\left(\cdots\right.}{\cdots}$$

$$= -\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(e+d(f+gx))\left(\cdots\right.}{\cdots}$$

$$= -\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(e+d(f+gx))\left(\cdots\right.}{\cdots}$$

Mathematica [B] time = 0.72, size = 415, normalized size = 2.47

$$\frac{3b^2p^2\left(2e\text{Li}_2\left(\frac{d(f+gx)}{e}+1\right)+d(f+gx)\log^2\left(d+\frac{e}{f+gx}\right)+e\left(2\log\left(-\frac{d(f+gx)}{e}\right)-\log(df+dgx+e)+2\log\left(d+\frac{e}{f+g\ldots}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^3,x]
```

```
[Out] (3*b*d*p*(f + g*x)*Log[d + e/(f + g*x)]*(a - b*p*Log[d + e/(f + g*x)] + b*L
og[c*(d + e/(f + g*x))^p])^2 + d*(f + g*x)*(a - b*p*Log[d + e/(f + g*x)] +
b*Log[c*(d + e/(f + g*x))^p])^3 + 3*b*e*p*(a - b*p*Log[d + e/(f + g*x)] + b
*Log[c*(d + e/(f + g*x))^p])^2*Log[e + d*(f + g*x)] + 3*b^2*p^2*(a - b*p*Lo
g[d + e/(f + g*x)] + b*Log[c*(d + e/(f + g*x))^p])*(d*(f + g*x)*Log[d + e/(
f + g*x)]^2 + e*(2*Log[-((d*(f + g*x))/e)] - Log[e + d*f + d*g*x] + 2*Log[d
 + e/(f + g*x)])*Log[e + d*(f + g*x)] + 2*e*PolyLog[2, 1 + (d*(f + g*x))/e]
) + b^3*p^3*(Log[d + e/(f + g*x)]^2*(-3*e*Log[-(e/(d*f + d*g*x))] + (e + d*
f + d*g*x)*Log[d + e/(f + g*x)]) - 6*e*Log[d + e/(f + g*x)]*PolyLog[2, 1 +
e/(d*f + d*g*x)] + 6*e*PolyLog[3, 1 + e/(d*f + d*g*x)]))/(d*g)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)^3+3\,ab^2\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)^2+3\,a^2b\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)+a^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^3 + 3*a*b^2*log(c*((d*g
*x + d*f + e)/(g*x + f))^p)^2 + 3*a^2*b*log(c*((d*g*x + d*f + e)/(g*x + f))
^p) + a^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^3, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\left(b\ln\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+1/(g*x+f)*e)^p)+a)^3,x)
```

```
[Out] int((b*ln(c*(d+1/(g*x+f)*e)^p)+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-3\,a^2begp\left(\frac{f\log\left(gx+f\right)}{eg^2}-\frac{\left(df+e\right)\log\left(dgx+df+e\right)}{deg^2}\right)+3\,a^2bx\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a^3x+\frac{b^3dgx\log\left(\left(dgx+\ldots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^3,x, algorithm="maxima")
```

```
[Out] -3*a^2*b*e*g*p*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d*
e*g^2)) + 3*a^2*b*x*log(c*(d + e/(g*x + f))^p) + a^3*x + (b^3*d*g*x*log((d*
g*x + d*f + e)^p)^3 - 3*(b^3*d*f*p*log(g*x + f) + b^3*d*g*x*log((g*x + f)^p
) - (d*f*p + e*p)*b^3*log(d*g*x + d*f + e) - (b^3*d*g*log(c) + a*b^2*d*g)*x
)*log((d*g*x + d*f + e)^p)^2)/(d*g) + integrate(((d*f + e)*b^3*log(c)^3 + 3
*(d*f + e)*a*b^2*log(c)^2 - (b^3*d*g*x + (d*f + e)*b^3)*log((g*x + f)^p)^3
+ 3*((d*f + e)*b^3*log(c) + (d*f + e)*a*b^2 + (b^3*d*g*log(c) + a*b^2*d*g)*
x)*log((g*x + f)^p)^2 + (b^3*d*g*log(c)^3 + 3*a*b^2*d*g*log(c)^2)*x + 3*(2*
b^3*d*f*p^2*log(g*x + f) + (d*f + e)*b^3*log(c)^2 - 2*(d*f*p^2 + e*p^2)*b^3
*log(d*g*x + d*f + e) + 2*(d*f + e)*a*b^2*log(c) + (b^3*d*g*x + (d*f + e)*b
^3)*log((g*x + f)^p)^2 - (2*(d*g*p - d*g*log(c))*a*b^2 + (2*d*g*p*log(c) -
d*g*log(c)^2)*b^3)*x - 2*((d*f + e)*b^3*log(c) + (d*f + e)*a*b^2 + (a*b^2*d
*g - (d*g*p - d*g*log(c))*b^3)*x)*log((g*x + f)^p))*log((d*g*x + d*f + e)^p
) - 3*((d*f + e)*b^3*log(c)^2 + 2*(d*f + e)*a*b^2*log(c) + (b^3*d*g*log(c)^
2 + 2*a*b^2*d*g*log(c))*x)*log((g*x + f)^p))/(d*g*x + d*f + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\ln\left(c\left(d+\frac{e}{f+g\,x}\right)^p\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/(f + g*x))^p))^3,x)
```

```
[Out] int((a + b*log(c*(d + e/(f + g*x))^p))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/(g*x+f))**p))**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e/(f + g*x))**p))**3, x)
```

3.638 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2\,dx$

Optimal. Leaf size=115

$$-\frac{2bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}+\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg}-\frac{2b^2ep^2\mathrm{Li}_2\left(\frac{e}{d(f+g}\right.}{dg}$$

```
[Out] -2*b*e*p*ln(-e/d/(g*x+f))*(a+b*ln(c*(d+e/(g*x+f))^p))/d/g+(e+d*(g*x+f))*(a+
b*ln(c*(d+e/(g*x+f))^p))^2/d/g-2*b^2*e*p^2*polylog(2,1+e/d/(g*x+f))/d/g
```

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2483, 2449, 2454, 2394, 2315}

$$-\frac{2b^2ep^2\text{PolyLog}\left(2,\frac{e}{d(f+gx)}+1\right)}{dg}-\frac{2bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}+\frac{(d(f+gx)+e)\left(a+b\log\left(c\right.\right.}{dg}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^2,x]
```

```
[Out] (-2*b*e*p*Log[-(e/(d*(f + g*x)))]*(a + b*Log[c*(d + e/(f + g*x))^p]))/(d*g)
 + ((e + d*(f + g*x))*(a + b*Log[c*(d + e/(f + g*x))^p])^2)/(d*g) - (2*b^2*
e*p^2*PolyLog[2, 1 + e/(d*(f + g*x))])/(d*g)
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2449

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[((e + d*x)*(a + b*Log[c*(d + e/x)^p])^q)/d, x] + Dist[(b*e*p*q)/d, In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
```

```
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2 dx = \frac{\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{e}{x}\right)^p\right)\right)^2 dx, x, f+gx\right)}{g}$$
$$= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(2bep)\,\text{Subst}\left(\int \frac{a+b\log(}{}\right.}{dg}$$
$$= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} - \frac{(2bep)\,\text{Subst}\left(\int \frac{a+b\log(}{}\right.}{dg}$$
$$= -\frac{2bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg} + \frac{(e+d(f+gx))\left(a\right.}{}$$
$$= -\frac{2bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg} + \frac{(e+d(f+gx))\left(a\right.}{}$$

Mathematica [A] time = 0.26, size = 219, normalized size = 1.90

$$x\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2 - \frac{bp\left(2df\log(f+gx)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right) - 2(df+e)\log(d(f+gx)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^2,x]
```

```
[Out] x*(a + b*Log[c*(d + e/(f + g*x))^p])^2 - (b*p*(2*d*f*Log[f + g*x]*(a + b*Lo
g[c*(d + e/(f + g*x))^p]) - 2*(e + d*f)*(a + b*Log[c*(d + e/(f + g*x))^p])*
Log[e + d*(f + g*x)] + b*d*f*p*(Log[f + g*x]*(Log[f + g*x] - 2*Log[(e + d*f
 + d*g*x)/e]) - 2*PolyLog[2, -((d*(f + g*x))/e)]) - b*(e + d*f)*p*((2*Log[-
((d*(f + g*x))/e)] - Log[e + d*f + d*g*x])*Log[e + d*(f + g*x)] + 2*PolyLog
[2, (e + d*f + d*g*x)/e])))/(d*g)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)^2 + 2\,ab\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right) + a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^2 + 2*a*b*log(c*((d*g*x
 + d*f + e)/(g*x + f))^p) + a^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^2, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \left(b \ln \left(c \left(d+\frac{e}{g x+f}\right)^{p}\right)+a\right)^{2}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(c*(d+1/(g*x+f)*e)^p)+a)^2,x)
```

```
[Out] int((b*ln(c*(d+1/(g*x+f)*e)^p)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-2\,abegp\left(\frac{f\log\left(gx+f\right)}{eg^{2}}-\frac{\left(df+e\right)\log\left(dgx+df+e\right)}{deg^{2}}\right)+2\,abx\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a^{2}x+b^{2}\left(\frac{dgx\log\left(\left(dgx+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*e*g*p*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d*e*
g^2)) + 2*a*b*x*log(c*(d + e/(g*x + f))^p) + a^2*x + b^2*((d*g*x*log((d*g*x
 + d*f + e)^p)^2 + d*g*x*log((g*x + f)^p)^2 - (d*f*p^2 + e*p^2)*log(d*g*x +
 d*f + e)^2 + 2*(d*f*p^2 + e*p^2)*log(d*g*x + d*f + e)*log(g*x + f) - 2*(d*
f*p*log(g*x + f) + d*g*x*log((g*x + f)^p) - d*g*x*log(c) - (d*f*p + e*p)*lo
g(d*g*x + d*f + e))*log((d*g*x + d*f + e)^p) + 2*(d*f*p*log(g*x + f) - d*g*
x*log(c) - (d*f*p + e*p)*log(d*g*x + d*f + e))*log((g*x + f)^p))/(d*g) - in
tegrate(-(d*g^2*x^2*log(c)^2 + (d*f^2 + e*f)*log(c)^2 + (2*e*g*p*log(c) + (
2*d*f*g + e*g)*log(c)^2)*x - 2*(d*f^2*p^2 + 2*e*f*p^2 + (d*f*g*p^2 + e*g*p^
2)*x)*log(g*x + f))/(d*g^2*x^2 + d*f^2 + e*f + (2*d*f*g + e*g)*x), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{f+g\,x}\right)}^p\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/(f + g*x))^p))^2,x)
```

```
[Out] int((a + b*log(c*(d + e/(f + g*x))^p))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \log{\left(c \left(d + \frac{e}{f + g x}\right)^{p} \right)}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/(g*x+f))**p))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/(f + g*x))**p))**2, x)
```

3.639 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)dx$

Optimal. Leaf size=50

$$ax+\frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{bep\log(d(f+gx)+e)}{dg}$$

```
[Out] a*x+b*(g*x+f)*ln(c*(d+e/(g*x+f))^p)/g+b*e*p*ln(e+d*(g*x+f))/d/g
```

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2483, 2448, 263, 31}

$$ax+\frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{bep\log(d(f+gx)+e)}{dg}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e/(f + g*x))^p],x]
```

```
[Out] a*x + (b*(f + g*x)*Log[c*(d + e/(f + g*x))^p])/g + (b*e*p*Log[e + d*(f + g*
x)])/(d*g)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)dx = ax+b\int \log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)dx$$
$$= ax+\frac{b\,\mathrm{Subst}\left(\int \log\left(c\left(d+\frac{e}{x}\right)^p\right)dx, x, f+gx\right)}{g}$$
$$= ax+\frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{(bep)\,\mathrm{Subst}\left(\int \frac{1}{\left(d+\frac{e}{x}\right)x}dx, x, f+gx\right)}{g}$$
$$= ax+\frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{(bep)\,\mathrm{Subst}\left(\int \frac{1}{e+dx}dx, x, f+gx\right)}{g}$$
$$= ax+\frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{bep\log(e+d(f+gx))}{dg}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 1.40

$$ax+bx\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)-begp\left(\frac{f\log(f+gx)}{eg^2}-\frac{(df+e)\log(df+dgx+e)}{deg^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e/(f + g*x))^p],x]
```

```
[Out] a*x - b*e*g*p*((f*Log[f + g*x])/(e*g^2) - ((e + d*f)*Log[e + d*f + d*g*x])/
(d*e*g^2)) + b*x*Log[c*(d + e/(f + g*x))^p]
```

fricas [A] time = 0.44, size = 76, normalized size = 1.52

$$\frac{bdgpx\log\left(\frac{dgx+df+e}{gx+f}\right)-bdfp\log\left(gx+f\right)+bdgx\log(c)+adgx+\left(bdf+be\right)p\log\left(dgx+df+e\right)}{dg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/(g*x+f))^p),x, algorithm="fricas")
```

```
[Out] (b*d*g*p*x*log((d*g*x + d*f + e)/(g*x + f)) - b*d*f*p*log(g*x + f) + b*d*g*
x*log(c) + a*d*g*x + (b*d*f + b*e)*p*log(d*g*x + d*f + e))/(d*g)
```

giac [B] time = 0.21, size = 177, normalized size = 3.54

$$\frac{\left(dfge^{(-2)}-\left(df+e\right)ge^{(-2)}\right)\left(dpe^2\log\left(-d+\frac{dgx+df+e}{gx+f}\right)+de^2\log(c)-\frac{\left(dgx+df+e\right)pe^2\log\left(-d+\frac{dgx+df+e}{gx+f}\right)}{gx+f}+\frac{\left(dgx+df+e\right)pe^2\log\left(\right.}{gx+f}\right.}{d^2g^2-\frac{\left(dgx+df+e\right)dg^2}{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/(g*x+f))^p),x, algorithm="giac")
```

```
[Out] (d*f*g*e^(-2) - (d*f + e)*g*e^(-2))*(d*p*e^2*log(-d + (d*g*x + d*f + e)/(g*
x + f)) + d*e^2*log(c) - (d*g*x + d*f + e)*p*e^2*log(-d + (d*g*x + d*f + e)
/(g*x + f))/(g*x + f) + (d*g*x + d*f + e)*p*e^2*log((d*g*x + d*f + e)/(g*x
+ f))/(g*x + f))*b/(d^2*g^2 - (d*g*x + d*f + e)*d*g^2/(g*x + f)) + a*x
```

maple [A] time = 0.10, size = 81, normalized size = 1.62

$$-\frac{bfp\ln\left(gx+f\right)}{g}+\frac{bfp\ln\left(dgx+df+e\right)}{g}+bx\ln\left(c\left(\frac{dgx+df+e}{gx+f}\right)^{p}\right)+ax+\frac{bep\ln\left(dgx+df+e\right)}{dg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*ln(c*(d+1/(g*x+f)*e)^p)+a,x)
```

```
[Out] a*x+b*x*ln(c*((d*g*x+d*f+e)/(g*x+f))^p)+b/g*p*ln(d*g*x+d*f+e)*f+b*e/g*p/d*l
n(d*g*x+d*f+e)-b/g*p*f*ln(g*x+f)
```

maxima [A] time = 0.67, size = 70, normalized size = 1.40

$$-begp\left(\frac{f\log\left(gx+f\right)}{eg^2}-\frac{\left(df+e\right)\log\left(dgx+df+e\right)}{deg^2}\right)+bx\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/(g*x+f))^p),x, algorithm="maxima")
```

```
[Out] -b*e*g*p*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d*e*g^2)
) + b*x*log(c*(d + e/(g*x + f))^p) + a*x
```

mupad [B] time = 0.42, size = 61, normalized size = 1.22

$$a\,x+b\,x\,\ln\left(c\,{\left(d+\frac{e}{f+g\,x}\right)}^p\right)-\frac{b\,f\,p\,\ln\left(f+g\,x\right)}{g}+\frac{b\,p\,\ln\left(e+d\,f+d\,g\,x\right)\,\left(e+d\,f\right)}{d\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*log(c*(d + e/(f + g*x))^p),x)
```

```
[Out] a*x + b*x*log(c*(d + e/(f + g*x))^p) - (b*f*p*log(f + g*x))/g + (b*p*log(e
+ d*f + d*g*x)*(e + d*f))/(d*g)
```

sympy [A] time = 2.05, size = 114, normalized size = 2.28

$$ax+b\left(\begin{cases} x\log\left(c\left(\frac{e}{f}\right)^{p}\right) & \text{for}\: d=0\wedge g=0\\ x\log\left(c\left(d+\frac{e}{f}\right)^{p}\right) & \text{for}\: g=0\\ -\frac{fp\log\left(f+gx\right)}{g}+px\log\left(e\right)-px\log\left(f+gx\right)+px+x\log\left(c\right) & \text{for}\: d=0\\ \frac{fp\log\left(d+\frac{e}{f+gx}\right)}{g}+px\log\left(d+\frac{e}{f+gx}\right)+x\log\left(c\right)+\frac{ep\log\left(df+dgx+e\right)}{dg} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e/(g*x+f))**p),x)
```

```
[Out] a*x + b*Piecewise((x*log(c*(e/f)**p), Eq(d, 0) & Eq(g, 0)), (x*log(c*(d + e
/f)**p), Eq(g, 0)), (-f*p*log(f + g*x)/g + p*x*log(e) - p*x*log(f + g*x) +
p*x + x*log(c), Eq(d, 0)), (f*p*log(d + e/(f + g*x))/g + p*x*log(d + e/(f +
 g*x)) + x*log(c) + e*p*log(d*f + d*g*x + e)/(d*g), True))
```

3.640 $\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/(a+b*ln(c*(d+e/(g*x+f))^p)),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^(-1),x]
[Out] Defer[Int][(a + b*Log[c*(d + e/(f + g*x))^p])^(-1), x]
```

Rubi steps

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx=\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$$

Mathematica [A] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-1),x]
[Out] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-1), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p)),x, algorithm="fricas")
[Out] integral(1/(b*log(c*((d*g*x + d*f + e)/(g*x + f))^p) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p)),x, algorithm="giac")
[Out] integrate(1/(b*log(c*(d + e/(g*x + f))^p) + a), x)
```

maple [A] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{b\ln\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*ln(c*(d+1/(g*x+f)*e)^p)+a),x)
[Out] int(1/(b*ln(c*(d+1/(g*x+f)*e)^p)+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p)),x, algorithm="maxima")
[Out] integrate(1/(b*log(c*(d + e/(g*x + f))^p) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{a+b\,\ln\left(c\left(d+\frac{e}{f+g\,x}\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*log(c*(d + e/(f + g*x))^p)),x)
[Out] int(1/(a + b*log(c*(d + e/(f + g*x))^p)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*ln(c*(d+e/(g*x+f))**p)),x)
[Out] Integral(1/(a + b*log(c*(d + e/(f + g*x))**p)), x)
```

3.641 $\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^(-2),x]
[Out] Defer[Int][(a + b*Log[c*(d + e/(f + g*x))^p])^(-2), x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$$

Mathematica [A] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-2),x]
[Out] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-2), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)^2+2\,ab\log\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^2 + 2*a*b*log(c*((d*
g*x + d*f + e)/(g*x + f))^p) + a^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^(-2), x)
```

maple [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\ln\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*ln(c*(d+1/(g*x+f)*e)^p)+a)^2,x)
```

```
[Out] int(1/(b*ln(c*(d+1/(g*x+f)*e)^p)+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{dg^{2}x^{2}+df^{2}+ef+\left(2\,dfg+eg\right)x}{b^{2}egp\log\left(\left(dgx+df+e\right)^{p}\right)-b^{2}egp\log\left(\left(gx+f\right)^{p}\right)+b^{2}egp\log(c)+abegp}-\int \frac{2}{b^{2}ep\log\left(\left(dgx+df+e\right)^{p}\right)-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="maxima")
```

```
[Out] (d*g^2*x^2 + d*f^2 + e*f + (2*d*f*g + e*g)*x)/(b^2*e*g*p*log((d*g*x + d*f +
 e)^p) - b^2*e*g*p*log((g*x + f)^p) + b^2*e*g*p*log(c) + a*b*e*g*p) - integ
rate((2*d*g*x + 2*d*f + e)/(b^2*e*p*log((d*g*x + d*f + e)^p) - b^2*e*p*log(
(g*x + f)^p) + b^2*e*p*log(c) + a*b*e*p), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{f+g\,x}\right)^{p}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*log(c*(d + e/(f + g*x))^p))^2,x)
```

```
[Out] int(1/(a + b*log(c*(d + e/(f + g*x))^p))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*ln(c*(d+e/(g*x+f))**p))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/(f + g*x))**p))**(-2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```